International Islamic University Chittagong

Department of Economics & Banking

Final Term Examination: Spring-2023

Program: MSS in Economics and Banking

Course Code:MATH-5203 Course Title: Mathematics: Static and Dynamic Optimization

Time: 3 Hours Full Marks: 50

Answer the following questions. All parts of a question must be answered sequentially. Figures in the right margin indicate full marks.

1. (a) Illustrate the Pontryagin's maximum principle of optimal control theory. 4
   (b) Find the optimal paths of the control, state, and co state variables that will 6

$$\text{Maximize} \quad \int_0^1 (yu - u^2 - y^2)dt$$
$$\text{subject to } y^t = u$$
$$\text{and } y(0) = 2 \quad y(1) \text{ free}$$

2. (a) Write out the characteristic equation for each of the following, and find the characteristic roots: 4

   i) $y_{t+2} + \frac{1}{2}y_{t+1} - \frac{1}{2}y_t = 5$

   ii) $y_{t+2} + 6y_{t+1} + 9y_t = 4$

   State on the basis of characteristic roots whether the time path involves oscillation or stepped fluctuation, and whether it is explosive,

   (b) Solve the following difference equations: 6

   i) $p_{t+2} + 3p_{t+1} - \frac{7}{4}p_t = 9 \quad (p_0 = 4;\ p_1 = 2)$

   ii) $y_{t+2} - 4y_{t+1} + 4y_t = 5 \quad (y_0 = 3;\ y_1 = 4)$

   Analyze the time path obtained on your solution.

3. (a) Explain the duality theorem. 2
   (b) Given the primal 8

$$\text{Min. } C = 3y_1 + 2y_2$$
$$\text{Subject to } \begin{bmatrix} 2 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \geq \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$
$$y_1, y_2 \geq 0$$

Find the dual problem. Solve via simplex method and obtain the results of primal problem from the optimum tableau.

4. A two product firm faces the following demand and cost functions 10

$$Q_1 = 40 - 2P_1 - P_2$$
$$Q_2 = 35 - P_1 - P_2$$
$$C = {Q_1}^2 + 2{Q_2}^2 + 10$$

   a) Find the output levels that satisfy the first-order condition for maximum profit.

   b) Check the second-order sufficient condition. Can you conclude

that this problem possesses a unique absolute maximum?

c) What is the maximal profit?

Or

(a) How can you define the strict concavity and strict convexity? 2

(b) Check whether the following functions are concave, convex, strictly concave, strictly convex, or neither: 8

i) $Z = {x_1}^2 + 2{x_2}^2$, by using differentiable rule

ii) $Z = (x + y)^2$, by using geometric rule

5. Find the extreme values of the following functions. Check whether they are maxima or minima by the Hessian determinant test. 10

i) $z = {x_1}^2 + 3{x_2}^2 - 3x_1x_2 + 4x_2x_3 + 6{x_3}^2$

ii) $z = -{x_1}^3 - {x_2}^2 + 3x_1x_2 + 2x_2 - 3{x_3}^2$

iii) $z = e^{2x} + e^{-y} + e^{w^2} - (2x + 2e^w - y)$

Or

(a) Mention the detrimental test for sign definiteness of a quadratic form. 3

(b) Find the characteristic equation, roots, and vectors for the following quadratic form equation: 7

$$q = -4uv + 3u^2 + 7v^2$$

What can you conclude about the sign of this equation?