A COMPARATIVE STUDY OF APPROXIMATE METHODS OF MULTISTORIED BUILDING FRAME ANALYSIS

A Thesis

by

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ABSTRACT

This thesis examines the accuracy of the different commonly used approximate methods of building frame analysis subjected to static loads (lateral and gravity), considering only the linear elastic behaviour.

Approximate procedures for the analysis of the building frames are of great value to structural engineers for determining the preliminary dimensions of the structural elements and to prepare data for a more exact analysis. Even in the final design stages, these approximate procedures are sometimes used to check the numerical accuracy of the solutions. The approximate methods of analysis available for building frames subjected to lateral and gravity loads are discussed and four methods, viz (i) Portal method, (ii) Cantilever method, (iii) Stiffness center method (iv) Point of contraflexure method, are investigated. For gravity load analysis, the exact point of contraflexures in the girders are calculated and an inference is made in this regard for reasonably accurate analysis of building frames subjected to gravity loads.

Exact analysis procedures form the basis of all approximate procedures. A thorough study of the exact methods of analysis is a vital prerequisite to the study of approximate methods of analysis. Exact solutions for lateral and gravity loads by stiffness method (using a recursion procedure) are compared with approximate solutions of the same problems and conclusions are made about the use of approximate procedures.

In the analysis of tall building frames the construction time must be treated as an additional variable, which implies that the vertical load will be considered as applied gradually story-by-story while, at the same time the framework is being progressively build up. A computer programme has been developed which treats construction time as a variable and it is observed that in unsymmetrical frame analysis, failure in considering construction time as a variable will cause over-stressing of some members of the frame in the lower stories at that level of construction.

The concrete shear wall has come to be accepted as a natural component of a multistory concrete building. The effect of interacting frames in planes perpendicular to the shear wall is studied. Since the interacting frame is connected to the shear wall only at one column, the analysis of the interaction will primarily involve solution for vertical load distribution in a grid wall and which has been done by the solution of frame for a linearly increasing vertical loads along a column line. When a shear wall interacts with a frame in a plane normal to its own, it behaves like an I-beam. Simplified graphs are presented to findout the effective numbers of columns acting as the flange of the assumed I-beam.

Computer programmes, based on the above mentioned methods of analysis, written in FORTRAN, are discussed and listings are included in Appendix.

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CHAPTER 1

GENERAL INTRODUCTION

1.1 Introduction

The present concept of highrise buildings resulted from the large-scale migration of agricultural population to the commercial and industrial districts, where the need for more working, living and other spaces in these cities can be efficiently solved by the construction of multistory buildings keeping much more open space on the ground level.

Multistory building frames represent one of the most complex types of structural systems encountered in ordinary civil engineering practice. With the increasing availability of large capacity digital computers, it is now possible to obtain 'exact' analysis of multistory buildings. However, in the initial planning and dimensioning phase of design of building frames, rapid methods of analysis using slide rules or calculators only are needed. Besides, these approximate methods of analysis and design can be used to measure the reliability of the exact stress analysis.

'Exact' analysis of building frames subjected to gravity loads are normally carried with the help of computer programmes. However, there is an inherent susceptibility of receiving misleading results when such an analysis is carriedout, since these programmes are based on the input of the entire structural system with its loading, and an analysis for the entire structure as one unit. The dead load in a real building, is built up gradually and; for example, in a 40-story building, a dead load of the 10th floor cannot be resisted by a 40 story frame, since at the time the 10th floor load is applied, there is only a 10-story frame available to resist this load, and not a 40-story frame. So, in exact analysis of a building frame, time i.e. the phase of construction should be considered as a variable.

Again in the context of Bangladesh in present days, it has become a common practice to have a building analyzed and designed for say 20-stories. But the construction will be carriedout upto say 10-stories. In this case the failure in assuming construction time as an additional variable may lead to under design of some of the elements. And here the building must be analyzed and designed for live and dead loads at all possible semi-complete stages.

The use of shear walls at suitable locations in a high-rise building has come to be a justified solution in a earthquake and lateral load resistant building. And many of these shear walls interact with frames in a plane normal to the shear wall. The effectiveness of the closely spaced exterior frame as flange of the shear wall (for equivalent I-beam, where shear wall is considered as web) can be studied by isolating the shear wall from the frame and investigating the vertical load distribution characteristic of the frame itself.

1.2 Scope of the Thesis

The purpose of this work is to investigate both the exact and approximate methods of analysis of building frames subjected to lateral and gravity loads. Only the linear elastic behaviour under static loads is considered. For the lateral load analysis three approximate methods and one exact method are discussed:

(a) Portal method

(b) Cantilever method

(c) Stiffness center method(5)

(d) Stiffness method

For the gravity load analysis two approximate methods and one exact method are discussed:

(a) Method suggested by Wilbur and Norris(10)

(b) Method suggested by Salvadori and Levy(7)

(c) Stiffness method

Although for exact analysis many of the comprehensive computer programmes can be used for gravity load analysis, disregard of time as an additional variable for this analysis will definitly introduce an error; specially when the frame is unsymmetrical. So gravity load analysis at different construction stage is essential for having a complete picture of stress level in the frame. A computer programme is developed taking time as an additional variable, which means the vertical load will be considered as applied gradually story-by-story, while, at the same time the frame is being progressively built up.

The behaviour of an interacting frame normal to a shear wall is approximated by analysing the frame for linearly varying concentrated loads along a column line and from the dispersion of this concentric loads an imaginary flange-width can be approximated for the shear wall (which acts as a web of an I-beam). Simplified graphs are presented to find out the effective number of columns acting as the flange of the equivalent I-beam.

Computer programmes are presented for each of these methods of analysis and these programmes are valid for any number of stories and any number bays.

CHAPTER 2

LITERATURE REVIEW

2.1 Review of Different Methods for Lateral Load Analysis

Analysis for lateral loads is a major problem faced in the design of multi-storied frames. Lateral loads on structures are produced by the action of wind, earthquake and blast. There are two very common approximate methods (a) portal and (b) cantilever, for analysis of building frames. Both the methods assume that there is a point of inflection at the mid-points of each girder and column. In addition, while the portal method assumes, that each interior column carries twice as much shear as each exterior column, the cantilever method varies the magnitude of axial force in each column according to the distance from the center of gravity of all columns of the story under consideration. Both assumptions are analogous to shear and bending stress distributions in a solid cross-section, which inturn follow the well known principle of Bernoulli i.e. "plane sections before bending remain plane after bending".

But when a system consists of hollow portions, or is made of beams and columns, the plane sections no longer remain plane after application of load due to the rotation of girders. So, the concept of center of gravity in cantilever methods is a wrong assumption because the column area has very little effect on the axial force distribution among the columns. A method has been developed which calculates the column shear and axial force by evaluating the stiffness coefficient of a structure in axial and transverse directions. This method appears in ASCE 1974 July, Structural Engineering Journal as Stiffness Center Method in the paper by Kardestuncer. The details of these methods are discussed in Chapter 3.

2.2 Review of Different Methods for Gravity Load Analysis

The girders in building frames carry vertical loads from the slab in addition to their own weights. As the girders are rigidly connected to the columns all the members are subjected to bending moments, shear and axial force and because of their rigid construction they form a highly indeterminate structure. For the solution of this indeterminancy two sets of assumptions are suggested, one by Norris and Wilbur(10) and other by Salvadori and Levy(7).

The details of these two methods are given in Chapter 3.

2.3 Review of Exact Method for Gravity and Lateral Load Analysis

The method for the analysis of large multistory building frames with any location of shear walls subjected to vertical or lateral loading or both is based on the development of a tri-diagonal stiffness matrix and its reduction by recursion relationships. This method can consider the axial and flexural distortions, as well as shear distortions of the members. The detail of this method is given in Chapter 4.

CHAPTER 3

APPROXIMATE METHODS FOR THE ANALYSIS OF BUILDING FRAMES

3.1 Introduction

The approximate methods of structural analysis were developed to enable the engineer to reach quick decisions regarding the dimensions and layout of structural members and to compare different schemes of structural systems. Although the availability of powerful and sophisticated computers encourages the engineers to go for exact analysis of the structures, the final design is often accomplished by shuttling between "analysis" and "design", the starting (i.e the preliminary design) and acceptance of the final design still require engineering judgement. In both cases, the engineer needs to make short hand calculations either to prepare "data for exact analysis" or to assure himself that the "computer results are at least realistic".

3.2 Analysis for Gravity Loads

A building frame is always subjected to its own dead load. Vertical live load may or may not be present. So building frames is to be analysed and designed for its full dead load and for any combinations of live load. Furthermore, a building frame must have resistance against horizontal forces. Therefore, it is to be constructed in such a way that the girders are rigidly connected to the columns as a result all the member of the frame would carry bending moment, shear, and axial force. Because of the rigid construction, a building frame is highly indeterminate (shown in Fig. 3.1). It can be made determinate by cutting each of the girder near the mid-span (Fig. 3.2). To arrive at this condition, however, it is necessary to remove the bending moment, shear, and axial force in each girder where it is cut. If 'n' is the number of girders in the bent, it is necessary to remove '3n' redundants to make the bent statically determinate; hence the bent is indeterminate to the '3n' th degree.

3.2.1 Assumptions

Since a rigid building frame is statically indeterminate to a degree equal to three times the number of girders, it will be necessary to make three stress assumptions for each girder, if an analysis is to be carried out on the basis of statics only. In Fig. 3.3, a girder is subjected to a uniform load of intensity w per unit length extending over the entire span. Both the joints A and B which are partially restrained against rotation will rotate as shown in Fig. 3.4. Had the supports at A and B been completely fixed against rotation, as shown in Fig. 3.5, it can easily be shown from a consideration of bending moment in a fixed-end beam that the points of inflection would be located at a distance of 0.21 L from each end. If the supports at A and B are hinged, as shown in Fig. 3.6, the points of zero moment would be at the ends of

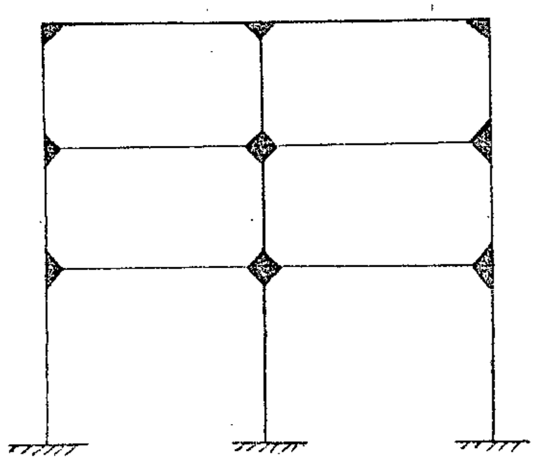


Fig. 3.1 Typical Building Frame

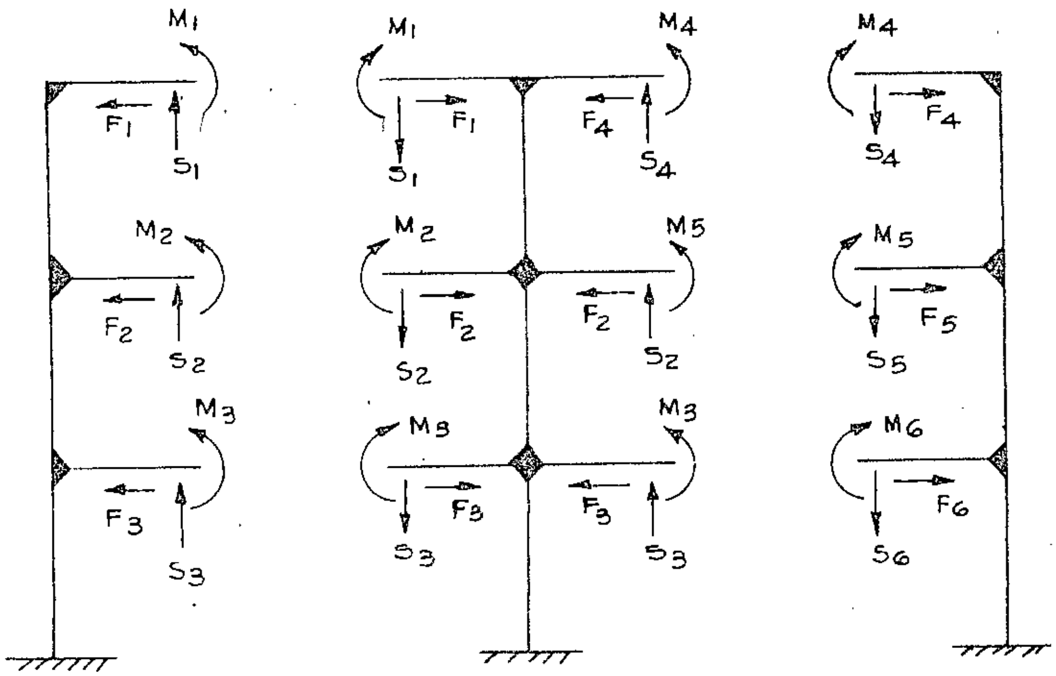


Fig. 3.2 A Possible release system for the frame in Fig 3.1

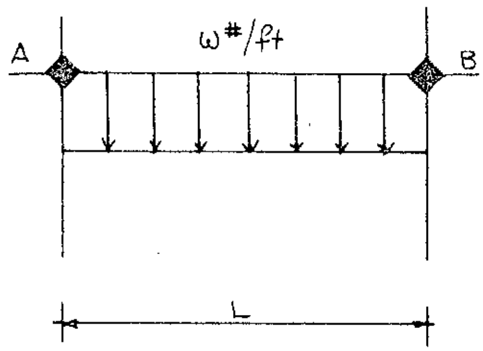


Fig. 3.3
Girder Subjected to Gravity Load

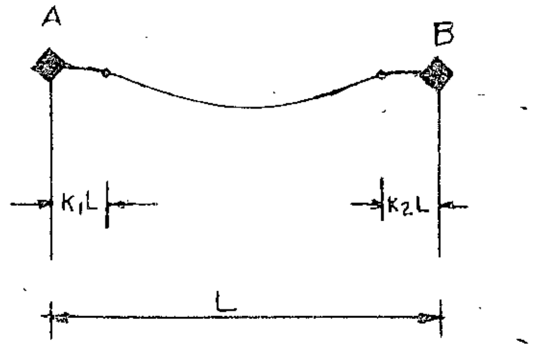


Fig. 3.4
Location of Girder Inflection Points

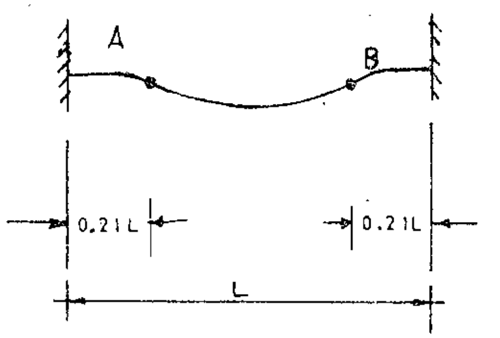


Fig. 3.5
Girder Inflection Points for fixed ended Girder

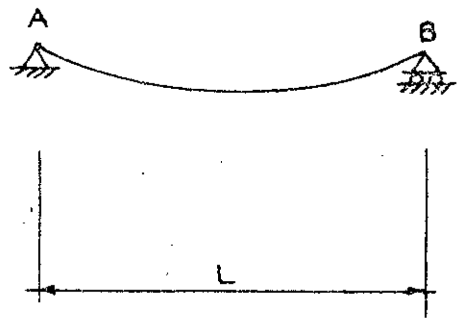


Fig. 3.6
Simply Supported Girder

beam. For the actual case of partial fixity, the points of inflection may be assumed to be somewhere between the two extremes of 0.21 L and 0.00L from the ends of the beams.

Solutions of building frames based on elastic action show that under vertical loads the axial force in the girders is usually very small.

Based on above discussions two different sets of assumptions are made, one by Norris and Wilbur(10) and the other by Salvadori and Levy(7), for each girder in analyzing a building frame acted upon by vertical loads.

Assumptions made by Norris and Wilbur(10):

(i) The axial force in the girder is zero.

(ii) A point of inflection occurs at the one-tenth point measured along the span from the left support.

(iii) A point of inflection occurs at the one-tenth point measured along the span from the right support.

Assumptions made by Salvadori and Levy(7):

(i) The axial force in the girder is zero.

(ii) A point of inflection occurs at the one-tenth point measured along the span from the exterior support.

(iii) A point of inflection occurs at the one-fifth point measured along the span from the interior support.

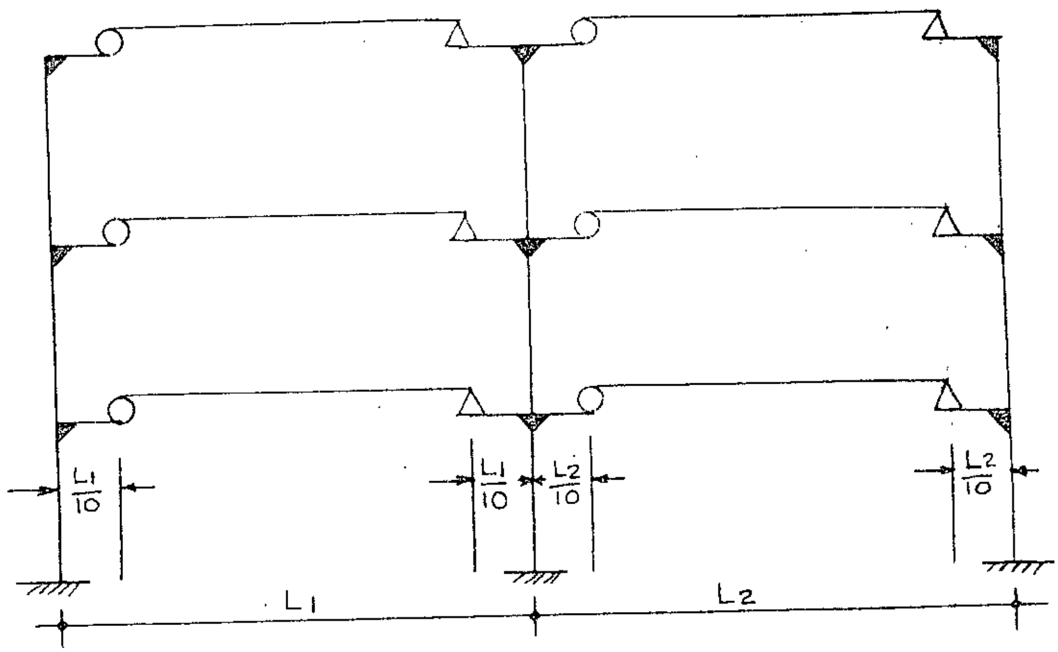


Fig. 3.7 Location of Inflection Points Suggested by Norris and Wilbur (10)

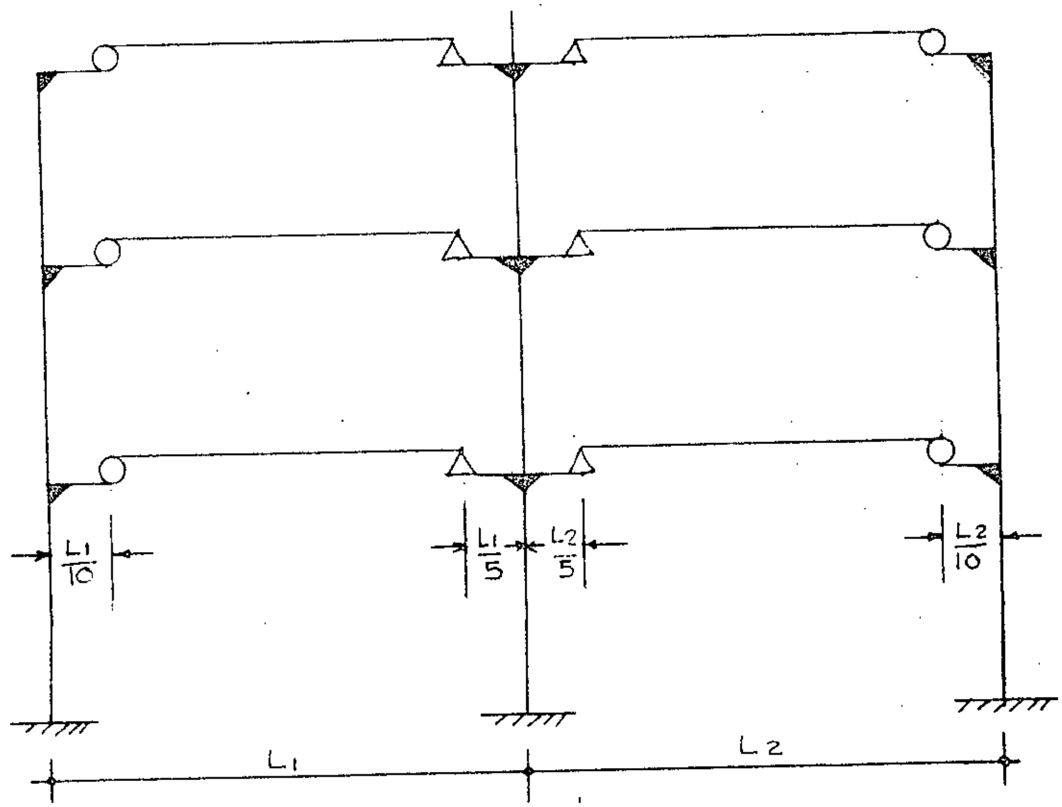


Fig. 3.8 Location of Inflection Points Suggested by Levy and Salvadori (7)

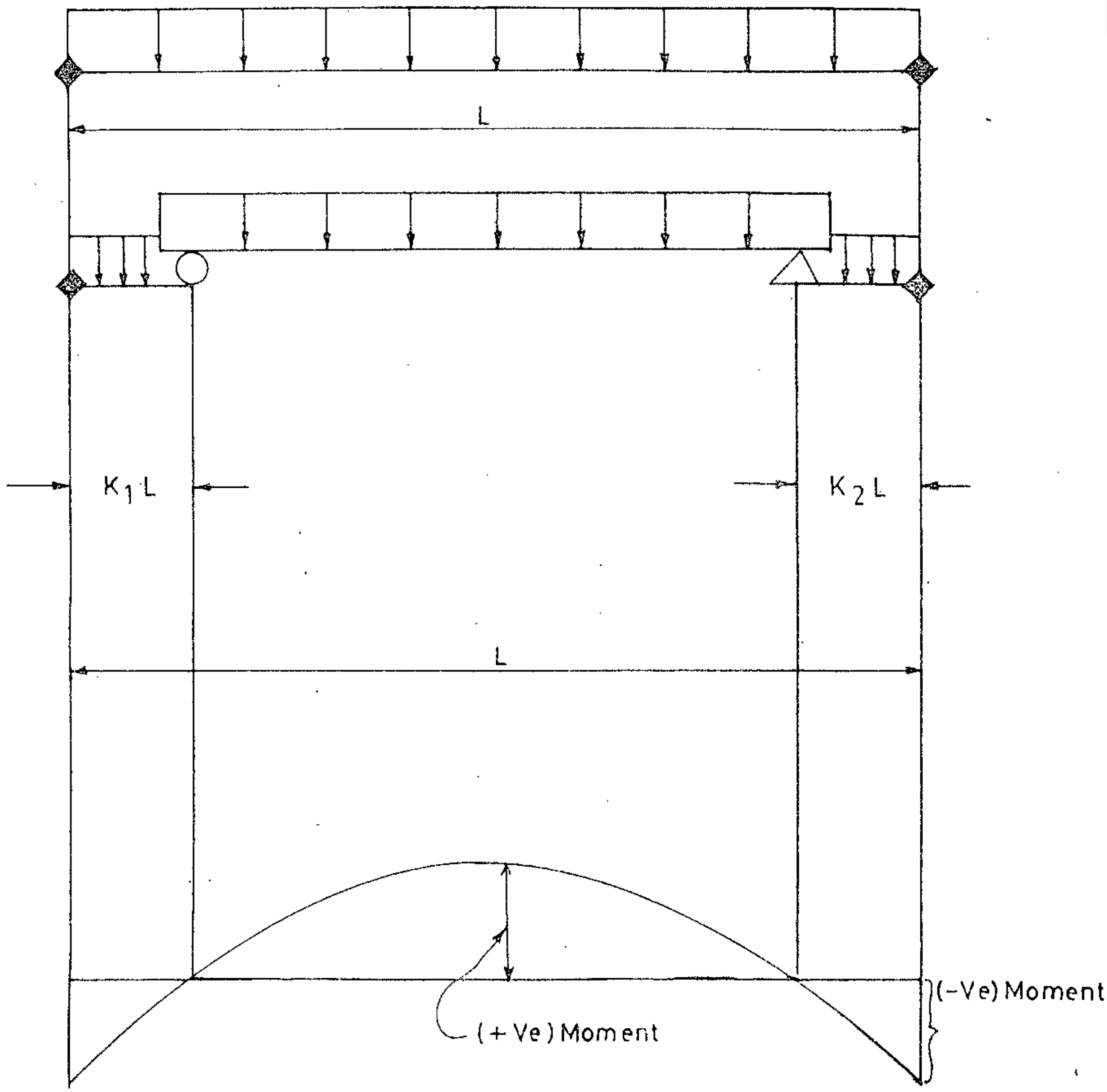


Fig. 3.9 Solution Procedure for Gravity Loading

3.2.2 Solutions

The analysis is equivalent to the solution of statically determinate bent of Fig. 3.9. Girders may then be analyzed by statics. Maximum positive moment occurs at the span (and at 0.45L from exterior ends in case of Salvadori and Levy solution) and the maximum negative moments occur at either ends of the span. The end shears acting on the girders are equal to the vertical forces applied to the columns by the girders and the axial forces in the columns are easily found by summing up the girder shears from the top of the column down to the column section under consideration. The exterior column moments come directly from the girder end moments and the interior column moments are obtained by summing the adjacent girder moments and the girder moments are distributed between the columns in proportion to their stiffnesses.

3.3 Approximate Analysis for Lateral Loads

Lateral loads on building frames are produced by the action of wind, earthquake, and blast. Exact analysis procedures are lengthy and largely depend on the availability of large capacity computers. Again it can be pointed out that considerable uncertainity prevails regarding the magnitude as well as the distribution of wind and earthquake and other lateral forces. It was pointed out in Art. 3.2 that the degree of statical indeterminacy for a building frame (with column bases fixed) equals three times the number of girders. Numbers of assumptions needed for its solution equals three times the number of girders, regardless of the type of loading. The assumptions made in analyzing building frames acted upon by vertical loads will not, however, be suitable for lateral-load analysis, for the structural action of building frames are entirely different when lateral loads are considered. This may be seen by a consideration of Fig. 3.10, which illustrates, to an exaggerated scale, the shape that a building frame takes under the action of lateral loads. It will be noted that, the location of points of inflection in the members are different from those occuring under the action of vertical loads. Actually, for lateral loads a point of inflection may occur near the center of each girder and each column.

3.3.1 Portal Method

3.3.1.1 Assumptions

In the portal method, the following assumptions are made:

(i) There is a point of inflection at the center of each girder.

(ii) There is a point of inflection at the center of each column.

(iii) The total horizontal shear on each story is divided between the columns of that story in a manner such that

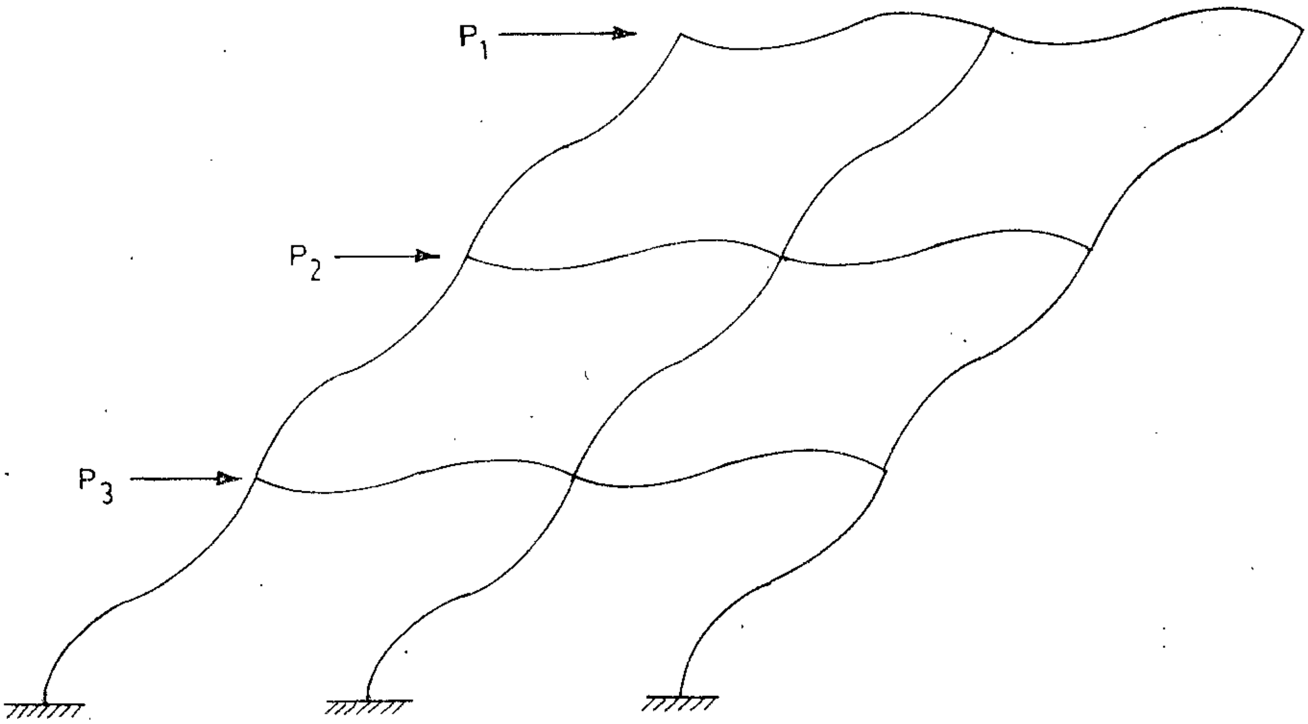


Fig. 3.10 Deflected Shape Under Lateral Loads

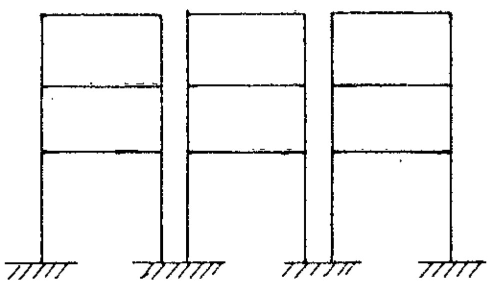


Fig. 3.11 Series of Portals Equivalent to Bldg. Frame

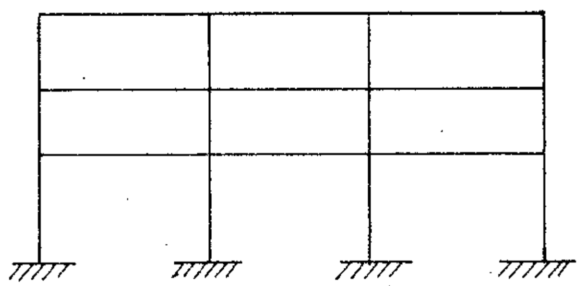


Fig. 3.12 A Building Frame

each interior column carries twice as much shear as each exterior column.

The last assumption is arrived at by considering each story to be made up of a series of portals, as shown in Fig. 3.11. Thus, while an exterior column corresponds to a single portal leg, an interior column corresponds to two portal legs, so that it becomes reasonable to assume interior columns to carry twice the shear of exterior columns.

With reference to the bent of Fig. 3.12, application of the portal methods results in making the following number of assumptions.

Inflection point in girders	2x3 = 6
Inflection point in columns	4x2 = 8
Column shear relations	2x3 = 6
Total	20

Since there are six girders in this frame, the frame is indeterminate to the eighteenth degree. Hence, the portal method makes more assumption than are necessary. However, it so happens that the additional assumptions are consistent with the necessary assumptions and no inconsistency of stresses, as computed by statics, results.

3.3.1.2 Notation

B_i	Exterior column shear, Story i
C_i	Interior column shear, Story i
$FC_{i,j}$	Column shear, Story i, Column j
P_i	Total lateral shear, Story i
$SG_{i,j,k}$	Girder shear, Story i, Girder j, Location k
XLC_i	Story height, Story i
XLG_j	Girder length, Girder j
$XMC_{i,j,k}$	Column moment, Story i, Column j, Location k
$XMG_{i,j,k}$	Girder moment, Story i, Girder j, Location k
Y_i	Lateral force, Story i

3.3.1.3 Computer Programme

Although in actual practice, it is unlikely that a digital computer would be used for approximate analysis of frames by Portal Method, a computer programme was developed for a rapid analysis of a large number of frames necessary for the comparativeevaluation made in this thesis.

The programme, based on the assumptions presented in Art. 3.3.1.1, has been written in FORTRAN and which is valid for any number of bays and any number of stories. The sequence of operations followed in the programme is outlined in the flow diagram shown in Fig. 3.13.

A listing of the programme, is given in Appendix.

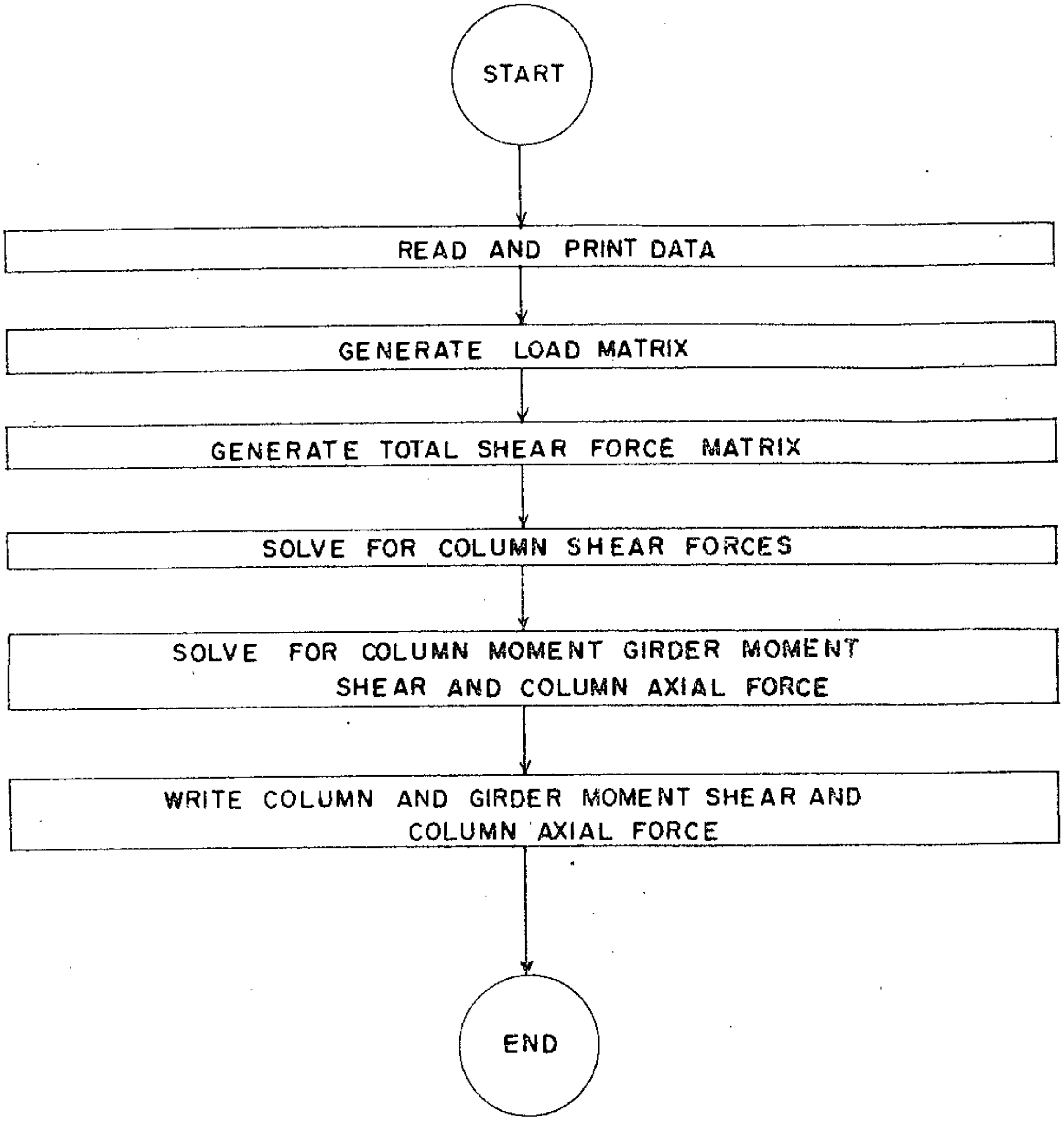


Fig. 3.13 Flow Diagram for the Computer Solution of Building Frame by Portal Method

3.3.2 Cantilever Method

3.3.2.1 Assumptions

In the cantilever method, the following assumptions are made:

(i) There is a point of inflection at the center of each girder.

(ii) There is a point of inflection at the center of each column.

(iii) The intensity of axial stress in each column of a story is proportional to the horizontal distance of that column from the center of gravity of all the columns of the story under consideration.

This last assumption is arrived at by considering that the column axial stress intensities can be obtained by a method analogous to that used for determining the distribution of normal stress intensities on a transverse section of a cantilever beam. For a building frame shown in Fig. 3.12, cantilever method makes the following number of assumptions.

Inflection points in girders	2x3 = 6
Inflection points in columns	2x4 = 8
Column axial force relation	2x3 = 6
Total	20

Since there are six girders in this frame, the total number of indeterminacy is eighteen. Hence, the cantilever method makes more assumptions than are needed, but again the additional assumptions prove to be consistent with the necessary assumptions.

3.3.2.2 Notation

$AC_{i,j}$	Column area, story i, column j
AM_i	Total moment of column area, Story i
AT_i	Total column area, Story i
CLG_j	Cumulative girder length, Girder j
$D_{i,j}$	Column distance from centroid, story i, Column j
DC_i	Distance of centroid of column area, Story i
F_i	Stress in column, Story i
$FC_{i,j}$	Force in column, Story i, Column j
P_i	Lateral shear, Story i
$SG_{i,j}$	Girder shear, Story i, Girder j
XLC_i	Story height, Story i
XLG_j	Girder length, Girder j
XM_i	Moment due to lateral force, Story i
$XMC_{i,j,k}$	Column moment, Story i, Girder j, Location k
$XMG_{i,j,k}$	Girder moment, Story i, girder j, location k

3.3.2.3 Computer Programme

A computer programme, based on the assumptions presented in Art. 3.3.2.1 has been written in FORTRAN and this programme is valid for any number of stories and any number of bays. The sequence of operations followed in the programme is outlined in the flowdiagram shown in Fig. 3.14.

A listing of the programme is given in Appendix.

3.3.3 Stiffness Center Method

3.3.3.1 Introduction

Both the commonly used approximate analysis for lateral loads, viz portal and cantilever methods, assume that there is a point of inflection at the mid point of each girder and each column. In addition while the portal method assumes that each interior column carries twice as much shear as each exterior column, the cantilever method varies the magnitude of axial force in each column according to the distance from the center of gravity of all columns of the story under consideration. Both assumptions are analogous to shear and bending stress distribution in a solid cross-section.

In a beam-column system, the rotation of ends of girders invalidates the assumption of the plane sections remaining

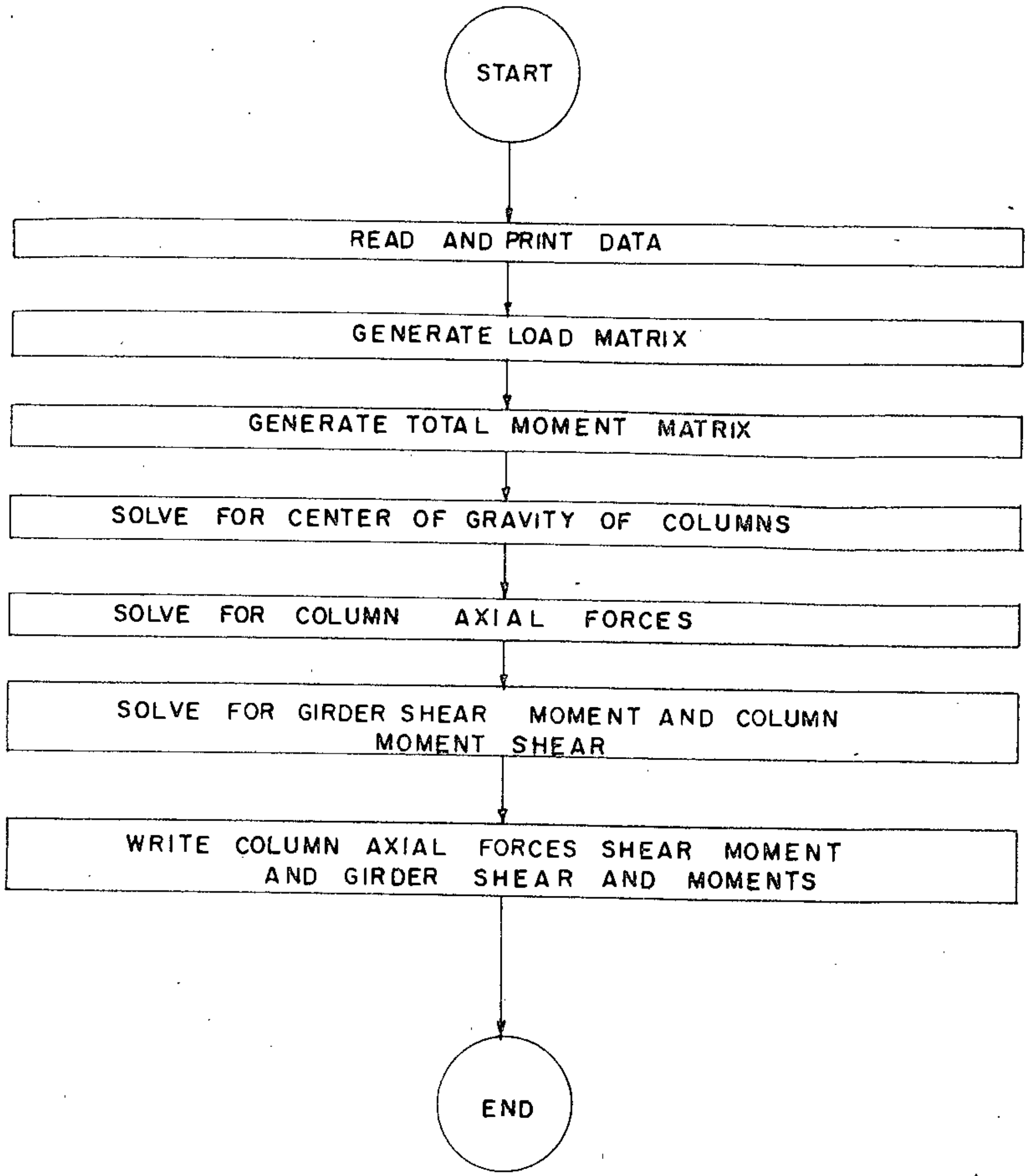


Fig. 3_14 Flow Diagram for the Computer Solution of the Building Frame by Cantilever Method

plane after the application of loads. So the concept of the lateral shear and the axial force distributions among the columns of portal and cantilever method is wrong. Rather the consideration of the axial and transverse stiffness of columns may give a better result and these stiffness consideration results in the "stiffness center method".

3.3.3.2 Assumptions

The method starts with the evaluation of the stiffness co-efficients of a structure in axial and transverse directions. And the computation of axial and transverse forces in columns are done after the calculation of stiffnesses. So, there will be three groups of assumptions: one for calculating axial and transverse stiffness, one for distributing axial and transverse forces and third for calculating the points of inflection in the columns.

Assumption Regarding Stiffness Computation

The axial stiffness is calculated by allowing the frame in a deflected shape shown in Fig. 3.15. Here the girders are assumed to be hinged at the far ends and the change in length of the column is neglected.

The transverse stiffness is measured by giving a transverse displacement as shown in Fig. 3.16 and the transverse stiffness computation includes both the shear stiffness of column and the rotational stiffness of girders.

Assumptions Regarding the Distribution of Axial and Transverse Column Forces

The axial force in a column is proportional both to the distance of the column from the stiffness center of the column and to the load carrying capacity of the column itself. And when a column is located at the stiffness center of the group it carries no load regardless of how large its capacity might be.

The transverse force in a column is directly proportional to its transverse stiffness. Therefore, it is equal to the product of the total lateral shear and the ratio of the transverse stiffness of the column in question to the summation of the stiffnesses of the group.

Assumptions Regarding the Location of Inflection Points in the Column

The location of the column inflection points depends on the column end-moments. And the column top moment is equal to moment due to side sway in column minus moments due to girder rotation, while the column bottom moment equals moment due to side sway minus half of girder rotational moments (Fig. 3.17).

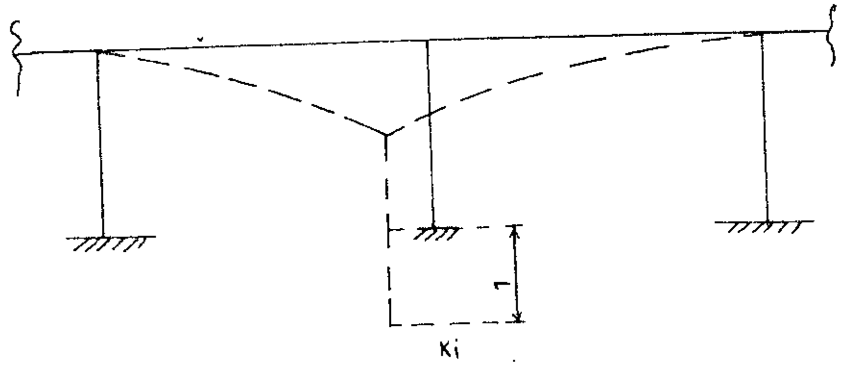


Fig. 3.15 Axial Stiffness Diagram

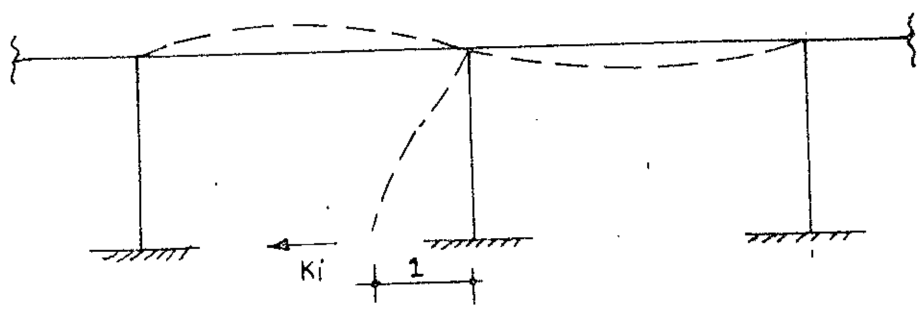


Fig. 3.16 Transverse Stiffness Diagram

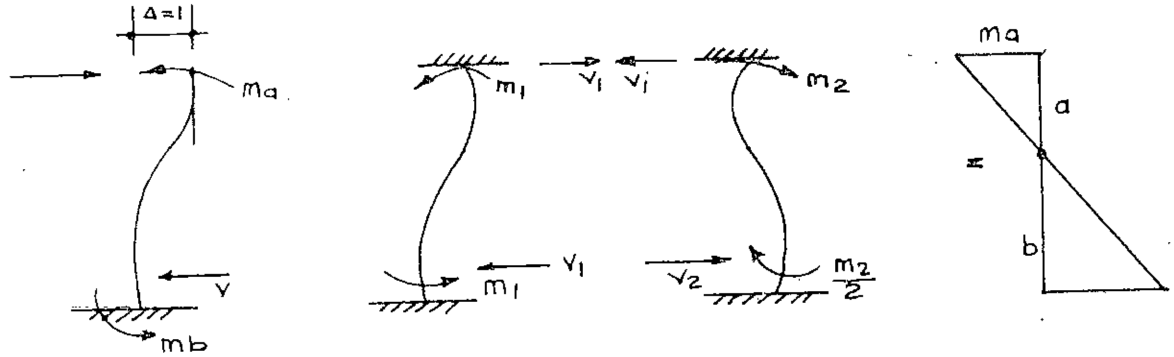


Fig. 3.17 Transverse Stiffness and Column Inflection Point Diagram

3.3.3.3 Notation

$AS_{i,j}$	Axial column stiffness, Story i, Column j
CLG_j	Cumulative girder length, Girder j
$D_{i,j}$	Distance of column from stiffness center, Story i, Column j
DC_i	Distance of Stiffness center, Story i
$F_{i,j}$	Column axial force, Story i, Column j
$GS_{i,j,k}$	Girder shear, Story i, Girder j, Location k
$H_{i,j}$	Column shear, Story i, Column j
P_i	Total lateral shear, Story i
$R_{i,j}$	Column inflection, Story i, Column j
$TS_{i,j}$	Column transverse stiffness, Story i, Column j
$XIC_{i,j}$	Column moment of inertia, Story i, Column j
$XIG_{i,j}$	Girder moment of inertia, Story i, Girder j
XLC_i	Story height, Story i
XLG_j	Girder length, Girder j
XM_i	Moment due to lateral force, Story i
$XMC_{i,j,k}$	Column moment, Story i, Column j, Location k
$XMG_{i,j,k}$	Girder moment, Story i, Girder j, Location k.

3.3.3.4 Computer Programme

A computer programme, based on the assumptions presented in Art. 3.3.3.2 has been written in FORTRAN and this programme is valid for any number of stories and any number of bays. The sequence of operations followed in the programme is outlined in the flow diagram shown in Fig. 3.18.

A listing of the programme is given in Appendix.

3.3.3.5 Solution of Building Frames

The solution of building frames in stiffness center method involves the following steps:

Solution Regarding Stiffness

The axial stiffness is calculated as

$$K_i = \sum 3EI_g/L_g^3$$

while the component contributed by AE/L_c is disregarded (shown in Fig. 3.15).

The transverse stiffass is calculated as in relative values

$$T_i = 1 - \tfrac{1}{4}(I_c/L_c)/(\sum I/L)$$

which comes from the expression $v = (v_1 - v_2)$

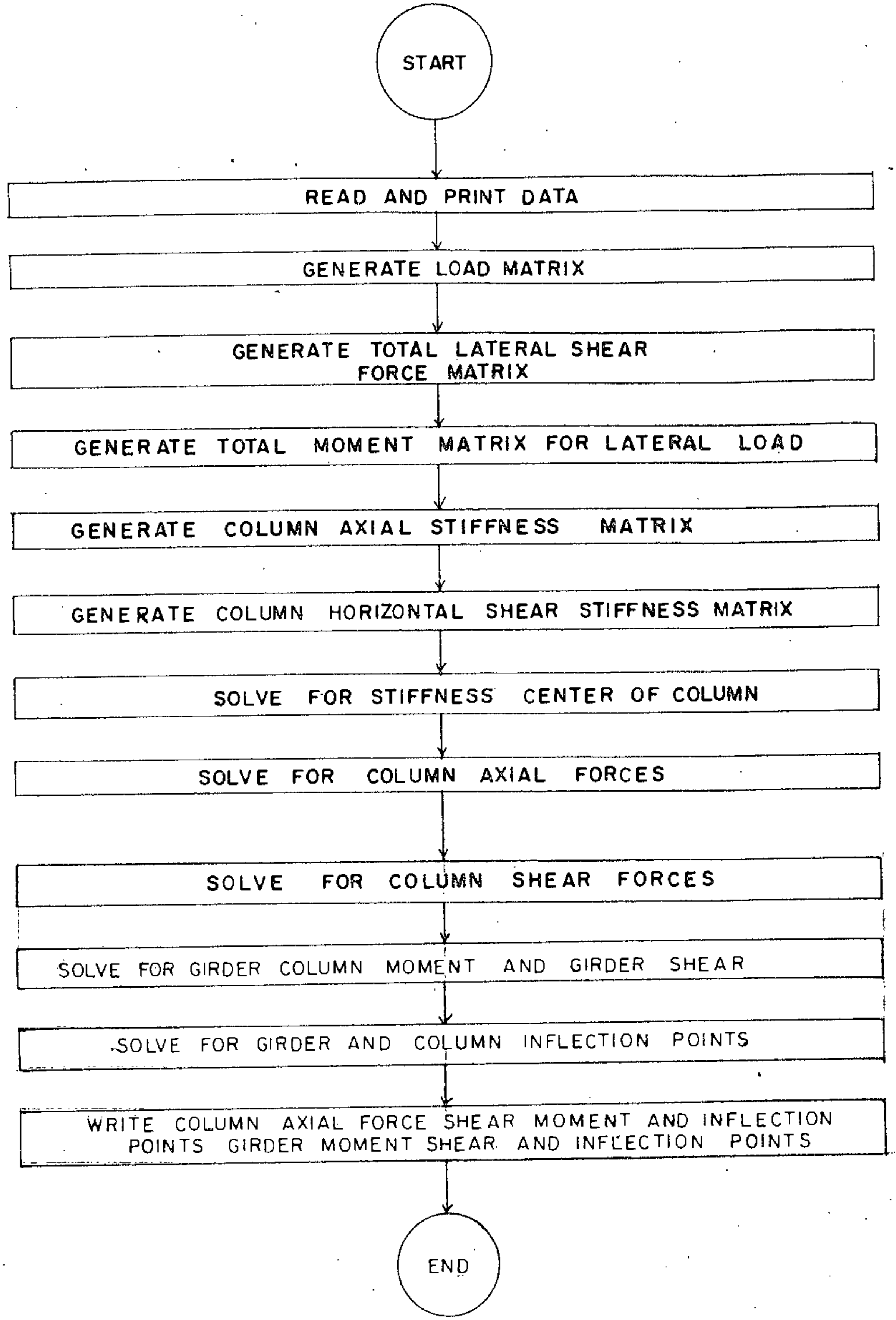


Fig. 3.18 Flow Diagram Computer Solution of Building Frame By Stiffness Center

where,

$$v_1 = 12EI_c/L_c^3$$

$$v_2 = 3m_2/(2L_c)$$

$$m_1 = 6EI_c/L_c^2$$

$$m_2 = (K_c/\sum K) \times m_1$$

$$v = 12EI_c/L_c^3 - 9EI_c/L_c^3 \times (K_c/\sum K)$$

These computations are shown in Fig. 3.16 and Fig. 3.17.

Solution Regarding Axial and Lateral Force Distribution in the Column

Axial force in any column in any story is equal to

$$p_i = K_i\ d_i\ M/(\sum K_j d_j^2)$$

where,

K_i = axial stiffness of that column

d_i = distance of that column from the centroid of axial stiffness of the columns

M = Bending moment of the lateral loads above that floor about the center point of that column

K_j = axial stiffness of any column

d_j = distance of any column from the stiffness center

And the location of the stiffness center of any story may be found from

$$D_c = \sum K_j \times D/\sum K_j$$

The lateral shear in any column in any story given by

$$H_i = T_i/\sum T_j \times V$$

where,

T_i = transverse stiffness of that column

T_j = transverse stiffness of any column

V = Total lateral shear upto that floor calculated from the top story.

Computation Regarding Column Inflection Points

The column inflection points are determined by calculating m_a and m_b for the deflected shape shown in Fig. 3.17. From figure,(3.17)

$$m_b/m_a = b/a = (m_1 - m_2/2)/(m_1 - m_2)$$

$$= \tfrac{1}{2}((2m_1 - m_2)/(m_1 - m_2))$$

where,

$$m_2 = (K_c/(\sum K)) \times m_1$$

$$b/a = \tfrac{1}{2}((2\sum K - K_c)/(\sum K - K_c))$$

$$= \tfrac{1}{2}((2\sum K_g + K_c)/(\sum K_g))$$

$$= 1 + K_c/(2\sum K_g)$$

And the complete solution of frame can easily be obtained from these known values.

CHAPTER 4

'EXACT' ANALYSIS FOR GRAVITY AND LATERAL LOADS BY STIFFNESS METHOD AND RECURSION PROCEDURE

4.1 Introduction

Multistory building frames represent one of the most complex structural systems encountered in ordinary civil engineering practice, the 'exact' analysis of which is only possible with the availability of large scale computers. In the chapter an efficient digital computer method for the structural analysis of large multistory building frames is presented. The structure may be subjected to both vertical and lateral loading and may include an arbitrary system of shear walls. Axial and shear distortion, as well as flexural distortions of the members, may be considered. The method, developed by Clough, Wilson and King[2], is based on the generation of a tri-diagonal system of stiffness equations and its solution by a recursion technique.

4.2 Assumptions and Limitations

Although the method of analysis is general, it is based on certain assumptions and limitation which are outlined below:

(a) Regular Rectangular Building Frame:

The building is laid out in a rectangular grid pattern. Story height may vary arbitrarily, as may the bay widths in each directions, but it is assumed that the complete structure is made up of series of parallel frames acting in transverse directions. Columns are assumed to extend continuously from base to top and girders from side to side. All members are assumed to be connected rigidly at the joints.

Minor variation in regularity resulting from ommission of members at arbitrary locations may be accommodated by assuming a very small value of stiffness for the omitted members.

(ii) Rigid Floor Diaphragms

The floor diaphragms are rigid in their own plane (but have no stiffness normal to this plane) and also the loading is such that it produces no rotations of the diaphragms in their plane. Thus each floor level is constrained to translate without rotation, and each parallel frame is subjected to the same displacement at any given floor level.

(iii) Shear Wall:

Shear walls may be incorporated into any of the frames. In general, it is assumed that each wall is of uniform width over the entire height of the building, but its stiffness may vary with height. Shear wall arrangement in various parallel frames may be specified arbitrarily but the assumption that the building deflects without twisting is consistent only with a reasonably symmetric distribution of stiffness.

(iv) Axial and Shear Deformations:

Axial deformations of the column and shear walls are considered in the analysis, but the girder axial deformations are neglected (as required by the rigid floor diaphragm). Shear deformations of, columns and shear walls and girder are neglected and flexural deformations are considered, of course, in all members.

4.3 Notation

A, B, C	-	Story stiffness matrices
D, E, F, G, BO, FO, FS, U		Operator matrices for recursion procedure
BU, CU	-	Stored matrices for recursion procedure
UU	-	Deformation matrix
FMAT	-	Force matrix
$GI_{i,j}$	-	Girder moment of inertia/Girder inflection, Story i, Girder j
GL_j	-	Girder length, Girder j
$GM_{i,j,k}$	-	Girder moment/Fixed end girder moment, story i, Girder j, Location k
$GS_{i,j,k}$	-	Girder shear/Fixed end girder shear, Story i, Girder j, Location k
$SA_{i,j}$	-	Column or shear wall area, story i, column or shear wall j
$SF_{i,j}$	-	Column or shear wall axial force, story i, column or shear wall j
SH_i	-	Story height, story i
$SI_{i,j}$	-	Column or shear wall moment of inertia/ Column or wall inflections, Story i, column or shear wall j
$SM_{i,j,k}$	-	Column or shear wall moment, story i, Column or Shear wall j, Location k
$SS_{i,j,k}$	-	Column or shear wall shears, Story i, Column or shear wall j, Location k
$ST_{i,j}$	-	Distances of shear wall ends from centroidal axes of wall, story i, Shear wall j.

4.4 Computer Programme

A computer programme, based on the assumption presented in Art. 4.2, has been written in FORTRAN and this programme is valid for any number of stories and any number of bays and also for any arbitrary location of shear walls. The sequence of operation followed in the programme is outlined in the flow-diagram shown in Fig. 4.1.

Four different listing of the programme for different building frames with different types of loadings are given in Appendix-A.

4.5 Development of Stiffness Matrix

The numbering system used in identifying the generalized forces and displacements is shown in Fig. 4.2. The stiffness matrix for the system is tridiagonal, as shown in Fig. 4.3. The elements of B_i and C_i matrices are developed by considering the deflected modes of the frame directly, as shown in Fig.4.4 through 4.8.

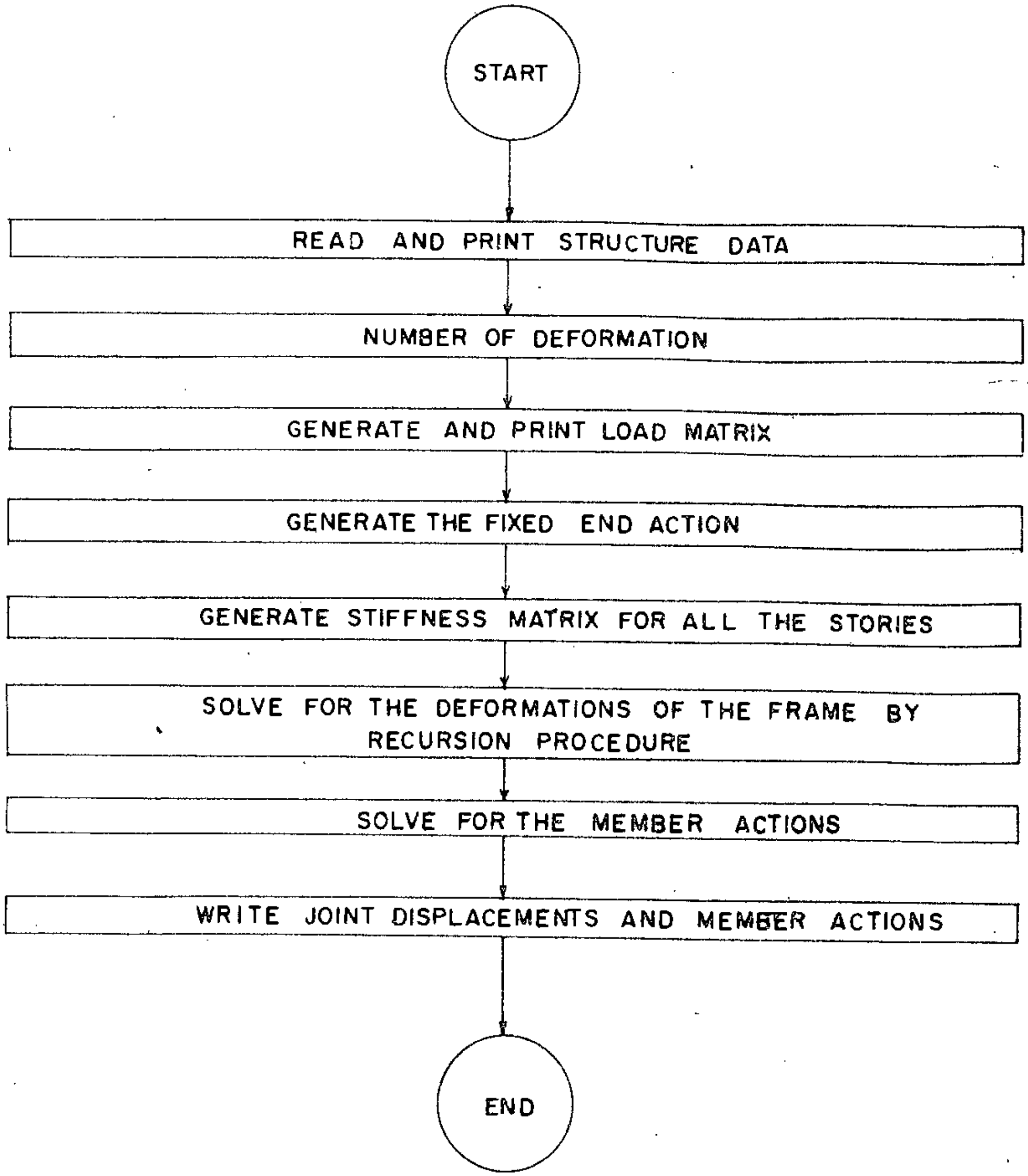


Fig. 4.1 Flow Diagram of the Computer Program for the exact Analysis of Plane Building Frame

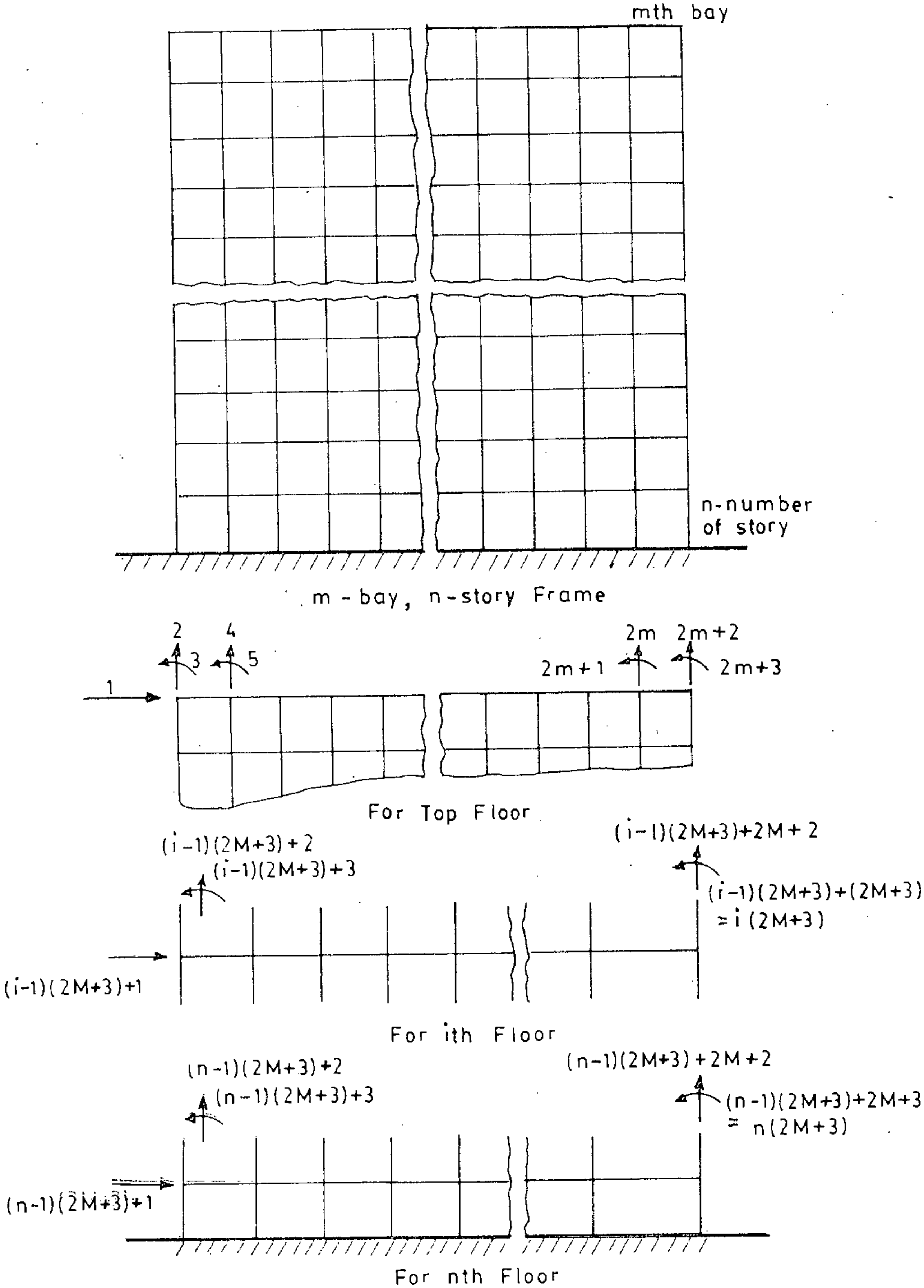


Fig. 4.2 Co ordinate Showing Joint Displacement

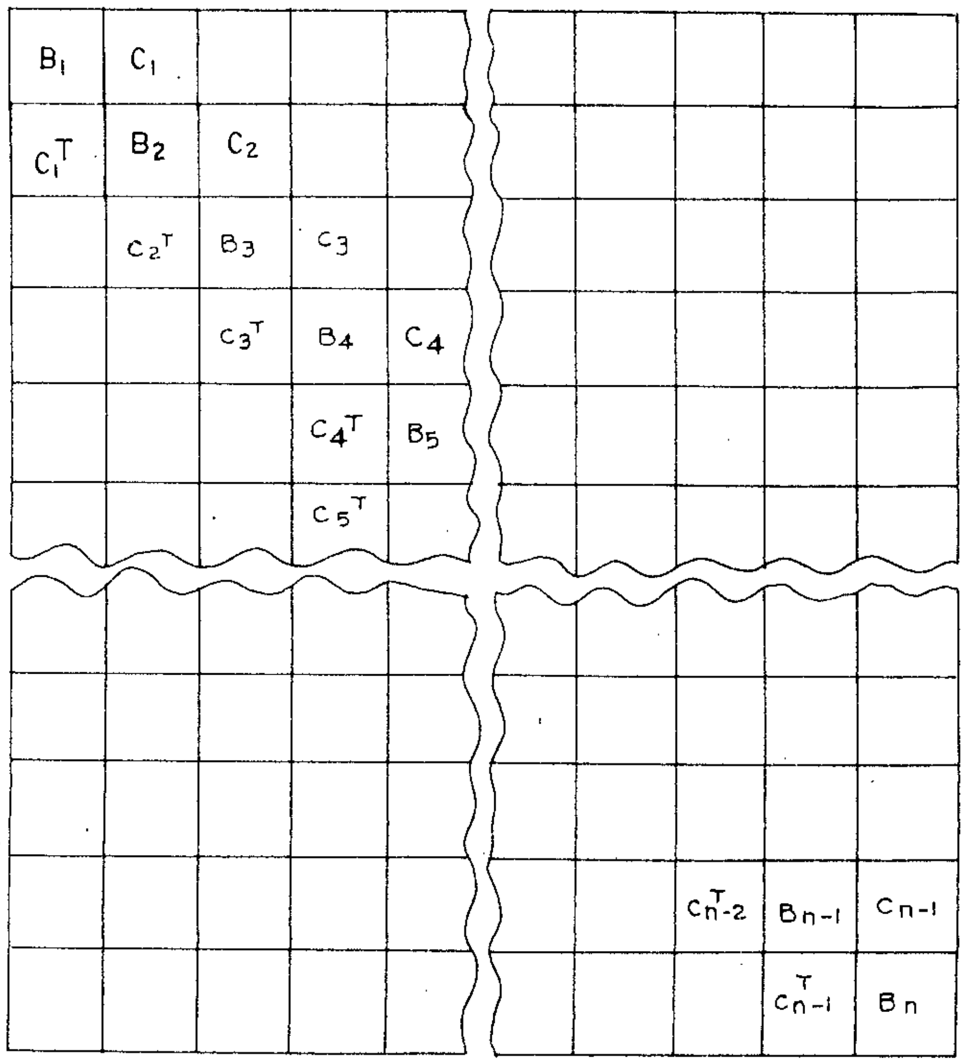


Fig. 4.3 Tri-diagonal Band Stiffness Matrix for the Frame

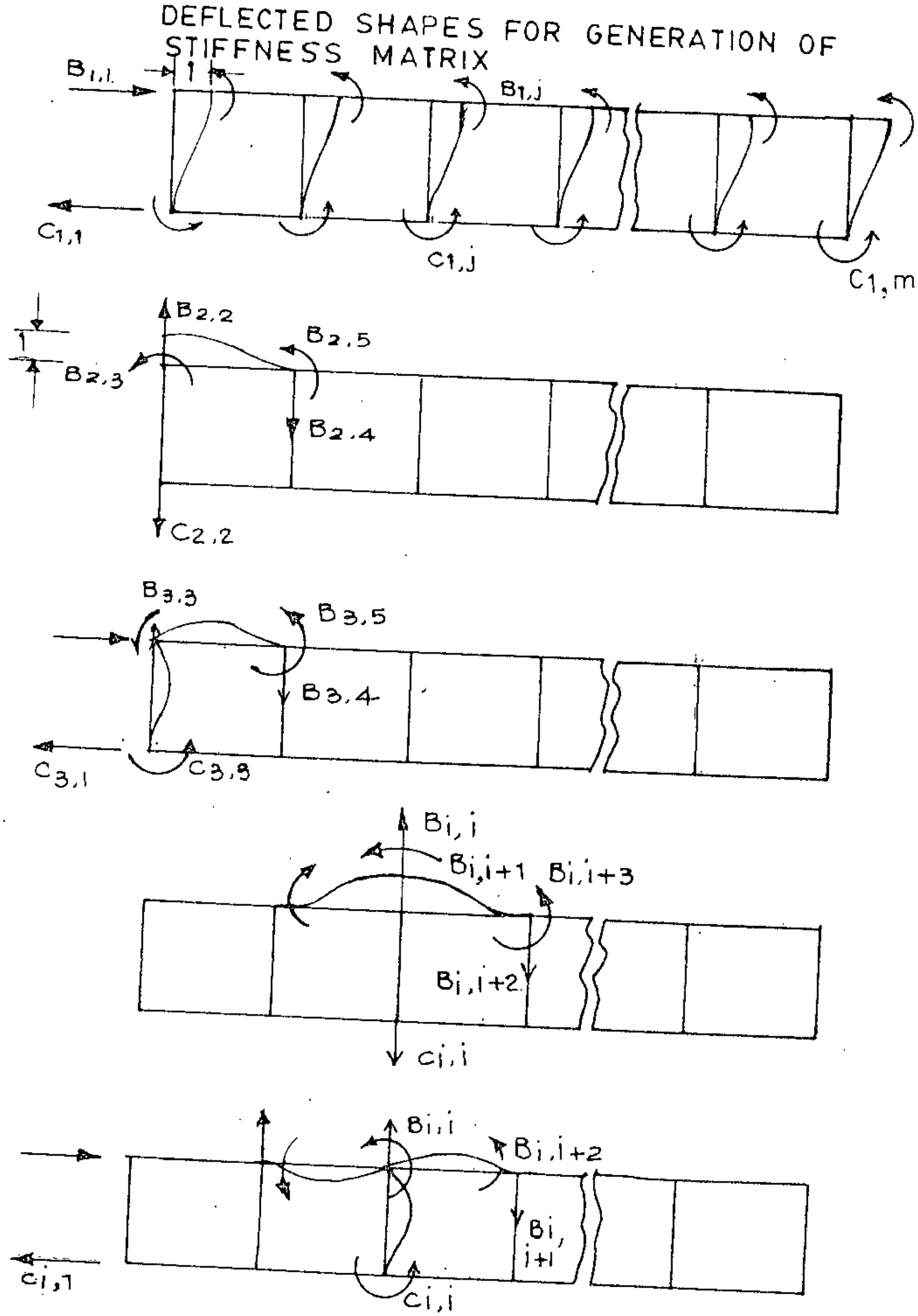


Fig. 4.4 Possible Deflected Shapes For Top Floor (Elements of B_1 Matrix)

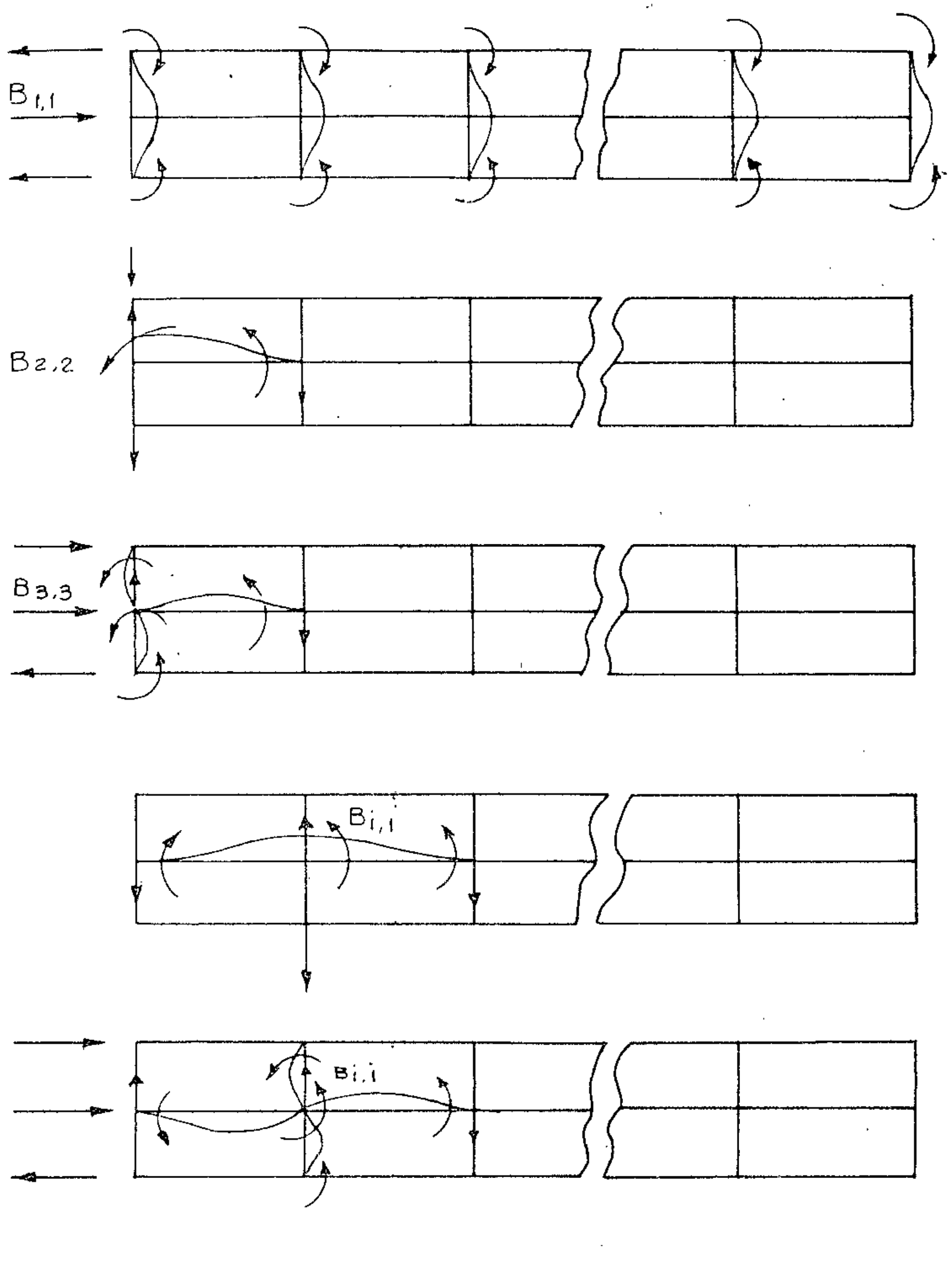


Fig. 4.5 Possible Deflected Shape for any floor Other than Top Story (Elements of B_i Matrix, $i \neq 1$)

$B_1 =$

s_{11}	s_{12}	s_{13}				$s_{1,2m+2}$	$s_{1,2m+3}$
s_{21}	s_{22}	s_{23}					$s_{2,2m+3}$
$s_{2M+2,1}$							$s_{2m+2,2m+3}$
$s_{2M+3,1}$						$s_{2m+3,2m+2}$	$s_{2m+3,2m+3}$

$C_1 =$

$s_{1,2M+4}$						$s_{1,2(2m+3)-1}$	$s_{1,2(2m+3)}$
$s_{2,2M+4}$							
$s_{2m+3,2M+4}$							$s_{2m+3,2(2m+3)}$

Fig. 4.6 Elements of Matrices B_1, C_1

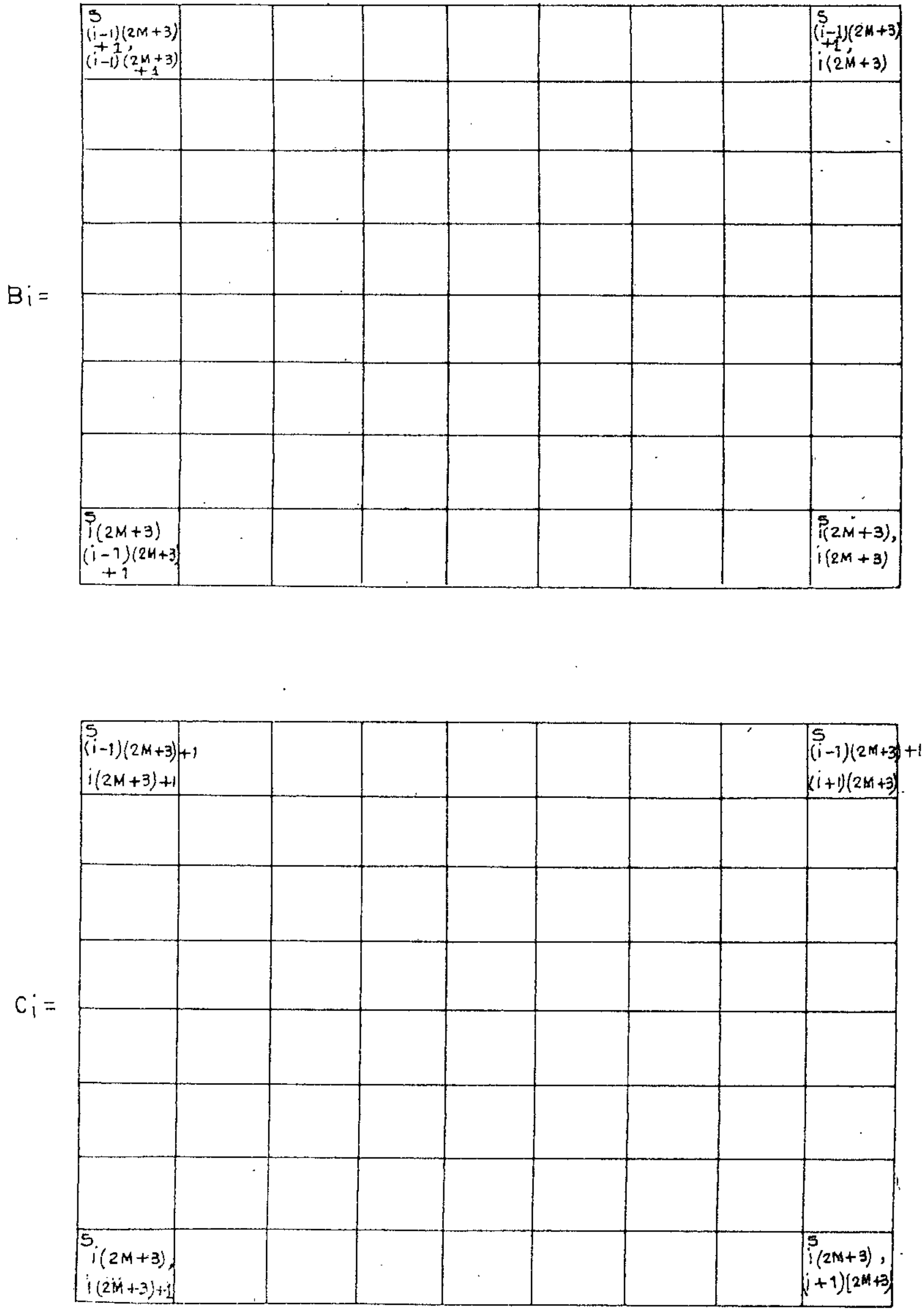


Fig 4.7 Differe... Element of Matrices B_i and C_i

BN =

$S_{(N-1)(2M+3)+1,\,(N-1)(2M+3)+1}$					$S_{(N+1)(2M+3)+1,\,N(2M+3)}$
$S_{N(2M+3),\,(N-1)(2M+3)+1}$					$S_{N(2M+3),\,N(2M+3)}$

C(N−1)

$S_{(N-2)(2M+3)+1,\,(N-2)(2M+3)+1}$					$S_{(N-2)(2M+3)+1,\,N(2M+3)}$
$S_{(N-1)(2M+3),\,(N-1)(2M+3)+1}$					$S_{(N-1)(2M+3),\,N(2M+3)}$

Fig. 4.8 Different Elements of Matrix B_n and C_{n-1}

DEVELOPMENT OF [B1]

M1 = M + 1

MF = 2M + 3

$$B(1,1) = \sum_{J=1}^{M1} 12 * SI(1,3) / SH(1)^3$$

$$B(1,J) \,/\, J = 3, MF, 2 = 6 * SI\left(1, \frac{J-1}{2}\right) / SH(1)^2$$

$$B(2,2) = (SA(1,1) / SH(1) + 12 * GI(1,1) / GL(1)^3)$$

$$B(2,3) = 6 * GI(1,1) / GL(1)^2 \, (1 + 2 * ST(1,2) / GL(1))$$

$$B(2,4) = -12 * GI(1,1) / GL(1)^3$$

$$B(2,5) = 6 * GI(1,1) / GL(1)^2 * (1 + 2 * ST(1,3) / GL(1))$$

$$B(3,3) = 4 (SI(1,1) / SH(1) + GI(1,1) / GL(1) * (1 + 3 * (ST(1,2) / GL(1)) * (1 + ST(1,2) / GL(1))))$$

$$B(3,4) = 6 * GI(1,1) / GL(1)^2 * (1 + 2 * ST(1,2) / GL(1))$$

$$B(3,5) = 2 * (GI(1,1) / GL(1) * (1 + (3 / GL(1)) * (ST(1,2) + ST(1,3) + 2 * ST(1,2) * ST(1,3) / GL(1))))$$

$$i = 4,\ 2M,\ 2$$
$$iT = i+2$$
$$iM = i-1$$
$$iN = i+1$$
$$iD = i/2$$
$$i2 = iD-1$$
$$iR = i+3$$

$$B(i,i) = SA\ (1,iD)/SL\ (1) + 12 * (GI(I,L2)/GL\ (i2)^3$$
$$GI(1,iD)/GL(iD)^3)$$

$$B\ (i,iT) = 12 * (GI(1,iD)/GL(iD)^3)$$

$$B(i,iN) = 6((GI(1,iD)/GL(iD)^2) * (1+2 * ST(1,i)/GL(iD))$$
$$-(GI(1,i2)/GL\ (i2)^2 * (1+2 * ST(1,im)/GL(i2))$$

$$B\ (i,iR) = 6 * GI(1,iD)/GL\ (iD)^2 * (1+2 * ST\ (1,iN)/GL(iD))$$

```
MR = 2 * M + 1
     i = 5 , MR , 2
     iD = ( i -1 ) /2
     i2 = iD - Z
     i1 = i +1
     iR = i +2
     i0 = i -1
     iL = i -2

B (i,i) = 4.0 * SI (1, iD) /SH (1) + 4.0 * (GI (1 , iD)
GL (iD)) + (1.0 + 3.0 * SH (1, i0) / GL (iD) * (1.0 + ST (1 , i0)
/GL (iD)) + 4.0 * ( GI (1, i2) / GL (i2) * (1.0+3.0 * ST (1, iL)
/GL (i2) * (1.0 + ST (1, iL) / GL (i2 )))

B (i,i1) = 6.0 * GI (1,iD)/GL (iD) ** 2 * (1.0 +2.0 * ST (1,i0) / GL (iD))

B (i, iR) = 2.0 * GI (1, iD) /GL (iD) * (1.0 + 3.0/ GL (iD) * (ST (1,i0)
            ST (1, i) + 2.0 * ST (1, i0) / ST (1, i) /GL (i D))
```

```
MR = 2*M + 2
M3 = 2*M+1
B (MR, MR) = SA (1, M1)/SH (1)+ 12.0 * GI (1,M)/GL(M)**3

B (MR, MR+1)=-6.0 * GI (1,M) / GL(M)** 3
              * (1.0 + 2.0 * ST(1, M3)/ GL (M))

B (MF, MF) = 4.0 * SI ( 1, M1 ) / SH (1)
        + 4.0 * GI (1,M )/GL(M) * (1.0 +3.0 * ST (1,M3)
         /GL(M) * (1.0 + ST (1, M3)/GL ( M )))
```

DEVELOPMENT OF [C1]

$C(1,1) = -B(1,1)$

$i = 3, MF, 2$

$iD = i - 1/2$

$C(1,i) = 6.0 * (SI(1,iD)/SH(1)**2)$

$M3 = 2 * M + 2$

$i = 2, M3, 2$

$iD = i/2$

$C(i,i) = -SA(1,iD)/SH(1)$

$i = 3, MF, 2$

$iD = i - 1/2$

$C(i,1) = -6.0 * SI(1,iD)/SH(1)**2$

$C(i,i) = 2.0 * SI(1,iD)/SH(1)$

All the Elements of the Matrix is to be Multiplied by Modulus of Elasticity. The Values of other Elements of the Matrix which are not Mentioned, are Zero

DEVELOPMENT OF STIFFNESS MATRIX FOR FLOOR NUMBER 2 TO N

Total Unknown above ith floor = (i-1) (2* M + 3)

```
i = 2,N
iM = (i - 1) * (2 * M + 3)
iI = (i -1 )
K = 1, MF
L = 1, MF
B (K, L) = 0 0
C (K, L) = 0 0

SUM = 0.0
j = 1, MF
SUM = SUM + 12.0*(SI(i1 , j)/SH (i1)** 3 + SI (i, j) SH(i)**3)

B (1,1) = SUM
j = 3, MF, 2
iD = ( j-1 ) / 2
B (1, j) = 6.0 * (SI (i, iD)/SH (i)* *2 - SI(i1, iD)/
   SH (iD) * * 2 )
```

```
B (2,2) = SA (i1,1)/SH(i1) + SA(i,1)/SH(i) +
          12.0 * GI(i,1)/GL(1)**3

B (2,3) = 6.0 * GI(i,1)/GL(1) ** 2 * (1.0 +
          2.0 * ST(i,2)/GL(1))

B (2,4) = -12.0 * GI(i,1)/GL(1)** 3

B (2,5) = 6.0 * GI(i,1)/GL(1) ** 2 * (1.0 +
          2.0 * ST(i,3)/GL(1))


B (3,3) = 4.0 * GI(i,1)/GL(1) * (1.0 + 3.0 *
ST(i,2)/GL(1) * (1.0 + ST(i,2)/GL(1))) +
 4.0 * (SI(i1,1)/SH(i1) + SI(i,1)/SH(i))

B (3,4) = (-6.0 * GI(i,1)/GL(1) ** 2) *
          (1.0 + 2.0 * ST(i,2)/GL(1))

B (3,5) = 2.0 * GI(i,1)/GL(1) * (1.0 + 3.0/GL(1) *
 (ST(i,2) + ST(i,3) + 2.0 * ST(i,2) * ST(i,3)/
  GL(1)))
```

```
M2 = 2*M
L = 4,M2,2
LD = L/2
L2 = LD-1
L1 = L-1

B(L,L) = SA(i,LD)/SH(i) + SA(i1,iD)/
SH(i1) + 12.0 * GI(i,L2)/GL(L2)**3 +
GI(i,LD)/GL(LD)**3)

B(L,L+1) = 6.0 * GI(i,LD)/GL(LD)**2*
(1.0+2.0*ST(i,L)/GL(LD)) - 6.0 * GI(i,L2)/
GL(L2)**2*(1.0 + 2.0 *ST(i,L)/GL(L2))

B(L,L+2) =-12.0* GI(i,LD)/GL(LD)**3

B(L,L+3)= 6.0* GI(i,LD)/GL(LD)**2
*(1.0 + 2.0 * ST(i,L+1)/GL(LD))
```

```
MB = 2 * M + 1
L = 5 , MB, 2
LD =(L   1)/2
L2 = LD - 1
L1 = L - 1

B (L, L) = 4.0 *(S1 (i1, iD)/SH  (i1) + SI (i, LD)/SH  (i)) +
        4.0 * GI (i, L2) / GL  (L2) * ( 1.0 + 3.0 * ST (i , L- 2 )/
        GL  (L2) *(1.0 + ST (i,L-2 )/GL   (L2) + 4.0  *
        GI ( i, LD ) / GL (LD)  * (1.0+3.0 * ST ( i, L1 ) / GL(LD)
        (1.0 + ST ( i, L1 ) / GL (LD )))

B (L , L+1 ) = - 6.0 * GI  ( i, LD )/GL  ( LD ) * * 2  *  (1.0 + 2.0  *
              ST  (i, L1)/GL  (LD ))

B  (L, L+2 ) =  ( 2.0  * GI ( i, LD ) /GL  ( LD )) * (1.0 + 3.0 /GL (LD) *
             ( ST   ( i, L1) + ST  ( i, L ) + 2.0 * ST  ( i, L1 ) *
             ST ( i , L ) / GL  ( i  D )))
```

```
MR = 2 * M + 2
NR = MR - 1
B (MR,MR) = (SA (i1, M1) /SH (i1)+SA(i,M1)/SH(i) +
   12.0 * (GI (i,M) / GL(M) ** 3)

B (MR,MF) = - 6.0 * GI (i,M)/GL (M) ** 2 *(1.0 +
            2.0 * ST (i,NR)/GL (M))

B (MF,MF) = 4.0 * GI (i,M) /GL (M) *(1.0 + 3.0 * ST
            (i,NR) /GL (M) *(1.0 + ST (i,NR)/GL
            (M))) + 4.0 *( SI (i1, M1)/SH (i1)+SI
            (i, M1) /SH (i) )
```

In B Matrix on the upper Triangular Matrix is Generated as Matrix B is Symmetrical

```
MR = 2*M + 2
L = 2 , MR , 2
LD = L/2
C (L,L)=-SA (i , LD)/SH(i)

L=3 , MF, 2
LD = L - 1/2
C (L,1)=-6.0 * SI ( i , LD )/
SH ( i ) ** 2

C (L, L) = (2.0 * SI (i , LD) /SH (i) )
```

```
i = 2 ,   N - 1, 1
SUM   00
J = 1 , MI
 SUM = SUM + 12.0 * (SI (i,J) / SH(i) ** 3)

 C (i, 1) = SUM

 K = 3 ,  MF , 2
 KD =(K-1)/2

 C (1 ,K)=(6.0 *SI (i,KD)/SH (i) ** 2)
```

4.6 SOLUTION OF THE FRAME

The equilibrium equation may be conveniently solved with the help of a recursion technique[2] as outlined below:

$$\begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_i \\ \vdots \\ F_n \end{Bmatrix} = \begin{bmatrix} B_1 & C_1 & & & & & \\ A_2 & B_2 & C_2 & & & & \\ & A_3 & B_3 & C_3 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & A_i & B_i & C_i & \\ & & & & \ddots & \ddots & \ddots \\ & & & & & A_n & B_n \end{bmatrix} * \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ \vdots \\ U_i \\ \vdots \\ U_n \end{Bmatrix}$$

$$\{F_1\} = \{B_1\}\{U_1\} + \{C_1\}\{U_2\} \tag{1}$$

$$\{F_2\} = \{A_2\}\{U_1\} + \{B_2\}\{U_2\} + \{C_2\}\{U_3\} \tag{2}$$

$$\{F_3\} = \{A_3\}\{U_2\} + \{B_3\}\{U_3\} + \{C_3\}\{U_4\} \tag{3}$$

$$\vdots$$

$$\{F_i\} = \{A_i\}\{U_{i-1}\} + \{B_i\}\{U_i\} + \{C_i\}\{U_{i+1}\} \tag{i}$$

$$\vdots$$

$$\{F_n\} = \{A_n\}\{U_{n-1}\} + \{B_n\}\{U_n\} \tag{n}$$

$$\{U_1\} = \{B_1\}^{-1}\left[\{F_1\} - \{C_1\}\{U_2\}\right] \tag{1(a)}$$

From Eq 1(a) and Eq 2 we can write

$$\{F_2\} = \{A_2\}\{B_1\}^{-1}\{F_1\} - \{A_2\}\{B_1\}^{-1}\{C_1\}\{U_2\} + \{B_2\}\{U_2\} + \{C_2\}\{U_3\}$$

$$\{F_2^*\} = \{F_2\} - \{A_2\}\{B_1\}^{-1}\{F_1\}$$

$$\{B_2^*\} = \{B_2\} - \{A_2\}\{B_1\}^{-1}\{C_1\}$$

$$\{F_2^*\} = \{B_2^*\}\{U_2\} + \{C_2\}\{U_3\}$$

$$\{U_2\} = \{B_2^*\}^{-1}\{F_2^*\} - \{B_2^*\}^{-1}\{C_2\}\{U_3\} \quad \cdots 2(a)$$

$$\vdots$$

$$\{U_i\} = \{B_i^*\}^{-1}\{F_i^*\} - \{B_i^*\}^{-1}\{C_i\}\{U_{i+1}\} \quad \cdots\cdots i(a)$$

$$\vdots$$

$$\{U_n\} = \{B_n^*\}^{-1}\{F_n^*\} \quad \cdots\cdots\cdots\cdots n(a)$$

From Eq^{n}s 1(a) 2(a) n(a)

We have

$$\{U_n\} = \{B_n^*\}^{-1}\{F_n^*\}$$

$$\{U_{n-1}\} = \{B_{n-1}^*\}^{-1}\{F_{n-1}^*\} - \{B_{n-1}^*\}^{-1}\{C_{n-1}\}\{U_n\}$$

$$\vdots$$

$$\{U_i\} = \{B_i^*\}^{-1}\{F_i^*\} - \{B_i^*\}^{-1}\{C_i\}\{U_{i+1}\}$$

$$\vdots$$

$$\{U_1\} = \{B_1^*\}^{-1}\{F_1^*\} - \{B_1^*\}^{-1}\{C_1\}\{U_2\}$$

In Computer Programme

$\{B_i^*\}^{-1} * \{F_i^*\}$ and $\{B_i^*\}^{-1} * \{C_i\}$ are Stored as $\{BU_i\}$ and $\{CU_i\}$ respectively for $i=1$ to $n-1$ and $\{U_i\}$ are Stored as $\{UU_i\}$ for $i = 1$ to n .

$$\{F_1^*\} = \{F_1\}$$

$$\{F_2^*\} = \{F_2\} - \{A_2\}\{B_1^*\}^{-1}\{F_1^*\}$$

$$\vdots$$

$$\{F_i^*\} = \{F_i\} - \{A_i\}\{B_{i-1}^*\}^{-1}\{F_{i-1}^*\}$$

$$\vdots$$

$$\{F_n^*\} = \{F_n\} - \{A_n\}\{B_{n-1}^*\}^{-1}\{F_{n-1}^*\}$$

$$\{B_1^*\} = \{B_1\}$$

$$\{B_2^*\} = \{B_2\} - \{A_2\}\{B_1^*\}^{-1}\{C_1\}$$

$$\vdots$$

$$\{B_i^*\} = \{B_i\} - \{A_i\}\{B_{i-1}^*\}^{-1}\{C_{i-1}\}$$

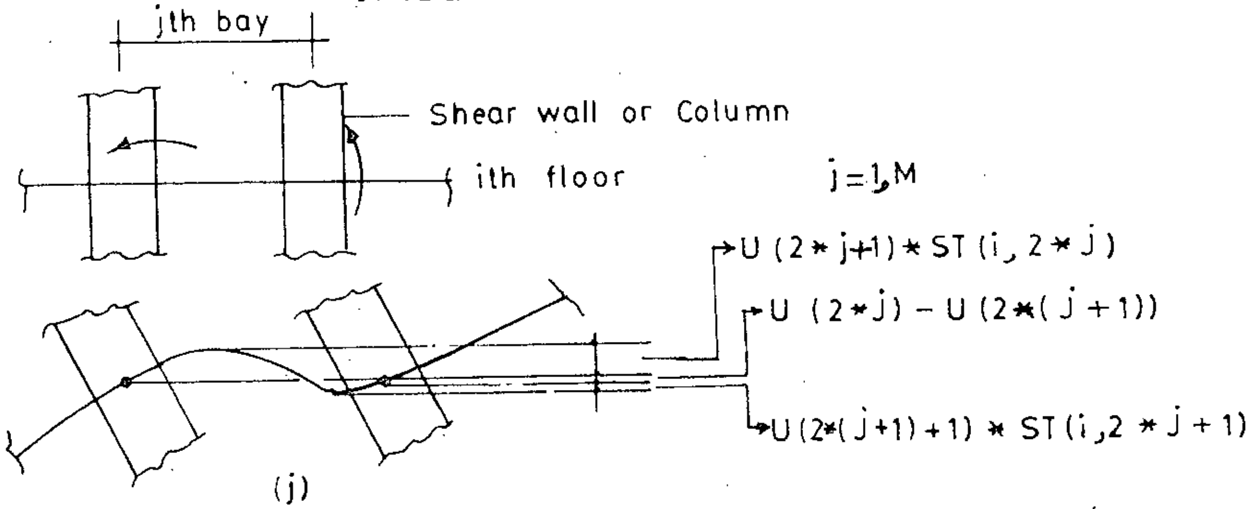
$$\vdots$$

$$\{B_n\} = \{B_n\} - \{A_n\}\{B_{n-1}^*\}^{-1}\{C_{n-1}\}$$

SOLUTION OF THE MEMBER END ACTIONS

$$i = 1, N$$
$$iN = (i-1) * MF$$
$$MK = 2 * MF$$
$$K = 1, MK$$
$$U(K) = UU(iN + K)$$

GIRDER MOMENTS AND SHEARS



$$DEL = U(2*J) - U(2*J+2) + U(2*J+1) * ST(i, 2*J) + U(2*J+3) * ST(i, 2*J+1)$$

Fig 4.9 Girder Deflection for Girder end action Computation

$$GM(i,j,1) = (2.0 * GI(i,j)/GL(j)) * (3.0 * DEL/GL(j) + 2.0 * U(2*j+1) + U(2*j+3)) + FGM(i,j,1)$$

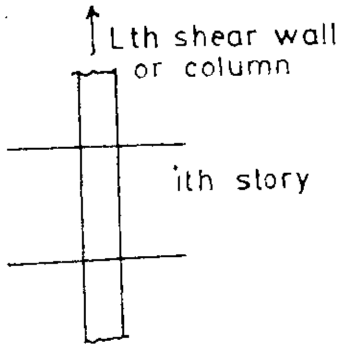
$$GM(i,j,2) = (2.0 * GI(i,j)/GL(j)) * (3.0 * DEL/GL(j) + U(2*j+1) + 2.0 * U(2*j+3)) + FGM(i,j,2)$$

$$O = (GM(i,j,1) + GM(i,J,2))/GL(j)$$

$$GS(i,j,1) = FGS(i,j,2) + O$$

$$GS(i,j,2) = FGS(i,j,2) - O$$

COLUMN MOMENTS AND SHEARS



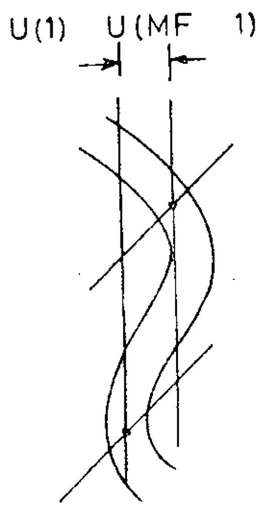
L = 1, M1

LM = 2 * L+1

LK = MF+2 * L+1

DEL = U (1) − U (MF+1)

SM (i,L,1) = 2.0 * SI (I,L)/SH (1) *
(3.0 * DEL/SH (I)+2.0 * U (LM) +
U (LK)) + FGM (I,L,1)



SM (I,L,1) = 2.0 * SI (I,L)/SH(I) *
(3.0 * DEL/SH (2) + U (LM)+2.0* U(LK)) +
FGM (I L 2)

0 = (SM (I,L,1) + SM(I,L,2))/SH (J)

SS(I,L,1) = FSS (I,L,1) + 0

SS (I,L,2) = FSS (I,L,2) − 0

Fig. 4.10 Deflected Shape for Column end Action Computation

COLUMN AXIAL FORCE

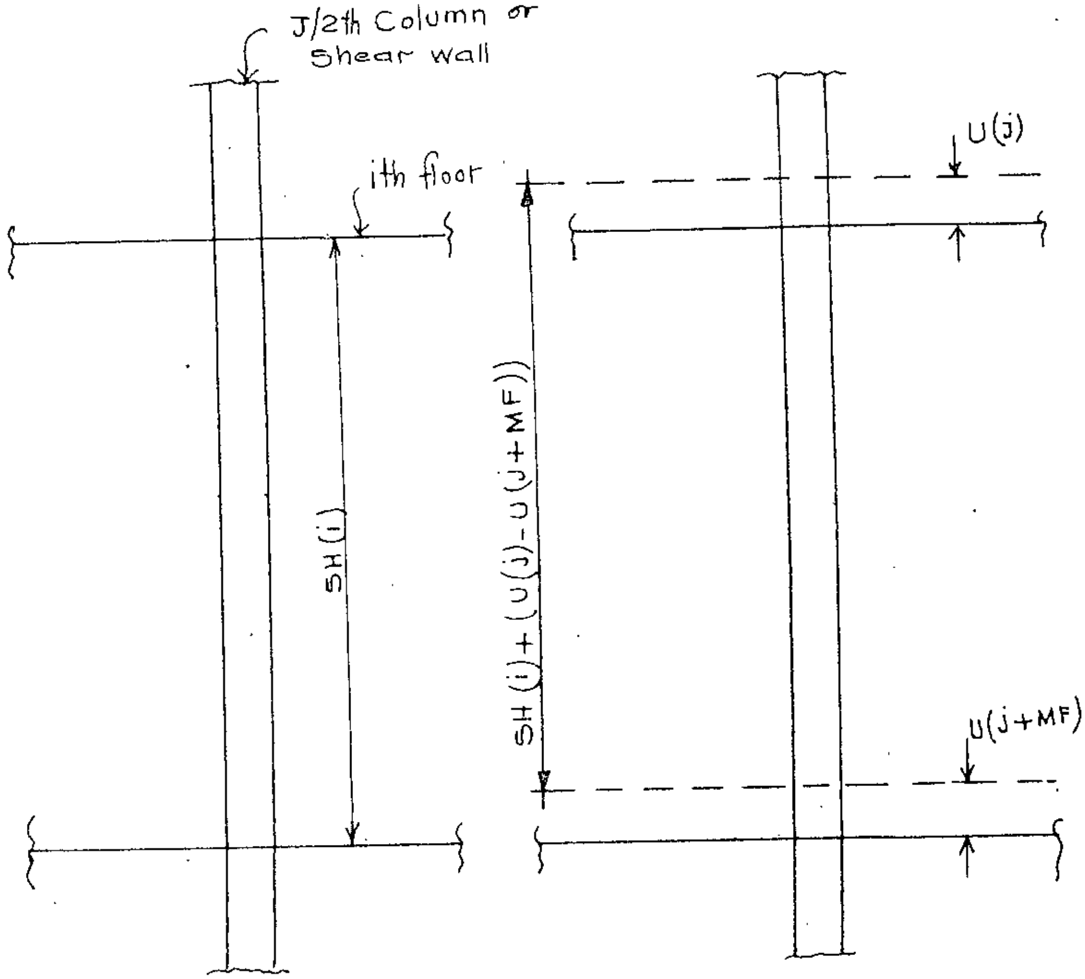


Fig 4.11 Column Deflection for column Axial Force Computation

$J = 2, MF, 2$

$SF(i, J/2) = -SA(i, J/2) / SH(i) * (U(J) - U(J+MF))$

CHAPTER 5

DISPERSION OF CONCENTRIC LOADS IN ORDINARY BUILDING FRAMES

5.1 Introduction

For a medium-rise concrete building upto say 20 stories, the use of shear wall is often a matter of choice. But for buildings taller than 20 stories, and particularly in the range of 30-60 stories, the use of shear walls in one form or other becomes imperative from the point of view of economy. And the use of shear walls results in interaction of frames (both parallel and perpendicular) with shear walls. Fig. 5.1 shows a plan of a high-rise building with shear walls running across the entire depth of the building, where the exterior columns are closely spaced and are connected with heavy spandrels. The wind on the long face in those buildings are carried primarily by the shear walls and it seems logical that the closely spaced column-spandrel system should interact with shear wall and will make up an equivalent flange for the wall.

The effectiveness of the closely spaced exterior frame to form an equivalent flange of the shear wall can be studied by isolating the shear wall from the frame and investigating the vertical load distribution characteristic of the frame itself.

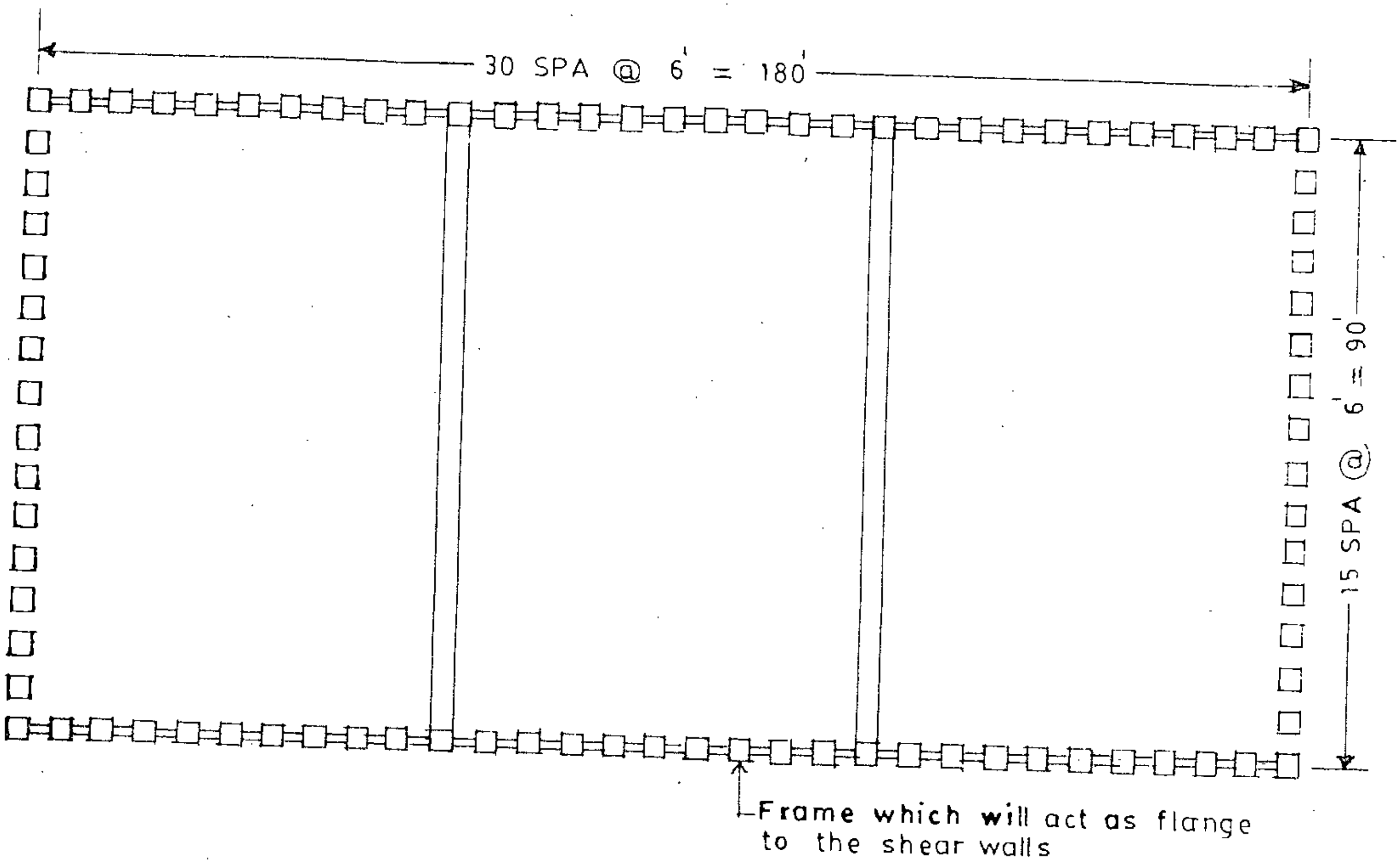


Fig. 5.1 A Common Plan of High Rise Building

5.2 Method of Analysis Suggested by Khan[6]

Since the interacting frame is connected to the shear wall only at one column, the analysis of the interaction will primarily involve solution for vertical load distribution in a grid wall system shown in Fig. 5.2. The interacting columns may be thought of to be composed of two systems: system I will consist of the column directly connected to the shear wall, system II will consist of remaining spandrel and columns as shown in Fig. 5.3. For uniform lateral load the vertical shear at each connecting point will be proportional to the total horizontal shear at each level. Therefore, a linearly increasing vertical loads should be applied to the system I. And after the solution of interacting frame for this increasing concentric load, the effective numbers of columns participating fully as flange of the shear wall at each floor is computed by the ratio of the total force initially applied at the column to the actual force computed at the end of analysis.

5.3 Analytical Varification

The behaviour of the interacting building frame normal to the plane of shear wall is studied by computer analysis of building frames subjected to a linearly varying concentric

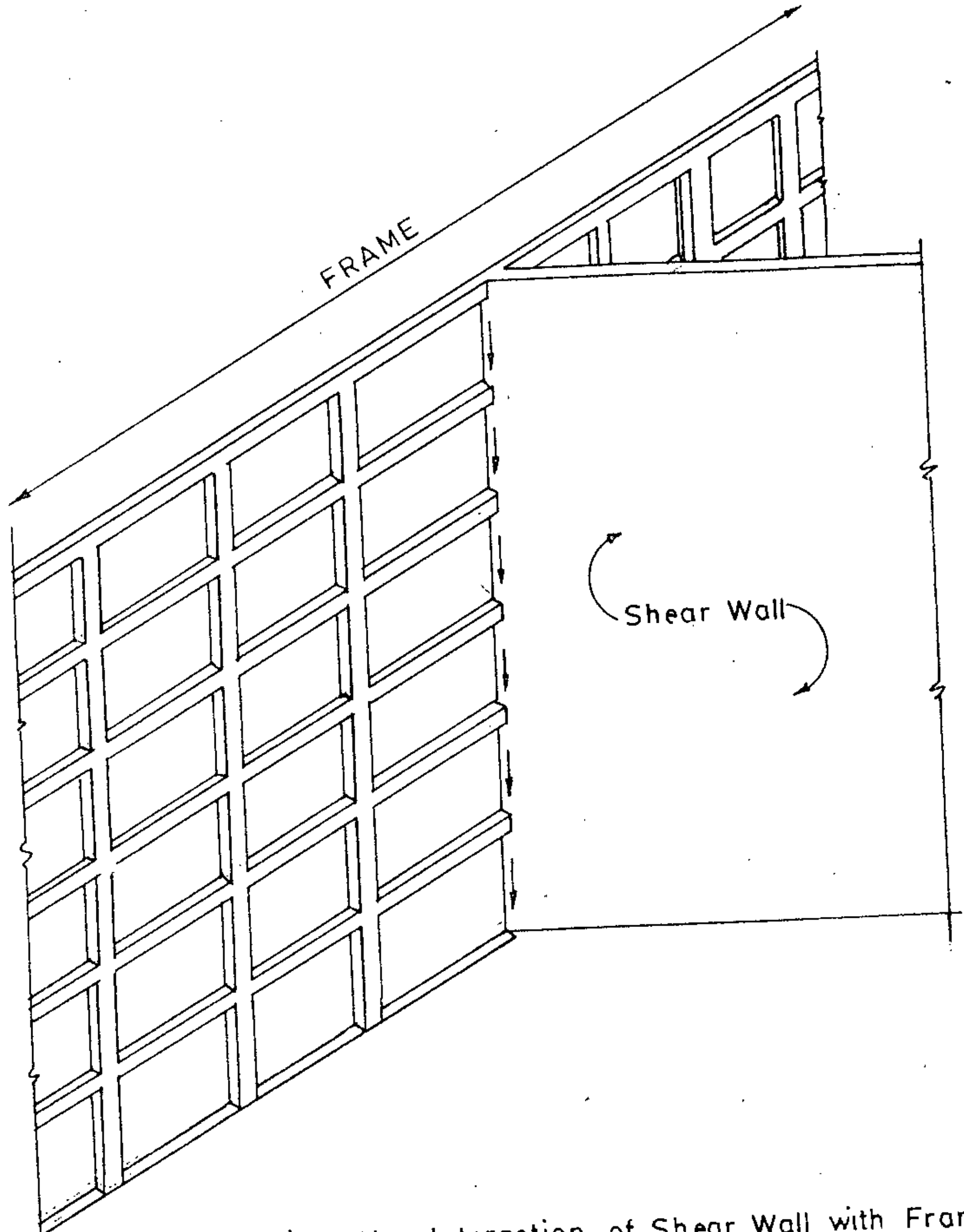


Fig: 5.2 Showing the Interaction of Shear Wall with Frame

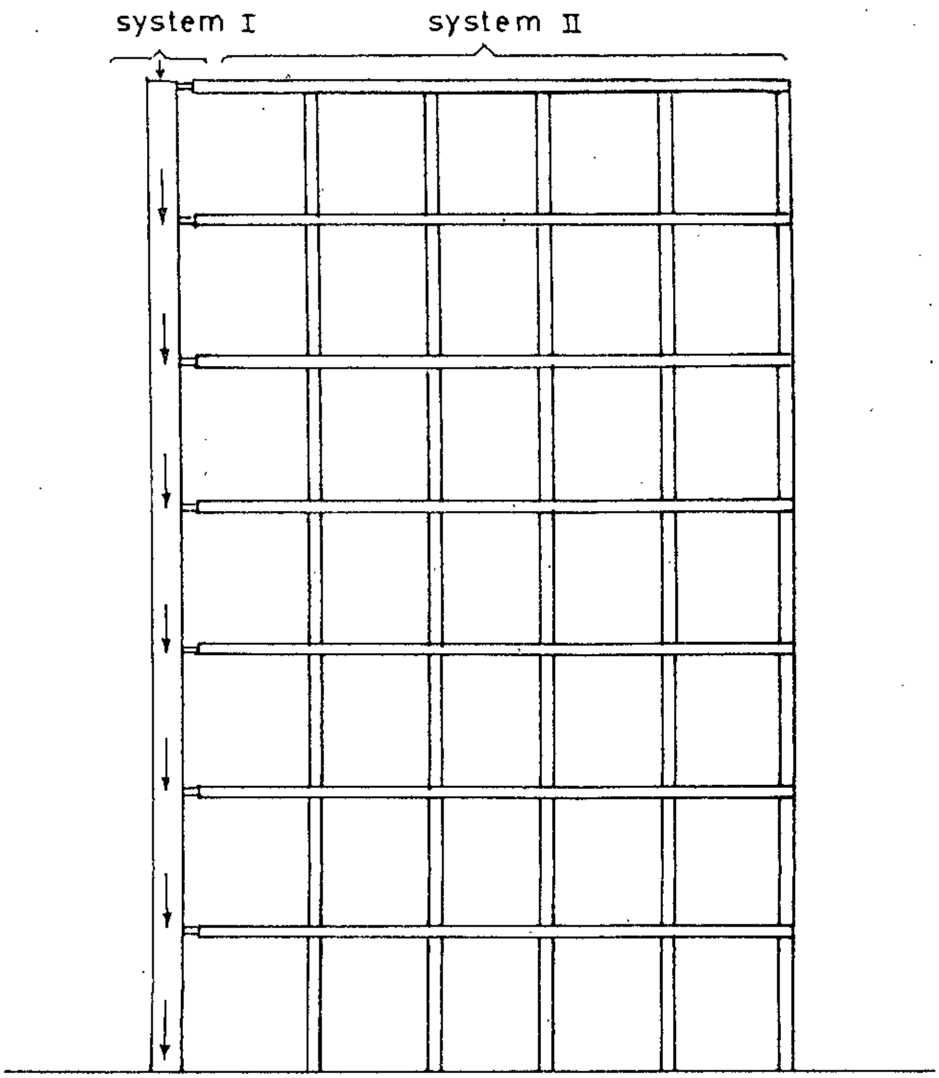


Fig. 5.3 Showing Two Different Column System

loads along the exterior column line. The programme generated and used can accommodate any number of bays and stories. For analytical varification of the method suggested by Khan, a set of five-bay five-story frames are analysed for the ratio of V_b/V_c varying from 0.01 to 10.0 for all levels (i.e. for stories varying from 2 to 5). V_b is the shear stiffness of the beam and is equal to $12*GI/GL^3$ (assuming equivalent shear stiffness of beam $C_v = 1.0$) and V_c is the axial stiffness of column and is equal to SA*E/SH.

CHAPTER 6

ANALYSIS OF BUILDING FRAMES FOR DIFFERENT STAGE OF CONSTRUCTION

6.1 Importance of this Analysis

In general the analysis of a building frame is done considering the entire building frame with its loading. But the construction of this frame is carriedout story-by-story which implies that the total number of stories varies at each level of construction. As an example, for a five story building frame there will be one, two, three four and five story frames and a dead load of the 2nd floor cannot be resisted by a 5-story frame, since at the time the 2nd floor load is applied there is only a 2-story frame available to resist this load, and not a 5-story frame. Therefore, the procedure of simultaneous analysis of an entire structure is correct only for live loads and for loads applied after the structure is completed.

Again the simultaneous loading of the structure can lead to large errors in case where neighboring columns are designed to different stress levels which results in differential shortening. For example if we consider a continuous beam in the 20th story, supported by a highly stressed column while the neighbouring columns are lightly stressed, the differential elastic shortening over the height of the structure will result in the beam not having any negative moments over the highly stressed column; on the contrary, the analysis will show substantial positive moments over the column. In reality, when the 20th floor slab is a cast, all the elastic shortening of its supports due to gravity from 19 story has already taken place and no elastic settlement stress will affect the beam when it is cast. However, the beams will be subjected to elastic differential support shortening for all the dead loads which will be applied above the 20th floor.

From these discussion it is clear that in the total analysis and design of a building frame the construction time must be considered as an additional variable.

6.2 Analytical Verification

The behaviour of a building frame discussed in Art. 6.1 is studied by developing a computer programme, considering the construction time as an additional variable, which means the vertical load will be considered as applied gradually story-by-story while at the same time the framework is being progressively built up. And this programme can be accomodated for any number of bays, stories and for all level of construction. For analytical verification of the behaviours mentioned in Art. 6.1 a set of three bay six story building frames are analysed for all levels of construction.

CHAPTER 7

DISCUSSION OF RESULTS

7.1 Introduction

This thesis deals with four differnt aspects of tall building frame design, viz.

(i) To check the validity of approximate methods for gravity load analysis.

(ii) To check the validity of approximate methods for lateral load analysis.

(iii) To study the behaviour of perimeter frames interconnected with shearwall, so as to act as flanges to the walls.

(iv) To study the behaviour of frames at different stages of construction.

7.2 Discussion on Exact and Approximate Analysis of Building Frames for Gravity Load

To check the validity of approximate methods for analyzing building frames subjected to gravity loading, a series of two bay ten storied building frames are analyzed by stiffness method for gravity loads. A non-dimensional parameter $\lambda = (\sum I_c/h)/(\sum I_g/L_g)$ has been selected and frames with λ varying from 0.48 to 4.8 have been analyzed. The points of inflections obtained from this exact analysis are shown in Fig. 7.1 to Fig. 7.20. It is observed from Fig. 7.1 to Fig. 7.6 and from Fig. 7.16 to Fig. 7.18 that the points of inflection in both the girders are located at approximately 0.2L from both the exterior and interior columns support. Another distinct feature is observed from the Fig. 7.7 to Fig. 7.15 and from Fig. 7.19 to Fig. 7.20 that in the left girder the location of inflection points lie at approximately 0.2L from exterior column and 0.15L from interior column support and in the right girder no definite location of inflection can be identified. From these observations the following conclusions are made.

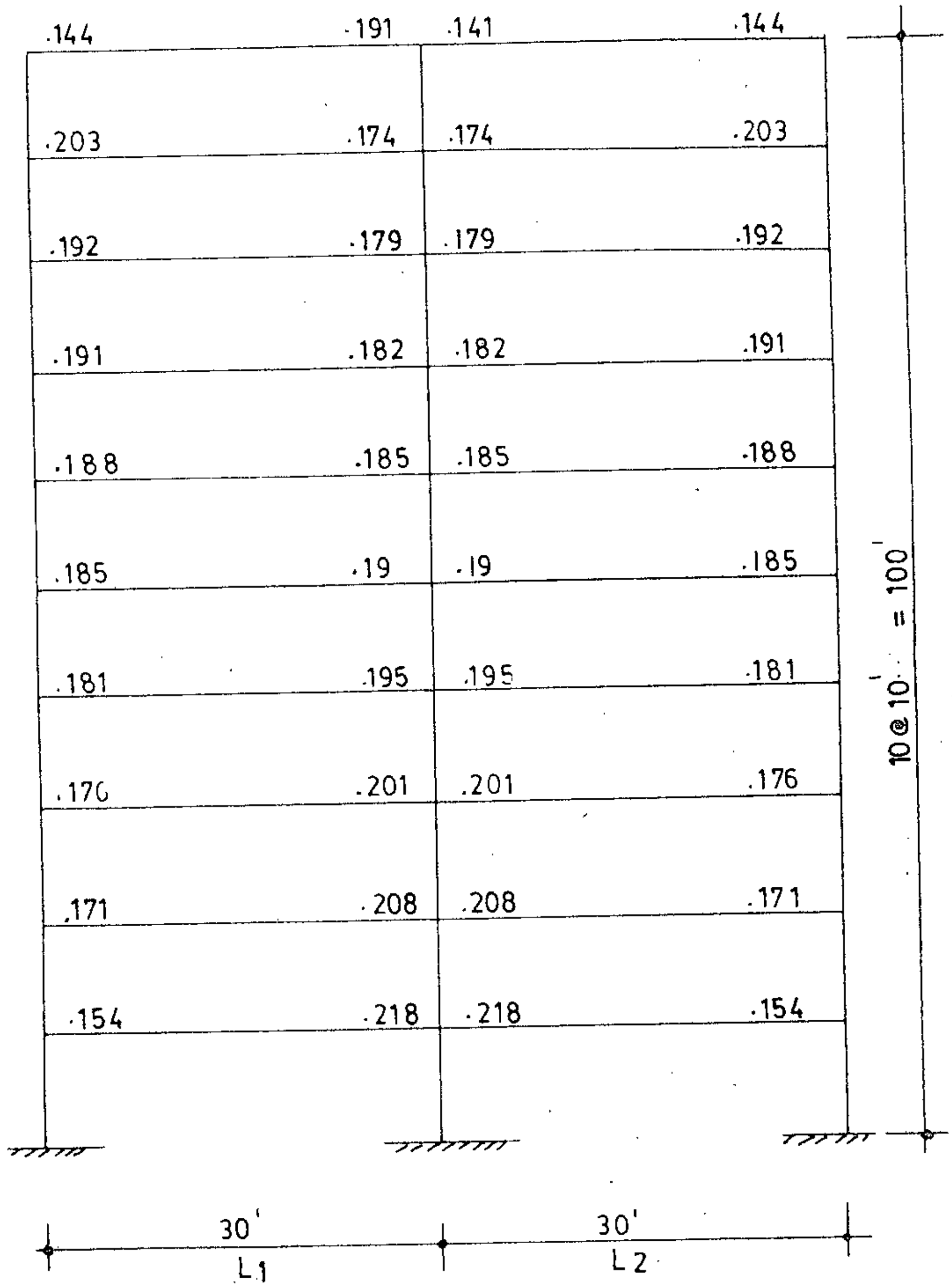
7.2.1 Concluding Remarks

(a) For two bay frames where ratio of the girders lengths (L_1/L_2) does not exceed 1.75 and the value of λ is within the realistic range (λ = 0.5 to 5.0),

(i) The points of inflection are located at a distance of 0.2L from the exterior column support.

(ii) The points of inflection are located at a distance of 0.2L from the interior column supports.

(b) For frames where L_1/L_2 ratio exceeds 1.75 the location of inflection points in the girder L_1 are at a distance of 0.2L and 0.15L from exterior and interior column supports respectively.



SI = 16875

Fig.7.1 Exact Locations of Points of inflections under Gravity Loading

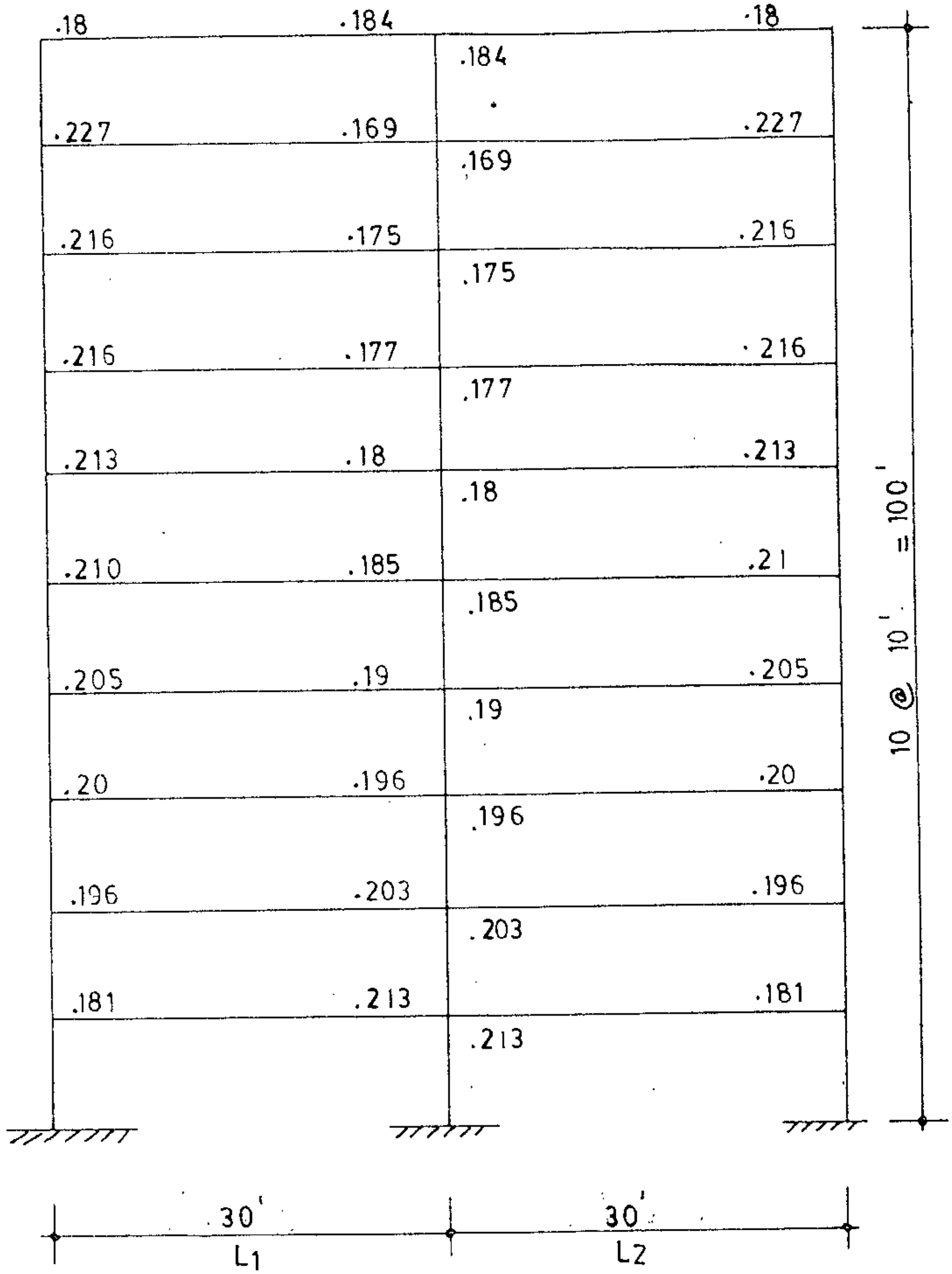


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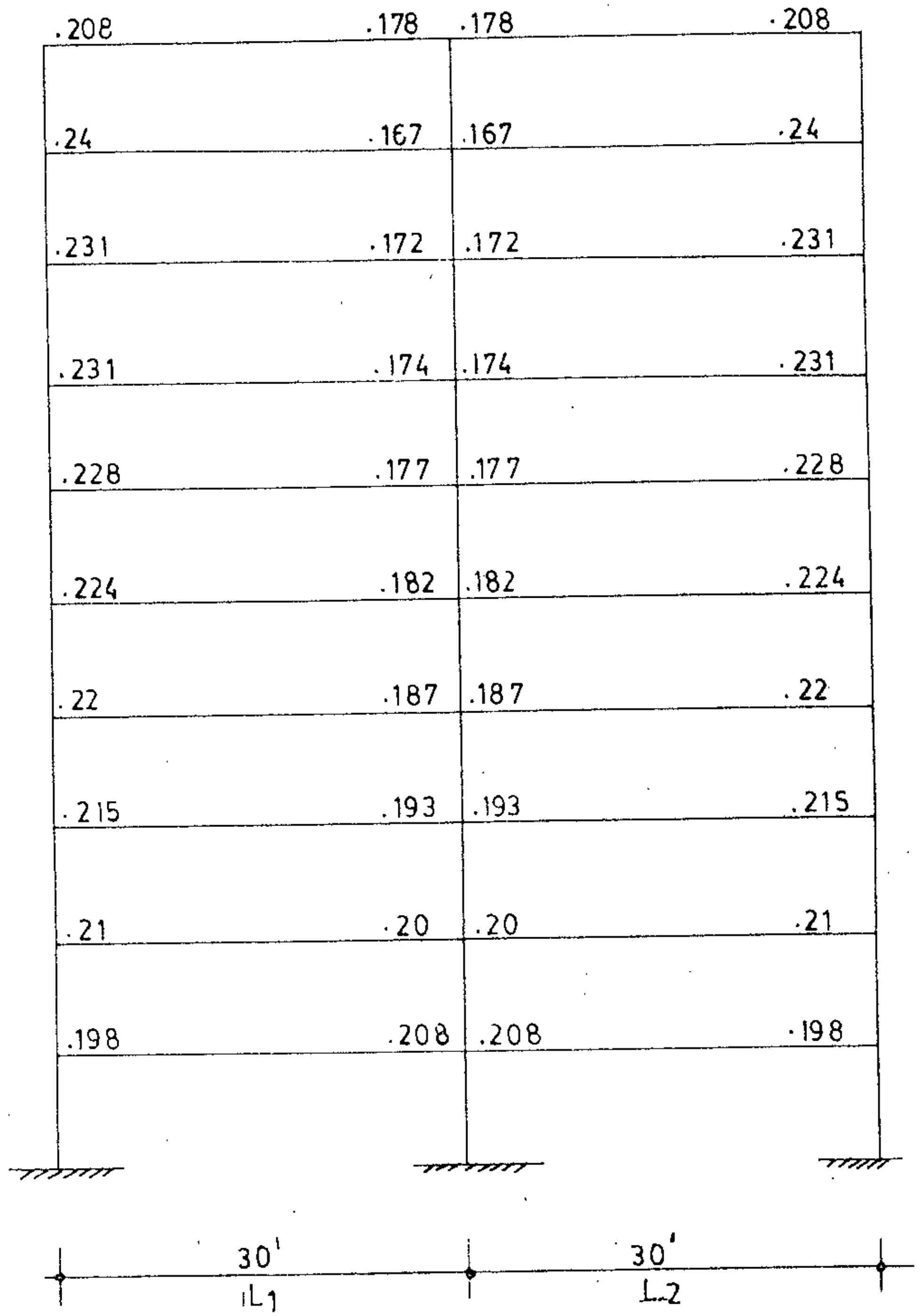
Fig. 7.2 Exact Locations of Points of inflections



SI = 67500

Fig. 7.3 Exact Locations of Points of inflections

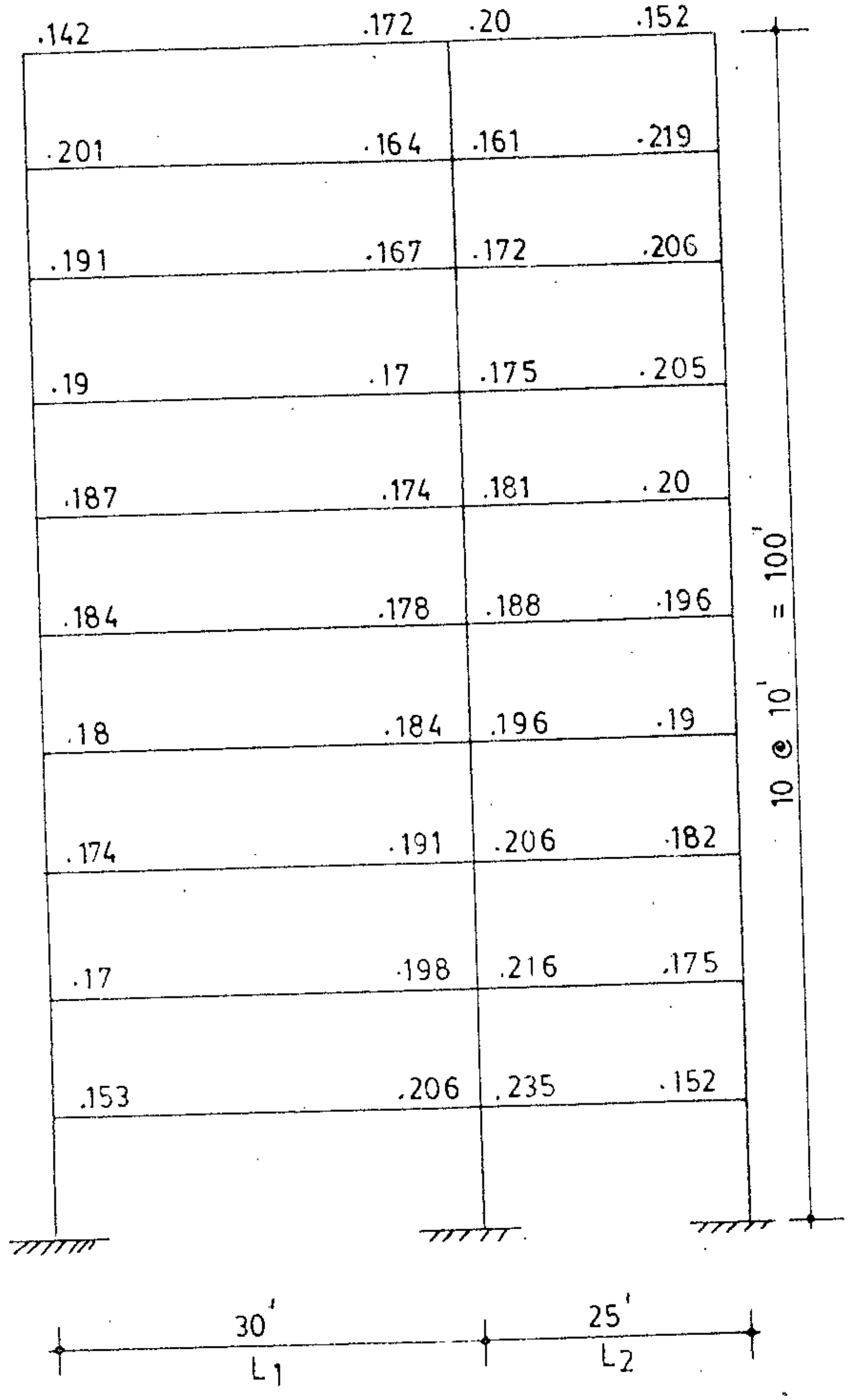


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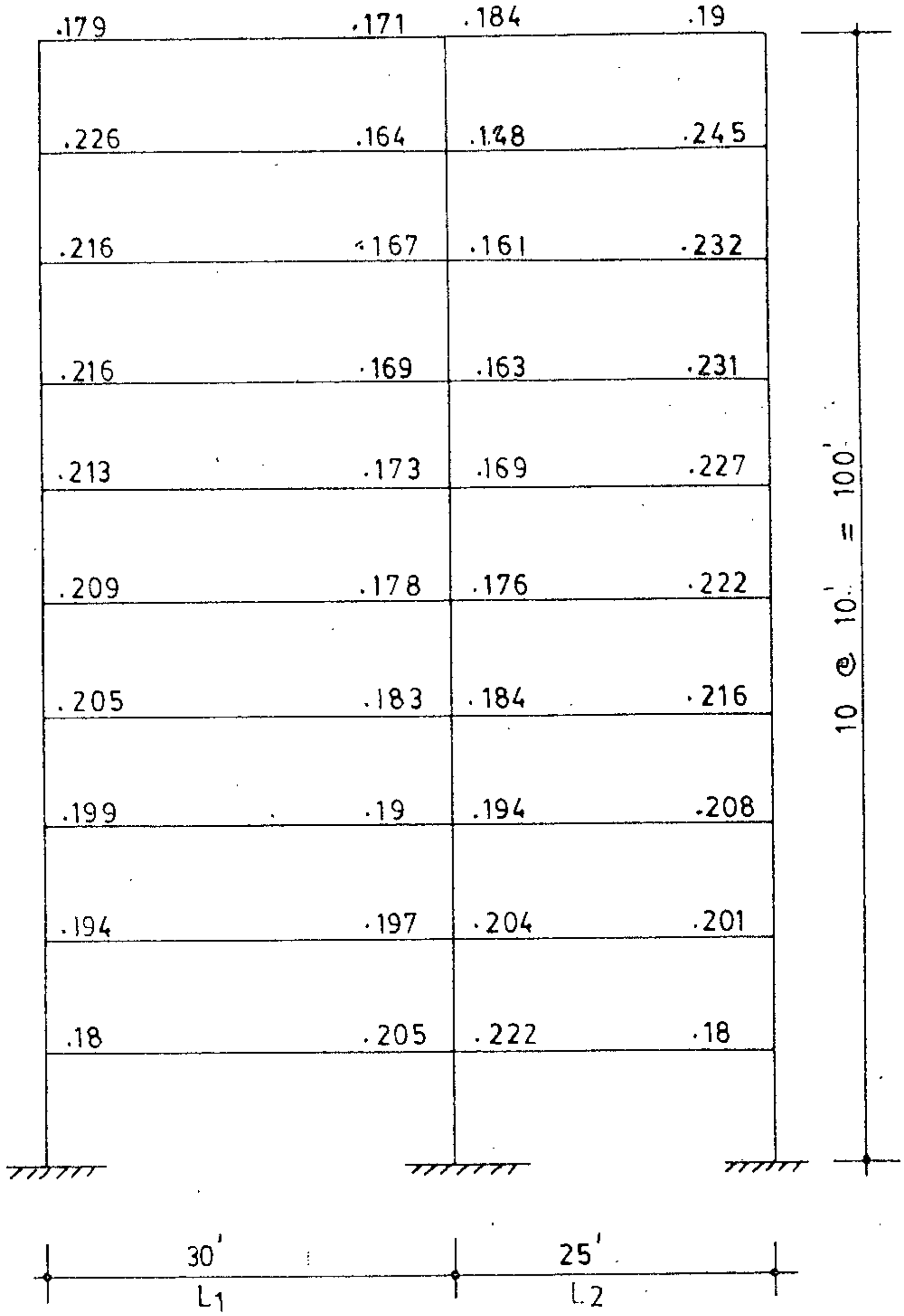
Fig. 7.4 Exact Locations of Points of inflections



S1 = 33750

Fig. 7.5 Exact Locations of Points of inflections

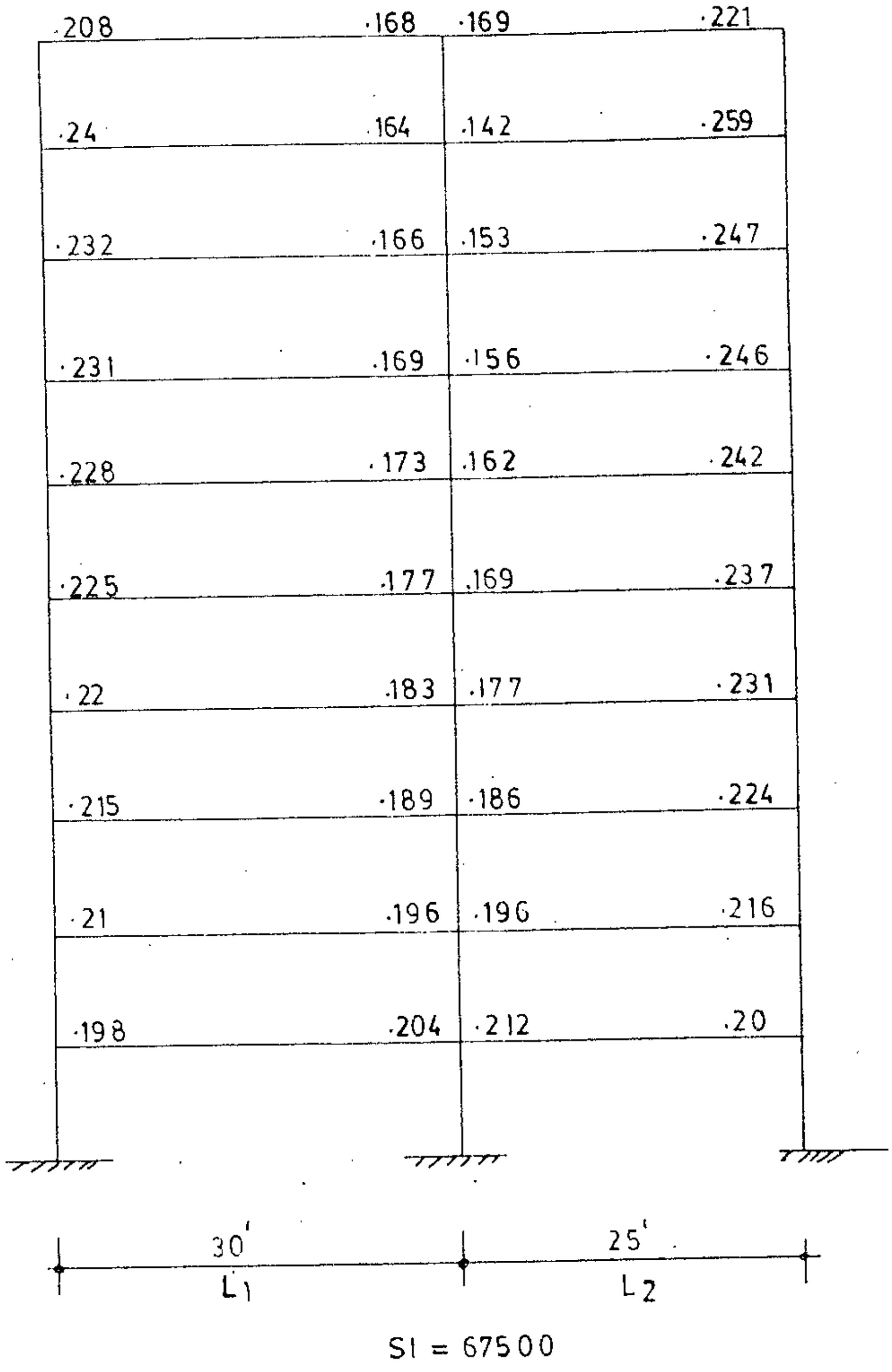


Fig. 7.6 Exact Locations of Points of inflections

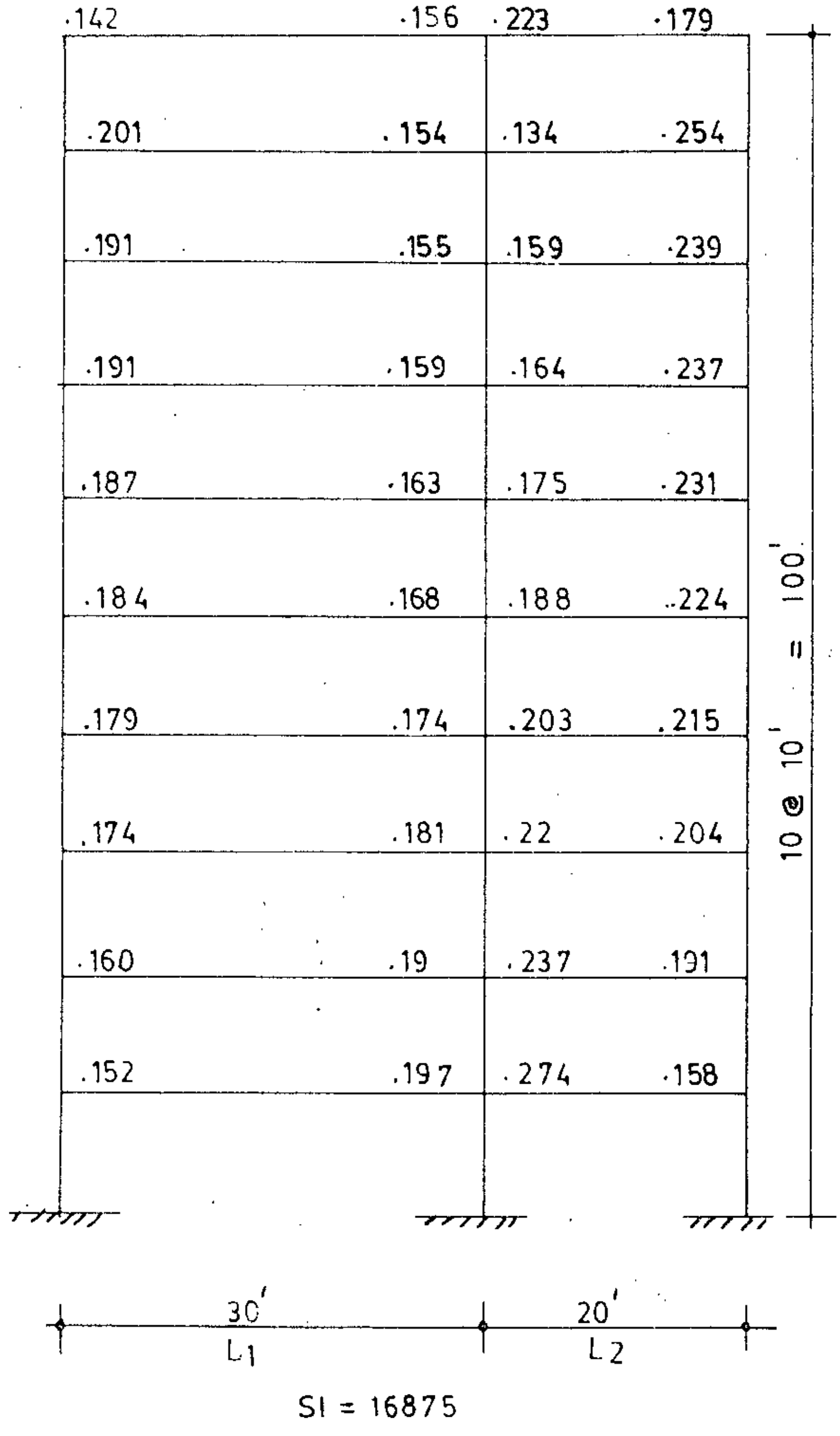
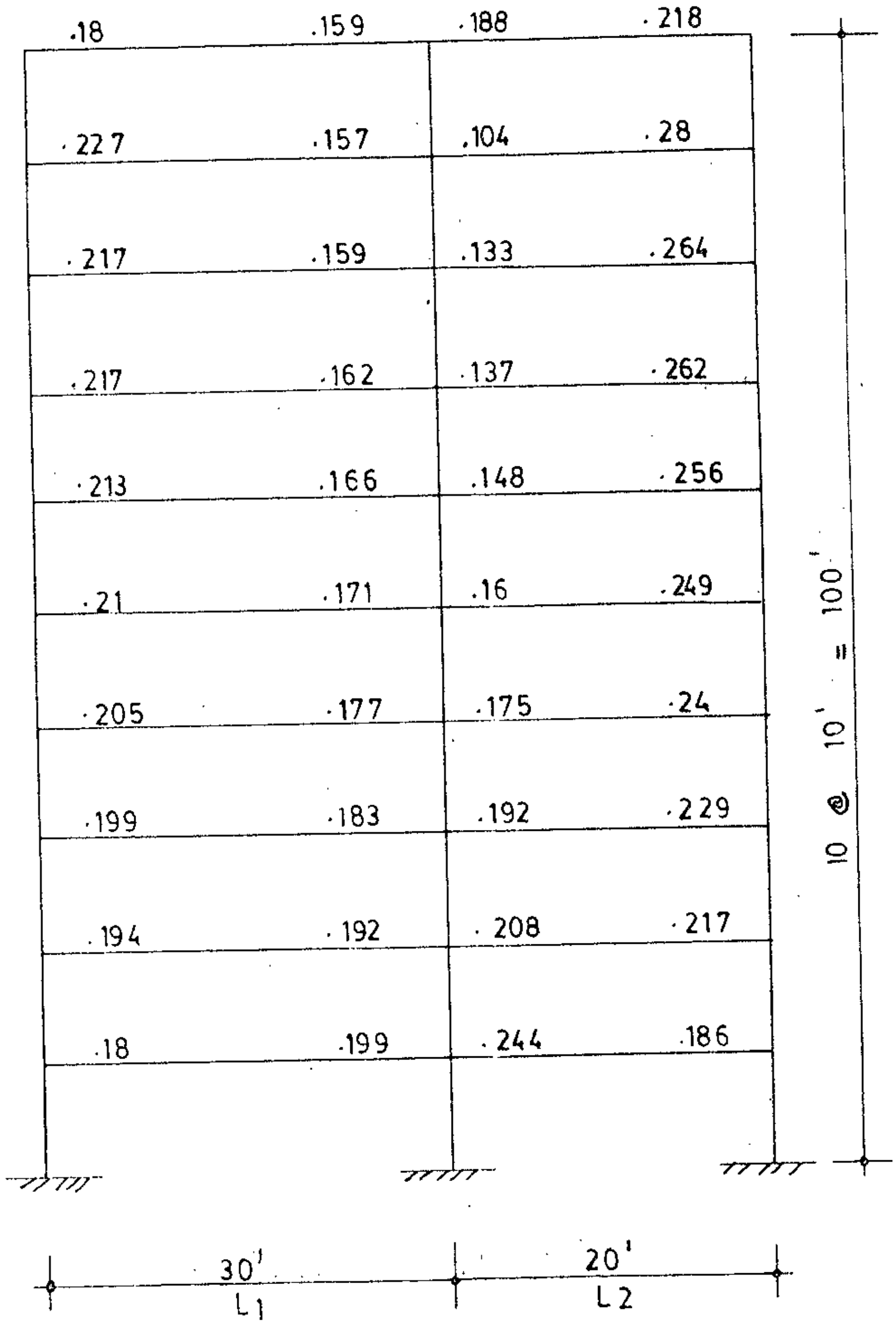


Fig. 7.7 Exact Locations of Points of inflections



SI = 33750

Fig. 7.8 Exact Locations of Points of inflections

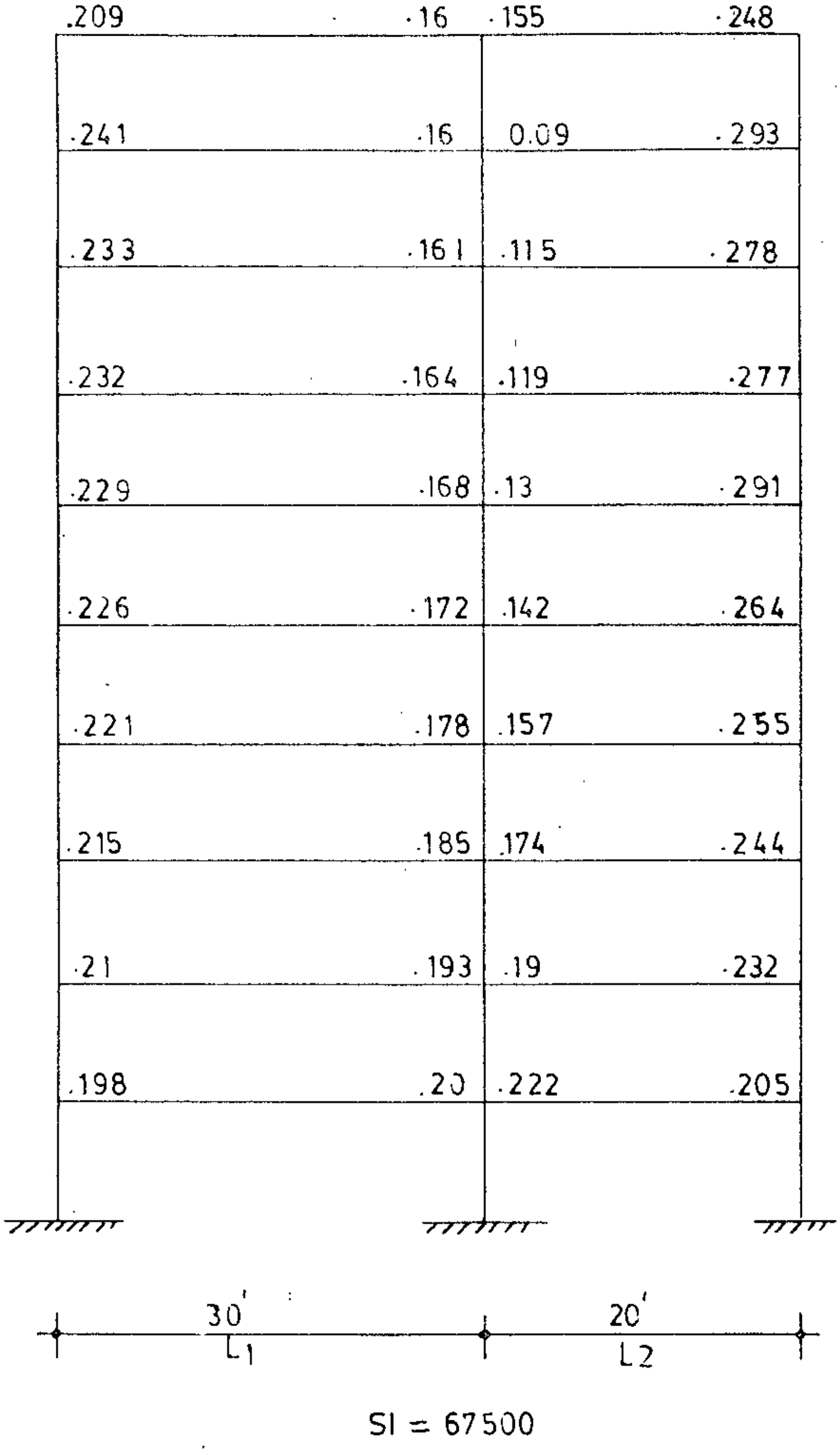


Fig. 7.9 Exact Locations of Points of inflections

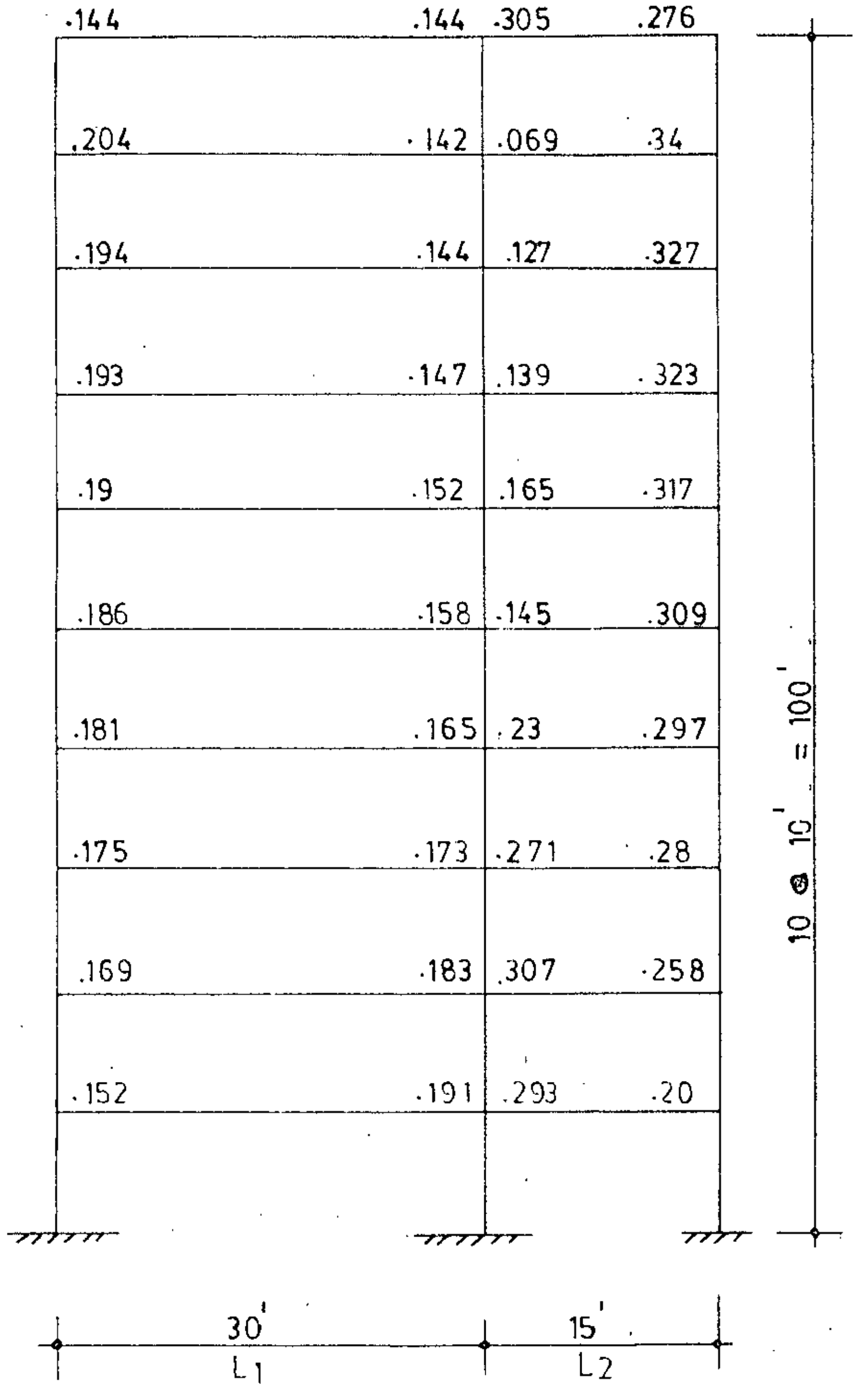


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Fig. 7.10 Exact Locotions of Points of inflections

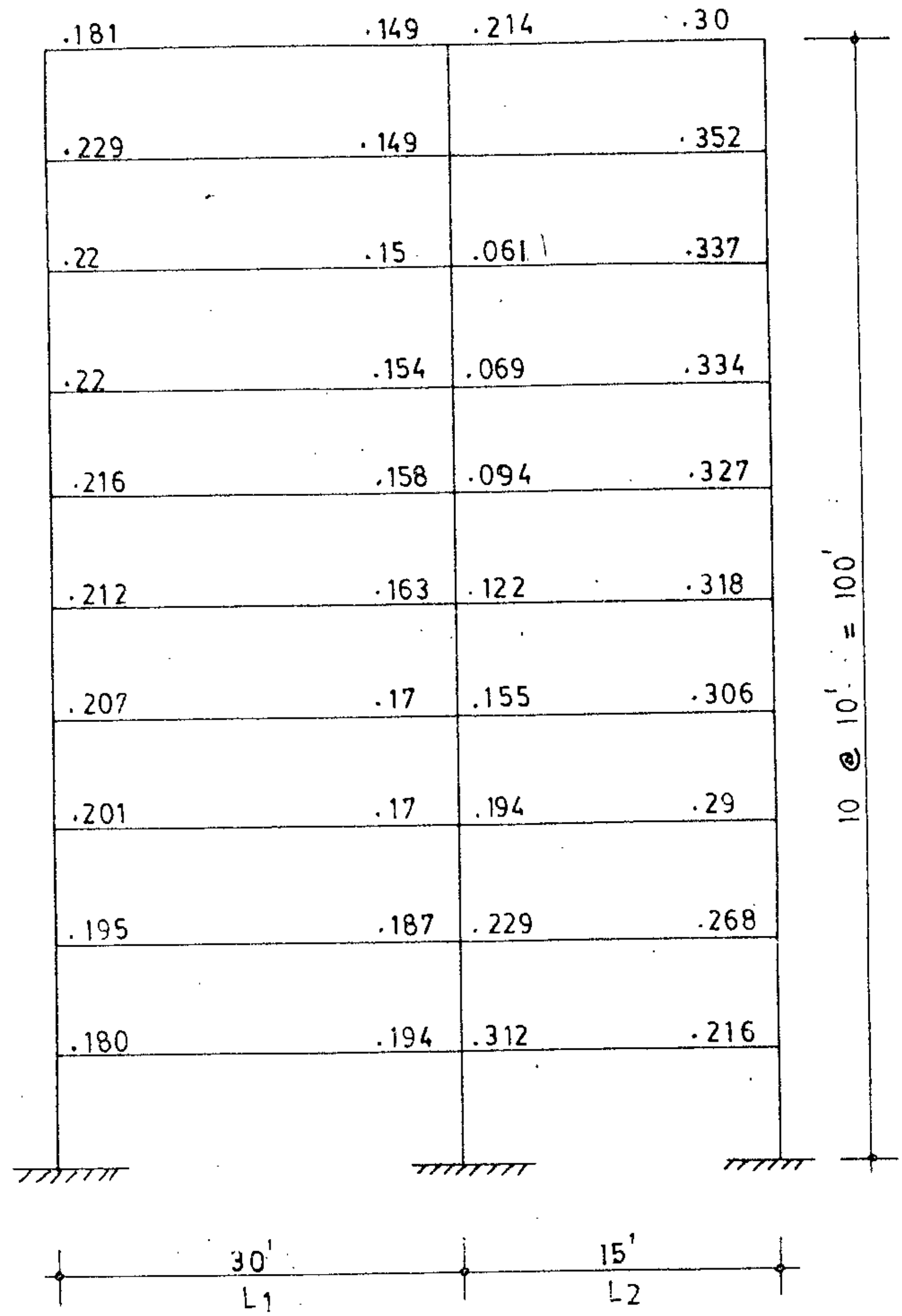

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Fig. 7.11 Exact Locations of Points of inflections

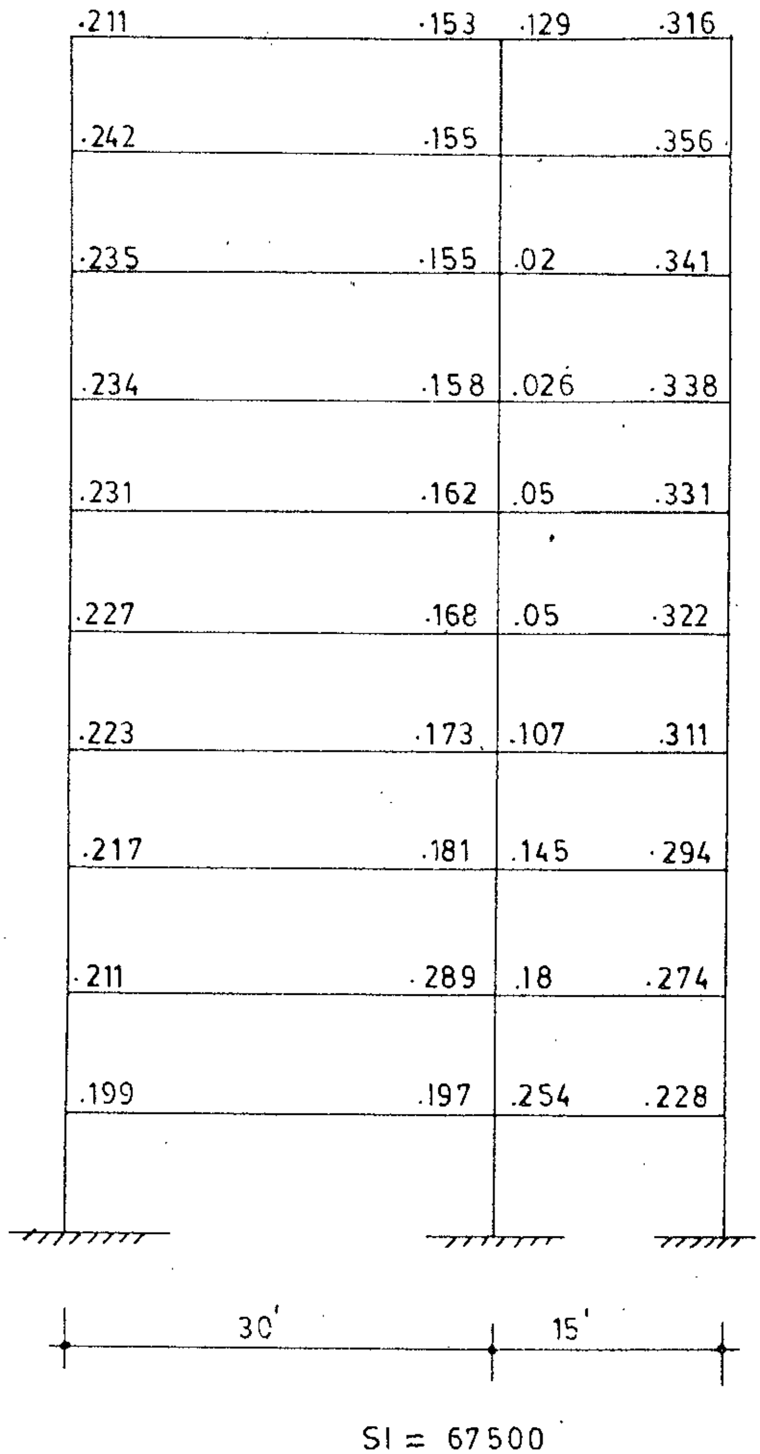

Fig. 7.12 Exact Locations of Points of inflections

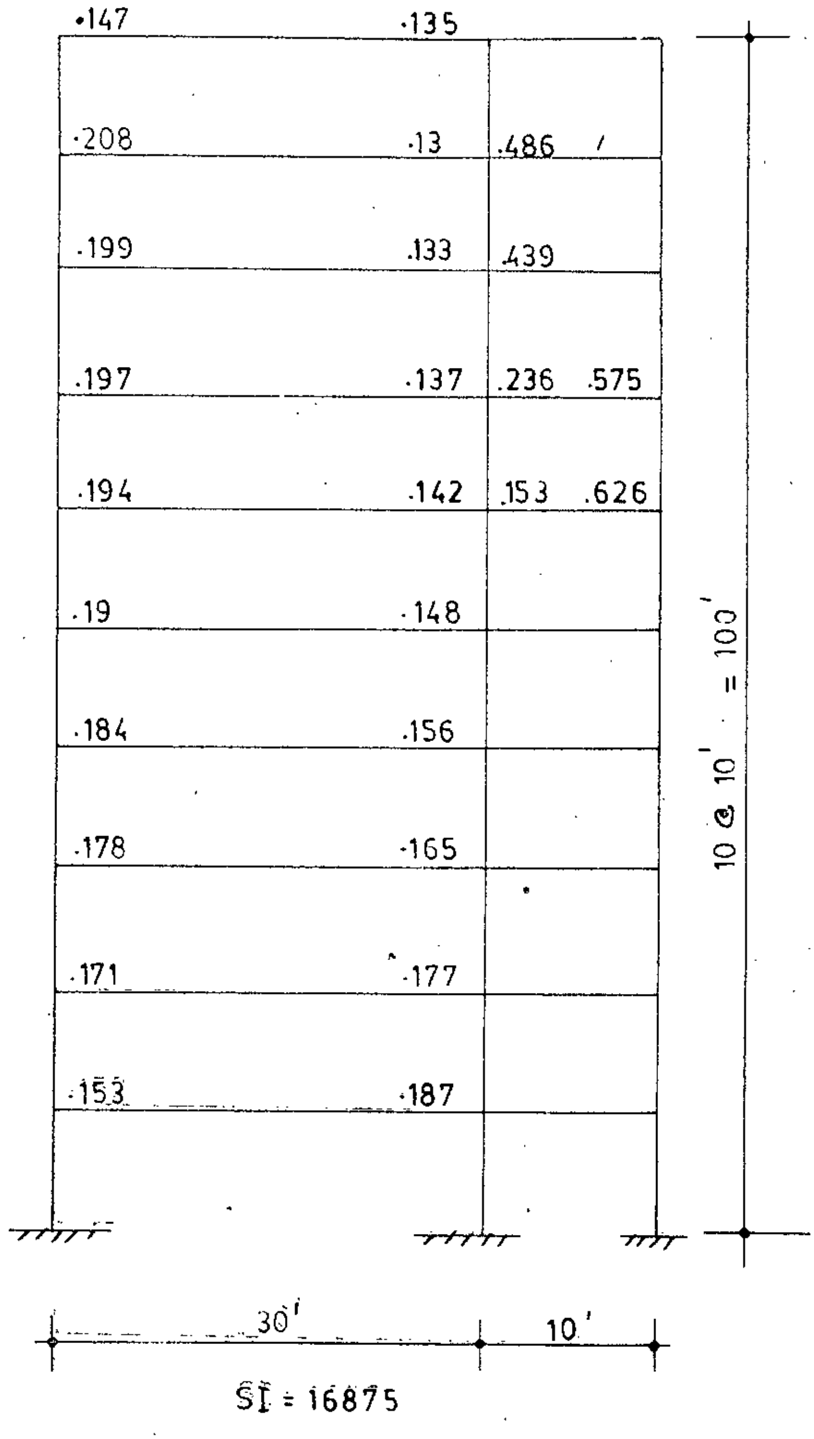

Fig. 7.13 Exact Locations of Points of inflections

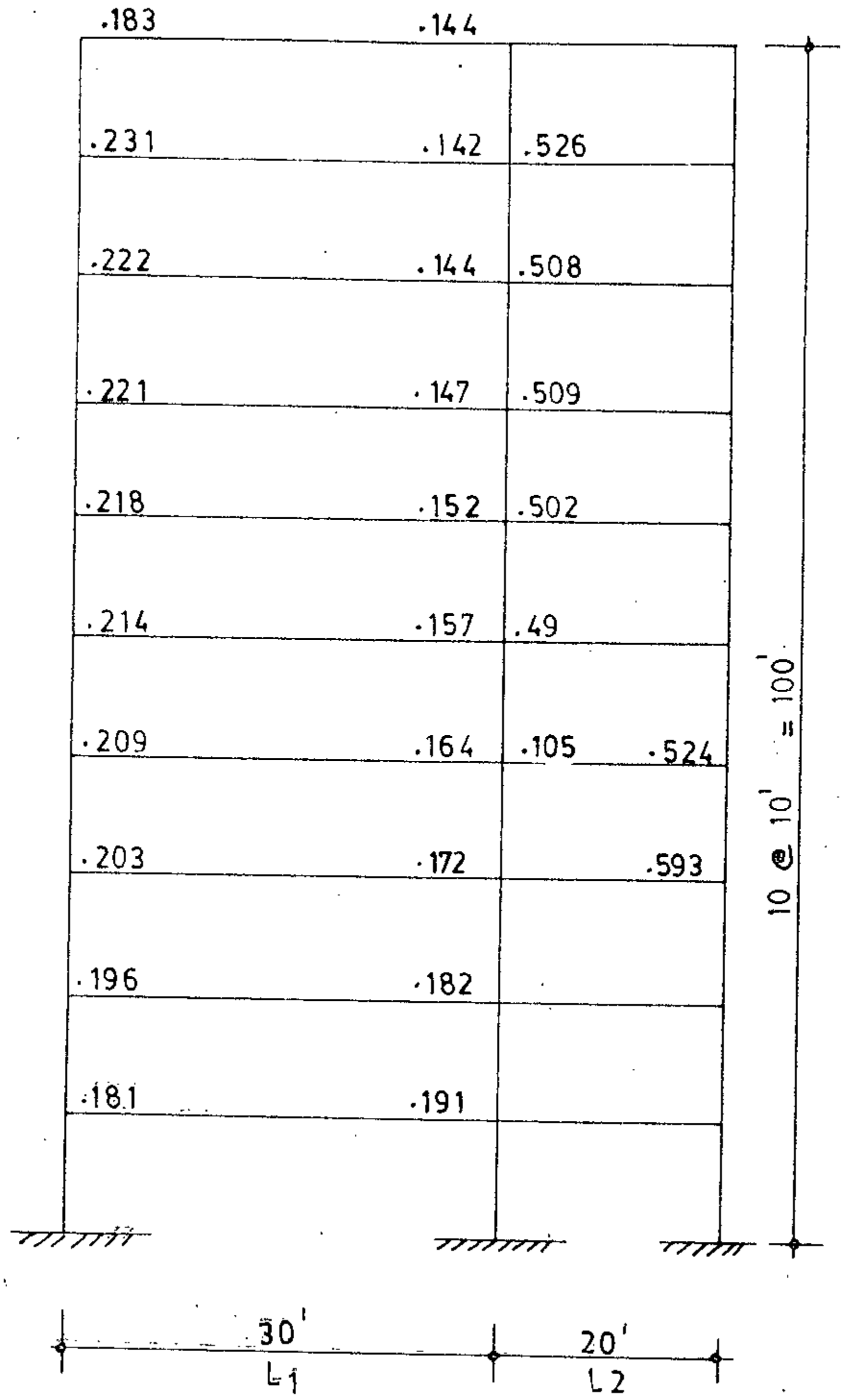

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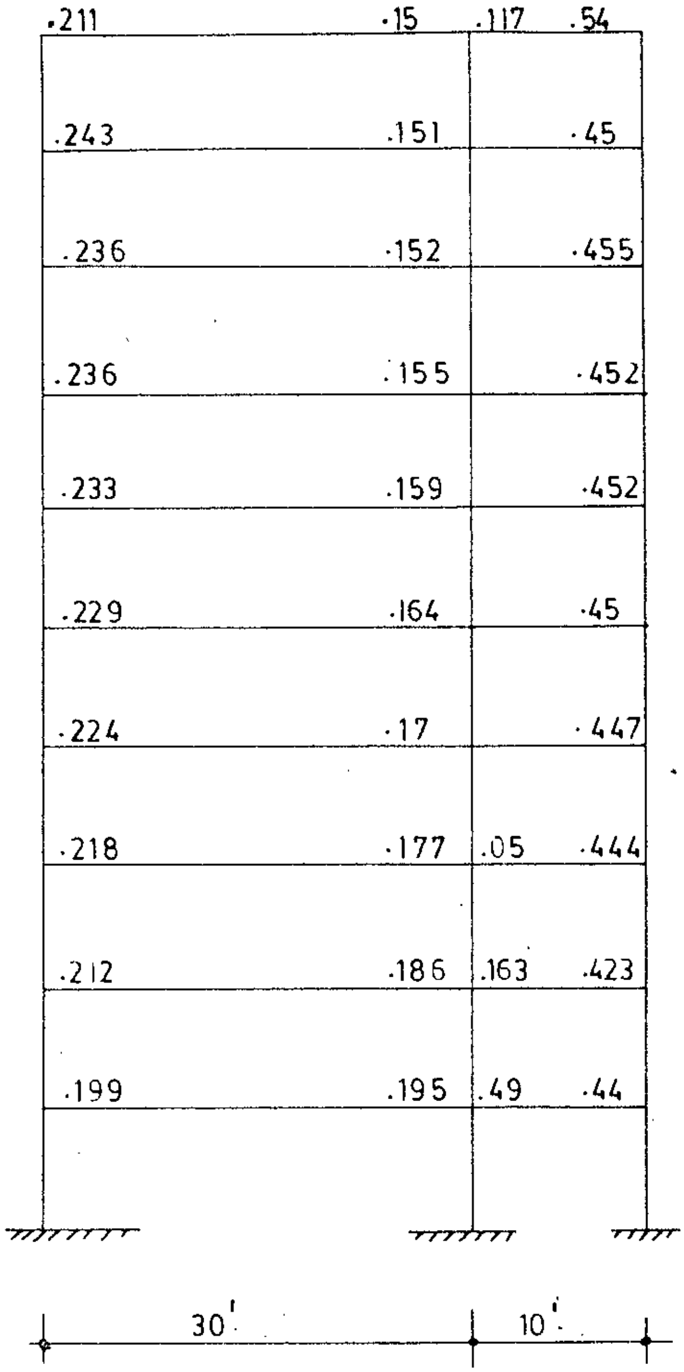
Fig.7.14 Exact Locations of Points of inflections



SI = 67500

Fig. 7.15 Exact Locations of Points of inflections

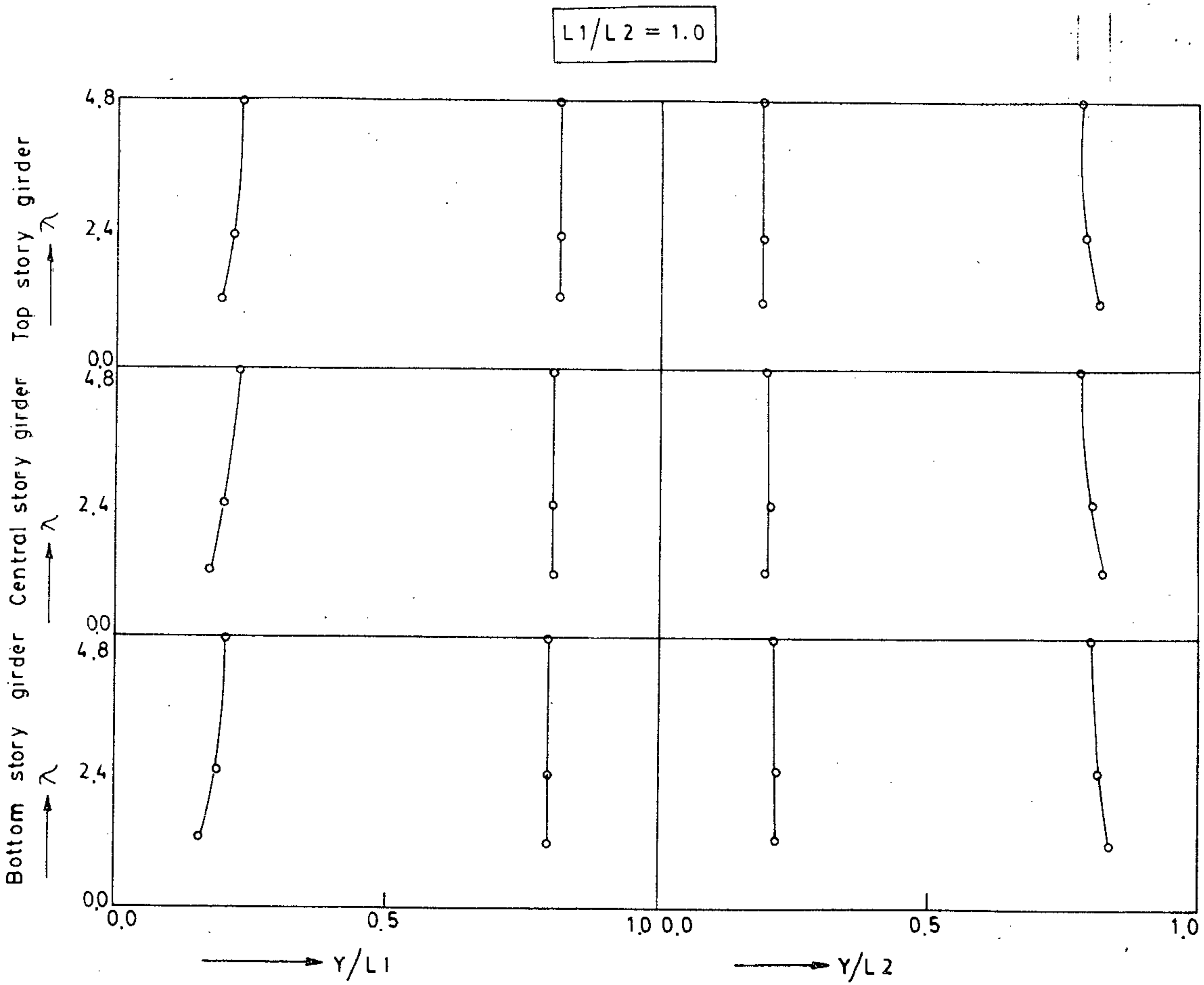


Fig. 7.16 Exact Locations of points of Inflections

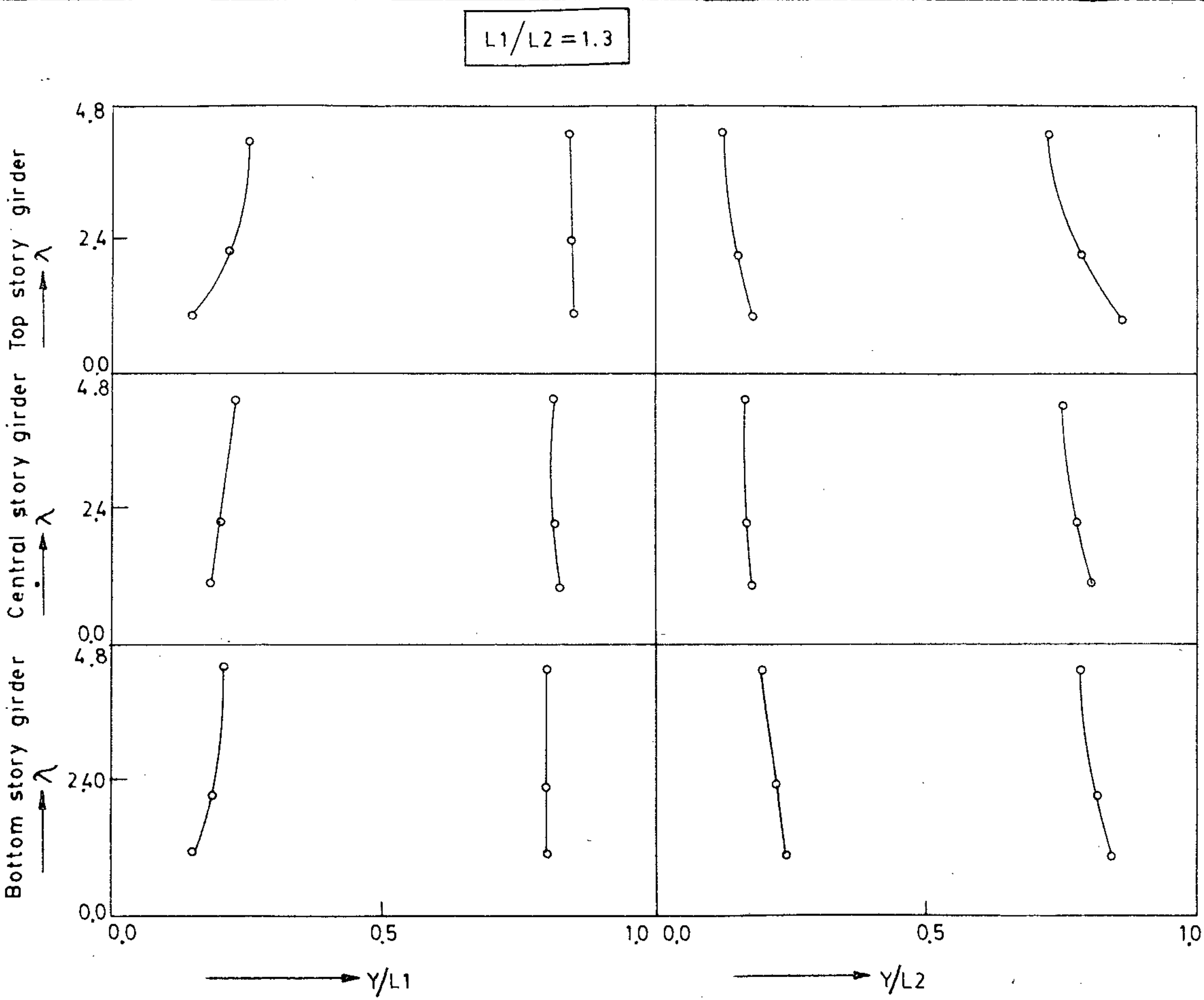


Fig. 7.17 Exact Location of Points of Inflection

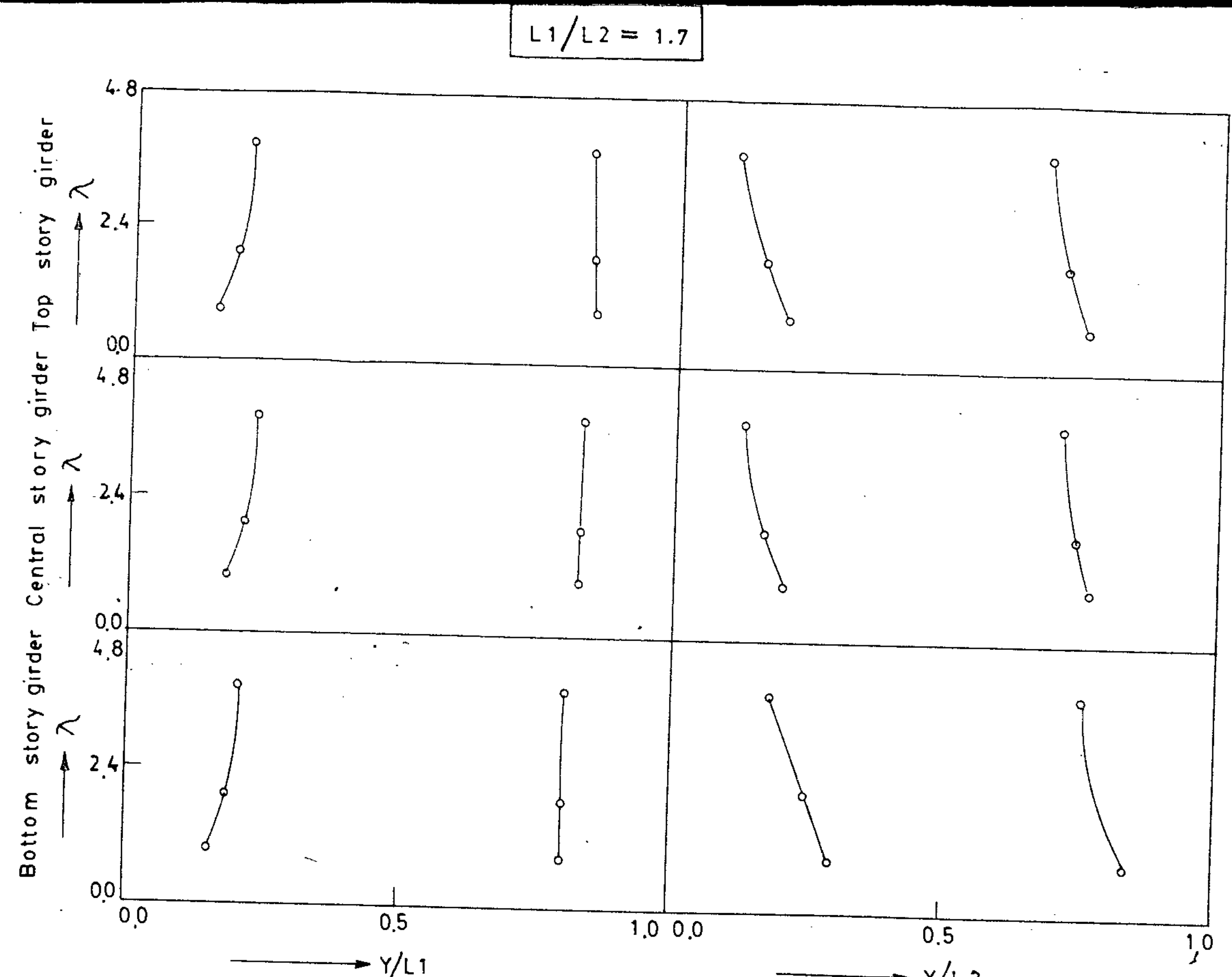


Fig. 7.18 Exact Location of points of Inflection

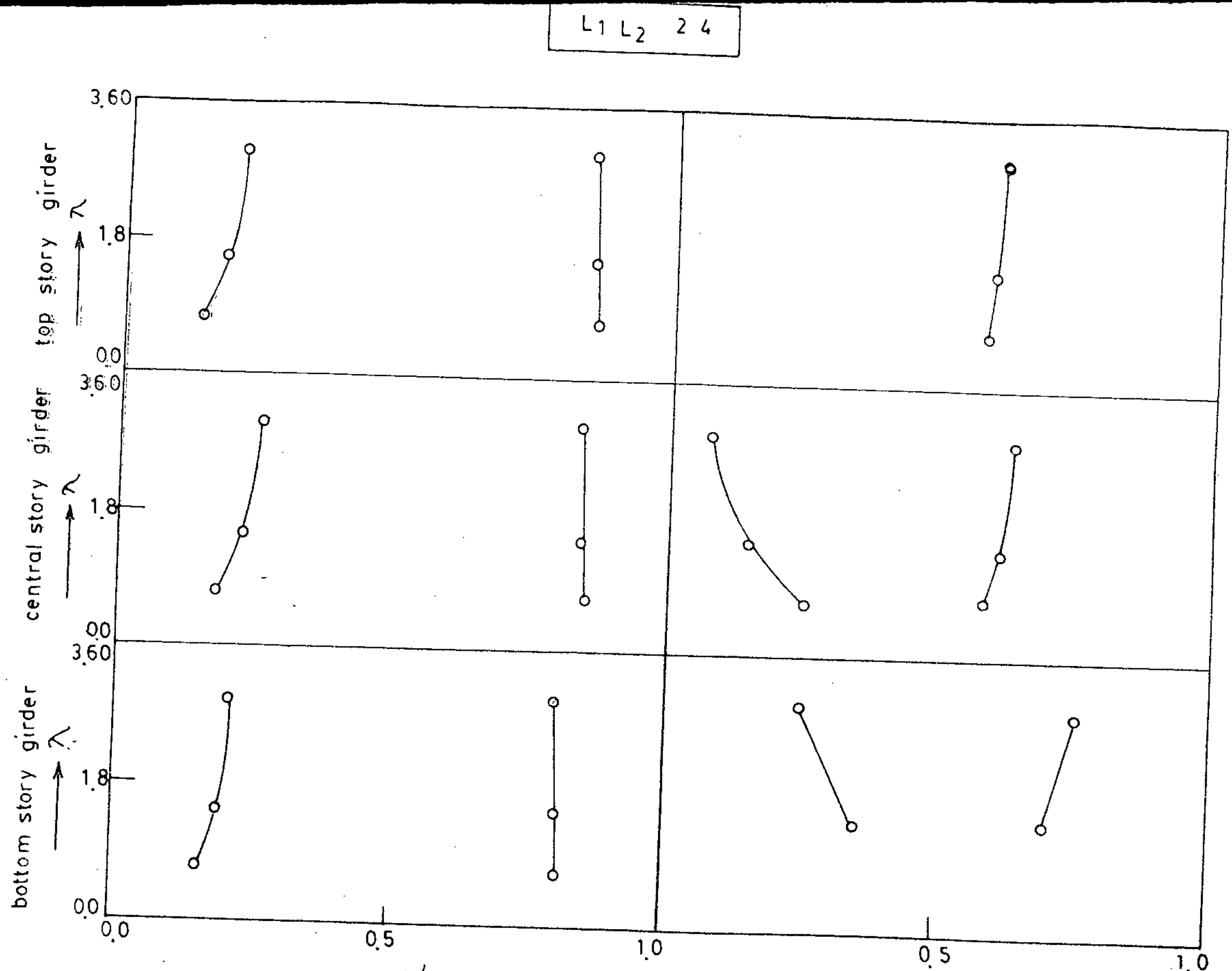


Fig. 7.19 Exact Location of Points of Inflection

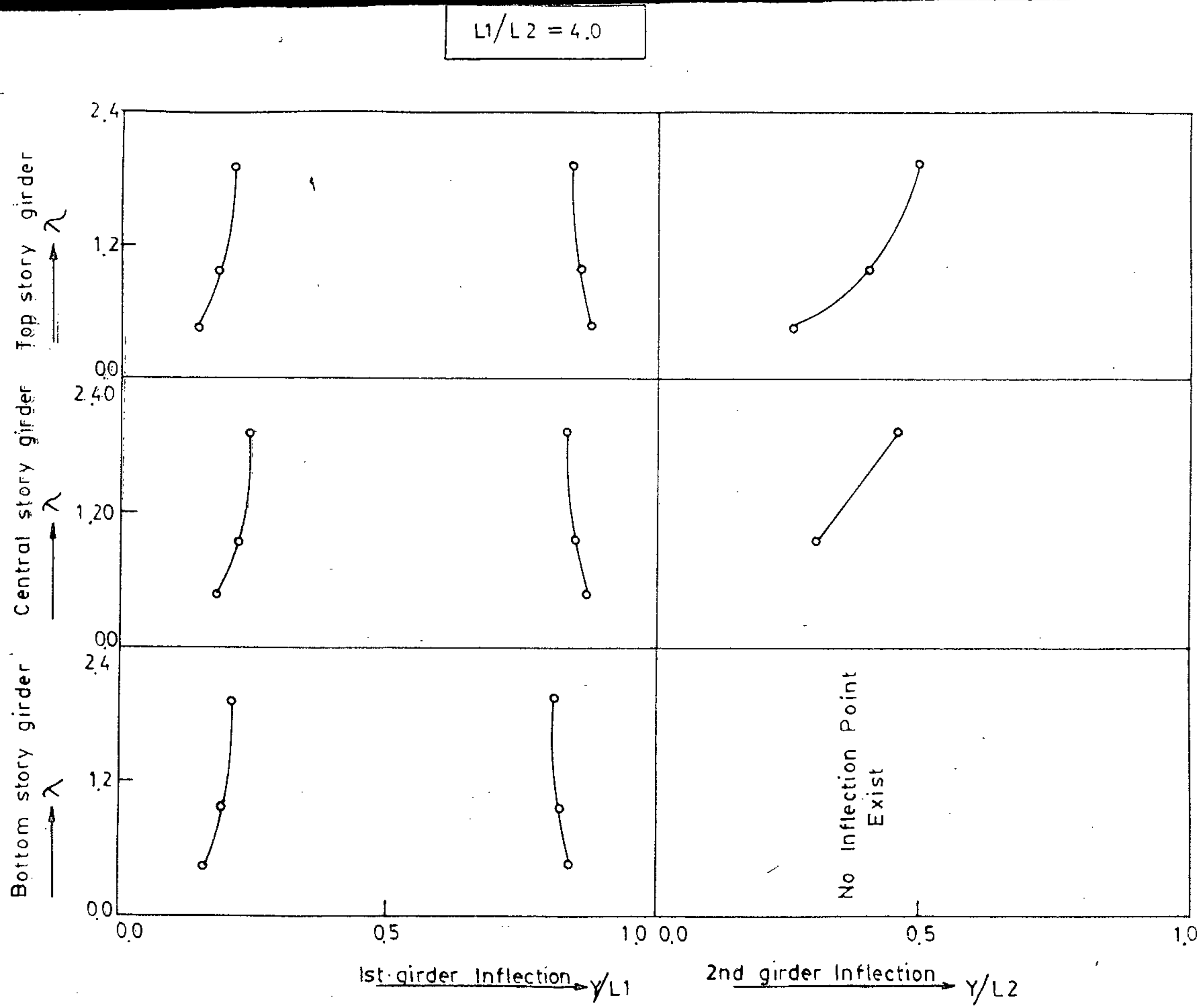


Fig. 7.20 Exact Location of points Inflection

Therefore, the approximate locations of inflection points suggested by LEVY[7] and NORRIS[10] for gravity load analysis of building frames are invalid.

7.3 Discussion on the Exact and Approximate Methods of Analysis of Building Frames for Lateral Loads.

To check the validity of the approximate methods for aalyzing building frames under lateral loads, a series of two bay ten-storied building frames are analyzed by portal, cantilever, stiffness center and exact methods. The value of λ is varied from 0.72 to 3.8. The results obtained from these analysis have been presented in Fig. 7.21 to Fig. 7.27. From the study of Fig. 7.22 to Fig. 7.26 it is seen that the exact location of inflection points in the girders and in the central story columns exist almost at the midspan of the girders and columns respectively. And the locations of inflection points obtained by stiffness center method in the top and bottom story columns almost coincide with the locations obtained by exact analysis. From these observations the following conclusions are made.

7.3.1 Concluding Remarks

(i) The assumption made in portal and cantilever method about the location of girder inflection is approximately correct for all the realistic values of λ.

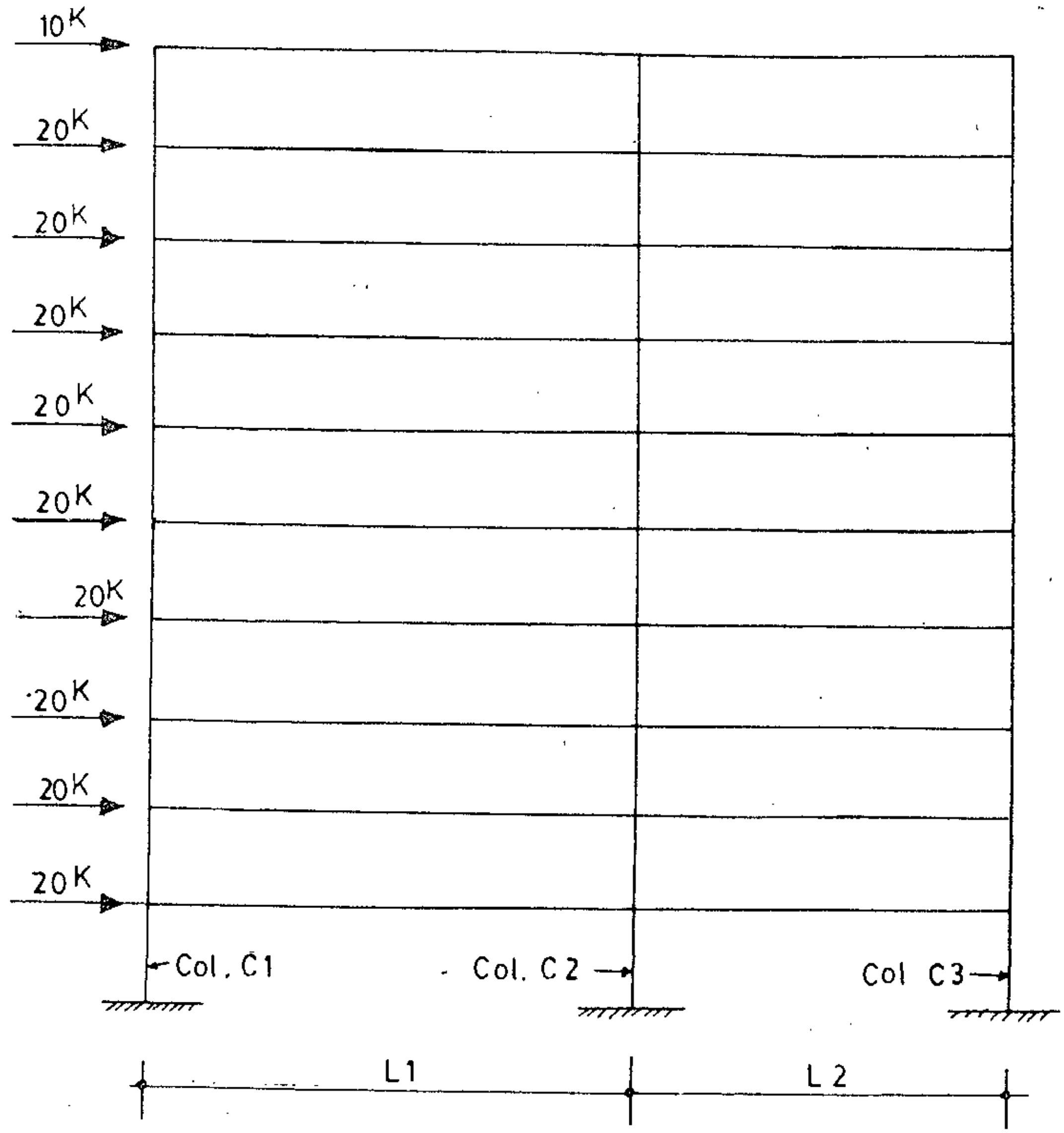


Fig. 7.21 The Frame Under Lateral Load

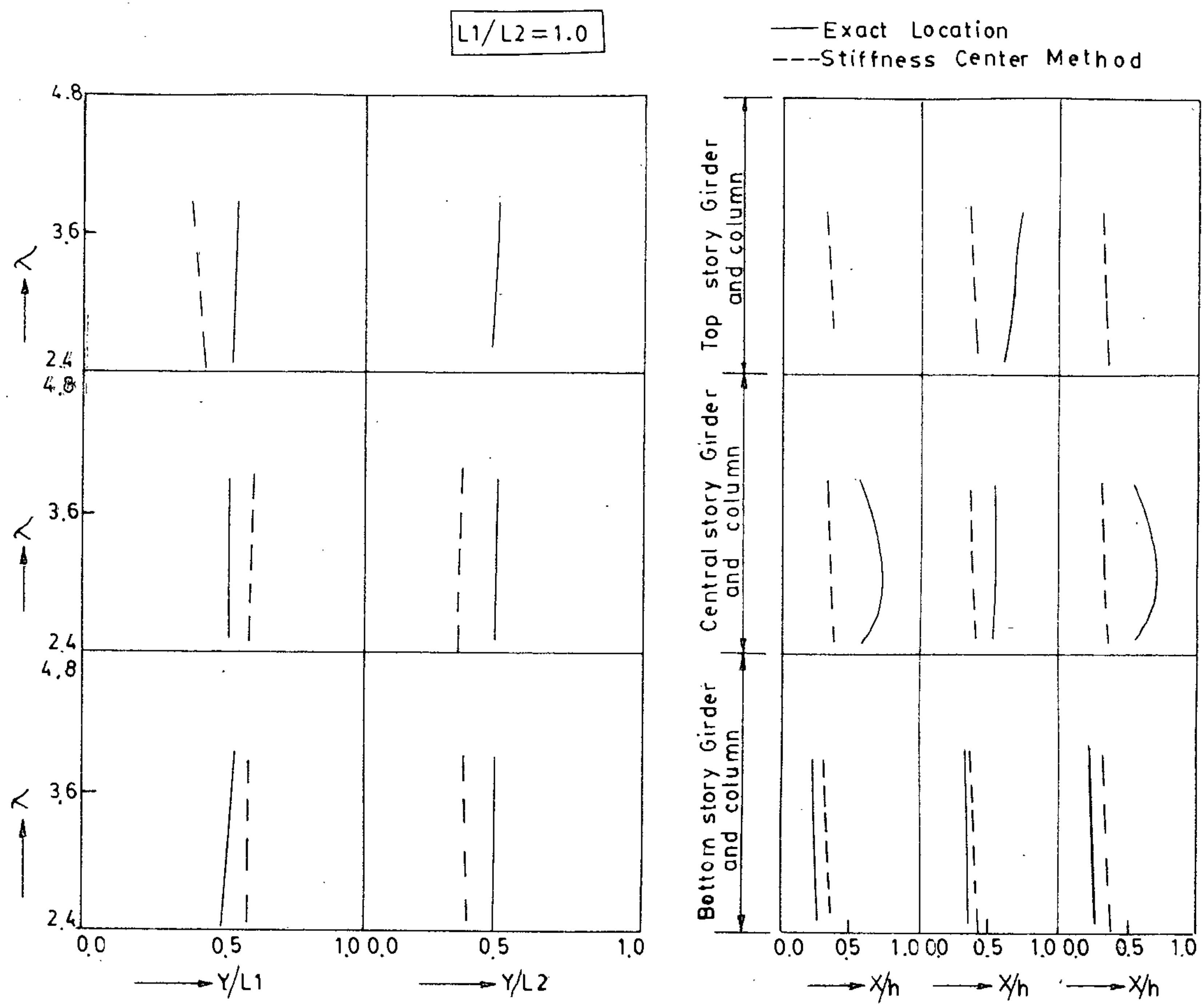


Fig. 7.22 Girder and Column Inflections

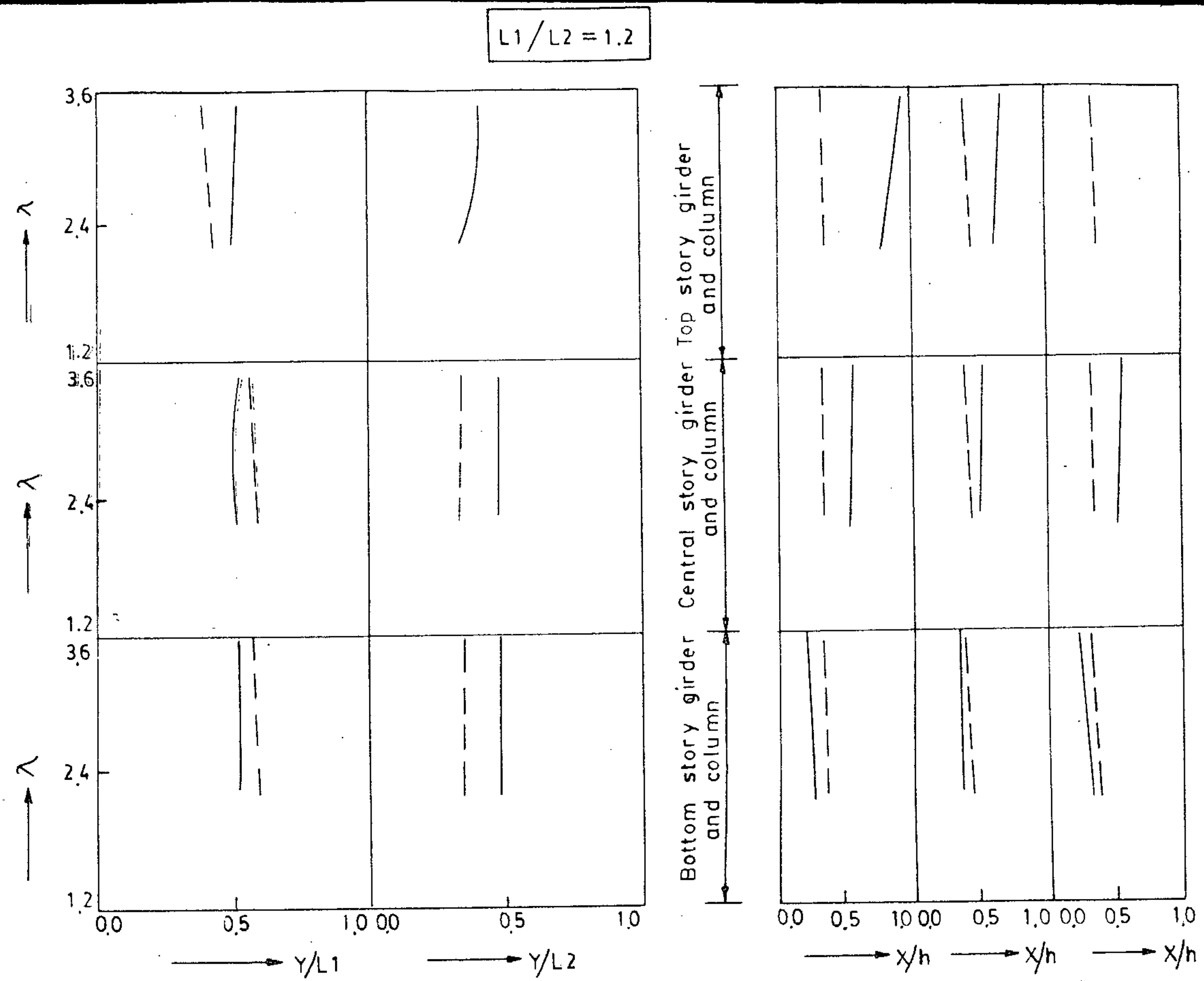


Fig. 7.23 Girder and Column Inflections

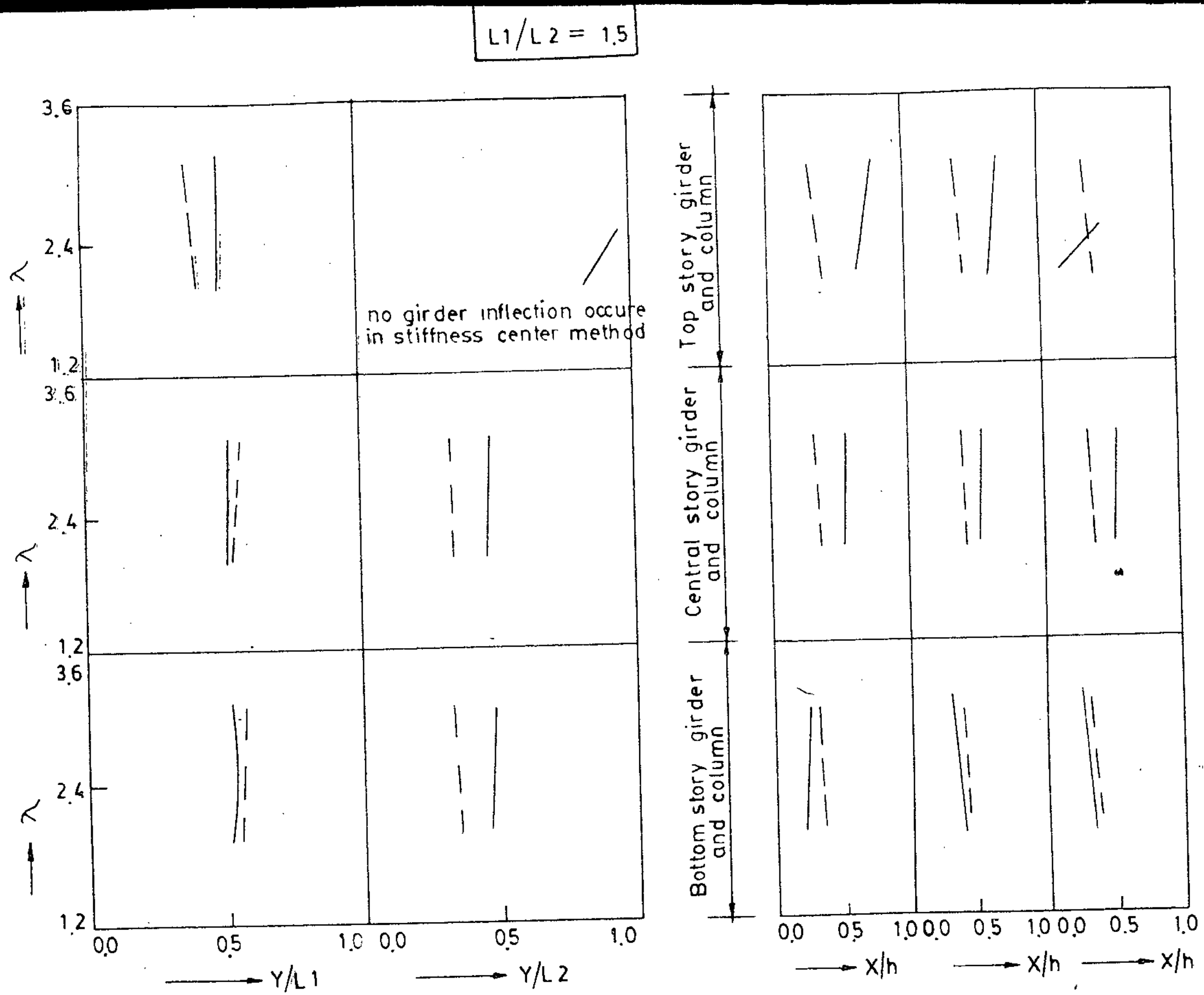


Fig 7.24 Girder and Column Inflections

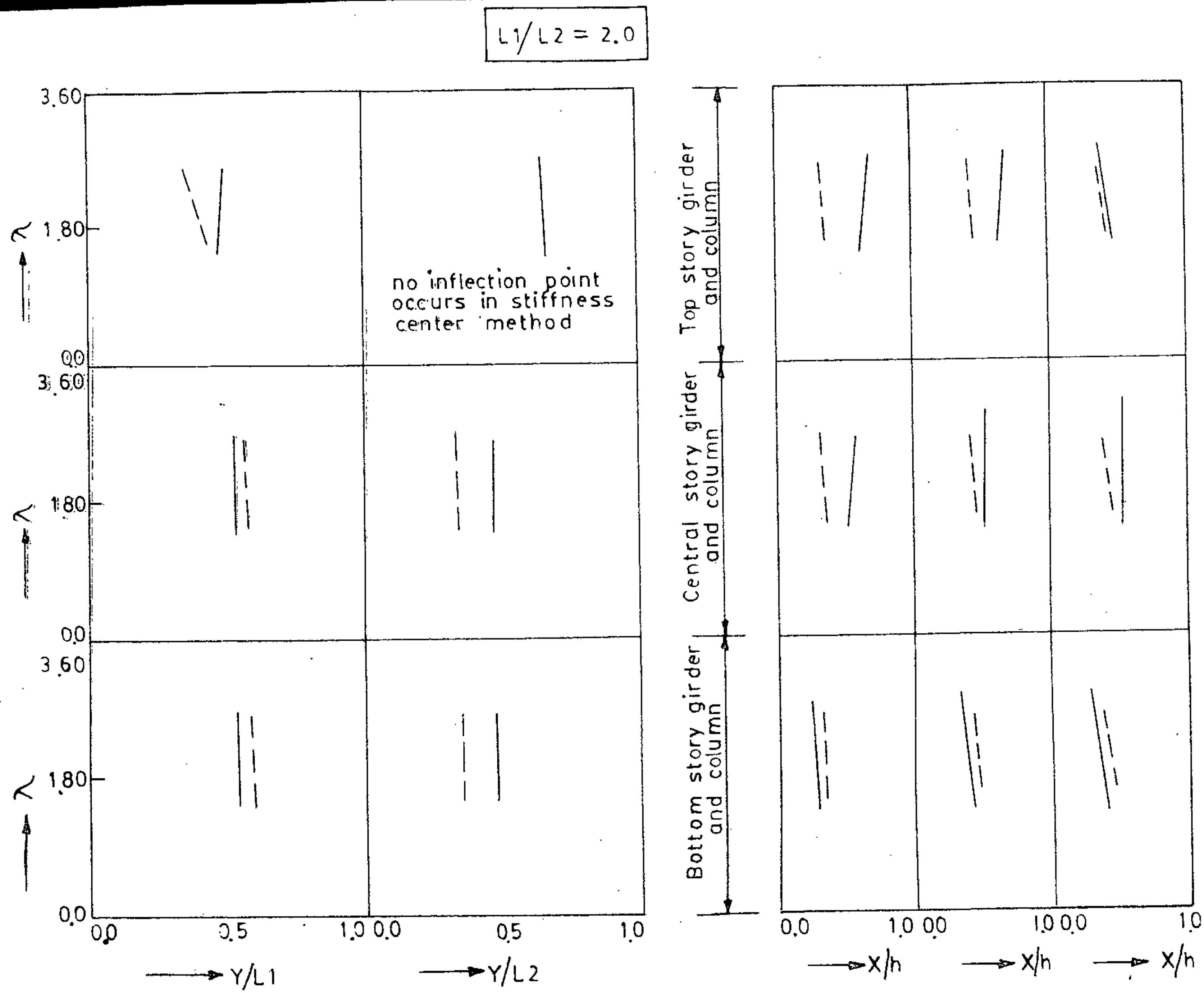


Fig. 7.25 Girder and Column Inflection

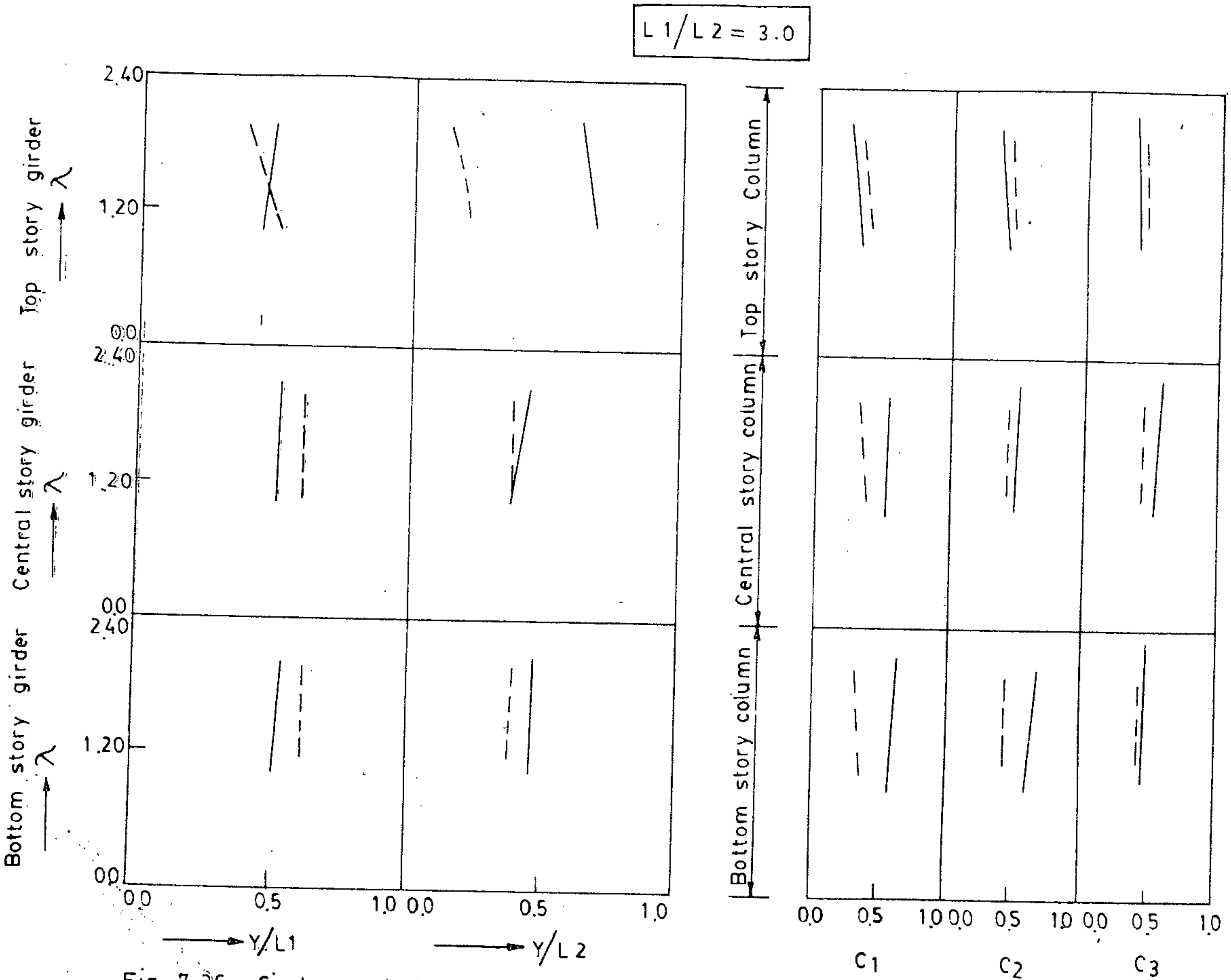


Fig. 7.26 Girder and Column Inflection

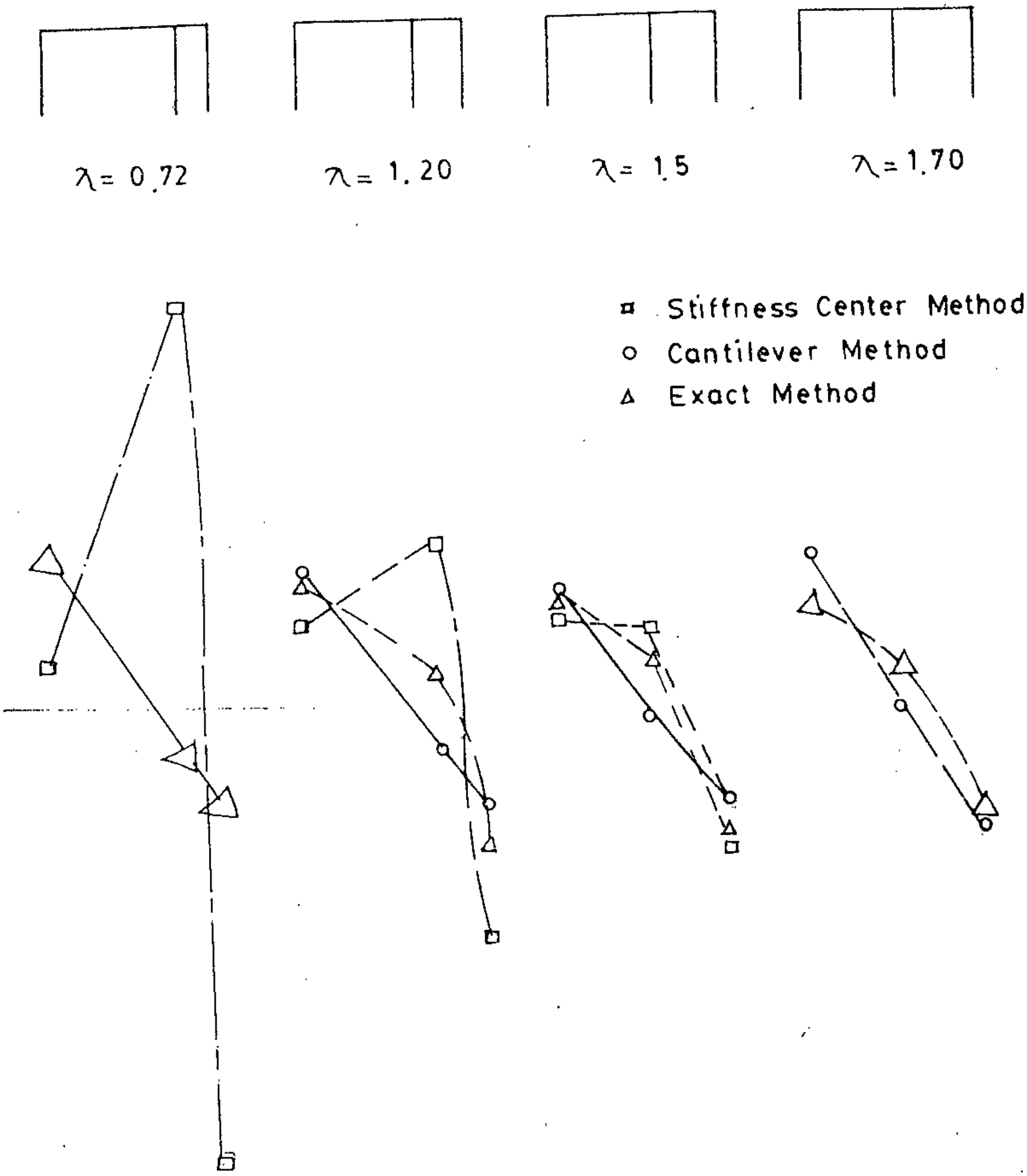


Fig. 7.27 Column Axial Force

(ii) The inflection points obtained for the top and bottom story column by the stiffness center method and for the central stories the inflection points obtained by portal and cantilever method are approximately correct for all realistic values of λ .

(iii) For calculating axial forces in the columns, the use of cantilever method will yield approximately correct results for the value of λ less than 1.0. And for the higher values of λ the stiffness center method is to be used.

7.4 Comparison of the Analytical Results with Khan's Method for the Effective Flange Area Computation of a Shearwell Interconnected with Frame

For comparing the results of Khan's method a series of 4-bay and 5-bay 5 storied building frames has been analyzed for the values of V_b/V_c 0.01 to 10.0 (V_b = eqivalent shear stiffness of beam = $12EI/L_g^3$ and V_c = axial stiffness of column = AE/L_c, shear co-efficient of beam C_v = 0.0) for a linearly increasing concentric loads along the exterior column line. This represents the case where a frame acts as a flange to a shear wall producing the L beam effect. For a frame acting as flange to a shear wall producing a T-beam effect, the analysis of the frame may be carried out by restraining the horizontal deflection throughout the centre line (Fig. 7.28)

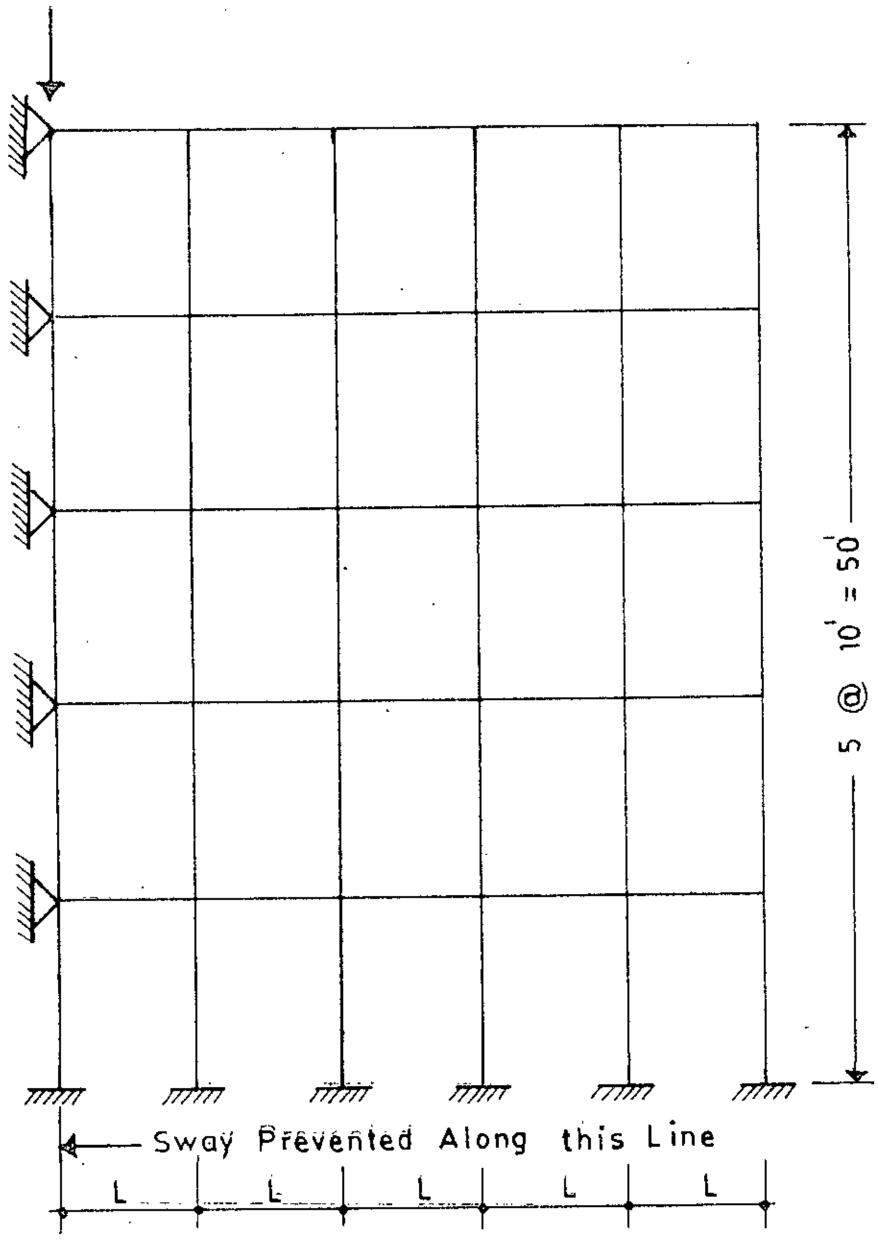


Fig. 7.28 Frame with vertical load, Sway prevented.

The load dissipation characteristics of each story columns are studied and graphs are presented in Fig. 7.29 to Fig. 7.34. From the study of the Fig. 7.31 to Fig. 7.34 it is clear that the load dissipation characteristics of a building frame is not uniform for all the stories and the dissipation of axial loads is maximum at the top story and minimum at the bottom story. From Fig. 7.31 for a 5-storied building for $V_b/V_c = 10.0$ the load dissipations at the 5th and 2nd story are 71 percent and 40 percent respectively. From these observations the following conclusions are made.

7.4.1 Concluding Remarks

(i) The effective flange area of the shear wall will be maximum at the top story level and minimum at the bottom story level and it will not be of uniform width throughout its whole height.

7.5 Discussion on the Stress Level at Different Phase of Construction

For determining the behaviour of a building frame at different construction stages a series of 3-bay six storied building frames have been analyzed at for gravity loading (self weight). The significance of considering construction time as a variable in a building frame analysis may be pointed out from a study

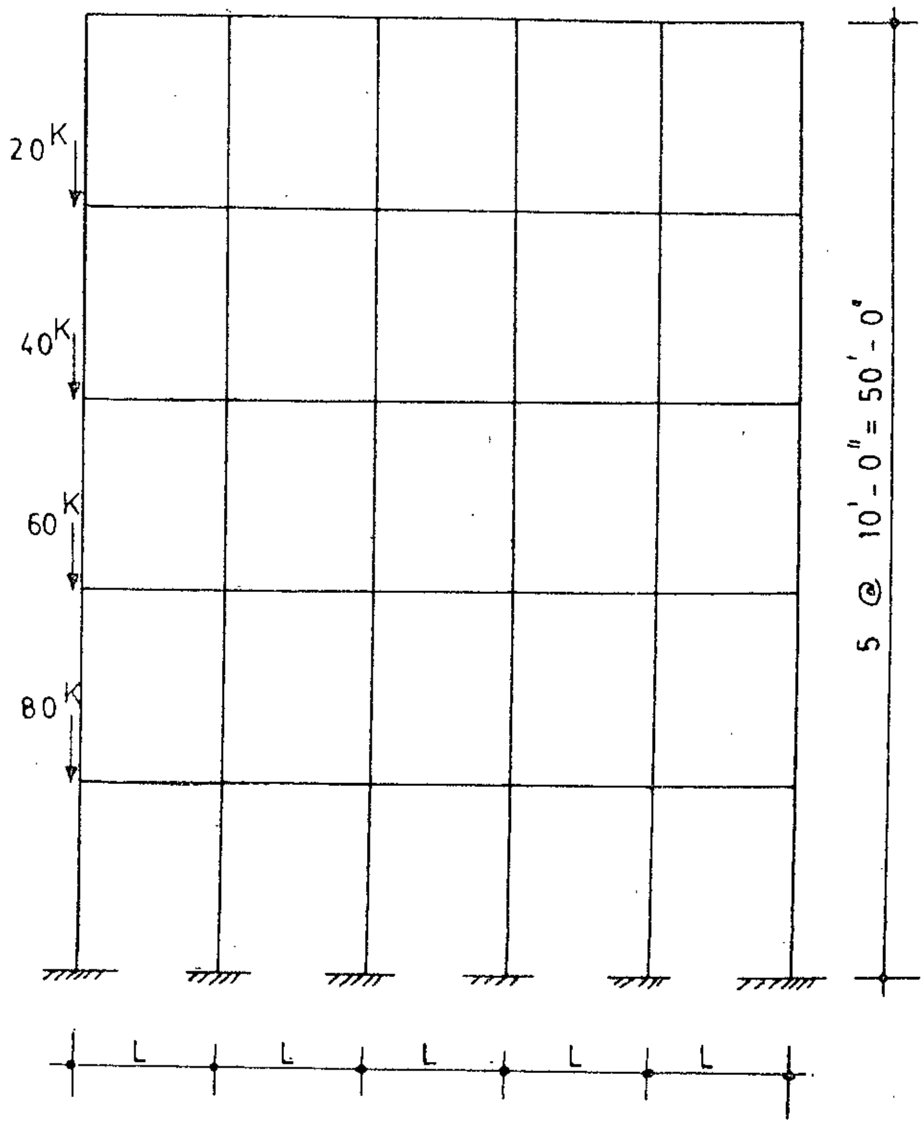


Fig. 7.29 Linearly Increasing Axial Loads in a Frame Interconnected Normal to the Plane of Shear wall

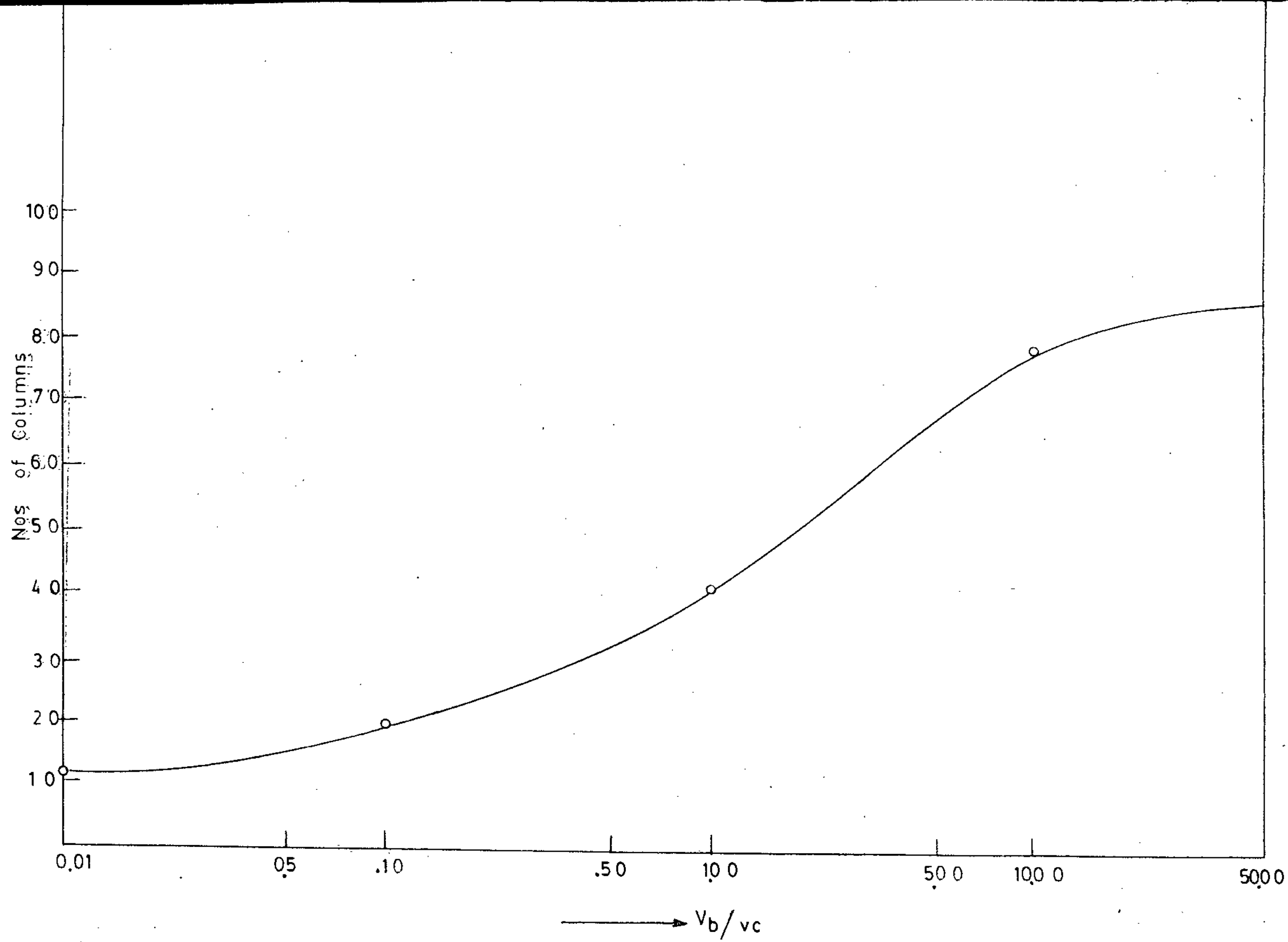


Fig. 7.30 Graph Suggested by Khan (6)

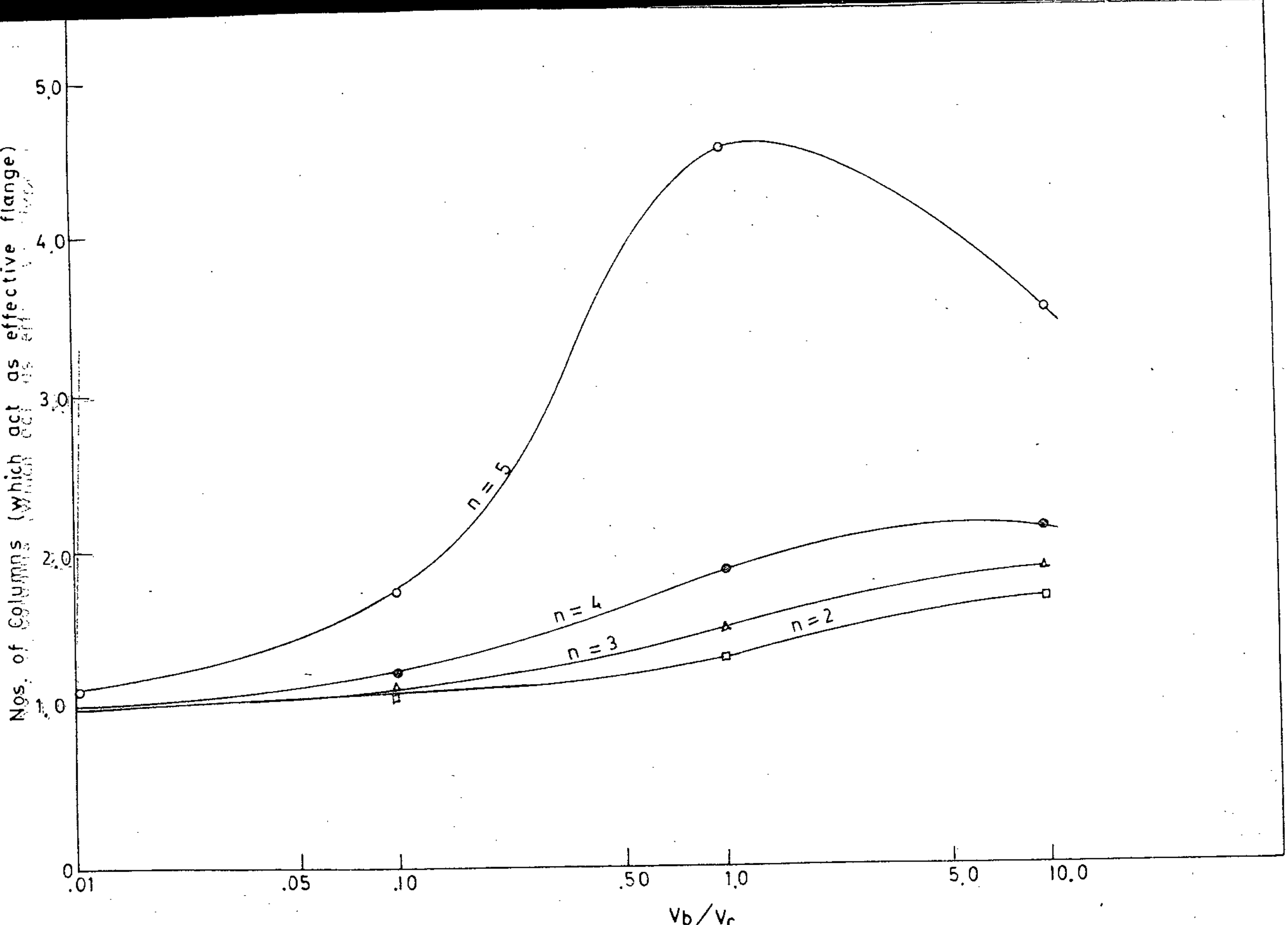


Fig. 7.31 Axial Load Dissipation of 5 storied 5 bay Frame. (n story number)

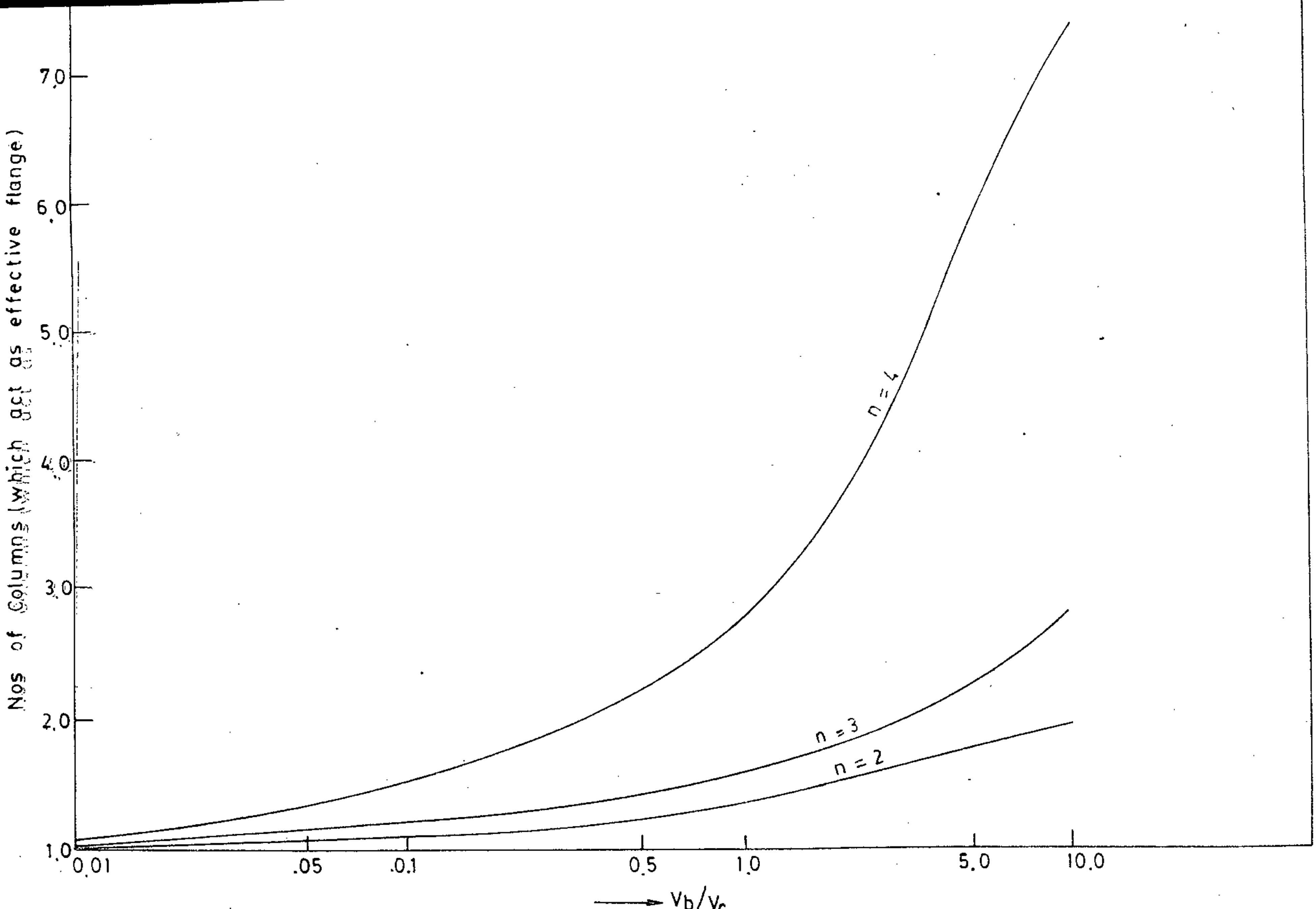


Fig. 7.32 Axial Load Dissipation of 4 Storied 5 bay Frame. (n story number)

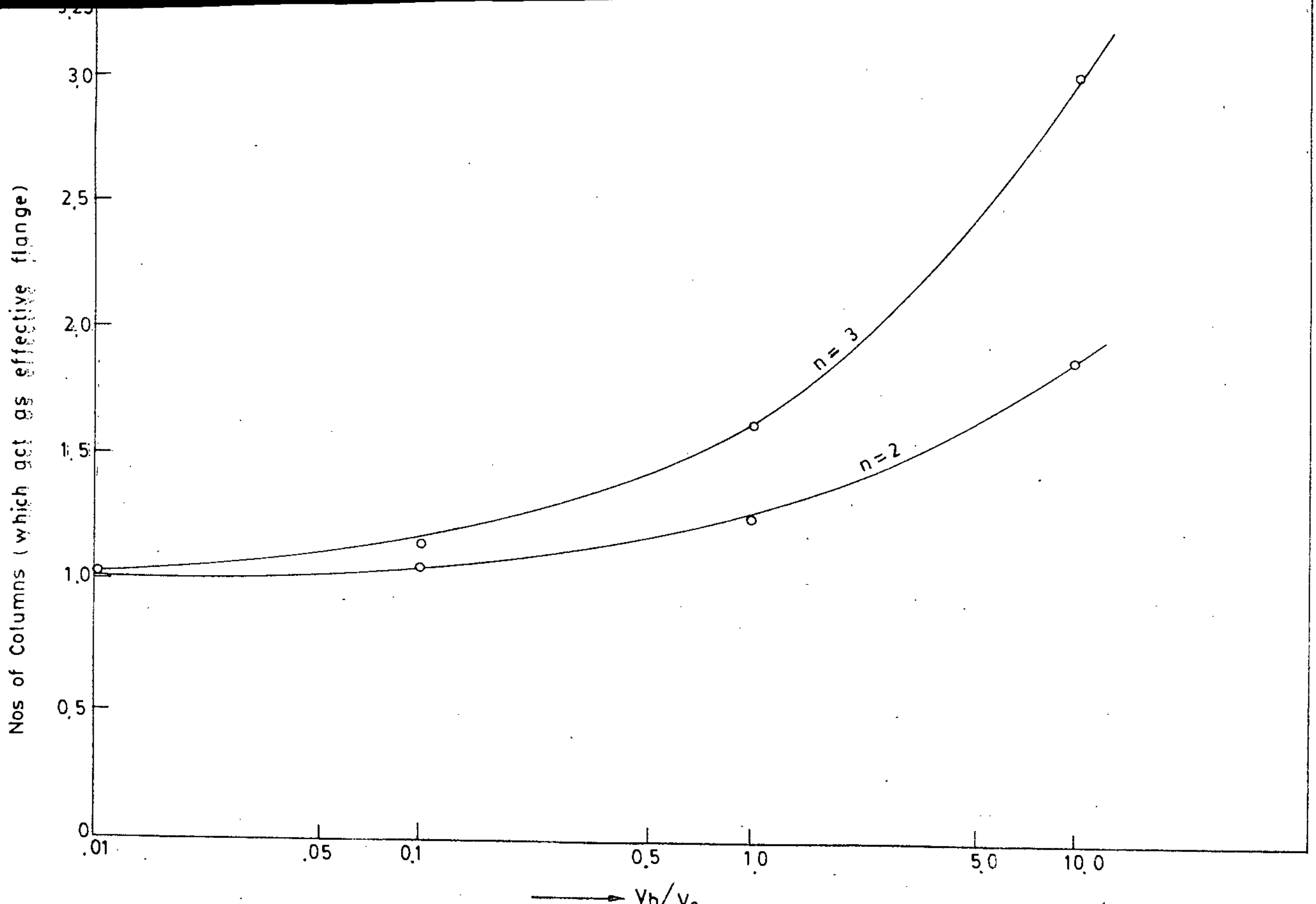


Fig.7.33 Axial Load Dissipation of 3 Storied 5 bay Frame (n story number)

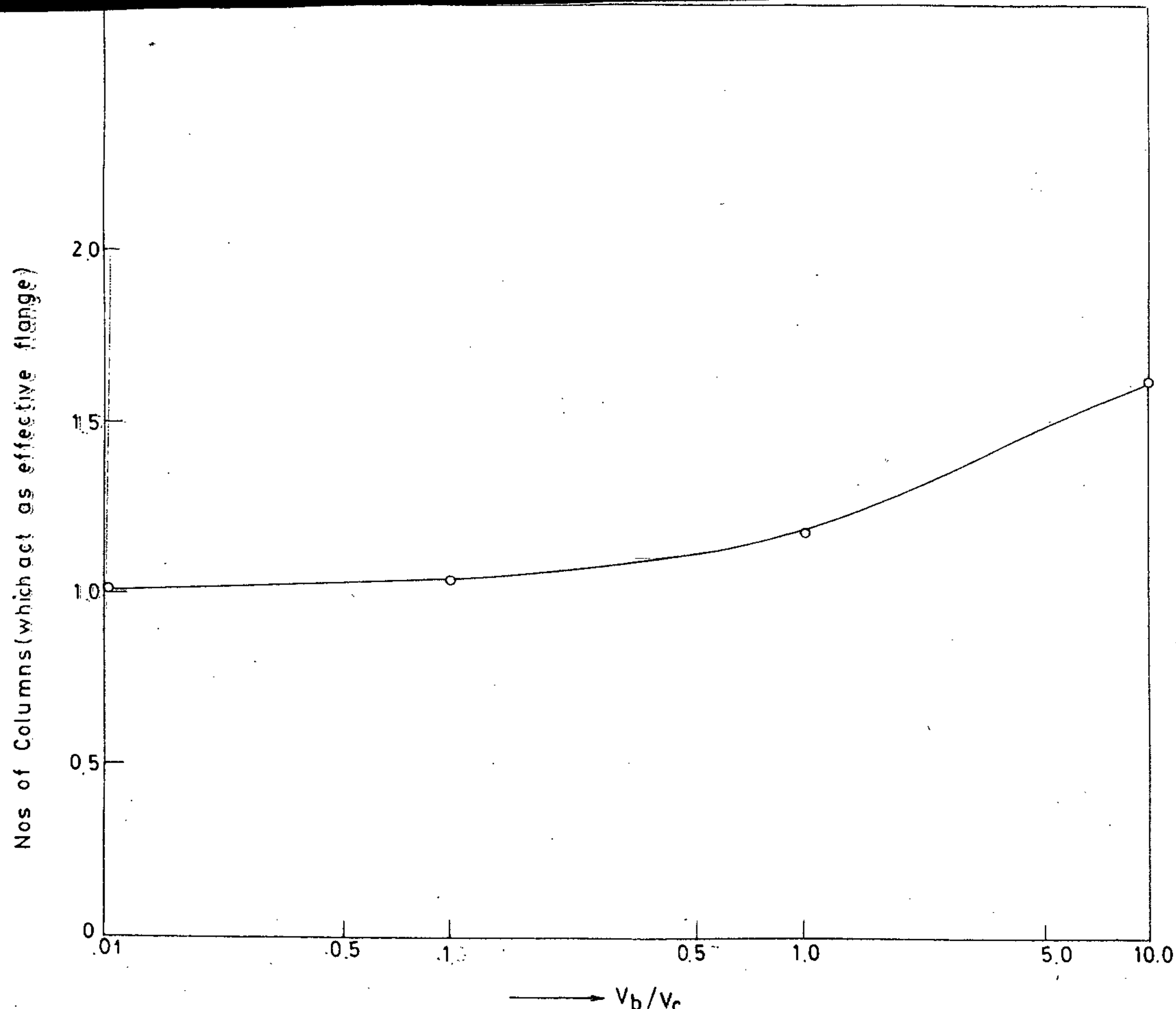


Fig. 7.34 Axial Load Dissipation of 2 Storied 5 bay Frame. (n Story Number)

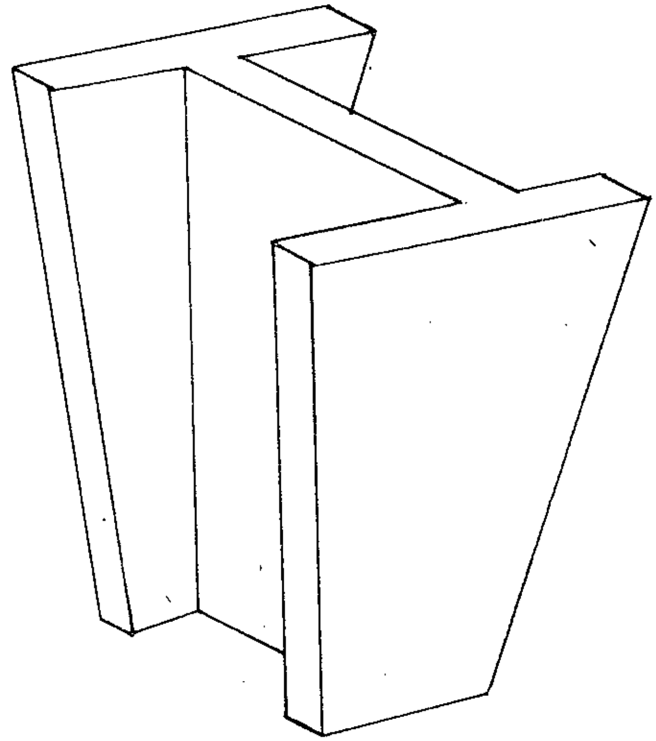


Fig. 7.35 Varying Effective Flange Area of the Shear wall at Different Story Level

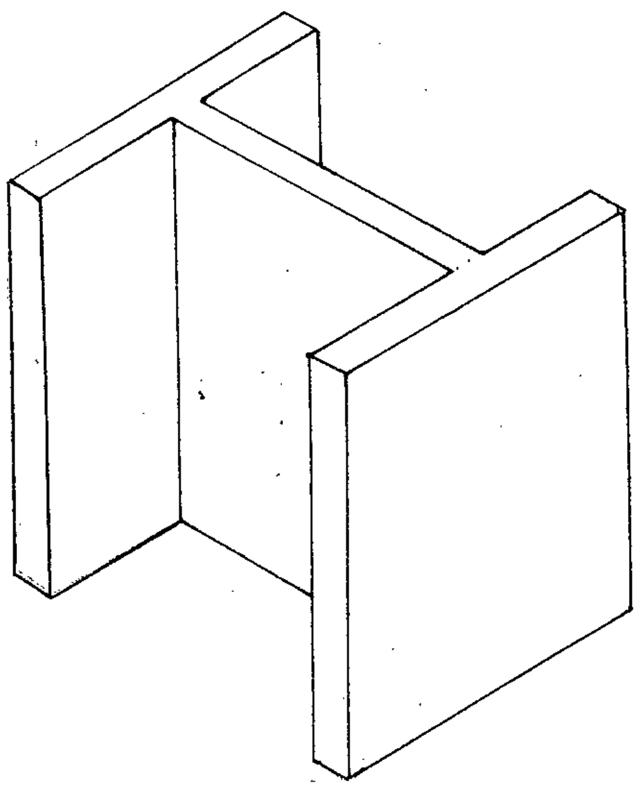


Fig. 7.36 Uniform Effective Flange Area of the Shear wall Suggested by Khan

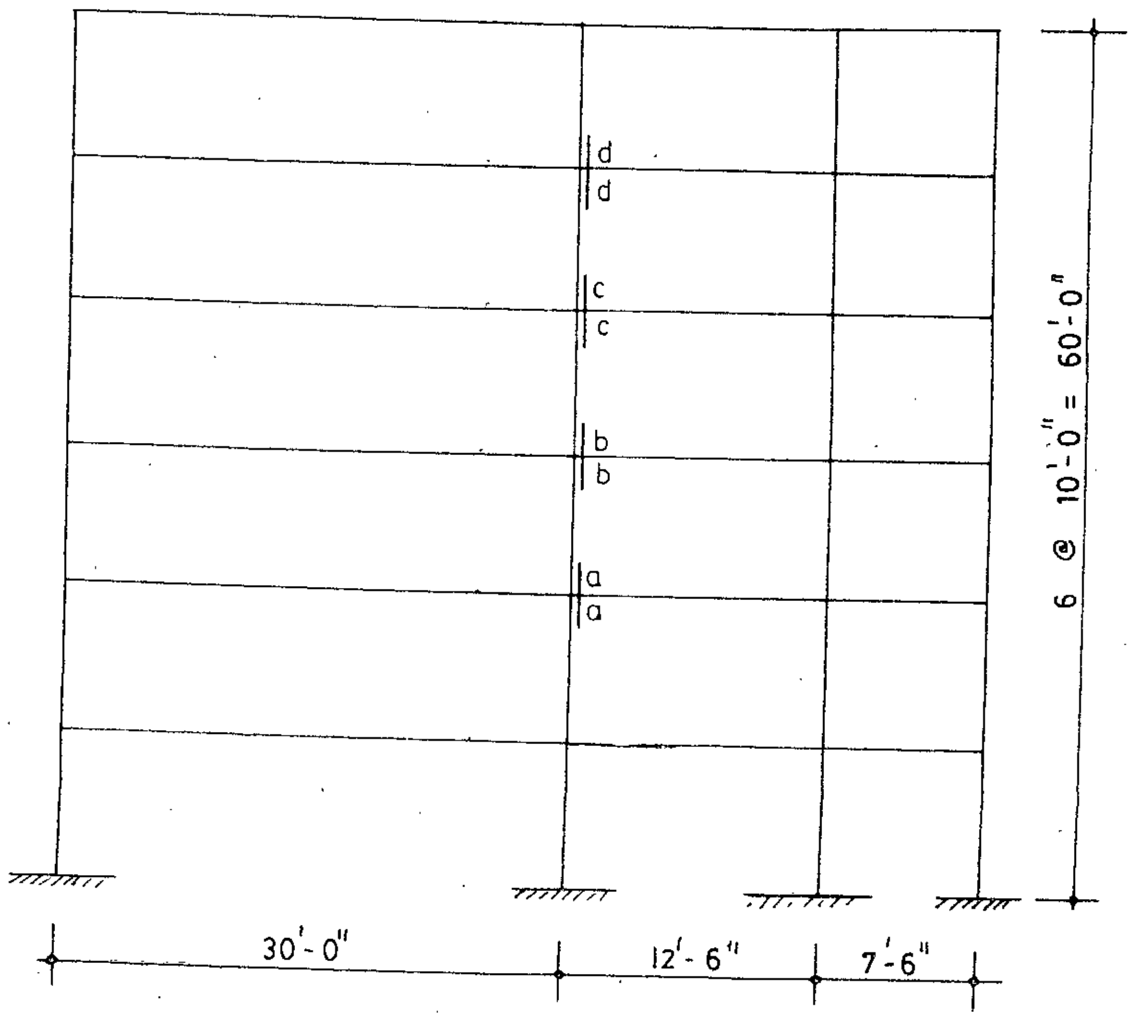
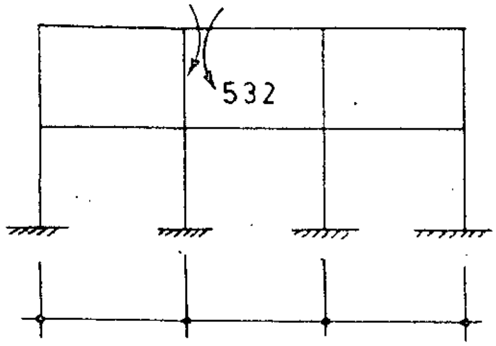
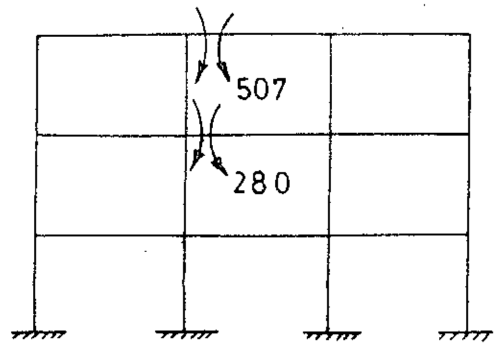


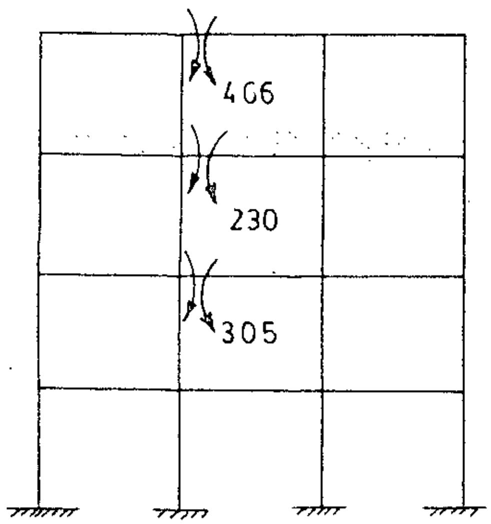
Fig. 7.37 A Typical Unsymmetrical Building Frame



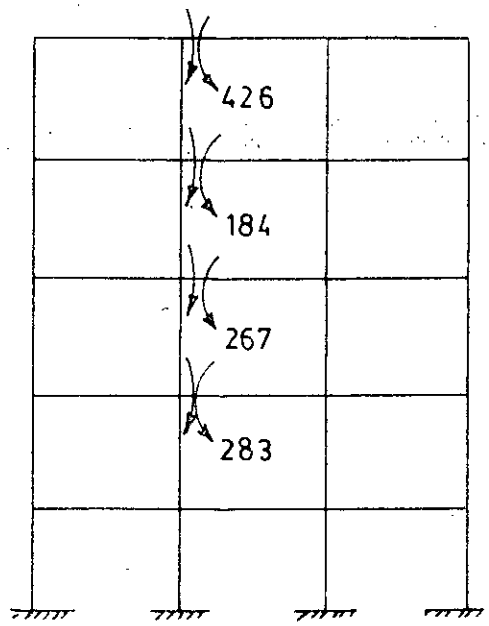
2nd story constructed



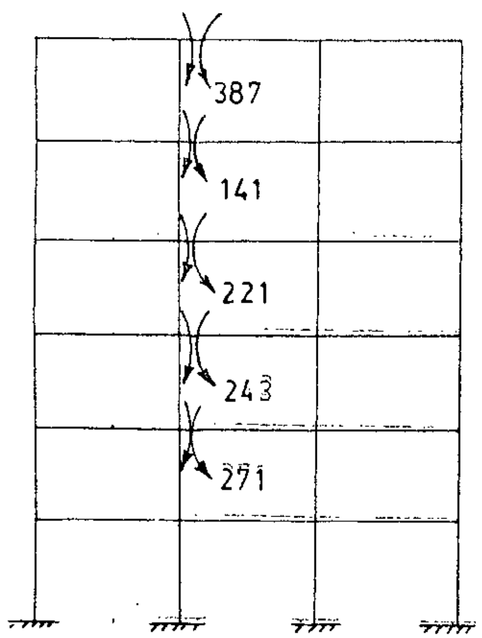
3rd story constructed



4th story constructed



5th story constructed



6th story constructed

Fig. 7.38 The Magnitudes of moments at some specified locations at different stages of its construction

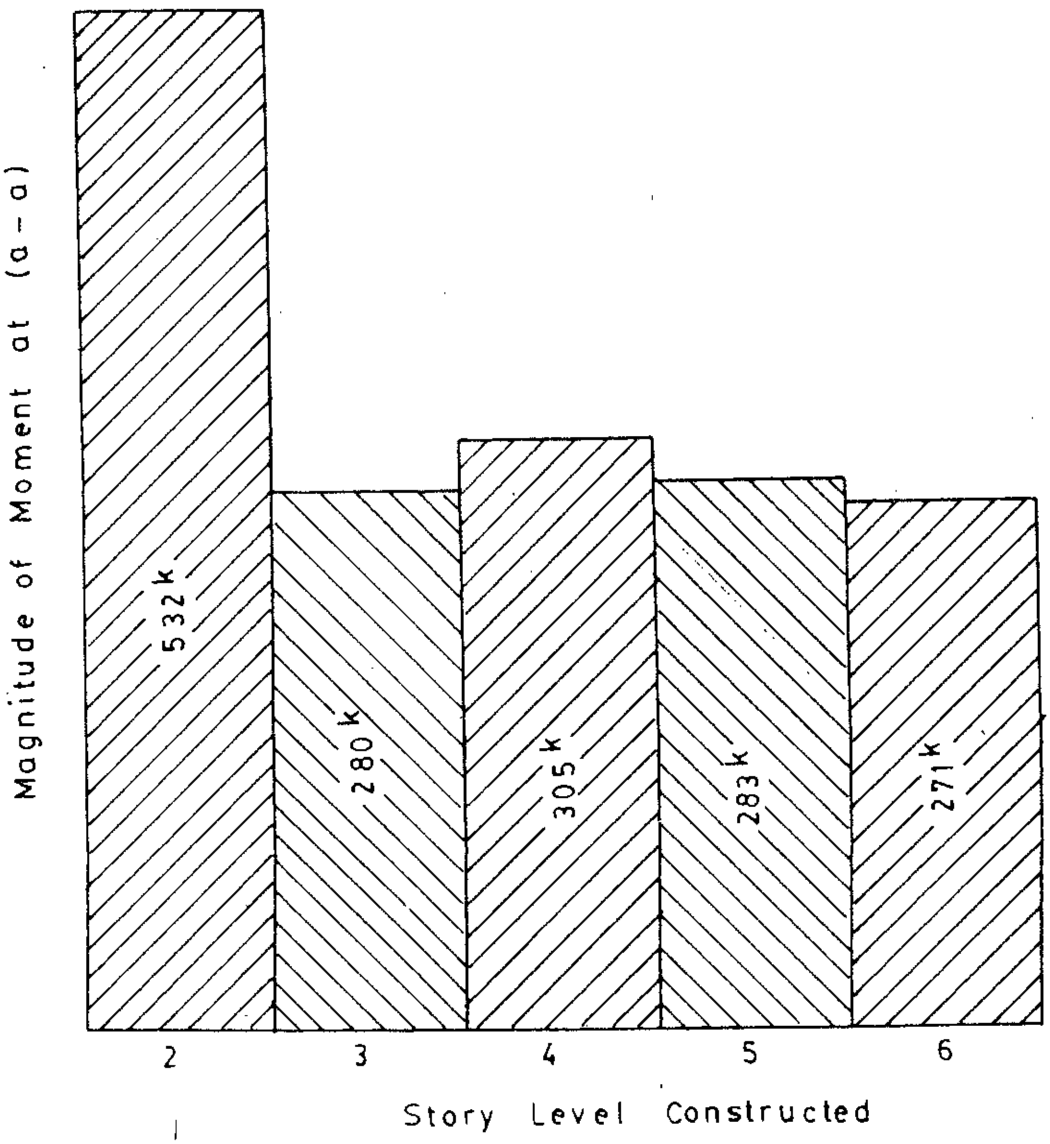


Fig. 7.39 The Variation of Moment Magnitude at a Specified Location at Different Construction Stages

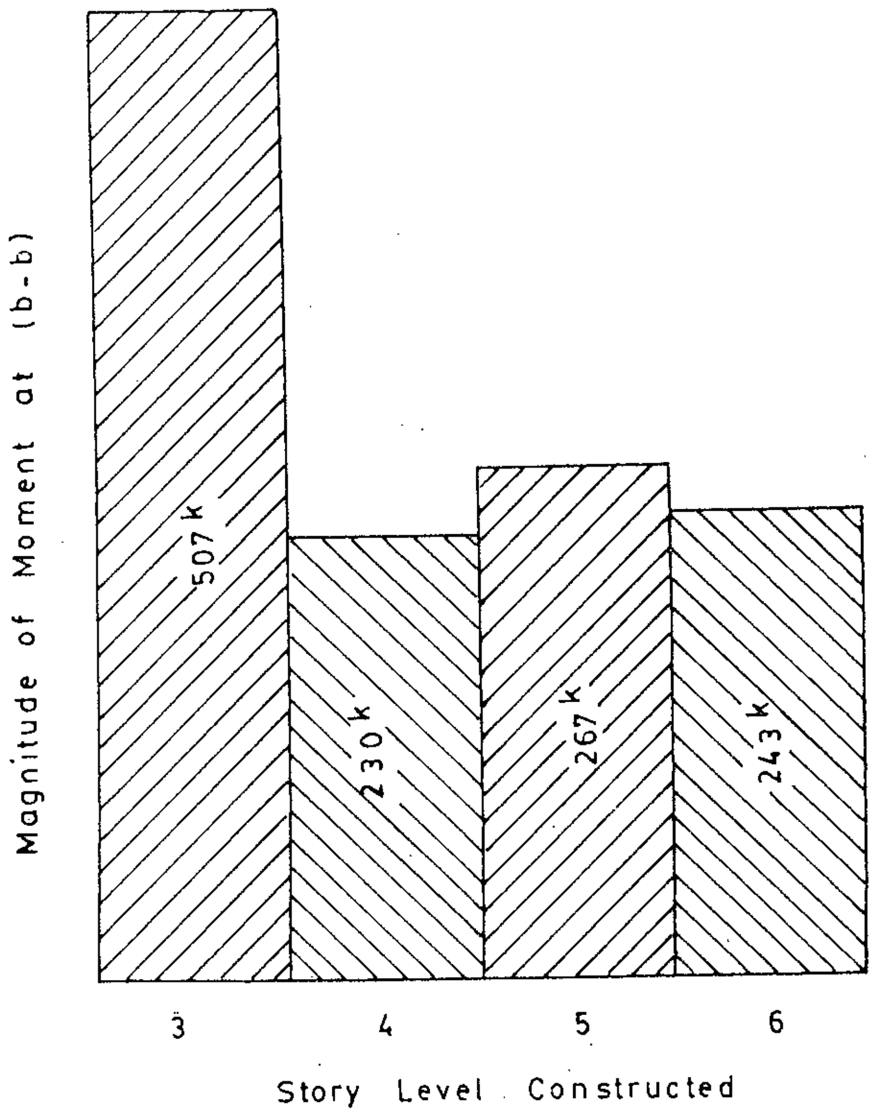


Fig. 7.40 The Variation of Moment Magnitude at a Specified Location at Different Construction Stages

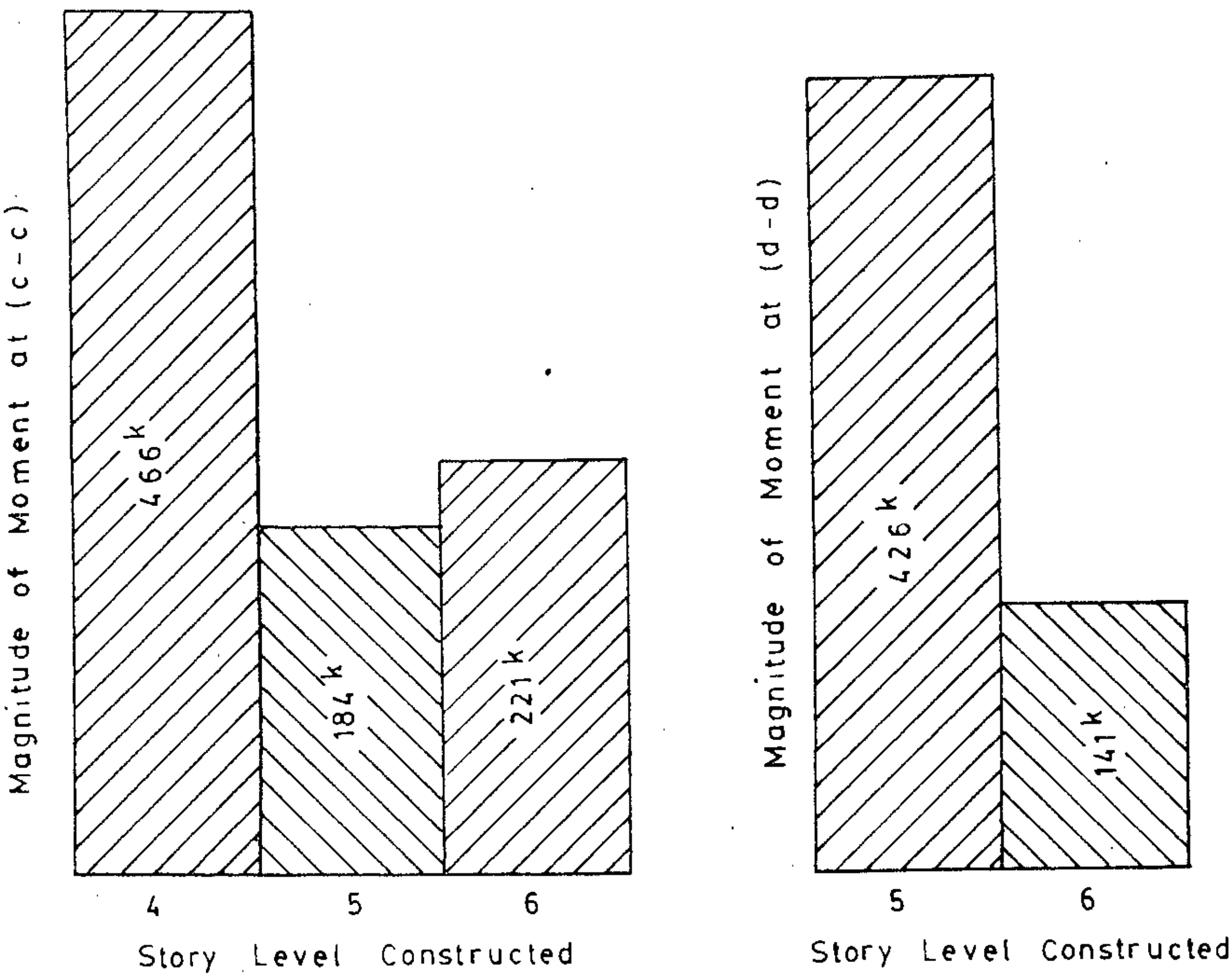


Fig. 7.41 The Variation of Moment Magnitude at (c-c) & (d-d) at different Stages of Construction

of the girder moments at the locations (a-a), (b-b), (c-c) and (d-d) for different levels of construction. Graphical diagrams are presented and at (d-d) a moment variation of 302 percent is observed between the 5th and 6th story level of construction. Assuming the dead load as 70 percent of design load a 302 percent moment variation during construction at lower story level will result in a 211 percent (302x.7) increase in the design stress.

7.5.1 Concluding Remarks

(i) In unsymmetrical building frames and also in the frames, where all the vertical elements of the structure have different stress levels, the construction time must be considered as a variable during the analysis and design, which means that the analysis is to be carriedout for every level of construction for its dead loads.

(ii) In case of a building frame, which will be constructed to a lower level than the designed level, the analysis and design is to be done for every probable final floor level with full design loads.

CHAPTER 8

CONCLUSIONS

From analysis performed in the context of this thesis the following conclusions are made:

(a) On Approximate Method for Gravity Load Analysis of Building Frame

(i) The locations of points of inflection in the girders as suggested by Salvadori and Levy[7] and Wilbur and Norris[10] are found invalid.

(ii) Inflection points in girders of a two bay building frame, where the girder spans ratio does not exceed 1.75 and for all realistic range of λ (0.5 to 5.0), exist at 0.2L from both exterior and interior column supports.

(iii) In two bay frames where girder spans ratio exceed 1.75 and for all realistic range of λ the inflection points in the longer girder will be located at 0.2L from the exterior column support and at 0.15L from the interior column support.

(b) On Approximate Methods for Lateral Load Analysis of Building Frame

(i) Inflection points in the girders and central story columns of a multi-storied building frame for all realistic values of λ, are located at the mid span of each girder and midheight of each central story column.

(ii) For locating of inflection points in the top and bottom story columns in a building frame for all realistic values of λ the use of stiffness center method will yield better result.

(iii) For axial force calculation in the columns the use of cantilever method will give better results when λ is less than 1.0 and for higher values of λ the stiffness center method should be used.

(c) On Axial Load Dissipation Characteristics of a Building Frame

(i) The concept of uniform dissipation of concentric load throughout its whole height has/been found invalid. Rather the dissipation is to be considered varying for whole of the building height and it is maximum at the top story level and minimum at the bottom story level.

(d) On Assuming Construction Time as an Additional Variable in designing Multi-story Building Frame

(i) In an unsymmetrical building frame where all the vertical elements have different stress levels the construction time must be considered as a variable during analysis and design.

(ii) In building frames, where the construction may be made upto a lower level than the level designed for, the analysis and design is to be done for its every probable final floor levels with full design loads.

CHAPTER 9

LIMITATIONS OF THE PRESENTED ANALYSIS AND SUGGESTION FOR FURTHER STUDY

The analysis presented in the thesis consider only the linear elastic behavior of the structure.

The main problem encountered during whole work is the availability of large capacity computer. Due to the smaller available computer store in every phase of this thesis smaller building frames have been analyzed although the programmes generated in all cases can accommodate any number of stories and any number of bays and can also incorporate any number of shear wall at any arbitrary locations.

The analysis presented here is based on the assumption that the bases of the columns are fixed. In practice, the foundation may have some flexibility. A little modification in the analysis will make possible the inclusion of effects of foundation deformations on the structures.

Further study may be made on the following topics:

(i) The validity of portal, cantilever and stiffness center methods for taller multibay buildings.

(ii) The effectiveness of the columns to form a flange area of the shear wall in a larger building frames with interconnected shear wall.

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APPENDIX-A

LIST OF COMPUTER PROGRAMS

1. General program for gravity load analysis by stiffness method.

2. General program for portal method.

3. General program for cantilever method.

4. General program for stiffness center method.

5. General program for lateral load analysis by stiffness method.

6. General program for analysis at different construction phase by stiffness method.

7. General program for concentric load dissipation by stiffness method.