STUDY ON APPROXIMATE LENGTHS OF SOME STEADY GRADUALLY VARIED FLOW PROFILES

BY

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In partial fulfillment of the requirements for the Degree of Master of Engineering (Water Resources)

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CERTIFICATE

This is to certify that this project work has been done by me and neither this project nor any part thereof has been submitted elsewhere for the award of any degree or diploma.

Countersigned

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We hereby recommend that the project prepared by Mohammad Zafar Ullah entitled **"Study on Approximate Lengths of Some Steady Gradually Varied Flow Profiles"** be accepted as fulfilling this part of the requirements for the degree of Master of Engineering (Water Resources).

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ABSTRACT

A simple formula for computing the approximate lengths of steady gradually varied flow profiles which are asymptotic to the normal depth line has been derived. This formula is valid for a channel of any shape, regular or irregular. According to this formula the profile length depends on the flow depth, the channel slope, the channel shape, the Froude number of the flow, the initial perturbation and the ratio between the actual perturbation and the original perturbation. The approach used in the derivation is similar to the work of Samuel (1989).

The validity of the formula has been determined by computing the lengths of M1, M2, S2 and S3 profiles in wide and rectangular channels and comparing the computed profile lengths with the actual profile lengths computed by the direct integration method.

The derived formula seem to be satisfactory for computing the lengths of M1, M2, S2 and S3 flow profiles, when the perturbation in depth above or below the normal depth line is less than about 10% of the normal depth and when the Froude number of the flow is less than 0.3 for subcritical flow and greater than 2 for supercritical flow. The accuracy in the computed profile length is increased when the longer flow profile is considered. The derived formula tends to underestimate the lengths of M1 and S3 profiles and overestimate the lengths of M2 and S2 profiles.

The validity of the de Vries formula for estimating profile length in wide channels has also been determined. This formula seems to be satisfactory for predicting the profile length in wide channels for small Froude numbers and for any perturbation in depth. The predictive capacity of this formula for higher values of the Froude number seem to improve considerably when the effect of Froude number is included in the formula.

Using the derived formula, the backwater lengths in the Ganges, the Jamuna, the Meghna and the Gorai rivers of Bangladesh at the Hardinge Bridge, Bahadurabad, Bhairab Bazar and Gorai Railway Station, respectively, have been determined. The computation of the backwater lengths is based on the bankful depth. Since the slopes of these alluvial rivers are very small, the backwater lengths in these rivers are found to be quite considerable.

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LIST OF SYMBOLS

The main symbols used in this paper are included in the following list. When other symbols have been used, they are defined in the text.

Symbol	Meaning	Unit(s)
A	area of cross-section	m^2
b	bottom width of rectangular and trapezoidal sections	m
C	Chezy's resistance factor	$m^{1/2}/s$
Fr	Froude number	-
g	acceleration due to gravity	m/s^2
K	conveyance	-
K_n	conveyance for uniform flow	-
L	length of backwater profile	m
L^+	non-dimensional profile length	m
M	hydraulic exponent for critical flow	-
n	Manning's roughness coefficient	-
N	hydraulic exponent for uniform flow	-
P	wetted perimeter	m
Q	discharge	m^3/s
R	hydraulic radius	m
S_f	friction slope	-
S_o	channel slope	-
T	top or free surface width	m
V	mean velocity of flow	m/s
x	longitudinal distance	m
x_o	value of x at the downstream control section	m
y	depth of flow	m
y_n	normal depth	m
y_c	critical depth	m
z	side slope of trapezoidal section	-
Z_o	vertical distance of the channel bottom above the datum	m
Z_c	section factor for critical flow	-
Z	section factor	-
α	energy coefficient	-

CHAPTER 1

INTRODUCTION

1.1 General

The differential equation that governs the free surface profile of steady gradually varied flow in open channels is expressed as (French, 1986)

$$\frac{dy}{dx}=\frac{S_o-S_f}{1-Fr^2}=S_o\frac{1-(y_n/y)^N}{1-Fr^2} \tag{1.1}$$

in which dy/dx = slope of the free surface at any location, S_o = slope of the channel bed, S_f = friction slope, Fr = Froude number of the flow, y = depth of flow, y_n = depth of uniform flow or normal depth, and N is the hydraulic exponent for uniform flow computation, respectively. Equation (1.1) is known as the dynamic equation of gradually varied flow.

The water surface profile computed by Eq. (1.1) represents a backwater curve when dy/dx >0, i.e. the flow depth y increases in the direction of flow and a drawdown curve if dy/dx <0, i.e. the depth of decreases in the direction of flow.

Equation (1.1) is not in general explicitly solvable because the right hand side of it cannot be expressed explicitly in terms of y for all types of channel section. If the channel geometry and the form of the resistance equation are both particularly simple, the equation may be solved explicitly. Many attempts were made either to solve the equation for a few special cases or to introduce assumptions that make the equation amenable to mathematical integration. So far the solution of Eq. (1.1) has been obtained for a horizontal channel, a wide channel (Bresse method) and a frictionless channel.

Since the gradually varied flow equation is not explicitly solvable for all channel sections, in most cases, the solution must be sought either graphically or by numerical integration. The gradually varied flow profiles can also be determined by the use of advanced numerical techniques (Subramanya, 1982). All of these methods are well - suited for computer applications. Some general case computer models, e.g. the HEC-2 model of the U.S. Army Corps of Engineers (1982), and a similar model of the U.S. Geological Survey (Shearman, 1977) were also developed for the computation of gradually varied flow profiles.

Engineers often require quick but approximate estimate of the effects of a proposal at the planning stage of an investigation. In the context of hydraulic and river engineering, one fundamental parameter is the length of the channel affected by works, e.g. construction of a dam or channel amendments, downstream. Samuels (1989) presented an approximate solution to Eq. (1.1) for computing backwater lengths in rivers. His formula for the backwater length in a wide river (b>>y), based on Manning's formula, is

$$L=-0.30\frac{y_n}{S_o}(1-Fr^2)\ln\eta/\eta_o \tag{1.2}$$

where y_n is the normal depth of flow, taken to be equal to the bankful depth, S_o is the bottom slope of the channel and L is the longitudinal distance at which the downstream perturbation η_o reduces to η.

De Vries (1994) also presented an equation for estimating the upstream distance at which the induced rising of the water level is 50% of the one at x = 0, x being positive upstream. This equation is given by

$$L_{1/2}= 0.24\eta_o^{4/3} \tag{1.3}$$

where

$$y_o= \eta_o y_n \tag{1.4}$$

is the depth at x = 0 and L is the dimensionless length scale given by

$$L=\frac{xS_o}{y_n} \tag{1.5}$$

The numerical values of 0.24 and 4/3 in Eq. (1.3) have been obtained by matching the real solution with an equation of the shape of Eq. (1.3). For small-induced water level changes $(\eta_0 \to 1)$, the value $L_{1/2}=\frac{1}{4}$ is obtained.

Samuels (1989) used Eq. (1.2) to compute the length of M1 profile in rivers which are assumed to be wide. However, his formula can also be used to compute the lengths of the flow profiles which are asymptotic to the normal depth line. Out of 12 gradually varied flow profiles, the profiles M1, M2, S2 and S3 are asymptotic to the normal depth line.

Equation (1.3) presented by de Vries (1994) is also intended to compute the approximate length of M1 profile in rivers. But this equation is also equally applicable for computing the lengths of M2, S2 and S3 profiles in wide channels.

Equations (1.2) and (1.3) are applicable only for wide rectangular channels, i.e. channels for which b>>y and R ≃ h. However, these equations are not applicable for channels which are not wide.

The aim of the present study is to extend the work of Samuels (1989) and derive a general equation for the approximate length of the flow profiles which are asymptotic to the normal depth line. This formula can be conveniently used for computing the lengths of flow profiles in a channel of any arbitrary shape, regular or irregular. Samuels equation, Eq. (1.2), is obtained as a special case of the general equation when the channel is wide.

1.2 Objective of the study

The specific objectives of the present study are:

(a) to derive a formula for computation of length of the flow profiles which are asymptotic to the normal depth line in a channel of any arbitrary shape,

(b) to determine the limitations of the derived formula for predicting the actual length of the flow profiles,

(c) to compare the profile lengths computed by the formula with those computed by Eq. (1.3) presented by de Vries (1984), and

(d) to determine the backwater lengths in some rivers of Bangladesh at selected stations.

CHAPTER 2

REVIEW OF LITERATURE

2.1 Introduction

Steady nonuniform or varied flows are flows in which the flow depth is constant with respect to time ($\partial y/\partial t = 0$), but not with respect to space ($\partial y/\partial x \neq 0$). In a long prismatic channel in which no controls are present, the flow tends to be uniform. If there are controls in the channel, they tend to pull the flow away from the uniform condition, and there will be a transition between the two uniform states of flow. In the transition region the flow will in general be varied or nonuniform. In a non-prismatic channel the flow may be varied or nonuniform also due to a change either in the direction or slope or cross-section of the channel.

Varied flow may be either gradually varied or rapidly varied. In a gradually varied flow, the water depth varies gradually along the channel length ($\partial y/\partial x \approx 0$). The streamlines are practically parallel so that the fluid pressure is hydrostatic. In gradually varied flows, friction plays a dominating role in determining the flow characteristics. Flow behind a dam and flow upstream of a sluice gate or an overflow weir are examples of steady gradually varied flow.

All the solution techniques developed for gradually varied flow are based on the assumption that the headloss at a channel section is the same as for uniform flow having the hydraulic radius and average velocity of the section (Chow, 1959). According to this assumption, the uniform flow formula may be used to evaluate the friction slope of a gradually varied flow. The friction slope S_f can be written as

$$S_f = \frac{V^2}{C^2 R} \tag{2.1}$$

when the Chezy formula is used, and

$$S_f = \frac{n^2 V^2}{R^{4/3}} \tag{2.2}$$

when the Manning formula is used.

The errors which arise from this assumption are believed to be small compared with those incurred in the normal use of the Chezy and Manning equations and in the estimation of n or C. This equation is probably more accurate for contracting flow fields than for expanding flow fields since in the former the headloss is primarily the result of frictional effects while in the latter there may be significant losses attributable to eddies.

2.2 Equation of steady gradually varied flow

With reference to Fig. 2.1, the total energy or head at any channel section is given by

$$H = Z_o + y + \alpha \frac{V^2}{2g} \tag{2.3}$$

where Z_o is the elevation of the channel bottom above a horizontal datum, y is the depth of flow, V is the cross-sectional mean velocity and α is the energy co-efficient.

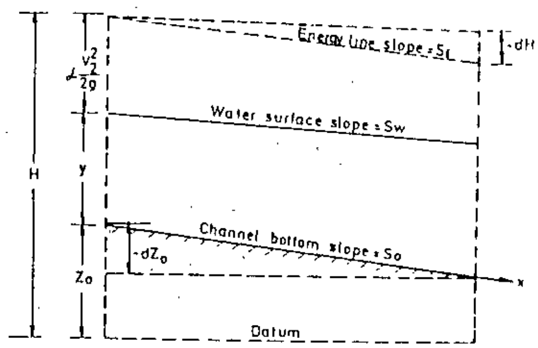

Fig. 2.1 Representation of different heads at a channel section

Differentiating of Eq. (2.3) with respect to x yields

$$\frac{dH}{dx}=\frac{dZ_o}{dx}+\frac{dy}{dx}+\frac{d}{dx}\left(\alpha\frac{V^2}{2g}\right) \tag{2.4}$$

Since $\frac{dH}{dx}=-S_f,\ \frac{dZ_o}{dx}=-S_o$, where S_f and S_o are the friction slope and bottom slope, respectively, and

$$\frac{d}{dx}\left(\alpha\frac{V^2}{2g}\right)=\frac{d}{dy}\left(\frac{\alpha Q^2}{2gA^2}\right)\frac{dy}{dx}=-\frac{\alpha Q^2}{gA^3}\frac{dA}{dy}\frac{dy}{dx}=-\frac{\alpha V^2}{gD}\frac{dy}{dx}=-Fr^2\frac{dy}{dx} \tag{2.5}$$

for a specified flow rate Q and channel section, Eq. (2.4) becomes

$$-S_f=-S_o+\frac{dy}{dx}-Fr^2\frac{dy}{dx}$$

or

$$\frac{dy}{dx}=\frac{S_o-S_f}{1-Fr^2} \tag{2.6}$$

Equation (2.6) is known as the **dynamic equation of steady gradually varied flow** for a channel of any arbitrary shape. It represents the slope of the water surface with respect to the channel bottom. The water surface profile represents a backwater curve when the depth of flow increases in the flow direction (dy/dx>0) and a drawdown curve when the depth of flow decreases in the flow direction (dy/dx<0). When dy/dx = 0, the water surface is parallel to the channel bottom and the flow is uniform.

It has been stated earlier in Art. 2.1 that the friction slope S_f of gradually varied flow is computed by either Eq. (2.1) or Eq. (2.2). In a general form expressed in terms of the conveyance K, the energy slope S_f may be written as

$$S_f = \frac{Q^2}{K^2} \tag{2.7}$$

Suppose that a uniform flow of a discharge Q occurs in the section. The energy slope would then be equal to the bottom slope and Eq. (2.7) becomes

$$S_o = \frac{Q^2}{K_n^2} \tag{2.8}$$

where K_n is the conveyance for uniform flow at a depth y_n. Then, dividing Eq. (2.7) by Eq. (2.8) one obtains

$$\frac{S_f}{S_o} = \frac{K_n^2}{K^2} \tag{2.9}$$

The section factor Z is given by

$$Z = \frac{\sqrt{A^3}}{T} \tag{2.10}$$

and if a critical flow of a discharge equal to Q occurs in the section, then

$$Q = Z_c\sqrt{g/\alpha} \tag{2.11}$$

where Z_c is the section factor for critical flow at a depth equal to y_c. Then

$$Fr^2 = \frac{\alpha V^2}{gD} = \frac{\alpha Q^2 T}{gA^3} = \left(\frac{Z_c}{Z}\right)^2 \tag{2.12}$$

Substituting Eqs. (2.9) and (2.12) into Eq. (2.6), one obtains

$$\frac{dy}{dx} = S_o \frac{1-(K_n/K)^2}{1-(Z_c/Z)^2} \tag{2.13}$$

Equation (2.13) is another form of the steady gradually varied flow equation.

Since the section factor and the conveyance are functions of the depth of flow only, it is assumed that

$$K^2 = C_1 \, y^N \tag{2.14}$$

$$K_n^2 = C_1 \, y_n^N \tag{2.15}$$

$$Z^2 = C_2 \, y^M \tag{2.16}$$

and

$$Z_c^2 = C_2 y_c^M \tag{2.17}$$

where C_1 and C_2 are co-efficients, and N and M are the hydraulic exponents for uniform and critical flow computations, respectively. If these expressions are now substituted into Eq. (2.13), the gradually varied flow equation becomes

$$\frac{dy}{dx} = S_o \frac{1-(y_n/y)^N}{1-(y_c/y)^M} \tag{2.18}$$

Equation (2.18) is still another form of the steady gradually varied flow equation.

2.3 Types of flow profiles

The channel slopes an conveniently classified as positive ($S_o>0$), zero ($S_o = 0$) and negative ($S_o>0$) slopes for the purpose of classification of flow profiles. A positive slope is the slope that falls in the direction of flow. It may be mild ($y_n>y_c$, $S_o<S_c$), critical ($y_n = y_c$, $S_o = S_c$) and steep ($y_n<y_c$, $S_o>S_c$). A horizontal slope is a zero slope and adverse slope is a negative slope that rises in the direction of flow.

For the given discharge and channel conditions the normal and critical depth lines divide the space in a channel into the following three zones:

Zone 1 Space above the upper line ($y>y_n$, $y>y_c$)

Zone 2 Space between the two lines ($y_n>y>y_c$ or $y_c>y>y_n$)

Zone 3 Space below the lower line ($y<y_n$, $y<y_c$)

The flow profiles are classified into thirteen different types according to the nature of the channel slope and the zone in which the flow surface lies. They are designated as M1, M2, M3, C1, C2, C3, S1, S2, S3, H2, H3, A2 and A3, where the letter is descriptive of the slope -- M for mild, C for critical, S for steep, H for horizontal and A for adverse, and the numbers 1,2 and 3 represent the zones. Of the thirteen flow profiles, twelve are for gradually varied flow and one, C_2, is for uniform flow. The flow profiles H_1 and A_1 are physically not possible since in channels with horizontal and adverse slopes the normal depth of flow is infinity and imaginary, respectively.

The general characteristics of the flow profiles are given in Table 2.1 and their shapes are shown in Fig. 2.2. Out of twelve flow profiles of gradually varied flow, the profiles M1, M3, S1, S3, C1, C3, H3 and A3 are backwater curves and the profiles M2, S2, H2 and A2 are drawdown curves.

The following principles regarding gradually varied flow profiles can be noted:

1. When the flow profile approaches the normal depth, it does so asymptotically ($y \rightarrow y_n$, $\partial y/\partial x \rightarrow 0$).
2. When the flow profile approaches the critical depth, it crosses this depth at a large but finite angle ($y \rightarrow y_c$, $\partial y/\partial x \rightarrow \infty$). This indicates a hydraulic jump or a hydraulic drop in the flow profile. If the flow depth decreases while crossing the critical depth line a hydraulic drop occurs. If the flow depth increases, then a hydraulic jump occurs. In both cases the flow becomes so curvilinear or rapidly varied that the theory and equations of gradually varied flow become inapplicable.

3. When $y \to \infty$ (large depth and small flow velocity), $\partial y/\partial x \to S_0$, i.e. the flow profile is horizontal.

4. As $y \to 0$ (small depth and large flow velocity), $\partial y/\partial x$ tends to a positive finite limit and the profile makes a certain acute angle with the channel bottom.

5. When $y_n = y_c$, it follows that $\partial y/\partial x \approx S_0$, i.e. the C1 and C2 profiles are practically horizontal.

Table 2.1 Flow profile characteristics

Channel slope	Zone designation			Relative relation of y to y_n and y_c	Type of curve	Type of flow
	1	2	3			
Mild $0<S_0<S_c$	M1	M2	M3	$y>y_n>y_c$ $y_n>y>y_c$ $y_n>y_c>y$	Backwater Drawdown Backwater	Subcritical Subcritical Supercritical
Critical $S_0=S_c>0$	C1	C2	C3	$y>y_c=y_n$ $y_c=y=y_n$ $y_c=y_n>y$	Backwater Parallel to channel bottom Backwater	Subcritical Uniform-critical Supercritical
Steep $S_0>S_c>0$	S1	S2	S3	$y>y_c>y_n$ $y_c>y>y_n$ $y_c>y_n>y$	Backwater Drawdown Backwater	Subcritical Supercritical Supercritical
Horizontal $S_0 = 0$	None	H2	H3	$y>y_n>y_c$ $y_n>y>y_c$ $y_n>y_c>y$	None Drawdown Backwater	None Subcritical Supercritical
Adverse $S_0 < 0$	None	A2	A3	$y>y_n>y_c$ $y_n>y>y_c$ $y_n>y_c>y$	None Drawdown Backwater	None Subcritical Supercritical

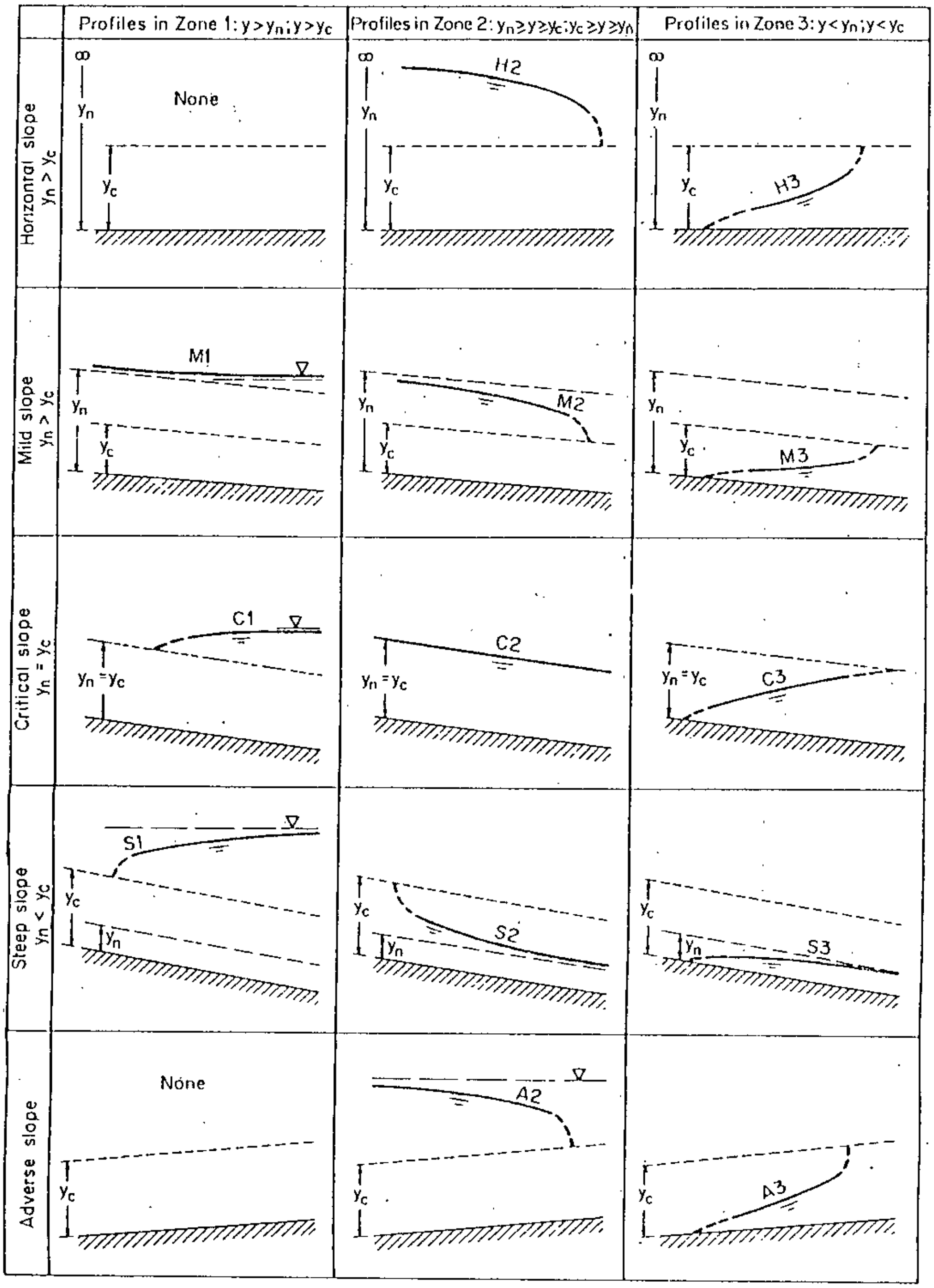

Fig. 2.2 Shape of flow profiles of gradually varied flow (Chow, 1959)

2.4 Computation of gradually varied flow profiles

The computation of steady gradually varied flow basically involves the integration of the dynamic equation of steady gradually varied flow, i.e. Eq. (2.18). This equation is a nonlinear ordinary differential equation and can be solved when Q, S_o, α, n (or C), the channel shape and the boundary condition at x = 0 are specified, However, a direct and exact integration of this equation is troublesome (Gill, 1976) except for a few simple cases (e.g. a wide channel), as the right hand side of this equation can not be expressed explicitly in terms of y for all types of channel sections. This is why in most cases the equation is solved numerically.

Although the basic form of the equation remains unchanged, it is the characteristics of the numerical methods that the differing circumstances of particular problems may require differing methods of solution. However, all numerical solutions of the gradually varied flow equation have this in common that the calculations must start at a control and proceed in the direction in which the control is being exercised. Thus, the computation of the subcritical flow profiles must be started from the downstream end and the supercritical flow profiles from the upstream end of the channel reach, since subcritical flow is subjected to downstream control and supercritical flow is subjected to upstream control.

The flow profile may approach the normal depth asymptotically at upstream or downstream boundary. As $y \rightarrow y_n$, the absolute magnitude of the numerator of Eq. (2.18) becomes small and cannot be computed accurately. This is, however, not a matter of great concern, since in practice we are usually not interested in details of the asymptotic approach of y to y_n; we are more concerned with the flow profile in regions where a control has drawn the depth substantially away from y_n (Henderson, 1966). The full length of the flow profiles which an asymptotic to the normal depth line are never determined, rather, in practice the computation is terminated at a location where the depth of flow is 1% or 10% higher or lower than the normal depth.

In computing a flow profile, the discharge Q, the depth of flow at the control section, the channel shape at various channel sections, the longitudinal slope S_o, the energy co-efficient α and the Manning's roughness co-efficient n or Chezy's C are generally required.

There are many methods of computing flow profiles in a channel. They may, however be broadly classified into the following two groups:

i) Methods suitable for regular channels

ii) Methods suitable for irregular channels

The methods suitable for regular channels compute x from y explicitly without the need for any trial. The direct step method and the direct integration method fall into this category. On the other hand, the methods used for irregular channels compute x from y and require a trial-and-error procedure. The standard step method is an example of this type.

The various methods for computing the steady gradually flow profile can be found in many text books (e.g. Chow, 1959; Henderson, 1966; Range Raju, 1981; Subramanya, 1982; French, 1986). Here the direct integration method of computing flow profile in a regular channel is presented, since it is relevant to the present study.

2.5 Direct integration method

The differential equation of gradually varied flow, Eq. (1.1), is not in general explicitly solvable because the right hand side of it cannot be expressed explicitly in terms of y for all types of channel section. If the channel geometry and the form of the resistance equation are both particularly simple the equation may be solved explicitly. Many attempts have been made to solve the equation for a few special cases that make the equation amenable to mathematical integration. Chow (1959) provided a chronological list of some methods used to integrate the gradually varied flow equation directly. Most of the early methods were developed for channels of a specified cross-section but that later solution, since Bakhmeteff's original work in 1912 (Bakhmeteff, 1932), were developed for channels of all shapes. Also, most early methods employed the Chezy formula, whereas later methods used Manning's formula.

In the Bakhmeteff method (Bakhmeteff, 1932) the channel length under consideration is divided into short reaches and the integration is carried out by short range steps and with the aid of a varied flow function. To improve Bakhmeteffs method, Mononobe (1938) introduced two assumptions for hydraulic exponents, which are (i) $P \propto y^{const}$, and (ii) $A^2 \propto y^{const}$. By these assumptions the effect of velocity changes and friction head are taken into account integrally without the necessity of dividing the channel length into short reaches. The Monobe method affords a more direct and accurate computation procedure whereby results can be obtained without recourse to successive steps. But the first assumption is not very satisfactory in many cases. Later, Lee (1947) and Von Seggern (1950) suggested new assumptions which result in more satisfactory solutions. Von Seggern introduced a new varied flow function in addition to the function used by Bakhmeteff but in Lee's method no new function is required.

The method of direct integration described here (French, 1986) is the outcome of a study of many existing methods. Putting $u = y/y_n$; Eq. (2.18) may be expressed for dx as

$$dx=\frac{y_n}{s_o}\left[1-\frac{1}{1-u^N}+\left(\frac{y_c}{y_n}\right)^M\frac{u^{N-M}}{1-u^N}\right]du \qquad (2.19)$$

Integration of Eq. (2.19) yields

$$x=\frac{y_n}{s_o}\left[u-\int_o^u\frac{du}{1-u^N}+\left(\frac{y_c}{y_n}\right)^M\int_o^u\frac{u^{N-M}}{1-u^N}\,du\right]+cosnt. \qquad (2.20)$$

The first integral on the right side of the above equation is designated by F (u, N), or

$$F(u,N)=\int_0^u \frac{du}{1-u^N} \tag{2.21}$$

which is known as **varied flow function**. The second integral in Eq. (2.20) may also be expressed in the form of the varied flow function. Putting $v = u^{N/J}$ and $J = N/(N - M + 1)$; this integral can be transformed into

$$\int_0^u \frac{u^{N-M}}{1-u^N}du=\frac{J}{N}\int_0^v \frac{dv}{1-v^J}=\frac{J}{N}\ F(v,J) \tag{2.22}$$

where

$$F(v,J)=\int_0^v \frac{dv}{1-\ v^J} \tag{2.23}$$

is another varied flow function like F (u, N), except that the variables u and N are now replaced by v and J, respectively. Using the notation for varied flow functions, Eq. (2.20) may be written as

$$x=\ \frac{y_n}{S_o}\left[u-\ F(u,N)+\left(\frac{y_c}{y_n}\right)^M\ \frac{J}{N}\ F(v,J)\right]\ +\ const. \tag{2.24}$$

or

$$x=\ A\ [\ u-F(u,N)+B\ \ F(v,J)]\ +\ const. \tag{2.25}$$

where

$$A=\frac{y_n}{S_o} \tag{2.26}$$

and

$$B\ =\ \left(\frac{y_c}{y_n}\right)^M\frac{J}{N} \tag{2.27}$$

The length of flow profile between two consecutive sections 1 and 2 is equal to

$$L = x_2 - x_1 = A\{(u_2 - u_1) - [F(u_2, N) - F(u_1, N)] + B[F(v_2, J) - F(v_1, J)]\} \quad (2.28)$$

where the subscripts 1 and 2 refer to sections 1 and 2, respectively.

2.6 Bresse Method (1860)

At this point it should be noted that Eq. (2.18) can be integrated exactly for the special case of N = M = 3, that is, a wide rectangular channel with the conveyance expressed in terms of the Chezy equation. In such a case

$$x = \frac{y_n}{S_o}\left[u - \left(1 - \frac{y_c^3}{y_n^3}\right)F(u, 3)\right] + const. \quad (2.29)$$

where

$$F(u,3) = \int_o^u \frac{du}{1-u^3} = \frac{1}{6}\ln\left[\frac{u^2+u+1}{(u-1)^2}\right] - \frac{1}{\sqrt{3}}\cot^{-1}\frac{2u+1}{\sqrt{3}} \quad (2.30)$$

which is known as the **Bresse function**. This integration was first performed by Bresse. A determination of the flow profile by this solution is widely known as the Bresse method.

The normal and critical depths in a wide rectangular channel are given respectively by

$$y_n = 3\sqrt{q^2/C^2S_o} \quad (2.31)$$

and

$$y_c = 3\sqrt{q^2/g} \quad (2.32)$$

where q is the discharge per unit width. Therefore,

$$\left(\frac{y_c}{y_n}\right)^3 = \frac{C^2 S_o}{g} \qquad (2.33)$$

Substituting this expression for $(y_c/y_n)^3$, Eq. (2.24) may also be written as

$$x = \frac{y_n}{S_o}\left[u - \left(1 - \frac{C^2 S_o}{g}\right) F(u,3)\right] + conts. \qquad (2.34)$$

or

$$x = A[u - BF(u, 3)] + \text{const.} \qquad (2.35)$$

where $A = y_n/S_o$ and $B = \left(1 - \frac{C^2 S_o}{g}\right)$

The length of the flow profile between two consecutive sections of depth y_1 and y_2 is

$$L = A[(u_2 - u_1) - B\{F(u_2, 3) - F(u_1, 3)\}] \qquad (2.36)$$

At this point it may be mentioned that many alluvial channels are wide, i.e. their width is very large compared to their depth and the Bresse method can be conveniently used to compute the flow profiles in these channels.

2.7 Approximate estimate of the profile length

The computation of gradually varied flow in artificial and natural channels is often tedious and time - consuming and the information obtained is not rewardingly proportional to the effort. Engineers often require quick but approximate and still reasonable estimate of the effects of a proposed work at the planning stage of an investigation. Such estimates may come from an accumulation of past experience, as empirical fits to sets of data, or from a simplified formulation of problem concerned with suitable simplying approximation. In the context of hydraulic and river engineering, one fundamental parameter is the length of the channel affected by works e.g. the construction of a dam or weir, channel amendments, downstream or upstream. Vallentine (1964, 1967) presented generalized non-dimensional profiles in wide, mild and steep channels for various values of y/y_n. Chen and Wang (1969) extended the work of Vallentine (1967) and developed the non-dimensional curves for all the gradually varied flow profiles in a wide channel using the critical depth and critical slope to normalize the vertical and horizontal co-ordinates of gradually varied flow. Samuels (1989) and de Vries (1994) gave two formulas for estimating the approximate lengths of flow profiles in wide channels.

2.7.1 Samuels formula

Samuels (1989) presented an approximate solution to Eq. (2.6) for computing flow profiles of steady gradually varied flow in wide channels. Since for given discharge, channel section and bottom slope, the right hand side of Eq. (2.6) is a function of the flow depth only, this equation can be expressed as

$$\frac{dy}{dx}=f(y) \tag{2.37}$$

where

$$f(y)=\frac{S_o-S_f}{1-Fr^2} \tag{2.38}$$

Let η be the perturbation to the normal depth such that $\eta<<y_n$ and $y = y_n + \eta$. Then by expanding the function $f(y_n + \eta)$ about $f(y_n)$ in a Taylor series and neglecting all the terms higher than the first order and evaluating the friction slope S_f using the Manning formula, Samuels gave the following formula for the length of the flow profile in a wide channel ($R \simeq y$)

$$L = -\ 0.30\frac{y_n}{S_o}(1-Fr^2)\ln\eta/\eta_o \quad (3.38a)$$

In the above equation L is the longitudinal distance along the channel in which the original perturbation η_o above or below the normal depth line reduces to η.

2.7.2 De Vries formula

De Vries (1994) also presented a formula for estimating the length of a flow profile in a wide channel. It has been shown in Art. 2.6 that the dynamic equation of steady gradually varied flow for a wide channel when the conveyance is expressed in terms of Chezy formula is given by

$$x=\frac{y_n}{S_o}\left[\ u-\ (1-\frac{y_c^3}{y_n^3})\ \ F(u,3)\right] + const. \quad (2.39)$$

For a wide channel it can be shown that

$$Fr^2=\frac{y_c^3}{y_n^3} \quad (2.40)$$

where Fr is the uniform flow Froude number. Then the solution of Eq. (2.39) can be expressed as

$$L=\frac{y_n}{S_o}[\ u-BF(u,3)] + const. \quad (2.41)$$

where

$$B = 1 - Fr^2 \quad (2.42)$$

De Vries introduced the dimensionless length scale

$$L^+ = \frac{LS_o}{y_n} \tag{2.43}$$

and expressed the solution of Eq. (2.41) as

$$L_2^+ - L_1^+ = (u_2 - u_1) - B\{F(u_2,3) - F(u_1,3)\} \tag{2.44}$$

By introducing a value $y_o = u_o y_n$ at $L = 0$ where L is measured positive upward and assuming Fr<<1 (and hence $B \rightarrow 1$), he determined the location upstream at which the induced perturbation in depth is 50% of the one at L = 0. With $L_1^+ = 0$ and $L_{1/2}^+ = L_2^+$, he presented the formula.

$$L_{1/2}^+ = 0.24 u_0^{4/3} \tag{2.45}$$

in which the numerical values of 0.24 and 4/3 have been obtained by matching the real solution with that of an equation of the shape of Eq. (2.45).

For small induced water level changes, $u_o \rightarrow 1$ and the value $L_{1/2}^+ = \frac{1}{4}$ is obtained.

CHAPTER-3

DERIVATION OF FORMULA FOR APPROXIMATE LENGTH OF FLOW PROFILE

3.1 Introduction

In the previous chapter two formulas for estimating the approximate length of steady gradually varied flow profiles, one due to Samuels (1989) and the other due to de Vries (1994), have been presented. These formula are applicable for wide channels for which b>10y. In this chapter, the work of Samuels (1989) is extended and a formula for approximate length of flow profiles is developed that can be used to compute the profile length in a channel of any arbitrary shape. It is also shown that the formula derived herein reduces to the formulas presented by Samuels and de Vries.

3.2 Derivation of the formula

Let us consider the dynamic equation of steady gradually varied flow in the form

$$\frac{dy}{dx} = S_o \frac{1-(y_n/y)^N}{1-Fr^2} \tag{3.1}$$

and let η be a small change in the depth of flow from the normal depth y_n (Fig. 3.1) i.e.

$$y = y_n + \eta \tag{3.2}$$

where $|\eta| << y_n$, Now for any function f(y) the Taylor series expansion about y_n is

$$f(y_n+\eta) = f(y_n) + \eta\frac{df}{dy} + \frac{\eta^2}{2!}\frac{d^2f}{dy^2} + \dots\dots\dots \tag{3.3}$$

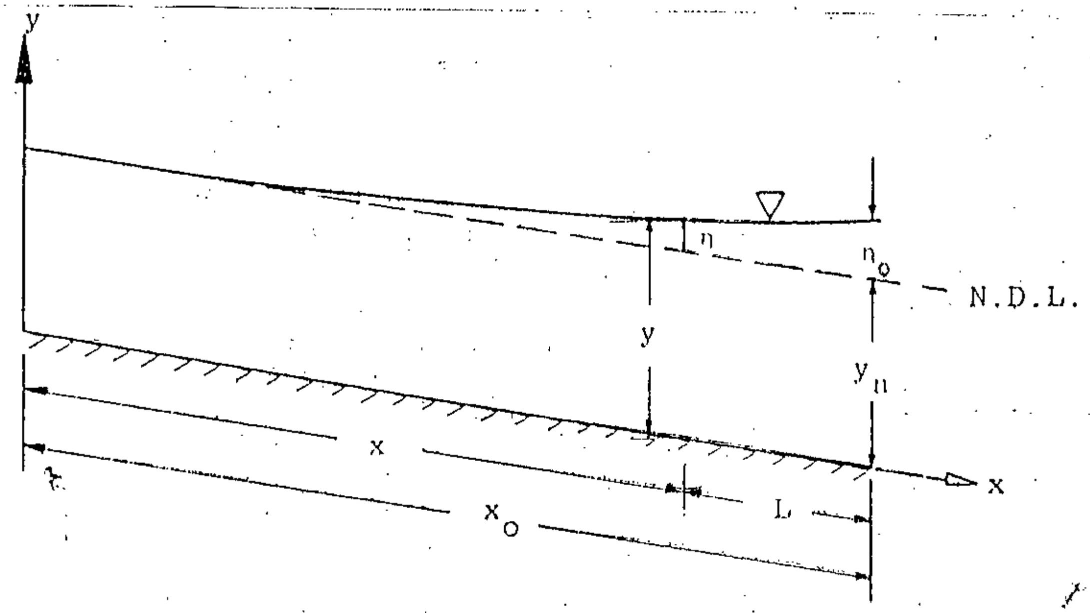

Fig. 3.1 Definition sketch

where the derivatives $\frac{df}{dy}, \frac{d^2f}{dy^2}, \ldots\ldots$, are evaluated at $y = y_n$.

In our case, $f(y_n) = 0$, since at normal depth $y = y_n$ by definition. Also, $\frac{dy_n}{dx} = 0$. Using the above relations into Eq. (3.1) one obtains

$$\frac{dy}{dx} = \frac{d}{dx}(y_n + \eta) = \frac{d\eta}{dx} = \frac{d}{dy}\left[S_o \frac{1 - \left(\frac{y_n}{y}\right)^N}{1 - Fr^2}\right]\eta + HOT \tag{3.4}$$

where HOT represents all higher order terms. Now,

$$\frac{d}{dy}\left[S_o \frac{1 - \left(\frac{y_n}{y}\right)^N}{1 - Fr^2}\right] = S_o\left[\left\{1 - \left(\frac{y_n}{y}\right)^N\right\}\frac{d}{dy}\left\{\frac{1}{1 - Fr^2}\right\} + \frac{1}{1 - Fr^2}\frac{d}{dy}\left\{1 - \left(\frac{y_n}{y}\right)^N\right\}\right] \tag{3.5}$$

where S_o is constant. Since $y = y_n$ as stated previously, Eq. (3.5) reduces to

$$\frac{d}{dy}\left[S_o\frac{1-\left(\frac{y_n}{y}\right)^N}{1-Fr^2}\right]=\frac{S_o}{1-Fr^2}\frac{N}{y_n} \tag{3.6}$$

Since when $y = y_n$, $\left(\frac{y_n}{y}\right)^N=1$, combining Eq. (3.6) with Eq. (3.4) and neglecting the higher order terms, we obtain the first order equation

$$\frac{d\eta}{dx}=\frac{S_o N\ \eta}{(1-Fr^2)y_n} \tag{3.7}$$

for the variation of the depth above or below the normal depth line with respect to x. Equation (3.7) has the solution

$$\eta=\eta_o\exp\left[-\frac{(x_o-x)}{1-Fr^2}\frac{S_o}{y_n}N\right] \tag{3.8}$$

and the length of the flow profile, $L = x_o - x$, is given by

$$L=-\frac{y_n(1-Fr^2)}{N\ S_o}\ln\frac{\eta}{\eta_o} \tag{3.9}$$

Equation (3.9) is the desired formula for the approximate length of the flow profile for a small perturbation above or below the normal depth in a channel of any arbitrary shape. Using this equation the approximate length of backwater profile can be computed if the values of N and Fr at y_n and the channel bottom slope S_o are known and η/η_o is assumed.

3.3 Hydraulic exponent N for prismatic channels

The computation of the length of flow profiles requires the numerical value of the hydraulic exponent for uniform flow computation (N). The value of N is different for different channel shapes and, for a given channel, it depends on the depth of flow y. It also depends on whether the conveyance is expressed in terms of Manning's n or Chezy's C.

The general equation for N for a channel of any arbitrary shape is given by (Chow, 1959; Henderson, 1966)

$$N = \frac{2y}{3A}\left(5T - 2R\frac{dP}{dy}\right) \tag{3.10}$$

when the Manning's equation is used, and

$$N = \frac{y}{A}\left(3T - R\frac{dP}{dy}\right) \tag{3.11}$$

when the Chezy equation is used. In Eqs. (3.10) and (3.11), A is the flow area, T is the free surface width, P is the wetted perimeter and R is the hydraulic radius, For computing flow profile, it is customary to adopt an average value of N in any given flow situation and then assume it to be constant. Values of N for different channel shapes are given in Table 3.1.

Table 3.1 Values of N for different channel sections

Channel section	Conveyance computed by Manning equation	Conveyance computed by Chezy equation
1. wide (b>>y)	3.33	3.00
2. Narrow (b<<y)	2.00	2.00
3. Triangular	5.33	5.00
4. Rectangular	$\frac{2}{3}\left[5-\frac{4(y/b)}{1+2(y/b)}\right]$	$3-\frac{2(y/b)}{1+2(y/b)}$
5. Trapezoidal (side slope z:1)	$\frac{10}{3}\left[\frac{1+2z(y/b)}{1+z(y/b)}\right]$	$3\frac{1+2z(y/b)}{1+z(y/b)}$
	$-\frac{8}{2}\left[\frac{\sqrt{1+z^2}(y/b)}{1+2\sqrt{1+z^2}(y/b)}\right]$	$-\frac{2\sqrt{1+z^2}(y/b)}{1+2\sqrt{1+z^2}(y/b)}$

3.4 Reduction to Samuels and de Vries formulas

For wide channel (b>>y) when the conveyance is expressed by the Manning formula, N = 3.33 and hence Eq. (3.9) reduces to

$$L= -\ 0.30\frac{y_n}{S_o}(1-Fr^2)\ln\eta/\eta_o \tag{3.12}$$

which is the Samuels formula (Eq. (2.38a)) for profile length in a wide channel.

As stated earlier in Art. 2.7.2, the de Vries (1994) formula for the nondimensional length of the flow profile in a wide channel between the locations where L = 0 and where the perturbation in water level is 50% of its value at L = 0, is given by

$$L^*_{1/2}= 0.24u_o^{4/3} \tag{3.13}$$

Since $L' = \frac{LS_o}{y_n}$, Eq. (3.13) reduces to

$$L_{1/2} = \frac{0.24 y_n}{S_o} u_o^{4/3} \tag{3.14}$$

For a small perturbation, $u_o = \left(\frac{y_o}{y_n}\right) \rightarrow 1$ and hence

$$L_{1/2} = \frac{0.24 y_n}{S_o} \tag{3.15}$$

Now, taking $L_{1/2}$, the profile length, as the distance at which the original perturbation reduces to 50%, i.e. taking $\eta = 0.5\ \eta_o$ and using N = 3 which is the numerical value of the hydraulic exponent for a wide channel, when the Chezy formula is used Eq. (3.9) reduces to

$$L_{1/2} = \frac{0.23 y_n (1 - Fr^2)}{S_o} \tag{3.16}$$

and when Fr<<1

$$L_{1/2} = \frac{0.23 y_n}{S_o} \tag{3.17}$$

Thus, for small Froude numbers and small induced rise or fall in the water surface, the de Vries (1994) formula and the formula derived in the present study are almost identical.

At this point it is worthy to mention that when the Manning formula is used, N = 3.33 for a wide channel and our formula reduces to the Samuels formula. This is because Samuels, in deriving his formula, also used the Manning equation to express the hydraulic exponent. On the other hand, when the Chezy formula is used, N = 3.0 for a wide channel and our formula becomes very close to the de Vries formula. This is because de Vries derived his formula empirically utilizing the profile lengths computed by the Bresse method in which the Chezy formula is used to express the hydraulic exponent N.

An inspection of Eq. (3.9) reveals that the profile lengths computed by N = 3.33 are about 11% higher than those computed by N = 3.0 when other conditions remain the same.

The difference between the length of M1 profile computed by present formula and that computed by de Vries formula expressed as a percentage of the former is shown in Table 3.2. From this table it is observed that for higher values of Froude number and perturbation, the difference between the two formula increases. De Vries formula overestimates the length of flow profiles for all values of Fr and y/y_n compared to the present formula.

Table 3.2 Percent difference between the profile lengths computed by de Vries (1994) and the derived formulas

For depth ratio ($u=y/y_n$) / Froude No. (Fr)	1.1	1.2	1.3	1.4	1.5
0.1	16.44	22.55	33.07	39.35	44.67
0.2	18.95	27.81	35.10	41.19	46.34
0.3	23.17	31.56	38.48	44.25	49.14
0.4	29.08	36.83	43.21	48.54	53.05
0.5	36.68	43.60	49.29	54.05	58.08

3.5 Profile lengths for different channel sections

Let us consider a flow with low Froude number (Fr<<1). Then Eqs. (3.8) and (3.9) reduce, respectively, to

$$\eta=\eta_o \exp\left[-(x_o- x)S_o\frac{N}{y_n}\right] \tag{3.18}$$

and

$$L=-\frac{y_n}{N\,S_o}\ln\frac{\eta}{\eta_o} \tag{3.19}$$

Let us take the backwater length as the distance at which $\eta = 0.01\eta_o$, we designate this length as L^*. Then Eq. (3.19) reduces to

$$L^*=\frac{4.60y_n}{NS_o} \tag{3.20}$$

We also limit out discussions hereafter on the values of N based on Manning's equation since this formula is mostly used in the computation of flow profiles.

Wide channel: For a wide channel (b>>y), N = 3.33 and hence

$$L^*=1.38\frac{y_n}{S_o} \tag{3.21}$$

Narrow channel: For a narrow channel ($b<<y$), $N = 2.00$ and one obtains

$$L^{*}=2.30\frac{y_n}{S_o} \tag{3.22}$$

Equations (3.21) and (3.22) indicate that the width of the channel compared to its depth has a significant effect on the profile length and as stated by Vallentine (1964), the profile are longer in narrow channels than those in wide channels.

Triangular channel: For a triangular channel, $N = 5.33$ and hence

$$L^{*}=0.86\frac{y_n}{S_o} \tag{3.23}$$

Rectangular channel: For a rectangular channel, the value of N varies from 3.33 when $y/b \to 0$ (wide channel condition) to 2.00 when $y/b \to \infty$ (narrow channel condition). The backwater length is minimum when $y/b \to 0$ and maximum when $y/b \to \infty$ and

$$L^{*} = (1.38 \text{ to } 2.30)\ y_n/S_o \tag{3.24}$$

Trapezoidal channel: For a trapezoidal channel having a side slope of z:1, N varies both with b/y and z. The values of N are higher for higher values of z and y/b and vice versa, and approaches the values of 3.33 when $y/b \to 0$ (wide channel condition) and 5.33 when $y/b \rightarrow \alpha$ (triangular channel condition) irrespective of the side slope. Thus for a trapezoidal channel

$$L^* = (1.38 \text{ to } 0.86)\ y_n/S_o \qquad (3.25)$$

Equations (3.21) to (3.25) provide a guide as to the length of the channel reach expected to be affected by works downstream or upstream.

3.6 Effect of different parameters on profile length

Effect of S_o: The backwater length, as expected, is inversely proportional to the channel bed slope S_o. Thus, for the same perturbation in water level, the length of the backwater curve will be shorter in a channel with steeper bed slope than that in a channel with milder bed slope.

Combined effect of y, Fr and N: When the depth of flow in a channel changes, the Froude number and the hydraulic exponent (except for wide, narrow and triangular channels) change simultaneously. Thus, it is necessary to consider the combined effect of y, Fr and N on L. The variation of the product $y\,(1 - Fr^2)$ with respect to depth for different channel sections is as follows.

Wide, narrow and rectangular channels

$$\frac{d}{dy}[y(1-Fr^2)]=1+2Fr^2 \qquad (3.26)$$

Triangular channel

$$\frac{d}{dy}[y(1-Fr^2)]=1+4Fr^2 \tag{3.27}$$

Trapezoidal channel

$$\frac{d}{dy}[y(1-Fr^2)]=1+Fr^2\frac{2b^2+7bzy+8z^2y^2}{b^2+3bzy+2z^2y^2} \tag{3.28}$$

Obviously, $d[y(1 - Fr^2)]/dy$ approaches $(1+2Fr^2)$ as $y/b \rightarrow 0$ and $(1 + 4Fr^2)$ as $y/b \rightarrow \infty$.

Thus, the product $y(1 - Fr^2)$ increases with an increase in y in all the channel sections considered. Now, for wide, narrow and triangular channels, N is invariant with respect to y, and N decreases with an increase in y in rectangular and circular channels. Thus, in wide, narrow, triangular and rectangular channels, the backwater length L increases with an increase in depth of flow y. For a trapezoidal channel, both N and $y(1 - Fr^2)$ increase with an increase in y, but numerical computations indicate that the rate of increase of $y(1 - Fr^2)$ with respect to y is somewhat higher than the variation of N with respect to y. As a result, L also increases in a trapezoidal channel with an increase in y, but the rate of increase in low. Thus, the backwater length increases with depth in all channel sections considered and the sensitivity of L to the variation of y seems to be maximum for a triangular channel, followed by rectangular, wide and narrow channels and minimum for a trapezoidal channel.

3.7 Decay of flow profile

When the effect of Froude number on the profile length is considered, Eq. (3.20) becomes

$$L^* = \frac{4.61 y_n (1 - Fr^2)}{N S_o} \tag{3.29}$$

The approximate decay of flow profile can be computed using Eq. (3.9) and is given in Table 3.3. In this table, the distances have been normalized using the length L^* given by Eq. (3.29).

Table 3.3 Approximate decay of backwater

η/η_o	L/L^*	η/η_o	L/L^*
1.00	0.00	0.40	0.20
0.90	0.02	0.30	0.26
0.80	0.05	0.20	0.35
0.70	0.08	0.10	0.50
0.60	0.11	0.05	0.65
0.50	0.15	0.01	1.00

From Table 3.1, it is obvious that the flow profile decays by 50% where $L/L^* = 0.15$ and by 90% when $L/L^* = 0.50$. As $y \rightarrow y_n$ the profile approaches the normal depth line asymptotically and the decay of the flow profiles occurs at a much slower rate. From Eq. (3.9) it is apparent that the decay of flow profiles which are asymptotic to the normal depth line is exponential.

CHAPTER 4

APPLICABILITY AND LIMITATIONS OF THE FORMULA

4.1 General

In the previous chapter a general formula for estimating the approximate length of flow profiles has been derived. This formula is applicable for any channel section, regular or irregular. However, it should, in principle, be valid for a small perturbation in depth above or below the normal depth line, since it is based upon an approximate solution of the dynamic equation of gradually varied flow. In this chapter, the applicability and limitations of the formula for estimating profile lengths under various flow conditions are considered.

4.2 Applicability of the formula

In this section, the flow profiles for which the formula is applicable, the information needed and the procedure for computing the profile length are considered.

4.2.1 Flow profiles

As stated earlier, the formula gives the length of the flow profiles which are asymptotic to the normal depth line. Four flow profiles which are asymptotic to the normal depth line are the M1, M2, S2 and S3 flow profiles.

These flow profiles are shown in Fig. 2.2 and their general characteristics are presented in Table 2.1.

The M1 backwater profile (Fig. 2.2) is created in a mild slope channel when the downstream depth of flow is more than the normal depth. It meets the normal depth asymptotically at its upstream end. Out of 12 gradually varied flow profiles, the M1 profile is the most well-known backwater curve; it is the most important of all the flow profiles from the practical point of view. It occurs in rivers and canals when the downstream depth is more than the normal depth. Typical examples of the M1 profiles are the profiles behind a dam in a natural river, upstream of a sluice gate in a canal and in a canal connecting two reservoirs.

The M2 drawdown profile (Fig. 2.2) occurs in a mild slope channel when the depth of flow is less than the normal depth but more than the critical depth. The upstream end of the profile is tangential to the normal depth line and the downstream end is normal to the critical depth line. Practical examples of the M2 profiles are the profiles in a mild slope channel terminating in a free overall and the profile at the upstream side of a sudden enlargement of a canal cross-section.

The S2 drawdown profile (Fig. 2.2) occurs in a steep slope channel when the depth of flow is more than the normal depth but less than the critical depth. It rather acts like a transition between a hydraulic drop and uniform flow since it starts upstream with a vertical slope at the critical depth and is tangent to the normal depth line at the downstream end. Examples of the S2 profiles are the profiles found on the downstream side of an enlargement of channel section and on the steep-slope side as the channel slope changes from steep to steeper.

The S3 backwater profile (Fig. 2.2) is created on a steep slope channel when the depth of flow is less than the normal depth. It also acts like a transition between an issuing supercritical flow at the upstream and the normal depth to which the profile is tangent at the downstream. Practical examples of the S3 profiles are the profile below of a sluice gate in a steep slope channel and the profile on the steep-slope side as the channel slope changes from steep to mild.

4.2.2 Data requirements

In computing the length of the flow profile using Eq. (3.9), the following information are generally needed:

1. The discharge Q for which the flow profile is derived.
2. The depths of flow y_0 at the initial or control section.
3. The channel shape of the different channel sections.
4. The longitudinal slope of the channel S_0.
5. The Manning's roughness co-efficient n or Chezy's C.
6. The energy coefficient α

When the above data are available, the flow profile can be computed accurately using the methods like the direct integration method, direct step method etc. This is the case here also, i.e. when the above data are available, Eq. (3.9) can be used to compute the flow profile in a channel of any shape, and it can be done in the following two ways:

i. by computing the length of the profile from given depth (L from y)

ii. by computing the depth of flow for a specified length (y from L).

i. Computation of profile length from given depth

The following procedure is to be adopted for computing the length between two sections (hereinafter called the initial and the end sections) when the end depths are known or assumed:

1. From the given discharge, channel section and roughness coefficient, compute the normal depth by using the Manning or the Chezy formula.
2. Compute the hydraulic exponent for uniform flow computation, N, based on Manning or Chezy formula, and y_n.
3. Compute the Froude number Fr corresponding to y_n.
4. Compute η/η_0 from the known assumed depths at the initial and the end sections.
5. Compute the length of the flow profile using Eq. (3.9).

The procedure is explicit and does not involve any trial. In this case the length of the flow profile is computed from given depth as is done by the graphical integration, direct integration and direct step methods (Chow, 1959; Henderson, 1966).

ii. Computation of depth from given profile length

The following steps are to be followed to compute the depth of flow (y) at the end section when the distance L between the initial and the end sections are known.

1. Compute the normal depth y_n
2. Compute the hydraulic exponent for uniform flow computation N.
3. Compute the Froude number Fr.
4. Compute η/η_0 using Eq. (3.9).
5. Compute the depth of flow y (= $y_n + \eta$) from the computed values of y_n and η.

In this case, the depth of flow y is computed from given L as is done in the standard step method (Henderson, 1966; French 1986). However, the standard step method is implicit, i.e. it requires a trial-and-error procedure to compute y from given L, but use of Eq. (3.9) does not involve any trial. This is because, in the standard step method the depth at the end section has to be assumed to determine the geometric elements, whereas the quantities N and Fr in Eq. (3.9) are computed using the normal depth y_n.

4.2.3 Use of the formula with minimum data

As stated earlier, the hydraulic and river engineers often require quick but approximate estimate of the length of the channel reach to be affected by some proposed engineering works with minimum amount of data, since at the initial stage it is quite unlikely that all the information or data necessary for the computation of a flow profile will be available. The minimum data requirements for obtaining a rough estimate of the profile length using Eq. (3.9) are the flow depth (taken to be equal to the normal depth y_n), the bed slope S_o, the geometric elements of the channel section like area, wetted perimeter and top width and the Manning's n or Chezy's C. The geometric elements of the channel section and the channel roughness together with the flow depth are required for computing the Froude number Fr and the hydraulic exponent N. For an existing channel in which a structure is to be constructed or some form of channel amendments are to be made, the geometric elements and the channel roughness may be available or easily determined. But if the channel is not existing, the channel dimensions are to be assumed and the roughness coefficient is to be roughly selected for the material in which the channel is to be constructed.

For flows in rivers and irrigation canals, the usual flow profiles are M1 and M2 and the Froude number of the flow is usually small. Equation (3.9) can then be written as

$$L=\frac{y_n}{NS_o}\ln\frac{\eta}{\eta_o} \qquad (4.1)$$

However, when the channel slope is steep and the flow profiles is S_2 or S_3, the effect of Froude number has to be considered.

Regarding the numerical value of N, there is no problem if the channel is wide, deep or triangular, since the value of N constant for these channel sections. But for rectangular and trapezoidal sections the value of N has to be estimated.

4.3 Validity of the formula

To investigate the validity of the formula, the lengths of M1, M2, S2 and S3 profiles in wide and rectangular channels have been computed using Eq. (3.9) and compared with those computed by the direct integration method. The direct integration method is presented in Art. 2.5. The application of the direct integration method for computing flow profiles in open channels involves determination of the varied flow functions F(u,N) and F(v,J) given by Eqs. (2.21) and (2.23), respectively. Tables for these functions for different values of u and N and v and J, originally prepared by Chow (1955), are available in the text books of Open Channel Hydraulics (e.g. French (1986)). These tables give the values of the functions F(u, N) and F(v, J) for some fixed values of u and N and v and J. As a result, most of the problems of profile computation need interpolation of the tabulated values. When these functions vary considerably with the depth of flow, the interpolated values of the functions involve significant error. This is particularly true as $y \to y_n$, since the above functions vary considerably when the flow depth approaches the normal depth. When $y = y_n$ the above functions become infinity.

The problem has been solved by evaluating the functions numerically using Romberg integration (Churchhouse, 1981). A FORTRAN program has been developed to compute the functions F(u,N) and F(v,J) for any values of u and N and v and J.

In Tables 4.1 and 4.2 the difference between the lengths of the flow profiles computed using Eq. (3.9) and the direct integration method, expressed as a percentage of the latter, is presented. In these tables a positive error indicates overestimation of the actual profile length and a negative error indicates underestimation. The error in profile length computed by Eq. (3.9) is found to depend on (i) the Froude number Fr of the flow, (ii) the ratio of perturbed depth to normal depth at the initial section y_0/y_n, and (iii) the ratio η/η_0.

To determine the individual effect of Fr, y_0/y_n and η/η_0, the numerical value of one of these parameters has been varied while keeping the numerical values of the other two fixed. The computation has been performed for (i) Fr varying from 0.1 to 0.5 for the M1 and M2 subcritical flow profiles and from 1.5 to 7.0 for S2 and S3 supercritical flow profiles, (ii) $|1-y_0/y_n|$ varying from 0.10 to 0.50, and (iii) η/η_0 equal to 0.01 and 0.1. The computations are based on Manning's roughness coefficient n = 0.025.

4.3.1 Effect of initial perturbation

From the results presented in Tables 4.1 and 4.2, it is readily seen that the initial perturbation, expressed in terms of y_0/y_n, has the most dominant influence on the accuracy of the computed profile length. In deriving Eq. (3.9), the function $f(y_n+\eta)$ has been expanded in a Taylor series about $f(y_n)$ and the terms higher than the first order terms have been omitted. The omission of the higher order terms has the effect of decreasing the lengths of M1 and S2 profiles and increasing the lengths of M2 and S3 profiles. For all the profiles, the magnitude of the error in the computed profile length increases systematically with an increase in $|1 - y_o/y_n|$. The errors are more for the M1 and M2 profiles and relatively less for S2 and S3 profiles. The derived formula, Eq. (3.9), seems to be applicable for small initial perturbation. For the estimation of the profile lengths using the derived formula, the initial to normal depth ratio y_o/y_n should be less than 1.1 for M1 and S2 profiles and greater than 0.9 for M2 and S3 profiles, i.e. $|1-y_o/y_n|$ should be less than 0.1.

4.3.2 Effect of Froude number

The accuracy with which the profile lengths can be computed using Eq. (3.9) also depends on the Froude number of the flow. From the results presented in Tables 4.1 and 4.2 it is seen that the magnitude of the error in the computed profile length is maximum when $Fr \rightarrow 1$. The error decreases when the numerical value of Fr departs from unity, i.e. decreases for the M1 and M2 profiles and increases for the S2 and S3 profiles. When $Fr = 1$, the flow is critical and from Eq. (1.1), $dy/dx = \infty$ which means that the profile become vertical. In this case, the flow becomes rapidly varied with substantial curvative of the streamlines and the theory of gradually varied flow becomes inapplicable. This is the reason why the error in profile computation is maximum when $Fr = 1$. Therefore, for an accurate computation of the profile length using Eq. (3.9), the Froude number of the flow should be as low as possible for the M1 and M2 profiles and as high as possible for the S2 and S3 profiles.

4.3.3 Effect of the ratio η/η_0

From the results presented in Tables 4.1 and 4.2, it is readily seen that the magnitude of error in the computed profile length decreases when the ratio η/η_0 increases. This means that the error is minimized when a larger profile length is computed. The desired formula, Eq. (3.9), represents the equation of a curve which decays exponentially with the distance along the channel. The rate of decay is high initially, but with an increase in distance, the rate decreases and finally the curve approaches a constant value asymptotically. The results presented in Tables 4.1 and 4.2 seem to indicate that the actual profile initially does not decay exponentially, but at a rate lower than this. This is possibly the reason why the error is large for η/η_0 = 0.1 and small for η/η_0 = 0.01.

4.3.4 Effect of channel shape

In this study the profile lengths have been computed in wide and rectangular channels only. Other sections have not been considered because of the time constraint since the computation of flow profiles for different values of y_o/y_n, η/η_0 and Fr involve a considerable volume of work and hence requires considerable amount of time. From the results presented in Tables 4.1 and 4.2 it is seen that the channel shape does not seem to have any noticeable effect on the computed profile length of M1 and M2 profiles. However, the effect of channel shape on the computed length of S2 and S3 profiles seem to be considerable. The magnitude of error in the computed profile length is more for a rectangular channel than for a wide channel.

Thus, the results presented in Tables 4.1 and 4.2 seem to indicate that the present formula is valid, as expected, for a small perturbation in depth above or below the normal depth line and when the Froude number of the flow is small for subcritical flow profiles and large for supercritical flow profiles. The initial perturbation, expressed in terms of y_0/y_n, should be less than 1.1 for M1 and S2 profiles and greater than 0.9 for M2 and S3 profiles. The Froude number of the flow should be small (Fr<0.3) for M1 and M2 profiles and large (Fr>2) for S2 and S3 profiles. To obtain better accuracy, a longer profile, preferably with η/η_0 = 0.01, should be computed.

Table 4.1 Percentage of error in computing the profile length by the derived formula as compared to the direct integration method for $\eta/\eta_0 = 0.01$

A. (i) M1 profile in wide channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	1.1	1.2	1.3	1.4	1.5
0.1	-4.33	-8.31	-12.09	-15.61	-18.99
0.2	-4.36	-8.57	-12.34	-15.95	-19.41
0.3	-4.62	-8.94	-13.00	-16.76	-20.35
0.4	-5.02	-9.67	-14.01	-18.00	-21.78
0.5	-5.62	-10.80	-15.56	-19.88	-23.93

- ve indicates underestimation by the derived formula

(ii) M1 profile in rectangular channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	1.1	1.2	1.3	1.4	1.5
0.1	-4.38	-8.58	-12.49	-16.14	-19.26
0.2	-4.40	-8.59	-12.59	-16.19	-19.38
0.3	-4.58	-8.93	-12.82	-16.43	-19.43
0.4	-5.00	-9.79	-13.70	-18.20	-21.73
0.5	-6.40	-11.69	-16.54	-20.97	-25.03

- ve indicates underestimation by the derived formula

B. (i) M2 profile in wide channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	0.9	0.8	0.7	0.6	0.5
0.1	+4.39	+9.03	+14.01	+19.02	+24.20
0.2	+4.57	+9.48	+14.69	+19.95	+25.40
0.3	+4.86	+10.12	+15.73	+21.45	+27.36
0.4	+5.34	+11.15	+17.43	+23.95	+30.77
0.5	+6.06	+12.87	+20.15	+27.91	+36.24

+ ve indicates overestimation by the derived formula

(ii) M2 profile in rectangular channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	0.9	0.8	0.7	0.6	0.5
0.1	+4.40	+9.70	+14.76	+20.15	+25.83
0.2	+4.55	+9.78	+14.83	+20.31	+26.41
0.3	+4.79	+10.36	+15.64	+16.03	+28.93
0.4	+5.01	+11.73	+17.54	+25.34	+33.51
0.5	+6.45	+13.74	+19.54	+29.94	+40.52

+ ve indicates overestimation by the derived formula

C. (i) S2 profile in wide channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	1.1	1.2	1.3	1.4	1.5
1.5	+4.52	+9.56	+15.19	+21.17	+28.61
2.0	+2.08	+4.35	+6.69	+7.12	+11.73
3.0	+1.18	+2.18	+3.29	+4.47	+5.55
5.0	+0.79	+1.31	+2.00	+2.69	+3.29
7.0	+0.67	+1.15	+1.66	+2.24	+2.88

+ ve indicates overestimation by the derived formula

(ii) S2 profile in rectangulr channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	1.1	1.2	1.3	1.4	1.5
1.5	+8.74	+17.48	+29.75	+38.02	+54.7
2.0	+5.69	+10.35	+15.71	+21.13	+26.68
3.0	+4.43	+6.19	+10.88	+14.24	+18.14
5.0	+3.01	+5.55	+8.76	+12.49	+15.48
7.0	+2.15	+4.71	+7.43	+10.40	+14.58

+ ve indicates overestimation by the derived formula

D. (i) S3 profile in wide channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	-0.9	0.8	0.7	0.6	0.5
1.5	-4.11	-7.72	-10.95	-13.88	-17.2
2.0	-2.05	-3.89	-5.57	-7.22	-8.86
3.0	-1.10	-2.00	-2.99	-4.02	-4.93
5.0	-0.74	-1.28	-1.92	-2.56	-3.28
7.0	-0.62	-1.04	-1.62	-2.16	-2.83

- ve indicates underestimation by the derived formula

(ii) S3 profile in rectangular channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	0.9	0.8	0.7	0.6	0.5
1.5	-5.9	-6.76	-19.14	-22.08	-29.89
2.0	-4.02	-8.28	-11.31	-14.45	-17.97
3.0	-2.90	-4.3	-8.70	-12.70	-15.40
5.0	-1.62	-3.19	-8.21	-11.11	-13.95
7.0	-1.50	-2.15	-7.08	-10.16	-12.43

- ve indicates underestimation by the derived formula

Table 4.2 Percentage of error in computing the profile length by the derived formula as compared to the direct integration method for $\eta/\eta_0 = 0.1$

A. (i) M1 profile in wide channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	1.1	1.2	1.3	1.4	1.5
0.1	-7.49	-14.1	-20.08	-25.29	-29.98
0.2	-7.65	-14.47	-20.60	-25.91	-30.68
0.3	-8.14	-15.31	-21.69	-27.19	-32.09
0.4	-8.91	-16.62	-23.37	-29.12	-34.20
0.5	-10.08	-18.58	-25.85	31.94	-37.24

- ve indicates underestimation by the derived formula

(ii) M1 profile in rectangular channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	1.1	1.2	1.3	1.4	1.5
0.1	-7.77	-14.62	-20.71	-26.06	-30.31
0.2	-7.82	-14.69	-20.82	-26.31	-30.47
0.3	-8.15	-15.06	-21.20	-26.38	-31.07
0.4	-9.02	-16.47	-23.26	-28.59	-32.9
0.5	-10.81	-19.31	-26.49	-34.82	-37.75

- ve indicates underestimation by the derived formula

B. (i) M2 profile in wide channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	0.9	0.8	0.7	0.6	0.5
0.1	+8.25	+17.68	+28.68	+40.24	+54.44
0.2	+8.73	+18.65	+30.32	+43.16	+58.11
0.3	+9.41	+20.24	+33.19	+47.74	+65.05
0.4	+10.48	+22.85	+31.99	+55.63	+77.39
0.5	+12.19	+27.10	+46.19	+69.75	+100.76

+ ve indicates overestimation by the derived formula

(ii) M2 profile in rectangulr channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	0.9	0.8	0.7	0.6	0.5
0.1	+8.81	+18.93	+29.83	+43.55	+58.21
0.2	+8.85	+18.96	+31.04	+44.06	+60.45
0.4	+9.20	+19.91	+32.40	+49.54	+67.25
0.4	+10.20	+23.36	+40.01	+57.00	+69.24
0.5	+12.63	+28.25	+52.1	+79.5	+122.6

+ ve indicates overestimation by the derived formula

C. (i) S2 profile in wide channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	1.1	1.2	1.3	1.4	1.5
1.0	+12.11	+25.35	+43.31	+68.03	+102.86
2.0	+6.51	+12.12	+19.02	+27.16	+36.30
3.0	+3.48	+7.08	+10.66	+14.70	+18.89
5.0	+2.58	+5.25	+7.60	+10.36	+13.08
7.0	+2.38	+4.72	+6.90	+9.36	+11.81

+ ve indicates overestimation by the derived formula

(ii) S2 profile in rectangular channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	1.1	1.2	1.3	1.4	1.5
1.5	+14.98	+36.60	+59.63	+111.48	+178.25
2.0	+10.56	+19.01	+31.31	+44.77	+59.79
3.0	+7.50	+15.92	+20.53	+27.57	+37.21
5.0	+6.65	+13.69	+15.96	+24.17	+30.33
7.0	+5.75	+12.07	+14.51	+23.60	+29.46

+ ve indicates overestimation by the derived formula

D. (i) S3 profile in wide channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	0.9	0.8	0.7	0.6	0.5
1.5	-9.08	-16.56	-22.69	-28.01	-32.35
2.0	-5.13	-9.78	-13.81	-17.62	-20.83
3.0	-3.27	-6.38	-9.14	-11.96	-14.32
5.0	-2.53	-4.97	-7.16	-9.49	-11.43
7.0	-2.30	-4.58	-6.62	-8.83	-10.65

- ve indicates underestimation by the derived formula

(ii) S3 profile in rectangulr channel

Froude No. (Fr)	For depth ratio (y/y_n)				
	0.9	0.8	0.7	0.6	0.5
1.5	-8.41	-15.10	-30.85	-33.82	-44.48
2.0	-5.08	-11.09	-21.17	-26.83	-29.88
3.0	-4.19	-9.30	-15.61	-21.61	-26.85
5.0	-3.40	-8.01	-14.51	-19.73	-25.31
7.0	-2.15	-7.43	-13.79	-17.37	-23.59

- ve indicates underestimation by the derived formula

4.4 Validity of the de Vries (1994) formula

One of the objectives of the present study is to compare between the profile lengths computed by the general formula and the de Vries formula in wide channels. It was shown in Art. 3.4 that the present formula for computing profile length in a wide channel is almost identical to the de Vries formula when the Froude number of the flow and the perturbation in depth are both small. At this point it is to be mentioned here that the de Vries formula is intended to compute the profile length in rivers only. Since rivers are mild-sloped channels, the expected flow profiles in rivers are M1 and M2.

The validity of the derived equation for computing the lengths of M1 and M2 flow profiles in a wide channel (e.g. a river) has been considered in Art. 4.3. It was shown that the general formula is satisfactory for computing the lengths of M1 and M2 profile when the perturbation in depth above or below the normal depth line is less than 10% and the Froude number of the flow is low (Fr<0.3).

Combining Eqs. (2.43) and 2.45), the de Vries formula can be written as

$$L_{1/2}=0.24\frac{y_n}{S_o}\left(\frac{y_o}{y_n}\right)^{1.33} \tag{4.2}$$

The difference between the profile lengths computed by the above formula and the actual profile lengths computed by Eq. (2.36), expressed as a percentage of the latter, is given in Table 4.3. It is readily seen that for small Froude numbers the de Vries formula seems to predict the actual profile length quite satisfactorily for any perturbation in depth above or below the normal depth line, i.e. for any value of y_0/y_n. The reason is quite obvious, since this formula has been derived empirically by a curve fitting procedure. This formula can be used when Fr ≈ 0.1 involving an error less than 10% for any value of y_0/y_n. For higher values of Fr, this formula overestimates the actual profile length and the magnitude of the error increases with an increase in the numerical value of the Froude number. It is to be noted here that the effect of increasing the numerical value of Fr is to decrease the profile length according to Eq. (2.36) or (3.9).

Table 4.3 Percent difference between the profile lengths computed by the de Vries formula and the Bresse method

For depth ratio (y_0/y_n)	For Froude No. (Fr)					
	0.0	0.1	0.2	0.3	0.4	0.5
0.5	+5.50	+10.46	+25.0	+61.01	+171.4	+1800
0.6	+7.00	+11.00	+21.00	+44.04	+95.10	+255.80
0.7	+7.19	+9.50	+17.30	+33.00	+61.95	+125.70
0.8	+5.90	+7.80	+13.30	+24.40	+42.40	+76.20
0.9	+4.50	+6.10	+10.00	+18.18	+30.80	+52.90
1.01	+3.40	+4.75	+8.00	+13.55	+22.72	+37.28
1.1	+2.64	+3.81	+6.25	+11.00	+18.26	+28.90
1.2	+1.66	+2.68	+4.79	+8.51	+14.17	+22.40
1.3	+0.59	+1.19	+3.00	+5.90	+10.74	+16.83
1.4	+0.26	+0.26	+1.90	+4.16	+8.06	+12.95
1.6	-1.5	-1.10	+0.00	+1.58	+4.18	+7.69
2.0	-3.21	-2.89	-1.95	-1.30	+0.00	+1.85
5.0	-0.24	+0.245	+0.29	+0.39	+0.49	+0.64

+ ve indicates overestimation by the de Vries formula

- ve indicates underestimation by the de Vries formula

4.4.1 Possible extension of the de Vries formula

The de Vries formula seems to be quite capable of predicting the actual profile length with a reasonable degree of accuracy for flows with small Froude numbers. The predictive capacity of this formula is improved if this formula is extended to include the effect of the Froude number. The possible extension of the formula is indicated by the form of Eq. (3.9), i.e. to multiply the right hand side of this formula by $(1-Fr^2)$. Equation (4.2) then becomes

$$L_{1/2}=0.24\frac{y_n}{S_o}\left(\frac{y_o}{y_n}\right)^{1.33}(1-Fr^2) \tag{4.3}$$

When Fr = 0, Eq. (4.3) reduces to the original equation, Eq. (4.2). Equation (4.3) predicts the actual profile lengths with reasonable accuracy for Fr<0.30 for $0.9 \le y_0/y_n \le 5.00$ as shown in Table 4.4. It is to be mentioned here that the Froude number of river flow generally varies from 0.1 to 0.3.

Table 4.4 Percent difference between the profile lengths computed by the modified de vries formula Eq. (4.3) and the Bresse method

For depth ratio (y_0/y_n)	For Froude No. (Fr)					
	0.0	0.1	0.2	0.3	0.4	0.5
0.50	+5.50	+9.36	+20.00	+45.76	+128	+132.40
0.60	+7.00	+9.17	+16.00	+30.95	+63.87	76.16
0.70	+7.19	+8.08	+17.30	+20.53	+35.86	68.18
0.80	+5.90	+7.80	+12.59	+12.58	+19.20	32.17
0.90	+4.50	+6.66	+8.28	+7.38	+9.43	+14.70
1.01	+3.40	+3.44	+3.55	+3.27	+3.00	+2.82
1.10	+2.64	+2.67	+1.95	+0.80	-0.86	-3.31
1.20	+1.66	+1.34	+0.34	-1.41	-4.10	-8.40
1.30	+0.59	+0.00	-1.21	-3.73	-7.16	-12.37
1.40	-0.26	-0.80	-2.17	-4.16	-9.22	-15.36
1.60	-1.53	-2.20	-4.01	-7.70	-12.55	-19.23
2.00	-3.21	-4.02	-6.01	-10.31	-16.08	-23.64
5.00	-0.25	-0.73	-3.68	-8.61	-15.56	-24.51

+ ve indicates overestimation by the de Vries formula

- ve indicates underestimation by the de Vries formula

4.5 Practical Use of the Formula

The simple estimate of the profile length, provided by Eq. (3.19), can be used for many practical circumstances as presented below. Some of these applications have been identified by Samuels (1989).

i. Upstream influence of a proposed work in a river

When any work on a river channel is proposed or planned, it is of paramount importance to identify the reach of the channel that may be affected by the influence of the proposed work. The extent of such a change is obviously the length of the flow profile upstream of the proposed scheme.

ii. Influence of control structures on a canal

The control structures like sluice gates, broad-crested weirs, flumes etc., which are usually provided in an irrigation canal, obviously affect the canal reach for some distance upstream from their locations. The channel reach under the influence of a control structure is obviously the length of the flow profile created by the structure.

iii. Locating hydraulic model boundary

If the impact of a river engineering project requires the use of a hydraulic model (either a scale or physical model or a computational or mathematical model), one question that usually arises is where to locate the upstream or downstream boundary. One criterion for selecting a boundary location is that any error in estimating water levels at that point should not materially affect the study area. The errors are simply the perturbations to the actual water surface profile, and hence, the appropriate distance between the project area and the model boundary is the backwater length for the river.

iv. Spacing of cross-sections in a computational model

In a computational or mathematical model, the spacing between the cross-sections should be such that the backwater effect due to a structure can be reproduced accurately. This means that the backwater profile from any control covers a number of cross-sections. Experience shows that the number of sections in a backwater profile should at least be five. The spacing between the cross-sections can easily be estimated using Eq. (3.9).

v. Location of gauging stations and analysis of gauging station records

Confluences are generally present in the upper reaches of rivers. At the confluence the strong backwater effect can be found due to difference between two river flows. The stage measuring or gauging stations have to be selected carefully due to backwater effect from a confluence.The backwater length indicates the distance which a gauging station on a tributary should be sited upstream of confluence of the tributary with the primary river,if it is required that the gauging station should not be influenced by backwater. If it is not feasible, a slope area method of gauging should be considered, or alternatively, the gauging station records are to be corrected for the backwater effect which can approximately be done by using Eq. (3.9).

CHAPTER 5

BACKWATER LENGTHS IN SOME RIVERS OF BANGLADESH

5.1 General

In the context of river Engineering backwater length is an important factor, because it is often necessary to investigate the probable damage caused by backwater due to an obstacle in a stream. Backwater effect is produced by the construction of structures like dam, barrage, sluice gate etc. in a stream. Strong backwater effect can also be produced upstream of a confluence. When a dam or barrage is constructed across a river the pool surface will be further raised for a long distance upstream from the pool. As a result, the river would drop a part of its sediment load, resulting in a rise in its bed level with a consequent rise in water level and increase in afflux. This would cause an increased length of backwater profile immediately upstream of the structure.

In this chapter the derived formula is applied to compute the backwater length in some rivers of Bangladesh.

5.2 Representative discharge of a river

The wide variation of river flow makes it difficult to choose a representative discharge in studying the river characteristics. Different methods have been proposed by different investigators at various times for the choice of representative discharge. This representative discharge is also referred to as dominant discharge in the literature and may be defined as that hypothetical discharge which would produce the same result in terms of average channel dimensions as the actual varying discharge.

Inglis (1947) assumed that the dominant discharge corresponds to the bankful stage and it can be used for any parameter such as width, depth, slope etc. Inglis (1968) suggested that for Indian rivers the dominant discharge was nearly the same as the bankful discharge which may be taken to be one-half to two-thirds of the maximum discharge. Nixon (1959) also preferred to use bankful discharge for studying the geometry of the alluvial channels in England and Wales.

In this study, the backwater lengths in rivers are computed based on the bankful depth as suggested by Samuels (1989). The bankful depth can be obtained if the bankful discharge is known or determined.

Following Nixon (1959), the bankful discharge may be defined as follows:

"A bankful flood of a river is the highest flood which can be contained within the incised channel of the river and which does not cause spilling of water on to the river's flood plain or washlands. The bankful discharge is the maximum discharge reached by a bankful flood at any point in the course of the river".

5.3 The rivers selected

In the present study, the backwater lengths in four rivers, namely (i) the Ganges at Hardinge Bridge, (ii) the Jamuna at Bahadurabad, (iii) the Meghna at Bhairab Bazar, and (iv) the Gorai at Gorai Railway Bridge, have been determined. The Padma, the Jamuna and the Meghna are the three major rivers of Bangladesh. The fourth river, i.e. the Gorai river, has been selected because of the availability of necessary data. The rivers selected and the locations of the stations are shown in Fig. 5.1.

5.4 Procedure

The length of backwater profile in the rivers has been computed using Eq. (3.9) assuming that N = 3.3, η/η_0 = 0.10 and Fr <<1. Under these simplifications Eq. (3.9) becomes

$$L=0.691\frac{y_n}{S_o} \quad (5.1)$$

where y_n to be used here is the bankful depth of the channel and S_o is the mean bed slope.

The bankful depth has been obtained by dividing the flow area with flow width at bankful condition. The bankful discharge is determined by two concepts, namely (1) The Inglis's concept

(Inglis, 1947), and (2) the flow duration concept (Nixon 1959; Hossain 1992). These two concepts are described below.

(1) Inglis's concept of bankful discharge: The concept utilizes the rating curve as the basis for determination of bankful discharge. The discharge which corresponds to bankful stage is directly read from the rating curve. Rating curves for a number of years are considered and an average value is adopted.

(2) Flow duration concept: According to this concept a curve with discharge on the ordinate against percent of time the discharge equalled or exceeded on the abscissa are drawn. The discharge that corresponds to about 25 percent of time is read and noted. Average discharge for a number of years may be used.

5.5 Data need and data collection

The following data are needed to obtain the bankful discharge:

(i) daily discharge

(ii) stage

(iii) width

(iv) cross-sectional area, and

(v) bankful stage

Data collected by Bangladesh Water Development Board were utilized in the present study. The bankful discharges for the Ganges at Hardinge Bridge and the Jamuna at Bahadurabad were taken from Hossain (1992). On the other hand, bankful discharge of the Meghna and the Gorai were determined in the study utilizing the data for the Meghna at Bairab Bazar between the years 1981-82 and 1988-89 and the Gorai at Gorai Railway Bridge between the years 1985-86 and 1991-92. The bankful stage for Bairab Bazar and Gorai railway bridge were also collected and utilized to obtain bankful discharge.

For the bankful depth, the cross-sectional area and width data of the Ganges at Hardinge Bridge between the years 1988 and 1992, the Gorai at Gorai Railway Bridge between the years 1986 and 1992, the Jamuna at Bahadurabad between the years 1988 and 1992 and the Meghna at Bairab Bazar between the years 1985 and 1989 were collected and these are utilized to obtain bankful depth for each of the stations.

5.6 Determination of bankful discharge and bankful depth for the rivers

Yearly rating curves are drawn for the years 1981 to 1989 at Bairab Bazar and shown in Fig. 5.2. The average value of bankful discharge for the years is considered. The discharge for almost all the years varied between 9600 m^3/s and 13900 m^3/s. An average value of the bankful discharge of 11650 m^3/s corresponding to bankful stage of 6.25 m PWD was obtained from Fig. 5.2.

Similarly, rating curve is drown for the Gorai at Gorai Railway Bridge and shown in Fig. 5.3. The average discharge of 4800

m^3/s corresponding to bankful stage of 12.19 m PWD was obtained from Fig. 5.3.

Flow duration curve for the Meghna at Bairab Bazar and the Gorai at Gorai railway bridge have been developed and shown in Figs. 5.4 and 5.5. From these figures the discharge that corresponds to 25 percent of time is found to be about 9900 m^3/s and 2600 m^3/s, respectively. The study also shows that the lowest and highest bankful discharges of the Meghna are 9900 m^3/s and 11650 m^3/s, the average discharge being 10500 m^3/s. Similarly, the average bankful discharge of the Gorai is 3700 m^3/s.

Thus the two methods produce a very close compliance in the value of bankful discharge, since the variation of estimated value from one method to another is not more than 10 percent. Therefore, the result obtained here may be dependable and average value is preferable.

The discharge versus average depth curve was drawn for the years from 1990-91 to 1994-95, 1988-89 to 1992-93, 1985-86 to 1991-92 and 1986-87 to 1991-92 at Hardinge Bridge, Bahadurabad, Bairab Bazar and Gorai Railway Bridge, respectively. These curves are shown in Figs. 5.6, 5.7, 5.8 and 5.9, respectively. From these curves the bankful depth that corresponds to bankful discharge are directly read. Thus the bankful depths for the respective stations were obtained and are presented in Table 5.1. On the other hand, the bankful depths are also computed by the Chezy uniform flow formula using the values of Chezy's C determined previously from an analysis of the streamflow data for the Ganges at Hardinge Bridge and the Jamuna at Bahadurabad (Ghosh, 1993; Rahman, 1996). The bankful depth of 8.95 m and 5.01 m for the two stations are obtained when the values of C are approximately 90 $m^{1/2}/s$ and 65 $m^{1/2}/s$, respectively. These values are well within the range of C determined by Ghosh (1993) and Rahman (1996). For the Meghna and the Gorai rivers, the values of C are not available. However, using the Chezy's formula the values of C for these two rivers have been determined using the bankful discharge, width, cross sectional area etc. at Bhairab Bazar and Gorai Railway Bridge. The values obtained are 47 $m^{1/2}/s$ and 83 m $m^{1/2}/s$ for the two stations, respectively. These values seem to be reasonable which in turn demonstrates that the normal depth predicted in this study is satisfactory.

Table 5.1 Bankful depth for the rivers

River Name	Gauge Station	Bankful discharge (m^3/s)	Bankful depth(m)
Ganges	Hardinge Bridge	30,500	8.95
Jamuna	Bahadurabad	38,000	5.01
Meghna	Bairab Bazar	10,500	16.0
Gorai	Gorai Railway Bridge	3,700	5.75

5.7 Determination of backwater length for the rivers

Using the bankful depth from Table 5.1 and the river slope as given in Table 5.2, the backwater length was determined using Eq.

5.1 and is shown in Table 5.2. From Table 5.2, it is readily seen that the backwater length can very widely from river to river due to river slope and bankful depth.Since the slopes of the alluvial rivers of Bangladesh are very small , the backwater effect is found to propagate considerable distance upstream.

Table 5.2 Profile length for the rivers

River	Gauge station	Slope (S_o)	Backwater length(L)
Ganges	Hardinge Bridge	1:21000	129.87 km
Jamuna	Bahadurabad	1:22000	76.16 km
Meghna	Bairab Bazar	1:50500	558.32 km
Gorai	Gorai Railway Bridge	1:23000	91.38 km

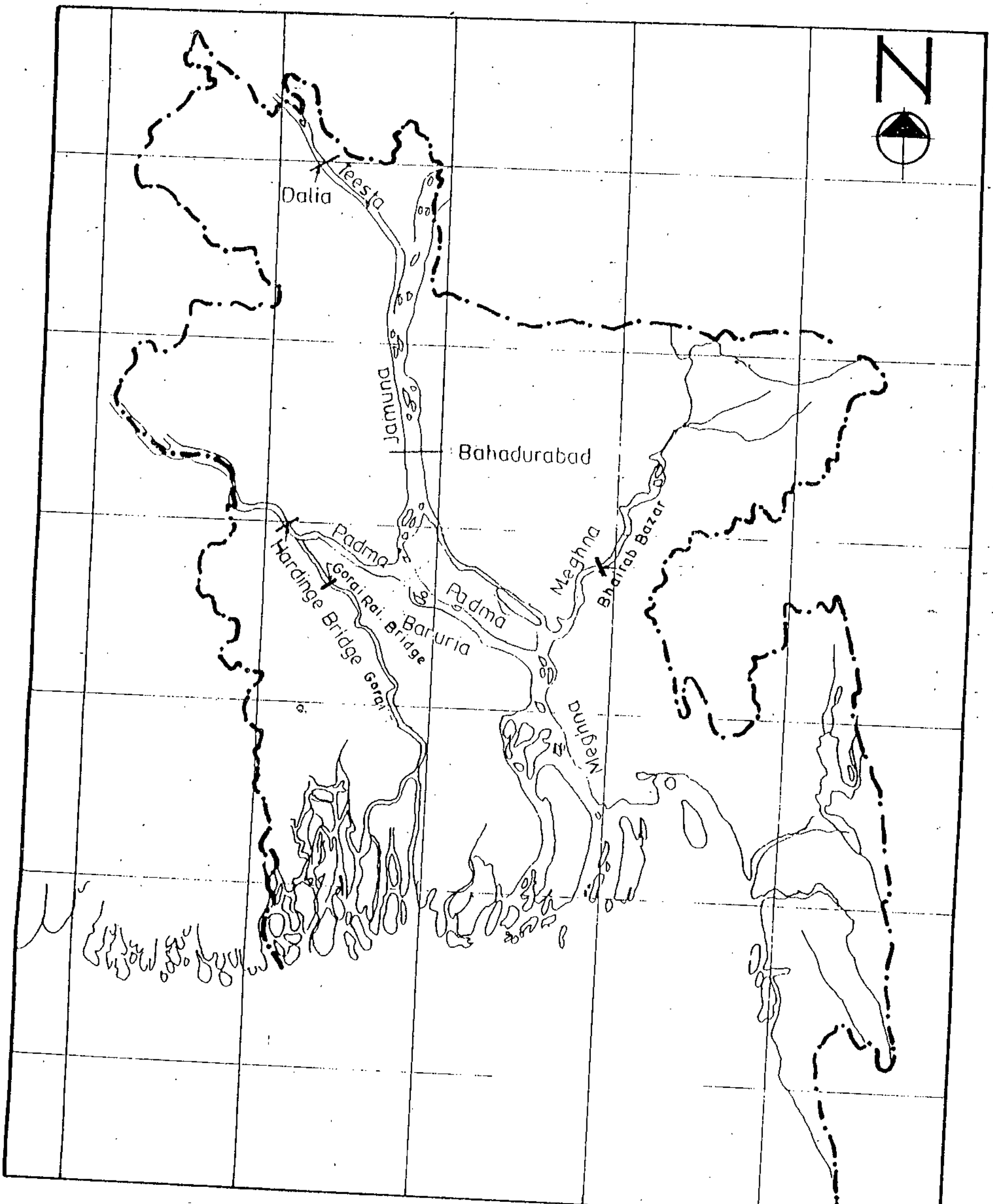

Fig.5.1 Index map of Bangladesh showing the rivers and the locations of the stations.

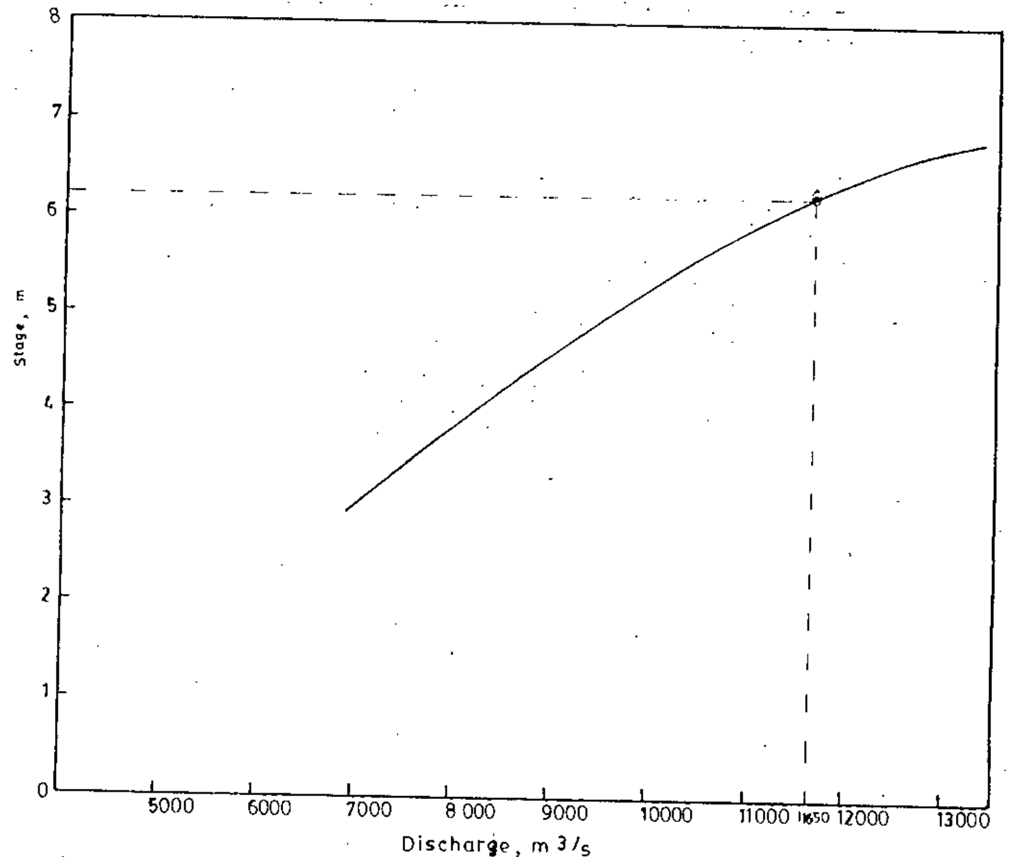

Fig. 5. 2 Rating-curve for the Meghna at Bhairab Bazar (1981-82 to 1988-89)

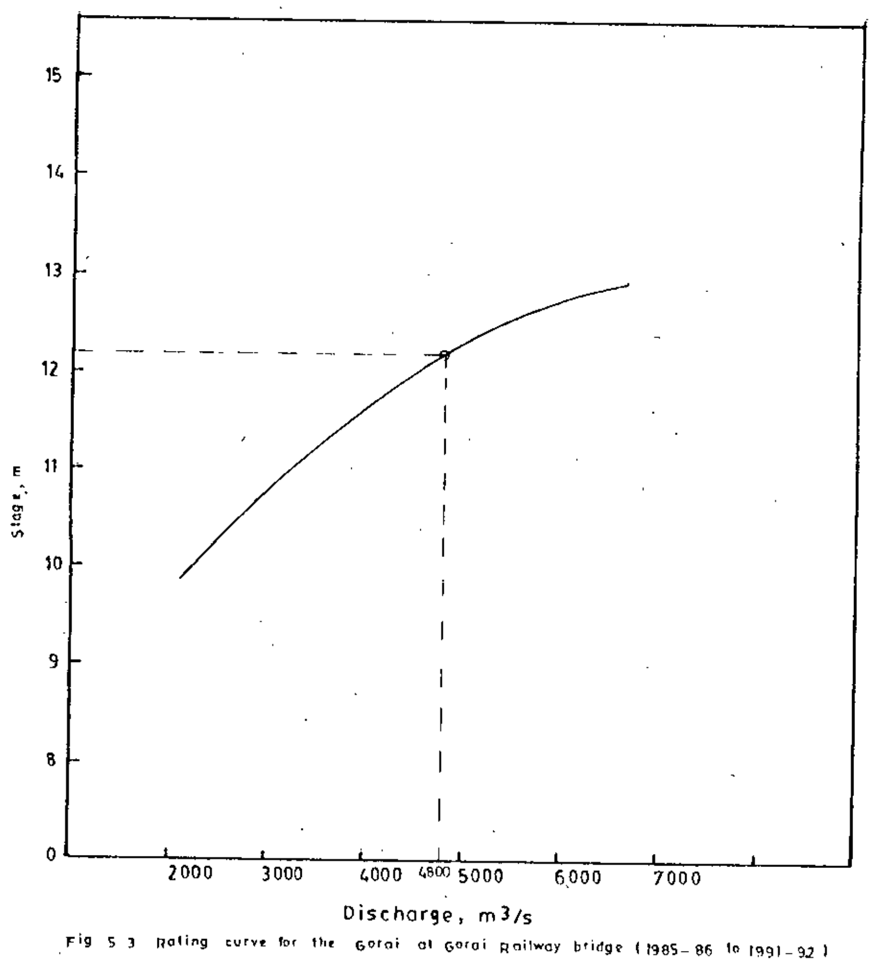

Fig 5 3 Rating curve for the Gorai at Gorai Railway bridge (1985-86 to 1991-92)

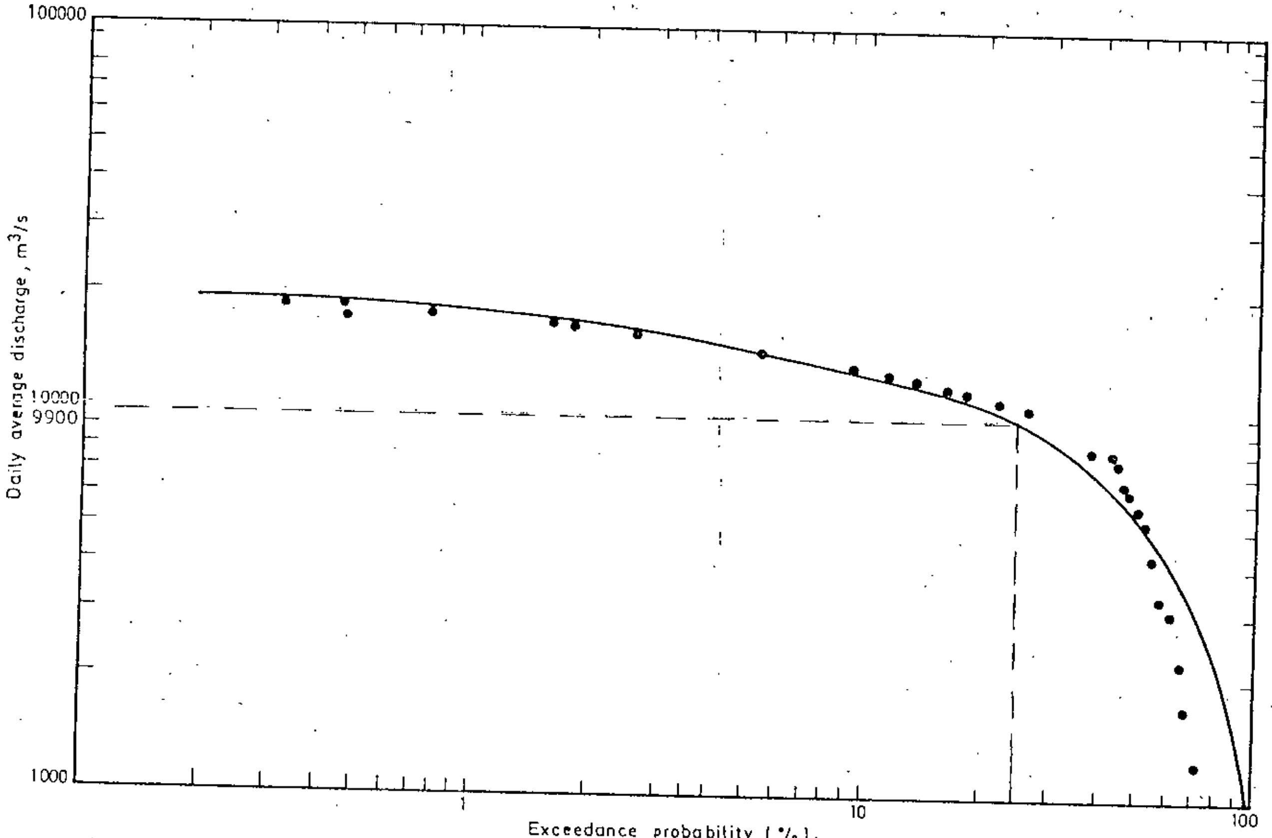

Fig 5.4 Flow–duration curve for the Meghna at Bhairab Bazar (1981–82 to 1988–89)

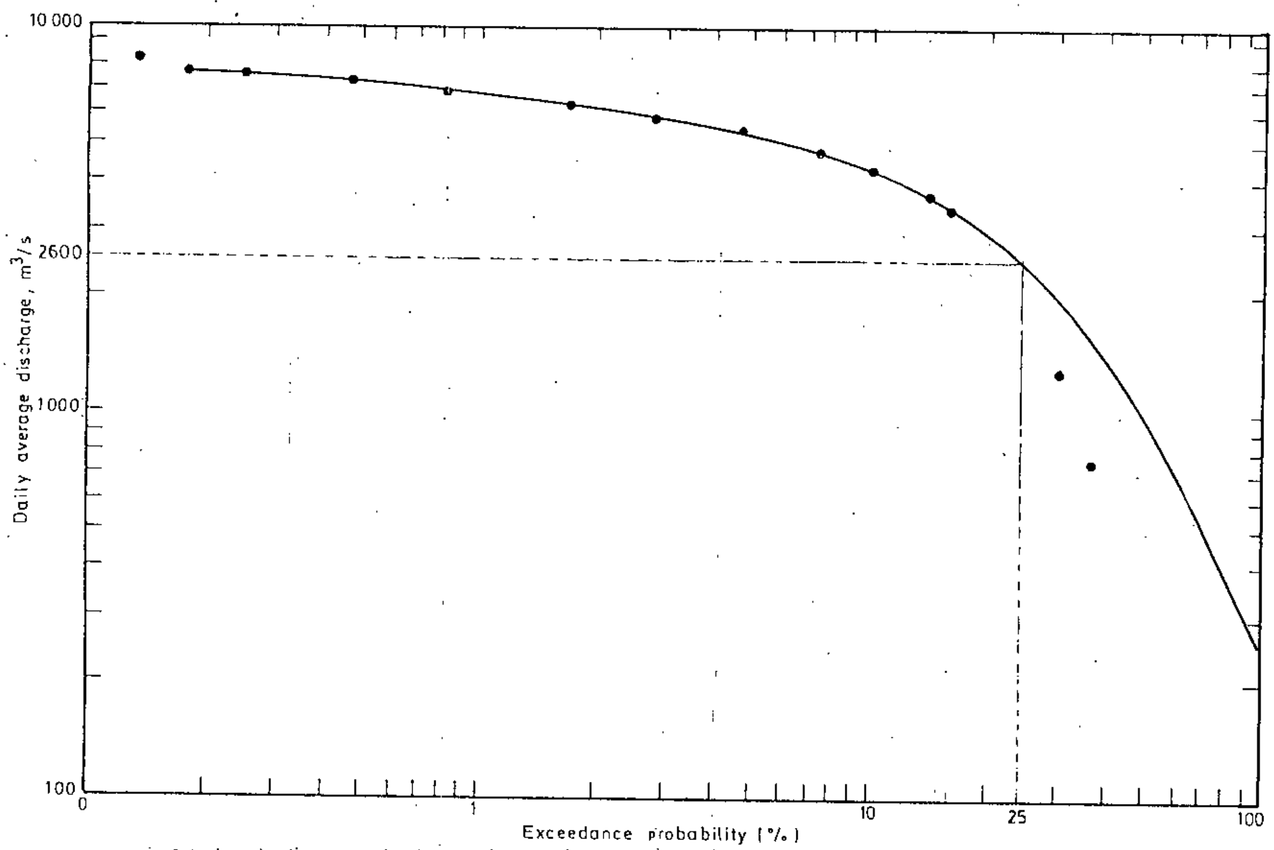

Fig. 5.5 Flow–duration curve for the Gorai at Gorai Railway Bridge (1984–85 to 1992–93)

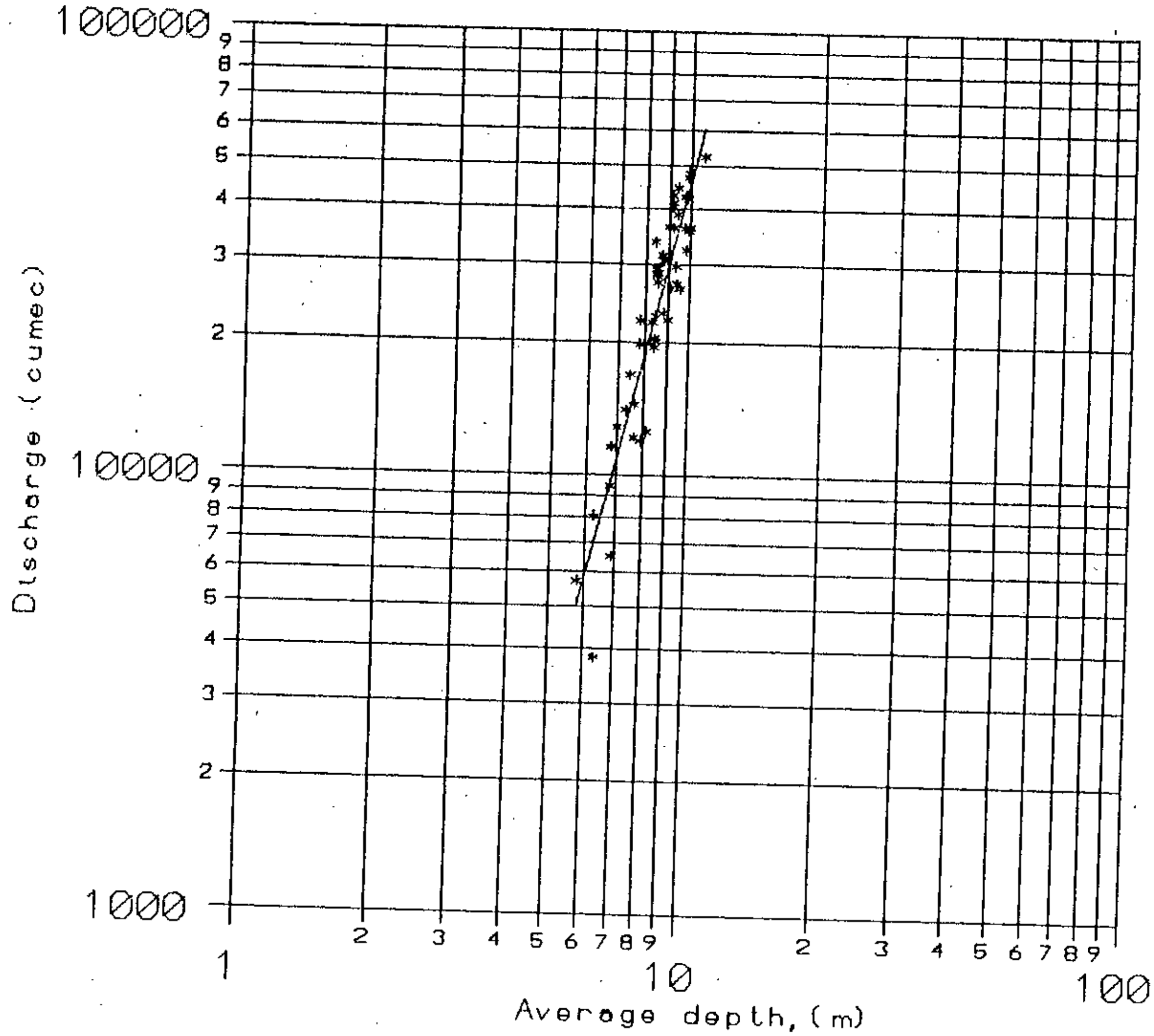

Fig. 5.6 Discharge - average depth curve for the Ganges at Hardinge Bridge

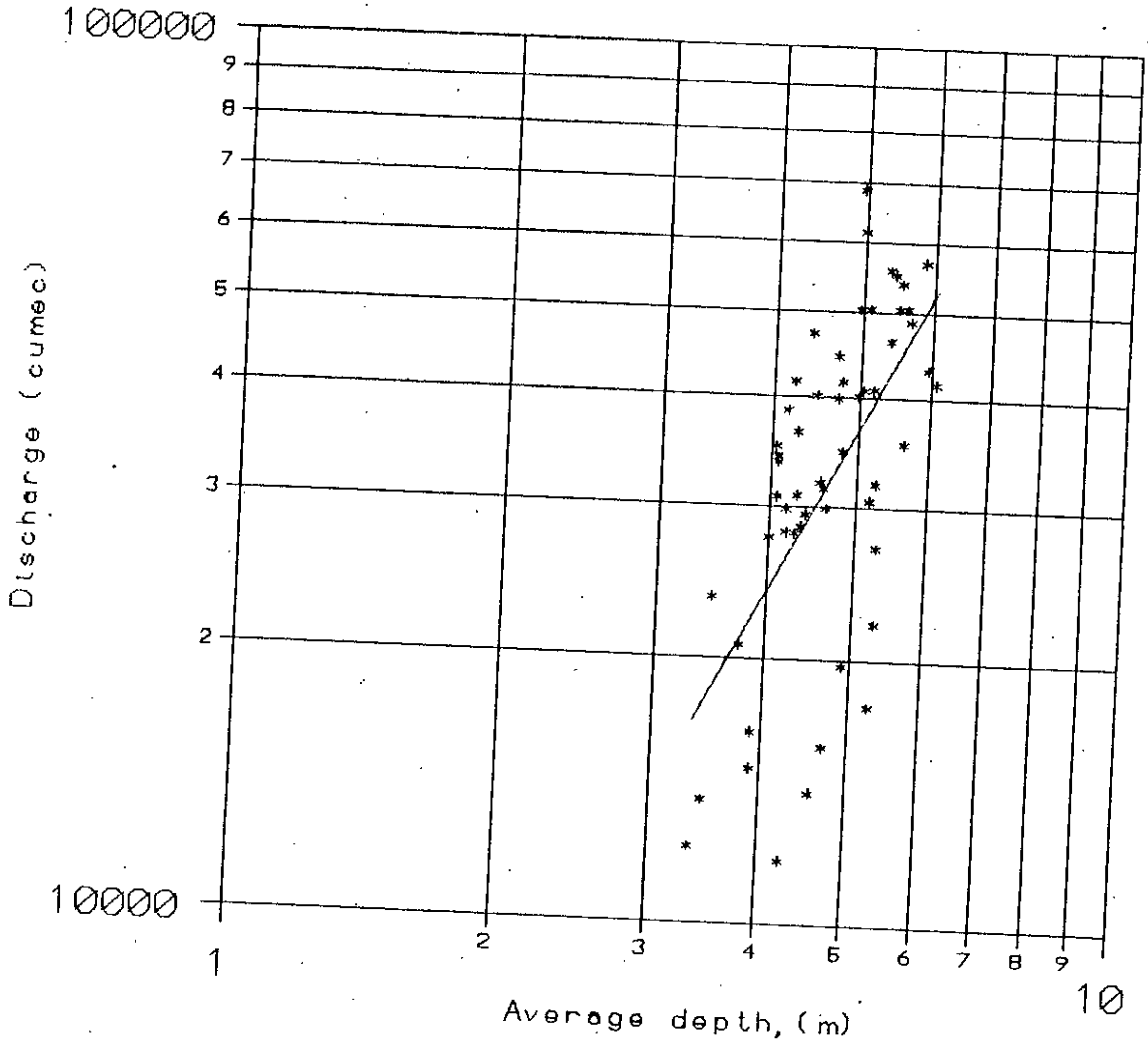

Fig. 5.7 Discharge - average depth curve for the Jamuna at Bahadurabad

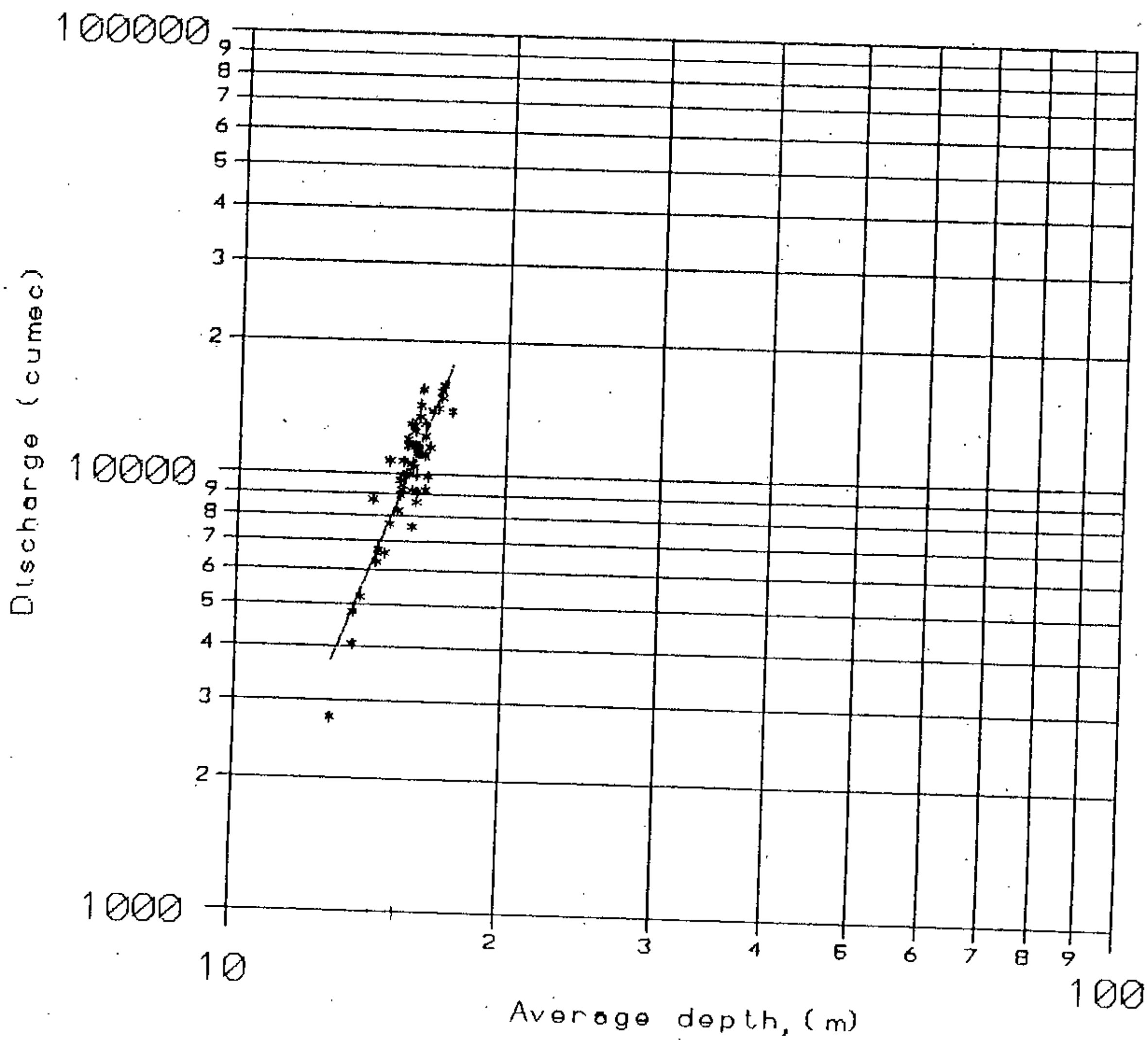

Fig. 5.8 Discharge - average depth curve for the Meghna at Bhairab Bazar

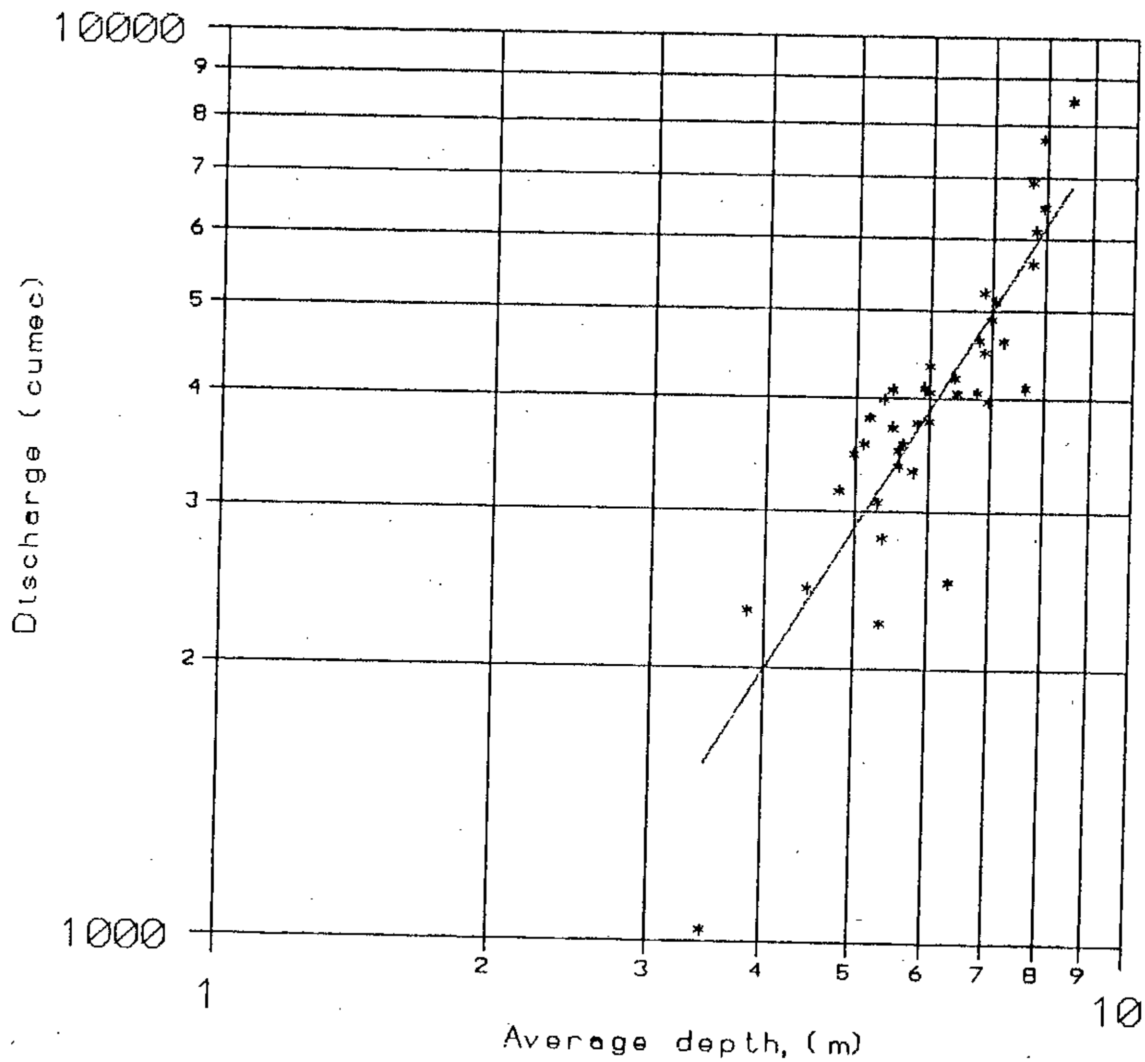

Fig. 5.9 Discharge - average depth curve for the Gorai at Gorai Railway Bridge

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The following conclusions could be drawn from the present study on approximate lengths of some steady gradually varied flow profiles:

1. The length of the flow profile depends on the depth of flow, the channel slope, the channel shape, the initial perturbation in depth, the ratio η/η_0 and the Froude number of the flow.
2. The derived formula, Eq. (3.9), seen to be satisfactory for computing the lengths of M1, M2, S2 and S3 flow profile in channels of any arbitrary shape for a small perturbation in depth above or below the normal depth line and when the Froude number of the flow is small for subcritical flow and large for supercritical flow.
3. The omission of the higher order terms in the Taylor series expansion of the dynamic equation of gradually varied flow has the effect of decreasing the lengths of M1 and S3 profiles and increasing the lengths of M2 and S2 profiles.
4. The initial perturbation in depth, expressed in terms of y_o/y_n, should be less than 1.1 for M1 and S2 profiles and greater than 0.9 for M2 and S2 profiles.
5. The Froude number of the flow should be small ($Fr \leq 0.3$) for M1 and M2 subcritical flow profiles and large ($Fr \geq 2$) for S2 and S3 supercritical flow profiles.
6. The accuracy in the computed profile length increases when a lower value of η/η_o is used. So for better accuracy a lower value of η/η_o, and preferably $\eta/\eta_o = 0.01$, should be used.
7. The de Vries (1994) formula seems to be quite satisfactory for predicting the profile length in wide channels (i.e. rivers) for small Froude numbers and for any perturbation in depth.

The predictive capacity of the de Vries formula is improved for low Froude no when the effect of the Froude number is included in the formula.

8. As the slopes of the alluvial rivers of Bangladesh are very small, the backwater lengths in these rivers are found to be quite considerable.

6.2 Recommendations

The present study has provided valuable information to plan and guide future studies relating to the lengths of gradually varied flow profiles. The following recommendations are made for further study:

1. The derived formula, Eq. (3.9), has been obtained by expanding $f(y_n+\eta)$ about $f(y_n)$ in a Taylor series and neglecting all the terms higher than the first order terms. As a result, the derived formula seem to be satisfactory for small perturbation in depth and when the Froude number of the flow is small ($Fr \leq 0.3$) for subcritical flow and large ($Fr \geq 2$) for supercritical flow. Attempts can be made to derive a formula retaining more terms in the Taylor series, at least upto the second order terms. In that case, the formula should be applicable for higher perturbation in depth and for a wider range of the Froude number.
2. The present study has been undertaken for wide and rectangular channels only. The validity of the derived formula for other channel sections, like triangular, trapezoidal, parabolic, circular etc., should also be determined.
3. In this study the backwater lengths of the Ganges, the Jamuna, the Meghna and the Gorai rivers at the Hardinge Bridge, Bahadurabad, Bhairab Bazar and Gorai Railway Bridge stations, respectively, have been determined. The backwater lengths of these rivers at other stations and also of other rivers at different stations should also be determined.

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