

THE BEHAVIOUR OF PRE-STRESSED CONCRETE BEAMS

WITH CIRCULAR PERFORATIONS

A thesis submitted for the degree of
Doctor of Philosophy

by

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ABSTRACT

The behaviour of prestressed rectangular beams containing a single hole or an array of circular holes is reported. Theoretical and experimental studies are described, attention being given to the elastic stress distribution around the holes, the deflection of beams containing multiple holes, the ultimate load carrying capacity and to possible methods of strengthening perforated beams by additional reinforcement.

Computer programmes are developed for the calculation of the stress distribution around holes for the application of prestress, pure bending and bending with shear.

The experimental results from tests on a total of thirty-seven beams show good agreement with the theoretical results.

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CONTENTS

Abstract

Acknowledgement

Page

NOTATION

1

CHAPTER

I

INTRODUCTION

5

1.1 General Introduction

5

1.2 The Problem

6

1.3 Scope of Research

7

1.4 Programme of Work

8

II

REVIEW OF RELATED WORK

12

2.1 Solutions for Plate Problems

12

2.2 Solutions for Steel Beams

21

2.3 Solutions for reinforced and prestressed
Concrete Beams

27

Figs. 2.1 - 2.6

III

ELASTIC STRESS ANALYSIS AND DEFLECTIONS

32

3.1 Elastic Stress Analysis

32

3.2 Deflection of perforated Beams

44

Table 3.1 - 3.4

Figs. 3.1 - 3.15

IV

ULTIMATE LOAD ANALYSIS AND DESIGN OF REINFORCEMENT

53

4.1 Ultimate Load Analysis

53

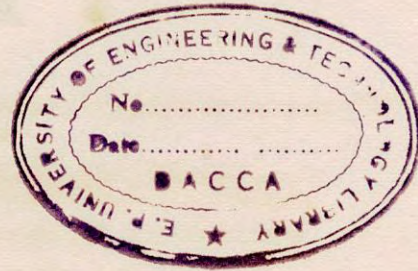
4.2 Design of Reinforcement

58

Figs. 4.1 - 4.2

<u>CHAPTER</u>		<u>Page</u>
V	<u>MATERIALS USED IN EXPERIMENTAL STUDY</u>	64
	5.1 Concrete	64
	5.2 Steel	68
	Figs. 5.1 - 5.6	
VI	<u>SPECIMEN PREPARATION AND TEST PROGRAMME</u>	71
	6.1 Specimen Preparation	71
	6.2 The Test	78
	6.3 Test Programme	80
	Table 6.1	
	Figs. 6.1 - 6.9	
	Plates 6.1 - 6.4	
VII	<u>PRESENTATION OF EXPERIMENTAL RESULTS</u>	90
	7.1 Beams containing single Holes	90
	7.2 Beams containing multiple Holes	105
	Table 7.1	
	Figs. 7.1 - 7.50	
	Plates 7.1 - 7.23	
VIII	<u>DISCUSSION</u>	115
	8.1 Elastic Stresses around the Hole	115
	8.2 Stress Distribution at minimum Section	119
	8.3 Diagonal Cracking	122
	8.4 Flexure Cracking	123
	8.5 Top Fibre Stress in a perforated Beam	124
	8.6 Steel Strain	125
	8.7 Deflection of perforated Beam	126
	8.8 Ultimate Load	128
	Figs. 8.1 - 8.2	

<u>CHAPTER</u>		Page
IX	<u>CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH</u>	131
	9.1 Elastic Stresses and Deflections	131
	9.2 Ultimate Strength of perforated Beams	133
	9.3 General Conclusions	134
	9.4 Suggestions for further Research	135
REFERENCES		
APPENDIX A	<u>ANALYSIS OF TEST BEAMS</u> Figs. A.1 - A.9	(i)
APPENDIX B	<u>COMPUTER PROGRAMMES</u>	(xviii)

NOTATION

A	a constant
a	radius of the hole
a_{sh}	shear moment arm
a'	the distance between the top of the beam and top of the hole
A_{st}	area of tension reinforcement
A'_{st}	cross-sectional area of compression steel
A_w	area of one pair of stirrups
b	half-depth of the beam
b_1	breadth of the rectangular beam
B	= $\frac{\text{diameter of the hole}}{\text{depth of the beam}}$
c	distance of the centre of the hole from the top fibre
c_1	distance of the bottom fibre from the neutral axis
d	diameter of the hole
d_1	depth of top flange at the minimum section
d'_1	the depth of the centroid of the compression steel
d_2	depth of bottom flange at the minimum section
D	depth of plate
e	distance between adjacent holes
E	Young's modulus of elasticity
e_a, e_b, e_c	strains in three directions of a rectangular rosette
F	stress function
ϕ	angle of σ_1 (anti-clockwise from the direction of e_a)

f'_c	compressive strength of concrete (cylinder strength)
f_u	the ultimate tensile strength of the tendon
f_{su}	stress in the tension reinforcement at failure
f_{pe}	compressive stress in concrete due to prestress only
f'_s	ultimate strength of prestressing steel
f_d	stress due to dead load at the extreme fibre (bottom)
F_a	stress concentration factor
G	shear modulus of elasticity (lb/sq.in.)
$2h$	depth of beam
I	second moment of area
I_e	equivalent second moment of area
I_n	second moment of area of the net section about its centroidal axis
I_t	second moment of area of the section above or below the hole about its centroid
l_1	distance of the loading point from the support reaction
l	distance of the hole centre from the near reaction
L	effective span of the beam
M	bending moment
M_{AB}	bending moment at A of the member AB
n	number of holes in a beam
n_p	depth of rectangular stress block = $A_{st} f_{su} / 0.85 f'_c b_1$
N.A.	neutral axis of a beam
p	uniform stress (tensile or compressive)
p_m	the remaining tensile stress in the compression steel

P	design load
P_1	prestressing force in cable
P_{cr}	cracking load of a beam
q	shearing force at the minimum section
r, θ	polar co-ordinates
R	reaction
s	pitch of stirrups
s.c.f.	stress concentration factor
t	thickness of plate
t_y	yield stress of steel in stirrups
T	total tensile force
V	shearing force
V_u	ultimate shearing force
V_d	shear due to dead load
V_p	vertical component of the effective prestress force
w	symmetrical load
x	longitudinal distance from the centre of the hole
y	transverse distance from the centroid
γ	shape factor
σ_r	radial stress around a hole
σ_1, σ_2	principal stresses
σ_θ	tangential stress around a hole
τ_{re}	shearing stress in polar co-ordinate system
σ_x	stress along the length of the beam

σ_y stress perpendicular to the longitudinal axis of the beam

τ_{xy} shearing stress

θ angle of inclination of the cable

$$\lambda = \frac{\text{diameter of hole}}{\text{depth of beam}}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

μ Poisson's ratio

CHAPTER I

INTRODUCTION

1.1 GENERAL INTRODUCTION

In modern, and particularly in industrial, building construction, there is an ever-increasing need for holes and openings at different positions in beams. These holes permit the passage of pipes and ducts for sanitation, heating, ventilation, air-conditioning and other electrical and mechanical services. Openings are often necessary to provide access to the inside of box beams. The dead space in false ceilings can be put to economical use by arranging services in two directions; without such openings, this would be possible only at the expense of additional headroom. Moreover, in the case of T-beams with monolithic slab construction, it is preferable to provide holes just below ceiling level.

Holes may also have to be provided in buildings for architectural requirements. Holes and perforations may be arranged in a beam in various patterns in order to provide aesthetic beauty.

Proper provision should initially be made in the design office for these openings instead of the later knocking out of holes on site to suit installations, which might possibly weaken the structure to a critical degree. The need for enlarging the holes in the field and the danger of accidentally reducing the structural

capacity of a beam can thus be eliminated. Lorentsen¹ reports in the Swedish Journal "Byggnästaren" that a hole was cut in a T-beam in such a way that the web was completely removed close to the support, where the shear capacity was critical.

The structural engineer is thus often faced with the problem of providing holes in structural elements. By providing suitable reinforcement in proper places, it is possible to have large perforations in reinforced and prestressed concrete beams without reducing the load-carrying capacity.

1.2 THE PROBLEM

The effect of perforations on beam strength and stiffness is at present imperfectly understood. The problem remains less dealt with from the point of view of the structural designer and is rarely mentioned in current specifications for design.

Only a limited amount of research work on prestressed beams containing perforations has been performed and published, and only a few of the possible parameters have been investigated. In practice the openings may be of different shapes and sizes and may cut the beam in various positions. The ultimate strength and deflection of such a beam may be affected depending on the position and relative size of the holes. The design of such openings has in the past been mainly intuitive and has not been adequately supported by tests.

The provision of holes in flexural members changes their simple mode of behaviour to a complex and highly indeterminate one. The exact mathematical solution of such a problem is very difficult and requires great time and effort which may not be justified in the structural design office. Research work should, therefore, explore the behaviour of such members under load and provide a reasonably simple procedure for their analysis and design.

1.3 SCOPE OF RESEARCH

The work described in this thesis involves a fundamental investigation of the web-opening problem in terms of stress concentrations and distribution for different sizes and positions of circular holes in rectangular prestressed beams, the effect on the cracking load, ultimate strength and the stiffness of the beams. A sensibly exact analytical solution is established as a part of the programme which will, with the help of the digital computer, give the stress pattern at the boundary of a circular hole either in pure bending or in bending with shear. A relatively simple procedure is developed that will indicate, in an approximate manner, the weakness in a prestressed concrete beam resulting from the presence of circular perforations and the way in which they can be possibly strengthened so that the beam retains its full strength.

The first part of the programme did not consider opening reinforcement. The need for reinforcing and the most appropriate

form which it should assume out of a number of probable methods was established in later parts of the work.

The main parameters considered in the study are:

- a) size of the hole (the hole diameter being 40%, 50% and 60% of the beam depth),
- b) location of the hole (either in a pure bending zone or in a combined bending and shear zone),
- c) position of the hole with respect to the centroidal axis of the beam (on, above or below the horizontal axis of symmetry),
- d) level of prestress,
- e) effect of bond (bonded or unbonded tendons),
- f) different methods of reinforcing the holes (bonded steel liners, bonded ring reinforcement around the hole or vertical stirrups at the sides of the holes).

The objective:

The programme had the objective of providing sufficient experimental information on the behaviour of rectangular prestressed beams containing one or an array of circular perforations, with or without reinforcement, by testing model beams in the laboratory in order to develop a design method for such members.

1.4 PROGRAMME OF WORK

This thesis consists of a review of related work, a theoretical investigation, a report on materials, instrumentation and test

programme, experimental verification, discussion and conclusions. In the appendices are given the analyses of test beams and computer programmes which calculate the stresses around the holes. Photographic plates showing crack and failure patterns of the beams are also presented.

The review of related work comprises the available methods and results of solutions given by different authors for the problem of holes and openings in plates, steel beams and reinforced and prestressed concrete beams subject to different kinds of loading.

The theoretical analysis deals with the development of the exact mathematical elastic solutions for beams with circular openings, the centre being situated on, above or below the centroidal axis. Computer programmes were developed to predict the stress trajectories at the boundary of the hole and the principal stresses around the hole for uniform compression and pure bending or bending with three different ratios of bending moment to shearing force, arising out of the prestressing force and loading. An "equivalent second moment of area" method for predicting the deflection of perforated beams due to bending moment and shearing force is developed.

The ultimate strength of beams having holes in the pure bending zone and in the shear span is examined on the basis of CP 115:1959 and ACI Building Code. A design method for reinforcement against splitting (due to tensile stress concentration at the

ends of the horizontal diameter of the hole arising out of pre-stress) and diagonal cracking due to bending with shear is evolved. A method of calculation of the reinforcement against failure by excessive principal tensile stress on top of a hole is presented.

Test Programme

The experimental work involved the testing of thirty-seven model beams, all rectangular, post-tensioned and of the same length (90 in. effective span) subject to third-point loading, with the exception of three beams tested with a central point load on a span of 48 in. Of these beams, eight had bonded and the rest unbonded tendons. A detailed report on the method of preparation and instrumentation, testing procedure and test results is presented. It was investigated in the tests whether the weakness of the beams containing perforations could be compensated by providing bonded steel liners on the inside surface of the holes, or by providing bonded ring reinforcement with the minimum cover. A third possibility was to provide straight stirrups at the sides of the holes. To investigate the effect of holes situated above the ultimate neutral axis, holes were located above the centroidal axis so as to lie just below the depth of the ultimate rectangular stress block. In two cases, the holes actually extended into the depth of the Whitney stress block. The deflection, ultimate load and the reduction in weight due to the perforations were studied.

The experimental stress trajectories on the boundary of the

circular hole, either in the pure bending zone or in the shear span are compared with the theoretical curves. The observed deflections, cracking loads and ultimate loads are compared with the theoretically predicted values.

CHAPTER II

REVIEW OF RELATED WORK

The study of the problem of holes and perforations in members subjected to different conditions of stress has been, until very recently, from the point of view of the machine designer rather than as a structural engineering problem. Only a limited amount of work has been carried out on the behaviour of concrete beams containing perforations, more work being devoted to steel beams and the use of castellated construction.

2.1 SOLUTIONS FOR PLATE PROBLEMS

The first important theoretical investigation² dealing with stress concentrations was made by Larmor³ at the end of the nineteenth century. Following him a number of mathematical solutions for stresses in plates containing various types of holes under different stress systems was made by different authors.

Experimental methods have been widely used in the study of stress concentrations around holes. To obtain the maximum stresses produced at holes, fillets and grooves, tests were made using models made of elastic materials, the strains being measured by mechanical extensometers, mirror extensometers and more recently by precise electronic gauges.⁴

The method of brittle coating has been used to detect the weakest spots in complicated cases.⁵ The magnitude of the stresses can be approximately determined if a preliminary test of a calibration bar coated with the same kind of lacquer is made.

Photo-elasticity^{6,7} provides a more accurate experimental method for determining the magnitude and distribution of stresses around a hole.

2.1.1 Analytical Solutions

Single hole

Kirsch⁸ investigated the stress distribution around a small circular hole in a wide plate subject to uniform longitudinal tension,⁹ p.

Using the principle of superposition on a Lamé thick cylinder subject to a stress of $P/2$ and a case obeying the stress function, $\phi = f(r) \cos 2\theta$, the radial, tangential and shearing stresses in polar co-ordinates were found to be:

$$\sigma_r = P/2 \left(1 - \frac{a^2}{r^2}\right) + P/2 \left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right) \cos 2\theta \quad (2.1)$$

$$\sigma_\theta = P/2 \left(1 + \frac{a^2}{r^2}\right) - \frac{P}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta \quad (2.2)$$

$$\tau_{r\theta} = -\frac{P}{2} \left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right) \sin 2\theta \quad (2.3)$$

At the edge of the hole, when $\theta = \pi/2$ and $\frac{3\pi}{2}$, i.e., at the ends of the diameter perpendicular to the direction of tension, the maximum tangential stress is equal to three times the uniform

stress applied. At the ends of the diameter parallel to the direction of the applied stress, there is a stress of opposite nature of magnitude equal to the applied stress. It is evident from the equations that the effect of the hole is very localised; as r increases the stress approaches the applied stress very rapidly.

Using the theory of complex variables, Kolosoff¹⁰ solved the problem of stress concentration produced in a plate by an elliptical hole. Inglis¹¹ discussed the same problem in a paper presented to the Naval Architects.

Muskhelishvili¹² further developed the method of analysing stress distributions around holes and solved the problem of stress concentrations for a variety of hole shapes.

Neuber¹³ applied the general methods of elasticity represented by three harmonic functions and obtained several results for two-dimensional problems.

Heller¹⁴ obtained an exact solution for the stress distribution at the boundary and in the neighbourhood of an ovaloid opening in the web of a beam subjected to bending with shear. He was able to predict the stress concentration factors for circular, elliptical, ovaloid and rectangular holes in a rectangular strip for the cases of tension, pure shear and bending with varying ratios of moment to shear.

Howland¹⁵ solved by successive approximation the problem of a uniform tension (p) applied to a plate bounded by two parallel

edges and containing a hole midway between the finite edges. The calculated values for the stresses across the minimum section for a strip whose width is twice that of the hole diameter is shown in Fig. 2.1. Curve I shows the stresses σ_θ/p along the contour of the circular hole, Curve II shows the stress distribution σ_x/p for the most highly stressed cross-section ($\theta = \pi/2$), Curve III shows σ_x/p for the cross-section $\theta = 0$, i.e., along the strip axis, and Curve IV illustrates the stress distribution σ_y/p along the cross-section $\theta = 0$.

The case of a circular hole near the straight boundary of a semi-infinite plate under tensile stress (p) parallel to the boundary was analysed by Jeffery.¹⁶ Mindlin¹⁷ corrected the solution given by Jeffery who made an error in his application. Mindlin found that the total force, H , passing between the hole and the edge of the plate, is given by

$$H = p \sqrt{c^2 - a^2} \quad (2.4)$$

From a study by photo-elasticity of the stress distribution around a circular hole on the centre line of a beam of uniform rectangular section subject to a uniform bending moment, Tuzi¹⁸ was able to derive a suitable governing stress function. A theoretical solution was worked out by him (Art. 3.1.2) on the assumption of an infinitely extended plane. The experimental values at several sections and boundaries compared well with the theoretical ones.

An approximate numerical approach is possible based on the

replacement of the governing differential equations by the equations in finite difference form and solving the resulting simultaneous equations by successive approximations, generally by electronic computation. The method of finite elements was used by Zienkiewicz, Cheung and Stagg¹⁹ to find the flow of stress around a circular hole in an isotropic and anisotropic material when the applied stress was uniform. A high degree of accuracy was achieved by this powerful method.

Single row of holes in plate

The maximum stresses due to a single row of circular holes in a plate which is subjected to tension in a direction normal to the line of holes was determined by Hutter,²⁰ (Fig. 2.2).

Howland²¹ determined the stresses in an elastic plate containing an infinite number of holes of equal size by successive approximations for the particular case where the holes are circles and the plate is stressed by uniform tensile stress (p) along the ox or oy - axis for the values of d/e less than 0.54 (Fig. 2.2).

In Fig. 2.3, Curve I shows the tangential stress concentration along the contour of the hole, Curve II, σ_x/p along the Oy -axis, Curve III, σ_x/p along the ox -axis and Curve IV, σ_x/p over the section $x = b/2$ for $d/e = 0.50$.

Stress relieving holes

The stress concentration due to a circular hole can be reduced by introducing two smaller holes on either side of it in the line

of the applied force as shown in Fig. 2.4. The lines of principal stress now tend to run more smoothly in circumventing the large central hole thereby reducing the stress concentration. A series of tests has been carried out by Heywood²² to estimate the reduction in stress that may be achieved by this method. The general dimensions of the model are shown in Fig. 2.4. For various sizes of the two relieving holes, stress concentration factors were calculated from the measurements at the central hole and relieving holes. The values are expressed in terms of the nominal stress across the minimum section in the figure. With the increase in the size of the relieving holes, the maximum stress in these holes increases, but that at the central hole is reduced. It is apparent that the optimum condition is obtained when the maximum stress in all three holes is the same and this condition is obtained when the relieving holes have diameters 80% of that of the central hole.

It is thus best to use large relieving holes placed close to the central hole. For this condition, the maximum stress is reduced to 84 percent of that due to a single hole.

It seems unlikely that a greater reduction in stress can be achieved by placing the relieving holes at a greater distance from the central hole. Further, the addition of two other relieving holes of smaller diameter placed in line with the three holes would only lead to a very small reduction in stress, since their effect on the central hole would be of a second order. The theoretical

condition for minimum stress is a row of infinite holes all of the same size and just touching one another.

The maximum stresses on the boundary of two circular holes of equal diameter in an infinite plate have been ascertained mathematically by Ling.²³

Holes in members subjected to combined loading

The solution for uniform tension or compression in one direction can be used to obtain, by superposition, the solution for tension or compression in two perpendicular directions. For example, tensile stresses in two perpendicular directions produce a maximum tensile stress of twice the applied stress.

Circular hole subjected to pure shear

By taking a tensile stress (p) in the x -direction and a compressive stress (p) in the y -direction, a case of pure shear is obtained. The tangential stresses at the boundary of the hole are

$$\sigma_{\theta} = p - 2p \cos 2\theta - [p - 2p \cos (2\theta - \pi)] \quad (2.5)$$

$$\text{At } \theta = \pi/2 \text{ or } 3\pi/2, \sigma_{\theta} = 4p$$

Hence for a large plate under pure shear, the maximum tangential stress at the boundary of the hole is four times the applied pure shearing stress.

Transverse circular hole in plate subjected to bending

Howland and Stevenson²⁴ have given the calculated stress distribution in a plate containing a central circular hole subject to bending in the plane of the plate. The results are expressed in

terms of a stress concentration factor which is based on the nominal stress, $6M/t_D^2$. He deduces that the stress concentration factor equals $2d/D$.

The photoelastic results of Ryan and Fischer²⁵ have demonstrated that this does not hold good for ratios of d/D above 0.5, the observed s.c.f. being appreciably greater than those derived from the formula.

2.1.2 Experimental Solutions

Tests on models of ductile material

Kirsch^{8 10} made a large number of tensile tests on steel plates with circular holes and found that in the case of ductile materials the hole usually increases the ultimate stress, the lateral contraction of the specimen at fracture being somewhat hindered by the presence of the hole.

Some tests were made by Wahl and Beeuwkes²⁶ on a large steel specimen containing a hole with a diameter almost equal to the width and of a size large enough so that accurate strain measurements could be made with Huggenberger extensometers. The stress concentration factor was found, by dividing the stress readings on the inside of the hole by the average value between the stress obtained on the outside and that obtained on the inside, to be 1.91.

Tests on models of brittle material

Peterson²⁷ used Plaster of Paris, a brittle material that follows practically a linear stress-strain diagram up to failure, in his tests.

He used two tension specimens, one containing a hole and the other having the same net cross-sectional area, but with no hole. The two specimens were tested to failure and the breaking load P_1 for the specimen without a hole and P_2 for the specimen with a hole obtained. The ultimate strength of the material was then computed ($s_1 = P_1/a$) based on the specimen without the hole. s_1 is assumed to be equal to S_h , the localised stress at the edge of the hole at failure since both s_1 and S_h represent the ultimate unit stress at which the particles of the plaster tear apart.

$$\text{Hence, } s_1 = S_h = P_1/a \quad (2.6)$$

The stress which would exist at the load P_2 if the specimen were of the net area and did not contain a hole is

$$s_2 = P_2/a \quad (2.7)$$

$$\text{From (2.6) and (2.7), } \frac{S_h}{s_2} = P_1/P_2 \quad (2.8)$$

The stress concentration factor which is the ratio of the stress S_h at the edge of the hole to the stress s_2 , which would exist in a member having the same net cross-sectional area and not containing a hole, is given by

$$F = S_h/s_2 = P_1/P_2 \quad (2.9)$$

Zelisko, Zyczowski and Grabczynski²⁸ established by means of the photoelastic method the distribution of stresses in a rectangular diaphragm (deep beam) with a central circular opening, simply supported at the ends and loaded by a single concentrated central load. The authors noticed that there was an increase in the intensity of the normal stresses at the edge of the opening with the increasing diameter of the hole. With the increase in the span of the diaphragm, these stresses showed reductions in their intensity in the upper edge of the opening while in the lower edge there appeared to be progressive increase in their intensity.

2.2 SOLUTIONS FOR STEEL BEAMS

The problem of a simply supported rectangular beam having a single circular hole for the case of bending with shear was investigated by Gibson and Jenkins.²⁹ An approximate mathematical analysis of the stress conditions in the neighbourhood of a circular hole carrying a central concentrated load along the vertical diameter of the hole was presented. The authors concluded that the theoretical values for the tangential stresses give reasonable agreement with experimental values determined photoelastically and from steel beam tests. The concentrated load just above the hole gives a localised stress distribution which is not predicted by their theory.

The tangential stress at the boundary of the hole is given by

$$\sigma_{\theta} = C a^2 \sin \theta (y_1 + y_2) \quad (2.10)$$

where $y_1 = \frac{m}{2} (4 \sin^2 \theta - 2),$

$$y_2 = 2 \cos \theta [(2n^2 + 1) - 4 \sin^2 \theta],$$

and $m = l/a, n = d/a, \quad (2.11)$

a = radius of the hole, l = beam span

$2d$ = depth of the beam

Approximately, $\sigma_{\theta} = C a^2 \sin \theta y_1. \quad (2.12)$

It is interesting to note that the value for σ_{θ} at $\theta = 90^\circ$ is precisely twice the value that would occur at the same distance from the neutral axis in a beam without a hole.

McCutcheon, So and Gersovitz³⁰ presented in their report to the Canadian Institute of Steel Construction the stress trajectories around a circular hole (the diameter to depth ratio being 0.6) in wide-flange beams with one or two point loading thereby keeping the hole under either pure bending or varying ratios of bending and shear. Measured strains for the four beam tests compare with the theoretical strains. The discrepancy between the theoretical and experimental curves is more pronounced when there is a relatively high shear force acting at the section.

Measured deflections indicate rather higher values than would be anticipated from elastic bending theory. In each case the ultimate strength of the beam was unchanged by the presence of the opening.

McCutcheon, Dickie and Cheng³¹ in their second report to the Canadian Institute of Steel Construction presented the experimental data and observations pertaining to the stress distribution around an elliptical hole in a wide-flange beam. The measured strains were compared to those obtained by the photoelastic technique. The distribution of boundary strains are somewhat different from those of the circular hole and are appreciably greater in relative magnitude. The greater stress level lowered the ultimate strength by 36% and shifted the plastic hinge from under the load to the opening.

Segner³² directed his work to reinforcement requirements around large rectangular openings in wide-flange beams. Segner tested six beams containing a total of twenty-three openings of various shapes and reinforcing details. A central concentrated load on a 10 ft. span was used throughout. Segner concluded from his work as follows:

- a) Reinforcement requirements can be determined from the consideration of ratios of applied moment to moment capacity, applied shear to shear capacity and the breadth to depth ratio of the opening.
- b) The most suitable reinforcement is an open or closed ring around the opening; in regions of high shear, this should be combined with oblique flange stiffeners.
- c) Elastic deflection for a beam with openings is approximately

one-third higher than that for the gross section.

d) Beam capacity at yield stress for a beam with reinforced opening is reduced approximately by 30%.

e) The ultimate capacity of the beam remains unchanged.

An analytical method for calculating stresses around holes in the web of wide-flange beams was presented by Bower.³³ The applicability of the analysis depends on the size of the web hole and on the magnitude of the moment-shear ratio at the hole. The analysis predicts stresses accurate in all respects for depth ratios of about 2 or greater at a shear-moment ratio of 0 and for depth ratios of about 6.0 or greater at a shear-moment ratio of $2.4/L$, where L is the half-length of the beam; the stress distribution shows that an increase in the shear-moment ratio shifts the location of the maximum normal stress at the boundary of the hole from the vertical apex towards the horizontal apex of the hole.

Bower³⁴ next presented experimental stresses in wide-flange beams with holes. He dealt with two types of holes, circular and rectangular, in beams subjected to concentrated loads. Tangential stresses at the boundary of a circular hole, as predicted by elastic analysis, were compared with the experimental stresses for bending-shear stress ratios of 7.2, 3.6 and 0. The correlation between the theoretical and measured compressive stresses is not as good as the correlation for tensile stresses; nevertheless, the measured stresses generally follow the predicted pattern. He

concluded that the theory-of-elasticity analysis adequately predicts the tangential stresses at the boundary of circular holes for all bending-shear stress ratios tested when the diameter of the hole equals one-half the beam depth. For rectangular holes, the author assumed Vierendeel behaviour and concluded that the analysis predicts stresses on transverse cross-sections with reasonable accuracy, but does not predict the stress concentrations at the hole corners.

Redwood³⁵ discussed the paper of Bower and reported that the theory of elasticity solution compares well with experimental tangential stress for an opening, either circular or rectangular with semi-circular ends for the case of pure bending, but the theory significantly under-estimates the stresses when shear is present.

Bower³⁶ also investigated the effect of rectangular holes on the ultimate strength of beams. He discussed two methods of predicting the ultimate strength - a lower bound solution neglecting the effect of strain hardening and an elastic Vierendeel analysis. The lower bound solution was found to predict loads 19% below the experimental load. Allowable design load based on an elastic Vierendeel analysis showed better agreement.

Gibson and Jenkins³⁷ predicted by an approximate analysis the behaviour of castellated I-beams with regularly spaced hexagonal holes in the webs for the case of a simply supported centrally loaded beam. An assumption that the beam behaves as a Vierendeel

girder was made and the stresses arrived at by this analysis were compared with those obtained by a photoelastic investigation. The authors compared the deflection as arrived at by their analysis with that given by elementary beam theory using the least second moment of area. As the span increases the agreement between these two formulae is more close and shows almost complete agreement for very large spans.

The beam theory gives the chord stresses as linear functions varying from zero at the supports to a maximum value at the load. The theory presented by Gibson and Jenkins, and also the experimental results, show great variation with the beam theory, but close to the centre-line where the critical stress occurs the agreement is quite good. Therefore, as far as determining the critical stress the beam theory as used by the manufacturers is sufficiently accurate for design purposes.

Kolosowski³⁸ proposed a practical design formula for castellated beams. He, too, assumed Vierendeel behaviour and compared stresses with those obtained by the simple theory of bending. He commented that the assumption that plane sections remain plane after bending, is not justified for castellated girders, because,

a) the stress distribution is not linear across either the solid section or the castellation,

b) the pattern of the stress distribution along the top and bottom flanges does not comply with that indicated by the bending

moment diagram (Fig. 2.5),

c) the actual deflections are much larger than those calculated by the simple theory of bending.

The experimental results indicate that the castellated girder behaves as a Vierendeel truss. However, the analogy is not exact owing to the fact that the openings are rather small compared with the overall dimensions of the girder; consequently, the members of the Vierendeel truss are short and secondary stresses are introduced.

In the course of discussion of a paper by Cheung, King and Zienkiewicz, Sawko³⁹ remarked that the relative importance of shear versus flexural effects is a function of the size and shape of openings. Several perspex beams with different ratios of hole size to depth were tested under third point loading and the relative importance of shear against bending deflection is shown in Fig. 2.6. For the shape of cells considered and for values of d/a commonly found in bridge decks the shear deformations were found to be dominant. Square holes exhibited a far greater shear flexibility than circular holes.

2.3 SOLUTIONS FOR REINFORCED AND PRESTRESSED CONCRETE BEAMS

Lorentsen¹ presented a design method for reinforced concrete beams with extended openings in their webs. In this method, the lower framing flange, in which the tensile reinforcement of the

beam is embedded, was considered as a tension rod, that is, unable to resist any moment. The tension rod is divided into short sections by primary flexural cracks and usually this chord is not able to transfer the shear corresponding to secondary moments. The beam is then reduced to a rigid frame with a tension rod. Lorentsen states that it is desirable to locate the hole near midspan because of low shearing forces. Near the ends the shearing force is usually large and the normal force small from which the principal tensile stress in the concrete becomes large and the potential danger of the development of diagonal tension cracks increases, since the shearing force must be carried by the upper framing flange only.

The author dealt with the detailed reinforcement requirements around an extended opening for the cases of simply supported and continuous beams. An approximate design method was proposed. Four beams were tested to verify the applicability of the analysis. The first two beams failed in a similar manner to that exhibited by beams without holes. The hole did not weaken the beam and the capacity of the main tensile reinforcement was fully developed. In the remaining two beams the sections at the edge of the hole were deciding factors in the magnitude of the failure load. The steel at the edge of the hole yielded. Since the main reinforcement was not fully developed in tension, the lower flange could be utilised in resisting part of the moment.

Nasser, Acavalos and Daniel⁴⁰ outlined a theoretical approach

for the behaviour of rectangular reinforced concrete beams with large openings. Full size beams were tested and the results were compared with the behaviour of an identical beam having no opening. Because only a limited amount of research work on large openings in beams was available, the authors made the following assumptions:

1. The top and bottom cross members of the opening are assumed to behave similarly to the chords of a Vierendeel panel.

2. The cross members of the openings have contra-flexure points at their midspan, provided they are not subjected to transverse loads.

3. The cross members, when they have adequate stirrups, carry the external shear in proportion to their cross-sectional areas.

4. There is a diagonal force concentration at the corners induced by the chord shear and its value is twice the simple shearing force.

All these assumptions were found valid for rectangular beams from test results of twelve beams having one or two holes in pure bending or bending with varying shear, the beam being subject to one or two concentrated loads. It was concluded that adequately reinforced large openings in rectangular beams do not reduce the ultimate capacity of the beam but reduce its stiffness.

Discussing the above mentioned paper by Nasser, Acavalos and Daniel, Barker⁴¹ reported the test results of three pairs of singly reinforced beams where he observed that when openings were made in

the tension region of beams without compression reinforcement, the ultimate capacity of the beams was not reduced. He pointed out that 'properly designed' means that the opening does not reduce the compression zone required to resist the ultimate moment capacity.

Kocsis⁴² made a series of tests on small unreinforced plaster beams with holes for the purpose of finding a stress concentration factor for use in design of beams with rectangular holes. The hole depths were 25, 50 and 75% of the beam depth. He proposed a design method based on the stress concentration factor.

Let C_1 be the distance of the outer compression fibre from the neutral axis of a beam without any hole,

and C_2 be the distance of the compression side of a hole in a beam.

Assuming that failure occurs at the hole boundary and that both beams are made of the same material,

$$\frac{M_1 C_1}{I_1} = \frac{M_2 C_2}{I_2} F_a \quad (2.13)$$

M_1 is obtained by test on a solid beam, M_2 by tests on beams with holes. I and I_2 (I_2 is the net I) are computed and F_a is found from equation (2.13).

Ragan and Warwaruk⁴³ made an experimental study of model and full-size prestressed concrete T-beams containing large web openings. These beams contained an array of rectangular holes with rounded corners; the holes were of different sizes, the holes near

the support being of lesser depth, since the curved tendon had to pass below the hole.

In all cases, immediately after the formation of the first flexural cracks, the tension lost by the concrete was transferred to the reinforcement. This resulted in a rapid change of slope in the load-deflection diagram as the stiffness of the beam was reduced. In the later stages of loading, the load-deflection diagram was nearly flat. This stage suggested that the main reinforcement stress was approaching or was in the inelastic range of the reinforcement stress-strain curve.

The mode of failure of all the model beams with openings was through the formation of mechanisms. The authors concluded from the tests that large web openings in T-beams could be provided while still meeting code requirements.

Ibrahim⁴⁴ tested a number of reinforced concrete beams containing an extended hole in the web. He proposed several methods for reinforcing the holes. The top and bottom cross members behaved like separate beams as assumed in Vierendeel analysis. He concluded that the beam did not lose its strength in bending so long as the hole did not cut the ultimate compression depth.

FIG. 2.2 Maximum stresses due to a single row of holes

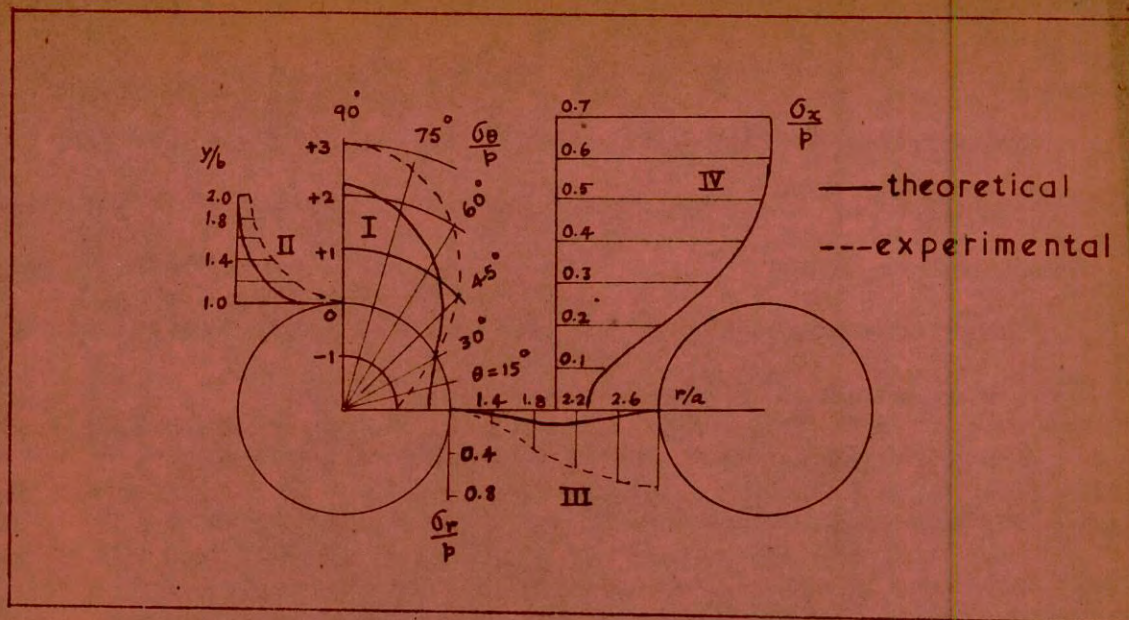


FIG. 2.3 Infinite number of holes in an elastic plate

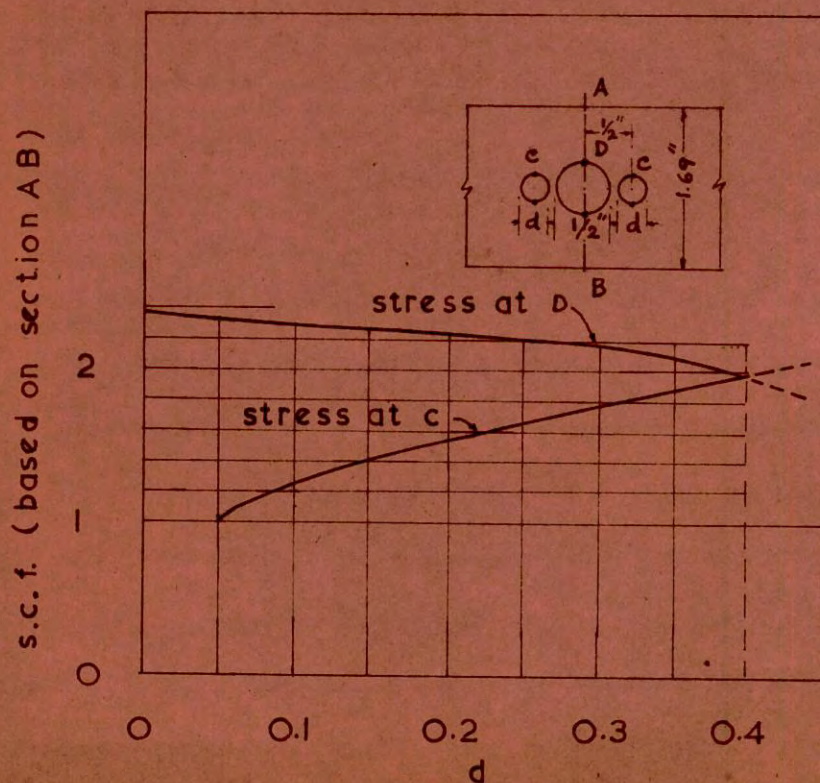


FIG. 2.4 Stress relieving holes

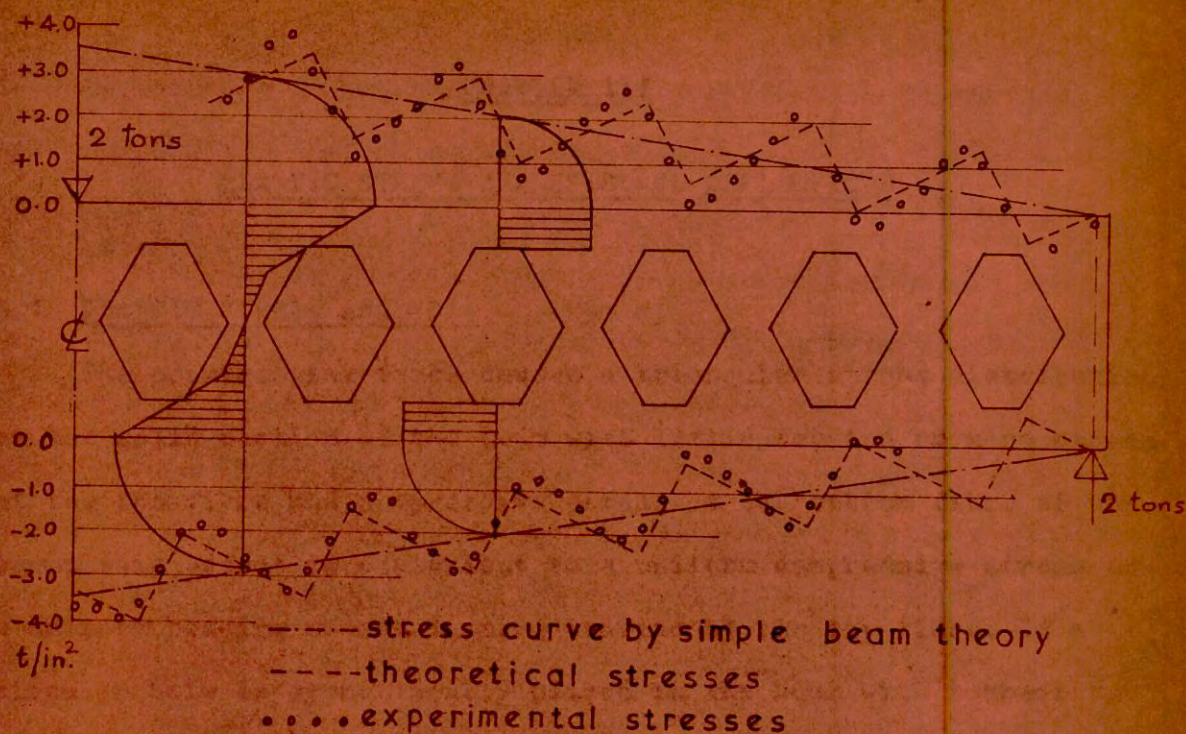


FIG. 2.5 Top fibre stress in a castellated beam

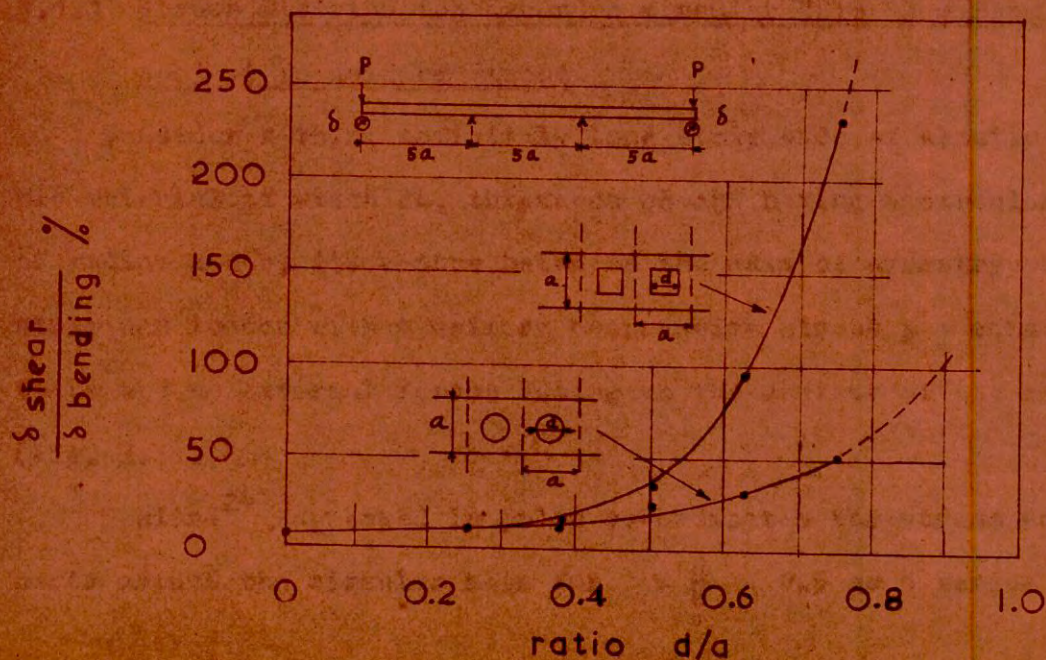


FIG. 2.6 Effect of shear on deflection of beams with holes

CHAPTER III

ELASTIC STRESS ANALYSIS AND DEFLECTIONS

3.1 ELASTIC STRESS ANALYSIS

The prestressing force causes a triangular stress distribution at the solid section of the beam with little tension or zero stress at the top fibre and compressive stress at the bottom fibre of the beam; this effect is equivalent to a uniform compressive stress and a uniform bending moment causing tension at the top fibre. If a circular hole is symmetrically placed in the beam with respect to its depth, the problem of finding the stress distribution on the boundary of such a hole reduces to the superposition of the stresses due to a uniform compressive stress and a bending moment.

3.1.1 Stress Distribution around a circular Hole in a Beam subject to uniform Stress

Consider a thin, infinitely long strip made of elastic isotropic material of width $2b$, thickness $2h$ and having a circular hole of radius $a < b$, its centre being on the axis of symmetry of the strip and loaded with a uniform compressive stress $p = \text{constant}$; there are no external forces acting on the contour of the hole (Fig. 3.1).

Howland²⁴ expressed in polar co-ordinates the stress components around the circular hole for $\lambda = \frac{a}{b} \leq 0.5$ as a series as

follows. The polar axis coincides with the Ox-axis and the angles are measured from the Ox to the (positive) Oy-axis.

$$\begin{aligned} \sigma_n = p \left\{ \frac{1}{2}(1 + \cos 2\theta) + 2m_0 - \frac{d_0}{\rho^2} + 2 \sum_{n=1}^{\infty} \left[\frac{n(2n+1) d_{2n}}{\rho^{2n+2}} \right. \right. \\ \left. \left. + \frac{(n+1)(2n-1) e_{2n}}{\rho^{2n}} + n(2n-1) \frac{1_{2n} \rho^{2n-2}}{\rho^{2n}} + (n-1)(2n+1) \frac{m_{2n} \rho^{2n}}{\rho^{2n}} \right] \cos 2n\theta \right\}; \end{aligned} \quad (3.1)$$

$$\begin{aligned} \sigma_\theta = p \left\{ \frac{1}{2}(1 - \cos 2\theta) + 2m_0 + \frac{d_0}{\rho^2} - 2 \sum_{n=1}^{\infty} \frac{n(2n+1) d_{2n}}{\rho^{2n+2}} \right. \\ \left. + \frac{(n-1)(2n-1) e_{2n}}{\rho^{2n}} + n(2n-1) \frac{1_{2n} \rho^{2n-2}}{\rho^{2n}} + (n+1)(2n+1) \frac{m_{2n} \rho^{2n}}{\rho^{2n}} \right] \cos 2n\theta \right\}; \end{aligned} \quad (3.2)$$

$$\begin{aligned} \tau_{r\theta} = -p \left\{ \frac{1}{2} \sin 2\theta + 2 \sum_{n=1}^{\infty} \left[n(2n-1) \frac{1_{2n} \rho^{2n-2}}{\rho^{2n}} - \frac{e_{2n}}{\rho^{2n+2}} \right] \right. \\ \left. + n(2n+1) \left(\frac{m_{2n} \rho^{2n}}{\rho^{2n+2}} - \frac{d_{2n}}{\rho^{2n+2}} \right) \right] \sin 2n\theta \right\} \end{aligned} \quad (3.3)$$

The values of the coefficients in the formulae for several values of λ are given in Table 3.1. Table 3.2 gives the values of σ_θ/p along the contour of a circular hole for various values of λ . Table 3.3 gives the values σ_θ/p along the transverse cross-section of the strip ($\theta = \frac{\pi}{2}$), passing through the centre of the circular hole (where $\rho = \frac{r}{b}$).

The values of σ_θ/p in Table 3.2 for $\lambda = 0.6$ have been obtained by extrapolation; from the trend of the curves for $\lambda = 0.1$ to 0.5, the suggested values of σ_θ/p are obtained. Table 3.2 is represented by the curves in Fig. 3.2.

3.1.2 Stress Distribution around a circular Hole in a Beam subject to pure Bending

A circular hole, of radius a , has its centre on an arbitrary origin through which the axes x and y are drawn (Fig. 3.3). In a beam without any hole in pure bending, $\sigma_x = Ay$ and $\sigma_y = \tau_{xy} = 0$.

If F be the stress function, $\nabla^2 \nabla^2 F = 0$,

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2} \text{ and } \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y}.$$

In polar co-ordinates, $\nabla^2 \nabla^2 F = 0$

$$\text{where } \nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2},$$

The boundary conditions are given by,

$$\sigma_x = Ay = Ar \sin \theta, \quad \sigma_y = 0, \quad \tau_{xy} = 0, \quad \text{at } x = \pm \infty \quad (3.4)$$

$$\sigma_r = 0, \quad \sigma_\theta = 0 \quad \text{at } r = a \quad (3.5)$$

$$\text{From (3.4), } F_0 = \frac{A}{6} y^3 = \frac{A}{6} r^3 \sin^3 \theta \quad (3.6)$$

Putting $F = F_0 + F_1$,

$$\sigma_r = \left(\frac{1}{r} \frac{\partial F_0}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F_0}{\partial \theta^2} \right) + \left(\frac{1}{r} \frac{\partial F_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F_1}{\partial \theta^2} \right)$$

$$\sigma_\theta = -\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial F_0}{\partial \theta} \right) - \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial F_1}{\partial \theta} \right)$$

At $r = a$, $\sigma_r = 0$ and $\tau_{r\theta} = 0$

$$\therefore \left[\frac{1}{r} \frac{\partial F_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F_1}{\partial \theta^2} \right]_{r=a} = - \left[\frac{1}{r} \frac{\partial F_0}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F_0}{\partial \theta^2} \right]_{r=a}$$

$$= -Aa (\sin \theta - \sin^3 \theta)$$

$$\left[-\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial F_1}{\partial \theta} \right) \right]_{r=a} = - \left[-\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial F_0}{\partial \theta} \right) \right]_{r=a} = Aa (\cos \theta - \cos^3 \theta)$$

(3.7)

Assuming $F_1 = b_1 r^{-1} \sin \theta + (b_3 r^{-3} + d_3 r^{-1}) \sin 3\theta$

gives $\left[\sigma_r \right]_{F_1, r=a} = - \left(\frac{2b_1}{a^3} + \frac{36b_3}{a^5} + \frac{30d_3}{a^3} \right) \sin \theta + \left(\frac{48b_3}{a^5} + \frac{40d_3}{a^3} \right) \sin^3 \theta$

$\left[\sigma_\theta \right]_{F_1, r=a} = \left(\frac{2b_1}{a^3} - \frac{36b_3}{a^5} - \frac{18d_3}{a^3} \right) \cos \theta + \left(\frac{48b_3}{a^5} + \frac{24d_3}{a^3} \right) \cos^3 \theta$

(3.8)

solving equations (3.7) and (3.8),

$$b_1 = \frac{Aa^4}{8}, \quad b_3 = -\frac{Aa^6}{12}, \quad d_3 = \frac{Aa^4}{8}$$

$$\therefore F = F_0 + F_1 = Aa^3 \left\{ \left(\frac{a}{2r} - \frac{a^3}{4r^3} \right) \sin \theta + \left(\frac{r^3}{6a^3} - \frac{a}{2r} + \frac{a^3}{3r^3} \right) \sin^3 \theta \right\} \quad (3.9)$$

Using this complete stress function, Tuzi¹⁸ derived equations, in polar co-ordinates, for radial, tangential and shear stresses at the boundary of the circular hole.

$$\sigma_r = Aa^3 \left\{ \left(\frac{r}{a^3} - \frac{4a}{r^3} + \frac{3a^3}{r^5} \right) \sin \theta - \left(\frac{r}{a^3} - \frac{5a}{r^3} + \frac{4a^3}{r^5} \right) \sin^3 \theta \right\} \quad (3.10)$$

$$\sigma_\theta = Aa^3 \left\{ \left(\frac{a}{r^3} - \frac{3a^3}{r^5} \right) \sin \theta + \left(\frac{r}{a^3} - \frac{a}{r^3} + \frac{4a^3}{r^5} \right) \sin^3 \theta \right\} \quad (3.11)$$

$$\tau_{r\theta} = -Aa^3 \left\{ \left(\frac{r}{a^3} + \frac{2a}{r^3} - \frac{3a^3}{r^5} \right) \cos \theta - \left(\frac{r}{a^3} + \frac{3a}{r^3} - \frac{4a^3}{r^5} \right) \cos^3 \theta \right\} \quad (3.12)$$

At the periphery of the hole,

$$\sigma_\theta = 2Aa \sin \theta (2 \sin^2 \theta - 1) \quad (3.13)$$

Following Tuzi's approach the stresses around a circular hole in a beam, the centre of the hole being situated at mid-depth of the beam under the action of pure bending, were expressed in polar co-ordinates by So³⁰ as follows.

$$\begin{aligned} \text{Radial stress, } \sigma_r = & \left(-\frac{wl_1 r}{4I} + \frac{wl_1 a^4}{4Ir^3} \right) \sin \theta \\ & + \left(\frac{5wl_1 a^4}{4Ir^3} - \frac{wl_1 a^6}{Ir^5} - \frac{wl_1 r}{4I} \right) \sin 3\theta \end{aligned} \quad (3.14)$$

$$\begin{aligned} \text{Tangential stress, } \sigma_\theta = & \left(-\frac{3wl_1 r}{4I} - \frac{wl_1 a^4}{4Ir^3} \right) \sin \theta \\ & - \left(\frac{wl_1 a^4}{4Ir^3} - \frac{wl_1 a^6}{Ir^5} - \frac{wl_1 r}{4I} \right) \sin 3\theta \end{aligned} \quad (3.15)$$

$$\begin{aligned} \text{Shearing stress, } \tau_{re} = & - \left(\frac{wl_1 a^4}{4Ir^3} - \frac{wl_1 r}{4I} \right) \cos \theta \\ & - \left(\frac{3wl_1 a^4}{4Ir^3} - \frac{wl_1 a^6}{Ir^5} + \frac{wl_1 r}{4I} \right) \cos 3\theta, \end{aligned} \quad (3.16)$$

the centre of the hole being the origin of co-ordinates and angle θ being measured anti-clockwise (positive) as shown in Fig. 3.1.

At the boundary of the hole, where $r = a$,

$$\sigma_r = \tau_{re} = 0 \quad \text{and} \quad \sigma_\theta = -\frac{wla}{I} (\sin \theta - \sin 3\theta) \quad (3.17)$$

The classical development by Tuzi implies an extension of the web substantially beyond the hole boundary.

when $\theta = \pi/2$ or $3\pi/2$ in eqn. (3.17), $\sigma_\theta = -\frac{2wla}{I}$, which is equal to twice the stress at the same point in a beam without a hole.

3.1.3 Stresses across the minimum Section

In order to investigate the distribution of stresses across the minimum section, i.e., at the section along the vertical diameter of the hole, the substitution of $\theta = 90^\circ$ in eqn. (3.15) is

made,

$$\text{Tangential stress, } \sigma_{\theta} = - \frac{wlr}{I} - \frac{wla^6}{Ir^5} \quad (3.18)$$

Defining the stress concentration factor as the ratio of the tangential stress at any point to the normal stress at the same point in an unperforated beam, and using the gross second moment of area,

$$\text{s.c.f.} = 1 + \frac{a^6}{r^6} \quad (3.19)$$

This equation shows that for all values of a , i.e., for all sizes of circular holes, the stress concentration factor at the hole boundary is equal to 2.

Using the exact mathematical solution by Howland and Stevenson²⁴ for stresses in a perforated beam under pure bending, the s.c.f.s can be found at the minimum section. Table 3.4 shows the s.c.f.s for two sizes of holes for diameter to depth ratios equal to 0.4 and 0.5. These values are represented graphically in Fig. 3.4. This shows that the stress at the top of the beam at this section is slightly less than that obtained from the simple solid beam consideration; the simple beam stress distribution is shown by dashed lines.

3.1.4 Hole near the Edge of a Beam under uniform Stress

When a circular hole is situated in a beam eccentrically with respect to the mid-depth of the beam, it is possible that the hole

will be near the edge of the beam. The stress distribution around a circular hole near the edge of a plate in compression will be different from that when the hole is centrally placed, because of the influence of the plate boundary. The stress concentration factors at the points n and ^Pp (Fig. 3.5) are more pronounced, and the stress at m approaches zero as the hole approaches the edge of the plate.

Mindlin¹⁷ solved such a problem where a hole was situated near the edge of a semi-infinite plate and a uniform tension was applied parallel to the straight edge. The set of curves in Fig. 3.5 shows the stress concentration factor at the hole boundary for different c/R_2 ratios.

3.1.5 Stresses around an eccentric Hole in a Beam

If the centre of the hole is not situated on the centroidal axis of the beam but at a distance d from it, the problem can be reduced to:

a) calculation of stresses around the hole due to the tension or compression in the direction of the neutral axis by the stress $p = \frac{Md}{I}$ by use of Mindlin's curve (Fig. 3.5), if necessary,

b) calculation of stresses around the hole due to pure bending of a strip having a hole, the centre of which is at the centroidal axis.

The final stress trajectories can be obtained by summation of

these two cases.

This method has a limitation; the solution for pure bending applied in (b) may be open to question where the hole boundary is close to the edge of the beam.

3.1.6 Circular Hole in a Beam subjected to Bending and Shear

Following the solutions by Tuzi¹⁸ and Wang,⁴⁵ the solution for stresses around a circular hole in a rectangular beam subjected to bending and shear can be developed.³⁰ The stresses are found as follows:

$$\begin{aligned} \sigma_r = & \frac{2ql}{I} \left(\frac{a^4}{8r^3} - \frac{r}{8} \right) \sin \theta + \frac{q}{I} \left[\frac{a^4}{r^4} \left(\frac{3h^2}{2} - \frac{a^2}{2} \right) - \frac{2a^2}{r^2} \left(h^2 - \frac{a^2}{4} \right) + \frac{h^2}{2} \right] \\ & \sin 2\theta + \frac{2ql}{I} \left[-\frac{a^6}{2r^5} + \frac{5a^4}{8r^3} - \frac{r}{8} \right] \sin 3\theta + \frac{q}{I} \left[\frac{5a^8}{4r^6} - \frac{3a^6}{2r^4} + \frac{r^2}{4} \right] \sin 4\theta \end{aligned} \quad (3.20)$$

$$\begin{aligned} \sigma_\theta = & \frac{2ql}{I} \left[-\frac{a^4}{8r^3} - \frac{3r}{8} \right] \sin \theta + \frac{q}{I} \left[\frac{a^4}{r^4} \left(-\frac{3h^2}{2} + \frac{a^2}{2} \right) - \left(\frac{h^2}{2} - \frac{r^2}{2} \right) \right] \sin 2\theta \\ & + \frac{2ql}{I} \left[\frac{a^6}{2r^5} - \frac{a^4}{8r^3} + \frac{r}{8} \right] \sin 3\theta + \frac{q}{I} \left[-\frac{5a^8}{4r^6} + \frac{a^6}{2r^4} - \frac{r^2}{4} \right] \sin 4\theta \end{aligned} \quad (3.21)$$

$$\begin{aligned} \tau_{r\theta} = & \frac{2ql}{I} \left[-\frac{a^4}{8r^3} + \frac{r}{8} \right] \cos \theta + \frac{q}{I} \left[-\frac{a^4}{r^4} \left(\frac{3h^2}{2} - \frac{a^2}{2} \right) + \frac{a^2}{r^2} \left(h^2 - \frac{a^2}{4} \right) \right. \\ & \left. + \left(\frac{h^2}{2} - \frac{r^2}{4} \right) \right] \cos 2\theta + \frac{2ql}{I} \left[\frac{a^6}{2r^5} - \frac{3a^4}{8r^3} - \frac{r}{8} \right] \cos 3\theta + \frac{q}{I} \left[-\frac{5a^8}{4r^6} + \frac{a^6}{r^4} + \frac{r^2}{4} \right] \\ & \cos 4\theta \end{aligned} \quad (3.22)$$

At the hole boundary where $r = a$,

$$\sigma_\theta = -\frac{qla}{I} (\sin \theta - \sin 3\theta) - \frac{qa^2}{I} (\sin 4\theta - \sin 2\theta) - \frac{2q}{I} h^2 \sin 2\theta \quad (3.23)$$

One drawback in using this solution for stresses around a hole placed in a region where the bending moment and/or the shearing force are changing magnitude is that these equations take the bending moment and shearing force at the section through the centre of the hole into consideration, whereas at any section at the left or right of the centre of the hole, the bending moment and shearing force may be greater or smaller depending on the distribution of these quantities along the length of the beam. This effect is more pronounced if the diameter of the hole is large in comparison with the shear span.

3.1.7 Distribution of Shear across the minimum Section

From the exact mathematical solution by Howland and Stevenson²⁴ for stresses in a beam containing circular holes under the action of bending moment and shearing force, the distribution of shearing stress at the minimum section can be calculated. Fig. 3.6 shows the distribution of shearing stress on top and bottom cross-sections of a beam for holes having diameter to depth ratios equal to 0.4 and 0.5. The dashed lines in the figure represent the average shearing stress at a section without any hole.

3.1.8 Applicability of Vierendeel Analysis

Stresses at the hole

The portion of the beam between the support reaction and the centre-line of a rectangular hole is in equilibrium with an

internal bending moment M and an internal shear force $2V$ balancing the bending moment and shearing force caused by the reaction R (Fig. 3.7). For a hole centred on the centroidal axis of the beam

$$V = 0.5 R \quad (3.24)$$

The stresses caused by V can be computed according to the elementary beam theory. A shear stress τ_{xy} and a bending stress σ_{x1} are caused by V :

$$\begin{aligned} \tau_{xy} &= \frac{VQ}{I_t b_1} \\ &= \frac{V}{2 I_t} (\bar{y}^2 - y_t^2) \end{aligned} \quad (3.25)$$

$$\sigma_{x1} = \frac{V x y_t}{I_t} \quad (3.26)$$

where $\bar{y}$ = distance from hole boundary to the centroid of the top section

y_t = transverse distance from the centroid of the section

According to eqn. (3.26), a point of contraflexure is assumed to occur at sections vertically above and below the centre of the hole.

$$\text{Moment } M = R(u+x)$$

$$\sigma_{x2} \text{ due to } M = \frac{My}{I_n}$$

$$\text{Total stress at D} = -\frac{V a \bar{y}}{I_t} - \frac{M h}{I_n} \quad (+ \text{ tensile stress}) \quad (3.27)$$

Application to circular holes

The extension of the Vierendeel analysis to beams with circular holes is questionable, because the boundary conditions at the hole are not satisfied, except on the section through the centre-line of the hole. τ_{xy} would, according to eqn. (3.25), always be zero at the hole boundary; but τ_{xy} would not be equal to zero at the boundary of circular holes, rather the shear stress tangent to the boundary would be zero. Also σ_x would according to eqn. (3.26) have a value even at the edge of the hole on the longitudinal axis; but the bending stress must be zero at these sections.

Stresses on top of the beam

For a beam having rectangular holes, a simplified formula can be derived for the top fibre stresses according to Kolosowski.³⁸ Consider a section through the centre-line of a hole, say AA, where there is a bending moment M and a shearing force V (Fig. 3.8). A bending moment, a shearing force and an axial force would exist at any section. At the section chosen, however, there is no bending moment in the chords because this is the point of contraflexure of the chord.

The compressive force C in the top chord and the tensile force T in the bottom chord are

$$C = T = \frac{M}{a} \quad (3.28)$$

where a is the lever arm which may be assumed to be = (d-2t)

The axial stress, $f_a = \frac{M}{axA_c}$ where A_c = area of the chord.

The shear force at this section may be assumed to be equally distributed over the top and bottom chords. The shearing stress does not affect the stress at the extreme fibres. The shear force V (Fig. 3.7) produces a bending moment M_0 at the corners of the castellation

$$M_0 = Vx \frac{b}{2} \text{ where } b = \text{length of castellation}$$

The bending stress at the extreme fibre due to moment M_0 is

$$f_b = \mp \frac{Vbt}{2I_c}$$

This stress produces peak values at the corners of the castellation. The maximum resultant stress is given by

$$f_{\text{resultant}} = f_a \mp f_b = \frac{M}{axA_c} \mp \frac{Vbt}{2I_c} \quad (3.29)$$

Application to circular holes

If a beam perforated with circular holes can be approximated to a beam having flat-top holes (Fig. 3.12), the above formula for extreme fibre stress should be applicable. From eqn. (3.29), it is clear that the stress at the middle of the flat surface, i.e., at AA, would be more than that at section BB, the low bending moment corner of the castellation (Fig. 3.8). The comparison between the experimental results of tests carried out in this research study and the results given by eqn. (3.29) will be discussed later.

3.1.9 Cracking Load of perforated Beam

Let p be the stress at the bottom fibre of the beam due to

prestress. p will have almost the same value at a solid section or at a section through a hole. If p_1 be the modulus of rupture of the concrete, the load at cracking, P_{cr} , can be calculated as follows:

$$\frac{P_{cr} \times l \times c}{I} = p + p_1$$

or, $P_{cr} = \frac{(p + p_1) \times I}{l \times c} \quad (3.30)$

3.2 DEFLECTION OF PERFORATED BEAMS

3.2.1 Methods of calculating Deflection of a perforated Beam

It is clear that the deflection of a perforated beam will be greater than that of a solid beam because of the reduction in stiffness of the beam. The following methods may be applied to predict the deflection.

1. Using the second moment of area for a solid beam.
2. Using the net second moment of area at the minimum section.
3. a) Using the slope deflection equations on an approximated linear form.
- b) Adding the shear deflection for the horizontal members.
4. Using the area-moment method
 - a) (i) By using the gross second moment of area in the solid portion and the net section second moment of area in the hole section of the beam
 - (ii) By the method of integration for the equivalent second moment of area,

b) Adding the shear deflection, arrived at by the method of integration.

Each of these methods will now be described in detail.

3.2.2 Deflection using the gross Section second Moment of Area

For a rectangular section of breadth b_1 and depth D , the gross second moment of area,

$$I = \frac{b_1 D^3}{12} \quad (3.31)$$

For third-point loading (as used in the experimental work) on a span L (Fig. 3.9), the central deflection δ at a load P , before cracking, is given by

$$\delta = \frac{23}{1296} \frac{PL^3}{EI} \quad (3.32)$$

3.2.3 Deflection using the net Section second Moment of Area

In this case, $I = \frac{b_1 D^3}{12} - \frac{b_1 d^3}{12}$ (Fig. 3.10) (3.33)

If the hole is placed above or below the centroidal axis,

$$I = \frac{b_1 D^3}{12} - \frac{b_1 d^3}{12} - b_1 x d x d_1^2 \quad (3.34)$$

where d = diameter of the hole

d_1 = distance of the centre of the hole from the neutral axis (Fig. 3.11).

Shear deflection

Average shear stress = $\frac{V}{b_1 D}$

Average shear strain = $\frac{\gamma V}{b_1 D x G}$

$$\text{Maximum shear deflection} = \frac{\gamma V}{b_1 D G} \frac{L}{3}$$

where γ = shape factor = 1.2 for a rectangular section,

$\frac{L}{3}$ = shear span

3.2.4 Deflection by slope Deflection Equations

If the beam containing circular perforations is simplified to one having square holes of side equal to the diameter and treated as a Vierendeel girder as shown in Fig. 3.12, it is possible to write slope-deflection equations for the horizontal and vertical members of the girder.

$$M_{AB} = \frac{2E I_{AB}}{L_1} \left(-2\theta_A - \theta_B + \frac{3\Delta_B}{L_1} \right) \quad (3.35)$$

$$M_{AL} = \frac{2E I_{AL}}{L_6} \left(-2\theta_A - \theta_L \right) \quad (3.36)$$

etc. where $\theta_A = \theta_L$ due to symmetry

I_{AB} = second moment of area of the minimum
section between AB

Δ_B, Δ_C etc. = deflections at points B, C etc.
relative to points A, B etc. due
to bending.

In the particular problem considered in the experimental work, there are nine unknowns, i.e., $\theta_A, \theta_B, \theta_C, \theta_D, \theta_E, \Delta_B, \Delta_C, \Delta_D$, and Δ_E . To determine these unknowns, nine simultaneous equations are necessary.

Moment conditions:

$$\sum M_A = 0, \sum M_B = 0, \sum M_C = 0, \sum M_D = 0, \sum M_E = 0,$$

Shear conditions:

$$\text{Span AB: } \frac{2M_{AB} + 2M_{BA}}{L_1} = P \quad (3.37)$$

Similarly, for spans BC, CD and DE.

These nine simultaneous equations can be solved by the use of the digital computer (an example is given later in Appendix A).

The total deflection at E due to bending = $\Delta_B + \Delta_C + \Delta_D + \Delta_E$

3.2.5 Deflection by equivalent second Moment of Area Method

Constant bending moment

Consider a beam subjected to pure bending moment M. For a beam with second moment of area I_e , $\theta = \frac{ML}{EI_e}$ (Fig. 3.13).

For a beam with varying I, $\theta = \text{area of } \frac{M}{EI} \text{ diagram.}$

$$\begin{aligned} \text{For the same stiffness, } \frac{ML}{EI_e} &= \text{area of } \frac{M}{EI} \text{ diagram} \\ &= \frac{M}{E} \times \text{area of } \frac{1}{I} \text{ diagram} \end{aligned}$$

$$\text{Therefore, } I_e = \frac{L}{\text{area of } \frac{1}{I} \text{ diagram}}$$

Referring to Fig. 3.13,

$$\text{in } A_1, \frac{1}{I} = \frac{12}{bd^3}; \quad \therefore A_1 = \frac{1 \cdot 12}{bd^3}$$

Let the number of holes of diameter d be equal to n.

$$\sum A_1 = (L - nd) \frac{12}{bd^3}.$$

$$\begin{aligned}\text{In } A_2, \text{ at any angle } \theta, \quad I &= \frac{bD^3}{12} - \frac{b}{12} (d^3 \cos^3 \theta) \\ &= \frac{b}{12} (D^3 - d^3 \cos^3 \theta)\end{aligned}$$

$$\therefore \frac{1}{I} = \frac{12}{b(D^3 - d^3 \cos^3 \theta)}$$

$$\text{Area in length } dx = \frac{12 dx}{b(D^3 - d^3 \cos^3 \theta)} = \frac{12 d \cos \theta d\theta}{2b(D^3 - d^3 \cos^3 \theta)}$$

$$\therefore A_2 = \frac{6d}{b} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos \theta d\theta}{D^3 - d^3 \cos^3 \theta}$$

$$\therefore A_2 = \frac{6nd}{b} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos \theta d\theta}{D^3 - d^3 \cos^3 \theta} = \frac{6nd}{bD^3} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos \theta d\theta}{1 - B^3 \cos^3 \theta} \quad \text{where } B = \frac{d}{D}$$

$$\text{Total area of } \frac{1}{I} \text{ diagram} = (L - nd) \frac{12}{bD^3} + \frac{6nd}{bD^3} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos \theta d\theta}{1 - B^3 \cos^3 \theta}$$

$$= \frac{12}{bD^3} \left[L - nd + \frac{nd}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos \theta d\theta}{1 - B^3 \cos^3 \theta} \right]$$

$$= \frac{1}{I} (L - nd + \frac{nd}{2} k) \quad \text{where } k = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos \theta d\theta}{1 - B^3 \cos^3 \theta}$$

$$\therefore I_e = I \times \frac{L}{(L - nd + \frac{nd}{2} k)}$$

$$\therefore \frac{I}{I_e} = \frac{L - nd + \frac{nd}{2} k}{L}$$

(3.38)

A set of curves for I/I_e can be drawn for different values of d/D and $\frac{nd}{L}$. These curves are given in Fig. A.5 of Appendix A. When I is known, I_e can be found from these curves for any given value of $\frac{d}{D}$ and $\frac{nd}{L}$. It should be noted that this method is applicable to

central holes only.

Varying bending moment

In the above derivation, it is assumed that the beam is subject to pure bending and that the change of slope at midspan is proportional to the central deflection.

Consider a general case where the moment is constantly varying, $M = f(x)$. Applying the second area-moment theorem:

For a beam with second moment of area I_e , central deflection,

$$\delta = \frac{1}{EI_e} \int_0^{\frac{L}{2}} f(x) x dx$$

For variable second moment of area, $\delta = \frac{1}{E} \int_0^{\frac{L}{2}} \frac{f(x) x dx}{I_x}$

Equating these two deflection equations,

$$I_e = \frac{\int_0^{\frac{L}{2}} f(x) x dx}{\int_0^{\frac{L}{2}} \frac{f(x) x dx}{I_x}} \quad (3.39)$$

In Fig. 3.14,

$$\text{in } B_1, \quad \frac{1}{I_x} = \frac{12}{bD^3}$$

$$\text{in } B_2, \quad \frac{1}{I_x} = \frac{12}{bD^3 (1 - B^3 \cos^3 \theta)} \quad \text{where } B = \frac{d}{D}$$

$$\text{Denominator of eqn. (3.39)} = \frac{12}{bD^3} \left[\int_0^{l_1} f(x) x dx + \sum_{n=1}^{n=n-1} \int_{l_1+nd+(n-1)l}^{l_1+nd+n l} f(x) x dx \right.$$

$$\left. + \int_{l_1+nd+(n-1)l}^{l_1+nd+(n-1)l+l_2} f(x) x dx + \frac{d}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ f(x) \right\} \frac{\cos \theta}{(1-B^3 \cos^3 \theta)} \sum_{n=0}^{n=n-1} \int_{l_1+nd+n l+\frac{d}{2}}^{l_1+nd+n l+\frac{d}{2}} f(x) x dx \right. \\ \left. (1+\sin \theta) \right\} d\theta \Big] = \frac{1}{I} (B_1 + \frac{d}{2} C_1) \\ \text{Therefore, } \frac{I}{I_g} = \frac{B_1 + \frac{d}{2} C_1}{A_1} \quad (3.40)$$

A set of curves could be plotted for I/I_g for different values of $\frac{d}{D}$, l_1 , l_2 and n . However, a complete set of curves is not presented because of the large number of parameters involved. The sample calculation given in Appendix A indicates that the result does not differ significantly from that employing a constant bending moment.

3.2.6 Shear Deflection

Referring to Fig. 3.15,

$$\text{Average shear stress}^{46} = 1.2 \times \frac{V}{b_1 D}$$

$$\text{Average shear strain} = 1.2 \frac{V}{b_1 D G}$$

a) Shear deflection for n number of holes in a shear span

$$\begin{aligned}
 &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1.2 \frac{V}{b_1(D-d \cos \theta) \times G} nd \, d\theta \\
 &= \frac{1.2 Vnd}{b_1 DG} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1 - \frac{d}{D} \cos \theta} d\theta \\
 &= \frac{1.2 Vnd}{b_1 DG} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\theta}{1 - B \cos \theta} \quad \text{where } B = \frac{d}{D}
 \end{aligned} \tag{3.41}$$

b) Shear deflection for the solid part

$$= 1.2 \frac{V}{b_1 DG} (1 - nd)$$

(3.42)

$$\text{Maximum shear deflection} = \frac{1.2 Vnd}{b_1 DG} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\theta}{(1 - B \cos \theta)} + \frac{1.2 V (1 - nd)}{b_1 DG}$$

$$\text{Shear defln in a perforated beam} = \frac{1.2 Vnd}{b_1 DG} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\theta}{(1 - B \cos \theta)} + \frac{1.2 V (1 - nd)}{b_1 DG}$$

Shear defln in a solid beam

$$= \frac{nd}{1} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\theta}{(1 - B \cos \theta)} + \frac{1 - nd}{1}$$

$$= 1 + \frac{nd}{1} \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\theta}{(1 - B \cos \theta)} - 1 \right)$$

(3.43)

Curves can be plotted for the ratios of the shear deflection in a perforated beam to the shear deflection in a solid beam for

different values of $\frac{d}{D}$ and $\frac{nd}{L}$. These curves are shown in Fig. A.6 of Appendix A.

Table 3.1
Coefficients

	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$
d_0	$5.01 \cdot 10^{-3}$	$2.02 \cdot 10^{-2}$	$4.59 \cdot 10^{-2}$	$8.28 \cdot 10^{-2}$	$1.32 \cdot 10^{-1}$
d_2	$2.54 \cdot 10^{-5}$	$4.25 \cdot 10^{-4}$	$2.34 \cdot 10^{-3}$	$8.28 \cdot 10^{-3}$	$2.33 \cdot 10^{-2}$
d_4	$3.15 \cdot 10^{-11}$	$3.05 \cdot 10^{-8}$	$1.85 \cdot 10^{-6}$	$3.32 \cdot 10^{-5}$	$3.14 \cdot 10^{-4}$
d_6	$1.40 \cdot 10^{-15}$	$2.13 \cdot 10^{-11}$	$6.32 \cdot 10^{-9}$	$3.42 \cdot 10^{-7}$	$7.51 \cdot 10^{-6}$
d_8	$5.00 \cdot 10^{-20}$	$1.19 \cdot 10^{-14}$	$1.72 \cdot 10^{-11}$	$2.82 \cdot 10^{-9}$	$1.45 \cdot 10^{-7}$
e_2	$-5.08 \cdot 10^{-3}$	$-2.13 \cdot 10^{-2}$	$-5.21 \cdot 10^{-2}$	$-1.04 \cdot 10^{-1}$	$-1.89 \cdot 10^{-1}$
e_4	$-4.21 \cdot 10^{-9}$	$-1.02 \cdot 10^{-6}$	$-2.75 \cdot 10^{-5}$	$-2.79 \cdot 10^{-4}$	$-1.70 \cdot 10^{-3}$
e_6	$-1.68 \cdot 10^{-12}$	$-6.41 \cdot 10^{-10}$	$-8.45 \cdot 10^{-8}$	$-2.58 \cdot 10^{-6}$	$-3.64 \cdot 10^{-5}$
e_8	$-5.72 \cdot 10^{-18}$	$-3.41 \cdot 10^{-13}$	$-2.20 \cdot 10^{-10}$	$-2.02 \cdot 10^{-8}$	$-6.7 \cdot 10^{-7}$
l_2	$4.13 \cdot 10^{-3}$	$1.62 \cdot 10^{-2}$	$4.09 \cdot 10^{-2}$	$8.03 \cdot 10^{-2}$	$1.40 \cdot 10^{-1}$
l_4	$1.06 \cdot 10^{-3}$	$4.10 \cdot 10^{-3}$	$1.01 \cdot 10^{-2}$	$1.92 \cdot 10^{-2}$	$3.27 \cdot 10^{-2}$
l_6	$2.83 \cdot 10^{-4}$	$1.08 \cdot 10^{-3}$	$2.61 \cdot 10^{-3}$	$4.82 \cdot 10^{-3}$	$8.09 \cdot 10^{-3}$
l_8	$7.24 \cdot 10^{-5}$	$2.74 \cdot 10^{-4}$	$6.44 \cdot 10^{-4}$	$1.17 \cdot 10^{-3}$	$1.95 \cdot 10^{-3}$
m_0	$5.69 \cdot 10^{-4}$	$2.46 \cdot 10^{-3}$	$5.15 \cdot 10^{-3}$	$8.89 \cdot 10^{-3}$	$1.40 \cdot 10^{-2}$
m_2	$-1.13 \cdot 10^{-3}$	$-4.35 \cdot 10^{-3}$	$-1.09 \cdot 10^{-2}$	$-2.11 \cdot 10^{-2}$	$-3.62 \cdot 10^{-2}$
m_4	$-5.95 \cdot 10^{-4}$	$-2.29 \cdot 10^{-3}$	$-5.60 \cdot 10^{-3}$	$-1.06 \cdot 10^{-2}$	$-1.78 \cdot 10^{-2}$
m_6	$-2.28 \cdot 10^{-4}$	$-8.70 \cdot 10^{-4}$	$-2.10 \cdot 10^{-3}$	$-3.88 \cdot 10^{-3}$	$-6.47 \cdot 10^{-3}$
m_8	$-7.65 \cdot 10^{-5}$	$-2.90 \cdot 10^{-4}$	$-6.88 \cdot 10^{-4}$	$-1.25 \cdot 10^{-3}$	$-2.08 \cdot 10^{-3}$

Table 3.2

 $\frac{\delta\theta}{p}$ values

θ°	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$
0	- 1.11	- 1.26	- 1.44	- 1.58	- 1.72
15	- 0.82	- 0.95	- 1.12	- 1.32	- 1.55
30	- 0.06	- 0.15	- 0.30	- 0.51	- 0.80
45	+ 1.00	+ 0.98	+ 0.91	+ 0.77	+ 0.56
60	+ 2.07	+ 2.15	+ 2.25	+ 2.32	+ 2.37
75	+ 2.85	+ 3.03	+ 3.32	+ 3.72	+ 4.32
90	+ 3.14	+ 3.36	+ 3.74	+ 4.32	+ 5.12

Table 3.3

 $\frac{\delta \epsilon}{p}$ values

ρ	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$
0.1	-	-	-	-
0.2	3.14	-	-	-
0.3	1.57	3.36	-	-
0.4	1.26	1.93	3.74	-
0.5	1.16	1.47	2.30	4.32
0.6	1.11	1.28	1.75	2.75
0.7	1.07	1.17	1.48	2.04
0.8	1.05	1.07	1.28	1.61
0.9	1.01	0.96	1.08	1.22
1.0	0.97	0.89	0.81	0.73

Table 3.4

s.c.f.s due to pure bending at the minimum section

Stress Concentration factor		
$\rho = \frac{r}{b}$	$\lambda = \frac{a}{b} = 0.4$	$\lambda = \frac{a}{b} = 0.5$
0.4	2.08	-
0.5	1.29	2.20
0.6	1.11	1.41
0.7	1.05	1.18
0.8	1.02	1.08
0.9	1.00	1.00
1.0	0.97	0.95

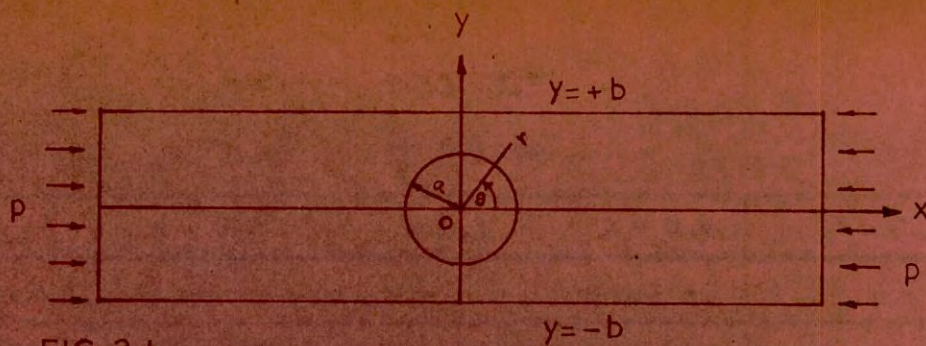


FIG. 3.1

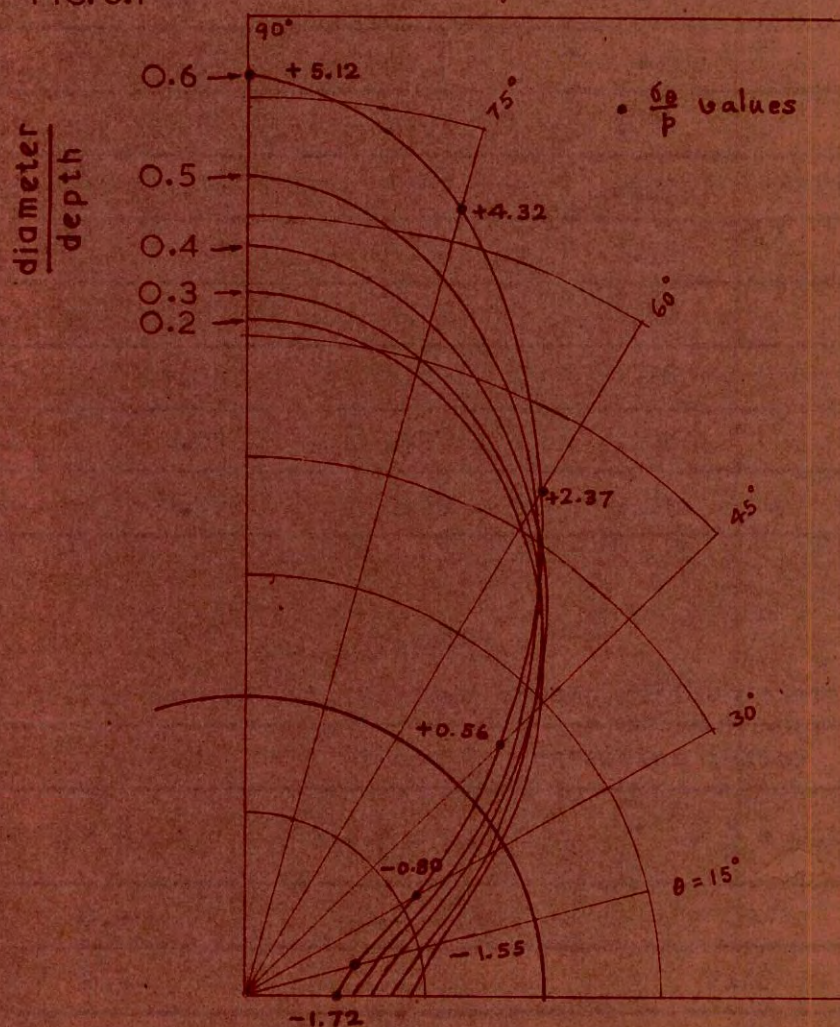


FIG. 3.2

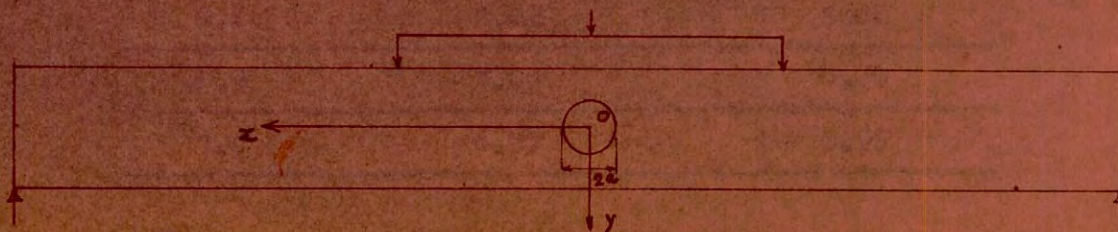


FIG. 3.3

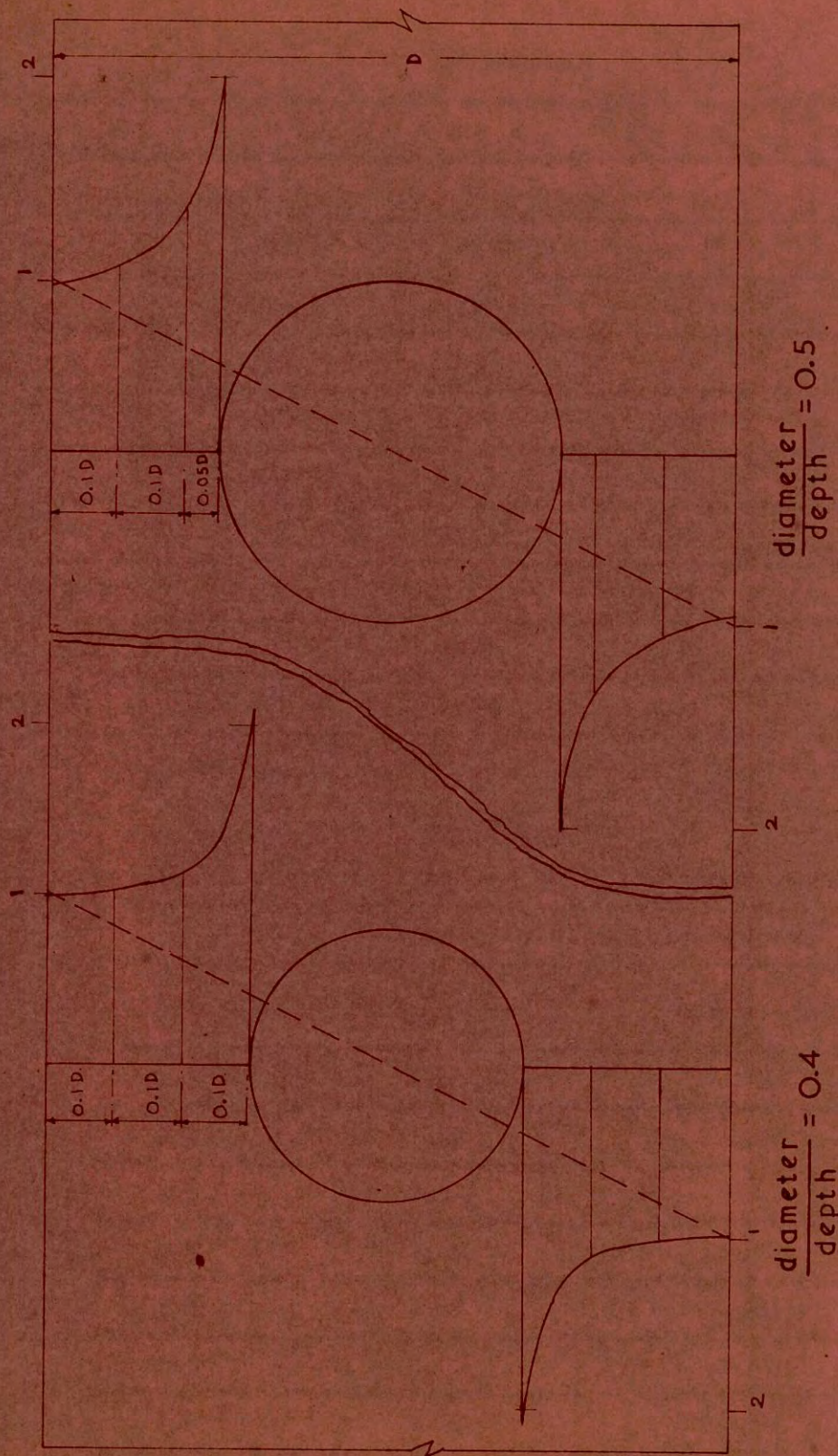


FIG. 3.4 Stress distribution at minimum section due to bending

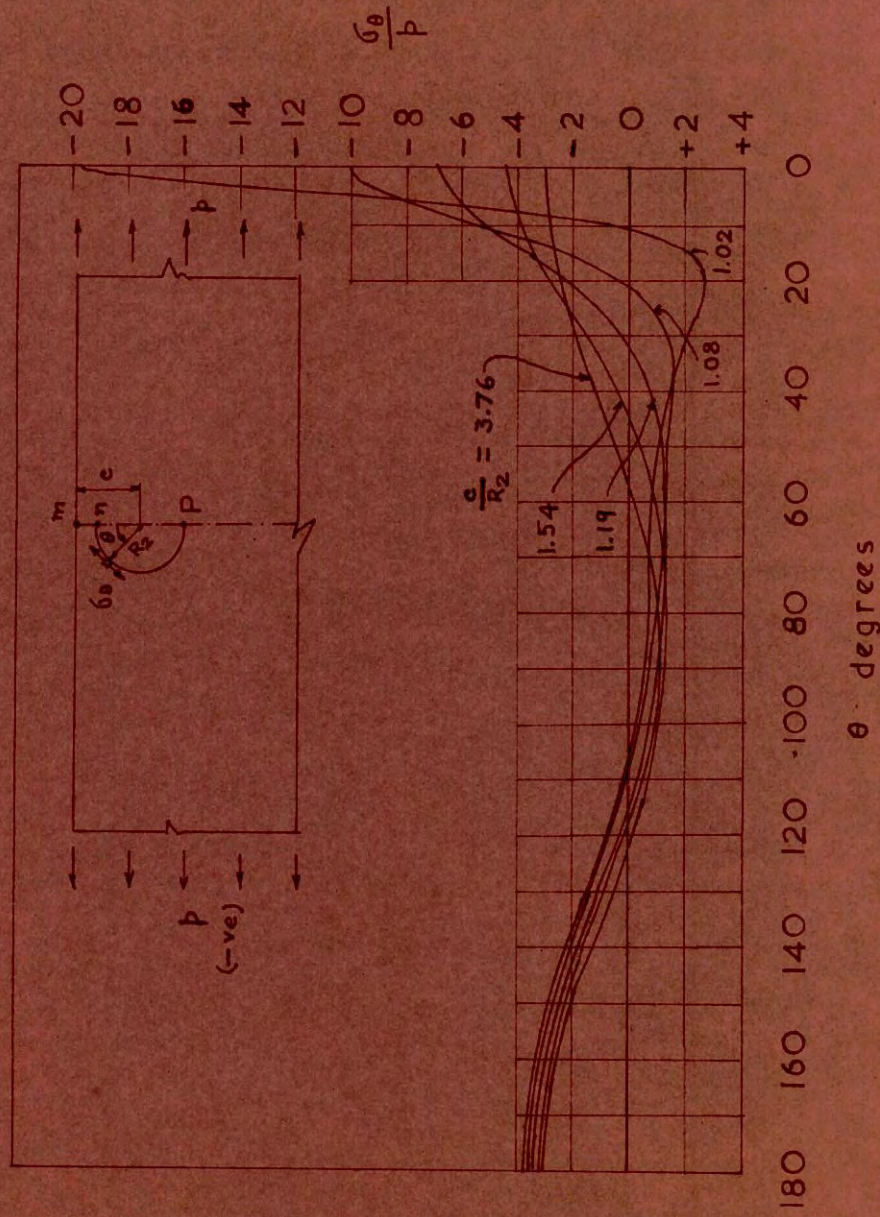


FIG. 3.5 Stress concentration at the boundary of a circular hole near the edge of a plate

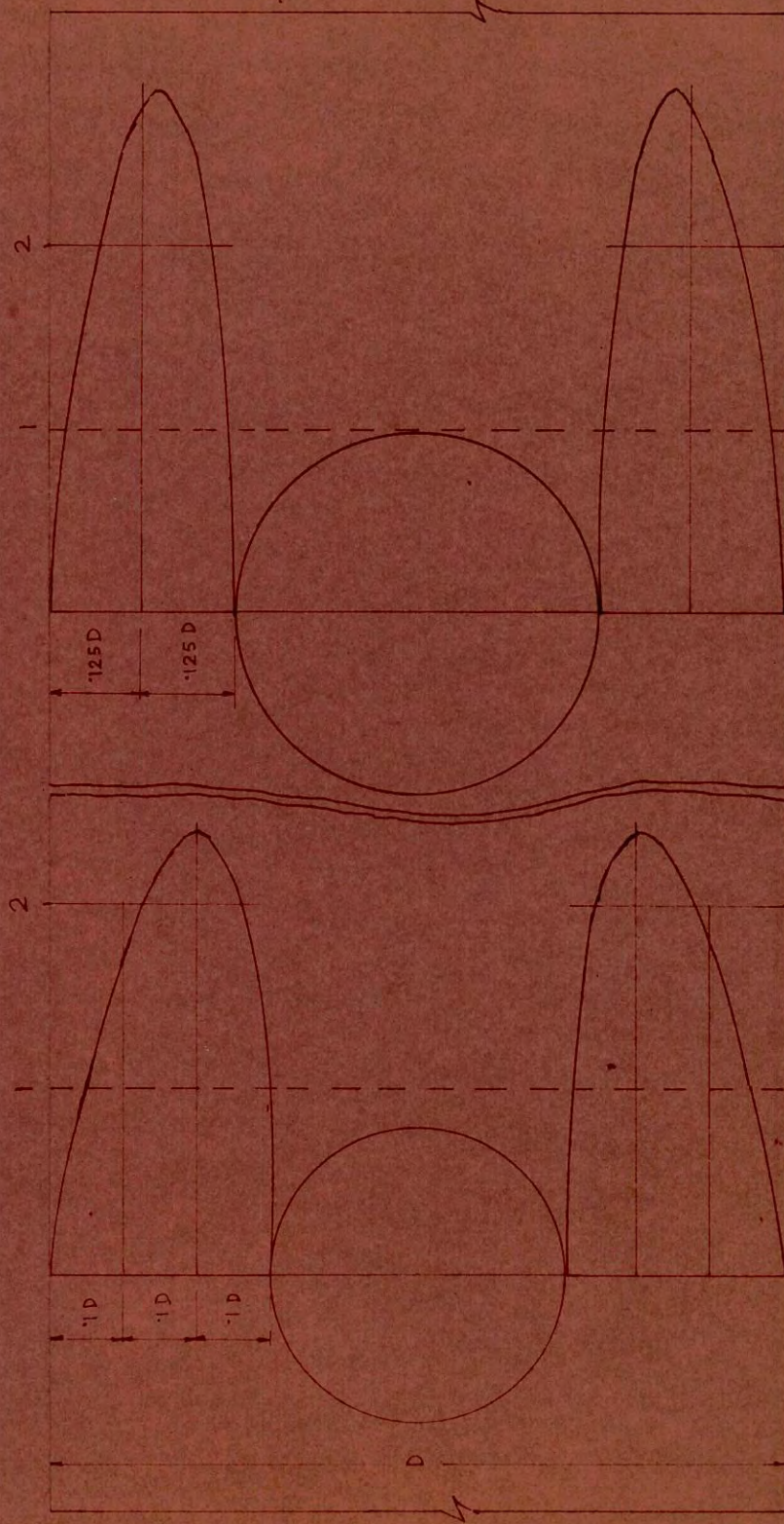


FIG. 3.6 Distribution of shearing stress at minimum section

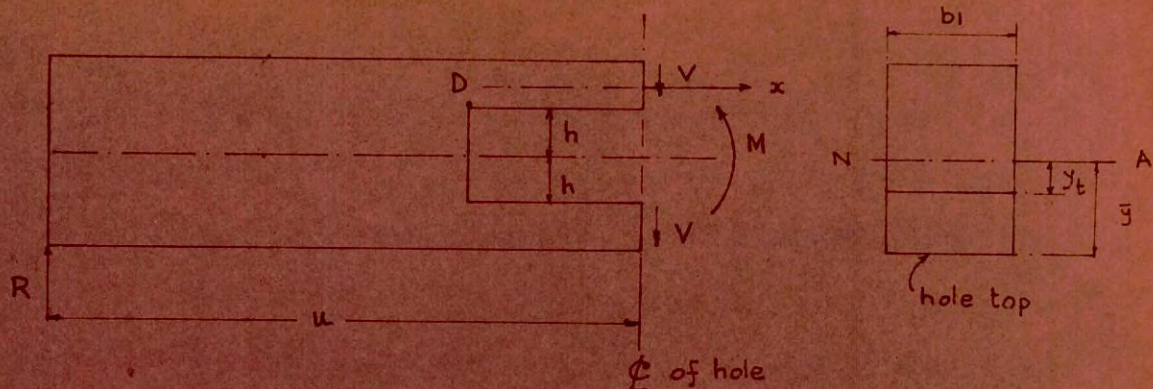


FIG. 3.7

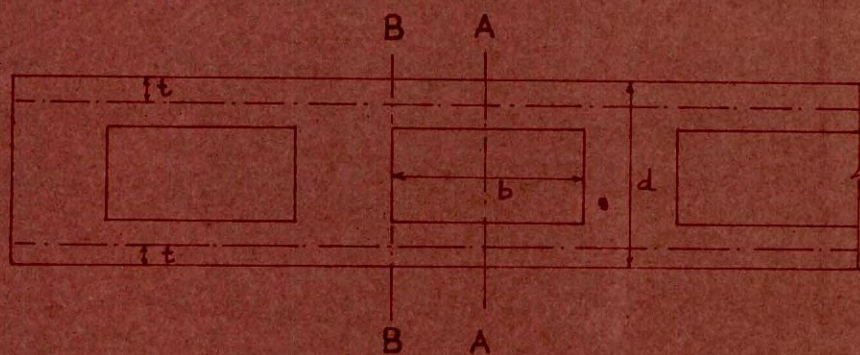


FIG. 3.8

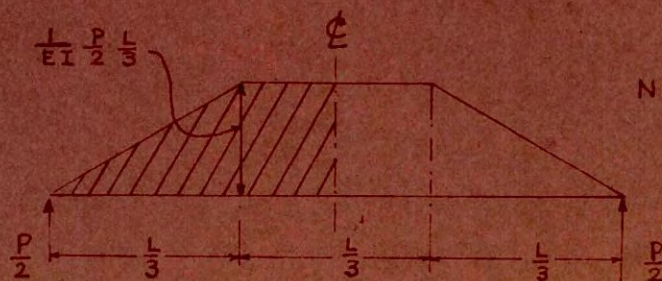


FIG. 3.9 $\frac{M}{EI}$ diagram

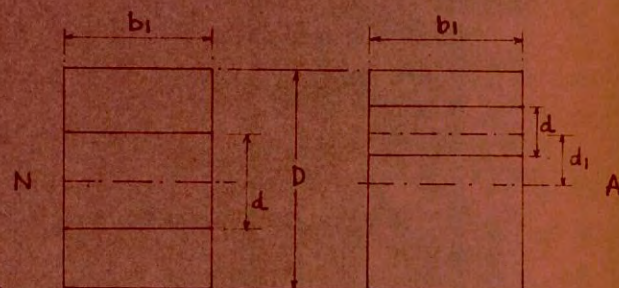


FIG. 3.10

FIG. 3.11

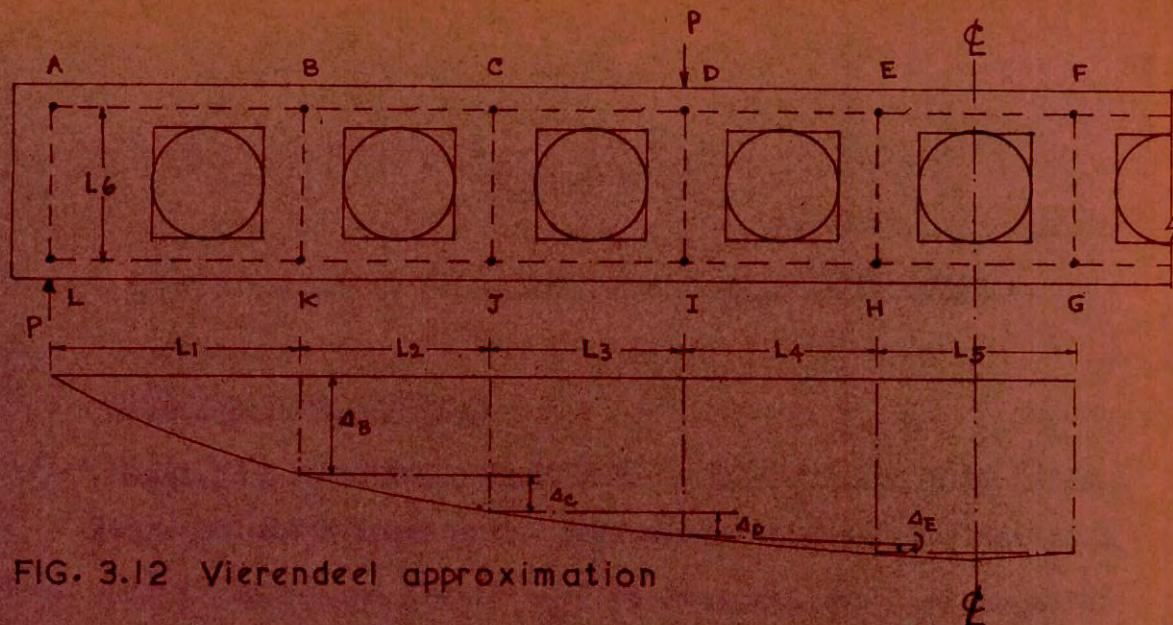


FIG. 3.12 Vierendeel approximation

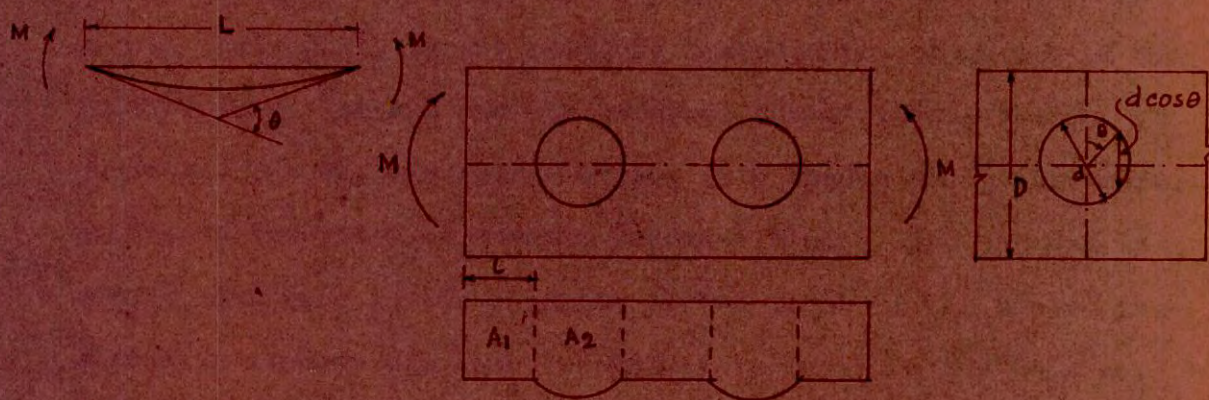


FIG. 3.13

$\frac{M}{EI}$ diagram

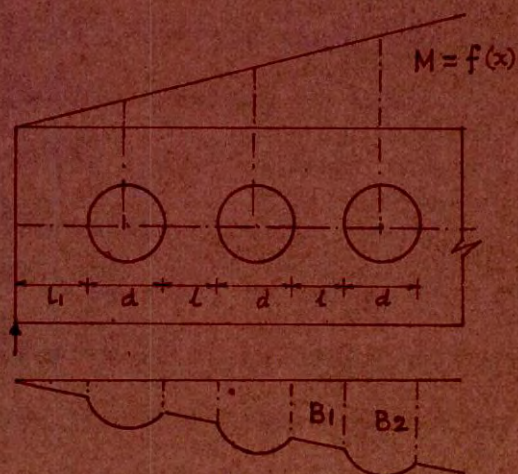


FIG. 3.14 $\frac{M}{EI}$ diagram

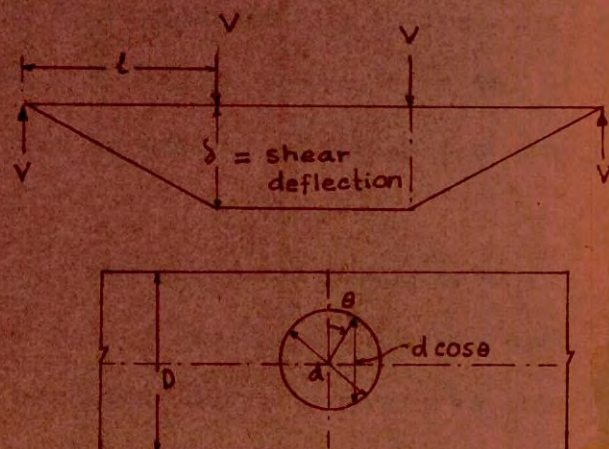


FIG. 3.15

CHAPTER IV

ULTIMATE LOAD ANALYSIS AND DESIGN OF REINFORCEMENT

4.1 ULTIMATE LOAD ANALYSIS

4.1.1 Difference in Behaviour between bonded and unbonded Beams

There exists a fundamental difference in the behaviour of a beam containing bonded prestressed tendons and one that contains unbonded tendons.⁴⁷ In a beam that contains bonded tendons the strain that exists at any point in a tendon is a function of the initial tendon strain and the external moment at the section in question. The maximum tendon stress in a bonded beam, therefore, occurs at the point of maximum external moment. The changes in strain occurring in the concrete at the level of the bonded prestressing tendons may also be assumed to occur in the tendons.

In an unbonded beam, the change in strain is a function of the external moment at all points along the beam. Any change in force occurring along the length of the tendon is due to friction forces. Assuming no friction forces, the tendon stress over the entire length between anchorages will be uniform, thereby reducing the tendon stress.

However, so far as the stress in the concrete is concerned, there will be no difference between bonded and unbonded beams upto the point of cracking. The stress distribution in concrete at any section of the beam and the deflection should, therefore, be the

same in both bonded and unbonded beams before cracking.

For identical conditions with the only variable being the bonded and unbonded tendon, the beam with the bonded tendon will develop many closely spaced small cracks, while the beam with the unbonded tendon will develop a few large cracks in the maximum moment zone.

In a perforated beam with unbonded tendons, it is expected that cracks will develop below each hole within the high moment zone. When these holes are close to one another, the unbonded beam might well develop a greater number of cracks than the solid unbonded beam.

4.1.2 Ultimate Strength of a bonded Beam

In order to introduce holes in a prestressed beam, the tendons may have to be placed below the middle-third of the beam section, thereby causing tension at the top fibre on pre-stressing. Compressive stress can be obtained at the top fibre by placing pre-stressing steel above the hole so that the resulting stress distribution at a solid section shows zero stress or little tension at the top and the desired compressive stress at the bottom fibre. For the calculation of the ultimate strength of such a beam, the stresses in both top and bottom prestressing wires at failure must be known.

According to CP115 : 1959⁴⁸

The following steps are recommended to determine the ultimate strength of a bonded beam with both top and bottom prestressing wires.

a) Calculate $\frac{f_u A_{st}}{U_w b_1 d_1}$

where U_w , works cube strength, at 28 days

d_1 , distance of the centroid of the tendon in the tensile zone from the compression surface.

b) From Table 7, determine f_m/f_u and d_n/d_1 and hence f_m and d_n ,

where f_m , stress in tension tendon at failure

d_n , depth of the neutral axis at failure

c) Determine the net prestressing force in the tendon in the compression zone after deducting all losses,

d) Calculate the loss in tension of the tendon in the compression zone at the ultimate condition. The compressive strain in the concrete is proportional to the distance from the neutral axis and has a maximum value of 0.002; the bonded steel strain will be the same as that of the concrete at the same level.

e) The ultimate moment is given by

$$M_u = f_m A_{st} (d_1 - 0.4 d_n) - p_m A'_{st} (0.4 d_n - d'_1). \quad (4.1)$$

According to ACI Building Code⁴⁹

The ultimate flexural strength for a rectangular beam is given

by

$$M_u = A_{st} f_{su} \left(d - \frac{n_p}{2} \right), \text{ provided } p f_{su} / f'_c \geq 0.30, \quad (4.2)$$

i.e., the beam should be under-reinforced.

where $f_{su} = f'_s (1 - 0.5 p f'_s / f'_c)$, provided $f_{se} \leq 0.5 f'_s$,

$$p = \frac{A_{st}}{bd}$$

f'_c , cylinder strength of concrete

f_{se} , effective prestress after losses

d , distance from extreme compression fibre
to centroid of the prestressing force.

In this formula, no mention is made of the depth of the neutral axis at failure and it is, therefore, difficult to determine the ultimate stress in the prestressing steel in the compression zone.

4.1.3 Ultimate Strength of an unbonded Beam

According to CP115 : 1959⁴⁸

This code gives, in Table 8, the stress in tendons for unbonded beams at failure as a proportion of the effective prestress, provided the effective pre-stress, p_e , does not exceed $0.55 f_u$ in the case of wires. The ratio of depth of neutral axis to that of the centroid of the tendons in the tensile zone for different values of $\frac{p_e A_{st}}{U_w b d_1}$ is also given. The effective depth is determined by assuming that the tendons are in contact with the top of the duct.

If the steel in the compression zone is also unbonded, then

the strain in the concrete at failure at the level of the reinforcement is not the strain in the unbonded steel; in this case the total strain in concrete at the level of the reinforcement is equal to the total strain in the whole length of the unbonded wire.

The same formula for bonded beams can be used with these modifications.

$$\text{Therefore, } M_u = f_m A_{st} (d_1 - 0.4 d_n) - p_m A'_{st} (0.4 d_n - d'_1) \quad (4.3)$$

According to ACI Building Code⁴⁹

The ultimate flexural strength is given by

$$M_u = A_{st} f_{su} \left(d - \frac{n_p}{2}\right) \quad \text{as in the case of a bonded beam,} \quad (4.4)$$

provided f_{su} for bonded beams is replaced by

$$f_{su} \text{ for unbonded beams} = f_{se} + 15000 \text{ lb/sq.in.} \quad (4.5)$$

4.1.4 Ultimate Strength of a Beam with Holes above the centroidal Axis

Considering the beam to be one without any hole, the depth of the rectangular stress block can be easily determined from the formula

$$n_p = A_{st} f_{su} / 0.85 f'_c b_1 \quad (4.6)$$

Due to the stress concentration on top of the hole, the stress distribution above the hole may be assumed to be rectangular. It is, therefore, expected that so long as the hole does not protrude into the depth n_p , the beam should not lose its strength.

If the hole does cut a portion from the depth n_p , the ultimate strength of such a beam can easily be calculated by replacing n_p by a' , and assuming C , the resultant compressive force, to act at $\frac{a'}{2}$ from the top of the beam (Fig. 4.1).

4.2 DESIGN OF REINFORCEMENT

4.2.1 Design of Reinforcement for high longitudinal Stresses

Due to stress concentration, the stress at the bottom of a hole due to prestress and that on top of the hole due to loading may be more than the allowable limits. No Code of Practice considers the increased stresses due to the effect of concentration, which may be subsequently relieved due to the creep property of concrete, but reinforcement can be provided at these localised areas to reduce the stresses.

The following simple method is suggested.

If f be the concentrated stress at the bottom or at the top of the hole, the excess stress for which reinforcement should be provided is $f - f_c$,

where f_c is the allowable stress in concrete.

The effect of concentration being local, the depth of excessive stress can be assumed to be one-third the depth of the flange and the distribution linear (Fig. 4.2).

The total force for which reinforcement is to be provided, is

$$C = (f - f_c) \times \frac{1}{2} \times \frac{d_1}{3} \times b_1 \quad (4.7)$$

If f_s be the allowable stress in steel,

$$\text{Area of reinforcement} = \frac{C}{f_s}$$

4.2.2 Design of splitting Reinforcement

The tensile stress concentration, due to the prestressing force, at the ends of the horizontal diameter of the hole can be readily obtained.

For a central hole, the tensile stress, $s = \text{s.c.f.} \times \text{compressive stress due to prestressing at the level of the centre of the hole,}$

(4.8)

For an eccentric hole, the tensile stress, $s = \text{s.c.f.} \times \text{compressive stress at the level of the centre of the hole} + \text{s.c.f.} \times \frac{MC}{I}$

(4.9)

where s.c.f. for central hole is as given in Table 3.2 or Fig. 3.2

s.c.f. (eccentric hole) given in Fig. 3.5

M is the moment at design load

C is the distance of the centre of the hole from the neutral axis

I second moment of area of the gross section

The distribution of the tensile stress is such that the maximum value occurs at the boundary of the hole and quickly reduces to zero at a distance approximately equal to the radius of the hole (Fig. 2.1). Hence the total force to be resisted by vertical stirrups can be assumed to be $r \times b_1 \times \frac{s}{2}$

If f_s be the allowable stress in the stirrup, then

$$A_s = \frac{r \times b_1 \times s}{2 \times f_s} \quad (4.10)$$

4.2.3 Design of Reinforcement against diagonal tensile Stress

The maximum tensile stress at the design load (due to the diagonal stress concentration) can be determined from eqn. (3.21). This stress when added to the stress due to the prestressing force will give the total stress.

This stress is very local in effect and quickly reduces to a nominal stress as found from solid beam considerations. Nominal stirrups can be provided in the solid section; in the design of the reinforcement, it can be assumed that the stress reduces to zero within a distance equal to the radius of the hole.

If s is the total tensile stress at the design load, the total force to be resisted by the stirrup = $\frac{s}{2} \times b_1 \times r$

If f_s be the allowable stress in the stirrup, then

$$A_s = \frac{s \times b_1 \times r}{2 \times f_s} \quad (4.11)$$

4.2.4 Vertical Shear at the minimum Section

In a fully prestressed beam at the design load, i.e., before any crack is formed at the section through the hole, it can be assumed that the top and bottom portions share the shearing force in proportion to their cross-sectional areas.

If V be the total shearing force at the minimum section,

the shear carried by the top flange is

$$V_1 = V \times \frac{d_1}{d_1 + d_2}, \quad (4.12)$$

the shear carried by the bottom flange is

$$V_2 = \frac{V \times d_2}{d_1 + d_2} \quad (4.13)$$

At the design load, P , the normal compressive stress in the top fibre can be calculated by the simple beam theory. Due to the stress concentration on top of the hole, the normal stress distribution in the top flange can be assumed to be rectangular.

$$\text{Normal stress at design load, } P_1 = \frac{P \times I_1 \times c}{I}$$

$$\text{Compressive stress due to the prestress} = P_2$$

$$\text{Maximum shearing stress, } P_3 = 1.5 \frac{V_1}{bd_1} \quad (4.14)$$

$$\text{The principal tensile stress, } t = (P_1 + P_2) / 2 - \sqrt{\left(\frac{P_1 + P_2}{2}\right)^2 + P_3^2} \quad (4.15)$$

If t be greater than the allowable tensile stress (as given in Table 5, CP115), stirrups have to be provided.

The same formula holds good for the bottom flange with proper signs for the various quantities.

4.2.5 Design of Stirrup

According to CP115:1959, where the principal tensile stress due to pre-stress, bending and shear at working loads exceeds that

given in Table 5, CP115, the proportion of shear to be resisted by the shear reinforcement provided should be assumed to vary linearly with the principal tensile stress from a value of 0 for the stress given in Table 5:CP115, to 1.0 for a stress of 1.5 times that given. When the principal tensile stress is greater than 1.5 times of that the whole of the shear should be carried by reinforcement.

4.2.6 Design for Shear at ultimate Load

In an uncracked section at ultimate conditions, the stirrups can be provided in the same way as at working load except that 1.37 at ultimate load Table 6:CP115 provides a higher allowable diagonal tensile stress and the yield stress of steel reinforcement is used instead of the allowable stress in the working load method.

Guyon^{50,51} has developed a formula based on the provision of stirrups to give the full shear resistance after the web has cracked. Extending this formula, based on the working load, to the ultimate load,

$$V_u = \frac{a_{sh} A_w t_y}{s \tan \psi} + P_1 \sin \theta \quad (4.16)$$

where ψ is the angle of inclination of the crack

$$= \frac{1}{2} \tan^{-1} \frac{2 f_{sh}}{f_h - f_v},$$

In the formula ψ should have a value twice this angle or 45° , whichever is greater.

When there is no vertical prestress and the cable is straight,

$$V_u = \frac{a_{sh} A_w t_y f_h}{2s_{sh} f_{sh}} \quad (4.17)$$

According to ACI Building Code,⁴⁹ the area of shear reinforcement placed perpendicular to the axis of a member shall be not less than

$$A_v = \frac{(V_u - \phi V_c) s}{\phi d f_y} \text{ nor less than } A_v = \frac{A_s}{80} \cdot \frac{f'_s}{f_y} \cdot \frac{s}{d} \sqrt{\frac{d}{b'}} \quad (4.18)$$

where V_c is the shear at diagonal cracking.

437

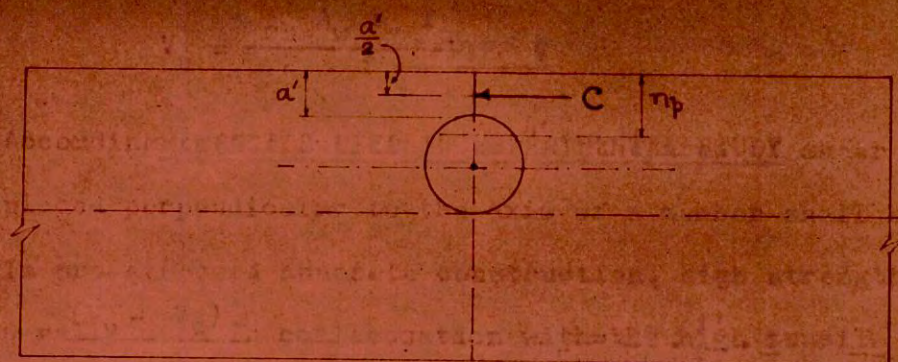


FIG. 4.1

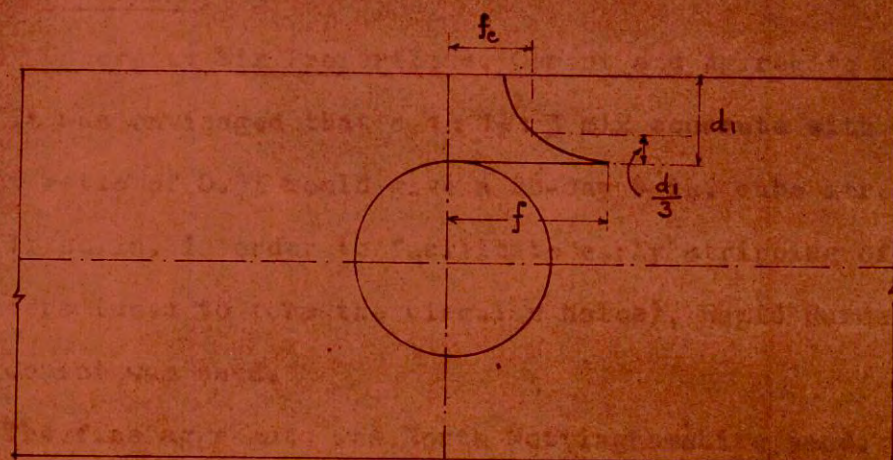


FIG. 4.2

CHAPTER V

MATERIALS USED IN EXPERIMENTAL STUDY

In pre-stressed concrete construction, high strength concrete is generally used in collaboration with the high tensile steel. In the present research, the concrete strength was not deliberately varied and the only type of high tensile steel used was 0.276 in. diameter plain stress relieved p.s.c. wire.

5.1 CONCRETE

5.1.1 Concrete: Mix Proportions, Cement and Aggregate

It was envisaged that a $1 : 1\frac{1}{2} : 3$ mix concrete with a water/cement ratio of 0.53 would give a 28-day 4 in. cube strength of 7000 lb/sq.in. In order to facilitate early stripping of the wooden cylinders (used to form the circular holes), Rapid Hardening Portland cement was used.

The fine aggregate was North Nottinghamshire sand. The coarse aggregate used was graded gravel of $\frac{3}{8}$ in. nominal size. From time to time, the aggregates were sieved and analysed according to Road Note No. 4.⁵² In Fig. 5.1, one such sample is shown.

5.1.2 Control Specimens

For quality control and prediction of the quantities such as modulus of elasticity, modulus of rupture, splitting stress and

strength of concrete in the test beams, control samples were cast and vibrated simultaneously with the beam, cured in the same condition as the beams and tested immediately before and after the beam test.

For each casting of two beams twelve representative 4 in. cubes, four cylinders (6 in. diameter and 12 in. long), four columns (4 in. x 4 in. x 20 in.) and sometimes one long column (3 in. x 3 in. x 36 in.) were cast.

Crushing test of cubes

At an age of 28 days, the cubes were brought out of the curing room along with the beam and dried for 24 hours. Next day, just before prestressing the beam, three cubes were tested to ensure that the strength of about 7000 lb/sq.in. had been attained. Three other cubes were crushed just after the test of the beam.

A 300 ton capacity Denison hydraulic compression testing machine was used to crush the cubes. This test was always performed according to BS 1881-1952.⁵³ According to the provision of the code, the average of three cube strengths was taken.

Column test for Young's modulus of elasticity

A static compression test on a 4 in. x 4 in. x 20 in. plain concrete prism was made to obtain the value of the modulus of elasticity of the concrete. This test was carried out according to the method specified in BS 1881.⁵³ A 50 ton Denison hydraulic machine was used to load the specimen and strain was recorded over

a gauge length of 18 in. The strain measuring device used is based on the principle of the Ewing's extensometer.⁵⁵ The dial gauge could read a minimum deformation of 0.0001 in. with a magnification factor of 2.

One sample of the stress-strain curve for 8000 lb/sq.in. concrete is shown in Fig. 5.2. The value of E in this case is 4.55×10^6 lb/sq.in. Fig. 5.3 shows a complete stress-strain curve of 8200 lb/sq.in. concrete. The 4 in. x 4 in. x 20 in. prism specimen used for E was tested to failure to give the complete curve. The strain is the average of four readings of resistance gauges fixed on four sides of the specimen.

If the concrete is allowed to creep at a particular level of constant stress, the subsequent stress-strain curve on loading may be assumed to follow the original curve. In Fig. 5.3, curve ABC'D' shows the complete stress-strain curve of concrete which has suffered a creep of 50 micro-strain at an assumed constant stress of 2800 lb/sq.in.

Beam test for modulus of rupture

As the column test for Young's modulus of elasticity is a non-destructive one, the same specimen was used for a beam test to give the tensile strength of plain concrete in flexure. This test was done according to BS 1881.⁵² The test consisted of third-point loading on a span of 16 in. The beam failed immediately on the appearance of a tensile crack at the bottom of the beam. The modulus

of rupture was calculated by means of the following formulae:⁵⁴

a) modulus of rupture = $\frac{PL}{b_1 d^2}$, if the fracture occurred within the central one-third of the beam,

b) modulus of rupture = $\frac{3 Pa}{b_1 d^2}$, if the fracture occurred outside the load-points; this is the maximum stress at the critical section.

If failure occurred at a section such that $\frac{L}{3} - a > 0.015 L$, the result was discarded, where

P = maximum load on the beam

d = depth of the beam

a = distance from the near support, measured along the tension surface of the beam.

Splitting test

A 6 in. diameter 12 in. long cylinder was placed with its axis horizontal between the platens of the 300 ton Denison hydraulic testing machine and the load applied until failure by splitting along the vertical diameter took place. As recommended by BS 1881-1952,⁵³ $\frac{1}{8}$ in. thick and $\frac{1}{2}$ in. broad plywood pieces were interposed between the cylinder and the platens of the machine.

The splitting stress was calculated as follows:

Splitting stress = $\frac{2P}{\pi LD}$ where P = load on the cylinder at failure

L = length of the cylinder

D = diameter

Test for Poisson's ratio

In this test, the longitudinal strain of the 3 in. x 3 in. x 36 in. plain concrete column was read on a gauge length of 30 in. The dial of the extensometer could be read upto a minimum 0.0001 in. with a magnification factor of 3. The lateral strain was read by means of mirror and scale arrangement.

If L = distance between the mirror and the illuminated scale,

l = distance between the parallel mirrors

s = scale reading

s' = actual scale reading (after correction to length)

d = diameter of the mirror rollers

b = width of the specimen

$$\text{Lateral deformation, } \delta x = \frac{s' \times d}{8L + 12l}$$

$$\text{Lateral strain} = \frac{\delta x}{b}$$

$$\text{Longitudinal strain} = \frac{\text{Dial reading}}{3 \times 30} \times 10^{-4};$$

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

5.2 STEEL

5.2.1 High tensile Steel Wires

0.276 in. diameter plain stress relieved wires were used for pre-stressing the beams. Tests were made in the laboratory for the strength and modulus of elasticity of the pre-stressing wires. Three samples were collected from three rolls and tested in tension

in a 50 ton capacity Denison hydraulically operated testing machine in order to explore the stress-strain curve till failure. The Ewing's extensometer⁵⁵ was fixed on punch marks on an 8 in. gauge length. One smallest division of the extensometer is 0.0002 in. considering the magnification factor of 2.

The stress-strain curve of such a wire is shown in Fig. 5.4. The value of Young's modulus as found from the curve is 29.6×10^6 lb/sq.in. and the 0.2% proof stress is 207,000 lb/sq.in. The wire failed by rupture at 6.49 tons equivalent to the ultimate nominal stress of 242,000 lb/sq.in. Out of three wires tested, two ruptured at 6.49 tons and the third at 6.50 tons.

Relaxation characteristics

The relaxation curve of 0.276 in. diameter p.s.c. wire as supplied by the manufacturers⁵⁶ is shown in Fig. 5.5. The curve representing the percentage loss in the initial stress in the wire with respect to the time elapsed shows that the wire has an instantaneous loss of 1% of the initial stress.

5.2.2 a) Stirrup reinforcement:

The stirrups used in the tests were of two types, $\frac{1}{4}$ in. diameter mild steel and $\frac{3}{8}$ in. diameter intermediate grade steel. The stress-strain curve of $\frac{1}{4}$ in. ϕ mild steel rod is shown in Fig. 5.6. Two specimens were tested and both failed at a load of 1.69 tons, i.e., at a nominal stress of 77,000 lb/sq.in., the yield stress being 55,000 lb/sq.in. and Young's modulus 30.6×10^6 lb/sq.in.

b) Steel liners:

Steel liners $\frac{1}{8}$ in. thick (of 4, 5 and 6 in. internal diameter and 6 in. long) were made by cutting and machining the internal surface of ordinary steel pipes (Plate 6.1).

c) Ring reinforcement:

Mild steel rods were bent in the form of rings and used as bonded reinforcement around the hole (Plate 6.1).

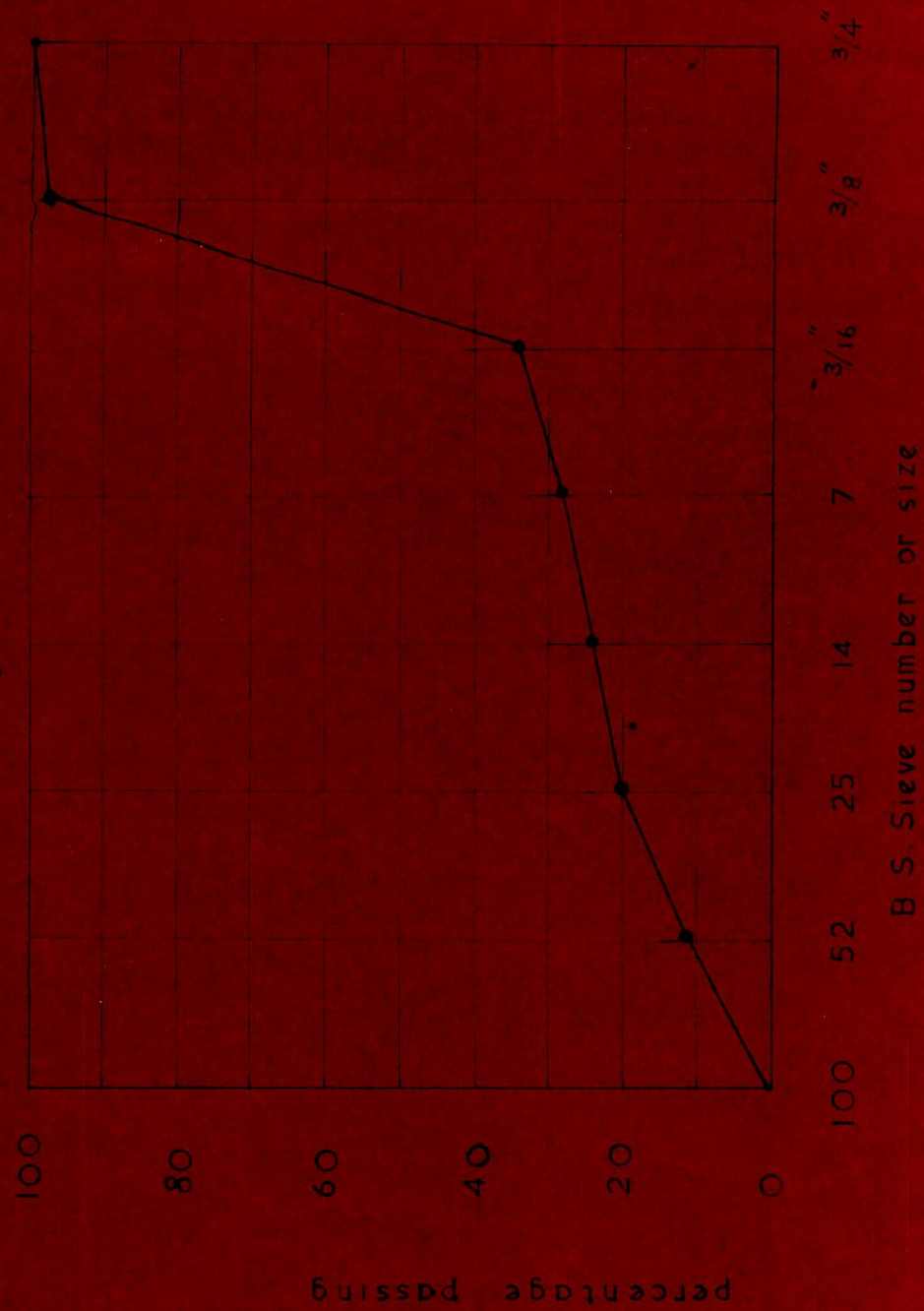


FIG. 5.1 Grading curve for 3/8" aggregate

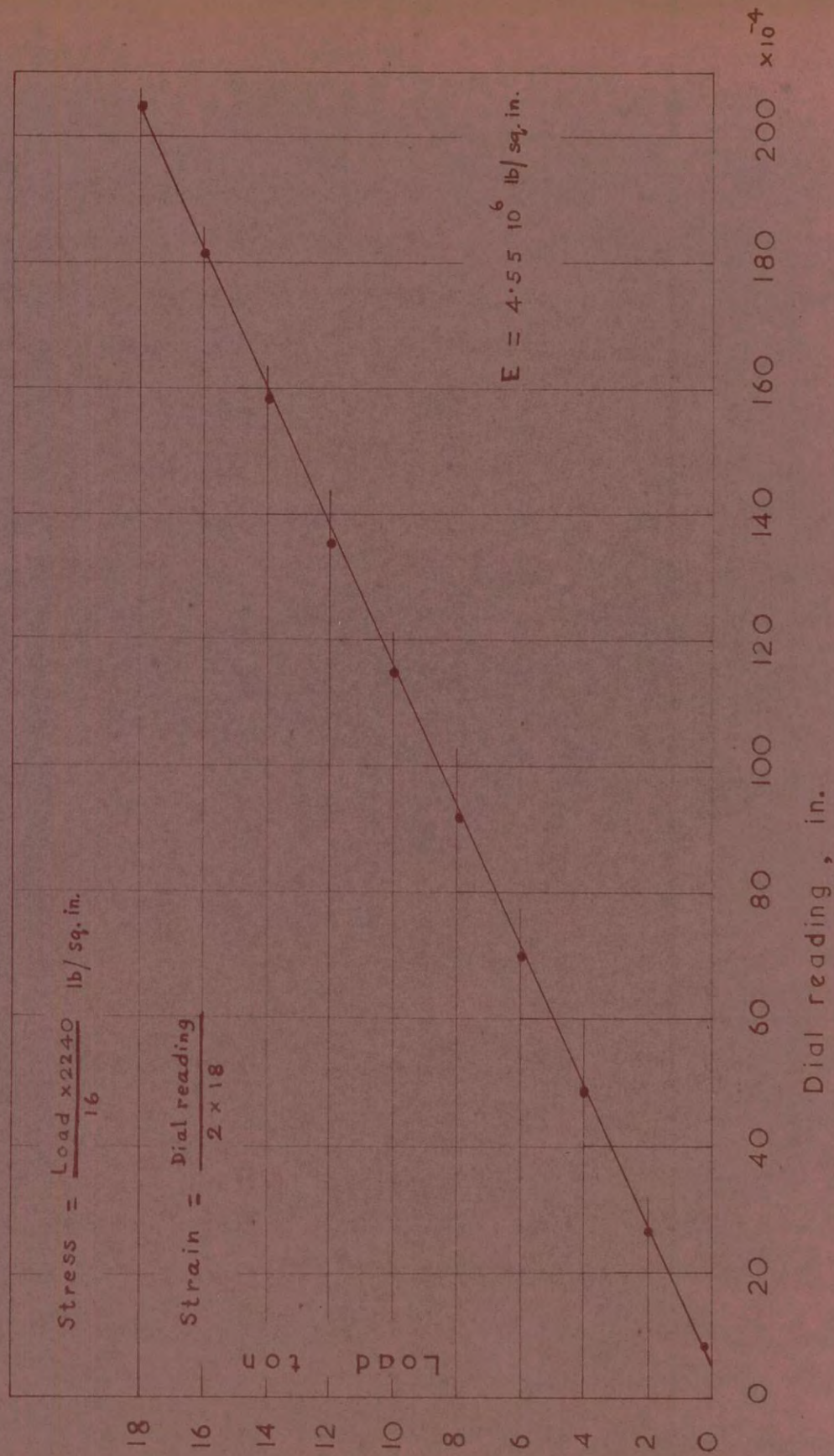


FIG. 5.2 Stress-strain curve of concrete .

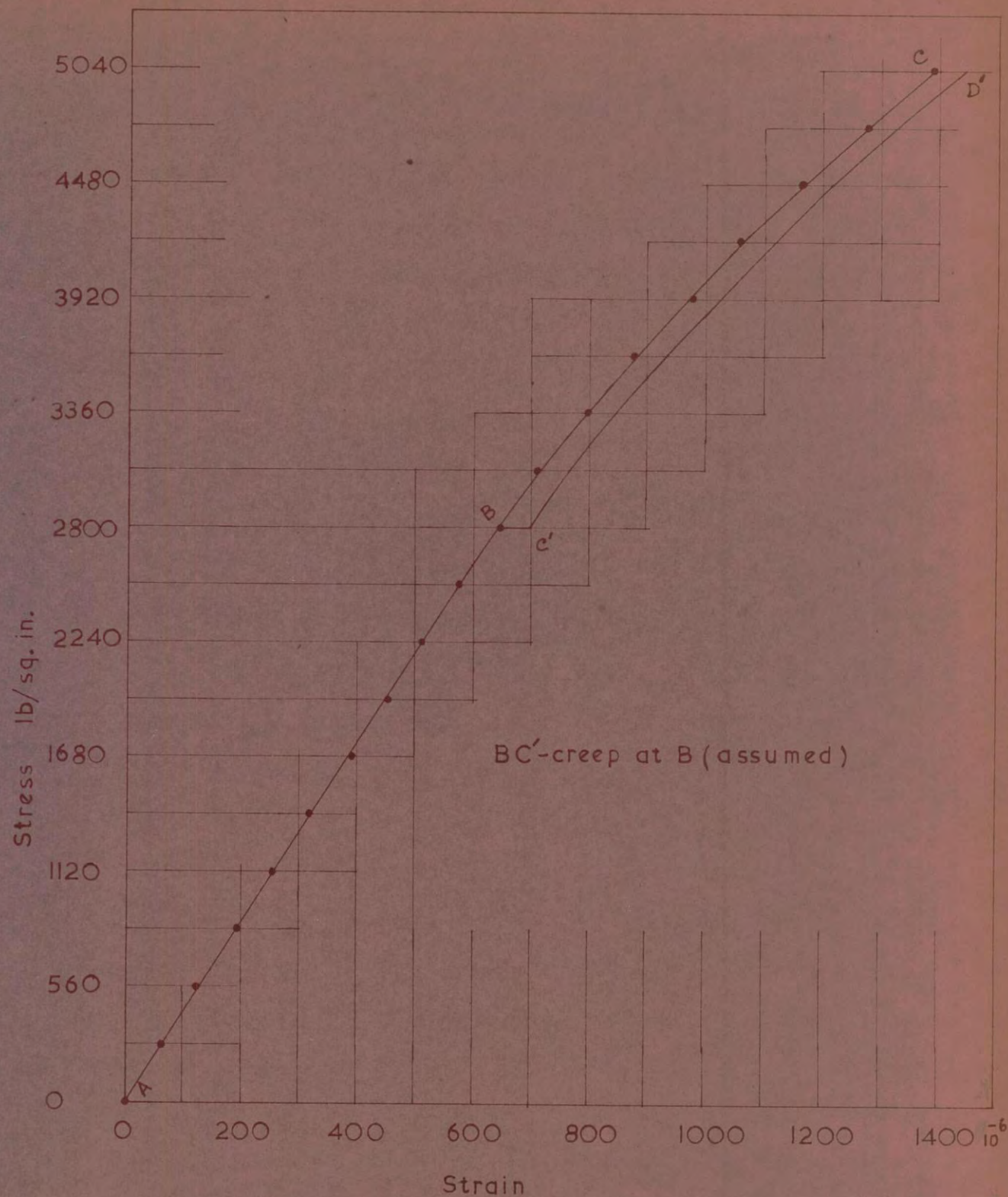


FIG. 5.3 ABC—complete stress-strain curve of concrete

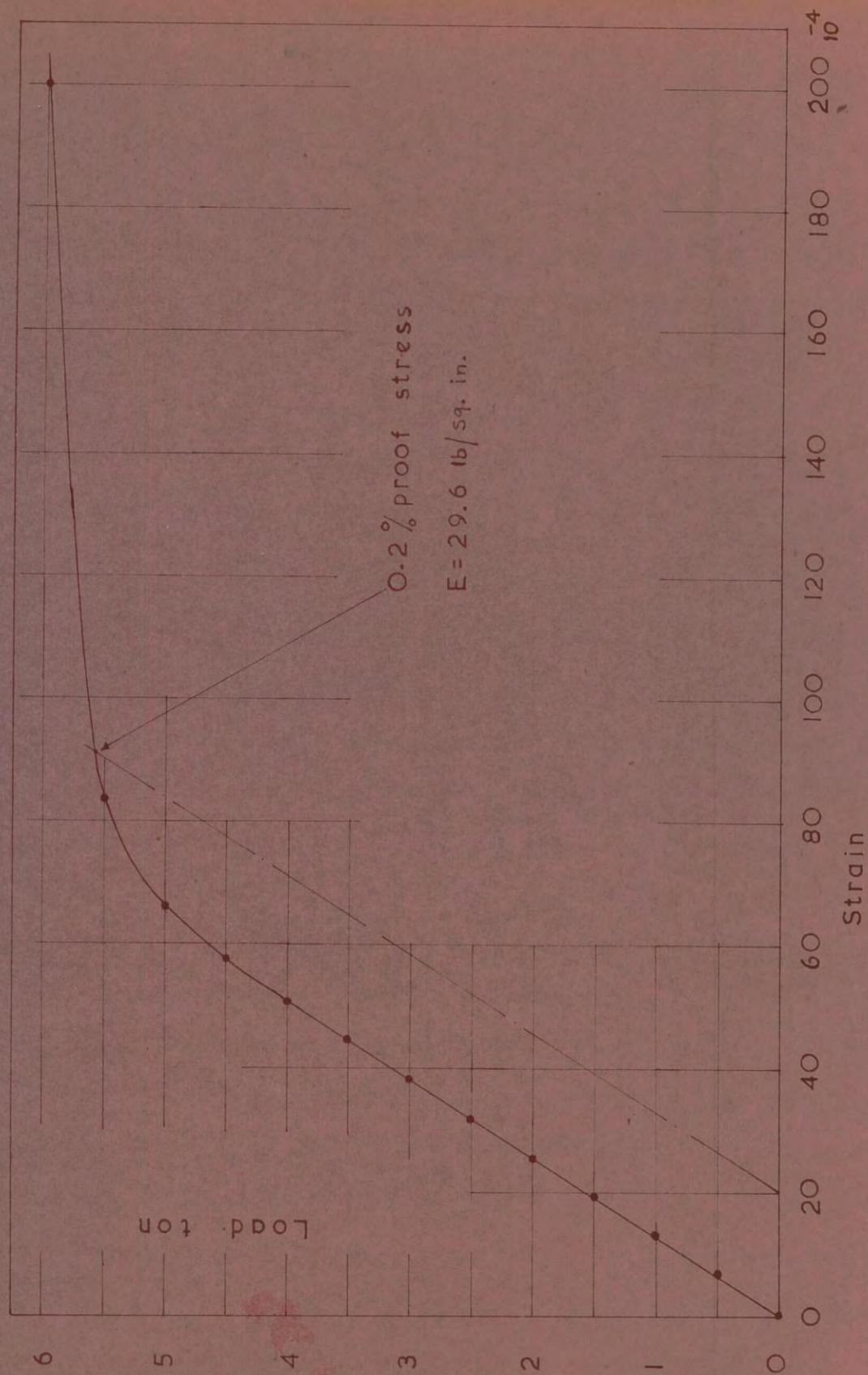


FIG. 5.4 Stress-strain curve of #276 dia. p.s.c. wire

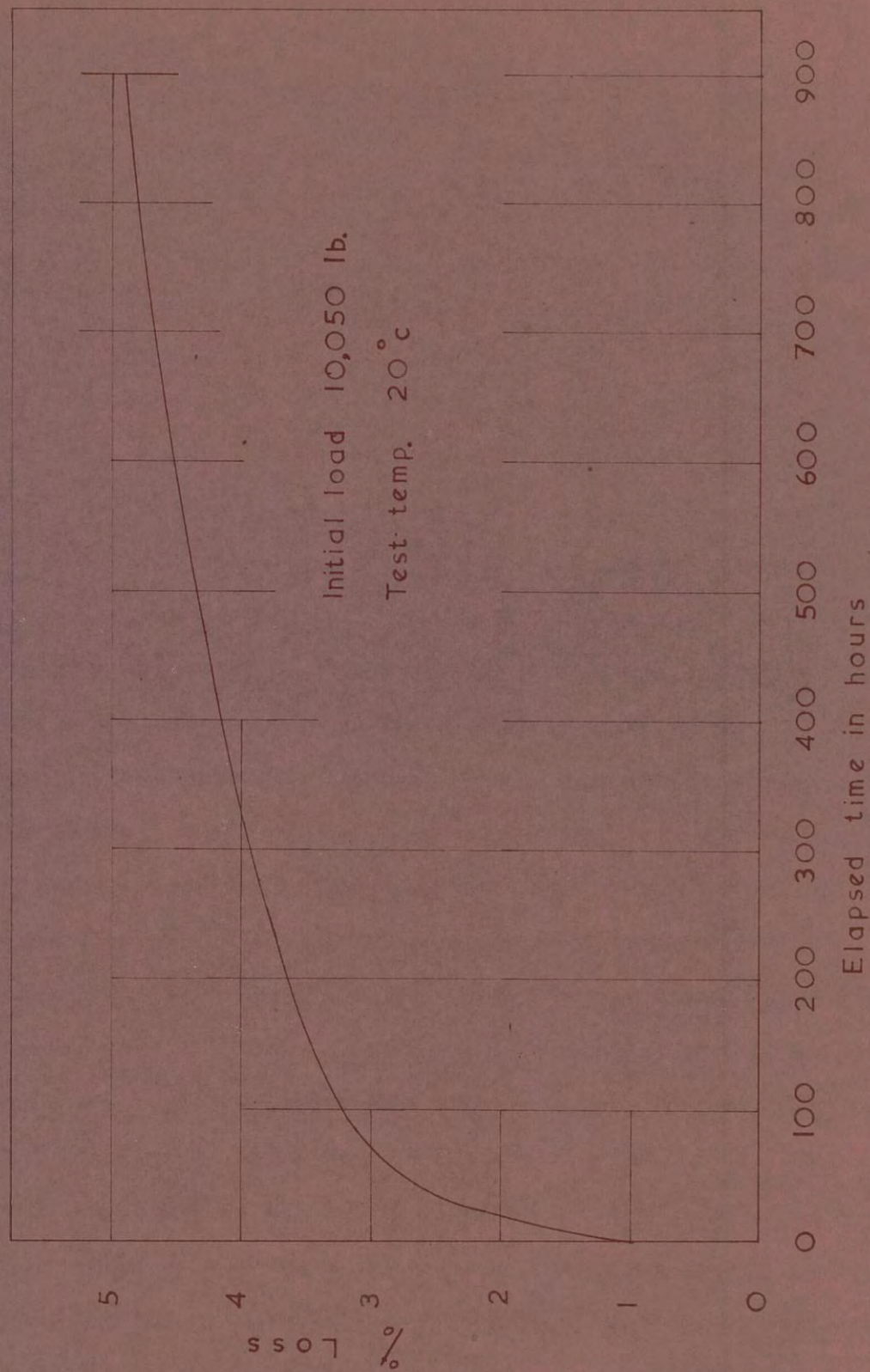


FIG. 5.5 Relaxation curve of .276 dia. p.s.c. wire

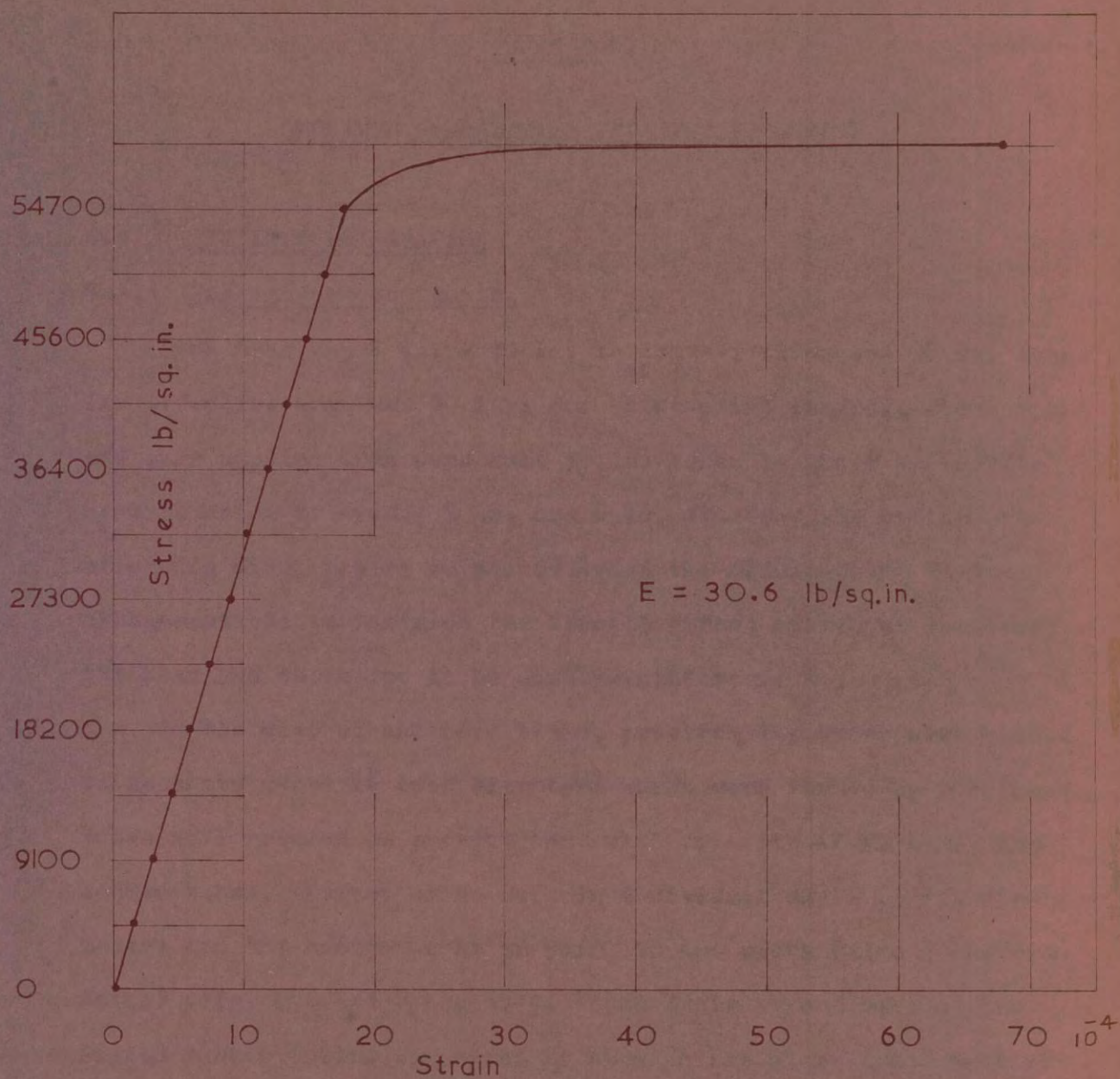


FIG. 5.6 Stress-strain curve of 1/4 in. dia. mild steel

CHAPTER VI

SPECIMEN PREPARATION AND TEST PROGRAMME

6.1 SPECIMEN PREPARATION

6.1.1 Preliminary to Casting

Each beam was 6 in. x 10 in. in cross-section and 96 in. long. The effective span was 90 in.; for third-point loading, shear spans and pure bending span were each 30 in. long. In order to provide central holes of 4 in., 5 in. and 6 in. diameter the bottom prestressing wires had to be placed below the middle-third of the cross-section; to restrict the tensile stress on top of the beam, the beam had therefore to be doubly-reinforced.

In the case of unbonded beams, prestressing wires were placed in separate ducts ($\frac{3}{8}$ inch diameter) which were formed by polythene tubes well greased to prevent bond with concrete (Fig. 6.1). For bonded beams, the top wires were in individual ducts ($\frac{3}{4}$ in. diameter) and the bottom wires in pairs in two ducts (also $\frac{3}{4}$ in. diameter) (Fig. 6.2 and Plate 6.1). These ducts were formed by inflated rubber tubing supported by steel rings along the length of the mould.

The mould consisted of a steel bed with a $1\frac{1}{2}$ in. thick timber base fixed on it; two 4 in. x 10 in. channels formed the sides of the mould which were bolted to the steel bed through the timber base. One of the side channels was kept permanently in position

while the other side channel was removed during positioning of hole formers and stripping.

The end plates for casting unbonded beams were provided with holes in the correct positions for the polythene tubes to pass through. The end plates were welded to a strong support so that the steel wires carrying the polythene tubes could be screwed tight against the end plates. In the case of bonded beams, the inflated rubber tubings passed straight along the length of the mould through small rings positioned at certain intervals and the end plates. The rubber tubes were inflated by compressed air at a pressure of 120 lb/sq.in.

6.1.2 Provision of Holes and Reinforcement

Solid wooden cylinders (4 in., 5 in. and 6 in. in diameter and 6 in. long) with central spindles were used as hole formers. When in position, the cylinders could be secured in position by screwing the spindles through the side channels. Before casting the beams the wooden cylinders were heavily greased and a light coat of mould oil was applied to facilitate stripping.

The vertical splitting reinforcement was in the form of four-legged stirrups (Fig. 6.3) which were made by welding two extra rods to the usual two-legged stirrups (Plate 6.1). The end block reinforcement consisted of four two-legged stirrups at each end. Plate 6.2 shows the arrangement of reinforcement ready for casting (one side channel removed).

The steel liners used for reinforcing the holes were $\frac{1}{8}$ in. thick pipes. The outside surface of the liners was left rough to give better bond with the concrete. Two $\frac{1}{2}$ in. x $\frac{1}{8}$ in. strips of steel were welded diametrically on two sides of the liner, and the spindle passed centrally through these strips.

The bonded reinforcement was in the form of multiple rings. Four rings were welded to two spacer rods at the desired spacing to form a circular cage (Plate 6.1).

6.1.3 Casting and Curing

Two batches of concrete were made in a Liner Cum Flow horizontal pan mixer of four cubic feet capacity. Concrete was placed in the mould for the beam and simultaneously in the moulds for control specimens. The control specimens were compacted on an Allam vibrating table. For beam compaction two Trillar clamp-on vibrators were used. Tamping rods were also used to help the concrete flow into the corners of the mould. When casting was complete, the beams were given a trowelled finish. After two to three hours, the moulds were covered with damp sacks.

The beams were left in the mould for a week when the sacks were hosed regularly twice a day. On the 7th day the beams were stripped and placed in the curing room along with the control specimens. The wooden cylinders were immediately removed by forcing with the help of the prestressing jack bearing against the case of beams having large holes, a small prestressing

force (about 1000 lb in each of four corner wires) was applied to the beam at the time of removal of the cylinders; the applied force was released immediately afterwards.

The curing room maintained a constant temperature of 64°F and relative humidity of 95 - 100%.

6.1.4 Pre-stressing

One of the two beams cast together was brought out of the curing room at an age of 28 days and the other beam at 35 days. They were left to dry for 24 hours. For prestressing the unbonded beams 6 in. x 10 in. x 1 in. thick end plates having holes in the correct positions for the prestressing wires to pass through were used. In the case of bonded beams $1\frac{1}{2}$ in. thick plates were used (Fig. 6.4). The ducts terminated in rectangular grooves near the ends of the beam to facilitate grouting. The ends of the beam were smoothed using a piece of carborundum. The end plates distribute the concentrated prestressing force and reduce any tensile stresses in the end block which might be caused by high spots.

In the C.C.L. system the monowire jack can draw one wire at a time. When the wire was drawn to the required load as recorded on the dial of the jack, wedges were driven home to bear against the end plates. When all the wires were drawn in the specified order, the whole jacking operation was repeated thus compensating for the effect of elastic shortening, a proportion of relaxation of the steel and a small amount of anchorage loss. The uniformity of

stress on two sides of the beam was checked and corrected by strain measurement on 8 in. demec gauges fixed on top and bottom of both sides.

6.1.5 Grouting

The bonded beams were grouted with a neat cement-water mix of 2 to 1 proportion. The grouting machine used compressed air to build up a pressure of 50 lb/sq.in. which is sufficient to force the grout through the ducts. Immediately after grouting, all the openings were closed by plaster. Two hours after grouting the beam was placed back in the curing room.

6.1.6 Fixing of mechanical Gauges

When the beam was dry, reaction, load and dial gauge positions were marked. The inside surfaces of all holes were wiped clean of all grease using paraffin and a rotating carbor^uandum disc used to remove all surface irregularities from the inside surfaces. Finally, the surface was cleaned with ethylene.

Stainless steel discs, for the mechanical strain gauges, were fixed to the concrete by a paste made of dental cement and Amco F88 solution. After half an hour to allow the paste to set Demec mechanical strain gauges could be used to measure strains. Rectangular rosettes were used to give principal stresses; strains were measured in three directions on 2 in. Demec gauge lengths.

6.1.7 Fixing of Resistance Gauges

According to the instructions supplied by the manufacturers, resistance strain gauges (PL-3 and PL-30 from Tokyosokki Kenkuyojo Co. Ltd.) could be fixed on a smooth concrete surface directly by the mixture of PS adhesive Drug A and Drug B. When the concrete surface is not completely dry, a thin water-proofing coat of the mixture of PS adhesive Drug A and Drug B is first applied, on this surface the gauge can be fixed by the admixture of P2 adhesive Drug A and Drug B. Half an hour should elapse before the gauges are used to take strain readings.

A similar strain gauge was fixed on a similar concrete surface, say a concrete cube, to act as dummy gauge.

Fig. 6.5 shows a typical arrangement of PL-30 gauges (gauge length 30 mm.) at the periphery of a circular hole.

6.1.8 Reading Strain from Resistance Gauges

The two ends of the strain gauges were soldered to connecting wires, the other ends of which were connected to an extension box; the dummy was connected to the terminals provided for them. The Peekel, the strain measuring instrument, was then connected to the extension box. The gauge factor as marked on the gauge container by the manufacturers was set on the Peekel. When the exact gauge factor could not be set, the nearest factor was used and the readings corrected afterwards. The correction is as follows:

$$\text{Correct strain} = \text{observed strain} \times \frac{\text{G.F. set}}{\text{G.F. of the gauge}}$$

Strain can be read directly on the Peekel at null position of the pointer. The smallest division in the Peekel is 10 micro-strain, but smaller strains can be estimated.

6.1.9 Accuracy of PL-30 Gauges

In order to test the accuracy of PL-30 resistance gauges, a control test was performed. A 4 in. x 4 in. x 20 in. plain concrete column was fitted with a mechanical extensometer on a gauge length of 18 in; the dial gauge used in this extensometer could record a minimum deformation of 0.0001 in. Four PL-30 gauges were fixed on the four sides of the column. Load was applied by a 50 ton Denison hydraulic testing machine. Readings were taken on all the gauges at 2 ton increments of load.

The difference between the maximum and minimum strain readings as recorded by the four resistance gauges was about 15%, but the average of the four readings was close (within 5% accuracy) to the strain calculated from the extensometer reading. Fig. 6.6 shows two load versus strain curves; one curve shows strains given by the average of four PL-30 gauges and the other as given by the extensometer. It is clear that the PL-30 gauges record strains very close to those given by the accurate extensometer over a larger gauge length.

6.1.10 Strain in the prestressing Wires

A special end plate was fabricated to facilitate measurement

of the strain in the prestressing wires. Two 6 in. x 6 in. x 1 in. vertical plates at a clear distance of 3 in. were welded to two cross pieces of 6 in. x 3 in. x 1 in. and two 4 in. x 3 in. x 1 in. steel plates (Plates 6.1 and 6.3). Both the vertical plates had holes in correct positions for the prestressing steel to pass through. As the tendons were wedged against the outer plate, loaded tendons were exposed to an extent of 3 in.

Resistance gauges (PL-3) of 3 mm. gauge length were directly fixed to the tendons on diametrically opposite sides of each tendon. By this means it was possible to determine the tendon stresses at any loading (prestress or transverse load). The total loss of prestress and the ultimate strain in the tension and compression tendons could also be readily obtained.

6.2 THE TEST

6.2.1 The Testing Machine

The machine used for testing the beams is of the screw type, having 50 tons capacity. The machine can be used for three different capacity scales, i.e., 2, 10 and 50 tons. For testing prestress type I beams which failed at more than 10 tons load, the 50 tons scale was used; in prestress type II beams where the ultimate load was less than 10 tons, the 10 tons scale was used. The minimum graduation on the 50 tons scale is 0.1 ton and that on the 10 tons scale 0.02 ton. Plate 6.4 shows the machine with a beam in position.

6.2.2 Setting of the Beam

In the case of unbonded beams, the first of the two beams in each casting was tested on the 29th day and the second beam on the 36th day after casting. In the case of bonded beams, the first beam was tested on the 36th day and the second on the 43rd day after casting including ~~seven~~^{seven} days of curing after grouting.

The beam was placed on two non-yielding supports resting on the bed of the machine. 6 in. x $1\frac{1}{2}$ in. x $\frac{1}{2}$ in. steel plates were plastered to the top of the beam at the load points and to the underside of the beam at the reaction points. 1 in. diameter and 6 in. long steel rollers were placed between the support and the steel plate (previously plastered) to carry the beam. Similar rollers were placed on the plates on top of the beam to carry the spreader beam which was a 3 ft. long 4 in. x 6 in. I section. Centrally on the spreader beam was placed a square steel piece (6 in. x 6 in. x 1 in.) with a central hemispherical seat to house the ball bearing interposed between this plate and a similar plate directly in contact with the loading head of the machine.

The beam was placed central with respect to the length as well as the breadth. It was checked each time that the loading head was just at the middle of the spreader beam. Before loading the beam, zero adjustment of the machine was carried out.

Deflection of the beam was measured at five points at distances of 15, 30, 45, 60 and 75 in. from one reaction. 2 in. travel

Baty magnetic dial gauges (minimum graduation 0.001 in.) were placed in position keeping the dial axis vertical. Small squares of flat aluminium $\frac{1}{16}$ in. thick were fixed to the underside of the beam for the spindles of the dial gauges to bear against.

A thin coat of white plaster was applied on both sides of the beam for easy detection of the first appearance and propagation of cracks. An ordinary magnifying glass was used for the detection of cracks.

6.2.3 Beam Testing

When everything was ready for the commencement of the test, all initial readings for demec, electrical and dial gauges at zero load were taken. Load was applied in $\frac{1}{2}$ or 1 ton increments and all the readings were repeated. The initiation and propagation of cracks ^{were} ~~was~~ marked and entered in the laboratory sheet. Just prior to failure the dial gauges were replaced by a central scale having $\frac{1}{20}$ in. divisions carried by a retort stand for further record of the central deflection. The load and mode of failure were noted and the beam photographed after the test.

6.3 TEST PROGRAMME

6.3.1 Purpose of Test

The purpose behind the test programme is:

- 1) to explore the pattern of stress concentration and distribution around a circular hole in a rectangular beam due to prestress

and loading, the hole being placed in

- a) the pure bending zone of the beam having the centre of the hole
 - (i) at the horizontal centroidal axis of the beam
 - (ii) above or below the centroidal axis in such a way that some holes will touch or cut the depth of the ultimate compression block calculated according to Whitney as in the ACI Building Code,
 - b) the shear span, where there is shear force and varying bending moment, the centre of the hole being
 - (i) at the horizontal centroidal axis of the beam
 - (ii) above the centroidal axis
- 2) to find the effect of the perforations on the deflection, cracking and ultimate load of the beam,
- 3) to explore the effect of additional reinforcement around the holes
- a) in the form of bonded steel liners
 - b) in the form of bonded ring reinforcement
 - c) in the form of straight stirrups at the sides of the hole, and study the effect on the deflection, cracking and ultimate load of the beam.

6.3.2 Constant Quantities in the Test

The following quantities were kept constant throughout the test:

- a) the size of the beam (rectangular 6 in. x 10 in.),
- b) the type of loading and beam span (third-point loading on 90 in. span) except in the case of three beams (central point loading on 48 in. span),
- c) the position of the main reinforcement with respect to the depth of the beam,
- d) concrete strength (4 in. cube strength of 7000 lb/sq.in. at 28 days). A degree of variation in concrete strength did, however, occur,
- e) hole type (circular hole).

6.3.3 Parameters of the Test

The following are the parameters treated in the programme:

1. diameter of the hole (4, 5 and 6 in., i.e., 40%, 50% and 60% of the beam depth),
2. stress field in which hole is situated (in a pure bending zone or in a combined bending and shear zone),
3. position of the hole (on, above or below the horizontal centroidal axis),
4. level of prestress,
5. effect of bond (bonded or unbonded tendons)
6. different methods of reinforcing the holes (bonded steel liners, bonded ring reinforcement around the hole and vertical stirrups).

6.3.4 Prestress and Loading

Prestress type I: each of the two top wires had a force of 7000 lb and each of the four bottom wires had 9000 lb. After deducting all losses, the stress distribution at a solid section was as shown in Fig. 6.7.

Prestress type II: each of two top wires had a force of 4000 and each of two bottom wires had 8000 lb gross. After deducting all losses, the stress distribution at a solid section was as shown in Fig. 6.7.

Third point loading on an effective span of 90 in. was used, i.e., two shear spans of 30 in. each and central bending span of 30 in. (Fig. 6.7).

6.3.5 Details of Beams tested

In order to explore the effect of changes in the parameters, it was decided to test seven series of beams. Each series explored certain aspects of the problem.

For the nomenclature of the beams, initial B indicates one hole in pure bending,

O ₁	:	circular hole of 4 in. in diameter
O ₂	:	circular hole of 5 in. in diameter
O ₃	:	circular hole of 6 in. in diameter
BS	:	a solid beam without any hole.
U	:	the hole is above the horizontal centroidal axis.

D indicates the hole is below the horizontal centroidal axis.

S : the hole is in the shear span

M : the beam contains eight holes

MM : the beam contains nine holes.

Table 6.1 gives a list of the beams tested in the programme.

6.3.6 Purpose of Individual Beam Test

Beams of Series I:

The first beam (BS-1) without any hole (Fig. 6.7) was tested for comparison purposes.

The stress distribution at the boundary of the 4 in. dia. hole in Beam BO_1-1 due to the prestressing force and loading could be obtained. As this central hole did not cut the depth of the ultimate neutral axis, it was expected that the beam would not suffer a reduction in strength. Moreover, the presence of only one hole would not appreciably increase the deflection of the beam.

Beam BO_2-2 was tested to observe whether the initial crack that could develop due to prestressing (owing to the tensile stress concentration at the ends of the horizontal diameter of the hole) would affect the stress distribution at the hole boundary and the behaviour of the beam.

Beam BO_1-U-3 had the hole top 1 in. below the top of the beam (Fig. 6.7) which ensured that the ultimate compression depth was cut into to an extent of 0.66 in. (see A.7.1).

Beams of Series II: (Fig. 6.7)

Beam BS-2 was tested for comparison with other beams containing holes in this series.

For Beam BO_3-4 , the prestressing force in this series being low, it was anticipated that no splitting crack would develop.

Beam BO_2-U-5 was tested to find the effect of a hole intruding into the depth of the ultimate neutral axis calculated according to CP115: 1959.

Beam BO_1-U-6 contained a 4 in. diameter hole near its edge. In this case, the hole went well within the depth of ultimate neutral axis and just touched the ultimate compression depth (0.99 in.), calculated according to ACI Building Code. The experimental stress distribution at the hole boundary could be compared with the theoretical values obtained by the mathematical solution which considers the influence of the hole near the beam boundary.

The stress distribution at a hole occupying 50% of the beam depth could be obtained from the test of Beam BO_2-7 . In the case of Beam BO_2-2 , also having dia./depth ratio of 0.5, the splitting crack due to higher prestress might have influenced the stress distribution.

Beam BO_1-D-8 had the hole completely below the centroidal axis; the stress distribution at a hole situated in the tension zone could thus be obtained. In so far as strength is concerned, the hole should have no effect.

Beam BO₃-U-9 has a 6 in. diameter hole just below the ultimate depth of compression, according to ACI Building Code. Though this was a large hole, the ultimate strength of the beam should not be affected.

Beams of Series III: (Fig. 6.7)

These beams were tested to explore the stress trajectories at the hole boundary when the holes were situated in the middle of the shear span with a moment-shear ratio of 15 in. All the beams in this series were unreinforced, the beams could, therefore, fail in shear due to the diagonal stress concentration.

In order to obtain distribution of stresses in the vicinity of the hole, rectangular strain rosettes were used all around the hole in Beam SO₂-U-4. From the strain data, principal stresses at all these points could be calculated.

Beams of Series IV: (Fig. 6.7)

This series, consisting of only three beams, was devised to study the stress distribution at a circular hole when the hole was near the reaction (bending moment/shearing force is equal to 8 in.) where the shearing force predominates. The beams of this series were tested under central point loading.

Beams of Series V: (Fig. 6.8)

This series comprised two bonded beams where the holes (5 in. and 6 in. in diameter) were placed centrally in the shear span. This series was devised to study the difference in behaviour

between bonded and unbonded beams. The stress trajectories of Beams SO_2 -B-1 and SO_3 -B-2 could be compared with those of unbonded Beams SO_2 -2 and SO_3 -1 of Series III.

Beams of Series VI:

After having experimental information on the behaviour of beams containing a single hole in bending or in bending with shear, this series consisting of eleven beams was devised to give information on the behaviour of beams having an array of circular holes. Eight beams in this series were reinforced against splitting and diagonal tensile stresses. Three beams of this series had reinforcement in the form of bonded steel liners and bonded circular reinforcement around the hole. The remaining five beams were provided with vertical four-legged stirrups on both sides of each hole. This series was used to study the effect of different types of hole reinforcement on the cracking, ultimate load and deflection of the beam.

Beam BS-3 had resistance gauges fixed to the top and bottom prestressing wires and was used for comparison purposes.

Beam MO_1 -1 (Fig. 6.8) was tested to find the effect of a number of unreinforced holes in the shear spans.

Beams MO_2 -2 and MO_2 -7 (Fig. 6.8) were reinforced against splitting and diagonal tensile stresses. The measured deflection could be compared with that calculated by the various suggested methods.

Beams MO_3-3 and MO_3-4 had reinforcement against splitting and diagonal tensile failure (Fig. 6.8). Holes in the shear spans were not provided with stirrups on top and bottom. If the cracked bottom flange were assumed not to carry any shear, the principal tensile stress on top of the hole at ultimate condition would be more than the ultimate allowable value.

In Beam MO_3-L-5 each of the eight holes were provided with $\frac{1}{8}$ in. thick steel liners bonded to the concrete (Fig. 6.8). The effect of the hole liners on the cracking and ultimate load and on the stiffness of the beam could be obtained from this test.

In Beam MO_2-R-6 hole reinforcement was provided in the form of a multiple ring bonded to the concrete around the hole with a cover of $\frac{1}{4}$ in. (Fig. 6.9). The cross-sectional area of this reinforcement was the same as that of the vertical stirrup in Beam MO_2-2 . The difference in behaviour between this beam and Beam MO_2-2 , if any, could then be attributed to the shape of the reinforcement.

For Beam MO_3-R-7 the bonded reinforcement around each hole consisted of four $\frac{1}{2}$ in. diameter steel rings. The cross-sectional area of this reinforcement was the same as that of the liner provided in Beam MO_3-L-5 (Fig. 6.9).

Beam MO_1-U-8 contained unreinforced holes. The behaviour of this beam could be compared with that of Beam MO_1-1 which had central holes.

Beam MMO_2-9 had nine holes arranged in such a way that there

were three holes in each of two shear spans and the bending span (Fig. 6.9). This beam could be compared with Beam MO₂-2 which had eight holes with two holes directly under the load points.

Beams of Series VII:

These beams were tested to explore the effect of bond on the behaviour of beams containing circular holes. It was noticed that Beams MO₃-3, MO₃-4, MMO₃-B-5 and MMO₃-B-6 (Fig. 6.9) could not attain the full flexural strength, because the beams failed at top of the hole in the shear span nearest to the load point. The principal tensile stress on top of the holes, already having flexural cracks at the bottom, was high enough to cause shear failure. In Beam MMO₃-B-7, short stirrups were provided on top and bottom of each hole in the shear span (Fig. 6.9). Moreover, the vertical $\frac{1}{4}$ in. diameter four-legged stirrups provided on the sides of each hole were replaced by $\frac{3}{8}$ in. diameter four-legged stirrups.

Table 6.1 Description of beams tested

Series I: (Unbonded beams) - Prestress type I:		
1.	BS-1	a beam without hole
2.	BO ₁ -1	one 4" diameter hole centrally placed in pure bending
3.	BO ₂ -2	one 5" diameter hole centrally placed in pure bending
4.	BO ₁ -U-3	one 4" diameter hole, the top of which is 1" below the top of the beam
Series II: (Unbonded beams) - Prestress type II:		
5.	BS-2	beam without hole
6.	BO ₃ -4	a 6" diameter hole placed centrally in the bending zone
7.	BO ₂ -U-5	a 5" diameter hole, the top of which was 1½" from the top of the beam
8.	BO ₁ -U-6	a 4" diameter hole, the top was 1" from the top of the beam
9.	BO ₂ -7	a 5" diameter hole placed at the centre of the beam
10.	BO ₁ -D-8	a 4" diameter hole, the bottom of which was 1" from the bottom of the beam
11.	BO ₃ -U-9	a 6" diameter hole, the top being 1" from the top of the beam
Series III: (Unbonded beams) - Prestress type III:		
12.	SO ₃ -1	a 6" diameter hole placed centrally in one shear span
13.	SO ₂ -2	a 5" diameter hole placed centrally in one shear span
14.	SO ₁ -3	a 4" diameter hole placed centrally in one shear span
15.	SO ₂ -U-4	a 5" diameter hole, placed at the middle of one shear span, the top of the hole being 1" from the top of the beam

Series IV: (Unbonded beams) - Prestress type II: central point loading:

16.	SO-1	a 4" diameter hole placed at the centroidal axis in the shear span at a distance of 8" from the near reaction
17.	SO-2	a 5" diameter hole placed as in Beam SO-1
18.	SO-3	a 6" diameter hole placed as in Beam SO-1

Series V: (Bonded beam) - Prestress type II:

19.	SO ₂ -B-1	a 5" diameter hole placed on the centroidal axis of a shear span
20.	SO ₃ -B-2	a 6" diameter hole placed as in Beam SO ₂ -B-1

Series VI: (Unbonded beams) - Prestress type I:

21.	BS-3	a beam without hole
22.	MO ₁ -1	8 unreinforced holes of 4" diameter placed on the centroidal axis
23.	MO ₂ -2	8 holes of 5" diameter (with vertical stirrups) placed on the centroidal axis
24.	MO ₂ -7	
25.	MO ₃ -3	8 holes of 6" diameter (with vertical stirrups) placed on the centroidal axis
26.	MO ₃ -4	
27.	MO ₃ -L-5	8 holes of 6" diameter (with steel liners) placed on the centroidal axis
28.	MO ₂ -R-6	8 holes of 5" diameter (with circular reinf.) on the centroidal axis
29.	MO ₃ -R-7	8 holes of 6" diameter (with circular reinf.) on the centroidal axis
30.	MO ₁ -U-8	8 unreinforced holes of 4" diameter, the tops being 1" from the top of the beam
31.	MMO ₂ -9	9 holes of 5" diameter (with vertical stirrups) on the centroidal axis

Series VII: (Bonded beams) - Prestress type I:

32.	BS-B-1	a beam without a hole
33.	MMO ₁ -U-B-2	9 holes of 4" diameter (with vertical stirrups), the tops being 1 $\frac{1}{2}$ " below the top of the beam
34.	MMO ₂ -B-4	9 holes of 5" diameter (with vertical stirrups) on the centroidal axis
35.	MMO ₃ -B-5	9 holes of 6" diameter (with vertical stirrups) on the centroidal axis
36.	MMO ₃ -B-6	
37.	MMO ₃ -B-7	9 holes of 6" diameter (with stirrups at sides and on ^{top} and bottom of the holes) on the centroidal axis.

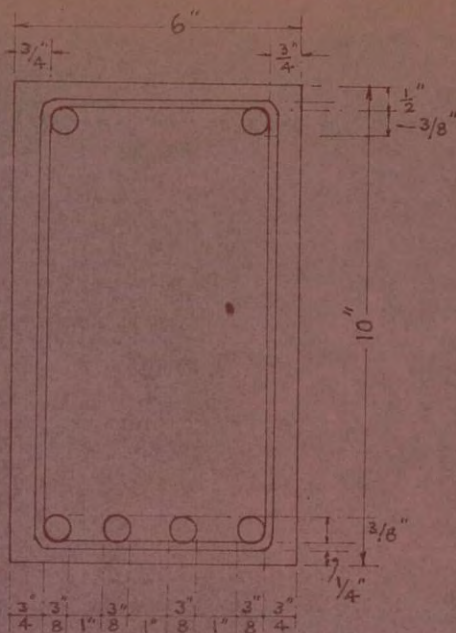


FIG. 6.1 Unbonded beam section

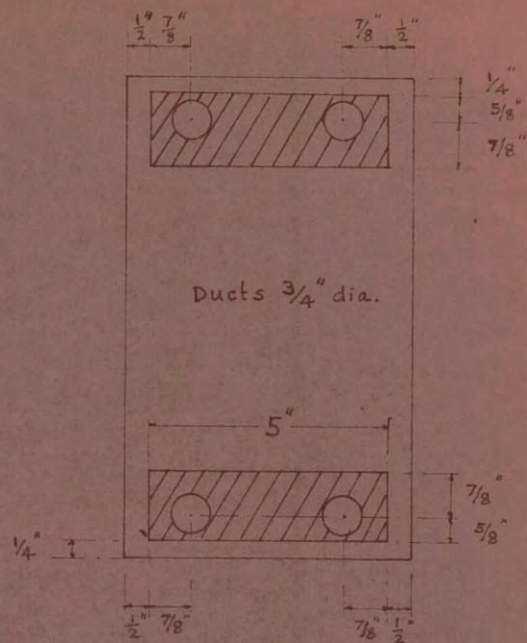


FIG. 6.2 End plate for casting bonded beam

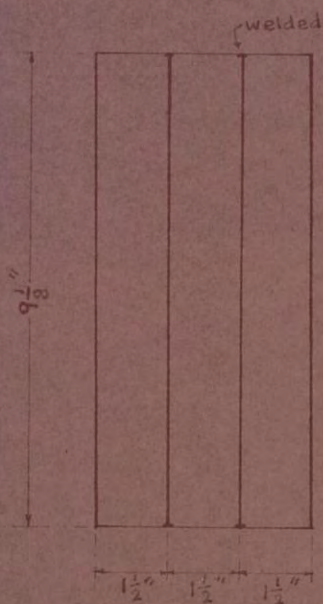


FIG. 6.3 Four-legged stirrup

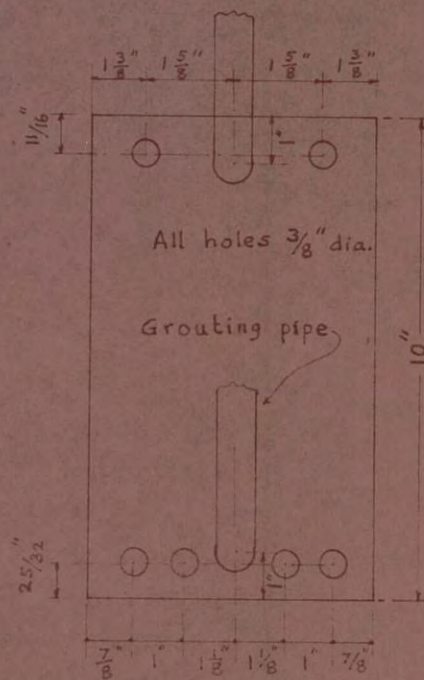


FIG. 6.4 End plate for stressing bonded beams

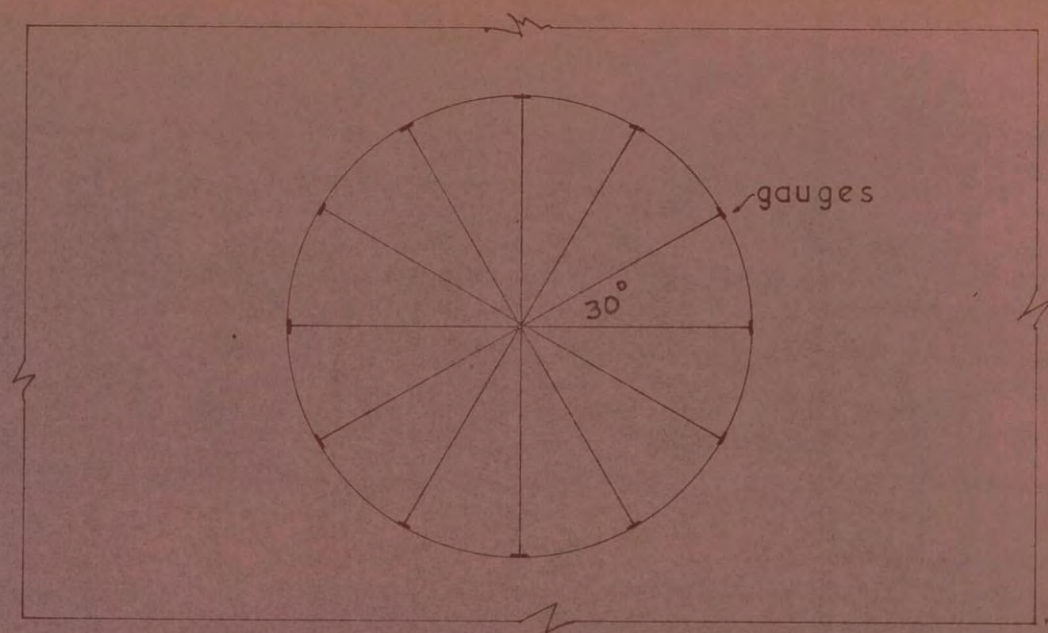


FIG. 6.5 Positions of PL-30 gauges at the boundary of a circular hole

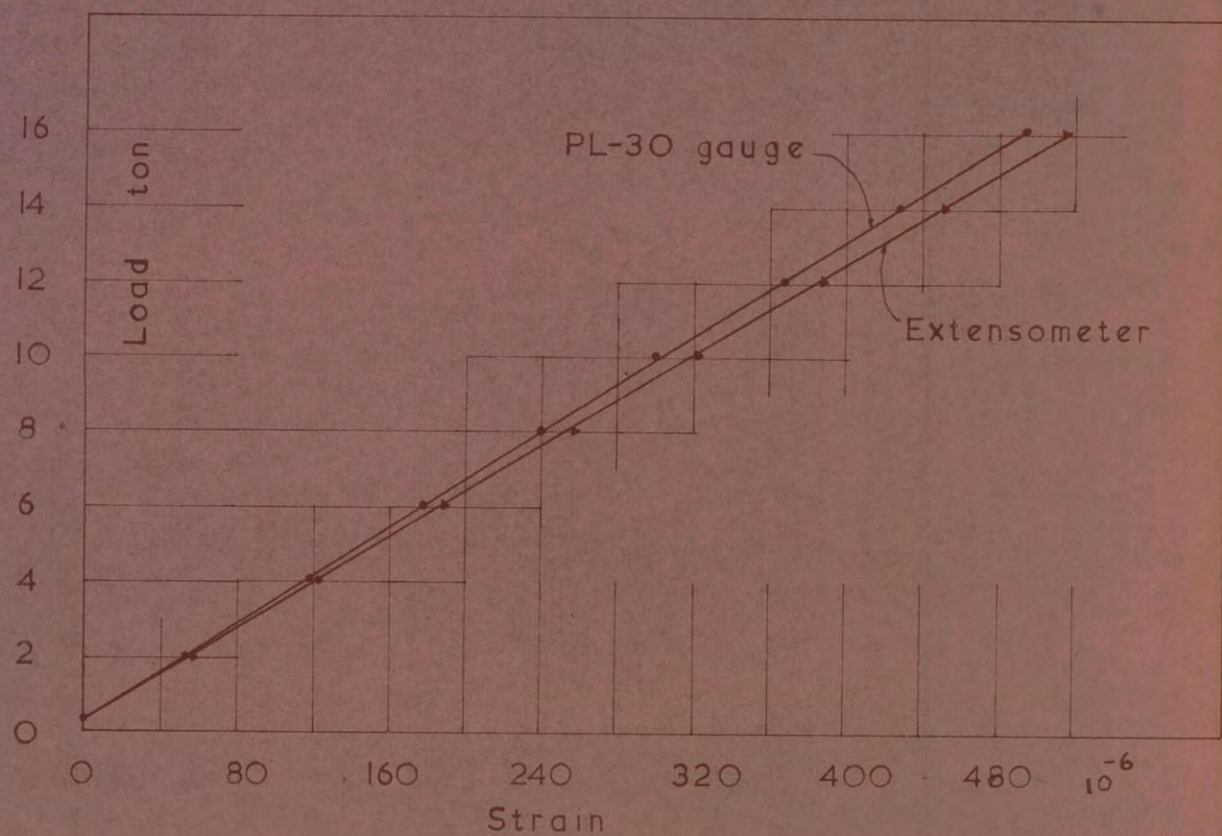


FIG. 6.6 Accuracy of PL-30 gauges

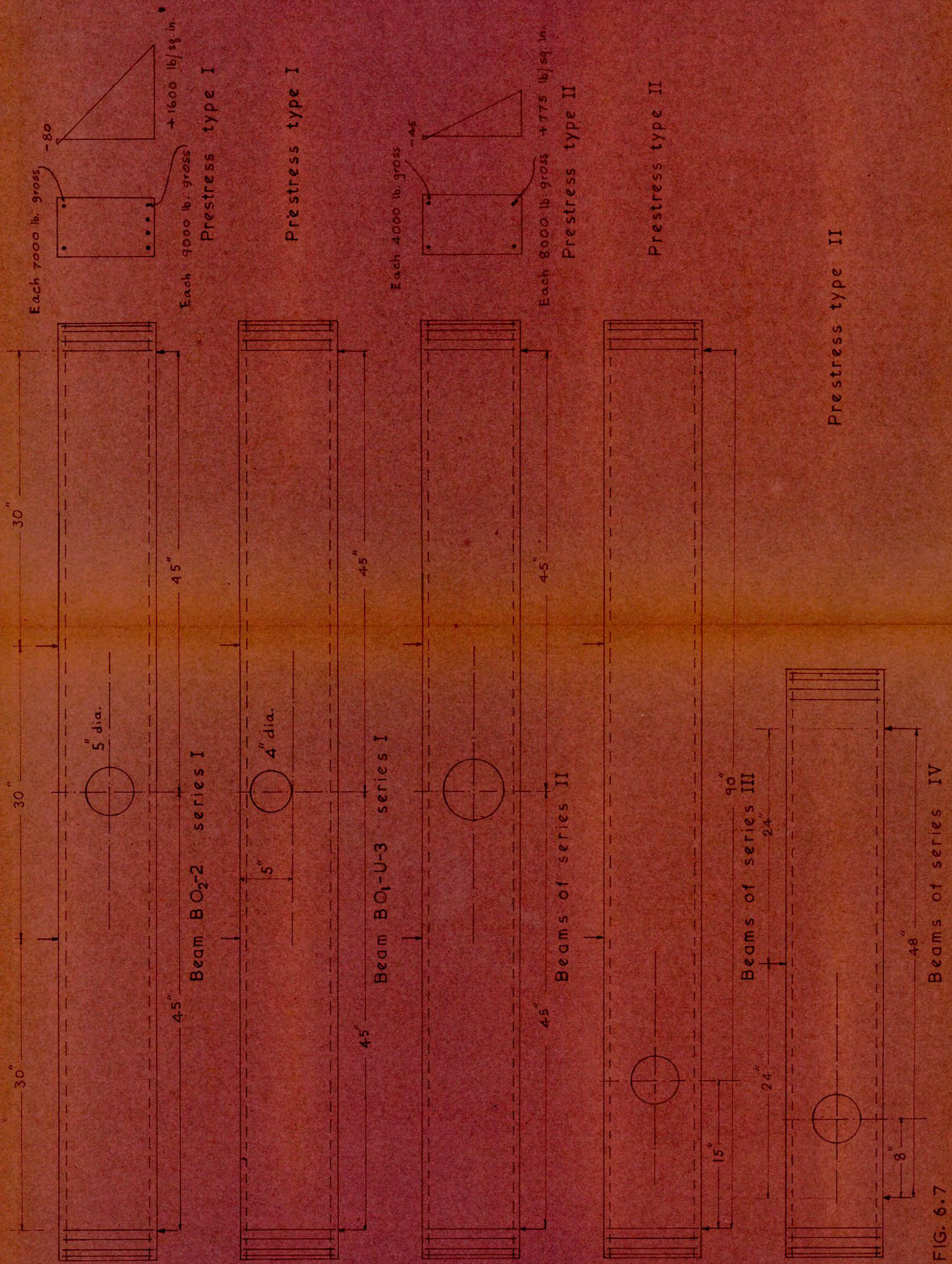
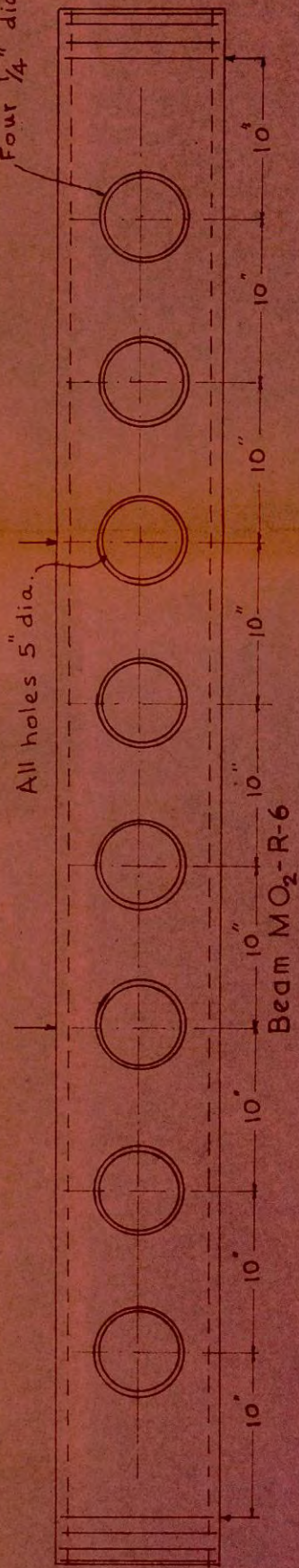


FIG. 6.7

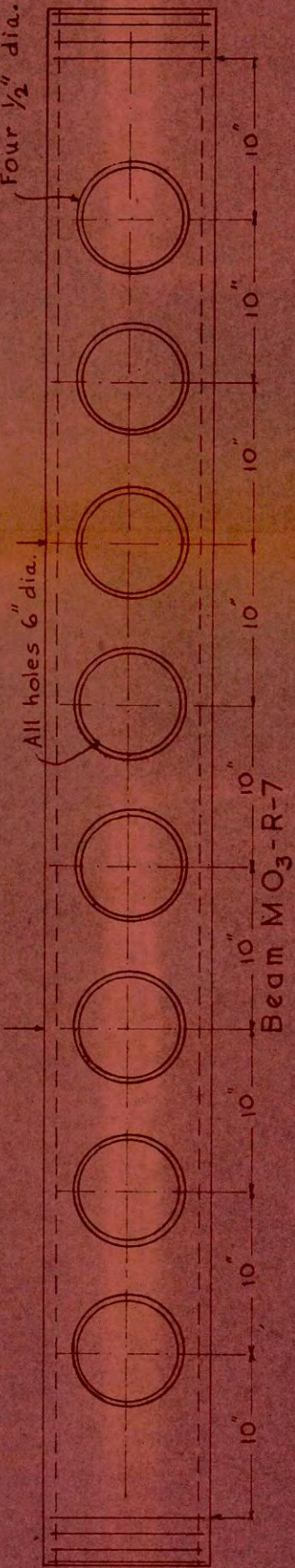
FIG. 6.8

Four $\frac{1}{4}$ " dia. ring reinf.



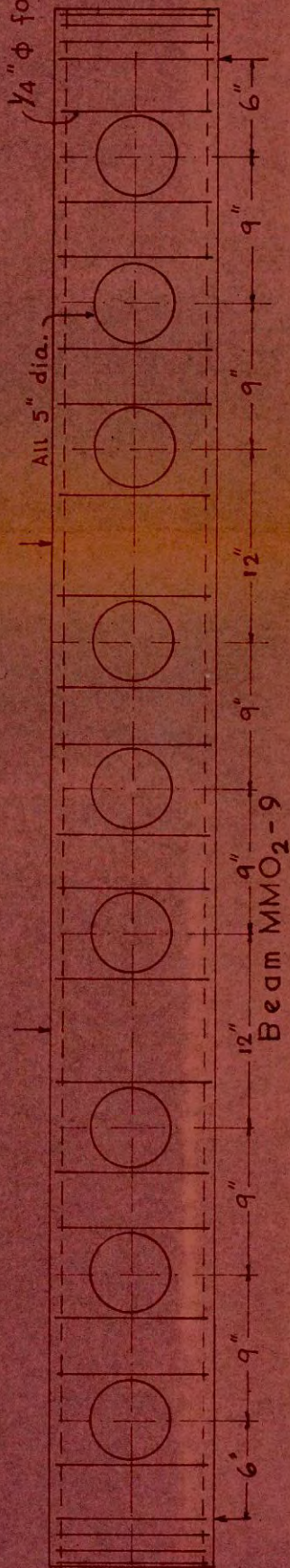
Prestress type I

Four $\frac{1}{2}$ " dia. ring reinf.



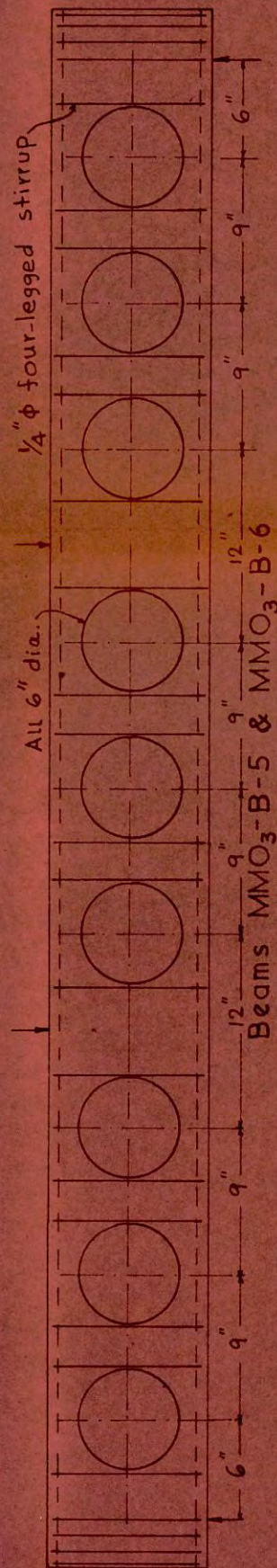
Prestress type I

$\frac{1}{4}$ " ϕ four-legged stirrup

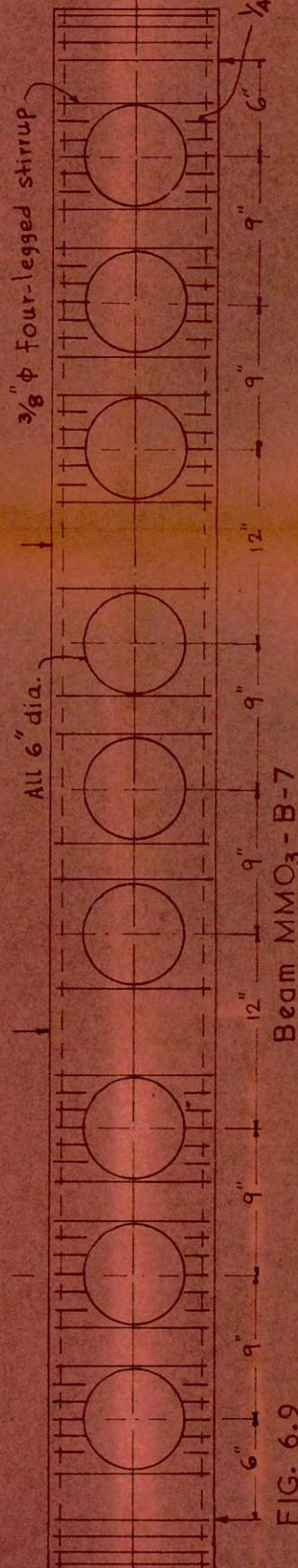


Prestress type I

$\frac{1}{4}$ " ϕ four-legged stirrup



$\frac{3}{8}$ " ϕ four-legged stirrup



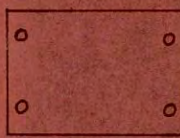
Prestress type I

$\frac{1}{4}$ " ϕ two-legged stirrup @ 1 in. c/c

One wire in each duct

Each 7000 lb gross

-80



Two wires in each duct 1600 lb/sq. in.
Each 9000 lb gross

Prestress type I

FIG. 6.9

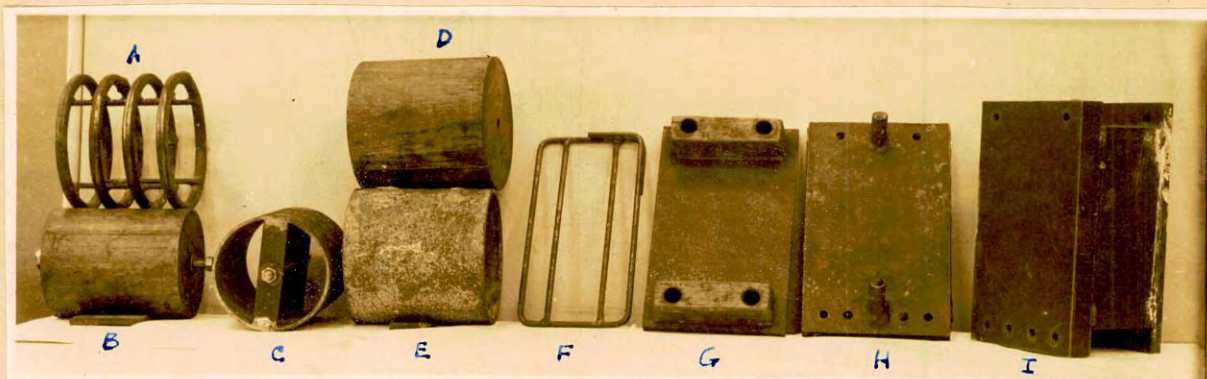


Plate 6.1

- A: Ring reinforcement around the hole
- B: 5" dia. wooden cylinder used to form holes
- C: 5" dia. bonded hole liner
- D: 6" dia. wooden cylinder to form holes
- E: 6" dia. hole liner
- F: 1/4" dia. four-legged stirrup
- G: End plate for casting bonded beams
- H: End plate for stressing and grouting bonded beams
- I: Special end plate for stressing unbonded beams



Plate 6.2

Mould ready for casting beam MO₃-4

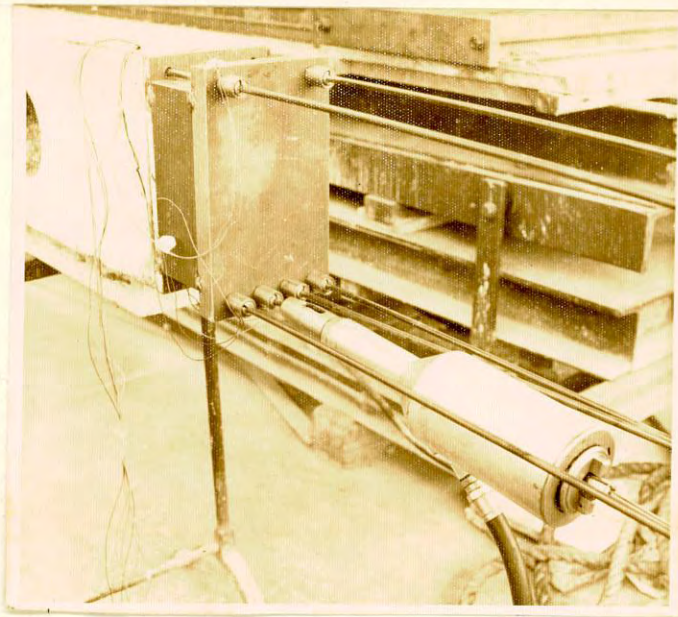


Plate 6.3
Stressing an unbonded beam, showing the special end plate

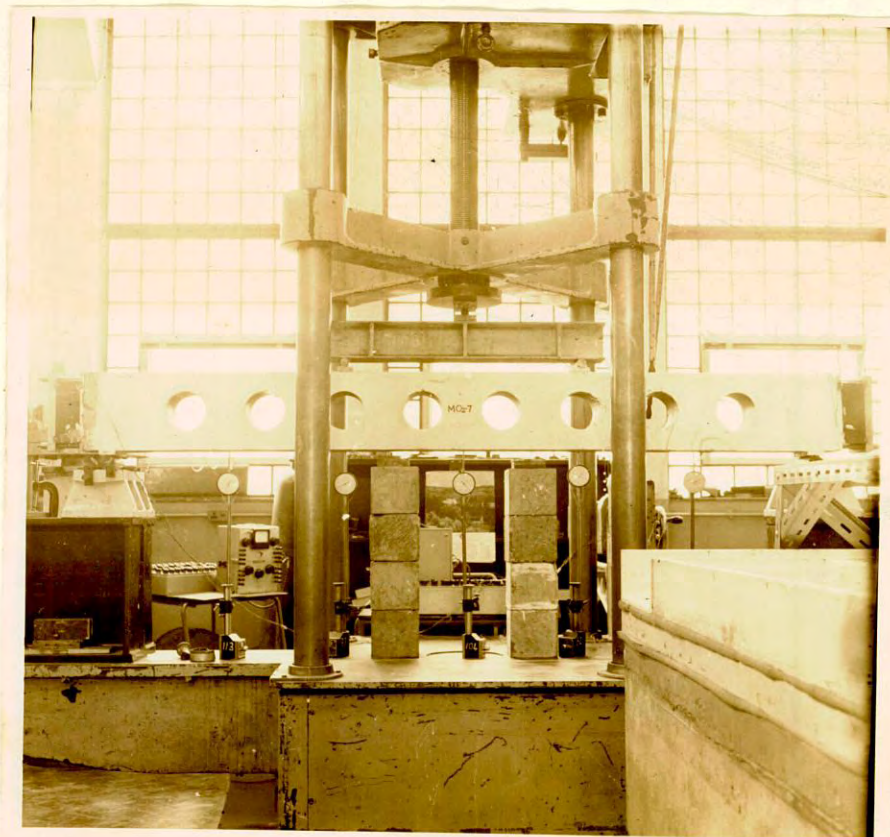


Plate 6.4
A beam in the machine ready for testing.

CHAPTER VII

PRESENTATION OF EXPERIMENTAL RESULTS

The information gained from the tests of individual beams is now discussed in relation to the elastic stresses, deflections, load at cracking, crack pattern, ultimate load and mode of failure. Details of the beams to which the designation letters and numbers apply are given in Table 6.1. Table 7.1 gives the results of control specimen tests of all the beams.

7.1 BEAMS CONTAINING SINGLE HOLES

Beam BS-1

The stress distribution within the depth of the beam due to the pre-stressing force was obtained by 8 in. Demec measurements. The stresses on top and bottom of the beam were found to be very close to the calculated values and the distribution was linear as in Fig. 6.7.

The recorded deflection at the working load of 5 tons was 0.079 in. (Fig. 7.44) as against a calculated value of 0.067 in. This shows that the actual deflection is greater by about 17%.

The first flexural crack was developed at about the middle of the beam at a load of 6.5 tons which is very close to the calculated value of 6.25 tons (Art. A. 3.2). According to Leonhardt,⁵⁸ the spacing of cracks in an unbonded beam would be approximately

equal to the depth of the beam. In this beam, four cracks developed over a total distance of 30 in. at approximately 10 in. spacing equal to the depth of the beam. The middle crack, the first to develop, propagated very quickly into the compression zone and the beam failed at 11 tons as a result of crushing of the concrete. This early failure of the beam as compared to the calculated value of 11.4 tons was probably due to the measured strength of the concrete (6800 lb/sq.in.) being slightly lower than that used in the calculation (Art. A. 7.1).

Beam BO₁-1

The splitting stress due to the tensile stress concentration at the ends of the horizontal diameter of the hole was calculated to be 1150 lb/sq.in. The high strain recorded in the PL-30 resistance gauges fixed at these points predicted a crack; however, no crack was visible.

The strains at the boundary of the hole, as recorded by the resistance strain gauges, were converted to stresses and curves were plotted on polar graph paper (Figs. 7.1 and 7.9). The continuous curves show the theoretical tangential stresses at the boundary of the hole and the dashed curves show their experimental values. Fig. 7.1 shows the stress trajectory due to the prestressing force and Fig. 7.9 shows the same for a bending load of 5 tons. The curves show close agreement, the maximum deviation in Fig. 7.1 being about 20% and that in Fig. 7.9 about 30%.

The distribution of stress at the minimum section is shown in Fig. 7.28. The stress at top and bottom of the beam can be obtained approximately by the simple beam theory (continuous curve).

The deflection at the working load of 5 tons was 0.076 in. (Fig. 7.44) as against the solid beam deflection of 0.079 in. There being only one hole in a beam of 90 in. span, the beam deflection was not greatly influenced.

The beam developed a crack almost below the hole at a load of 7 tons. At 10 tons load, two cracks developed on two sides on top of the hole and progressed towards the load points. The beam failed at 11.5 tons by crushing of concrete on top of the hole, bounded by these two cracks. The ultimate strength was not affected by the presence of the hole.

Beam BO₂-2

The splitting stress in this case was 1265 lb/sq.in. A local visible crack developed at the boundary of the hole and extended 0.5 in. along the horizontal axis. Fig. 7.2 shows the tangential stresses at the boundary of the hole due to the prestressing force and Fig. 7.10 shows the same due to a bending load of 5 tons. The experimental ^{dashed} ~~continuous~~ curves show good agreement with the theoretical ^{continuous} ~~dashed~~ curves. The experimental curves are discontinuous at the ends of the horizontal diameter because of the initial splitting crack.

In Fig. 7.44 is shown the load-central deflection curve which

is very similar to that of the solid beam BS-1.

The first flexural crack developed near the minimum section at a load of 5.5 tons. The crack pattern of this beam is shown in Plate 7.1.

The load at which the beam failed was 13.5 tons which is rather high probably because of the higher concrete strength of 7400 lb/sq.in. (Table 7.1).

Beam BO₁-U-3

This beam had the hole situated above the centroidal axis. The splitting stress was only 690 lb/sq.in.; no crack developed.

The load-central deflection curve is shown in Fig. 7.44 which shows that the deflection is almost the same as that of the solid beam BS-1.

The only flexural crack developed at a load of 4.5 tons just below the hole. Two more cracks developed and progressed almost along the horizontal diameter of the hole (Plate 7.2). The concrete above the hole crushed and the beam failed at 7.4 tons.

The calculated ultimate load (Art. A. 7.2) was 6.65 tons. The proposed method of calculation given in Art. 4. 1.4 is found to under-estimate the ultimate load by about 12%.

The measured strains above the hole (Fig. 7.29) show that the top portion of the beam was separated from the rest by the development of the two almost horizontal cracks and some tension developed on top of the beam due to the haunching effect. The compressive

strain on top of the hole at failure was as high as 7700 micro-strain. In Fig. 5.3, the strain in concrete at failure of the prism was only 1400 micro-strain.

Beam BS-2

The resistance strain gauges, fixed directly to the compression and tension reinforcement, showed a total loss of 15% of prestressing force. The stress distribution within the depth of the beam as obtained by 8 in. Demec measurements conformed closely to the one obtained by calculation.

The recorded deflection at 3 tons was 0.047 in. (Fig. 7.45) against the calculated value of 0.041 in. As in beam BS-1, the calculated deflection under-estimates the actual deflection by 15%.

The initiation of a flexural crack at 3.5 tons agrees well with the calculated value of 3.6 tons. Four cracks developed simultaneously, two below the load points and two between them. The beam failed at 7.3 tons against the calculated value, according to CP115 : 1959, of 6.21 tons (Art. A. 7.3). CP115 : 1959 under-estimates the actual failure load by 17% in this case (Table 7.1).

Beam BO₃-4

The prestressing force produces a stress of 365 lb/sq.in. at the mid-depth of a solid section of the solid beam. For a 6 in. diameter central hole, the splitting stress was 630 lb/sq.in. The concrete did not crack as confirmed by the reading on the strain gauge fixed at that point.

The tangential stresses at the boundary of the hole are given in Figs. 7.3 and 7.11. In Fig. 7.3 the theoretical continuous curve due to the prestressing force compares well with the experimental dashed curve. In Fig. 7.11, the theoretical curve due to a bending load of 3 tons compares well with the experimental curve.

The recorded deflection at 3 tons was 0.049 in. (Fig. 7.4) which is close to that of the solid beam (0.047 in.).

The strains measured at three points above the hole at the minimum section (Fig. 7.30) increase almost linearly upto a load of 5.5 tons, which is about 80% of the ultimate load, when the strain on top of the hole suddenly changes sign to tension and that on top of the beam increases to high compression. This is due to the propagation of cracks around the hole.

One crack developed just below the hole at a load of 4 tons (calculated value = 3.8 tons). At 6 tons two cracks developed near the top of the hole and progressed towards the load-points, as shown in Plate 7.3. The beam failed at 7.5 tons by crushing of the concrete.

Beam BQ₂-U-5

The theoretical stresses at the boundary of the hole due to the prestressing force were obtained by using the coefficients given in Fig. 3.5 for uniform compression in a beam where the hole is situated near the edge of the beam. Fig. 7.4 shows the theoretical (continuous) and experimental (dashed) curves due to pre-

stressing. Fig. 7.12 shows the theoretical and experimental curves of stresses at the hole boundary due to a bending load of 3 tons. These curves exhibit good agreement except at the bottom of the hole, where the experimental stress is low.

The measured strains at three points above the hole at the minimum section show that the strains increase almost linearly upto the cracking load after which strain reduction takes place.

The deflection at the design load of 3 tons was 0.050 in. which is very close to that of the solid beam.

Three cracks developed simultaneously at a load of 3.5 tons, two below the load points and one almost below the hole (Plate 7.4). As characteristic in unbonded beams the cracks that developed below the load points were branched at the top. As in other beams of this series, two cracks developed at both sides on top of the hole at a load of 6.5 tons and progressed towards the load points. The beam failed by crushing of the concrete above the hole at 7.4 tons.

Beam BO₁-U-6

In this case the theoretical stresses at the boundary of the hole were calculated according to Art. 3. 1.5, by using the stress concentration factors as given in Fig. 3.5 (Mindlin's curves). In Fig. 7.5 the theoretical curve of stresses due to prestressing shows good agreement with the experimental curve except at top of the hole where the experimental stress is 35% greater than the

theoretical value. Fig. 7.13 compares the theoretical and experimental tangential stresses at the boundary of the hole due to a bending load of 3 tons. The experimental curve shows the pattern of stress, but the value on top of the hole is much greater than the theoretical value. This is probably because the bending stresses around the hole were calculated according to Art. 3. 1.5, ignoring the effect of hole boundary being close to the beam boundary.

The load-central deflection curve (Fig. 7.45) shows that this beam has 20% higher deflection than that of a solid beam.

Fig. 7.31 shows the concrete strains at the tops of the hole and the beam at the minimum section. Upto the point of cracking the strains increase almost linearly after which the top fibre strain reduces and subsequently increases to high values. The strain on top of the hole steadily reduces after cracking.

The crack propagation and distribution are shown in Plate 7.5. The beam failed at 7.8 tons. This higher load than calculated can be justified on the basis of a higher concrete strength (7600 lb/sq.in.). The hole was situated within the depth of ultimate neutral axis, but the strength was not affected, because it did not cut the depth of the ultimate compression block.

Beam BO₂-7

As the prestressing force was low this beam, having a 5 in. diameter central hole, did not develop any splitting crack. The

complete stress diagrams around the hole boundary were obtained and compared with the theoretical curves. These stresses due to the prestressing force exhibit excellent agreement. Fig. 7.14 shows its experimental and theoretical stress trajectories for a bending load of 3 tons to be in very close agreement.

The measured deflection of 0.049 in. at the design load of 3 tons is very close to that of the solid beam. The load-deflection diagram is very similar to that of the solid beam BS-2.

CP115 : 1959 gives the minimum percentage of reinforcement to avoid failure by fracture of the tension steel as $\frac{15}{f_u} = 0.14\%$. In this beam the percentage actually supplied was 0.20%. However, the beam failed with the fracture of one of the tension wires within the anchorage at a load of 7.5 tons.

Beam BO₁-D-8

The solid beam stress at the level of the horizontal diameter of the hole due to the prestressing force was 525 lb/sq.in. The splitting stress at the hole in this beam was sufficient to cause cracking of the concrete. Cracks were visible on both sides of the hole.

The theoretical stresses at the hole boundary show good agreement with those obtained by experiment due to prestress and a bending load of 3 tons. The figures showing the stress distribution are, however, not shown.

The deflection at the design load (0.047 in.) was the same as

that of the solid beam (Fig. 7.45).

The crack pattern is shown in Plate 7.6. The cracks were branched above the hole and the beam failed at 7.4 tons due to secondary crushing of the concrete on top of the beam above the hole. This hole, being placed completely below the centroidal axis, did not have any effect on the strength of the beam (similar results have been reported by Barker⁴¹⁴⁰).

Beam BO₃-U-9

The stress trajectory at the boundary of the hole due to the prestressing force is shown in Fig. 7.6. The experimental curve shows close agreement to the continuous theoretical curve. In Fig. 7.15 are drawn theoretical and experimental curves due to a bending load of 3 tons. The experimental stress on top of the hole (Fig. 7.15) is high in comparison to the theoretical stress. This may be due to two reasons; the stresses were calculated according to Art. 3. 1.5 which does not consider the effect of the proximity of the hole to the boundary of the beam. At the high total stress (about 3800 lb/sq.in.) at the top of the hole, creep of the concrete must occur.

After cracking, the strain in concrete on top of the beam at the minimum section rapidly increases while the strain on top of the hole reverses to tension. This is rather contradictory to the assumed rectangular stress block on top of the hole at failure.

The crack pattern is shown in Plate 7.7. The hole in this beam

was situated in such a way that it cut the ultimate neutral axis depth, but did not cut the depth of the ultimate rectangular stress block. The beam failed at a load of 7.2 tons, which is very close to the failure load of the solid beam.

Beam SO₃-1

The tangential stresses at the boundary of the 6 in. diameter hole due to bending moment and shearing force corresponding to a total third-point load of 3 tons were calculated from Art. 3. 1.6 and the experimental stresses were obtained from strain measurements. The continuous theoretical curve and dashed experimental curve are shown in Fig. 7.19. The experimental curve is discontinuous at the centroidal axis because of the splitting crack that developed due to prestressing. The discrepancy is large which can be explained as follows.

1) The theoretical solution for stresses around the hole considers only the shearing force and bending moment at the section through the centre of the hole. In the present case, the hole being large in comparison with the shear span, the varying bending moment was different at different sections of the beam through the hole, there being low bending moment towards the reaction and high bending moment towards the load point.

2) The gauge length of the resistance gauge (PL-30) was 30 mm., i.e., the strain recorded by the gauge was an average of strain over that length (Fig. 6.5), whereas the theoretical solution gives

the peak stress at the point considered.

3) The theoretical solution assumes the material as elastic and the medium continuous; in the present case, the beam had initially developed splitting cracks due to prestressing.

The first diagonal splitting crack developed at a load of 2.5 tons when the calculated maximum tensile stress at the hole boundary was 750 lb/sq.in. The diagonal cracks developed approximately along the line joining the reaction and load points. As the hole had no reinforcement, the cracks propagated quickly (Plate 7.8) and the beam failed at 4.8 tons.

The load deflection curve is shown in Fig. 7.45. One hole in the shear span does not appear to influence the deflection to any great extent.

Beam SO₂-2

The theoretical and experimental curves for the tangential stresses at the boundary of the 5 in. diameter hole due to a load of 3 tons are shown in Fig. 7.20. In this case, the experimental stresses are found to be close to the theoretical values. The curves in Fig. 7.20 show that the experimental (dashed) curve is above the theoretical curve on the right top quadrant showing greater compressive stresses and below the theoretical curve on the left top quadrant showing greater tensile stresses. This phenomenon can be explained on the basis of the varying bending moment, i.e., the bending moment at the left of the hole is greater and

that on the right of the hole is smaller than that at the centre of the hole (as considered in the theoretical calculation).

The load-deflection curve is shown in Fig. 7.45 and it is similar to that of the solid beam BS-2.

The total theoretical principal stresses all around the hole due to prestressing and a load of 3 tons have been plotted in Fig. 7.25. In Fig. 7.26 are shown the principal stresses at some selected points as obtained by rosette measurements. The rosette stresses are different from the theoretical stresses due to the reasons discussed later in Art. 8. 1.5.

The first diagonal splitting crack developed at 5 tons load when the maximum theoretical tensile stress at about that point was 900 lb/sq.in. The crack pattern is shown in Plate 7.9. The beam failed at 6 tons.

Beam SO₁-3

The stresses around the hole due to the prestressing force are shown in Fig. 7.7. The experimental curve compares well with the theoretical continuous curve. In Fig. 7.21 are shown the theoretical and experimental curves for tangential stress at a load of 3 tons.

The total principal stresses, due to the prestressing force and a load of 7 tons, around the hole are plotted in Fig. 7.24. This figure gives a complete picture of stresses around the hole when a crack was just visible at the boundary of the hole. The

maximum tensile stress at the boundary was about 1000 lb/sq.in.

The beam developed the first diagonal splitting crack at a load of 7 tons. The beam failed in flexure at a load of 7.5 tons prior to its shear mode of collapse.

Beam SO₂-U-4

The stresses at the hole boundary at a load of 3 tons are shown in Fig. 7.22 where the experimental curve is found to be in agreement with the theoretical curve except at the top of the hole where the discrepancy is about 35%.

The deflection of this beam is very similar to that of the solid beam BS-2.

The principal stresses around the hole due to the prestressing force and a load of 5 tons as obtained by rosette measurements are plotted in Fig. 7.27. The tensile stresses at points L1, L6, R1, R2, R3 and R4 justify the propagation of the observed cracks (Plate 7.10).

The first crack developed at the hole boundary as shown in Plate 7.10 at a load of 5 tons when the total tensile stress at that point was about 1100 lb/sq.in. The beam failed at a load of 6 tons.

Beam SO₂-2 behaved in the same way as this beam. Both developed diagonal splitting cracks at 5 tons and failed at 6 tons. It seems that the position of the hole with respect to the depth may not affect the behaviour and strength of the beam.

Beams SO-1, SO-2 and SO-3

The tangential stresses at the boundary of 4, 5 and 6 in. diameter holes (Fig. 6.7) in centrally loaded beams SO-1, SO-2 and SO-3 respectively at a load of 3 tons are shown in Figs. 7.16, 7.17 and 7.18 respectively. The dashed experimental curve is found to be close to the theoretical curve in the case of the small hole (4 in. dia.). In beams SO-2 and SO-3, the experimental stresses are greater than the theoretical values probably due to the reasons given in the case of beam SO₃-1.

The hole stresses in all three beams due to the prestressing force were close to those predicted by the theoretical analysis. These curves are, however, not shown.

The experimental curves in Figs. 7.16, 7.17 and 7.18 are not complete because of the absence of strain measurement in one quadrant.

Beams SO₂-B-1 and SO₃-B-2

The experimental stress trajectory at the hole boundary of the bonded beam SO₂-B-1 is very similar to that of the unbonded beam SO₂-2.

Beam SO₂-B-1 developed a diagonal crack at a load of 5 tons (as in the unbonded beam SO₂-2) as predicted by the theoretical analysis and the beam failed at 5.5 tons.

Beam SO₃-B-2 behaved almost in the same way as that of the unbonded beam SO₃-1. The first diagonal crack developed at the same

Load of 2.5 tons and the beam failed at 4.5 tons (Plate 7.11).

Beam BS-3

This beam is identical with beam BS-1 except that the concrete strength of this beam was higher, i.e., 7200 lb/sq.in. (Table 7.1).

The average of strains recorded by four resistance strain gauges fixed directly on the tensile reinforcement of the beam recorded a strain of 4400 micro-strain after prestressing. This shows a total loss of 12%. The further loss of strain due to creep and shrinkage upto the time of testing the beam was about 3%.

The increase in strain in the tension reinforcement with the increase in load is shown in Fig. 7.40. The ultimate strain at failure was 7753 micro-strain, equivalent to a stress of 202,000 lb/sq.in. According to the calculations based on Table 7 of CP115 : 1959 the stress at failure should be taken as 198,200 lb/sq.in. (Art. A. 7.1). As the ultimate stress in the tension steel was underestimated, the beam strength was higher than calculated. This beam failed at 12.8 tons against the calculated load of 11.4 tons (Art. A. 7.1).

7.2 BEAMS CONTAINING MULTIPLE HOLES

Beam MO₁-1

According to the theoretical analysis, the diagonal crack was expected to be visible at a load of 6.5 tons. In this beam, diagonal cracks actually developed at a load of 6.5 tons at the two

holes near the supports. The presence of a number of holes in the shear span does not seem to influence the cracking of the beam. The cracks quickly progressed and the unreinforced beam failed at 7 tons as shown in Plate 7.12.

The Load-central deflection curve is shown in Fig. 7.46. The central deflection at a Load of 5 tons is 0.078 in. which is close to that of a beam without holes, i.e., 0.079 in.

Beams MO₂-2 and MO₂-7

Resistance strain gauges were fixed directly to the prestressing wires of the beam MO₂-7. The change of strain with the change in load is shown in Fig. 7.41. The change in strain is almost linear up to the point of cracking after which the strain increases rapidly. The load-strain curve of the tension steel is very similar to that of the solid beam BS-3; in both beams, the ultimate stress reached a value of 202,000 lb/sq.in. This beam failed at a load of 12.6 tons, compared to a calculated load of 11.4 tons.

The net stress in the top prestressing steel at failure was 3750 lb/sq.in. This is less than the value of 5760 lb/sq.in., calculated according to CP115: 1959.

The Load-central deflection curves are shown in Figs. 7.47 and 7.48. The experimental deflection at the design load was 0.083 in. The theoretical deflection, calculated with the help of the curves given in Figs. A.5 and A.6 is 0.075 in. The actual deflection is higher than the calculated which was also noticed in

the case of the solid beam BS-1.

The crack pattern of the beam MO₂-7 is shown in Plate 7.13. Diagonal cracks developed on both sides of each hole in the two shear spans at a load of 5.5 to 6.5 tons. It appears that the initiation of diagonal cracks was somewhat delayed probably due to the reinforcement around the holes. The diagonal cracks were all arrested by the hole reinforcement.

Beams MO₃-3 and MO₃-4

The load/central deflection curve of beam MO₃-3 is shown in Fig. 7.49. The deflection at a load of 5 tons, calculated by the equivalent second moment of area method (Art. 3. 2.5 and 3. 2.6) is 0.081 in. and the recorded deflection was 0.092 in. This shows that the actual deflection is 15% more than the calculated value. In the case of the solid beam BS-1 this discrepancy was 16%. The complete deflection curve of beam MO₃-3 at one ton increments of load has been plotted in Fig. 7.50. This figure shows that the deflection curve was symmetrical about the centre of the beam up to the design load. When the load further increased and the cracks developed unequally on the two sides, the deflection on the left of the beam was more than that on the right.

The strains in the concrete on top of the hole and at the top of the beam at the minimum section are plotted in Fig. 7.32. This shows that after cracking the strain on top of the hole increases at a greater rate and that at top of the beam decreases.

The crack pattern of the beam MO₃-4 is shown in Plate 7.14. This beam failed at 11 tons due to the excessive principal tensile stress on top of the second hole from the support point (Art. A.5.2).

The strain in the vertical four-legged stirrup placed by the side of the second hole in the shear span of beam MO₃-4 has been plotted in Fig. 7.42. This shows that the total strain in the stirrup at the failure load of 11 tons was 1475 micro-strain. Referring to Fig. 5.6, the yield strain of $\frac{1}{4}$ in. ϕ mild steel was 1650 micro-strain. This shows that the stirrup was very close to its yield point at the time of the failure of the beam.

Beam MO₃-L-5

The bonded hole liner helped the beam against the development of the splitting cracks at the time of prestressing. The first diagonal crack developed at 5 tons, the design load; but the beam failed instantly on the development of the diagonal cracks, because the bond between the liner and concrete was destroyed and the beam behaved as one having unreinforced holes (Plate 7.15).

The load-deflection curve is shown in Fig. 7.46. The deflection was less than that of beam MO₃-3, because the liner increased the stiffness of the beam to some extent.

The top fibre stresses, calculated from the resistance strain gauges fixed above holes and midway between adjacent holes, are plotted in Fig. 7.35. From the distribution of stress along the length of the beam, the assumption of Vierendeel behaviour would

not appear to be applicable. The continuous line giving the top fibre stresses based on the net section second moment of area can be used to give the maximum stress.

Fig. 7.33(a) shows the distribution of strain at a section midway between two adjacent holes in pure bending, as measured by PL-30 gauges. The distribution of strain is not linear even at a solid section in a perforated beam. The probable physical reasons will be given later in Art. 8.2.

Beam MO₂-R-6

The reinforcement provided in the form of bonded rings around the holes in this beam was not as effective as the vertical stirrup of the same cross-sectional area in beam MO₂-2 or MO₂-7. The initiation of diagonal cracking was independent of the form of reinforcement around the hole. Two diagonal cracks developed at a load of 5 tons on both sides of a hole in the shear span. These cracks quickly destroyed the bond between the ring reinforcement and concrete. As soon as the bond was destroyed the beam behaved as one having no hole reinforcement and failed prematurely at a load of 8.5 tons. The complete crack-pattern is shown in Plate 7.16.

The load-deflection curve is shown in Fig. 7.47. The deflection upto the cracking load is similar to that of Beam MO₂-7.

Beam MO₃-R-7

The bonded ring reinforcement, having the same cross-sectional area as that of the liner in beam MO₃-L-5, was better than the

Liner. The first diagonal crack developed at a load of 5 tons, the same as beam MO₃-L-5. The ring reinforcement bonded round the hole did not lose bond as quickly as the liner. However, the beam failed prematurely at a load of 8 tons (Plate 7.17).

The load-deflection curve is shown in Fig. 7.46. The deflection is similar to that of beam MO₃-L-5 which had lined holes.

Beam MO₁-U-8

The experimental tangential stress trajectory at the boundary of a hole at a load of 3 tons is compared with the theoretical curve in Fig. 7.23. These two curves are found to be close to each other, the maximum discrepancy, on top of the hole, being about 20%.

The beam developed diagonal cracks at a load of 4.5 tons at a theoretical tensile stress of about 1100 lb/sq.in. The beam failed at 7.2 tons. The crack pattern is shown in Plate 7.18.

Beam MMO₂-9

Resistance strain gauges were fixed on top of the beam, just above the holes and midway between adjacent holes. The stresses at a bending load of 5 tons as obtained from these gauges are plotted in Fig. 7.36. The points obtained by experiments have been joined by dashed straight lines. The continuous line, Curve I, gives the stresses on top of the beam as calculated from the beam theory, considering the net section second moment of area. Curve II gives the stress due to the simple beam action.

The experimental curve shows that the stress just above a hole is less than that given by the simple beam theory and the stress on top of the beam midway between adjacent holes is higher than that given by the simple beam theory (Curve II).

The stresses given by Curve I are approximately equal to the maximum stresses on top of the beam. For all design purposes, the stress as given by Curve I can be safely used.

The increase in the tensile strain, recorded in a PL-3 gauge fixed at the maximum tensile stress location of a four-legged stirrup is shown in Fig. 7.43. This is similar to the curve in Fig. 7.42. This also shows that the maximum tensile stress in the stirrup at failure was near the yield stress.

The stress trajectory at the hole boundary of this beam due to the prestressing force, plotted in Fig. 7.8, shows that the peak stresses are lower than the theoretical values in contrast to that of beam BO₂-2. This is due to the stress relieving influence of adjacent holes in the beam (see Art. 2. 1.1 - Stress relieving holes).

The calculated minimum load of initiation of flexural cracks is 6.25 tons (Art. A. 3.2). This beam, however, developed five flexural cracks under five holes as shown in Plate 7.19 at 8 - 10 tons of load.

The load-central deflection curve is given in Fig. 7.47. The beam failed in flexure at a load of 12.6 tons.

Beam BS-B-1

This solid bonded beam failed at 13 tons exactly as calculated using CP115 : 1959 (Art. A. 7.4). The deflection diagram is shown in Fig. 7.49. The central deflection up to cracking was almost the same as that of the unbonded beam BS-1. After cracks developed, the deflection was less than that of the bonded beam probably because of the development of a number of distributed cracks.

Beam MMO₁-U-B-2

The stresses on top of the beam, just above the holes and midway between adjacent holes, have been plotted in Fig. 7.37. It is noticed that the difference of stresses between two adjacent points is more pronounced in this beam having eccentric holes than in beam MMO₂-9. This is due to the influence of the proximity of the hole to the beam boundary.

The complete crack pattern at failure is shown in Plate 7.20.

Beam MMO₂-B-4

In Fig. 7.38 is drawn a dashed curve joining points representing stresses on top of the beam at a load of 5 tons, the points being just above holes and midway between adjacent holes. The continuous line curve gives the stresses on top of the beam using a constant net section second moment of area. This figure is similar to Fig. 7.36 of beam MMO₂-9. The continuous line curve gives the maximum stress on top of the beam.

The load-central deflection curve is shown in Fig. 7.48 which

shows that after cracking the curve is steeper than that of the similar unbonded beam MMO_2-9 .

This beam developed diagonal cracks at a load of 9 tons as in beam MMO_2-9 . The complete crack pattern is shown in Plate 7.21.

Beams MMO_3-B-5 and MMO_3-B-6

The top fibre stresses of beam MMO_3-B-5 have been plotted in Fig. 7.39 at a load of 4 tons. The continuous line curve giving the top fibre stress calculated using a constant minimum section second moment of area predicts the maximum stress with reasonable accuracy.

The load-central deflection curve of beam MMO_3-B-5 has been plotted in Fig. 7.49.

The complete crack-pattern of beam MMO_3-B-5 is shown in Plate 7.22. Beams MMO_3-B-5 and MMO_3-B-6 failed at loads of 10 tons and 9 tons respectively. The failure pattern is similar to that of beam MO_3-3 .

Beam MMO_3-B-7

This beam was provided with short stirrups above and below each hole in the shear spans. The failure of this beam took place on top of the third hole from one support point at a load of 12.3 tons. Though this was a shear failure, the load at failure was very near to the flexural failure load of 13 tons. After the examination of the failure it was noticed that there was a fault in the position of a stirrup where the failure took place. Difficulties

in positioning the stirrups, which were as short as 1.5 in., had been experienced at the time of casting.

The load-deflection curve is shown in Fig. 7.49 and the failure pattern is shown in Plate 7.23.

Table 7.1 Summary of beams

1	2	3	4	5	6
Sl. No.	Beam Designation	Average cube strength lb/sq.in.	Modulus of rupture lb/sq.in.	Cylinder splitting stress lb/sq.in.	Poisson's ratio
1	BS-1	6800	616	300	
2	BO ₁ -1	7250	490	290	
3	BO ₂ -2	7400	620	310	
4	BO ₁ -U-3	7200	550	317	
5	BS-2	6750	486	287	
6	BO ₃ -4	7300	590	325	
7	BO ₂ -U-5	7100	550	300	
8	BO ₁ -U-6	7600	590	298	0.195
9	BO ₂ -7	7550	580	298	
10	BO ₁ -D-8	8350	620	325	
11	BO ₃ -U-9	8950	660	317	
12	SO ₃ -1	7450	550	295	
13	SO ₂ -2	7600	628	327	
14	SO ₁ -3	7750	630	327	
15	SO ₂ -U-4	7800	770	413	
16	SO-1	8400	550	410	
17	SO-2	8100	610	417	
18	SO-3	7550	590	298	0.19
19	SO ₂ -B-1	7750	550	417	
20	SO ₃ -B-2	7800	600	435	
21	BS-3	7200	550	317	
22	MO ₁ -1	7400	512	357	
23	MO ₂ -2	7400	520	317	
24	MO ₂ -7	6900	490	295	
25	MO ₃ -3	7900	525	352	
26	MO ₃ -4	8000	610	352	
27	MO ₃ -L-5	8000	630	428	
28	MO ₂ -R-6	7750	600	350	0.195
29	MO ₃ -R-7	8050	490	314	
30	MO ₁ -U-8	7850	610	350	
31	MMO ₂ -9	8000	770	413	
32	BS-B-1	6900	590	387	
33	MMO ₁ -U-B-2	8200	525	405	
34	MMO ₂ -B-4	6850	505	467	
35	MMO ₃ -B-5	8450	525	405	
36	MMO ₃ -B-6	7550	540	435	
37	MMO ₃ -B-7	7850	616	420	

1	7	8	9	10
Sl. No.	Modulus of Elasticity $E, \times (10^6)$	Ultimate load (Tons)	Mode of failure	Ultimate load expressed as percentage of the calculated load
1	4.42	11.0	A	96.5
2	4.57	11.5	A	101
3	4.55	13.5	A	118
4	4.50	7.4	B	119
5	4.37	7.3	A	117
6	4.38	7.5	A	120
7	4.35	7.4	A	119
8	4.46	7.8	A	125
9	4.46	7.5 *	A	-
10	4.70	7.4	A	119
11	4.88	7.2	A	116
12	4.30	4.8	C	77
13	4.52	6.0	C	96.5
14	4.55	7.2	A	116
15	4.54	6.0	C	96.5
16	4.62	10.7 *	A	-
17	4.60	9.0	C	-
18	4.55	5.4	C	-
19	4.32	5.5	C	88.5
20	4.32	4.5	C	72.5
21	4.50	12.8	A	112
22	4.46	7.0	C	61
23	4.35	12.6	A	110
24	4.36	12.6	A	110
25	4.35	11.0	D	96.5
26	4.55	10.5	D	92
27	4.65	5.0	C	44
28	4.50	8.5	C	74.5
29	4.64	8.0	C	70
30	4.60	7.2	C	63
31	4.54	12.8	A	112
32	4.43	13.0	A	100
33	4.46	11.8	D	91
34	4.46	12.2	D	94
35	4.60	10.0	D	77
36	4.32	9.0	D	69
37	4.44	12.3	D	95

* Failure by rupture of tension reinforcement

A: Tension failure

B: Compression failure

C: Diagonal tensile failure

D: Principal tensile failure above hole

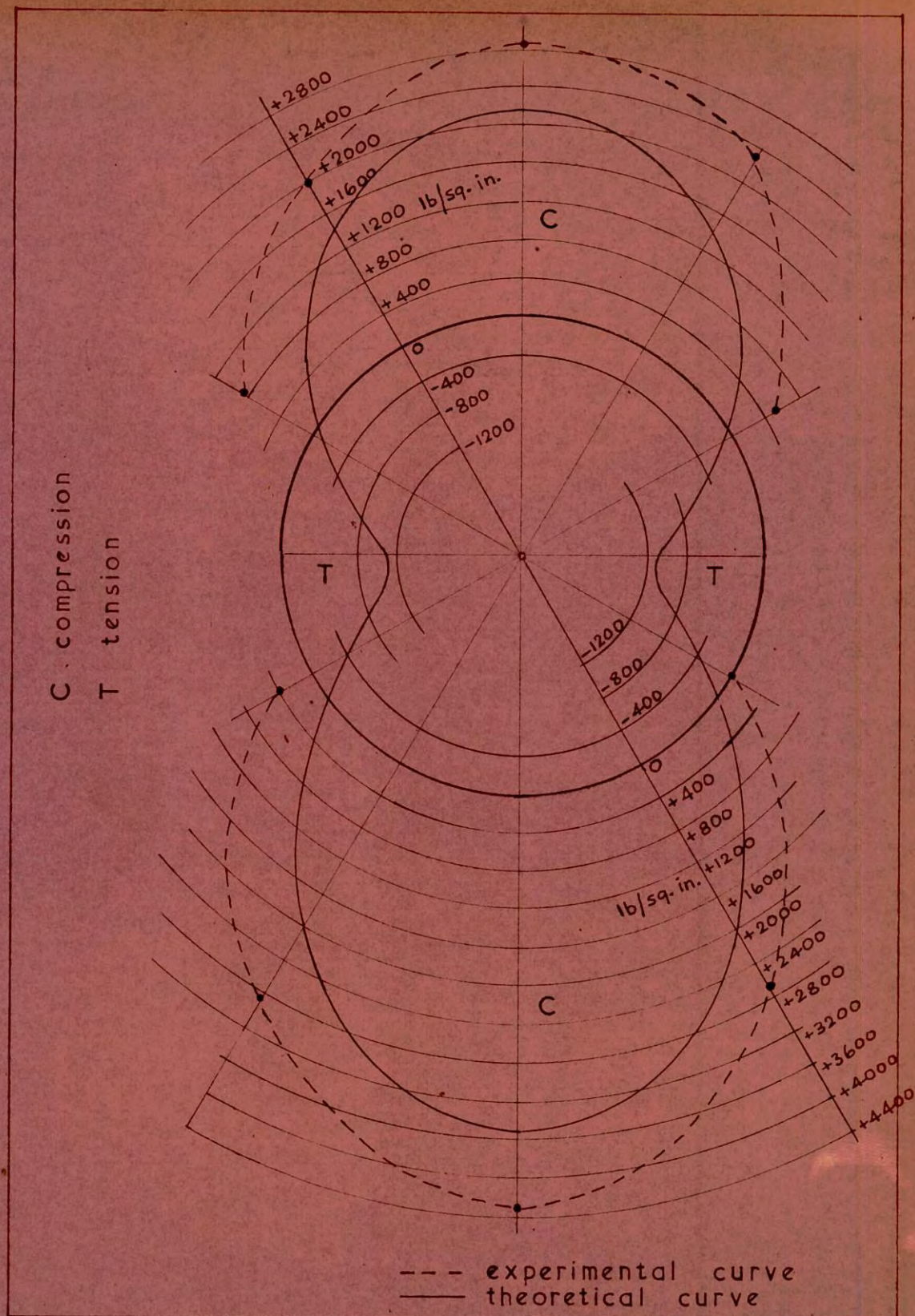


FIG. 7.1 Stress trajectory due to prestress Beam BO₁-1

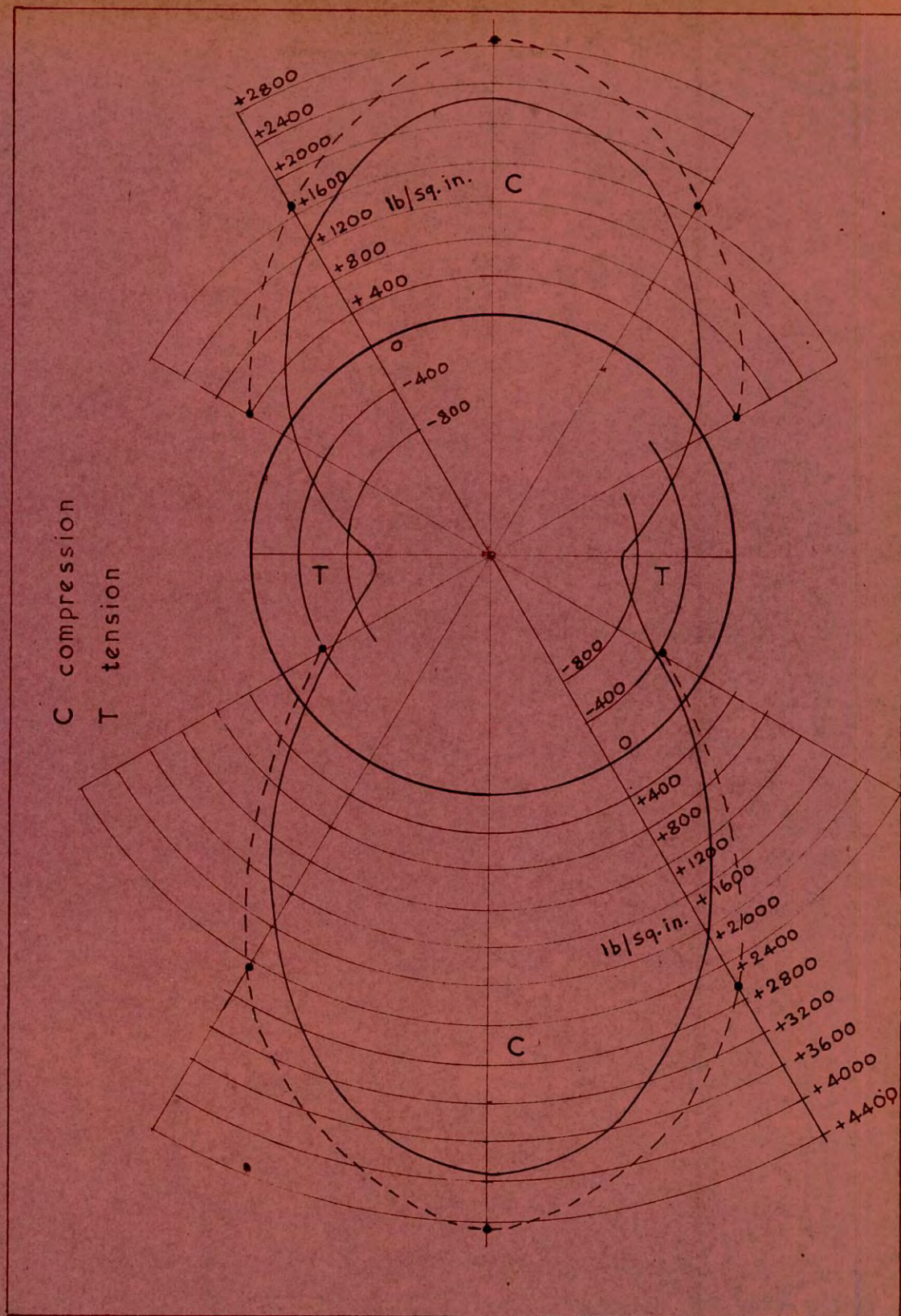


FIG. 7.2 Stress trajectory due to prestress Beam BO₂-2

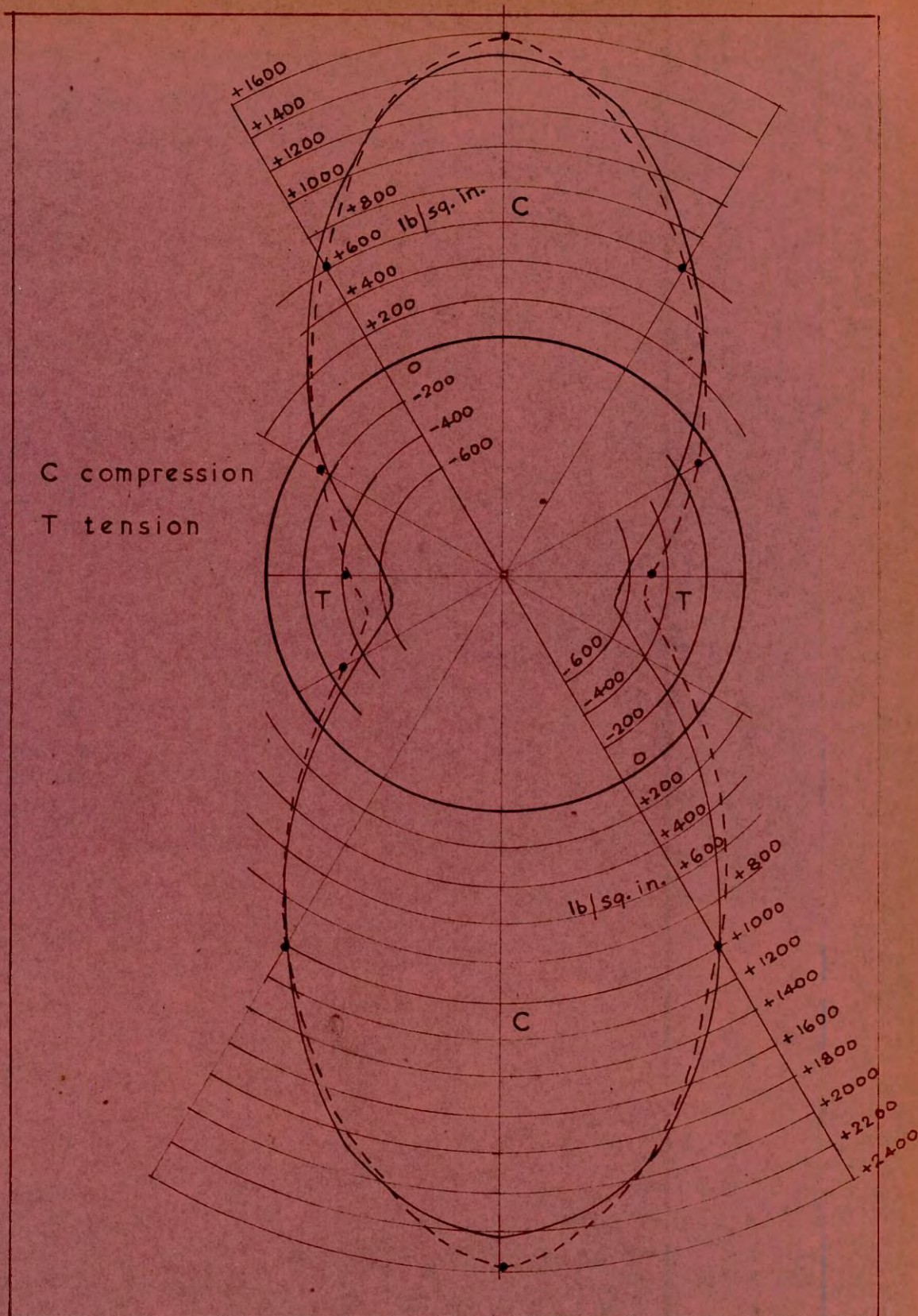


FIG.7.3 Stress trajectory due to prestress Beam BO_3-4

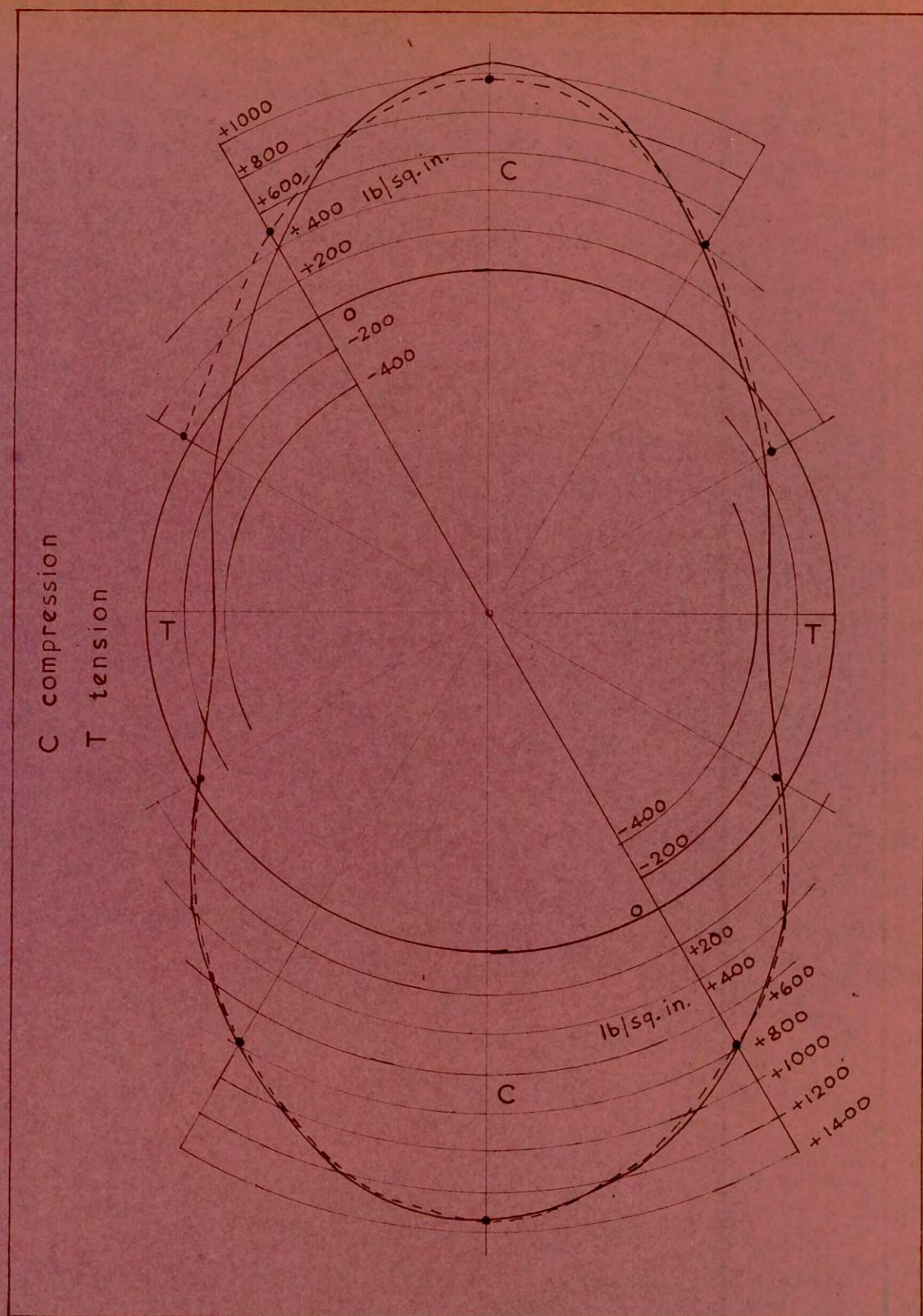


FIG. 7.4 Stress trajectory due to prestress Beam BO₂-U-5

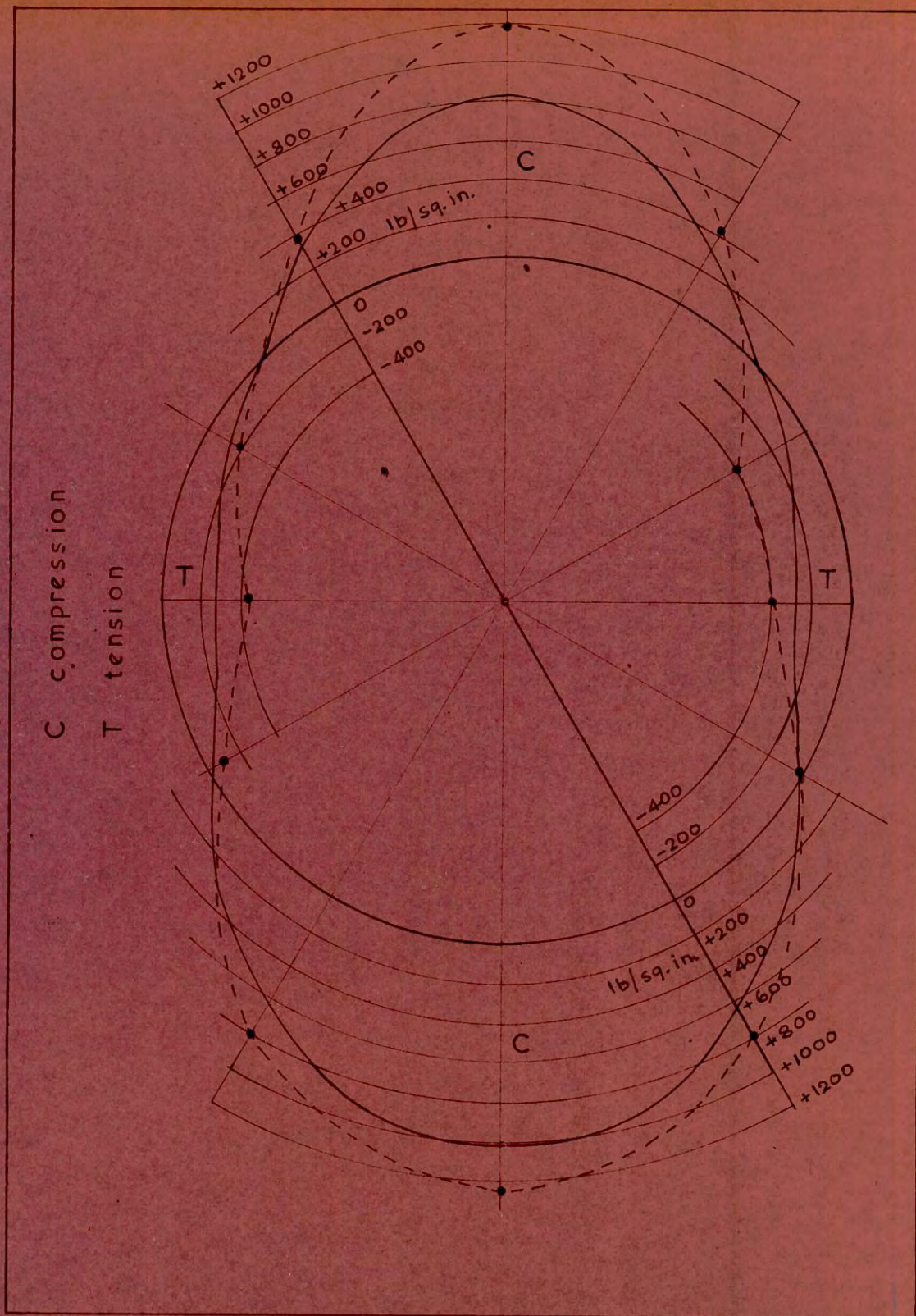


FIG. 7.5 Stress trajectory due to prestress Beam BO₁-U-6

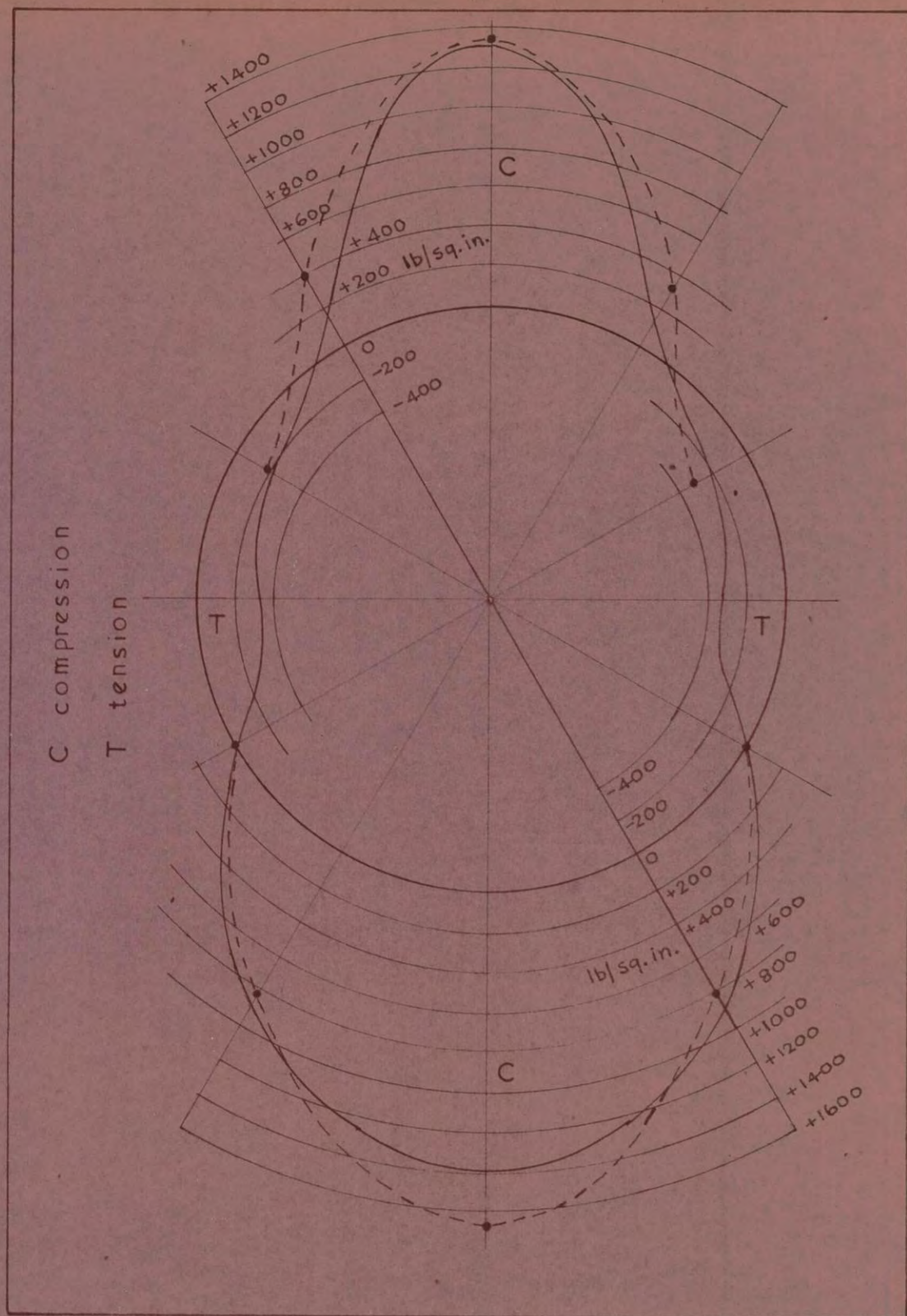


FIG. 7.6 Stress trajectory due to prestress Beam BO_3-U-9

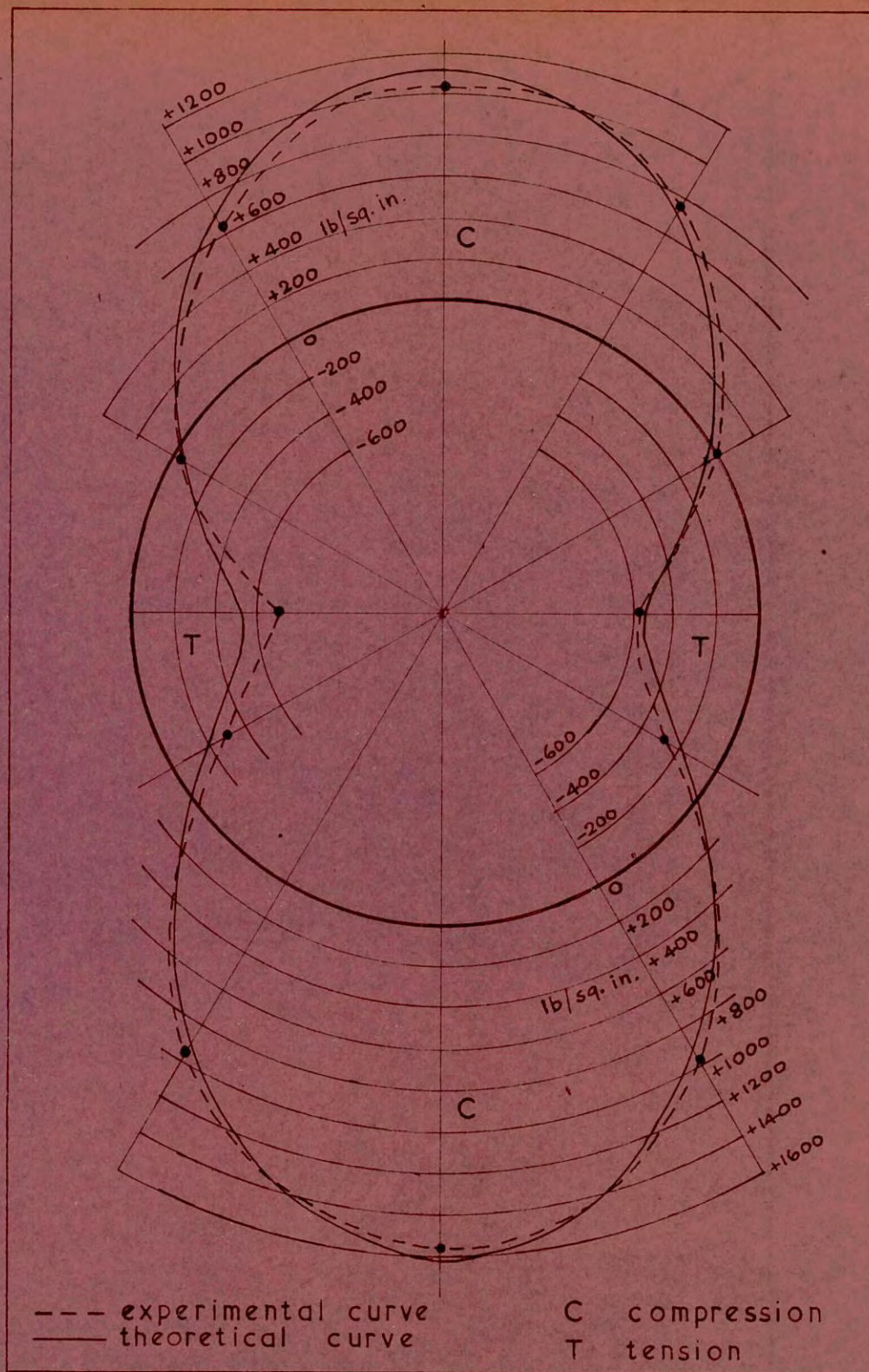


FIG. 7.7 Stress trajectory due to prestress Beam SO₁-3

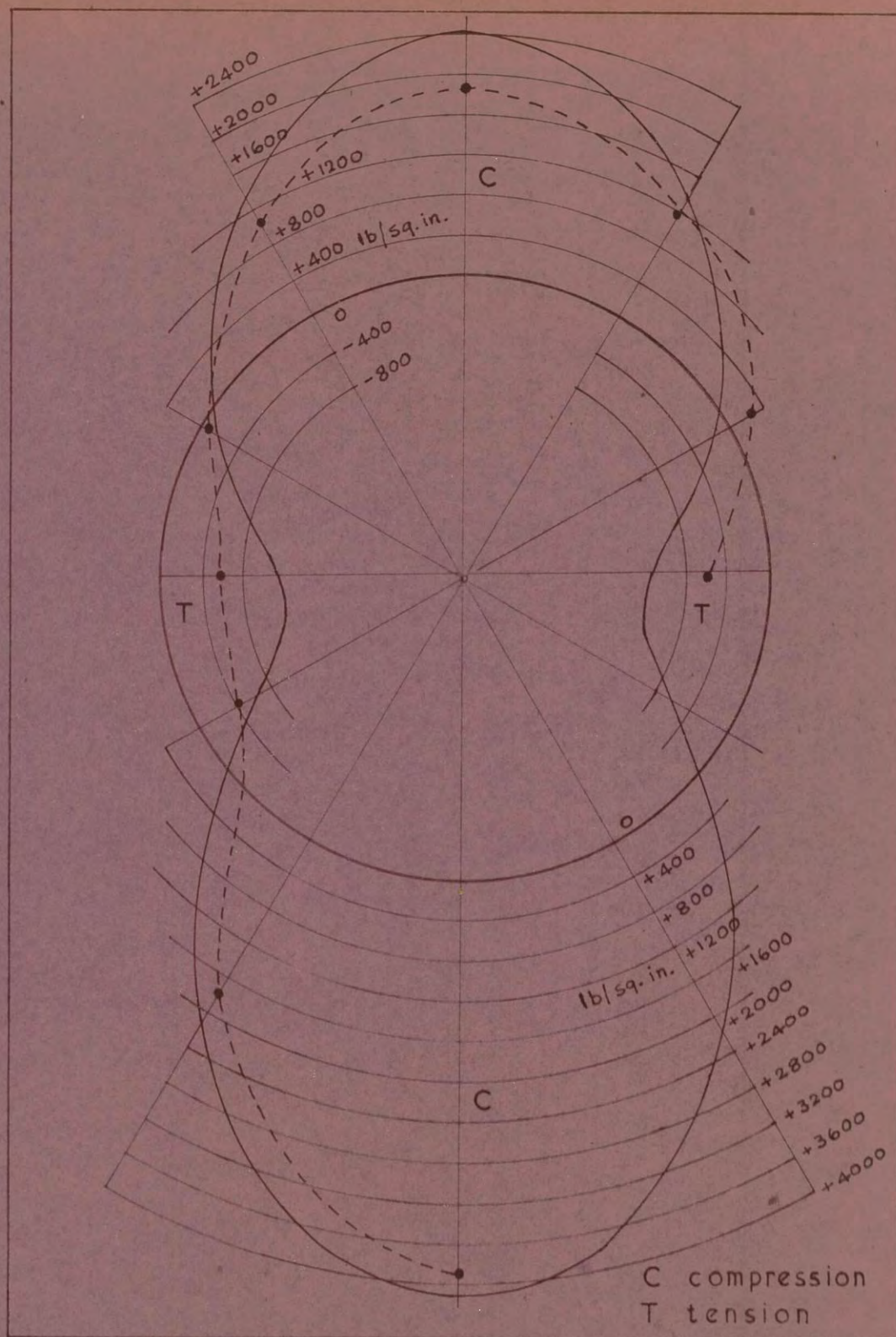


FIG. 7.8 Stress trajectory due to prestress Beam MMO₂-9

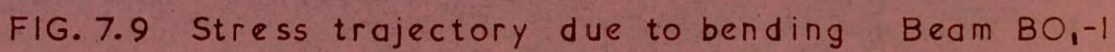


FIG. 7.9 Stress trajectory due to bending Beam BO_1-I

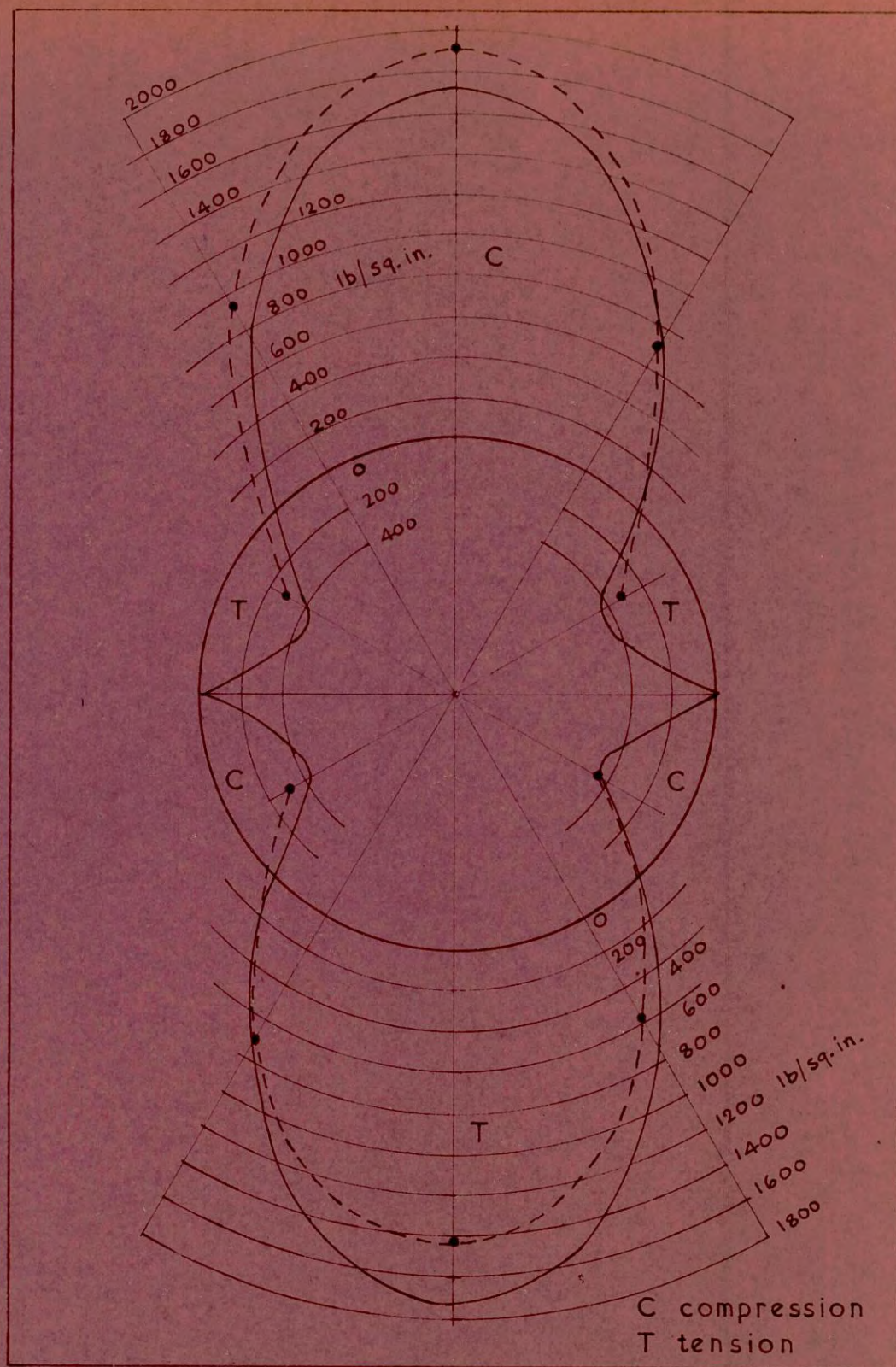


FIG. 7.10 Stress trajectory due to bending Beam BO_2-2

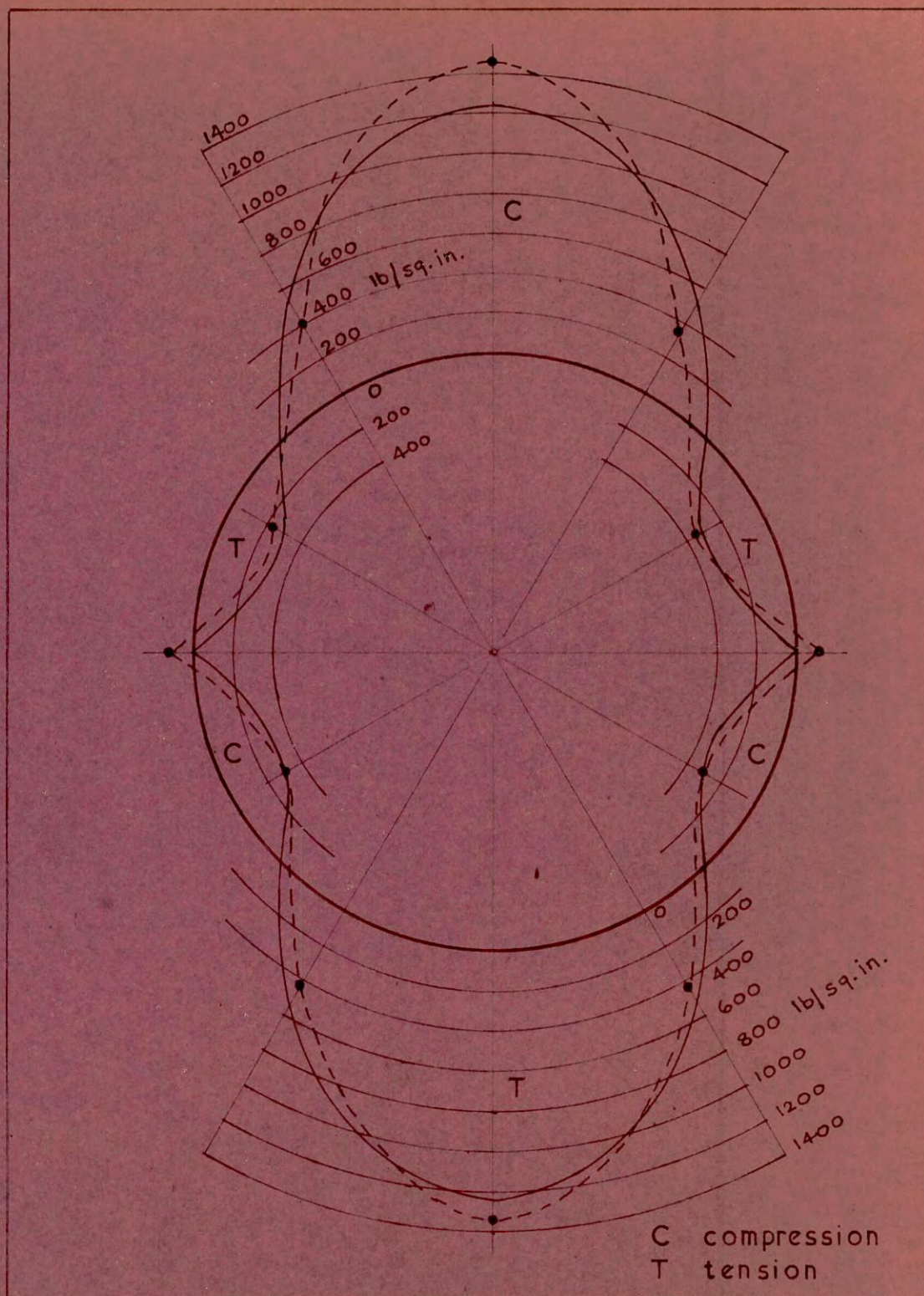


FIG. 7.11 Stress trajectory due to bending Beam BO₃-4

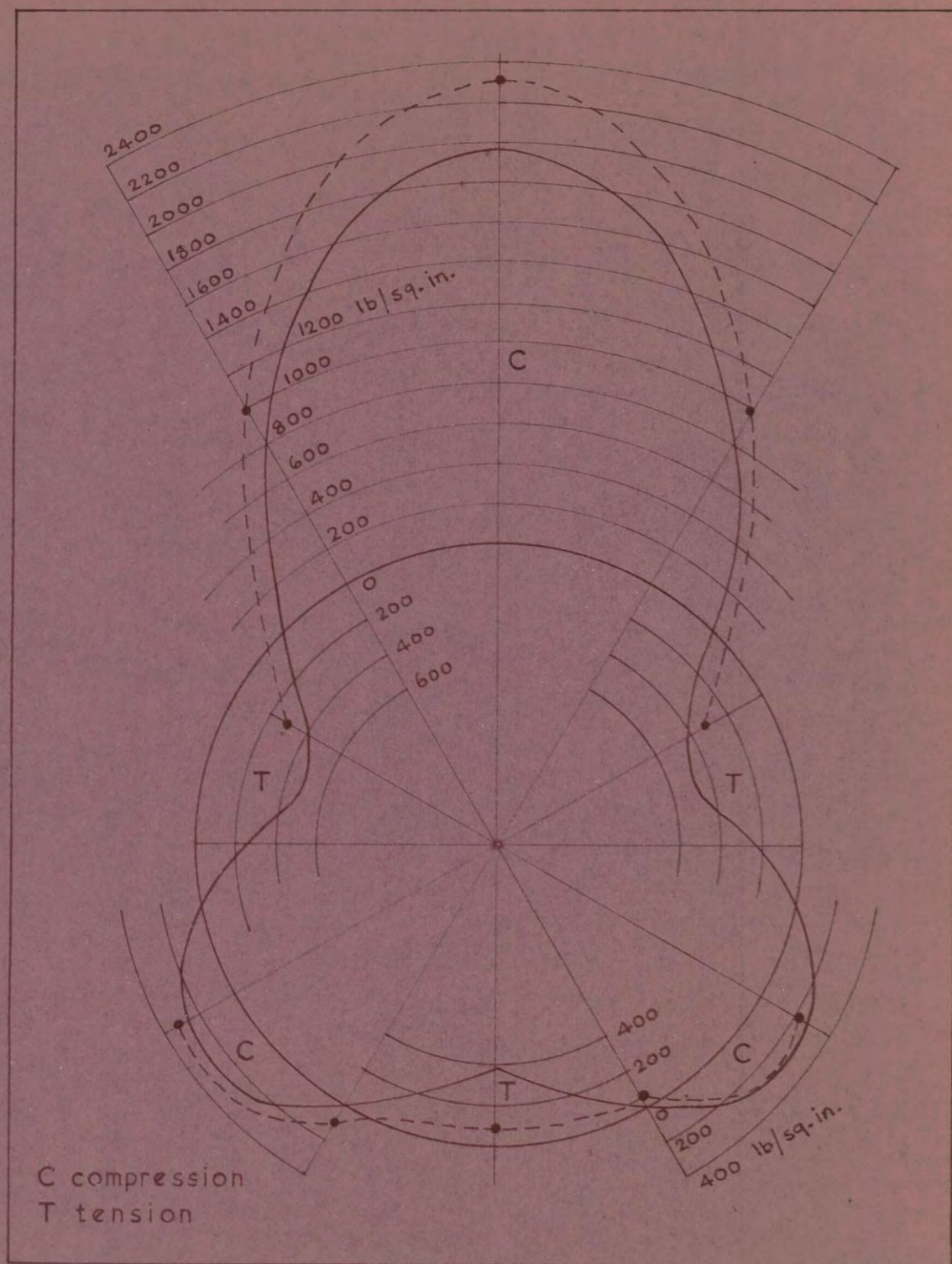


FIG. 7.12 Stress trajectory due to bending Beam BO₂U-5

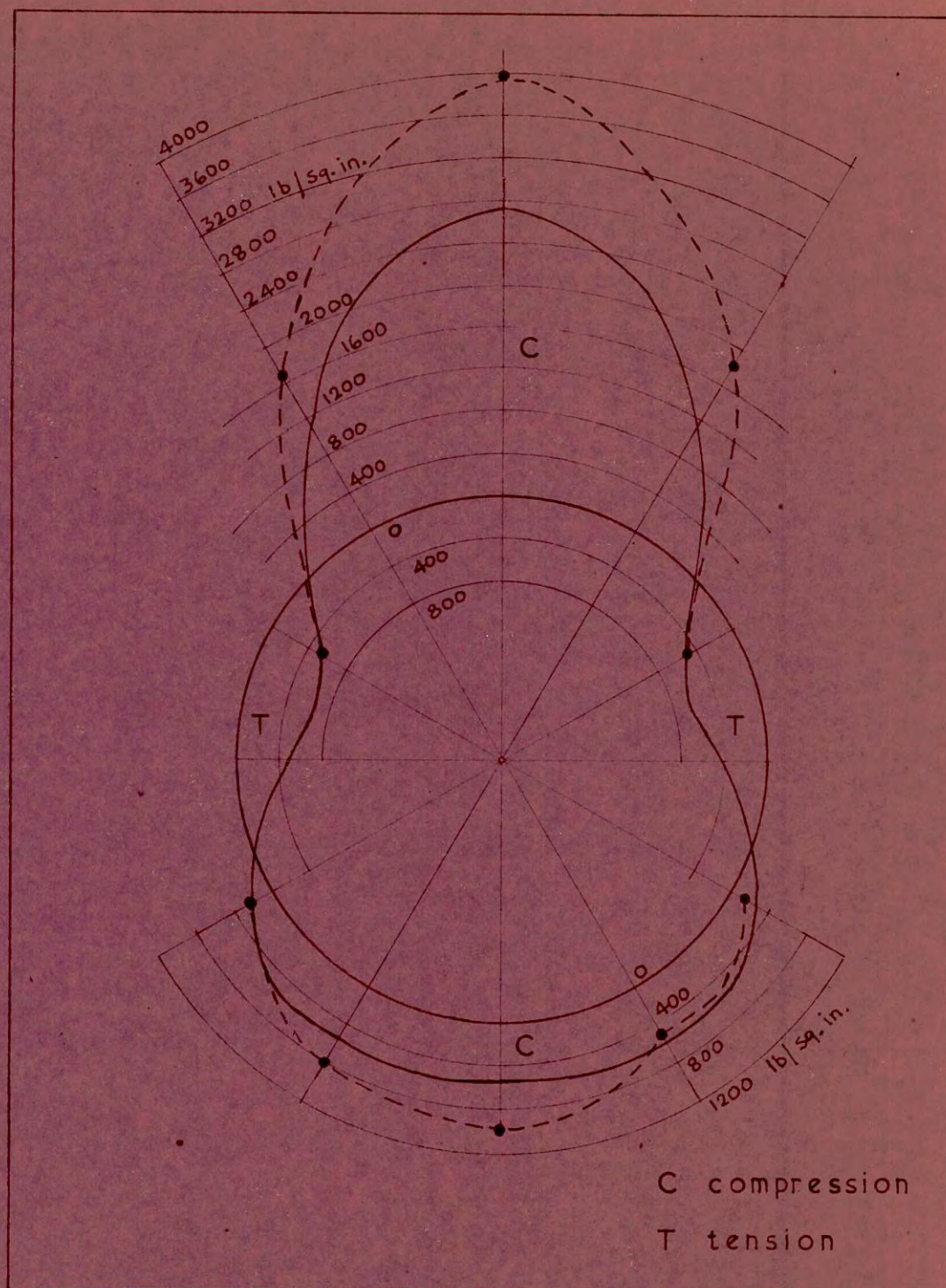


FIG. 7.13 Stress trajectory due to bending Beam BO₁-U-6

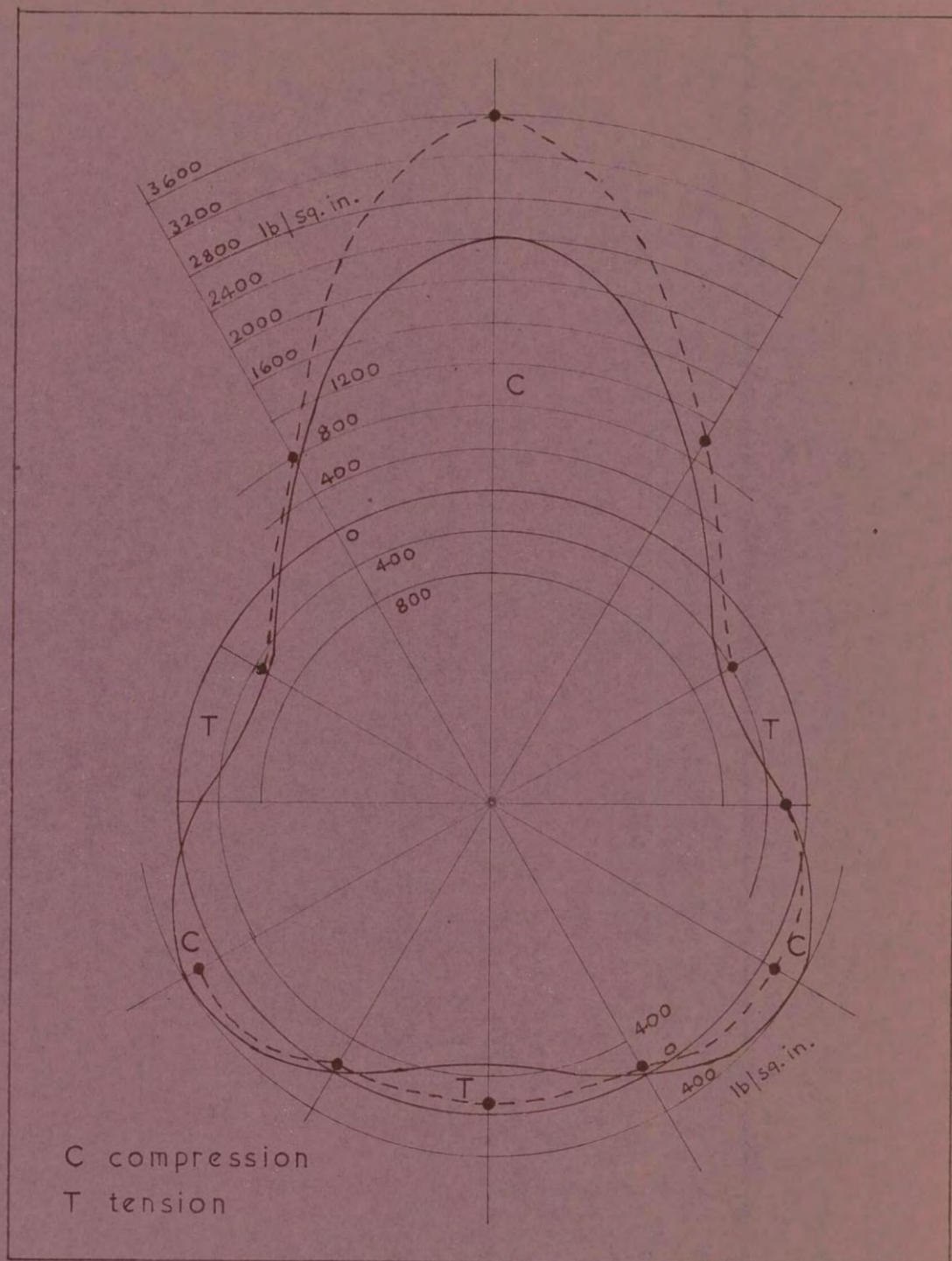


FIG. 7.15 Stress trajectory due to bending Beam BO₃-U-9

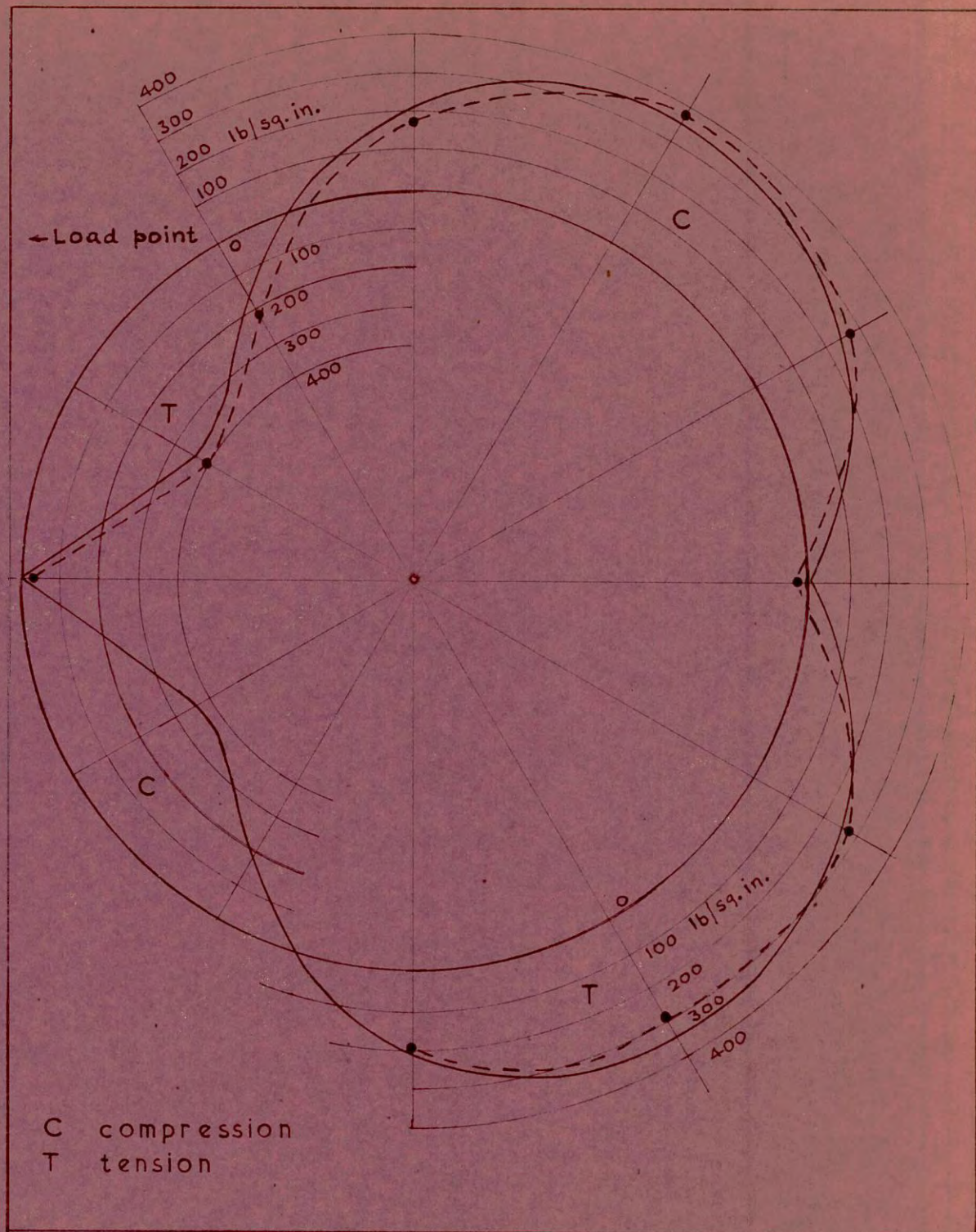


FIG. 7.16 Stress trajectory due to bending with shear
Beam SO-1

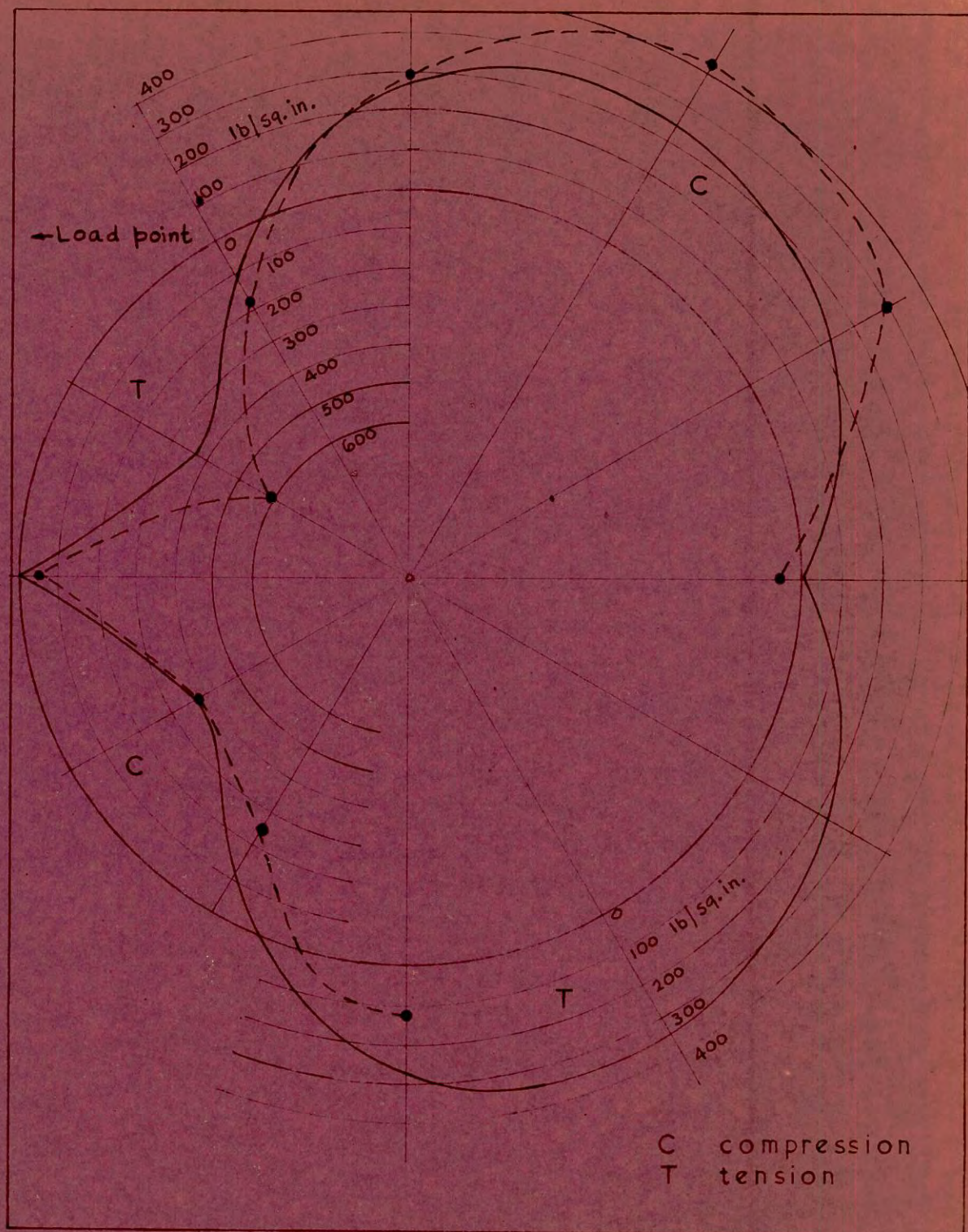


FIG. 7. 17 Stress trajectory due to bending with shear Beam SO-2

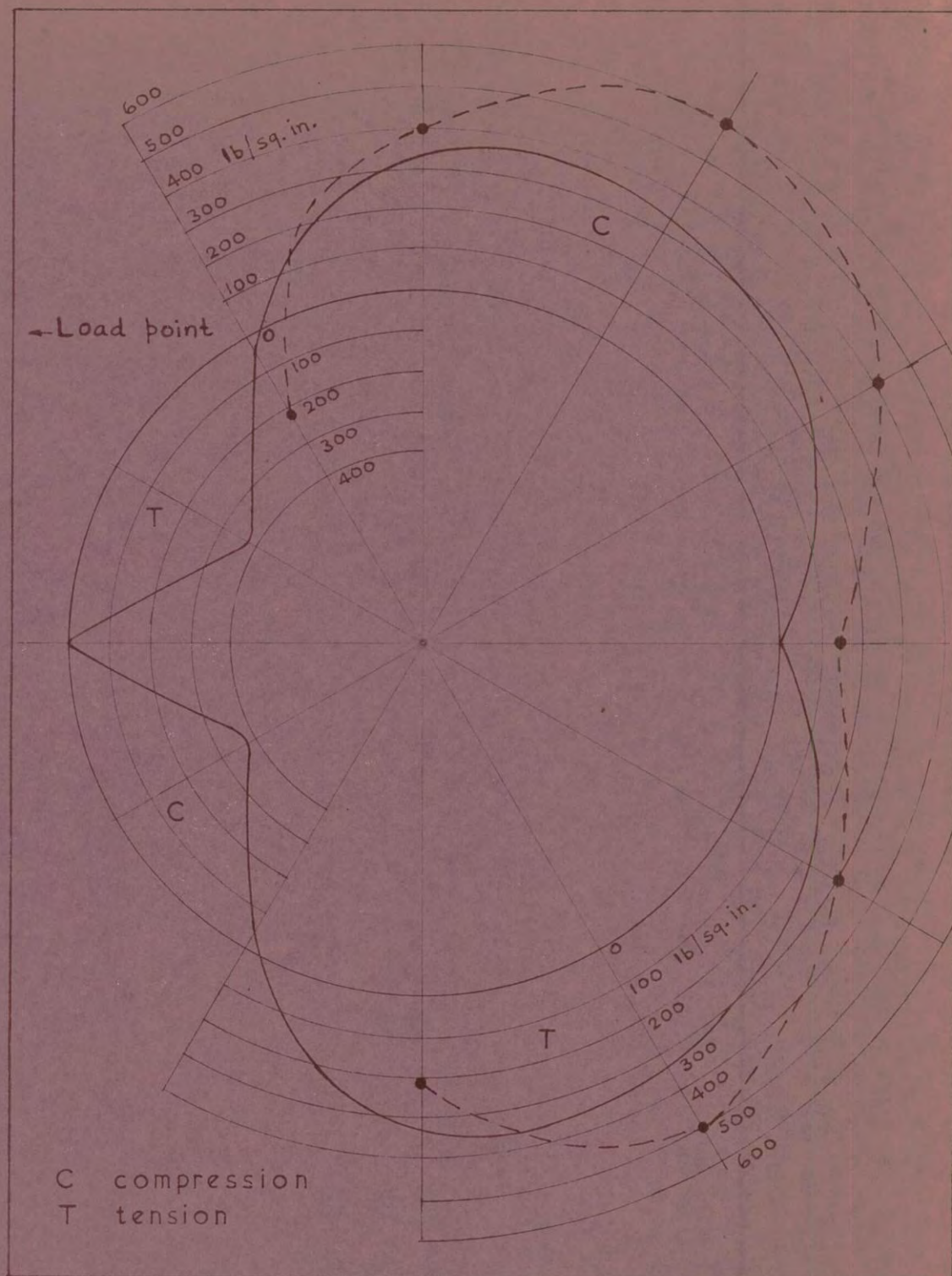


FIG. 7.18 Stress trajectory due to bending with shear Beam SO-3

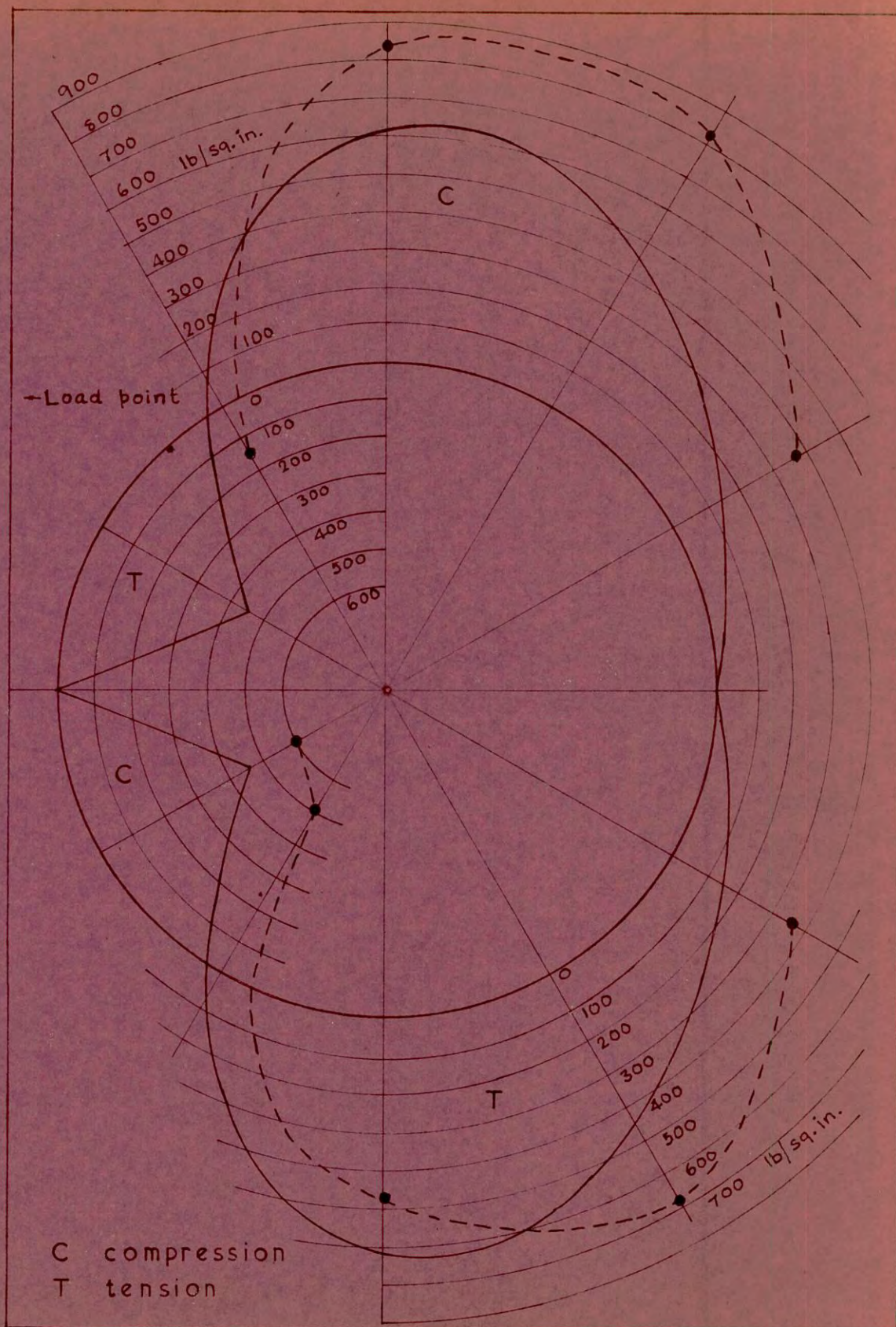


FIG. 7.19 Stress trajectory due to bending with shear
Beam SO₃-I

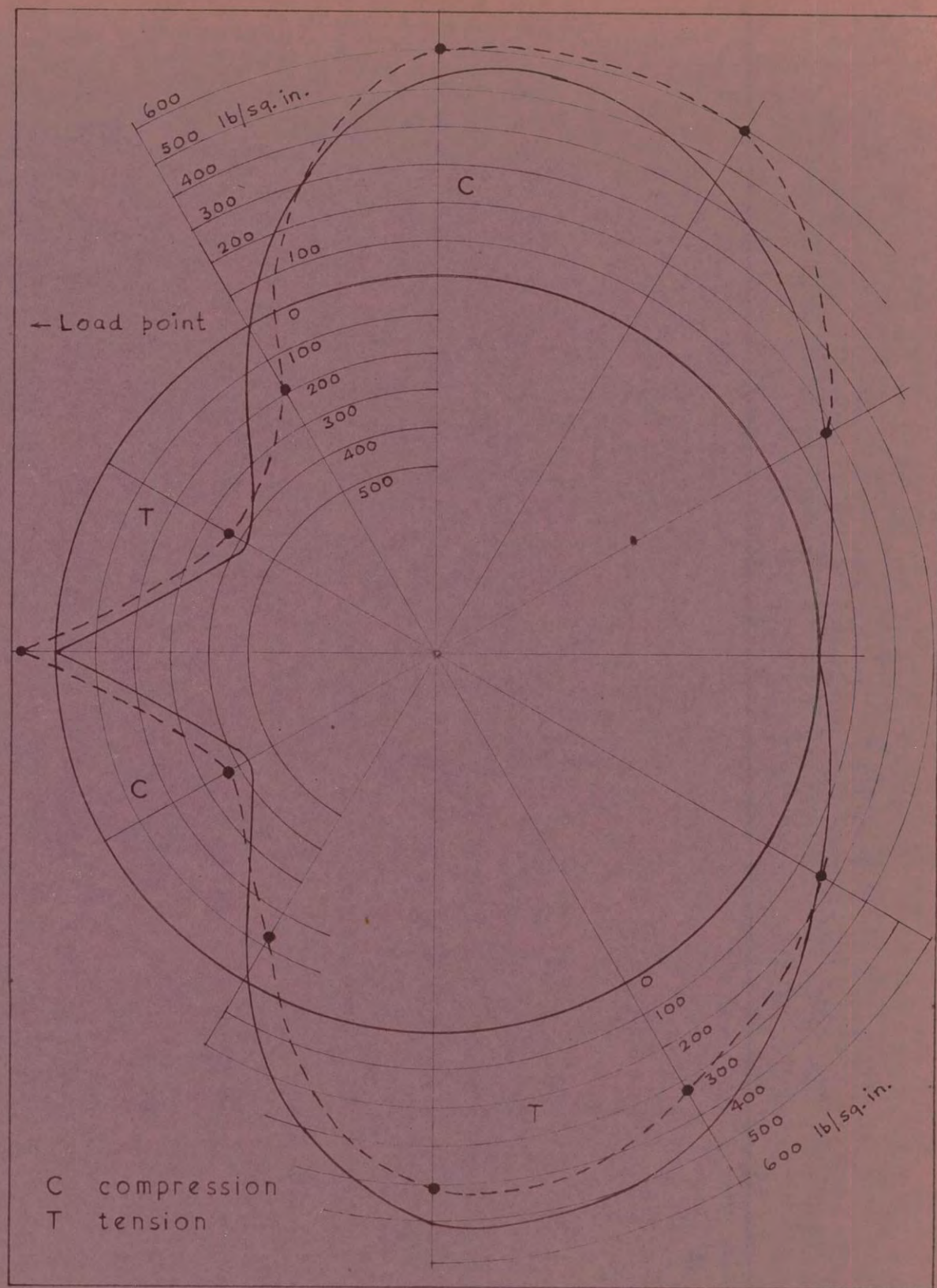


FIG. 7. 20 Stress trajectory due to bending with shear
Beam SO₂-2

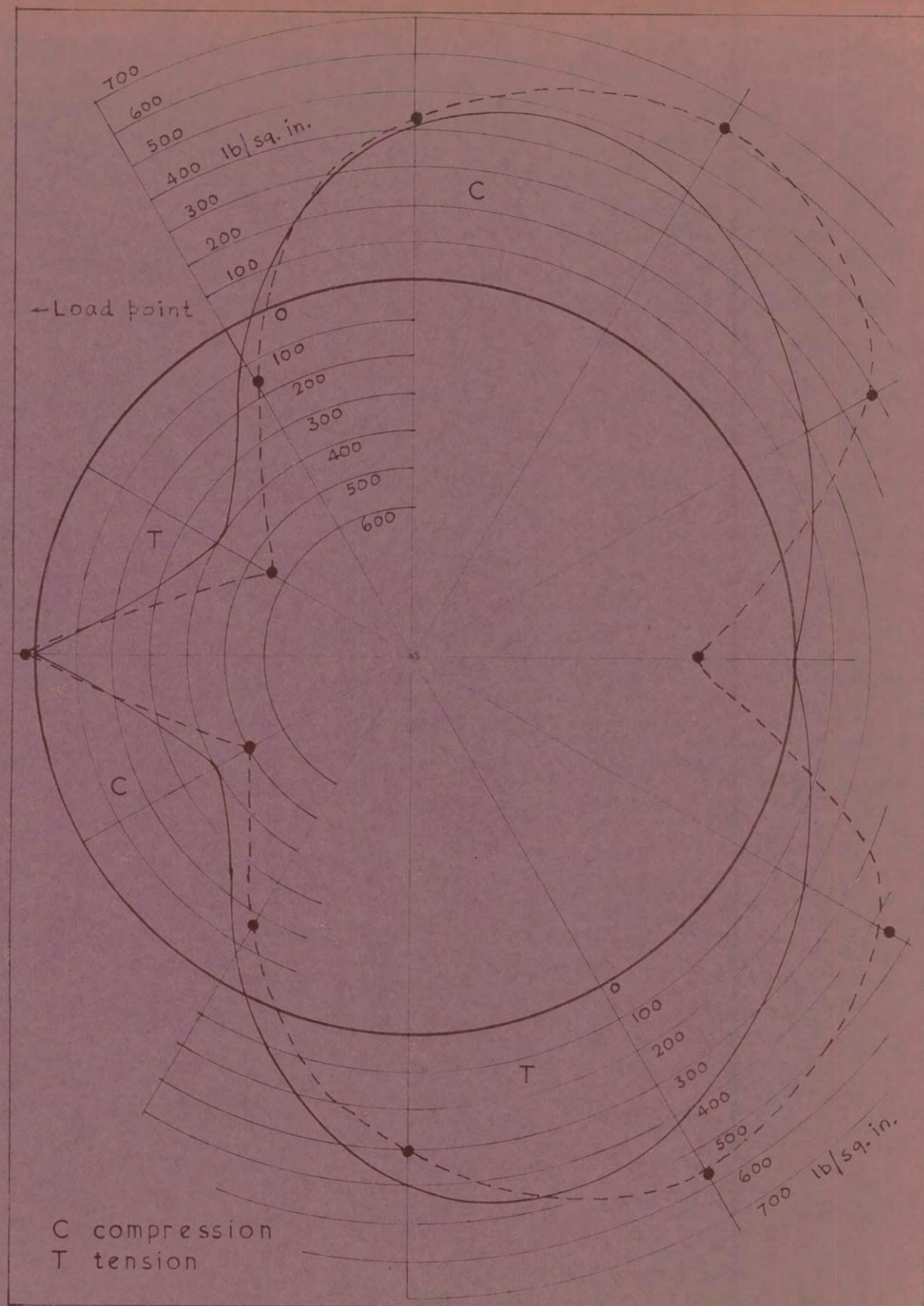


FIG.7. 21 Stress trajectory due to bending with shear
Beam S O₁-3

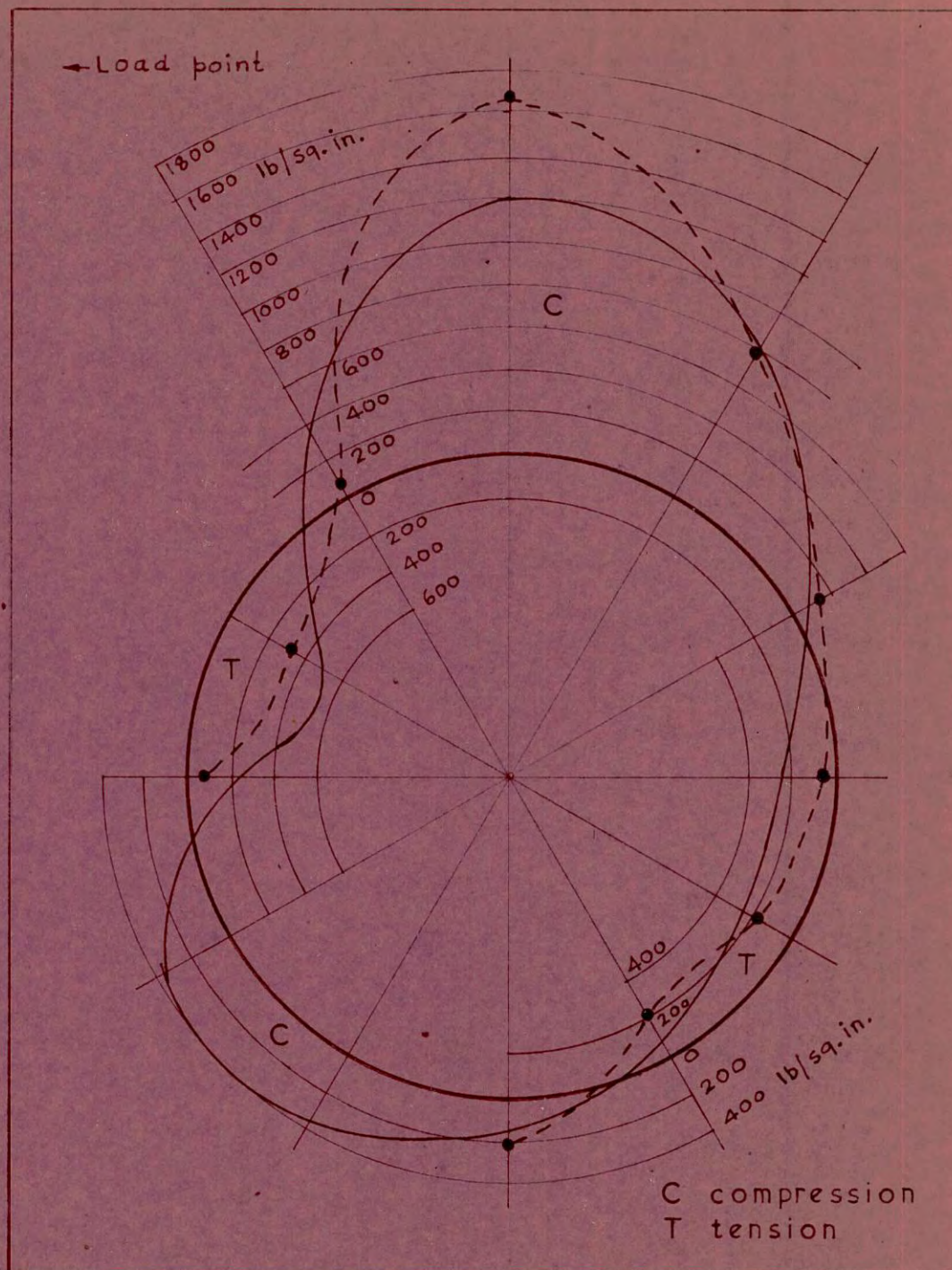


FIG. 7. 22 Stress trajectory due to bending with shear
Beam SO₂-U-4

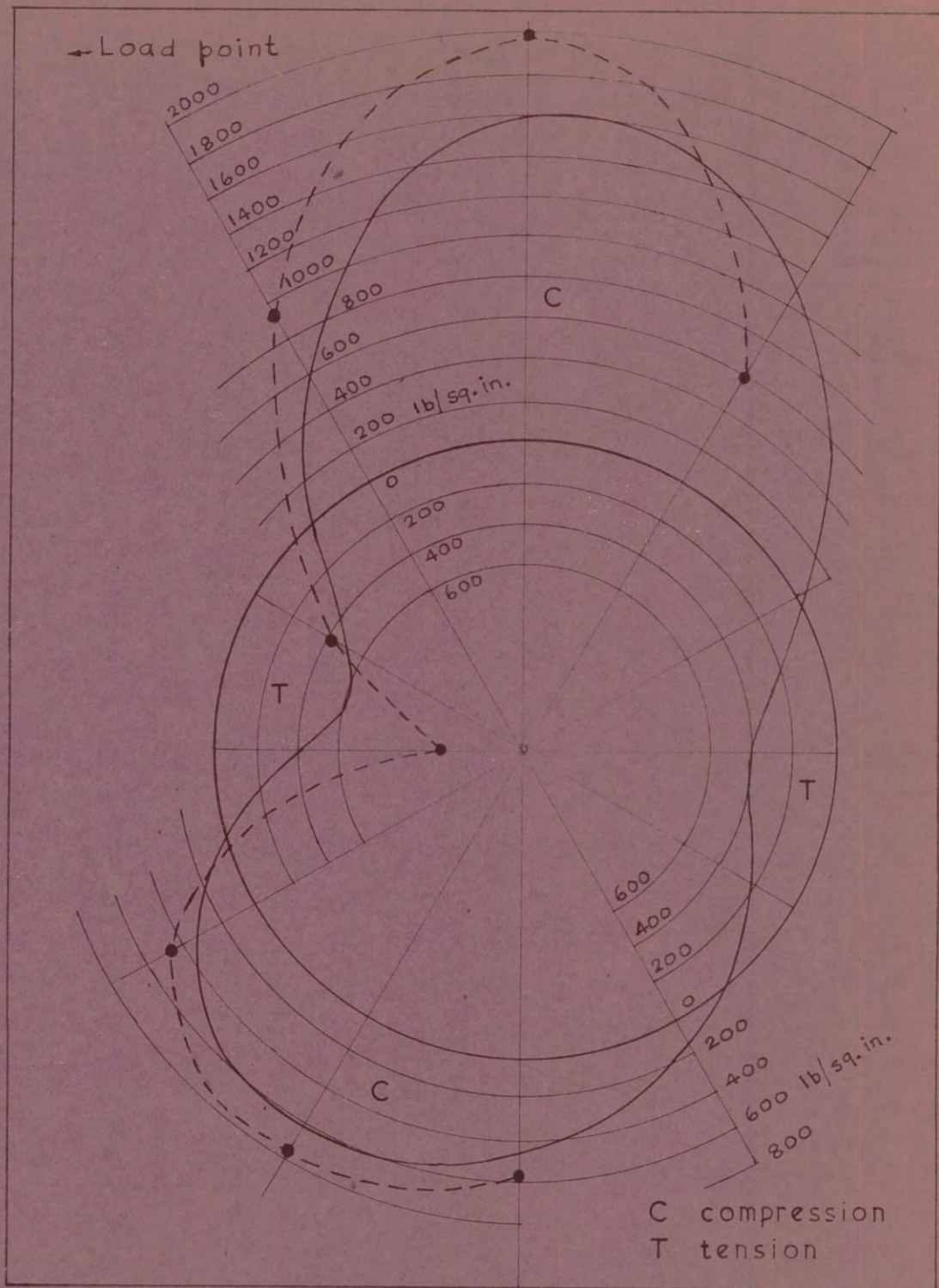
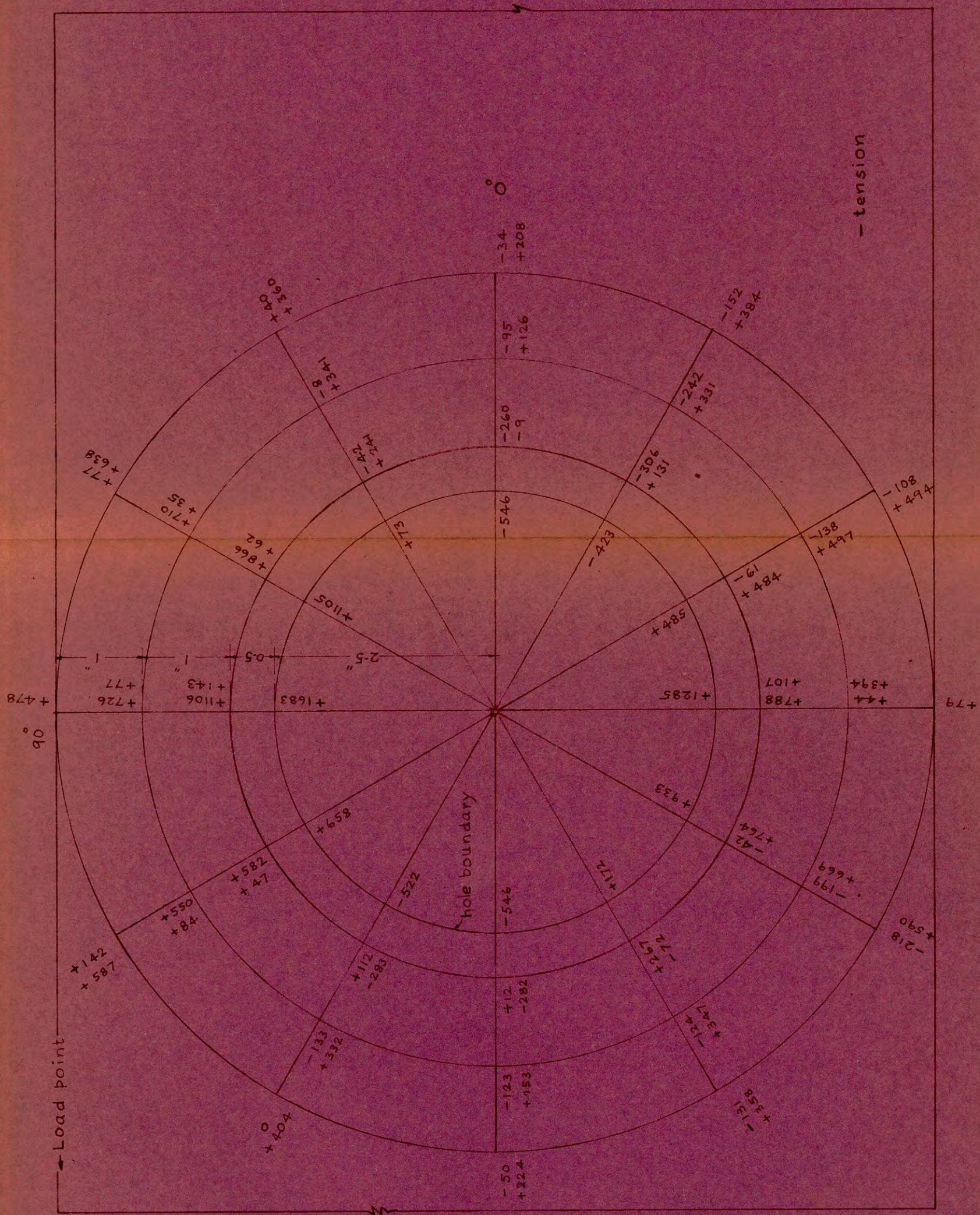


FIG.7. 23 Stress trajectory due to bending with shear
Beam MO₁-U-8



Load point →

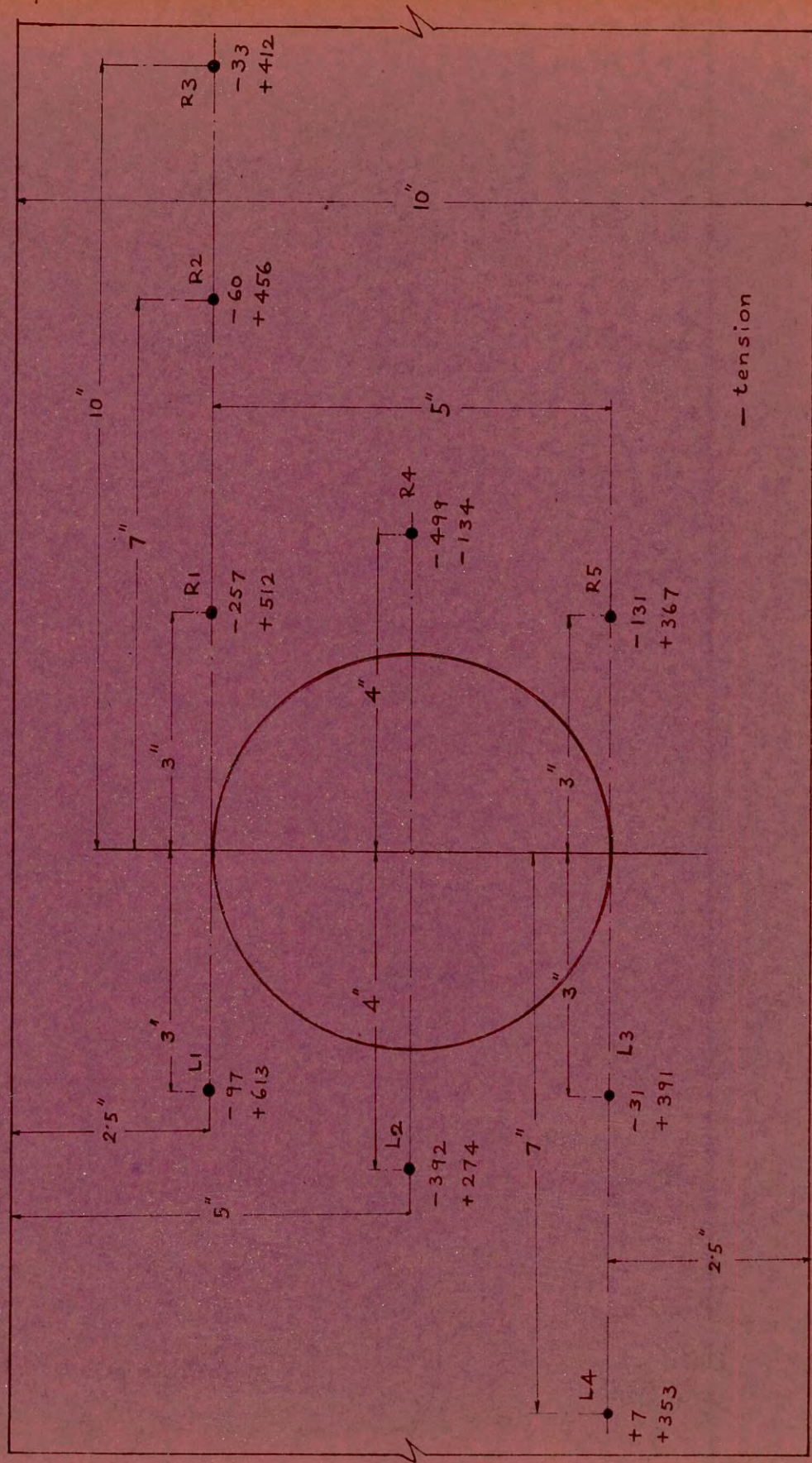
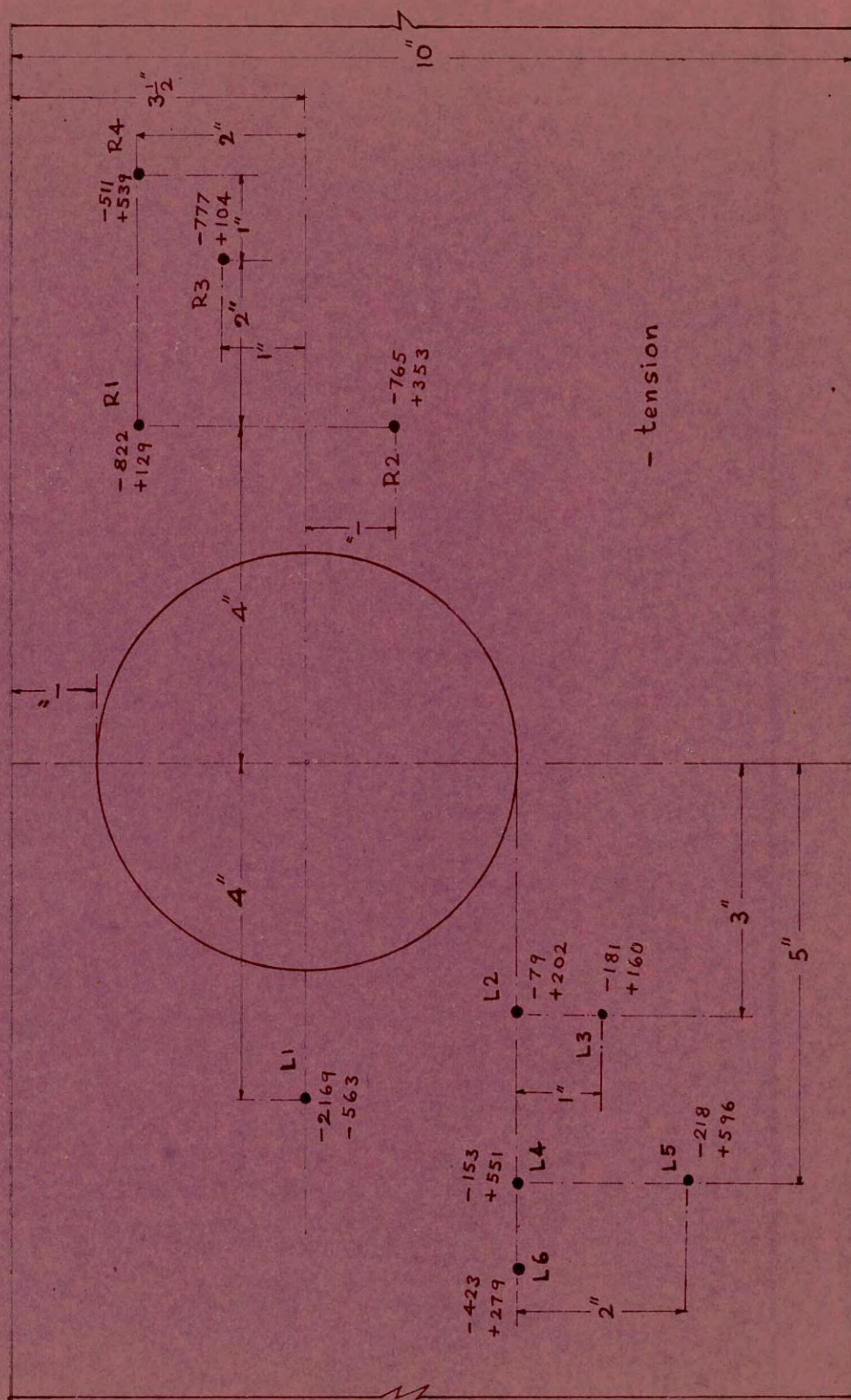


FIG. 7. 26 Principal stresses from 2" rosettes Beam SO₂-2 lb/sq. in.

Load point →



- tension

FIG. 7.27 Principal stresses from rosettes lb/sq.in. Beam SO₂-U-4

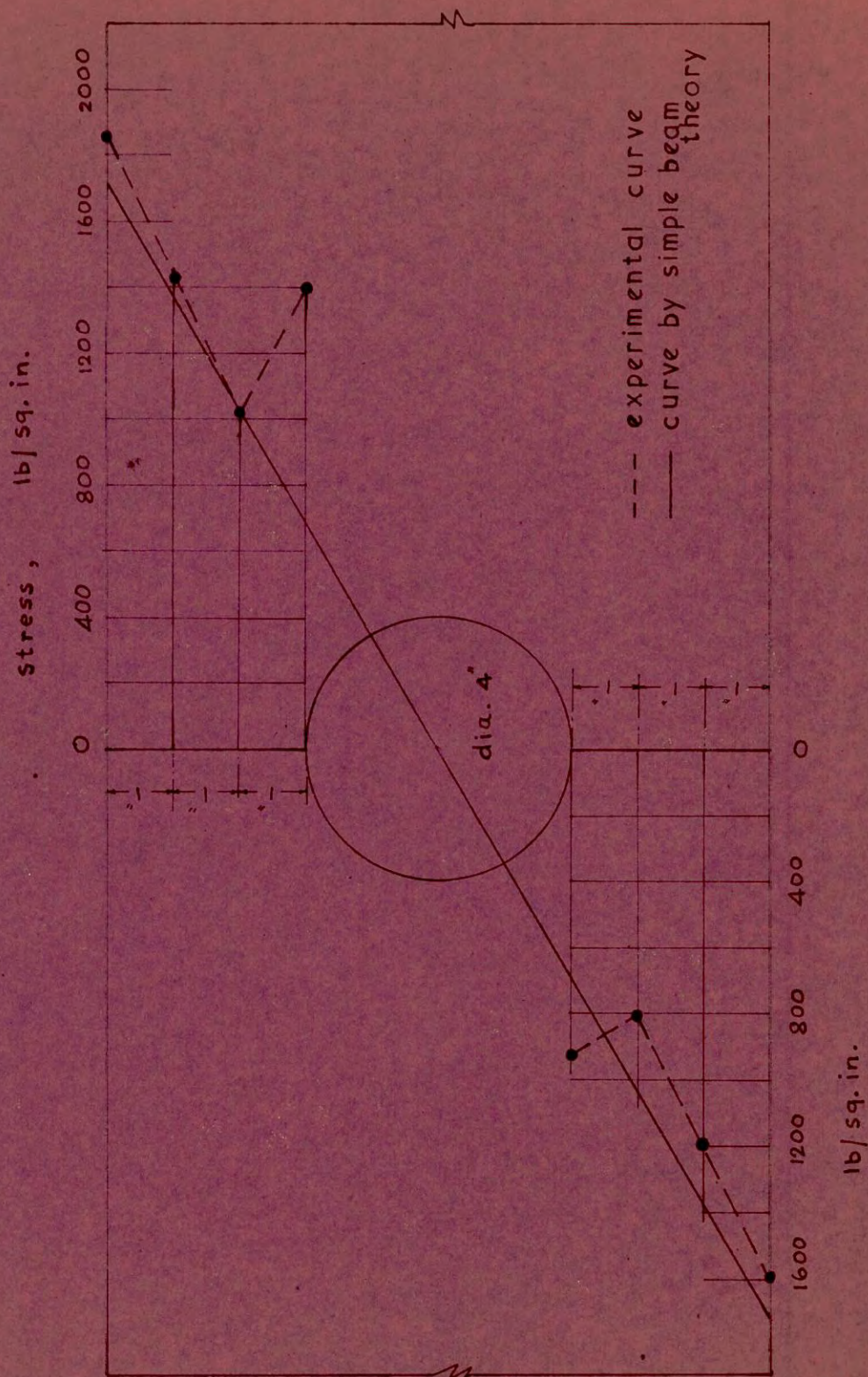


FIG. 7.28 Stress distribution at minimum section Beam BO₁-1

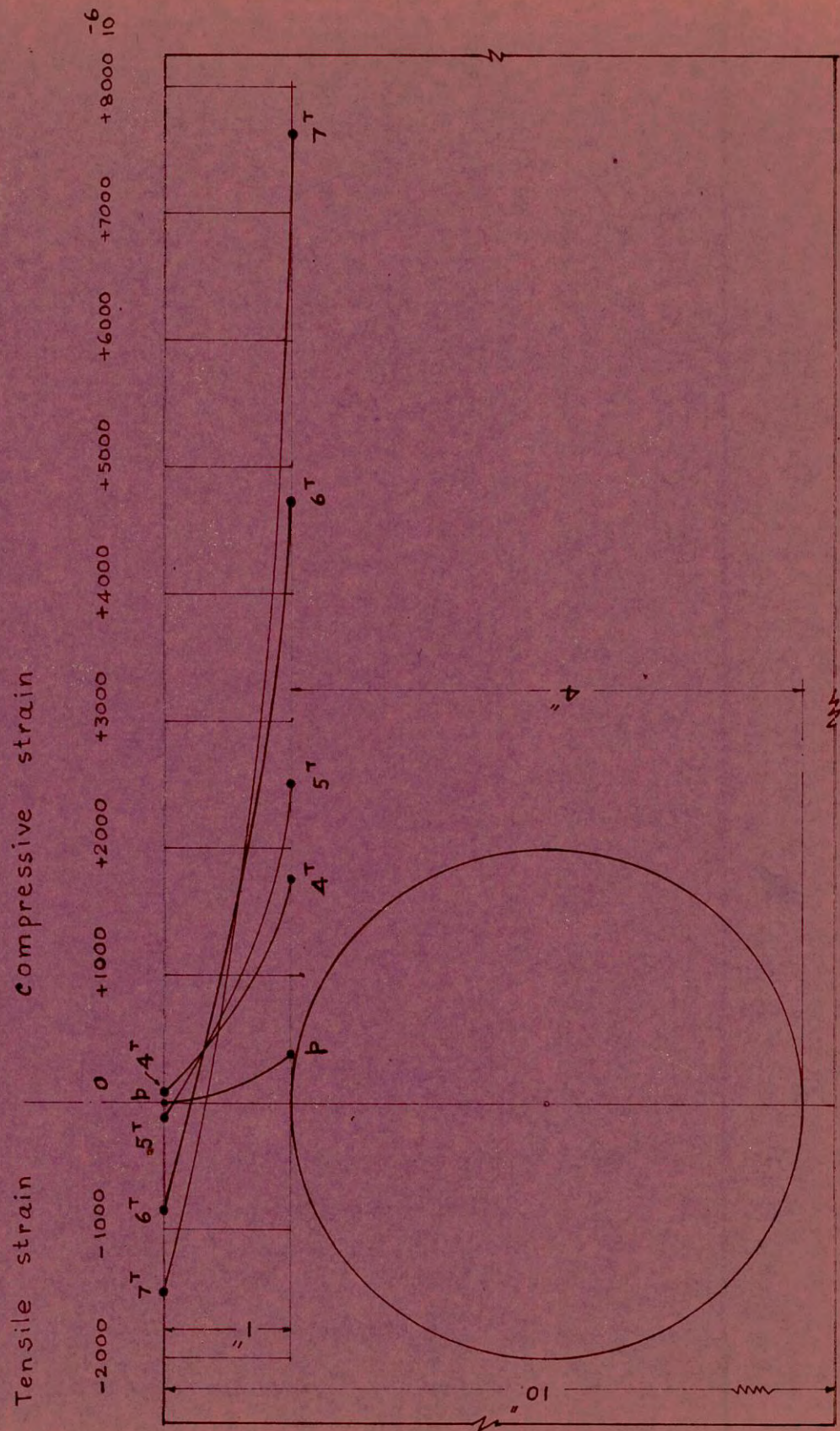


FIG. 7.29 Strain on top of hole Beam BO₁-U-3

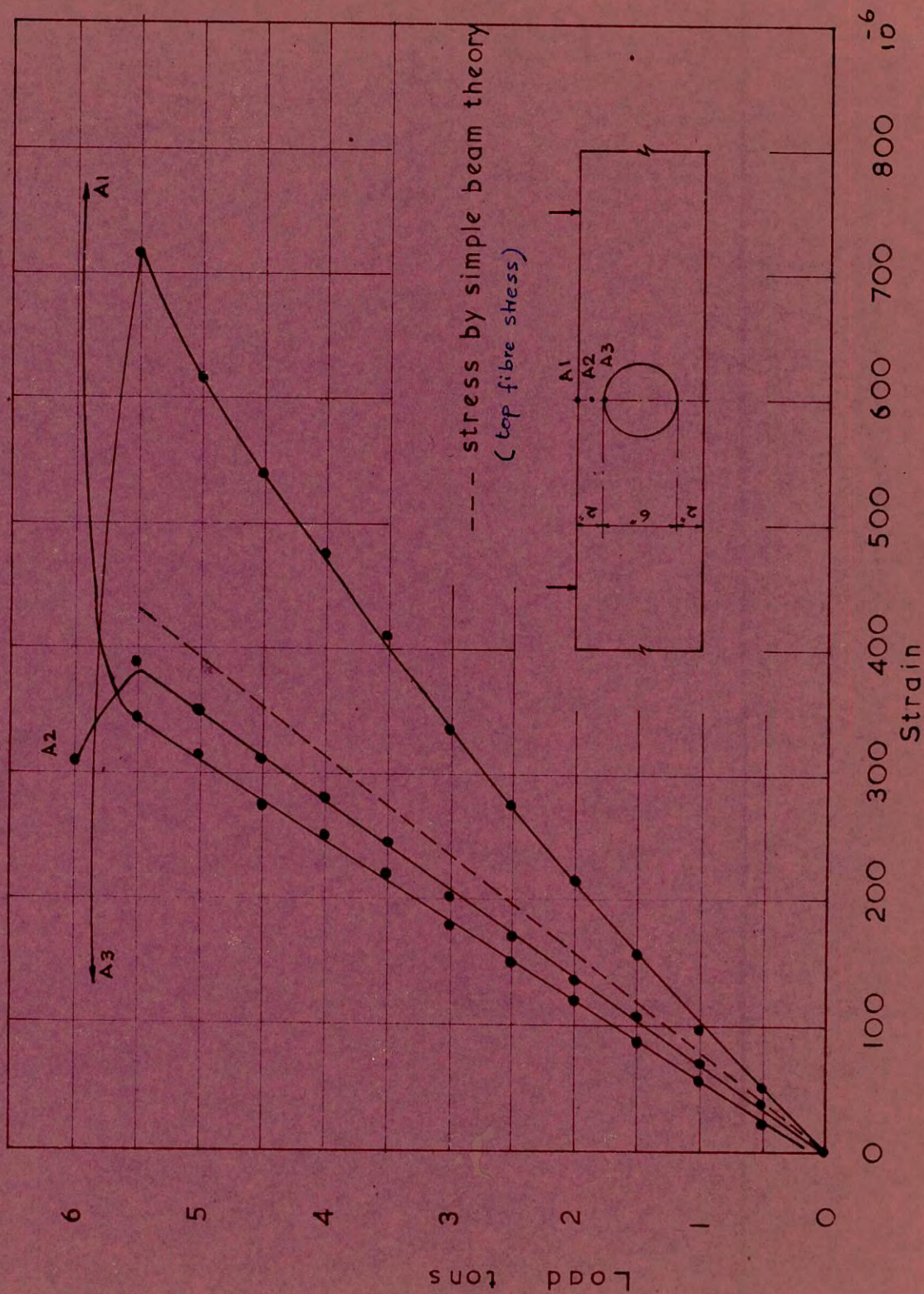


FIG. 7.30 Curve, load-concrete strain above hole Beam BO₃-4

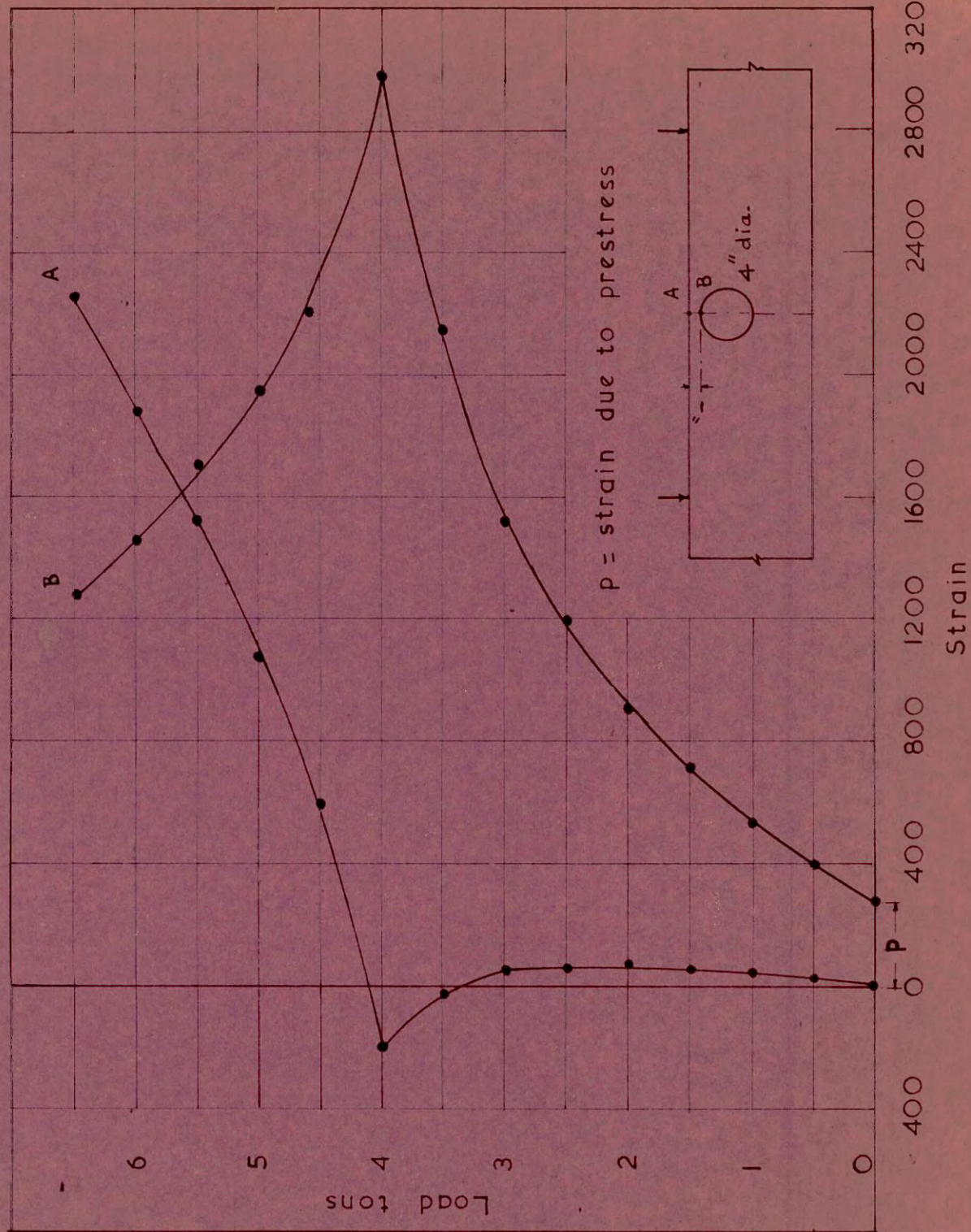


FIG. 7.31 Concrete strain above hole Beam BO₁-U-6

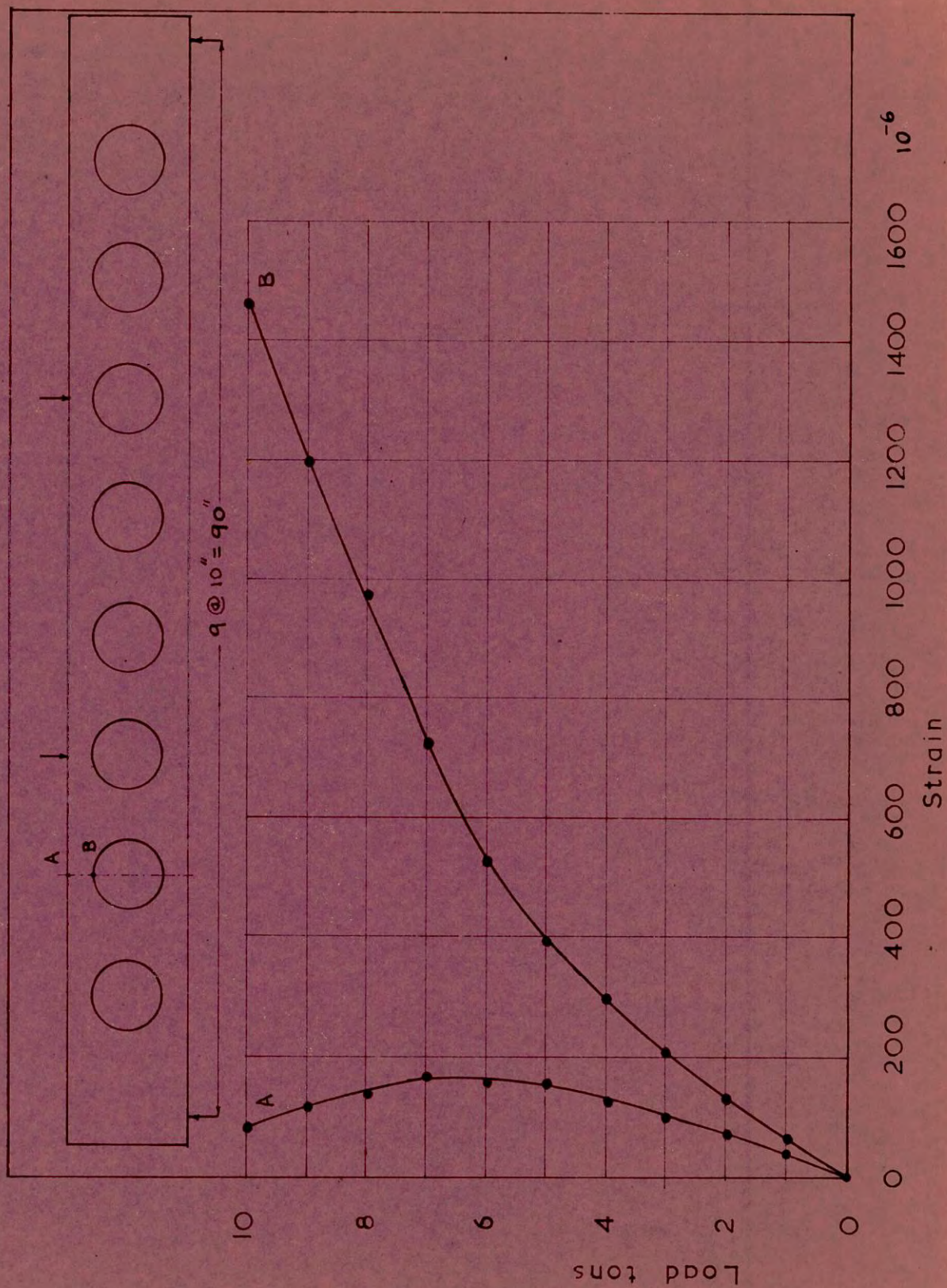


FIG.7.32 Load-concrete strain above hole Beam MO₃-3

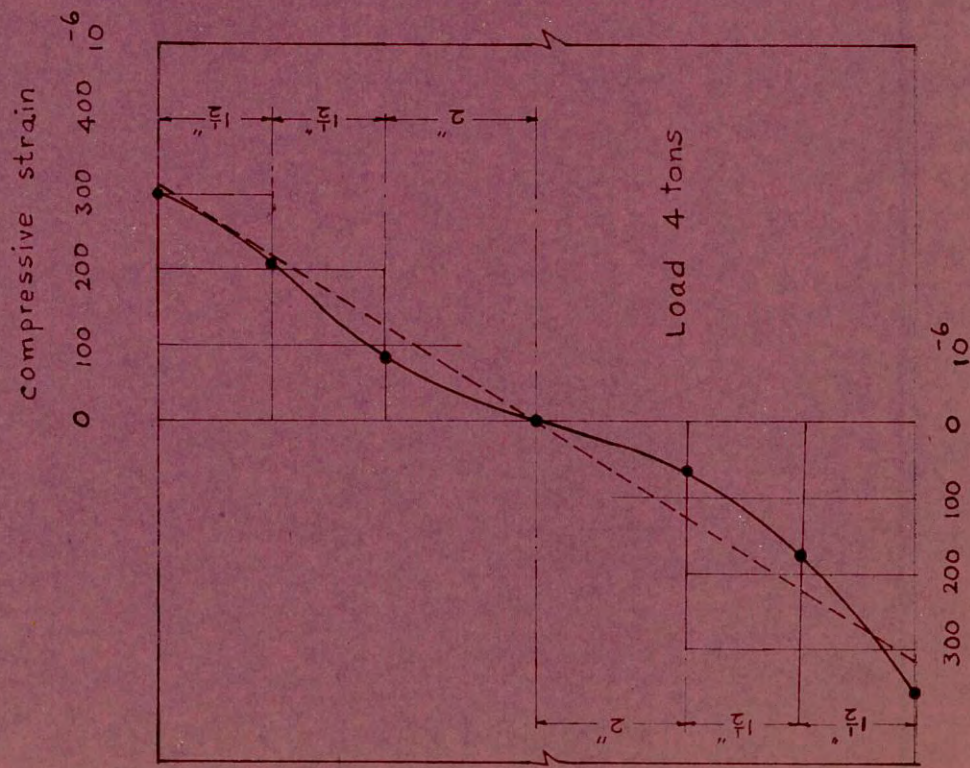


FIG. 7.33(a) Strain distribution midway between adjacent holes in bending

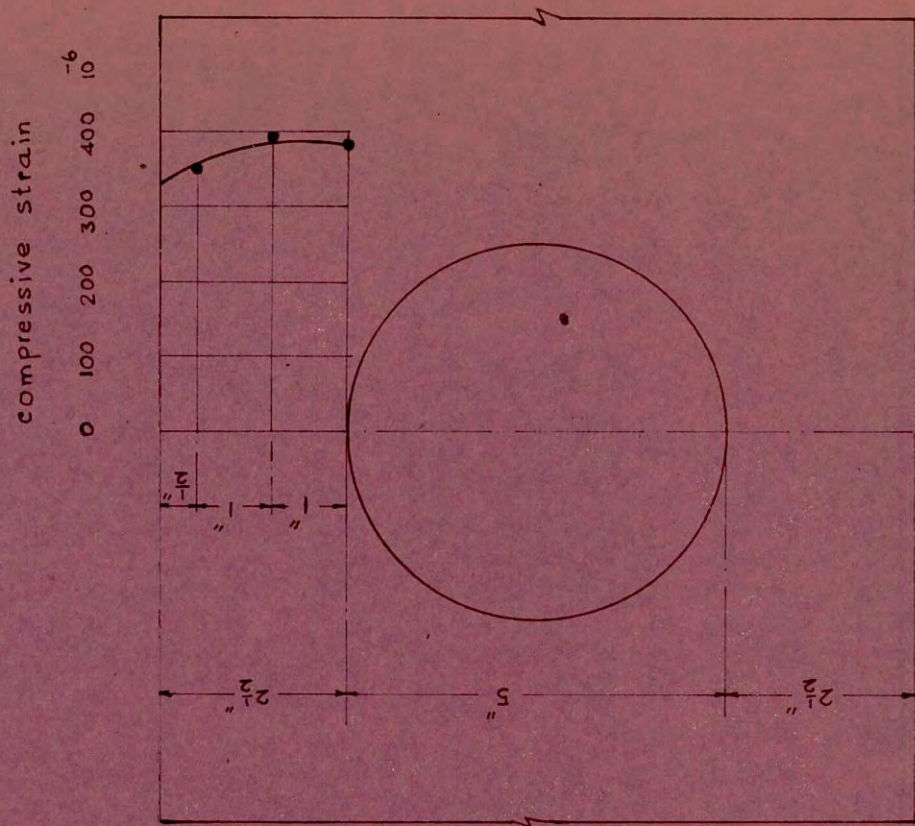


FIG. 7.33(b) Strain distribution above hole Beam $\text{MMO}_2\text{-B-4}$

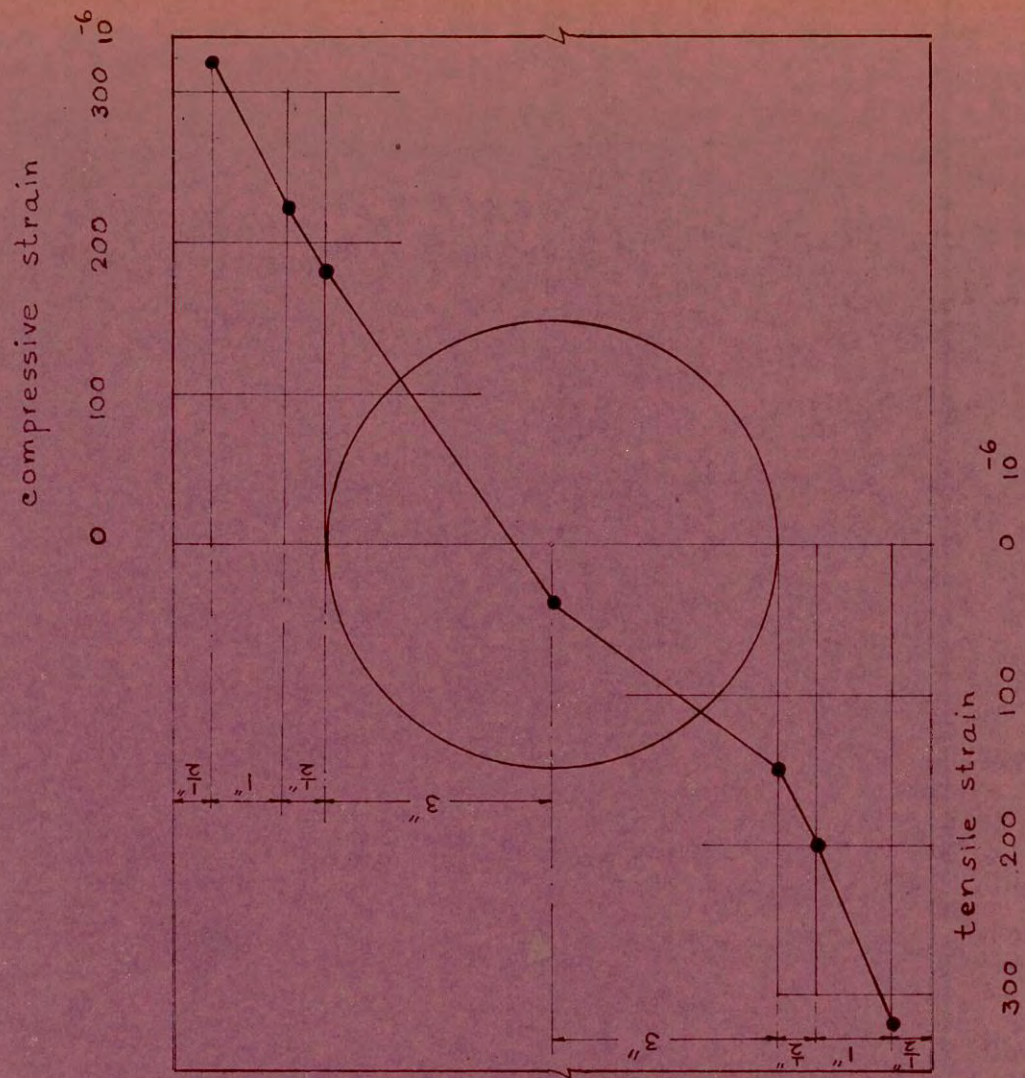


FIG. 7. 34(b) Concrete strain due to bending
with shear Beam MMO₃-B-5

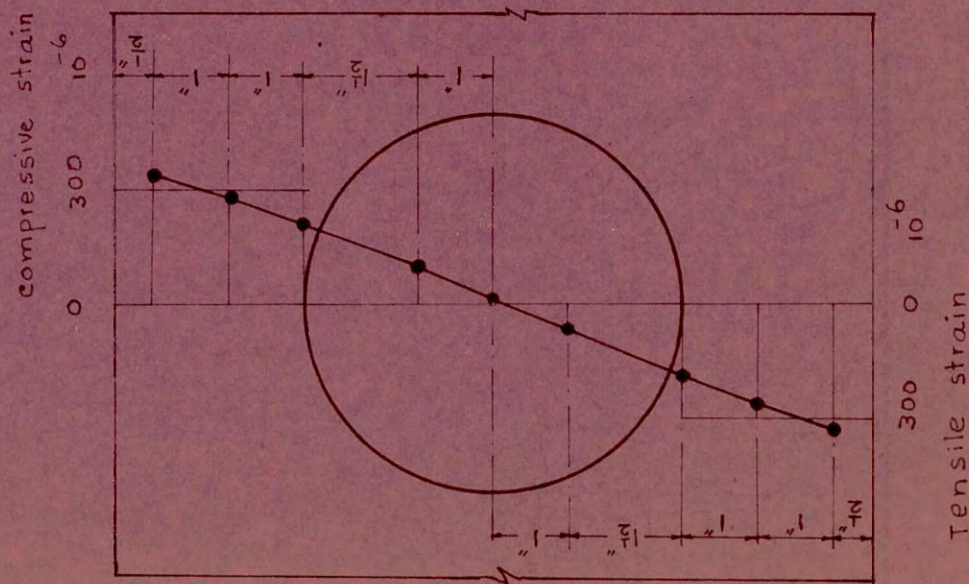


FIG. 7. 34(a) Concrete strain in
pure bending Beam MMO₂-B-4

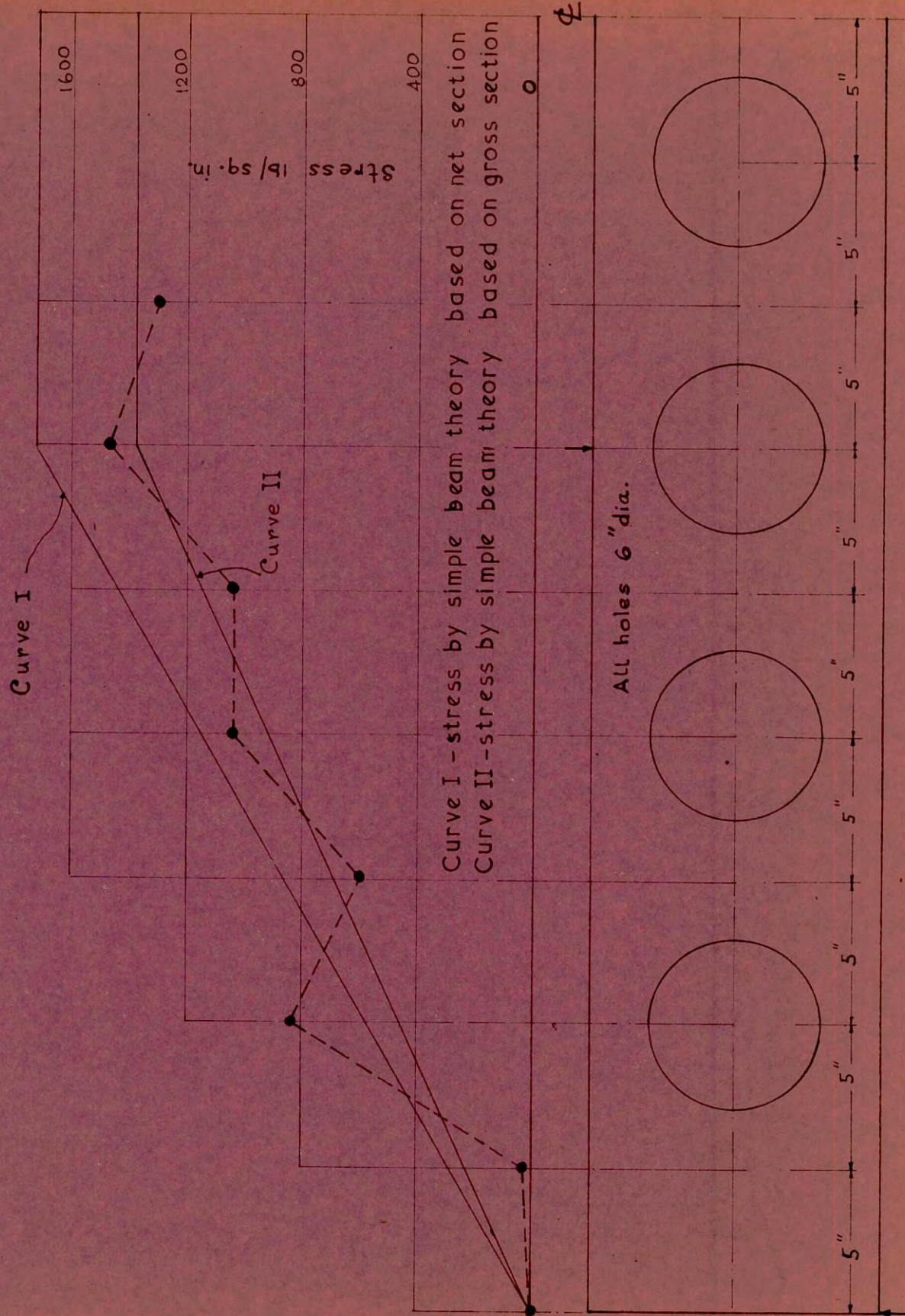


FIG. 7.35 Top fibre stresses Beam MO₃-L-5

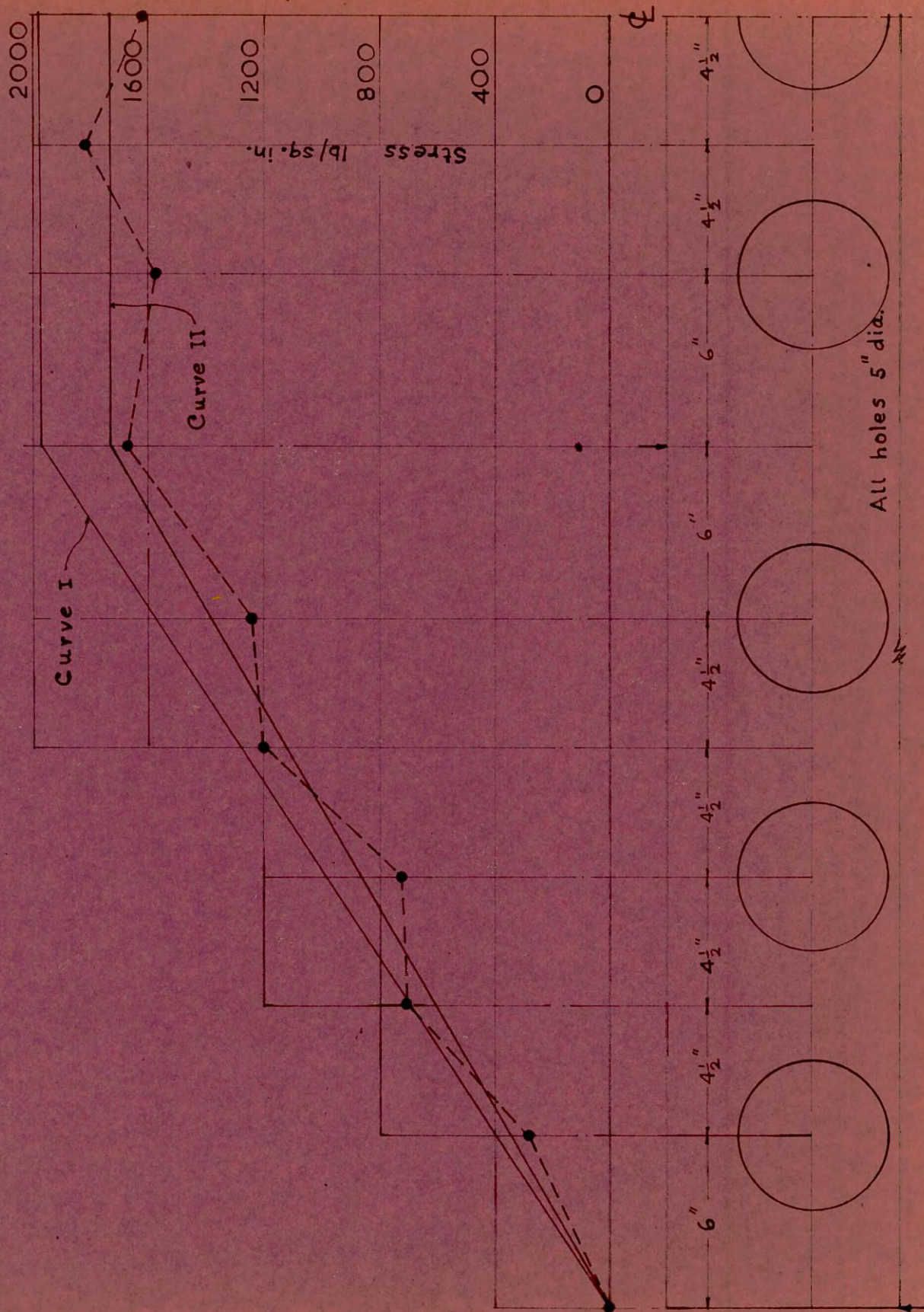


FIG. 7.36 Top fibre stresses Beam MMO₂-9

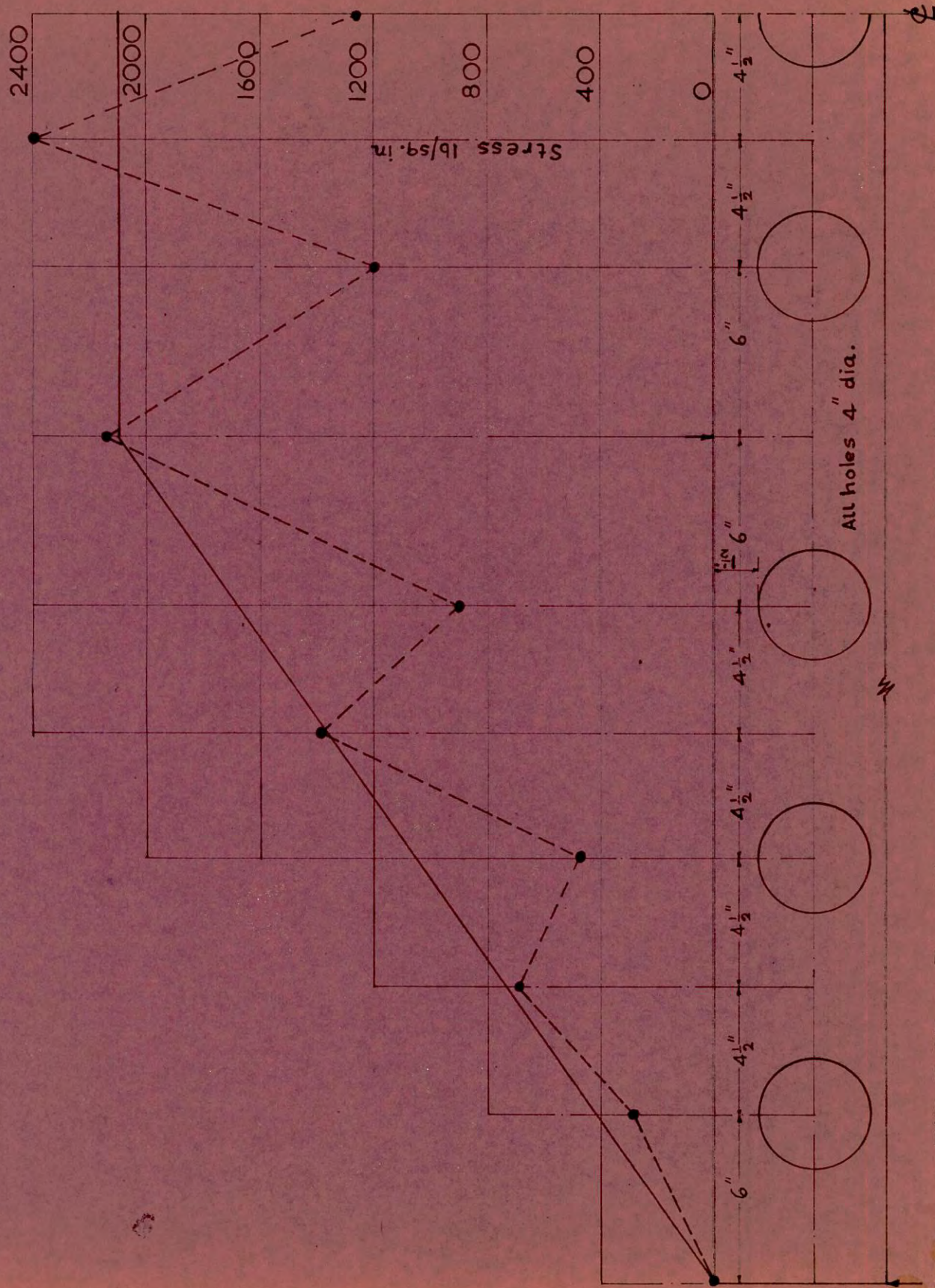


FIG. 7.37 Top fibre stresses Beam MMO1-U-B-2

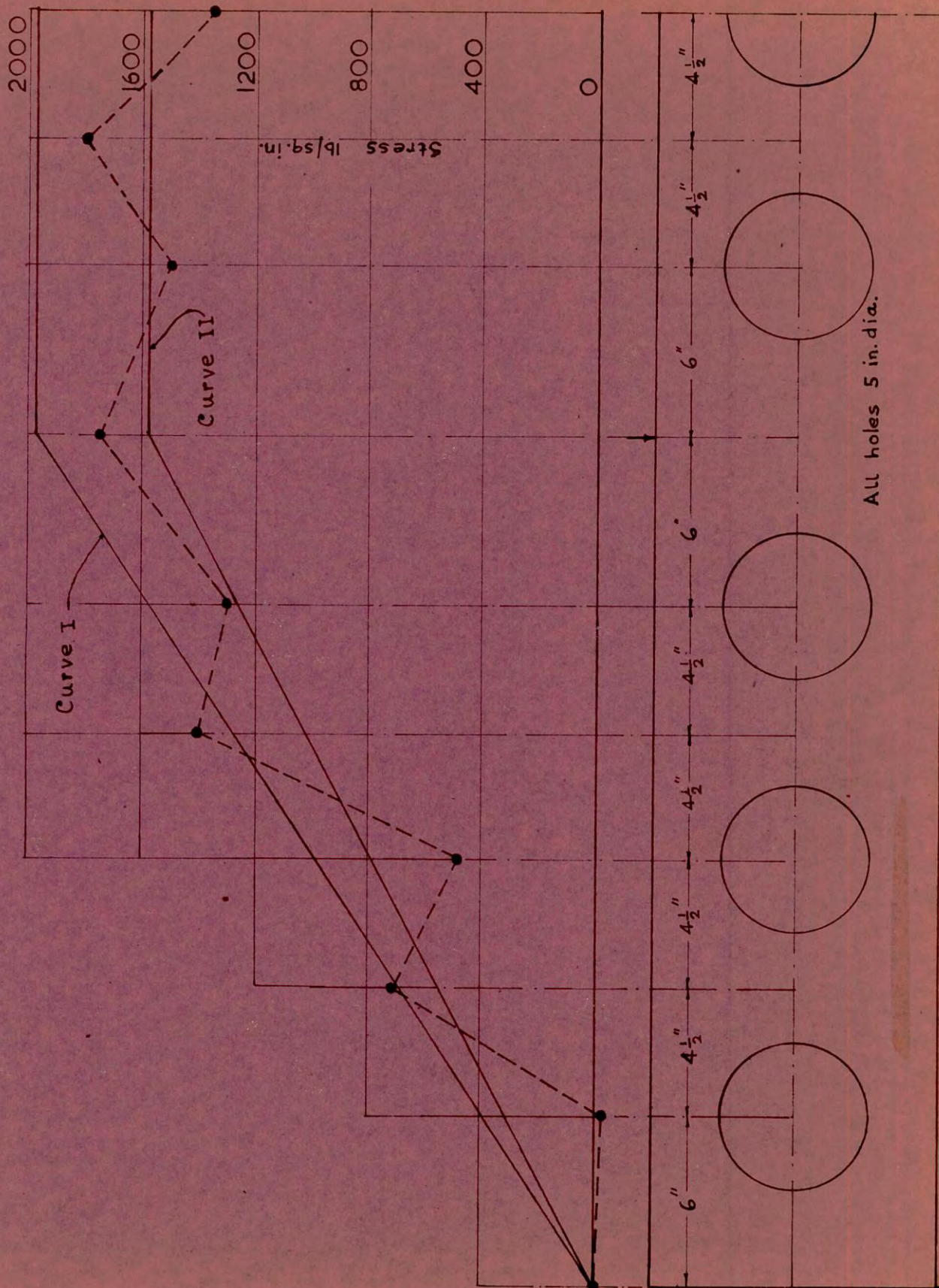


FIG. 7.38 Top fibre stresses

Beam MMO₂-B-4

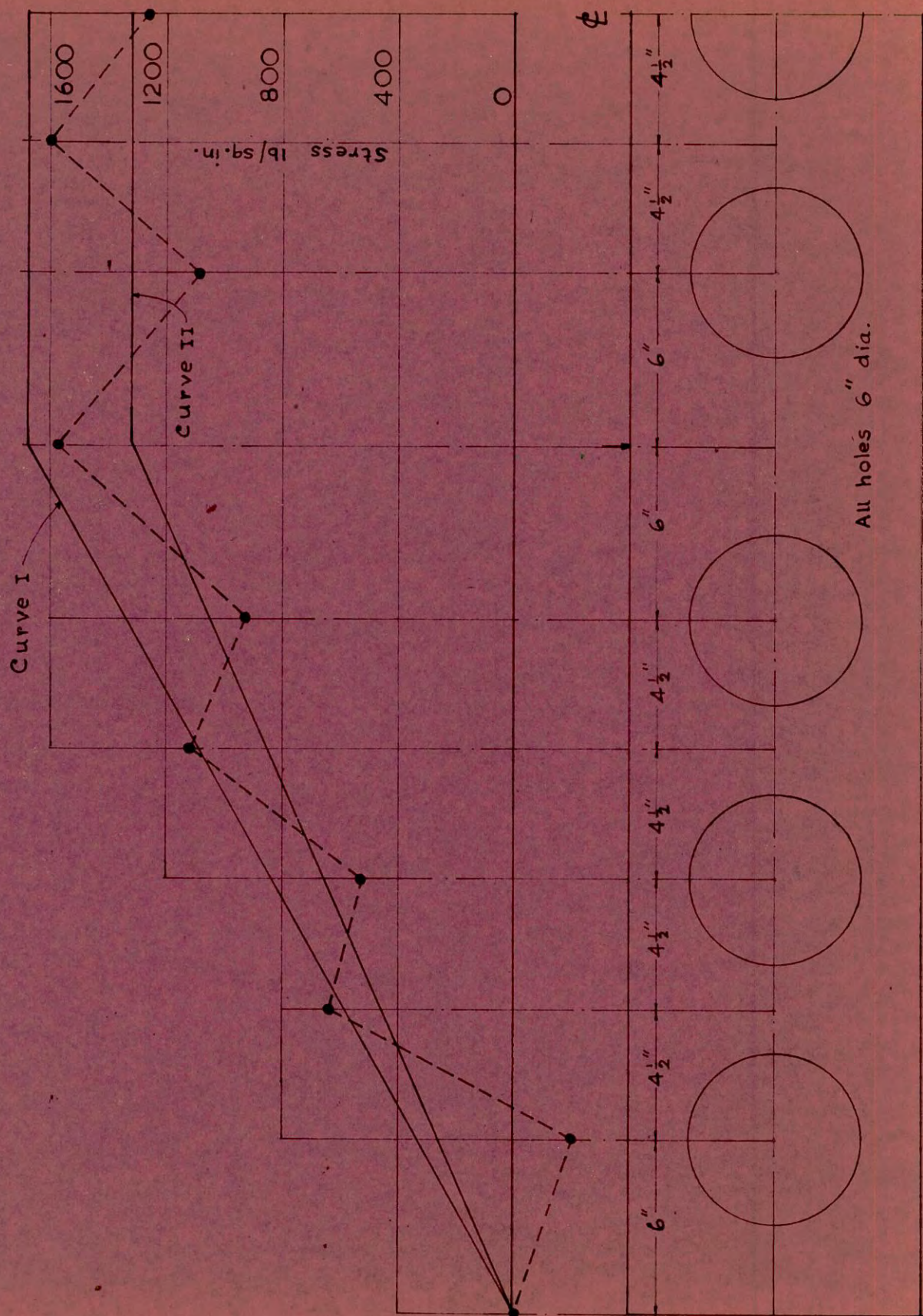
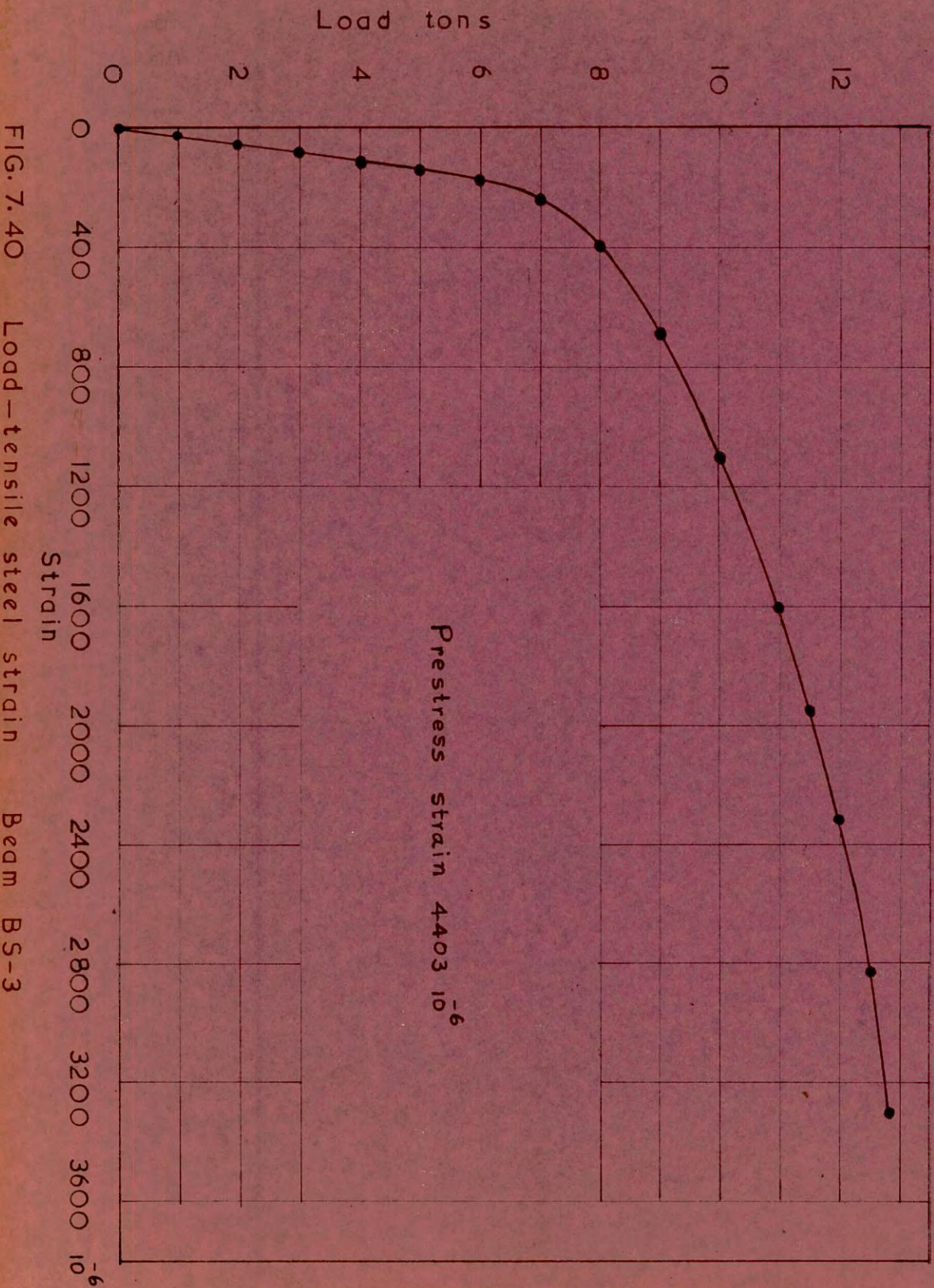
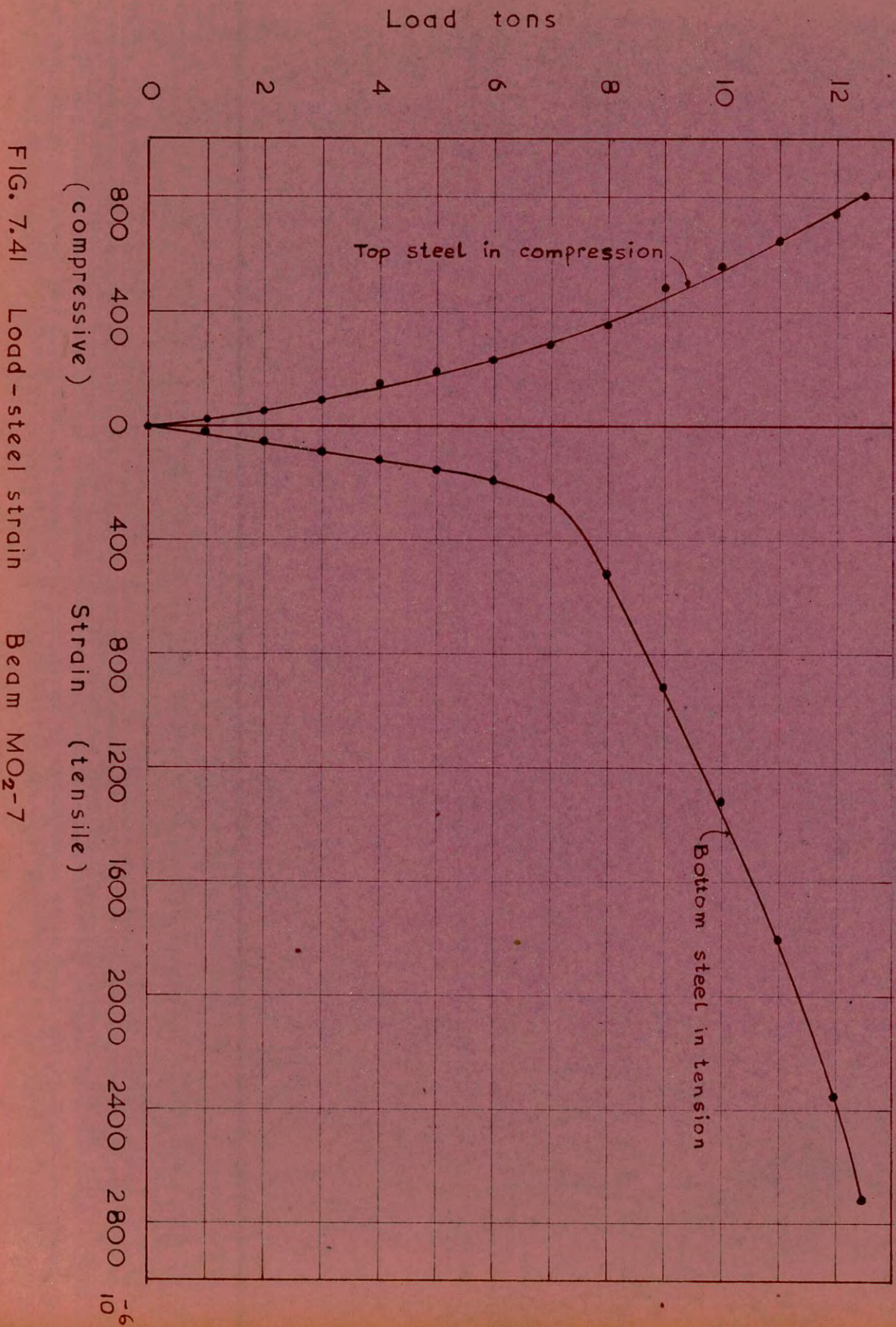


FIG. 7.39 Top fibre stresses Beam MMO₃-B-5





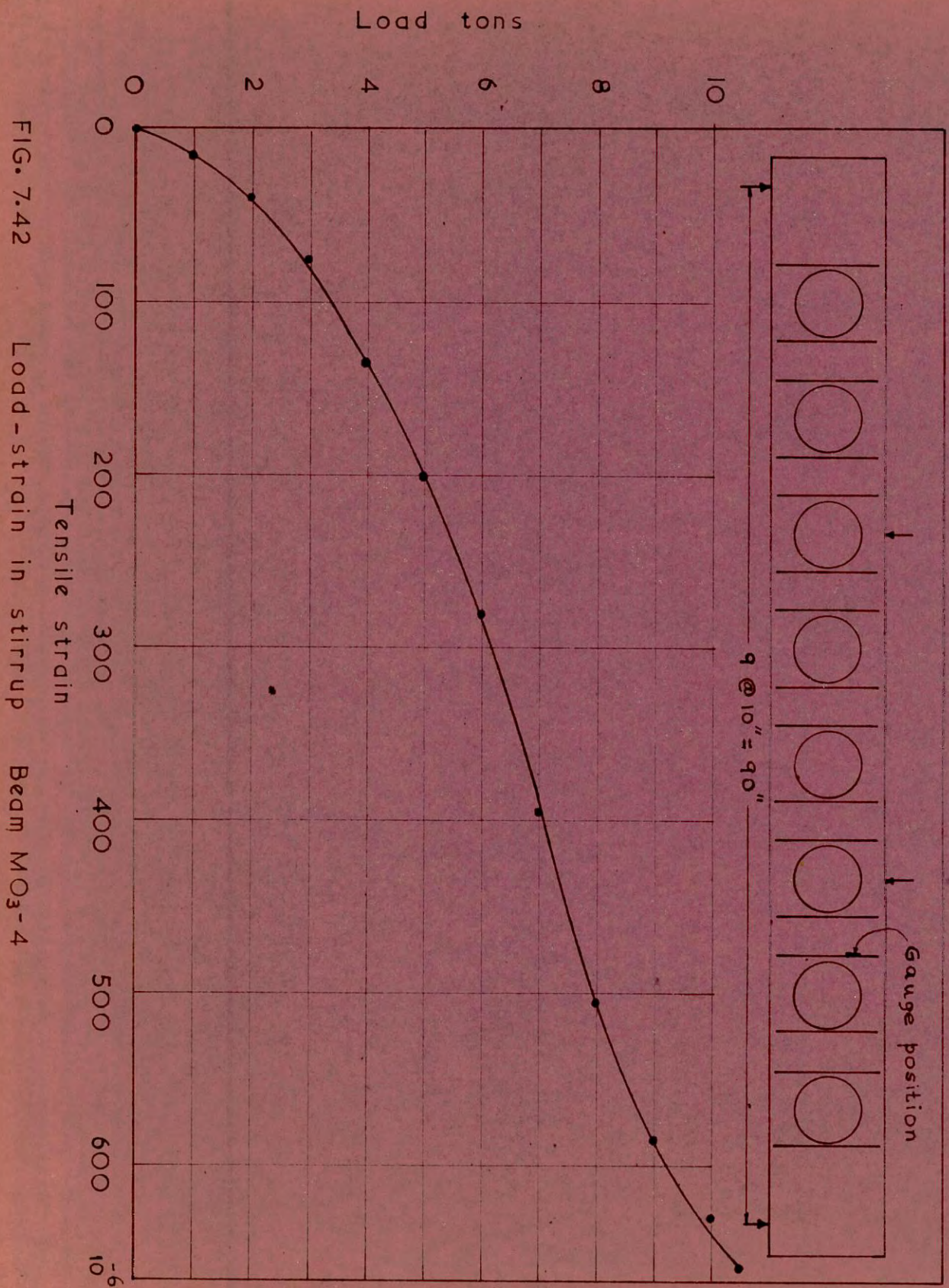


FIG. 7.42

Load-strain in stirrup

Beam MO₃-4

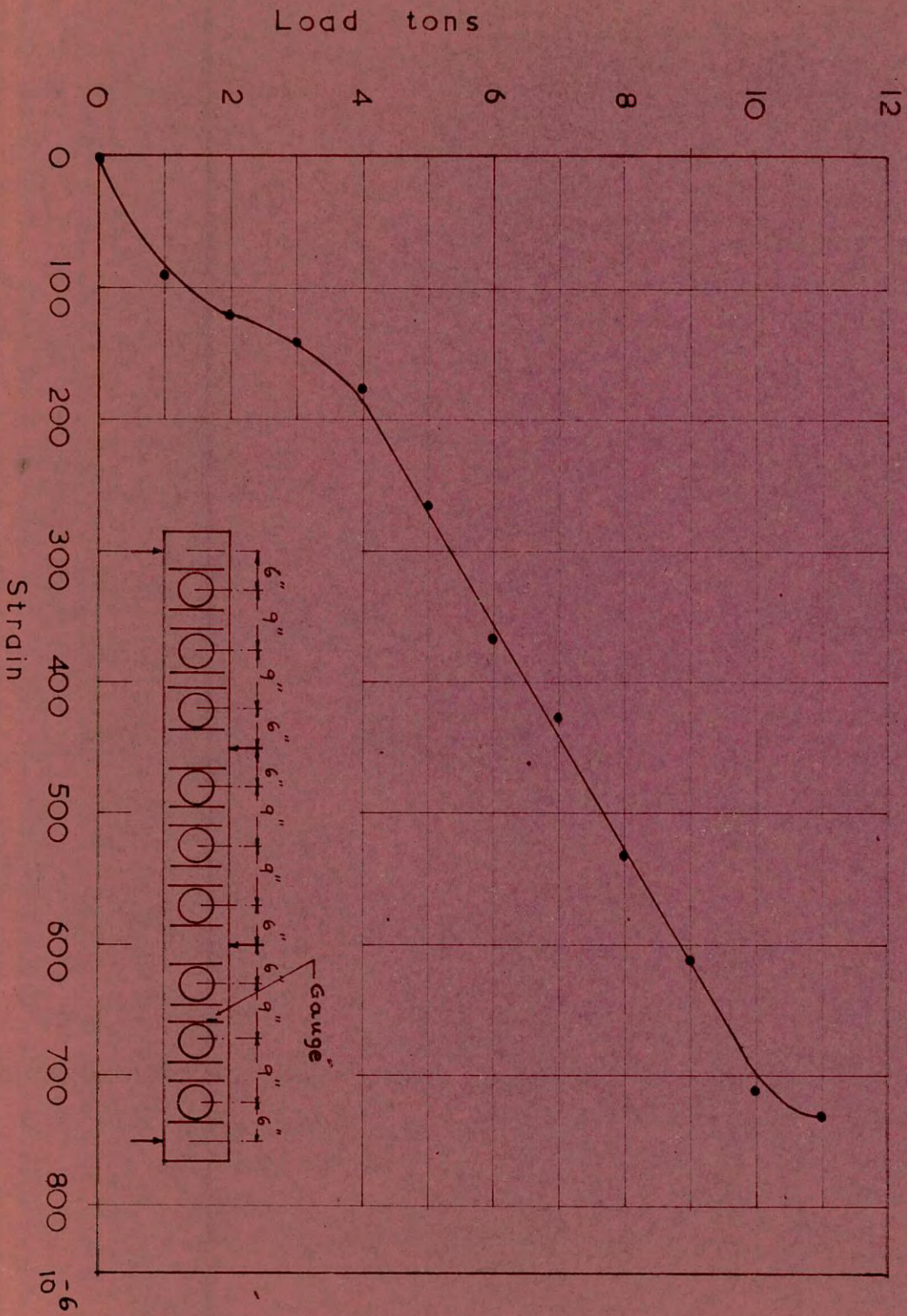


FIG. 7.43 Load - tensile strain in stirrup Beam MMO₂-9

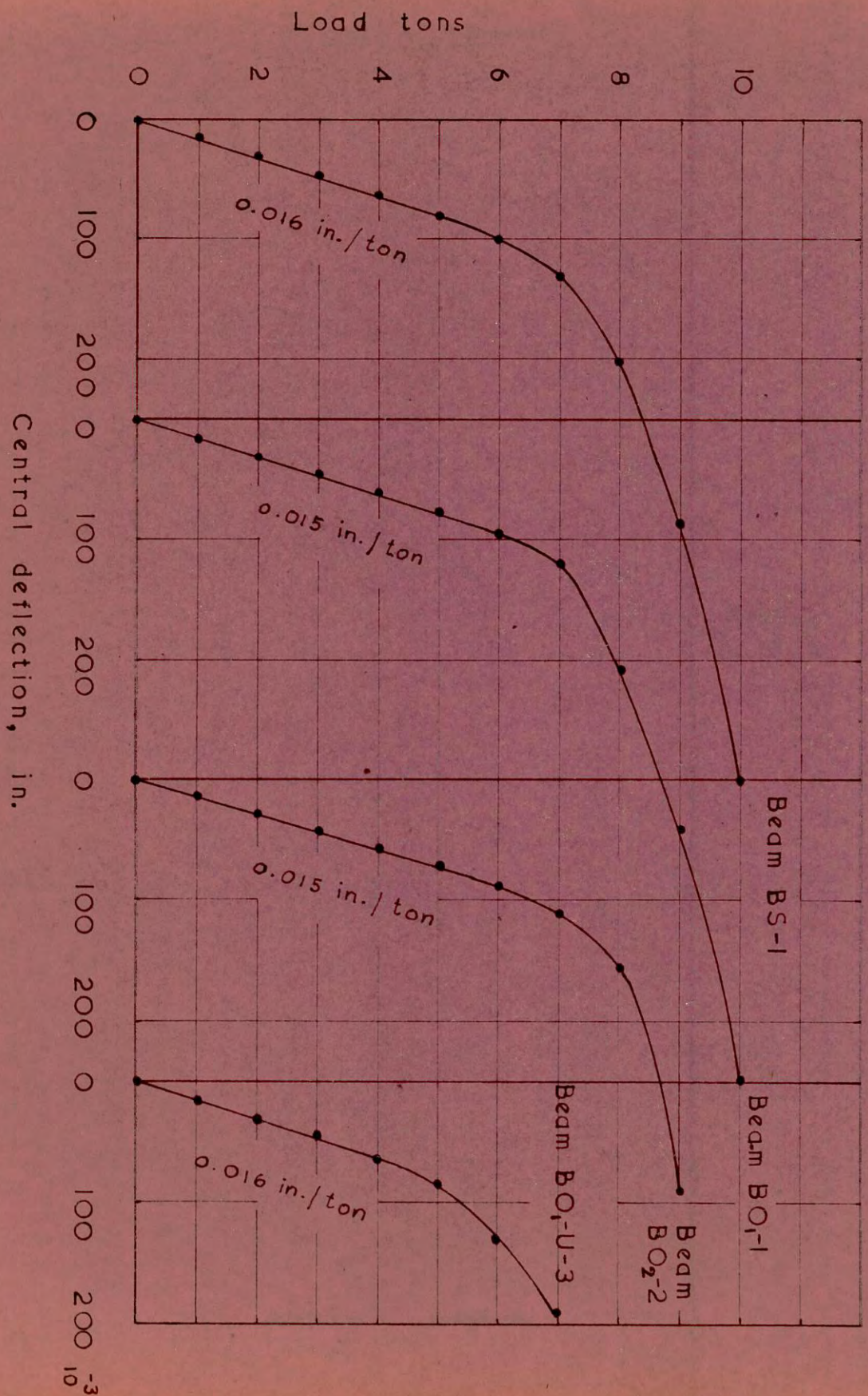


FIG. 7.44

Load - central deflection curves

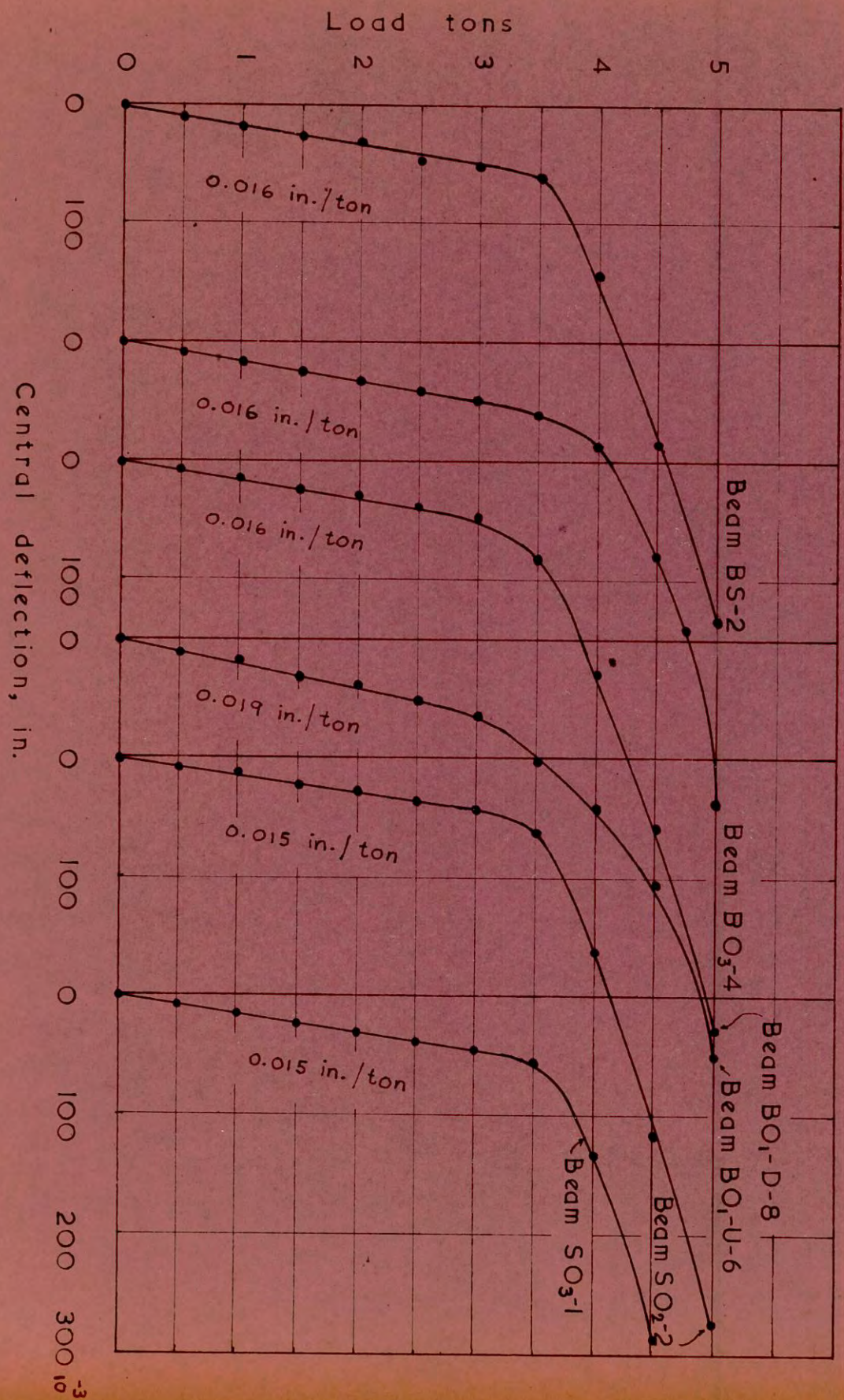


FIG. 7.45

Load - central deflection curves

Load tons

0 1 2 3 4 5 6 7

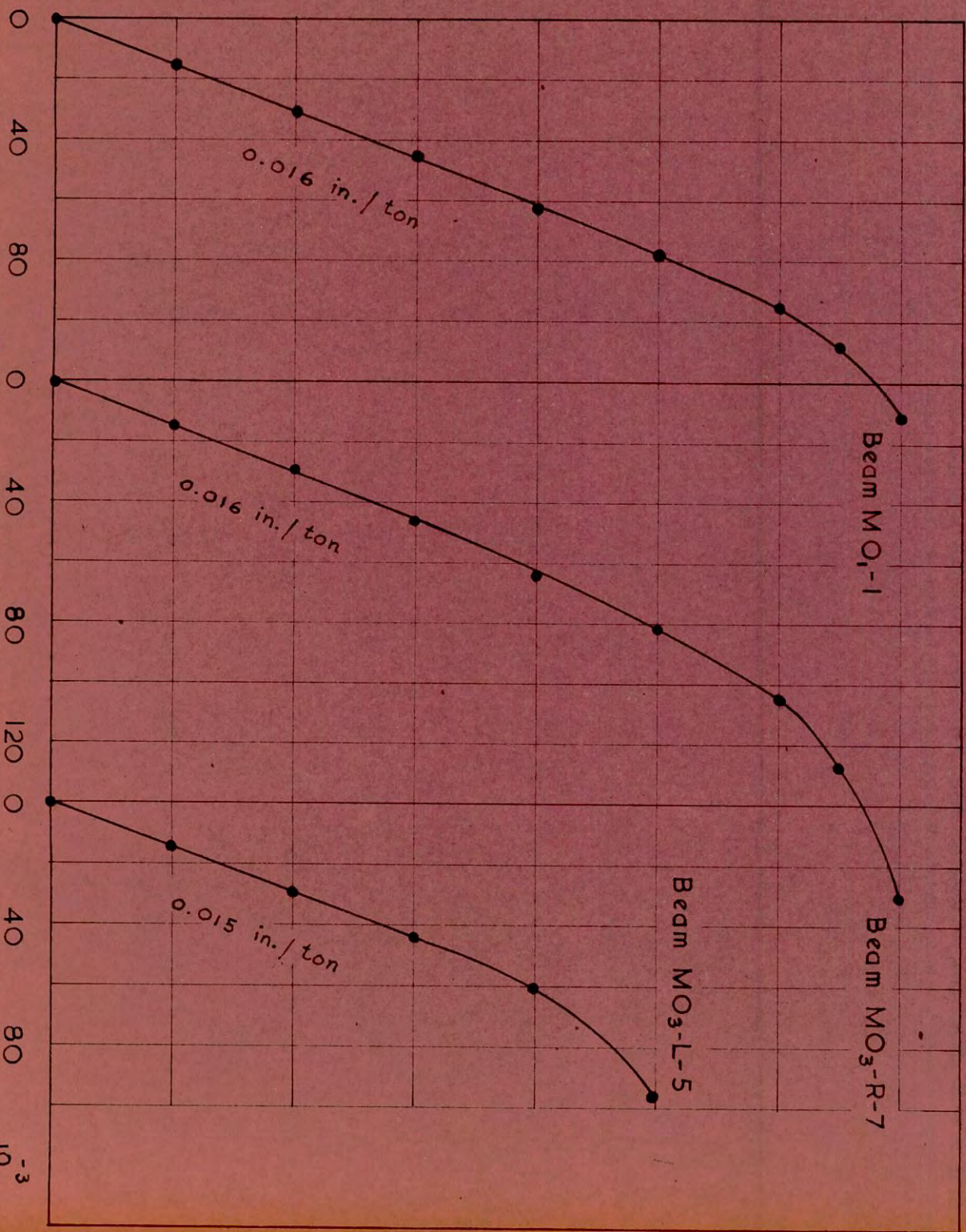


FIG. 7.46 Load-central deflection curves

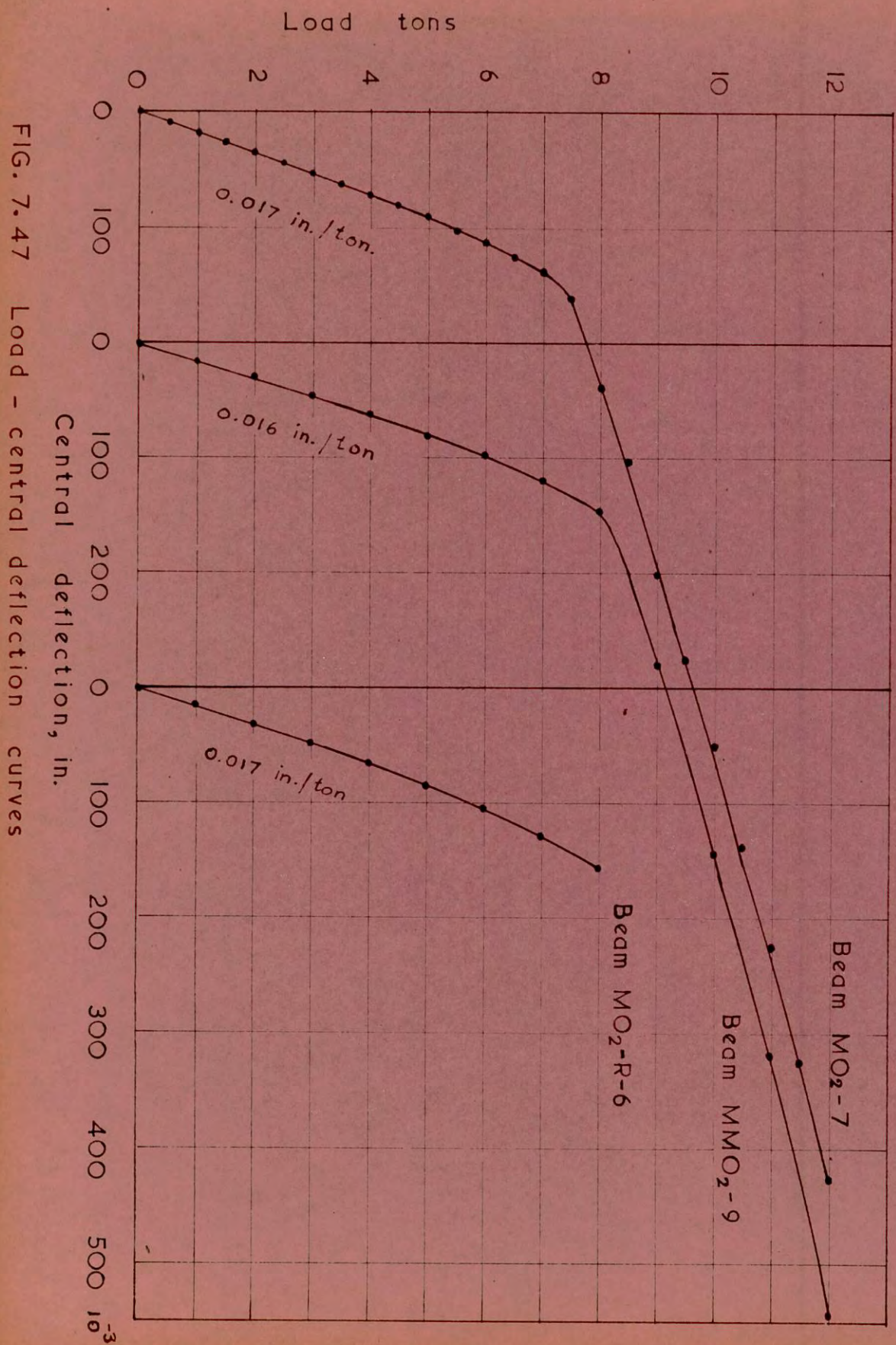


FIG. 7.47 Load - central deflection curves

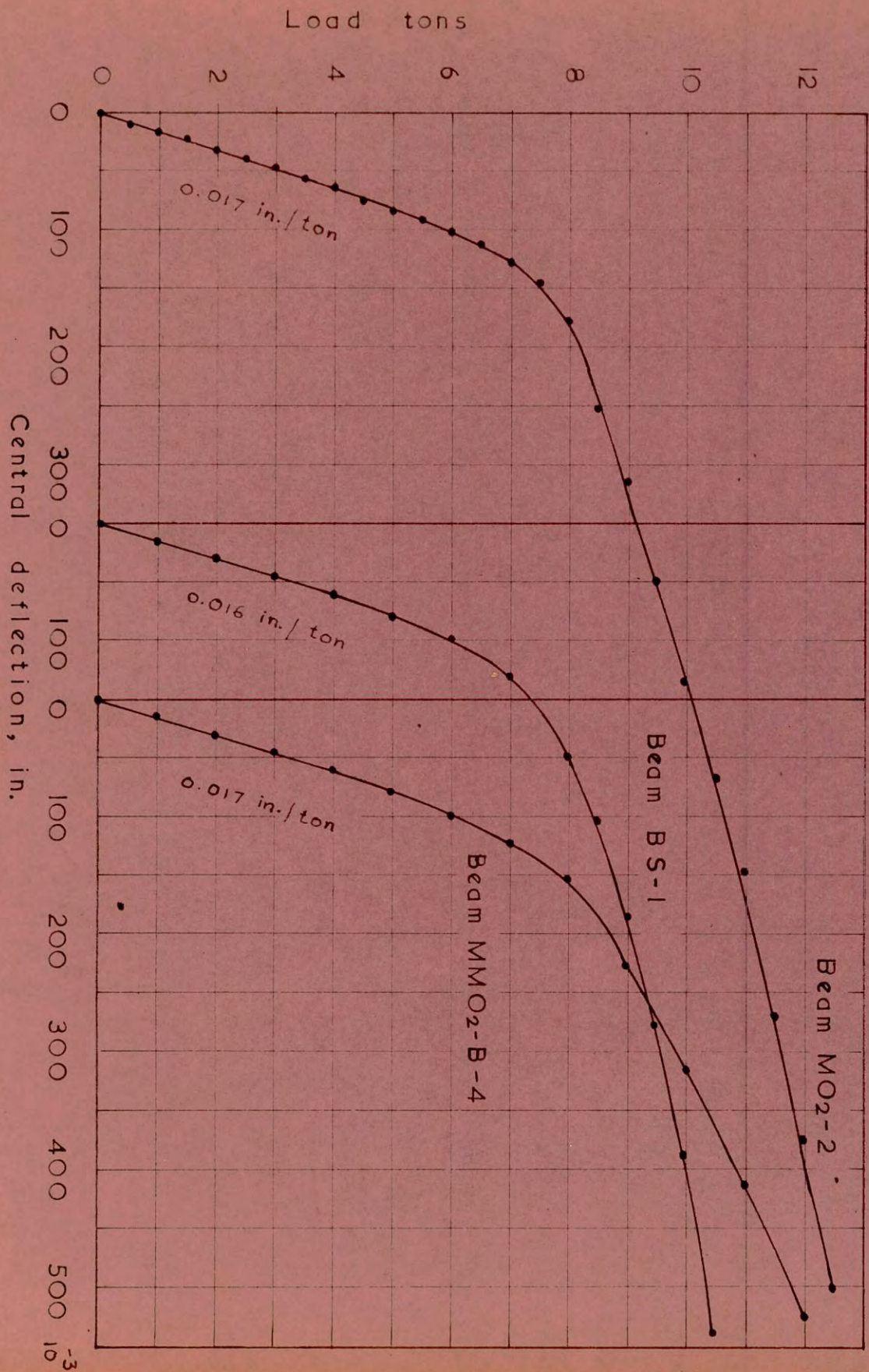


FIG. 7.48 Load - central deflection curves

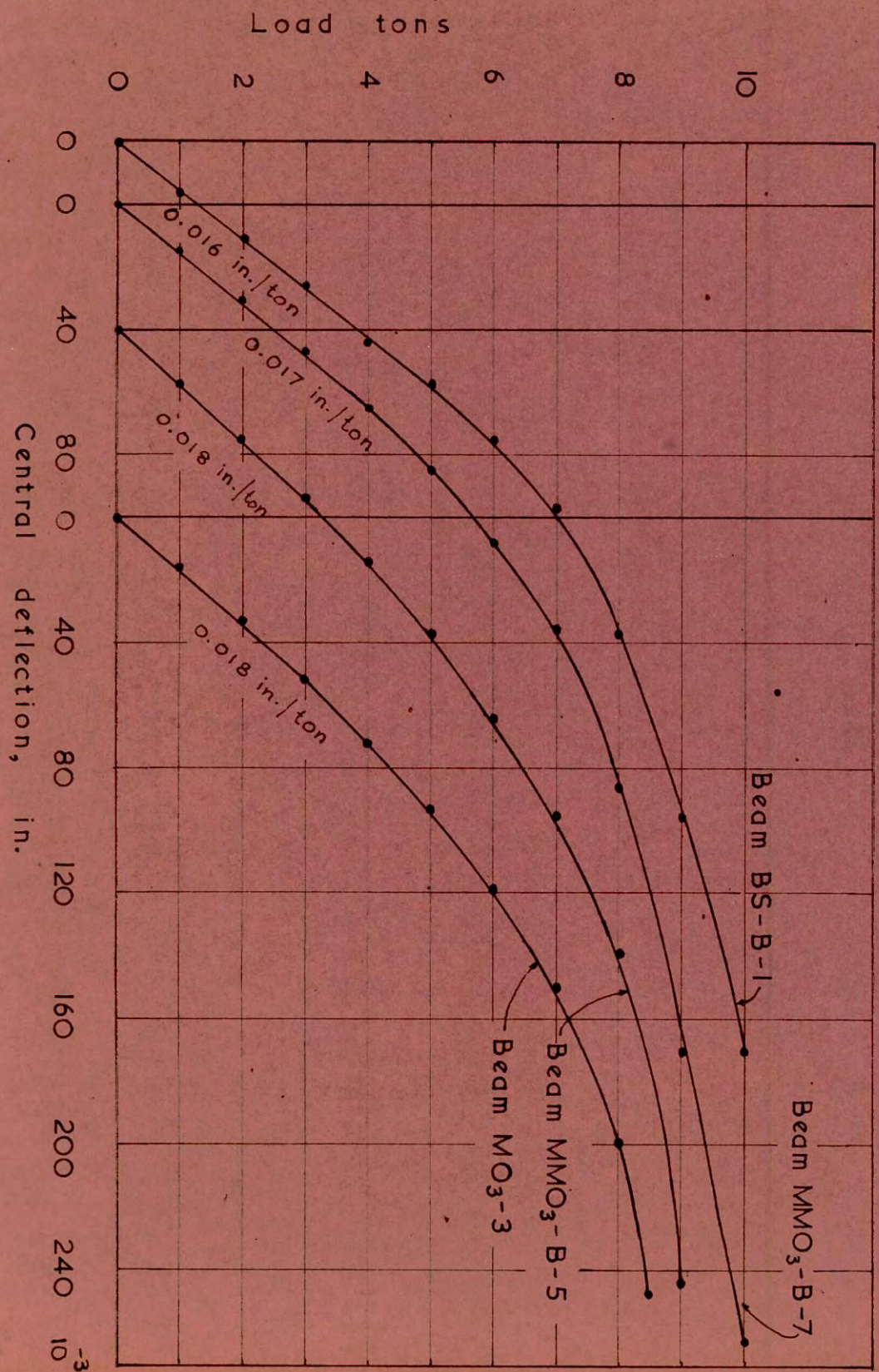


FIG. 7.49 Load-central deflection curves

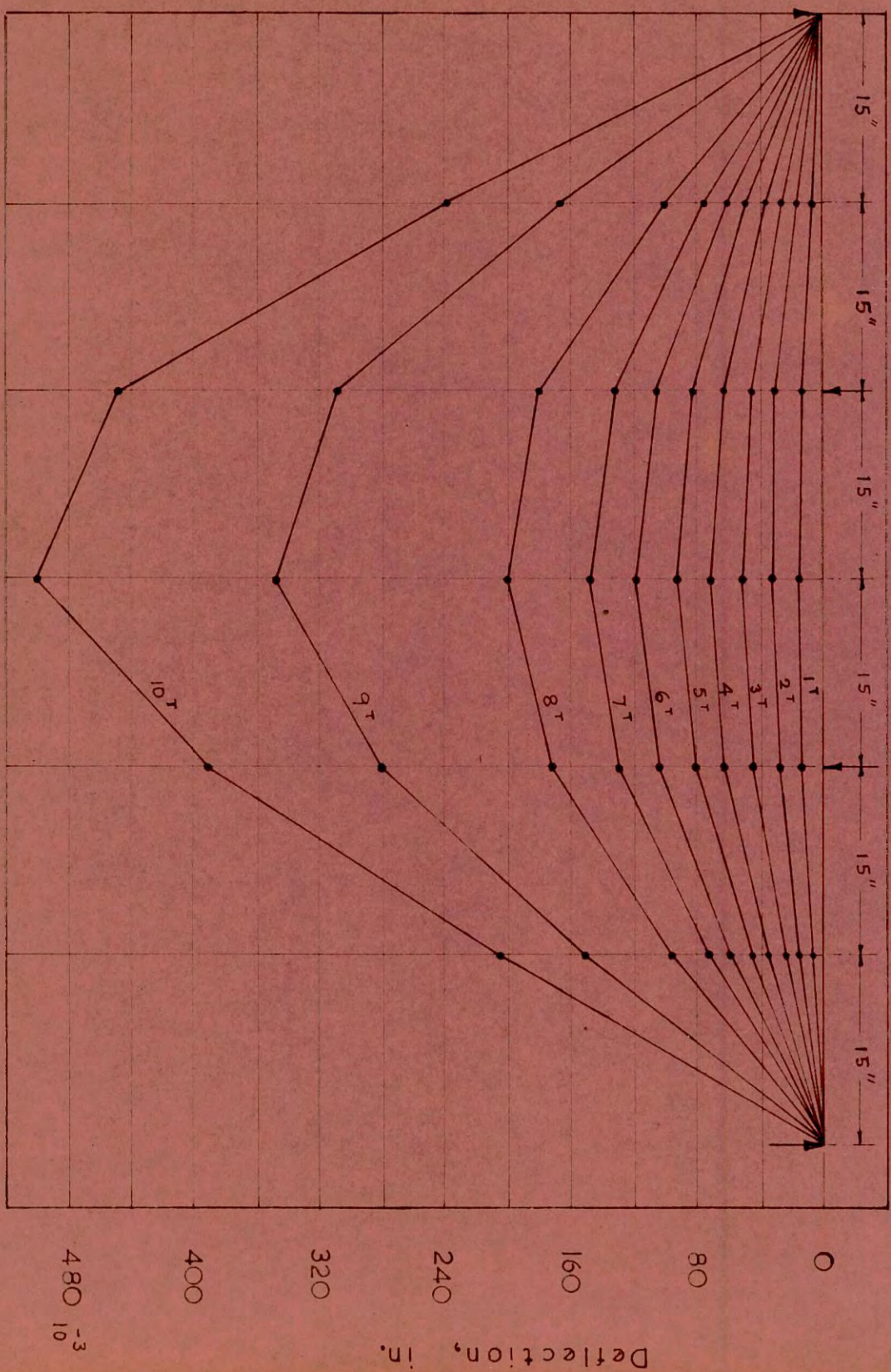


FIG. 7.50 Deflection curves of Beam MO₃-3

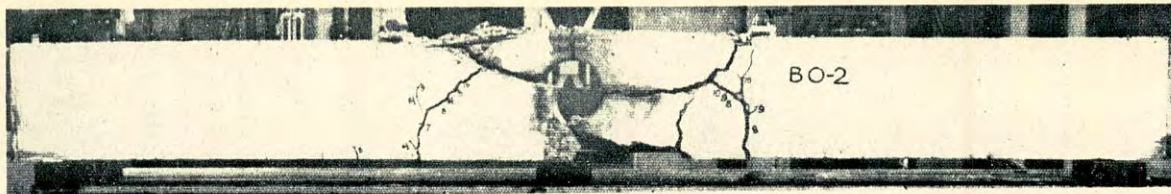


Plate 7.1

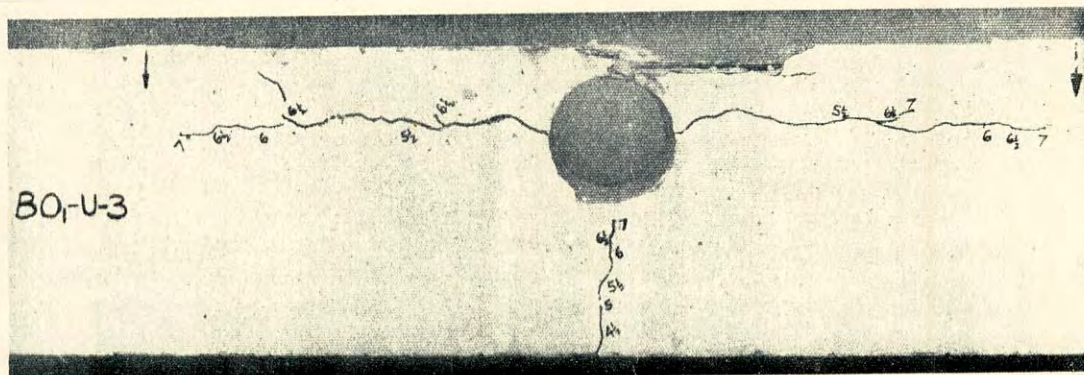


Plate 7.2



Plate 7.3



Plate 7.4

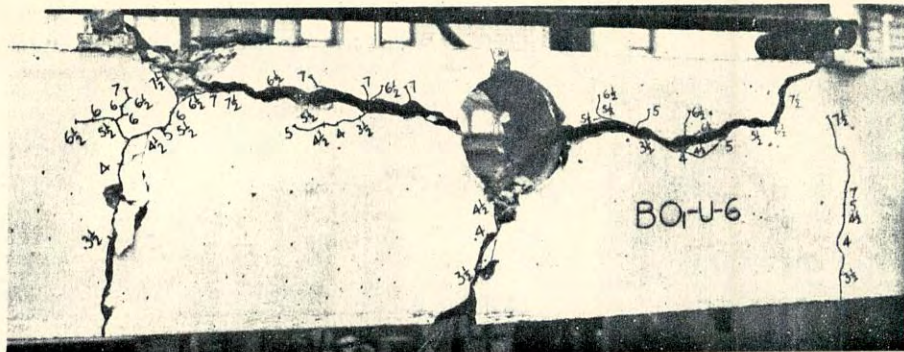


Plate 7.5

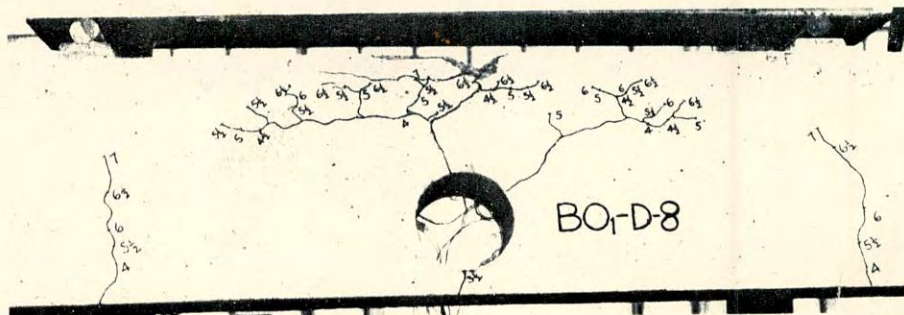


Plate 7.6

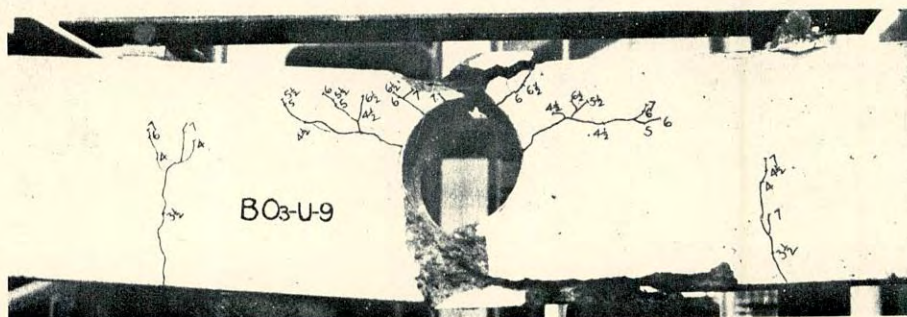


Plate 7.7



Plate 7.8

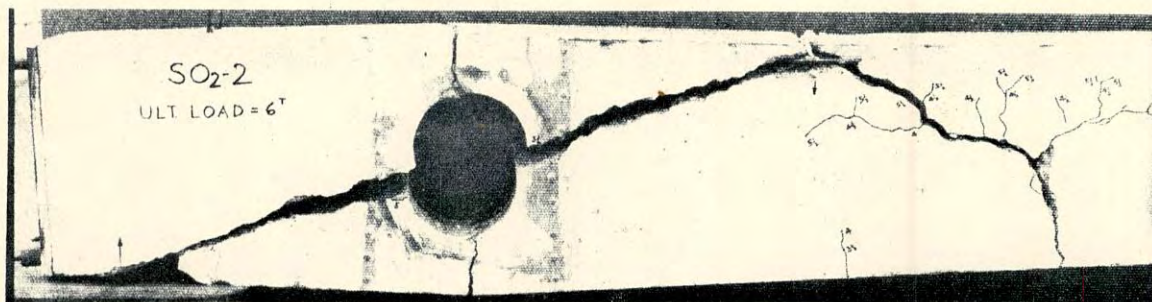


Plate 7.9

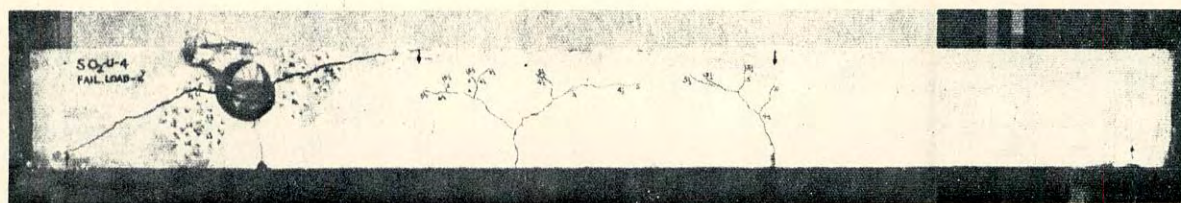


Plate 7.10

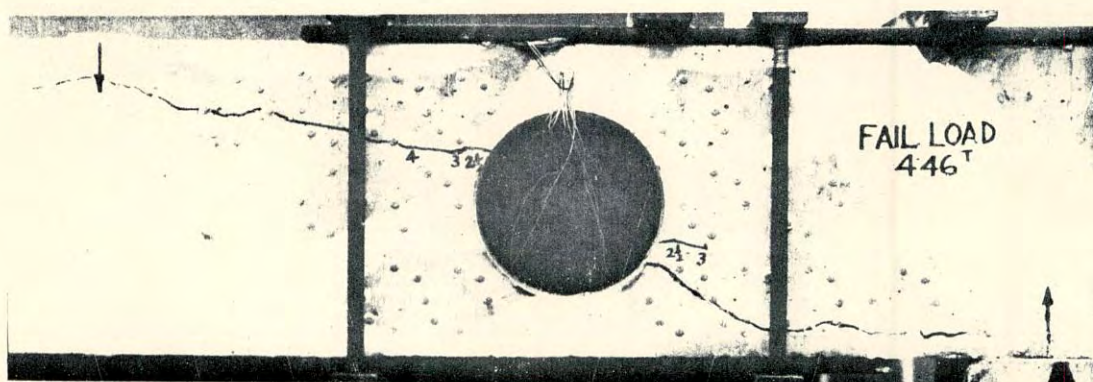


Plate 7.11



Plate 7.12

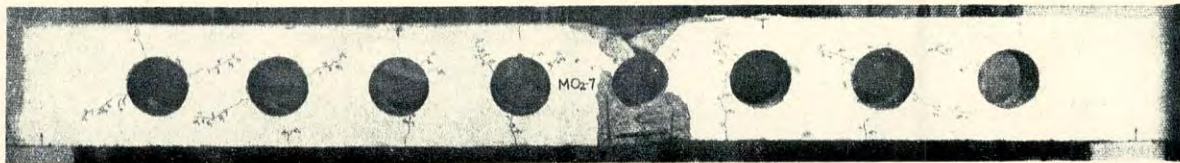


Plate 7.13

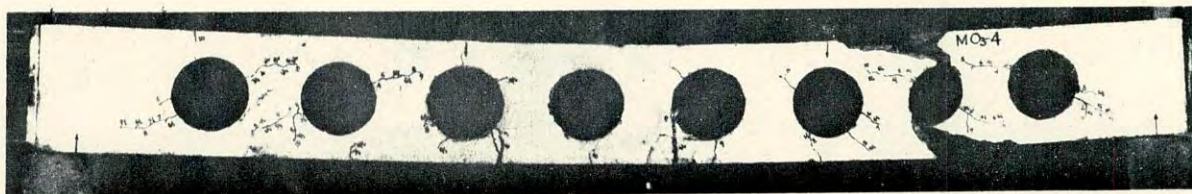


Plate 7.14

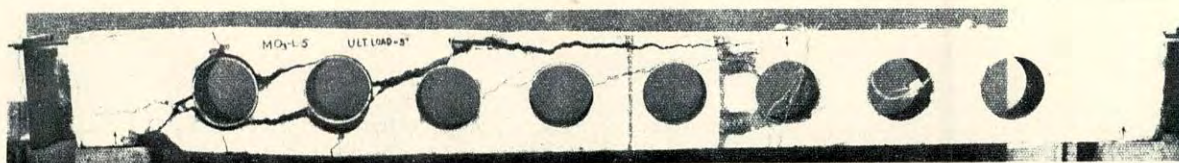


Plate 7.15

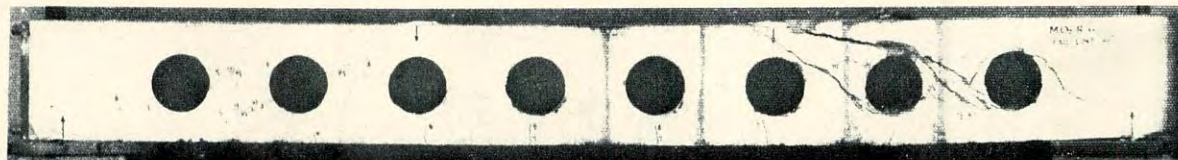


Plate 7.16

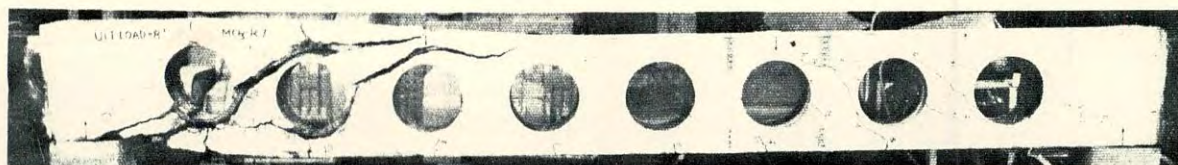


Plate 7.17

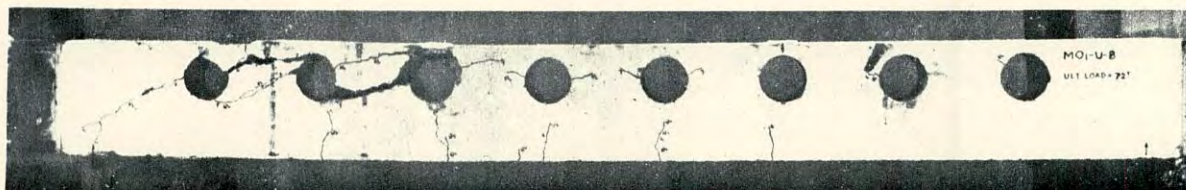


Plate 7.18

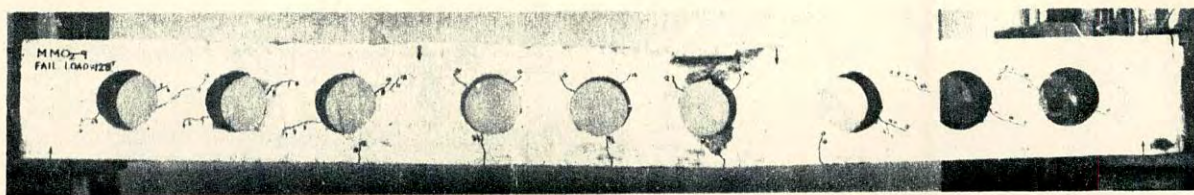


Plate 7.19

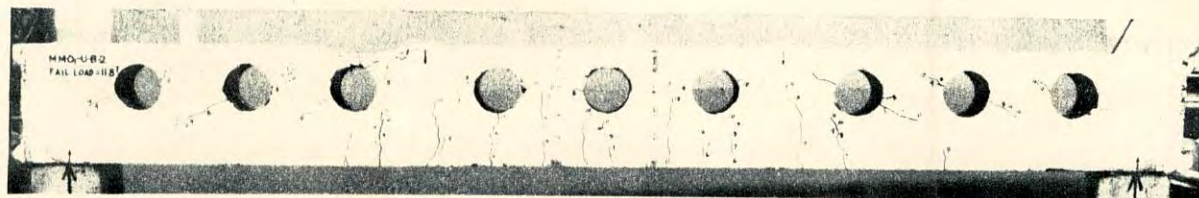


Plate 7.20

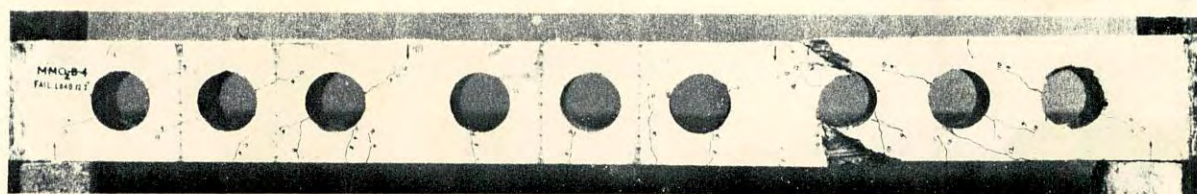


Plate 7.21

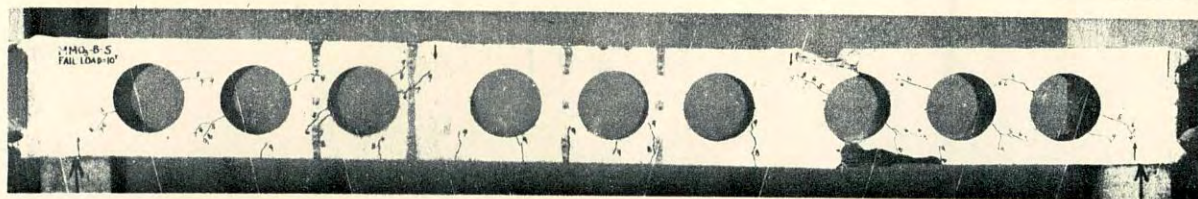


Plate 7.22

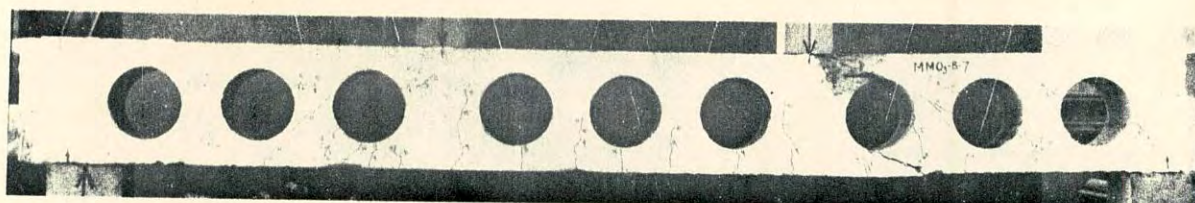


Plate 7.23

CHAPTER VIII

DISCUSSION

From the theoretical analyses and the experimental behaviour some important aspects can now be discussed.

8.1 ELASTIC STRESSES AROUND THE HOLE

8.1.1 Stress Trajectory due to Prestress

Due to the high tensile stress at the ends of the horizontal diameter of the hole, the concrete cracked locally in many cases and it was not possible to obtain the complete trajectory in the cases of high prestress, i.e., beams BO₁-1, BO₂-2 (Figs. 7.1 and 7.2) and others with prestress type I. In beam BO₃-4 with prestress type II the complete trajectory was obtained because of the low prestress force (Fig. 7.3). The trajectory of stresses in the cases of eccentric holes was obtained in beams BO₂-U-5, BO₁-U-6 and BO₃-U-9 (Figs. 7.4 - 7.6).

In all these cases the experimental curves are close to the theoretical and the theoretical analysis can be used for practical purposes. In some cases the peak stresses are greater than the theoretical values, because of the limitations of the method of analysis suggested in Chapter III.

The concentration factors for stresses at the boundary of a

6 in. diameter hole ($\text{diameter/depth} = 60\%$) due to uniform stress as suggested in Table 3.2 and Fig. 3.2 are found to give good results.

The presence of a number of holes theoretically lowers the peak stresses (Art. 2. 1.1: stress relieving holes). Referring to Fig. 7.8, the peak stresses on the top and bottom of the hole in beam MMO₂-9 containing nine holes are found experimentally to be lower than the theoretical value, whereas in the case of a similar beam (BO₂-2) having one hole (Fig. 7.2) the experimental curve is above the theoretical one.

The theoretical calculation of tangential stresses in a hole situated above or below the centroidal axis of the beam can be based on Art. 3. 1.5. As evident from Figs. 7.4 - 7.6, the peak stresses are higher due to the reasons given under "Beam BO₁-U-6" in Art. 7.1.

8.1.2 Splitting Crack

In Art. 4. 2.2 the design of splitting reinforcement was based on the theoretical values of tensile stresses at the centroidal axis in the vicinity of a hole in a direction perpendicular to the axis of the beam. Fig. 8.1 shows the dashed experimental curve and continuous theoretical curve for tensile stresses due to a uniform compression. As these two curves are close to each other, the design procedure is justified.

From the tests performed, it seems difficult to predict the stress at which the splitting crack will appear. This depends on whether the hole is unreinforced or adequately reinforced against splitting. It also depends on the presence of neighbouring holes. In the case of beam MMO_2-9 (Fig. 7.8) there was no visible splitting crack whereas in beam BO_2-2 (Fig. 7.2) there was a visible crack. The presence of a number of holes in beam MMO_2-9 would have reduced the tensile stresses perpendicular to the axis.

The appearance of the first crack due to the tensile stress may be expected at a stress of 900 - 1300 lb/sq.in.

8.1.3 Hole Stresses due to pure Bending

The measured tangential stresses at the boundary of a hole due to a pure bending load (Figs. 7.9 - 7.11) are found to be very close to the theoretical stresses calculated according to eqn. (3.17).

The stresses at the boundary of eccentric holes are plotted in Figs. 7.12, 7.13 and 7.15. They show good agreement except at the peak points. This is probably due to the application of the theorem given in Art. 3. 1.2 in the cases of holes which are close to the boundary.

8.1.4 Hole Stresses due to Bending and Shear

Figs. 7.16 - 7.18 show the stress trajectories at the boundary of holes in a zone where the bending moment/shearing force ratio is

equal to 8 in. due to a central load of 3 tons. The discrepancy between the theoretical and experimental values of stress increases with the hole size. In Figs. 7.19 - 7.21 are shown stress curves at holes where the bending moment/shearing force ratio is 15 in. Here also the larger holes show a greater discrepancy. The reasons for this are as given under "Beam SO₃-1" in Art. 7.1. As reported by McCutcheon, So and Gersovitz³⁰ experimental stresses at the boundary of a circular hole in a wide-flange steel beam measured by SR-4 gauges had a discrepancy of 16 - 20%.

Fig. 7.23 shows the stresses at a hole in the shear span of a beam containing a number of adjacent holes. The proximity of other holes seems to have influenced the stress pattern to some extent.

8.1.5 Principal Stresses around the Hole

Figs. 7.24 and 7.25 show theoretical principal stresses around 4 in. and 5 in. diameter holes respectively due to the combined effect of prestress and loading. In beam SO₁-3 the visible crack appeared at a stress of about 1000 lb/sq.in. (Fig. 7.24). Fig. 7.25 shows the principal stresses at the design load of beam SO₂-2 and indicates that the stress is concentrated at the boundary and quickly drops to nominal values.

The principal stresses around a 5 in. diameter hole as obtained by the 2 in. rectangular rosettes are shown in Fig. 7.26. Comparing these with the theoretical stresses at similar positions of the beam (Fig. 7.25), the rosette strains are found to be different.

This is due to the following reasons.

1. In the rosette, strain is measured over a 2 in. gauge length; but the theoretical analysis gives the stress at the point considered.

2. The minimum reading of strain in the 2 in. demec gauge is 24.5 micro-strain. One division of error means an appreciable change in the stress, particularly when the measured strain is small.

In Fig. 7.27 are shown the principal stresses as obtained by rosette measurements at the moment of the development of a diagonal crack in a beam containing an eccentric hole.

8.1.6 Longitudinal Stresses at Top and Bottom of the Hole

Due to stress concentration, the stress at the bottom of a hole due to prestress and that at the top of the hole due to loading can exceed the allowable limits. No code of practice comments on the increased stresses due to the effect of concentration; reinforcement can be provided at these localised areas to reduce the stresses. This high concentration of stress is subsequently relieved due to the creep property of concrete and does not affect the ultimate strength of the beam.

8.2 STRESS DISTRIBUTION AT MINIMUM SECTION

The elastic stress distribution at the minimum section is very important, because it is the most highly stressed cross-section.

Fig. 7.28 shows the stress distribution at the minimum section of a beam containing a 4 in. diameter hole. The diagram shows that theoretically the stress at the top or bottom of the hole is concentrated to a limited distance; beyond that the stress is almost linear. The stresses at the top and bottom are close to those given by the elementary beam theory.

The distribution of strain at the minimum section measured by demec gauges over an 8 in. gauge length shows that the strain is linear when the hole is in pure bending (Fig. 7.34(a)); the concentration of stress cannot be obtained by demec measurement. The linearity of strain distribution is disturbed at the hole when the hole is subject to shearing force (Fig. 7.34(b)).

In the calculation of the ultimate load of beam BO₁-U-3 it was assumed that the stress distribution above the hole at ultimate condition was rectangular (Art. 4. 1.4). Fig. 7.29 shows the strain distribution above the hole. Instead of showing a rectangular compressive stress distribution, it indicates that there is some tension on top of the beam above the hole which may be explained on the basis of the crack pattern. Due to the almost horizontal cracks on the two sides of the hole (Plate 7.2) the portion of the beam on top of the hole tends to lift. The strain recorded on top of the hole is unusually high.

Fig. 7.30 shows the gradual increase of stress above the hole until a stage is reached when the strain at the top of the hole

becomes tensile and that on top of the beam compressive. This behaviour seems contradictory to that of beam BO₁-U-3.

In Fig. 7.31, the strain at the minimum section on top of the hole of beam BO₁-U-6 shows that both the stresses on top of the beam and on top of the hole remain compressive until failure.

The strains on top of a hole in the shear span of beam MO₃-3 (Fig. 7.32) show that the strain on top of the beam above the hole gradually reduces and that on top of the hole gradually builds up until failure.

It is, therefore, difficult to verify the rectangular stress distribution above a hole from resistance gauge readings. Fig. 7.33(b) shows the strain above the hole near the ultimate load of 12 tons as obtained by 8 in. Demec measurements. This shows a more or less rectangular pattern.

When a beam contains an array of circular holes the strain distribution at the mid section between two adjacent holes is not linear as shown in Fig. 7.33(a). The strain is more or less parabolic having zero stress at the neutral axis. The parabolic diagram can be physically explained as follows.

In a solid section of a beam having no hole, a linear strain diagram holds good. The stress above the neutral axis is compressive and below the neutral axis tensile. Two sections are taken along the vertical diameters of two adjacent holes and a freebody diagram can be drawn as in Fig. 8.2. As there is no force along

the periphery of the hole, there must be equal and opposite stresses induced in the solid part in order to maintain force equilibrium. The compressive stress above the neutral axis is somewhat reduced due to the induced tensile stress. The resulting stress distribution can, therefore, take the shape as shown in Fig. 7.33(a).

8.3 DIAGONAL CRACKING

From the stress trajectories around holes in bending with shear, it is found that the tensile stress builds up at the hole boundary at the two sides of the hole. When developed, diagonal cracks weaken the beam to the extent that it will suffer a very premature failure in so far as the bending strength is concerned.

The experimental results given in Chapter VII show that the tensile stress at which the crack starts and the beam fails is dependent on many factors, such as the hole diameter and the bending to shear ratio.

It is found that when the maximum resulting tensile stress due to prestress and loading is between 700 - 1100 lb/sq.in., diagonal cracks become visible. The higher stress is applicable to smaller diameter holes and the lower value for large holes. Moreover, the cracking and failing stresses are close to each other in the case of small holes, whereas in large holes, the beam can carry further load after the crack is formed.

When there is reinforcement provided against diagonal failure, the cracks are arrested, but it seems unlikely that the reinforcement can delay the initiation of cracking.

The vertical stirrups provided serve as a good form of reinforcement against diagonal cracking failure. The reinforcement provided in the form of a bonded steel liner or bonded rings around the hole cannot help to stop the crack; this is because the cracking destroys the bond, and the beam behaves as one having unreinforced holes.

8.4 FLEXURE CRACKING

The distribution of stress at the minimum section of a perforated beam due to uniform compression and bending is such that the top and bottom fibre stresses can be approximately obtained by the simple beam theory. Therefore, the effective prestress at the bottom of the beam would be the same at the hole section as that at the solid section. The initiation of cracking due to flexure is, therefore, essentially independent of the holes.

Plates (7.1 - 7.23) show the load at which the first flexural crack developed. It is clear from these that the initiation of the flexural crack is very uncertain and can lie within wide limits. However, the load at first flexural cracking calculated on the basis of the modulus of rupture gives a lower limit.

In an unbonded beam, a few major cracks develop roughly at a

distance equal to the depth of the beam, progress quickly, branch at the top, increase the deflection and cause premature failure. When the beam contains a number of perforations, the number of cracks increases, there being at least one crack under each hole in the high bending moment area.

8.5 TOP FIBRE STRESS IN A PERFORATED BEAM

The experimental stresses on top of perforated beams (Figs. 7.35 to 7.39) show that the stress on top of each hole is lower than the stress at the same point as given by the elementary beam theory, and the stress on top of the beam midway between two adjacent holes is greater than the stress given by the elementary beam theory. This proves that the approximation of a beam containing circular holes to a Vierendeel beam is not valid, because of the development of appreciable secondary stresses in short but thick members. Kolosowski's³⁷ approximation of a castellated T-beam to a Vierendeel girder was not exact due to similar reasons. The actual stress pattern on top of a Vierendeel beam will be as shown by the dashed curve in Fig. 2.5.

However, the stress on top of the beam as given by the elementary beam theory, considering the net (reduced) second moment of area, gives approximately the maximum value. For all design purposes, this approach can readily be used. (Gibson and Jenkins³⁷ reported similar results from their tests on wide-flange castellated

steel beams).

8.6 STEEL STRAIN

8.6.1 Strain in Bottom Prestressing Steel

The load-strain curves for the lower steel (Figs. 7.40 and 7.41) are found to be similar to the load-central deflection curves of the beams. The curves depart from linearity after flexural cracks have formed; the strain then rapidly increases.

The strain as given by the resistance gauges fixed to the main prestressing steel shows that the ultimate strain in the case of the unbonded beams is under-estimated by CP115 : 1959. In Fig. 7.40 the total stress at the failure load of 12.8 tons is 202,000 lb/sq.in. whereas the stress as given by CP115 : 1959 at the ultimate condition is 198,200 lb/sq.in. Hence CP115 underestimates the actual ultimate load.

8.6.2 Strain in the Top Prestressing Wire

The loss of stress in the top steel at the ultimate condition is found to be more than that calculated according to CP115. However, the change of the stress in the top steel has little influence on the ultimate strength of the beam.

8.6.3 Strain in Stirrups

Fig. 7.42 shows the load-strain curve for a stirrup situated at the probable maximum tensile stress position of beam MO₃-4. It

was found that when the ultimate load was reached, the total strain in the stirrup (1480 micro-strain) was very near to the yield point (Fig. 5.6). This would suggest that the area of stirrup reinforcement should be increased.

In the case of beam $\text{MMO}_3\text{-B-7}$, the stirrup reinforcement was increased and provided according to Art. 4. 2.3. The development of the first diagonal crack was delayed to 6 tons and the diagonal crack at failure was not as wide as in beams $\text{MMO}_3\text{-B-5}$ and $\text{MMO}_3\text{-B-6}$.

8.7 DEFLECTION OF PERFORATED BEAM

The load-central deflection curves of beams having a single hole either in the pure bending zone or in the shear zone are shown in Figs. 7.44 and 7.45. Both figures show the deflection curves of similar solid beams for comparison. It is clear that the presence of one hole in the beam does not influence the deflection to any great extent. Before the beam is cracked, its deflection can be calculated by using the gross section second moment of area (Art. 3. 2.2).

The recorded deflection of beam $\text{MMO}_2\text{-9}$ containing nine 5 in. diameter holes at the design load is 0.083 in. (Fig. 7.⁴⁷~~48~~). This can be compared to the value of deflection, δ , calculated by the various methods suggested in Art. 3. 2.1.

Using minimum section second moment of area (Art. A. 6.2)

$$\delta = 0.077 \text{ in.}$$

Using the slope-deflection equations and adding shear deflection (Art. A. 6.3), $\delta = 0.058$ in.

Using both gross and net section second moment of area (Art. A. 6.4), $\delta = 0.072$ in.

Using the equivalent second moment of area method (Art. A. 6.5), $\delta = 0.075$ in.

It is clear that all these methods except the one using the slope-deflection method, give more or less close values. The deflection as given by the slope-deflection solution for the equivalent structure (Art. A. 6.3) seriously under-estimates the actual deflection. Consideration of an equivalent rigid jointed structure with members of varying cross-section might well improve the correlation. The actual deflection of the solid beam BS-1 was 17% more than calculated. The greater deflection of beam MMO₂-9 can thus be justified.

The deflection of beams containing 6 in. diameter perforations (Fig. 7.49) has been similarly calculated in Art. A.6 by the suggested methods. Here also the deflection as calculated by the equivalent second moment of area method gives good results. For all practical purposes, the deflection of perforated beams containing large holes (diameter/depth = 60%) can be approximately obtained by considering a simple beam with the constant second moment of area of the minimum section.

The deflections of both bonded and unbonded beams are the same

before cracking (Figs. 7.48 and 7.49). After the cracks have developed, the deflection curve is flatter in the case of unbonded beams due to a few but wide cracks.

8.8 ULTIMATE LOAD

In column 10 of Table 7.1 experimental failure loads are compared with the calculated ultimate load of a similar solid beam. Except for beam BS-1, all beams that failed in flexure carried more load than calculated. CP115 : 1959 under-estimates the ultimate load by upto 20%.

The strength of the beams failing in flexure is not affected so long as the hole does not cut the ultimate depth of compression, calculated according to ACI Code (Arts. 4. 1.2 and 4. 1.3). The method of calculation, as presented in Art. 4. 1.4 to evaluate the ultimate load when the hole is within the ultimate depth of compression, is found to give good results in the case of beam BO₁-U-3.

All the perforated beams having 6 in. diameter holes failed in shear. Beams MO₃-3, MO₃-4, MMO₃-B-5 and MMO₃-B-6 had no stirrup reinforcement on top and bottom of the holes. Due to the development of flexural cracks below holes in the high moment zone, the portion of concrete below the hole was unable to carry any shear. At the minimum section, the top concrete had to carry all the shear. Though there was a high bending stress on top of the hole, this high shearing stress developed a high principal tensile stress

(Art. A. 5.2) and the beams failed due to excessive principal tensile stress above the hole.

In the case of beam MMO_3 -B-7, short stirrups were provided on top and bottom of each hole in both the shear spans. Though this beam failed in shear as other beams, the load was very near to the ultimate flexural failure load. Moreover, it was detected that the short (only 1.5 in. deep) stirrups were not placed properly. From this it is clear that with the provision of proper stirrup reinforcement, it is possible to provide large holes without weakening the beam in shear.

Nasser, Acavalos and Daniel,⁴⁰ from their tests of reinforced concrete beams containing rectangular holes, suggest that when adequately reinforced, the top and bottom flanges carry shear in proportion to their cross-sectional areas. The results of the test of beam MMO_3 -B-7 contradict this at the ultimate condition. As both top and bottom flanges were provided with stirrups, the external shear should be divided between them; if so, the principal tensile stress on top of the hole would be within allowable limits.

Of the various hole reinforcements, the liner is found to be the weakest. The bond between the liner and the concrete is quickly destroyed and the beam fails as an unreinforced one. The ring reinforcement bonded inside the concrete around the hole is better than the liner; however, here also the bond is destroyed and the crack propagates quickly to bring about a premature failure. The

reinforcement provided in the form of vertical stirrups by the side of the hole is the best form of reinforcement. The same reinforcement can be used against both splitting and diagonal stresses.

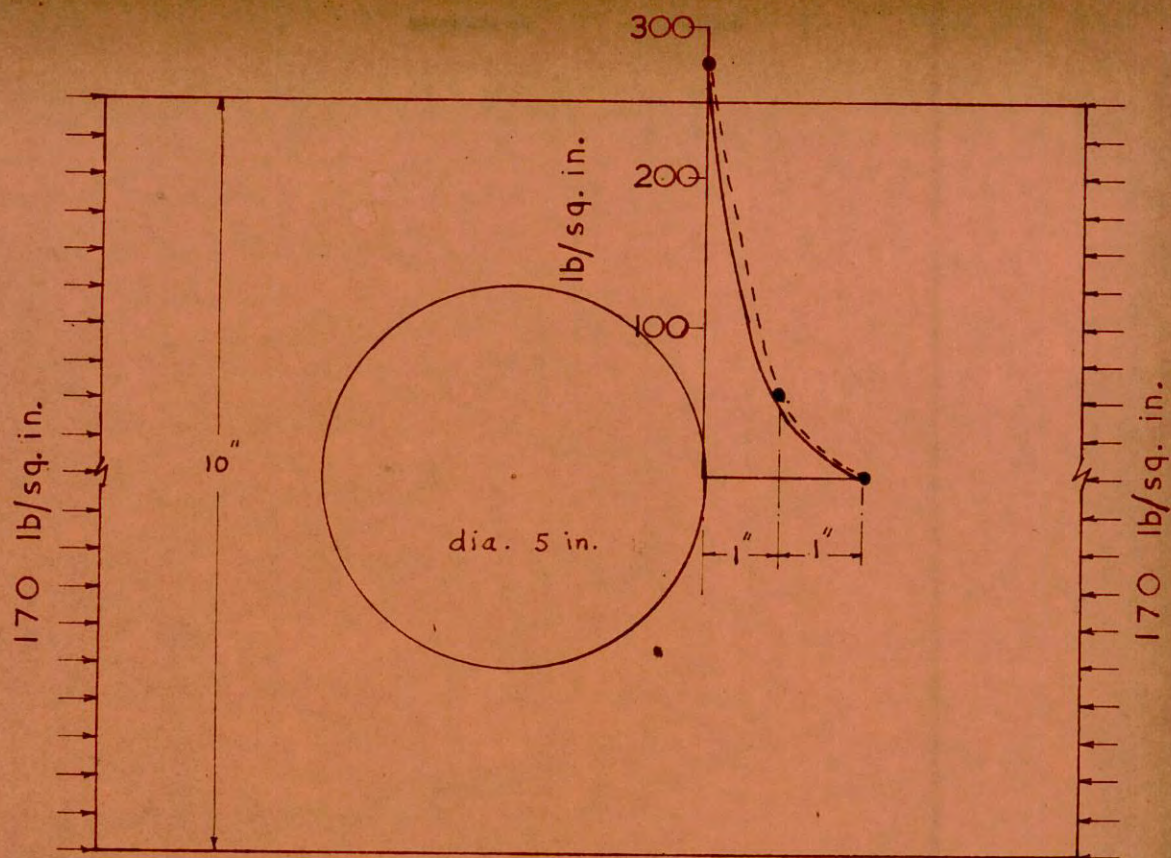


FIG. 8.1 Tensile stress induced at hole due to compression

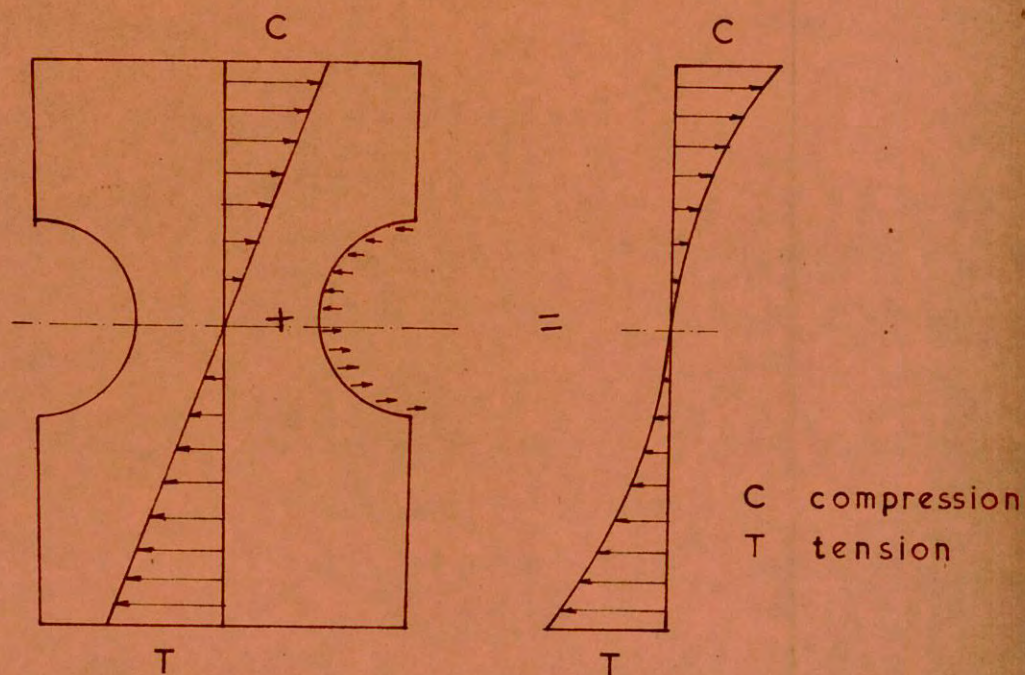


FIG. 8.2

CHAPTER IX

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

From the analyses and experimental investigations of the limited number of tests of beams containing circular holes presented in this thesis, the following conclusions can be drawn.

9.1 ELASTIC STRESSES AND DEFLECTIONS

Beams containing single holes

1. The stress concentration and distribution around a circular hole, situated on the centroidal axis of the beam (diameter being upto 60% of the depth) due to prestress and bending or bending with shear, can reasonably be predicted from the mathematical solutions presented in Chapter III.

2. When the hole is situated above or below the centroidal axis of the beam, the tangential stresses at the boundary of a circular hole, as given by the superposition of compression (using Mindlin's curves for holes near the edge of the beam) and bending solutions (given in Chapter III) show reasonable agreement with the experimental values except at some peak points.

3. The effect of stress concentration at the boundary of a circular hole in a beam is confined to a limited distance; the extreme fibre stresses at the minimum section can be predicted

reasonably by the elementary beam theory.

4. The presence of only one hole, large or small, in the pure bending zone or in the shear span of a beam, does not influence the deflection. Also a number of small holes (diameter upto 40% of the depth) on the centroidal axis of a beam does not appreciably increase the deflection.

Beams containing multiple holes

5. Conclusions 1. and 2. are equally applicable with reasonable accuracy to beams containing multiple holes except that due to the presence of other holes in the vicinity, the peak stresses at a hole are reduced.

6. The distribution of stress within the depth of a perforated beam is not linear, either at the hole or between two adjacent holes.

7. The approximation of a beam containing circular perforations to one with flat-top holes is not valid, i.e., the Vierendeel analysis is not applicable, either for the top fibre stresses or for the deflection of the beam.

8. The maximum top fibre stress in a perforated beam can be predicted by the elementary beam theory using the net section second moment of area.

9. The deflection of a perforated beam can be determined with reasonable accuracy from the sets of curves presented in Fig. A.5 and Fig. A.6, which are based on the equivalent second moment of

area method.

10. The deflection of beams containing an array of large holes, (diameter being equal to or more than 60% of the beam depth) spaced close to one another, can be approximately obtained by using the elementary beam theory by substitution of the minimum section second moment of area.

9.2 ULTIMATE STRENGTH OF PERFORATED BEAMS

11. The provision of a hole in a pure bending zone, subject to the condition contained in 12., does not reduce the strength of the beam, but ensures failure at the hole.

12. In bending, the ultimate strength of the beam is not reduced until the hole cuts material from within the ultimate depth of the rectangular stress block according to ACI Building Code, even though the hole may go within the depth of the ultimate neutral axis calculated according to CP115 : 1959.

13. When the hole is situated in such a way that it goes within the ultimate depth of the rectangular compression block, as mentioned in 12., the ultimate strength of such a beam can be calculated by taking the reduced depth and considering the line of action of the compressive force in concrete to act at mid-depth of this reduced ultimate depth.

14. When no reinforcement is provided at a hole subject to bending with shear, the strength is seriously reduced depending on

the magnitude of the shearing force.

15. The provision of suitably proportioned reinforcement can ensure that failure of the beam takes place in a flexural mode even though holes occur in the critical shear zone.

9.3 GENERAL CONCLUSIONS

16. The stress at which splitting or diagonal cracking becomes visible at the boundary of a circular hole due to prestress or bending with shear is much higher than either the cylinder splitting stress or the modulus of rupture.

17. The initiation of flexural cracking, which appears to be independent of the presence of holes, is uncertain and can lie within wide limits. However, the load at initial flexural cracking, calculated on the basis of the modulus of rupture, gives a lower limit.

18. The assumption that the top and bottom flanges at a hole, if properly reinforced, share the shearing force in proportion to their depth, is true at the design load, but not at the ultimate condition.

19. Of the different methods of reinforcing the holes attempted in the tests (bonded hole liners, bonded ring reinforcement and vertical stirrups by the sides of the hole), vertical stirrups appear to offer the best solution.

20. The methods of design of the stirrups against splitting,

diagonal cracking and excessive principal tensile stress above the hole, as given in Chapter IV, are found to give good results.

21. Prestressed concrete beams with circular perforations, if properly reinforced, can be as strong as a solid beam, but the deflection may be greater, the amount depending on the hole characteristics.

9.4 SUGGESTIONS FOR FURTHER RESEARCH

The following topics concerning perforations in beams are suggested for further investigation:

1. The structural behaviour of reinforced concrete beams containing circular perforations.
2. The behaviour of reinforced and prestressed concrete beams with extended holes, i.e., rectangular or elliptical holes.
3. The extension of the present problem of rectangular beams to I or T-section beams containing holes.
4. A study on the initial appearance, width and propagation of the crack around a hole due to the induced tensile stresses.
5. The effect of perforations in deep beams.
6. The weakening effect of holes in a beam which fails in shear even without any hole being present.
7. The extension of the present problem to the case where the beam carries the load of installations passing through these web perforations.

8. The investigation of the effect of distributed load on the strength and behaviour of beams containing holes.

9. The effect of creep in highly stressed areas around the holes on the behaviour of the beam.

10. The effect of shrinkage on the behaviour of a beam containing an array of holes. The shrinkage will be more pronounced in this case because more surface area will be exposed.

11. The effect of sustained load on the behaviour of beams containing holes.

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APPENDIX AANALYSIS OF TEST BEAMSA.1 General

According to CP115 : 1959, the permissible compressive stress at transfer is the lesser of $0.5 U_t$ i.e. 3500 lb/sq.in. and 3000 lb/sq.in. In a rectangular beam without holes where there will be a triangular distribution of prestress, the bottom fibre stress can be up to 3000 lb/sq.in. for the maximum utilisation of concrete strength. The critical point in a beam with a large circular hole will be at the bottom of the vertical diameter of the hole where the stress will be very high due to stress concentration. As this effect will be very localised, the stress at that limited area was allowed to exceed the permissible value. However, the bottom fibre stress at the solid section was limited to a value below the permissible stress. In the prestress type I beams containing holes, the bottom fibre stress was restricted to 1600 lb/sq.in., the stress at the critical point then being about 3500 lb/sq.in. For this local high stress, reinforcement could be provided at the bottom of the hole.

A.2 Loss of Prestress

a) Loss due to relaxation of steel: The relaxation curve of 0.276 diameter high tensile steel wire as supplied by the manufac-

turers (Fig. 5.5) shows that due to an initial load of 10,050 lb the loss at four days is about 3%. An instantaneous loss of 1% can be compensated by stressing the wires for a second time. In the present calculations, relaxation loss will be taken as 2%.

b) Loss due to the elastic shortening of concrete: In the present tests, the prestressing operation was made a second time. No loss due to elastic shortening need then be taken into consideration.

c) Loss due to shrinkage: The beams were cured for 28 days at a constant temperature of 64°F and relative humidity of 95 - 100% and stressed twenty-four hours after removal from the curing room. In the case of unbonded beams, they were tested within twenty-four hours of stressing; the effect of shrinkage can be considered as negligible. In the case of bonded beams, they were returned to the curing room after grouting and brought out 24 hours before testing. The effect of shrinkage can also be considered as negligible in this case.

d) Loss due to creep of concrete: In a beam without a hole, this loss would be very low because of the low bottom fibre stress at prestressing and the short time lapse between prestressing and testing. For beams containing an array of holes, the loss might be significant because there exist areas at the boundary of the holes where high concentrations of stress occur.

e) Loss due to anchorage: This loss is due to slip of the

tendons during the driving of the anchorage wedges. The slip is roughly proportional to the applied force and is a variable depending on the length of the beam.

The loss due to anchorage slip was determined on an experimental basis. Two blocks of steel each containing a hole $\frac{3}{8}$ in. diameter were welded to the top of a 6 in. x 12 in. I-section at a clear distance of 8 feet (equal to the length of the beam). One 0.276 in. diameter high tensile steel wire was anchored against one block and force was applied to the wire by jacking against the other block. When the wire was just taut, an initial reading was taken on four PL-3 strain gauges fixed at two positions of the wire, one gauge being diametrically opposite to the other at the same position. At a load as read on the jack dial, readings were taken on the strain gauges just before and after driving the wedges home. The difference between these two readings gave the loss due to anchorage slip. A curve of load as given by the jack dial versus the load as calculated from the resistance gauges is plotted in Fig. A.1. The dashed curve represents the load after anchorage loss. Above 5000 lb the anchorage loss is about 8%.

f) Loss due to friction: As the ducts were quite large ($\frac{3}{8}$ in. diameter for a single wire and $\frac{3}{4}$ in. for two wires) and the profile straight, this loss can be neglected.

Total Loss: The total loss for beams having no hole or a single hole can be taken as 12%. The measured strains on the

resistance gauges fixed directly on the main stressing wire of a solid beam showed a total loss of 12%.

For beams containing a series of holes, the total loss increases to 15% as is supported by strain measurement, the increased loss being due to the creep of concrete at high stress areas.

A.3 Stress Distribution, Design Load and Cracking Load

A.3.1 Stress distribution on prestressing

For prestress type I:

For a gross force of 7000 lb in each top wire and 9000 lb in each bottom wire, the net distribution of stress after losses is as follows:

Top fibre stress for a solid beam with unbonded tendons =
- 80 lb/sq.in.

Bottom fibre stress = 1600 lb/sq.in. (Fig. 6.7).

The area and second moment of area of the cross-section were calculated neglecting the ducts and reinforcement.

For prestress type II:

Each top wire was subjected to a gross force of 4000 lb and each bottom wire to 8000 lb

Top fibre stress = - 45 lb/sq.in.

Bottom fibre stress = 775 lb/sq.in. (Fig. 6.7).

A.3.2 Cracking load

For beams with prestress type I:

The self weight of the beam produces a maximum stress of 52 lb/sq.in. Taking 600 lb/sq.in. as the modulus of rupture, the cracking load, P_{cr} is given by $\frac{P_{cr} \times 30 \times 5}{2 \times 488} = 1600 + 600 - 52 = 2148$
Hence, $P_{cr} = 6.25$ tons.

For beams with pre-stress type II:

$$\frac{P_{cr} \times 30 \times 5}{2 \times 488} = 775 + 600 - 52 = 1323$$

Hence, $P_{cr} = 3.8$ tons.

A.4 Elastic Stresses around the Hole

A.4.1 Stresses at the hole boundary

Computer programme no. 1, given in Appendix B, was used to calculate the elastic stresses at the boundary of a hole. This programme was used to calculate tangential stresses due to bending moment and shearing force. Curves showing the boundary stresses due to prestressing and loading for a number of beams are shown in Figs. 7.1 to 7.23.

A.4.2 Principal stresses around the hole

a) Theoretical stresses:

Computer programme no. 2, given in Appendix B, was used to calculate the theoretical principal stresses and their directions around the hole due to prestress and loading.

b) Experimental stresses from rosette readings:

Computer programme no. 3 calculates principal stresses and their directions around the hole when the measured strains along the three directions of the rectangular rosette are supplied.

The expressions⁵⁹ for the principal stresses in terms of the strains in three directions of the rectangular rosette are as follows:

$$\sigma_1 = \frac{E}{2(1-\mu^2)} \left[(1+\mu) (e_a + e_c) + (1-\mu) \sqrt{2(e_a - e_b)^2 + 2(e_b - e_c)^2} \right]$$

$$\sigma_2 = \frac{E}{2(1-\mu^2)} \left[(1+\mu) (e_a + e_c) - (1-\mu) \sqrt{2(e_a - e_b)^2 + 2(e_b - e_c)^2} \right]$$

$$\phi = \frac{1}{2} \tan^{-1} \left[\frac{2 e_b - e_a - e_c}{e_a - e_c} \right]$$

A.5 Analysis for Shearing ForceA.5.1 At working load of 5 ton

At a section without a hole, the maximum shearing stress =

$$1.5 \times \frac{2.5 \times 2240}{6 \times 10} = 140 \text{ lb/sq.in.}$$

Average prestress at centroid = 800 lb/sq.in.

The principal tensile stress is within allowable limits. No stirrup is, therefore, necessary.

At the bottom flange at a section through the centre of a 6 in. diameter central hole, the maximum shearing stress =

$$\frac{1.5 \times 2.5 \times 2240}{6 \times 4} = 350 \text{ lb/in.}^2$$

Pre-stress at the middle of bottom flange = 1600 lb/sq.in.

Tensile stress at the middle of the bottom flange at 5 tons
= 1146 lb/sq.in.

$$\begin{aligned}\text{Principal tensile stress} &= \frac{1600 - 1146}{2} - \sqrt{\left(\frac{1600 - 1146}{2}\right)^2 + 350^2} \\ &= -188 \text{ lb/sq.in.}\end{aligned}$$

This is slightly greater than the allowable principal tensile stress. Nominal $\frac{1}{4}$ in. ϕ stirrups @ 2 in. c/c will suffice.

A.5.2 At ultimate load condition

a) Uncracked section without hole,

maximum shearing stress at the ultimate load of 11.4 tons =

$$1.5 \times \frac{11.4 \times 2240}{2 \times 6 \times 10} = 320 \text{ lb/sq.in.}$$

The ultimate allowable principal tensile stress = 380 lb/sq.in.
for 7000 lb/sq.in. concrete.

b) Cracked section,

if no stirrup reinforcement is supplied above or below a hole in the shear span, all the shearing force must be carried by the top flange only at the ultimate condition, because the bottom flange develops flexural cracks and ceases to carry any shear (as supported by Lorentsen¹ in his tests).

For a 6 in. diameter central hole, the maximum shear stress at the ultimate load = $1.5 \times \frac{11.4 \times 2240}{2 \times 2 \times 6} = 1600 \text{ lb/sq.in.}$

The maximum bending stress = $\frac{2}{3} \times 0.85 \times 8 \times 7000 = 3500 \text{ lb/sq.in.}$

Principal tensile stress = -630 lb/sq.in.; this high stress would cause the beam to fail in shear before flexural failure of the beam.

Design of stirrup:

$$V_u - V_c = A_v f_y \frac{d}{s} \quad \text{from eqn. 4.18}$$

$$\text{Neglecting } V_c, \quad V_u = A_v f_y \frac{d}{s}$$

$$s = \frac{A_v f_y d}{V_u} = 1 \text{ in.}$$

Stirrups would be provided at a spacing of 1 in. near the top of the hole.

A.5.3 Design of pre-stress splitting reinforcement

In a beam having a 5 in. diameter hole situated on the centroidal axis, the tensile stress concentration factor at the ends of the horizontal diameter is 1.58.

For prestress type I, tensile stress = $800 \times 1.58 = 1264 \text{ lb/sq.in.}$

$$\text{Area of stirrup}^{60} = \frac{1264 \times 2.5 \times 6}{2 \times 20000} = 0.46 \text{ sq.in.}$$

For a four-legged $\frac{3}{8}$ in. diameter stirrup, $A_s = 0.45 \text{ sq.in.}$

A.5.4 Reinforcement against diagonal splitting stresses

At the design load of 5 tons, the greatest tensile stress at the boundary of a 5 in. diameter hole due to the combined effect of prestress and loading (bending with shear) = 1377 lb/sq.in.

$$\text{Area of reinforcement required} = \frac{1377}{2} \times \frac{2.5 \times 6}{20,000} = 0.49 \text{ sq.in.}$$

Area of four-legged $\frac{3}{8}$ in. diameter stirrup = 0.45 sq.in.

A.6 Calculation of Deflection

A.6.1 Using gross section second moment of area

The second moment of area of an unbonded beam at a solid section = 488 in.⁴

The central deflection at 5 tons load = 0.067 in.

The central deflection at 3 tons load = 0.041 in.

A.6.2 Using minimum section second moment of area

The second moment of area at the minimum section at a 5 in. diameter central hole = 426 in.⁴

The second moment of area at the minimum section at a 6 in. diameter central hole = 380 in.⁴

The central deflection (5 in. diameter hole) at 5⁺ load =
0.077 in.

The central deflection at 5 tons load in a beam (having 6 in. diameter holes) = 0.086 in.

A.6.3 Using the slope deflection equations

In Fig. A.2 is shown the approximate linear form for the beam containing nine 6 in. diameter holes. The slope-deflection equations for the structure are:

$$\text{Joint A: } M_{AB} = \frac{2Ex^4}{10.5} \left(-2\theta_A - \theta_B + \frac{3\Delta_B}{10.5} \right)$$

$$M_{AL} = \frac{2Ex \cdot 108}{8} \left(-2\theta_A - \theta_L \right) \text{ where } \theta_L = \theta_A \text{ due to symmetry.}$$

For moment condition, $M_{AB} + M_{AL} = 0$

$$\text{i.e., } -41.262 \theta_A - 0.381 \theta_B + 0.109 \Delta_B = 0 \quad (\text{A.1})$$

Similarly, for joint B:

$$-6.712 \theta_B - 0.444 \theta_C - 0.381 \theta_A + .109 \Delta_B + .148 \Delta_C = 0 \quad (\text{A.2})$$

$$\text{Joint C: } -6.712 \theta_C - 0.444 \theta_B - 0.381 \theta_D + .148 \Delta_C + .109 \Delta_D = 0 \quad (\text{A.3})$$

$$\text{Joint D: } -42.024 \theta_D - 0.381 \theta_C - 0.381 \theta_E + .109 \Delta_D + .109 \Delta_E = 0 \quad (\text{A.4})$$

$$\text{Joint E: } -6.267 \theta_E - 0.381 \theta_D + 0.109 \Delta_E = 0 \quad (\text{A.5})$$

Shear Conditions:

$$\text{Span AB: } \frac{2 M_{AB} + 2 M_{BA}}{I_{AB}} = \text{shearing force}$$

$$\text{i.e., } -3 \theta_A - 3 \theta_B + 0.571 \Delta_B = 0.0069 \quad (\text{A.6})$$

$$\text{Span BC: } -3 \theta_B - 3 \theta_C + 0.667 \Delta_C = 0.0050 \quad (\text{A.7})$$

$$\text{Span CD: } -3 \theta_C - 3 \theta_D + 0.571 \Delta_D = 0.0067 \quad (\text{A.8})$$

$$\text{Span DE: } -3 \theta_D - 3 \theta_E + 0.571 \Delta_E = 0 \quad (\text{A.9})$$

These nine simultaneous equations (A.1 - A.9) were fed into the computer and the unknown quantities were determined.

The total deflection at E due to a bending load of 5 tons =
0.053 in.

Shear deflection of cross-members =

$$(10.5 + 9 + 10.5) 1.2 \times \frac{1.25 \times 2240}{6 \times 2 \times 1.9 \times 10^6} = .005 \text{ in.}$$

Total deflection = .058 in.

A.6.4 Using both gross and net second moment of area

a) Using the gross section second moment of area at the solid portion and minimum section second moment of area at the perforated portion of the beam, the central deflection for a beam containing eight central holes of 5 in. diameter

$$\begin{aligned}
 &= \text{moment of } \frac{M}{EI} \text{ diagram (Fig. A.3)} \\
 &= \frac{1}{E} \left[\left(\frac{Px7.5 \times 7.5}{2 \times 488} \times \frac{2}{3} \times 7.5 \right) + \left(\frac{px12.5 + px7.5}{2} \times \frac{5}{425} \times 10 \right) \right. \\
 &+ \left(\frac{12.5 + 17.5}{2} \times \frac{Px5}{488} \times 15 \right) + \left(\frac{17.5 + 22.5}{2} \times \frac{Px5}{425} \times 20 \right) \\
 &+ \left(\frac{22.5 + 27.5}{2} \times \frac{Px5}{488} \times 25 \right) + \left(\frac{27.5 + 30}{2} \times \frac{Px2.5}{425} \times 28.75 \right) \\
 &+ \left(\frac{30 \times 2.5}{425} p \times 31.25 \right) + \left(\frac{Px30 \times 5}{488} \times 35 \right) + \left(\frac{30 Px5}{425} \times 40 \right) \\
 &\left. + \left(\frac{30 Px2.5}{488} \times 43.75 \right) \right] = 56.84 \frac{P}{E}
 \end{aligned}$$

where I for solid portion = 488 in.⁴

and I for hole portion = 425 in.⁴

$$\text{At a load of 5 tons, the central deflection} = \frac{56.84 \times 2.5 \times 2240}{446 \times 10^6} =$$

0.072 in.

b) For 8 central holes of 6 in. diameter, the central deflection = moment of $\frac{M}{EI}$ diagram (Fig. A.4)

$$= 61.88 \frac{P}{E}$$

$$\text{At a load of 5 tons, the maximum deflection} = 61.88 \times \frac{5 \times 2240}{2 \times 44 \times 10^6} =$$

0.079 in.

A.6.5 Using the equivalent second moment of area

a) Moment deflection:

Following the analysis proposed in Art. 3. 2.5, a set of values for the ratio I/I_e for various ratios of d/D and $\frac{nd}{L}$ has been calculated with the help of the digital computer and the set of curves shown in Fig. A.5 obtained.

b) Shear deflection:

According to the analysis presented in Art. 3. 2.6 the deflection due to the shearing force in a perforated beam has been calculated with the help of the computer. A set of curves is drawn in Fig. A.6 for the ratio of the deflection in a perforated beam to that in a similar solid beam for different values of $\frac{d}{D}$ and $\frac{nd}{L}$.

From Fig. A.5, the moment deflection of a beam containing nine 5 in. diameter holes is found to be only 4% greater than that of an equivalent solid beam. The shear deflection, according to Fig. A.6 is about three times greater than that of a solid beam.

The total central deflection of a solid beam at 5 tons

$$= .067 \times \frac{104}{100} + 1.8 \times 3 \times 10^{-3} = .076 \text{ in.}$$

For nine holes of 6 in. diameter, the total maximum deflection at 5 tons

$$= .067 \times \frac{109}{100} + 4 \times .0018 = .081 \text{ in.}$$

A.6.6 Using the equivalent second moment of area and considering the variable moment

Using the exact solution as given in Art. 3. 2.5, the deflection due to variable moment can be calculated as follows.

Fig. A.7 shows the bending moment diagram due to third-point loading in a beam having nine 6 in. diameter holes. The second area-moment theorem can be applied, i.e., taking moment of the area of the $\frac{M}{EI}$ diagram about A ($\frac{M}{EI}$ diagram is divided into A1, A2, A3, A4, A5 and A6 solid portions and B1, B2, B3, B4 and B5 perforated portions),

$$A1: \int_0^{n1} \frac{P}{2} x - \frac{x \cdot x}{I} dx$$

$$\text{where } n1 = 3, d = 6, n2 = 3, n3 = 3, n4 = 3, n5 = 3 \text{ and } n6 = 3 \\ = \frac{P}{2I} (9)$$

$$A2: \int_0^{n2} \frac{P}{2} \frac{(n1 + d + x)(n1 + d + x)}{I} dx = \frac{P}{2I} (333)$$

$$A3: \int_0^{n3} \frac{P}{2} \frac{(n1 + d + n2 + d + x)}{I} x (n1 + d + n2 + d + x) dx = \frac{P}{2I} (1143)$$

$$A4: \int_0^{n4} \frac{P}{2} \frac{(n1 + d + n2 + d + n3 + d + x)}{I} x (n1 + d + n2 + d + n3 + d + x) dx \\ = \frac{P}{2I} (2439)$$

$$A5: \int_0^{n5} \frac{P}{2} x \frac{L}{3I} \left(\frac{L}{3} + x \right) dx = \frac{P}{2I} (2835)$$

$$A6: \int_0^{n6} \frac{P}{2} x \frac{L}{3I} \left(\frac{L}{3} + n5 + d + x \right) dx = \frac{P}{2I} (3645)$$

(xiv)

$$B1: \int_{-\pi/2}^{\pi/2} \frac{P}{2} \frac{(n1 + \frac{d}{2} + \frac{d}{2} \sin \theta) 12}{b(D^3 - d^3 \cos^3 \theta)} \times (n1 + \frac{d}{2} + \frac{d}{2} \sin \theta) \times \frac{d}{2} \cos \theta d\theta = \frac{P}{2I} \quad (81)$$

$$B2: \int_{-\pi/2}^{\pi/2} \frac{P}{2} \frac{(n1 + d + n2 + \frac{d}{2} + \frac{d}{2} \sin \theta) 12}{b(D^3 - d^3 \cos^3 \theta)} \times (n1 + d + n2 + \frac{d}{2} + \frac{d}{2} \sin \theta) \times \frac{d}{2} \cos \theta d\theta = \frac{P}{2I} \quad (1579)$$

$$B3: \int_{-\pi/2}^{\pi/2} \frac{P}{2} \frac{(n1 + d + n2 + d + n3 + \frac{d}{2} + \frac{d}{2} \sin \theta) 12}{b(D^3 - d^3 \cos^3 \theta)} \times (n1 + d + n2 + d + n3 + \frac{d}{2} + \frac{d}{2} \sin \theta) \times \frac{d}{2} \cos \theta d\theta = \frac{P}{2I} \quad (4019)$$

$$B4: \int_{-\pi/2}^{\pi/2} \frac{P}{2} \times \frac{L}{3} \times \frac{12}{b(D^3 - d^3 \cos^3 \theta)} \left(\frac{L}{3} + n5 + \frac{d}{2} + \frac{d}{2} \sin \theta \right) \times \frac{d}{2} \cos \theta d\theta = \frac{P}{2I} \quad (7500)$$

$$B5: \int_{-\pi/2}^{\pi/2} \frac{P}{2} \times \frac{L}{3} \times \frac{12}{b(D^3 - d^3 \cos^3 \theta)} \left(\frac{L}{3} + n5 + d + n6 + \frac{d}{2} + \frac{d}{2} \sin \theta \right) \times \frac{d}{2} \cos \theta d\theta = \frac{P}{2I} \quad (4665)$$

$$\leq \text{moment about the reaction} = \frac{P}{2I} \quad (28248)$$

Equating the central deflection of a solid beam having an equivalent second moment of area I_e and this beam,

$$\frac{23}{1296} \frac{PL^3}{EI_e} = \frac{P}{2EI_e} \quad (28248)$$

$$\text{i.e., } I/I_e = 1.092$$

It is clear from Fig. A.5 that the exact solution considering the variable moment does not appear to influence the I/I_e ratio for the particular configuration of perforations used here.

A.7 Ultimate Load Calculations

A.7.1 Unbonded beam (Prestress Type I)

From the properties of the beam,

$$\frac{p_e A_{st}}{U_w b d_1} = 0.083 \quad \text{Table 7; CP115: 1959}$$

$$\frac{f_m}{p_e} = 1.47 \quad \text{after interpolation}$$

$$f_m = 1.47 \times 135000 = 198200 \text{ lb/sq.in.}$$

$$\frac{d_n}{d_1} = 0.30$$

$$d_n = 0.30 \times 9.3 = 2.79 \text{ in.}$$

Ultimate stress in compression steel:

$$\text{Loss of strain in compression steel at failure} = .002 \times \frac{.638}{2.79} \times \frac{2}{3}$$

where the factor $\frac{2}{3}$ is for an unbonded beam;

$$\text{Total loss of stress} = 9030 \text{ lb/sq.in. in each wire}$$

Total load in the compression wires at failure

$$= 2 (7000 \times .88 - 9030 \times .06) = 11520 \text{ lb}$$

Fig. A.8 shows the force diagram at the ultimate load condition

$$\text{Ultimate moment, } M_u = f_m A_{st} (d_1 - 0.4 d_n) = 11520 \times 0.478 \text{ lb-in.}$$

$$\text{Ultimate load} = \frac{M_u \times 2}{2240 \times 30} = 11.4 \text{ tons}$$

Depth of ultimate compression block (n_p):

According to ACI Building Code (1963),

$$6 \times n_p \times 0.85 \times 7000 = 4 \times 0.06 \times 198\,200 + 11520$$

$$\text{i.e., } n_p = 1.66 \text{ in.}$$

A.7.2 Ingress of hole into compression block at ultimate load

When a hole is situated in the pure bending zone of the beam in such a way that the top of the hole is 1 in. below the top of the beam, then:

$$(\text{Fig. A.9}) \quad C_c = 6 \times 1 \times 0.85 \times 7000$$

$$C_s = 2 \times 7000 \times 0.8 = 11200 \text{ lb}$$

The ultimate moment, M_u , by taking moment about tension steel, is

$$M_u = C_c \times 8.81 - C_s \times 8.67$$

$$P_u = \frac{M_u \times 2}{30 \times 2240} \text{ tons} = 6.65 \text{ tons}$$

A.7.3 Unbonded beam (Prestress Type II)

From the properties of the beam,

$$\frac{p_e A_{st}}{U_w b d_1} = 0.0368$$

From Table 7 of CP115 : 1959,

$$\frac{f_m}{p_e} = 1.6528, \quad \frac{d_n}{d_1} = 0.1525$$

$$f_m = 1.6528 \times 120\,000$$

$$d_n = 0.1525 \times 9.31 = 1.42$$

$$\text{Loss of strain in compression steel at failure} = 0.002 \times \frac{0.638}{1.42} \times \frac{2}{3}$$

Loss of force per wire = 1065 lb

Remaining force in compression steel = 5070 lb

$$M_u = f_m A_{st} (d_1 - 0.4 d_n) + 5070 \times 0.07$$

$$P_u = \frac{M_u \times 2}{30 \times 2240} = 6.21 \text{ tons}$$

A.7.4 Bonded beam (Prestress Type I)

From the properties of the beam and reinforcement,

$$\frac{f_u A_{st}}{U_w b d_1} = 0.149$$

Referring to Table 7 of CP115 : 1959,

$$\frac{f_m}{f_u} = 1 \quad \text{and} \quad \frac{d_n}{d_1} = 0.3725$$

$$f_m = f_u; \quad d_n = 3.46 \text{ in.}$$

Loss of strain at failure in the compression steel is

$$= 0.002 \times \frac{0.638}{3.46} = 0.000368$$

Loss of stress per wire = 10900 lb/sq.in.

Remaining force at failure per wire = 5646 lb

$$M_u = f_m A_{st} (d_1 - 0.4 d_n) - 5646 \times 2 \times 0.746$$

$$P_u = \frac{M_u \times 2}{30 \times 2240} = 13 \text{ tons}$$

Load as given by strain gauge, lb.

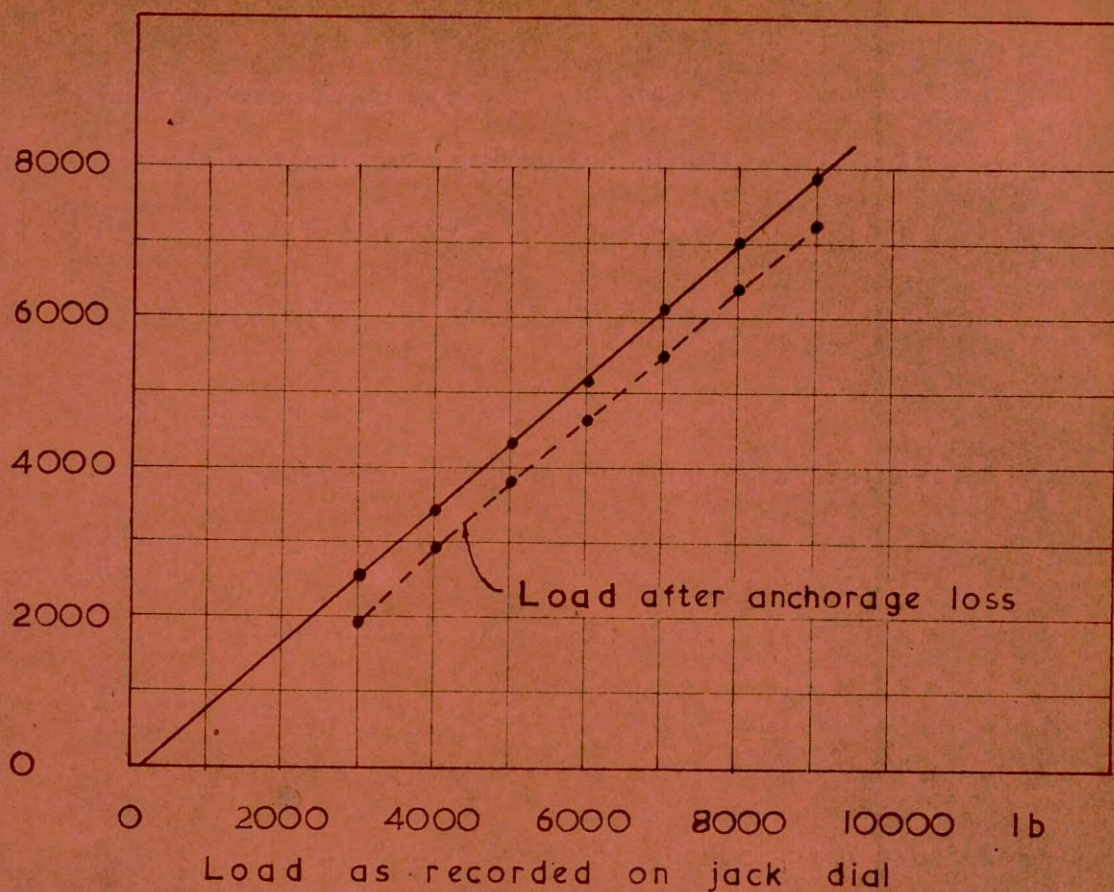


FIG. A.1

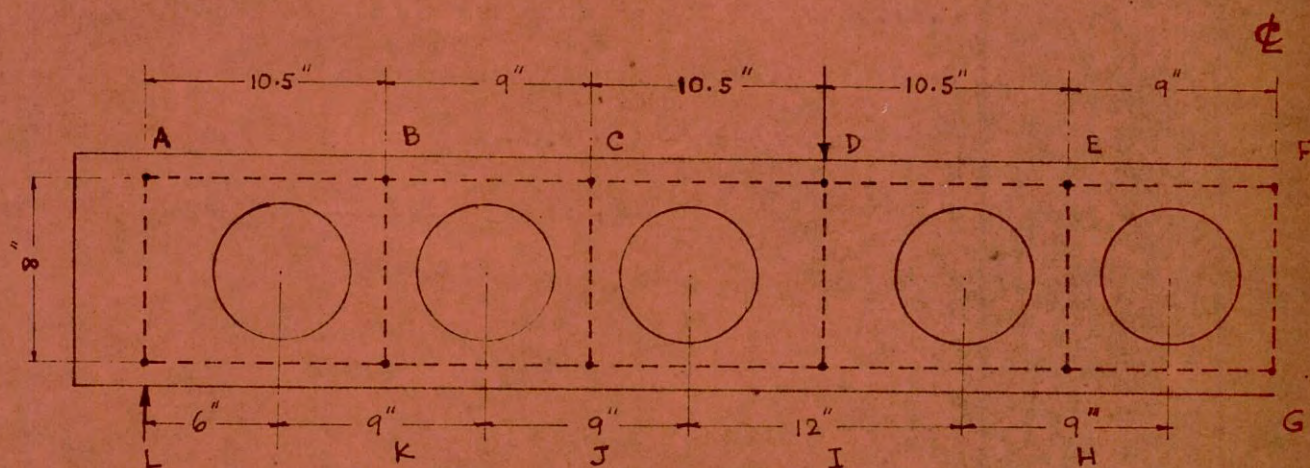


FIG. A.2

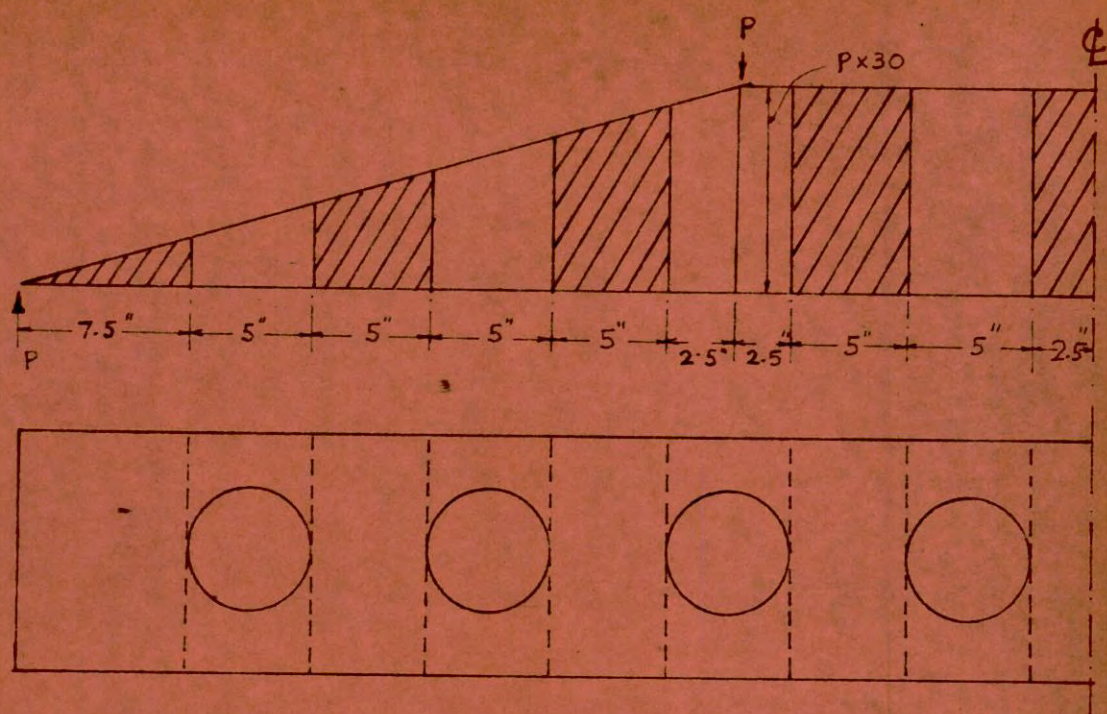


FIG. A.3

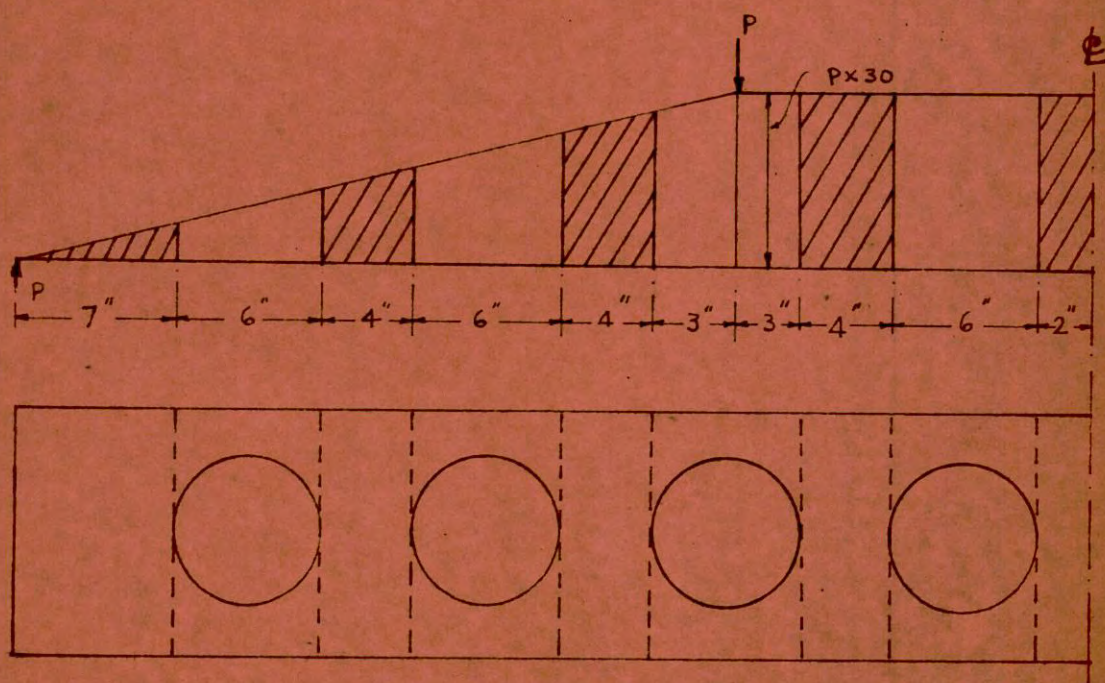


FIG. A.4

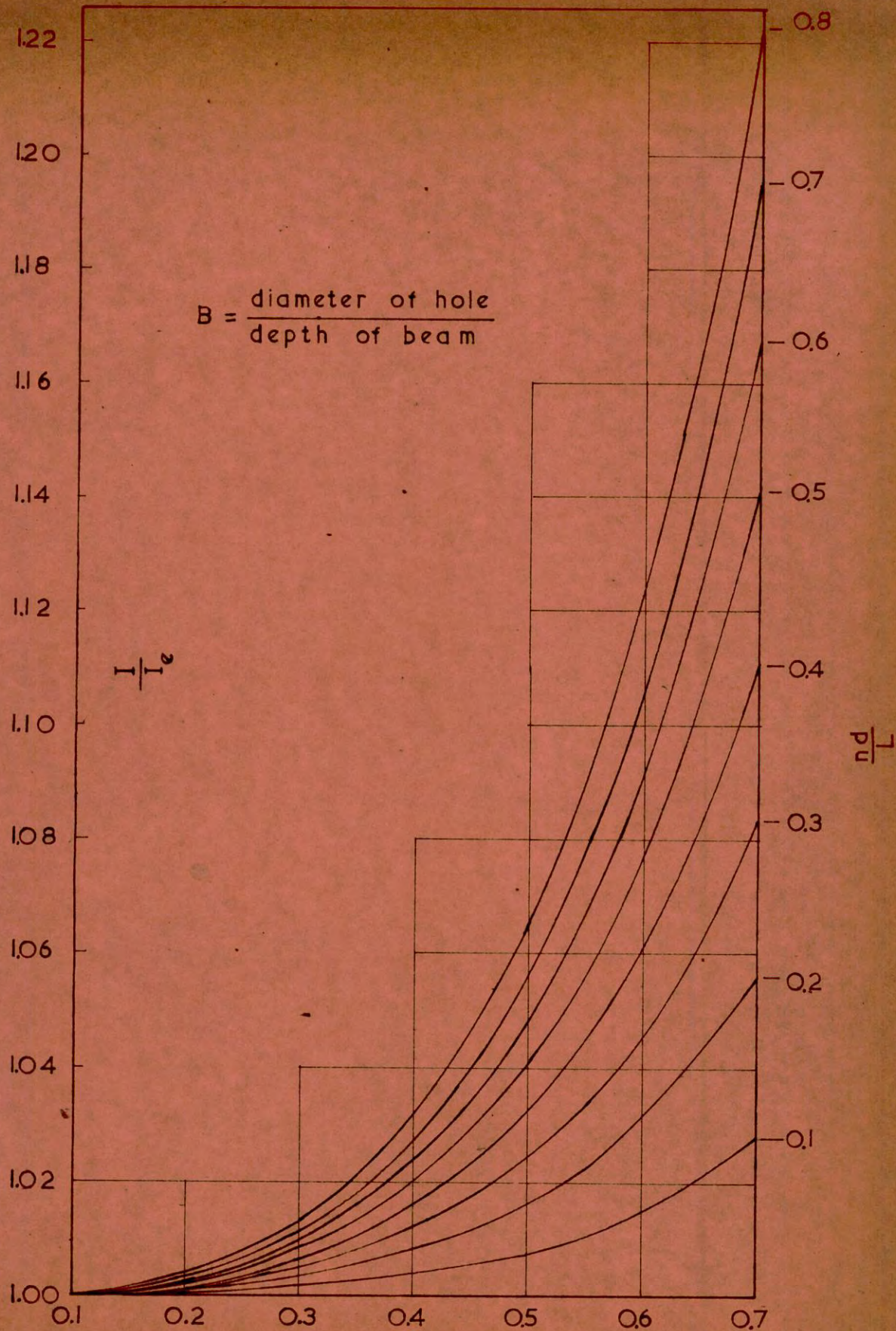


FIG. A.5

B

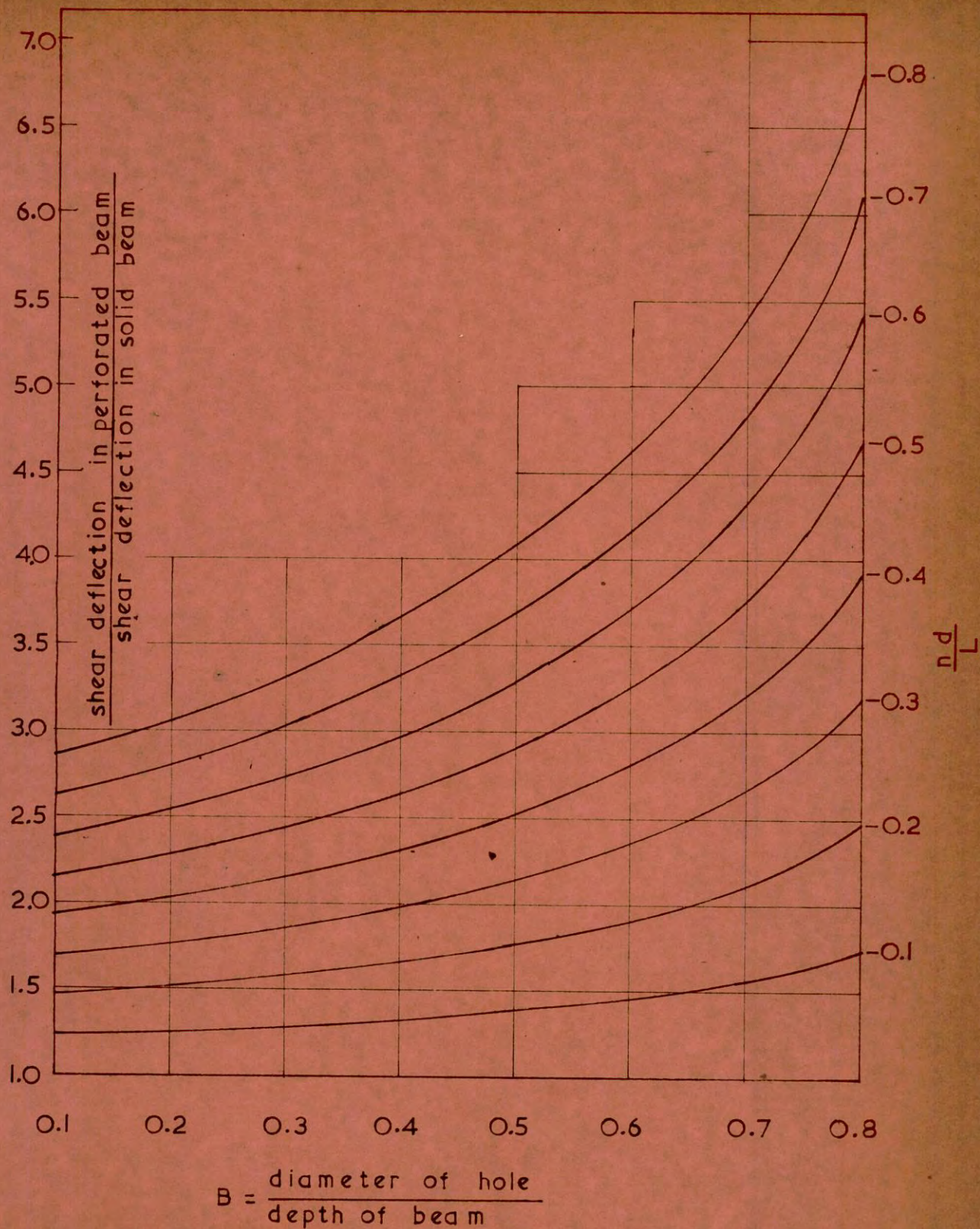


FIG. A.6

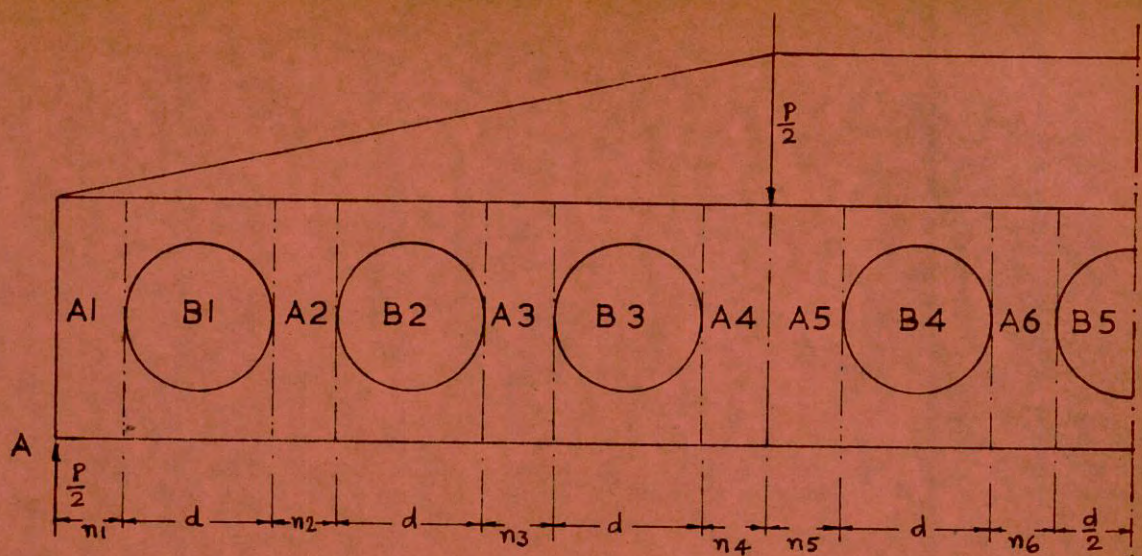


FIG. A.7

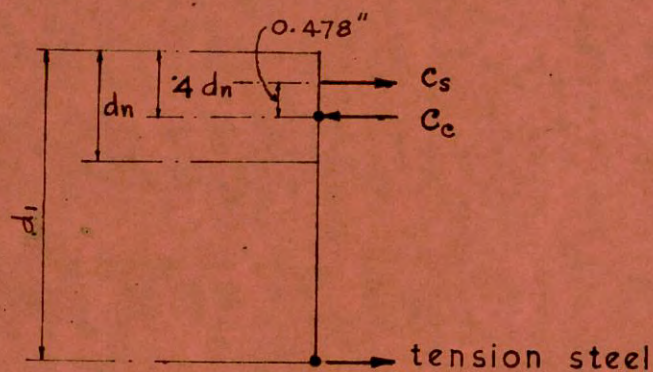


FIG. A.8

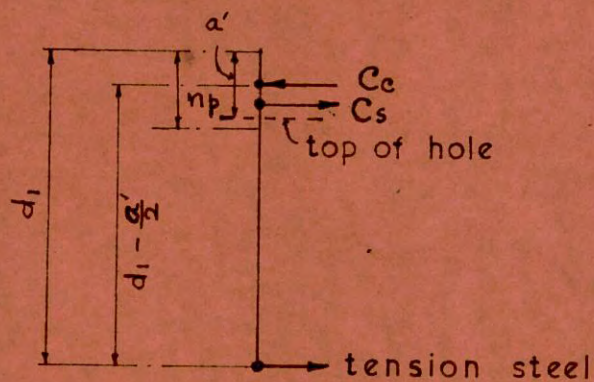


FIG. A.9

PROGRAMME NO.1.

begin comment to evaluate tangential stresses and strains at the boundary of a circular hole placed centrally or eccentrically in a beam by (a) prestressing force, (b) pure bending or bending with shear;

integer f1,f2,f3,f4,f5,m,t,L,u,I,w,i,p,f,q;
real a,E,P,Q,c,pe,fe,qe;
real array A[1:24], k[1:24];

open (30);
open (20);

f1:=format ([3s nd.d]);
f2:=format ([3s nd.dddd]);
f3:=format ([4s +ndddd]);
f4:=format ([5s +nd.dddddd 1]);
f5:=format ([4s +nd.dddddd c]);

write text (30,[[4s] tangential*stresses*and*strains*at*the*
boundary* of*a*circular*hole*placed*centrally*or*eccentrically
*in*a*beam*by*prestress* and*pure*bending *or*bending*with
*shear [2c] [4s] ANGLE [3s] PRESTRESS[6s] PRESTRAIN [3s]
LOAD[4s] ANGLE [6s] STRESS [6s] STRAIN [3s] TOTALSTRESS [3s]
TOTALSTRAIN [2c]]);

for m:=1 step 1 until 24 do
A[m]:=read (20);
for m:=1 step 1 until 24 do
k[m]:= mx0.2618-0.2618;

a:= read(20); t:= read(20); E:= read(20); P:= read(20);
Q:= read(20); L:= read(20); u:= read(20); I:= read(20);
w:= read(20); c:= read(20);

for i:=1 step 1 until t do
for m:=1 step 1 until 24 do

begin
if w=1 then begin
p:= A[m]xP/59.34-A[m]xQx4.26xc/I-Qx4.26xa/Ix(sin(k[m])
-sin(3xk[m]));
pe:=p/E;


```

if u=1 then
    f:= A[m]*i*1120*Lxc/I+i*1120*Lxa/Ix(sin(k[m])
        -sin(3*k[m]))+i*1120*xa^2/Ix(sin(4*k[m])-sin(2*k[m]))
        +2*i*1120/Ix5^2*xsin(2*k[m])
else
    f:= A[m]*i*1120*Lxc/I+i*1120*Lxa/Ix(sin(k[m])
        -sin(3*k[m]));
    fe:=f/E;
    q:=(p+f); qe:=q/E;
end
else begin
    p:= A[m]*P/59.34-Q*4.26*xa/Ix(sin(k[m])-sin(3*k[m]));
    pe:=p/E;
    if u=1 then
        f:=i*1120*Lxa/Ix(sin(k[m])-sin(3*k[m]))+i*1120*xa^2/Ix
            (sin(4*k[m])-sin(2*k[m]))+2*i*1120/Ix5^2*xsin(2*k[m])
    else
        f:=i*1120*xa*L/Ix(sin(k[m])-sin(3*k[m]));
        fe:=f/E;
        q:=(p+f); qe:=q/E;
    end;
write (30,f2,k[m]); write (30,f3,p); write (30,f4,pe);
write (30,f1,i); write (30,f2,k[m]); write (30,f3,f);
write (30,f4,fe); write (30,f3,q); write (30,f5,qe);
end;
close(20);
close(30)
end->

```

Sample data for beam SO₂-2

```

-1.58;-1.32;-0.51;+0.77;+2.32;+3.72;+4.32;+3.72;+2.32;
+0.77;-0.51;-1.32;-1.58;-1.32;-0.51;+0.77;+2.32;+3.72;
+4.32;+3.72;+2.32;+0.77;-0.51;-1.32;

2.5;
6;
4.52e6;
21600;
7200;
15;
1;
488;
2;
0;->

```


PROGRAMME NO.2

```
begin comment  A program to evaluate principal stresses and
                  their directions around a circular hole in a
                  beam due to prestress and pure bending or
                  bending with shear;

library AO,A6;

real z,n,m,mo,do,P,w,a1,b1,c1;

open(20); open(30);
z:= read(20);

begin
real array d,e,l,M,a2,b2,c2[1:4],ro[1:z],A[1:24],d1,d2,
            d3[1:z,1:24];
integer F1,F2,F3;

F1:=format([2s d.ddd]);
F2:=format([2s +nd dddd.d]);
F3:=format([2s +nd dddd.d c]);

for n:=1 step 1 until 4 do
begin  d[n]:= read(20);
        e[n]:= read(20);
        l[n]:= read(20);
        m[n]:= read(20);
end;

for m:=1 step 1 until z do
        ro[m]:= read(20);
        mo:= read(20);do:= read(20); P:= read(20);

for w:=1 step 1 until 24 do
        A[w]:= read(20);

write text (30,[[4s] stress*due*to*uniform*compression [2c]]);

for m:=1 step 1 until z do
for w:=1 step 1 until 24 do
```



```

begin a1:=PX(1/2X(1-cos(2XA[w]))+2xmo-do/ro[m]↑2);

for n:=1 step 1 until 4 do
begin a2[n]:=-PX(2X(nX(2xn+1)xd[n]/ro[m]↑(2xn+2)
+(n+1)X(2xn-1)xe[n]/ro[m]↑(2xn)+nX(2xn-1)x1[n]xro[m]↑(2xn-2)
+(n-1)X(2xn+1)xM[n]xro[m]↑(2xn))xcos(2xnXA[w]));
end;

b1:=PX(1/2X(1+cos(2XA[w]))+2xmo+do/ro[m]↑2);

for n:=1 step 1 until 4 do
begin b2[n]:=PX(2X(nX(2xn+1)xd[n]/ro[m]↑(2xn+2)
+(n-1)X(2xn-1)xe[n]/ro[m]↑(2xn)+nX(2xn-1)x1[n]xro[m]↑(2xn-2)
+(n+1)X(2xn+1)xM[n]xro[m]↑(2xn))xcos(2xnXA[w]));
end;

c1:=PX(1/2Xsin(2XA[w]));

for n:=1 step 1 until 4 do
begin c2[n]:=PX(2X(nX(2xn-1)X(1[n]xro[m]↑(2xn-2)
-e[n]/ro[m]↑(2xn))+nX(2xn+1)X(M[n]xro[m]↑
(2xn)-d[n]/ro[m]↑(2xn+2)))Xsin(2xnXA[w]));
end;

d1[m,w]:=a1+a2[1]+a2[2]+a2[3]+a2[4];
d2[m,w]:=b1+b2[1]+b2[2]+b2[3]+b2[4];
d3[m,w]:=c1+c2[1]+c2[2]+c2[3]+c2[4];

write(30,F2,ro[m]); write(30,F1,A[w]);write(30,F2,d1[m,w]);
write(30,F2,d2[m,w]); write(30,F3,d3[m,w]);
end;

begin real k,o,MO,I,a,h;
real array r[1:z],e1,e2,e3,f1,f2,f3,P1,P2,phi1[1:z,1:24];

for o:=1 step 1 until z do
r[o]:= read(20);

for k:=1 step 1 until 24 do
A[k]:=kx0.2618-0.2618;

write text(30,[[4s] distribution*of*total*stress*
due*to*prestress[2c]]);

MO:= read(20);I:=488; a:= read(20); h:=5.0;

for o:=1 step 1 until z do
for k:=1 step 1 until 24 do
begin e1[o,k]:=(-MOxr[o]/(4xI)+MOxa↑4/(4xIxr[o]↑3))
Xsin(A[k])+(5xMOxa↑4/(4xIxr[o]↑3)-MOx
a↑6/(Ixr[o]↑5)-MOxr[o]/(4xI))Xsin(3XA[k]);

```



```
e2[o,k]:=(-3*MOxr[o]/(4*I)-MOxa↑4/(4*IXr[o]↑3))*
sin(A[k])-(MOxa↑4/(4*IXr[o]↑3)-MOxa↑6/(IXr[o]↑5)+
-MOxr[o]/(4*I))*sin(3*A[k]);
```

```
e3[o,k]:=-(MOxa↑4/(4*IXr[o]↑3)-MOxr[o]/(4*I))*cos(A[k])
-(3*MOxa↑4/(4*IXr[o]↑3)-MOxa↑6/(IXr[o]↑5)+
MOxr[o]/(4*I))*cos(3*A[k]);
```

end;

```
for m:=1 step 1 until z do
for k:= 1 step 1 until 24 do
```

begin

o:=m; w:=k;

f1[o,k]:=d1[m,w]+e1[o,k];

f2[o,k]:=d2[m,w]+e2[o,k];

f3[o,k]:=d3[m,w]+e3[o,k];

P1[o,k]:=(f1[o,k]+f2[o,k])/2+sqrt(((f1[o,k]-f2[o,k])/2)↑2
+f3[o,k]↑2);

P2[o,k]:=(f1[o,k]+f2[o,k])/2-sqrt(((f1[o,k]-f2[o,k])/2)↑2
+f3[o,k]↑2);

phi1[o,k]:=1/2*arctan(2*f3[o,k]/(f1[o,k]-f2[o,k]))*57.32;

write(30,F2,r[o]); write(30,F1,A[k]); write(30,F2,f1[o,k]);

write(30,F2,f2[o,k]); write(30,F2,f3[o,k]);

write(30,F2,P1[o,k]); write(30,F2,P2[o,k]);

write(30,F3,phi1[o,k]);

end;

begin real t,u,p,Q,L;

Q:=1120; L:= read(20); t:= read(20); u:= read(20);

begin

real array g1,g2,g3,h1,h2,h3,C1,C2,phi2[1:z,1:24,1:t];

write text(30,[[4s] stresses*due*to*pure*bending*or*bending
*with*shear[2c]]);

for o:=1 step 1 until z do

for k:=1 step 1 until 24 do

for p:=1 step 1 until t do


```

begin
  if u=1 then

```

```

    begin g1[o,k,p] := -(2*QxpL/Ix(a↑4/(8xr[o]↑3)-r[o]/8)*
      sin(A[k])+Qxp/Ix(a↑4/r[o]↑4*(3*ht2/2-a↑2/2)-2*at2/r[o]↑2*
      (ht2-a↑2/4)+ht2/2)*sin(2*A[k])+2*QxpL/Ix(-a↑6/(2xr[o]↑5)+5*
      a↑4/(8xr[o]↑3)-r[o]/8)*sin(3*A[k])+Qxp/Ix(5*at8/(4xr[o]↑6)-
      3*at6/(2xr[o]↑4)+r[o]↑2/4)*sin(4*A[k]));

```

```

    g2[o,k,p] := -(2*QxpL/Ix(-a↑4/(8xr[o]↑3)-3xr[o]/8)*
      sin(A[k])+Qxp/Ix(a↑4/r[o]↑4*(-3*ht2/2+a↑2/2)-(ht2/2-r[o]↑2/2))
      *sin(2*A[k])+2*QxpL/Ix(a↑6/(2xr[o]↑5)-a↑4/(8xr[o]↑3)+
      r[o]/8)*sin(3*A[k])+Qxp/Ix(-5*at8/(4xr[o]↑6)+a↑6/(2xr[o]↑4)-
      r[o]↑2/4)*sin(4*A[k]));

```

```

    g3[o,k,p] := -(2*QxpL/Ix(-a↑4/(8xr[o]↑3)+r[o]/8)*
      cos(A[k])+Qxp/Ix(-a↑4/r[o]↑4*(3*ht2/2-a↑2/2)+a↑2/r[o]↑2
      *(ht2-a↑2/4)+(ht2/2-r[o]↑2/4))*cos(2*A[k])+2*QxpL/Ix
      (a↑6/(2xr[o]↑5)-3*at4/(8xr[o]↑3)-r[o]/8)*cos(3*A[k])+Qxp/Ix
      (-5*at8/(4xr[o]↑6)+a↑6/r[o]↑4+r[o]↑2/4)*cos(4*A[k]));

```

```

  end

```

```

  else

```

```

    begin

```

```

      g1[o,k,p] := -( -QxpLxr[o]/(4*I)+QxpLxa↑4/(4*Ixr[o]↑3))
      *sin(A[k])-(5*QxpLxa↑4/(4*Ixr[o]↑3)-QxpLxa↑6/(Ixr[o]↑5)-
      -QxpLxr[o]/(4*I))*sin(3*A[k]);

```

```

      g2[o,k,p] := (3*QxpLxr[o]/(4*I)+QxpLxa↑4/(4*Ixr[o]↑3))
      *sin(A[k])+(QxpLxa↑4/(4*Ixr[o]↑3)-QxpLxa↑6/(Ixr[o]↑5)-
      -QxpLxr[o]/(4*I))*sin(3*A[k]);

```

```

      g3[o,k,p] := (QxpLxa↑4/(4*Ixr[o]↑3)-QxpLxr[o]/(4*I))
      *cos(A[k])+(3*QxpLxa↑4/(4*Ixr[o]↑3)-QxpLxa↑6/(Ixr[o]↑5)-
      +QxpLxr[o]/(4*I))*cos(3*A[k]);

```

```

    end;

```

```

  write(30,F2,p); write(30,F2,r[o]); write(30,F1,A[k]);

```

```

  write(30,F2,g1[o,k,p]); write(30,F2,g2[o,k,p]);

```

```

  write(30,F3,g3[o,k,p]);

```

```

  end;

```

```

  write text(30,[[4s] principal*stresses*around*the*hole*due*
  to*prestress* and*pure*bending*or*bending*and*shear [2c]]);

```

```

  for o:=1 step 1 until z do

```

```

  for k:=1 step 1 until 24 do

```

```

  for p:=1 step 1 until t do

```

```

  begin

```

```

    h1[o,k,p] := f1[o,k]+g1[o,k,p];

```

```

    h2[o,k,p] := f2[o,k]+g2[o,k,p];

```

```

    h3[o,k,p] := f3[o,k]+g3[o,k,p];

```



```

C1[o,k,p]:=(h1[o,k,p]+h2[o,k,p])/2+sqrt(((h1[o,k,p]
-h2[o,k,p])/2)2+h3[o,k,p]2);
C2[o,k,p]:=(h1[o,k,p]+h2[o,k,p])/2-sqrt(((h1[o,k,p]
-h2[o,k,p])/2)2+h3[o,k,p]2);
phi2[o,k,p]:=1/2*arctan(2*h3[o,k,p]/(h1[o,k,p]-h2[o,k,p]))
x57.32;

write(30,F2,p); write(30,F2,r[o]); write(30,F1,A[k]);
write(30,F2,C1[o,k,p]); write(30,F2,C2[o,k,p]);
write(30,F3,phi2[o,k,p]);
end;
close(20);
close(30);
end;
end;
end;
end;
end→

```

Sample data for beam SO₂-2

4;

2.33₁₀⁻²; -1.89₁₀⁻¹; 1.40₁₀⁻¹; -3.62₁₀⁻²;
 3.14₁₀⁻⁴; -1.70₁₀⁻³; 3.27₁₀⁻²; -1.78₁₀⁻²;
 7.51₁₀⁻⁶; -3.64₁₀⁻⁵; 8.09₁₀⁻³; -6.47₁₀⁻³;
 1.45₁₀⁻⁷; -6.7₁₀⁻⁷; 1.95₁₀⁻³; -2.08₁₀⁻³;

0.5; 0.6; 0.8; 1.0;

1.40₁₀⁻²;
 1.32₁₀⁻¹;
 344;

1.5708; 1.3090; 1.0472; 0.7854; 0.5236; 0.2618; 0.0000; 6.0214;
 5.7596; 5.4978; 5.2360; 4.9742; 4.7124; 4.4506; 4.1888; 3.9270;
 3.6652; 3.4034; 3.1416; 2.8798; 2.6180; 2.3562; 2.0944; 1.8326;

2.5; 3.0; 4.0; 5.0;

31000;
 2.5;
 15;
 6;
 1;
 →

PROGRAMME NO.3

begin comment to evaluate principal stresses and their directions, given the strains of a rectangular rosette;

real m,n,mew,E;
integer f1,f2,t,w;

open(20); open(30);
t:= read(20); w:= read(20);

begin
real array ga, gb, gc[1:t,1:w], pa,pb,pc[1:t-2,1:w],
 P1,P2,phi1[1:w], C1,C2,phi2 [1:t-3,1:w];

f1:= format([5s +nd ddddd]);
f2:= format([5s +nd dddd.d c]);

for m:=1 step 1 until t do
for n:=1 step 1 until w do
 begin ga[m,n]:=read(20) x2.45₁₀⁻⁵;
 gb[m,n]:=read(20) x2.45₁₀⁻⁵;
 gc[m,n]:=read(20) x2.45₁₀⁻⁵;

end;
mew:=0.19; E:= read(20);

write text(30, [[4s] principal*stresses*due*to*
 prestressing*force [2c]]);

for m:=4 step 1 until t do
for n:=1 step 1 until w do

begin pa[1,n]:=ga[2,n]-ga[1,n];
 pa[m-2,n]:=0.95xpa[1,n]+ga[m,n]-ga[3,n];
 pb[1,n]:=gb[2,n]-gb[1,n];
 pb[m-2,n]:=0.95xpb[1,n]+gb[m,n]-gb[3,n];
 pc[1,n]:=gc[2,n]-gc[1,n];
 pc[m-2,n]:=0.95xpc[1,n]+gc[m,n]-gc[3,n];

end;

comment tensile stresses are +ve in this program;

for n:=1 step 1 until w do

begin P1[n]:= E/(2X(1-mew↑2))X((1+mew)X(pa[1,n]+
pc[1,n])+(1-mew)X sqrt(2X(pa[1,n]-
pb[1,n])↑2+2X(pb[1,n]-pc[1,n])↑2));

P2[n]:=E/(2X(1-mew↑2))X((1+mew)X(pa[1,n]+
pc[1,n])*(1-mew)X sqrt(2X(pa[1,n]-
pb[1,n])↑2+2X(pb[1,n]-pc[1,n])↑2));

if (pa[1,n]-pc[1,n])=0 then phi1[n]:=45.0 else
phi1[n]:=(1/2X arctan((2Xpb[1,n]-pa[1,n]-pc[1,n])/
(pa[1,n]-pc[1,n])))X57.32;

write(30,f1,P1[n]); write(30,f1,P2[n]); write(30,f2,phi1[n]);
end;

write text(30,[[4s] principal*stresses*due*to*prestress*and*
bending*with*shear[2c]]);

for m:=4 step 1 until t do
for n:=1 step 1 until w do

begin C1[m-3,n]:=E/(2X(1-mew↑2))X((1+mew)X(pa[m-2,
n]+pc[m-2,n])+(1-mew)X sqrt(2X(pa[m-2,n]-
pb[m-2,n])↑2+2X(pb[m-2,n]-pc[m-2,n])↑2));

C2[m-3,n]:=E/(2X(1-mew↑2))X((1+mew)X(pa[m-2,n]+
pc[m-2,n])-(1-mew)X sqrt(2X(pa[m-2,n]-pb
[m-2,n])↑2+2X(pb[m-2,n]-pc[m-2,n])↑2));

if (pa[m-2,n]-pc[m-2,n])=0 then phi2[m-3,n]:=45.0

else

phi2[m-3,n]:=(1/2X arctan((2Xpb[m-2,n]-
pa[m-2,n]-pc[m-2,n])/(pa[m-2,n]
-pc[m-2,n])))X57.32;

comment phi gives angles in degrees for P1 and C1 clockwise
from the horizontal direction of ga;

write(30,f1,C1[m-3,n]); write(30,f1,C2[m-3,n]);
write(30,f2,phi2[m-3,n]);

end;
close(20);
close(30);
end
end→

