OPTIMIZATION OF MULTIECHELON INVENTORY SYSTEM FOR COST EFFECTIVE SUPPLY CHAIN

BY

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It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree.

A.B.M. Abdul Malek

Dedication

To my mother
To my family

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ABSTRACT

The objective of this study is to find an optimal inventory distribution policy in a multi echelon supply chain environment (manufacturing plant, distribution center and retailer) by collaborative approach considering a target service level. An inventory model is developed by replicating echelons periodic, order up to level policy. The total inventory cost is set as an objective function, while the retailer's service level and the retailer's minimum order up -to-levels are set as constraints. The model tries to find out an optimal allocation of inventory at different echelons. This optimal solution sets target service levels at the same time the total inventory cost is significantly reduced.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

Inventory plays a major role in the business. The inventory cost takes a lion share of the total cost of goods sold. In manufacturing organizations its stake holding raises up to 70-80% of the total cost. Selection of appropriate inventory policy is a tenet in industrial arena. With the development of low cost, high speed computing techniques and the existence of widely used data communication systems, it is now easy to group business enteritis to form network of organizations. The very common name given to these coordinated business group is supply chain. Why the individual business units try to form a chain? They tries to reduce the overall cost of doing business with a major objective set before them are to make the product available to the customer with desired quality but with a competitive price. So business units try to find out some competitive advantages through the formation of a chain. A supply chain comprises of suppliers, manufacturers, distributors, retailers and customers. The activities included in a traditional supply chain are purchase of raw materials and manufacturing items at one or more factories, shipping the items to various warehouses for storage and in turn distribution of the same to the respective retailers or to the customers.

1.2 COST EFFECTIVE SUPPLY CHAIN

All of the trading partners target a better surplus to share among themselves so that they can sustain in the business with a proper growth of market share. Market growth is ensured by proper customer service levels. Hence, a beneficial coordination and integration of organizations with individual objectives to achieve a common goal can be referred to as a supply chain. Processes that support the effective integration of suppliers, manufacturers, warehouses and stores to guarantee suitable production and distribution of right quantities to the right location in right time and decreasing the total supply chain eventually in addition to satisfying service level requirements are incorporated in Supply chain management. The manufacturer procures raw materials, produce the end product and distribute the products to through the channel to reach the end customer.

A problem that a majority of the supply chains are aiming at is to reduce the total costs of inventory. But keeping inventory at a lower level will pull down the customer service level. So, a trade off must be made sacrificing a customer service level and thus gain a cost benefit in inventory.

Shorter product lifecycles that lead to higher demand uncertainty and their effect on global markets accordingly increasing the supply chain complexity, results in severe problems in the management of the supply chain [2]. The research on shorter product life cycle tends to address four problems from the operational perspective: Inventory management and control; production, planning and scheduling; information sharing, coordination, monitoring and operation tools [23].

The supply chain cost was immensely influenced by the overload or shortage of inventories. Thus inventory optimization has transpired into one of the most recent topics as far as supply chain management is considered [3], [11], [12]. Inventory is defined as the ensemble of items stored by an enterprise for future use and a set of procedures called inventory systems aid in examination and control of the inventory. It is possible to stock the inventory through various stages along the production and distribution supply chain [30]. The inventory system supports the estimation of amount of each item to be stored, when the low items should be restocked and the number of items that must be ordered or manufactured as soon as restocking becomes essential [4]. It is the responsibility of a supply manager to make a decision on the offers that need to be accepted in addition to updating the anticipated future inventory replacement costs. Inventory optimization application organizes the latest techniques and technologies, thereby assisting the enhancement of inventory control and its management across an extended supply network. Some of the design objectives of inventory optimization are to optimize inventory strategies, thereby enhancing customer service, reducing lead times and costs and meeting market demand [3]. The design and management of the storage policies and procedures for raw materials, work-in-process inventories and typically, final products [13] are illustrated by the inventory control. The costs and lead times can be reduced and the responsiveness to the changing customer demands can be significantly improved and subsequently inventory can be optimized by the effective handling of the supply chain [3]. The inventory and supply chain managers are mainly concerned about the estimation of the exact amount of inventory at each point in the supply chain free of excesses and shortages although the total supply chain cost is minimized. Owing to the fact that shortage of inventory yields to lost sales, whereas excess of inventory may result in extra costs, the precise estimation of optimal inventory is indispensable [3]. In other words, there is a cost involved in manufacturing any product in the factory as well as in holding any product in the ware houses, distribution center, and in the retail outlets. More the products manufactured or held, higher will be the holding cost. Meanwhile, there is possibility for the shortage of products. For the shortage of each product there will be a shortage cost. Holding excess stock levels as well as the occurrence of shortage for products lead to the increase in the supply chain cost. Thus the complexity of the problem has been increased. Thus, the maximum occurrences of stock level should be considered in order to optimize effectively.

1.3 BACKGROUND OF THE STUDY

The key idea that drives the study to undergo is the (R,s,S) policy for finished goods inventory replenishment and base stock policy. The (R,s,S) policy says that the inventory would be reviewed at R interval and when inventory falls below reorder level an order is to be place to fill the inventory to order up to level. The model is modified with integer review periods between replenishments. Thus the complexity of the model is relaxed.

The reorder point is reached at a random time between reviews. At the instant the inventory position drops to reorder point there is a time denoted by τ, remaining until the next review instant. Thus it is continuous review model with effective lead time (τ+L), where the τ is a random variable. Under the assumption of stationary, independent, normally distributed demands in non overlapping time intervals, the behavior of the inventory position can be modeled by a diffusion process which, in turn, permits us to develop estimates of the first two moments of τ as a function of the distance S-s and a measure of variability called coefficient of variation of the demand process. These moments are used as building blocks in choosing S and s so as to meet the two constraints in the following manner.

First, we recognize that the expected time between consecutive replenishments in the (R,s,S) system is the average size of a replenishment divided by the demand rate. But the average size of replenishment is (S-s) plus the average size of back orders. The average size of back order can be predicted from the expected value of τ. Thus we can select S-s so as to target the desired expected time between replenishments.

Second, the moments of tau also lead to expressions for the mean and variance of total demand over the effective lead time (τ +L). Assuming the total demand is approximately normally distributed, we can then determine a value of s appropriate to meet the fill rate constraint.

1.4 OBJECTIVES OF THE THESIS

The principal objective of this research is to find a solution to the problem of inventory replenishment policies for a serial supply chain having multiple echelons. The question that is addressed in this research is how to optimize inventory levels at different echelons so that the total inventory cost of the supply chain can be minimized to a near optimal condition. With an additional aim to achieve specific fill rate and so as to maintain a reorder level of inventory and a target order up to level inventory. Specific question is-

- Where do we set the re-order level considering possible stock out during lead time?
- How much should be ordered at a time to reach an order up to level?

To achieve a specific fill rate and meet the lead time demand uncertainty we are to find reorder point in the time line and at the same time we have to ascertain a maximum order up to level of inventory so that the demands in between the replenishments can be handled successfully with a minimum amount of stock out possibility. So the following objectives can be set:

- To develop a multi-echelon supply chain model
- To develop the mathematical formulation for the model
- To find the reorder level and order up to level with a target fill rate.
- To find ways to optimize the inventory cost
- To develop an algorithm for inventory replenishment policies
- To generate some solutions from the output of the computer program

1.5 OUTLINE OF METHODOLOGY

This thesis is outlined as follows:

- Develop a multi-echelon supply chain model where inventory replenishment policy can be formulated.
- Develop the inventory replenishment policy for different stages.
- Make necessary relaxation of the mathematical formulation so target parameters can be set.
- Determine the appropriate re-order level and order up to level for a target fill rate.
- Study the case and make analysis of the case study.
- Develop an algorithm for optimization of inventory cost.
- Code the simulation program
- Simulate the problem for different parameter sets.
- Conduct a post simulation analysis to verify the results of simulation.

1.6 POSSIBLE OUTCOMS OF THE STUDY

Guaranteed customer service combined with low inventory investment is a target for the most supply chain networks. This study tries to formulate an appropriate inventory replenishment policy that will highlight both the targets. Appropriate fill rate maintenance through possible minimum inventory level can help the trading partners to drive out the possibility of extreme level bullwhip effect. As in this formulated policy the inventory is reviewed periodically and fixes the order limit so bullwhip effect can be minimized to a lower level. The ultimate target of maintaining a high service level with a lower level of overall finished goods inventory will make the way to gain a high inventory turnover. Thus tied up of idle capital in the form of finished goods at different stages can be lowered to an acceptable level. The capacity of the storing facilities and the operating cost can be augmented as the specific size of inventory to hold can be minimized to a level appropriate for the agent. The transportation cost will be minimized also, as the inbound and outbound number of trips will be regularized with appropriate inventory levels. The routing of the transportation modes can be designed properly with the knowledge of the size of orders at specific time interval. The tracking of the orders can be done easily if the information and telecommunication network can be developed in accordance with the policy framing.

1.7 LIMITATIONS OF THE STUDY

Supply chain is a very complicated system. It includes many parameters and critical partnership phenomena. The stake holding matters are there to affect the operation of the supply chain. Collective decision making along with local targets of individual partner affects everyday's decisions. The trading partners are located at different socio-economic environment. The trading policies framed at corporate level are sometimes overlooked. Hundreds of conflicts that arises in the network cannot be addressed with mathematical formulations only. The demand distribution of this particular product is greatly affected by seasonality factors. There are peak demand seasons and off peak demand and moderate demand seasons exist practically. Market share of this particular brand fluctuate with competitors policy. There exist some uncertainty in principal raw material procurement (lime stone) and this produces some capacity limitations for the production unit but we assumed the capacity is ample to support the distribution supply chain.

1.8 ORGANIZATON OF THE THESIS

Chapter 1 illustrates about objectives of the thesis. Chapter 2 gives some overview of the existing inventory optimization models. The benefits of multi-echelon inventory optimization are highlighted with previous works. Chapter 3 shows about construction of supply chain inventory model and mathematical formulation. Chapter 4 elaborates about the data collection and case study. Chapter 5 shows about calculation of reorder level and order up to level and total inventory cost of supply chain network. Chapter 6 illustrates the optimization methodology and. Chapter 7 discusses the results of the simulation output, limitations and chapter 8 contains conclusion and recommendations for future work.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

Single echelon single location inventory models have been extensively studied in literature. Assuming linear holding and fixed ordering costs, the policies in periodic review (R, s, S) is proven. Because of their simple structure, these policies are widely applied in practice and have been implemented in many organizations. These topics are inherently included in the business information systems, such as Enterprise Resource Planning (ERP) and Advanced Planning Systems (APS) systems. However, Multi echelon inventory management system is a complex phenomenon as each echelon imposes some additional restrictions for coordinated ordering policy.

The first concept in the notion of *echelon inventory levels* and *echelon holding costs* and *back order costs* is the Clark and Scarf (1960) model. The echelon inventory level is the total of the inventory that is aggregated from stage to stages, also includes the inventory is in transit from the parent stage to the last stage. The echelon holding cost at a stage is the difference between the cost of holding a unit of inventory at the stage for a time period, and the cost of holding it at the parent stage. Without loss of generality it is assumed that the echelon holding costs are nonnegative; if this is not the case then no inventory will be stored at the parent stage. Holding costs can be correctly accounted for either in the conventional manner, or using echelon inventories and echelon holding costs.

The echelon inventory level provides a compact description of the structure of the optimal policies. In each period, as a function of the current state of the system (but independent of the inventory control policy used in previous periods), there are target echelon inventory levels for each stage, also called echelon base stock levels. The optimal policy aims to keep the echelon inventory levels as close as possible to the target base-stock levels. That is, at the beginning of the period, if the echelon inventory level at a stage is below the target, the stage orders enough to bring the echelon inventory up to the target, or it orders the entire inventory that is currently on hand at the parent stage. If the echelon inventory level is above the target, no order is placed.

A number of researches have been done using different inventory models to optimize inventory level. Their intension was to set optimum inventory at different stages of supply chain network considering desired service level. Liana Napalkovav[12] developed a simulation based inventory model considering service level. Guillermo Gallego, Özalp Özer[13] used advanced demand information for optimal replenishment policies. One ware house and multi retailer supply chain proposed by Chumpol Monthatipkul and Pisal Yenrade [14] focuses on stochastic demand process. The model illustrates the installation stock and echelon stock inventory control systems. The installation stock inventory control system refers to a situation, in which each entity makes a decision based on its own current inventory status. In contrast echelon stock policy considers its own inventory as well as all downstream and pipe line inventories. So, echelon stocking policy call for a coordinated approach of replenishment.

This research focus on order up to level at each echelon with a desired service level. In this paper, order up to level calculated by periodic review policy.

2.2 INVENTORY MANAGEMENT PHILOSOPHY: PULL, PUSH AND MULTI ECHELON

Inventory management is developed around two basic philosophies. First, there is the pull approach and second, is the push approach. Pull approach views each stocking point , for example, a ware house, independent of all others in the chain. Forecasting demand and determining, replenishment quantities are accomplished by taking into account only local conditions. No direct consideration is given to the effect that the replenishment quantities, each with their different levels and timing, will have on the economics of the sourcing plant. However, this approach does give precise control over inventory levels at each location. Pull methods are particularly popular at the retail level in the supply chain where over 60 percent of the hard goods and almost 40 percent of the soft goods are under replenishment program (Tom Andel, 1996)

Alternatively, there is the push approach to inventory management. When decisions about each inventory are made independently, the timing and replenishment order sizes are not necessarily well coordinated with product lot sizes, economical purchase quantities, or order size concern. Therefore, many firms choose to allocate replenishment quantities to inventories based on projected needs for inventories at each location, available space, or some other criteria. Inventory levels are set collectively across the entire distribution system. Typically push method is used when purchasing or production economies of scale outweigh the benefits of minimum collective inventory levels as achieved by the pull method. Collaborative replenishment can be used as a hybrid of the pull and push method. In this case, the channel members representing the source point and the stocking point jointly agree on the replenishment quantities and their timing. The result can be order replenishment that is more economical for the supply chain than if either party alone were to make the replenishment decision. This is called multi-echelon inventory planning

A study by Aberdeen Group shows that the increasing pressure on inventory reduction can be represented as:

Table 1: Top Pressures Causing Inventory Strategies to be Re-examined
(**Source**: Aberdeen Group, December 2004)

Pressure factor	**Factor rating**
Pressure to reduce stock outs and customer service failures	72%
Corporate need to improve return on invested capital	65%
Market pressure to reduce product price and increase responsiveness	64%
Global sourcing and/or manufacturing make lead time to increase	44%
Variability in demand and shorter product life cycle	44%

It is found from the study that, the factors that drive reduction in inventory

- Top management emphasis on inventory reduction (19%)
- Reduce the Number of SKUs in the warehouse (10%)
- Improved forecasting (7%)
- Use of sophisticated inventory management software (6%)
- Coordination among supply chain members (6%)
- Other

Factors that Drive Inventory Turns Increase

- Better software for inventory management (16.2%)
- Reduced lead time (15%)
- Improved forecasting (10.7%)
- Application of SCM principals (9.6%)
- More attention to inventory management (6.6%)
- Reduction in SKU (5.1%)
- Others

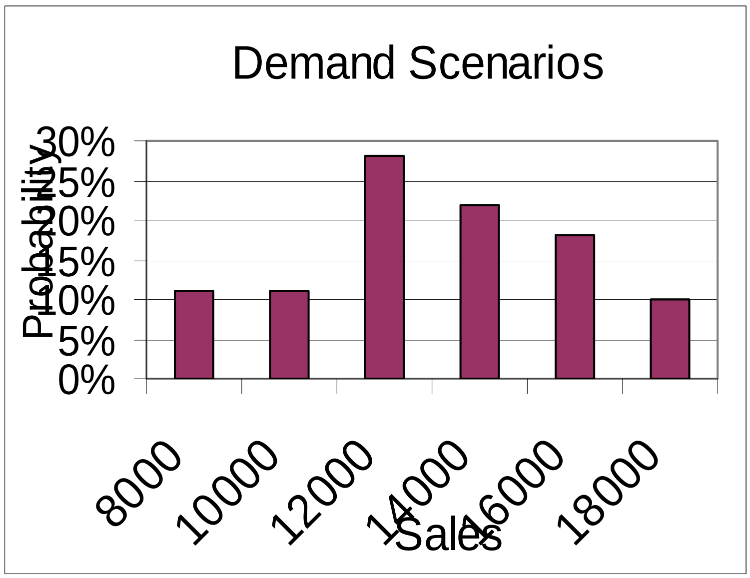


Fig. 2.1 Probabilistic demand scenario

2.3 REVIEW OF DISTRIBUTION SUPPLY CHAIN

A distribution supply chain (DSC) is a part of the SC, which has attracted considerable attention in the literature. Both academic researchers and industrial practitioners alike have expended much effort to manage the overall distribution system more efficiently through various coordination and cooperation strategies. The models for DSC coordination developed so far can be categorized into two main groups: (a) models that consider integrated DSCs in order to find joint optimal inventory policies that minimize the total cost of the DSC, and (b) models that propose some coordination mechanisms which help every member within a DSC to work together harmoniously.

One of the earliest models, developed by Schwarz (1973), considered a continuous review policy for a single-warehouse multi-retailer inventory system, and minimized the average system cost per unit time interval. Goyal (1977) suggested a joint economic lot size model with the objective being to minimize the total cost of a single supplier – single buyer system. Yang and Wee (2002) used a heuristic procedure to determine the optimal number of deliveries favoured by the buyers in such a way to reduce the overall cost of the supply chain.

Lu (1995) developed a one-vendor multi-buyer integrated inventory model, where the vendor seeks to minimize total annual cost subject to the maximum costs which buyers are prepared to incur.

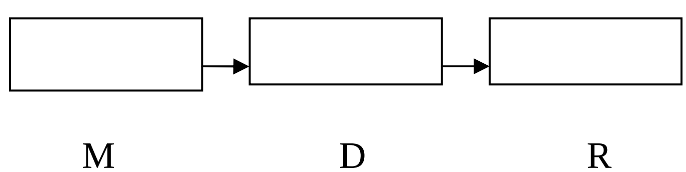
Monahan (1984) developed a coordination strategy in which discount is offered to buyers so as to induce them to order in the quantity that maximizes the vendor's profit. In Viswanathan and Piplani (2001), the vendor specifies common replenishment periods and requires all buyers to replenish only at suggested time periods. As compensation, the vendor provides a price discount to entice the buyers to accept the common replenishment periods. A conclusion was made that the proposed coordination mechanism was beneficial only in the case when the order processing cost was larger than a given limit value. In this paper, a single-warehouse, multi-retailer DSC is considered to be cost effective. It is supposed that customer demand is uncertain and is represented by discrete fuzzy sets. Retailers are supplied periodically from the warehouse and the warehouse is replenished periodically from an external source. All inventories in the DSC apply an order-up-to level replenishment policy. It is assumed that the DSC operates 'under one roof' in the sense that a single measure of performance of DSC control is defined; it is the total relevant cost per unit time interval, which includes order/set up cost, holding cost and shortage cost. The problem is to determine the review periods and order-up-to levels of all the inventories in the DSC which give a satisfactory small total cost of the DSC.

In the paper, the complex control problem of the DSC as a whole is decomposed in to a number of simpler sub problems of controlling the retailers and the warehouse independently. The solutions obtained in such a way can be improved by coordination actions. An iterative coordination mechanism is developed which coordinates the retailers and the warehouse to get satisfactory control of the DSC as a whole.

2.4 FINDING AN APPROPRIATE SIZE OF ECHELON STOCK WITH A TARGET FILL RATE

Looking through the literature, we found two seemingly contradictory views for setting internal fill rates. More often than not, researchers and practitioners assume upstream locations should achieve higher fill rates to guarantee a desired system fill rate (see Bollapragada et al. 2004b, Paschalidis and Liu 2000). However, using a two-stage model, Choi et al. (2004) show that this rule of thumb is not necessarily accurate. Moreover, drawing from empirical observations based on the optimal system behavior in multi-echelon inventory models, Axsater (2003b p. 555) concludes, "What is typical is that upstream installations should have very low stocks compared to downstream installations." This implies that internal fill rates for upstream stages may be lower than those for downstream stages. From Graves et al. (1998) it can also be noted, from a system perspective, it may be often better to have low levels of internal service. Likewise, the numerical examples in Bollapragada et al. (2004b) contain optimal internal fill rates much lower than the system fill rate.

2.5 INVENTORY MODELS WITH MULTI ECHELONS



Here, M= Manufacturer, D= Distributor, R= Retailer

Fig.2.2 A simple serial system with three echelons

Despite this common practice, there has been little theoretical investigation on the linkages of different stages. For example, while commenting on their internal fill-rate assumptions, Graves et al. (1998, p. S48) wrote: *In extending the single-stage model to a multistage setting,... we assume that the service levels will be set to assure a high level of service, and in the model* *analysis, we ignore the downstream consequences of an upstream stock out; i.e., starvation of inputs. These assumptions raise two questions. One is what are the consequences of ignoring internal stock outs, and the second is what should the internal service level be ... simulation tests ... confirm that ignoring the internal stock outs in the analysis does not distort the results of the model.*

Indeed, looking through the literature, we found two seemingly contradictory views for setting internal fill rates. More often than not, researchers and practitioners assume upstream locations should achieve higher fill rates to guarantee a desired system fill rate (see Bollapragada et al. 2004b, Paschalidis and Liu 2000). However, using a two-stage model, Choi et al. (2004) show that this rule of thumb is not necessarily accurate. Moreover, drawing from empirical observations based on the optimal system behavior in multi-echelon inventory models, Axsater (2003b p. 555) concludes, *"What is typical is that upstream installations should have very low stocks compared to downstream installations."* This implies that internal fill rates for upstream stages may be lower than those for downstream stages. Graves et al. (1998 p. S48) also note that *"[t]he literature on multiechelon distribution systems suggests that, from a system perspective, it often may be better to have low levels of internal service."* Likewise, the numerical examples in Bollapragada et al. (2004b) contain optimal internal fill rates much lower than the system fill rate.

However, computing the optimal policies is tractable only under rather strong assumptions on the demand distributions and there are many important scenarios where the required computations seem unlikely to be tractable. In particular, these models become computationally intractable in the presence of correlated and evolving demands. Such demand structures are typical in scenarios where dynamic forecasts are incorporated into the supply-chain management.

The details of the single-item, periodic-review, serial system picturesque is as follows. A single commodity moves through a supply-chain that consists of a customer, distinct inventory-storage locations called stages, and an external supplier. The stages are organized in a serial system.

The first stage in the series (at the bottom of the series) is facing a sequence of stochastic demands over a planning horizon of finitely many discrete periods. Each stage is supplied by the next stage in the series, and the last stage is supplied by an external supplier with infinite capacity. There are lead times between each two consecutive stages in the system that correspond to the number of periods that it takes to ship the commodity from one stage to the next.

In each period several types of costs are incurred, per-unit ordering costs for ordering inventory at each stage, per-unit holding costs that are incurred for each unit of inventory that is at a stage or in transit between stages at the end of a time period, and a per-unit backordering penalty cost for each unit of demand that is not yet satisfied at the end of a period. Unsatisfied demand is fully backordered, i.e., it stays in the system until it is satisfied. In the more general assembly system, the stages in the supply-chain are organized as a directed tree. Each stage produces a different subassembly, and the *root stage* of the tree produces a final product and is facing stochastic demands. Each unit in each stage is assembled from one unit from each of the *parent stages*. There are again lead times between each stage and its *child stage* in the tree. As in the serial system there are per-unit ordering costs and holding costs in each one of the stages, as well as a per-unit backordering penalty cost that is incurred only in the root stage. In both models the goal is to find an ordering policy that respects the system constrains and minimizes the overall expected cost over the entire planning horizon.

As we have already mentioned these fundamental models have attracted a lot of attention from many researchers. The most commonly used paradigm to address these models has been the dynamic programming framework. This approach has been very effective in characterizing the structure of the optimal policies for these models. There are several dynamic-programming-based proofs for the optimality of *echelon state dependent base-stock policies* in the serial system.

The first proof has been established in the seminal paper of Clark and Scarf [4] for a model where the demands in different periods are assumed to be independent and identically distributed. Several subsequent papers (see, for example, [2, 3, 5, 6, 18]) by different authors have established simpler proofs of the initial result of Clark and Scarf and extended it to models with more general assumptions (e.g., more general assumptions on the demand distributions). In particular, the optimality of echelon base-stock policies has been established for models in which demands follow an-exogenous Markov-modulated process or an autoregressive process.

All of the above-mentioned proofs are based on several concepts regarding the cost accounting scheme. The first major concept is the notion of *echelon inventory levels* and *echelon holding costs*. The echelon inventory level of a stage is the total inventory that is at the stage, that is in transit from the parent stage to the stage, and that is at or in transit to *any* of the downstream stages in the system. The echelon holding cost at a stage is the difference between the cost of holding a unit of inventory at the stage for a time period, and the cost of holding it at the parent stage. Without loss of generality we assume that the echelon holding costs are nonnegative; if this is not the case then no inventory will be stored at the parent stage. Holding costs can be correctly accounted for either in the conventional manner, or using echelon inventories and echelon holding costs [12] and Section 3 below.

The echelon inventory level provides a compact description of the structure of the optimal policies. In each period, as a function of the current state of the system (but independent of the inventory control policy used in previous periods), there are target echelon inventory levels for each stage, also called echelon base stock levels. The optimal policy aims to keep the echelon inventory levels as close as possible to the target base-stock levels. That is, at the beginning of a replenishment cycle, if the echelon inventory level at a stage is below the target, the stage orders enough to bring the echelon inventory up to the target, or it orders the entire inventory that is currently on hand at the parent stage. If the echelon inventory level is above the target, no order is placed.

The research literature on stochastic serial inventory systems has two cost re-allocation schemes, which effectively shift costs from one stage to another. Both of these schemes re-allocate costs between stages for a single time period only, but they do not shift costs from one time period to another. The first published cost re-allocation scheme is the classical, dynamic-programming-based proof of the optimality of an echelon base-stock policy.

This proof decomposes the problem into a sequence of single-stage problems, and then proves the optimality of an echelon base-stock policy recursively, stage by stage. Each stage passes an *implicit penalty function* to the parent of the stage. The penalty function captures the impact that decisions made by upper stages will have on the stage in question. This approach was introduced by Clark and Scarf [4], was simplified by Zipkin [5], and has been extended by Dong et.al. [6]. Computationally, a series of single-stage problems are solved to optimality, one for each stage, with each stage assuming that the lower stages have already been optimized. Part of the computation done at each stage is to compute the implicit penalty function that will be passed to the parent stage.

The second cost re-allocation scheme is due to Chen and Zheng [8], for a model in which each stage also has a *fixed set-up cost* that is incurred whenever the stage places an order. In addition to using induced penalty functions, they propose *cost parameter allocation* bounds, in which they create a ”component” for each stage in the original system, and a separate multi-stage inventory model for each component. They then allocate the backordering penalty cost parameter and the holding cost parameters to the different components.

Unfortunately**,** the rather simple state-dependent base-stock form of the optimal policies does not always lead to efficient algorithms for computing the optimal policies. The corresponding dynamic programs are tractable in cases where the demands in different periods are independent and do not evolve over time, although even in such simple scenarios the computations involved can be tedious. The standard dynamic programming approach can still be tractable in models with exogenous Markov-modulated demand but under rather strong assumptions on the size of the state space of the underlying Markov process [17, 2].

In particular, they have established the optimality of echelon state-dependent base-stock policies under general exogenous Markov-modulated demand and stochastic lead times with no order crossing. Using this unit decomposition, they have also suggested new methods to compute the optimal policies. However, their computational methods are essentially dynamic programming approaches applied to the unit sub problems, and hence they suffer from similar problems in the presence of correlated and evolving demands. Kathryn, Peter Jackson and Muckstadt [18] have extended their approach to systems with capacity constraints on the size of the order. Although our approach is very different from theirs, we use their unit-decomposition approach extensively as a technical and descriptive tool.

As a result of this apparent computational intractability, many researchers have attempted to construct computationally efficient (but suboptimal) heuristics for these problems [7, 1, 5, 16, 14]. However, we are aware of no attempts to analyze the worst-case performance of these heuristics. Moreover, we are aware of no computationally efficient policies for which there exists a worst-case analysis that establishes constant performance guarantees. Rosling [19] has shown that the optimality of echelon base-stock policies holds also in the more general assembly system. Moreover, he has shown that under this condition an assembly system can be transformed into an equivalent serial system, in that policies in one system can be mapped to policies in the other system with the same cost. In particular, echelon base-stock policies in the serial system are mapped to echelon base-stock policies in the assembly system. However, since the assembly system is a generalization of the serial system, it is clear that the above-mentioned computational challenges still exist, and might be even hard.

This research tries extends the recent work by Levi, P´al, Roundy and Shmoys [20] who have considered, respectively, the un capacitated and the capacitated *single-item, single-stage periodic-review system*, with general demand structures that allow correlation and evolution over time. They have proposed a class of computationally efficient policies that they call dual-balancing policies and shown that these policies admit a worst-case performance guarantee of 2. Their work is based on two novel ideas.

Firstly, they have introduced a new *marginal cost accounting scheme*, in which they assign to a decision *all* present and future costs that, as a result of this decision, become independent of *any* future decision, and are only a function of future demands. This is in contrast with standard approaches that directly associate with each decision only the costs incurred in that period (or more generally a lead time ahead). Secondly, they have used cost balancing techniques to construct dual-balancing policies. In each period, the dual-balancing policy orders such that the (conditional) expected marginal holding costs incurred by the units ordered is equal to the (conditional) expected marginal backordering cost that will be incurred one lead time into the future. These two costs oppose each other, in that the expected marginal holding cost is an increasing function of the size of the order, while the expected marginal backordering costs is a decreasing function.

Thus, it is always possible to order a quantity that will make these two costs equal to each other. The worst-case analysis for the single-item, single-stage models is based on simple, yet powerful, amortization of the costs incurred by the dual-balancing policy and by an optimal policy. In particular, it can be shown that the optimal policy incurs on expectation at least half of the costs incurred by the dual-balancing policy, which implies that the dual-balancing policy has a worst-case performance guarantee of 2.

The models considered in this paper are direct generalizations of the un-capacitated single-stage models in [20]. We use cost balancing techniques, and an innovative, multi-stage version of marginal cost accounting scheme, also called *cause-effect cost accounting.* The goal is to associate each element of cost with the decision that directly causes this cost, where decisions are indexed by the time period and the stage at which the ordering decision arose. Our approach is based on the notion of the *critical chain*. Focus on a single unit of demand and suppose that we know in advance when demand for this unit will occur. It is then clear that we would like to order and deliver this unit of inventory in a just-in-time fashion, so that will arrive at stage 1 exactly on time (incurring no backordering penalty cost) and will not be delayed at any intermediate stage (avoiding unnecessary holding costs). The critical chain is the just-in-time path that the unit would ideally follow through the inventory system, and the *critical time periods* are the time periods when orders for the unit would ideally be placed, at each stage in the system. We use the critical chain and the critical time periods as points of comparison. There is a critical time period for each unit and each stage.

We categorize all holding costs into three categories, and we assign the holding and backorder costs to ordering decisions, in the following manner. Holding costs that are incurred while units are in transit between stages are called *pipeline costs*. The pipeline costs are inevitable - they will be incurred by every unit that is demanded, regardless of the policy followed. For a given unit, the echelon-n pipeline cost is assigned to stage n, and to the time period in which the unit was ordered by stage n. whenever we order a unit at a stage earlier than the critical time we cause early holding costs to be incurred. These are extra holding costs, in addition to the pipeline costs.

The early holding costs incurred by a given unit are assigned to the stages and the time periods in which the unit was ordered early, relative to the critical time period.

For a given ordering decision, taken at a given stage and in a given time period, we consider the expectation of all of the costs assigned to this order. We separate these expected costs into two opposing functions.

The expected pipeline and early holding costs are increasing in the size of the order placed by the stage in the time period. On the other hand, the expected backordering and late holding costs are decreasing in the size of the order. This leads to a balancing policy that, at each stage in each period, selects an order quantity to balance these two opposing cost functions. This policy is computationally efficient, and can be implemented in an on-line manner, that is, regardless of any future decisions.

This study focus on a very basic model: a serial inventory system with a normal demand process, linear holding costs, and a fill-rate constraint. A periodic review order up to policy is employed at each stage.

2.6 A MODEL BASED ON DESIRED FILL RATE

There is a great deal of literature on multi-echelon supply chain inventory models. However, many inventory models are not well suited for some of today's complex supply networks. In much of the literature, demand is considered to be stationery (Graves and Willems, 2000; Cachon and Zipkin, 1999) while many products experience non-stationery demand. Addressing non-stationery demand is one of the important extensions to this research. In the cases of non-stationary demand situations there always exist a chance of stock out. The non stationary demand is associated with the cycle service level and order fill rate.

The service level generally called cycle service level (CSL) is different from order fill rate. The cycle service level is the fraction of replenishment cycles that end with all the customer demand being met. A replenishment cycle is the interval between two successive replenishment deliveries. The CSL is equal to the probability of not having a stock out in a replenishment cycle. CSL should be measured over a specified number of replenishment cycles.

On the other hand order fill rate is the fraction of orders that are filled from available inventory. Order fill rate is to be measured over a specified number of orders. So, for the high level of product availability thus a high fill rate can be achieved through maintaining a larger value of safety factor. But the scenario is critical where facility cost is higher. Thus a trade of should be achieved based on inventory holding cost and probable stock out cost. [21]

As in this model the clear view is absent and the replenishment occurs at predetermined review instants so, it has an objective to minimize costs in the supply chain subject to order fill rate constraint. The model developed tries to make a sensitivity analysis regarding target fill rates. In this model polynomial regression model is developed and then simulated with historical data. Here fractional polynomial regression is used to determine the expected demand during review period and lead time. Order up to level and reorder point is estimated from the forecasted data. Demand of the end stage is satisfied with the appropriate level of s and S. All other stages give the back up for the end stage only.

The objective of the problem is set to find a feasible ordering policy (i.e., one that respects the system constraints) that minimizes the overall expected ordering cost, holding cost and other costs involved. Backorders fully accumulate over time until they are satisfied. That is, each unit of unsatisfied demand will stay in the system until it is satisfied from the incoming order.

CHAPTER THREE

MODEL DEVELOPMENT

3.1 INTRODUCTION

The supply chain model developed in this study is a single-product, three stage serial supply chain consisting of a retailer, distributor, and manufacturer. All the members in the supply chain operate under a periodic-review (R, s, S) policy, where the review periods are preferably in unit time (or a day). Procurement lead time and transportation lead time are combined accordingly at each stage and considered together as one component, called replenishment lead time for that stage. S,s values takes discrete integer values. The manufacturer has ample capacity. Even though the replenishment lead time is assumed to be deterministic, there exists supply uncertainty in the sense that the customer or any member will be replenished, provided that the immediate upstream member has sufficient on-hand inventory to meet the replenishment order. Unmet demands will be back ordered and imposing a backorder cost.

3.2 THEORETICAL BACKGROUND

An inventory model provides the organizational structure and the operating policies for maintaining and controlling goods to be stocked. The model is responsible for ordering and receipt of goods: timing the order placement and keeping track of what has been ordered, how much and from whom.

Multi period inventory models are of two types.

- Fixed order quantity models (Q model)
- Fixed time period models (P model)

The Basics of Economic order quantity (EOQ):

The discussions about deriving the optimal order quantity are based on the following assumptions.

- Demand for the product is constant and uniform throughout the period.
- The lead time (time from order to receipt) is constant.
- Price per unit of product is constant.
- Inventory holding cost is based on average inventory.
- Ordering or setup costs are constant.
- All demands for the product will be satisfy (no back orders are allowed).

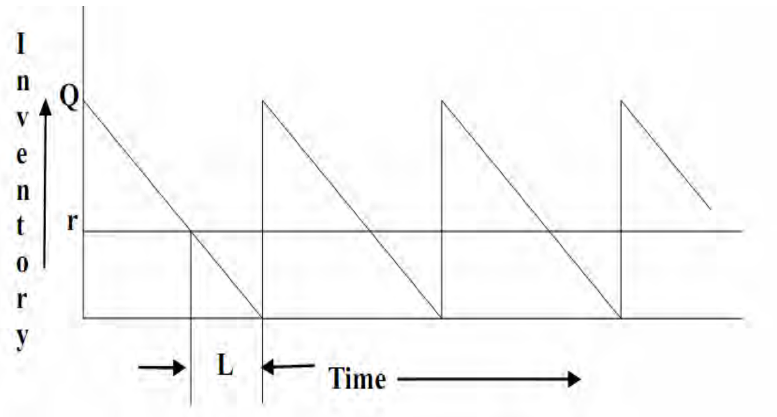


Fig.3.1(a) Time pattern for inventory model for simple EOQ model

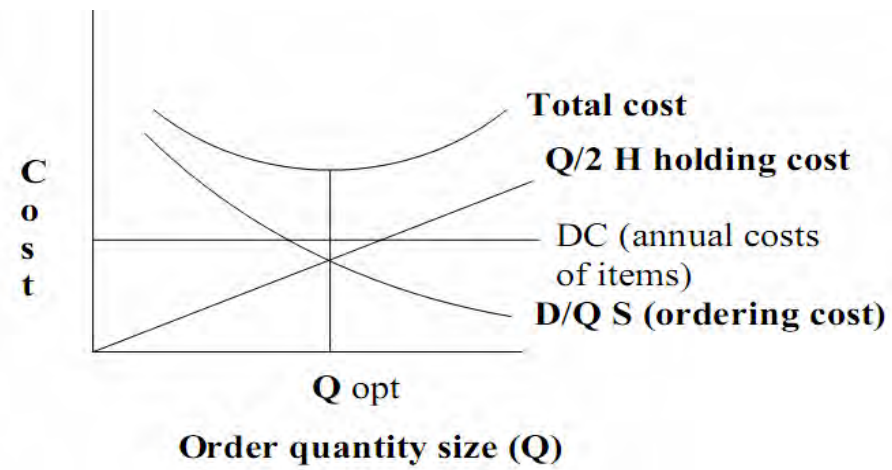


Fig3.1(b). Annual product cost based on size of order.

The parameters used in deriving the EOQ:

TC = Total annual cost of inventory

D = Annual demand

C = Cost per unit

Q = Quantities to be ordered

S = Set up cost or cost of placing an order

R = Recorder point

L = Lead time

H = Annual holding and storage cost per unit of average inventory.

DC= Cost of Materials or finished goods

$\frac{D}{Q}S$ = The annual ordering cost.

$\frac{Q}{2}H$ = The annual holding cost

Now, Total annual cost of Inventory,

$$TC = DC + \frac{D}{Q}S + \frac{Q}{2}H$$

To minimize the total cost, differentiate TC with respect to Q and put it equal to zero.

$$\frac{dTC}{dQ} = 0$$

$$\Rightarrow \frac{d}{dQ}\left(DC + \frac{DS}{Q} + \frac{QH}{2} \right)$$

$$\Rightarrow 0 - \frac{DS}{Q^2} + \frac{H}{2}$$

$$setting,$$

$$\frac{d(TC)}{dQ} = 0$$

$$Q_{opt} = \sqrt{\frac{2DS}{H}}$$

This is the expression for Economic Order Quantity.

Calculation of Reorder Level Inventory

If the demand during lead time is only the matter of consideration and no safety stock is needed then the reorder level inventory is the demand to be fulfilled during lead time,i.e.,

$$ROP = \overline{d} * L$$

Where,

$\overline{d}$ = Average daily demand (constant)

L = Lead time in days (constant)

Fixed-Order Quantity Model with Specific Service Level

A fixed-order quantity system perpetually monitors the inventory level and places a new order when stock reaches some level, R. The danger of stock out in this model occurs only during the lead time.

The amount of safety stock depends on the service level desired. The quantity to be ordered, q, is calculated in the usual way considering the demand, shortage cost, ordering cost and so forth. A fixed order quantity model can be used to compute q. The reorder point is then set to cover the expected demand during the lead time plus a safety stock determined by the desired service level. Thus the key difference between a fixed-order quantity model, where demand is known and one where demand is uncertain is in computing the reorder point. The order quantity is the same in both cases. The uncertainty element is taken into account in the safety stock.

The reorder point is [4]

$$ROP = \bar{d}L + z_{\alpha}.\sigma_L$$

z_{α} = number of standard deviation for a specific service level(1-α)

σ_L = standard deviation of demand during lead time.

$z_{\alpha}*\sigma_L$ = safety stock.

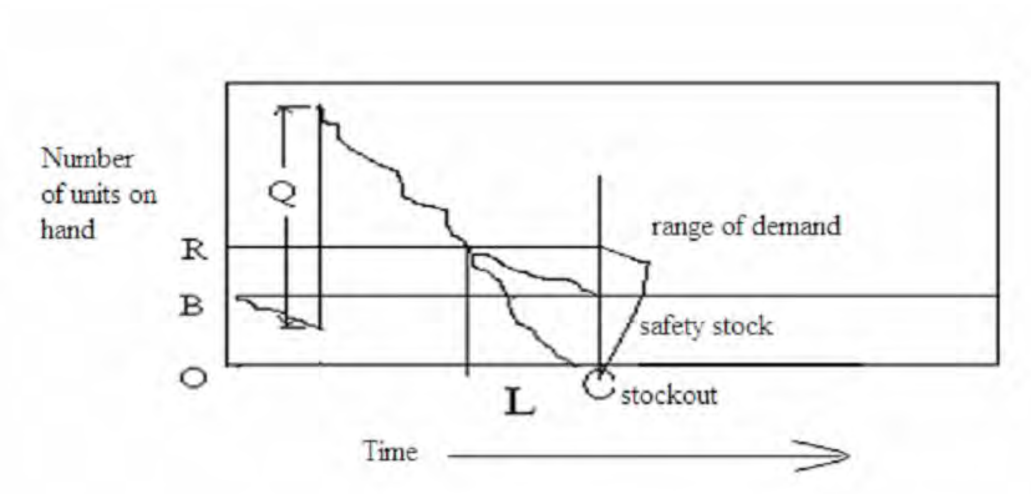


Fig3.2: Fixed order quantity model [7]

R = Reorder point in units, B = Base stock level, Q = Quantity to be ordered, L = Lead time

Fixed Time Period Model with Specified Service Level

In a fixed-time period system, reorders are placed at the time of review instants with a lead time, L and the safety stock that is to be maintained is

$$\text{Safety stock} = z_{\alpha}.\sigma_{T+L}$$

Fig shows a fixed time period system with a review time of T and a constant lead time of L

In this case, demand is randomly distributed about a mean, $\bar{d}$

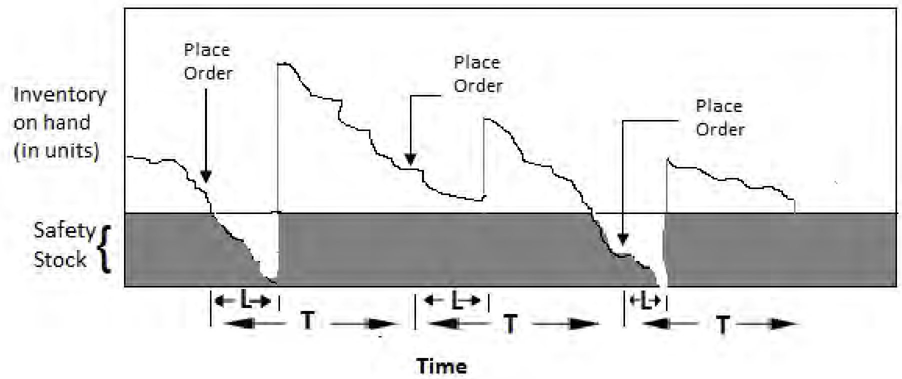


Fig.3.3 Fixed period quantity demand

The quantity to order, q, is

$$q = \bar{d}(T + L) + z_{\alpha}\sigma_{(T+L)} - I$$

Where,

q= Order quantity,

(T+L) = Average demand over the vulnerable period,

z_{α} = Safety factor, number of standard deviations for a specified service level,(1-α)

I = Inventory currently on hand (plus on order, if any)

q= Quantity to be ordered

T= The number of days between reviews

L= Lead time in days (time between placing an order and receiving it)

$\bar{d}$ = Forecasted average daily demand

$\sigma_{(T+L)}$ =Standard deviation of demand over the review period and lead time

Note: the demand, lead time, review period, and so forth can be any time units such as days, weeks or months, consistent throughout the equation.

In this model, demand $\bar{d}$ can be forecast and revised each review period if desired, or the yearly average may be used if appropriate. We assume that demand is normally distributed.

$$E(z) = \frac{\bar{d}T(1-p)}{\sigma_{(t+L)}}$$

Where,

E(z) = Expected number units short from a normalized table

p = Service level desired expressed as a fraction (e.g., satisfying 95% expressed as the fraction .95 of demand from items in stock)

$\bar{d}$ T = Demand during the review period

$\sigma_{(T+L)}$ = Standard deviation over the review period and lead time.

3.2. MODEL DEVELOPMENT

In this model there are three echelons involved i.e., one manufacturer, one distribution center and retailers of three categories (low volume retailers, medium volume retailers and high volume retailers). The manufacturer has sufficient capacity to supply the downstream members and instantaneous supply can be ensured to the distribution center. The target is to determine the order up to level(S) and reorder level(s) for the retailers first. The construction of model given below:

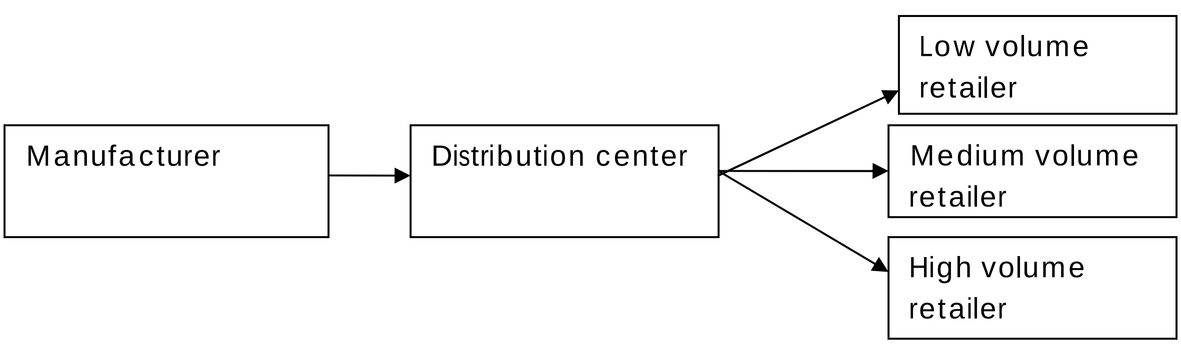


Fig.3.4 Model of the supply chain network.

3.3 NOTATIONS AND PARAMETERS

Assumptions

The following assumptions are made for the mathematical formulation

i) The inventory position is reviewed every R units of time, where R is pre-specified, and for convenience in the development we will set R=1, which means that the review interval is redefined as unit time.

ii) There is a constant replenishment lead time (L) from when replenishment is triggered (at a review instant) until it is available in stock.

iii) Demands in disjoint intervals of time are independent, stationary, normally distributed variables.

iv) There is complete backordering of any demand during a stock out situation.

v) The service measure used is the fill rate, the fraction of demand to be routinely met from stock.

vi) A target average time between consecutive replenishments is specified rather than explicitly incorporating setup and carrying costs.

vii) τ follows a uniform distribution and expected value of τ is .5 with a very small variance.

Parameters

P_2 – desired (fractional) value of the fill rate

n – Desired (integer) average number of review intervals between consecutive replenishments.

s – Reorder point

S – order-up-to-level

Q – Size of a replenishment

L – Replenishment lead time

μ – Average demand in a unit time interval

σ – Standard deviation of demand in a unit time interval

$CV = \sigma/\mu$ – coefficient of variation of demand.

τ – a random variable representing the time from when the inventory position hits s until the moment of the next review.

$E(\tau)$ – the mean value of τ

$Var(\tau)$ – the variance of τ

X – The total demand in $L + \tau$

σ_x – the standard deviation of X

k – Safety factor

m = (S - s) / µ

u – The first passage time for the inventory position to drop from S to s

I -Current inventory level (includes items on order)

h- holding cost per year as a fraction of product cost.

O - fixed ordering cost.

D-Forecasted demand per year

3.4 MATHEMATICAL FORMULATION

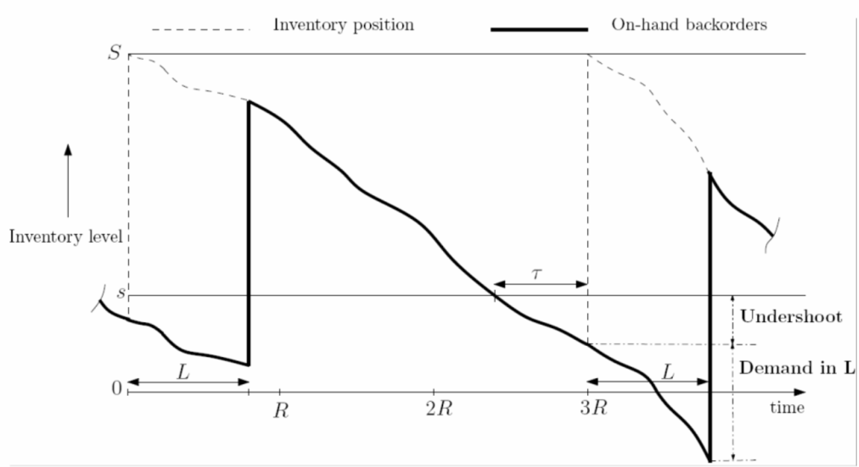


Fig.3.5 Periodic review inventory (R,s,S) with moment of τ.

As indicated in Fig. , τ is the time from the instant that is inventory position first reaches reorder level until the moment of the next review. The behavior of cumulative normally distributed demand can be modeled as a continuous time diffusion process (Miltenburg and Silver, 1984). Moreover, the probability density function of the first passage time, u, for the inventory position to drop from 'Order up Level, S) to 'Reorder Level,s' is given by (Cox and Millerr,1965)

$$f_u(u_o) = \frac{S-s}{\sigma\sqrt{2\pi u_o{}^3}} \exp\left[-\frac{\left(S-s-\mu u_o\right)^2}{2\sigma^2 u_o}\right] \qquad 0 < u_o$$

Setting $$m = \frac{S - s}{\mu} \tag{3.1}$$

and $$CV = \frac{\sigma}{\mu} \tag{3.2}$$

$$f_u(u_o) = \frac{m}{CV\sqrt{2\pi u_o^{\,3}}} \exp\left[-\frac{(m - u_o)^2}{2(CV)^2 u_o} \right] \qquad 0 < u_o \tag{3.3}$$

As an aside, Burgain(1969) developed an analytic expression for the expected value of u.
Now, multiple values of u can produce the same values of τ. Specifically $\tau = \tau_o$ will result from any of u = 1- τ_o , 2- τ_o, etc. Hence the density function of τ is given by

$$f_\tau(\tau_o) = \sum_{i=1}^{\infty} f_u(i - \tau_o)$$

Using (3) $$f_\tau(\tau_o) = \frac{m}{CV\sqrt{2\pi}} \sum_{i=1}^{\infty} \frac{1}{\sqrt{(i - \tau_o)^3}} \exp\left[-\frac{(m + \tau_o - i)^2}{2(CV)^2 (1 - \tau_o)} \right] \qquad 0<\tau_o<1$$

(3.4)

Unfortunately, it is not possible to analytically develop expressions for the moments of τ. However, it is worth noting that the density function, hence the moments, depends on only two parameters namely, m and CV. For given values of these parameters τ_o can be discretized on a fine grid and accurate estimates of E(τ) and Var(τ) can be found by numerical integration. If τ have a uniform distribution, then E(τ) and Var(τ) could be analytically determined from a known distribution.

E (τ) and Var (τ) for a given value of (S-s) interval.

Specifically, in an inventory control system commonly used in practice, the status of an item is examined at equi-spaced (review) intervals and, if the inventory position (on-hand plus on-order minus backorders) is at or below the reorder point (denoted by *s*), then a replenishment, that raises it to the order-up-to-level (denoted by *S*), is initiated. The review interval is preferably set to unit time.
We want to target an integer *n* for the average time between replenishments E(*t*).

Thus we require,

$$m + E(\tau) = n \tag{3.5}$$

For a given value of τ the expected size of the undershoot

E(undershoot) = $\mu E(\tau)$

Average order size, E(Q) = S-s + E(undershoot)

= S-s + $\mu E(\tau)$

The average time between replenishments,

$$E(t) = E(Q) / \mu$$

$$= [S - s + \mu E(\tau)] / \mu \tag{3.6}$$

Thus, $n = m + E(\tau)$ (3.7)

The target is an integer value of *n*, with the help of *m* and *E (τ)*

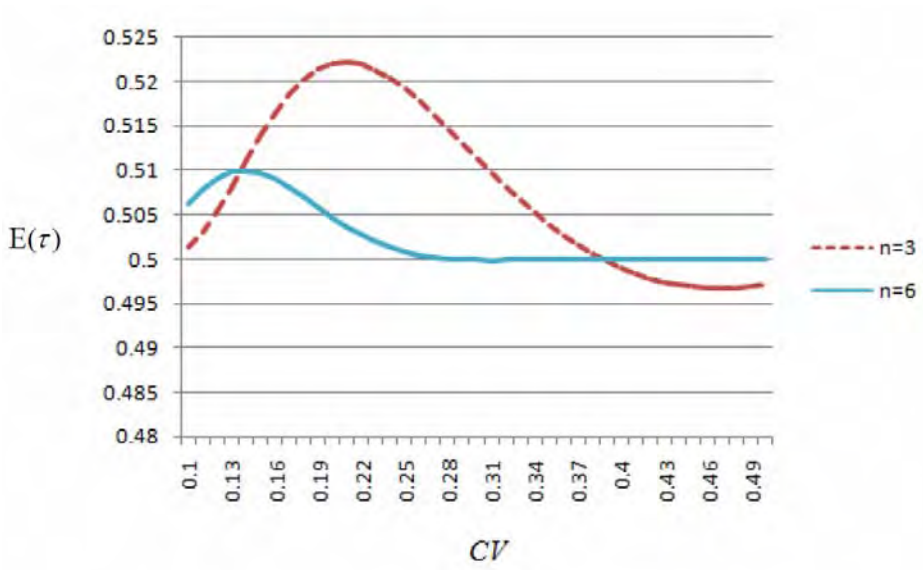


Fig.3.6 (a) The figure shows E(τ) as a functions of CV for n=3 and 6.

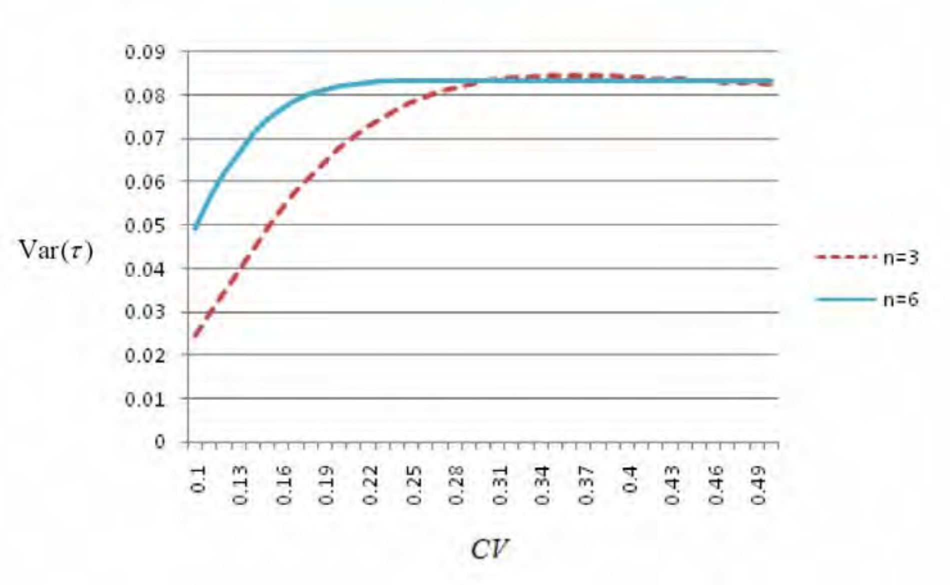


Fig.3.6 (b)The figure shows Var(τ) as a functions of CV for n=3 and 6.

For vary small values of CV the first passage time distribution shows a significant variation of E(τ). As CV increases, the distribution skewed to the right, leading to a apparent uniformity and the Value of E(τ) tends back to 0.5. Rather than providing detailed tables of E(τ) vs. CV and Var(τ) vs. CV, for several practical values of *n* , here for better approximation the fractional polynomial functions of degree 3 with very high values of coefficient of determination(Stata Corp. 2003) was used.

Determining *s* to give a desired fill rate.

For given values of *n* and *CV* we are able to determine E(τ) and Var(τ). Let *X* represent the total demand in the effective lead time $L + \tau$. A shortage will occur if *X* exceeds the reorder point *s*. We can model the situation as a continuous review, reorder point system where the effective lead time $L + \tau$ is a random variable with mean E(τ) + L and variance Var(τ). Under such circumstances *X* has moments (Silver et al., 1998, p. 283)

$$E(X) = [E(\tau) + L]\mu \tag{3.8}$$

and

$$Var(X) = [E(\tau) + L]\,\sigma^2 + \mu^2\, Var(\tau)\,\mu \tag{3.9}$$

In the usual fashion, set

$$s = E(X) + k\sigma_x \tag{3.10}$$

It is convenient to divide through by μ to obtain

$$s/\mu = E(X)/\mu + k\sigma_x/\mu \quad (3.11)$$

Using 8) and (9), this reduces to

$$s = [E(\tau) + L + k\sqrt{[E(\tau)+L](CV)^2 + Var(\tau)}\,]*\mu \quad (3.12)$$

If we assume that X is normally distributed (which we know is an approximation because τ is a random variable; however, Tyworth and O'Neill, 1997, and Silver et al., 1998, pp. 272-3, argue that as long as $\sigma_x/E(X) \leq 0.5$, there is little risk in making the normality assumption), then use of (10) leads to the expected units short per replenishment cycle (Silver et al., 1998)

$$EUSPRC \approx \sigma_x G_u(k) \quad (3.13)$$

Where

$$G_u(k) = \int_k^{\infty} (zo - k)\, \varphi(z_0)\, dz_o \quad (3.14)$$

is the unit normal loss function and $\varphi(z_0)$ is the unit normal density function. There are tables, spreadsheet functions (e.g., in Excel), and other accurate, rational approximations for the *G* function.

The target allowed (average) units short per replenishment cycle

$$AUSPRC = (1 - P_2)E(Q) \quad (3.15)$$

From (13) and (15) and noting that $E(Q) = n\mu$, we thus require

$$G_u(k) = \frac{(1-P_2)n\mu}{\sigma_x} \quad (3.16)$$

Also, using (9), we can express

$$\frac{\sigma_x}{\mu} = \sqrt{[E(\tau)+L](CV)^2 + Var(\tau)} \quad (3.17)$$

Hence, the choice of *k(safety factor)* depends only upon the values of four parameters, namely *CV*, *n*, *L* and *P*2.

Finally, to obtain *S* we proceed as follows. Then from equation 1

$(S - s)/\mu + E(\tau) = n$

Hence

$$S/\mu = s/\mu + n - E(\tau) \tag{3.18}$$

It is seen from (12), (17) and (18) that the key results are normalized with respect to μ, the average demand per unit time.

For Retailer Stage:

Order up to level

$$s = [E(\tau) + L + k\sqrt{[E(\tau)+L](CV)^2 + Var(\tau)}] * \mu$$

Quantity to be demanded at retailer's stage,

$Q_{r} = S - I$

Total OUL at retailer stage= OUL at LVR+ OUL at MVR + OUL at HVR.

Annual cost of inventory= material cost + holding cost + ordering cost.

Material Cost= C_i*D

Where, C_i is the unit price of the product.

Annual inventory cost,

$$= \sum_{j=1}^{n} CD_j + [\sum_{j=1}^{n} h_{Rj} \times \sum_{j=1}^{n} q_{Rj}/2] + \sum_{j=1}^{n} h_R \times SS_{Rj} + \sum_{j=1}^{n} (D_j/q_{Rj}) \times S$$

For distribution center:

Reorder level for the distribution center

$$s_D = [E(\tau) + L + k\sqrt{[E(\tau)+L](CV)^2 + Var(\tau)}] * \mu_D$$

CHAPTER FOUR

ANALYSIS OF THE CASE

4.1 INTRODUCTION

In this section, a case analysis of particular finished good, cement is presented. The case study has been conducted in various retailers of Lafarge Surma Cement Manufacturing Company Ltd. The objective of this case study has been to search for different aspects of inventory control, more specifically their current policy for inventory control and cost involved.

4.2 DATA COLLECTION

Data were collected from various retailers of cement sellers and distribution centre of specific cement manufacturing company at Sylhet. These retailers are well known in the Sylhet metropolitan area. The data were from the ninety-one retailers and one distribution center. Data are presented in the appendix **A**.

4.3 DATA VALIDATION

The collected data were tested for normal distribution. So that it can have a valid ground for further analysis. Normal distributions are symmetrical with a single central peak at the mean (average) of the data. The shape of the curve is described as bell-shaped with the graph falling off evenly on either side of the mean. Fifty percent of the distribution lies to the left of the mean and fifty percent lies to the right of the mean. The spread of a normal distribution is controlled by the standard deviation, σ. The smaller the standard deviation, the more concentrated the data is. It is approximated that,

- 68% of the distribution lies within one standard deviation of the mean.
- 95% of the distribution lies within two standard deviations of the mean.
- 99.7% of the distribution lies within three standard deviations of the mean.

Table 4.1 Demand data frequency distribution for low volume retailer

Observed frequency	Calculated	Expected
6	3.57	12.75
8	.041	6.375
9	1.08	6.375
9	1..08	6.375
7	.06	6.375
7	.06	6.375
5	.30	6.375
	6.56	

X^2_{cal}=6.56

$X_{.05}$=12.592

Degree of freedom=6

Which means the data is accepted in 95% confidence interval.

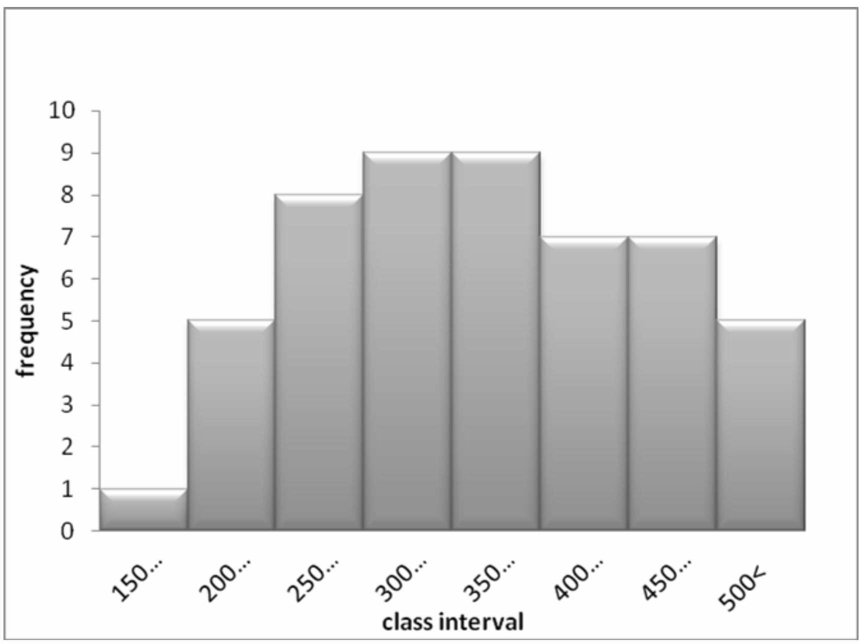


Fig. 4.1 Distribution of the demand data [Appendix A]

4.3.1 INVENTORY SYSTEM OF RETAILER SHOPS

The retailers shop focus on finish goods inventory. They forecast their demand from previous sales data. They don’t follow any scientific method for controlling inventory.. They control inventory of finished goods by setting target for a month’s forecasted demand which is collected from their previous sale experience. For this cause, supply network can’t optimize their inventory level and network carry large amount of inventory cost. At retailer level, we divide the retailer demand according to their demand volume. These are:

1. Low volume retailer 0<low volume retailer<500
2. Medium volume retailer 500< medium volume retailer<1000
3. High volume retailer high volume retailer>1000

Fig.4.2 Actual demand versus forecasted demand at low volume retailer

Fig.4.3 Actual demand versus forecasted demand at medium volume retailer

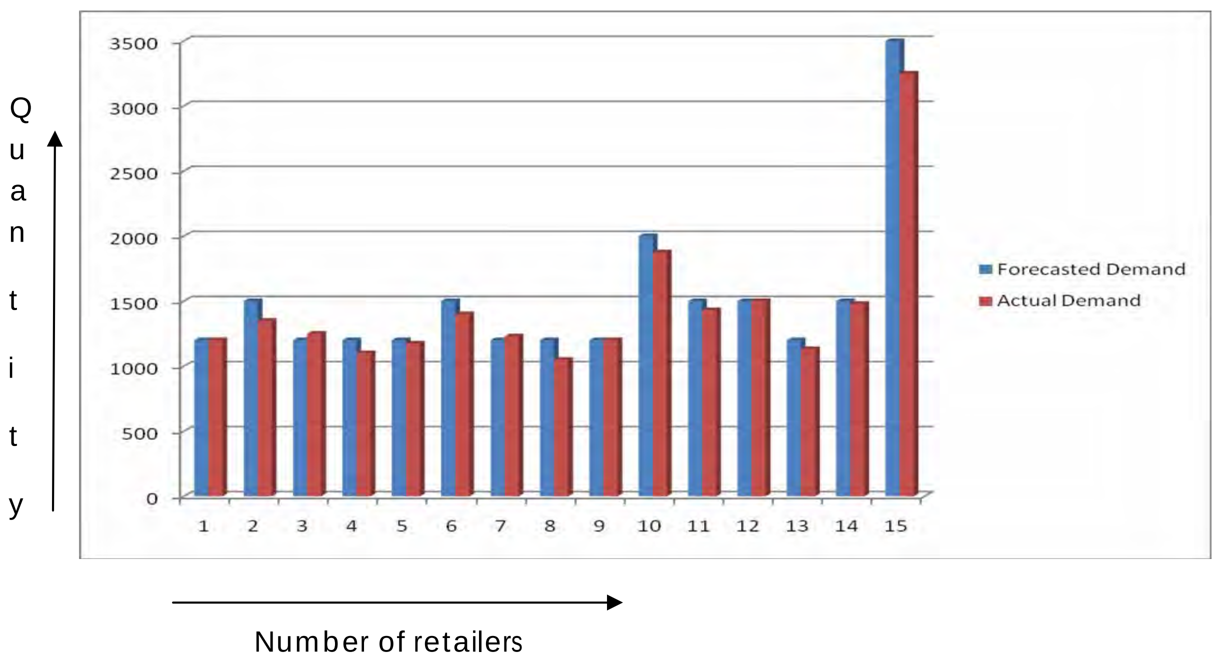


Fig. 4.4 Actual demand versus forecasted demand at high volume retailer

4.3.2 INVENTORY SYSTEM OF THE DISTRIBUTION CENTER

At Sylhet Metropolitan area there is one distribution center for 'Supercrete Brand Cement' named Kibria Complex. This distribution center has a capacity of 125,000 bags of cement. This inventory is carried to fulfill the retailers demand.

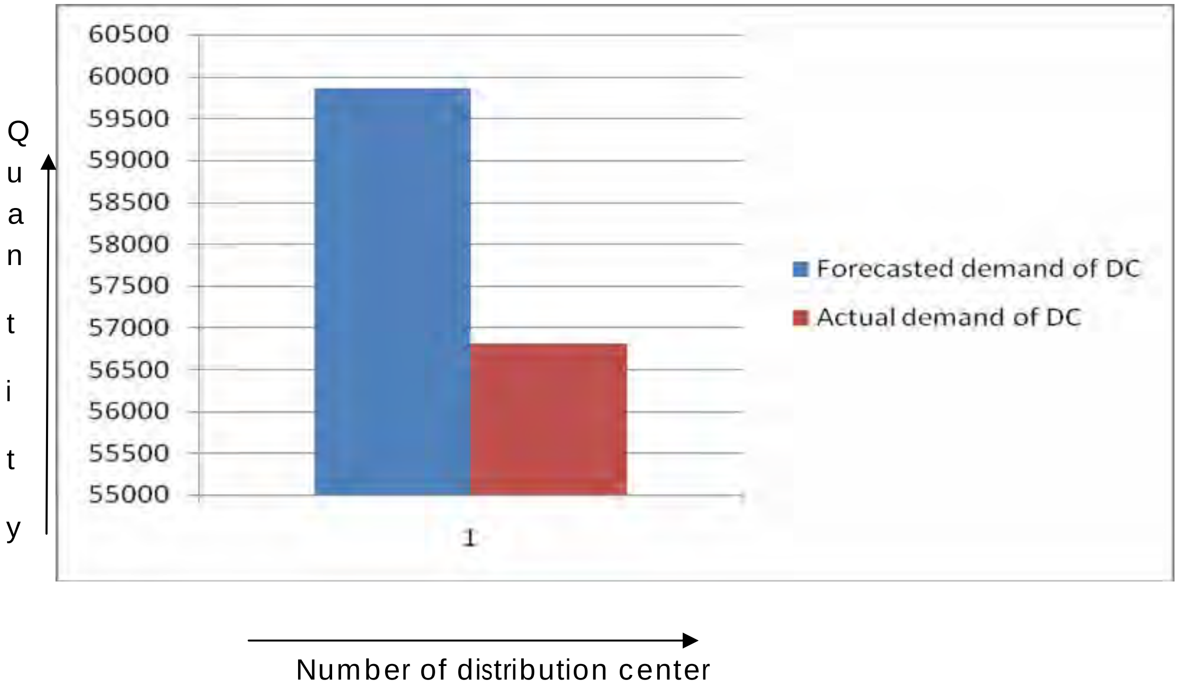


Fig.4.5 Forecasted demand vs. actual demand at DC

CHAPTER FIVE

RESULTS OF CASE ANALYSIS

5.1 ASSUMPTIONS

- Demand for the product is constant and uniform throughout the month.
- The service level is constant (96%) for finished goods.
- Cost per unit of product is constant.
- The manufacturer has ample capacity for instant supply.
- All production related resources (e.g. equipment, machineries, labor etc.) are available for meeting the demand.
- Transportation facilities are available without any disturbance.

5.2 SAMPLE CALCULATION OF ORDER UP TO LEVEL FOR RETAILER

Calculation of low volume retailer data considering 96% service level.

P = Target fill rate= 96%=0.96

n = desired average number of review intervals between consecutive replenishments (1,2,3,4………

L= replenishment lead time=2

σ= standard deviation of demand in unit interval at different type of retailer level

μ= average demand in unit interval at different type of retailer level

CV=σ/ μ= coefficient of variation of demand in a unit time interval at different type of retailer.

N=number of retailer

R_i= Retailers demand.

$$\mu= \quad R_1+R_2+R_3+R_4|R_5+R_6+\ldots\ldots\ldots\ldots\ldots+R_N /N$$

$$\sigma = \sqrt{(R_1 - \mu)^2 + (R_2 - \mu)^2 + (R_3 - \mu)^2 + (R_4 - \mu)^2 + (R_5 - \mu)^2 + \ldots\ldots\ldots\ldots\ldots\ldots / N}$$

From a sample having,

n = 2, L=2, σ= 96, μ=376

Thus, CV=σ/ μ= 96/376=0.255

From the fractional polynomial approximations[App-C] , E(τ)= 0.5349 and Var (τ)=0.0782

From Eq-17

$$\sigma_x / \mu = \sqrt{\{(0.5349+2)*(0.255)^2\}+0.0782}$$

=0.4154

$$G_u(k) = \frac{(1-P_2)n}{\sigma_x / \mu}$$

= (1-.96)*2/0.4154

=0.1925

K = 0.605 from standard Unit Loss Integral Function Table.[Appedix D]

Re-order level,

$$s = [E(\tau) + L + k\sqrt{[E(\tau)+L](CV)^2 + Var(\tau)}] * \mu$$

s =(0.5349+2+0.605*.5349)*376 bags

=1075 bags

Order up to level, S = s+[n-*E(τ)*]μ

=1075 +(2-0.5349)

=1575 bags

Calculation of medium volume retailer considering 96% service level.

P=0.96

n = 2

$L=2$

$\sigma = 151$

$\mu=722$

$CV=\sigma/\mu = 151/722=.209$

From the fractional polynomial approximations[App-C], $E(\tau)= 0.5277$ and $Var(\tau)=0.0782$

From Equation- 17

$$\sigma_x/\mu = \sqrt{\{(0.5277 +2)*(0.209)^2\}+0.0782}$$

$$=0.4344$$

$$G_u(k) = \frac{(1-P_2)n\mu}{\sigma_x}$$

$$= (1-.96)*2/0.4344$$

$$=0.184$$

From unit normal loss integral Table, K = 0.585[Appendix D]

Re Order Point Inventory Level,

$$s = [E(\tau) + L + k\sqrt{[E(\tau) + L](CV)^2 + Var(\tau)}] * \mu$$

$$=(0.5277+2+0.585*0.4350)722$$

$$= \quad 2008 \text{ bags}$$

Order up to level, $S = s+[n-E(\tau)]\mu$

$$=2008 +(2-0.5277)722$$

$$=3071 \text{ bags}$$

Calculation of high volume retailer considering 96% service level.

P=0.96

n = 2

L=2

σ= 180

μ=1200

CV=σ/ μ= 180/1200=0.15

From the fractional polynomial approximations[App-C], E(τ)= 0.2672 and Var (τ)=0.0332

From Equation-17,

$$\sigma_x / \mu = \sqrt{\{(0.2672+2)*(0.15)^2\}+0.0332}$$

$$=0.290$$

$$G_u(k) = \frac{(1-P_2)n\mu}{\sigma_x}$$

$$= (1-.96)*2/0.290$$

$$=0.2757$$

K = 0.275 [from the Unit Normal Loss Integral table. Appendix D]

Reorder Point Inventory Level, $s = [E(\tau) + L + k\sqrt{[E(\tau)+L](CV)^2 + Var(\tau)}] * \mu$

$$=(0.2672+2 +0.275*0.290)*1200$$

$$=2816$$

Order up to Level, S = s+[n-$E(\tau)$]μ

$$=2816 +(2-0.2672)1200$$

$$= 4880$$

Table:5.1 Order up to level for different retailers considering target service level of 96%

Rev. periods between replenishments, n	Order up to level at low volume retailer	Order up to level at medium volume retailer	Order up to level at high volume retailer
2	1575	3071	4840
4	2285	4347	7145
6	2977	5686	9400

Average on hand inventory for low volume retailer shop = 50

Average on hand inventory for medium volume retailer shop = 120

Average on hand inventory for high volume retailer shop = 180

Order quantity (in bags) q **=** Order up to level **–** Avg. on hand inventory (I)

Table: 5.2 Order Quantities for Different Types of Retailers.

Number of periods between replenishments	Quantity to be ordered at LVR(bags)	Quantity to be ordered at MVR(bags)	Quantity to be ordered at HVR(bags)
2	1525	2951	4660
4	2235	4227	6965
6	2927	5566	9220

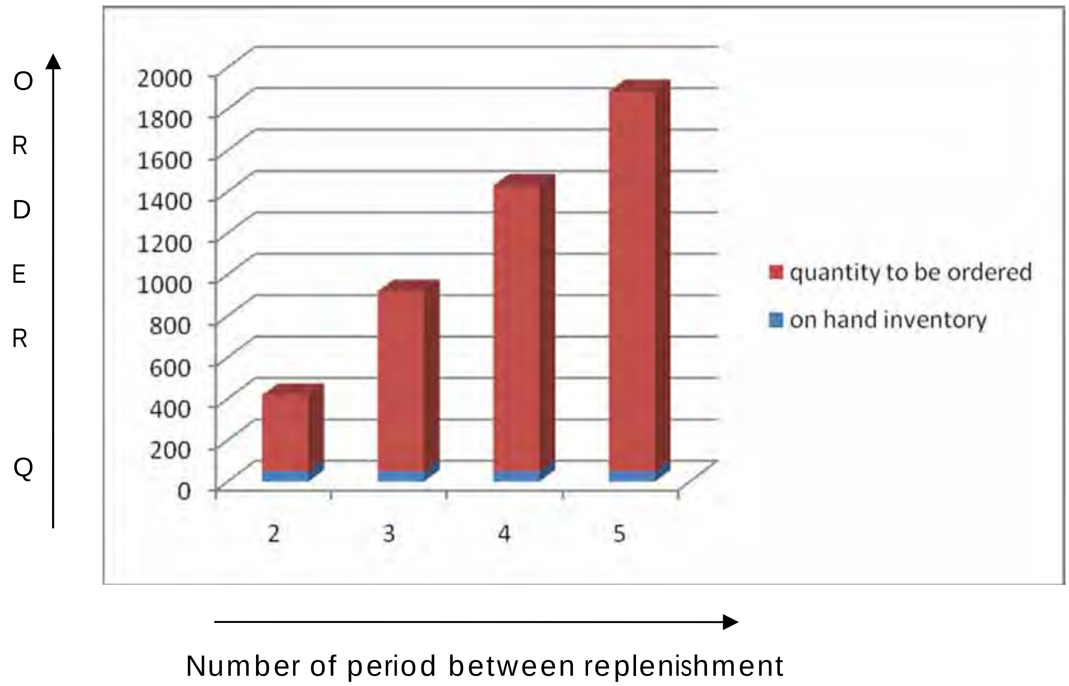


Fig.5.1 Quantity of demand for LVR retailer

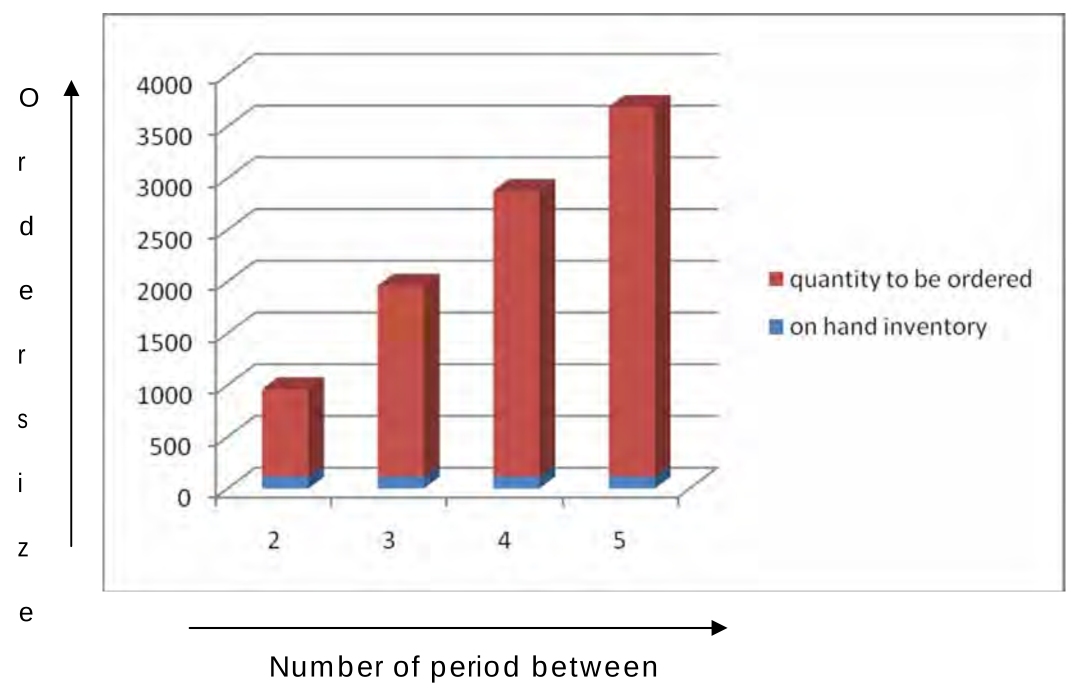


Fig.5.2 quantity of demand for MVR retailer

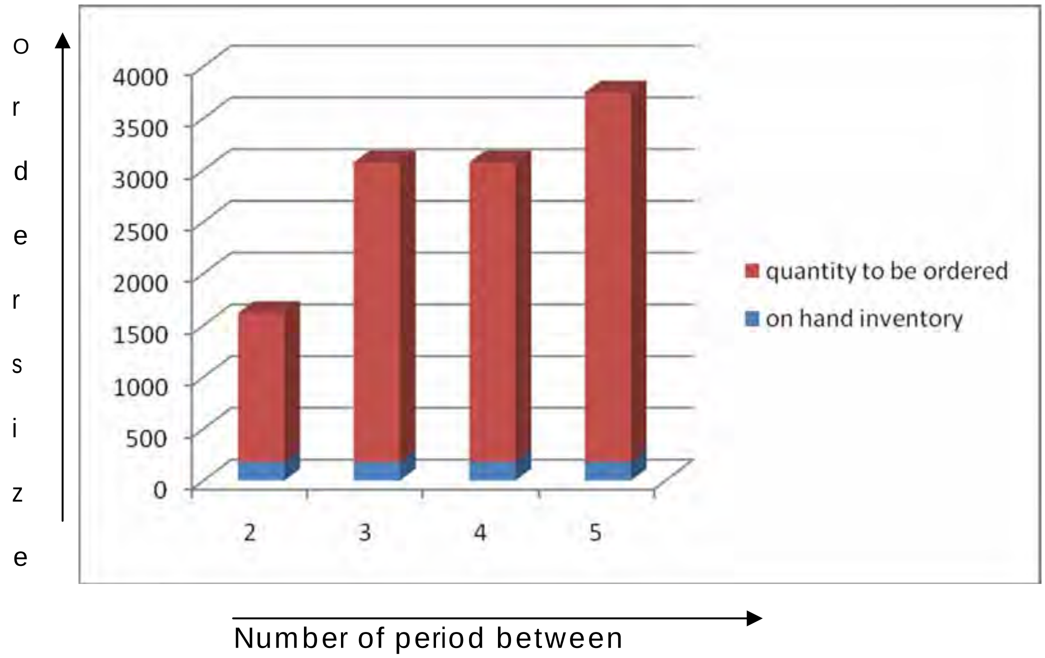


Fig.5.3 quantity of demand for HVR retailer

Calculation of Retailer Annual Inventory Cost:

Total cost= material cost +loading/ unloading cost +holding cost +transportation cost+ other miscellaneous cost+ ordering cost.

Table.5.3 All retailers accumulated monthly inventory cost with different review periods time at target service level

Number of period between replenishments, n	Inventory cost per year (taka)
2	2,71,50,900
4	2,53,74,280
6	2,79,68,287

5.3 CALCULATING ORDER QUANTITY FOR THE DISTRIBUTION CENTER

On hand inventory is fixed for distribution center and the amount of on hand inventory=10,000

Table 5.4 Determining monthly order size for the distribution center

Review periods between replenishments	Quantity to be ordered (bags)	Minimum Monthly Cost involved (TK)
2	66,135	
4	62,200	2,45,96,604
6	65,205	

Total Monthly Cost for the Chain

TCC= D_C+ $\sum_{i=1}^{N} R_i$

TCC= Total Chain Cost.

Dc= cost of distribution centers.

R_i= Cost of retailer shop.

Total Cost of Inventory for the model supply chain with service level 96%

= 4,96,65,133 TK

Table 5.5 Total supply chain inventory cost with different review periods and 96% service level

Number of period between replenishments, n	Inventory cost per year (Tk)
2	33,96,69,360
4	31,94,59,200
6	33,48,92,880

So, from the study of the case we found that the minimum cost occurs for n=4, that is the periods between consecutive replenishments is to be 4.

Field data shows that the distribution center orders for a higher value than the study generated. The company follows the historical data for forecasting then add a growth rate to forecast the next event of replenishment. The aforesaid distribution center has a capacity of 1,25,000 bags.

This study tries to find out a specific ordering system based on periodic review re-order level and order up to level policy. After optimization methodology developed all the echelon members will have a clear view of upstream and downstream inventory positions and then optimize the overall chain.

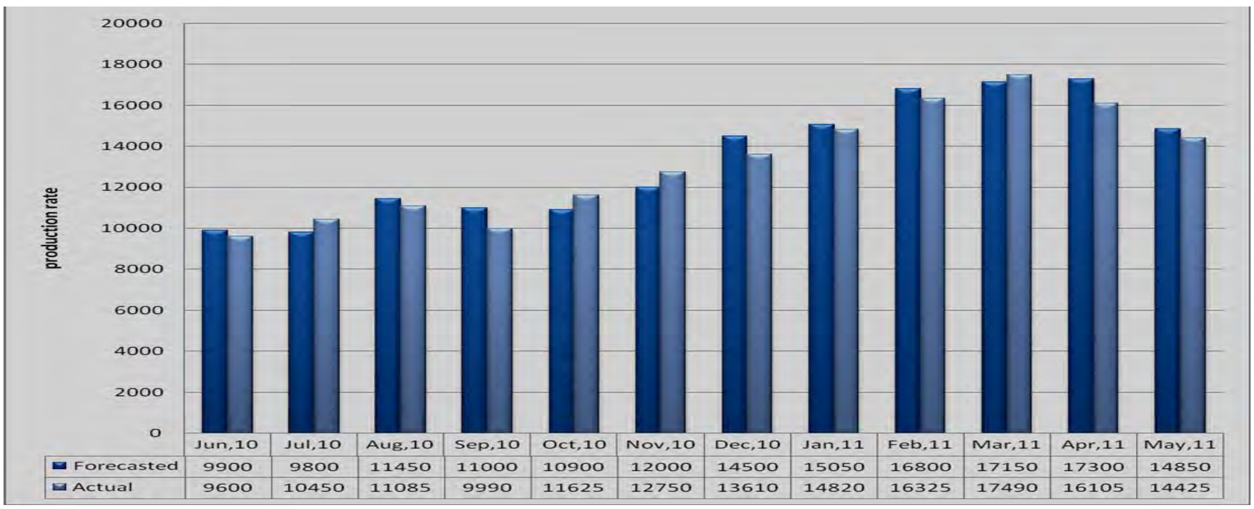


Fig.5.5 Manufacturers inventory forecasted vs. actual(in metric tons)

CHAPTER SIX

OPTIMIZATION METHODOLOGY

6.1 INTRODUCTION

Supply chain inventory replenishment policy studied earlier give a glimpse of the scenario but not the whole picture. Once the model of a system is developed and then have mathematical formulation of the problem then an optimization methodology should be developed. The optimization can be carried out in different ways. One of the methods of optimization is to simulate the whole process. By simulating the process, the whole system can be studied easily without doing it practically. Thus, numerically exercising the model for the inputs in question to see how they affect the output of the system. Discrete event simulation is developed in this study.

6.2 STEPS IN THE SIMULATION STUDY

A simulation study contains different steps that do a sequential process. As one proceeds with a study new insight add up to his understanding. Generally a simulation study involves the following steps:

- Formulate the problem and plan the study
- Collect data and define a model
- Construct a computer program
- Make pilot runs
- Check the validity
- Analyze output data
- Implement results

Multi-echelon inventory system is very complex to be replicated. The events that occur in a supply chain got the complexity as because different partners work with different views. The trading partners involve themselves have diversified goals. The model developed in this research got the agents like retailers, distribution centers and manufacturer. The inventory replenishment in this model starts in the retailer stage and finally ends up with the manufacturer provide the necessary supply to the downstream. The process works as follows

i. Generating demand data for the retailers

ii. Setting objective functions.

iii. Setting inventory polices and costs.

iv Develop the Simulation Algorithm

v. Coding of the simulation program

vi. Analysis of the output.

A random function generator built in was used to generate demand data within a range. The objective function set was to find the optimum order size for different echelon members and corresponding minimum cost of inventory. The simulation program was run for a month. The policy in the simulation was the desired fill rate that an echelon partner wants to full fill. The coding was done in C++ .

6.3 REPLENISHMENT POLICY

The simulation replicates what would happen in the application of (R, s, S) inventory policy. The inventory position is only updated at each review instant. The review period for this simulation study got unit value i.e. the inventory is reviewed on a daily basis. But the replenishment only occurs at the designated, *n* values. And the periods between two consecutive replenishment can vary from a minimum 2 to maximum 6.

Inventory position at a review instant = Inventory position after a possible order at previous review – total demand in the review interval.

If the updated inventory position, I is less than or equal to s, then an order of size Q= (S-s) is placed. The total demand is randomly generated from a normal distribution of forecasted demand data with parameters μ, and σ. For a particular set of demand data, the coefficient of variation simulated under different service level conditions. The simulation then verifies the different policies for minimum overall cost of inventory and an adapted order size is given as an output at the same time this adapted order size is kept in memory to be accumulated for the total amount to be ordered by the distribution center that serve these retailers. The distribution center procures the total of the retailers demand from the manufacturer and it keeps its inventory position up to that level. The manufacturer in turn fulfills the total demand of the distribution centers.

6.4 SELECTION OF POLICY AND PARAMETER SETS

A set of parameters are set in the program to generate different solution sets, and hence search for a minimum cost criterion. The service level of set for the sample run got a range from 80 % to 98%. The normal integral loss function table was provided with the program to get the desired k factor. The coefficient of variation is selected at an arithmetic progression with a step size of 0.05. The minimum value set to 0.10 and the maximum one at 0.50. The values for other parameters i.e. unit holding cost, loading and unloading cost, transportation cost, other miscellaneous cost was found from the field survey.

6.5 CODING OF THE SIMULATION PROGRAM

The simulation is programmed in C++ platform. The parameters are set which suit the system most and can find the optimum ordering policy for the lowest value of the objective function which is minimization of the total supply chain cost. The codes are provided in the appendix F.

6.6 SIMULATION INPUT/OUTPUT FORM

The input file included in the program get the input parameters from a text file called ‘in.txt’ and the out puts are preserved in two files, one is inventory status file and the other is ‘out.txt’. Both these files are formatted properly so the reader of the file can have clear idea of the simulation output.

The input /output of the simulation is presented in the Appendix F.

CHAPTER SEVEN

OPTIMIZATION RESULTS

7.1 INTRODUCTION

The optimization engine worked for the different policies and the output is presented in two files and attached in the Appendix. F

The outputs found shows that if we review the inventory in every 4 periods then, low volume retailer's inventory cost is augmented. For high volume retailers we should make frequent replenishments to avoid stock out. The details are presented in the following sections.

7.2 SIMULATION INPUT STRUCTURE

For a Sample Run:

The annotations used here,

Pc = Unit Product cost

hc = Holding cost per unit/month

lc = loading and unloading cost per unit

tc = transportation cost per unit

mc = other miscellaneous cost (obsolence, insurance,repair etc.) per unit

cpo = cost per order.

No. of Distribution Center: 02

No. of Retailers under the Distribution Center 01: 03

2 /No of Distribution center under the manufacturer

3 /No of retailers under Distribution center 1

576 60 /Mean daily demand and standard deviation for retailer 1

pc	hc	lc	tc	mc	oc	
400	4.5	5.0	4.5	1.0	1000	/Associated costs for retailer 1
3	0.92	0.94	0.96			/policies with retailer 1: target order fill rates
1200	100				/Mean daily demand and standard deviation for retailer 02	
395	5.0	5.5	5.5	1.5	1500	/Associated costs for retailer 02:
2	0.93	0.95				/policies with retailer 01: target order fill rates
2500	200				/Mean daily demand and standard deviation for retailer 03	
390	4.8	5.0	4.5	1.0	2500	/Associated costs for retailer 03
3	0.94	0.95	0.96			/policies with retailer 03: target order fill rates.
380	4.5	3.0	3.5	1.5	5500	/Associated costs for Distribution center 1:

7.3 SIMULATION OUTPUTS

7.3.1 OUTPUT FORMATS

- **The out put from the program is in three formats**
  - The direct program out put
  - Out put in text file
  - Inventory status report in text file

- **Sought parameters:**
  - Monthly Optimum Order Size for the Retailers and Distribution Centers
  - Monthly Minimum Inventory costs for the Echelons
  - Minimum Chain Cost

- **Inventory Status File Shows the Ordering Follow ups**
  - When replenishment is taking place
  - When the inventory falls below the re-order level
  - The stock out positions.

7.3.2 SAMPLE OUT PUT TEXT FILE

DISTRIBUTION CENTER NO: 1

RETAILER NO 1:

Policy 1 (for P2 = 0.92):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
2	0.104	0.5001	0.7745	-0.61	16443	6838845.00
2	0.204	0.5263	0.4047	-0.02	16933	7042195.00
2	0.304	0.5354	0.2877	0.24	16704	6947160.00
2	0.404	0.5220	0.2265	0.41	16641	6921015.00
4	0.104	0.5040	1.2773	-1.23	15597	6479755.00
4	0.204	0.5129	0.7525	-0.58	15575	6470625.00
4	0.304	0.5029	0.5696	-0.31	15435	6412525.00
4	0.404	0.4983	0.4562	-0.11	15601	6481415.00
6	0.104	0.5075	1.7074	-1.69	16804	6978660.00
6	0.204	0.5047	1.1137	-1.04	16638	6909770.00
6	0.304	0.5003	0.8554	-0.72	16633	6907695.00
6	0.404	0.4999	0.6847	-0.49	16560	6877400.00

minimum cost: 6412525.00   order size: 15435

Policy 2 (for P2 = 0.94):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
2	0.104	0.5001	0.5809	-0.33	16740	6962100.00
2	0.204	0.5263	0.3035	0.20	16527	6873705.00
2	0.304	0.5354	0.2158	0.44	16640	6920600.00
2	0.404	0.5220	0.1699	0.69	16716	6952140.00
4	0.104	0.5040	0.9580	-0.85	15590	6476850.00
4	0.204	0.5129	0.5644	-0.30	15412	6402980.00
4	0.304	0.5029	0.4272	-0.06	15759	6546985.00
4	**0.404**	**0.4983**	**0.3422**	**0.11**	**15335**	**6371025.00***
6	0.104	0.5075	1.2806	-1.23	16694	6933010.00
6	0.204	0.5047	0.8353	-0.69	16745	6954175.00
6	0.304	0.5003	0.6416	-0.42	16948	7038420.00
6	0.404	0.4999	0.5135	-0.22	16681	6927615.00

minimum cost: 6371025.00 order size: 15335

Policy 3 (for P2 = 0.96):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
2	0.104	0.5001	0.3872	0.02	16514	6868310.00
2	0.204	0.5263	0.2024	0.48	16692	6942180.00
2	0.304	0.5354	0.1438	0.79	16794	6984510.00
2	0.404	0.5220	0.1133	0.83	17110	7115650.00
4	0.104	0.5040	0.6387	-0.42	15387	6392605.00
4	0.204	0.5129	0.3763	0.04	15600	6481000.00
4	0.304	0.5029	0.2848	0.25	15498	6438670.00
4	0.404	0.4983	0.2281	0.40	15712	6527480.00

6	0.104	0.5075	0.8537	-0.72	16470	6840050.00
6	0.204	0.5047	0.5569	-0.29	16917	7025555.00
6	0.304	0.5003	0.4277	-0.06	16773	6965795.00
6	0.404	0.4999	0.3423	0.11	16820	6985300.00

minimum cost: 6392605.00 order size: 15387

Optimum Monthly order size for retailer 1: 15335 and corresponding cost: 6371025.00

The optimum order size resulted for *n*= 4 i.e. inventory should be replenished at an interval of 4 review periods.

So, Minimum cost is achieved by maintaining a target fill rate of 94%.

It is worth noting that 92% fill rate did not give optimality for minimum cost criteria what general sense of less fill rate means less cost. So optimality criterion set in for this policy, is found to be true.

7.3.3 SUMMARIZED OUTPUT FOR THE ECHELONS TOGETHER

DISTRIBUTION CENTER NO: 1

Retailer 1	***n***	**CV**	**Exp. Valu (τ)**	**G(k)**	**k Factor**	**Monthly Order**	**Total Cost**
Policy 1 (for P2 = 0.92):	4	0.304	0.5029	0.5696	-0.31	15435	6412525.00
Policy 2 (for P2 = 0.94):	4	0.404	0.4983	0.3422	0.11	15335	6371025.00
Policy 3 (for P2 = 0.96):	4	0.104	0.5040	0.6387	-0.42	15387	6392605.00

Optimum Monthly order size for retailer 1: 15335 and corresponding cost: 6371025.00

Retailer 2

Policy 1 (for P2 = 0.93)	4	0.383	0.4986	0.4168	-0.04	31769	13115212.50
Policy 2 (for P2 = 0.95):	4	0.283	0.5048	0.3750	0.04	31966	13196475.00

Optimum Monthly order size for retailer 2: 31769 and corresponding cost: 13115212.50

Retailer 3

Policy 1 (for P2 = 0.94):	4	0.180	0.5145	0.6140	-0.38	67270	27282030.18
Policy 2 (for P2 = 0.95):	4	0.080	0.4863	1.2899	-1.24	66344	26906722.39
Policy 3 (for P2 = 0.96):	4	0.380	0.4987	0.2398	0.37	66923	27141391.08

Optimum Monthly order size for retailer 3: 66344 and corresponding cost: 26906722.39

Total retailers	Total order size	Total Cost (from manufacturer to DC 1)
3	113448	44544840.00

DISTRIBUTION CENTER NO: 2

Retailer 1

Policy 1 (for P2 = 0.9):	4	0.083	0.4896	2.3033	-2.30	16108	6799743.80
Policy 2 (for P2 = 0.95):	4	0.083	0.4896	1.1517	-1.09	15972	6742392.59

Optimum Monthly order size for retailer 1: 15972 and corresponding cost: 6742392.59

Retailer 2

Policy 1 (for P2 = 0.9):	4	0.320	0.5017	0.6855	-0.49	26806	11273714.00
Policy 2 (for P2 = 0.92):	4	0.320	0.5017	0.5484	-0.28	26638	11203322.00
Policy 3 (for P2 = 0.95):	4	0.120	0.5100	0.6920	-0.50	26676	11219244.00

Optimum Monthly order size for retailer 2: 26638 and corresponding cost: 11203322.00

Total retailers	Total order size	Total Cost (from manufacturer to DC 2)
2	42610	17137220.00

Total Chain Cost: 126020734.49

7.3.4 SAMPLE INVENTORY STATUS TEXT FILE

The simulation was done for a span of a month comprising with 30 review periods of equal size. The replenishment initiated at every 4^{th} period and received after 2 days after (L=2 days). Here only a part of the run is presented.

For retailer 1 and Policy no 1(0.92): n=4, S=3355 s=1341:

Review period	inventory status	random demand
1	3355	612

2	2743		618
3	2125		541
4	1584	Order Placed	529
5	1055		582
6	473		
6	2244	Order Received	517
7	1727		529
	Inventory below reorder level / order placed		
8 chk stock out?:			570
9	2785	Order Received	606
10	2179		598
11	1581		581
	Inventory below reorder level /order placed		
12 chk stock out?:			524
13	2831	Order Received	568
14	2263		540
15	1723		599
	Inventory below reorder level/order placed		
16 chk stock out?:			550
17	2805	Order Received	568
18	2237		543
19	1694		585
	Inventory below reorder level /order placed		
20 chk stock out?:			579
21	2776	Order received	591

22 2185 535

23 1650 576

Inventory below reorder level /order placed

24 chk stock out?: 631

25 2724 Ordered received 599

26 2125 626

27 1499 538

Inventory below reorder level /order placed

28 chk stock out?: 556

29 2799 Order received 537

30 2262 635

n=4, S=3410 s=1394:

Rev periods	inventory status	random demand
1	3410	570
2	2840	635
3	2205	527
4	1678 Order Placed	591
5	1087	527
6	560	
6	2292 Order received	568
7	1724	627

Inventory below reorder level /order placed

8 chk stock out?: 521

9 2889Order received 623

10	2266	550
11	1716	543

Inventory below reorder level /order placed

12	chk stock out?:	608
13	2802	565
14	2237	540
15	1697	591

Inventory below reorder level /order placed

16	chk stock out?:	588
17	2822	570
18	2252	571
19	1681	587

Inventory below reorder level /order placed

20	chk stock out?:	592
21	2818	618
22	2200	615
23	1585	590

Inventory below reorder level /order placed

24	chk stock out?:	602
25	2808	583
26	2225	561
27	1664	538

Inventory below reorder level /order placed

28	chk stock out?:	604
29	2806	582

30	2284	624(end stock 1660)

7.4 SIMULATION OUTPUT ANALYSIS

To optimize inventory at different echelons of supply chain, initial task is to develop coordination with all the echelon members of the network. Due to lack of coordination, wrong information passes through the chain and true customer demand is distorted. Result is the bullwhip effect that increases inventory level at different echelons in the supply chain unnecessarily. The network carries large amount of inventory without proper justification. So, coordination is an important factor for inventory optimization of supply chains. Another important factor is service level. If service level is low customers show dissatisfaction about product and chain can loss the market share of the product. On the contrary high service level will overburden the chain carrying large amount of inventory and associated costs. The study in this thesis tries to develop a relation between these two opposing characteristics' i.e., between service level and order size. It is shown how much of the finished product is needed for a specific service level and what is happening after as demands are being placed at random.

7.5 LIMITATIONS OF THE STUDY

Due to lack of available resources especially the relevant information, the case study has not reached the perfection level that was expected. The demand distribution is based on historical data, but no seasonal effect and other abnormal conditions were included. This type of analysis needs de-seasonalized and trend corrected with the available forecasting models like 'Hault's Model' or 'Winter's Model' but it was not done in this study. The other limitations are as follows:

- Manufacturer inventory level was not calculated, as it was assumed that manufacturer has ample capacity.
- It was assumed that the manufacturer can instantly supply the distribution center, it is not possible.
- Back order cost was not calculated.
- The lead time was set to be constant but lead time for different echelons may vary.
- The optimization proposed here needs that demand data be updated daily this can

only be possible when the echelons are in real time reliable data communication network.

CHAPTER EIGHT

CONCLUSION AND RECOMMENDATION

8.1 CONCLUSION

In this thesis, inventory cost optimization was used as the main object and it is proved that by the adapted policies the inventory cost is locally optimized first then the optimization works for the remaining echelons located upwards. The study identified order up to level and reorder level for each echelon considering a target fill rate at the retailer stage. The inventory cost for total chain was the final output of the optimization process. It is proved that order fill rate is an important parameter for the optimization of supply chain inventory cost. The study also proved that the chain can altogether achieve a high fill rate at an optimal cost if the members interact.

8.2 RECOMMENDATIONS FOR FUTURE WORK

There is a broad scope for future research in the arena of multi-echelon inventory system optimization. This research can be extended to consider price discount in different echelons, so as to facilitate the economy of scale. Joint ordering policy consideration holds a significant area for low cost supply chain development. There exists a wide scope to analyze the demand for other distribution like exponential distribution and gamma distribution. The collection of raw material for the finished product is an area of extreme importance for a process industry like cement, paper, fertilizer, etc. The scope for working with an extended chain that incorporates the raw material sourcing will get an elaborate scope for further study. This optimization method can be applied in other networks like retail chain or other service oriented nets.

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APPENDICES

Appendix A

DEMAND DATA SHEET

DATA 01: DAILY DEMAND FOR LOW VOLUME RETAILER

Actual demand	Forecasted demand	Transportation cost per bag (load &unload)
300	300	4.5
475	500	4.5
350	300	4.5
450	500	4.5
250	300	4.5
425	400	4.5
450	500	4.5
375	350	4
450	500	4
360	400	4
500	500	4
300	300	4
320	350	4
250	250	4
300	350	4
450	400	4
150	250	4
340	400	4
225	250	3.5

250	300	3.5
250	250	3.5
470	500	3.5
375	400	3.5
280	250	3.5
500	500	3.5
250	300	4
380	400	4
320	300	4
500	500	4
250	300	4
250	250	4
490	500	4
400	450	4
220	250	4
400	400	4
475	500	4
400	400	4
375	350	4
500	500	4
375	400	4
275	250	4
500	450	4
475	500	4
300	300	3.5

400	500	3.5
300	250	3.5
250	300	3.5
300	350	3.5
500	450	3.5
200	250	3.5
400	500	3.5

DATA 02: DAILY DEMAND FOR MEDIUM VOLUME RETAILER

Actual demand	Forecasted demand	Transportation cost per bag (load &unload)
800	750	4
1000	860	4
600	750	4
1000	920	4
600	550	4
600	600	4
600	570	4
600	540	4
800	750	4
700	750	4
600	600	4
900	850	4
650	600	4
900	830	4

600	650	4
600	560	4
900	900	4
600	640	4
600	600	3.5
700	650	3.5
600	530	3.5
1000	900	3.5
900	820	3.5
600	560	3.5
600	575	3.5

DATA 03: DAILY DEMAND FOR LOW VOLUME RETAILER

Actual demand	Forecasted demand	Transportation cost per bag (load &unload)
1200	1200	4
1500	1350	4
1200	1250	4
1200	1100	4
1200	1175	4
1500	1400	4
1200	1230	4
1200	1050	4
1200	1200	4
2000	1875	4
1500	1430	4
1500	1500	4
1200	1130	4
1500	1480	4
3500	3250	4

Appendix B

QUESTIONNAIRE FOR DATA COLLECTION

For retailers

1. How many bags of cement order from distribution centre?
2. How many bags of cement sales at each period?
3. What is the transportation mode for conveying cement?
4. How many bags of cement storage as a on hand inventory at each period?
5. What is the lead time (time between placing order and receive goods)?

For distribution center

1. Is there any planning dept. in distribution center?
2. Is there any forecasting technique which they follow?
3. What is the highest capacity (in terms of bag) of the distribution center?
4. Location of the distribution centre?
5. How many bags are kept as the safety stock in each distribution centre?
6. How DC maintain the inventory?
7. If yes, how Lafarge manage it to reach the product to DC?
8. What are the transportation modes?
9. How they deliver product to retailers?
10. Who bear the cost of the transportation, DC or retailer?
11. How many bags are transported daily to different retailer shop?

Appendix C

Fractional polynomial approximations of E(τ) as a function of CV for selected values of review periods

n	Adj. R^2	Approximation (note C =CV)
2	0.9863	$0.53608 + 0.44271(C^{-1} - 3.333) + 1.7634(C^{-½} - 1.826) + 1.0508(C^{-½} \ln C + 2.198)$
3	0.9835	$0.51211 + 1.8652(C - .3) - 1.1430(C^{2} - .09) + 3.1367(C^{2} \ln C + 0.1084)$
4	0.9740	$0.50325 + 2.1455(C^{½} - .5477) - .65943(C^{½} \ln C + .6594) + .50973(C^{½} (\ln C)^{2} - .794)$
5	0.9726	$0.50079 - 0.13438(C^{-1} - 3.333) - 0.39946(C^{-½} - 1.826) - .28188(C^{-½} \ln C + 2.198)$
6	0.9784	$0.50004 - 0.00237(C^{-2} - 11.11) - 0.03307(C^{-1} - 3.333) - 0.02296 (C^{-1} \ln C = 4.013)$

Fractional polynomial approximations of Var(τ) as a function of CV for selected values of review periods

n	Adj. R^2	Approximation (note C =CV)
2	0.9993	$0.07401+0.53380(C - 0.3) -0.58217(C^{2} -0.09)$
3	0.9965	$0.08283+ 0.36794(C^{½} - 0.5477) - 0.35809(C^{½} \ln C+.6594)$
4	0.9933	$0.08387-0.12888(C^{-½} - 1.826)-0.10939(\ln C +1.204)$
5	0.9931	$0.8371 + 0.1876 (C^{-1} - 3.333) + 0.00887(C^{-1} \ln C = 4.013)$
6	0.9936	$0.08352 - 0.00078(C^{-2} - 11.11) + 0.00503(C^{-1} - 3.333)$

APPENDIX D

Table Unit Normal Loss Integrals

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.4	2.4027	2.4126	2.4226	2.4325	2.4424	2.4523	2.4623	2.4722	2.4821	2.4921
-2.3	2.3037	2.3136	2.3235	2.3334	2.3433	2.3532	2.3631	2.3730	2.3829	2.3928
-2.2	2.2049	2.2148	2.2246	2.2345	2.2444	2.2542	2.2641	2.2740	2.2839	2.2938
-2.1	2.1065	2.1163	2.1261	2.1360	2.1458	2.1556	2.1655	2.1753	2.1852	2.1950
-2.0	2.0085	2.0183	2.0280	2.0378	2.0476	2.0574	2.0672	2.0770	2.0868	2.0966
-1.9	1.9111	1.9208	1.9305	1.9402	1.9500	1.9597	1.9694	1.9792	1.9890	1.9987
-1.8	1.8143	1.8239	1.8336	1.8432	1.8529	1.8626	1.8723	1.8819	1.8916	1.9013
-1.7	1.7183	1.7278	1.7374	1.7470	1.7566	1.7662	1.7758	1.7854	1.7950	1.8046
-1.6	1.6232	1.6327	1.6422	1.6516	1.6611	1.6706	1.6801	1.6897	1.6992	1.7087
-1.5	1.5293	1.5386	1.5480	1.5574	1.5667	1.5761	1.5855	1.5949	1.6044	1.6138
-1.4	1.4367	1.4459	1.4551	1.4643	1.4736	1.4828	1.4921	1.5014	1.5107	1.5200
-1.3	1.3455	1.3546	1.3636	1.3727	1.3818	1.3909	1.4000	1.4092	1.4118	1.4275
-1.2	1.2561	1.2650	1.2738	1.2827	1.2917	1.3006	1.3095	1.3185	1.3275	1.3365
-1.1	1.1686	1.1773	1.1859	1.1946	1.2034	1.2121	1.2209	1.2296	1.2384	1.2473
-1.0	1.0883	1.0917	1.1002	1.1087	1.1172	1.1257	1.1342	1.1428	1.1514	1.1600
-0.9	1.0004	1.0086	1.0168	1.0250	1.0333	1.0416	1.0499	1.0582	1.0665	1.0749
-0.8	0.9202	0.9281	0.9361	0.9440	0.9520	0.9600	0.9680	0.9761	0.9842	0.9923
-0.7	0.8429	0.8505	0.8581	0.8658	0.8734	0.8812	0.8889	0.8967	0.9045	0.9123
-0.6	0.7687	0.7759	0.7833	0.7906	0.7980	0.8054	0.8128	0.8203	0.8278	0.8353
-0.5	0.6978	0.7047	0.7117	0.7187	0.7257	0.7328	0.7399	0.7471	0.7542	0.7614
-0.4	0.6304	0.6370	0.6436	0.6503	0.6569	0.6637	0.6704	0.6772	0.6840	0.6909
-0.3	0.5668	0.5730	0.5792	0.5855	0.5918	0.5981	0.6045	0.6109	0.6174	0.6239
-0.2	0.5069	0.5127	0.5186	0.5244	0.5304	0.5363	0.5424	0.5484	0.5545	0.5606
-0.1	0.4509	0.4564	0.4618	0.4673	0.4728	0.4784	0.4840	0.4897	0.4954	0.5011
-0.0	0.3989	0.4040	0.4090	0.4141	0.4193	0.4244	0.4297	0.4349	0.4402	0.4456
0.0	0.3989	0.3940	0.3890	0.3841	0.3793	0.3744	0.3697	0.3649	0.3602	0.3556
0.1	0.3509	0.3464	0.3418	0.3373	0.3328	0.3284	0.3240	0.3197	0.3154	0.3111
0.2	0.3069	0.3027	0.2986	0.2944	0.2904	0.2863	0.2824	0.2784	0.2745	0.2706
0.3	0.2668	0.2630	0.2592	0.2555	0.2518	0.2481	0.2445	0.2409	0.2374	0.2339
0.4	0.2304	0.2270	0.2236	0.2203	0.2169	0.2137	0.2104	0.2072	0.2040	0.2009
0.5	0.1978	0.1947	0.1917	0.1887	0.1857	0.1828	0.1799	0.1771	0.1742	0.1714
0.6	0.1697	0.1659	0.1633	0.1606	0.1580	0.1554	0.1528	0.1503	0.1478	0.1453
0.7	0.1429	0.1405	0.1381	0.1358	0.1334	0.1312	0.1289	0.1267	0.1245	0.1223
0.8	0.1202	0.1181	0.1160	0.1140	0.1120	0.1100	0.1080	0.1061	0.1042	0.1023
0.9	0.1004	0.0986	0.0968	0.0950	0.0933	0.0916	0.0899	0.0882	0.0865	0.0849
1.0	0.0833	0.0817	0.0802	0.0787	0.0772	0.0757	0.0742	0.0728	0.0714	0.0700
1.1	0.0686	0.0673	0.0660	0.0647	0.0634	0.0621	0.0609	0.0596	0.0584	0.0573
1.2	0.0561	0.0550	0.0538	0.0527	0.0517	0.0506	0.0495	0.0485	0.0475	0.0465
1.3	0.0455	0.0446	0.0436	0.0427	0.0418	0.0409	0.0400	0.0392	0.0383	0.0375

1.4	0.0367	0.0359	0.0351	0.0343	0.0336	0.0328	0.0321	0.0314	0.0307	0.0300
1.5	0.0293	0.0287	0.0280	0.0274	0.0267	0.0261	0.0255	0.0249	0.0244	0.0238
1.6	0.0232	0.0227	0.0222	0.0217	0.0211	0.0206	0.0202	0.0197	0.0192	0.0187
1.7	0.0183	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146
1.8	0.0143	0.0139	0.0136	0.0132	0.0129	0.0126	0.0123	0.0120	0.0116	0.0113
1.9	0.0111	0.0108	0.0105	0.0102	0.0100	0.0097	0.0094	0.0092	0.0090	0.0087
2.0	0.0085	0.0083	0.0081	0.0078	0.0076	0.0074	0.0072	0.0070	0.0068	0.0067
2.1	0.0065	0.0063	0.0061	0.0060	0.0058	0.0056	0.0055	0.0053	0.0052	0.0050
2.2	0.0049	0.0048	0.0046	0.0045	0.0044	0.0042	0.0041	0.0040	0.0039	0.0038
2.3	0.0037	0.0036	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028
2.4	0.0027	0.0026	0.0026	0.0025	0.0024	0.0024	0.0023	0.0022	0.0021	0.0021

APPENDIX E

C Plus Plus CODING FOR THE OPTIMIZATION PROGRAM

```
#include<stdio.h>

#include<conio.h>

#include<math.h>

#include<stdlib.h>

#include<time.h>


int avg_demand, sd_demand, low, high, n, L, s, S, total_order, policy_first_time;

long total_order_size, policy_adapted_order_size;

float first_CV, CV, P2, evT, varT, del_x_by_mu, k, cpb, cpo, range, nlf;

double total_cost, policy_min_cost, chain_total_cost;


FILE *inputfile,*outfile, *inventoryfile;


void cost_per_bag()

{

  float pc, lc, hc, vc, mc;


  printf("\nProduct cost per unit: ");

  fscanf(inputfile,"%f",&pc);

  printf("%g\n",pc);


  printf("Loading and unloading cost per unit: ");

  fscanf(inputfile,"%f",&lc);
```

```
  printf("%g\n",lc);

  printf("Holding cost per unit: ");
  fscanf(inputfile,"%f",&hc);
  printf("%g\n",hc);

  printf("Vehicle cost per unit: ");
  fscanf(inputfile,"%f",&vc);
  printf("%g\n",vc);

  printf("Miscellenious cost per unit: ");
  fscanf(inputfile,"%f",&mc);
  printf("%g\n",mc);

  cpb = (pc + lc + hc + vc + mc);
}

void cost_per_order()
{
  printf("Cost per order: ");
  fscanf(inputfile,"%f",&cpo);
  printf("%g\n",cpo);
}

void calculate_total_cost()
{
```

```
  total_cost = total_order_size * cpb + total_order * cpo;
}

int data_for_retailer(int j)
{
  int N;

  printf("\n\nDATA FOR RETAILER %d:\n",j+1);

  n = 2; //desired (integer) average number of review intervals between consecutive replenishments
  L = 2; //replenishment lead time

  fscanf(inputfile,"%d",&avg_demand);
  printf("\nAverage demand in a unit time interval(mu): %d\n",avg_demand);

  fscanf(inputfile,"%d",&sd_demand);
  printf("Standard deviation of demand in a unit time of interval(sigma): %d",sd_demand);

  first_CV = float(sd_demand) / avg_demand;
  low = avg_demand - sd_demand;
  high = avg_demand + sd_demand;
  range = (float)(high - low);

  cost_per_bag();
  cost_per_order();
```

```
  fscanf(inputfile,"%d",&N);
  printf("\nHow many policies(different P2)?: %d",N);


  return N;
}


void calculate_expected_value_of_T()
{
  if(n == 2)
    evT = 0.53608 + 0.44271 * (1/CV - 3.333) + 1.7634 * (1/sqrt(CV) - 1.826) + 1.0508 * ((1/sqrt(CV)) *
log(CV) + 2.198);
  if(n == 3)
    evT = 0.51211 + 1.8652 * (CV - 0.3) - 1.1430 * (CV*CV - 0.09) + 3.1367 * (CV*CV * log(CV) +
0.1084);
  if(n == 4)
    evT = 0.50325 + 2.1455 * (sqrt(CV) - 0.5477) - 0.65943 * (sqrt(CV) * log(CV) + 0.6594) + 0.50973 *
(sqrt(CV) * log(CV)*log(CV) - 0.794);
  if(n == 5)
    evT = 0.50079 - 0.13438 * (1/CV - 3.333) - 0.39946 * (1/sqrt(CV) - 1.826) - 0.28188 * ((1/sqrt(CV)) *
log(CV) + 2.198);
  if(n == 6)
    evT = 0.5004 - 0.00237 * (1/(CV*CV) - 11.11) - 0.03307 * (1/CV - 3.333) - 0.02296 * (1/CV *
log(CV) + 4.013);
}


void calculate_variance_of_T()
{
```

```
 if(n == 2)
   varT = 0.07401 + 0.53380 * (CV - .3) - 0.58217 * (CV*CV - 0.09);
 if(n == 3)
   varT = 0.08283 + 0.36794 * (sqrt(CV) - 0.5477) - 0.35809 * (sqrt(CV) * log(CV) + 0.6594);
 if(n == 4)
   varT = 0.08387 - 0.12888 * (1/sqrt(CV) - 1.826) - 0.10939 * (log(CV) + 1.204);
 if(n == 5)
   varT = 0.08371 + 0.01876 * (1/CV - 3.333) + 0.00887 * (1/CV * log(CV) + 4.013);
 if(n == 6)
   varT = 0.08352 - 0.00078 * (1/(CV*CV) - 11.11) + 0.00503 * (1/CV - 3.333);
}

float calculate_safety_factor()
{
 float normal_loss_function, value, z, sf = 0.5302, first_value, prev_track[9];
 int i, j;
 FILE *infile;

 infile = fopen("unli.txt", "r");
 del_x_by_mu = sqrt((evT + L) * CV*CV + varT);
 normal_loss_function = (1 - P2) * n / del_x_by_mu;
 nlf = normal_loss_function;
// printf("\nnormal loss function: %f",normal_loss_function);
 for(i=0;i<=9;i++)
 {
   fscanf(infile,"%f",&value);
```

```
}

for(j=1;j<=25;j++)
{
  fscanf(infile,"%f",&z);
 // printf("\n%f    ",z);

  fscanf(infile,"%f",&first_value);
 // printf("%f",first_value);

  if(first_value<=normal_loss_function)
  {
    if(first_value==normal_loss_function)
    {
      sf = z;
     // printf("\n%d",i);
     // printf("\n%f",sf);
      fclose(infile);
      return sf;
    }
    for(i=1;i<=9;i++)
    {
      fscanf(infile,"%f",&value);
     // printf("%f    ",value);
      if(value>normal_loss_function)
      {
```

```
       sf = z - i * 0.01;
      // printf("\n%d",i);
       //printf("\n%f",sf);
       fclose(infile);
       return sf;
      }
    }
    sf = z - 0.1;
    fclose(infile);
    return sf;
  }
  else
  {
    for(i=1;i<=9;i++)
      fscanf(infile,"%f",&value);
  }
}

for(j=1;j<=25;j++)
{
  fscanf(infile,"%f",&z);
 // printf("\n%f    ",z);

  fscanf(infile,"%f",&first_value);
 // printf("%f",first_value);
  if(first_value<=normal_loss_function)
```

```
{
 if(first_value==normal_loss_function)
 {
   sf = z;
  // printf("\n%d",i);
  // printf("\n%f",sf);
   fclose(infile);
   return sf;
 }
 for(i=0;i<=8;i++)
 {
   //fscanf(infile,"%f",&value);
  // printf("%f    ",value);
   if(prev_track[i]<normal_loss_function)
   {
    sf = (z-0.1) + i * 0.01;
   // printf("\n%d",i);
    //printf("\n%f",sf);
    fclose(infile);
    return sf;
   }
 }
 sf = z + 0.09;
 fclose(infile);
 return sf;
}
```

```
  else
  {
    for(i=0;i<=8;i++)
      fscanf(infile,"%f",&prev_track[i]);
  }
 }

 fclose(infile);
 return sf;
}

void calculate_s()
{
  s = (int)(avg_demand * (evT + L + k * del_x_by_mu));
}

void calculate_S()
{
 S = (int)(s + (n - evT) * avg_demand);
}

void calculate_s_S()
{
  calculate_expected_value_of_T();
  calculate_variance_of_T();
  k = calculate_safety_factor();
```

```
  calculate_s();
  calculate_S();
}

int random_demand()
{
 int r;

//low = 1;
//range = 900;
 r = low + (int)(range * rand()/(RAND_MAX + 1.0));
// printf("\n%d",r);
 return r;
}

void place_order(int I)
{
  int order_size;
  total_order++;
  order_size = S - I;
  //printf("  %d   %d",total_order,order_size);
  total_order_size += order_size;
}

int replenish(int prev, int R)
{
```

```
  int i, j, I, rd;

  j = prev;

  for(i=1;i<=L;i++)
  {
    rd = random_demand();
    fprintf(inventoryfile,"  %d\n",rd);
    I = prev - rd;
    //fprintf(outfile, " I%d: %d %d",R,I,rd);
    prev = I;
    R++;
   fprintf(inventoryfile,"\n                    %d    %d\n",R,prev);
  }
  place_order(j);
  prev = prev + (S - j);
  return prev;
}

void inventory_update()
{
 int month_length = 30, R, rd, previous_position, I;

 total_order = 0;
 total_order_size = 0;
 //s = 1515;
```

```
//S = 3000;
//n = 4;
//L = 2;
previous_position = S;
fprintf(inventoryfile,"\n\n        n=%d,  S=%d  s=%d:\n",n,S,s);
fprintf(inventoryfile,"\n                    review period        inventory status    random demand\n");
fprintf(inventoryfile,"\n                  ----------------        ----------------    -------------\n");
for (R=1;R<=month_length;R++)
{
  fprintf(inventoryfile,"\n                        %d    %d\n",R,previous_position);
  if((R%n)==0)
  {
    fprintf(inventoryfile,"   Replenish ");
    previous_position = replenish(previous_position, R);
    R = R + L - 1;
  }
  else
  {
    rd = random_demand();
    fprintf(inventoryfile,"%d\n",rd);
    I = previous_position - rd;
   // fprintf(outfile, " I%d: %d %d",R,I,rd);
    if(I<=s)
    {
      fprintf(inventoryfile,"     below reorder level  ");
      fprintf(inventoryfile,"\n                        %d   no action",R+1);
```

```
      place_order(I);

      R = R + L - 1;

      previous_position = S;

     }

     else

      previous_position = I;

   }

 }

 //printf("\n%d ,total_order_size");

}


void report()

{

  fprintf(outfile, "\n                    %d      %.3f      %10.4f      %.4f      %5.2f      %ld
%.2lf",n,CV,evT,nlf,k,total_order_size,total_cost);

}


void minimum_cost()

{

  if (policy_first_time)

  {

    policy_first_time = 0;

    policy_min_cost = total_cost;

    policy_adapted_order_size = total_order_size;

  }

  else

  {
```

```
    if (policy_min_cost>total_cost)
    {
      policy_min_cost = total_cost;
      policy_adapted_order_size = total_order_size;
    }
  }
}

int main()
{
    int dc, i, r, j, N, l, DC_Q = 0, first_time;
    long  adapted_order_size;
    float total, cpb_DC, cpo_DC;
    double min_cost;

    chain_total_cost = 0;

    inputfile = fopen("input.txt", "r");
            outfile = fopen("out.txt", "w");
            inventoryfile = fopen("inventory.txt","w");
    //for(i=0;i<5;i++)
      // j = random_demand();
      //calculate_safety_factor();
      //inventory_update();

            fscanf(inputfile,"%d",&dc);
```

```
printf("\nNumber of Distribution center: %d",dc);


      for(i=0;i<dc;i++)
      {


        fscanf(inputfile,"%d",&r);
        printf("\nFor distribution center %d, no. of retailers: %d",i+1,r);


  fprintf(outfile, "\n\n                         DISTRIBUTION CENTER NO: %d",i+1);
  fprintf(outfile, "\n                         --------------------------");


  for(j=0;j<r;j++)
  {
    first_time = 1;
    N = data_for_retailer(j);
    fprintf(outfile, "\n\n\n\nRETAILER NO %d:",j+1);
    fprintf(outfile, "\n---------------");
    if (N<=3)
    {
      for(l=0;l<N;l++)
      {
            fscanf(inputfile,"%f",&P2);
            printf("\nFor policy %d value of the fill rate (P2): %g",l+1,P2);
        fprintf(outfile, "\n     Policy %d (for P2 = %g):",l+1,P2);
        fprintf(inventoryfile, "\n\n\nFor retailer %d and Policy no %d :\n",j+1,l+1);
        fprintf(outfile, "\n\n                 Period     CV     Expected Value of T  G(u)    k Factor
Total Order Size    Total Cost  ");
```

```
fprintf(outfile, "\n    ------      ----  -----------------------  -----  --------------- ----- ----------------\n");
policy_first_time = 1;
for(n=2;n<=6;n+=2)
{
 CV = first_CV;
 do{
   calculate_s_S();
   inventory_update();
   calculate_total_cost();
   report();
   minimum_cost();
   CV += 0.1;
 }while(CV<=0.5);
}
fprintf(outfile, "\n\n                              minimum cost: %.2lf   order size:
%ld\n",policy_min_cost,policy_adapted_order_size);
//fprintf(outfile, "\n\n          ------------------------------------------------------------------");
if (first_time)
{
  first_time = 0;
  min_cost = policy_min_cost;
  adapted_order_size = policy_adapted_order_size;
}
else
{
  if (min_cost>policy_min_cost)
  {
```

```
          min_cost = policy_min_cost;

          adapted_order_size = policy_adapted_order_size;

        }

      }

    }

    printf("\nOptimum Monthly order size for retailer %d:  %ld\n",j+1,adapted_order_size);

    fprintf(outfile, "\n                              Optimum Monthly order size for retailer %d:  %ld         and
corresponding cost: %.2lf\n\n",j+1,adapted_order_size,min_cost);

    chain_total_cost += min_cost;

    DC_Q += adapted_order_size;

  }

  else

  {

    printf("\nPolicies should not be more than three");

    j--;

  }

}


printf("\n\nDATA FOR DISTRIBUTION CENTER %d:\n",i+1);

printf("\nTotal order size: %d",DC_Q);

total_order_size = DC_Q;

total_order = r;

cost_per_bag();

cost_per_order();

calculate_total_cost();

chain_total_cost += total_cost;

printf("\nTotal cost from manufacturer to distribution center %d is = %.2lf,\n",i+1,total_cost);
```

```
      fprintf(outfile, "\n\nDISTRIBUTION CENTER %d:",i+1);
      fprintf(outfile, "\n------------------------");
      fprintf(outfile, "\n              -----------------------------------------------------------------");
      fprintf(outfile, "\n\n                  Total retailers   Total order size       Total Cost  ");
      fprintf(outfile, "\n                  -----------------  ------------------  -----------------  \n");
      fprintf(outfile, "\n                      %d            %ld             %.2lf  ",r,total_order_size,total_cost);
      fprintf(outfile, "\n\n              -----------------------------------------------------------------\n\n\n");
    }
    printf("\nTotal chain cost is = %.2lf,\n",chain_total_cost);
    fprintf(outfile, "\n\n\n          Total Chain Cost: %.2lf  ",chain_total_cost);
   fclose(inputfile);
   fclose(outfile);
   fclose(inventoryfile);
   getch();
}
```

APPENDIX F

INPUT DATA FOR OPTIMIZATION PROGRAM

2 /No of Distribution center under the manufacturer

3 /No of retailers under Distribution center 1

576 60 /Forcasted mean daily demand and standard deviation for retailer 1

400 4.5 5.0 4.5 1.0 1000 /Associated costs for retailer 1: unit product cost, unit holding cost, unit transportation cost, unit other miscellaneous cost, ordering cost

3 0.92 0.94 0.96 /policies associated with retailer 1: target order fill rates

1200 100 /Forcasted mean daily demand and standard deviation for retailer 02

395 5.0 5.5 5.5 1.5 1500 /Associated costs for retailer 02: unit product cost, unit holding cost, unit transportation cost, unit other miscellaneous cost, ordering cost

2 0.93 0.95 /policies associated with retailer 01: target order fill rates

2500 200 /Forcasted mean daily demand and standard deviation for retailer 03

390 4.8 5.0 4.5 1.0 2500 /Associated costs for retailer 03: unit product cost, unit holding cost, unit transportation cost, unit other miscellaneous cost, ordering cost

3 0.94 0.95 0.96 /policies associated with retailer 03: target order fill rates

380 4.5 3.0 3.5 1.5 5500 /Associated costs for Distribution center 1: unit product cost, unit holding cost, unit transportation cost, unit other miscellaneous cost, ordering cost

2 /No of retailers under Distribution center 2

600 50 /Forcasted mean daily demand and standard deviation for retailer 01

410 4.5 3.0 3.0 1.2 1000 /Associated costs for retailer 01: unit product cost, unit holding cost, unit transportation cost, unit other miscellaneous cost, ordering cost

2 0.90 0.95 /policies associated with retailer 01: target order fill rates

1000 120 /Forcasted mean daily demand and standard deviation for retailer 02

405 4 5 4 1 6000 /Associated costs for retailer 02: unit product cost, unit holding cost, unit transportation cost, unit other miscellaneous cost, ordering cost

3 0.90 0.92 0.95 /policies associated with retailer 02: target order fill rates

390 4 3.0 3.5 1.5 4000 /Associated costs for Distribution center 2: unit product cost, unit holding cost, unit transportation cost, unit other miscellaneous cost, ordering cost

OPTIMIZATIOM OUTPUT

DISTRIBUTION CENTER NO: 1

RETAILER NO 1:

Policy 1 (for P2 = 0.92):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
2	0.104	0.5001	0.7745	-0.61	16443	6838845.00
2	0.204	0.5263	0.4047	-0.02	16933	7042195.00
2	0.304	0.5354	0.2877	0.24	16704	6947160.00
2	0.404	0.5220	0.2265	0.41	16641	6921015.00
4	0.104	0.5040	1.2773	-1.23	15597	6479755.00
4	0.204	0.5129	0.7525	-0.58	15575	6470625.00
4	0.304	0.5029	0.5696	-0.31	15435	6412525.00
4	0.404	0.4983	0.4562	-0.11	15601	6481415.00
6	0.104	0.5075	1.7074	-1.69	16804	6978660.00
6	0.204	0.5047	1.1137	-1.04	16638	6909770.00
6	0.304	0.5003	0.8554	-0.72	16633	6907695.00

6	0.404	0.4999	0.6847	-0.49	16560	6877400.00

minimum cost: 6412525.00 order size: 15435

Policy 2 (for P2 = 0.94):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
------	----	-----------------	-----	---------	------------	--------
2	0.104	0.5001	0.5809	-0.33	16740	6962100.00
2	0.204	0.5263	0.3035	0.20	16527	6873705.00
2	0.304	0.5354	0.2158	0.44	16640	6920600.00
2	0.404	0.5220	0.1699	0.69	16716	6952140.00
4	0.104	0.5040	0.9580	-0.85	15590	6476850.00
4	0.204	0.5129	0.5644	-0.30	15412	6402980.00

4	0.304	0.5029	0.4272	-0.06	15759	6546985.00
4	0.404	0.4983	0.3422	0.11	15335	6371025.00
6	0.104	0.5075	1.2806	-1.23	16694	6933010.00
6	0.204	0.5047	0.8353	-0.69	16745	6954175.00
6	0.304	0.5003	0.6416	-0.42	16948	7038420.00
6	0.404	0.4999	0.5135	-0.22	16681	6927615.00

minimum cost: 6371025.00 order size: 15335

Policy 3 (for P2 = 0.96):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
------	----	-----------------	-----	---------	------------	--------
2	0.104	0.5001	0.3872	0.02	16514	6868310.00

6942180.00	2	0.204	0.5263	0.2024	0.48	16692
6984510.00	2	0.304	0.5354	0.1438	0.79	16794
7115650.00	2	0.404	0.5220	0.1133	0.83	17110
6392605.00	4	0.104	0.5040	0.6387	-0.42	15387
6481000.00	4	0.204	0.5129	0.3763	0.04	15600
6438670.00	4	0.304	0.5029	0.2848	0.25	15498
6527480.00	4	0.404	0.4983	0.2281	0.40	15712
6840050.00	6	0.104	0.5075	0.8537	-0.72	16470
7025555.00	6	0.204	0.5047	0.5569	-0.29	16917
6965795.00	6	0.304	0.5003	0.4277	-0.06	16773
6985300.00	6	0.404	0.4999	0.3423	0.11	16820

minimum cost: 6392605.00 order size: 15387

Optimum Monthly order size for retailer 1: 15335 and corresponding cost: 6371025.00

RETAILER NO 2:

Policy 1 (for P2 = 0.93):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
------	----	-----------------	-----	---------	------------	--------
2	0.083	0.5261	0.8991	-0.78	34752	14357700.00
2	0.183	0.5201	0.3894	0.01	34912	14423700.00
2	0.283	0.5360	0.2673	0.39	34936	14433600.00
2	0.383	0.5259	0.2072	0.46	34779	14368837.50
2	0.483	0.5037	0.1706	0.69	34527	14264887.50
4	0.083	0.4896	1.6123	-1.59	32233	13306612.50
4	0.183	0.5144	0.7076	-0.52	32584	13451400.00
4	0.283	0.5048	0.5250	-0.24	32680	13491000.00
4	0.383	0.4986	0.4168	-0.04	31769	13115212.50
4	0.483	0.5008	0.3431	0.11	32337	13349512.50
6	0.083	0.4913	2.0803	-2.08	34727	14332387.50

6	0.183	0.5064	1.0375	-0.95	34751	14342287.50
6	0.283	0.5008	0.7878	-0.63	34717	14328262.50
6	0.383	0.4998	0.6257	-0.40	34545	14257312.50
6	0.483	0.5008	0.5144	-0.22	34754	14343525.00

minimum cost: 13115212.50 order size: 31769

Policy 2 (for P2 = 0.95):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
------	----	-----------------	-----	---------	------------	--------
2	0.083	0.5261	0.6422	-0.42	34271	14159287.50
2	0.183	0.5201	0.2781	0.27	34588	14290050.00
2	0.283	0.5360	0.1909	0.52	34842	14394825.00
2	0.383	0.5259	0.1480	0.67	34874	14408025.00
2	0.483	0.5037	0.1219	0.89	34421	14221162.50

4	0.083	0.4896	1.1517	-1.09	32643	13475737.50
4	0.183	0.5144	0.5054	-0.20	32135	13266187.50
4	0.283	0.5048	0.3750	0.04	31966	13196475.00
4	0.383	0.4986	0.2977	0.22	32431	13388287.50
4	0.483	0.5008	0.2451	0.35	32140	13268250.00
6	0.083	0.4913	1.4859	-1.46	34638	14295675.00
6	0.183	0.5064	0.7410	-0.57	34920	14412000.00
6	0.283	0.5008	0.5627	-0.30	35327	14579887.50
6	0.383	0.4998	0.4469	-0.10	34532	14251950.00
6	0.483	0.5008	0.3674	0.06	34883	14396737.50

minimum cost: 13196475.00 order size: 31966

Optimum Monthly order size for retailer 2: 31769 and corresponding cost: 13115212.50

RETAILER NO 3:

Policy 1 (for P2 = 0.94):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
------	----	-----------------	-----	---------	------------	--------
2	0.080	0.5352	0.8190	-0.67	73507	29829886.20
2	0.180	0.5189	0.3393	0.12	73277	29736667.21
2	0.280	0.5359	0.2314	0.49	72011	29223557.42
2	0.380	0.5265	0.1789	0.56	72987	29619130.21
2	0.480	0.5046	0.1471	0.68	73235	29719644.61
4	0.080	0.4863	1.5479	-1.52	68113	27623698.07
4	0.180	0.5145	0.6140	-0.38	67270	27282030.18
4	0.280	0.5052	0.4539	-0.11	67868	27524399.57
4	0.380	0.4987	0.3597	0.08	67695	27454282.67
4	0.480	0.5006	0.2959	0.22	68079	27609917.87
6	0.080	0.4860	2.0021	-2.00	72524	29406476.31
6	0.180	0.5067	0.8986	-0.78	73514	29807723.30
6	0.280	0.5009	0.6809	-0.48	71693	29069672.02
6	0.380	0.4998	0.5401	-0.26	72659	29461191.81

6 0.480 0.5007 0.4436 -0.09 72537
29411745.21

minimum cost: 27282030.18 order size: 67270

Policy 2 (for P2 = 0.95):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
------	----	-----------------	-----	---------	------------	--------
2	0.080	0.5352	0.6825	-0.48	72765	29529153.61
2	0.180	0.5189	0.2827	0.25	73284	29739504.31
2	0.280	0.5359	0.1928	0.51	72035	29233284.62
2	0.380	0.5265	0.1491	0.67	71471	29004695.43
2	0.480	0.5046	0.1226	0.78	72839	29559145.81
4	0.080	0.4863	1.2899	-1.24	66344	26906722.39
4	0.180	0.5145	0.5117	-0.21	66858	27115046.58
4	0.280	0.5052	0.3782	0.04	67786	27491164.97
4	0.380	0.4987	0.2998	0.21	67525	27385381.68

4	0.480	0.5006	0.2466	0.35	66582	27003183.79
6	0.080	0.4860	1.6684	-1.65	72974	29588861.31
6	0.180	0.5067	0.7488	-0.58	72269	29303124.82
6	0.280	0.5009	0.5675	-0.31	73822	29932555.70
6	0.380	0.4998	0.4501	-0.10	72348	29335143.52
6	0.480	0.5007	0.3696	0.06	71963	29179103.02

minimum cost: 26906722.39 order size: 66344

Policy 3 (for P2 = 0.96):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
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2	0.080	0.5352	0.5460	-0.27	71935	29192754.62
2	0.180	0.5189	0.2262	0.41	72924	29593596.31
2	0.280	0.5359	0.1543	0.65	72905	29585895.61

2	0.380	0.5265	0.1193	0.80	71729	29109262.82
2	0.480	0.5046	0.0981	0.91	72781	29535638.41
4	0.080	0.4863	1.0319	-0.94	67322	27303105.78
4	0.180	0.5145	0.4093	-0.03	67491	27371601.48
4	0.280	0.5052	0.3026	0.21	68366	27726238.97
4	0.380	0.4987	0.2398	0.37	66923	27141391.08
4	0.480	0.5006	0.1973	0.50	68333	27712864.07
6	0.080	0.4860	1.3348	-1.29	72521	29405260.41
6	0.180	0.5067	0.5991	-0.36	73102	29640739.71
6	0.280	0.5009	0.4540	-0.11	73183	29673569.01
6	0.380	0.4998	0.3601	0.08	72497	29395533.22
6	0.480	0.5007	0.2957	0.22	72216	29281643.92

minimum cost: 27141391.08 order size: 66923

Optimum Monthly order size for retailer 3: 66344 and corresponding cost: 26906722.39

DISTRIBUTION CENTER 1:

Total retailers	Total order size	Total Cost
3	113448	44544840.00

DISTRIBUTION CENTER NO: 2

RETAILER NO 1:

Policy 1 (for P2 = 0.9):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
2	0.083	0.5261	1.2844	-1.24	17345	7329386.71
2	0.183	0.5201	0.5563	-0.29	17418	7360170.81
2	0.283	0.5360	0.3818	0.03	17238	7284264.81
2	0.383	0.5259	0.2960	0.22	17180	7259806.21
2	0.483	0.5037	0.2438	0.36	17399	7352158.51

4	0.083	0.4896	2.3033	-2.30	16108	6799743.80
4	0.183	0.5144	1.0108	-0.92	16177	6828841.10
4	0.283	0.5048	0.7500	-0.58	16256	6862155.40
4	0.383	0.4986	0.5954	-0.35	16507	6968002.10
4	0.483	0.5008	0.4902	-0.18	16161	6822093.90
6	0.083	0.4913	2.9718	-2.50	17440	7359448.21
6	0.183	0.5064	1.4821	-1.45	17538	7400774.81
6	0.283	0.5008	1.1255	-1.05	17273	7289024.31
6	0.383	0.4998	0.8939	-0.77	17291	7296614.91
6	0.483	0.5008	0.7348	-0.56	17173	7246854.31

minimum cost: 6799743.80 order size: 16108

Policy 2 (for P2 = 0.95):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
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7428486.21	2	0.083	0.5261	0.6422	-0.42	17580
7264444.91	2	0.183	0.5201	0.2781	0.27	17191
7406557.81	2	0.283	0.5360	0.1909	0.52	17528
7310410.21	2	0.383	0.5259	0.1480	0.67	17300
7340350.91	2	0.483	0.5037	0.1219	0.89	17371
6742392.59	4	0.083	0.4896	1.1517	-1.09	15972
6855829.90	4	0.183	0.5144	0.5054	-0.20	16241
6888300.80	4	0.283	0.5048	0.3750	0.04	16318
6897999.90	4	0.383	0.4986	0.2977	0.22	16341
6780767.30	4	0.483	0.5008	0.2451	0.35	16063
7293663.01	6	0.083	0.4913	1.4859	-1.46	17284
7470355.32	6	0.183	0.5064	0.7410	-0.57	17703
7367460.51	6	0.283	0.5008	0.5627	-0.30	17459
7421016.41	6	0.383	0.4998	0.4469	-0.10	17586
7313482.91	6	0.483	0.5008	0.3674	0.06	17331

minimum cost: 6742392.59 order size: 15972

Optimum Monthly order size for retailer 1: 15972 and corresponding cost: 6742392.59

RETAILER NO 2:

Policy 1 (for P2 = 0.9):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
2	0.120	0.4984	0.8311	-0.69	29012	12246028.00
2	0.220	0.5300	0.4741	-0.15	29392	12405248.00
2	0.320	0.5344	0.3445	0.11	28716	12122004.00
2	0.420	0.5187	0.2742	0.28	29210	12328990.00
4	0.120	0.5100	1.3840	-1.35	27638	11622322.00
4	0.220	0.5114	0.8946	-0.77	27124	11406956.00
4	0.320	0.5017	0.6855	-0.49	26806	11273714.00
4	0.420	0.4984	0.5525	-0.28	27184	11432096.00
6	0.120	0.5103	1.9039	-1.90	29018	12188542.00
6	0.220	0.5035	1.3306	-1.29	28877	12129463.00
6	0.320	0.5001	1.0297	-0.94	28859	12121921.00

6	0.420	0.5001	0.8289	-0.69	29448	12368712.00

minimum cost: 11273714.00 order size: 26806

Policy 2 (for P2 = 0.92):

Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order	Total Cost
2	0.120	0.4984	0.6649	-0.46	29061	12266559.00
2	0.220	0.5300	0.3793	0.04	28872	12187368.00
2	0.320	0.5344	0.2756	0.27	29740	12551060.00
2	0.420	0.5187	0.2194	0.43	30008	12663352.00
4	0.120	0.5100	1.1072	-1.03	26879	11304301.00
4	0.220	0.5114	0.7157	-0.53	27553	11586707.00
4	0.320	0.5017	0.5484	-0.28	26638	11203322.00
4	0.420	0.4984	0.4420	-0.09	27690	11644110.00
6	0.120	0.5103	1.5231	-1.50	28564	11998316.00
6	0.220	0.5035	1.0645	-0.98	28928	12150832.00

	6	0.320	0.5001	0.8238	-0.68	29385
12342315.00						
	6	0.420	0.5001	0.6631	-0.45	28625
12023875.00						

minimum cost: 11203322.00 order size: 26638

Policy 3 (for P2 = 0.95):

Total Cost	Period	CV	Exp. Value(Tau)	G(k)	k Factor	Monthly Order
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	2	0.120	0.4984	0.4155	-0.04	28760
12140440.00						
	2	0.220	0.5300	0.2370	0.38	29238
12340722.00						
	2	0.320	0.5344	0.1723	0.58	28789
12152591.00						
	2	0.420	0.5187	0.1371	0.72	29077
12273263.00						
	4	0.120	0.5100	0.6920	-0.50	26676
11219244.00						
	4	0.220	0.5114	0.4473	-0.10	27398
11521762.00						
	4	0.320	0.5017	0.3428	0.11	26862
11297178.00						
	4	0.420	0.4984	0.2762	0.27	27107
11399833.00						
	6	0.120	0.5103	0.9519	-0.84	28847
12116893.00						

11864236.00	6	0.220	0.5035	0.6653	-0.46	28244
12206559.00	6	0.320	0.5001	0.5149	-0.22	29061
12106837.00	6	0.420	0.5001	0.4144	-0.04	28823

minimum cost: 11219244.00 order size: 26676

Optimum Monthly order size for retailer 2: 26638 and corresponding cost: 11203322.00

DISTRIBUTION CENTER 2:

Total retailers	Total order size	Total Cost
2	42610	17137220.00

Total Chain Cost: 126020734.49