RESILIENCE-BASED NETWORK DESIGN OPTIMIZATION UNDER EPISTEMIC UNCERTAINTY

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ABSTRACT

Design optimization of infrastructure networks is a crucial task due to its huge impacts in socio-economic sectors and consideration of resilience in the design phase of a network ensures a higher level of performance even after the occurrence of any disruptive event. Due to the lack of knowledge regarding the effects of future disruptive events and the exact values of the resilience parameters, uncertainty consideration is essential for this type of optimization formulation. In this research, resilience-based network design optimization models are formulated with a view to minimizing design cost, maximizing the level of performance in the post-disrupted state, and maximizing resilience. The models are formulated for both deterministic and stochastic cases. The stochastic model, formulated in this study, is able to deal with epistemic uncertainty arising from interval data of the resilience parameters. The solution methodology generates the optimal network topology and the design capacities of each link present in the optimal solution. Finally, the formulations are solved to generate the optimal network that can satisfactorily perform even after multiple disruptive events.

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LIST OF ABBREVIATIONS

STP	:	Shortest Processing Time
PMF	:	Probability Mass Function
GA	:	Genetic Algorithm
PSDA	:	Probabilistic Solution Discovery Algorithm

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Traditionally, risk and reliability management strategies have focused on reducing the likelihood of disruptive events and their potential consequences. However, various past events have suggested that not all undesired events can be predicted and prevented [1].Various natural and man-made disasters often occur and cause serious disruptions in various infrastructures. In order to diminish the consequences of disruptive events, recent efforts have been expanded from reliability engineering to a new paradigm–resilience engineering [2, 3]. In general, while reliability engineering focuses on enhancing the ability of a system to work properly for a specified period of time by reducing its probability of failure, resilience engineering emphasizes improving the capability of the system to bounce back quickly from the disrupted state to offer a desired level of performance after the disruption [3].

Consideration of resilience in the design phase can have a great impact on the design parameters of a system and its performance. While reliability engineering tries to reduce the probability of system failure due to disruptions by increasing the system redundancy [4, 5], resilience engineering focuses on designing system components with the capability of restoring performance in a timely manner after disruptions [3]. Therefore, the goal in resilience-based design is to develop systems with components that can return to an acceptable level of performance after the disruption considering minimization of cost through reducing system redundancy.

Most of the existing studies on resilience engineering focus on resilient design of industrial process/system (e.g., [6]). Researchers have recently focused on infrastructure resilience due to its huge impact on society and economy (e.g., power transmission network [7], and traffic network [8]). In practice, many infrastructure systems, e.g., transportation networks, power grids, and telecommunication networks, are organized in the form of networks [3].

Another important point that has significant impact in resilience-based design is the consideration of uncertainty in system parameters that may arise due to insufficient knowledge for different specific conditions. Components of a system may show different levels of performance reacting to different natures of disruptions. At the design phase, there remains a high possibility of having a lack of information regarding these performance levels of system components. Although the consideration of this epistemic uncertainty in resilience-based design is highly important, most of the existing methods modeled system parameters with respect to aleatory uncertainty (i.e., physical variability) only. Inclusion of epistemic uncertainty (i.e., imprecise probabilistic information due to sparse and interval data) in the design of a network adds another level of complexity in the computational framework. Therefore, development of a resilience-based network design optimization model including epistemic uncertainty in parameter values is still an open problem and thereby yields the scope of the proposed thesis.

1.2 Objectives with Specific Aims

The specific objectives of this research are as follows:

(i) To develop a resilience-based network design optimization formulation with a view to minimizing the design cost, maximizing the recovery state level of performance, and maximizing resilience for a directed network under epistemic uncertainty arising from interval data.

(ii) To propose a modified probabilistic solution discovery algorithm for solving resilience-based network design problems under epistemic uncertainty.

1.3 Possible Outcomes

The proposed research develops and demonstrates generalized methodologies for resilience-based design and performance analysis of networks under epistemic uncertainty. The methodologies developed in this research can be used to solve network design problems in a variety of cases, e.g., road traffic network, power transmission network, data transmission network, railway network, airline network, etc.

1.4 Outline of Methodologies Used

The research methodologies used in this thesis are outlined below:

(i) The uncertainties in parameters related to resilience (e.g., absorptive capacity, restorative ability, restoration speed, etc.) for each link of the network are quantified using interval data.

(ii) Optimization problems are formulated with the objectives being minimization of the design cost, maximization of the level of performance in the recovery state, and maximization of system resilience for both deterministic and stochastic cases.

(iii) A modified *Probabilistic Solution Discovery Algorithm* is proposed to solve the optimization formulation developed in Step (ii). The proposed probabilistic solution discovery algorithm consists of three steps:

   a. Strategy development: In this stage, a number of random sample networks are generated using the link probabilities by *Monte Carlo simulation*.

   b. Strategy analysis: In this stage, optimization problems are solved for each random network using linear programming and genetic algorithm for integer variables in order to minimize the design cost and maximize of the level of performance in the recovery state. The values of resilience for the optimal networks are evaluated. Then these values of the optimal design cost and resilience are used to rank the sample networks using the Stochastic Ranking algorithm.

   c. Solution Discovery: In this stage, the probability of each link to be present in the optimal network is updated using the information of a number of top-ranked optimal sample networks from the strategy analysis stage. When all the probability becomes 0 or 1, the algorithm terminates and links with the probability value of 1 constitute the ultimate optimum network.

(iv) The proposed resilient-based network design optimization methodology is illustrated for two different network design problems with varying numbers of network nodes and links.

(v) A brute-force method along with Monte Carlo simulation (MCS) is used to verify the accuracy and efficiency of the proposed resilient-based network design optimization method.

1.5 Contributions of the Present Study

This thesis contributes to the formulation and optimization of the network design cost as a function of capacity and length of the links which is more appropriate than previously developed formulations.

The consideration of epistemic uncertainty in parameter values is also a new addition to resilience-based network design optimization. Incorporation of epistemic uncertainty using interval data has enabled the model to be more robust and applicable to a variety of situations in various domains.

Finally, the proposed model is evaluated for satisfactory performance even after multiple disruptive events which is not considered in most of the existing studies.

1.6 Organization of the Thesis

This thesis report is organized in the following manner:

Chapter 1 contains the necessary background of the thesis, specifically defined objectives, possible outcomes, a summary of the developed methodology, and the contributions of this study in the field of resilience-based network design optimization. Chapter 2 contains a brief review of all relevant literature. Chapter 3 provides the necessary theoretical background regarding definitions and measures of resilience, epistemic uncertainty, its quantification, and network design formulations. Chapter 4 contains the formulation of the proposed resilience-based network design optimization problem, methods to quantify epistemic uncertainty in parameter values, and modified probabilistic solution discovery algorithm to achieve the optimum network design. In Chapter 5, the proposed methodology is illustrated using a mathematical problem and a practical engineering problem. Finally, in Chapter 6, the thesis is concluded with recommendations for future work for the practitioners and researchers.

CHAPTER 2

LITERATURE REVIEW

The concept of resilience was first introduced by Holling [9]. Although he explained resilience as an important factor in maintaining the stability of ecological balance, researchers of other domains also became interested in its importance since its introduction [3]. Resilience has been defined in several ways in literature. Rose and Liao [10] defined resilience as the inherent ability and adaptive responses of systems that enable them to avoid potential losses. Hollnagel et al. [11] defined resilience as the result of a system (i) preventing adverse consequences, (ii) minimizing adverse consequences, and (iii) recovering quickly from adverse consequences. Haimes [12] defined the resilience of a system as the ability to withstand a major disruption within acceptable degradation of parameters and to recover within an acceptable time. He compared resilience with preparedness, vulnerability, reliability, and risk. Henry and Ramirez-Marquez [13] first defined resilience as a time-dependent function and described the key parameters necessary to analyze system resilience (e.g., disruptive events, component restoration, and overall resilience strategy). They proposed metrics of network and system resilience and total cost of resilience.

There is now an extensive volume of methods and applications available for resilient-based design. Hosseini et al. [1] reviewed some of the recent research articles related to defining and quantifying resilience in various engineering systems. Most of the existing studies focus on resilient design in industrial process/system (e.g., [6]). Researchers have recently focused on infrastructure resilience due to its huge impact on society and economy and, in practice, many infrastructure systems, e.g., transportation networks, power transmission networks, and telecommunication networks, are organized in the form of networks.

Attoh-Okine et al. [14] developed a resilience index for urban infrastructure using a belief function framework and presented the steps of the analysis using a prototype urban highway infrastructure network. Ouyang et al. [15] proposed a multi-stage framework to analyze infrastructure resilience for both single hazards and concurrent multiple hazard types. Barker et al. [16] developed two resilience-based component importance measures quantifying (i) the potential adverse impact on system resilience from a disruption affecting a particular link and (ii) the potential positive impact on system resilience when the link cannot be disrupted. Fang et al. [17] proposed two metrics, i.e., the optimal repair time and the resilience reduction worth, to measure the criticality of the components of a network system from the perspective of their contribution to system resilience. Adjetey-Bahun et al. [18] proposed a simulation-based model for quantifying resilience in mass railway transportation systems by quantifying passenger delay and passenger load as the system's performance indicators. In order to connect resilience assessment to engineering design by investigating the effects of failure, recovery, and the system failure paths on system resilience, Fotouhi et al. [8] developed a bi-level, mixed-integer, stochastic program for quantifying the resilience of a coupled traffic-power network under a host of potential natural or anthropogenic hazard-impact scenario.

However, very few studies have investigated the methods for resilient system design. Youn et al. [19] proposed a resilience-driven system design (RDSD) framework with the goal of designing complex engineered systems with resilience characteristics and minimizing the system's life-cycle cost (LCC) that generally incurs due to a high level of system redundancy. Yodo and Wang [20] developed a resilience allocation framework for resilience analysis in the early design stage of complex engineering systems. Most of these studies have focused on engineering system design and there are only a few studies that have worked on resilience-based infrastructure network design [3].

Chen and Miller-Hooks [21] developed a resilience-based optimization of an intermodal freight transport network in order to maximize the expected number of shipments. They proposed a stochastic mixed-integer program for quantifying network resilience and identifying an optimal post-event course of action. Faturechi et al. [22] proposed a model for quantifying and optimizing the resilience of an airport's runway and taxiway network under multiple potential damage-meteorological scenarios. They formulated a stochastic integer program taking into account operational, budgetary, time, space, and physical resource limitations. Faturechi and Miller-Hooks [23] then proposed a bi-level, three-stage Stochastic Mathematical Program with Equilibrium Constraints (SMPEC) for quantifying and optimizing travel time resilience in roadway networks under non-recurring natural or human-induced disastrous events.

Fang et al. [24] developed a combinatorial multi-objective optimization for power transmission networks using a Nondominated Sorting Binary Differential Evolution (NSBDE) algorithm. They considered the problem of allocation of generation to distributors by rewiring links under the objectives of maximizing network resilience to cascading failure and minimizing investment costs. Bhatia et al. [25] developed a quantitative framework for measuring, comparing and interpreting hazard responses and recovery strategies for the Indian Railway network. However, they did not address the *priori* analysis during the design phase to optimize the topology of the current network to strengthen its resilience against these disruptions [3]. Fang and Sansavini [26] used a planner-attacker-defender model to minimize investment and operating costs, and functionality loss after disruptions in power transmission networks in order to develop decisions of capacity expansion and switch installation in the systems that ensure optimum performance under nominal operations and attacks. Asadabadi and Miller-Hooks [27] proposed a multi-stage, stochastic, bi-level, mixed-integer program to figure out the optimal long-term transportation investment plan in order to protect from and mitigate impacts of climate change on roadway performance.

Zhang et al. [3], in 2018, figured out some of the common shortcomings of these studies which cannot be ignored in resilience-based infrastructure design. As mention by them, the shortcomings are listed below:

i. Most of the studies on resilience-based design did not characterize the significant parameters related to resilience (e.g., absorptive ability, restorative ability, and restoration speed) which actually play important roles in quantification of the system resilience after the occurrence of the disruptive events.
ii. Most of the studies did not model the system resilience as a time-varying variable, whereas, in reality, the restoration and recovery process of a disrupted system usually consumes a certain amount of time to bounce back to the original performance and during this period the system performance varies with the time-lapse.
iii. Most of the existing methods modeled system parameters with respect to aleatory uncertainty (i.e., physical variability) only. However, uncertainty in system parameters may also arise from other contributing sources, for example, epistemic uncertainty arising due to lack of knowledge.

Zhang et al. [3] developed a time-dependent equation for the evaluation of system performance after the occurrence of any disruptive event including the absorptive ability, restorative ability, and restoration speed of the system components. They modeled a resilience-based network design optimization problem to maximize the flow through the network and minimize the cost incurred to build the network for both deterministic and stochastic cases. However, in their formulation, they only make decision on whether a link should be present or not in the optimal network. Their model cannot decide on the optimal capacities for each link to be present in the optimal network. Also, in their stochastic model formulation, although they considered interval data for parameter values, they only used uniform distribution for the estimation of values within these intervals which actually cannot incorporate the actual stochastic nature in the parameters. Their model was not designed for integer link capacities. While solving the optimization problem they allowed fractional capacity values, which is totally inconsistent with reality.

Later, Matelli and Goebel [28] proposed method to quantify resilience for a cogeneration plant and developed a resilience-based conceptual design of the cogeneration plant. Nogal and Honfi [29] proposed a dynamic, stochastic traffic assignment model and performed the assessment of road traffic resilience for the traffic network between Luxembourg and Metz. Cerqueti et al. [30] proposed a new method for measuring network resilience based on the study of propagation of shock along the patterns of connections among different nodes. None of these recently published research considered the above-mentioned concerns in resilience-based network design optimization formulation. Also, most of the studies on resilience-based design did not consider the consequences of the occurrence of multiple consecutive disruptive events. Therefore, development of an improved resilience-based network design optimization model including epistemic uncertainty in parameter values is still a requirement and thereby yields the scope of this thesis.

CHAPTER 3

THEORETICAL BACKGROUND

3.1 Infrastructure Networks and their Performance Matrices

In practice, most of the infrastructure systems are built in the form of networks and these infrastructure networks have a great impact in different socio-economic sectors. Examples of infrastructure networks include power transmission network, telecommunication network, transportation network, inland waterway network, road traffic network, airline network, railway network, etc. Every network consists of some nodes and links. In practice, each link in the network is bounded to have a limit on maximum flow capacity through that link and cost is incurred to set up these nodes and links. The nodes of a network where the flow out of the network exceeds the flow into the network are known as the *supply nodes* or *source nodes* and the nodes of a network where the flow into the network exceeds the flow out of the network are known as the *demand nodes* or *sink nodes*. The nodes satisfying the conversion of flow through the network are known as the *intermediate nodes* or the *transshipment nodes*. When the flow through the network is controlled using the direction of movements, the network is known as a *directed network*.

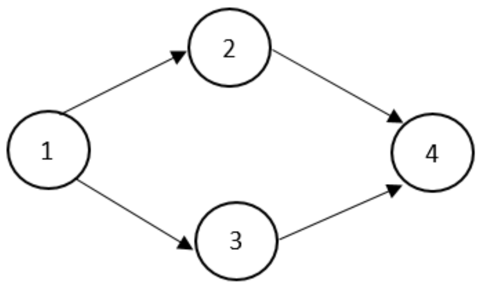

Figure 3.1: A 4-node directed network.

The above Figure 3.1, is an example of a 4-node directed network. Here, node 1 is the supply node and node 4 is the demand node. Node 2 and node 3 are the intermediate nodes.

Performance matrices of infrastructure networks can be described in several ways including network capacity, travel time, traffic flow, system throughput, network connectivity, etc. [3]. These performance matrices as a whole depend on the individual capacities of the links.

3.2 System Resilience

System resilience is commonly known as the capability of a system to bounce back quickly to its desired level of performance after the occurrence of any disruptive event. Although resilience has been defined in several ways in literature as discussed in the previous chapter, the definition provided by Henry and Ramirez-Marquez [13] is widely accepted (e.g.,[3, 16]). According to Henry and Ramirez-Marquez [13], system resilience is a time-dependent function and resilience at a certain time can be defined as the ratio of the amount of performance recovery at that point of time after the occurrence of the disruptive event to the amount of total performance loss due to the disruption. Therefore, if a disruptive event occurs at a previous time t_d, the amount of resilience $R(t)$ at time t after the disruption can be defined using the following equation [3, 13, 16]:

$$R(t) = \frac{\mathrm{Re}\,\mathrm{cov}ery(t)}{Loss(t)}, t \geq t_d \tag{3.1}$$

The system state transition due to the occurrence of a disruptive event can be explained using the following diagram:

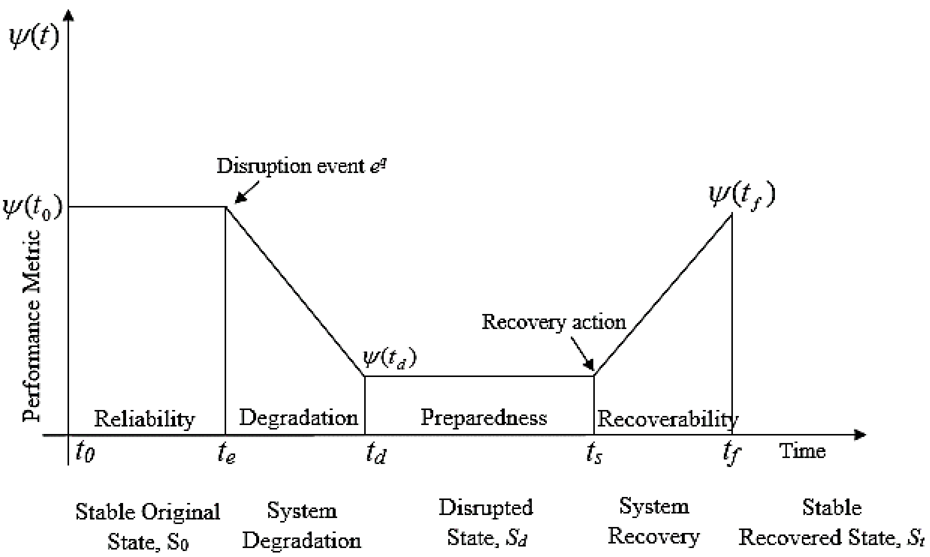

Fig. 3.2: System state transitions in terms of performance matric over time [3, 13, 16].

Here, the system state transitions are classified into five different stages and $\psi(t)$ is the performance matric of this infrastructure system. The original state is defined as S_0. $\psi(t_0)$ is the original level of system performance that was available before the occurrence of the disruptive event e^q at the time t_e. The effect of e^q continues until the time t_d which is the system degradation state and the performance level at the end of this stage eventually degrades to $\psi(t_d)$.

At the disrupted state S_d, preparations are taken to recover the system which include identifying system malfunctions and preparing for repairing or replacing the damaged components. At the recovery state, when the recovery actions are taken at time t_s, the level of performance starts to increase again. Finally, at time t_f, the system regains the performance level $\psi(t_f)$ which is the maximum achievable level of performance after the recovery actions been taken. From then, the system starts running at $\psi(t_f)$ in the stable recovered state S_f.

From this scenario, the amount of loss due to disruption is $(\psi(t_0) - \psi(t_d|e^q))$ and the amount of recovery at time t after the disruption is $(\psi(t|e^q) - \psi(t_d|e^q))$, where $\psi(t_d|e^q)$ is the disrupted state performance level and $\psi(t|e^q)$ is the recovered state performance level at time t given that the disruptive event has occurred. Therefore, according to Eq. (3.1), the resilience for the infrastructure system can be quantified using the following equation:

$$R(t_f|e^q) = \frac{\psi\left(t|e^q\right) - \psi\left(t_d|e^q\right)}{\psi\left(t_0\right) - \psi\left(t_d|e^q\right)} \tag{3.2}$$

3.3 Characteristic Parameters related to System Resilience

As quantification of resilience requires identifying disrupted and recovered state level of performance (i.e., $\psi(t_d|e^q)$ and $\psi(t|e^q)$), it is essential to incorporate some system characterizing parameters related to resilience. Disruptive events decrease the performance level of infrastructure networks by damaging network components and eventually lowering the link capacities. Therefore, in order to identify the disrupted and recovered state performance level, it is required to calculate the disrupted and recovered state link capacities. Zhang et al. [3] identified three characteristic parameters related to resilience as discussed below:

i. **Absorptive ability:** Absorptive ability (a) is the percentage of original capacity that can be retained after the occurrence of the disruptive event.
ii. **Restoration speed:** Restoration speed (b) is the amount of capacity recovered per unit time after the recovery actions been taken.

iii. **Restorative ability:** Restorative ability (λ) is the maximum percentage of the lost capacity that can be regained after the completion of the recovery actions.

Using these three parameters, Zhang et al. [3] developed the following nonlinear equation for the calculation of link capacities at any time after the disruption:

$$u_{ij}(t|e^q) = u_{ij}[a_{ij} + \lambda_{ij}(1-a_{ij})(1-e^{-b_{ij}t})] \tag{3.3}$$

where,

u_{ij} = original capacity of the link (i, j),

$u_{ij}(t|e^q)$ = capacity of the link (i, j) at any time t after the occurrence of the disruptive event,

a_{ij} = absorptive ability of the link (i, j),

b_{ij}= restoration speed of the link (i, j), and

λ_{ij}= restorative ability of the link (i, j).

Eq. (3.3) incorporates the resilient characteristics of the system by incorporating the parameters a, b, and λ, and the time-dependent nature of the restoration process. In general, restoration of various components of a complex network follows Shortest Processing Time (STP) algorithm while selecting the sequence of components to be restored first [3] and the time-varying nature of the capacity restoration process of Eq. (3.3) is highly consistent with the circumstance.

3.4 Uncertainty Estimation in Resilience Parameter Values (i.e., *a*, *b*, and *λ*)

3.4.1 Uncertainty and its Classification

Uncertainty can be defined as the state of having limited knowledge where it is almost impossible to exactly describe the existing state or estimate a future outcome. Uncertainty arises from both quantitative and qualitative sources. Quantitative sources may include inherent randomness of a system, statistical uncertainty (i.e., imprecise information regarding variability), and modeling uncertainty. Qualitative sources of uncertainty may arise from definitions of parameters, human factors, environmental impacts, etc.

All these uncertainties can be classified as either *aleatory uncertainty* or *epistemic uncertainty*. Uncertainty resulting from inherent variation or randomness is known as aleatory uncertainty. Examples of aleatory uncertainty may include manufacturing variation due to limited precision in machine tools and processes, environmental variations in temperature, humidity, wind speed, etc. On the other hand, the uncertainty resulting from a lack of information is known as epistemic uncertainty. It is hardly an inherent property of a system. Rather it represents the state of knowledge of system parameters and may be reduced if more knowledge about the system is acquired. Examples of epistemic uncertainty may include interval data of parameters gathered from expert opinions.

As consideration of resilience in network design optimization is relatively a new topic, in practice, the values of the parameters a, b, and λ are impossible to obtain as deterministic values for most of the networks. The best possible values that can be obtained for these parameters are expert opinions which may include upper bounds and lower bounds of these parameters. Therefore, for estimating parameter values from these interval data, the bounds of moments for these intervals must be estimated first. The bounds on moments thus obtained are then used to estimate the distribution parameters of the resilience characteristic parameters.

3.4.2 Moment Estimation Technique from Interval Data

Zaman et al. [31-32] proposed method for calculating bounds of moments for both single and multiple interval data. This method can work with both overlapping and non-overlapping sets of data. The bounds of moments generated using this method can completely enclose the possible moments that can be generated from various combinations of interval data. The moment bounding methods for single and multiple interval data are summarized in Tables 3.1 and 3.2, respectively.

Table 3.1: Methods for Calculating Moment Bounds for Single Interval Data [31, 32].

Moment	**Condition**		**Formula**
	Lower Bound	**Upper bound**	
1	$PMF = 1$ at lower endpoint = 0elsewhere	$PMF = 1$ at upper endpoint = 0 elsewhere	$M_1 = E(X)$
2	$PMF = 1$ at any point = 0 elsewhere	$PMF = 0.5$ at each endpoint	$M_2 = E(X^2) - (E(X))^2$
3	$PMF = 0.2113$ at lower endpoint = 0.7887 at upper endpoint	$PMF = 0.7887$ at lower endpoint = 0.2113at upper endpoint	$M_3 = E(X^3) - 3E(X^2)E(X) + 2(E(X))^3$

Moment	Condition		Formula
	Lower Bound	Upper bound	
4	$PMF = 1$ at any point $= 0$ elsewhere	$PMF = 0.7887$ at one of the endpoints $= 0.2113$ at the other endpoint	$M_4 = E(X^4) - 4E(X^3)E(X)$ $+ 6E(X^2)(E(X))^2 - 3(E(X))^4$

Note: $E(X^k) = \sum_{i=1}^{2} x_i^k p(x_i)$, where $p(x_i) =$ Probability Mass Function (PMF) and k = 1, or, 2, or, 3, or 4.

Table 3.2: Methods for Calculating Moment Bounds for Multiple Interval Data [32].

Moment	Formula
1	$[\underline{M}, \overline{M}] = [\frac{1}{n}\sum_{i}^{n} lb_i, \frac{1}{n}\sum_{i}^{n} ub_i]$ Where, $lb_i \le x_i \le ub_i$ for i= 1, 2,….., n and $[\underline{M}, \overline{M}]$ are the lower and upper bounds on the mean, respectively.
2, 3, and 4	$\min/\max_{x_1, x_2, \dots x_n} M_k = \frac{1}{n}\sum_{i=1}^{n}(x_i - \frac{1}{n}\sum_{j=1}^{n} x_j)^k$ Where, $lb_i \le x_i \le ub_i$ for i= 1, 2,….., n and minimizing or maximizing for k = 2, or, 3, or, 4 provides the lower or upper bound on the second, or, third, or, fourth moment.

Note: n = number of intervals.

3.4.2 Representation of Interval Uncertainty

As mentioned earlier, the bounds of the moments can completely enclose all the possible moment values, randomly generated instances between these bounds help to estimate the parameters of the distributions followed by the resilience parameters *a*, *b*, and *λ*.

We observe that beta distribution can be the best choice to interpret data for *a*, *b*, and *λ*, as beta distribution is a bounded distribution and can be used when a random variable is known to be bounded by an upper and a lower limit [33]. The normal distribution is valid between -∞ and +∞, and the lognormal distribution is valid between 0 and +∞. Many random variables of engineering significance are bounded within two limits and the beta distribution is quite appropriate for them. If a random variable *X* follows a beta distribution bounded between the lower bound *lb* and the upper bound *ub*, and the first and second moments, i.e., mean and variance for this interval data are found as m_1 and m_2, respectively, the conversion of this distribution to standard beta distribution involves the following equations to convert the mean and variance:

$$\overline{X} = E(X) = \frac{m_1 - lb}{ub - lb} \tag{3.4}$$

$$\sigma^2 = \text{var}(X) = \frac{m_2}{(ub - lb)^2} \tag{3.5}$$

Using the values of mean and variance, the shape parameters α and β of the standard beta distribution is evaluated by the following equations:

$$\alpha = \mu[\frac{\mu(1-\mu)}{\sigma^2} - 1] \tag{3.6}$$

$$\beta = (1-\mu)[\frac{\mu(1-\mu)}{\sigma^2} - 1] \tag{3.7}$$

The information of the standard beta distribution parameters help to generate the values of random variable X between [0, 1] and using these values, random variables of the original beta distribution between [lb, ub] can be easily generated.

3.5 Resilience-based Design Optimization of Network Infrastructures

There are several types of network optimization models commonly used for solving various issues related to networks such as shortest path problem, spanning tree problem, maximum flow problem, minimum cost flow problem, etc. [34]. However, these models are basically developed for solving problems for existing networks with fixed link capacities and costs associated with these models are basically the transportation costs. Network design optimization is somewhat different from these problems.

Designing any infrastructure network involves the long-term decision making procedure about the topology and the design capacity of the links of the network. This is the most crucial part of network design because deciding on these two issues eventually controls the net level of performance of the network and the cost of building the network. In practice, the objective of design optimization of any network is maximizing the level of performance while minimizing the design cost of the network. To solve this problem, the decision variable must be defined as the capacities of the links and the optimum solution would be those values of link capacities ensuring maximum level of performance of the network while incurring minimum cost.

Along with these objectives, when the target design optimization is focused on resilience of the network, another objective comes at hand which is maximizing the network resilience and ensuring a minimum level of performance at the recovery state after the disruption. This problem also has the same decision variable, i.e., the link capacities. Designing the network topology by selecting the links with higher capability to quickly restore the lost capacity due to any disruptive event and selecting the optimum values of the design capacities of these links can solve this issue.

Zhang et al. [3] solved the resilience-based network design optimization problem by minimizing the design cost considering a minimum level of performance at the recovery state and a minimum amount of resilience as the constraints. They expressed the level of performance of the network in terms of flow through the network. They proposed two formulations for solving the problem-a deterministic formulation and a stochastic formulation as shown in Eqs. (3.8) and (3.9), respectively.

$$\begin{aligned} &\min \sum_{(i,j)\in E} \delta_{ij} c_{ij} \\ &s.t.\ \frac{\psi\left(u^*(t)\middle|e^q\right)-\psi\left(t_d\middle|e^q\right)}{\psi(t_0)-\psi\left(t_d\middle|e^q\right)} > \kappa \\ &\qquad \psi(u^*(t)|e^q) \geq \xi \end{aligned} \tag{3.8}$$

$$\begin{aligned} &\min \sum_{(i,j)\in E} \delta_{ij} c_{ij} \\ &s.t.\ P\left(\frac{\psi\left(u^*(t)\middle|e^q\right)-\psi\left(t_d\middle|e^q\right)}{\psi(t_0)-\psi\left(t_d\middle|e^q\right)} > \kappa\right) > \beta \\ &\qquad P(\psi(u^*(t)|e^q) \geq \xi) > \theta \end{aligned} \tag{3.9}$$

In the above formulations,

$$\delta_{ij} = \begin{cases} 1, \text{if link } (i, j) \text{ is included in the network} \\ \qquad\qquad 0, \text{otherwise} \end{cases}$$

c_{ij} = cost incurred to build the link (i, j)

$\psi(t_0)$ = level of performance of the network before any disruption

$\psi(t_d|e^q)$ = level of performance of the network at the disrupted state

$\psi(u^*(t \mid e^q))$ = level of performance of the network at time t after the disruption

$\frac{\psi\left(u^{*}(t)\middle|e^{q}\right)-\psi\left(t_{d}\middle|e^{q}\right)}{\psi(t_{0})-\psi\left(t_{d}\middle|e^{q}\right)}$ = resilience at time t

κ = predefined threshold for system resilience

ξ = predefined threshold for the level of performance at time t after the disruption

β = predefined threshold for probability of system resilience to be greater than κ at time t after the disruption

θ = predefined threshold for probability of level of performance at time t after the disruption to be greater than ξ

In these formulations, the decision variable is δ_{ij} and the decision is made on whether a link would be present or not in the optimum solution. However, this formulation does not provide any choice on the values of the required link capacities that must be kept to satisfy the constraints. The optimum solution found using this formulation sticks to the originally declared maximum possible capacity of the links. In practical cases, these amount of capacities may not be required to satisfy the constraints, and as a result, these extra amount of capacities would incur a greater amount of design cost. Also, this formulation cannot ensure integer values for capacities in the disrupted and recovery states.

In this research, an improved optimization formulation to this resilience-based network design problem is proposed that can not only decide on the presence of a link in the optimum solution but also can decide on the required design capacities of these links.

CHAPTER 4

MODEL FORMULATION AND SOLUTION METHODOLOGY

4.1 Model Formulation

As discussed in Chapter 3, the objectives of this resilience-based network design optimization are as follows:

i. Minimization of the network design cost
ii. Maximization of the level of performance of the network at the recovery state
iii. Maximization of the amount of resilience

In order to satisfy all these three objectives, in this research, the optimization problem is formulated using two optimization models- an inner optimization model to operate within an outer optimization model. The outer optimization model minimizes the design cost and ensures a maximum level of resilience attainable within this design cost, whereas, the inner optimization model maximizes the level of performance of the network at the recovery state. The formulations are developed for both deterministic and stochastic cases.

4.1.1 Deterministic Model Formulation

The outer optimization problem for the deterministic case (where all the parameter values are assumed to be known with certainty) is formulated as shown below:

$$\begin{aligned}
&\underset{u_{ij}}{Min} \quad f = \sum_{(i,j)\in E} c_{ij} l_{ij} u_{ij} \\
&s.t. \quad \frac{\psi\left(t \middle| e^q\right) - \psi\left(t_d \middle| e^q\right)}{\psi\left(t_0\right) - \psi\left(t_d \middle| e^q\right)} \geq \kappa \\
&\qquad \psi\left(t \middle| e^q\right) \geq \xi \\
&\qquad lb_{ij} \leq u_{ij} \leq ub_{ij} \quad \forall (i,j) \in E \\
&\qquad u_{ij} \text{ are integers} \, \forall (i,j)
\end{aligned} \tag{4.1}$$

where,

u_{ij} = capacity of the link (i, j) (the decision variable)
c_{ij} = cost per unit length per unit capacity for the link (i, j)
l_{ij} = length of the link (i, j)

lb_{ij} = lower bound for u_{ij}
ub_{ij} = upper bound for u_{ij}

All other symbols carry the same meaning as mentioned in section 3.5.

In this outer optimization model, the link capacities u_{ij} are defined as the decision variables. Cost of any link (i, j) is defined in terms of capacity and length of the link (i, j) which is a more realistic definition for design cost as compared to [3]. The objective function is taken as the minimization of the network design cost. Although ensuring at least a minimum level of performance ξ and at least a minimum amount of resilience κ at the recovery state are taken as the constraints, these two quantities are basically maximized in other ways. The level of performance is maximized using an inner optimization model and the resilience is maximized using *Probabilistic Solution Discovery Algorithm* by achieving a balance between design cost and resilience.

Similar to Zhang et al. [3], in this research, the level of performance of the network is defined as the maximum amount of flow through the network. In the outer optimization model formulated in Eq. (4.1), the level of performance is maximized by using a maximum flow problem as the inner optimization model while evaluating the constraints. The inner optimization model is formulated as below:

$$\begin{aligned} \max \quad & f_{od} \\ s.t. \quad & \sum_{(i,j)\in E} f_{ij} - \sum_{(j,i)\in E} f_{ji} = 0 \quad \forall j \notin \{o, d\} \\ & 0 \le f_{ij} \le u_{ij} \qquad\qquad \forall (i,j) \in E \\ & f_{ij} \text{ are integers } \forall (i,j) \end{aligned} \tag{4.2}$$

where,

o = origin node
d = destination node
f_{od} = flow from the origin o to destination d
f_{ij} = flow through the link (i, j),
u_{ij} = the maximum capacity of the link (i, j) obtained from the optimization problem in Eq. (4.1).

In order to evaluate the constraints in Eq. (4.1), the optimization problem formulated in Eq. (4.2) is solved for three times to obtain the original state maximum flow at time t_0, i.e., $\psi(t_0)$, the disrupted state maximum flow at time t_d i.e., $\psi\left(t_d \middle| e^q\right)$, and the recovered state maximum flow at time t i.e., $\psi\left(t \middle| e^q\right)$. To obtain $\psi(t_0)$, the capacities obtained from the optimizer in Eq. (4.1) as the values of the decision variables u_{ij} are directly used as the maximum link capacities in the optimization in Eq. (4.2). To calculate $\psi\left(t_d \middle| e^q\right)$ and $\psi\left(t \middle| e^q\right)$, the disrupted state link capacities and the recovered state link capacities are calculated first. For this purpose, Eq. (3.3) is modified as below:

$$u_{ij}^d\left(t_d \middle| e^q\right) = a_{ij}u_{ij} \tag{4.3}$$

$$u_{ij}^r(t|e^q) = u_{ij}[a_{ij} + \lambda_{ij}(1-a_{ij})(1-e^{-b_{ij}t})] \tag{4.4}$$

where,

$u_{ij}^d\left(t_d \middle| e^q\right)$ = disrupted state capacity of the link (i, j)

$u_{ij}^r\left(t \middle| e^q\right)$ = recovered state capacity of the link (i, j)

t_d = time at which the system reaches its maximum disrupted state

t = time elapsed after disruption

All other symbols carry the same meaning as mentioned in section 3.3.

4.1.2 Stochastic Model Formulation

In order to encounter the uncertainty in the disruptive event, the resilience constraint and the recovery state flow constraint are formulated in a probabilistic manner in the stochastic model formulation as shown below:

$$\begin{aligned}
&\underset{u_{ij}}{Min} \quad f = \sum_{(i,j)\in E} c_{ij} l_{ij} u_{ij} \\
&s.t. \quad P\left(\frac{\psi\left(t \middle| e^q\right) - \psi\left(t_d \middle| e^q\right)}{\psi(t_0) - \psi\left(t_d \middle| e^q\right)} \geq \kappa \right) \geq \upsilon \\
&\qquad P\left(\psi\left(t \middle| e^q\right) \geq \xi\right) \geq \theta \\
&\qquad lb_{ij} \leq u_{ij} \leq ub_{ij} \quad \forall (i,j) \in E \\
&\qquad u_{ij} \text{ are integers} \, \forall (i,j)
\end{aligned} \tag{4.5}$$

where,

v = predefined threshold for probability of network resilience

θ = predefined threshold for probability of $\psi\left(t\middle|e^{q}\right)\geq\xi$

In this formulation, all other symbols have the same meaning as formulated in Eq. (4.1). The maximization of the level of performances in different states is also ensured using the formulation in Eq. (4.2) as the inner optimization model.

4.2 Solution Methodology

In order to solve this type of network optimization formulations, both combinatorial and heuristic algorithms can be used. However, the combinatorial algorithms require searching the entire solution space to find the optimal solution and therefore, they are computationally expensive. As a result, the use of combinatorial techniques in the literature is restricted to small networks [3]. On the other hand, heuristic algorithms are able to provide near-optimal solutions within an acceptable computational time, even for large-scale problems.

4.2.1 Solution Methodology for the Deterministic Model

In this research, the optimization problems formulated in Eqs. (4.1) and (4.2) are solved using a combination of three different methods. The inner optimization problem formulated in Eq. (4.2) is a linear optimization problem and therefore, it is solved using *Linear Programing* algorithm for integer variables. The optimization formulation in Eq. (4.2) is solved in two steps.

In the first step, the formulation is converted into a smaller problem eliminating the resilience constraint as follows:

$$
\begin{aligned}
&\underset{u_{ij}}{Min}\quad f=\sum_{(i,j)\in E}c_{ij}l_{ij}u_{ij}\\
&s.t.\quad \psi\left(t\middle|e^{q}\right)\geq\xi\\
&\qquad lb_{ij}\leq u_{ij}\leq ub_{ij}\qquad \forall(i,j)\in E\\
&\qquad u_{ij}\ \text{are integers}\qquad \forall(i,j)
\end{aligned}
\tag{4.6}
$$

The smaller problem formulated in Eq. (4.6) involves nonlinear constraints, linear equality constraints in the inner optimization problem, and integer decision variables. This problem is solved using *Genetic Algorithm* (*GA*), which is capable of handling all these three issues. Genetic Algorithm (GA) is a commonly used meta-heuristic method [35-37], which is able to generate high-quality solutions to optimization and search problems compared to general heuristic methods.

In the second step, the resilience constraint is evaluated using *Probabilistic Solution Discovery Algorithm* (*PSDA*) due to two reasons- i) its efficiency in solving optimization problems and ii) its ability to maximize the value of resilience in the optimal solution by achieving a balance between the design cost and resilience. This algorithm was originally developed by Ramirez-Marquez and Rocco [38] and widely used in literature to address various network optimization problems, e.g., all-terminal network reliability allocation [38], network interdiction [39], network protection [40], etc. Instead of dealing with the original large network, this algorithm randomly generates smaller sample networks from the original one and eventually identifies the optimal network. However, Zhang et al. [3] identified that the evaluation of the optimal value of the penalty term introduced in the original probabilistic solution discovery algorithm to transform a constrained optimization problem into an unconstrained one results in a difficult optimization problem by itself. To overcome this challenge, Zhang et al. [3] utilized the *Stochastic Ranking* technique developed by Runarsson and Yao [41] and integrated it with the original probabilistic solution discovery algorithm and proposed an *Improved Probabilistic Solution Discovery Algorithm* to deal with the resilience-based network design problem. Therefore, in this study, this improved probabilistic solution discovery algorithm is modified according to the requirement of this research and is used to maximize the resilience while minimizing the design cost.

The use of the stochastic ranking technique has assisted in achieving the balance between the design cost and resilience of randomly generated networks. The algorithm compares each pair of adjacent individuals in a bubble sort manner and eventually helps to achieve a balance between the objective function and the constraint. In this research methodology, the balance between the objective function, i.e., the design cost f_s and the resilience constraint R_s ($\forall S \in SAMPLE$, *SAMPLE* is the set of all sample networks randomly generated in the probabilistic solution discovery algorithm) is achieved using the stochastic ranking technique. This algorithm sorts the more desirable solutions in an ascending order as given below:

Algorithm 1: Stochastic Ranking Algorithm
Initialize the set of all sample networks $S \in SAMPLE$, f_s, R_s, and $P_{f_s} = 0.5 \ \forall S \in SAMPLE$
for p = 1 to SAMPLE **do**
for k = 1 to (SAMPLE-1) **do**
Randomly generate $w \in W(0,1)$
if $[R_s(k) \geq \kappa \ \& \ R_s(k+1) \geq \kappa]$ or ($w > P_f$) **then**
if $f_s(k) < f_s(k+1)$ **then**
swap (S_k, S_{k+1})
end if
else if $R_s(k) > R_s(k+1)$ **then**
swap (S_k, S_{k+1})
end if
end for
Update P_{f_s}
end for

The following algorithm shows the overall proposed methodology for solving the deterministic resilience-based network design optimization formulation mentioned in Eq. (4.1). The algorithm includes the following iteration parameters:

SAMPLE = number of sample networks taken in each iteration of the improved probabilistic solution discovery algorithm

$\gamma_{ij_{iter}}$ = probability of link (i, j) to be present in the optimal network in the $iter^{th}$ iteration $\forall (i, j) \in E$

T = number of top-ranked networks taken for the evaluation of $\gamma_{ij_{iter}}$ for the next iteration

itermax = maximum number of iterations to proceed for the improved probabilistic solution discovery algorithm

Algorithm 2: Solution Methodology for the Deterministic Resilience-based Network Design Optimization using Improved Probabilistic Solution Discovery Algorithm.	
1:	Initialize DECISION VARIABLE u_{ij}, $\forall(i, j) \in E$, OPTIMIZATION PARAMETERS c_{ij}, lb_{ij}, ub_{ij}, κ, ξ, t, RESILIENCE PARAMETERS a_{ij}, b_{ij}, λ_{ij}, and ITERATION PARAMETERS *SAMPLE*, $\gamma_1 = 0.5$, T, $iter = 1$, *itermax*
2:	**while** $iter \le itermax$ **do**
	STEP 1: Strategy Development
3:	**for** $s = 1$ to *SAMPLE* **do**
4:	Using Monte Carlo Simulation randomly generate s^{th} sample network: $U_{iter}^{s} = \left(\ldots, u_{ij_{iter}}^{s}, \ldots\right), \forall(i,j) \in E$ Based on $\gamma_{ietr} = \left(\ldots, \gamma_{ij_{iter}}, \ldots\right), \forall(i,j) \in E$
5:	**end for**
6:	**if** $\gamma_{ij_{iter}} = 1$ or $\gamma_{ij_{iter}} = 0, \forall(i,j) \in E$ **then**
7:	$U^{*} = \arg\max\{T\}$
8:	break;
9:	**end if**
	STEP 2: Strategy Analysis
10:	(a) Evaluate the optimization problem formulated in Eq. (4.6) for all sample networks: $\underset{u_{ij}}{Min} \quad f_s = \sum_{(i,j)\in E} c_{ij} l_{ij} u_{ij} \qquad \forall s \in SAMPLE$ $s.t. \quad \psi\left(t \middle\| e^{q}\right) \ge \xi$ $lb_{ij} \le u_{ij} \le ub_{ij} \qquad \forall(i,j) \in E$ u_{ij} are integers $\qquad \forall(i,j)$
11:	(b) Calculate the resilience for all optimal sample networks given by the following equation: $R_s = \dfrac{\psi\left(t \middle\| e^{q}\right) - \psi\left(t_d \middle\| e^{q}\right)}{\psi\left(t_0\right) - \psi\left(t_d \middle\| e^{q}\right)} \qquad \forall s \in SAMPLE$

12:	(c) Based on design cost f_s and resilience R_s, list all the generated solutions in an ascending order (i.e., solution having lower cost and higher resilience achieves higher position) in set S using the stochastic ranking technique as described in Algorithm 1, $\forall s \in SAMPLE$ $S \rightarrow \{S_{iter}^{1} \leq S_{iter}^{2} \leq \ldots \leq S_{iter}^{s} \leq \ldots \leq S_{iter}^{SAMPLE}\}$ A higher position in S means a more desirable solution.
	STEP 3: Solution Discovery
13:	(a) Take a set of T top-ranked samples from the set of ordered samples $T \rightarrow T \cup \left\{S_{iter}^{1}, S_{iter}^{2}, \ldots, S_{iter}^{T}\right\}$
14:	(b) $iter = iter + 1$
15:	(c) Update the values of γ_{iter} $\gamma_{ij_{iter}} = \dfrac{\sum_{x=1}^{T} \text{no. of times}\left(u_{ij_{(iter-1)}}^{*x}\right) > 0}{\sum_{x=1}^{T} \text{no. of times}\left(u_{ij_{(iter-1)}}^{x}\right) > 0} \quad \forall (i,j) \in E$
16:	**end while**

4.2.2 Solution Methodology for the Stochastic Model

The stochastic model as shown in Eq. (4.5) deals with uncertainty in the following ways:

i. Estimation of epistemic uncertainty in resilience parameters (*a, b,* and *λ*) from interval data
ii. Evaluation of the values of the probabilistic constraints

In order to estimate epistemic uncertainty in resilience parameters (*a, b,* and *λ*) from interval data, the bounds of the first and second moments for these data are derived first using the methods provided in Table 3.1 and Table 3.2. Within these bounds, moment values are randomly estimated to proceed with further calculations.

The values of the parameters *a, b,* and *λ* are estimated using beta distributions as explained in Chapter 3. Using Eqs. (3.4) and (3.5), the moment values i.e., the means and variances for the beta distributions between upper and lower bounds are converted to the mean and variance for the standard beta distributions between [0,1]. Using Eqs. (3.6) and (3.7), the parameters for these standard beta distributions are evaluated. Finally, using these standard beta distribution parameters, the values of *a, b,* and *λ* are generated randomly within their respective intervals.

In order to evaluate the probabilistic constraints, the values of *a, b,* and λ are estimated randomly for *n* times for *n* different types of disruptive events and this incorporates the uncertainty in the nature of the disruptive event into the model. Using these values for *n* types of disrupted scenarios, the probability of maximum level of performance in the recovery state greater than the threshold, i.e., $P(\psi(t|e^q) \geq \xi)$ and the probability of resilience greater than the threshold, i.e., $P\left(\frac{\psi(t|e^q)-\psi(t_d|e^q)}{\psi(t_0)-\psi(t_d|e^q)} \geq \kappa\right)$ are counted. And finally, the optimal network is derived using the improved probabilistic solution discovery algorithm as described in the previous section. The algorithm for the overall proposed methodology for solving the stochastic resilience-based network design optimization formulation mentioned in Eq. (4.5) is shown below (*n* = no. of different disruptive event scenarios):

Algorithm 3: Solution Methodology for the Stochastic Resilience-based Network Design Optimization Model	
Part 1: Representation of Epistemic Uncertainty	
1:	Initialize DECISION VARIABLE u_{ij}, $\forall(i, j) \in E$, OPTIMIZATION PARAMETERS c_{ij}, lb_{ij}, ub_{ij}, κ, ξ, v, θ, n, t, intervals for RESILIENCE PARAMETERS $[a_{lb_{ij}}, a_{ub_{ij}}]$, $[b_{lb_{ij}}, b_{ub_{ij}}]$, $[\lambda_{lb_{ij}}, \lambda_{ub_{ij}}]$, and ITERATION PARAMETERS *SAMPLE*, $\gamma_1 = 0.5$, T, $iter = 1$, *itermax*
2:	Estimate the bounds of first and second moments (i.e., mean and variance) for the RESILIENCE PARAMETERS using the methods in Tables 3.1 and 3.2 i.e., $M_{1_{a_{ij}}} = [M_{1_{lba_{ij}}}, M_{1_{uba_{ij}}}], M_{1_{b_{ij}}} = [M_{1_{lbb_{ij}}}, M_{1_{ubb_{ij}}}], M_{1_{\lambda_{ij}}} = [M_{1_{lb\lambda_{ij}}}, M_{1_{ub\lambda_{ij}}}]$, and $M_{2_{a_{ij}}} = [M_{2_{lba_{ij}}}, M_{2_{uba_{ij}}}], M_{2_{b_{ij}}} = [M_{2_{lbb_{ij}}}, M_{2_{ubb_{ij}}}], M_{2_{\lambda_{ij}}} = [M_{2_{lb\lambda_{ij}}}, M_{2_{ub\lambda_{ij}}}]$
3:	Randomly generate values for first and second moments within the moment bounds i.e., $m_{1_{a_{ij}}}$, $m_{2_{a_{ij}}}$; $m_{1_{b_{ij}}}, m_{2_{b_{ij}}}$; and $m_{1_{\lambda_{ij}}}, m_{2_{\lambda_{ij}}}$
4:	Convert the moment values into moments for standard beta distributions (i.e., $\overline{X}_{a_{ij}}, \sigma^2_{a_{ij}}$; $\overline{X}_{b_{ij}}, \sigma^2_{b_{ij}}$; and $\overline{X}_{\lambda_{ij}}, \sigma^2_{\lambda_{ij}}$) using Eqs. (3.4) and (3.5)
5:	Calculate the parameters of the standard beta distributions (i.e., $\alpha_{a_{ij}}, \beta_{a_{ij}}$; $\alpha_{b_{ij}}, \beta_{b_{ij}}$; and $\alpha_{\lambda_{ij}}, \beta_{\lambda_{ij}}$) using Eqs. (3.6) and (3.7)

6:	Randomly generate n values of a_{ij}, b_{ij}, and λ_{ij} within their intervals using the distribution parameters
Part 2: Implementation of Improved Probabilistic Solution Discovery Algorithm	
7:	**while** $iter \le itermax$ **do**
	STEP 1: Strategy Development
8:	**for** $s = 1$ to $SAMPLE$ **do**
9:	Using Monte Carlo Simulation randomly generate s^{th} sample network: $U^s_{iter} = \left(\ldots, u^s_{ij_{iter}}, \ldots\right), \forall (i,j) \in E$ Based on $\gamma_{ietr} = \left(\ldots, \gamma_{ij_{iter}}, \ldots\right), \forall (i,j) \in E$
10:	**end for**
11:	**if** $\gamma_{ij_{iter}} = 1$ or $\gamma_{ij_{iter}} = 0, \forall (i,j) \in E$ **then**
12:	$U^* = \arg\max\{T\}$
13:	break;
14:	**end if**
	STEP 2: Strategy Analysis
15:	(a) Evaluate the optimization problem formulated in Eq. (4.5) without considering the resilience constraint for all sample networks: $\underset{u_{ij}}{Min} \quad f_s = \sum_{(i,j)\in E} c_{ij} l_{ij} u_{ij} \qquad \forall s \in SAMPLE$ $s.t. \quad P\left(\psi\left(t \middle| e^q\right) \ge \xi\right) \ge \theta$ $lb_{ij} \le u_{ij} \le ub_{ij} \qquad \forall (i,j) \in E$ u_{ij} are integers $\qquad \forall (i,j)$
16:	(b) Calculate the resilience for n scenarios and the probability of resilience greater that the threshold for all optimal sample networks given by the following equation: $R_{n_s} = \frac{\psi_n\left(t \middle| e^q\right) - \psi_n\left(t_d \middle| e^q\right)}{\psi_n(t_0) - \psi_n\left(t_d \middle| e^q\right)} \qquad \forall s \in SAMPLE$ $P(R_s) = P\left(\frac{\psi\left(t \middle| e^q\right) - \psi\left(t_d \middle| e^q\right)}{\psi(t_0) - \psi\left(t_d \middle| e^q\right)} \ge \kappa\right)$

17:	(c) Based on design cost f_s and resilience $P(R_s)$, list all the generated solutions in an ascending order (i.e., solution having lower cost and higher resilience achieves higher position) in set S using stochastic ranking method, $\forall s \in SAMPLE$ $S \rightarrow \{S_{iter}^{1} \leq S_{iter}^{2} \leq \ldots \leq S_{iter}^{s} \leq \ldots \leq S_{iter}^{SAMPLE}\}$ Higher position in S means more desirable solution.
	STEP 3: Solution Discovery
18:	(a) Take a set of T top ranked samples from the set of ordered samples $T \rightarrow T \cup \left\{S_{iter}^{1}, S_{iter}^{2}, \ldots, S_{iter}^{T}\right\}$
19:	(b) $iter = iter + 1$
20:	(c) Update the values of γ_{iter} $\gamma_{ij_{iter}} = \dfrac{\sum_{x=1}^{T} \text{no. of times} \left(u_{ij_{(iter-1)}}^{*x}\right) > 0}{\sum_{x=1}^{T} \text{no. of times} \left(u_{ij_{(iter-1)}}^{x}\right) > 0} \quad \forall (i, j) \in E$
21:	**end while**

In the above algorithm, m_1 and m_2 represents the first and second moment values respectively.

4.3 Methodology for Consideration of Multiple Disruptive Events

Requirement of network design that can perform satisfactorily even after multiple disruptive events is an important practical requirement while performing design optimization based on resilience. In order to calculate the link capacities after N disruptive events, Eq. (4.4) can be modified as bellow:

$$u_{ij_N}(t) = u_{ij_0}\left[\prod_{r=1}^{N-1}\left[a_{ij} + \lambda_{ij}(1-a_{ij})\right] \times \left[\left[a_{ij} + \lambda_{ij}(1-a_{ij})(1-e^{-b_{ij}t})\right]\right]\right] \tag{4.7}$$

where, u_{ij_0} is the original capacity of the link (i, j) before the occurrence of any disruption.

Eq. (4.7) can be used in both the deterministic and stochastic models formulated in this study to evaluate the design parameter of the optimal network considering N disruptive events.

CHAPTER 5

NUMERICAL EXPERIMENTATION AND A CASE STUDY ON A DATA TRANSMISSION NETWORK

5.1 Numerical Example

In this section, a numerical example, similar to Zhang et al. [3], is solved using the proposed methodology. The network diagram was originally provided by Hillier and Liberman [34].

5.1.1 Problem Description

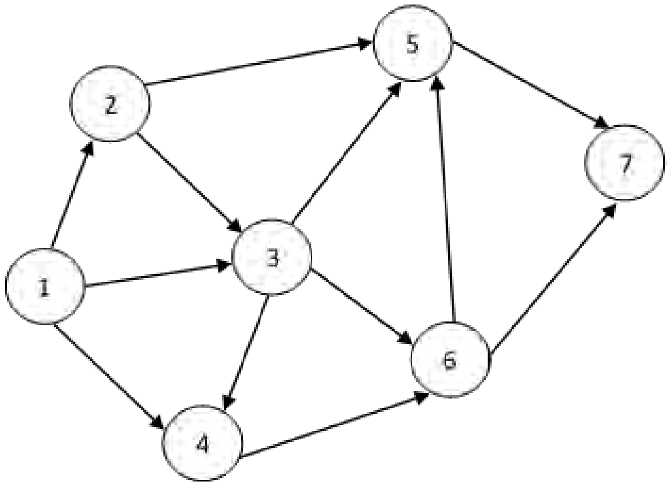

Fig. 5.1: Diagram of a 7-node directed network.

In Figure 5.1, node 1 is the supply node and node 7 is the demand node. Different parameters of this 7-node directed network of Figure 5.1 is listed below in Table 5.1.

Table 5.1: Link Characteristics for the 7-node directed network of Figure 5.1

Link	**Maximum Limit on Link Capacity, ub_{ij}**	**Length, l_{ij}**	**Cost, c_{ij}**	**Absorptive Ability (a)**	**Restoration Speed (b)**	**Restorative Ability (λ)**
(1, 2)	5	7	4	[0.28, 0.43]	[0.82, 4.95]	[0.80, 0.85]
(1, 3)	12	6	2	[0.15,0.31]	[0.78, 3.36]	[0.90, 0.95]
(1, 4)	10	5	3	[0.10,0.13]	[0.80, 1.16]	[0.97, 1.00]
(2, 3)	1	3	4	[0.37, 0.49]	[0.94, 1.16]	[0.60, 0.75]
(2, 5)	3	6	4	[0.33,0.45]	[0.82, 4.51]	[0.74, 0.85]
(3, 4)	13	5	3	[0.26,0.39]	[0.85, 2.26]	[0.97, 0.99]
(3, 5)	12	6	2	[0.22, 0.38]	[0.90, 2.67]	[0.96, 0.99]
(3, 6)	7	7	2	[0.25,0.42]	[0.96, 2.21]	[0.90, 0.95]
(4, 6)	14	6	3	[0.28,0.29]	[0.84, 1.97]	[0.97, 1.00]
(5, 7)	12	5	2	[0.27,0.33]	[0.76, 2.98]	[0.97, 1.00]
(6, 5)	12	9	3	[0.31,0.37]	[0.81, 3.39]	[0.97, 1.00]
(6, 7)	12	6	2	[0.24,0.42]	[0.88, 4.90]	[0.89, 0.92]

Our objective is to develop a resilience-based design optimization solution to this problem.

5.1.2 Problem solving

The problem is solved using the following values of the other parameters:

Optimization parameters: ξ = 9, κ = 0.8, v = 0.9, θ = 0.9, lb_{ij} = 0, n =20, and t = 6.

Iteration parameters: *SAMPLE* = 25, *T* = 40% of *SAMPLE*, and *itermax* = 15.

5.1.2.1 Results for Single Disruptive Event (i.e., *N* = 1)

i) Deterministic Results for Single Disruptive Event

The results found using the Algorithm 2 for the deterministic formulation is shown in the following tables. Here, Table 5.2 shows the convergence of the algorithm and Table 5.3 contains the results for the optimal network.

Table 5.2: Table for probabilities of the links to be present in the optimal network in each iteration for single disruptive event using deterministic formulation and proposed methodology.

Link	(1, 2)	(1, 3)	(1, 4)	(2, 3)	(2, 5)	(3, 4)	(3, 5)	(3, 6)	(4, 6)	(5, 7)	(6, 5)	(6, 7)
Iteration 0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Iteration 1	0	1.00	0.833	0	0	0.40	0.75	0.800	0.778	0.571	0.167	0.857
Iteration 2	0	0.90	0.556	0	0	0.25	0.571	0.5	0.667	0.8	0	0.667
Iteration 3	0	0.90	0.5	0	0	0	1.0	0.375	0.571	0.75	0	0.667
Iteration 4	0	0.90	0.2	0	0	0	0.9	0	0.250	1.0	0	0.167
Iteration 5	0	**1.00**	0	0	0	0	**1.0**	0	0	**1.0**	0	0

Table 5.3: Results for single disruptive event using deterministic formulation and proposed methodology.

Link	(1, 2)	(1, 3)	(1, 4)	(2, 3)	(2, 5)	(3, 4)	(3, 5)	(3, 6)	(4, 6)	(5, 7)	(6, 5)	(6, 7)
Optimal Capacities	0	10	0	0	0	0	10	0	0	10	0	0
Resilience	0.875											
Maximum flow in the recovery state	9											
Design Cost	340											

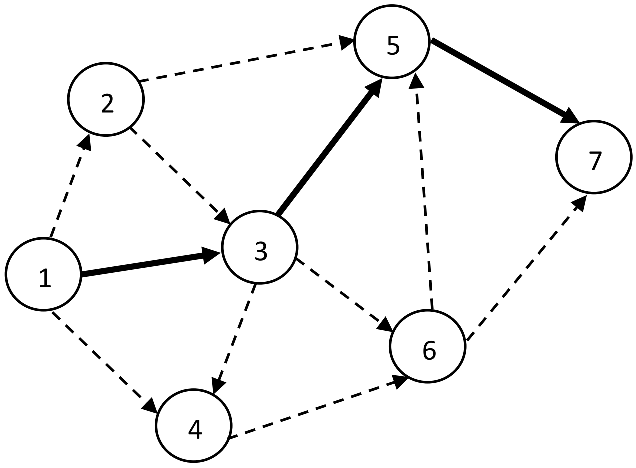

Fig. 5.2: Diagram of the optimal network for single disruptive event using deterministic formulation and proposed methodology.

In order to compare the result with another mathematical solution method the same problem is solved using only genetic algorithm as the solution methodology for the deterministic formulation. The results using this method is shown in the following table:

Table 5.4: Results for single disruptive event using mathematical solution method for deterministic formulation.

Link	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(2, 3)**	**(2, 5)**	**(3, 4)**	**(3, 5)**	**(3, 6)**	**(4, 6)**	**(5, 7)**	**(6, 5)**	**(6, 7)**
Optimal Capacities	0	12	0	0	0	0	10	0	0	10	0	0
Resilience	0.875											
Maximum flow in the recovery state	9											
Design Cost	364											

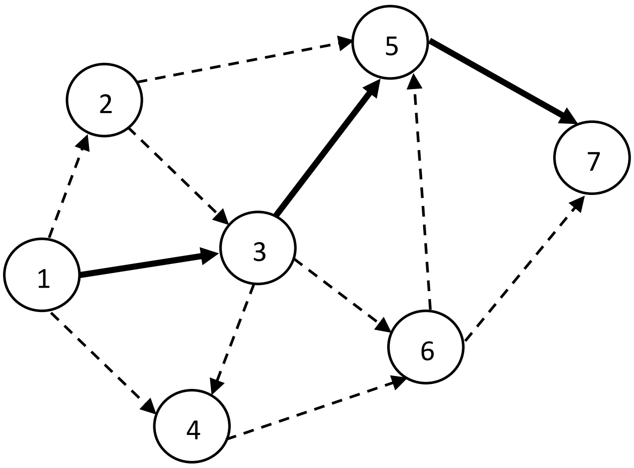

Figure 5.3: Diagram of the optimal network for single disruptive event using mathematical solution method for deterministic case.

Comparing the results in Tables 5.3 and 5.4 it is clear that, the proposed methodology ensures better solution in network design optimization for single disruptive event in deterministic case.

ii) Stochastic Results for Single Disruptive Event

The results found using the Algorithm 3 for the stochastic formulation is shown in the following tables. Here, Table 5.5 shows the convergence of the algorithm and Table 5.6 contains the results for the optimal network.

Table 5.5: Table for probabilities of the links to be present in the optimal network in each iteration for single disruptive event using stochastic formulation and proposed methodology.

Link	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(2, 3)**	**(2, 5)**	**(3, 4)**	**(3, 5)**	**(3, 6)**	**(4, 6)**	**(5, 7)**	**(6, 5)**	**(6, 7)**
Iteration 0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Iteration 1	0	1.0	0.75	0	0	0.5	0.5	0.667	0.8	0.75	0.333	0.6
Iteration 2	0	0.333	0.667	0	0	0	0.5	0	0.8	0.4	0	0.8
Iteration 3	0	0	**1.0**	0	0	0	0	0	**1.0**	0	0	**1.0**

Table 5.6: Results for single disruptive event using stochastic formulation and proposed methodology.

Link	(1, 2)	(1, 3)	(1, 4)	(2, 3)	(2, 5)	(3, 4)	(3, 5)	(3, 6)	(4, 6)	(5, 7)	(6, 5)	(6, 7)
Optimal Capacities	0	0	10	0	0	0	0	0	10	0	0	10
Probability of Resilience $\geq \kappa$	1											
Probability of Maximum flow in the recovery state $\geq \xi$	1											
Design Cost	340											

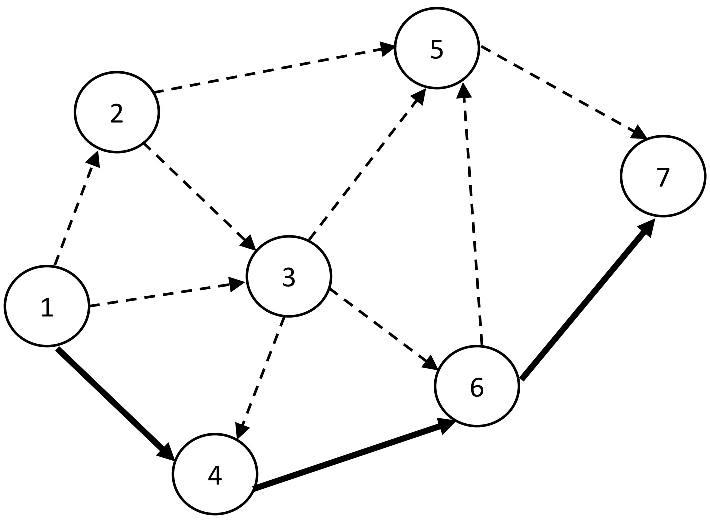

Figure 5.4: Diagram of the optimal network for single disruptive event using stochastic formulation and proposed methodology.

The results generated with the mathematical solution method of the same problem using genetic algorithm for the stochastic formulation is shown in the following table:

Table 5.7: Results for single disruptive event using mathematical solution method for stochastic formulation.

Link	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(2, 3)**	**(2, 5)**	**(3, 4)**	**(3, 5)**	**(3, 6)**	**(4, 6)**	**(5, 7)**	**(6, 5)**	**(6, 7)**
Optimal Capacities	0	4	7	0	0	0	4	0	7	4	0	7
Probability of Resilience $\geq \kappa$	1											
Probability of Maximum flow in the recovery state $\geq \xi$	1											
Design Cost	451											

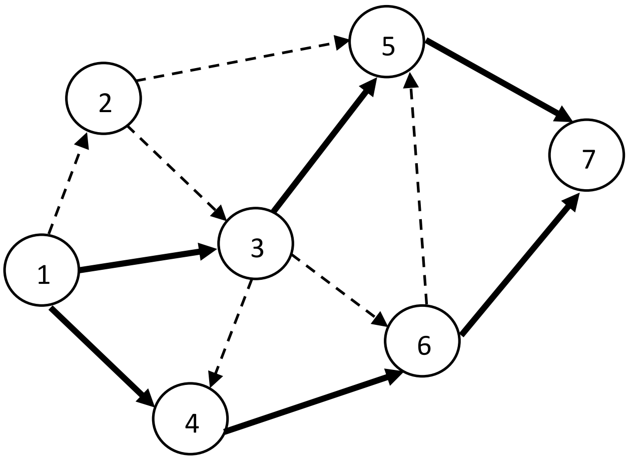

Fig. 5.5: Diagram of the optimal network for single disruptive event using mathematical solution method for stochastic case.

Comparing the results in Tables 5.6 and 5.7 it can be easily understood that, the proposed methodology also ensures better solution in network design optimization for single disruptive event in stochastic case.

5.1.2.1 Results for Multiple Disruptive Events (i.e., $N = 5$)

i) Deterministic Results for Multiple Disruptive Events

In this section, the optimal network designs are developed in order to withstand multiple disruptive events. The solution methodologies are performed for five disruptive events (i.e., $N = 5$). The results found using the Algorithm 2 for the deterministic formulation is shown in the following tables. Here, Table 5.8 shows the convergence of the algorithm and Table 5.9 contains the results for the optimal network.

Table 5.8: Table for probabilities of the links to be present in the optimal network in each iteration for multiple disruptive events using deterministic formulation and proposed methodology.

Link	(1, 2)	(1, 3)	(1, 4)	(2, 3)	(2, 5)	(3, 4)	(3, 5)	(3, 6)	(4, 6)	(5, 7)	(6, 5)	(6, 7)
Iteration 0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Iteration 1	0	1.0	1.0	1.0	0	0.143	1.0	0	1.0	1.0	0.125	1.0
Iteration 2	0	1.0	1.0	0.167	0	0	1.0	0	1.0	1.0	0	1.0
Iteration 3	0	**1.0**	**1.0**	0	0	0	**1.0**	0	**1.0**	**1.0**	0	**1.0**

Table 5.9: Results for multiple disruptive events using deterministic formulation and proposed methodology.

Link	(1, 2)	(1, 3)	(1, 4)	(2, 3)	(2, 5)	(3, 4)	(3, 5)	(3, 6)	(4, 6)	(5, 7)	(6, 5)	(6, 7)
Optimal Capacities	0	12	7	0	0	0	12	0	7	12	0	7
Resilience	0.8000											
Maximum flow in the recovery state	9											
Design Cost	723											

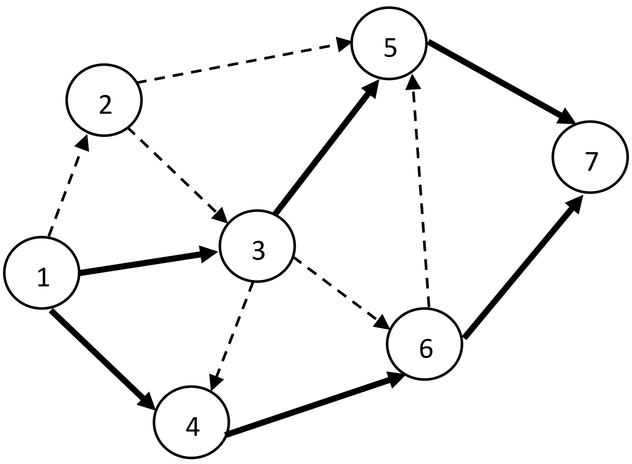

Fig. 5.6: Diagram of the optimal network for multiple disruptive events using deterministic formulation and proposed methodology.

In order to compare the result with the mathematical solution method the same problem is solved using genetic algorithm as the solution methodology for the deterministic formulation. The results using this method is shown in the following table:

Table 5.10: Results for multiple disruptive event using mathematical solution method for deterministic formulation.

Link	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(2, 3)**	**(2, 5)**	**(3, 4)**	**(3, 5)**	**(3, 6)**	**(4, 6)**	**(5, 7)**	**(6, 5)**	**(6, 7)**
Optimal Capacities	0	11	10	0	0	0	9	0	10	9	0	12
Resilience	0.8000											
Maximum flow in the recovery state	9											
Design Cost	804											

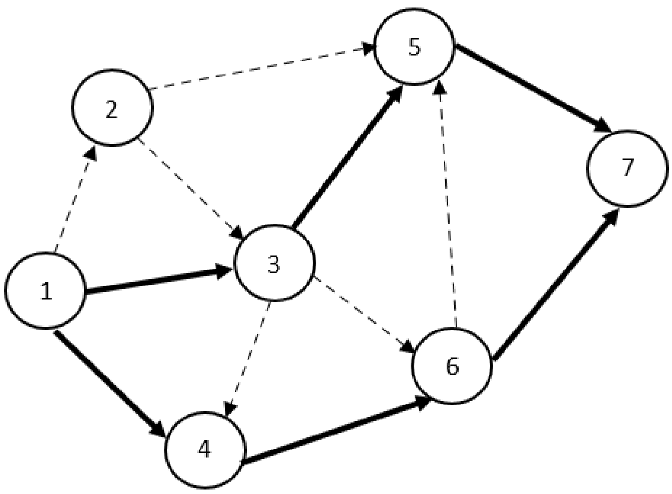

Fig. 5.7: Diagram of the optimal network for multiple disruptive event using mathematical solution method for deterministic case.

Comparing the results in Tables 5.9 and 5.10 it is visible that, the proposed methodology can ensure better solutions in network design optimization for multiple disruptive events in deterministic case.

ii) Stochastic Results for Multiple Disruptive Events

For multiple disruptive events, as the uncertainty is increased in the stochastic case, the maximum limits of the link capacities are required to be increased. The values of the other parameters are kept as before. The revised set of link capacities are shown in the following table.

Table 5.11: Table for revised maximum limits of link capacities for multiple disruptive event in the stochastic case.

Link	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(2, 3)**	**(2, 5)**	**(3, 4)**	**(3, 5)**	**(3, 6)**	**(4, 6)**	**(5, 7)**	**(6, 5)**	**(6, 7)**
Maximum Limit of Link Capacity, ub_{ij}	5	20	18	5	3	13	18	7	19	18	12	18

The results found using the Algorithm 3 for the stochastic formulation is shown in the following tables. Here, Table 5.12 shows the convergence of the algorithm and Table 5.13 contains the results for the optimal network.

Table 5.12: Table for probabilities of the links to be present in the optimal network in each iteration for multiple disruptive events using stochastic formulation and proposed methodology.

Link	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(2, 3)**	**(2, 5)**	**(3, 4)**	**(3, 5)**	**(3, 6)**	**(4, 6)**	**(5, 7)**	**(6, 5)**	**(6, 7)**
Iteration 0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Iteration 1	0	**1**	0	0	0	0	**1**	0	0	**1**	0	0

Table 5.13: Results for multiple disruptive events using stochastic formulation and proposed methodology.

Link	**(1, 2)**	**(1, 3)**	**(1, 4)**	**(2, 3)**	**(2, 5)**	**(3, 4)**	**(3, 5)**	**(3, 6)**	**(4, 6)**	**(5, 7)**	**(6, 5)**	**(6, 7)**
Optimal Capacities	0	16	0	0	0	0	14	0	0	14	0	0
Probability of Resilience $\geq \kappa$	1.0											
Probability of Maximum flow in the recovery state $\geq \xi$	1.0											
Design Cost	500											

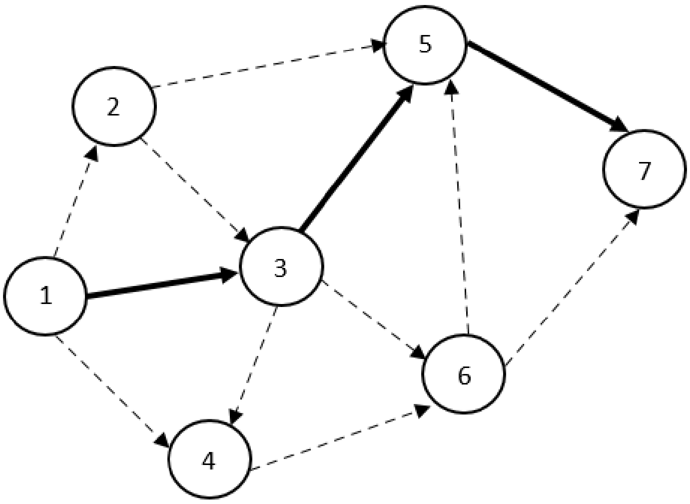

Figure 5.8: Diagram of the optimal network for multiple disruptive events using stochastic formulation and proposed methodology.

The results generated with the mathematical solution method of the same problem using genetic algorithm for the stochastic formulation is shown in the following table:

Table 5.14: Results for multiple disruptive events using mathematical solution method for stochastic formulation.

Link	(1, 2)	(1, 3)	(1, 4)	(2, 3)	(2, 5)	(3, 4)	(3, 5)	(3, 6)	(4, 6)	(5, 7)	(6, 5)	(6, 7)
Optimal Capacities	1	15	17	0	0	3	12	6	10	14	4	14
Probability of Resilience $\geq \kappa$	0											
Probability of Maximum flow in the recovery state $\geq \xi$	1											
Design Cost	1332											

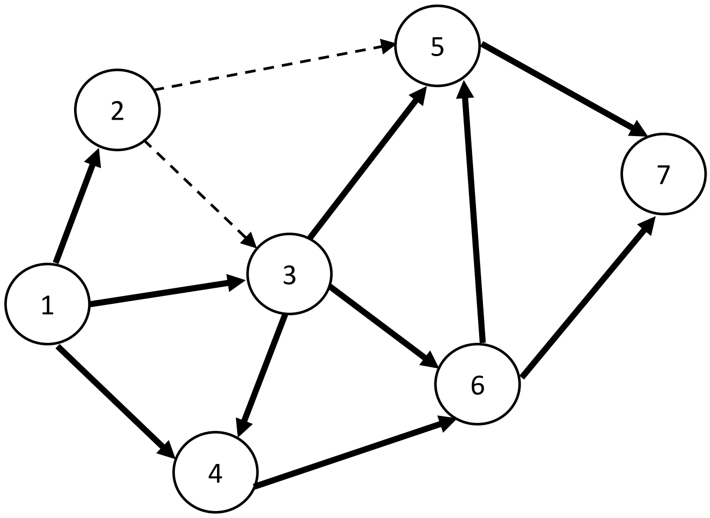

Figure 5.9: Diagram of the optimal network for multiple disruptive events using mathematical solution method for stochastic formulation.

As seen in Table 5.14, the mathematical solution method could not satisfy the constraints. Increasing the number of random scenarios of disruptive events might be able to improve the solution.

Comparing the results in Tables 5.13 and 5.14, it is justified that the proposed methodology ensures better solution in network design optimization for multiple disruptive events in stochastic case also.

5.1.3 Result Discussion

The results found in the previous section 5.1.2 are summarized in the following table:

Table 5.15: Result comparison between the proposed methodology and the mathematical method for the deterministic formulation.

		Deterministic Solution using Proposed Methodology	**Deterministic Solution using Mathematical Method**
Single Disruptive Event	**Optimal Design Cost**	340	364
	Resilience	0.875	0.874
	Maximum flow in the recovery state	9	9
Multiple Disruptive Events	**Optimal Design Cost**	723	804
	Resilience	0.8	0.8
	Maximum flow in the recovery state	9	9

Table 5.16: Result comparison between the proposed methodology and the mathematical method for the stochastic formulation.

		Stochastic Solution using Proposed Methodology	**Stochastic Solution using Mathematical Method**
Single Disruptive Event	**Optimal Design Cost**	340	506
	Probability of Resilience $\geq \kappa$	0.875	0.889
	Probability of Maximum flow in the recovery state $\geq \xi$	9	9
Multiple Disruptive Event	**Optimal Design Cost**	723	
	Probability of Resilience $\geq \kappa$	0.8	
	Probability of Maximum flow in the recovery state $\geq \xi$	9	

In the above Tables 5.15 and 5.16, it is evident that the proposed methodology can generate better solutions i.e., solutions with lower design cost while satisfying the constraints, for both deterministic and stochastic cases. The solutions are also better for both single disruptive event and for multiple disruptive events.

5.2 Case study on a Data Transmission Network

A practical experimentation of the proposed methodology is performed on a small segment of a large data transmission network. This practical scenario is provided by one of the few authorized companies in Bangladesh that have the license to build physical data transmission network all over the country.

5.2.1 Problem Description

The following diagram shows the existing data transmission network in Chandpur, Laksmipur, and Noakhali districts. The company is assumed to be in a contract with a bank to build a separate dark core connection from node 12 to node 24. Although data transmission networks are bidirectional in nature, In this case, the problem is formulated for only unidirectional transmission where node 12 is the supply node and node 24 is the demand node. As per mentioned by the bank, the capacity of this connection is required to be kept 1GBps. It is convenient for the data transmission company to build this connection within their existing network path as shown in the following Figure 5.10. In this problem, the proposed models and methodologies are going to be applied in order to figure out the optimal network topology and the link capacities for the bank that would be built among these nodes to ensure the resilience characteristics while minimizing the design cost.

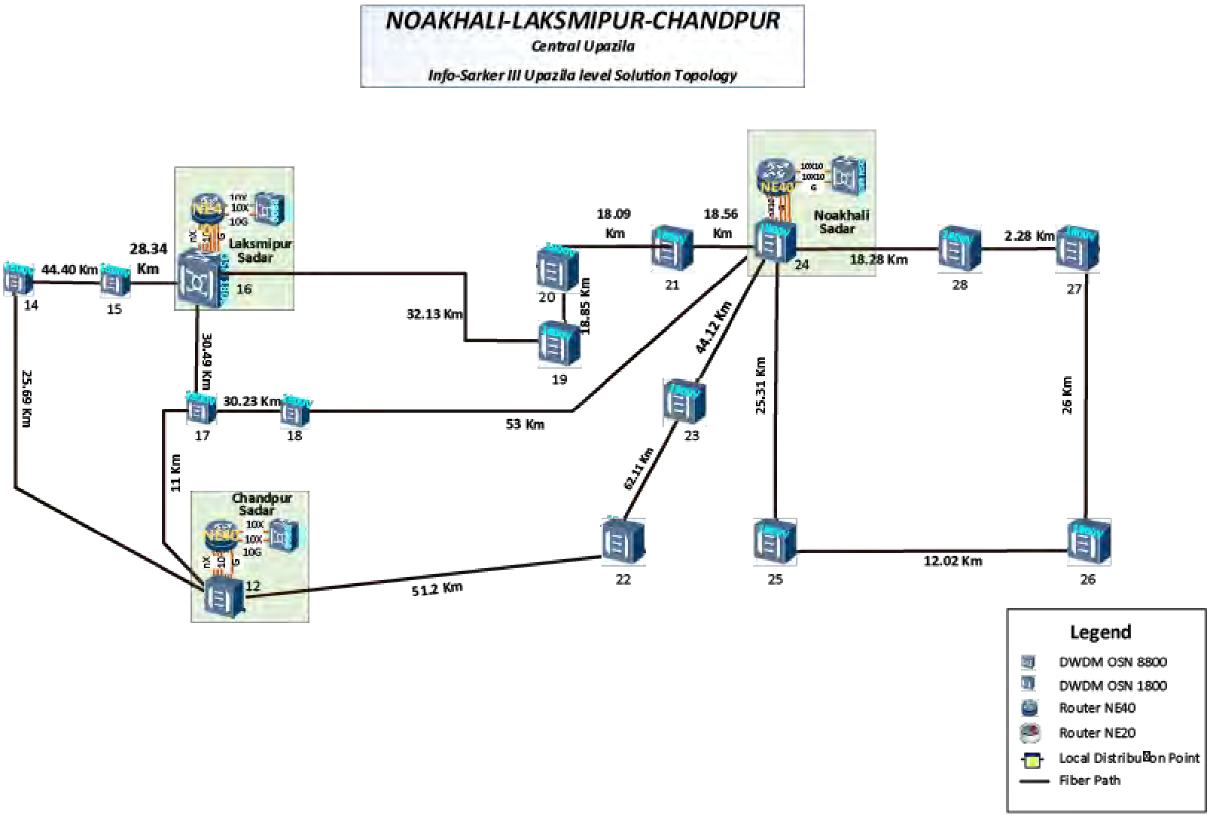

Figure 5.10: Existing data transmission network in Chandpur, Laksmipur, and Noakhali districts.

In order to find the optimal network from node 12 node 24, the figure 5.10 is simplified as below:

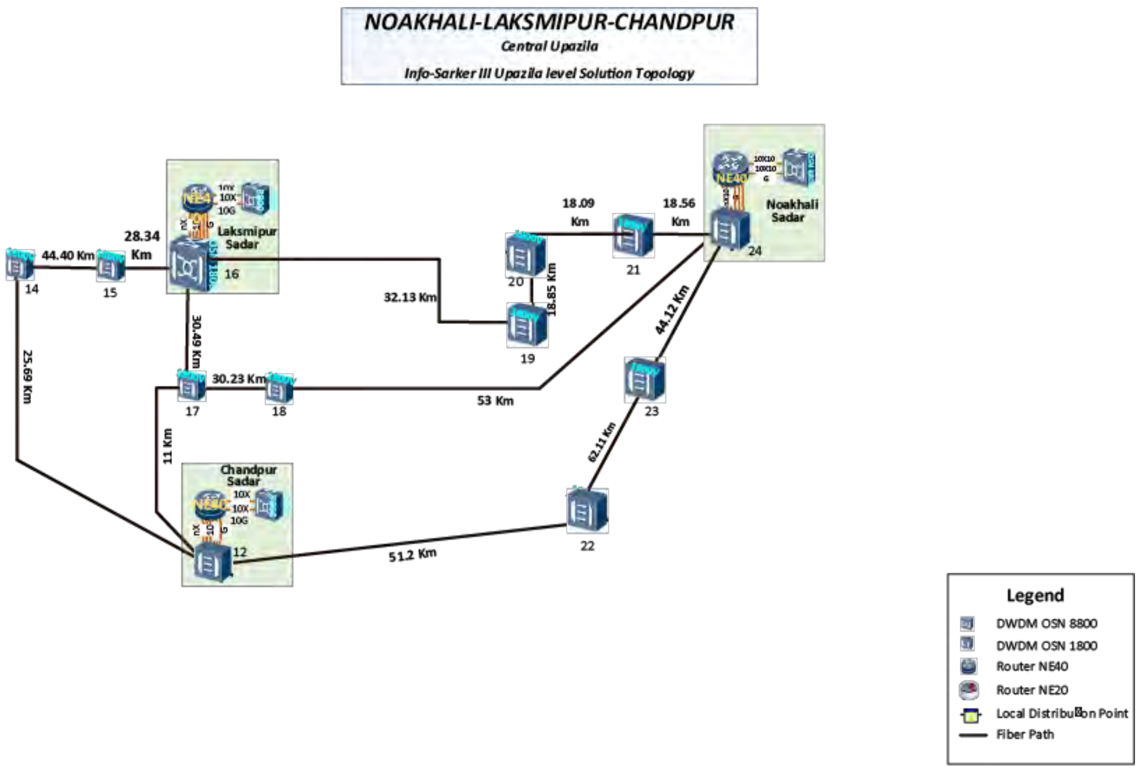

Figure 5.11: Existing data transmission network from node 12 to node 24.

The parameters related to this network are given below in Table 5.17.

Table 5.17: List of link parameters for Figure 5.11.

Link	Maximum Design Capacity (GB) ub_{ij}	Cost (Tk/Km.GB) c_{ij}	Distance (Km) l_{ij}	Absorptive Ability (a_{ij})	Restoration Speed GBps/hr (b_{ij})	Restorative Ability (λ_{ij})	Network Type
(12,14)	100	48,000	25.69	[0, 0.60]	[0.05,1.0]	1	Underground
(12,22)	10	150,000	51.2	[0,0. 30]	[0.05,1.0]	1	Overhead
(12,17)	100	48,000	11	[0, 0.60]	[0.05,1.0]	1	Underground
(14,15)	100	48,000	44.4	[0, 0.60]	[0.05,1.0]	1	Underground
(15,16)	100	48,000	28.34	[0, 0.60]	[0.05,1.0]	1	Underground
(17,16)	10	150,000	30.49	[0, 0.30]	[0.05,1.0]	1	Overhead
(17,18)	10	150,000	30.23	[0,0.30]	[0.05,1.0]	1	Overhead
(18,24)	10	150,000	38.3	[0, 0.30]	[0.05,1.0]	1	Overhead
(16,19)	10	48,000	32.13	[0, 0.60]	[0.05,1.0]	1	Underground
(19,20)	10	48,000	18.85	[0, 0.60]	[0.05,1.0]	1	Underground
(20,21)	10	48,000	18.09	[0, 0.60]	[0.05,1.0]	1	Underground
(21,24)	10	48,000	18.56	[0, 0.60]	[0.05,1.0]	1	Underground
(22,23)	10	150,000	62.11	[0, 0.30]	[0.05,1.0]	1	Overhead
(23,24)	10	150,000	44.12	[0, 0.30]	[0.05,1.0]	1	Overhead

5.2.2 Problem Solving

In this problem, the restorative ability of the links are 100%. Therefore, design optimization is not required to be performed for multiple disruptive events. The problem is solved for ξ = 1 GBps, κ = 0.8, v = 0.9, θ = 0.9, lb_{ij} =0, t = 4 hour, *SAMPLE* = 25, T = 40% of *SAMPLE* and n = 10.

The results for this problem are shown below:

i) Deterministic result

The deterministic result is shown in the following table:

Table 5.18: Deterministic result for the case study using proposed methodology.

Link	Link Probabilities			Optimal Design Capacity (GBps)	Cost (Tk/Km.GBps) c_{ij}	Resilience	Maximum flow at the Recovery State (GBps)
	Iteration 0	Iteration 1	Iteration 2				
(12,14)	0.5	0	0	0	2,60,25,000	1.0	1.0
(12,22)	0.5	0.5	0	0			
(12,17)	0.5	1	1	2			
(14,15)	0.5	0	0	0			
(15,16)	0.5	0	0	0			
(17,16)	0.5	0.125	0	0			
(17,18)	0.5	0.778	1	2			
(18,24)	0.5	0.778	1	2			
(16,19)	0.5	0.167	0	0			
(19,20)	0.5	0.167	0	0			
(20,21)	0.5	0.167	0	0			
(21,24)	0.5	0.25	0	0			
(22,23)	0.5	0.2	0	0			
(23,24)	0.5	0.333	0	0			

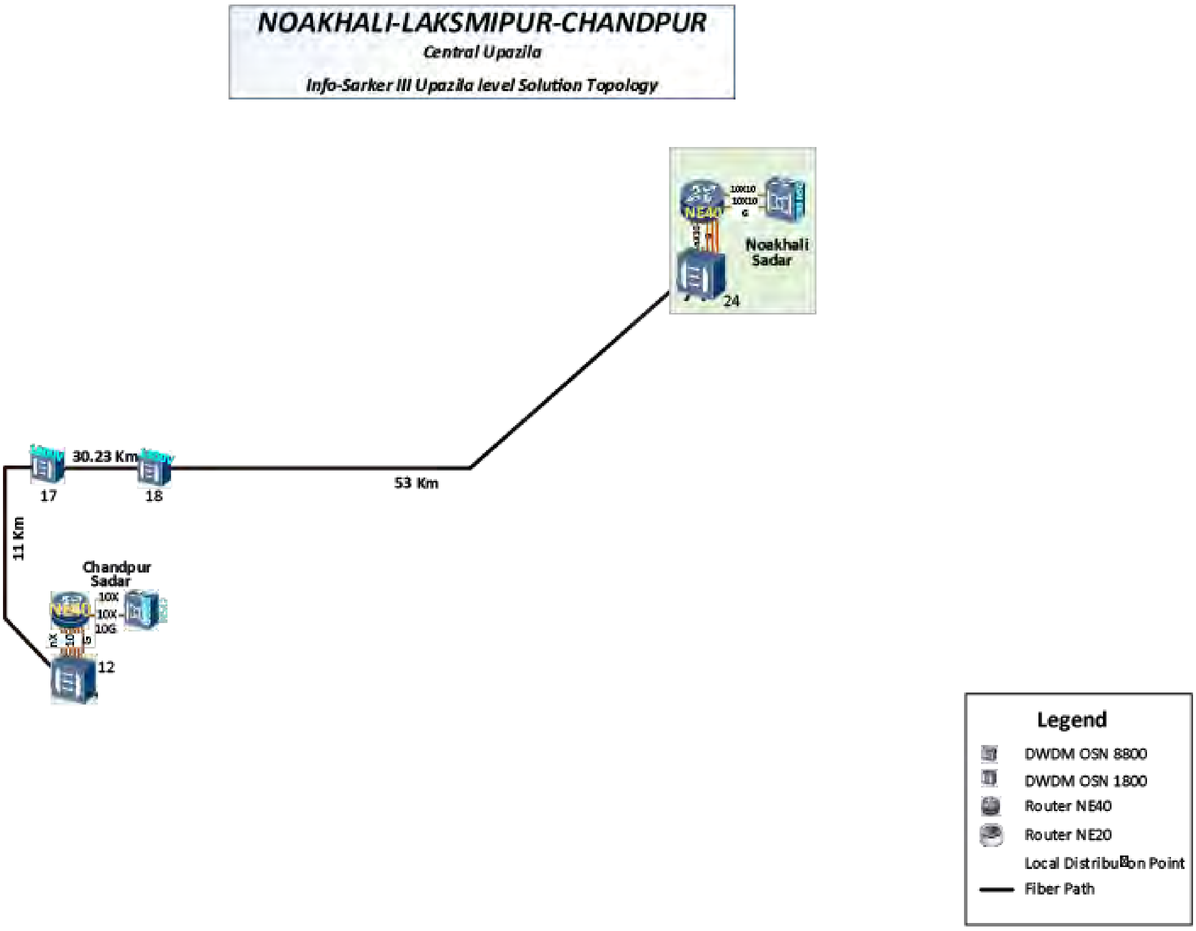

Figure 5.12: Diagram for the optimal network using deterministic formulation.

ii) Stochastic result

The stochastic result is shown in the following table:

Table 5.19: Stochastic result for the case study using proposed methodology.

Link	Link Probabilities			Optimal Design Capacity (GBps)	Cost (Tk/Km.GBps) c_{ij}	Probability of Resilience ≥ 0.8	Probability of Maximum flow in the recovery state ≥ 1 GBps
	Iteration 0	Iteration 1	Iteration 2				
(12,14)	0.5	0	0	0	7,80,75,000	1.0	1.0
(12,22)	0.5	0.333	0	0			
(12,17)	0.5	0.75	1.0	6			
(14,15)	0.5	0	0	0			
(15,16)	0.5	0	0	0			
(17,16)	0.5	0	0	0			
(17,18)	0.5	1.0	1.0	6			
(18,24)	0.5	0.75	1.0	6			
(16,19)	0.5	0	0	0			
(19,20)	0.5	0	0	0			
(20,21)	0.5	0	0	0			
(21,24)	0.5	0	0	0			
(22,23)	0.5	0.5	0	0			
(23,24)	0.5	0.5	0	0			

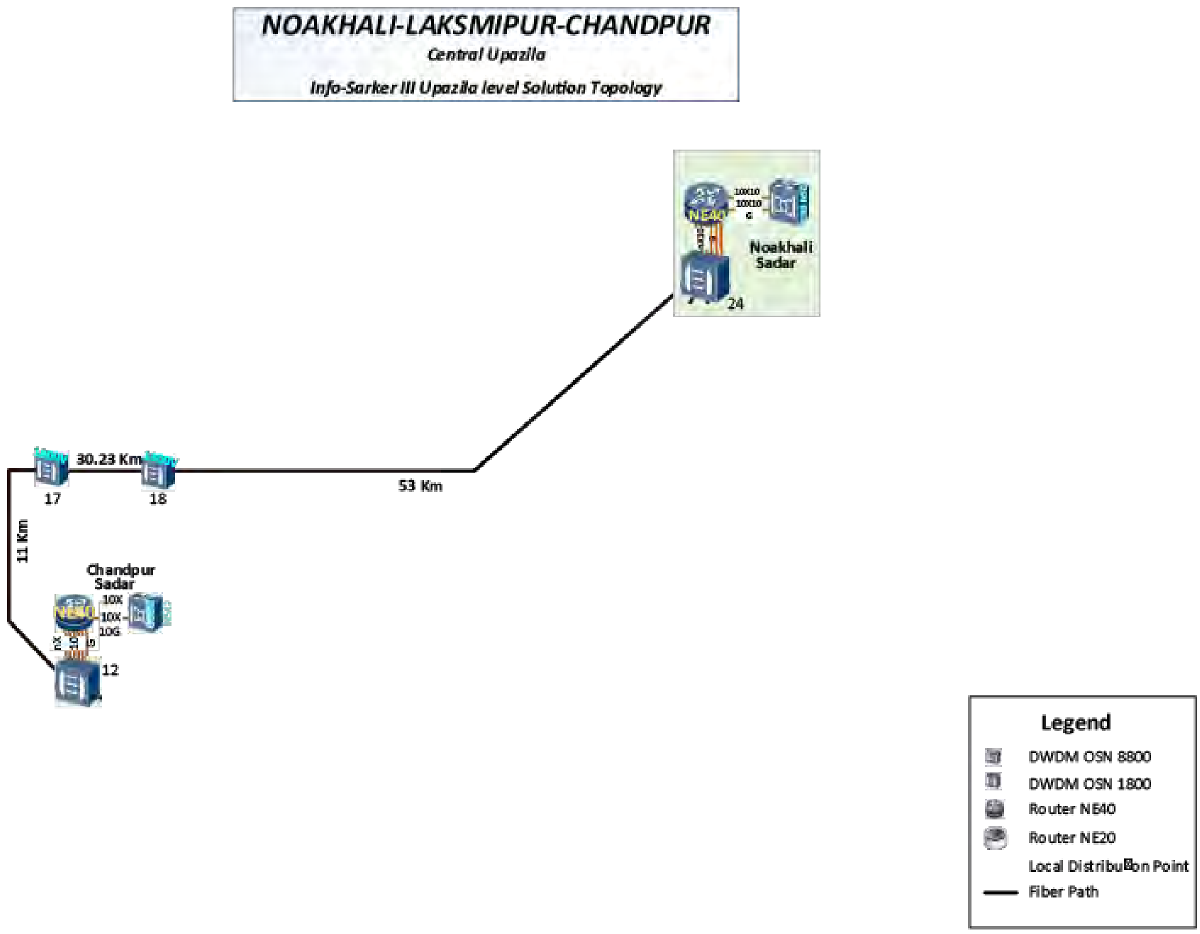

Figure 5.13: Diagram for the optimal network using stochastic formulation.

From Tables 5.18 and 5.19, it can be observed that the optimal design cost is greater in the stochastic formulation. But as the uncertainty in the resilience parameters are comparatively higher this case study, the stochastic result can better ensure the reliability of the optimal solution.

CHAPTER 6

CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

Consideration of resilience in the design phase of a network can significantly improve system performance even after the occurrence of any disruptive event. Design optimization of infrastructure networks based on resilience can derive optimal network topology while maximizing the resilience of the network and minimizing the design cost. In this research, both deterministic and stochastic models are formulated for resilience-based network design optimization where the design cost is minimized, the level of performance and resilience in the recovery state are maximized. This study provides the following contributions in this field of research:

i. Development of models for generating optimal network topology for both deterministic and stochastic cases where the optimal link capacities are decided for minimizing the design cost and maximizing the level of performance and resilience in the recovery state.
ii. Formulation of a more realistic definition of the design cost and consideration of capacity as an integer variable in all three states- original state, disrupted state, and recovery state.
iii. Quantification of epistemic uncertainty in resilience parameters.
iv. Consideration of the level of performance of the network after multiple consecutive disruptive events.

Most of the existing studies modeled the resilience-based design optimization formulations without considering these practical issues. The proposed formulation is solved for a numerical example and a practical problem based on a data transmission network. The proposed formulation is generalized for multidisciplinary applications and hopefully, the application of this study will significantly improve the design of future infrastructure networks.

6.2 Future Work

Several aspects of this research can be extended for future development of resilience-based network design optimization formulation. In this study, the length of the links are taken as parameters with deterministic values. Advanced model can be derived with the lengths as a decision variable. The values of the resilience parameters are obtained from expert opinions as interval data. Methods to calculate the exact values for these parameters must be developed to reduce the uncertainties and for these purpose, different types of networks must be studied thoroughly and data must be gathered through experimental results.

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