THERMOPHORESIS EFFECT ON THE MHD FLOW OVER AN INCLINED STRETCHING SHEET WITH HEAT GENERATION AND MAGNETIC FIELD

[1]M Enamul Karim and [2]Md. Alamgir Kabir

[1]Department of Mathematics, Comilla University, Comilla, Bangladesh and [2]Faculty of Science and Information Technology, Daffodil International University, Dhaka, Bangladesh

Email: ekarim_du@yahoo.com, akabir@daffodilvarsity.edu.bd

Abstract: *The thermophoretic effect of particle deposition on an electrically conducting, viscous, incompressible fluid past an inclined stretching porous plate in the presence of a uniform magnetic field with heat generation was studied experimentally and numerically. The equations governing the flow, temperature and concentration fields are reduced to a system of joined non-linear ordinary differential equations by similarity transformation. Non-linear differential equations are integrated numerically by using Nachtsheim-Swigert shooting iteration technique along with sixth order Runge-Kutta integration scheme. Finally the significance of physical parameters which are of engineering interest are examined both in graphical and tabular form.*

Keywords: *Boundary Layer, Heat generation, Magnetic field, Nusselt number and Thermophoresis.*

1. Introduction

Laminar boundary layer flow, a significant type of flow, in presence of magnetic field and radiation over a moving continuous and linearly stretched surface has been receiving wide attention due to its applications in engineering, electrochemistry and polymer processing. For examples, lots of metallurgical processes occupy the system of cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. By drawing such strips in an electrically conducting fluid subjected to a magnetic field, the rate of cooling can be controlled and a final product of desired characteristics can be achieved. The study of heat and mass transfer is necessary for determining the quality of the final product.

Ostrach [1] analyzed a similarity solution of transient free convection flow past a semi infinite vertical plate by an integral method. Goody [2], one of the initiator of free convection problem, considered a neutral fluid. Sakiadis [3] analyzed the boundary layer flow over a solid surface moving with a constant velocity. This boundary layer flow situation is quite different from the classical Blasius problem of boundary flow over a semi-infinite flat plate due to entrainment of ambient fluid. Crane [4] noted that usually the sheet is assumed to be inextensible, but situations may arise in the polymer industry in which it is necessary to deal with a stretching plastic sheet. Sparrow [5] explained a parameter named Rosseland approximation to describe the radiation heat flux in the energy equation in his book. Raptis and Perdikis [6] considered the viscous flow over a non-linear stretching sheet in the presence of a chemical reaction and magnetic field. Samad and Mohebujjaman [7] exposed the effect of a chemical reaction on the flow over a linearly stretching vertical sheet in the presence of heat and mass transfer as well as a uniform magnetic field with heat generation/ absorption. Liu and Andersson [8] examined the heat transfer in a liquid film driven by a horizontal sheet.

Thermophoresis, thermo-diffusion, or Ludwig-Soret effect, is a phenomenon observed when a mixture of two or more types of motile particles (particles able to move) are subjected to the force of a temperature gradient and the different types of particles respond to it differently. An example that may be observed by the naked eye with good lighting is when the hot rod of an electric heater is surrounded

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by tobacco smoke: the smoke goes away from the immediate vicinity of the hot rod. As the small particles of air nearest the hot rod are heated, they create a fast flow away from the rod, down the temperature gradient. They have acquired higher kinetic energy with their higher temperature. When they collide with the large, slower-moving particles of the tobacco smoke they push the latter away from the rod. The force that has pushed the smoke particles away from the rod is an example of a thermophoretic force.

Goren [9] studied thermophoresis in laminar flow over a horizontal flat plate. He found the deposition of particles on cold plate and particles free layer thickness in hot plate case. Homsy *et al.* [10] solved Blasius series solution. Epstein *et al.* [11] analyzed the thermophoresis in natural convection for a cold vertical surface. Lin *et al.* [12] investigated the suppression of particle deposition from flow through a tube with circular cross-section when the wall temperature exceeds that of the gas. Alam *et al.* [13] considered the effects of heat generation and thermophoresis on steady, laminar, hydromagnetic, two-dimensional flow with heat and mass transfer along a semi-infinite, permeable inclined flat surface. Loganathan and Arasu [14] analyzed the effects of thermophoresis particle deposition on non-Darcy MHD mixed convective heat and mass transfer past a porous wedge in the presence of suction or injection.

Thermophoretic force has been used in commercial precipitators for applications similar to electrostatic precipitators. It is exploited in the manufacturing of optical fiber in vapor deposition processes. It can be important as a transport mechanism in fouling. Thermophoresis has also been shown to have potential in facilitating drug discovery by allowing the detection of aptamer binding by comparison of the bound versus unbound motion of the target molecule. This approach has been termed micro-scale thermophoresis. Furthermore, thermophoresis has been demonstrated as a versatile technique for manipulating single biological macromolecules, such as genomic-length DNA, in micro- and nano-channels by means of light-induced local heating. Thermophoresis is one of the methods used to separate different polymer particles in field flow fractionation.

Very recently, Ferdows *et al.* [15] descried that in the presence of uniform magnetic field with viscous dissipation at the wall, the thermophoretic parameter is one of the most useful parameter to control the boundary layer of the fluid. Zueco *et al.* [16] discussed the thermophoretic effect on particle transport characteristics over a horizontal wall from of an electrically conducting, heat generating/absorbing fluid in the presence viscous and Joule heating.

This current paper is the investigation of the thermophoresis effect on the steady flow of a viscous incompressible fluid over an inclined linearly stretched sheet in the presence of heat and mass transfer permitted by a transversely applied uniform magnetic field with heat generation taking into account the Rosseland diffusion approximation [5]. The investigation is based on known similarity analysis and the local similarity solutions are obtained numerically.

2. Mathematical Analysis

A steady-state two-dimensional heat and mass transfer flow of an electrically conducting viscous incompressible fluid along an isothermal stretching permeable inclined sheet with an angle α to the vertical embedded in a porous medium with heat generation/absorption is considered. A strong magnetic field is applied in the *y*- axis direction. Here the effect of the induced magnetic field is neglected in comparison to the applied magnetic field. The electrical current flowing in the fluid gives rise to an induced magnetic field if the fluid were an electrical insulator, but here we have taken the fluid to be electrically conducting. Hence, only the applied magnetic field of strength B_0 plays a role which gives rise to magnetic forces $F = \dfrac{\sigma B_0^2 u}{\rho}$ in *x*-direction, σ where is the electrical conductivity assumed to be directly proportional to the *x*- translational velocity (u) of the fluid found by Helmy [17] and ρ is the density of the fluid. Two equal and opposite forces are introduced along the *x*- axis so that the sheet is stretched keeping the origin fixed as shown in Fig. 1.

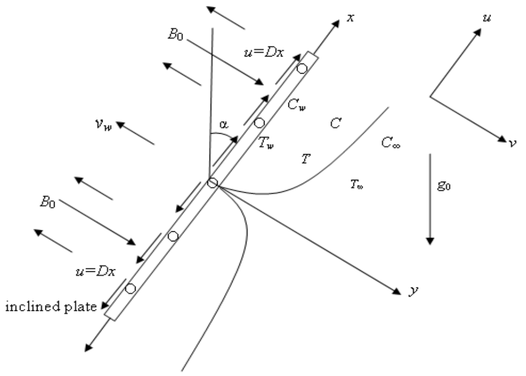

Fig. 1: Flow Configuration and Coordinate System

The fluid is considered to be gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiation heat flux in the energy equation. The radiative heat flux in the *x*-direction is negligible to the flux in the *y*-direction. The plate temperature and concentration are initially raised to T_w and C_w respectively which are thereafter maintained constant. The ambient temperature of the flow is T_∞ and the concentration of the uniform flow is C_∞.

Under the usual boundary layer and Boussinesq approximations and using the Darcy-Forchhemier model, the flow and heat transfer in the presence of radiation are governed by the following equations.

Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum Equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g_0\,\beta\,(T-T_\infty)\cos\alpha -\frac{\sigma B_0^{\,2}\,u}{\rho}-\frac{\upsilon\,u}{k}-\frac{b\,u^2}{k} \tag{2}$$

Energy Equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty) \tag{3}$$

Concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-\frac{\partial}{\partial y}\left(V_T C\right) \tag{4}$$

where *u* and *v* are the velocity components in the *x*- and *y*- directions respectively, υ is the kinematic viscosity, g_0 is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, α` is the angle of inclination, k is the Darcy permeability constant, T and T_∞ are the fluid temperature within the boundary layer and in the free-stream respectively, while C is the concentration of the fluid within the boundary layer, κ is the thermal conductivity of the fluid, c_p is the specific heat at constant pressure, Q_0 is the volumetric rate of heat generation/absorption and D_m is the chemical molecular diffusivity and V_T is the thermophoresis velocity, following Talbot *et al.* [18] given by $V_T=-k_v\frac{\nabla T}{T_{ref}}=-k_v\frac{1}{T_{ref}}\frac{\partial T}{\partial y}$

where k_v is the thermophoretic diffusivity.

The corresponding boundary conditions for the model are

$$\left.\begin{array}{lll} u=D\,x, v=v_w(x), T=T_w, C=C_w=0 & \text{at} & y=0 \\ u=0, \quad\quad T=T_\infty, C=C_\infty & \text{as} & y\to\infty \end{array}\right\} \tag{5}$$

where $D(>0)$ a constant, vw (x) is is a velocity component at the wall having positive value to indicate suction, T_w is the uniform sheet temperature and C_w is the concentration of the fluid at the sheet.

3. Similarity Analysis

In order to obtain similarity solution for the problem under consideration, we may take the following suitable similarity variables

$$\eta=y\sqrt{\frac{D}{\upsilon}},\ \psi=\sqrt{D\upsilon}\,x\,f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$

$$\text{and } \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}. \tag{6}$$

where ψ is the stream function, η is the dimensionless distance normal to the sheet, f is the dimensionless stream function, θ is the dimensionless fluid temperature and ϕ is the dimensionless concentration.

Since $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ we have the velocity components from equation (6) given by

$$u=Dx\,f'(\eta) \text{ and } v=-\sqrt{D\upsilon}\,f(\eta) \tag{7}$$

where prime denotes the derivative with respect to η. Now introducing the similarity variables from equation (6) and using equation (7), equations (2) to (4) are reduced to the dimensionless equations given by

$$f''' + ff'' + \gamma\cos\alpha\theta - \left(M + \frac{1}{Da.\mathrm{Re}}\right)f' - \left(1+\frac{Fs}{Da}\right)(f')^2 = 0 \tag{8}$$

$$\theta'' + \Pr f\theta' + \Pr Q\theta = 0 \tag{9}$$

$$\phi'' + Sc(f-\tau\theta')\phi' - Sc\tau\phi\theta'' = 0 \tag{10}$$

where $\gamma=\frac{Gr_x}{\mathrm{Re}_x^2}=\frac{g_0\beta(T_w-T_\infty)}{D^2x}$ is the buoyancy parameter,, $M=\frac{\sigma B_0^2}{\rho D}$ is the magnetic field parameter, $Da=\frac{k}{x^2}$ is the local Darcy number, $\mathrm{Re}=\frac{x^2D}{\upsilon}$ is the Reynolds number, $Fs=\frac{b}{x}$ is the Forchhemier number, $\Pr=\frac{\mu c_p}{\kappa}$ is the Prandtl number, $Q=\frac{Q_0}{\rho c_p D}$ is the heat source (Q > 0) / sink (Q < 0) parameter, $Sc=\frac{\upsilon}{D_m}$ is the Schmidt number and $\tau=-k_v\frac{(T_w-T_\infty)}{T_{ref}}$ is the thermophoretic parameter. The transformed boundary conditions are

$$\left.\begin{aligned} f=f_w,\ f'=1,\theta=1,\phi=1 \quad &\text{at} \quad \eta=0 \\ f'=0,\theta=0,\phi=0 \quad &\text{as} \quad \eta\to\infty \end{aligned}\right\} \tag{11}$$

where $f_w=-\frac{v_w}{\sqrt{D\upsilon}}$ is the suction parameter for f_w >0. The nonlinear ordinary differential equations (8), (9) and (10) under the boundary conditions (11) are solved numerically for various values of the parameters entering into the problems.

Skin friction, rate of heat transfer and mass flux: The parameters of engineering interest for the present problem are the skin friction coefficient (C_f), local Nusselt number (Nu_x) and Sherwood number (Sh) indicating physically the wall shear stress, the rate of heat transfer and the local surface mass flux respectively. From the following definitions

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{12}$$

$$q_w=-\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{13}$$

$$M_w=-D_m\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{14}$$

The dimensionless local wall shear stress, local surface heat flux and the local surface mass flux for an impulsively started plate are respectively obtained as

$$C_f=\frac{2\tau_w}{\rho U^2}=2(\mathrm{Re}_x)^{-1/2}f''(0) \tag{15}$$

$$Nu_x=\frac{q_w x}{\kappa(T_w-T_\infty)}=-(\mathrm{Re}_x)^{1/2}\theta'(0) \tag{16}$$

$$Sh=\frac{M_w x}{D_m(C_w-C_\infty)}=-(\mathrm{Re}_x)^{1/2}\phi'(0) \tag{17}$$

And hence the values proportional to the skin-friction coefficient, Nusselt number and Sherwood number are $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ respectively.

4. Numerical Computation

The numerical solutions of the non-linear differential equations (8) - (10) under the boundary conditions (11) have been performed by applying a shooting method namely Nachtsheim and Swigert [19] iteration technique (guessing the missing values) along with sixth order Runge-Kutta iteration scheme. We have chosen a step size $\Delta\eta=0.01$ to satisfy the convergence criterion of 10^{-6} in all cases. The value of η_∞ has been found to each iteration loop by $\eta_\infty=\eta_\infty+\Delta\eta$. The maximum value of η_∞ to each group of parameters f_w, γ,

α, *M*, *Da*, Re, *Fs*, Pr, *Q*, *Sc* and τ has been determined when the values of the unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-6}.

In order to verify the effects of the step size $\Delta\eta$, we have run the code for our model with three different step sizes as $\Delta\eta = 0.01$, $\Delta\eta = 0.001$ and $\Delta\eta = 0.005$ and in each case we have found excellent agreement among them shown Fig. 2.

5. Results and Discussion

For the purpose of discussing the results of the flow field represented in the Fig.1, the numerical calculations are presented in the form of non-dimensional velocity, temperature and concentration profiles. The value of buoyancy parameter γ is taken to be positive to represent cooling of the plate. The parameters are chosen arbitrarily where Pr = 0.71 corresponds physically to air at 20°C, Pr = 1.0 corresponds to electrolyte solution such as salt water and Pr = 7.0 corresponds to water, and *Sc* = 0.22, 0.6 and 1.0 correspond to hydrogen, water vapor and methanol respectively at approximate 25°C and 1 atmosphere.

Due to free convection problem positive large values of Re is kept 10, *Da* equal to 0.20, Inclination of the plate is set at 30°, *Fs* is 0.1, Pr is 0.71 and *Sc* is equal to 0.2. However, numerical computations have been carried out for different values of the suction parameter (f_w), the buoyancy parameter (γ), magnetic field parameter (*M*), heat source parameter (*Q*) and the thermophoretic parameter (τ). The numerical results for the velocity, temperature and concentration profiles are displayed in Figs. 5 – 19.

Figs. 5 – 7 display the effects of the suction parameter f_w on the velocity, temperature and concentration profiles respectively. It is observed that, when suction f_w increases, the velocity and temperature decrease monotonically but the concentration increase. These figures indicate that, since the boundary layer thickness and concentration gradient at the stretching sheet getting smaller but the temperature getting steeper, the increasing suction enhance the heat transfer coefficient.

The effect of the buoyancy parameter γ on the velocity field is shown in Fig. 8. From this figure we see that the velocity decreases with the increase of γ swiftly up to η = 1.83. Fig. 9 shows that the thermal boundary layer thickens and the temperatures drop off hastily with the raise of γ. Finally, we observe that γ affects the concentration very slowly far away the plate surface.

For different values of heat source parameter *Q* the velocity, temperature and concentration profiles are exposed in Figs. 11 – 13. Here we have illustrated non-dimensional velocity, temperature and concentration profiles against η for some representative values of the heat source parameter *Q* = 0.3, 0.5, 1.0, 1.5, 2.5. The positive value of *Q* represents source i.e., heat generation in the fluid. For heat generation, the peak velocity occurs near the surface of the stretching plate. This is corroborated by Fig. 12 where it is seen that the temperatures do indeed rapid increase as *Q* increases. We see that when *Q* = 0.3 the temperature is transferred from the sheet, but for increasing *Q*, the heat transfer rate is decreasing. Fig. 13 shows that the concentration profiles increase with the increase of heat source parameter.

The effects of magnetic field on the dimensionless velocity, temperature and concentration profiles are shown in Figs. 14 – 16. From Fig. 14, we observe that the velocity decreases with the increase of *M*. This is due to the effect of magnetic field on the stretching sheet as well as the temperature field. But the concentration increases with the increase of magnetic field very slowly. The magnetic field lines act like a string and tend to retard the motion of the fluid. The consequence of which is to increase the rate of heat transfer temperature variation is prominent as seen in Fig. 15.

In Figs. 17 – 19 we have plotted the dimensionless velocity, temperature and concentration profiles showing the effect of thermophoretic parameter τ. It is seen that the effect of τ on velocity and temperature profiles are very negligible. We observe from Fig. 17 that the thermophoretic parameter τ affects the concentration profiles very considerably. The concentration profiles increase with the increase of τ .

Finally, the effects of various parameters on the skin friction C_f, local Nusselt number Nu_x and local Sherwood number Sh are shown in the Table 1.

6. Conclusion

From the present study we can make the following conclusions:

1. The effect of suction parameter is dominating on the velocity, temperature and concentration profiles. So, using suction boundary layer growth can be stabilized.
2. The buoyancy parameter has a significant effect.
3. The effect of heat source parameter is very noteworthy.
4. Using magnetic field we can control the heat and mass transfer flow characteristics.
5. Thermophoretic parameter plays a considerable role on concentration profiles.

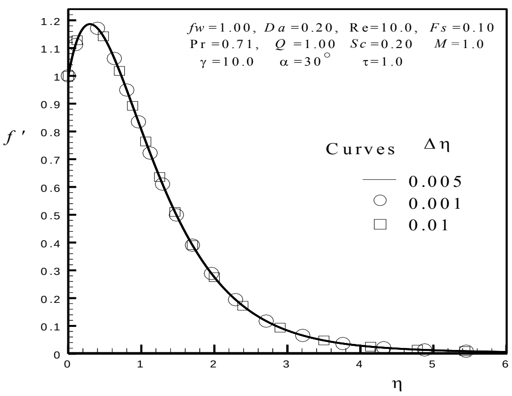

Fig. 2: Velocity Profiles for Different Step Sizes

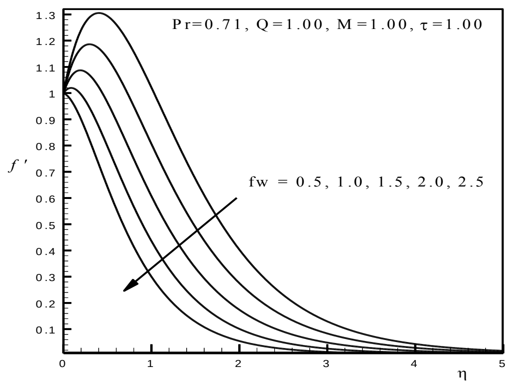

Fig. 4: Velocity Profiles for Different values of f_w.

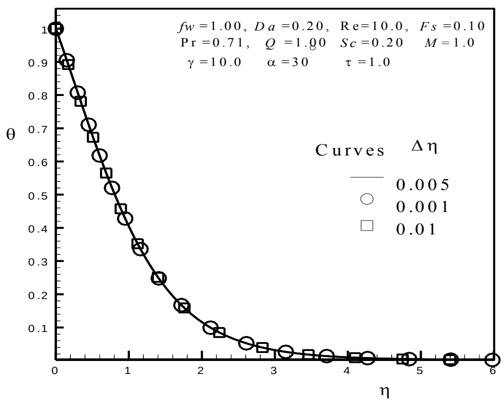

Fig. 3: Temperature Profiles for Different Step Sizes.

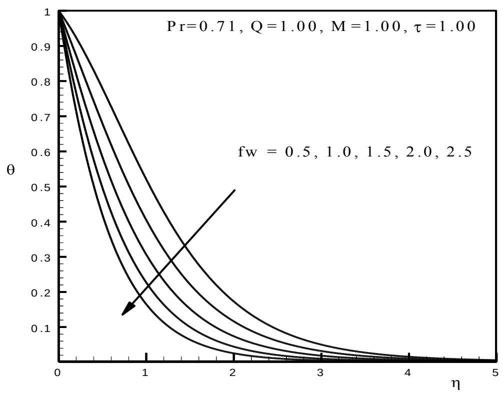

Fig. 5: Temperature Profiles for Different Values of f_w.

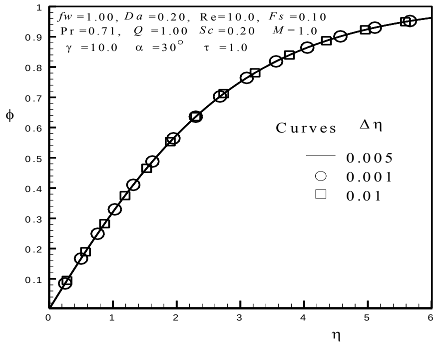

Fig. 6: Concentration profiles for different step sizes.

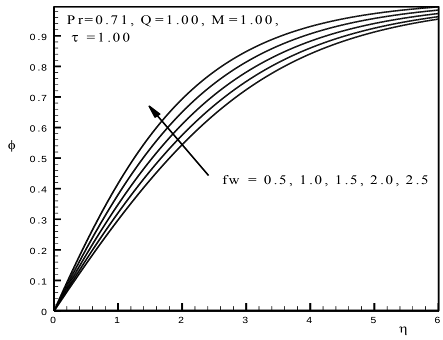

Fig. 7: Concentration profiles for different values of f_w.

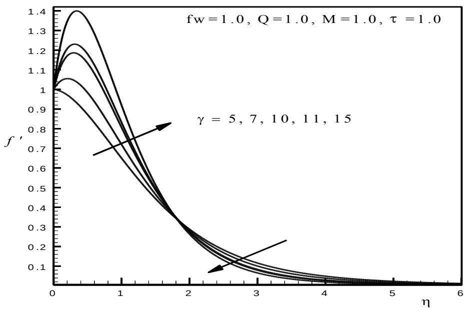

Fig. 8: Velocity profiles for different values of γ.

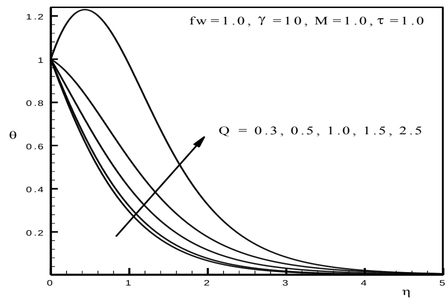

Fig. 10: Temperature profiles for different values of *Q*.

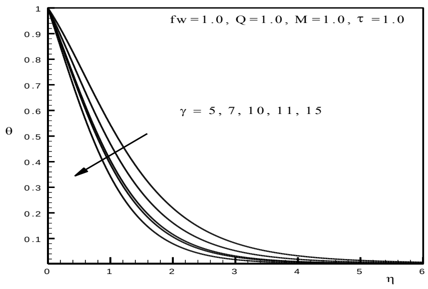

Fig. 9: Temperature profiles for different values of γ.

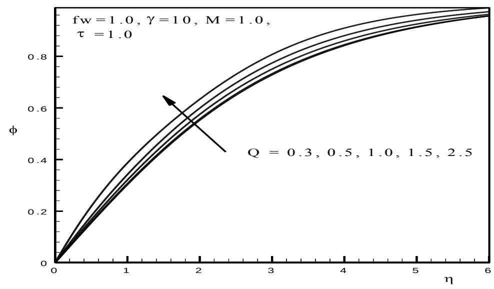

Fig. 11: Concentration profiles for different values of *Q*.

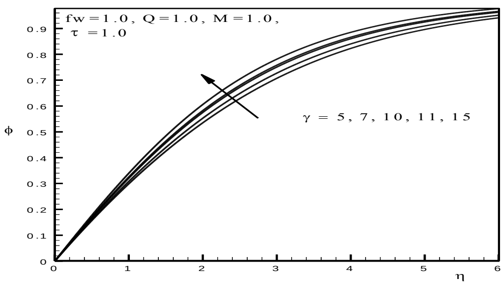

Fig. 12: Concentration profiles for different values of γ.

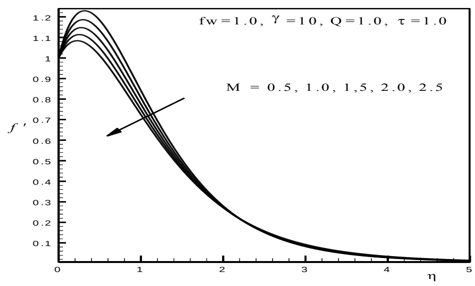

Fig. 14: Velocity profiles for different values of *M*.

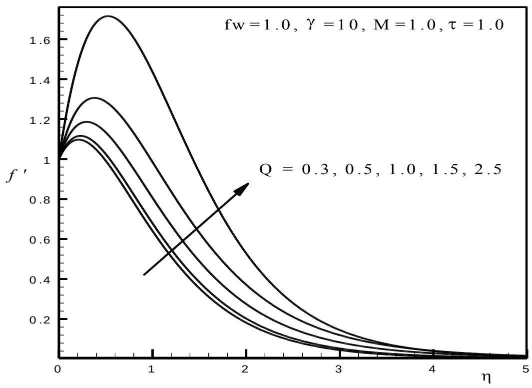

Fig. 13: Velocity profiles for different values of *Q*.

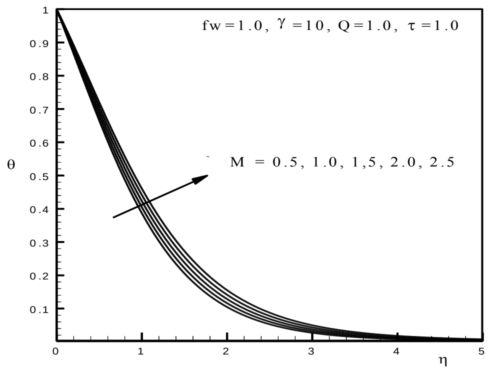

Fig. 15: Temperature profiles for different values of *M*.

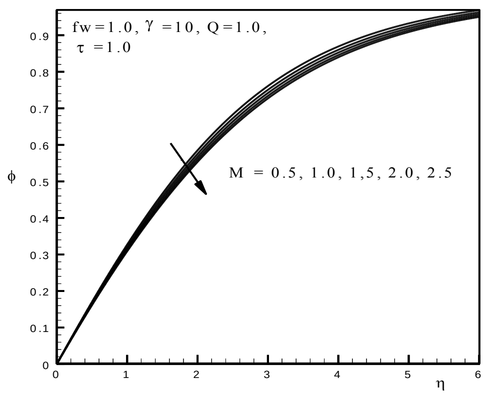

Fig. 16: Concentration profiles for different values of M.

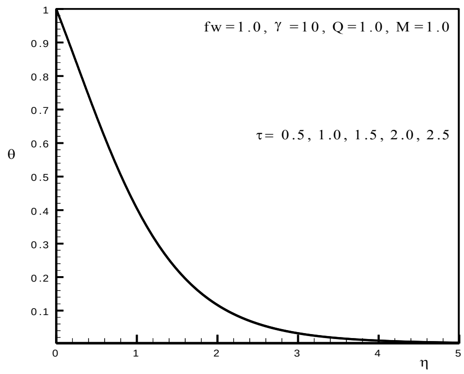

Fig. 18: Temperature profiles for different values of τ.

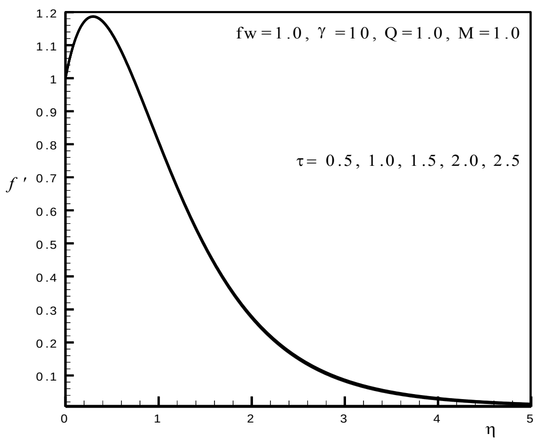

Fig. 17: Velocity profiles for different values of τ.

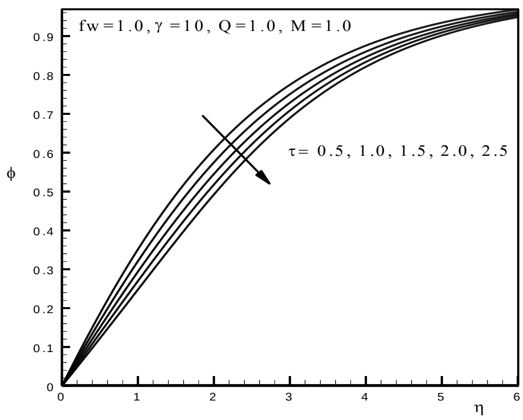

Fig. 19: Concentration profiles for different values of τ.

Table 1: C_f , Nu_x and Sh for different values of f_w, γ , Q, M and τ .

f_w	γ	Q	M	τ	C_f	Nu_x	Sh
0.5	10.0	1.0	1.0	1.0	1.77941542	0.32726104	-0.31286674
1.5	10.0	1.0	1.0	1.0	1.02056517	0.94059538	-0.37928641
1.0	5.0	1.0	1.0	1.0	-0.02736700	0.36286017	-0.32644859
1.0	10	1.0	1.0	1.0	1.45721565	0.61824354	-0.34337357
1.0	10	0.3	1.0	1.0	1.04209515	1.00556583	-0.31294979
1.0	10	2.5	1.0	1.0	3.19234760	-1.08255650	-0.48993495
1.0	10	1.0	0.5	1.0	1.69495014	0.65245694	-0.34865085
1.0	10	1.0	2.0	1.0	1.02844330	0.54928962	-0.33563746
1.0	10	1.0	1.0	0.5	1.45498830	0.61922207	-0.38842664
1.0	10	1.0	1.0	1.5	1.45905174	0.61743551	-0.30416951

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