INVESTIGATIONS ON REINFORCED AND PRESTRESSED CONCRETE BEAMS UNDER COMBINED BENDING, TORSION & SHEAR

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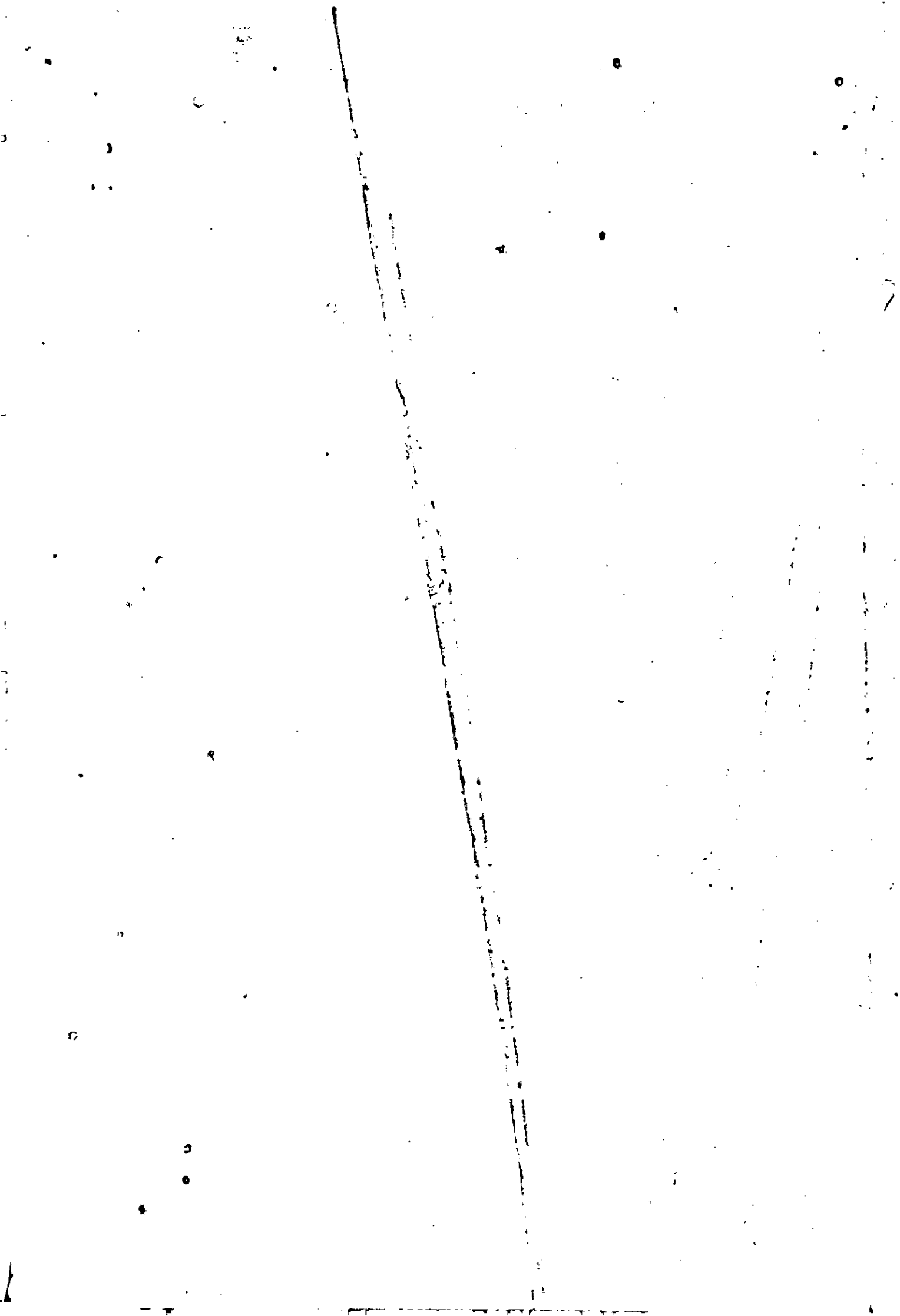

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PREFACE

Most of the Civil Engineering structures made out of concrete, either reinforced or prestressed, are subjected to combined loadings. This problem has been engaging the attention of research workers for many years in the past and it continues to do so because of its practical importance. Even though the contributions to the literature by many investigators are considerable, this problem is not yet fully solved because of its complex nature. The main aim of the present investigation is to give a physical interpretation to the behaviour of Reinforced and Prestressed Concrete Beams under combined loadings and hence to formulate a general solution to the problem and to use these results in giving a simple and straight-forward procedure for the design of these members. In particular, this thesis deals with the studies on reinforced and prestressed concrete beams under combined bending, torsion and shear. This more general problem has been further split into two parts (i.e. Bending and Torsion and Bending and Shear) so as to get a better understanding of it and finally the results of these two parts are combined together to discuss this general problem.

The thesis consists of six parts. The first part discusses the studies carried out on reinforced concrete beams under combined bending and torsion. In all, fifty five

rectangular and twenty tee beams have been tested, the main variables being ratio of twisting moment to bending moment, strength of concrete, longitudinal and transverse reinforcements. The general behaviour of beams and the influence of various physical quantities on the ultimate strength of beams have been discussed in detail in the earlier portions of this part. Based on certain mechanisms of failure and a failure criterion for concrete under compressive-tensile stresses, equations have been derived to predict the ultimate strength of reinforced concrete beams under combined bending and torsion. Later the theoretical results are studied in the light of experimental observations and it has been found that the analytical results are in good agreement with over three hundred test results available in the literature. Further a method has been suggested for the design of these members. This part also contains an exact solution for the elastic torsional properties of Tee and other alied flanged sections and approximate equations to calculate plastic torque of these sections.

The second part deals with the strength of reinforced concrete beams under combined bending and shear. The experimental work included testing of twenty one rectangular and nine tee beams with different shear span/depth ratios. In general, the behaviour of beams has been interpreted as two different mechanisms of failure and equations have been

derived for these two cases. Later the theory has been extended to the case of restrained beams and beams with uniformly distributed loads. On comparison with over one thousand test results, it has been found that the theoretical results agree fairly with the test observations. This part ends with the discussion of a method for the design of beams under combined bending and shear.

The analysis of the results on the strength of prestressed concrete beams under combined bending and torsion is discussed in the third part. The test results available in the literature have been carefully studied and the theory developed in the first part for the case of reinforced concrete beams has been extended to the case of prestressed concrete beams. Later the analytical results have been compared with the results of over two hundred tests available in the literature.

In the fourth part, the theory developed in the second part has been extended to the case of prestressed concrete beams under combined bending and shear. Realizing the importance of restricting the useful strength of prestressed concrete beams to its diagonal cracking strength, equations have been derived for predicting this. Results of about four hundred tests available in the literature have been found to give good agreement with the theoretical predictions.

Based on these results, a design method has been suggested.

The more general problem of reinforced concrete beams under combined torsion, bending and shear is discussed in the fifth part. The experimental work is of minor nature consisted of testing of ten reinforced concrete rectangular beams - with different ratios of torque/bending moment/shear force. Based on certain assumptions, equations have been derived for three modes of failure and the analytical results show good correlation with the tests results reported in the literature. This part ends with the illustration of the suggested method for the design of reinforced concrete beams under combined bending, shear and torsion.

The last part i.e. the sixth, summarises the results with respect to the deformation characteristics of reinforced concrete beams under combined bending and torsion and under combined bending and shear. Based on experimental results, an approximate method has been suggested to calculate deflection and angle of twist of beams under working loads determined using different load factors.

The thesis is concluded with a bibliography and three appendices. In the first appendix, a set of charts is given to calculate the elastic torsional properties of Tee

section as per the exact solution given in the first part of the thesis. The second appendix briefly reviews the various theories of failure of concrete under compressive-tensile stresses. The details of stress-strain relation in the compression zone of a concrete beam are given in the last appendix.

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C O N T E N T S

Acknowledgments

Preface

Contents

Notation

PART I : REINFORCED CONCRETE BEAMS UNDER COMBINED BENDING AND TORSION

			Page No.
CHAPTER I.1	: Introduction	...	1
I.2	: Review of Earlier Research Work	...	7
I.3	: Materials, Mixes used and Description of Specimens	...	57
I.4	: Making and Curing of Specimens	...	64
I.5	: Deformations and Behaviour of Beams	...	75
I.6	: Observed Modes of Failures	...	85
I.7	: Discussion of Experimental Results	...	91
I.8	: Torsion of Tee, Ell and I-Sections	...	118
I.9	: Theoretical Analysis of Reinforced Concrete Beams under Combined Bending and Torsion	...	149
I.10	: Discussion of Analytical Results	...	174
I.11	: Design Recommendations	...	213
I.12	: Conclusions	...	229

Contents (Contd.)

PART II : REINFORCED CONCRETE BEAMS UNDER COMBINED BENDING AND SHEAR

			Page No.
CHAPTER II.1	:	Introduction ...	241
II.2	:	Review of Earlier Research ...	246
II.3	:	Fabrication and Testing of Beams ...	256
II.4	:	General Behaviour and Observed Modes of Failure of Beams ...	260
II.5	:	Mechanism of Shear Failure ...	269
II.6	:	Theoretical Analysis of Shear Strength of Reinforced Concrete Beams ...	281
II.7	:	Discussion of Analytical and Test Results ...	301
II.8	:	Recommendations for Design ...	344
II.9	:	Concluding Remarks ...	358

PART III : TORSIONAL STRENGTH OF CONCRETE BEAMS UNDER AXIAL COMPRESSION AND OF PRESTRESSED CONCRETE BEAMS 365

PART IV : SHEAR STRENGTH OF PRESTRESSED CONCRETE BEAMS

CHAPTER IV.1	:	Introduction ...	390
IV.2	:	Review of Earlier Work ...	394
IV.3	:	Theoretical Analysis of Prestressed Concrete Beams under Combined Bending and Shear ...	398
IV.4	:	Discussion of Results ...	419
IV.5	:	Recommendations for Design and Concluding Remarks ..	435

Contents (Contd.)

		Page No.
PART V :	COMBINED TORSION, BENDING AND SHEAR IN REINFORCED CONCRETE BEAMS ...	451
PART VI :	STIFFNESS OF REINFORCED CONCRETE BEAMS UNDER COMBINED BENDING AND TORSION AND BENDING AND SHEAR ...	504
BIBLIOGRAPHY	...	528
APPENDIX 1 :	CHARTS FOR THE CALCULATION OF ELASTIC TORSIONAL PROPERTIES OF TEE SECTION.	
APPENDIX 2 :	THEORIES OF FAILURE OF CONCRETE UNDER COMPRESSIVE-TENSILE STRESSES.	
APPENDIX 3 :	STRESS-STRAIN RELATION IN THE COMPRESSION ZONE OF A CONCRETE BEAM.	

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NOTATION

In general the following notations are used throughout this thesis:

a	-	length of shear span
A_s	-	area of longitudinal steel
A_w	-	area of cross section of transverse reinforcing rods.
b	-	breadth of a rectangular beam or width of rib in a flanged beam.
b'	-	breadth of transverse hoops
B	-	breadth of flange in a flanged beam
c	-	depth of compression zone in a concrete beam
C	-	total compressive force acting at the compressi zone of a concrete beam
C_u	-	cube crushing strength of concrete
d	-	effective depth of a concrete beam
d'	-	depth of transverse hoops
D	-	overall depth of a concrete beam
e	-	eccentricity of the prestressing force with respect to centroid of the cross section in a prestressed concrete beam.
E, E_o	-	Modulus of Elasticity of Concrete
f, f'	-	normal compressive stress in concrete
f'_c, f'_t	-	uniaxial compressive and tensile strengths of concrete, respectively.
f_p	-	prestress in the prestressing wires

Notation (Contd.)

f_{pc}	-	prestress in the concrete
f_s	-	stress in longitudinal steel caused by bending moment.
f_r	-	modulus of rupture strength of concrete
f_y, f_{yw}	-	yield stress of longitudinal and transverse reinforcing steels, respectively.
G, G_o	-	modulus of rigidity of concrete
I, I_o	-	moment of inertia of the cross section
J, J_o	-	polar moment of inertia of the cross section
k	-	biaxial coefficient used in the theory
k'	=	f'_c/f'_t
K	=	τ/f'_c
$\bar{K}$	-	parameter defined in the theory
L	-	length of effective span of a simply supported beam.
M	-	bending moment
M_b	-	bending moment of a beam subjected to either bending and torsion or bending, shear and torsion.
M_{bo}, M_{FL}	-	flexural capacity of a beam subjected to pure bending.
M_{CR}	-	bending moment at failure of a beam subjected to combined bending and shear
M_t	-	twisting moment in a beam
M_{cf}	-	bending moment at flexural cracking
M_c	-	bending moment at diagonal cracking

Notation (Contd.)

M_{to}	-	torque capacity of a beam under pure torsion
$(M_t)_{max}$	-	maximum torque capacity of a beam under combined bending and torsion.
N, $\bar{N}$, N_w, $\bar{N}_w$		parameters defined in the theory
p	-	percentage of longitudinal steel (= A_s/bd)
P	-	prestressing force
q_p	=	pf_p/f
q_s	=	pf_s/f
q	=	$q_p + q_s$
Q	-	first moment of the area below or above centroid taken about the centroid of the cross section
r	=	A_w/bs
R	=	$M/bd^2f'_c$
s	-	spacing of transverse hoops
t	-	thickness of flange in a flanged beam or the thickness of wall in a hollow beam.
T	-	total tensile force in the longitudinal steel
V	-	shear force
$V_c, V_{cw}, V_{cd}, V_{cf}$	-	shear force at diagonal cracking
y	-	distance of the extreme fibre in the tension zone from the centroid of the cross section
μ	-	Poisson's Ratio
α	=	$M/Vd = a/d$
α_{TR}, α_m α_{cw}, α_{cd}	-	parameters defined in the theory

Notation (contd.)

ϕ = M_t/M_b

λ, β_a, β_e, β_c, β_e^{fl}, β_a^{fl}, β_f, β_θ – parameters as defined in the thesis.

δ – deflection of a beam

θ – angle of twist per unit length of a beam

τ – shear stress

τ_{xz}, τ_{yz} – shear stress components in a beam subjected to twisting moment

Other notations as defined in the thesis wherever they occur.

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PART I : REINFORCED CONCRETE BEAMS UNDER COMBINED BENDING AND TORSION

CHAPTER I.1

INTRODUCTION

I.1.1 The need for research in this problem:

Torsion is considered invariably as a secondary effect in Civil Engineering Structures. This may be true only in such cases when the torsional stresses are caused by the loads applied with an eccentricity to the plane containing the bending axis of the member or when they are produced by the skewness of the structure. But it is of primary importance in the case of members in a monolithic grid system or in the case of girders curved in plan. In general torsion is accompanied by direct shear and bending moment. In such cases, even if the torsional stresses are of small magnitude it might increase the direct shear stress and normal stress caused by the shear force and bending moment. This might reduce the ultimate and cracking strengths of members thus reducing the safety factor.

By going through the existing codes and specifications for reinforced concrete in many countries, one can understand that the design recommendations for torsion are based on any one of the classical elastic theories given by Rausch[1] or Cowan[2]. It seems that these recommendations are based on the feeling

that the magnitude of torsional stresses are secondary and the factors of safety involved in the codes will account for these secondary effects. It can be argued that even though these recommendations have not considered the interaction of stresses, the members designed based on them have not suffe: any serious damage or failure. This argument may not be strictly correct as the members designed as per these recommendations are over-reinforced to a great extent and hence no failure or damage to the structure can be expected. But over-reinforcing the members to a great extent might affect the economy of the structure. Further these recomm tions are applicable for the working load conditions. It very much emphasised in the recent years that these convent methods of designing reinforced concrete members are not al satisfactory and they do not give a consistent factor of sa in regard to failure. So design methods based on ultimate load philosophy of these members are critically needed.

A 'crude' estimate of the importance of any problem be obtained by considering the amount of research work conc on that particular problem. The problem of torsion has b tackled even as early as 1912[3]. The immensity of the res work increased after Nylander[4] reported his exhaustive investigation on concrete under combined stresses in 1945. In recent years we find that extensive research work is be carried out on this problem throughout the globe. This i

clearly indicated by the summary of research activities compiled by ASCE Task Committee[5] and the number of papers published on this subject in recent years.

The above discussion indicates that torsion is an important problem in reinforced concrete construction. But torsion combined with bending moment and shear force is an extremely difficult problem because our knowledge regarding the behaviour and strength of concrete under combined stresses is limited. Attempts have already been made to evolve certain solution to predict the strength of members in combined torsion, but again these are accompanied by a new awareness of gaps in our present knowledge. So extensive experimental and analytical research is needed to fully understand the fundamental behaviour of members under the combined action of torsion, bending moment and shear. Once this knowledge is acquired, it will then be possible to design these members using ultimate load methods of design.

I.1.2 Object and scope of the present investigation:

The importance of the problem of combined torsion has been stressed in the preceding article. In reinforced concrete, the simple superposition of effects of pure bending, pure torsion and pure shear may not give realistic results as there is considerable interaction among these stresses. But the author felt that to understand the behaviour of members under combined bending, shear and torsion,

it is necessary to study the individual effects of shear force and torsion on the bending moment. So in the earlier parts of this thesis (Parts I and II), the behaviour of beams under combined bending and torsion and under combined bending and shear are studied in detail. These individual studies are very useful in understanding the more general and complicated problem of combined bending, shear and torsion, which is discussed in Part V of this thesis.

In this part of the thesis, (i.e. Part I) the experimental and analytical investigations carried out on beams under combined bending and torsion are discussed. A fairly large amount of experimental work has been done with regard to rectangular beams only. But most of the structural members are of Tee or Ell sections. Therefore, in practical research both Tee and Ell beams should be tested and investigated along with rectangular ones. In his earlier investigation[6] the present author has tested rectangular beams. In the present investigation, rectangular beams were tested in the first instance so as to get clarification regarding certain points and to have better understanding of the problem. Later the experimental investigation was extended to the case of tee beams. The object of this work is to make a contribution to a better understanding of the effect of torsional stresses on reinforced concrete sections, the modes of failures and other related problems and hence to arrive at a suitable method on a more

rational basis to design such sections using ultimate load theory.

The investigation has been conducted in four phases. In the first phase of the work, an extensive study of all the literature available to the author has been made and all known factors that seem to influence the behaviour and stren of beams have been listed. The experimental investigation has been planned utilising this knowledge.

In the second phase, the experimental work has been carried out as programmed. The experimental work has been carried out to investigate the following factors:

i. Properties of concrete
ii. Longitudinal reinforcements
iii. Transverse reinforcements
iv. Ratio of twisting moment to bending moment
v. Presence of flexural cracks
vi. Shape of the cross section

In all, fifty five rectangular beams and twenty tee beams hav been tested to yield information regarding the influence of these factors on the strength of beams. The test results ha been studied in detail and appropriate conclusions have been drawn.

In the third phase of the work, analytical investigation has been done. Using a Fourier Series Technique and Nadai's[7] Sand-Heap analogy, both elastic and rigid plastic solutions have been obtained for the torsion of Tee and other allied sections. Then using certain failure mechanisms and a failure criterion for concrete under compressive-tensile stresses, equations have been derived for two causes of failure. The predictions of the theory have been compared with the tests results of about three hundred beams and it has been found that the theory is fairly good in predicting the test results. Further certain interesting results given by the theory have been examined in the light of the experimental results.

In the last phase, the provisions of the codes for torsion design have been studied critically and some new recommendations have been suggested for the design of members subjected to combined bending and torsion.

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CHAPTER I.2

REVIEW OF EARLIER RESEARCH WORK

A detailed literature survey of the earlier work done on this problem has revealed that this field has drawn attention of research workers from 1912 onwards. The contributions of the previous investigations are reviewed critically in the next few pages.

Bach and Graf[3] in 1912 carried out experiments on plain concrete as well as reinforced concrete specimens of various cross sections subjected to pure torsion. This was the first investigation taken up to study the effect of torsional stresses on concrete members. The torsional computations were based on Bach's Approximate Theory[8] which can be stated as thus:

The torque capacity, M_t of a narrow rectangular section = $\frac{1}{3}$ b^3DG and the maximum torsional stress is

$$\tau_{max.} = G\Theta b$$

where b and D are the breadth and depth of rectangular section, respectively, G is the shear modulus and Θ is the angle of twist per unit length of the beam.

If the section is composed of a number of narrow rectangles, then

$$\frac{M_t}{G\theta} = \Sigma \frac{1}{3} b^3 D$$

and

$$\tau_{max.} = G\theta t' \qquad \ldots \text{ I.2.1}$$

where t' is the largest thickness of the component rectangles.

They studied the influence of various types of reinforcements and finally concluded that only with helical type of reinforcements the torque capacity of members can be considerably increased. This is evident as the stress distribution produced by the twisting moment always caused a helical fracture. They concluded that the increase in the torque capacity of members by different types of reinforcements is as follows:

Type and Amount of Reinforcement	M_t / M_{tc}
2.88 % longitudinal steel	1.09
2.88 % longitudinal steel + 0.49 % of 45° spirals	2.34
1.44 % inclined hoops	1.27

where M_t is the ultimate torque capacity of members with

specified reinforcements and M_{tc} is the ultimate torque capacity of corresponding member of plain concrete.

Bach's Approximate Theory is valid only for sections composed of thin rectangles and requires modifications for the normal sections encountered in reinforced concrete construction.

In 1922, Graf and Mörsch[9] reported experiments on plain and reinforced concrete specimens of circular cross-section subjected to pure torsion. Their investigation confirmed the conclusions of Bach and Graf[3].

Young, Sagar and Hughes[10] in 1922 conducted 'pure torsion' experiments on square as well as rectangular specimens reinforced with longitudinal and spiral steel. They concluded that the increase in the ultimate torque due to the increase in the longitudinal reinforcement alone is virtually nil, but the increase due to spiral steel is paramount.

In 1927, Miyamato[11] reported test results of 66 circular specimens tested in torsion. His conclusions regarding the influence of various types of reinforcements on the torque capacity of beams are summarised below:-

Type and Amount of Reinforcement	M_t / M_{tc}
0.71 % of 45° spirals	1.59
0.72 % of 60° spirals	1.57

0.75 % of 30° spirals	1.47
1.43 % of 45° spirals + longitudinal bars	1.66
1.42 % of 60° spirals + longitudinal bars	1.66
1.42 % of 45° spirals + lateral rings	1.74
1.45 % spirals at 30° + lateral rings	1.73
1.43 % longitudinal bars + lateral rings	1.12
1.22 % two layers of spirals at 45°	2.07
2.08 % two layers of spirals at 45° + longitudinal bars	2.18

In addition, he reported that the modulus of rigidity was 2.09×10^6 psi at low torsional loads and 1.74×10^6 psi at 83 per cent of ultimate torque. The relation of modulus of rigidity to the torque capacity of beams may not express the proper physical behaviour of beams. Strictly, the torsional rigidity GJ depends on the torque capacity of beams. From the present investigation, it has been found that the torsional rigidity GJ goes on decreasing as the torque was increased and reached a minimum value when failure occurred.

Turner and Davies[12] in 1934 conducted pure torsion experiments on plain and reinforced concrete. They investigated the problem of Tee section in the case of a

concrete section and found that the torque taken by the section is given by dividing the section into a number of rectangles and using the formula,

$$M_t = \frac{4A}{P} (R_1 + R_2 + \ldots\ldots\ldots R_n) \qquad \ldots \text{I.2.2}$$

where

A - area of cross section

P - perimeter of the cross section

D - diameter of inscribed circle

and R_1, R_2....etc. - torque taken by the individual rectangles alone. Later this formula has been substantiated by the test results of Marshall and Tembe[15].

Andersen[13] in 1935 reported test results of members subjected to pure torsion. He presented a formula for the torsional strength of reinforced concrete circular members derived from theoretical considerations. The final formula can be put in the form

$$M_t = \frac{A_w}{s} \lambda' \sqrt{2} \; f_w . A' \qquad \ldots \text{I.2.3}$$

where

M_t = torsional strength of the member

A_w = cross-section area of one spiral wire

s = pitch of the spiral reinforcement

f_w = maximum allowable stress in spiral steel

A' = effective cross-sectional area of the section

λ' = numerical factor

($\lambda' = 2.00$ for circular sections

$\lambda' = 1.33$ for rectangular sections)

For circular sections his formula coincided with the formula given by Rausch[1]. For rectangular sections he arbitrarily recommended that the torsional strength of these sections can be taken as two-thirds of a circular section thus deducing the value of λ' to be equal to 1.33. He further deduced that the spiral reinforcement can be assumed to take practically all tensile stresses in excess of the ultimate tensile strength of the unreinforced concrete.

Further, he[14] reported that the ratio of torsional strength to compressive strength decreased as the compressive strength increased. If f'_c is the compressive strength, f'_t is the tensile strength and M_{to} is the torsional strength of a beam under pure torsion it follows that:

$\frac{f'_t}{f'_c}$ decreases as f'_c increases

M_{to} may directly vary with f'_t and hence

$\frac{M_t}{f'_c}$ decreases as f'_c increases. Thus his deduction follows from the simple logic presented above.

In 1941, Marshall and Tembe[15] reported test results of 26 plain concrete specimens of circular, rectangular, T shaped and L shaped cross sections and 24 rectangular and tee beams with mild steel reinforcements. They noticed instantaneous failure of plain concrete members. Rausch's[1] spiral reinforcement formula, which is valid for nearly circular sections, gave consistent values for the steel stress at failure. They proposed an empirical formula to calculate the torque capacity of beams i.e.

$$M_t = M_{tc} (1.33 + 0.1p) \qquad \ldots \text{ I.2.4}$$

where

M_t = torque resisted by a reinforced beam

M_{tc} = torque resisted by a plain concrete beam calculated using Turner and Davies formula (Eqn. I.2.2)

p = percentage of transverse steel

($p \neq 1.5$ %)

Marshall[16] in 1944 analysed all the results of torsion tests on plain concrete which were available at that time. The results are shown in Table I.2.1.

TABLE I.2.1

Comparison of Results (after Marshall[16])

Investigator	Section tested	Max. shearing stress (calc) psi		Direct tensile strength psi
		Elastic theory	Plastic theory	
Turner and Davis[12] (1934)	Square	395	247	301
		422	264	-
	2:1 Rectangle	528	309	-
		381	223	219
Bach and Graf[3] (1912)	Square	425	265	Not given
	2:1 Rectangle	450	263	
	Circle	350	261	
Bach[8] (1911)	Square	430	269	264
	2:1 Rectangle	460	269	
	Circle	360	269	
Young, Sagar and Hughes[10] (1922)	Square	539	336	Not given
	1.5:1 Rectangle	467	279	
	2:1 Rectangle	602	352	
Anderson[13] (1935)	Square	539	336	Not given
	1.25:1 Rectangle	505	306	
	1.5:1 Rectangle	575	342	
Anderson[14] (1937)	Square	640	400	Not given
	1.25:1 Rectangle	609	369	
	1.5:1 Rectangle	658	392	
Graf and Morsch[9] (1922)	Circle	202	151	164
Marshall and Tembe[15] (1941)	Circle	325	243	282
	Square	508	318	
	1.5:1 Rectangle	472	287	
	1.7:1 Rectangle	530	313	
	2:1 Rectangle	516	302	
	3:1 Rectangle	580	349	

Instead of calculating the ultimate torque he calculated the maximum shearing stress and compared the same with the tensile strength of concrete. From this he concluded that the plastic theory gives results which are much nearer the truth than those obtained by the elastic theory. From the table it can be seen that the comparison is not complete as the tensile strength of concrete used is not given for many cases. Also the plasticity of concrete decreases as the strength increases and the results analysed by Marshall have been obtained from specimens of low strength concrete. Hence his conclusion requires certain limitation.

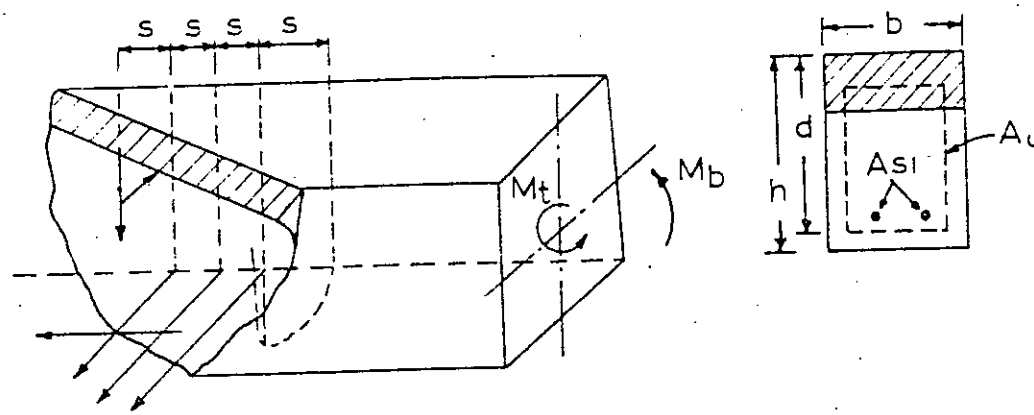
Nylander[4] in 1945 reported tests on about sixty beams carried out at the Royal Technical College, Stockholm. The beams have been tested in pure torsion, combined bending and torsion, combined shear and torsion, and combined bending and compression. Nylander concluded that the stress distribution at failure is such that the maximum shearing stress over the whole cross section has a constant value equal to the tensile strength of concrete, i.e. that concrete for the purpose of assessing the ultimate torsional strength may be treated as an ideal plastic material. This basic assumption of Nylander seems to have certain limitation and perhaps his assumption is more suitable for members having low concrete strength.

Nylander further concluded that the addition of direct compressive stress produced an increase in torsional strength of members whereas the addition of direct shearing stresses decreased the torsional strength.

During 1948-1956 Lessig[17-19] obtained a theoretical solution for predicting the ultimate strength of reinforced concrete rectangular beams subjected to combined bending and torsion and combined shear and torsion. Her analysis has been based on the following assumptions:

a. all reinforcements yield before crushing occurs
b. tensile strength of concrete is neglected
c. transverse reinforcement is uniformly distributed throughout the beam.
d. the zone of failure is free from local applied loads, and
e. plane of shear force and of bending moment is parallel to the vertical edge of the rectangular beam.

The analysis has been based on the criterion that when shear force is relatively small, failure has to occur due to the combined effects of bending and torsion and when the bending moment is relatively small, failure has to occur due to the combined effects of shear and torsion. Based on the hypothesis that when shear force was negligible, failure occurred by rotation about an axis in a horizontal plane



$$\phi = M_t / M_b$$

$$\bar{a} = b/h$$

$$p_1 = A_{sl}/bd$$

$$\lambda_1 = \frac{1+\alpha}{2(1+r^2)}$$

$$r = \sqrt{\left(\frac{1}{\phi}\right)^2 + \frac{f_y A_{sl}}{f_{y\omega} A_\omega} \frac{S}{b}\left(1+\frac{2}{a}\right)} - \frac{1}{\phi} \approx \left(1+\frac{2}{a}\right)$$

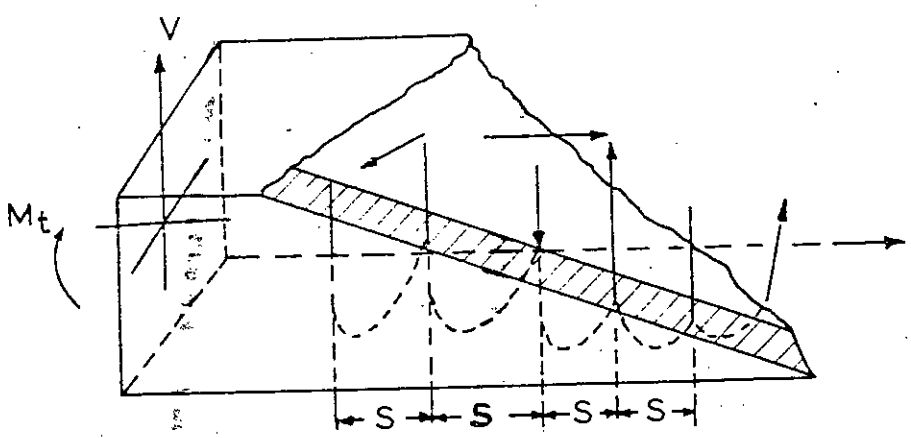
$$\bar{\alpha} = \frac{f_{y\omega} A_\omega}{f_y A_{sl}} \frac{b}{s} \frac{r^2}{\left(1+\frac{2}{a}\right)}$$

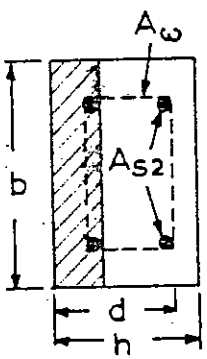
$$K_b = \phi \left\{ \frac{1+\bar{\alpha}}{1+r\phi} \right\}$$

f_y, f_{yw} — Yield stress of longitudinal and transverse reinforcements.

f'_c — Crushing strength of concrete prism

FIG. I.1. FAILURE MODES AS DEFINED IN LESSIG'S THEORY.





$$\bar{Q} = M_t / Vh$$

$$\bar{a} = b/h$$

$$p = A_{s2}/bd$$

$$\bar{v} = V/bd$$

$$r = 1 + 2/a$$

$$\bar{\alpha} = \frac{1 + f_{y\omega} A_\omega}{f_y A_{s2}} \frac{b}{s} \left(1 + \frac{2}{a}\right)$$

$$\lambda_2 = \frac{\bar{\alpha} - \bar{v}/p_2 r f_y}{2(1+r^2)}$$

$$K_v = \bar{\alpha}/r\left(1 - \frac{2}{\phi}\right)$$

FIG. I. 2. FAILURE MODES AS DEFINED IN LESSIG'S THEORY.

normal to the plane of bending and when the bending moment was negligible, failure occurred by the rotation about an axis which can be closely approximated by one in a vertical plane parallel to the shearing force, equations have been derived.

Omitting the intervening derivations, the final equations of this analysis can be put in the following form:

a. For combined bending and torsional moments

M_b and M_t respectively :- (Fig. I.1)

$$\frac{M_t}{bd^2 f'_c} = K_b \frac{p_1 f_y}{f'_c} \left(1 - \lambda_1 p_1 \frac{f_y}{f'_c}\right) \qquad \ldots \text{I.2.5}$$

b. For combined shear V and torsional moment

M_t :- (Fig. I.2)

$$\frac{M_t}{bd^2 f'_c} = K_v \frac{p_2 f_y}{f'_c} \left(1 - \lambda_2 p_2 \frac{f_y}{f'_c}\right) \qquad \ldots \text{I.2.6}$$

Lessig's analysis is based on the concept of plastic hinges and two modes of failure shown in Figs. I.1 and I.2. In a historical review of the problem, Collins et al[20] have shown that her theoretical equations give better result

when compared to those given by other investigators[42,53]. The basic concept suggested by Lessig has been used by most of the later investigators[42,53,54,56,57].

In 1950, Cowan[2,21] proposed an elastic theory for the torsional strength of rectangular reinforced concrete beams. After examining the theories proposed by Hsu[1] and Anderson[13], Cowan derived certain new formulae for the design of shear reinforcement for beams under pure torsion.

According to his theory, the amount of shear reinforcement required in a circular or rectangular section, when the permissible shear stress in the concrete has been exceeded is given by

$$\frac{A_w}{s} = \frac{M_t}{\lambda' \sqrt{2}\, f_w . A' \sin(\psi + 45^\circ)} \qquad \ldots \text{ I.2.7}$$

In particular, if the shear reinforcement is in the form of 45° spirals,

$$\frac{A_w}{s} = \frac{M_t}{\lambda' . \sqrt{2}\ f_w . A'} \qquad \ldots \text{ I.2.8}$$

and only nominal longitudinal reinforcement is required. If the shear reinforcement is in the form of vertical hoops,

$$\frac{A_w}{s} = \frac{M_t}{\lambda' \cdot f_w \cdot A'}$$

and an equal or greater volume of longitudinal reinforcement must be provided.

Numerical value of λ'

	Circular section	Rectangular section
Rausch's Theory[1]	2.00	2.00
Anderson's Theory[3]	2.00	1.33
Cowan's Theory[2,21]	2.00	1.60

During 1953-55 Cowan and Armstrong[22,25] conducted experiments on reinforced as well as prestressed concrete beams and also concrete-encased steel joists in combined bending and torsion. In all, thirty-three beams have been tested in this investigation and they observed two distinct types of failures in beams subjected to combined bending and torsion - one a primary torsion failure with a cleavage fracture normal to the direction of principal tension and the other primary bending failure resulting in the crushing of the concrete on the compression face. Finally, they have

given the following recommendations for the design of member subjected to torsion under working loads:

1. The torsional stiffness of a rectangular plain concrete section is given by

$$\frac{M_t}{\theta} = \beta' G b^3 D$$

2. The maximum permissible twisting moment (working moment) of a rectangular concrete section without shear reinforcement is

$$M_t = \alpha' b^2 D F'_{ct}$$

3. The torsional strength of a concrete-encased steel beam is approximately the same as that of plain concrete beam of the same external dimensions, provided there is a nominal amount of secondary reinforcement to ensure that the two materials act together.

4. The maximum permissible twisting moment (working moment) of the reinforced section is

$$M_t = \alpha' b^2 D F'_{ct} + \lambda' \sqrt{2} b' d' \frac{A_w}{s} \sin(\gamma + 45^\circ) f_w$$

... I.2.9

5. The maximum permissible twisting moment for a rectangular prestressed concrete beam is

$$M_t = M_{tc}\sqrt{1 + (f_p/F'_{ct})} \qquad \ldots \text{ I.2.10}$$

In these α', β' and λ' refer to numerical coefficients shown in Table I.2.2 and F'_{ct} denotes the maximum permissible stress (working stress) for the concrete in diagonal tension.

TABLE I.2.2

Values of α', β' and λ' defined in Cowan's Theory

D/b or d'/b'	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0
α'	0.208	0.219	0.227	0.234	0.240	0.246	0.258	0.267
β'	0.141	0.166	0.187	0.204	0.217	0.229	0.249	0.264
λ'	1.669	1.623	1.602	1.597	1.603	1.614	1.654	1.689

It is to be pointed out here that the equations proposed by Cowan are suitable to calculate the working moments. So it may not be possible to have a direct check of the validity of these equations using the results obtained from the ultimate load tests. However the safety factor given by the equations can be obtained. Table I.2.3

presents such a comparison. In this table, the torque capacities of beams as given by Cowan's theory are compared with the ultimate torques resisted by the beams and the factor of safety obtained are given in the last column. All the beams listed in Table I.2.3 have been tested in pure torsion and the values of F'_{ct} and f_w used in the calculations are taken as recommended in the I.S.Code[26]. From Table I.2.3 the following conclusions can be drawn:

i. Cowan's theory gives a factor of safety ranging between 1.5 and 2.0 for beams with both longitudinal and transverse reinforcements.

ii. For beams with longitudinal reinforcement only, his theory seems to give very conservative results, the factor of safety being over 4.0 in many cases.

They have also recommended to design reinforced concrete sections and concrete-encased steel joists independently for bending and torsion, without reduction or adjustment in the maximum permissible concrete and steel stresses appropriate to pure bending and torsion. Once again it may not be possible to check the validity of this statement using the results obtained from ultimate load tests. However the results obtained from ultimate load tests have shown that the ratio of twisting moment to bending moment has considerable influence on the strength of beams.

TABLE I.2.3

COMPARISON OF TEST RESULTS WITH THE THEORETICAL TORQUE CAPACITIES CALCULATED AS PER COWAN'S THEORY

Investigator	Specimen mark	M_t (ultimate) test in-kips	Cowan's Theory M_{tc} in-kips	Cowan's Theory M_{ts} in-kips	Cowan's Theory M'_t (working) in-kips	Factor of Safety (M_t/M'_t)
Ernst (Ref.No.30)	3TR- 0	37.6	14.6	0	14.6	2.58
	1	35.0	14.6	2.6	17.2	2.04
	3	34.3	14.6	5.2	19.8	1.73
	7	49.7	14.6	10.3	24.9	2.00
	15	61.7	14.6	18.0	32.6	1.89
	30	76.0	14.6	36.0	50.6	1.50
	4TR- 0	34.4	14.6	0	14.6	2.36
	1	32.1	14.6	2.6	17.2	1.87
	3	35.0	14.6	5.2	19.8	1.77
	7	54.8	14.6	10.3	24.9	2.20
	15	74.0	14.6	18.0	32.6	2.27
	30	85.0	14.6	36.0	50.6	1.68
	5TR- 0	33.8	14.6	0	14.6	2.32
	1	33.4	14.6	2.6	17.2	1.94
	3	43.0	14.6	5.2	19.8	2.18
	7	59.7	14.6	10.3	24.9	2.40
	15	76.5	14.6	18.0	32.6	2.34
	30	92.6	14.6	36.0	50.6	1.83

Table I.2.3 (contd.)

Investigator	Specimen mark	M_t (ultimate test) in-kips	Cowan's Theory			Factor of Safety (M_t/M'_t)
			M_{to} in-kips	M_{ts} in-kips	M'_t (working) in-kips	
Author	V4 I	33	7.1	10.8	17.9	1.84
	S6	30	6.9	8.1	15.0	2.00
	R4	30	6.7	10.8	17.5	1.71
	L1-1	29	5.9	8.1	14.0	2.07
	L6-1 II	25	4.1	8.1	12.2	2.05
	L6-2 II	25	4.1	8.1	12.2	2.05
	A1	23	5.8	0	5.8	3.97
	A2	22	5.7	0	5.7	3.86
	A3	22	6.0	0	6.0	3.67
	A4	23	6.6	0	6.6	3.48
	A5	26	6.0	0	6.0	4.34
	A6	22	5.2	0	5.2	4.24
	A7	23	6.0	0	6.0	3.83
	A8	24	7.0	0	7.0	3.43
	A9	24	5.9	0	5.9	4.06
	A10	23	6.1	0	6.1	3.78
	A11	26	6.0	0	6.0	4.34
	A12	26	6.3	0	6.3	4.13

(continued)

Table I.2.3 (contd.)

Investigator	Specimen mark	M_t (ultimate test) in-kips	Cowan's Theory			Factor of Safety (M_t/M'_t)
			M_{tc} in-kips	M_{ts} in-kips	M'_t (working) in-kips	
Author	A13	22	5.7	0	5.7	3.86
	A14	26	6.3	0	6.3	4.13
	A15	26	5.9	0	5.9	4.40
	B1	23	5.5	6.5	12.0	1.92
	B2 II	22	5.5	6.5	12.0	1.83
	B3	25	5.5	6.5	12.0	2.08
	B4 II	23	5.5	6.5	12.0	1.92
	B5	24	5.7	6.5	12.2	1.97
	B6 II	23	5.7	6.5	12.2	1.89
	B7 II	18	4.3	6.5	10.8	1.67
	B8	18	4.3	6.5	10.8	1.67
	B9 II	23	5.5	6.5	12.0	1.92
	B10	23	5.5	6.5	12.0	1.92
	T4	16	4.8	0	4.8	3.34
	T6	17	5.1	0	5.1	3.34
	T11 I	17	4.5	4.7	9.2	1.85
	T19	23	5.8	4.7	10.5	2.19
	T20 II	23	5.8	4.7	10.5	2.19

Table I.2.3 (contd.)

Investigator	Specimen mark	M_t (ultimate test) in-kips	Cowan's Theory M_{tc} in-kips	Cowan's Theory M_{ts} in-kips	Cowan's Theory M'_t (working) in-kips	Factor of Safety (M_t/M'_t)
Nylander (Ref.No.4)	III 1.1	13	3.2	0	3.2	4.06
	1.2	13	3.2	0	3.2	4.06
	V 2b	36	10.0	0	10.0	3.60
	3a	34	9.1	0	9.1	3.74
	I 1	13	2.8	0	2.8	4.65
	2	13	2.8	0	2.8	4.65
	3	15	3.2	0	3.2	4.70
	4	15	3.2	0	3.2	4.70
	IV 5a	16	3.3	0	3.3	4.85
	5b	15	3.3	0	3.3	4.85
Ramakrishnan and [illegible]jaya [illegible]ngun (Author) (Ref.No.6)	A5	29	5.5	0	5.5	5.27
	A6	29	5.5	0	5.5	5.27
	B2	26	4.8	0	4.8	5.42
	C1I	23	4.1	0	4.1	5.62
	C6I	22	3.7	0	3.7	5.95
	D1	28	5.2	10.8	16.0	1.75
	E1	28	5.1	15.3	20.4	1.37

Table I.2.3 (contd.)

Investigator	Specimen mark	M_t (ultimate test) in-kips	M_{tc} in-kips	M_{ts} in-kips	M'_t (working) in-kips	Factor of safety (M_t/M'_t)
Arockiaswamy (Ref. No.51)	R1	58	11.9	0	11.9	4.88
	R2	55	11.9	9.4	21.3	2.58
	R3	60	10.5	22.2	32.7	1.84
	R6	61	11.1	0	11.1	5.40
	T1	43	9.2	0	9.2	4.68
	T2	44	9.2	4.8	14.0	3.14
	T3	43	9.7	5.5	15.2	2.83
	T4	45	9.7	11.0	20.7	2.18
Lessig (Ref.No.17)	BK 1	122	12.0	58.0	70.0	1.74
	1a	104	8.8	58.0	66.8	1.56
	2	148	9.5	73.0	82.5	1.79
	2a	153	10.8	73.0	83.8	1.83
	3	176	11.3	58.0	69.3	2.54
	3a	149	11.6	58.0	69.6	2.14

Table I.2.3 (contd.)

Investigator	Specimen mark	M_t (ultimate test in-kips	Cowan's Theory M_{tc} in-kips	Cowan's Theory M_{ts} in-kips	Cowan's Theory M'_t (working) in-kips	Factor of safety (M_t/M'_t)
Lyalin (Ref. No.34)	B 8K	125	8.0	55.2	63.2	1.98
	8Ka	153	15.2	55.2	70.4	2.18
	B 7	221	15.5	148.0	163.5	1.35
	7a	208	15.5	148.0	163.5	1.35
Cowan and Armstrong (Ref.No.23)	R 3	72	13.5	12.3	25.8	2.80
Chinenkov (Ref.No.33)	B2.1-K	111	21.2	26.0	47.2	2.36
Rajagopal (Ref.No.55)	OA4	47	9.6	12.3	21.9	2.14

In 1953, Brown[27] reported tests on twenty six longitudinally reinforced concrete Tee, Ell and rectangular beams without web reinforcement under combined direct shear and torsion.

Direct shearing stress has been calculated using the formula V/bjd where V is the direct shear, b is the flange width of Tee beam and jd is the lever arm. The torsional shear stress has been calculated based upon the rigid plastic theory and using the relation

$$\text{Unit torsional shear stress} = \frac{\text{Torque}}{\frac{t^2}{2}\left(b-\frac{t}{3}\right) + \frac{(b')^2}{2}\left(h-t-\frac{b'}{3}\right) + \frac{(b')^4}{2.3}\,\frac{t}{(b')^2+t^2}} \quad \ldots \text{I.2.11}$$

This differs only slightly from the relation suggested by Nylander[4], which is of the following form:

$$\text{Unit torsional shear stress} = \frac{\text{Torque}}{\frac{t^2}{2}\left(b-\frac{t}{3}\right) + \frac{1}{2}(b')^2\left(h-t-\frac{b'}{3}\right) + \frac{(b')^4}{2.32}\,\frac{t}{(b')^2+t^2}} \quad \ldots \text{I.2.12}$$

624.1834
N674

56521

where h = overall depth of Tee beam

b = width of flange

b' = rib width and

t = thickness of flange

On comparison with test results it has been found that Nylander equation gives better results than that of Brown's. The mean value and maximum deviation by Nylander's formula are 1.09 and +25 % respectively while those for Brown's equation are 1.15 and + 32 % respectively.

Grassam[28] in 1956 reported test results of plain concrete circular specimens subjected to combined bending and torsion tested by him and Fisher[29]. He suggested a modification to the maximum stress theory. That is, under combined bending and torsion, the equation for interaction of moments can be expressed in the form:

$$\left(\frac{M}{M'}\right) + \left(\frac{T}{T'}\right)^2 = 1 \qquad \ldots \text{ I.2.13}$$

where M and T are respectively the bending moment and torque which if applied together would cause failure, M' and T' are respectively bending moment and torque calculated on the semi-plastic idea (i.e. mean value of torque or bending moment between the calculated values assuming complete plasticity and assuming complete elasticity) and which if applied separately would cause failure.

He further concluded that

1. the torsional stiffness of the section has not been affected by the application of 50% of bending moment, and

2. no increase in the torsional strength was found when a small bending moment was applied.

His observations are valid only for plain concrete members but not for reinforced concrete structures as the behaviour of the former under combined loadings is different from that of the latter.

Ernst[30] in 1957 conducted pure torsion tests on eighteen rectangular reinforced concrete beams to determine the interaction between the longitudinal and transverse steel and the concrete at ultimate torque. His experiments have shown that in general the transverse reinforcement yields before the final collapse of the beam. For the analysis part, Ernst used Cowan's[2] equations for elastic range and for the ultimate stage he suggested the following equation:

$$M_t = (K_v + K_h) \frac{A_w f_{yw} b'd'}{s} \qquad \ldots \text{I.2.14}$$

where $(K_v + K_h)$ is a numerical factor to get the lever arms of internal forces in terms of b' and d'. Ernst did not suggest any method to determine $(K_v + K_h)$. On the

other hand, he calculated the values of $(K_v + K_h)$ using Eqn. I.2.14 and test results.

He also suggested one more equation to determine the angle of twist per unit length in radians:

$$\theta = \bar{C} \cdot \frac{M_{tc}}{b^3 D} \qquad \ldots \text{I.2.15}$$

where M_{tc} - diagonal cracking moment

$\bar{C}$ - numerical coefficient which Ernst determined using experimental data.

Some of the conclusions of Ernst's investigation are interesting in knowing the interaction between longitudinal and transverse steels.

Based on his own elastic theory[2,21] and Bach's Approximate Theory[8], Cowan[31,32] in 1960, gave design formulas. Most of these recommendations have been included in the Australian Building Code. Bach's Approximate Formulae are valid only if the section is composed of narrow rectangles as suggested by Bach himself.

Chinenkov in 1959[33] carried out combined bending and torsion tests on rectangular beams in cooperation with Lessig and Korotkova. In all, thirty six beams have been tested with different ratios of bending to twisting moments to examin

the feasibility of reinforcing beams only by two plane welded skeletons placed near the sides and to verify the analysis proposed by Lessig[17,19]. Following are some of the conclusions Chinenkov has given which are in close agreement with the present author's observations.

1. In general, when failure occurred diagonal cracks opened on the side of the beam in flexural compression. In most of the beams the concrete on the side in flexural compression failed through splitting along a line connecting the ends of the cracks in the side surfaces.

2. In specimens, reinforced only with longitudinal reinforcement, failure took place with the appearance of first cracks.

3. Opening of cracks in torsion brings about an intensive increase in deformations and with spirals inclined at 45° greatly reduced the crack opening and development of deformations, compared to other types of reinforcement.

4. Deflections in similar beams increase with increasing ratio of torsional to bending moment. For equal flexural moments deflections under the combined action of bending and torsion were higher in almost all the beams than deflections in pure bending.

5. For equal torsional moments and otherwise identical conditions the angles of twist are the smaller, the smaller

the bending moment and the higher the percentage of longitudinal and transverse steel and the strength of the concrete.

6. Failure always occurred along the cracks formed under load increases thus showing the influence of concrete on the behaviour and strength of beams.

He also demonstrated a fair agreement between his experimental results and the analytical results of Lessig. From the test observations, Chinenkov concluded that the flexural capacity of the beam decreases with the presence of twisting moment as follows:

Value of ϕ ($= \frac{M_t}{M_b}$)	Reduction in the Flexural Capacity
0.10	0 to 1 %
0.20	7 to 19 %
0.30	23 to 25 %
0.40	29 to 33 %

He also observed that the transverse reinforcements yield before the final collapse of the beam.

From Chinenkov's test results, it has been found that replacing closed stirrups with 'open' ones (i.e. stirrups with vertical legs only) has got no noticeable effect (in some cases nil) on strength of beams. In the same year,

Lyalin[34], reported the results of an experimental investigation. The experimental results have been compared with the results of Lessig's theory and the conclusions are in similar line with those of Chinenkov.

Humphreys[35] in 1957 conducted torsion tests on a total of 94 concentrically prestressed beams of different cross-sectional sizes. Based on his test results he proposed the following equations:

1. Ultimate twisting moment

$$M_t = \alpha'(\tau + 0.30\, f_p) b^2 D \qquad \ldots \text{I.2.16}$$

where

τ - ultimate torsional stress in plain concrete

f_p - concentric prestress remaining after deduction for all losses.

α' - St. Venant numerical factor depending on D/b ratio.

2. The working stress may be increased to 120 psi for diagonal tension due to torsion.

3. The torsional rigidity of a rectangular concrete section can be calculated using elastic theory formula i.e.

$$\left(\frac{M_t}{\theta}\right) = K_2' b^3 D\, G \qquad \ldots \text{I.2.17}$$

where

G - modulus of rigidity of the concrete

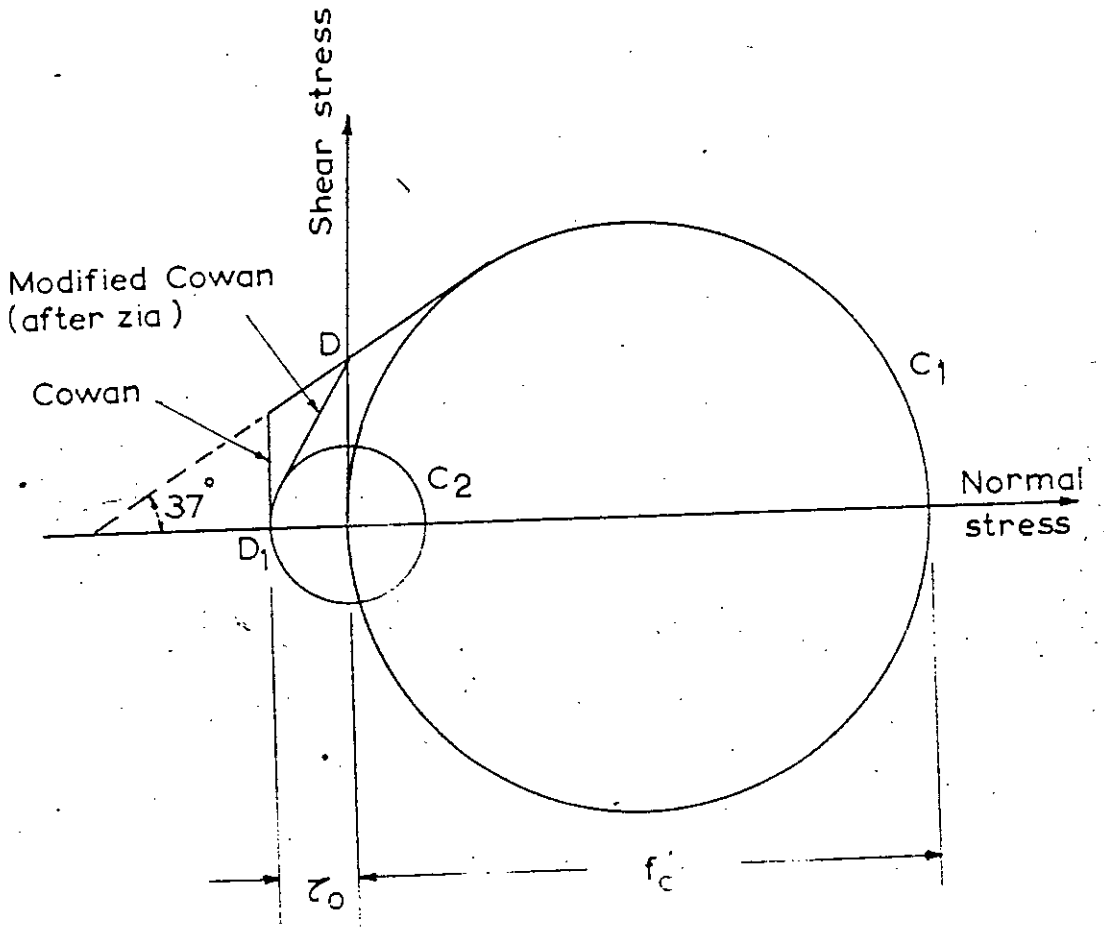
K_2' - St. Venant numerical coefficient depending on D/b ratio.

It is to be pointed out here that the torsional rigidity is not a constant as given by elastic theory, but varies with the increase of torque.

In 1960, Gardner[40] described an experimental investigation consisting of sixteen prestressed concrete I beams under combined bending and torsion. The beams were tested by the application of a bending moment which varied from 10 to 80% of the ultimate bending moment; the beams were then subjected to torsion until failure occurred.

From the results, Gardner indicated that the applied bending moment produced no significant change in the ultimate twisting moment. A good estimate of the test results has been obtained using rigid plastic theory for torsion.

Paul Zia[36,37] in 1961 reported test results of prestressed as well as plain concrete specimens in pure torsion. He has tested rectangular, Tee and I-sections. Most of his conclusions are in good agreement with those of previous investigations thus leading to the conclusion that the part played by longitudinal and transverse steels in resisting torsional stresses is same both for prestressed and reinforced concrete. He suggested a modification to Cowan's theory. It is



f_c – Uniaxial compressive strength of concrete

τ_o – Pure shear strength of concrete

Note :- Line DD_1 is drawn as a tangent to circle C_2 from the point D.

FIG. 1.3. FAILURE ENVELOPES AS GIVEN BY COWAN AND PAUL ZIA.

diagrammatically represented in Fig. I.3.

While calculating the torsional stresses, Paul Zia has adopted the formulae suggested by Lyse and Johnston[38] obtained from soap-film experiments in refinement to Bach's narrow rectangle formula[8]. In this context, the approximation suggested by Weinberger[39] can also be thought of as a better approximation for the normal non-rectangular sections encountered in practice.

In 1962, Reeves[41] reported the results of an experimental investigation on forty two prestressed concrete tee beams under combined bending and torsion. The results show that the torque capacity of beams is materially affected by the magnitude of the applied bending moment. For applied bending moments upto 80% of the flexural capacity of the beam, the torque capacity increased; but the amount of increase is interdependent on the physical properties of the beams. The conclusion of Reeves that the rotation capacity of beams is increased with the increase in the applied bending moment is in agreement with the conclusion of the present author.

Reeves made three different 'trial' analysis:

i. using the failure criterion for concrete suggested by Bresler and Pister;

ii. using rigid plastic theory for torsion and equating the principal tensile stress to the cylinder split strength;

iii. using elastic theory for torsion and equating the principal tensile stress to the tensile strength of concrete as given by modulus of rupture test.

From the comparison of results it appears that none of the above analyses gives good results. However, Reeves concluded that method (ii) gives better results than the other two methods of analyses.

Further he suggested an empirical equation to the interaction between bending and twisting moments. It is of the following form:

$$\left(\frac{M_t}{M_{to}}\right) = 1.0 + \bar{b}\left(\frac{M_b}{M_{bo}}\right) + \bar{c}\left(\frac{M_b}{M_{bo}}\right)^2 + \bar{d}\left(\frac{M_b}{M_{bo}}\right)^3 \qquad \ldots \text{I.2.18}$$

where $\bar{b}$, $\bar{c}$, $\bar{d}$ are coefficients obtained from experiments.

Yudin[42] in 1962, criticised the method of analysis proposed by Lessig. Instead of taking the moment of the internal forces about the axis of a failure hinge, he considered the moments of forces about the longitudinal axis

of the beam and an axis perpendicular to it. Using the same failure modes as given by Lessig and assuming that the failure crack always spirals around three sides of the beam at 45° to the beam axis, he derived the following equations:

$$M_t = 2 \frac{A_w f_{yw}}{s} b'd' \qquad \ldots \text{ I.2.19}$$

$$A_s = \frac{1}{f_y} \cdot \frac{M_b}{(d - \frac{c}{2})} \cdot \left[1 + (\frac{b'+d'}{2b'}) \phi\right] \qquad \ldots \text{ I.2.20}$$

Yudin has not made a comparison with experimental results of beams designed by this method. But the experimental comparison given by Collins et al[20] at a later date indicates that Yudin's theory is less satisfactory than that of Lessig in predicting the test results.

Further Yudin's equation for calculating M_t is essentially the same as that proposed by Rausch[1] for the elastic range. This equation suffers from the limitation of neglecting the effect of concrete and the interaction between moments on the torque capacity of beams.

In 1962, Narayan Swamy[43] summarised the results of an investigation into the behaviour and ultimate strength of post-tensioned prestressed concrete hollow beams in

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combined bending and torsion. Twenty-four beams have been tested with ratios of bending moment to twisting moments varying from o to ∞ . The size and shape of the hollow and the shear reinforcement have been varied.

The observations of Swamy with regard to the modes of failures and the influence of bending to twisting moment ratio over the strength of specimens are in good agreement with the observations of present investigation thus confirming that the behaviour of prestressed and reinforced concrete beams is same under combined bending and torsion. From the experimental results, it has been found that the Maximum-Stress Theory gives satisfactory results for certain ranges of twisting to bending moment ratio, $\emptyset$.

He further added an upper and lower bound solutions to the interaction curve of the bending and twisting moments using the inequality suggested by Hill and Siebel[44] and further developed by Gaydon, Nuttall and Steele[45,46].

During 1961-63, the present author[6] conducted experiments under the direction of Professor Ramakrishnan on rectangular beams subjected to combined bending and torsion. In all, thirty-two beams were tested and the individual influence of each factor on the general behaviour of beams, mode of failure and ultimate strength has been studied. Factors investigated were (1) concrete properties (2) percentage of tensile steel

(3) compression steel (4) angle of twist (5) ratio of twisting to bending moments and (6) contribution of transverse reinforcement in the form of ordinary hoops (both vertical and inclined) and spirals. As no rational analysis was attempted at that time, the following empirical equations were obtained.

(a) Ultimate Twisting Moment for beams without web reinforcement[47]:-

$$M_t = \alpha' b^2 D F_{ct} \qquad \ldots \text{I.2.21}$$

where

i. $F_{ct} = 2.60 F_c^{2/3}$

ii. $F_{ct} = 256 + C_t$

C_t - cylinder split strength

F_c - cube crushing strength

α' - St. Venant's torsion constant

(b) Contribution of Web reinforcement[48] :-

i. For vertical and inclined hoops:

$$\left(\frac{M'_t}{M_t}\right) = 1 + 0.663\,\emptyset + \left(\frac{4.50 \times 10^{-2}}{M_t} b'd' \frac{A_w}{s} \frac{f_{yw}}{\sin\psi}\right) \qquad \ldots \text{I.2.22}$$

ii. For spirals:

$$\left(\frac{M'_t}{M_t}\right) = 1 + \phi + \left(\frac{19.40 \times 10^{-2}}{M_t} b'd' \frac{A_w}{s} \frac{f_{yw}}{\sin\psi}\right)$$

... I.2.23

where

M'_t - ultimate twisting moment for the beam with web reinforcement.

M_t - ultimate twisting moment for the corresponding beam without web reinforcement.

ψ - angle of inclination of shear reinforcement to the axis of the beam.

Other notations have the same meaning as those used throughout this section of the thesis.

Limitations :-

1. These equations are valid only if $\phi > 0.20$
2. For $\phi \geqslant 0.40$, the term having the factor ϕ has to be omitted in calculating (M'_t / M_t).

Within the limitations, the equations predict the test results reasonably well. Many of the experimental observations[49,50] of this investigation are in good agreement with those of present investigation.

Arockiasamy[51] in 1964 reported an experimental investigation on the behaviour and strength of reinforced concrete members of various shapes subject to torsion. Test results of sixteen beams with reinforcements arranged in three different combinations were reported.

The three distinct stages in the torque-twist curves namely the first straight line portion corresponding to the elastic stage, secondly the transition stage from elastic to plastic and thirdly the plastic stage as observed by this investigator is in line with the results reported by earlier investigators[23,47]. It is to be added here that the second stage of the curve is not so well defined in many cases as the transition taking place is very smooth deviating only slightly from the straight line portion. He also observed that the modulus of rigidity decreases with the increase in the twisting moment. Strictly, the torsional rigidity GJ depends on the ultimate torque capacity of beams. The quantity GJ goes on decreasing as the torque is increased after cracking and reaches a minimum value when failure occurs.

The formula given by this investigator for predicting the maximum stress at failure (elastic) and hence the torsional moment of a Tee section is due to Bach[8] which is applicable

only if the Tee section is composed of narrow rectangles; but it gives erroneous results for normal reinforced concrete sections encountered in practice. Moreover very thin sections seldom occur in reinforced concrete structures. So the method suggested by Weinberger[39] can be thought of as a better approximate solution.

For a Tee section (as per Weinberger)

Torsional Rigidity $\frac{M_t}{\theta} \geqslant D_1+D_2+D_3+\cdots$ ... I.2.24

where D_1, D_2 and D_3 are torsional rigidities of component rectangles and the maximum shearing stress $\tau_{max.} \simeq K'G\theta b$ where K' is St. Venant's numerical factor depending on D/b ratio.

In 1964, Gesund and Boston[52] tested rectangular reinforced concrete beams to destruction under combined bending and torsion. The beams contained only longitudinal reinforcement, and concrete strength, amount of reinforcement and moment-torque ratios were varied. A theoretical model has been developed and it has been used to check the test results.

The part played by longitudinal steel even with little amount of transverse steel in increasing the torque carrying capacity of concrete beams is more predominent; but the increase in the ultimate torque due to increase in longitudinal

steel alone is virtually nil. This conclusion is in conformity with the test results of some of the previous investigations[3,8,9,10] and hence the conclusion of these investigators that if there is no transverse reinforcement, the dowel action of the longitudinal reinforcement is of paramount importance in resisting torsion is not very much convincing.

Gesund et al[53] in 1964, reported tests on rectangular reinforced concrete beams under combined bending and torsional loads. The beams contained both longitudinal and transverse reinforcement, and concrete strength, amount and spacing of reinforcement, and moment-torque ratios have been varied. A theoretical model has been developed from the observed failure mechanisms and it has been used to check the results of these tests and also the results of tests reported by other investigators. The final equations of their investigation are summarised below:

(a) <u>For predominent flexure failure</u>:

$$\frac{M_b}{M_{bo}} = \frac{1}{1 + C_1 \phi} \qquad \ldots \text{ I.2.25}$$

where

$$C_1 = \frac{d' (D + b \cot \theta)}{d'b' + db' \cot \theta}$$

$$\cot \theta = 0.5 \text{ for } \phi \geqslant 0.25$$

$$= 0 \quad \text{for } \phi < 0.25$$

(b) For predominant torsion failure:

M_t is larger of any one of the following:

$$\text{i.} \quad M_t = \frac{A_w f_{yw}}{s} (b'd' + b'd \cot \theta) \qquad \text{... I.2.26}$$

$$\text{ii.} \quad M_t = P_c \left[r_c + \frac{1}{r_c} \left(\frac{2d' r_t^2}{s} + \Sigma r_i^2 \right) \right]$$

where

$$P_c = \pm \frac{F_{ct} \cdot e \left(\frac{a_1}{\cos\beta} + \frac{a_2}{\sin\beta} \right)}{(6 - 12\, K_1) \sin(\beta - \phi')}$$

$$e = \frac{(3/16) bD}{\Sigma O}$$

ΣO = sum of the perimeters of the longitudinal tension reinforcement.

β must be such that P_c is a minimum

r_c = distance from the hinge to the bar pressing against the assumed failure plane i.e. the critical bar.

r_i = distance from the hinge to the ith bar.

r_t = distance from the hinge to the ties at their mid-height.

ϕ' = angle between r_c and the vertical beam axis.

The failure surface considered by these investigators seem to the author to be too idealized. The main conclusion the investigators arrive at regarding the transformation of torque on a reinforced concrete beam into additional bending moment would have been further supported if they had tested beams in pure bending and pure torsion.

Comparing with the experimental results with those predicted by the equations proposed by the investigators it has been found that the prediction is not upto the satisfaction in many cases[20,55].

The equation for predicting the ultimate bending resistance of beams under-estimates very much the capacity of beams. In addition, this equation always predicts the value of M_b to be less than M_{bo} since the quantity $(1+C_1\phi)$ can never approach unity unless ϕ is made equal to zero which is in the 'pure bending' case. Even though the bending resistance of a beam subjected to combined bending and torsio gets reduced by the presence of twisting moment, there is a limiting value of ϕ below which the bending resistance of the beam is not reduced. In other words, from the test results of the present author as well as those reported in the literature[33,6] it has been found that for $\phi < 0.10$ there is practically no reduction in the bending resistance of beams.

Previous investigations as well as author's investigation have shown that the torque capacity of beams is increased due to the presence of bending moment in certain ranges of ø. However, indefinite increase is not possible and there may be a limiting value of ø below which the torque capacity of beams may not increase. This behaviour of beams is not represented by the equation suggested by Gesund et al for predicting the ultimate torque capacity of beams. Further the investigators have not defined the ranges of applicability of the respective equations derived for the two types of failure. Since the equation derived for the predominent flexure failure is based on the assumption that the transverse hoops transfers the torque as an additional bending moment, its applicability for the case of beams containing only longitudinal steel is not justified.

In 1965, Evans and Sarkar[54] gave a method for the ultimate strength design of reinforced concrete beams in combined bending and torsion. The analysis is based on the concept of plastic hinge. But Evans and Sarkar made a refinement to the types of analysis given by earlier investigators by presenting a method to obtain the inclination of the crack which has been assumed as a constant value in the past.

By equating the moments of various forces about the failure plane, they derived the following equation:

$$M_b\,(1+\emptyset) = \Big[f'_c \cdot c^2\, b + f_y\, A_s\,(d_1-c) + \frac{A_w}{s} f_{yw} \Big\{(d-d_s-c)(b-2b_s)\cot\alpha - (d-2d_s)(0.6\cot\alpha + 0.4)\,(b\cot\alpha + d'-b)\Big\}\Big] \quad \ldots \text{I.2.27}$$

where $\cot\alpha$ gives the inclination of the crack calculated considering the normal and shearing stresses.

For simplicity the investigators proposed the following relation:

$$\frac{M_b}{M_{bo}} = \left(\frac{1}{1+\emptyset}\right) \quad \ldots \text{I.2.28}$$

Some of the conclusions of the investigation are summarised below:

1. The expression $\left(\frac{1}{1+\emptyset}\right) \times M_{bo}$ is a reliable guide for the preliminary assessment of the strength of a section under combined bending and torsion.

2. A load factor of 2.0 on the ultimate moments, for use at working load design, is suggested, which restricts

the crack widths to less than 1/100 in.

3. A factor 3.0 is suggested over the elastic torsional stiffness of the section, for the design of reinforced concrete grids and frame works.

The experimental work of the investigators included eighteen hollow beams of three different sizes, which is first of its kind in the field of reinforced concrete beams under combined bending and torsion.

Rajagopal[55] reported the results of an experimental investigation in 1965. The experimental work consisted of fourteen rectangular beams with transverse reinforcement tested to destruction varying the ratio between bending and twisting moments. By comparing the test results with the predictions of various theoretical solutions, he concluded that none of the existing theories predicts the test results to a reasonable degree of accuracy.

In 1966, Collins et al[56,57] reported the analytical and experimental results of their investigation on beams subjected to combined torsion, bending and shear. They considered three idealized failure modes and derived equations by considering the equilibrium of forces about the compression hinge. Omitting the intervening derivations,

the final equations are summarised below:

I Beams with transverse reinforcement:

Mode 1:

$$\frac{M_{t1}}{M_{bo}} = \frac{2r}{1+2(\frac{D}{b})}\left[\sqrt{(\frac{1}{\phi})^2 + \frac{1+2(\frac{D}{b})}{r}} - \frac{1}{\phi}\right] \quad \ldots \text{ I.2.29}$$

Mode 2:

$$\frac{M_{t2}}{M_{bo}} = \frac{1}{1+(\frac{Vb}{2M_t})} \cdot \frac{(D/b)}{\beta'} \sqrt{\frac{2(1+R)r}{2+(D/b)}} \quad \ldots \text{ I.2.30}$$

Mode 3:

$$\frac{M_{t3}}{M_{bo}} = \frac{2r}{1+2(\frac{D}{b})} \cdot \left[\sqrt{(\frac{1}{\phi})^2 + \frac{(1+2\frac{D}{b})R}{r}} + \frac{1}{\phi}\right] \quad \ldots \text{ I.2.31}$$

where

$$r = \frac{0.8\, A_w f_{yw}}{s} \cdot \frac{b}{A_s f_y}$$

β' = ratio of effective height to effective width of the section

$$R = \frac{A_s' f_y'}{A_s f_y}$$

The twisting moment at failure is the smallest of the values given by Eqns. I.2.29, I.2.30 and I.2.31. On comparison

with experimental results, they laid the following limitations on the beam proportions:

i. $\frac{M_t}{b^2 D f'_c} \leq 0.07$. This relation is the same as given by Lessig[17-19].

ii. $\frac{p f_y}{f'_c} \leq 0.40$

iii. $\frac{r}{r_o} \geq 1$ where $r_o = \frac{1}{4 + \frac{4}{\phi \sqrt{1+2(D/b)}}}$

iv. The formula developed for Mode 2 is applicable if $\frac{V}{V_o} < 0.5$ where V_o is the shear strength of the beam as given by ACI Building Code formula. For $\frac{V}{V_o} > 0.5$, they recommended the following equation:

$$\left(\frac{M_t}{M_{to}}\right) + \left(\frac{V}{V_o}\right) = 1 \qquad \ldots \text{ I.2.32}$$

where

$$M_{to} = 2 M_{bo} \sqrt{\frac{r}{1+2(a/b)}}$$

II Beams without transverse reinforcement:

$$M_t = 1.75 b^2 (D - \frac{b}{3}) \sqrt{f'_c} \left(1 - \frac{V}{V_o}\right) \qquad \ldots \text{ I.2.33}$$

The experimental work consisted of sixty-seven beams in total comprising twenty plain concrete specimens, twenty two containing only longitudinal steel and twenty-five containing both longitudinal and transverse steel. The investigators demonstrated a fair agreement between the analytical and experimental results. From the comparison it has been found that Eqns. I.2.32 and I.2.33 seem to be very conservative in predicting the test results. Finally based on their results, these investigators have given a set of recommendations for the design of beams under combined torsion, bending and shear.

The review of code requirements for torsion by various countries is discussed in Chapter I.11.

Concluding Remarks:

From the detailed review of earlier works on the problem, the following conclusions can be drawn:

1. The experimental study with regard to the influence of ratio of twisting moment to bending moment on the ultimate strength and behaviour of beams has been given considerable attention in the recent years.

2. The influence of properties of concrete on the ultimate strength of beams has not been fully studied.

3. The recommendations given by Cowan for torsion design give a safety factor ranging between 1.5 and 2.0 for the case

of beams with both longitudinal and transverse reinforcements; but they seem to be very conservative for beams with longitudinal steel only.

4. No investigator has given a complete procedure to calculate deformations of beams such as deflection, angle of twist etc.

5. The work on reinforced concrete Tee beams in torsion is very much limited.

6. Most of the investigators have used Bach's approximate theory to calculate the torsional rigidity of sections composed of rectangles. As this theory may be valid only for such sections composed of narrow rectangles, which is rarely satisfied in reinforced concrete structures, the error involved in using Bach's Theory is evident.

7. The influence of transverse reinforcement on the capacity of beams at different ranges of Torque/Moment ratio, ϕ has not been well established.

8. All ultimate strength analysis of reinforced concrete beams are based on only one concept i.e. by considering the equilibrium of forces (both internal and external) about an imaginary failure surface.

9. Except Gesund and others, most of the other investigators have considered the torque resisted by the longitudinal

reinforcement alone as insignificant. It is necessary to establish the part played by the longitudinal steel in the analysis of beams under combined bending and torsion.

10. It has been generally concluded that the addition of twisting moment reduced the flexural capacity of beams while the addition of bending moment increased the torque capacity of beams. The limitations of this conclusion is yet to be known.

The experimental and analytical work of the present investigation has been programmed in the light of the above remarks and to clarify some of these points so as to get a better understanding of the problem.

Experimental Work:

In all, seventy-five beams (fifty-five rectangular beams and twenty tee beams) were tested. The main variables were concrete strength, percentage of longitudinal steel, percentage of transverse steel and ratio of twisting to bending moment. The experimental work has been programmed in such a manner so that it attempts to clarify certain points raised in the concluding remarks (i.e. 2, 5, 7, 9 and 10). These results are discussed in Chapter I.7 in detail.

Analytical Work:

In contrast to the existing type of analysis, the author has attempted to solve the problem of strength of beams

under combined bending and torsion using a certain failure criterion for concrete under combined stresses. This opens up a new line of thought. Further, exact solution for the calculation of torsional rigidities of sections composed of rectangles such as Tee, Ell and I-sections have been given. Based on experimental observations, an approximate procedure for the calculation of deformations of beams under combined loadings has been given. Thus, Chapters 8 and 9 of this part and part VI attempt to answer the points (4) and (6) and suggests a different method of approach for the analysis of beams under combined bending and torsion.

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EXPERIMENTAL INVESTIGATION

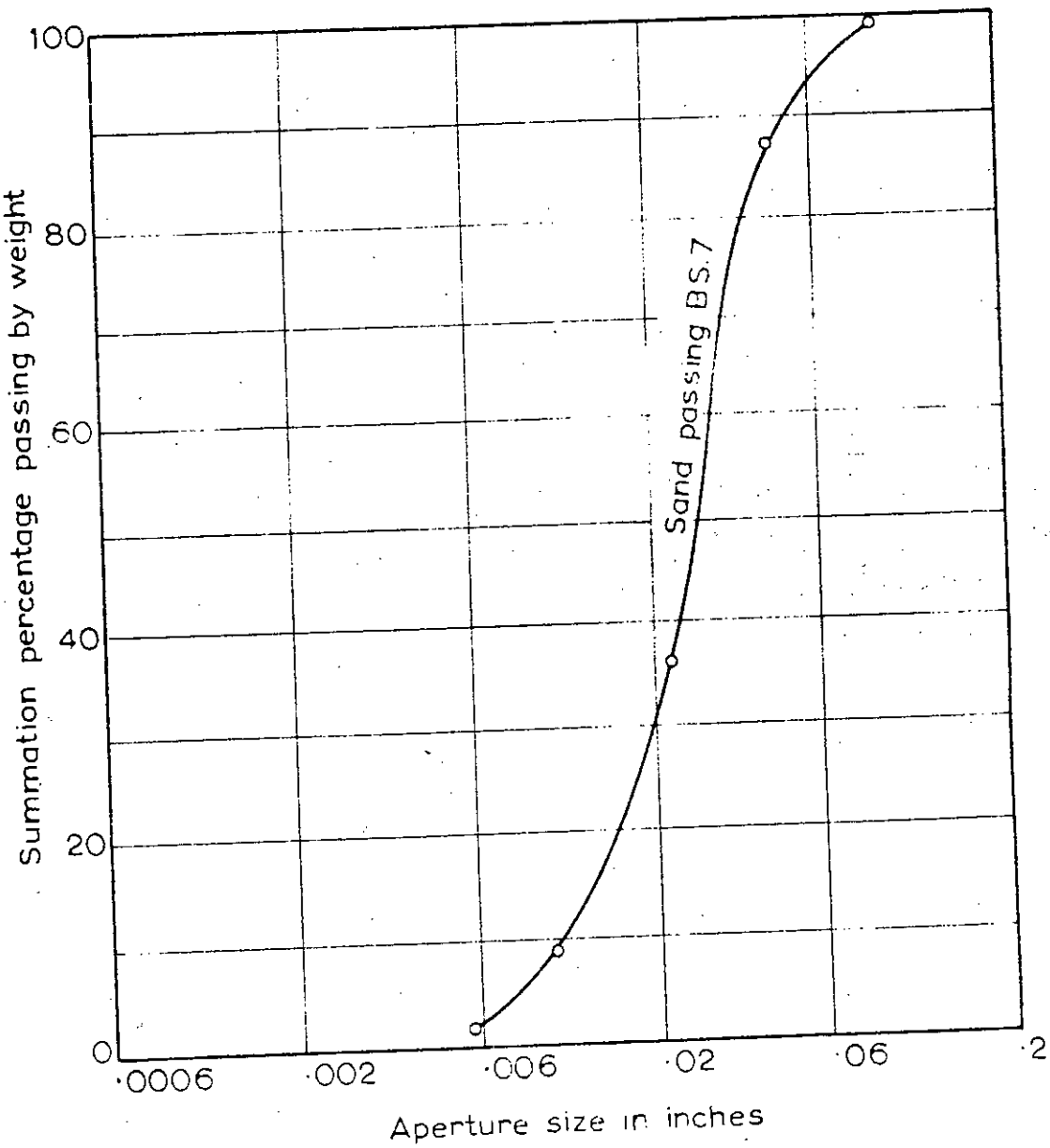


FIG. I.4. GRADING CURVE FOR SAND - SEMI LOG PLOTTING

CHAPTER I.3

MATERIALS, MIXES USED AND DESCRIPTION OF SPECIMENS

I.3.1 Materials:

(a) Cement:

The cement used was ordinary portland cement purchased in lumpsum from two different sources of supply throughout. Periodical tests on cement confirmed their standards as per Indian Standard Specifications.

(b) Water:

Water used in mixing the concrete was of drinking quality supplied by the Bangalore Corporation.

(c) Aggregates:

1. Sand

Sand used was got from local sources. The sieve analysis was done from time to time to ascertain the fineness modulus of sand used. The grading curve of the sand used i this investigation is shown in Fig. I.4, the average finenes modulus being 2.635.

ii. Coarse Aggregate

The maximum size of coarse aggregate has been limited to less than 1/2" due to the small dimensions of the cross section, the narrow spacing of reinforcing rods and to avoid seating of strain measuring devices on one particular aggregate The aggregates were got from local quarries and sieved to respective sizes, just before casting, from the lots.

Mainly the following size aggregates were used:

Coarse aggregate:

i. 3/16" to 3/8"

ii. 3/8" to 1/2"

Sand Passing B.S.No.7

(d) Steel:

The reinforcing rods used in all the beams were plain mild steel round bars. Specimens were cut from all the rods and tested in tension to determine the physical properties of the steel. The reinforcements were purchased in lots and reinforcements for any one series of beams were usually cut from a single lot.

All the rods were tested in Amsler's Universal Testing Machine of 100 tons capacity. A Whitmore Extensometer was used to measure the strains. The extensometer had a gauge

length of 10" and was fixed in the centre of the test piece to avoid any disturbances due to the grips of the machine. The least count of the extensometer was 0.0001 of an inch. Physical properties of reinforcing rods like modulus of elasticity, yield stress, ultimate stress etc. were determined. The results are given in Table I.3.1.

TABLE I.3.1

PHYSICAL PROPERTIES OF REINFORCING RODS

Diameter (in)	Area of cross section (in^2)	Yield stress (ksi)	Ultimate stress (ksi)	Modulus of Elasticity (psi x 10^6)	Percentage elongation over 10" gauge length	Percentage reduction in area
1/4	0.049	50	76	28	15	43
3/8	0.110	43	60	27	16	40
1/2	0.196	43	63	30	20	51
5/8	0.306	45	65	32	57	39
3/4	0.440	45	68	29	25	45

I.3.2 Mixes used:

Since the investigation was planned to study the influence of concrete strength, six trial mixes were made so a

to get a variation in strength between 3000 psi and 6000 psi. The following two basic mix proportions were then selected to be best suitable ones:

Mix. A

Coarse aggregate 3/16" to 1/2"	...	60 %
Sand passing B.S.No.7	...	40 %
Aggregate per cement ratio	...	3.50
Water per cement ratio	...	0.40 (variable)

Mix. B

Coarse aggregate 3/16" to 1/2"	...	55 %
Sand passing B.S.No.7	...	45 %
Aggregate/Cement Ratio	...	5.00
Water Cement Ratio	...	0.50 (variable)

Keeping these two mixes as basis, the water-cement ratio was slightly varied for each batch of concrete so as to get a variation in concrete strength which is a factor under investigation.

I.3.3 Description of Specimens:

In all, fifty-five rectangular and twenty tee beams were designed, cast and tested. All the rectangular beams were 5" x 8" in cross section. The beams are classified under different groups as follows:

Rectangular Beams:

V Series: These beams were provided with 1.73% of tensile reinforcement, 1.17% of compression reinforcement and 1/4" diameter vertical closed hoops spaced at 3" centres. Six beams were cast and tested with different ratios of twisting to bending moments.

S Series: Totally six beams were cast under this group. This group consisted of 1.49% of tension steel, 1.17% of compression steel and 1/4" diameter vertical stirrups at 4" centres.

R Series: This group of beams were provided with equal amount of tension and compression steels. Two beams had 1.17% while the other four beams had 1.79% of longitudinal steel on either face of the beam. All the beams were reinforced transversely by 1/4" diameter hoops at 3" centres.

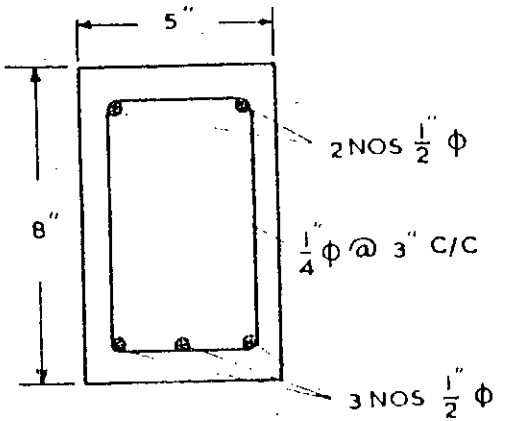
L Series: Twelve beams were cast in total. All the beams consisted of 1.79% tension reinforcement, 1.17% of compression steel and 1/4" diameter vertical hoops at 4" centres. In this group, beams were cast in six sets so as to get two identical beams in each set. These identical beams were tested under identical conditions so as to get reproducability of test results.

A Series: This group of fifteen beams were tested in pure torsion to study the influence of longitudinal steel on the torque capacity of beams. The beams had no transverse reinforcement and were cast in five sets with different amounts of longitudinal steel.

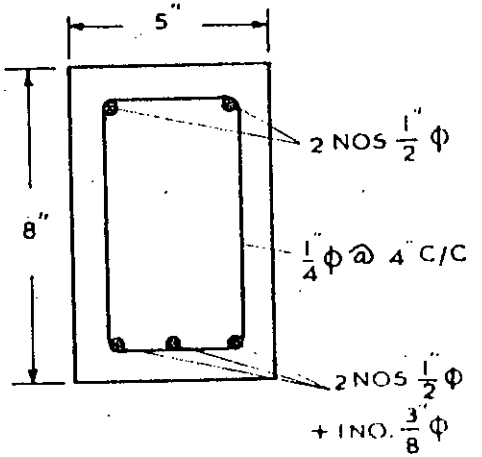
B Series: These beams were provided with 1.17% of tensile reinforcement, 0.29% of compression steel and 1/4" diameter vertical closed hoops at 5" centres. This group of beams were cast in five sets and each set consisted of two beams. These beams were tested to study the effect of presence of flexural cracks on the torque capacity of beams. For this purpose, one beam of each set was tested under pure torsion and the other beam was initially tested in pure bending till failure and then a retest was conducted on the same beam by subjecting it to pure torsion.

Tee Beams:

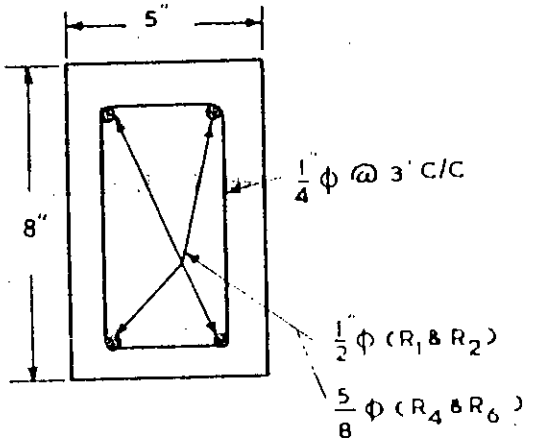
T Series: Totally twenty tee beams were designed, cast and tested. All the tee beams had flange width of 10" and flange thickness of 1 1/2", the rib dimensions being 4" x 5 1/2". Ten beams of this group had closed ties made up of 1/4" diameter rods at 4" centres while the others had only longitudinal steel.



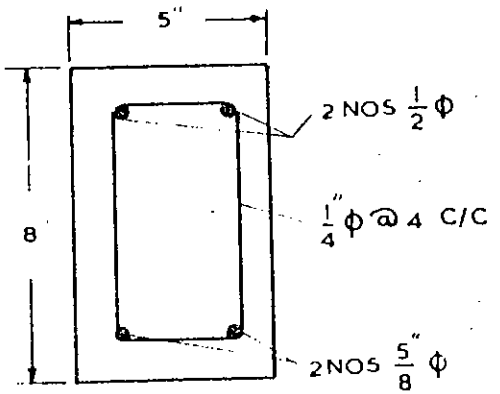
V Series



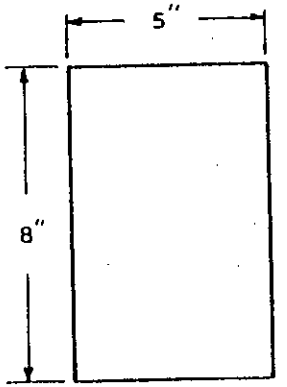
S Series



R Series

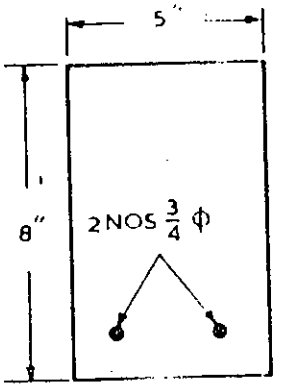


L Series



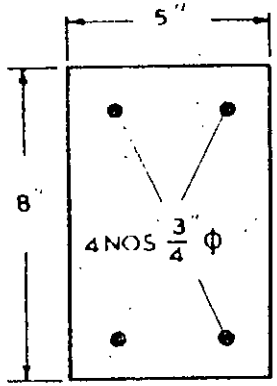
Plain concrete

A 1 - 3

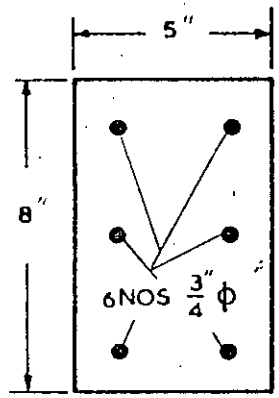


A 4 - 6

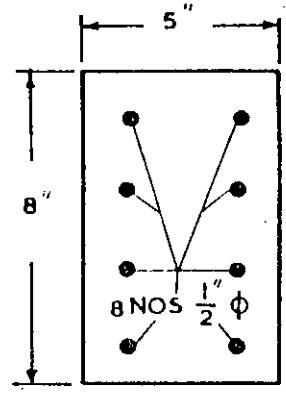
Fig. I.5 contd



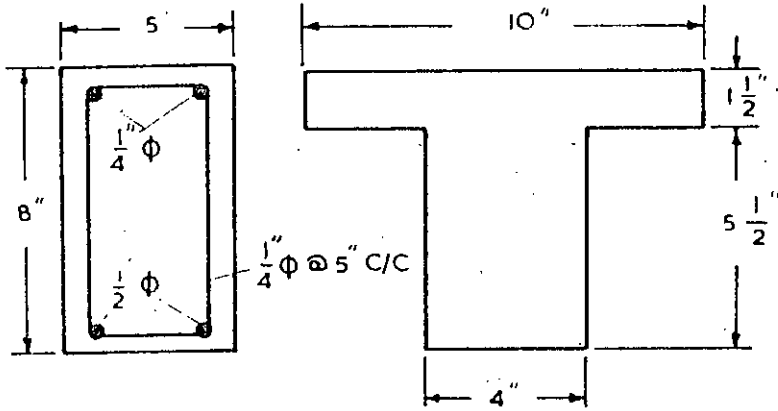
A 7 - 9



A 10 - 12



A 13 - 15



B Series

T1 - 20

Beam	Tension steel	Compression steel	Transverse steel
T1 - 5	5/8" ϕ - 2 nos	nil	nil
T6 - 10	5/8" ϕ - 2 nos	nil	nil
T11 - 15	1/2" ϕ - 2 nos	1/2" ϕ - 2 nos	1/4" ϕ @ 4" c/c
T16 - 20	5/8" ϕ - 2 nos	1/2" ϕ - 2 nos	1/4" ϕ @ 4" c/c

FIG. I. 5. CROSS SECTION PROPERTIES OF BEAMS.

The cross sectional properties of all the seventy-five beams tested in this investigation are summarised in Fig. I.5

Auxiliary Specimens:

6" cubes were used to determine the compressive strength of concrete. Four cubes were made with each beam. Cylinders of 6" diameter and 12" high were used to determine the splitting strength of concrete. Four cylinders were cast with each beam. From periodical testing of prisms of 6" x 6" x 12" size, it has been found that the prism strength is about 85% of the cube strength.

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CHAPTER I.4

MAKING AND CURING OF SPECIMENS

I.4.1 Preparation of Reinforcement:

All reinforcing rods were purchased in long lengths. The rods were cut to the required lengths and hooks were cold bent using metal jigs. These jigs were similar to the standard types of jigs normally used for this type of work. After bending, the rods were welded together using the correct spacing pieces.

Mould:

Two types of moulds were used: one made up of 8" x 4" mild-steel channel sections and the other wooden mould made from 3/4" thick plywood. Before concreting, the mould was fully tightened at all joints to check the leakage of mortar. The inside of the mould was oiled to prevent adhesion of the concrete.

All the auxiliary specimens were cast in standard steel moulds.

I.4.2 Preparation of Concrete and Casting:

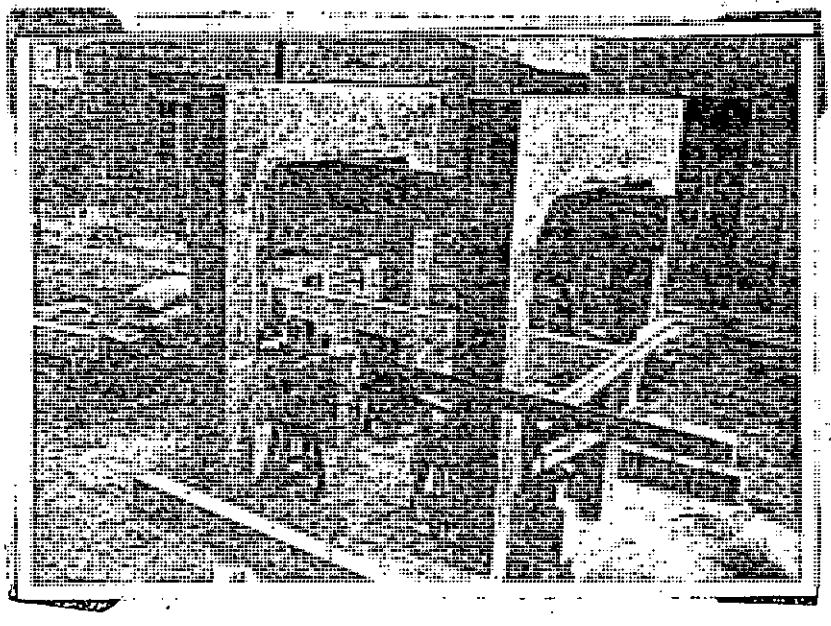
The mix proportion was designed and the exact quantities of the various materials were calculated. These

quantities were weighed for each of the batches of concrete and kept ready before the mixing was started.

Usually the fine aggregate (sand) and cement were initially mixed together. Then the coarse aggregate was added to this mixture and then the dry materials were properly mixed. Then water was added carefully so that no splashing occurred. The concrete was mixed for about five minutes and continued until the resulting concrete was uniform in appearance.

The beam mould was kept ready and packing pieces (previously made mortar specimens) were placed in position to give proper cover to the reinforcing bars. These packing pieces were placed at 2 or 3 points. Then a layer of concrete was spread just to the level of packing pieces and well compacted using Kango hammer. The reinforcement was then placed in position on the packing pieces. The concrete immediately after mixing, was filled in 2" deep layers in the beam mould and packed well using tamping rods. The concrete was also vibrated using needle vibrators. The auxiliary specimens were also filled in 2" layers in the steel moulds (lightly oiled beforehand) and compacted using Kango Hammer similar to the beams. Then the top of the beam and auxiliary specimens were floated off smoothly with a trawel. With each beam four 6" cubes and four 6" x 12" cylinders were cast.

FIG. I.6 : Beams under combined bending and torsion:
Details of Test Set-Up



I.4.3 Curing:

The beam and the auxiliary specimens were kept in the moulds for one day. Then they were removed from the moulds and kept under water until the time of testing for curing. The auxiliary specimens were always placed near the beams throughout the period of curing. The conditions of curing for the beams as well as for the specimens were kept the same as far as possible. The identification marks were marked on the surfaces of the specimens using black paint or bitumen so as to avoid confusion in identifying them. Usually the beams were cured in the normal position.

I.4.4 Loading Arrangements and Testing Procedures:

1. Loading Apparatus

a. Loading Frame

All the beams were tested in the 25 tons capacity reinforced concrete loading frame. The testing frame used can be seen in Fig. I.6 (photograph). The frame consisted of two portal frames with 12" x 12" size columns, the beam size being 12" x 24". These two portals are connected to a RSJ girder (8" x 16" size) at the bottom of their beams. The steel girder is used to butt against the loading jacks.

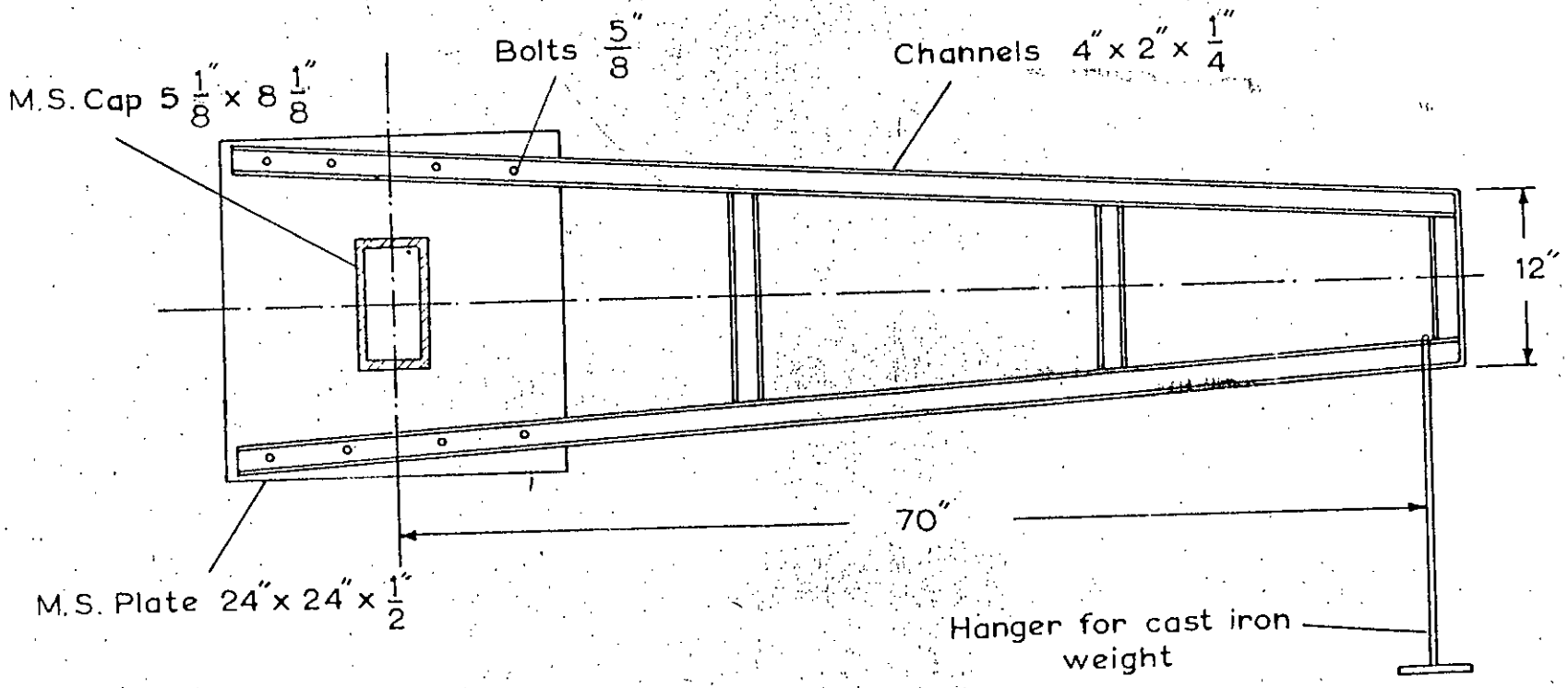


FIG. 1.7. DETAILS OF TORSION BRACKET.

b. End Support for the Beams

The beams were supported at a level comfortable enough for taking strain readings and tracing the cracks sitting in a comfortable position. The arrangement was made such that the loading point always remained at the same place and the supports could be arranged to the required span lengths. The supports were made using 5" x 2½" channels and ½" thick mild steel plate. Sometimes permanent concrete pillars were used as end supports.

c. Load Distribution Beams

The load distribution beam used for the beams was made from 4" x 2" channel sections and ½" thick mild steel plates. Two pieces of the channel, each 36" long, were arranged with their vertical faces budding against each other and welded together to form an I-beam. To this arrangement ½" thick mild steel plates were welded to each flange of the joined beam. The bottom and top surfaces of this built-in beam were machined so that it could slide smoothly in a horizontal direction on the loading rollers and also to present a levelled seating for the loading jack.

d. Torsion Brackets

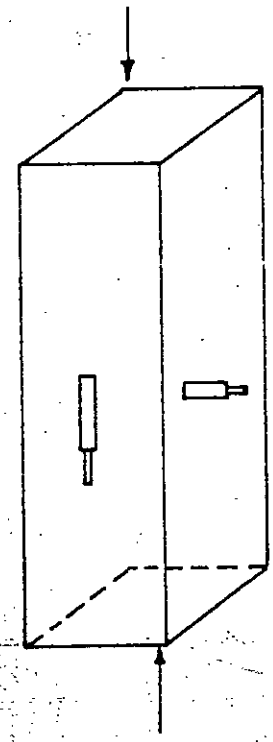
The torsion bracket used is shown in Fig. I.7. The bracket was 70" long from the centre of the beam cap to the

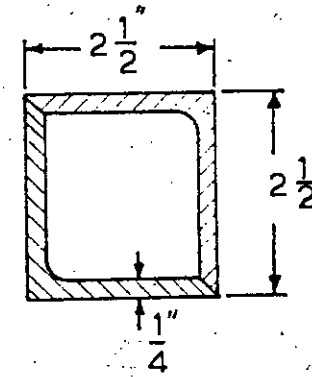
centre of the hanger rod and was made using 4" x 2" x ¼" channels and ¼" thick mild steel plates. The channels were arranged in the form of a 'truss' and welded. This was then joined to the bracket plate, 24" x 24" square, by means of eight 5/8" diameter bolts and nuts. The bracket plate was welded to the beam cap of 5 1/8" wide, 8 1/8" deep and 6" long made from 1/2" thick mild steel plates into which the beam was inserted during testing. Cowan and Armstrong[23] have connected the torsion brackets directly to the reinforcements of the beam. Since in this method there is every possibility of reinforcing rods 'slipping' from concrete, the author has thought of an alternate arrangement which led to the present arrangement. This method of putting a cap to the beam was proved to be quite satisfactory. The failure at the grips was always avoided by providing necessary reinforcements at the ends of the beam.

e. Loading units

i. Loading Jack

The loading jack used was Duff-Norton Hydraulic Jack and had a capacity of 12 tons. The pumping unit and jack was in single compact unit. The pump was operated by a hand lever and the load was measured on a direct reading pressure gauge with a total capacity of 24000 lbs. in 500 lb divisions. The jack readings were periodically checked by calibrating it in a universal testing machine which in turn has been calibrated





Section made up of two M S angles

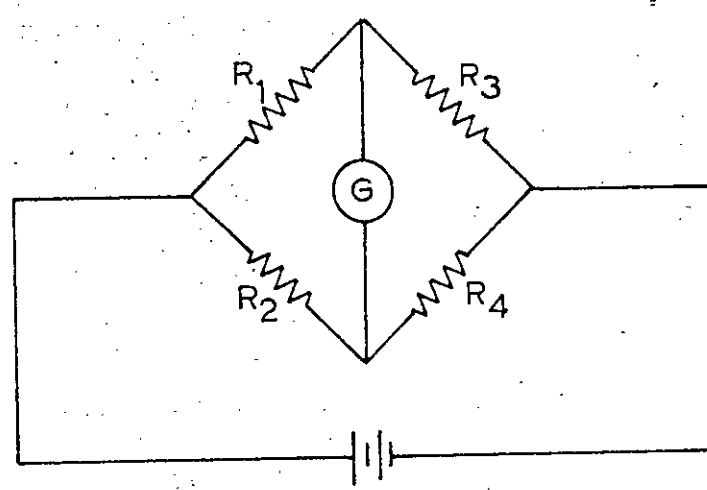


FIG.1.8. LOAD CELL AND LOAD CELL CIRCUIT.

using a Standard Calibrating Box. The jack was fixed at the centre of the frame at the point it was desired to apply the load. The load remained fairly constant at any particular stage.

ii. Load Cell

For some beams, the load cell designed and calibrated by the author was used to measure the applied load. It had a capacity of 25 tons and using the same it has been possible to measure the applied load upto an accuracy of 300 lb. The load cell consisted of a steel column of box section, 6" high attached with two Phillips 120 ohms strain gauges one vertically and the other horizontally so that both the vertical compression and the horizontal expansion of the column are detected. The general view of the load cell and the standard load cell circuit are shown in Fig. I.8. Two strain gauges were used to provide temperature compensation and increased electrical response.

f. Safety Support

During the test, a safety support was always kept at abou 3/4" below the torsion bracket ends as well as below the beam a all stages of loading. The safety support helped to prevent further damage to the beam after the final collapse had taken place. Specially fabricated steel stools made out of 6" x 3" channels with 6" cubes over them were used as safety supports f the beam. 6" x 6" x 30" concrete beams placed vertically were

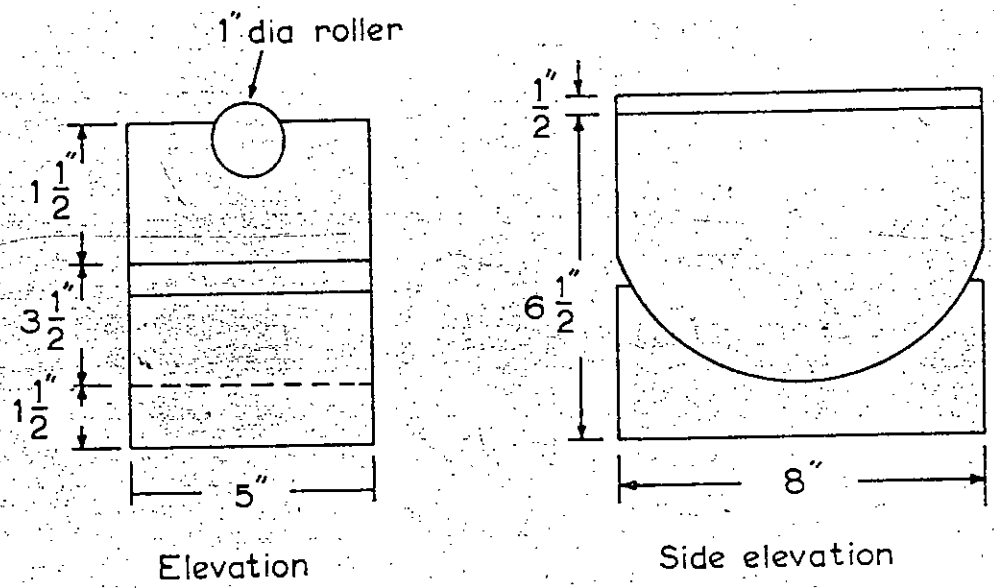


FIG. I. 9. DETAILS OF SADDLE.

used as safety supports for torsion brackets.

2. Loading Arrangements

The loading arrangements used for the beams is shown in Fig. I.6. The beams were supported on special type of saddles having a spherical seating (Fig. I.9). This arrangement made the beam statically determinate and allowed free rotation of the ends as well as free horizontal movements. The well oiled spherical seating permitted only vertical reactions and it was self adjusting to the bearing plane when the bearing plane was not perfectly horizontal. Only vertical loads were possible. All the beams (except the beams tested in pure torsion) were tested under two point loads placed symmetrically over the span. For this purpose, the total load was applied at one point and then dispersed to the two points equally by the load distributing beam. When the beams were tested in pure torsion or combined bending and torsion, torsional moment was induced by applying cast iron weights at the ends of torsion brackets.

3. Preparation of the beam for testing

After the beam had been cured for a period of 28 days from casting date, it was ready for testing. One day earlier than the testing date, it was removed from the curing tank and the mirrors used for measuring angles of twist and the Demec Gauge points were fixed to the beam. On the date of

testing, the beam was arranged in the loading frame and the torsion brackets were attached to the beam using plaster of Paris. Then these brackets were supported on safety supports so as to avoid initial twisting of the beam. At least four hours were allowed between fixing the brackets with plaster of Paris and starting the tests.

The beam was then whitewashed and horizontal and vertical lines were drawn at 2" intervals on the sides of the beam to aid in mapping the crack pattern. These lines were marked with red pencil to distinguish them from the black and blue pencils used for tracing the cracks. The loading points were also marked with red pencil. Then the load distributing beam was placed on the rollers. The jack was then introduced between the freme and the load distributing beam. Finally the correct position of the beam was checked. Then the telescopes carrying inverted scales were arranged at known distances from the mirrors for measuring the angles of twist. The dial gauge for measuring deflection was arranged at mid-span using stand fixed with magnetic base. After this stage the beam was ready for testing.

4. Testing

The maximum time taken for a single test did not exceed two hours and hence all the tests in this investigation are classified as short term tests.

a. Angle of Twist Measurements

A detailed description of the procedure followed in recording the angles of twist is given later.

b. Deflection Measurements

Deflections were measured at the bottom of the mid-span by the use of simple Baty Dial Gauges. The dial gauge used for the measurement of deflection read 0.001" per division and had a maximum travel of 2". Deflections were noted for every increment of load. Readings were taken both before and after application of torque so as to overcome any error in experimentation caused by the lateral rotation of the beam by twisting moment.

c. Strain Measurements

Surface strains were measured using 8" gauge length Demec gauge. The gauge had a least count of 1×10^{-5} inch/inch. Strain readings were taken for every increment in load.

d. Tracing of Cracks

The growth of the cracks was recorded by drawing lines on the crack. Only the visible cracks are shown in the drawings. Cracks were marked at every increase in the load and the development of each crack was recorded by marking the load at the end of the crack. Finally photographs were made of the beam showing the crack pattern.

e. Testing Procedure for the Beam

After the beam had been prepared for testing as explained before, the initial readings of the scales, deflection gauges and strain measuring units were recorded. The torsion brackets were released simultaneously on either side of the beam. Then the pump of the jack was operated for the first increment of bending load. Even though an approximate failure load was known before-hand in all cases, the increments of loads were kept as low as possible so as to get the exact load at which ultimate failure took place. After an increment of load or moment was applied, the beam was allowed to stabilize for sometime until there was no movement of the deflection dial gauge or of the scale reading. Then scale readings, deflection gauge readings and strain readings were recorded. The beam was examined minutely in search for cracks using magnifying lens. The cracks were marked by pencil and the moment at which the diagonal crack appeared was noted. The beams were tested until they broke down or until the concrete was crushing at the load point. If the beam collapsed during an increment, the maximum load reached during the increment was taken as the ultimate load. At collapse, the broken beam was supported by the safety supports. Each test usually took one to two hours.

After the failure, the final state of the cracks was marked with black paint or ink. Then the crack pattern was

mapped on graph paper. Finally a photograph of the beam showing the crack pattern was taken. Usually one or two close-up views of the failure were also photographed so that a complete record of the type of failure could be obtained. In cases where the beams were tested twice, the crack pattern was marked in full lines for one test and in broken lines for the other test.

f. Testing of Auxiliary Specimens

The standard 6 inch cubes were tested in the 200 Tons capacity compression testing machine on the same day as the main beams. The load was applied to the opposite sides of the cube as cast and was increased continuously at a constant rate. No packing was used between the faces of the test cube and the steel platens of the machine. In order to find the splitting strength of concrete, the cylinder splitting test was selected.

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CHAPTER I.5

DEFORMATIONS AND BEHAVIOUR OF BEAMS

I.5.1 Measurement of Deformations

Deformations of beams measured during the experiments included:-

i. angle of twist

ii. central deflection

iii. strains in concrete and steel

The methods and experimentations adopted to measure the above three quantities are briefly described in the following pages.

I.5.1.1 Measurement of Angle of Twist

Angles of twist were measured to yield information on the physical behaviour of beams at different stages of loading which is a valuable contribution to the problem under investigation. Angles of twist were measured on both the sides of the beam so as to check the experimental results. Measurement of angles of twist was made upto the moment of final collapse for all modes of failure and modes of testing so as to study the effect of each factor on the behaviour of beams.

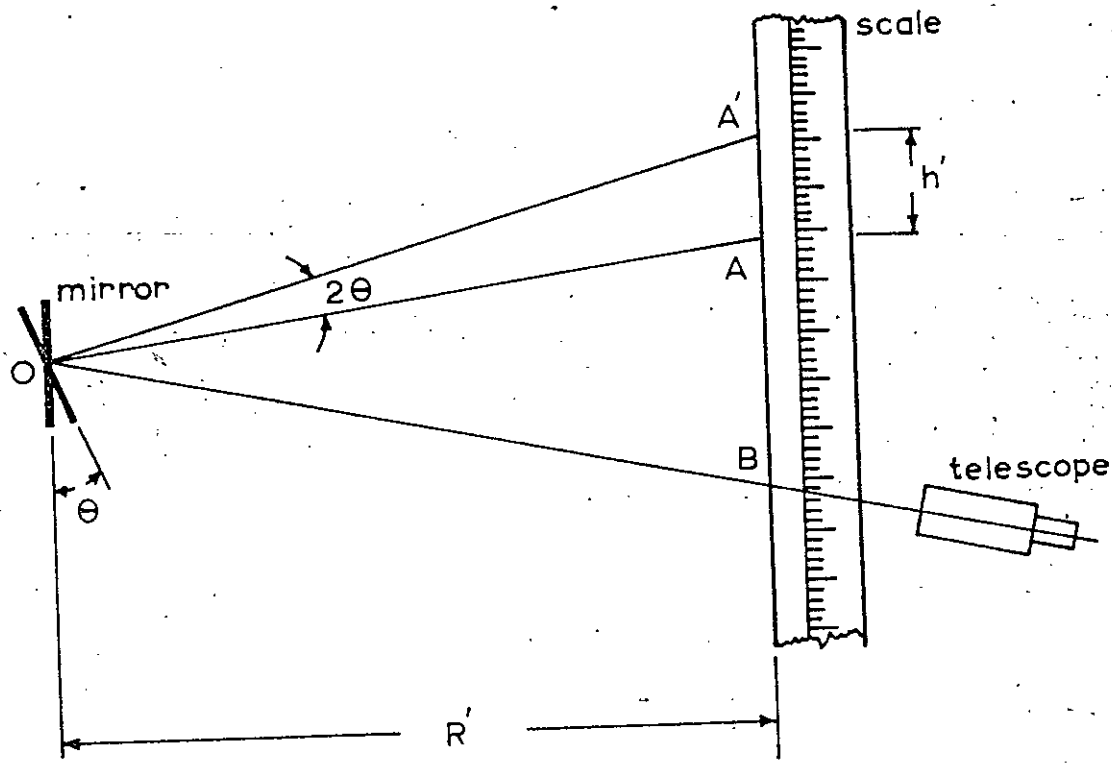


FIG. 1. 10. MEASUREMENT OF ANGLE OF TWIST.

The angular movements of both ends of the beam were measured by means of two mirrors, scales and telescopes. On each side of the beam a small mercury coated mirror of 1" x 4" size was fixed and two scales and telescopes were arranged at a known distance from the mirror on either side of the beam. Each mirror is so oriented that it will reflect a particular reading of one scale into the corresponding telescope.

Principles of operation

Referring to Fig. I.10, AO is the incident ray from the scale, which is reflected along OB into the telescope. Since the path of the light is reversible, we can assume that BO is the incident ray and OA is the reflected ray. Now, if the mirror is turned through an angle θ, then according to the principle of the optics, the reflected ray will move through 2 θ. Since this angle is reasonably very small, then, in radian measure $h' = R' \times 2\theta$, where h' is the scale intercept and R' is the distance between the mirror and the scale. Knowing the values of R' and h', the angular movement of beam on both sides were calculated. Ample illumination was provided for the scales.

Method of Fixing the Mirror to the Concrete Surface

The exact position of the mirror was marked with a pencil. The surface was thoroughly cleaned and then a

groove was cut on the surface to suit the size of the mirror. A thin layer of Plaster of Paris was applied to the surface of the groove and the mirror fixed by pressing it with the finger using a rolling movement so that surplus Plaster of Paris was rolled out at the ends of the mirrors. At least one day was allowed between fixing the mirrors and using them. This method was found to be successful in fixing the mirrors.

Recording of Angles of Twist

The scale readings were noted by visual observation through telescopes and angles of twist were calculated as mentioned before. Readings were taken every time for each increment in the torque-load applied. The initial reading was taken as soon as the load was applied and the final reading was taken just before next load was applied, so as to avoid any error due to disturbance of the loading arms.

The measurement of angles of twist by the method described above proved to be easy, cheap and accurate enough.

I.5.1.2 Measurement of Deflection

Central deflections of all the beams were measured using the usual type of dial gauge. The dial gauge had a least count of 1×10^{-3} inch, the maximum travel being 2". The dial gauges were held in position using stands fixed with

magnetic base. Special care was always taken to keep the dial gauge exactly at the centre of span and the travelling stem of the dial was kept at bottom of the beam in a vertical position. Deflection readings were taken for each increment in the load. The dial reading was noted as soon as the load was applied and the final reading was taken just before the application of next load, so as to avoid any experimental error.

I.5.1.3 Measurement of Strains

i, Concrete Strains

The concrete strains at selected points were measured using Demec Demountable Strain Gauges of 8" gauge lengths. The gauge had a least count of 1×10^{-5} inch/inch.

The general layout of the points where the discs were to be located was first drawn on the beam surface. The individual locations were then sand-papered and cleaned with Acetone. A small quantity of quick fix was then applied to the point and the locating disc was oriented on the point with a little pressure. The gauge length between two consecutive discs were corrected each time using correcting bars. The Demec discs were fixed at least one day earlier to the date of testing so as to allow them to set permanently with the surface of the concrete.

Normally, the bending strains were measured only in the compression zone of the beam. The discs were located starting from the point just below the top surface of the beam. They were laid in two or three lines and each line was separated by 1" intervals. Strains were measured for each increment in moment till the final collapse of the beam.

ii. Steel Strains

The strains developed both in longitudinal and transverse steels were measured using electrical strain gauges. The strain gauges were fixed to the surface of rods adopting standard procedures using Araldite. They were water-proofed using one coat of wax and another coat of Bitumen and always perfect water proofing was ensured by measuring the resistance accurately using a Wheatstone Bridge Circuit.

Measurement of Strains

The strains were measured at all stages of loading using a Direct Reading Measuring Bridge calibrated directly to give the strains. Using this bridge it had been possible to measure strains upto an accuracy of 2×10^{-5} inch/inch.

I.5.2 General Behaviour of Beams

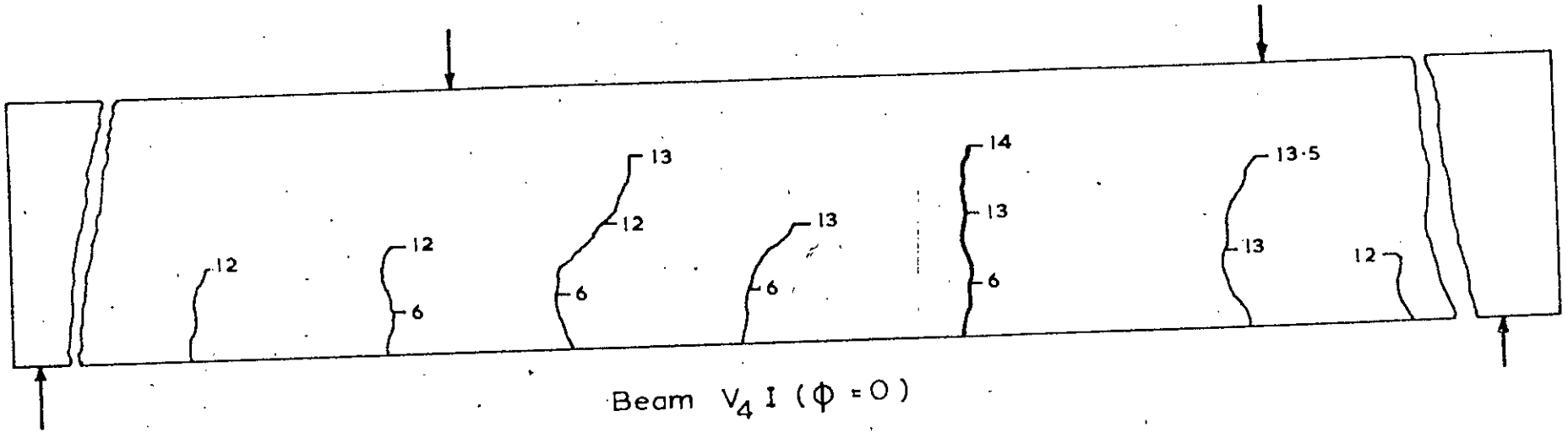
I.5.2.1 The understanding of the mechanism of failures of beams in combined bending and torsion is an important aspect

of this investigation. To do so, it is essential to study the actual behaviour of the beam at all stages of loading, both in the elastic and inelastic stages. In this article, the formation and development of flexural cracks due to bending moment and diagonal cracks due to the applied torque are briefly described.

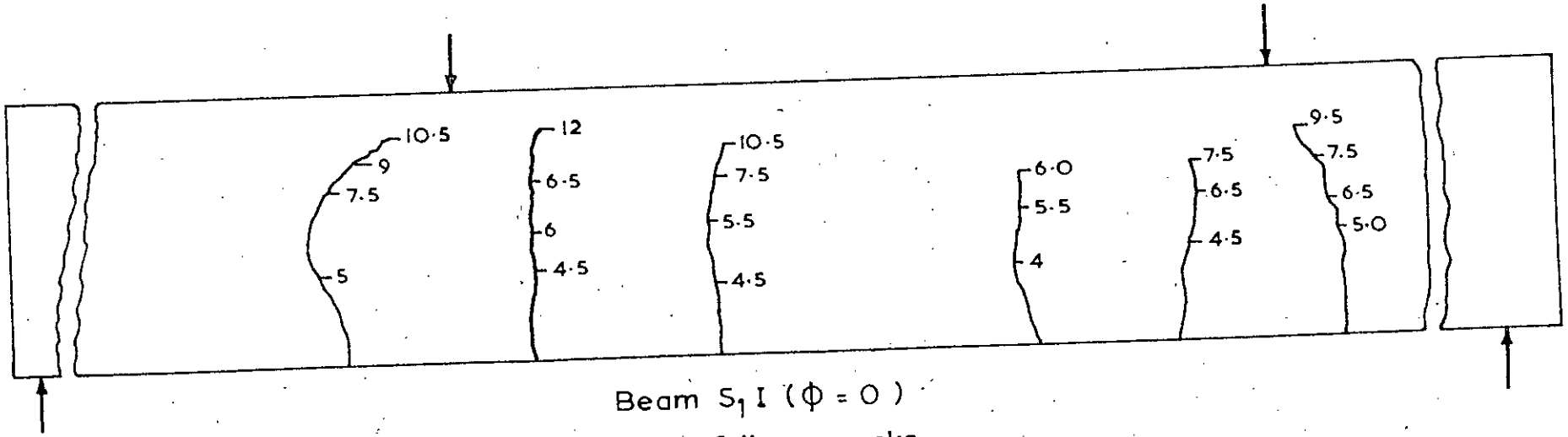
I.5.2.2 Flexural Cracks

The initial flexural cracks appeared in the maximum bending moment region. The moment at which the first flexural crack appeared depended mainly on the concrete properties of the beam. The initial cracks were usually hair cracks, detected by means of magnifying lens, starting from the bottom and penetrating upto the reinforcement level. As the moment was increased these cracks extended vertically upwards and new flexural cracks formed in between the existing cracks. In the maximum bending moment zone there were normally 4 or 5 cracks and the distance between two adjacent cracks varied from 3" to 5".

At later stages of loading, the penetration and rate of movement of these flexural cracks were depended on the ratio of twisting moment to bending moment, ϕ. For very low values of ϕ, these flexural cracks extended more or less vertically similar to those in beams subjected to pure bending. The effect of torsional stresses is practically nil on the



Beam V_4 I ($\phi = 0$)

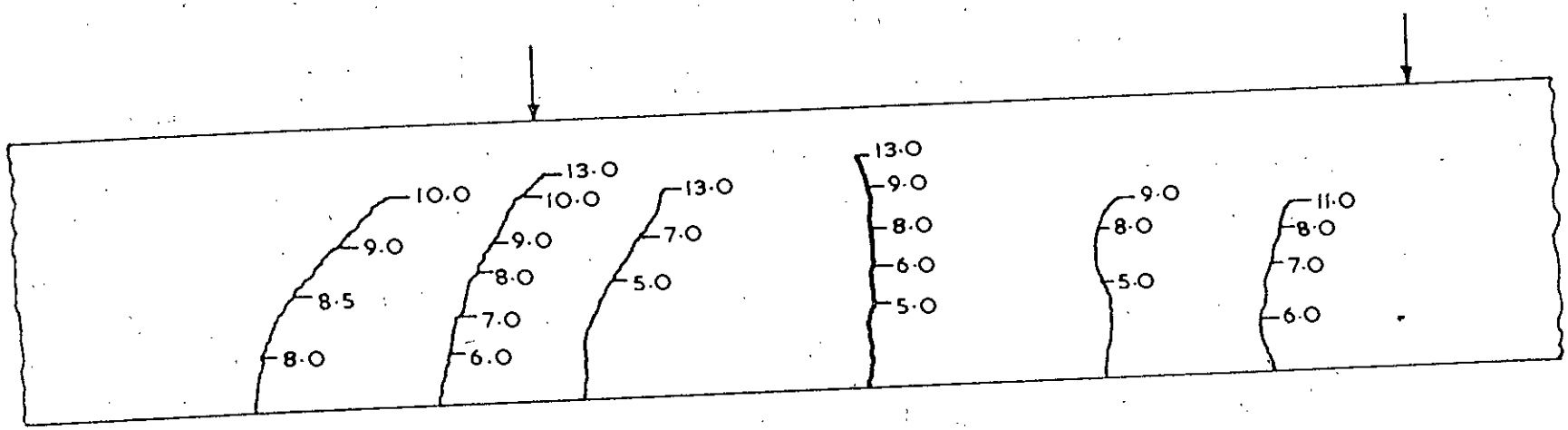


Beam S_1 I ($\phi = 0$)

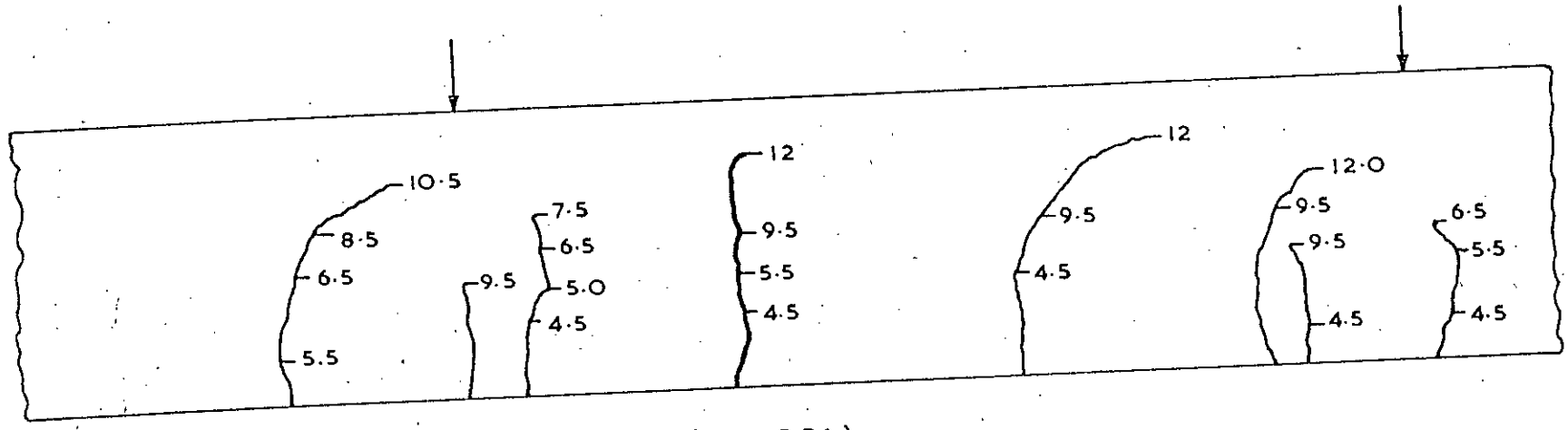
Note :— i Thick lines indicate failure cracks.

ii Numbers in the crack pattern correspond to jack loads in kips.

FIG. I. 11.

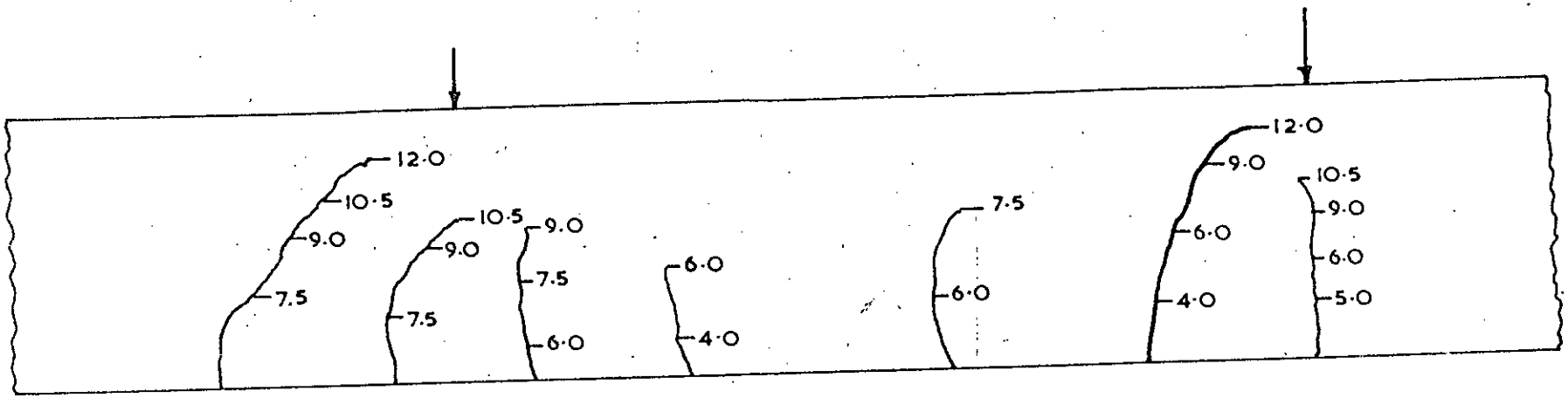


Beam S_2 I (ϕ = 0·104)

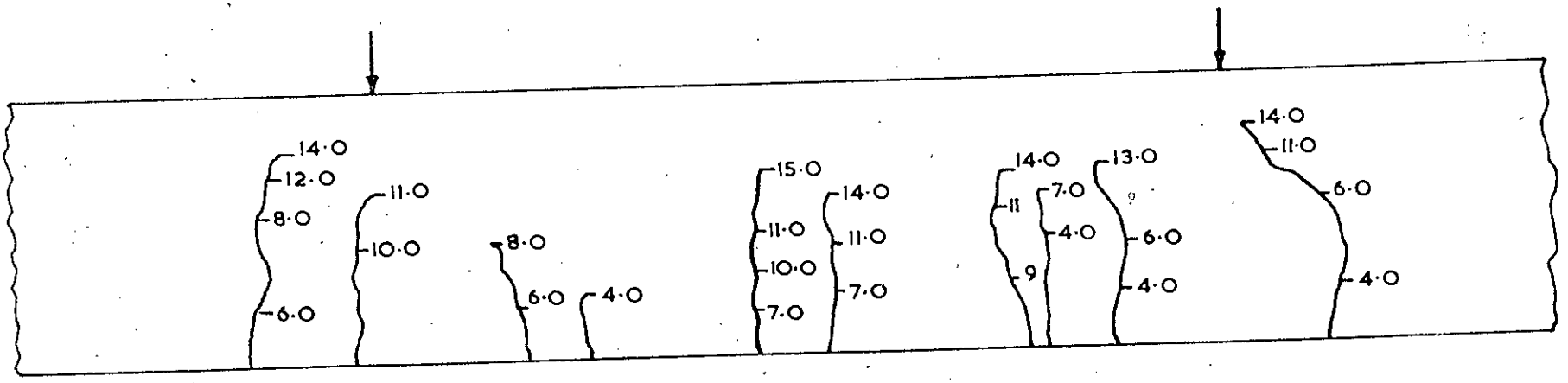


Beam S_4 I (ϕ = 0·081)

FIG. 1. 12.

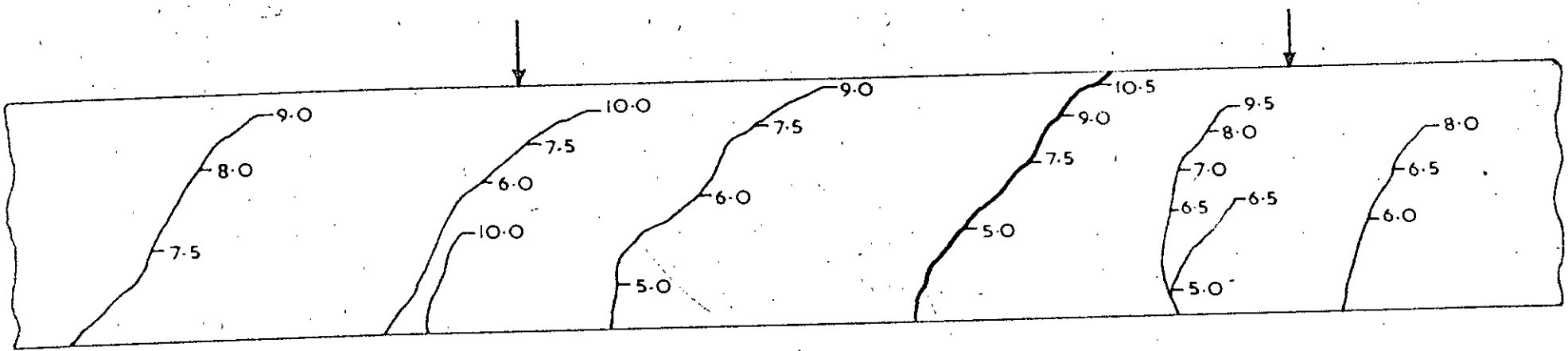


Beam S_5 I (ϕ = 0·135)

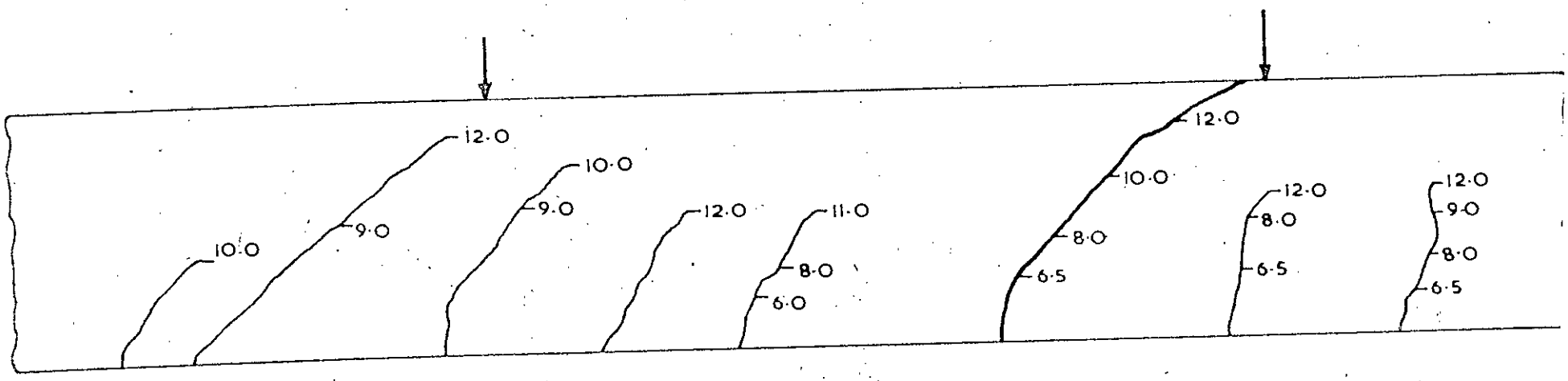


Beam R_3 I (ϕ = 0)

FIG. 1. 13.



Beam V_2 (ϕ = 0·298)



Beam V_3 (ϕ = 0·179)

FIG. 14

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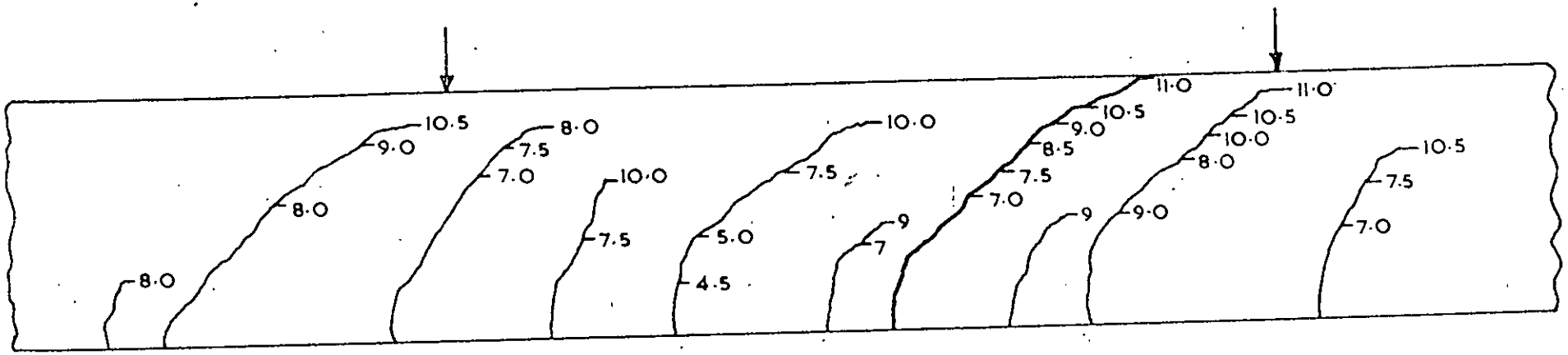
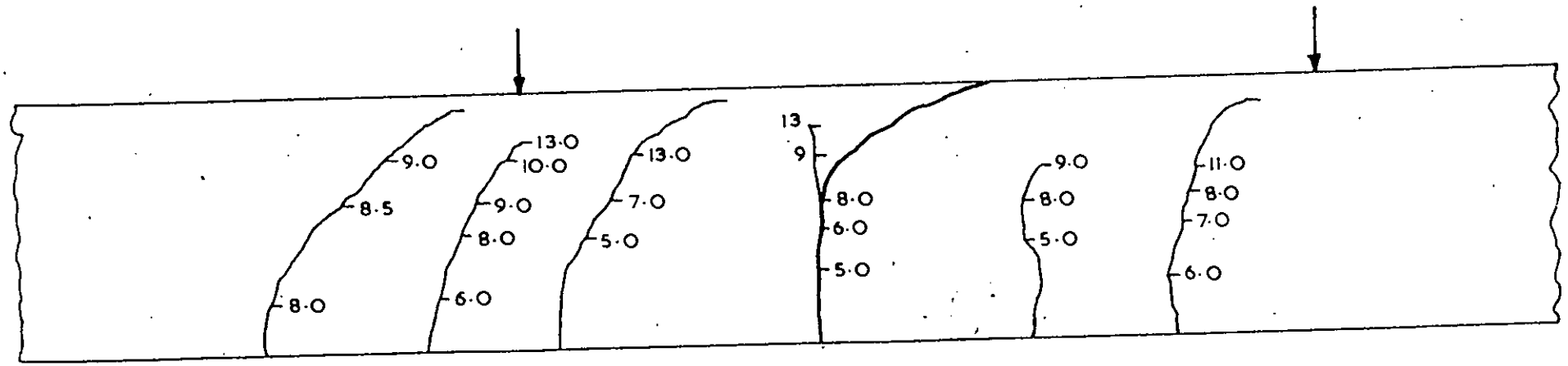
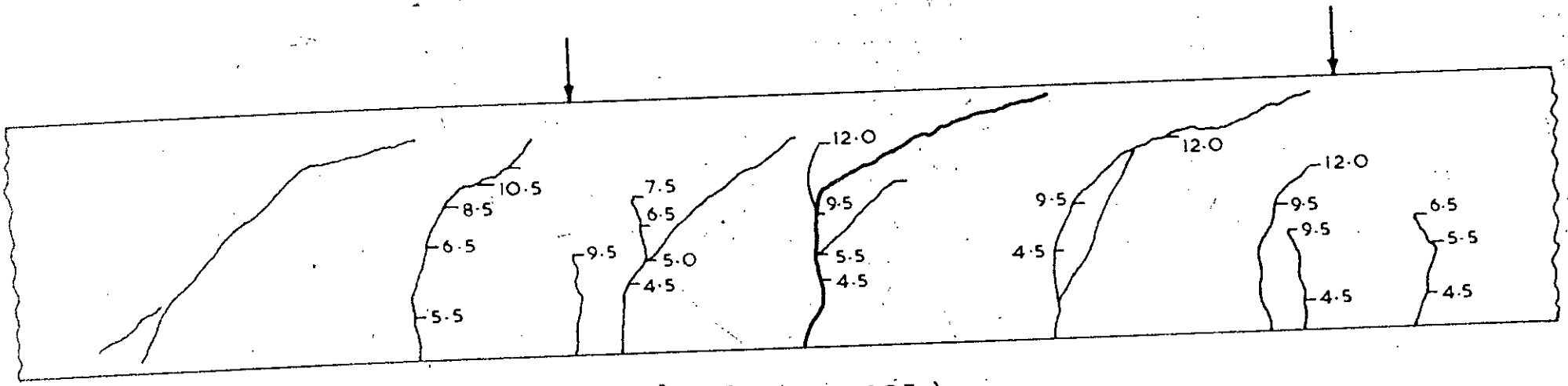


Beam V_5 (ϕ = 0·275)

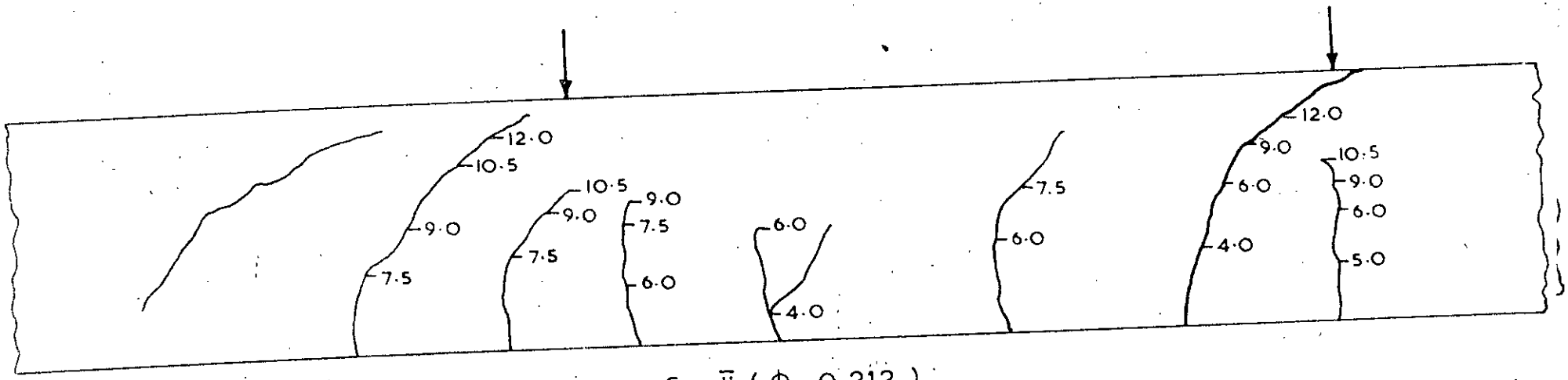


Beam S_2 II (ϕ = 0·238)

FIG. 1. 15.

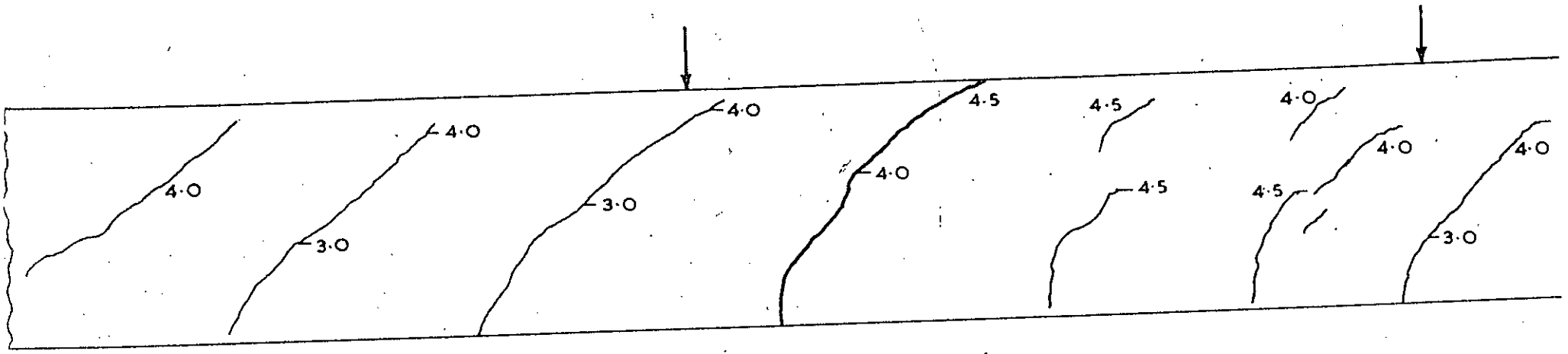


Beam S_4 II (ϕ = 0·235)

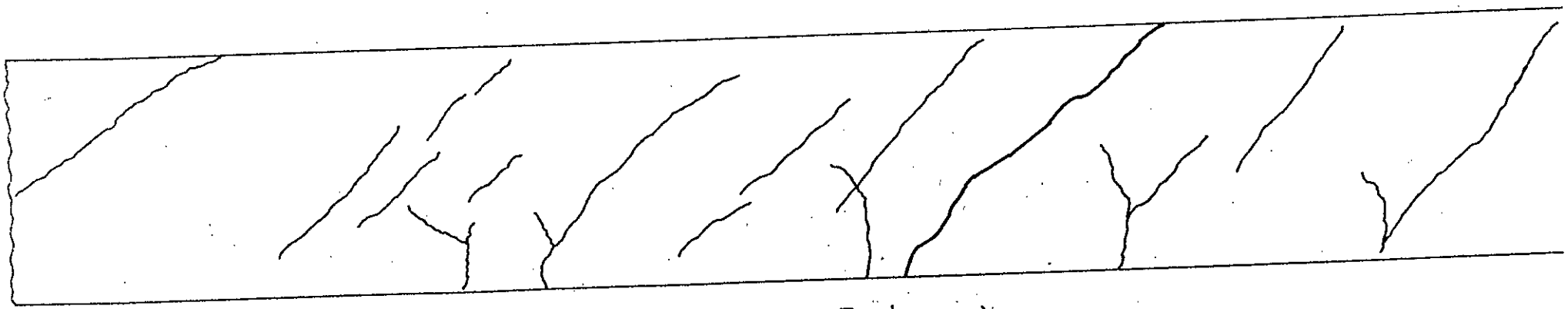


Beam S_5 II (ϕ = 0·212)

FIG. 1.16.

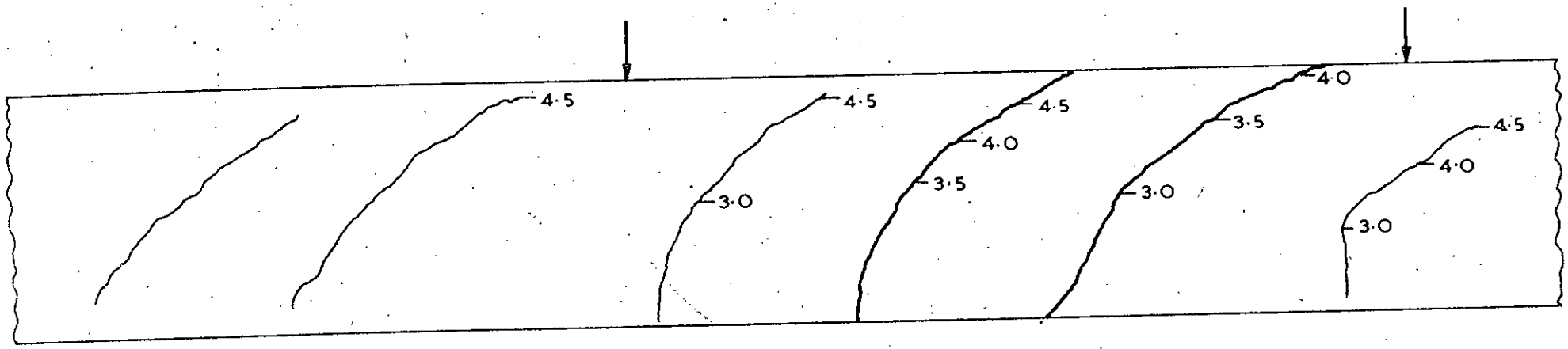


Beam V_1 ($\phi = 0.745$)

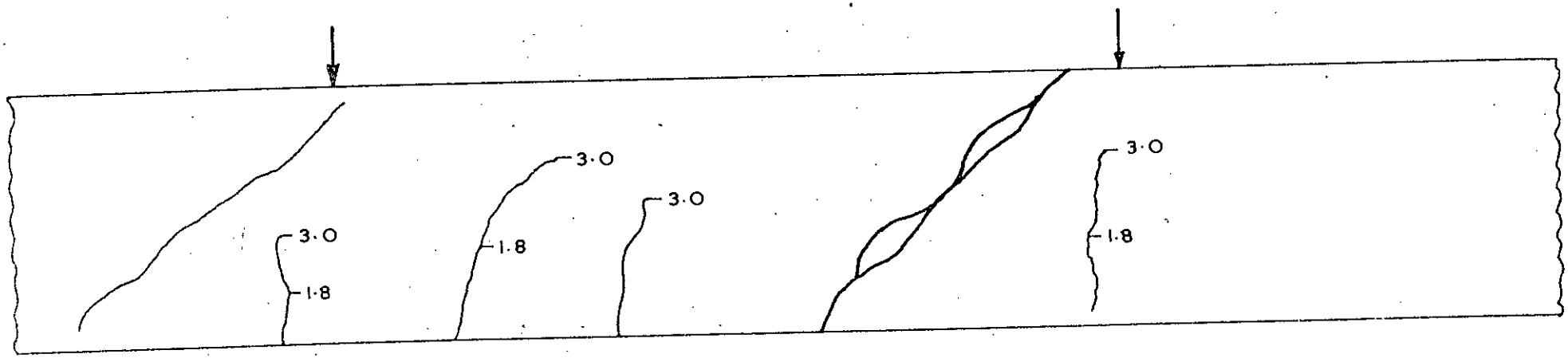


Beam V_4 II ($\phi = \infty$)

FIG. 1.18.



Beam V_6 ($\phi = 0.655$)



Beam S_3 ($\phi = 0.935$)

FIG. 1. 19.

movement and direction of these cracks. Meanwhile the width of these cracks gradually increased as the moment was increased. The final collapse of the beam occurred by opening of one or two cracks (Figs. I.11-13).

For medium values of $\emptyset$, the flexural cracks deviated from their vertical direction as they moved upwards showing evidence of torsional stresses. The deviation from the vertical is more and more as they moved further and normally they coincided with the existing diagonal cracks at mid depth of the beam. The flexure failure occurred by opening of the inclined crack (Figs. I.14-17).

For high values of $\emptyset$, all the beams prematurally failed in torsion. In this range of $\emptyset$, the flexural cracks extended rapidly in the beginning, but the rate of extension became rather slow as they approached the mid depth of the beam. Finally they just stopped at about mid depth of the beam. Before further extension of them, torsion failure occurred. The three types of development of flexural cracks described above are shown in Figs. I.11 to I.19.

I.5.2.3 Diagonal Cracks

At first a very short diagonal crack appeared at about mid-depth of the beam inclined at an angle of 45° to the longitudinal axis of the beam. For further increments of

moments, the development of diagonal crack depended on the value of ϕ. For high values of ϕ, the first crack extended straight both ways for further increments in torque. The crack that has extended upwards reached the top edge of the beam. For further increments, this crack reached the top surface of the beam and joined with the other crack developed from the other face of the beam. Similarly the crack that has extended downwards crossed the bottom face and joined the crack on the other vertical face of the beam. Thus the continuous chain of the cracks was in the form of a helix. The cracks on the two vertical faces were perpendicular to each other and the crack joining them on the top and bottom faces were inclined at about 45° to the longitudinal axis of the beam. The final collapse of the beam took place by wide opening of the crack on the top face of the beam.

For other ranges of ϕ, the upper end of the crack (formed at mid-depth of the beam) extended flatter and the lower end joined with the flexural crack that has already formed. As the moment increased the diagonal crack that has extended upwards became very flat and reached the top edge approximately parallel to the longitudinal axis of the beam. For further increments in moments, this crack extended into top surface of the beam and joined with the similar crack on the opposite face of the beam. The crack joining the

two cracks on the vertical faces on the top surface of the beam is very flat and almost parallel to the longitudinal axis of the beam in many cases. The crack (that has joined the flexure crack) at the bottom joined with the similar crack on the other face in a similar manner as described for the previous case. The final collapse occurred due to the wide opening of the crack on the top surface of the beam. This type of development of diagonal crack is shown in Figs. I. 14-17.

In some beams, an additional diagonal crack also formed mostly parallel to the original diagonal crack, before or simultaneous with failure. When the second crack formed before failure, it became as wide as the original crack when the torque was increased.

In many beams several diagonal cracks appeared before the beam finally collapsed, but beams usually failed due to the widening of one or two of these cracks, the other remaining as hair cracks (Fig. I.19). The developments of some diagonal cracks were sudden. The general pattern of cracking was symmetrical on ~~the~~ both the faces of the beam in many cases.

The concrete covering the stirrups in the top surface was extensively cracked in some cases and spalling of

concrete cover to the tension steel was observed. The failure cracks were sometimes accompanied by some secondary cracks.

I.5.2.4 Diagonal Cracking Moment

The first visual appearance of a diagonal crack of short length near the mid-depth of the beam could not be detected by any significant change in the torque-twist curves. But the development of this crack into a full length diagonal crack can be easily detected as this was followed by an increase in the deformations of the beam, particularly a significant change in the torque-twist curves. For the purpose of this investigation, the first visually observed full length inclined crack was taken as the diagonal crack and the torque at which this crack was observed was taken as the diagonal cracking torque. In the complete absence of a new inclined crack, the torque at which an already existing flexural crack changes its direction to develop into a diagonal crack, is taken as the diagonal cracking torque. Further, the diagonal cracking torque was independent of the presence of stirrups; it depended mainly on the strength of concrete and cross sectional dimensions of the beam.

CHAPTER I.6

OBSERVED MODES OF FAILURES

I.6.1 A knowledge of the modes of failures is one of the most important results to be obtained from ultimate load tests; one of the primary objects of this investigation was to study the behaviour and establish the modes of failures of beams subjected to combined bending and torsion. In this investigation, all the modes of failures were induced by the combined action of bending and torsional stresses. But the extent of influence of one stress on the other depended on the ratio of twisting moment to bending moment, ϕ.

There are mainly two causes for final collapse of a beam subjected to combined bending and torsion:

1. Flexure failure by crushing of compression zone concrete.
2. Torsion failure by diagonal splitting of compression block concrete.

In no case bond or anchorage stresses constituted a major cause of failure of beams; but they caused a secondary

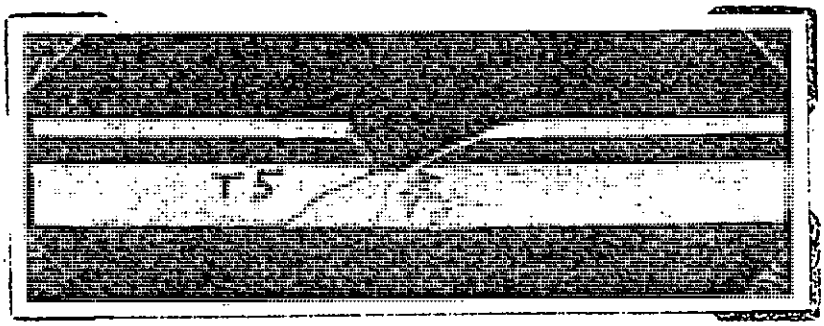
FIG. I.20 : Premature Flexure Failure

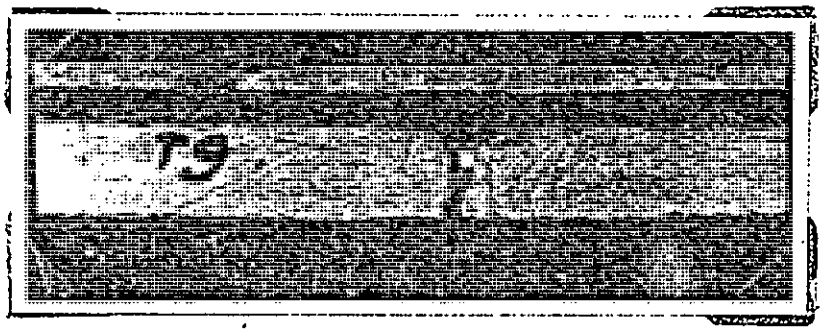
Beam T5 I ($\emptyset$ = 0.098)

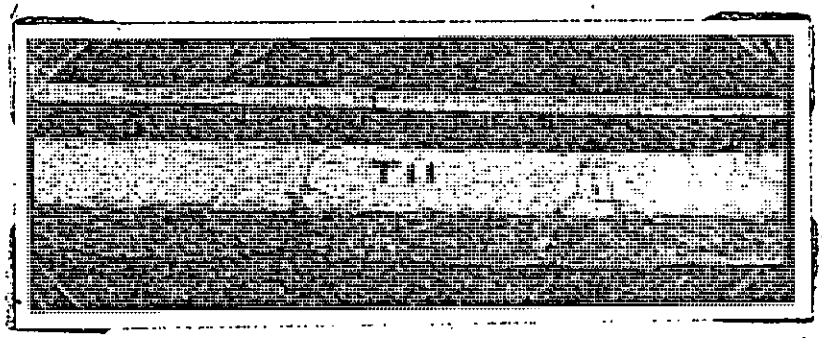
Beam T9 ($\emptyset$ = 0.103)

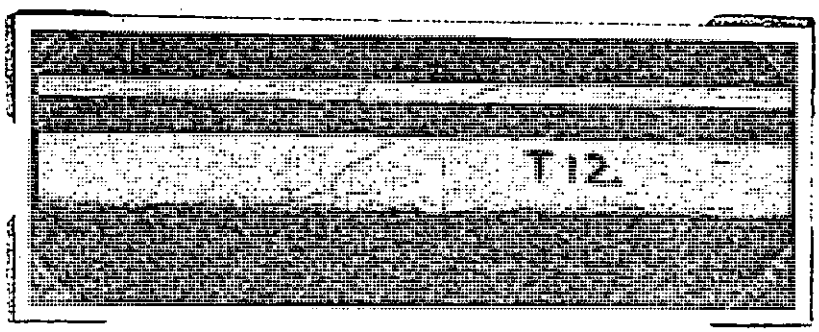
Beam T11 II ($\emptyset$ = 0)

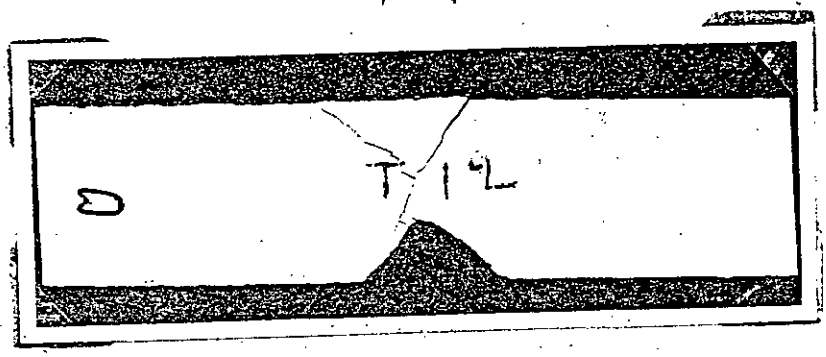
Beam T12 I ($\emptyset$ = 0.105)

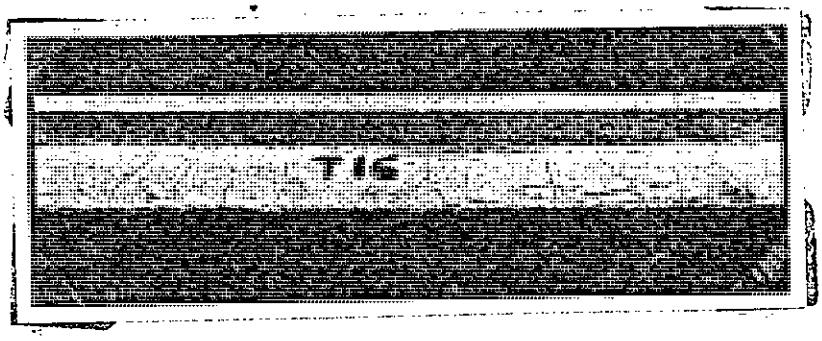

T5

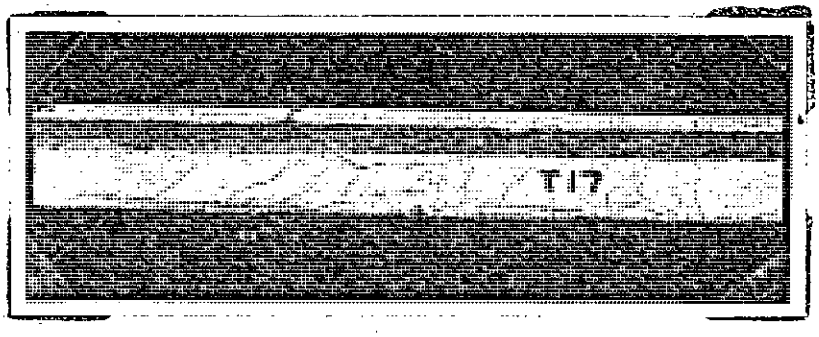

T9

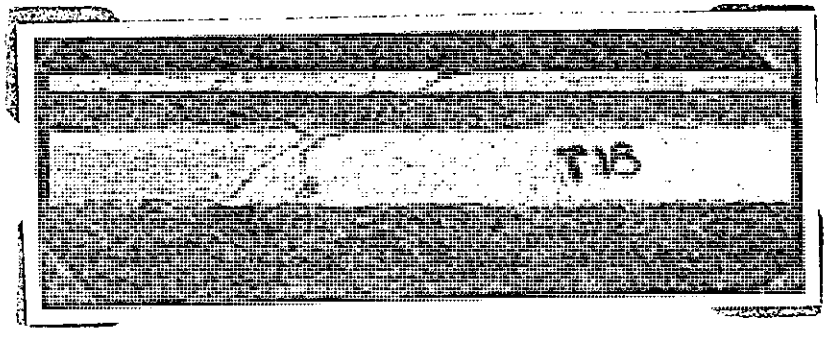

T11


T12


D
T 14


T 16


T 17


T 15

failure in few cases. Yielding of the transverse steel does not by itself constitute a failure, but it is always accompanied by torsion failure. Mainly three types of failures were observed in beams subjected to combined bending and torsion: (1) premature flexure failure (2) premature torsion failure (cleavage fracture) and (3) 'diagonal compression' failure.

I.6.2 Premature Flexure Failure

A flexure failure takes place by crushing of the concrete in the compression zone of the beam either before or after the yielding of the tensile reinforcement, and rarely by breaking of the tension steel. A beam is said to have failed in flexure if it fails by crushing of the concrete or fracture of the reinforcement as a result of normal stresses in case of pure bending or due to the interaction of bending and torsional stresses in the case of combined bending and torsion. According to this definition, beams S2, S4, S5, L2-1, L2-2, L3-1, L3-2, L6-1, L6-2, T5, T9, T10, T17 and T20 failed prematurally in flexure. For values of $\emptyset$ less than 0.150, the beams prematurally failed in flexure even before the formation of diagonal crack. The failure normally occurred due to the opening of one or two cracks. The cracks were approximately 1/4" to 3/8" wide and were spaced at 4" to 5" intervals. Figs. I.20 and I.21 represent this type of failure.

I.6.3 Premature Torsion Failure

A torsion failure takes place by diagonal splitting of the concrete. In general, a torsion failure is defined as a failure the primary cause for which is the formation of an inclined tension - diagonal - crack due to the combined effects of bending and torsional stresses. In other words, a torsion failure is preceded by and caused by a diagonal crack formed all along the periphery of the cross-section. Unlike flexural failures, which take place gradually giving warning, torsion failures are mostly sudden and catastrophic. Hence they are more dangerous and less desirable.

This type of failure was usually sudden and took place in beams tested with high value of twisting moment to bending moment ratio, $\emptyset$. After the formation of the diagonal crack the ultimate failure of the beam took place due to the splitting of concrete. The width of opening on top face of the beam varied from 1/2" to 3/4" while the lateral displacement at failure was between 1/4" to 1/2". The failure crack was approximately at 45° to the longitudinal axis of the beam. There was no sign of crushing of concrete in any part of the beam and the failure crack exhibited a granular appearance. This evidently shows that the failure is mainly due to torsional stresses.

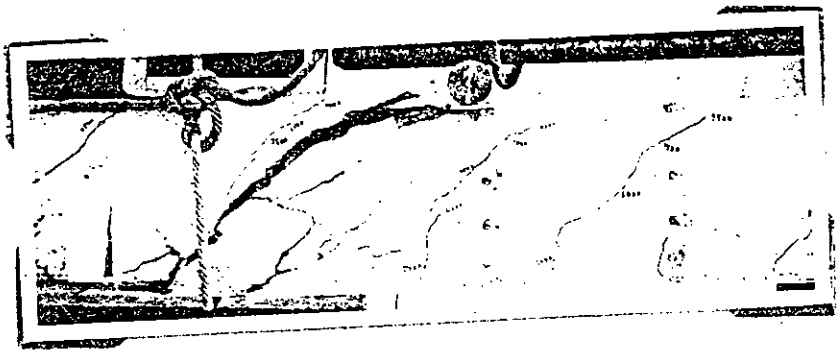
Fig. I.22 : Premature Torsion Failure

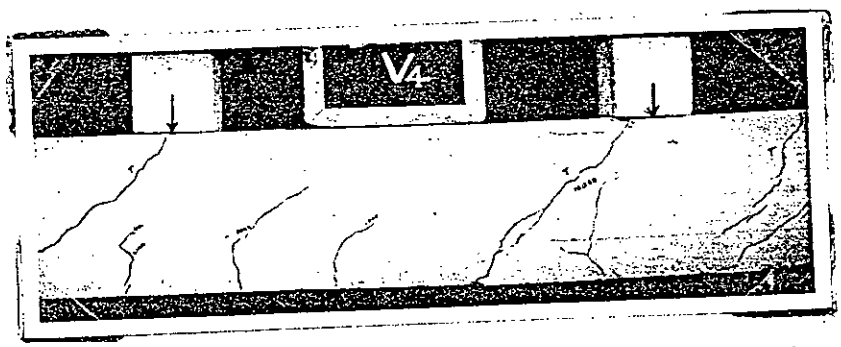
Beam V1 ($\phi = 0.745$)

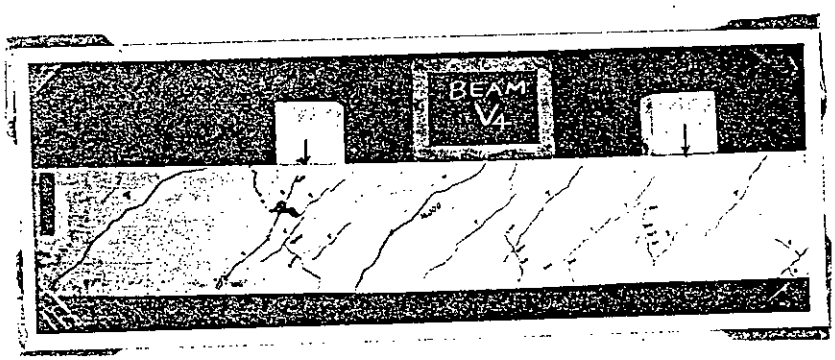
Beam V4 II ($\phi = \infty$)

Beam V4 II ($\phi = \infty$)

Beam A4 ($\phi = \infty$)




V4


BEAM
V4

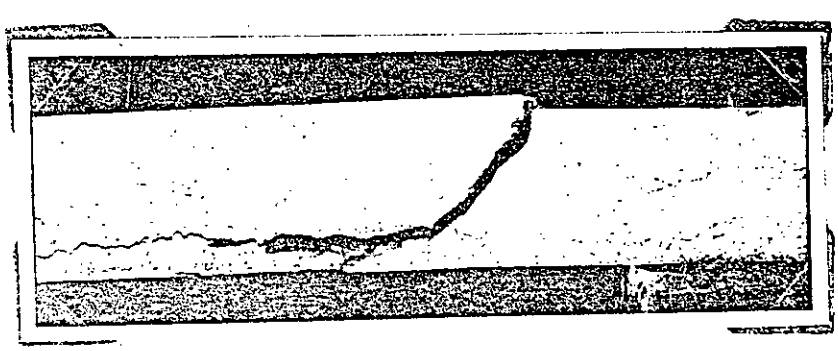
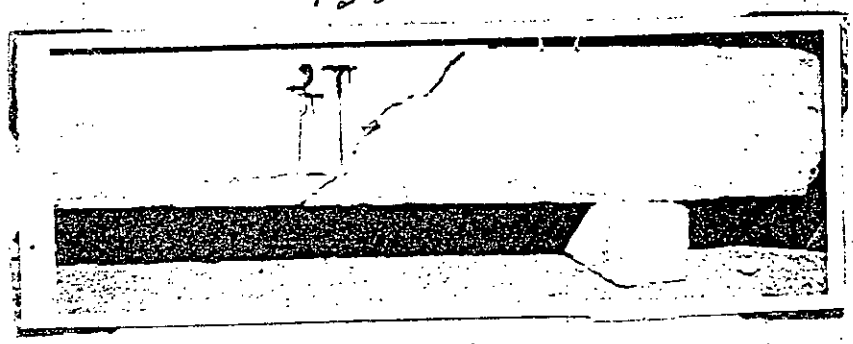
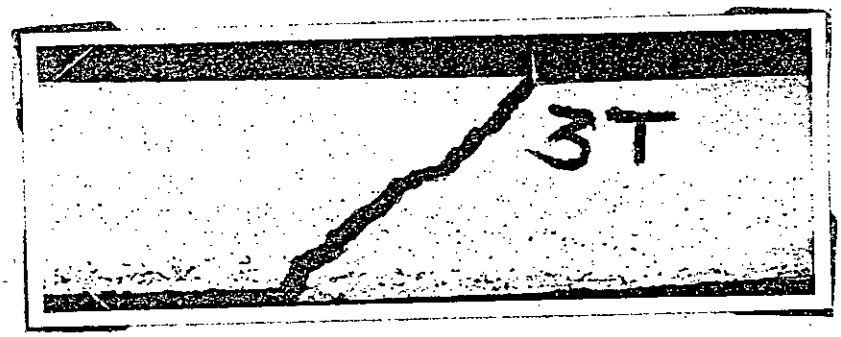
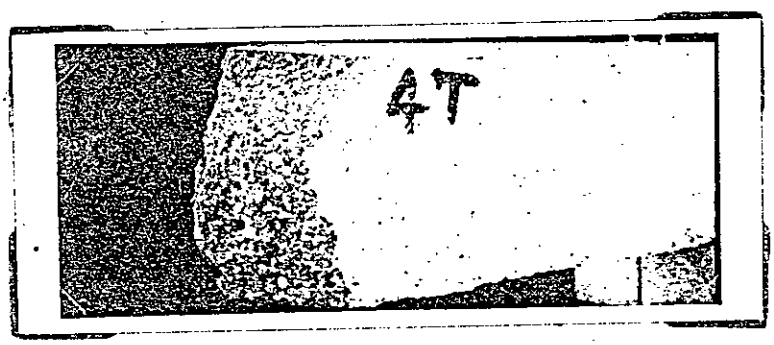


Fig. I.23 : Premature Torsion Failure - Beams of A Series


2T


3T


4T

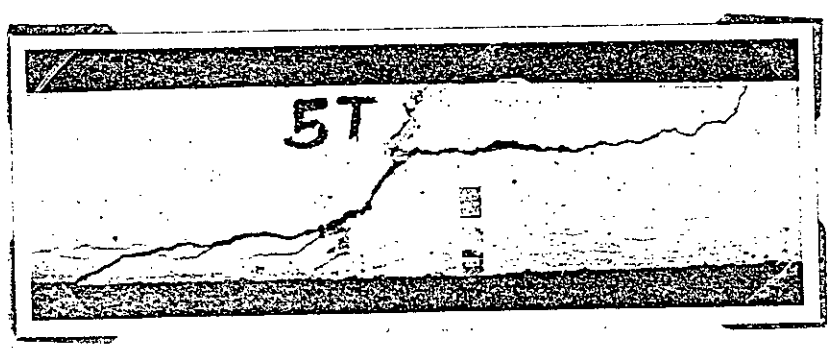
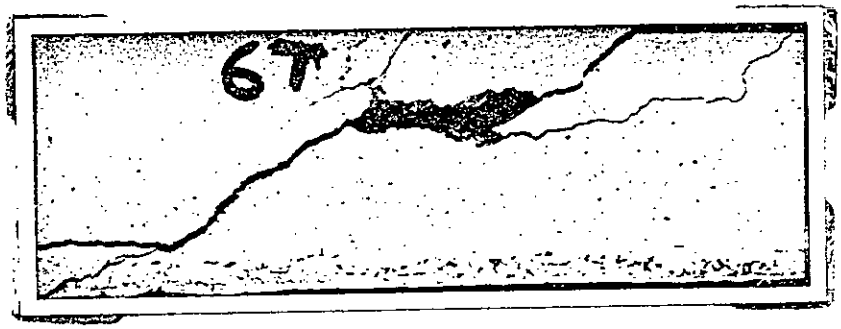
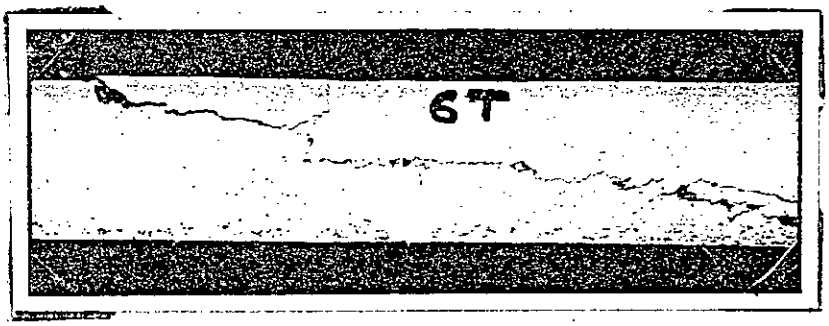
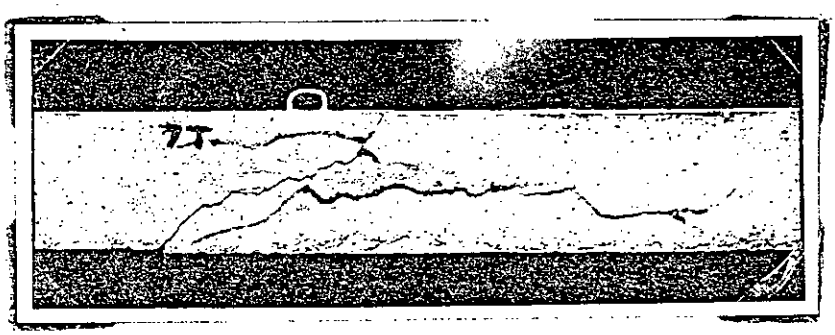

5T

Fig. I.24 : Premature Torsion Failure - Beams of A Series


6T


6T


7T

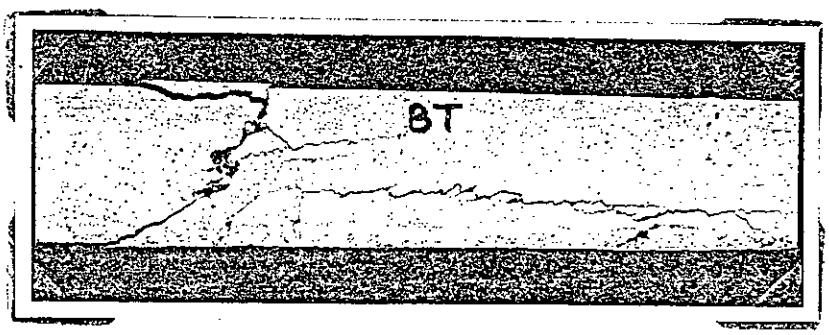
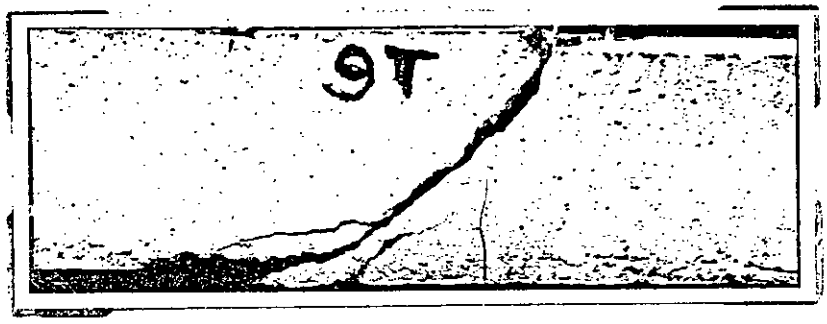
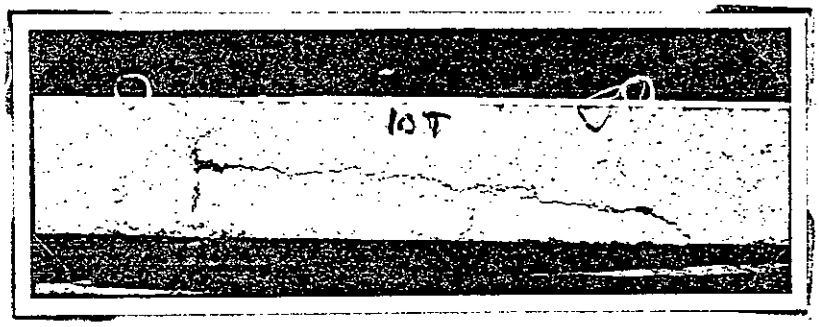
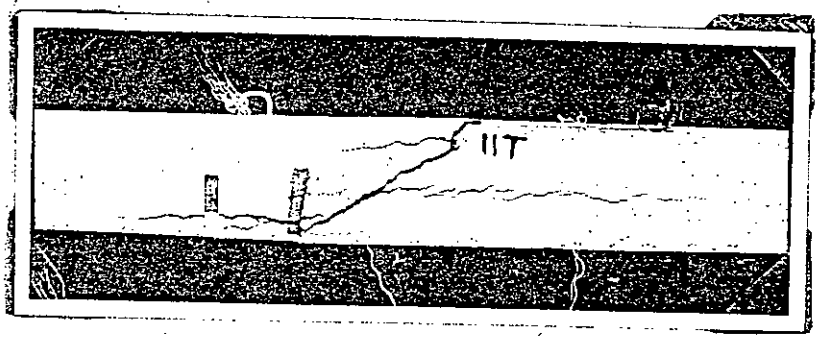

8T

Fig. I.25 : Premature Torsion Failure - Beams of A Series


9T


10T


11T

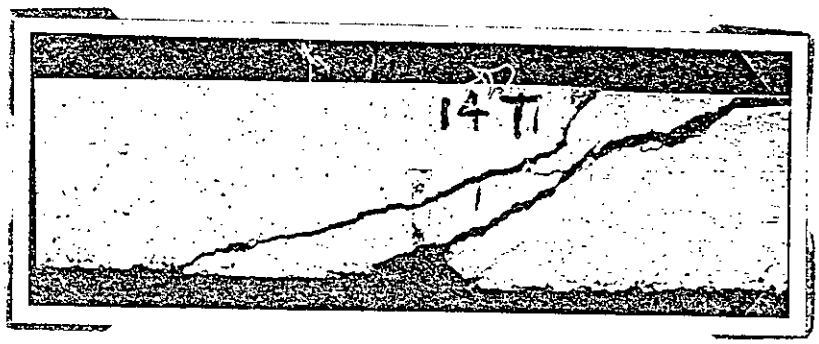

14T

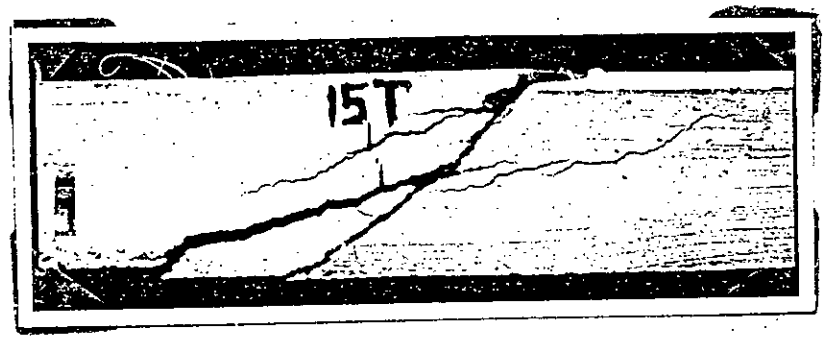
Fig. I.26 : Premature Torsion Failure

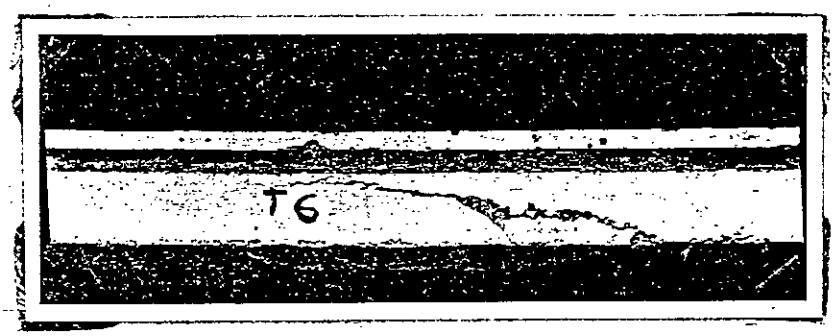
Beam A 15 ($\phi = \infty$)

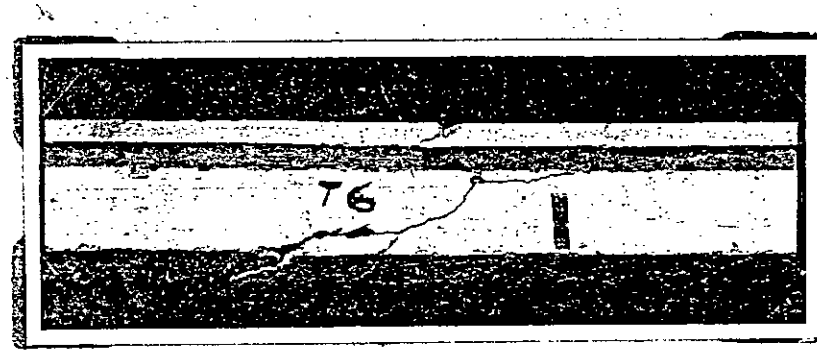
Beam T 6 ($\phi = \infty$)

Beam T 6 ($\phi = \infty$)

Beam T 6 ($\phi = \infty$) - Top view of beam


15T


T6


T6

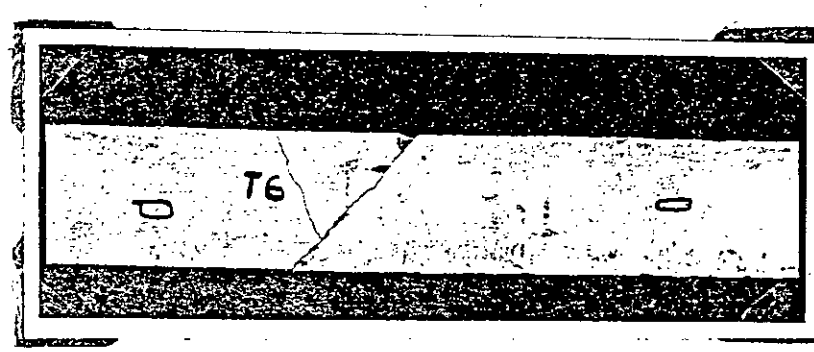

T6

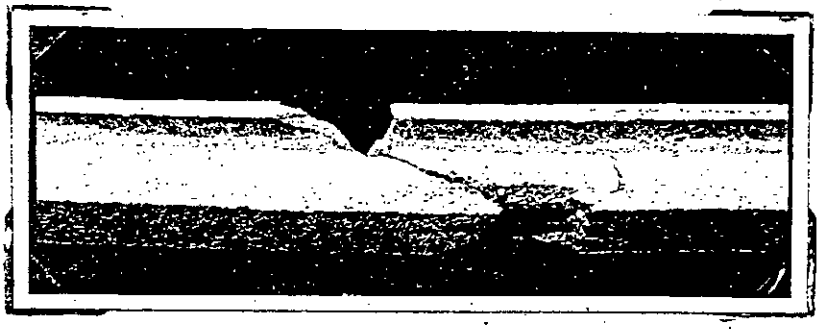
Fig. I.27 : Premature Torsion Failure

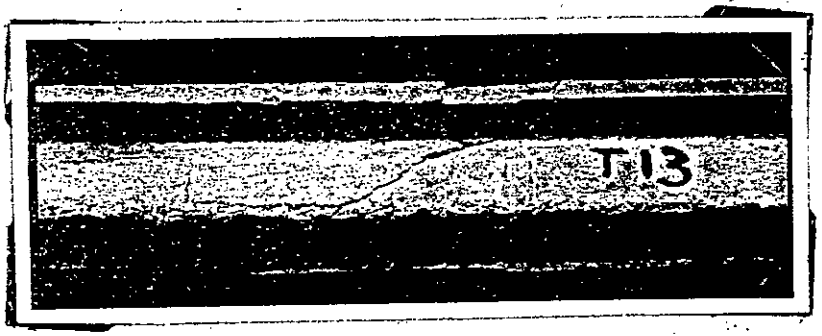
Beam T 11 ($\phi = \infty$)

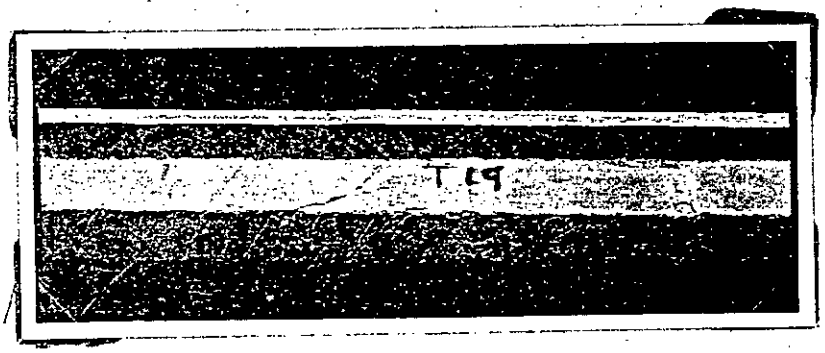
Beam T 13 ($\phi = 1.000$)

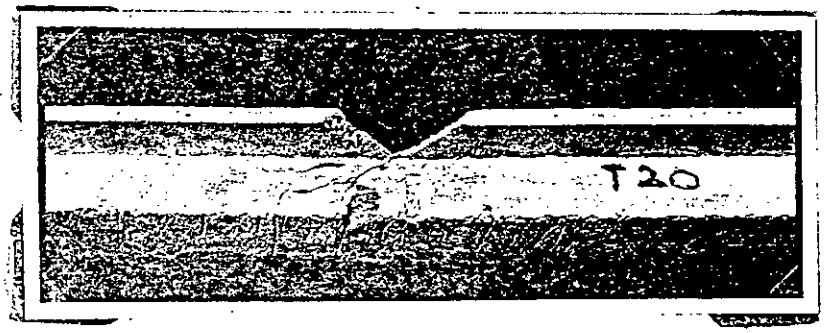
Beam T 19 ($\phi = \infty$)

Beam T 20$\underline{II}$($\phi = \infty$)




T13


T19


T20

The beams which failed as described above are generally termed as 'Cleavage Type Failure'. Beams V1, S3, L5-1, T4, T13 and T19 are some of the beams failed in this manner. Some views of this type of failure are shown in Fig. I.22 to I.27. The failed beams showed flexural cracks inclined slightly to the vertical in the zone of maximum bending moment with no evidence of crushing. The strains measured at the failure section were far less than the strains that are normally associated with concrete crushing. The maximum compressive strain in concrete at the top surface of the beam varied from 30×10^{-5} to 50×10^{-5} inch/inch.

This mode of failure was sometimes accompanied by splitting of concrete along the tension steel. The peeling of compression zone concrete was observed in few cases (See Figs. I.22 - 27).

I.6.4 'Diagonal Compression' Failure

In this investigation, this type of failure was observed mainly for the values of $\emptyset$ higher than 0.15 but below 0.65.

In this mode of failure, initially flexural cracks were noticed in addition to inclined cracks formed at mid-depth of the beam. As the moments were increased these flexural cracks extended upwards and new flexural

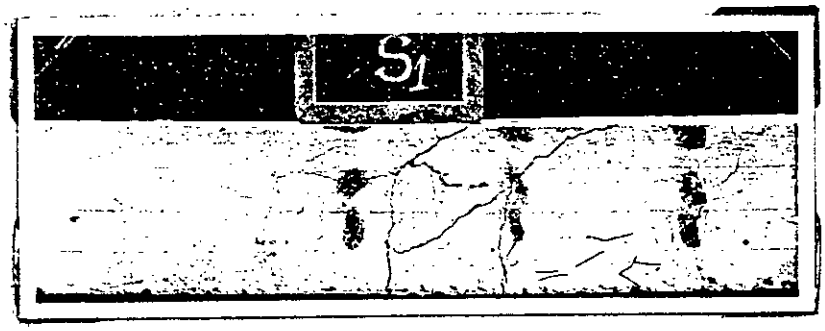
Fig. I.28 : Diagonal Compression Failure

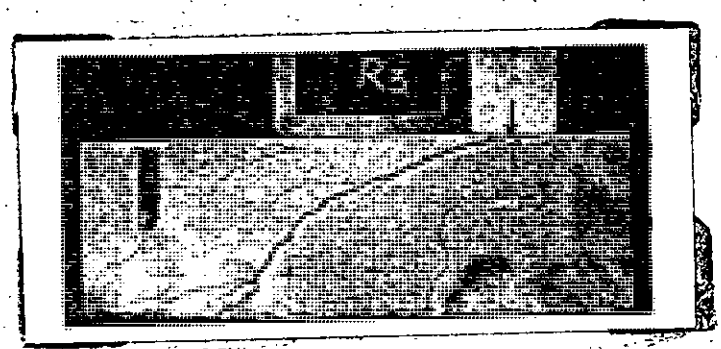
Beam S 1 II (ϕ = 0.156)

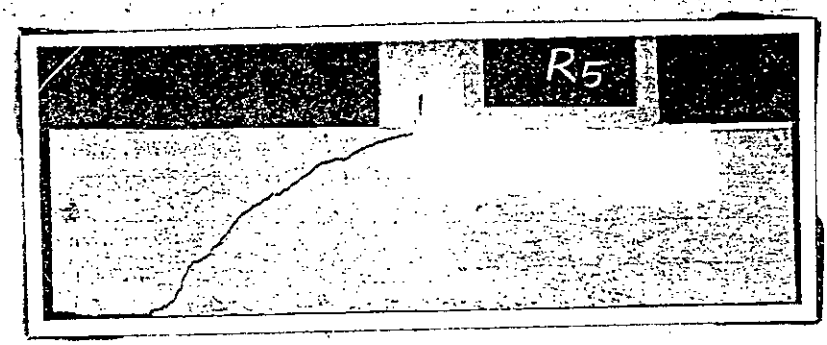
Beam R 3 II (ϕ = 0.162)

Beam R 5 II (ϕ = 0.162)

Beam T 1 (ϕ = 0.370)


S1


R4


R5

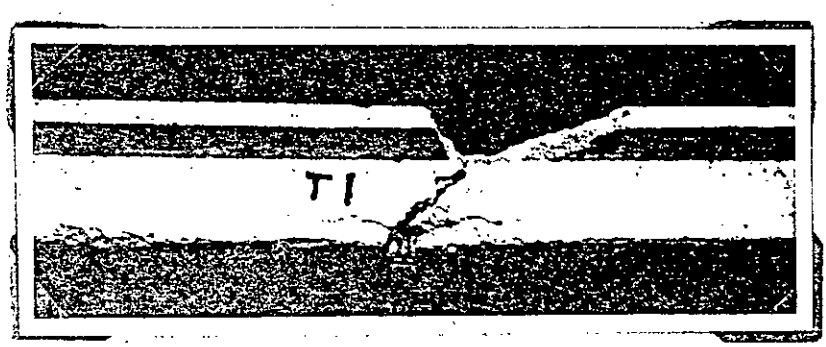

T1

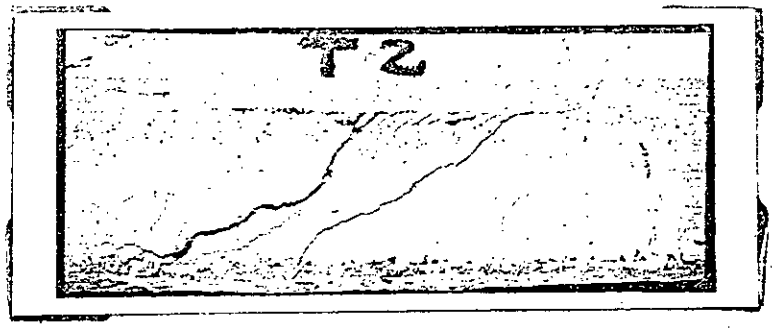
Fig. I. 29 : Diagonal Compression Failure

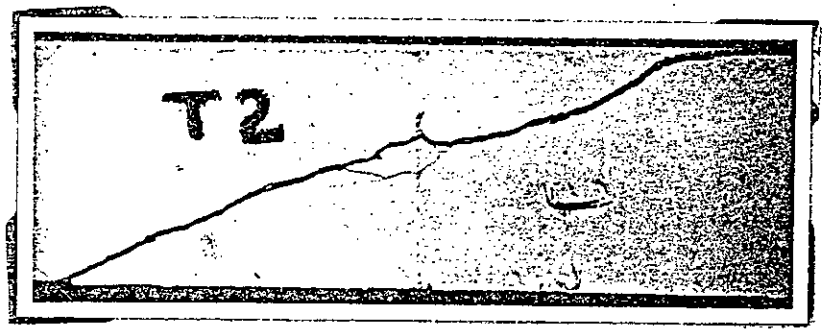
Beam T 2 (ϕ = 0.216)

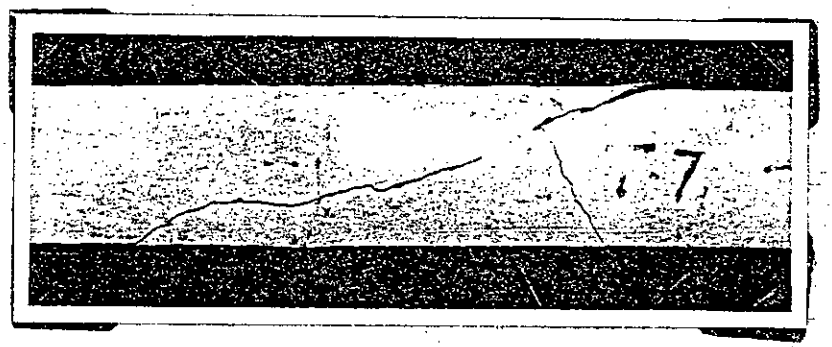
Beam T 2 (ϕ = 0.216) - Top view of beam

Beam T 7 II (ϕ = 157) - Top view of beam

Beam T 7II(ϕ = 0.157)


T2


T2


T7

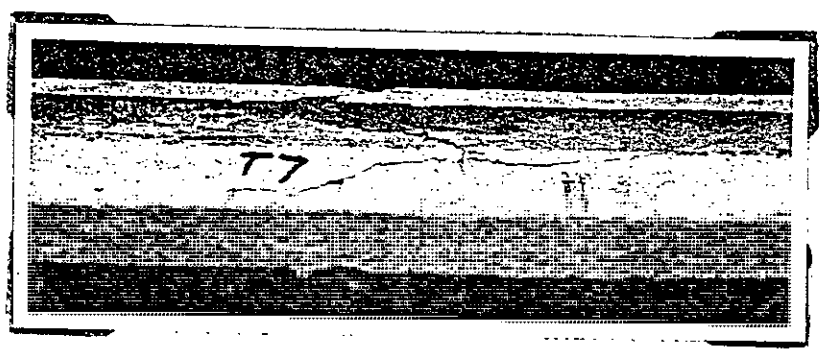

T7

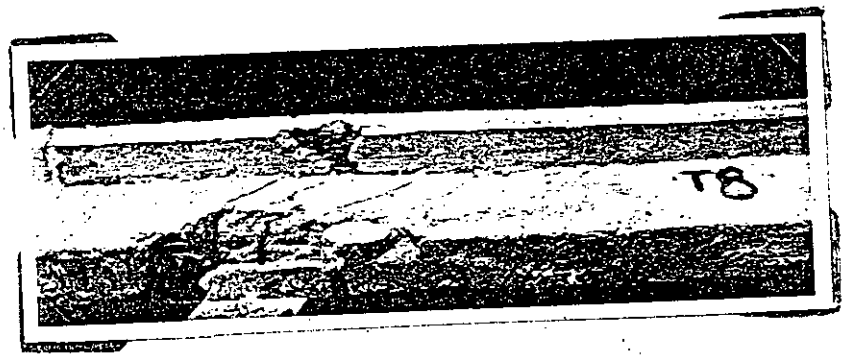
Fig. I.30 : Diagonal Compression Failure

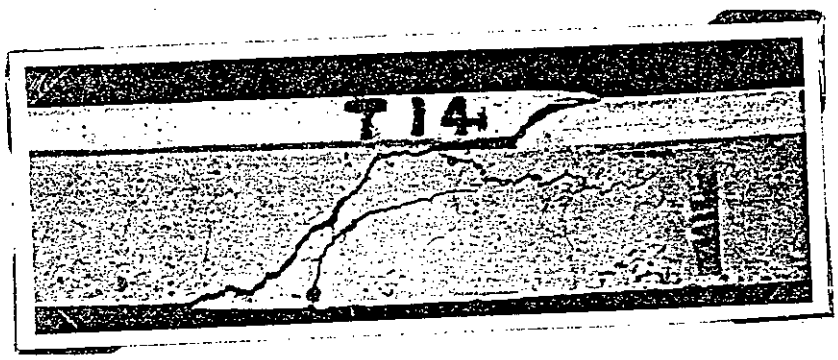
Beam T 8 (ϕ = 0.30C)

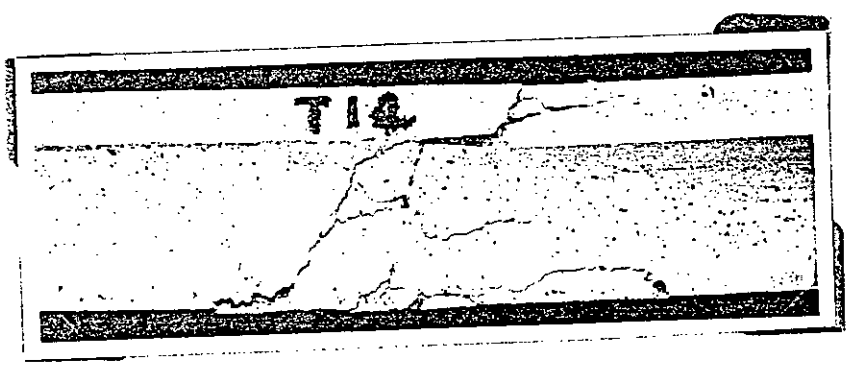
Beam T 14 (ϕ = 0.208)

Beam T 14 (ϕ = 0.208)

Beam T 14 (ϕ = 0.208) - Top view of beam


T8


T14


T14

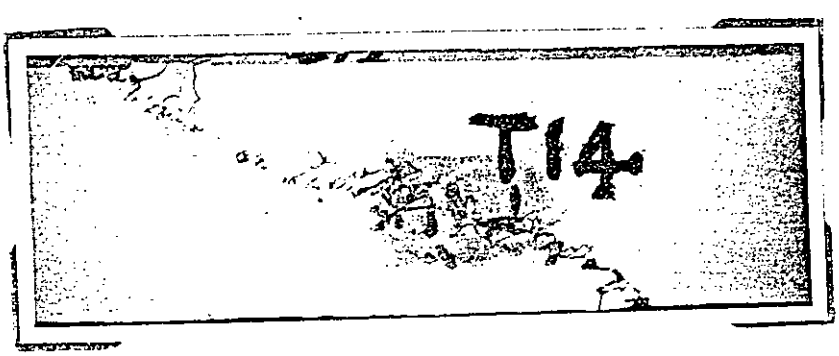

T14

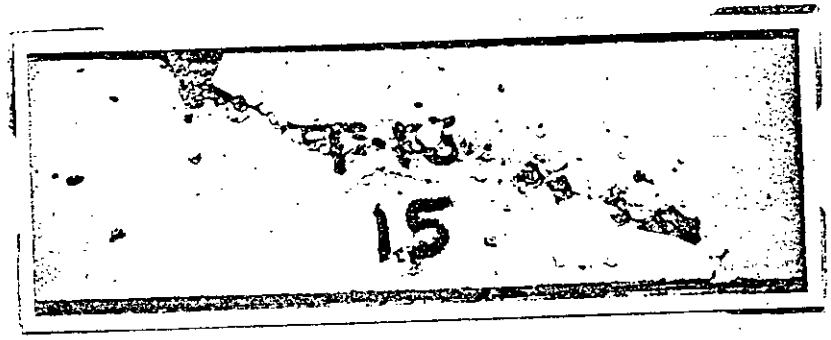
Fig. I.31 : Diagonal Compression Failure

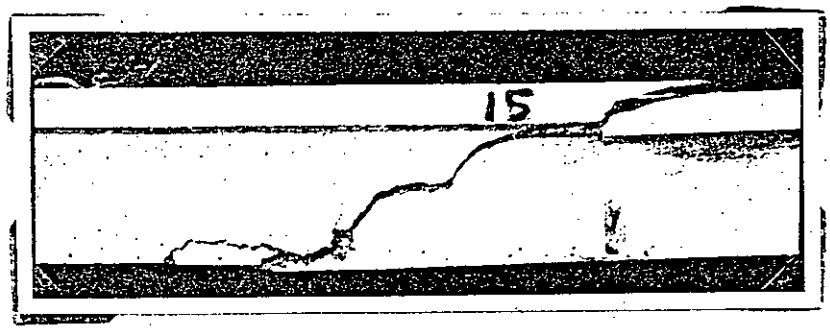
Beam T 15 (ϕ = 0.219) - Top view of beam

Beam T 15 (ϕ = 0.219)

Beam T 15 (ϕ = 0.219)

View showing peeling of projecting portions of flanges in a Tee Beam.


15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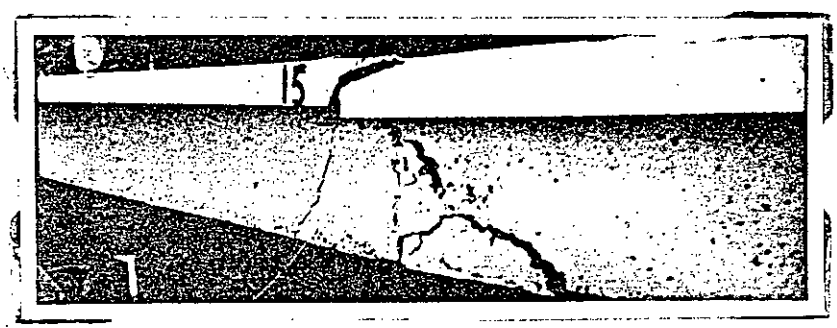
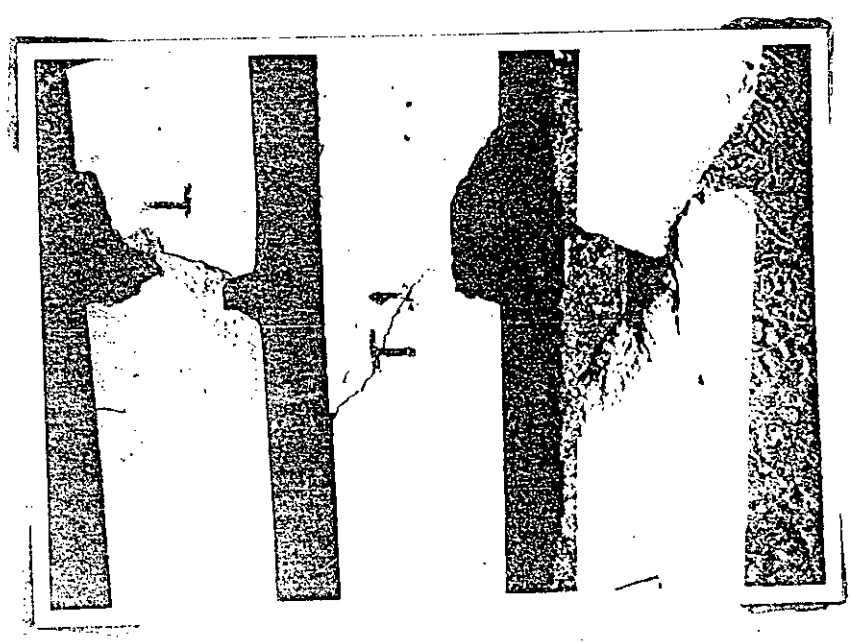

15



cracks were also formed. The flexural cracks deviated from the vertical as they extended upwards showing evidence of combined effect of bending and torsional stresses. Finally they joined with the diagonal cracks at mid depth of the beam. For further increments in moments, this inclined crack penetrated into the compression zone of the beam deviating more and more from the vertical. Finally they became very flat almost parallel to the longitudinal axis of the beam when they reached the top surface of the beam. When the moments were further increased, the flexural crack at the bottom of the beam slightly got widened. The inclined crack that has reached the top surface of the beam travelled further and joined with the similar crack on the other face of the beam. Finally the beam collapsed by the opening of the compression zone concrete with signs of crushing of concrete. This type of failure is not sudden as the previous one; but it is somewhat gradual.

Beams V2, R2, R6, L4-1, T8, T14 and T15 are typical examples of this type of failure. Figs. I28 to 31 illustrate this type of failure. In most of the cases the maximum compressive strains measured at failure were more or less equal to the strains that are normally associated with concrete crushing. The maximum compressive strain in concrete

of 180 x 10^{-5} inch/inch measured in the Beam V3 (ϕ = 0.179) is a typical example.

The beams gave greater warning of impending failure. The failure cracks were sometimes accompanied by some secondary cracks at the level of compression and tension reinforcements. From the behaviour of the beam, this mode of failure in some respects, can be thought of as analogous to the 'shear compression' failure of a beam subjected to combined bending and shear. The beams failed according to this mode resisted torques that were greater than the torque capacity of such beams under pure torsion. In the case of tee beams failing as per this mode, the projecting portions of the flanges had a tendency to separate out from the rest of the cross section. Such types of failures are shown in Fig. I.31.

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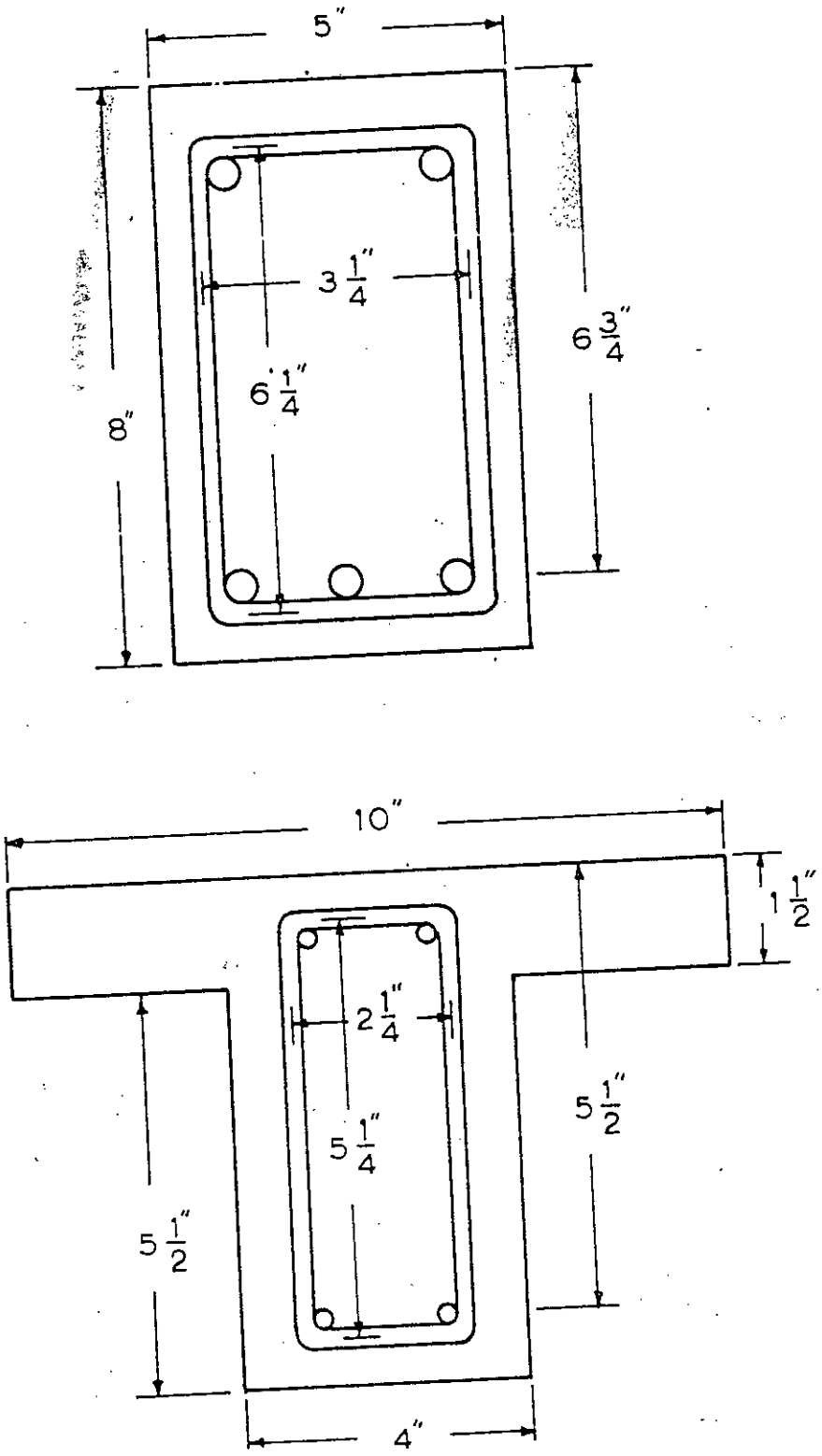

FIG. I. 32. CROSS SECTIONS OF BEAMS.

CHAPTER I.7

DISCUSSION OF EXPERIMENTAL RESULTS

I.7.1 The experimental work has been programmed to clarify certain aspects so as to get a better understanding of the problem. In this chapter, the experimental results are carefully presented and analyzed. Further, the influence of various parameters on the strength of beams under combined bending and torsion is studied. These detailed discussion of experimental results attempts to answer certain points raised at the end of Chapter 2.

I.7.2 Presentation of Experimental Results

The experimental programme included testing of seventy-five beams in total. The results are presented in Table I.7.1. The cross-sections of the beams have already been given in Chapter I.3 (See Fig. I.5). The dimensions of transverse hoops were same in all the beams and they are given in Fig. I.32. The beams of Series V, S, R and L had an effective span length of 75 ins and the other beams were tested with an effective span length of 100 ins. In all the

TABLE I.7.1

SUMMARY OF TEST RESULTS

Specimen Mark	Cube strength	Cylinder split strength	f'_c	Diagonal cracking torque	At Failure			Mode* of Failur
					(M_t/M_b)	M_b	M_t	
	psi	psi	psi	in-kips		in-kips	in-kips	
1	2	3	4	5	6	7	8	9
RECTANGULAR BEAMS								
V1	6650	476	5650	31	0.745	55	41	PT
V2	5750	400	4900	-	0.298	121	36	DC
V3	4920	338	4200	-	0.179	140	25	DC
V4 I	5650	390	4800	30	0	173	0	PF
II	5650	390	4800	-	∞	0	33	PT
V5	5400	380	4600	-	0.275	142	39	DC
V6	5040	365	4300	27	0.655	52	34	PT
S1 I	4900	340	4200	-	0	154	0	PF
II	4900	340	4200	-	0.156	154	24	DC
S2 I	5400	385	4600	-	0.104	154	16	PF
II	5400	385	4600	-	0.238	130	31	DC
S3	4800	340	4100	24	0.935	32	29	PT
S4 I	5600	395	4800	-	0.081	148	12	PF
II	5600	395	4800	-	0.235	132	31	DC
S5 I	5250	365	4500	-	0.135	148	20	PF
II	5250	365	4500	-	0.212	132	28	DC

(contd.)

Table I.7.1 (contd.)

1	2	3	4	5	6	7	8	9
S6	5350	380	4600	26	∞	0	30	PT
R1	3550	250	3000	19	0.715	35	25	PT
R2	6000	425	5100	-	0.252	111	28	DC
R3 I	5600	400	4800	-	0	185	0	PF
II	5600	400	4800	-	0.162	185	30	DC
R4	5160	350	4400	26	∞	0	30	PT
R5 I	6350	456	5400	-	0.045	200	9	PF
II	6350	456	5400	-	0.162	185	30	DC
R6	5160	355	4400	24	0.515	64	33	DC
L1-1	4100	300	3500	24	∞	0	29	PT
L1-2I	5160	380	4400	-	0	169	0	PF
II	5160	380	4400	-	0.535	60	32	DC
L2-1I	5725	400	4900	-	0.055	181	10	PF
II	5725	400	4900	-	0.458	72	33	DC
L2-2I	5725	400	4900	-	0.055	181	10	PF
II	5725	400	4900	-	0.458	72	33	DC
L3-1I	4230	350	3600	-	0.123	163	20	PF
II	4230	350	3600	-	0.286	105	30	DC
L3-2I	4230	350	3600	-	0.123	163	20	PF
II	4230	350	3600	-	0.294	102	30	DC

(contd.)

Table I.7.1 (contd.)

1	2	3	4	5	6	7	8	9
L4-1	4660	376	4000	-	0.353	85	30	DC
L4-2	3800	324	3200	22	0.795	34	27	PT
L5-1	3600	310	3100	20	0.820	33	27	PT
L5-2	3650	320	3200	20	0.500	58	29	DC
L6-1I	3600	250	3100	-	0.123	163	20	PT
II	3600	250	3100	-	∞	0	25	PT
L6-2I	3600	250	3100	-	0.141	156	22	PT
II	3600	250	3100	-	∞	0	25	PT
A1	3980	343	3400	23	∞	0	23	PT
A2	3785	336	3300	22	∞	0	22	PT
A3	4460	350	3600	22	∞	0	22	PT
A4	4940	363	4200	23	∞	0	23	PT
A5	4210	357	3600	26	∞	0	26	PT
A6	3360	331	3000	22	∞	0	22	PT
A7	4200	355	3600	23	∞	0	23	PT
A8	4965	380	4000	24	∞	0	24	PT
A9	4150	366	3600	24	∞	0	24	PT
A10	4310	379	3800	23	∞	0	23	PT
A11	4200	377	3700	26	∞	0	26	PT

(contd.)

Table I.7.1 (contd.)

1	2	3	4	5	6	7	8	9
A12	4470	362	3700	26	∞	0	26	PT
A13	3860	346	3400	22	∞	0	22	PT
A14	4460	383	3800	26	∞	0	26	PT
A15	4135	381	3600	26	∞	0	26	PT
B1	3700	250	3100	15	∞	0	23	PT
B2 I	3700	250	3100	-	0	134	0	PP
II	3700	250	3100	15	∞	0	22	PT
B3	3700	250	3100	15	∞	0	25	PT
B4 I	3700	250	3100	-	0	134	0	PP
II	3700	250	3100	-	∞	0	23	PT
B5	3850	270	3300	16	∞	0	24	PT
B6 I	3850	270	3300	-	0	138	0	PP
II	3850	270	3300	15	∞	0	23	PT
B7 I	2680	200	2300	-	0	126	0	PP
II	2680	200	2300	12	∞	0	18	PT
B8	2680	200	2300	12	∞	0	18	PT
B9 I	3650	260	3100	-	0	138	0	PP
II	3650	260	3100	15	∞	0	23	PT
B10	3650	260	3100	15	∞	0	23	PT

(contd.)

Table I.7.1 (contd)

1	2	3	4	5	6	7	8	9
				TEE BEAMS				
T1	3420	330	3200	-	0.370	57	21	DC
T2	3450	350	3300	-	0.216	98	21	DC
T3	3500	330	3300	-	0	163	0	PF
T4	3860	350	3600	16	∞	0	16	PT
T5 I	4800	450	4400	-	0.098	163	16	PF
II	4800	450	4400	-	0.127	157	20	DC
T6	4200	410	4000	16	∞	0	17	PT
T7 I	5160	480	4900	-	0	158	0	PF
II	5160	480	4900	-	0.157	140	22	DC
T8	3500	340	3300	16	0.300	60	18	DC
T9	3900	380	3700	-	0.103	175	18	PF
T10	2210	210	2100	-	0.083	158	13	PF
T11 I	3500	320	3300	13	∞	0	17	PT
II	3500	320	3300	-	0	114	0	PF
T12 I	3500	330	3300	-	0.105	114	12	PF
II	3500	330	3300	-	0.143	105	15	DC
T13	3700	350	3500	13	1.000	20	20	PT
T14	5800	500	5300	-	0.208	96	20	DC

(contd.)

Table I.7.1 (contd.)

1	2	3	4	5	6	7	8	9
T15	4200	350	3800	-	0.219	105	23	DC
T16	4650	370	4000	-	0	175	0	PF
T17 I	3750	310	3200	-	0.097	175	17	PF
II	3750	310	3200	-	0.114	158	18	DC
T18	5200	470	4600	-	0.146	158	23	PF
T19	5300	475	4700	18	∞	0	23	PT
T20 I	5200	465	4600	-	0.114	175	20	PF
II	5200	465	4600	-	∞	0	23	PT

PF - Premature Flexure Failure

PT - Premature Torsion Failure

DC - Diagonal Compression Failure

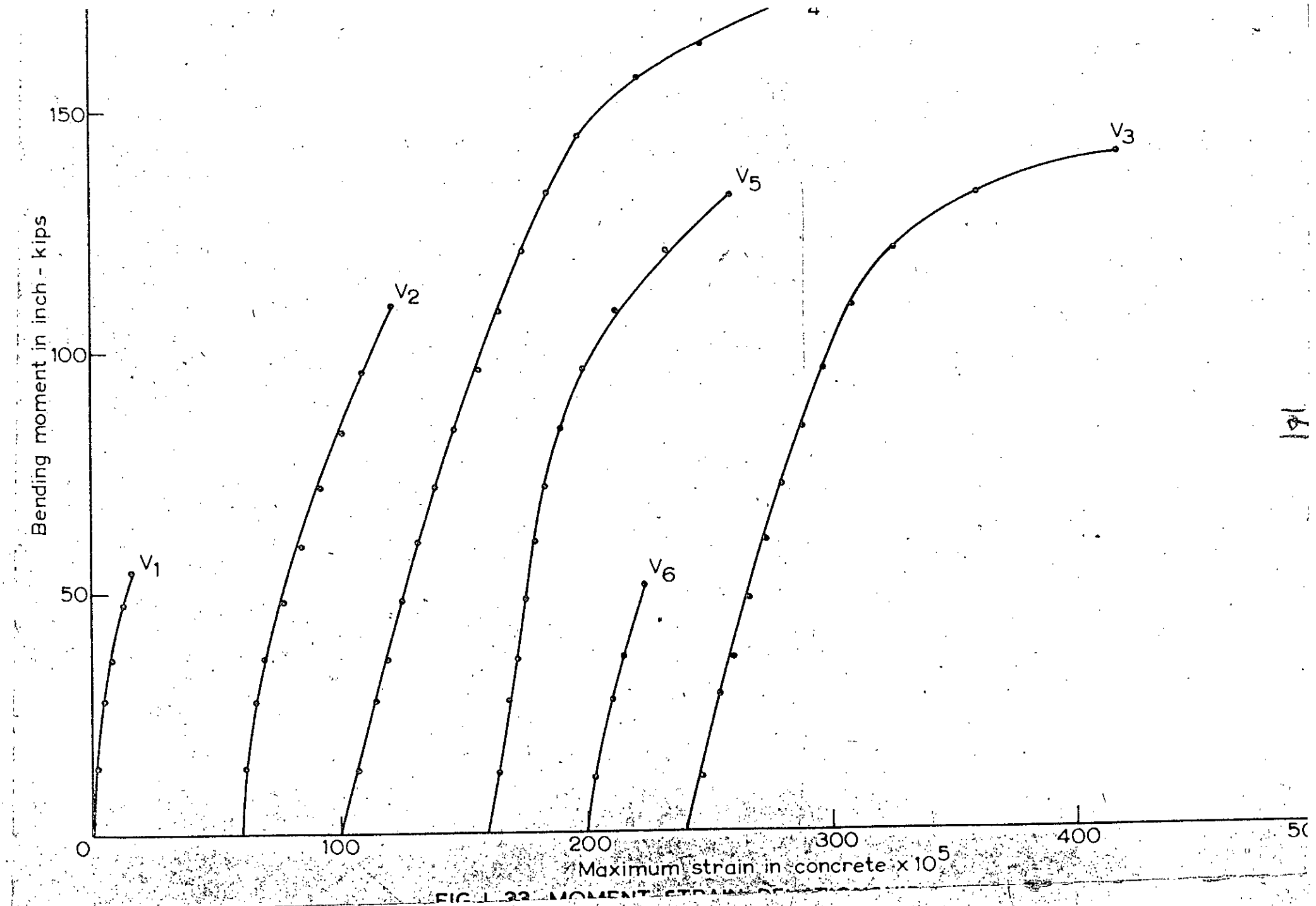

FIG. 33 MOMENT-STRAIN CURVES

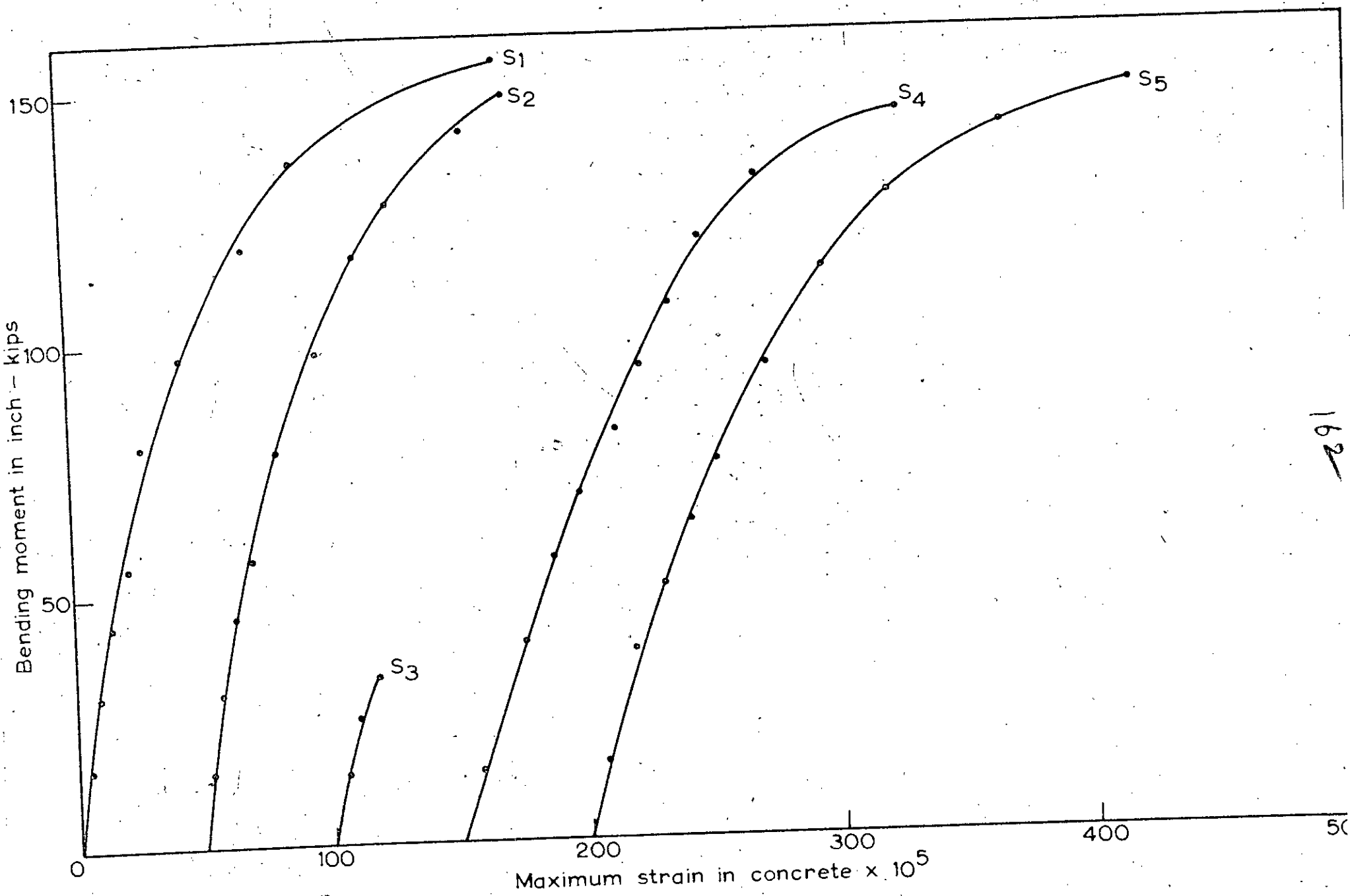

FIG. 1.34. MOMENT STRAIN RELATIONSHIP-S SERIES.

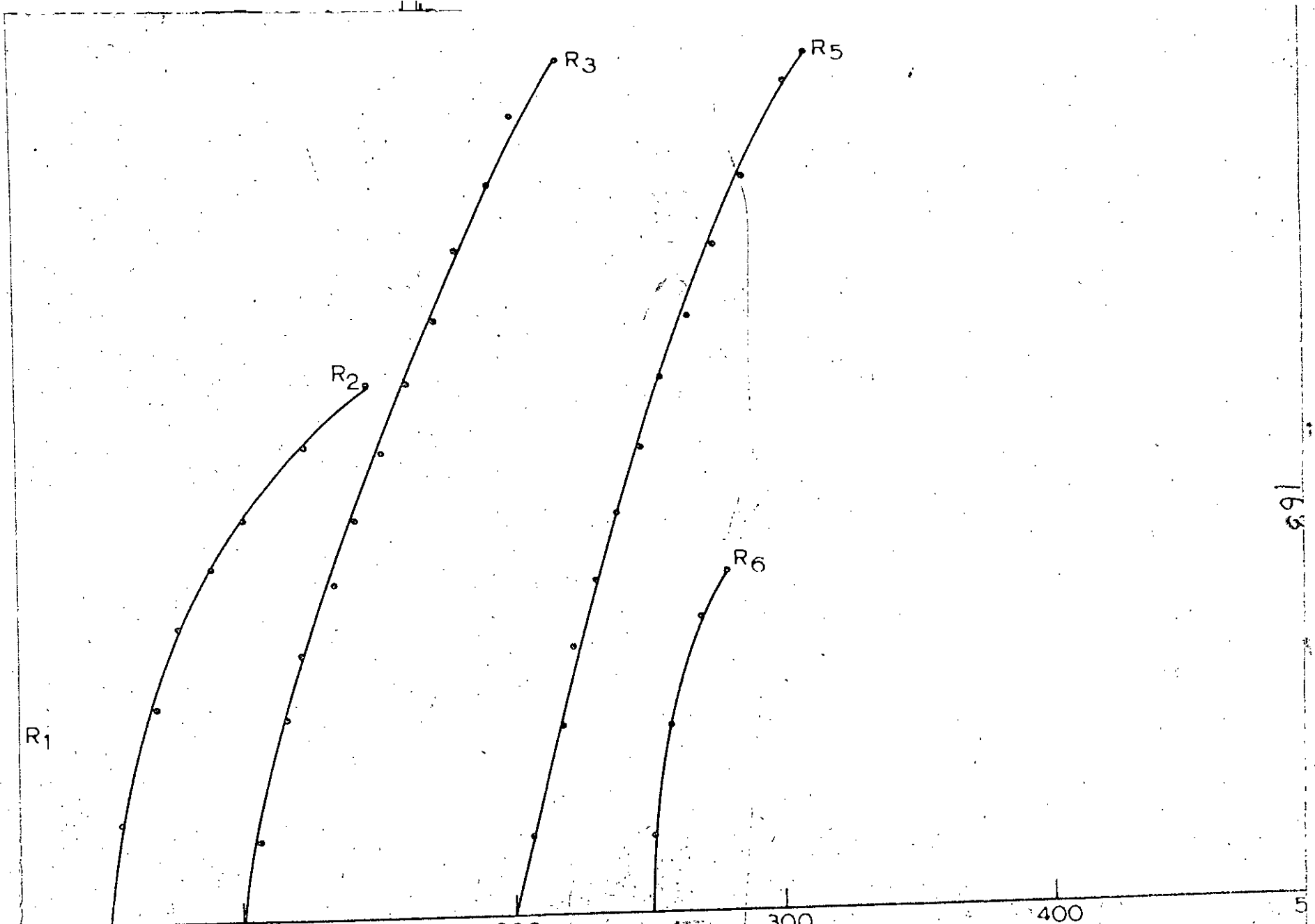

FIG. I. 35 MOMENT STRAIN RELATIONSHIP - R SERIES

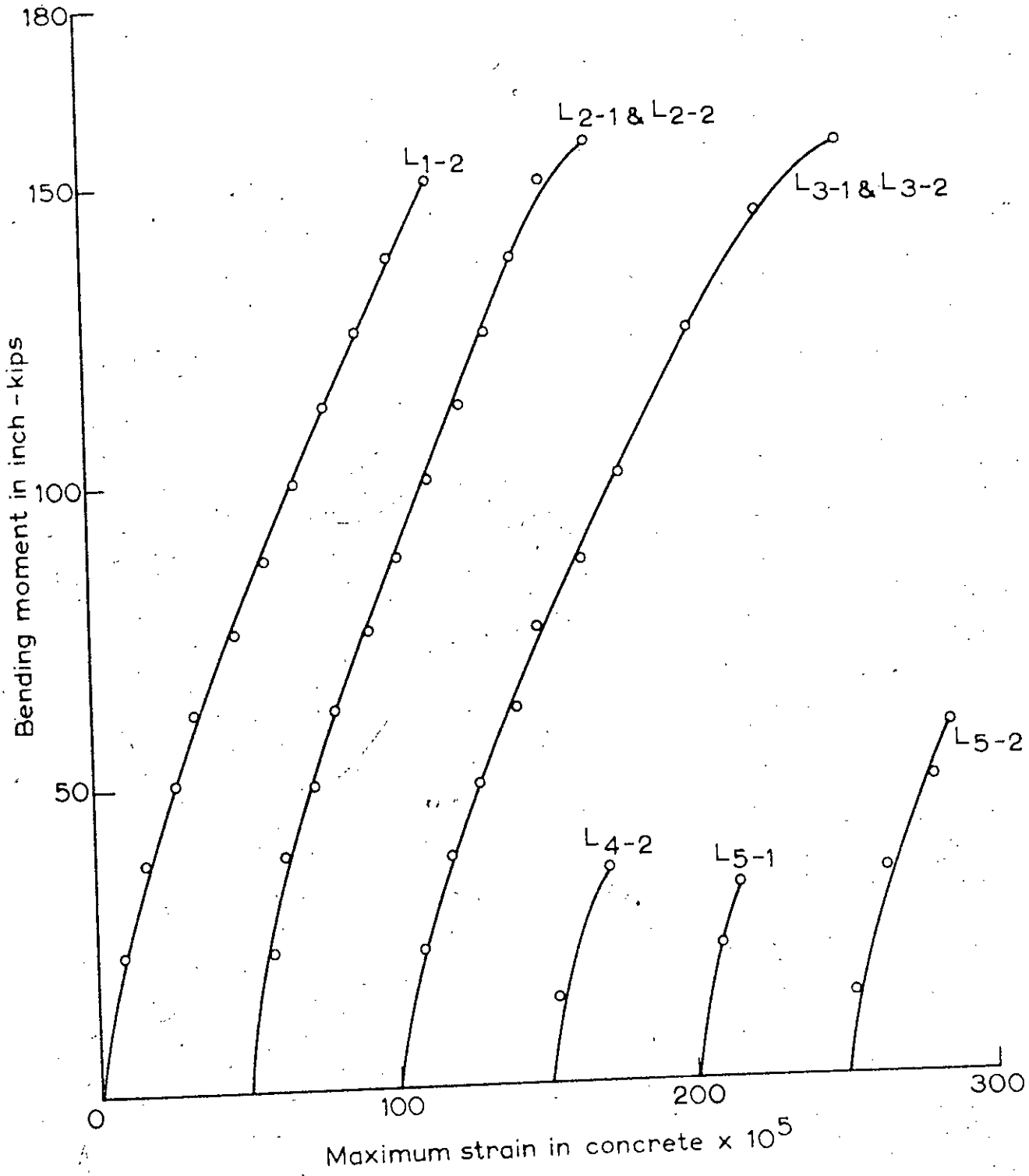

FIG. I. 36. MOMENT STRAIN RELATIONSHIP—L SERIES.

beams tested under combined bending and torsion, two equal vertical loads placed symmetrically at one-third spans applied the bending moment and the beam length in between the vertical loads was the test-length. On some beams, which failed prematurally in flexure, a retest was conducted so as to yield another result with a different value of Torque/Moment ratio, $\emptyset$. Such results are marked as " I " and " II " in Table I.7.1. This method of testing has yielded certain additional results and hence the total number of results obtained from the experimental programme is one hundred. In all the beams tested under combined loading, both the moments were increased simultaneously so that the final collapse of the beams is due to the combined action of both the moments.

I.7.3 Discussion of Strain Data

In order to understand the behaviour of beams, the author has measured both surface strains in concrete as well as strains in reinforcing rods. The qualitative picture obtained from strain measurement is briefly discussed in this article:

Surface Strains in Concrete

In Figs. I 33 to I 36, the variations of maximum compressive strain in concrete with bending moment are shown

for the beams of Series V, S, R and L. From the general trend of these curves the following conclusions can be drawn:

i. For beams failing prematurally in torsion the maximum compressive strain is very low (e.g. in beam V_6, strain = 30×10^{-5}).

ii. For beams failing in diagonal compression, the maximum strain noted is approximately equal to that of a beam failing in pure flexure (e.g. in beam V_3 strain = 180×10^{-5} which is equal to the ultimate strain observed in the case of beam V_4 tested in pure flexure). This indicates that this type of failures are compression failures in contrast to the premature torsion failures.

iii. For beams failing prematurally in flexure, the moment-strain diagrams are very similar to those of beams tested in pure bending. The moment-strain curves of beams S1 and S5 are typical example of this case. This shows that the behaviour of beams failing prematurally in flexure is same as that of beams tested in pure flexure. The torsional stresses influence the behaviour of beams only to a small extent.

iv. The moment-strain diagrams for the beams failing prematurally in flexure or in diagonal compression indicate

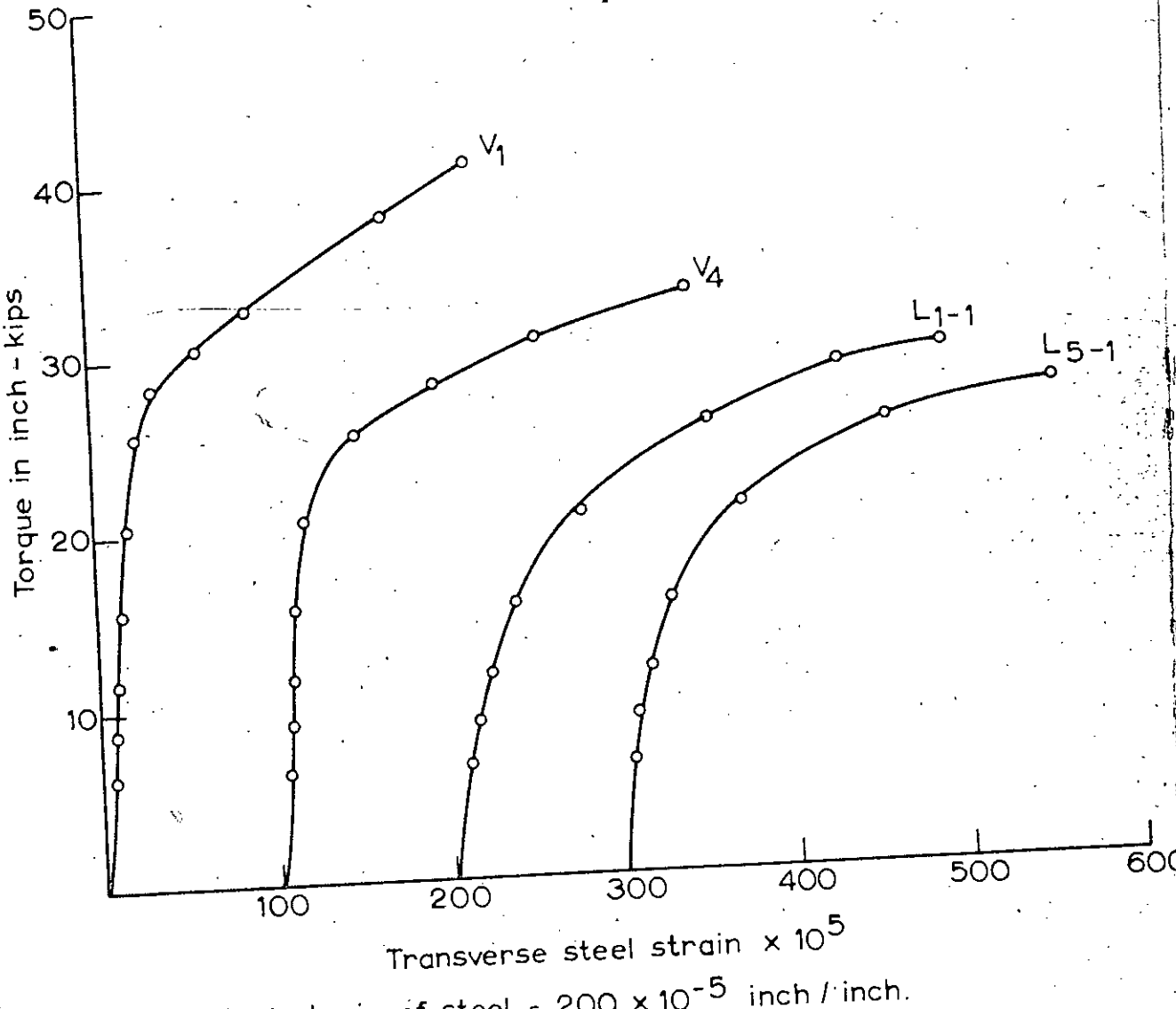

FIG. I. 37. TORQUE Vs TRANSVERSE STEEL STRAIN RELATIONSHIP.

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that the compressive strains increase approximately linearly upto 80 to 85% of the collapse moment. After this stage the curves take a sharp change in slope and the strains increase more rapidly with the moment. This type of behaviour suggests the expected non-linear behaviour of the beam at or near failure.

Strains in Transverse Steel

The variation of strains in the transverse steel with the torque is shown in Fig. I.37 for the beams V1, V4, L1-1 and L5-1. All these beams failed prematurally in torsion. From Fig. I.37 it can be seen that just before the formation of a diagonal crack, the strains in the transverse steel were very low. Immediately after the formation of diagonal crack, the strains increased rapidly and attained yield strain of steel at the final collapse of the beam. This indicates that for the beams failing prematurally in torsion the transverse steel yields at failure of the beam. The beams V1 and V4 had higher percentage of transverse steel than the other two beams. This is reflected in the types of torque-strain curves as can be seen from Fig. I.37. Also it has been found from the experiments that those stirrups which were not traversed by the failure crack have not registered any appreciable strain.

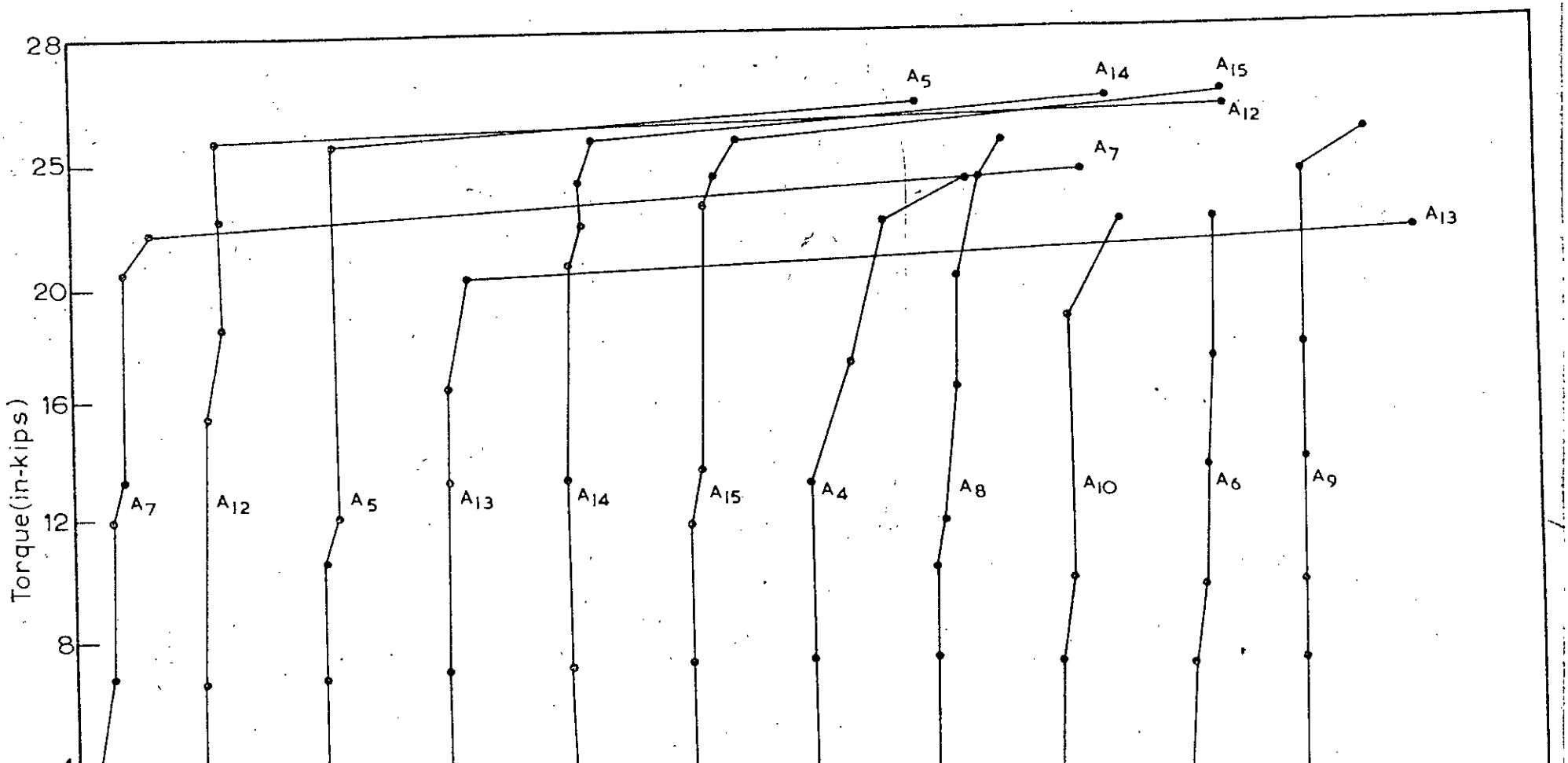

FIG.1.38. TORQUE Vs STRAIN IN LONGITUDINAL STEEL RELATIONSHIP

Strains in Longitudinal Steel

The variation of strains in the longitudinal steel in the case of beams under pure flexure is well known. In the present investigation strains in the longitudinal reinforcements were measured in the case of beams under pure torsion so as to get a qualitative picture about the influence of longitudinal steel on the torque capacity of beems. The strain measurements were made in the case of beams of Series A. Such observations are summarised in Fig. I.38.

In the initial stages of loading and prior to failure, the strains in the reinforcement were of small magnitude. But at the collapse of the beam, there was a sharp increase in the strains and they were of the order of about 80 x 10^{-5} inch/inch as can be seen from Fig. I.38. In all the cases the maximum strains were far less then the yield strain of steel and in some cases when the strain gauges were away from the failure section there was no appreciable strain in the steel. The sudden increase in the steel strain can possibly be explained as follows: Since the concrete covering the steel fails suddenly, the strain in the concrete at failure is being transferred to the adjoining steel thus resulting in a sudden increase in the magnitude of steel strain. This observation also indicates that the failure of beams under

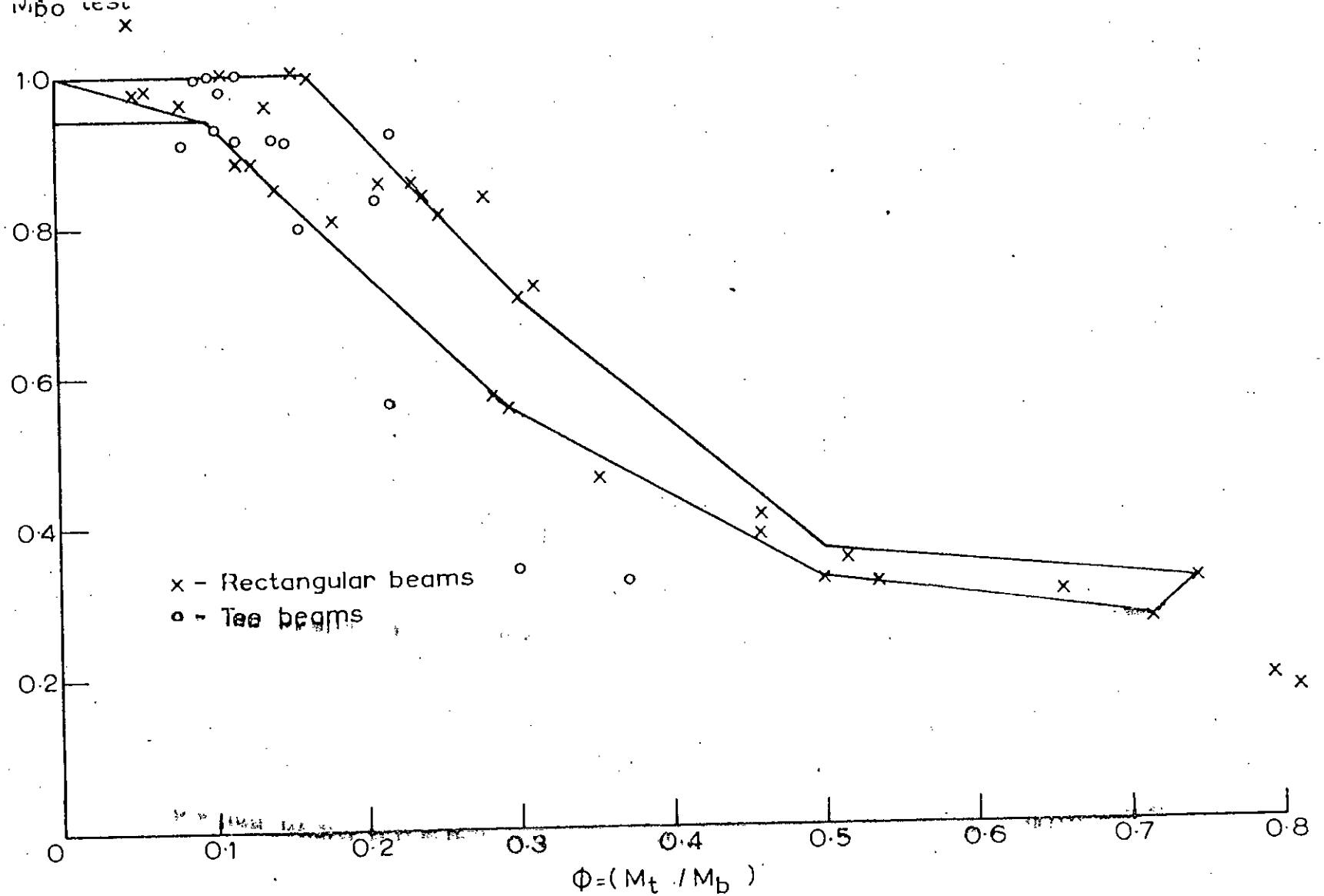

FIG. I. 39. VARIATION OF $\left(\frac{M_b}{M_{bo}}\right)_{test}$ WITH ϕ.

pure torsion is always supplemented by the failure of the concrete covering the longitudinal steel.

I.7.4 Interaction of Moments

An important aspect of the present investigation is to know the interaction of moments for all the ranges of Moment/Torque ratio. For this purpose the author has conducted tests on beams with different values of Moment/Torque ratio, ϕ varying from pure bending to pure torsion.

i. Influence of ϕ on Flexural Capacity of Beams

In general the flexural capacity of a beam subjected to combined bending and torsion is reduced by the presence of twisting moment. But the percentage reduction in the strength varies with the value of ϕ. In order to get a physical picture of this variation in strength, the test results of this investigation have been summarised in Fig.I.39. From Fig. I.39 it can be seen that:

a. for $\phi \leq 0.10$, the reduction in the flexural capacity of beams is very small varying between 0 and 4%;

b. for $0.10 \leq \phi \leq 0.30$, the flexural capacity reduces at a rapid rate and at $\phi = 0.30$ the reduction in the capacity varies from 30% to 45% ;

c. for $0.30 \leq \phi \leq 0.50$, the reduction in the capacity is at a lesser rate and beyond $\phi = 0.50$ it is approximately constant.

The reason for the reduction in the flexural capacity of beams can be explained as follows: By the interaction of internal stresses in a beam subjected to combined bending and torsion, compressive-tensile principal stresses are produced in the compression zone of the beam. The principal compressive stress, thus produced, is greater in magnitude than the normal stress caused by the applied bending moment alone. Further the presence of tensile principal stress will reduce the strength of concrete under uniaxial compression. Due to these reasons failure is brought out earlier thus resulting in the reduction in the flexural capacity of beams.

The increase in the magnitude of principal compressive stress depends on the magnitude of shear stress produced by the applied twisting moment. As the intensity of shear stress varies with the value of $\emptyset$, the reduction in the flexural capacity of the beam also varies with $\emptyset$. For instance, for $\emptyset \leq 0.10$, the increase in the magnitude of compressive principal stress over that of normal stress caused by the applied bending moment is very small. The compressive principal stress is, therefore, approximately equal to the normal stress caused by the applied bending moment and hence the reduction in the flexural capacity of beams in such cases is negligible.

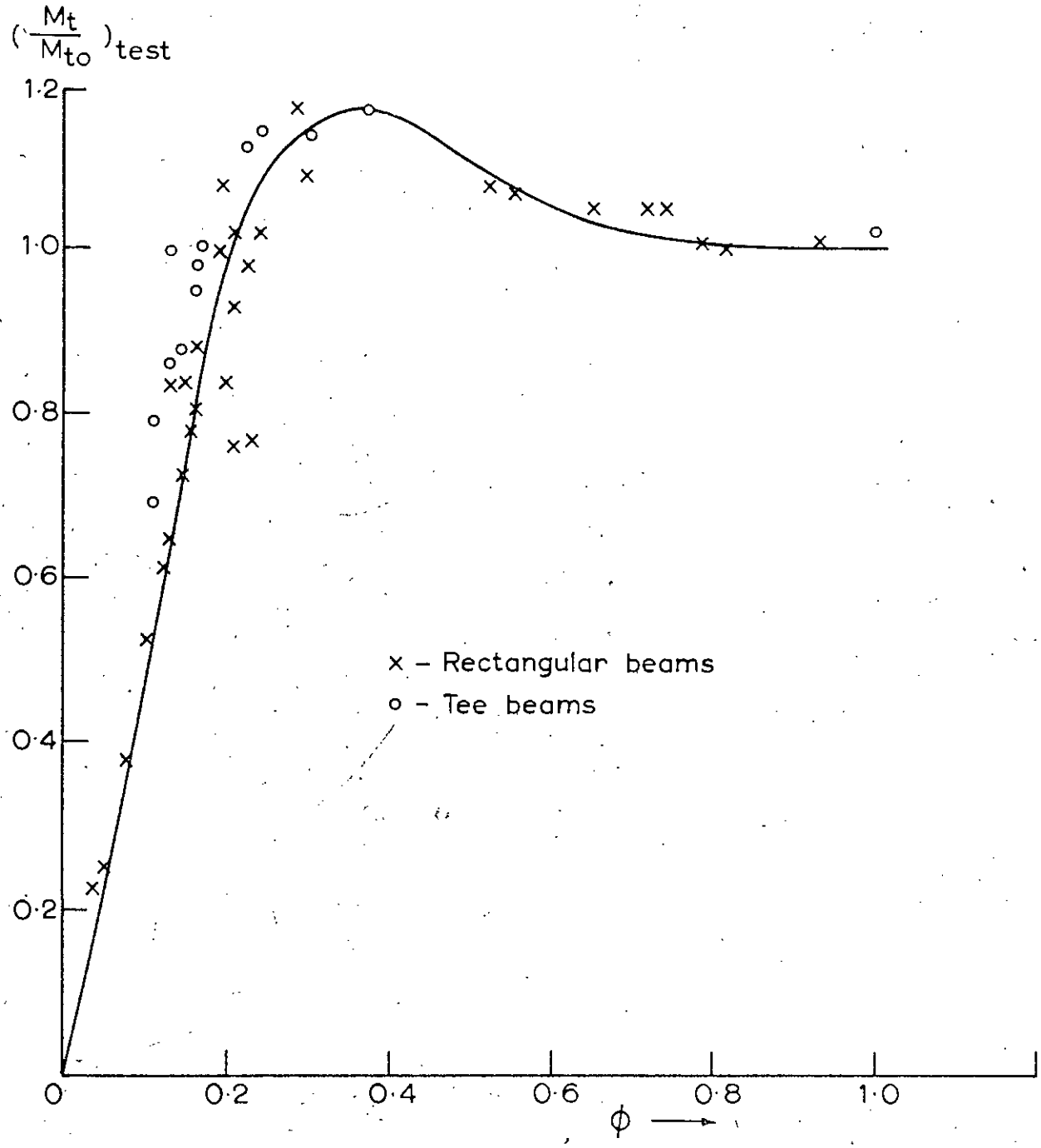

FIG.1.40. INFLUENCE OF ϕ ON TORQUE CAPACITY OF BEAMS.

ii. Influence of ϕ on Torque Capacity of Beams

Fig. I.40 shows the relationship between M_t/M_{to} and ϕ, where M_t is the torque resisted by a beam under combined bending and torsion and M_{to} is the torque capacity of the corresponding beam under pure torsion. Three different stages can be observed in this curve:

a. For $\phi \leq 0.20$, the ratio M_t/M_{to} is less than unity and most of the beams in this range have failed prematurally in flexure.

b. For $0.20 \leq \phi \leq 0.35$, the value of M_t/M_{to} increases with the increase in the value of ϕ and it reaches the maximum value of 1.18 at $\phi = 0.35$. This part of the curve represents the well-known conclusion that the addition of bending moment increases the torque capacity of beams. The behaviour of beams in this stage is due to the inherent property of concrete under combined stresses namely the presence of a normal stress enhances the shear strength of concrete.

c. For $\phi > 0.35$, the ratio (M_t/M_{to}) decreases and reaches unity at $\phi = \infty$ which is the 'pure torsion' case. From Fig. I.40, it can be seen that this part of the curve is some-what flatter beyond $\phi = 0.60$ and the value of M_t/M_{to} is approximately equal to unity at

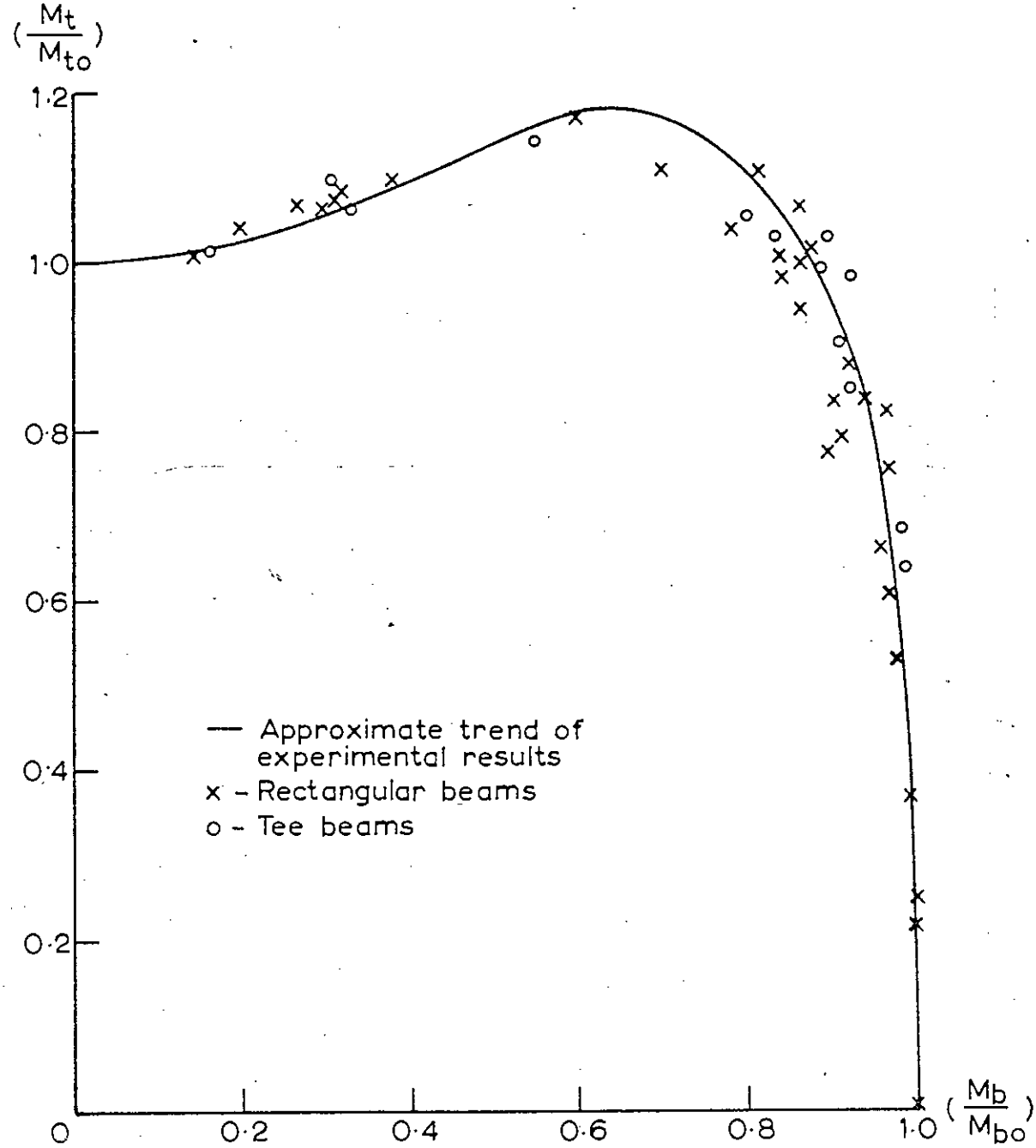

FIG. I. 41. INTERACTION DIAGRAM BASED ON TEST RESULTS.

$\phi = 1.0$. From this it is evident that the presence of small amount of bending moment may not have any significant influence on the torque capacity of beams.

iii. Interaction Diagram

In Fig. I.41, the results of Figs. I.39 and I.40 are combined together to show the interaction between the moments. The points discussed in the preceding articles (i) and (ii) are well brought out in the interaction diagram shown in Fig. I.41.

Condensing the above discussion the following conclusions can be drawn:

a. In general, the flexural capacity of a beam under combined bending and torsion decreases with the increase in the value of ϕ. However, for low values of ϕ (say $\phi \leq 0.10$), the reduction in the flexural capacity is insignificant.

b. The presence of bending moment increases the torque capacity of beams only in certain ranges of ϕ. For higher values of ϕ (say $\phi > 0.75$) the increase in the torque capacity of beams over the torque capacity of corresponding beams under pure torsion is very small and can be ignored for all practical purposes.

I.7.5 Factors influencing the strength of beams under combined bending and torsion

The influence of the following factors on the ultimate strength of beams is discussed below:

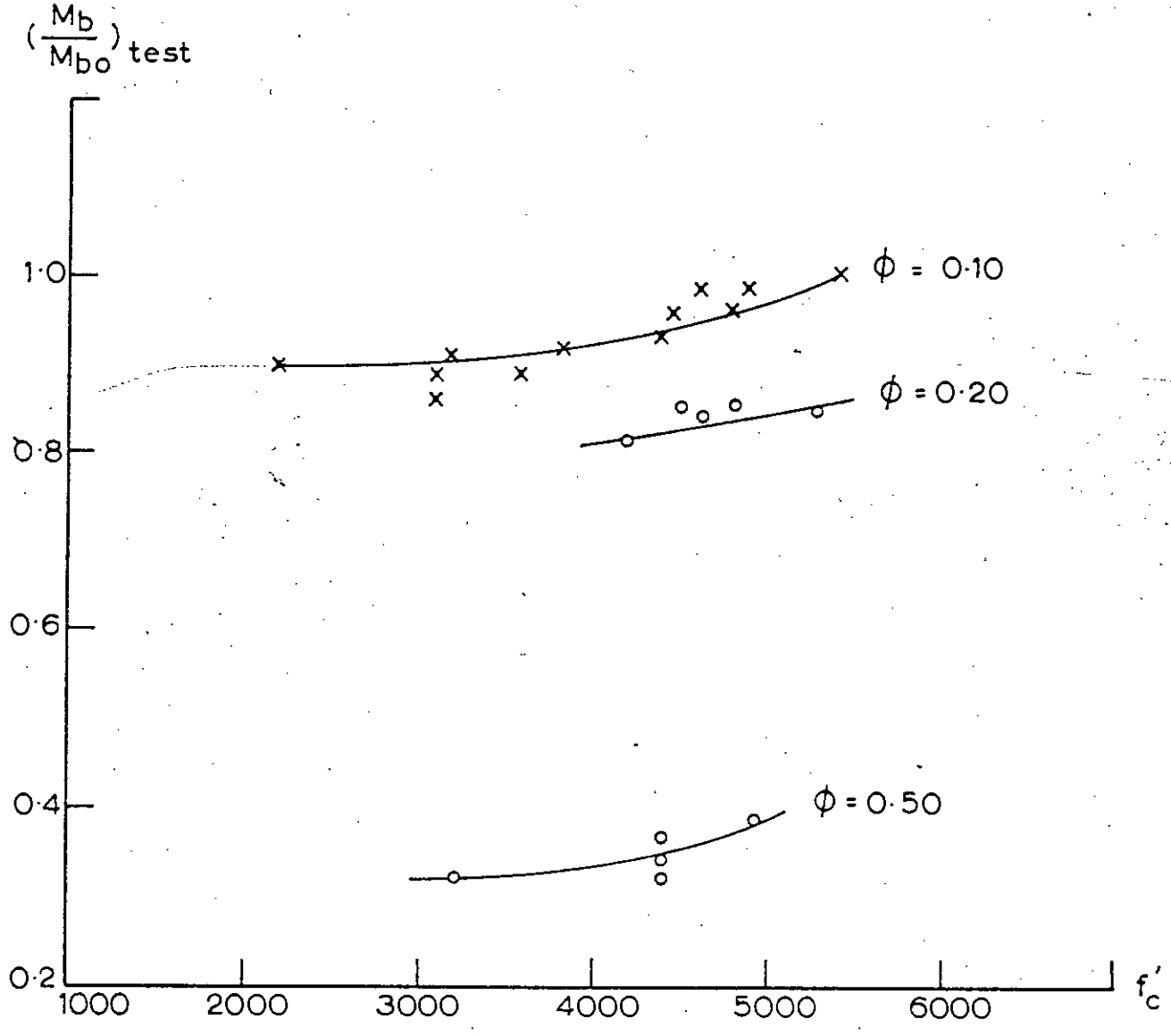

FIG.1.42. INFLUENCE OF STRENGTH OF CONCRETE ON $(\frac{M_b}{M_{bo}})_{test}$

1. Concrete strength
2. Longitudinal reinforcements
3. Transverse reinforcements
4. Ratio of twisting to bending moment, $\emptyset$
5. Presence of flexural cracks
6. Shape of the cross section

I.7.5.1 Concrete Strength

As expected the ultimate strength of beams subjected to combined bending and torsion is influenced by the concrete properties. But the degree of influence depended on the mode of failure and the value of $\emptyset$. In Fig. I.42, the influence of concrete strength on the value M_b/M_{bo} is studied for three values of $\emptyset$. All these beams had failed either in premature flexure or in diagonal compression. The main variables in these beams were concrete strength and the value of $\emptyset$. From Fig. I.42, it can be said that in the case of under-reinforced beams the reduction in the flexural capacity is some-what insignificant with the variation in the concrete strength. Due to the variation in the individual parameters, it has not been possible for the author to give a similar quantitative picture for the case of beams failed prematurally in torsion. But from the analysis of individual results, it has been found that in such cases the concrete strength influences the torque

capacity of beams to a considerable extent. Experiments have also shown that the torque capacity of a beam with transverse reinforcement is interdependent on the torque capacity of the corresponding beam without transverse reinforcement. Increase in the concrete strength increased the torque capacity as well as the diagonal cracking torque of the beams. Hence it is necessary to consider the torque carried by the concrete in the evaluation of torque capacity of beams. Since in the present investigation the value of $\emptyset$ has been selected as an important parameter for study, it has been not possible to get exhaustive results to know completely about the influence of concrete strength on the capacity of beams. Further experimentation on this line is recommended.

I.7.5.2 Longitudinal Reinforcements

Some of the early investigations[3,9,10] in this field of research have indicated that the increase in the torque capacity of beams by increasing the longitudinal steel alone is practically nil. But in a recent investigation Gesund and Boston[52] have considered the contribution from the longitudinal steel to the torque capacity of beams as considerable even in the absence of transverse steel. So it is necessary to know the limitations of these two contradicting conclusions.

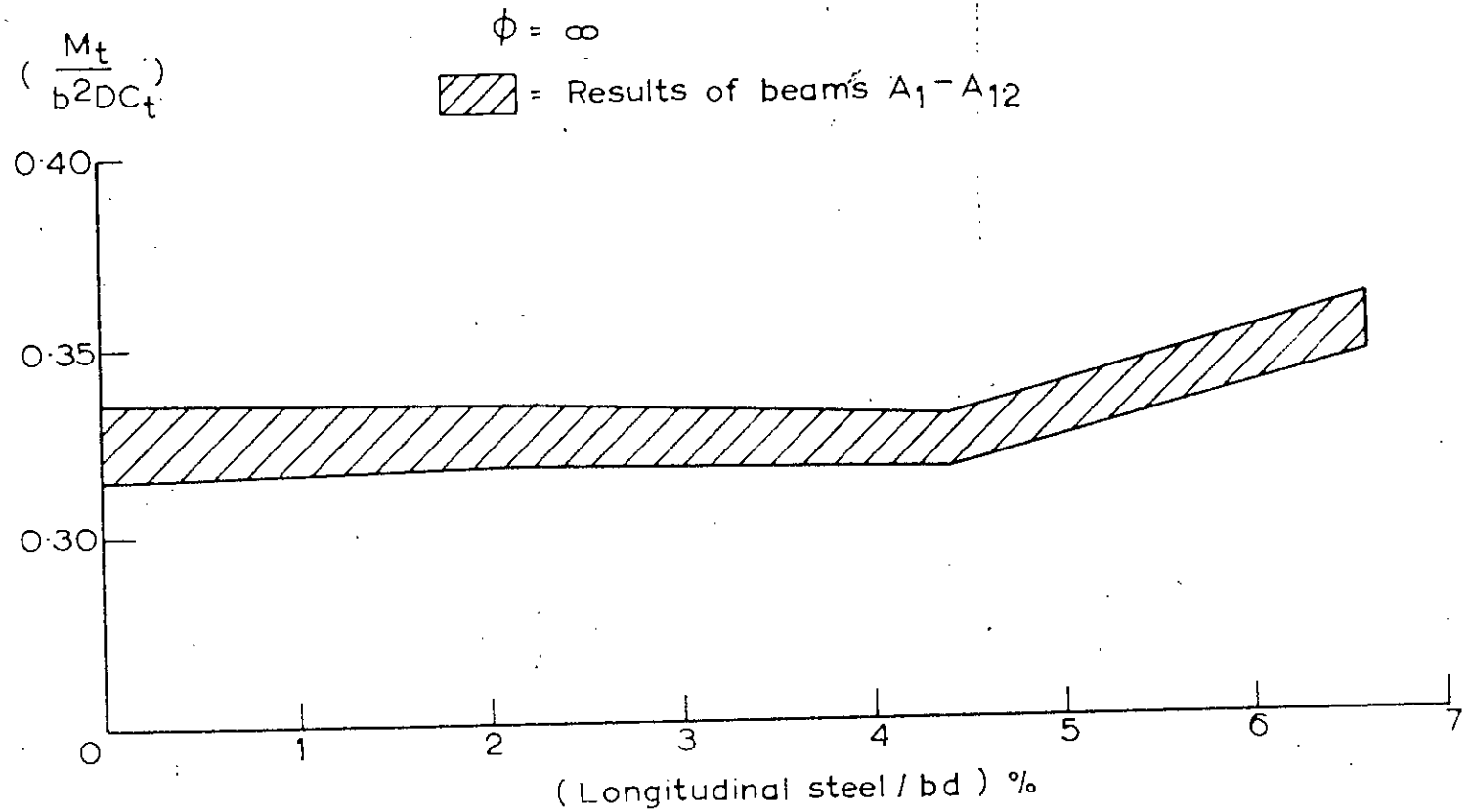

FIG. I. 43. INFLUENCE OF LONGITUDINAL STEEL ON TORQUE CAPACITY OF BEAMS WITHOUT TRANSVERSE STEEL.

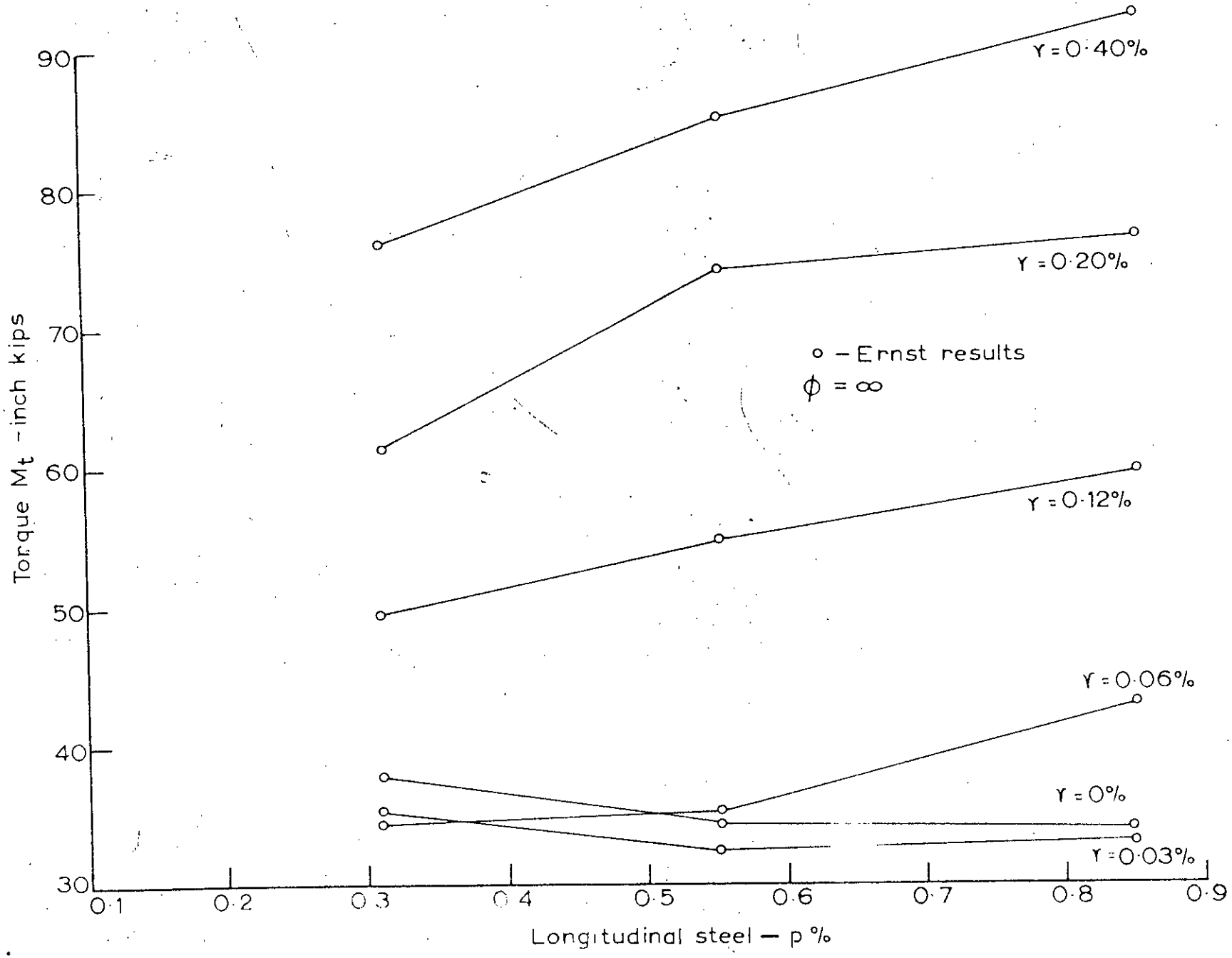

FIG. I. 44. INFLUENCE OF LONGITUDINAL STEEL ON TORQUE CAPACITY OF BEAMS WITH TRANSVERSE STEEL.

Since there is considerable interaction between bending and twisting moments, it is necessary to analyse the test results of beams under pure torsion so as to know the exact influence of longitudinal steel on the torque capacity of beams. The fifteen rectangular beams of Series A of author's investigation have been tested to know the increase in the torque capacity of beams by increasing the longitudinal steel alone. For this purpose the beams were provided with no transverse stee and the amount of longitudinal steel has been varied from 0 to 6.6 % with different positioning of reinforcing rods (See Fig. I.5). The test results of beams A1-12 are summarised in Fig. I.43. From Fig. I.43, it can be seen that the torque capacity increased only by 9% over the corresponding plain concrete beam when the longitudinal steel ratio was increased to 6.6%. The band of variation shown in Fig. I.43 suggests that the increase in the torque capacity, in the absence of transverse steel, by increasing the longitudinal steel alone is insignificant.

The contribution of longitudinal steel in the presence of transverse steel can be known by analysing the test results reported by Ernst[30]. The results are given in Fig. I.44. From Fig. I.44, it can be seen that for very low percentages of longitudinal steel, with moderate amount of transverse steel, the torque capacity increases to a certain extent by increasing the longitudinal steel/alone; but with the

increase in the percentage of longitudinal steel, this increase becomes negligible. For instance, for transverse steel ratio of 0.20% the increase in the torque capacity of the beam between p = 0.31 % and p = 0.55% is about 15% while the increase in the torque capacity is only 4% in between p = 0.55% and p = 0.85%. This experimental observation suggests that in the case of beams with vertical stirrups, a minimum amount of longitudinal steel may be required to resist the horizontal component of the diagonal tension as the vertical component is resisted by the stirrups themselves. The increase in the longitudinal steel alone beyond this minimum requirement may not have any significant contribution to the strength of the beam. Based on Ernst's experimental results, this minimum requirement of longitudinal steel can be fixed at about 0.55%.

As expected, the longitudinal steel increased the flexural capacity of the beam under combined bending and torsion in the same manner as in the case of a beam under pure bending.

I.7.5.3 Transverse Reinforcements

The influence of transverse reinforcement on the capacity of a beam under combined bending and torsion is two-fold:

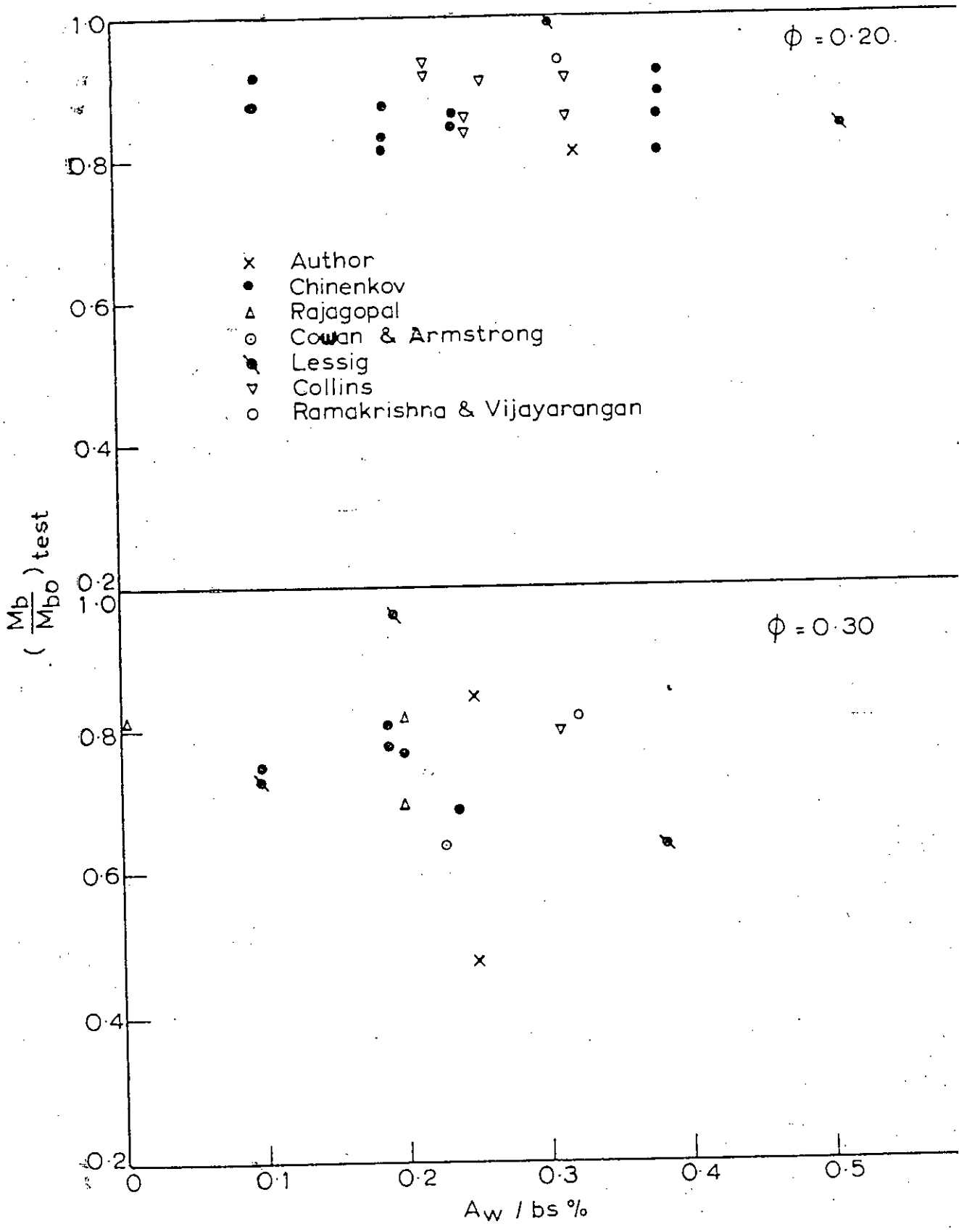
φ = 0·20
φ = 0·30
Author
Chinenkov
Rajagopal
Cowan & Armstrong
Lessig
Collins
Ramakrishna & Vijayarangan
(Mb/Mbo) test
Aw / bs %
1·0
0·8
0·6
0·4
0·2
0
0·1
0·2
0·3
0·4
0·5

FIG. I. 45.

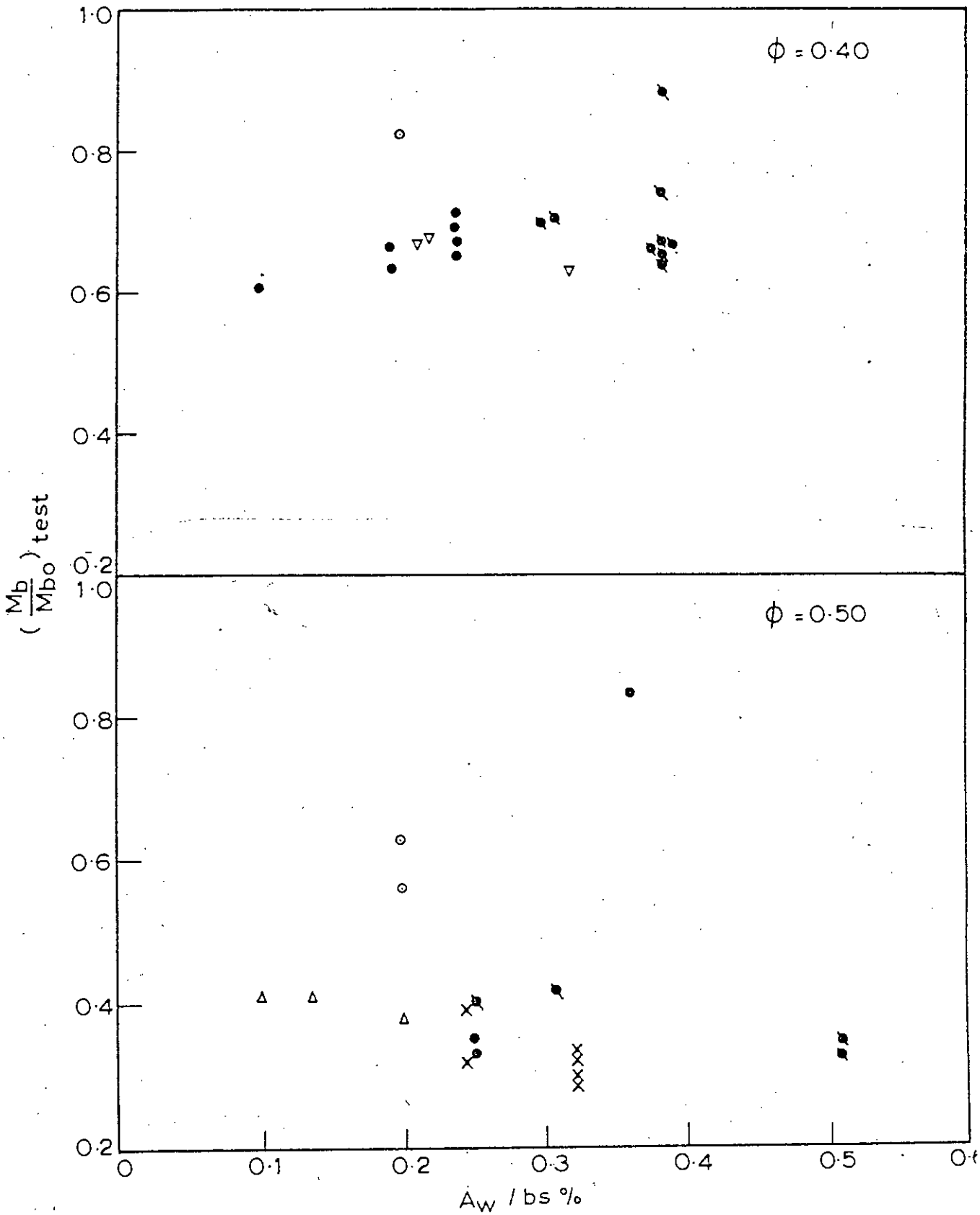

FIG. I. 45 & 46. INFLUENCE OF TRANSVERSE STEEL ON $(M_b/M_{bo})_{test}$

i. its influence on the flexural capacity of the beam; and

ii. its influence on the torque capacity of the beam.

In order to know the influence of transverse steel on the flexural capacity of beams failing either in premature flexure or in diagonal compression, the variation of M_b/M_{bo} with the transverse steel ratio A_w/bs has been studied. The results are shown in Fig. I.46, for four values of ϕ i.e. ϕ = 0.50, 0.40, 0.30 and 0.20. Besides author's own test results, the experimental results of some of the other investigators[6,17,23,33,55,56] are also shown in Fig. I.46. The D/b ratio of the beams of these investigations is approximately equal to that of beams of author's investigation. Having in view the scatter in the experimental results themselves, it can be seen from Fig. I.46 that the influence of transverse reinforcement on the flexural capacity of beams is not considerable. For the four values of ϕ the quantities M_b/M_{bo} lie approximately within a constant band for all the ranges of A_w/bs. Hence it can be concluded that the effect of transverse steel is not significant in considering the flexural capacity of beams failing either in premature flexure or in diagonal compression.

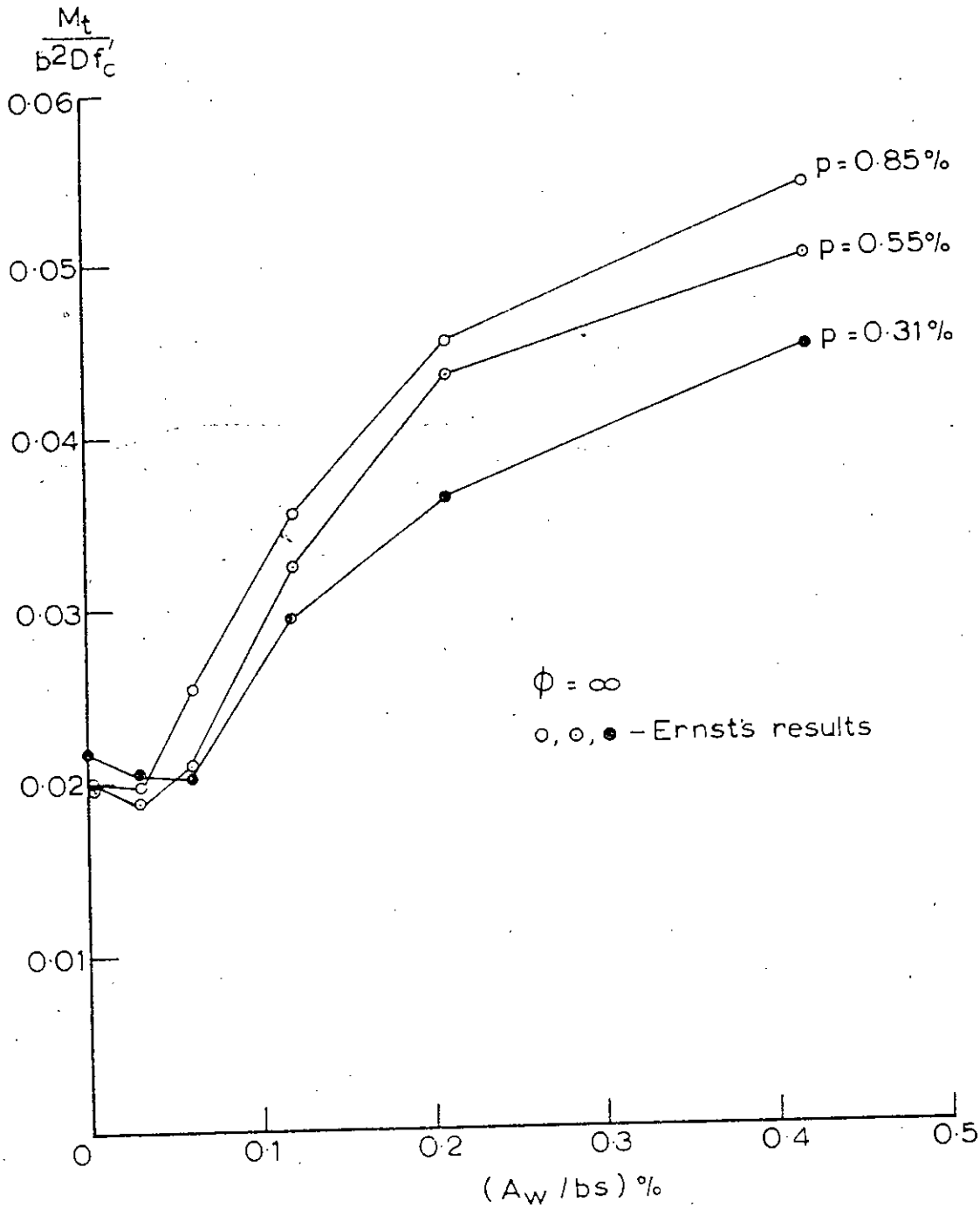

FIG.1.47. INFLUENCE OF TRANSVERSE STEEL ON TORQUE CAPACITY OF BEAMS.

For the case of beams failed in premature torsion, the experiments of the author have indicated that the torque capacity increased with the increase in the quantity of transverse steel. To get a quantitative picture about the influence of transverse steel, the results of Ernst's[30] investigation are summarised in Fig. I.47. The quantities M_t/b^2Df_c' have been plotted against $(A_w/bs)\%$ for various values of p%. From Fig. I.47 it can be seen that the torque capacity increases approximately in direct proportion with the increase in the quantity of transverse steel only in certain cases. But there seems to be a minimum amount of transverse reinforcement below which the torque capacity is not enhanced by the presence of transverse steel. For instance, in the case of Ernst's beams, when the spacing of transverse hoops is greater than the depth of the beam there is no increase in the torque capacity of the beam. In such cases, since the spacing is so large, a potential diagonal crack may form in between two adjacent hoops causing immediate failure of the beam. Hence it is necessary to restrict the spacing of the transverse hoops such that at least one hoop is traversed by the failure crack.

Similarly there is no proportionate increase in the torque capacity for values of A_w/bs higher than 0.21%. Perhaps in such cases the transverse steel may not yield to

give its maximum contribution to the capacity of beams. The maximum value of $M_t/b^2Df'_c$ obtained for Ernst's beams varies between 0.045 and 0.055. From the analysis of her own test results, Lessig[17] has given that the quantity $M_t/b^2Df'_c$ was not greater than 0.07 for her test results. Hence it is necessary to find an optimum value of $M_t/b^2Df'_c$ beyond which the torque capacity may not increase by the addition of transverse reinforcement. This will provide a basis for restricting the amount of transverse reinforcement. Further experimentation on this line is recommended.

I.7.5.4 Ratio of Twisting Moment to Bending Moment

The main factor which influences the strength of beams under combined bending and torsion is the ratio of twisting moment to bending moment, ϕ. Its influence has already been discussed in detail under Art. I.7.4.

I.7.5.5 Presence of Flexural Cracks

Some of the beams of author's investigation have been tested to yield results regarding the influence of flexural crack on the torque capacity of beams. For instance, the beam L6-1 was initially tested in combined bending and torsion upto failure and the beam failed in premature flexure. After failure, the applied bending moment was completely removed and the same beam was later subjected to pure torsion.

The beam collapsed for the second time at the same section where it failed in the first test and it is interesting to note that the torque resisted by the beam is equal to the "pure torsion" capacity of the beam. Similar test observations were noted in tests on beams V_4, L6-2 and T20 also. It is to be pointed out here that beams L6-1 and L6-2 were identical beams and their results coincided remarkably well.

To have further confirmation on this aspect, the author tested a special group of beams (Series B). This group consisted of five sets of beams and each set in turn consisted of two identical beams. One of the identical beams was tested in pure torsion so as to know the torque capacity of beams under pure torsion. The other beam was first tested in pure bending till failure. On the failed beam, a retest was conducted by subjecting it to pure torsion. The test results of this special group of beams, along with the results of some other beams, are presented in Table I.7.2. From Table I.7.2 it can be seen that the presence of flexural cracks has no significant influence on the torque capacity of beams. In most of the cases the torque resisted by a flexurally cracked beam is equal to the torque capacity of an uncracked beam.

TABLE I.7.2

INFLUENCE OF PRESENCE OF FLEXURAL CRACKS ON TORQUE CAPACITY OF BEAMS

Specimen Mark	Torque resisted by a flexurally cracked beam (Exptl.) in-kips	Torque resisted by an uncracked beam (Exptl.) in-kips	Ratio Col.(2)/Col.(3)
1	2	3	4
B2 II	22	23	0.96
B4 II	23	25	0.92
B6 II	23	24	0.96
B7 II	18	18	1.00
B9 II	23	23	1.00
V4 II	33	33	1.00
L6-1 II	25	25	1.00
L6-2 II	25	25	1.00
T20 II	23	23	1.00

The above conclusion is further supported by the test observations given by Ersoy[58] in which he reported that the presence of flexural cracks affects the torque capacity of beams by 6 to 7 per cent only. So even though we normally neglect the tension zone concrete in the calculation of flexural strength

of beams, it seems reasonable to consider the entire cross-section in the calculation of torque capacity of beams.

I.7.5.6 Shape of the Cross Section

In order to study the influence of shape of the cross section on the strength of beams, both rectangular and tee beams were tested. The torsional properties of the cross sections are given in Table I.7.3 below:

TABLE I.7.3

TORSIONAL PROPERTIES OF BEAMS

Cross Section	Elastic Theory: M_t/τ_m	Rigid Plastic Theory: $\frac{M_t}{\tau_m}$
Rectangle	46.8	79.0
Tee	31.6	57.2
	Ratio: $\frac{\text{Tee}}{\text{Rect.}} = 0.675$	Ratio: $\frac{\text{Tee}}{\text{Rect.}} = 0.725$

From the Table I.7.3, it can be seen that the tee beams have about (as an average) 70 per cent torsional strength of rectangular beams. So, as expected, in the case of beams

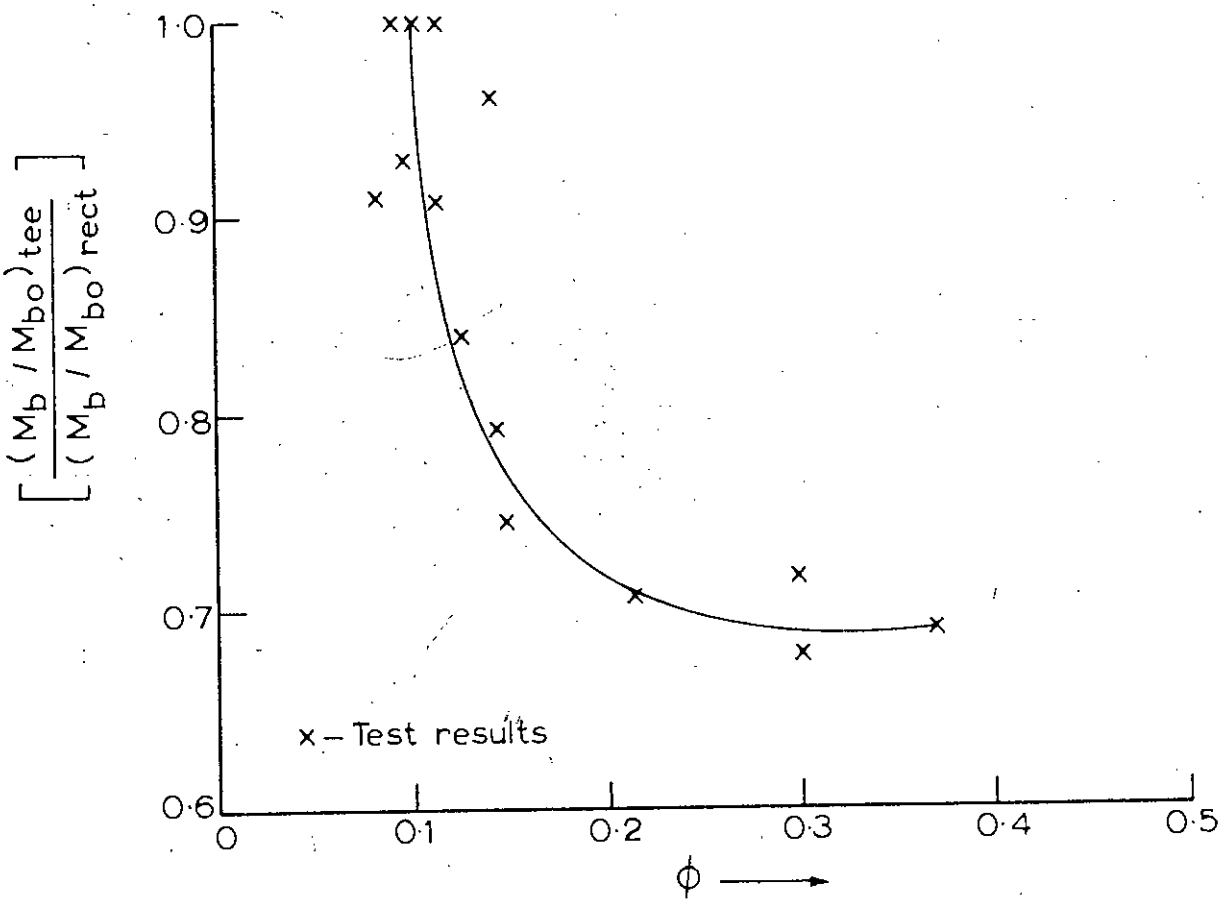

FIG.1.48. INFLUENCE OF SHAPE OF THE SECTION ON CAPACITY OF BEAMS.

failed in premature torsion, the tee beams resisted approximately 70 percent of the torque resisted by the rectangular beams. This is evident by comparing the results of Beams R4, T4 and T6. Further Fig. I.48 has been plotted to study the effect of shape of the cross section for the case of beams failed either in premature flexure or in diagonal compression. From Fig. I.48 it can be seen that for very low values of $\emptyset$ (i.e. in the case of beams failed in premature flexure), the difference in the flexural capacities of the beams is small; but it increases with the increase in the value of $\emptyset$. For values of $\emptyset$ greater than 0.20, the ratio of strengths is approximately constant equal to 0.70 which corresponds to the average ratio of torsional properties of the cross sections (See Table I.7.3). This observation suggests that the shape of the cross section may not have significant influence on the capacity of beam in the case of very low values of $\emptyset$; but it has got very significant influence at all other ranges of $\emptyset$.

I.7.6 Diagonal Cracking Torque

In the present investigation, the diagonal cracking torque was measured mostly in the case of beams failed in premature torsion. From the analysis of test results it has been found that the diagonal cracking torque was independent of the value of $\emptyset$ and the amount of transverse

steel. It depends mostly on the cross sectional dimensions and the concrete strength of the beam. In the case of beams without transverse reinforcement, the diagonal cracking torque is equal to the ultimate torque capacity of the beam (see the results of A series). In the case of beams with transverse steel, the strains in the hoops increased rapidly only after the formation of diagonal crack (see Fig. I.37). This observation suggests that it seems logical to add the torque contributed by the transverse steel to the diagonal cracking torque so as to evaluate the torque capacity of a beam with transverse steel.

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ANALYTICAL INVESTIGATION

CHAPTER I.8

TORSION OF TEE, ELL AND I-SECTIONS

I.8.1 Torsional characteristics of Tee, Ell, I and other allied sections are required in the analysis of beams under pure torsion and under combined bending and torsion. In the past, most of the previous investigators have used an approximate method, which was originally suggested by Bach[8] for rolled profile sections, to determine the torsional rigidities of compound sections consisting of rectangles by considering the section as made up of narrow rectangles and the torsional rigidity is given by the sum of the torsional rigidities of these rectangles. The formula thus obtained gives good results when the breadth of the rectangles forming the section is very small when compared to the length. But in reinforced concrete sections this assumption is rarely satisfied and a more rigorous solution is necessary in this case.

In this chapter, a solution using Fourier Series is given to solve the problem of elastic torsion of these

sections. Since in the ultimate strength design we may require the torsional rigidity of these sections under full plastic conditions, expressions are given for this case using the sand-heap analogy developed by Nadai[7].

I.8.2 Elastic Torsion

The problem of torsion consists in the determination of the stress function $\varphi(x,y)$ satisfying the differential equation[59]

$$\nabla^2 \varphi + 2G\theta = 0 \qquad \ldots \text{ I.8.1}$$

with the boundary condition

$$\varphi = \text{constant} \qquad \ldots \text{ I.8.2}$$

on the boundary.

In Eqn. I.8.1, G and θ are the shear modulus and angle of twist per unit length respectively. In solid sections we can take in Eq. I.8.2

$$\varphi = 0 \quad \text{on the boundary} \qquad \ldots \text{ I.8.3}$$

The shear stress components are given by

$$\tau_{xz} = \frac{\partial \varphi}{\partial y}$$

$$\tau_{yz} = -\frac{\partial \varphi}{\partial x} \qquad \ldots \text{ I.8.4}$$

and the torque is given by

$$M_t = 2 \iint \varphi \, dxdy \qquad \ldots \text{ I.8.5}$$

Torsional rigidity of the section is given by

$$C = \frac{M_t}{\theta} \qquad \ldots \text{ I.8.6}$$

I.8.2.1 Tee Section

The co-ordinates are shown in Fig. I.49. The Tee section can be considered as made up of two rectangles KLMN and PQRS which are shown as regions (1) and (2) respectively in Fig. I.49.

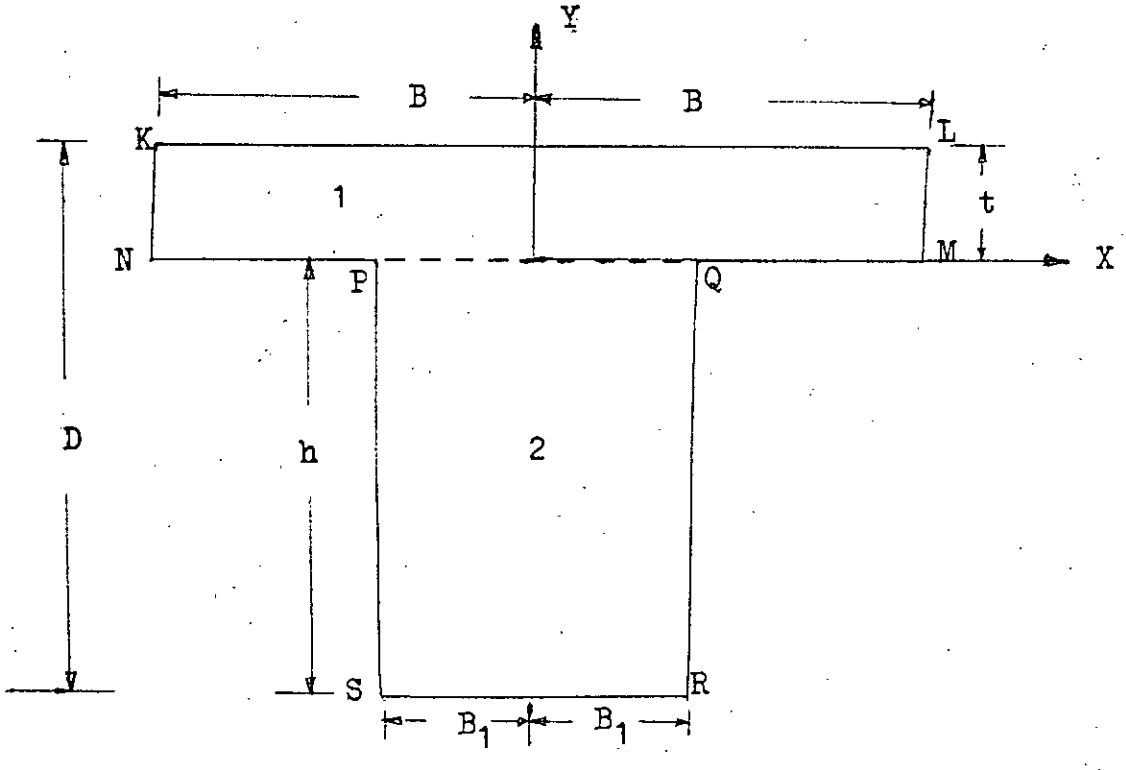


Fig.I.49

The functions φ can be found out separately for the two regions satisfying the boundary conditions and the continuity condition expressing that the stresses are continuous along the line of contact of the two regions i.e. PQ.

For Region 1:-

$$\varphi_1 = \sum_{n=1,3,5} \left[A_n \sinh \frac{n\pi y}{2B} + B_n \cosh \frac{n\pi y}{2B} + \beta_n\right] \cos \frac{n\pi x}{2B} \qquad \ldots \text{ I.8.7}$$

For Region 2:-

$$\varphi_2 = \sum_{m=1,3,5} \left[C_m \sinh \frac{m\pi y}{2B_1} + D_m \cosh \frac{m\pi y}{2B_1} + \beta_m\right] \cos \frac{m\pi x}{2B_1} \qquad \ldots \text{ I.8.8}$$

where

$$\beta_n = \frac{32GQB^2(-1)^{\frac{n-1}{2}}}{n^3}$$

$$\beta_m = \frac{32GQB^2{}_1(-1)^{\frac{m-1}{2}}}{m^3} \qquad \ldots \text{ I.8.9}$$

The boundary conditions are:

$$
\begin{aligned}
&\text{on } y = t, \quad \varphi_1 = 0 \qquad \ldots \text{(i)} \\
&\quad x = \pm B, \varphi_1 = 0 \qquad \ldots \text{(ii)} \\
&\text{on } y = o, \; B_1 \leq x \leq B_2, \; \varphi_1 = 0 \qquad \ldots \text{(iii)} \\
&\text{on } y = -h, \; \varphi_2 = 0 \qquad \ldots \text{(iv)} \\
&\quad x = \pm B_1, \; \varphi_2 = 0 \; \ldots \text{(v)}
\end{aligned}
\qquad \ldots \text{I.8.10}
$$

The continuity conditions are:

$$
\text{on } y = o, \quad -B_1 \leq x \leq + B_1,
$$

$$
\begin{aligned}
&\varphi_1 = \varphi_2 \qquad \ldots \text{(i)} \\
&\text{and} \quad \frac{\partial \varphi_1}{\partial y} = \frac{\partial \varphi_2}{\partial y} \qquad \ldots \text{(ii)}
\end{aligned}
\qquad \ldots \text{I.8.11}
$$

It can be seen from the Eqns. I.8.7 and I.8.8 that φ_1 and φ_2 satisfy the differential Eqn. I.8.1 when β_n and β_m are given by Eqn. I.8.9 and that the boundary conditions I.8.10 (ii) and I.8.10 (v) are also satisfied. Satisfying the boundary conditions in I.8.10 (i) and I.8.10 (iv) we get

$$A_n = \frac{-1}{\text{Sinh } \frac{n\pi t}{2B}} \left\{ B_n \cosh \frac{n\pi t}{2B} + \beta_n \right\}$$

$$C_m = \frac{1}{\text{Sinh } \frac{m\pi h}{2B_1}} \left\{ D_m \cosh \frac{m\pi h}{2B_1} + \beta_m \right\} \qquad \ldots I.8.12$$

We have

$$\varphi_1\Big]_{y=o} = \sum_{n=1,3,5} (B_n + \beta_n) \cos \frac{n\pi x}{2B} \qquad \ldots I.8.13$$

Hence, taking finite Fourier Transform we get,

$$B_n + \beta_n = \frac{2}{B} \int_0^B \varphi_1\Big]_{y=o} \cos \frac{n\pi x}{2B}\, dx$$

$$= \frac{2}{B} \left[\int_0^{B_1} \varphi_1\Big]_{y=o} \cos \frac{n\pi x}{2B}\, dx + \int_{B_1}^{B} \varphi_1\Big]_{y=o} \cos \frac{n\pi x}{2B}\, dx \right]$$

$\ldots I.8.14$

The second integral in the r.h.s. of Eqn. I.8.14 is zero from I.8.10 (iii). Using Eqn. I.8.11, we have

$$B_n + \beta_n = \frac{2}{B} \int_0^{B_1} \varphi_2\Big]_{y=o} \cos \frac{n\pi x}{2B}\, dx$$

$$= \frac{2}{B} \int_0^{B_1} \sum_{m=1,3,5..} (D_m + \beta_m) \cos \frac{m\pi x}{2B_1} \cos \frac{n\pi x}{2B}\, dx$$

$$= \frac{4}{BB_1} \cos \frac{n\pi B_1}{2B} \sum_{m=1,3,5..} \frac{m\,(D_m + \beta_m)(-1)^{\frac{m-1}{2}}}{\left(\frac{m^2}{B_1^2} - \frac{n^2}{B^2}\right)}$$

To determine the coefficients B_n and D_m, apart from eqn. I.8.15 we require one more equation. This is to be got from the condition I.8.11(ii).

We have

$$\left.\frac{\partial \varphi_1}{\partial y}\right]_{y=o} = \sum_{n=1,3,5..} A_n \frac{n\pi}{2B} \cos\frac{n\pi x}{2B} \quad \ldots (i)$$

$$\left.\frac{\partial \varphi_2}{\partial y}\right]_{y=o} = \sum_{m=1,3,5..} C_m \frac{m\pi}{2B_1} \cos\frac{m\pi x}{2B_1} \quad \ldots (ii)$$

$$\ldots \text{I.8.16}$$

Hence taking finite Fourier Transform of Eqn. I.8.16(i) we get,

$$\frac{n\pi}{2B} A_n = \frac{1}{B}\int_{-B}^{+B} \left.\frac{\partial \varphi_1}{\partial y}\right]_{y=o} \cos\frac{n\pi x}{2B}\, dx$$

$$= \frac{2}{B}\left[\int_{0}^{B_1} \left.\frac{\partial \varphi_1}{\partial y}\right]_{y=o} \cos\frac{n\pi x}{2B} dx + \int_{B_1}^{B} \left.\frac{\partial \varphi_1}{\partial y}\right]_{y=o} \cos\frac{n\pi x}{2B} d\right.$$

$$\ldots \text{I.8.17}$$

Using equations I.8.11(ii) and I.8.16(ii) we get,

$$\frac{n\pi}{2B} A_n = \frac{2}{B}\left[\int_0^{B_1} \sum_{m=1,3,5..} C_m \frac{m\pi}{2B_1} \cos\frac{m\pi x}{2B_1} \cos\frac{n\pi x}{2B}\, dx + \int_{B_1}^{B} \frac{\partial \Phi_1}{\partial y}\Big]_{y=o} \cos\frac{n\pi x}{2B}\, dx\right] \quad \ldots \text{ I.8.18}$$

For the second integral, we can use again Equation I.8.16(1) and we get,

$$\int_{B_1}^{B} \frac{\partial \Phi_1}{\partial y}\Big]_{y=o} \cos\frac{n\pi x}{2B}\, dx$$

$$= \int_{B_1}^{B} \sum_{r=1,3,5..} A_r \frac{r\pi}{2B} \cos\frac{r\pi x}{2B} \cos\frac{n\pi x}{2B}\, dx \quad \ldots \text{ I.8.19}$$

Substituting I.8.19 in I.8.18 and after simplification we get, when $r \neq n$,

$$A_n = \frac{4}{n\pi}\left[\cos\frac{n\pi B_1}{2B} \sum_{m=1,3,5\ldots} \frac{C_m(-1)^{\frac{m-1}{2}}}{\left\{1-\left(\frac{nB_1}{mB}\right)^2\right\}} + \sum_{r=1,3,5\ldots} \frac{A_r}{1-\left(\frac{n}{r}\right)^2}\left\{\frac{n}{r}\cos\frac{r\pi B_1}{2B}\sin\frac{n\pi B_1}{2B} - \sin\frac{r\pi B_1}{2B}\cos\frac{n\pi B_1}{2B}\right\}\right] \quad \ldots \text{ I.8.20(a)}$$

when $r = n$,

$$A_n = \frac{4}{n\Pi}\left[\cos\frac{n\Pi B_1}{2B}\sum_{m=1,3,5\ldots}\frac{C_m(-1)^{\frac{m-1}{2}}}{\left\{1-\left(\frac{n B_1}{mB}\right)^2\right\}} + A_n\left(\frac{n\Pi}{4} - \frac{n\Pi B_1}{4B} - \frac{1}{4}\sin\frac{n\Pi B_1}{B}\right)\right] \quad \ldots \text{I.8.20(b)}$$

Equations I.8.15 and I.8.20 with the relations given in I.8.12 are theoretically sufficient to determine B_n and D_m.

Moments:-

$$M_1 = 2\int_{-B}^{+B}\int_0^t \sigma_1 \, dxdy \quad \ldots \text{I.8.21}$$

$$= \frac{4(2B)^2}{\Pi^2}\sum_{n=1,3,5\ldots}\frac{(-1)^{\frac{n-1}{2}}}{n^2}B_n\left(\coth\frac{n\Pi t}{2B} - \frac{1}{\operatorname{Sinh}\frac{n\Pi t}{2B}}\right)$$

$$- \frac{32G\Theta(2B)^4}{\Pi^5}\sum_{n=1,3,5\ldots}\frac{1}{n^5}\left(\coth\frac{n\Pi t}{2B} - \frac{1}{\operatorname{Sinh}\frac{n\Pi t}{2B}}\right)$$

$$+ \frac{32G\Theta(2B)^3 t}{\Pi^4}\sum_{n=1,3,5\ldots}\frac{1}{n^4} \quad \ldots \text{I.8.22}$$

or observing

$$1 + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots = \frac{\pi^4}{96} \qquad \ldots \text{ I.8.23}$$

$$M_1 = \frac{4(2B)^2}{\pi^2} \sum_{n=1,3,5\ldots} \frac{(-1)^{\frac{n-1}{2}}}{n^2} B_n \left(\coth \frac{n\pi t}{2B} - \frac{1}{\sinh \frac{n\pi t}{2B}}\right)$$

$$- \frac{32G\Theta(2B)^4}{\pi^5} \sum_{n=1,3,5\ldots} \frac{1}{n^5} \left(\coth \frac{n\pi t}{2B} - \frac{1}{\sinh \frac{n\pi t}{2B}}\right)$$

$$+ \frac{G\Theta(2B)^3 t}{3} \qquad \ldots \text{ I.8.24}$$

Similarly

$$M_2 = 2 \int_{-B_1}^{+B_1} \int_{-h}^{0} \sigma_{z2} \, dxdy \qquad \ldots \text{ I.8.25}$$

$$= \frac{4(2B_1)^2}{\pi^2} \sum_{m=1,3,5\ldots} \frac{(-1)^{\frac{m-1}{2}}}{m^2} D_m \left(\coth \frac{m\pi h}{2B_1} - \frac{1}{\sinh \frac{m\pi h}{2B_1}}\right)$$

$$- \frac{32G\Theta(2B_1)^4}{\pi^5} \sum_{m=1,3,5\ldots} \frac{1}{m^5} \left(\coth \frac{m\pi h}{2B_1} - \frac{1}{\sinh \frac{m\pi h}{2B_1}}\right)$$

$$+ \frac{G\Theta(2B_1)^3 h}{3} \qquad \ldots \text{ I.8.26}$$

Shearing Stresses:-

(i) $(\tau_{yz})_2 = -\frac{\partial \varphi_2}{\partial x}$

$$= \sum_{m=1,3,5\ldots}\left[\frac{m\pi}{2B_1}(C_m \sinh \frac{m\pi y}{2B_1} + D_m \cosh \frac{m\pi y}{2B_1} + \beta_m)\sin \frac{m\pi x}{2B_1}\right]$$

... I.8.27

$\tau_m =$

$$\tau_{yz}\Big]_{\substack{x = \pm B_1 \\ y = -h/2}}$$

$$= \sum_{m=1,3,5\ldots}\left[\frac{m\pi}{2B_1} D_m(-1)^{\frac{m-1}{2}}\left\{\cosh \frac{m\pi h}{4B_1} - \coth \frac{m\pi h}{2B_1} \sinh \frac{m\pi h}{4B_1}\right\}\right.$$

$$\left. + \frac{16G\Theta B_1}{m^2\pi^2}\left(1 - \frac{1}{2\cosh \frac{m\pi h}{4B_1}}\right)\right] \qquad \ldots \text{ I.8.28}$$

(ii) $(\tau_{xz})_1 = \frac{\partial \varphi_1}{\partial y}$

$$= \sum_{n=1,3,5\ldots} \frac{n\pi}{2B} \cos \frac{n\pi x}{2B}(A_n \cosh \frac{n\pi y}{2B} + B_n \sinh \frac{n\pi y}{2B})$$

... I.8.29

$$\tau_{xz}\Big]_{\substack{x = o \\ y = t}}$$

$$= \sum_{n=1,3,5\ldots}\left[\frac{n\pi}{2B} B_n(\sinh \frac{n\pi t}{2B} - \coth \frac{n\pi t}{2B} \cosh \frac{n\pi t}{2B})\right.$$

$$\left. - \frac{16G\Theta B(-1)^{\frac{n-1}{2}}}{n^2\pi^2} \coth \frac{n\pi t}{2B}\right] \qquad \ldots \text{ I.8.30}$$

It is to be mentioned here that the torsional stresses are maximum at the re-entrant corners (theoretically equal to infinity). But this occurs only for a small region and hence does not affect the twisting moment of the section and shear stresses at other points.

Numerical Results:-

To indicate the accuracy of the results given by the solution given here, numerical results are obtained for a particular Tee section with the following relationships:

$$\left.\begin{aligned} B &= 5t \\ B_1 &= \tfrac{1}{4}B \\ h &= 4t \end{aligned}\right\} \qquad \ldots \text{ I.8.31}$$

Torsional rigidity of the section is given by

$$C = \frac{2}{\theta}\iint \Phi_1 \, dxdy + \frac{2}{\theta}\iint \Phi_2 \, dxdy \qquad \ldots \text{ I.8.32}$$

The first and the second integrals are evaluated over the regions (1) and (2) respectively. To get an idea of the convergence of the procedure adopted, results have been obtained using 2, 3 and 4 terms in each series. The results for the torsional rigidity are given in Table I.8.1.

TABLE I.8.1

TORSIONAL RIGIDITY OF TEE SECTION

Quantity	With 2 terms	With 3 terms	With 4 terms
$\frac{C}{Gt^4}$	21.319	20.731	20.702
$\frac{\tau_m}{G\,\theta\,t}$	2.340	2.345	2.345

Similarly the maximum shearing stress which occurs at

$x = \pm B_1$ and $y = -h/2$ is given in Table I.8.1.

From Table I.8.1, it can be seen that the convergence of the series used in this method is good and we can take only three terms in each series. It would be interesting to compare the results of the 4-term solution as obtained above with those given by other approximate methods. Hence the problem was solved using the Relaxation procedure. Approximate values for torsional rigidity and maximum shearing stress were determined as suggested by Bach[8] using the results of narrow rectangle. Saint Venant had observed an upper bound solution using the results of elliptic cross

section and this can be expressed by the inequality

$$C \leq \frac{GA^4}{4\pi^2 J} \qquad \ldots \text{I.8.33}$$

where A is the area of cross section and J is the polar moment of inertia about the centroid of the cross section. The upper bound given by this method has also been calculated and the results are shown in Table I.8.2. Weinberger[39] has suggested a lower bound for the torsional rigidity. His inequality for the torsional rigidity of two or more disjoint regions is

$$C_{12.....n} \geqslant C_1 + C_2 + C_3 + ... C_n \qquad \ldots \text{I.8.34}$$

For the Tee section, using the tables given by Timoshenko and Goodier[59], exact torsional rigidities for the regions 1 and 2 can be found out. The lower bound as given by this method for the Tee section is also given in Table I.8.2

TABLE I.8.2

COMPARISON OF TORSIONAL RIGIDITY FOR TEE SECTION AS GIVEN BY VARIOUS INVESTIGATORS

Quantity	By present analysis	By Relaxation procedure	By Weinberger's method (Lower bound)	By Narrow-rectangle formula	By St.Venant (Upper bound)	By Turner and Davis[12]
$\frac{C}{Gt^4}$	20.702	20.643	20.069	24.167	29.980	21.407
$\frac{\tau_m}{G\theta t}$	2.345	2.334	-	2.500	-	-

From the results given in Table I.8.2 it can be seen that the relaxation procedure gives good results when compared to those given by the present method. The method suggested by Bach using narrow rectangles formula is very much in error.

I.8.2.2 I and Ell Sections

The procedure for solving the torsion problem for these sections is exactly similar to the one indicated for the Tee section.

I.8.2.2.1- I - section

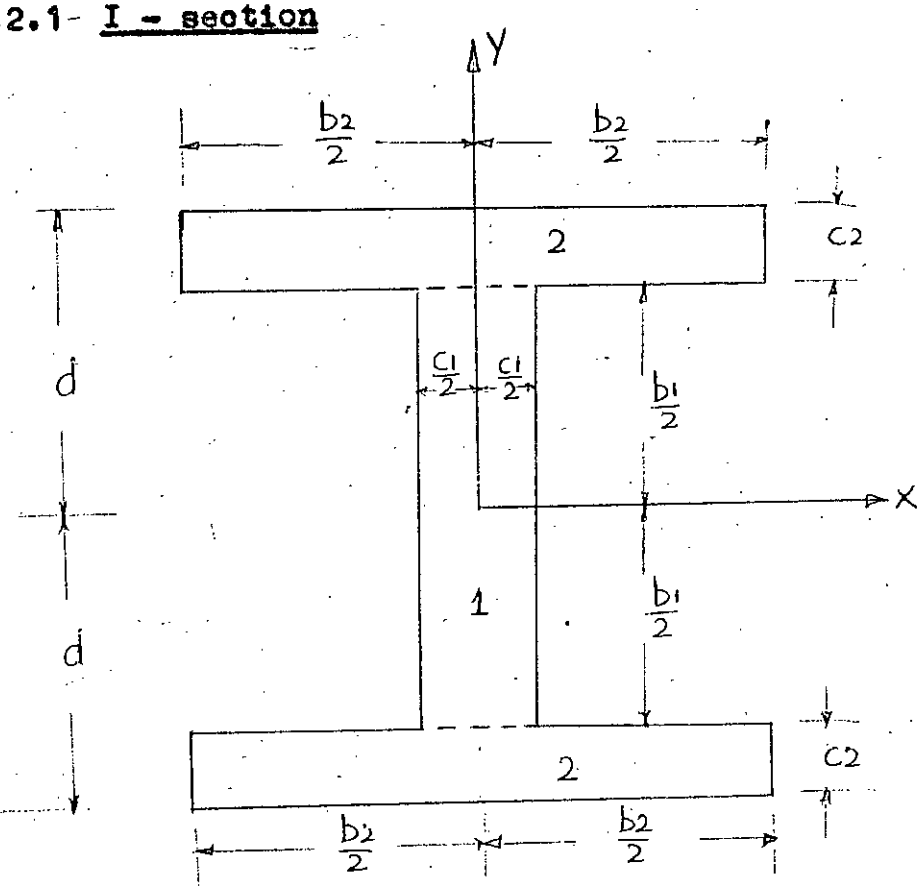


Fig. I.50

Referring to Fig. I.50,

For the region 1,

$$\varphi_1 = \sum_{n=1,3,5\ldots} \frac{c_1}{n\pi \sinh \frac{n\pi b_1}{2c_1}} \left[A_n \sinh \frac{n\pi y}{c_1} + B_n \cosh \frac{n\pi y}{c_1} + \beta_n \right] \cos \frac{n\pi x}{c_1} \qquad \ldots \text{ I.8.35}$$

For the region 2,

$$\varphi_2 = \sum_{m=1,3,5\ldots} \frac{b_2}{m\pi \sinh \frac{m\pi b_1}{2b_2}} \left[C_m \sinh \frac{m\pi y}{b_2} + D_m \cosh \frac{m\pi y}{b_2} + \beta_m \right] \cos \frac{m\pi x}{b_2} \qquad \ldots \text{ I.8.36}$$

where

$$\beta_n = \frac{8G\Theta c_1 (-1)^{\frac{n-1}{2}}}{n^2 \pi^2} \sinh \frac{n\pi b_1}{2c_1} \qquad \ldots \text{ I.8.37}$$

and $$\beta_m = \frac{8G\Theta b_2 (-1)^{\frac{m-1}{2}}}{m^2 \pi^2} \sinh \frac{m\pi b_1}{2b_2}$$

Applying the boundary conditions and continuity condition the expressions to determine the coefficients B_n and D_m reduce to

$$\frac{\pi b_2^2}{2m}\left[D_m\left(\coth\frac{m\pi b_1}{2b_2} - \coth\frac{m\pi d}{b_2}\right) + \right.$$

$$\left.\frac{8G\Theta b_2(-1)^{\frac{m-1}{2}}}{m^2\pi^2}\left(1 - \frac{\sinh\frac{m\pi b_1}{2b_2}}{\sinh\frac{m\pi d}{b_2}}\right)\right]$$

$$= \sum_{n=1,3,5\ldots}\left[(-1)^{\frac{n-1}{2}} B_n \coth\frac{n\pi b_1}{2c_1} + \frac{8G\Theta c_1}{n^2\pi^2}\frac{\cos\frac{m\pi c_1}{2b_2}}{\left(\frac{n^2}{c_1^2} - \frac{m^2}{b_2^2}\right)}\right] \quad \ldots\text{I.8.38}$$

$$C_m = -\left[D_m \coth\frac{m\pi d}{b_2} + \frac{\beta_m}{\sinh\frac{m\pi d}{b_2}}\right] \quad \ldots\text{I.8.39}$$

$$\left[C_m \coth\frac{m\pi b_1}{2b_2} + D_m\right]$$

$$= \frac{4}{\pi}\left[\frac{c_1}{b_2}\sum_{n=1,3,5\ldots}\frac{B_n(-1)^{\frac{n-1}{2}}\cos\frac{m\pi c_1}{2b_2}}{n\left\{1 - \left(\frac{mc_1}{nb_2}\right)^2\right\}} + \right.$$

$$\sum_{r=1,3,5\ldots}\left(C_r \coth\frac{r\pi b_1}{2b_2} + D_r\right)\left(\frac{m}{r}\cos\frac{r\pi c_1}{2b_2}\sin\frac{m\pi c_1}{2b_2}\right.$$

$$\left.\left.\frac{-\sin\frac{r\pi c_1}{2b_2}\cos\frac{m\pi c_1}{2b_2})}{r\left\{1 - \left(\frac{m}{r}\right)^2\right\}}\right]\right. \quad \ldots\text{I.8.40}$$

Equations I.8.38, I.8.39 and I.8.40 are used to evaluate the coefficients B_n and D_m.

Moments:-

$$M_1 = 2 \int_{-\frac{c_1}{2}}^{+\frac{c_1}{2}} \int_{-\frac{b_1}{2}}^{+\frac{b_1}{2}} \phi_1 \, dxdy$$

$$= \sum_{n=1,3,5\ldots} \left[\frac{8c_1^3 (-1)^{\frac{n-1}{2}}}{n^3 \pi^3} B_n \right] + \frac{1}{3} G\theta b_1 c_1^3 \qquad \ldots \text{ I.8.41}$$

$$M_2 = 2 \int_{-\frac{b_2}{2}}^{+\frac{b_2}{2}} \int_{\frac{b_1}{2}}^{d} \phi_2 \, dxdy$$

$$= \sum_{m=1,3,5\ldots} \left[\frac{4b_2^3}{m^3 \pi^3} (-1)^{\frac{m-1}{2}} D_m \left(\coth \frac{m\pi d}{b_2} \coth \frac{m\pi b_1}{2b_2} - \frac{1}{\sinh \frac{m\pi d}{b_2} \sinh \frac{m\pi b_1}{2b_2}} - 1 \right) \right.$$

$$\left. - \frac{32G\theta b_2^4}{m^5 \pi^5} \left(\coth \frac{m\pi d}{b_2} - \frac{\cosh \frac{m\pi b_1}{2b_2}}{\sinh \frac{m\pi d}{b_2}} \right) \right] + \frac{1}{3} G\theta c_2 b_2^3 \qquad \ldots \text{ I.8.42}$$

Torsional rigidity

$$C = \left(\frac{M_1}{\theta} + \frac{2M_2}{\theta} \right) \qquad \ldots \text{ I.8.43}$$

Shearing stress

$$(\tau_{yz})_1 = -\frac{\partial \phi_1}{\partial x}$$

$$= \sum_{n=1,3,5\ldots} \left[B_n \cosh \frac{n\pi y}{c_1} + \beta_n \right] \frac{\sin \frac{n\pi x}{c_1}}{\sinh \frac{n\pi b_1}{2c_1}} \qquad \ldots \text{ I.8.44}$$

$$\tau_{yz} \Big]_{\substack{x = c_1/2 \\ y = 0}}$$

$$= \sum_{n=1,3,5\ldots} \left[\frac{(-1)^{\frac{n-1}{2}} B_n}{\sinh \frac{n\pi b_1}{2c_1}} \right] + G\theta c_1$$

when $b_1 >> c_1$,

$$\tau_{yz} = G\theta c_1 \qquad \ldots \text{ I.8.45}$$

This coincides with the result of Bach's[8] approximate formula for rolled sections.

Numerical Results:-

Numerical results are obtained for the following two cases and the results are compared with the results of

approximate formulae in Table I.8.3

i. $d = b_2$; $c_2 = 2c_1$; $b_2 = 5c_2$

ii. $d = b_2$; $c_2 = 2c_1$; $b_2 = 10c_2$ ... I.8.46

TABLE I.8.3

COMPARISON OF RESULTS FOR I - SECTION

b_2 / c_2 Ratio	C / Gc_2^4		
	By present analysis	By Weinberger's approx.	By narrow-rectangle formula
5	4.075	3.243	3.667
10	7.529	6.990	7.417

I.8.2.2.2 Ell Section

Referring to Fig. I.51,

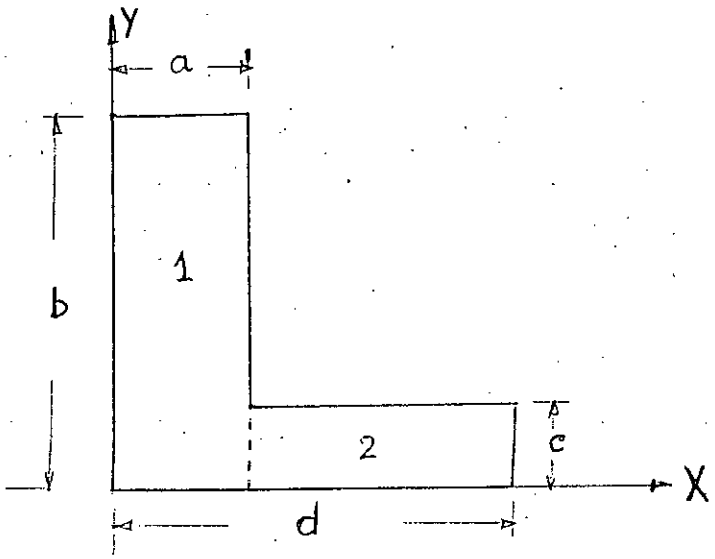


Fig. I.51

For the region 1,

$$\varphi_1 = \sum_{n=1,3,5\ldots} \frac{b}{n\pi \sinh \frac{n\pi a}{b}} \left[A_n \sinh \frac{n\pi x}{b} + B_n \cosh \frac{n\pi x}{b} + \beta_n \right] \sin \frac{n\pi y}{b} \qquad \ldots \text{ I.8.47}$$

For the region 2,

$$\varphi_2 = \sum_{m=1,3,5\ldots} \frac{c}{m\pi \sinh \frac{m\pi a}{c}} \left[C_m \sinh \frac{m\pi x}{c} + D_m \cosh \frac{m\pi x}{c} + \beta_m \right] \sin \frac{m\pi y}{c} \qquad \ldots \text{ I.8.48}$$

where

$$\beta_n = \frac{8G\Theta b(-1)^{\frac{n-1}{2}}}{n^2\pi^2} \sinh \frac{n\pi a}{b}$$

$$\beta_m = \frac{8G\Theta c(-1)^{\frac{m-1}{2}}}{m^2\pi^2} \sinh \frac{m\pi a}{c} \qquad \ldots \text{ I.8.49}$$

Satisfying the boundary conditions and continuity conditions the coefficients A_n and D_m can be evaluated using the following expressions:

$$\frac{\pi b^2}{n}\left[A_n + \frac{8G\Theta b(-1)^{\frac{n-1}{2}}}{n^2\pi^2}\sinh\frac{n\pi a}{b}\left(\frac{1}{\sinh\frac{n\pi a}{b}} - \coth\frac{n\pi a}{b}\right)\right]$$

$$= \sum_{m=1,3,5\ldots}\left[D_m\left(\coth\frac{m\pi a}{c} - \coth\frac{m\pi d}{c}\right) + \right.$$

$$\left.\frac{8G\Theta c(-1)^{\frac{m-1}{2}}}{m^2\pi^2}\left(1 - \frac{\sinh\frac{m\pi a}{c}}{\sinh\frac{m\pi d}{c}}\right)\right]\frac{\sin\frac{n\pi c}{b}}{\left(\frac{m^2}{c^2} - \frac{n^2}{b^2}\right)} \quad \ldots \text{ I.8.50}$$

$$\left[A_n \coth\frac{n\pi a}{b} - \frac{8G\Theta b}{n^2\pi^2}(-1)^{\frac{n-1}{2}}\sinh\frac{n\pi a}{b}\right] =$$

$$\frac{1}{\pi}\left[\sum_{m=1,3,5\ldots}-\sin\frac{n\pi c}{b}\left\{D_m\left(1-\coth\frac{m\pi d}{c}\coth\frac{m\pi a}{c}\right) - \frac{8G\Theta c(-1)^{\frac{m-1}{2}}}{m^2\pi^2}\frac{\cosh\frac{m\pi a}{c}}{\sinh\frac{m\pi d}{c}}\right\}\right.$$

$$\frac{}{n\left\{1 - \left(\frac{bm}{nc}\right)^2\right\}}$$

$$\left\{A_r \coth\frac{r\pi a}{b} - \frac{8G\Theta b(-1)^{\frac{r-1}{2}}}{r^2\pi^2}\sinh\frac{r\pi a}{b}\right\}\left\{\frac{n}{r}\cos\frac{r\pi c}{b}\sin\frac{n\pi c}{b}\right.$$

$$\left.+ \frac{- \sin\frac{r\pi c}{b}\cos\frac{n\pi c}{b}\right\}}{r\left\{\left(1 - \left(\frac{n}{r}\right)^2\right\}}\right]$$

$$\ldots \text{ I.8.51}$$

Moments:-

$$M_1 = 2\int_0^a \int_0^b \varphi_1 \, dxdy$$

$$= \sum_{n=1,3,5\ldots} \left[\frac{4b^3}{n^3\pi^3} A_n \left(\coth \frac{n\pi a}{b} - \frac{1}{\sinh \frac{n\pi a}{b}} \right) \right.$$

$$\left. + \frac{32G\Theta b^3}{n^4\pi^4} (-1)^{\frac{n-1}{2}} - \frac{32G\Theta b^4 (-1)^{\frac{n-1}{2}}}{n^5\pi^5} \sinh \frac{n\pi a}{b} \right]$$

... I.8.52

$$M_2 = 2\int_a^d \int_0^c \varphi_2 \, dxdy$$

$$= \sum_{m=1,3,5..} \left[\frac{4c^3}{m^3\pi^3} D_m \left(\coth \frac{m\pi d}{c} \coth \frac{m\pi a}{c} - 1 \right. \right.$$

$$\left. - \frac{1}{\sinh \frac{m\pi d}{c} \sinh \frac{m\pi a}{c}} \right) + \frac{32G\Theta c^3 (d-a)(-1)^{\frac{m-1}{2}}}{m^4\pi^4}$$

$$\left. - \frac{32G\Theta c^4 (-1)^{\frac{m-1}{2}}}{m^5\pi^5} \left(\coth \frac{m\pi d}{c} - \frac{\cosh \frac{m\pi a}{c}}{\sinh \frac{m\pi d}{c}} \right) \right]$$

... I.8.53

Torsional rigidity

$$C = \frac{M_1}{\Theta} + \frac{M_2}{\Theta}$$

... I.8.54

Shearing Stress:-

$$(\tau_{yz})_1 = -\frac{\partial \varphi_1}{\partial x}$$

$$= \sum_{n=1,3,5\ldots} \frac{1}{\sinh \frac{n\pi a}{b}} \left[A_n \cosh \frac{n\pi x}{b} - \beta_n \sinh \frac{n\pi x}{b} \right] \sin \frac{n\pi y}{b}$$

$$\tau_{yz}\Big]_{\substack{x=0 \\ y=+b/2}} = \sum_{n=1,3,5\ldots} \frac{(-1)^{\frac{n-1}{2}} A_n}{\sinh \frac{n\pi a}{b}} \quad \ldots \text{ I.8.55}$$

Numerical Example:-

Numerical example is done for the following two cases and the results are compared with the results of approximate methods in Table I.8.4.

i. b = d ; a = c ; b = 5c

ii. b = d ; a = c ; b = 10c

TABLE I.8.4

COMPARISON OF RESULTS FOR L - SECTION

b/c ratio	G/Gc^4		
	By present analysis	By Weinberger's approx.	By narrow-rectangle formula
5	3.275	2.579	3.000
10	6.405	5.892	6.333

It is to be pointed out here that the procedure suggested herein is general and can be applied to any other sections composed of rectangles.

I.8.3. Plastic Torsion

In the previous article, torsional characteristics of Tee, Ell and I sections in the elastic range were evaluated. The plastic torsion of these sections is considered in this article.

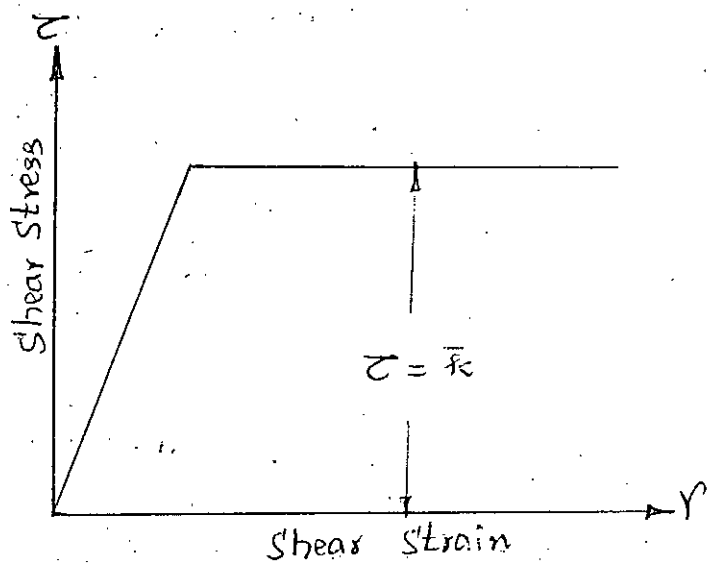


Fig. I.52

Any prismatic bar undergoes plastic deformation in certain parts of its cross section when it is twisted severly. An idealized stress-strain diagram as shown in Fig. I.52 may be assumed and as long as the shearing strains remain quite small, the states of stress in the bar subjected to pure torsion are taken as states of simple shear only. This state of simple shear has two components τ_{xz} and τ_{yz} whose resultant has a constant value $\bar{k}$ in those parts of the cross section where plastic state of stress exists. Therefore by the condition of plasticity τ_{xz} and τ_{yz} should satisfy:

$$\tau_{xz}^2 + \tau_{yz}^2 = \bar{k}^2 = \text{constant} \qquad \ldots \text{I.8.56}$$

These components τ_{xz} and τ_{yz} must further satisfy the equilibrium condition:

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0 \qquad \ldots \text{I.8.57}$$

This equation is satisfied by putting $\tau_{xz} = \frac{\partial F}{\partial x}$ and

$\tau_{yz} = -\frac{\partial F}{\partial x}$ where $F(x,y)$ is the plastic stress function of the cross section.

Thus substituting in Eqn. I.8.56

$$\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 = \bar{k}^2 \qquad \ldots \text{ I.8.58}$$

The left hand side of Eqn. I.8.58 represents square of the absolute value of |grad. F|. Therefore we have |grad. F| = $\bar{k}$ = constant.

I.8.3.1 Sand-Hill Analogy[7]:-

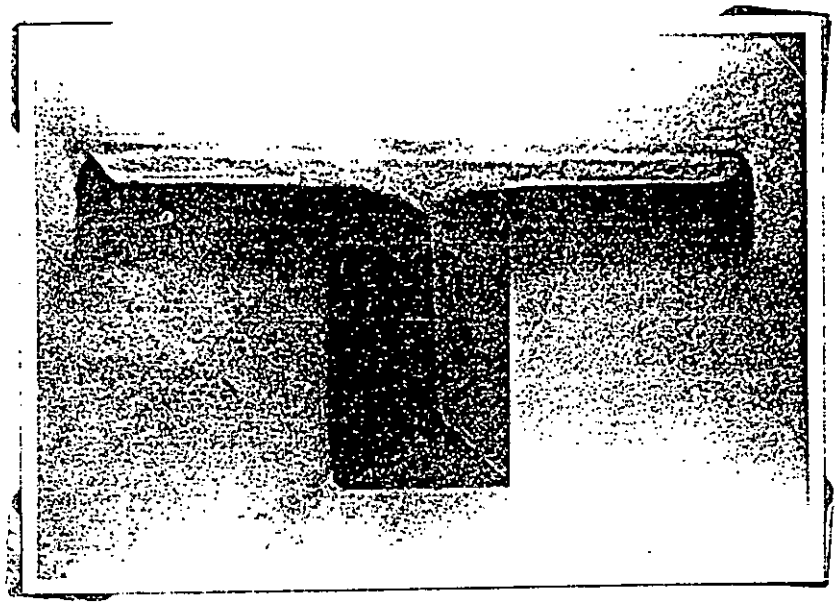
From the above property, plastic stress function F(x,y) may be thought of as a surface of constant maximum slope which one may construct over the edge of cross section. This leads to what is known as Sand-hill Analogy. That is, if the contour of the cross section is cut out of a piece of stiff pad or plywood and covered with sand while lying horizontally, there results a hill whose natural slope represents the picture of the plastic stress function F(x,y) and the volume of the surface formed is analogous to the torsional moment of the cross section in the complete plastic range.

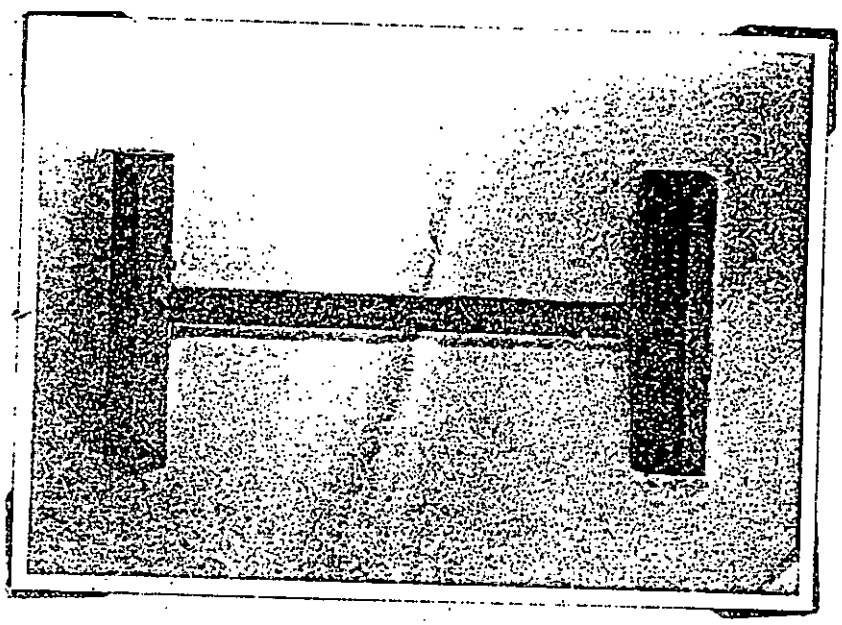
The rules of elastic torsion still hold good after the yield point has been reached i.e. the resulting shearing stress at any point in the cross section is the gradient of the stress surface F(x,y) and the twisting moment M_t of the bar is equal to twice the volume enclosed by this surface.

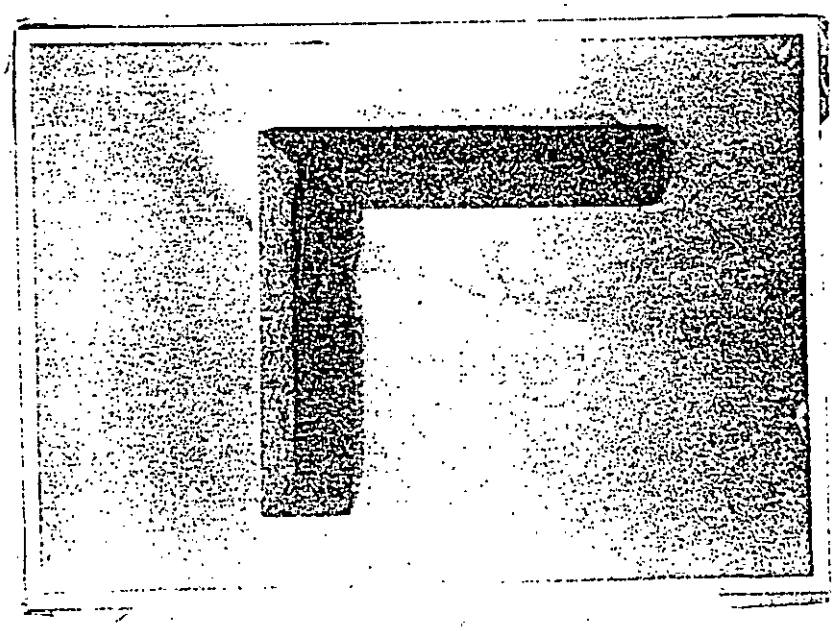
I.8.3.2 Twisting Moment for various Cross Sections

The shapes of the plastic stress function F(x,y) formed

Fig. I.53 : Plastic Torsion - Sand Hills on Tee, I and Ell Sections







as sand-hills on various cross-sections are shown in Fig. 53 (Photograph) and M_{pl} are calculated below:

(a) For Tee Section:-

$$V \simeq \bar{k}\left\{\frac{4}{3}B_1^3 + B_1^2(D-2B_1)\right\} + 2\bar{k}\left\{\frac{t^3}{6} + \frac{t^2}{4}(B-B_1-t) + \frac{t^3}{12}\right\}$$

$$M_t = \bar{k}\left\{2DB_1^2 + (B-B_1)t^2 - \frac{4}{3}B_1^3\right\} \quad \ldots \text{ I.8.59}$$

(b) For Ell Section:-

$$V \simeq \bar{k}\left\{\frac{a^3}{6} + \frac{a^2}{4}(b-a) + \frac{c^2}{4}(d-a)\right\}$$

$$M_t = \bar{k}\left\{\frac{a^3}{3} + \frac{a^2}{2}(b-a) + \frac{c^2}{2}(d-a)\right\} \quad \ldots \text{ I.8.60}$$

(c) For I Section:-

$$V \simeq 2\bar{k}\left\{\frac{c_2^3}{6} + \frac{c_2^2}{4}(b_2-c_2)\right\} + \bar{k}\left\{\frac{c_1^2}{2}(d-c_2) + \frac{c_1^3}{6}\right\}$$

$$M_t = 2\bar{k}\left\{\frac{c_2^3}{3} + \frac{c_2^2}{2}(b_2-c_2) + \frac{c_1^2}{2}(d-c_2) + \frac{c_1^3}{6}\right\}$$

$$\ldots \text{ I.8.61}$$

Numerical Results:-

Numerical results are obtained for the same sections for which elastic solutions were obtained and a comparative statement of maximum M_{el} and M_{pl} for various sections is shown in Table I.8.5. The plastic moments for Tee and

I-sections suggested by other investigators[4,27,60] are compared with the results of present analysis in Table I.8.6

TABLE I.8.5

TORSIONAL RIGIDITIES OF DIFFERENT SECTIONS AS PER ELASTIC AND RIGID PLASTIC THEORY

Section	Sectional Properties	M_{el} (Maximum)	M_{pl}	Ratio $\frac{M_{el}}{M_{pl}}$
Tee	$B = 5t$ $B_1 = \frac{1}{4}B$ $h = 4t$	$8.883\ \tau_m \cdot t^3$	$16.771\ \bar{k}t^3$	1.900
Ell	(i) $b = d$ $a = c$ $b = 5c$	$3.275\ \tau_m \cdot c^3$	$4.333\ \bar{k}c^3$	1.322
	(ii) $b = d$ $a = c$ $b = 10c$	$6.405\ \tau_m \cdot c^3$	$9.333\ \bar{k}c^3$	1.458
I	(i) $d = b_2$ $c_2 = 2c_1$ $b_2 = 5c_2$	$4.075\ \tau_m \cdot c_2^3$	$5.706\ \bar{k}c_2^3$	1.400
	(ii) $b_2 = d$ $c_2 = 2c_1$ $b_2 = 10c_2$	$7.529\ \tau_m \cdot c_2^3$	$11.956\ \bar{k}c_2^3$	1.590

TABLE I.8.6

COMPARISON OF RESULTS : TORSIONAL RIGIDITY BY RIGID PLASTIC THEORY

Section	Quantity	Sectional properties	By present analysis	By Nylander[4]	By Brown[27]	By Christopherson[60]
Tee	$\frac{M_{pl}}{kt^3}$	$B = 5t$ $B_1 = \frac{1}{2}B$ $h = 4t$	16.771	16.875	15.631	-
I	$\frac{M_{pl}}{kc_2^3}$	$d = b_2$ $c_2 = 2c_1$ $b_2 = 5c_2$	5.706	-	-	5.697
		$d = b_2$ $c_2 = 2c_1$ $b_2 = 10c_2$	11.956	-	-	11.929

From the above derivations, it is clear that the resulting equations derived for the elastic torsion are complicated and it may be cumbersome to use them directly to calculate the torsional rigidity of the section. In order to help quick calculation of rigidities, the solutions are worked out in a digital computer for Tee sections and the results are

given in the form of charts in the Appendix 1, for certain values of B/t and B_1/t. These charts are useful in the calculation of torsional properties of tee sections encountered in reinforced concrete construction as the contribution of fillets to the torsional rigidity in such cases is negligible and can be ignored for all practical purposes. The only limitation of these charts is that it cannot be directly used to find out the torsional rigidity of steel sections used in practice since it does not make any provision to include the contribution of fillets. However a better value of torsional rigidity of steel sections can still be obtained by adding to the value given by the charts the contribution of fillets which has been given by Darwish and Johnston[61]. Thus these charts can be directly used in the design of reinforced concrete structures and also in structural steel design with suitable modifications to account for the fillets.

CHAPTER I.9

THEORETICAL ANALYSIS OF REINFORCED CONCRETE BEAMS UNDER COMBINED BENDING AND TORSION

I.9.1 This chapter is devoted to the analytical study of reinforced concrete rectangular as well as flanged and hollow beams subjected to pure torsion and combined bending and torsion. All the analytical equations derived in this chapter are for the determination of the ultimate load carrying capacities of the beams. The analysis is done for the beams with both longitudinal and transverse reinforcements such that the capacities of beams with longitudinal steel only can be easily obtained from the general expressions derived for the former case.

As indicated at the end of Chapter I.2, theories for calculating the capacity of beams under combined bending and torsion are based on the concept of formation of plastic hinges at or near failure. The difficulty of this method of approach lies in the assumption to be made regarding the position of plastic hinge and the inclination of failure surface. Moreover the failure surface seems to vary with the ratio of bending to twisting moment and also with the type of

cross section and the manner of loading.

In this chapter the author attempts to establish a solution to the problem using a different method of approach. The analysis is based on the state of stress existing at the critical point of failure and a failure criterion for concrete under compressive-tensile stresses. Thus the author's theory presents a new line of thought. Moreover it offers a general solution both for beams with and without transverse reinforcement.

I.9.2 Assumptions:

The proposed theory is based on the following assumptions:

1. Concrete in the tension zone of the beam is neglected in the calculation of bending stresses. This assumption is made to keep the analysis of beams under combined bending and torsion similar to that of members subjected to pure bending.

2. The torsional rigidity of the beam is obtained using a semi-plastic concept in the case of beams failing in torsion. This assumption seems reasonable from the non-linear nature of deformations measured during experiments in such cases of failures.

3. The transverse steel yields before torsion failure of the beam and hence the axial stress in it is assumed to be equal to its yield stress. Experiments of author as well as of some other investigators[23,30] confirm this assumption.

4. The contribution of the horizontal (shorter) legs of the transverse hoops towards the torque capacity of the beam is neglected. This assumption is confirmed by the experiments conducted by Chinenkov[33] and Collins et al[56] on beams reinforced with steel skeletons. Their investigations have shown that practically there is no difference in the ultimate strengths of beams reinforced with steel skeletons (only vertical legs of stirrups) or closed hoops.

5. Contribution from dowel action of longitudinal steel is negligible. This is in confirmation with the experimental observations of previous investigators[3,9,10] and those of author (see Chapter I.7).

6. Concrete element at the critical point for failure is assumed to fail under the combined action of compressive-tensile stresses.

RECTANGULAR BEAMS

I.9.3.1 Beams subjected to pure torsion:

Failure Criterion:

In beams subjected to pure torsion, torsional shear

stresses are produced by the applied torque. At any point in the cross section, there are two shear stress components, say τ_{xz} and τ_{yz}. According to the plasticity condition, if the resultant of these shear stress components attains a constant value, the material at that point attains plastic state. In general every element in the cross section is subjected to a state of simple shear. The resultant shear stress gives rise to two unlike principal stresses which are equal in magnitude to the shear stress. Failure is caused by the destruction of concrete when these principal stresses satisfy the failure criterion for concrete under compressive-tensile stresses. This mechanism of failure is general and hence it is applicable both for beams with and without transverse hoops.

The above mechanism of failure results in the evaluation of strength of concrete under compressive-tensile principal stresses. So knowledge regarding a failure criterion for concrete under compressive-tensile stresses is essential at this stage. In the past many investigators[62-68] have suggested different types of failure criteria mostly in terms of normal and shear stresses. In 1964, Krishnaswamy[69] has developed a failure criterion for concrete under compressive-tensile stresses. This was later confirmed by large number of test results got from concrete specimens subjected to

compression, splitting and torsion. The author preferred to use this criterion because it is in terms of principal stresses and it is simple in form. A detailed review of different failure criteria given by various investigators is given in Appendix 2.

The failure criterion suggested by Krishnaswamy can be put in the following form:

$$\left(\frac{\sigma_1}{f'_c}\right) + \left(\frac{\sigma_2}{f'_t}\right)^2 = 1 \qquad \ldots \text{ I.9.1}$$

where

σ_1 - principal compressive stress

σ_2 - principal tensile stress

f'_c , f'_t - uniaxial compressive and tensile strengths of concrete, respectively.

In the absence of normal stresses,

$$\sigma_1 = -\sigma_2 = \tau \qquad \ldots \text{ I.9.2}$$

where τ is the shear stress caused by the applied torque. Substituting for τ from Eqn. I.9.2 in Eqn. I.9.1,

$$\left(\frac{\tau}{f'_c}\right) + \left(\frac{\tau}{f'_t}\right)^2 = 1 \qquad \ldots \text{ I.9.3}$$

Putting $f'_c/f'_t = k'$ in Eqn. I.9.3 and solving for τ we get,

$$\tau \simeq \left(\frac{2k'-1}{2k'^2}\right) f'_c \qquad \ldots \text{ I.9.4}$$

In beams with transverse reinforcement, the torque will be resisted both by concrete and the steel. The mechanism of failure of such beams considering the exact interaction between concrete and transverse steel may lead to highly complicated equations. So approximately the torque capacity of such beams is taken as follows:

Total torque M_{to} = Torque resisted by concrete M_{tc} + Torque resisted by transverse steel, M_{ts}

... I.9.5

(The negligible part contributed by the longitudinal steel has been ignored in Eqn. I.9.5)

The above approximation has been considered by Cowan[22] also in the evaluation of torque capacity of such beams. Moreover, such analogous approximation has normally been made by various investigators in the analysis of beams under bending and shear and it has been found to be sufficiently accurate for all practical purposes.

Torque Resisted by Concrete:

From the analysis of existing test results, it has

been found that neither the elastic nor the rigid plastic theories gives consistent results in the calculation of torque capacity of plain concrete beams. This is clearly brought out from the historical review of the problem by Collins et al[20]. From the analysis of test results, it seems that the elastic theory gives fairly good results at higher ranges of concrete strength and the rigid plastic theory gives good results when the concrete strength is very low. The reason for this variation is obvious due to the fact that for low strengths concrete the plasticity ratio* is high[70] so as to make major portion of the cross section to attain plastic state at failure. On the other hand, for high strengths concrete the plasticity ratio is very small such that major portion of the cross section remains at elastic state.

So, for the normal strengths of concrete used in the reinforced concrete construction, both the above theories may not be applicable to calculate M_{tc}. Hence the contribution by the concrete is obtained using a semi-plastic approach. This has also been clearly indicated by Ernst[30] and Grassam[28].

Hence in the present analysis, the torque resisted by the concrete is taken as a mean value of the torque capacities as given by elastic and rigid plastic theories.

---

* Plasticity ratio is defined as the ratio of plastic deformation to the total deformation of concrete at failure.

$$\text{i.e. } M_{tc} = \frac{1}{2}\left(M^{e}_{tc} + M^{p}_{tc}\right) \quad \ldots \text{ I.9.6}$$

$$\text{or } M_{tc} = \beta_s \, b^2 D \tau \quad \ldots \text{ I.9.7}$$

where M^{e}_{tc} = torque given by Elastic Theory[59] ;

M^{p}_{tc} = torque given by rigid plastic theory[7] ;

and β_s = semi plastic coefficient.

The values of the semi-plastic coefficient for different (D/b) ratios for rectangular beams are given in Table I.9.1.

TABLE I.9.1

VALUES OF COEFFICIENT β_s FOR RECTANGULAR BEAMS

D/b	1.0	1.2	1.5	2.0	2.5	3.0	4.0	5.0	10.0
β_s	0.271	0.285	0.310	0.332	0.346	0.356	0.370	0.379	0.400

Torque resisted by transverse hoops

(Only vertical hoops are considered in the analysis)

Experiments have shown that the transverse steel yields at failure of the beam. So the axial stress in the vertical legs of the transverse hoops can be assumed to be equal to the

yield stress of transverse steel.

Neglecting the effect of shorter legs of the transverse hoops,

Torque resisted by one hoop = $(A_w f_{yw}) \times b'$ ... I.9.8

where

A_w - area of cross section of <u>one</u> leg of the transverse hoop;

f_{yw} - yield stress of transverse steel;

and b' - breadth of the transverse hoop (See Fig. I.54)

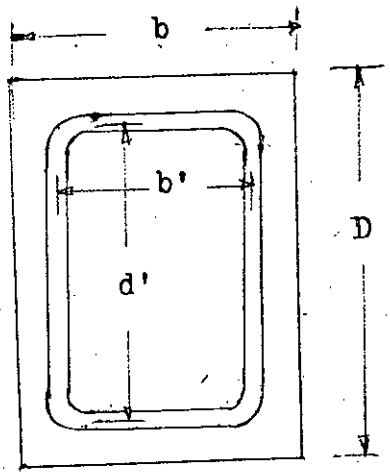


Fig. I.54

For beams subjected to pure torsion, the failure crack travels along the periphery of the cross section like a helix and its inclination to the longitudinal axis of the

beam is approximately equal to 45°. Considering the depth (centre to centre distance) of transverse hoop d', the projected length of the crack along the longitudinal axis of the beam is equal to d' cot ψ' where ψ' is the inclination of the crack.

Putting $\psi' = 45°$, we can get the projected length of crack along the longitudinal axis of the beam as equal to d'.

Let s be the spacing of transverse hoops. Then number of hoops resisting the torque will be equal to (d'/s).

Therefore total torque resisted by transverse hoops

$$= (A_w f_{yw} b') \frac{d'}{s}$$

$$\text{or } M_{ts} = A_w f_{yw} \frac{b'd'}{s} \qquad \ldots \text{ I.9.9}$$

Now substituting the values of M_{tc} and M_{ts} from Eqns. I.9.7 and I.9.9 in Eqn. I.9.5, we have,

$$M_{to} = \beta s\, b^2 D \tau + \frac{A_w f_{yw} b'd'}{s} \qquad \ldots \text{ I.9.10}$$

If failure is to occur, the shear stress τ in Eqn. I.9.10 should satisfy the failure criterion for concrete discussed earlier.

So substituting the value of τ from Eqn. I.9.4 in the Eqn. I.9.10 we can deduce that

$$M_{to} = \left(\frac{2k'-1}{2k'^2}\right) \beta_s \, b^2 D \, f'_c + \frac{A_w f_{yw} b'd'}{s} \qquad \ldots \text{ I.9.11}$$

I.9.3.2 Beams subjected to combined bending and torsion:

In a beam subjected to combined bending and torsion, there are two possible causes for failure:

i. Flexure or compression failure by crushing of concrete in the compression zone after yielding of longitudinal steel;

ii. Torsion failure by diagonal splitting.

Now it is necessary to derive expressions for these two causes of failure. The critical points of failure for these two cases will obviously be different from one another because of the basic difference in the resulting effect of normal stress on shearing stress. In other words, the shear stress decreases the carrying capacity of an element in taking up the normal stress whereas the presence of a compressive normal stress enhances its capacity in resisting the shear stresses.

i. Flexure Failure:

At the critical point for failure, let f be the horizontal normal stress (compressive) due to the applied bending moment and τ be the shear stress due to the applied

torque at the same point. By interaction, these stresses will yield two principal stresses:

$$\sigma_1 = \frac{f}{2}\left[\sqrt{1 + \left(\frac{2\tau}{f}\right)^2} + 1\right] \qquad \text{(compressive)}$$

$$\sigma_2 = \frac{f}{2}\left[\sqrt{1 + \left(\frac{2\tau}{f}\right)^2} - 1\right] \qquad \text{(tensile)}$$

... I.9.12

Designating $\left(\frac{2\tau}{f}\right) = \lambda\phi$

and $\sqrt{1 + \lambda^2 \phi^2} = \gamma$ We have

$$\sigma_1 = \frac{f}{2}(\gamma + 1)$$

$$\sigma_2 = \frac{f}{2}(\gamma - 1) \qquad \text{... I.9.13}$$

Substituting the values of σ_1 and σ_2 in the failure criterion (Eqn. I.9.1) and solving for f we get,

$$\left(\frac{f}{f'_c}\right) = \left(\frac{f'_t}{f'_c}\right)^2 \frac{(\gamma + 1)}{(\gamma - 1)^2}\left[\sqrt{1 + \left\{\left(\frac{2f'_c}{f'_t}\right)\left(\frac{\gamma - 1}{\gamma + 1}\right)\right\}^2} - 1\right]$$

... I.9.14

Neglecting the small difference in the values of depths of compression blocks of a beam subjected to pure bending and a beam under combined bending and torsion, we can approximately equate the ratio (f/f'_c) to (M_b / M_{bo}) where

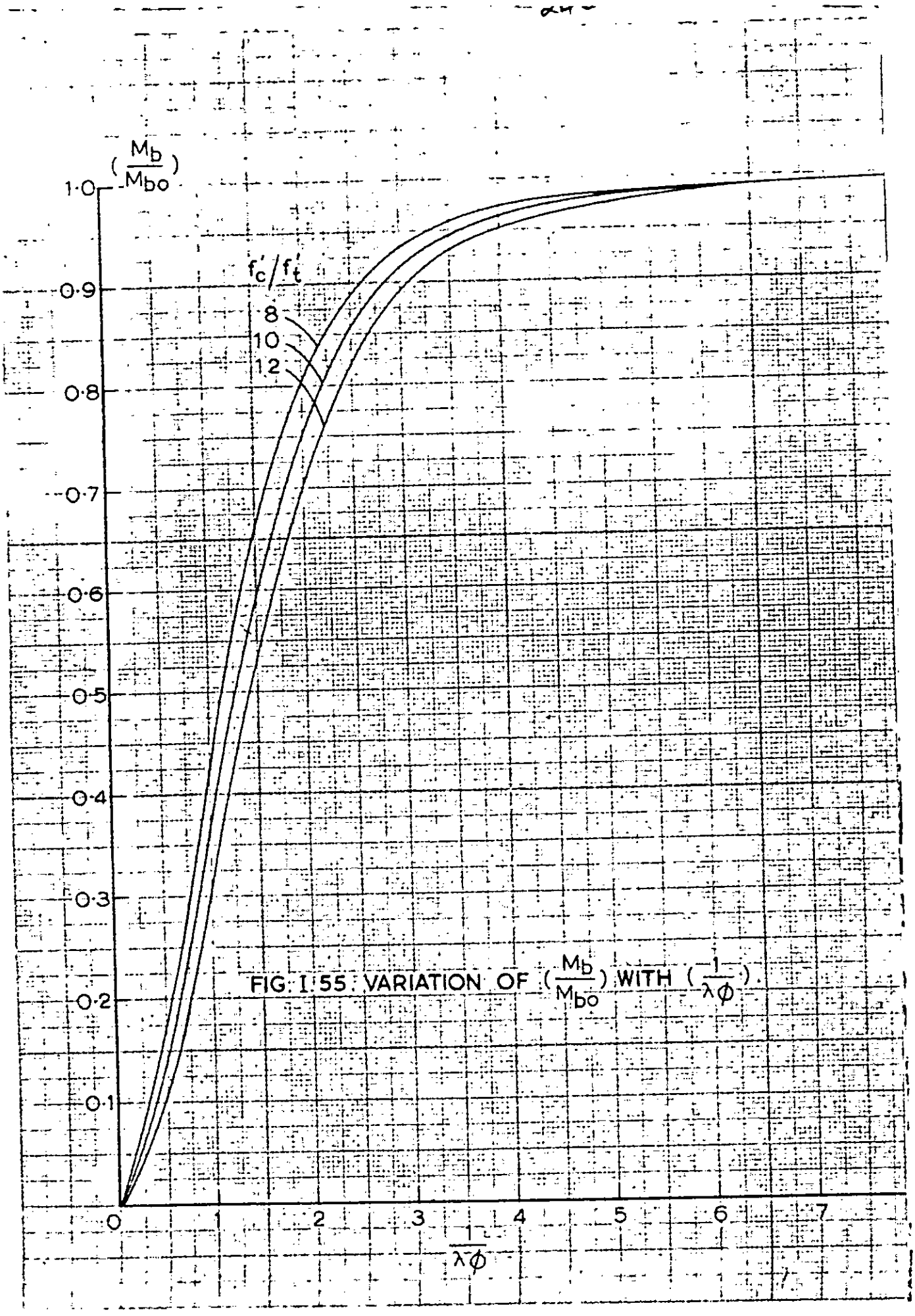


FIG. I 55 VARIATION OF $\left(\frac{M_b}{M_{bo}}\right)$ WITH $\left(\frac{1}{\lambda\phi}\right)$.

M_b is the ultimate flexural capacity of a beam under combined bending and torsion and

M_{bo} is the flexural capacity of the same beam under pure bending.

Thus Eqn. I.9.14 reduces to,

$$\left(\frac{M_b}{M_{bo}}\right) = \left(\frac{f'_t}{f'_c}\right)^2 \frac{(\gamma+1)}{(\gamma-1)^2}\left[\sqrt{1+\left\{\left(\frac{2f'_c}{f'_t}\right)\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^2} - 1\right] \quad \ldots \text{ I.9.15}$$

Eqn. I.9.15 directly gives the value of the quantity (M_b/M_{bo}), which expresses the reduction in the flexural capacity of the beam under combined bending and torsion, for different values of f'_c/f'_t in terms of the parameter $\lambda\phi$. This is diagramatically shown in Fig. I.55.

Determination of $\lambda\phi$:

As defined earlier, $\lambda\phi$ is equal to $\left(\frac{2\tau}{f}\right)$ where τ and f correspond to the magnitude of shear and normal stresses respectively at the critical point for flexure failure. The most critical point for flexure failure is the extreme fibre in the compression zone of the beam (ie. top of the beam) where the effect of torsional shear stresses on the bending stresses (compressive) is maximum. This is clear and evident

from the observed modes of failure (See Chapter I.6). If the beam has to fail in flexure due to crushing of concrete, the torsional stresses will have to be necessarily in the elastic range and the axial stresses in the transverse hoops will be small. So the torsional shear stresses can be calculated using the elastic theory. Even though the maximum shear stress occurs at the middle of longer sides of the rectangle according to the elastic theory, this point is not the critical one for flexure failure. So the shear stress corresponding to the middle of shorter sides is considered in the analysis. Then,

$$M_t = \beta_e \; b^2 \; D\tau \qquad \ldots \text{ I.9.16}$$

where

M_t - applied torque;

and β_e - elastic torsional shear stress coefficient corresponding to the middle of shorter sides of the rectangular beam[71].

(Since the axial stress in the transverse hoops will be small in a beam failing in flexure, its contribution to the torque M_t is neglected in Eqn. I.9.16).

In Eqn. I.9.16, even though the section might have been traversed by flexural cracks, the entire section is assumed

to be effective in the calculation of shear stress. One may question the validity of this assumption. This can be explained by using the test observations discussed in Chapter I.7 (vide Art. I.7.5.5). From the test observations it has been found that the presence of flexural cracks affect the torque capacity of beams to a small extent only. The difference between the torque capacities of flexurally cracked beams and uncracked beams varied from 0 to 8 per cent. Also the test results of Ersoy[58] on semi-continuous Ell beams have shown that the flexural cracks affect the torque capacity by 6 to 7 percent only. Hence it seems reasonable to consider the gross section in the calculation of torsional shear stress, τ.

Assuming a non-linear stress block for the compression zone of the beam (Fig. I.56), we have,

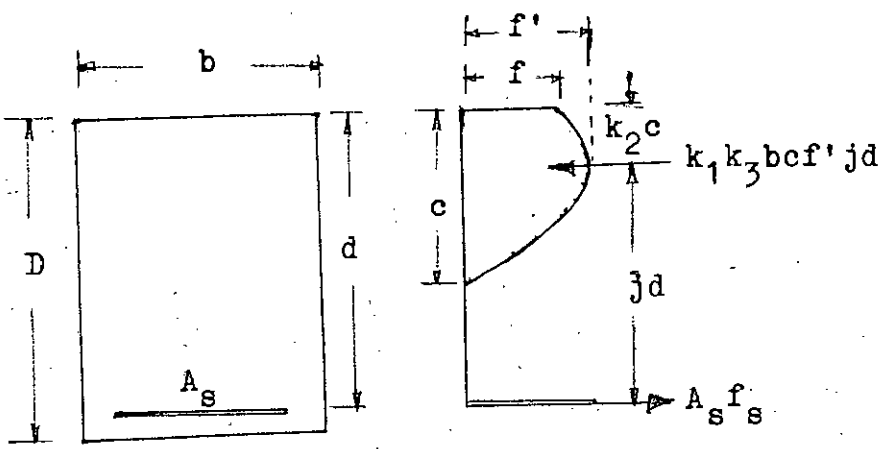


Fig. I.56

$$M_b = k_1 k_3 \, bc \, f' \, jd$$

$$\text{or} \quad f' = \frac{M_b}{k_1 k_3 bcjd} \qquad \ldots \text{ I.9.17}$$

where f' is the maximum stress.
(Contribution from the tension zone concrete is conservatively neglected).

Assuming the stress at the extreme fibre as 90% of the maximum stress f' (based on the "ideal" stress-strain curve of concrete reported by Whitney[72] corresponding to a maximum concrete strain of 0.3%) we have,

$$f = 0.90 f' = \frac{0.90 M_b}{k_1 k_3 \, bc \, jd} \qquad \ldots \text{ I.9.18}$$

Now $\lambda\phi = \frac{2\tau}{f}$

Substituting for τ and f from Eqns. I.9.16 and I.9.18 and observing that $\phi = M_t/M_b$ we get

$$\lambda = \frac{2k_1 k_3 \, bd^2 j(1-j)}{0.90 \beta_e \, b^2 D \, k_2} \qquad \ldots \text{ I.9.19}$$

Further, putting $D \simeq 1.2\ d$ and $j = \frac{7}{8}$, Eqn. I.9.19 reduces to:

$$\lambda = \frac{D}{b} \frac{1}{6 \beta_e (k_2/k_1 k_3)} \qquad \ldots \text{ I.9.20}$$

Observing $(k_2/k_1 k_3) = 0.525$ (see Appendix 3 for details) the

value of λ is necessarily a function of D/b ratio and hence it can be easily tabulated for different values of D/b ratio. Such values are given in Table I.9.2.

TABLE I.9.2

VALUES OF COEFFICIENT λ FOR RECTANGULAR BEAMS

D/b	1.0	1.2	1.5	2.0	2.5	3.0	4.0	5.0	10.0
λ	1.528	1.640	1.762	2.060	2.400	2.690	3.355	4.060	7.600

Thus knowing the value of λ from Table I.9.2, the reduction in the flexural capacity of the beam under combined bending and torsion can be easily obtained from the Fig.I.55.

(ii) Torsion Failure:

Torsion failure occurs in the case of large values of ϕ. The critical point for failure in torsion is that point where the influence of bending stresses (compressive) on shear stresses is minimum. So the critical point for torsion failure is at the level of neutral axis where the bending stresses are zero. The concrete element at the level of neutral axis is subjected to a state of simple shear. This

will result in two unlike principal stresses of equal magnitude and hence the problem is same as that of a beam under pure torsion. As in the case of beams under pure torsion, the torsional stresses are assumed to attain a semi-plastic state and the transverse steel will yield at failure. Hence the Eqn. I.9.11 derived for the 'pure torsion' case is valid here also. Hence the torque capacity of a beam failing prematurally in torsion, say M_t, is equal to its torque capacity under pure torsion i.e. $M_t = M_{to}$.

FLANGED AND HOLLOW BEAMS

I.9.4 The equations derived hitherto can be extended to reinforced concrete beams of other sections also. This can be easily achieved because the general mechanism and condition for failure is independent of the cross sections of beams.

The two important parameters introduced in the author's theory are the semi-plastic coefficient β_s and the parameter λ. The semi-plastic coefficient β_s depends on the elastic and rigid plastic torsional rigidities of the cross section. The parameter λ depends on the effective area in the compression zone of the beam resisting the bending stresses and the elastic torsional shear stress at

the point where the effect of interaction of stresses is maximum. The method of calculation of bending stresses for any type of cross section is well-known and the elastic and rigid plastic solutions for the torsional properties of flanged beams have been given in Chapter I.8. Using these results the values of β_s and λ can be calculated.

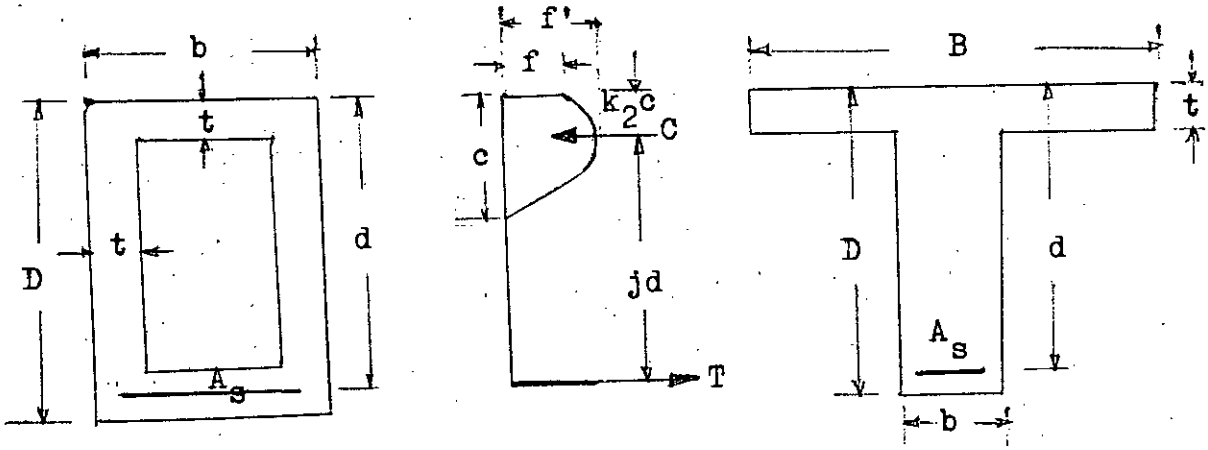


Fig. I.57

Tee Beams:

Value of λ:- Referring to Fig.I.57,

$$M_b = k_1 k_3 f' \, jd \left[bc + (B-b)t \right] \qquad \ldots \text{ I.9.21}$$

$$\text{or} \quad f = 0.90f' = \frac{0.90 \, M_b \, (k_2/k_1k_3)}{bd^2 \, j(1-j)\left[1 + \dfrac{(B-b)\, tk_2}{b(1-j)d}\right]} \qquad \ldots \text{ I.9.22}$$

Let $M_t = \beta_e^t \, t^3 \tau$

where β_e^t - elastic torsional shear stress coefficient for tee beam corresponding to the mid point of the flange.

Using Eqns. I.9.21 and I.9.22, the value of λ can be written as follows:

$$\lambda = \frac{bD^2\left[1 + 9.6\left(\frac{B}{b} - 1\right)\frac{t}{D}k_2\right]}{6\,\beta_e^t\left(\frac{k_2}{k_1 k_3}\right)t^3} \qquad \ldots \text{ I.9.23}$$

Observing $k_2 = 0.42$ and $k_2/k_1k_3 = 0.525$ (See Appendix 3 for details)

$$\lambda = \frac{\left(\frac{b}{D}\right)\left[1 + \left(\frac{B}{b} - 1\right)4.04\,\frac{t}{D}\right]}{3.15\,\beta_e^t\left(\frac{t}{D}\right)^3} \qquad \ldots \text{ I.9.24}$$

Value of β_s :

Let $M_{tc}^e = \beta_e\, b^2 D\tau$

$M_{tc}^p = \beta_p\, b^2 D\tau$

Then $M_{tc} = \beta_s^t\, b^2\, D\tau$ ... I.9.25

where $\beta_s^t = \frac{(\beta_e + \beta_p)^{tee}}{2}$

For example, these values for the tee sections tested in author's investigation can be obtained as follows:

For author's tee beams,

$$M_t^e = 21.132\ G\theta t^4$$

$$M_t^e = 9.40\ \tau_m t^3$$

$$M_t^e = 10.05\ \tau_f t^3$$

where τ_m - maximum shear stress at middle of web

τ_f - shear stress at mid point of the flange.

Using these results, the value of λ reduces to 4.28.

Similarly, $M_{tc}^e = 0.283\ b^2 D\tau$

$M_{tc}^p = 0.465\ b^2 D\tau$

Therefore $M_{tc} = \beta_s^t\ b^2 D\tau$

where $\beta_s^t = \frac{0.283 + 0.465}{2} = 0.374.$

Hollow Beams:-

Value of λ:

Referring to Fig. I.57,

$$M_b = k_1 k_3 f' jd \left\{bt + (c-t)2t\right\} \qquad \ldots \text{ I.9.26}$$

where $c = (1-j)\frac{d}{k_2}$

$$\text{or } M_b = k_1 k_3 f' jd \left[bt + 2t\left\{\frac{(1-i)d}{k_2} - t\right\}\right] \quad \ldots \text{ I.9.26}$$

$$\text{or } f = 0.90f' = \frac{0.90 M_b}{k_1 k_3 jd\left[bt + 2t\left\{\frac{(1-i)d}{k_2} - t\right\}\right]} \quad \ldots \text{ I.9.27}$$

Since in the case of hollow beams the wall thickness will be comparitively small, the torsional rigidities calculated using elastic and rigid plastic theories may give approximately same results.

Sadowsky[73] and Gurtin[74] have given rigid plastic solution for multiply connected sections. Using their solution the torsional strength of a hollow beam at plastic state can be put in the following form:

$$\frac{M^p_{tc}}{\tau} = \left[\frac{1}{6} b^2 D \left(3 - \frac{b}{D}\right)\right] - \left[\frac{1}{6} (b-2t)^2 (D-2t) \left\{3 - \frac{b-2t}{D-2t}\right\}\right] \quad \ldots \text{ I.9.28}$$

$$\text{or } M^p_{tc} = \beta_p b^2 D \tau \quad \ldots \text{ I.9.29}$$

where β_p is the plastic torsional coefficient depending on the cross sectional dimensions of the hollow beam. (Vide Table I.9.3)

TABLE I.9.3

VALUES OF COEFFICIENT β_p FOR HOLLOW BEAMS

D/b	β_p			
	t/b = 0.1	t/b = 0.2	t/b = 0.3	t/b = 0.4
1.0	0.163	0.262	0.312	0.331
1.2	0.166	0.271	0.330	0.356
1.5	0.168	0.281	0.348	0.381
2.0	0.171	0.291	0.366	0.405
2.5	0.173	0.296	0.377	0.420
3.0	0.174	0.300	0.384	0.430
4.0	0.176	0.306	0.393	0.444
5.0	0.177	0.310	0.400	0.450
10.0	0.178	0.314	0.408	0.465

The author is not aware of any exact solution for the elastic torsion of hollow beams except the solution given by Quinlan[75] for square hollow beams. The approximate solution given in the book by Timoshenko and Goodier[59] for such sections is based on the membrane analogy. In the absence of an exact solution, this approximate solution can be used.

The torsional moment was calculated using these methods for the beams reported by Evans and Sarkar.[54] The results are summarised in Table I.9.4.

TABLE I.9.4

COMPARISON OF RESULTS : TORSIONAL RIGIDITY OF HOLLOW BEAMS

Section	M_t/τ	
	Elastic Theory	Rigid Plastic Theory
6" x 7 1/2"	81.00	83.25
6" x 9"	101.25	103.50
6" x 12"	141.75	144.00

From Table I.9.4, it can be seen that the results obtained using the two theories are approximately the same. So it may be reasonable to use the rigid plastic theory itself for the calculation λ and βs .

Now, $\lambda\phi = \frac{2\tau}{f}$

Substituting for τ and f from Eqns. I.9.27 and I.9.29, we get

$$\lambda = \frac{2k_1k_3\ jd\left[bt + 2t\left\{\frac{(1-j)d}{k_2} - t\right\}\right]}{0.90\ \beta_p\ b^2D} \qquad \ldots\ I.9.30$$

putting $j = \frac{7}{8}$; $k_1k_3 = 0.800$ and $k_2 = 0.42$, we get,

$$\lambda = \frac{1.555d}{\beta_p\ b^2D}\left[bt + 2t\ (0.30d - t)\right] \qquad \ldots\ I.9.31$$

Using Eqn. I.9.31, the value of λ can be calculated.

Value of β_s:-

In the case of hollow beams $\beta_s = \beta_p$ and the torque contributed by the concrete can be known using Eqn. I.9.29.

I.9.5 Adopting the procedure indicated for tee and hollow beams, the values of λ and β_s can be known for beams of other cross sections. Further, as the theory developed in this chapter is general, it is applicable for beams without transverse reinforcement also.

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CHAPTER I.10

DISCUSSION OF ANALYTICAL RESULTS

I.10.1 In the previous chapter, a theory has been proposed for predicting the strength of reinforced concrete beams both with longitudinal and transverse reinforcements under combined bending and torsion. In this chapter, the results given by the theory are discussed in the light of the experimental results. Before actual comparison of results, it is necessary to define the ranges of applicability of different equations derived in the previous chapter.

I.10.2 Determination of Failure Zones:-

In beams subjected to combined bending and torsion, as stated earlier, there are two causes of failure and expressions have been derived for these. It is desirable to define the ranges of applicability of these. Even though the experiments of the author as well as those of previous investigators[17,23,33,34,53,54,56] have shown that flexure failure can occur over larger ranges of ϕ, there should be a limit beyond which flexure failure may not be possible.

By Eqn. I.9.14 we have,

$$\frac{f}{f'_c} = \left(\frac{f'_t}{f'_c}\right)^2 \frac{(\gamma+1)}{(\gamma-1)^2}\left[\sqrt{1+\left\{\left(\frac{2f'_c}{f'_t}\right)\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^2}-1\right] \quad \ldots \text{ I.10.1}$$

and $\lambda\phi = \frac{2\tau}{f}$

Transforming Eqn. I.10.1 in terms of τ/f'_c we get,

$$\left(\frac{\tau}{f'_c}\right) = \left(\frac{f'_t}{f'_c}\right)^2 \frac{(\gamma+1)}{(\gamma-1)^2}\left[\sqrt{1+\left\{\left(\frac{2f'_c}{f'_t}\right)\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^2}-1\right]\left(\frac{\lambda\phi}{2}\right) \quad \ldots \text{ I.10.2}$$

In the case of beams failing in flexure we have,

$$M_t = \beta_e\, b^2 D\tau \quad \text{or} \quad M_t = \beta_e\, b^2 D f'_c \left(\frac{\tau}{f'_c}\right) \quad \ldots \text{ I.10.3}$$

Substituting the value of τ/f'_c from Eqn. I.10.2, Eqn. I.10.3 reduces to,

$$M_t = \beta_e\, b^2 D f'_c \cdot \left(\frac{f'_t}{f'_c}\right)^2 \frac{(\gamma+1)}{(\gamma-1)^2}\left[\sqrt{1+\left\{\left(\frac{2f'_c}{f'_t}\right)\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^2}-1\right]\frac{\lambda\phi}{2} \quad \ldots \text{ I.10.4}$$

$$\text{or} \quad \left(\frac{M_t}{\beta_e b^2 D f'_c}\right) = \left(\frac{\lambda\phi}{2}\right)\left(\frac{f'_t}{f'_c}\right)^2 \frac{\gamma+1}{(\gamma-1)^2}\left[\sqrt{1+\left\{\frac{2f'_c}{f'_t}\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^2}-1\right] \quad \ldots \text{ I.10.5}$$

The Eqn. I.10.5 gives the torque capacity of beams failing in flexure and it can be seen that it is similar to the Eqn. I.9.15 and the quantity $(M_t/\beta_e b^2 D f'_c)$ depends on the value of $\lambda\phi$ and f'_c/f'_t.

At the transition point, the torque capacities of beam as given by the equations derived for the two causes of failure should be equal. So at the transition point, M_t as given by the Eqn. I.10.5 should be equal to M_{to} which corresponds to the torque capacity of the beam as per the torsion failure equation. The value of M_t as given by Eqn. I.10.5 varies with the value of $(1/\lambda\phi)$. Designating that particular value of $1/\lambda\phi$ at which the transition occurs as $(1/\lambda\phi)_{TR}$ we have,

$$\left(\frac{M_{to}}{\beta_e\, b^2 D f'_c}\right) = \frac{1}{2}\,\frac{1}{(1/\lambda\phi)_{TR}}\left(\frac{f'_t}{f'_c}\right)^2 \frac{\gamma'+1}{(\gamma'-1)^2}\left[\sqrt{1+\left\{\frac{2f'_c}{f'_t}\left(\frac{\gamma'-1}{\gamma'+1}\right)\right\}^2}-1\right] \quad \ldots \text{ I.10.6}$$

$$\text{where} \quad \gamma' = \frac{\sqrt{1+(1/\lambda\phi)^2_{TR}}}{(1/\lambda\phi)_{TR}}$$

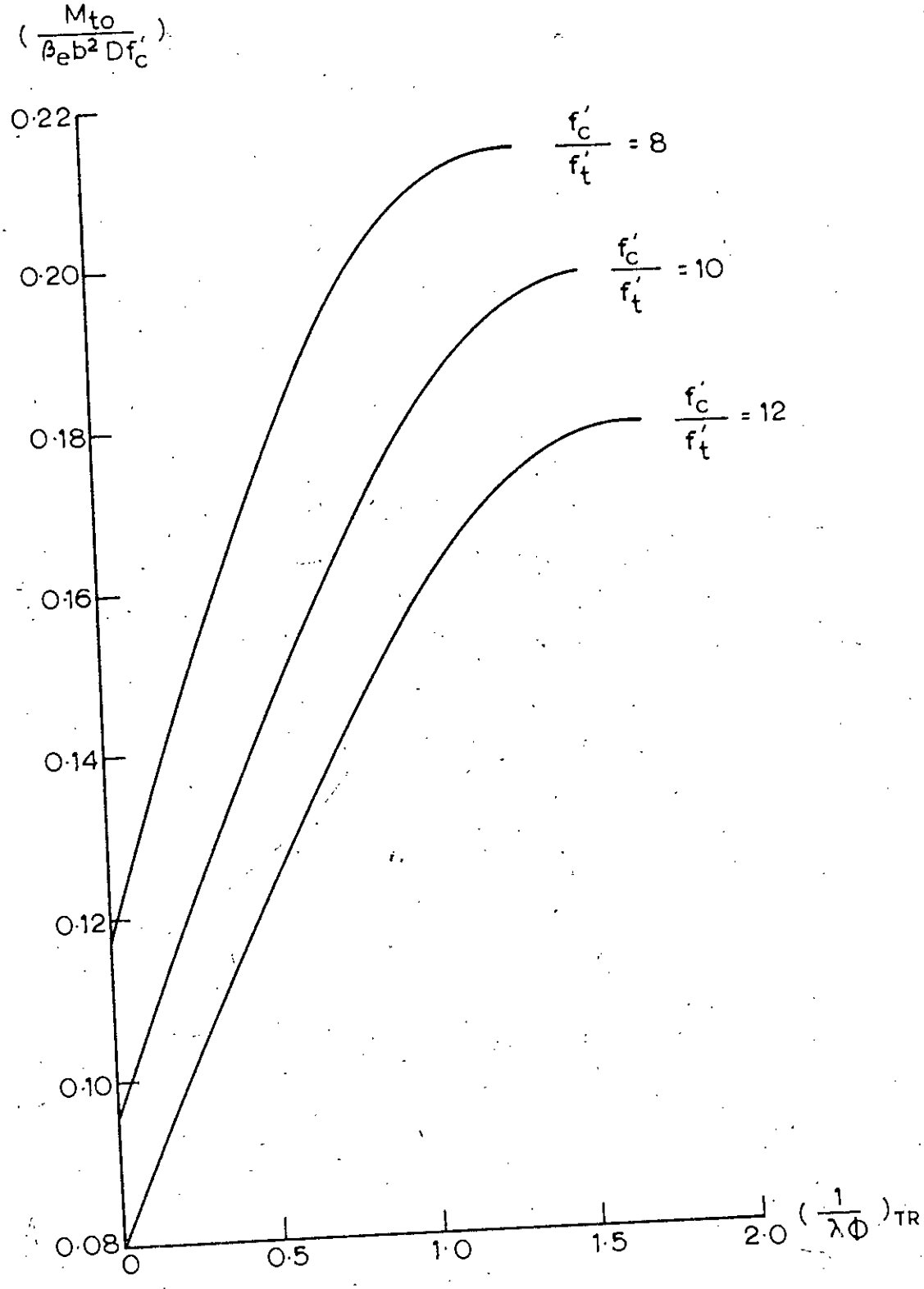


FIG. I.58. VARIATION OF $\left(\frac{1}{\lambda\Phi}\right)_{TR}$ WITH $\left(\frac{M_{to}}{\beta_e b^2 D f_c'}\right)$

The Eqn. I.10.6 can be graphically represented for various values of f'_c/f'_t . Such a diagram is given in Fig. I.58. Using Fig.I.58, for the known values of $(M_{to}/\beta_e b^2 D f'_c)$ and f'_c/f'_t , the value of $(1/\lambda\phi)_{TR}$ can be easily known. Then,

i. if a particular value of $1/\lambda\phi$ is greater than $(1/\lambda\phi)_{TR}$, the failure is termed as flexure failure and the reduction in the flexural capacity of the beam can be known using Fig. I.55;

ii. if the value of $1/\lambda\phi$ is less than $(1/\lambda\phi)_{TR}$, the failure is termed as torsion failure and the torque capacity of the beam is equal to M_{to} and it is given by the Eqn.I.9.11.;

iii. if the value of $1/\lambda\phi$ is equal to $(1/\lambda\phi)_{TR}$, it represents the transition point and the capacity of the beam can be known using either flexure failure equation or torsion failure equation.

I.10.3 Comparison of Results:

In Tables I.10.1 and I.10.2 the experimental results of about three hundred beams are compared with the theoretical predictions. In Table I.10.1 the results are compared for beams with $(1/\lambda\phi) > (1/\lambda\phi)_{TR}$ and Table I.10.2 provides a comparison for the case of beams with

TABLE I.10.1

COMPARISON OF RESULTS FOR BEAMS WITH $(1/\lambda\phi) > (1/\lambda\phi)_{TR}$

(FLEXURE FAILURE)

Investigator	Specimen Mark	$(\frac{f'_c}{f'_t})$	ϕ	λ	M_b in-kips	M_{bo} in-kips	M_b / M_{bo} Test	M_b / M_{bo} Calc.	Ratio Test/Calc.
1	2	3	4	5	6	7	8	9	10
Author	V2	12	0.298	1.83	121	173	0.70	0.66	1.06
	V3	12	0.179	1.83	140	173	0.81	0.89	0.91
	V5	12	0.275	1.83	142	173	0.82	0.71	1.15
	S1 II	12	0.156	1.83	154	154	1.00	0.93	1.08
	S2 I	12	0.104	1.83	154	154	1.00	0.98	1.02
	S2 II	12	0.238	1.83	130	154	0.84	0.79	1.06
	S4 I	12	0.081	1.83	148	154	0.96	0.99	0.97
	S4 II	12	0.235	1.83	132	154	0.86	0.79	1.09
	S5 I	12	0.135	1.83	148	154	0.96	0.95	1.01
	S5 II	12	0.212	1.83	132	154	0.86	0.84	1.02
	R1	12	0.715	1.83	35	135	0.26	0.22	1.18
	R2	12	0.252	1.83	111	135	0.82	0.76	1.08
	R3 II	12	0.162	1.83	185	185	1.00	0.92	1.09
	R5 I	12	0.045	1.83	200	185	1.08	1.00	1.08
	R5 II	12	0.162	1.83	185	185	1.00	0.92	1.09
	R6	12	0.515	1.83	64	185	0.35	0.34	1.03

(continued)

Table I.10.1 (contd.)

1	2	3	4	5	6	7	8	9	10
Author	L1-2 II	12	0.535	1.83	60	185	0.32	0.33	0.97
	L2-1 I	12	0.055	1.83	181	185	0.98	0.99	0.99
	L2-1 II	12	0.458	1.83	72	185	0.39	0.40	0.98
	L2-2 I	12	0.055	1.83	181	185	0.98	0.99	0.99
	L2-2 II	12	0.458	1.83	72	185	0.39	0.40	0.98
	L3-1 I	12	0.123	1.83	163	185	0.88	0.97	0.91
	L3-1 II	12	0.286	1.83	105	185	0.57	0.68	0.84
	L3-2 I	12	0.123	1.83	163	185	0.88	0.97	0.91
	L3-2 II	12	0.294	1.83	102	185	0.55	0.66	0.84
	L4-1	12	0.353	1.83	85	185	0.46	0.56	0.82
	L5-2	12	0.500	1.83	58	185	0.32	0.35	0.92
	L6-1 I	12	0.123	1.83	163	185	0.88	0.97	0.91
	L6-2 I	12	0.141	1.83	156	185	0.85	0.94	0.91
	T 1	10	0.370	4.28	57	175	0.32	0.25	1.28
	T 2	10	0.216	4.28	98	175	0.56	0.49	1.14
	T 5 I	10	0.098	4.28	163	175	0.93	0.87	1.07
	T 5 II	10	0.127	4.28	157	175	0.90	0.77	1.17
	T 7 II	10	0.157	4.28	140	175	0.80	0.67	1.19
	T 8	10	0.300	4.28	60	175	0.34	0.33	1.03
	T 9	10	0.103	4.28	175	175	1.00	0.87	1.15
	T 10	10	0.083	4.28	158	175	0.91	0.92	0.99

(contimued)

Table I.10.1 (contd.)

1	2	3	4	5	6	7	8	9	10
Author	T12 I	10	0.105	4.28	114	114	1.00	0.87	1.15
	T12 II	10	0.143	4.28	105	114	0.92	0.71	1.29
	T14	10	0.208	4.28	96	114	0.84	0.50	1.68
	T15	10	0.219	4.28	105	114	0.92	0.49	1.88
	T17 I	10	0.097	4.28	175	175	1.00	0.87	1.15
	T17 II	10	0.114	4.28	158	175	0.91	0.84	1.08
	T18	10	0.146	4.28	158	175	0.91	0.71	1.28
	T20 I	10	0.114	4.28	175	175	1.00	0.84	1.19
Cowan & Armstrong (Ref. No.23)	S2	12	0.250	1.76	258	258	1.00	0.78	1.28
	R2	12	0.500	1.76	158	258	0.61	0.38	1.60
	R1	12	0.167	1.76	258	258	1.00	0.92	1.09
	S1	12	0.400	1.76	206	258	0.80	0.52	1.54
Gesund and Boston (Ref. No.52)	3	10	1.000	1.53	58	180	0.32	0.21	1.52
	4	10	1.000	1.53	64	180	0.35	0.21	1.67
	5	10	0.500	1.53	86	180	0.48	0.50	0.96
	6	10	0.333	1.53	108	180	0.60	0.74	0.81
	7	10	0.333	1.53	177	180	0.98	0.74	1.32
	8	10	0.250	1.53	195	180	1.08	0.85	1.27
	9	12	0.500	2.06	83	276	0.30	0.31	0.97
	10	12	0.250	2.06	156	276	0.57	0.69	0.83

(continued)

Table I.10.1 (contd.)

1	2	3	4	5	6	7	8	9	10
Gesund et al (Ref. No.53)	3	12	0.500	1.53	122	180	0.68	0.45	1.51
	5	12	0.333	1.53	147	180	0.82	0.70	1.17
	7	12	0.250	1.53	173	180	0.96	0.85	1.13
	9	12	0.500	2.06	120	276	0.44	0.30	1.47
	10	12	0.250	2.06	176	276	0.64	0.70	0.92
	11	12	0.500	2.06	138	276	0.50	0.30	1.67
	12	12	0.250	2.06	213	276	0.77	0.70	1.10
** Chinenkov (Ref. No.33)	B2-8-a1	8	0.100	1.76	486	486	1.00	0.99	1.01
	0-1a	8	0.100	1.76	470	475	0.99	0.99	1.00
	0.2	8	0.200	1.76	417	490	0.85	0.92	0.93
	0.2a	8	0.200	1.76	417	485	0.86	0.92	0.94
	0.4B	8	0.280	1.76	348	505	0.69	0.81	0.85
	0.4	8	0.400	1.76	365	515	0.71	0.63	1.13
	0.4a	8	0.400	1.76	348	505	0.69	0.63	1.10
	0.4b	8	0.400	1.76	365	530	0.69	0.63	1.10
	0.4c	8	0.400	1.76	382	570	0.67	0.63	1.06
	0.4d	8	0.400	1.76	312	480	0.65	0.63	1.03
	0.4e	8	0.400	1.76	330	505	0.65	0.63	1.03
	0.4f	8	0.400	1.76	348	520	0.67	0.63	1.06
	0.4g	8	0.224	1.76	435	485	0.90	0.89	1.01

(continued)

Table I.10.1 (contd.)

1	2	3	4	5	6	7	8	9	10
** Chinenkov (Ref. No.33)	B-2-20-02	12	0.200	1.76	486	525	0.93	0.87	1.07
	02a	12	0.200	1.76	486	554	0.88	0.87	1.01
	10-02	12	0.200	1.76	417	515	0.81	0.87	0.93
	02a	12	0.200	1.76	452	545	0.83	0.87	0.96
	02b	12	0.200	1.76	452	515	0.88	0.87	1.01
	5-0.2	12	0.200	1.76	452	485	0.93	0.87	1.07
	-0.2a	12	0.200	1.76	452	560	0.81	0.87	0.93
	B-2.4-5-02	12	0.200	1.76	556	625	0.89	0.87	1.02
	0.2a	12	0.200	1.76	556	648	0.86	0.87	0.99
	0.2b	12	0.200	1.76	556	670	0.83	0.87	0.96
	B2-20-0.3	12	0.300	1.76	382	510	0.75	0.68	1.10
	0.3a	12	0.235	1.76	486	515	0.94	0.81	1.16
	0.3b	12	0.300	1.76	417	540	0.77	0.68	1.13
	0.3c	12	0.300	1.76	348	515	0.68	0.68	1.00
	0.3d	12	0.300	1.76	365	515	0.71	0.68	1.04
	B2-5-0.3	12	0.300	1.76	348	590	0.59	0.68	0.87
	B2-20-0.4	12	0.400	1.76	330	525	0.63	0.51	1.24
	10-0.4	12	0.400	1.76	287	530	0.54	0.51	1.06
	0-4-a	12	0.400	1.76	312	510	0.61	0.51	1.20
	0-4 b	12	0.400	1.76	295	500	0.59	0.51	1.16
	B2-5-0.4	12	0.400	1.76	312	540	0.58	0.51	1.14
	B2.4-5-04	12	0.400	1.76	417	720	0.58	0.51	1.14

** $(M_b/M_{bo})_{test}$ as given by the investigator

Table I.10.1 (contd.)

1	2	3	4	5	6	7	8	9	10
Rajagopal (Ref. No.55)	2A4	10	0.500	1.76	97	222	0.44	0.44	1.00
	3A4	10	0.333	1.76	146	222	0.67	0.66	1.02
	4A4	10	0.250	1.76	211	222	0.96	0.82	1.17
	5A4	10	0.200	1.76	228	222	1.04	0.88	1.18
	6A4	10	0.167	1.76	207	222	0.94	0.94	1.00
	2B0	10	0.500	1.76	91	222	0.42	0.44	0.95
	4B0	10	0.250	1.76	154	222	0.70	0.82	0.86
	2B6	10	0.500	1.76	91	222	0.42	0.44	0.95
	4B6	10	0.250	1.76	182	222	0.83	0.82	1.01
	2B8	10	0.500	1.76	91	222	0.42	0.44	0.95
	4B8	10	0.250	1.76	154	222	0.70	0.82	0.86
Rama-krishnan and Vijaya Rangan (Author) (Ref.No.6)	B6I	8	0.100	1.83	108	108	1.00	0.99	1.01
	B6II	8	0.216	1.83	108	108	1.00	0.89	1.12
	D3	10	0.193	1.83	146	180	0.81	0.90	0.90
	D4	10	0.286	1.83	111	180	0.62	0.72	0.86
	D5	10	0.187	1.83	161	180	0.90	0.90	1.00
	E4	10	0.205	1.83	156	180	0.87	0.87	1.00
	E5	10	0.302	1.83	111	180	0.62	0.69	0.90
	F1	10	0.236	1.83	146	180	0.81	0.82	0.99
	G2	10	0.300	1.83	131	180	0.73	0.69	1.06
	G3	10	0.305	1.83	131	180	0.73	0.69	1.06

Table I.10.1 (contd.)

1	2	3	4	5	6	7	8	9	10
Nylander (Ref. No.4)	III 3.1	10	0.300	2.06	48	75	0.64	0.64	1.00
	3.2	10	0.290	2.06	48	75	0.64	0.65	0.99
	4.1	10	0.230	2.06	73	75	0.97	0.78	1.24
	4.2	10	0.215	2.06	73	75	0.97	0.80	1.21
	VIII 1	10	0.750	1.53	52	75	0.69	0.32	2.16
	2	10	0.600	1.53	52	75	0.69	0.42	1.64
	3	10	0.675	1.53	58	75	0.77	0.37	2.08
	4	10	0.608	1.53	58	75	0.77	0.42	1.83
	5	10	0.725	1.53	75	150	0.50	0.32	1.56
	6	10	0.673	1.53	75	150	0.50	0.37	1.35
	7	10	0.462	1.53	110	150	0.73	0.56	1.30
	8	10	0.496	1.53	110	150	0.73	0.52	1.40
	9	10	0.540	1.53	58	75	0.77	0.46	1.67
	10	10	0.336	1.53	58	75	0.77	0.75	1.03
	V 1a	10	0.534	2.57	76	250	0.30	0.27	1.11
	1b	10	0.375	2.57	113	250	0.45	0.42	1.07
	2a	10	0.372	2.57	113	250	0.45	0.42	1.07
	VII 3	10	0.195	2.57	100	100	1.00	0.76	1.32
	4	10	0.206	2.57	95	100	0.95	0.73	1.30
	5	10	0.360	2.57	76	100	0.76	0.42	1.81
	6	10	0.284	2.57	82	100	0.82	0.54	1.52

Table I.10.1 (contd.)

1	2	3	4	5	6	7	8	9	10
Lessig (Ref. No.17)	BU 4	8	0.190	1.76	556	555	1.00	0.93	1.08
	4a	8	0.200	1.76	522	555	0.94	0.92	1.02
	5a	8	0.390	1.76	174	260	0.67	0.64	1.05
	6	8	0.400	1.76	174	274	0.64	0.62	1.03
	6a	8	0.700	1.76	104	274	0.38	0.34	1.12
Collins, et al (Ref. No.56)	L2	12	0.293	1.76	206	276	0.75	0.70	1.07
	L7	12	0.595	1.76	81	276	0.30	0.30	1.00
	L8	12	0.189	1.76	264	276	0.95	0.89	1.07
	LB1	12	0.383	1.78	142	300	0.48	0.52	0.93
	LB3	12	0.376	1.78	162	300	0.54	0.53	1.02
	36T5.5	12	0.500	1.78	173	300	0.58	0.38	1.53
	7704	12	0.490	1.78	201	425	0.47	0.39	1.20
	RU3A	10	0.630	1.78	141	300	0.47	0.32	1.47
Evans & Sarkar (Ref. No.54)	HB 2	12	0.507	1.34	67	85	0.79	0.52	1.52
	3	12	0.271	1.34	75	85	0.88	0.86	1.02
	4	12	0.192	1.34	82	85	0.97	0.95	1.02
	5	12	0.162	1.34	82	85	0.97	0.97	1.00
	8	12	0.269	1.50	80	106	0.76	0.63	1.20
	9	12	0.215	1.50	85	106	0.80	0.90	0.89

(continued)

Table I.10.1 (contd.)

1	2	3	4	5	6	7	8	9	10
Evans & Sarkar (Ref. No.54)	10	12	0.190	1.50	91	106	0.86	0.93	0.93
	11	12	0.150	1.50	94	106	0.89	0.97	0.92
	14	12	0.507	1.82	82	147	0.56	0.36	1.55
	15	12	0.269	1.82	111	147	0.75	0.73	1.03
	16	12	0.182	1.82	129	147	0.88	0.89	0.99
	17	12	0.142	1.82	137	147	0.93	0.94	0.99

Total number of Results = 166

Test/Calc. Mean = 1.08

Coe. of variation = 10%

Note: Test Results of Ref. 17 are collected from Ref. 20

TABLE I.10.2

COMPARISON OF RESULTS FOR BEAMS WITH $(1/\lambda\phi) < (1/\lambda\phi)_{TR}$

(TORSION FAILURE)

Investigator	Specimen Mark	$(\frac{f'_c}{f'_t})$	ϕ	β_s	M_{tc} in-kips	M_{ts} in-kips	M_t in-kips		Ratio:
							Test	Calc	Test/Calc
	2	3	4	5	6	7	8	9	10
Author	V1	12	0.745	0.314	28.3	15.2	41	43.5	0.94
	V4II	12	∞	0.314	24.0	15.2	33	39.2	0.84
	V6	12	0.655	0.314	21.6	15.2	34	36.8	0.93
	S3	12	0.935	0.314	20.4	11.4	29	31.8	0.91
	S6	12	∞	0.314	22.8	11.4	30	34.2	0.88
	R4	12	∞	0.314	22.0	15.2	30	37.2	0.81
	L1-1	12	∞	0.314	17.4	11.4	29	28.8	1.01
	L4-2	12	0.795	0.314	16.2	11.4	27	27.6	0.98
	L5-1	12	0.820	0.314	15.3	11.4	27	26.7	1.01
	L6-1 II	12	∞	0.314	11.1	11.4	25	22.5	1.11
	L6-2 II	12	∞	0.314	11.1	11.4	25	22.5	1.11
	A1	10	∞	0.314	20.2	-	23	20.2	1.14
	A2	10	∞	0.314	19.2	-	22	19.2	1.14
	A3	10	∞	0.314	22.6	-	22	22.6	0.98
	A4	10	∞	0.314	25.0	-	23	25.0	0.92
	A5	10	∞	0.314	21.4	-	26	21.4	1.21
	A6	10	∞	0.314	17.0	-	22	17.0	1.29 *

(continued)

Table I.10.2 (contd.)

1	2	3	4	5	6	7	8	9	10
Author	A7	10	∞	0.314	21.3	-	23	21.3	1.08
	A8	10	∞	0.314	25.2	-	24	25.2	0.95
	A9	10	∞	0.314	21.0	-	24	21.0	1.14
	A10	10	∞	0.314	21.8	-	23	21.8	1.06
	A11	10	∞	0.314	21.3	-	26	21.3	1.22*
	A12	10	∞	0.314	22.6	-	26	22.6	1.15
	A13	10	∞	0.314	19.6	-	22	19.6	1.12
	A14	10	∞	0.314	22.6	-	26	22.6	1.15
	A15	10	∞	0.314	21.0	-	26	21.0	1.24*
	B1	12	∞	0.314	15.7	9.1	23	24.8	0.93
	B2 II	12	∞	0.314	15.7	9.1	22	24.8	0.89
	B3	12	∞	0.314	15.7	9.1	25	24.8	1.01
	B4 II	12	∞	0.314	15.7	9.1	23	24.8	0.93
	B5	12	∞	0.314	16.4	9.1	24	25.5	0.94
	B6 II	12	∞	0.314	16.4	9.1	23	25.5	0.90
	B7 II	12	∞	0.314	11.4	9.1	18	20.5	0.88
	B8	12	∞	0.314	11.4	9.1	18	20.5	0.88
	B9 II	12	∞	0.314	15.5	9.1	23	24.6	0.94
	B10	12	∞	0.314	15.5	9.1	23	24.6	0.94
	T4	10	∞	0.374	13.4	-	16	13.4	1.19
	T6	10	∞	0.374	14.3	-	17	14.3	1.19
	T11 I	10	∞	0.374	11.8	6.9	17	18.7	0.91

(continued)

Table I.10.2 (contd.)

1	2	3	4	5	6	7	8	9	10
Author	T13	10	1.000	0.374	12.6	6.9	20	19.5	1.02
	T19	10	∞	0.374	18.0	6.9	23	24.9	0.92
	T20 II	10	∞	0.374	17.6	6.9	23	24.5	0.94
Cowan and Armstrong[23]	R3	12	∞	0.310	62.0	9.0	72	71.0	1.01
	R5	12	1.000	0.310	68.4	9.0	75	77.4	0.97
Gesund et al[53]	1	12	1.000	0.271	56.0	25.0	79	81.0	0.98
Chinenkov[33]	B2.1-k	12	∞	0.310	70.0	40.4	111	110.4	1.01
Rajagopal[55]	1A4	10	1.000	0.310	37.5	15.8	49	53.3	0.92
	0A4	10	∞	0.310	40.0	15.8	47	55.8	0.84
Ramakrishnan and Vijaya Rangan[6] (Author)	A5	8	∞	0.314	23.0	-	29	23.0	1.26
	A6	8	∞	0.314	23.0	-	29	23.0	1.26
	B2	8	∞	0.314	19.2	-	26	19.2	1.35
	C1 I	8	∞	0.314	16.0	-	23	16.0	1.44
	C6 I	8	∞	0.314	14.6	-	22	14.6	1.50
	D1	10	∞	0.314	17.2	13.9	28	31.1	0.90
	E1	10	∞	0.314	16.7	13.9	28	30.6	0.92

Table I.10.2 (contd.)

1	2	3	4	5	6	7	8	9	10
Nylander[4]	III.1.1	10	∞	0.332	12.5	-	13	12.5	1.04
	1.2	10	∞	0.332	12.5	-	13	12.5	1.04
	2.1	10	1.41	0.332	13.1	-	13	13.1	0.99
	2.2	10	1.41	0.332	13.1	-	13	13.1	0.99
	V 2b	10	∞	0.562	33.2	-	36	33.2	1.08
	3a	10	∞	0.562	27.0	-	34	27.0	1.26
	-	10	∞	0.336	10.4	-	13	10.4	1.25
	-	10	∞	0.336	10.4	-	13	10.4	1.25
	-	10	∞	0.336	12.5	-	15	12.5	1.20
	-	10	∞	0.336	12.5	-	15	12.5	1.20
	IV : 5a	10	∞	0.335	13.0	-	16	13.0	1.23
	5b	10	∞	0.335	13.0	-	15	13.0	1.15
Ernst[30]	3TR-0	12	∞	0.332	43.0	-	38	43.0	0.89
	1	12	∞	0.332	43.0	4.0	35	47.0	0.75 *
	3	12	∞	0.332	43.0	8.0	34	51.0	0.67 *
	7	12	∞	0.332	43.0	16.0	50	59.0	0.85
	15	12	∞	0.332	43.0	28.0	62	71.0	0.88
	30	12	∞	0.332	43.0	56.0	76	99.0	0.77
	4TR-0	12	∞	0.332	43.0	-	34	43.0	0.79 *
	1	12	∞	0.332	43.0	4.0	32	47.0	0.68 *
	3	12	∞	0.332	43.0	8.0	35	51.0	0.69 *

(Continued)

Table I.10.2 (contd.)

1	2	3	4	5	6	7	8	9	10
Ernst[30]	7	12	∞	0.332	43.0	16.0	55	59.0	0.93
	15	12	∞	0.332	43.0	28.0	74	71.0	1.04
	30	12	∞	0.332	43.0	56.0	85	99.0	0.86
	5TR- 0	12	∞	0.332	43.0	-	34	43.0	0.79 *
	1	12	∞	0.332	43.0	4.0	33	47.0	0.70 *
	3	12	∞	0.332	43.0	8.0	43	51.0	0.84
	7	12	∞	0.332	43.0	16.0	60	59.0	1.02
	15	12	∞	0.332	43.0	28.0	77	71.0	1.08
	30	12	∞	0.332	43.0	56.0	93	99.0	0.94
Arockia Swamy[51]	R1	12	∞	0.310	52.0	-	58	52.0	1.12
	R2	12	∞	0.310	51.6	17.4	55	69.0	0.80
	R3	12	∞	0.310	39.3	46.7	60	86.0	0.70 *
	R6	12	∞	0.310	43.8	-	61	43.8	1.39 *
	T1	12	∞	0.840	49.0	-	43	49.0	0.88
	T2	12	∞	0.840	49.0	5.0	44	54.0	0.82
	T3	12	∞	0.840	47.0	5.7	43	52.7	0.82
	T4	12	∞	0.840	47.0	11.2	45	58.2	0.78 *
Lessig[17]	BK1	8	∞	0.310	56.0	87.5	122	143.5	0.85
	1a	8	∞	0.310	43.8	89.0	104	132.8	0.79
	2	8	∞	0.310	47.6	99.5	148	147.1	1.01
	2a	8	∞	0.310	51.5	104.0	153	155.5	0.98

(continued)

Table I.10.2 (contd.)

1	2	3	4	5	6	7	8	9	10
Lessig[17]	3	8	∞	0.342	54.5	97.8	176	152.3	1.15
	3a	8	∞	0.342	56.2	97.8	149	154.0	0.97
Lyalin[34]	B 8k	8	∞	0.310	40.5	75.6	125	116.1	1.08
	8ka	8	∞	0.310	71.0	75.6	153	146.6	1.05
	B 7	10	∞	0.310	61.0	171.0	221	232.0	0.95
	7a	10	∞	0.314	55.0	171.0	208	226.0	0.92
Collins et al[56]	L 1	12	1.77	0.310	51.2	-	57	51.2	1.11
	L 3	12	11.10	0.310	58.1	-	61	58.1	1.05
	LB2	12	8.90	0.312	47.5	-	55	47.5	1.16
	36T4c	12	1.54	0.312	45.6	43.4	94	89.0	1.06
	24T 3	12	1.52	0.312	45.6	50.5	71	96.1	0.74 *
	RU 2	10	1.66	0.312	46.0	43.5	85	89.5	0.90
	RU 3	10	1.25	0.312	45.0	43.5	105	88.5	1.19
	LS 2	10	10.10	0.310	39.0	-	53	39.0	1.36
	S 3	10	10.20	0.310	51.0	-	64	51.0	1.26

Total No. of test Results = 112.

Test/Calc. Mean = 0.95
Coe. of variation = 13 %

* Discrepancy in the Experimental Result

Note: Test results of Ref. 17 and 34 are collected from Ref.20.

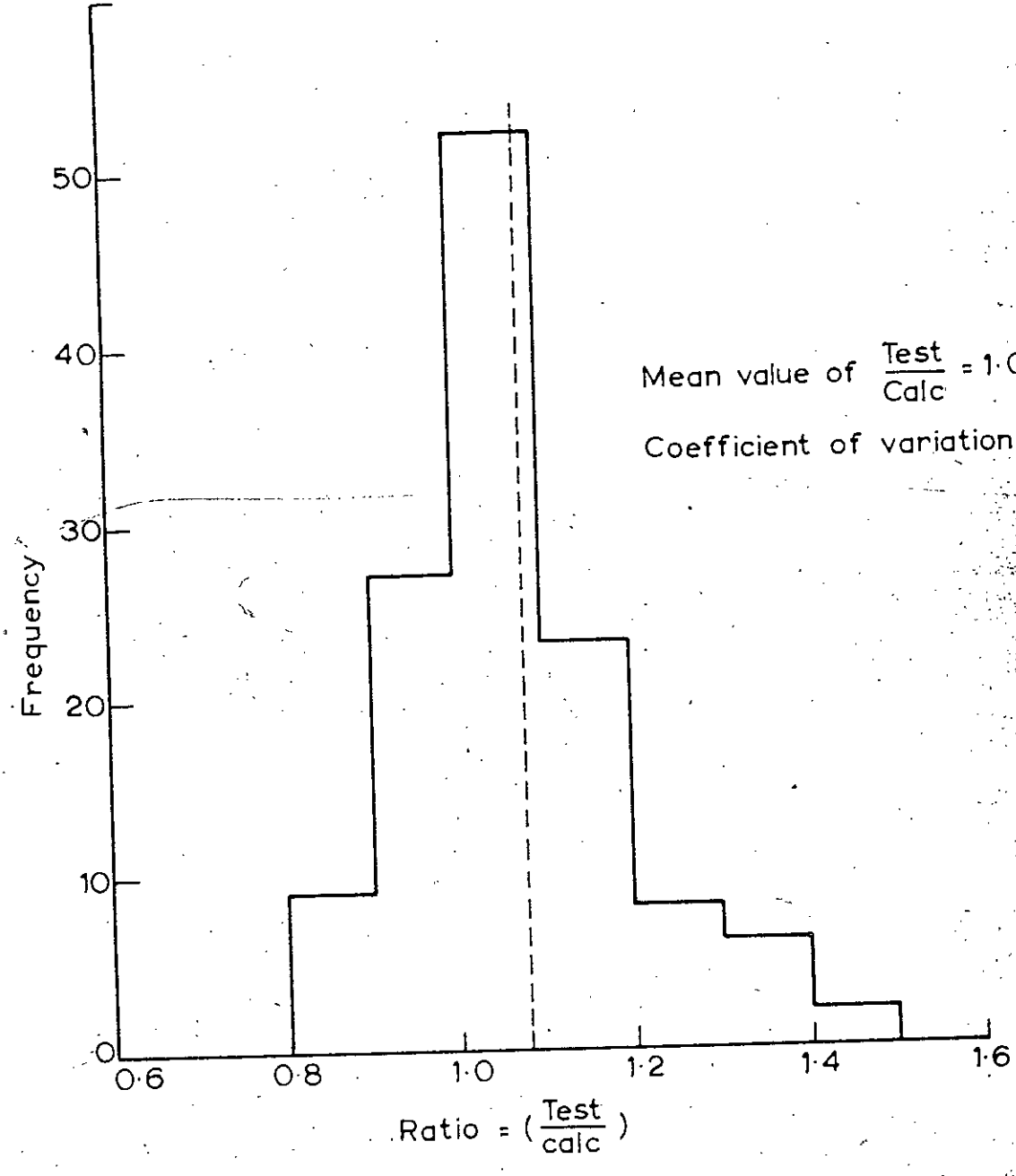


FIG. 1.59. HISTOGRAM FOR BEAM WITH $\frac{1}{\lambda\phi} > (\frac{1}{\lambda\phi})_{TR}$

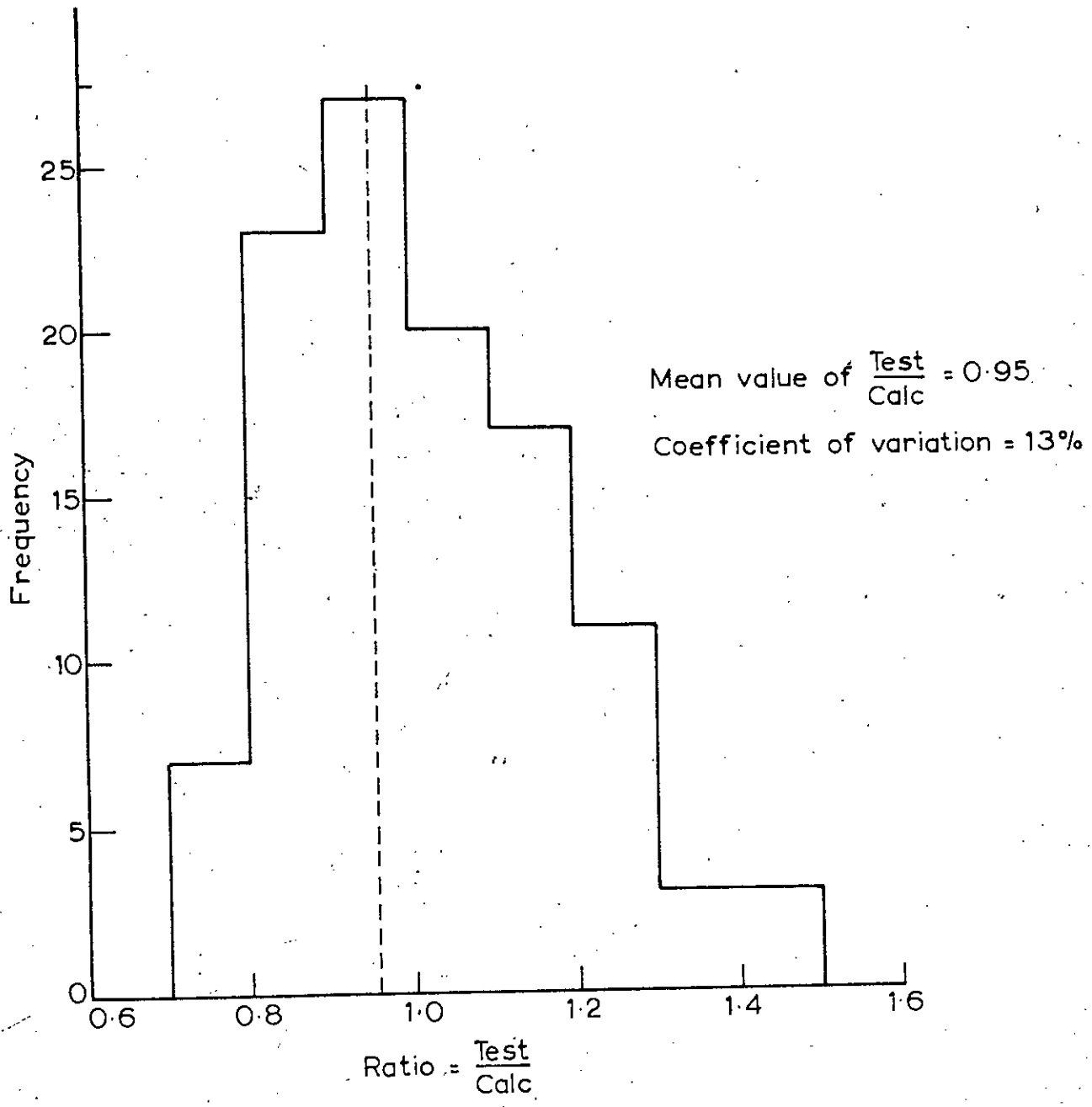
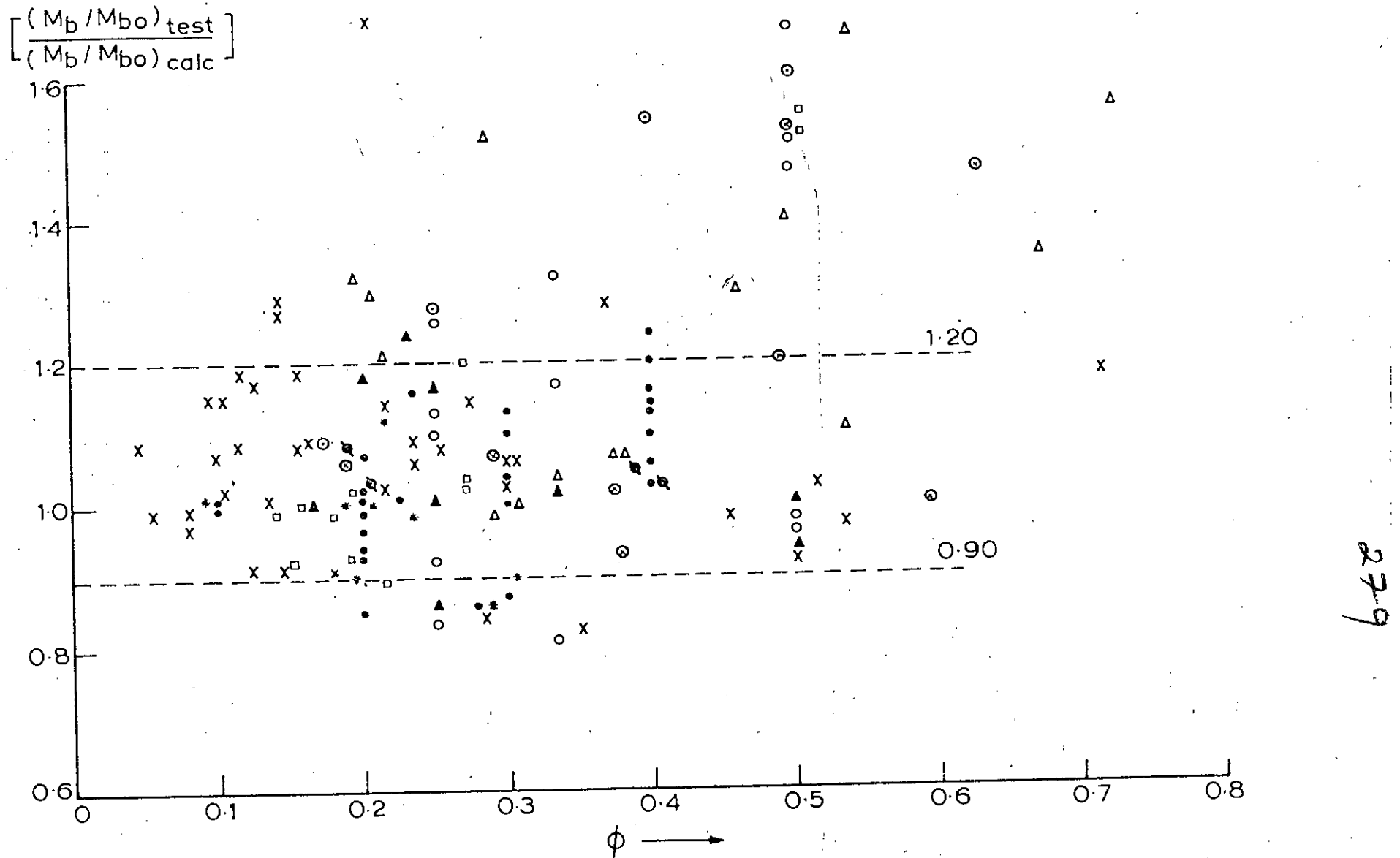


FIG.1.60. HISTOGRAM FOR BEAMS WITH $\frac{1}{\lambda\phi} < (\frac{1}{\lambda\phi})_{TR}$



× — Author, • — Chinenkov, ▲ — Rajagopal, ⊙ — Cowan & Armstrong, ○ — Gesund et.al.

⊛ — Lessig, □ — Evans & Sarkar, * — Ramakrishnan & Vijayarangan, ⊗ - Collins et.al., △ - Nylander.

FIG. I. 61. ACCURACY OF THEORETICAL RESULTS WITH VARIATION IN ϕ.

$(1/\lambda\phi) < (1/\lambda\phi)_{TR}$. The results of comparison are summarised below:-

Type of Failure	No. of test results	Ratio: Test/Calc. Mean value	Coefficient of variation.
Flexure Failure $1/\lambda\phi > (1/\lambda\phi)_{TR}$	166	1.08	10 %
Torsion Failure $1/\lambda\phi < (1/\lambda\phi)_{TR}$	112	0.95	13 %

Figs. I.59 and I.60 show the histogram for the two cases of failure. From the comparison it can be seen that the predictions of the theoretical equations are fairly good in the majority of the cases.

To check the accuracy of the flexure failure equation (Eqn. I.9.15) for all ranges of ϕ, the ratios $\frac{(M_b/M_{bo})_{test}}{(M_b/M_{bo})_{calc.}}$ were plotted versus ϕ as shown in Fig. I.61. From the Fig. I.61, it can be noted that the $\frac{Test}{Calc}$ ratios for the majority of the cases lie between 0.90 and 1.20. This "band" of variation is approximately constant for all the ranges of ϕ. However in few cases the deviations between experimental and theoretical results are large. It

is worth noting that in many such cases the value of ϕ is greater than 0.40. One of the reasons for these discrepancies might be due to the effect of transverse steel. That is, in the development of the equation for flexure failure the effect of transverse steel has been neglected assuming its contribution to the capacity of the beam as negligible. In certain beams with particular amounts of transverse steel tested with particular values of ϕ the transverse steel might have contributed to the strength of the beam. So for some beams which show large discrepancy with the theory, the results were recalculated considering the effect of transverse steel. The axial stress in the transverse hoops has been assumed to be equal to its yield stress and its contribution to the torque has been taken equal to $A_w f_{yw} \frac{b'd'}{s}$. The revised calculation is illustrated using the results of a beam of Gesund's[53] investigation below:-

Investigator: Gesund et al.

Specimen Mark : Beam 3

M_b = 122 in kips; M_t = 61 inch kips; M_{bo}= 180 inch kips

A_w=0.10 in^2; f_{yw}= 50 ksi; b'=d' = 5 in.; s = 5 in.

b = D = 8 in; = 1.53; f'_c/f'_t = 12; f'_c = 5310 psi

Calculations as per the theory:-

$$1/\lambda\phi = 1.31., \left(\frac{M_b}{M_{bo}}\right)_{test} = \frac{122}{180} = 0.68$$

$$M_{to} = \left(\frac{2k'-1}{2k'^2}\right) \beta_e b^2 D f'_c + A_w f_{yw} \frac{b'd'}{s}$$

$$= (0.08 \times 0.271 \times 8^3 \times 5.31) + \left(\frac{0.10 \times 50 \times 5 \times 5}{5}\right)$$

$$= \underline{84.0} \text{ in kips}$$

$$\beta_e \, b^2 D f'_c = (0.208 \times 8^3 \times 5.31)$$

$$= \underline{566} \text{ in kips}$$

$$\therefore \frac{M_{to}}{\beta_e b^2 D f'_c} = \underline{0.148}$$

Using Fig. I.58, $(1/\lambda\phi)_{TR} = \underline{0.85}$.

Since $1/\lambda\phi > (1/\lambda\phi)_{TR}$, flexure failure equation governs the calculations.

From Fig. I.55, $(M_b/M_{bo})_{calc} = 0.45$;

$$\text{Test/Calc. Ratio} = \frac{0.68}{0.45} = \underline{1.51}.$$

Revised Calculations:-

Contribution to torque by the transverse steel

$$= A_w f_{yw} \frac{b'd'}{s}$$

$$= \frac{0.10 \times 50 \times 5 \times 5}{5} = 25 \text{ in kips}$$

$\therefore$ Net torque to be resisted by the concrete

$$= (61-25) = 36 \text{ in kips}$$

Modified value of ϕ to be used in flexure failure equation

$$= \frac{36}{122} = 0.295$$

Modified value of $1/\lambda\phi = 2.22$.

From Fig. I.55, modified value of $M_b/M_{bo} = 0.77$

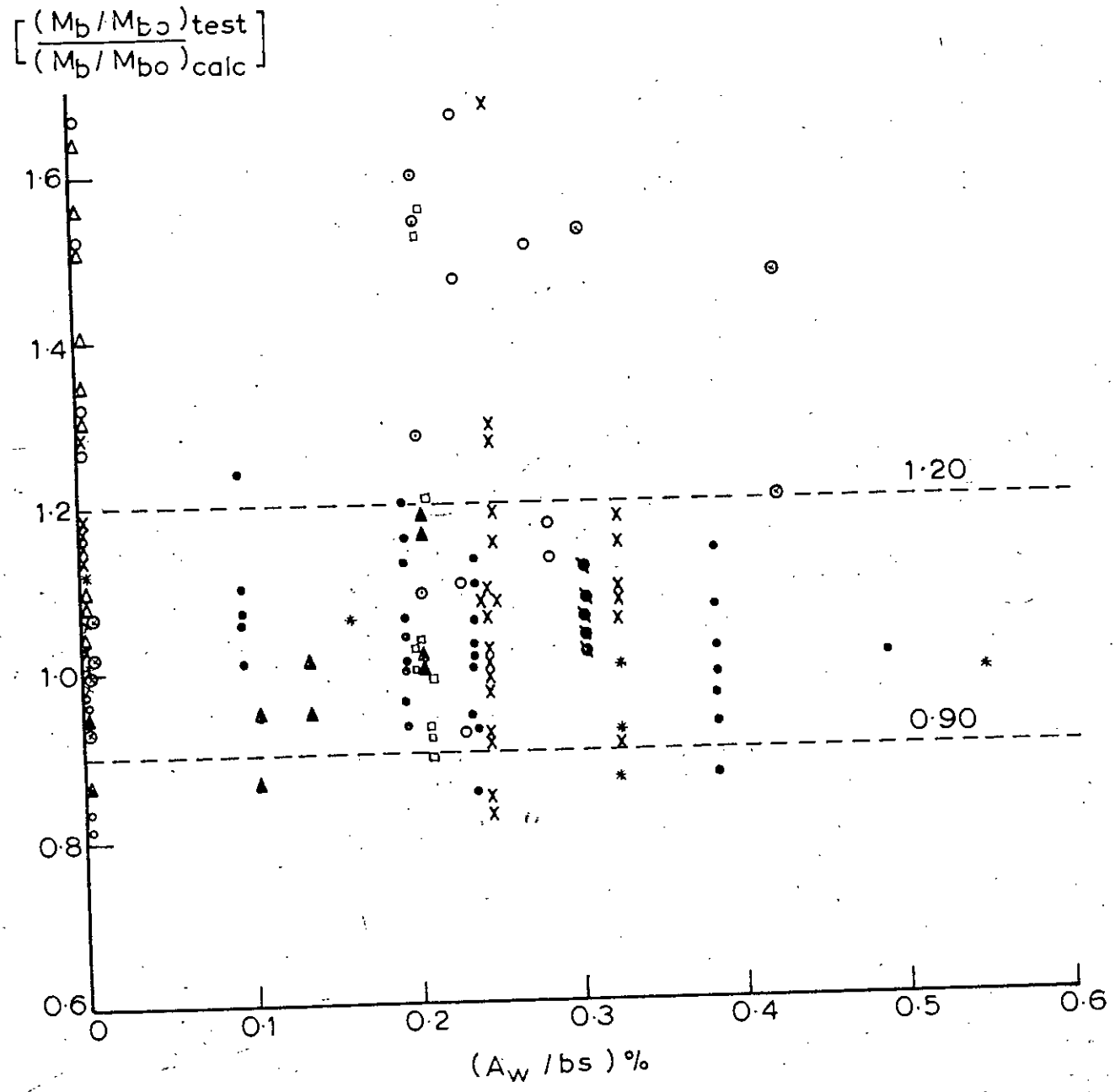
$$\left(\frac{\text{Test}}{\text{Calc}}\right) \text{ Ratio} = \frac{0.68}{0.77} = \underline{0.88}.$$

Similarly the calculations were revised for some other beams. The results are given in Table I.10.3.

TABLE I.10.3

COMPARISON OF RESULTS

Investigator	Specimen Mark	$\left(\frac{M_b}{M_{bo}}\right)$ test	$(M_b/M_{bo})_{calc}$		Ratio : Test/Calc.	
			Neglecting the effect of transverse steel	Considering the effect of transverse steel	Neglecting the effect of transverse steel	Considering the effect of transverse steel.
Besund et al[53]	3	0.68	0.45	0.77	1.51	0.88
	9	0.44	0.30	0.45	1.47	0.98
	11	0.50	0.30	0.62	1.67	0.81
Cowan[23]	R2	0.61	0.38	0.48	1.60	1.27
	S1	0.80	0.52	0.56	1.54	1.43
Chinenkov[33]	B2-20-0.4	0.63	0.51	0.62	1.24	1.02
Collins et al[56]	36T5.5	0.58	0.38	0.63	1.53	0.92
	RU3A	0.47	0.32	0.63	1.47	0.75
Evans and Sarkar[54]	HB2	0.79	0.52	0.88	1.52	0.90
	14	0.56	0.36	0.83	1.55	0.68



x — Author, • — Chinenkov, ▲ — Rajagopal, ⊙ — Cowan & Armstrong,
o — Gesund et.al., ✱ — Lessig, □ — Evans & Sarkar, △ — Nylander
* — Ramakrishnan & Vijayarangan, ⊙ — Collins et.al.

FIG.1.62. ACCURACY OF THEORETICAL RESULTS WITH VARIATION IN $\left(\frac{A_w}{bs}\right)$.

From Table I.10.3 it can be seen that the revised calculations, as expected, offer better-fit to the experimental results. In most of these beams the value of ϕ was equal to 0.50 and the transverse steel ratio A_w/bs was varied between 0.20% and 0.30%. The predicted values as per the revised calculations are higher than the test values in most of the cases. This may be due to the fact that the transverse steel might not have yielded at this stage and the stress in it may be below yield stress. Also it is to be noted here that even in the ranges of ϕ in which there are large discrepancies between experimental and computed results, there are many more results which fall within the "band" of variation (See Fig. I.61). Hence the method indicated in the revised calculations may be applicable only to certain cases and is not supported by many results. So the author is not encouraged to modify his theory and equations to suit these few cases.

The above discussion is once again well brought-in in Fig. I.62 in which the test/calc. ratios have been plotted against the transverse steel ratio, A_w/bs. From Fig. I.62 it can be said that the "band" of variation is approximately constant between 0.90 and 1.20 for different values of A_w/bs except in few cases, which have already been discussed. Hence it may be pointed out here that the value of A_w/bs has no

considerable influence on the predictions of the theory for the case of beams with $1/\lambda\phi > (1/\lambda\phi)_{TR}$.

The reason for the large discrepancies between the results in few other cases (i.e. some of the beams of Series VIII of Nylander's[4] investigation and beams 3 and 4 of Gesund and Boston[52]) is due to the fact that the values of $1/\lambda\phi$ for these beams were in the neighbourhood of $(1/\lambda\phi)_{TR}$. In such cases, the bending moment present will be small and the relationship $f/f'_c = M_b/M_{bo}$ used in the derivation of flexure failure equation may be a poor approximation resulting in the large discrepancies between the computed and test results. Hence it is to be indicated here that although the flexure failure equation can be used to give fairly good results over a wide range of $1/\lambda\phi$, its application for the beams with $1/\lambda\phi$ values in the neighbourhood of $(1/\lambda\phi)_{TR}$ is limited. However the results are conservative even in such cases.

As can be seen from Table I.10.2, the torsion failure equation gives satisfactory results in most of the cases. The main reason for the discrepancy in the results in few cases is due to experimental scatter only. Such cases have been indicated in Table I.10.2.

I.10.4 Interaction of Moments:-

As per the theory the individual variation of the dimensionless parameters M_b/M_{bo} and M_t/M_{to} are given by the

following equations:

For flexure failure zone,

$$\frac{M_b}{M_{bo}} = \left(\frac{f'_t}{f'_c}\right)^2 \frac{(\gamma+1)}{(\gamma-1)^2}\left[\sqrt{1+\left\{\left(\frac{2f'_c}{f'_t}\right)\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^2}-1\right] \quad \ldots \text{ I.10.7}$$

$$\frac{M_t}{M_{to}} = \frac{1}{(M_{to}/\beta eb^2Df'_c)}\left(\frac{\lambda\phi}{2}\right)\left(\frac{f'_t}{f'_c}\right)^2 \frac{(\gamma+1)}{(\gamma-1)^2}\left[\sqrt{1+\left\{\left(\frac{2f'_c}{f'_t}\right)\frac{\gamma-1}{\gamma+1}\right\}^2}-1\right] \quad \ldots \text{ I.10.8}$$

For torsion failure zone,

$$\frac{M_t}{M_{to}} = 1.0, \text{ for all ranges of } \frac{M_b}{M_{bo}} \quad \ldots \text{ I.10.9}$$

The Eqn. I.10.7 depends on the values of $1/\lambda\phi$ and f'_c/f'_t and Eqn. I.10.8 depends on the values of $1/\lambda\phi$, f'_c/f'_t and $M_{to}/\beta eb^2Df'_c$. The quantity $M_{to}/\beta eb^2Df'_c$ is a function of $(1/\lambda\phi)_{TR}$ and can be known with the help of Fig. I.58.

Combining Eqns. I.10.7 and I.10.8 we have, for flexure failure zone,

$$\frac{M_t}{M_{to}} = \frac{(M_b / M_{bo})}{2(M_{to}/\beta eb^2Df'_c)(1/\lambda\phi)} \quad \ldots \text{ I.10.10}$$

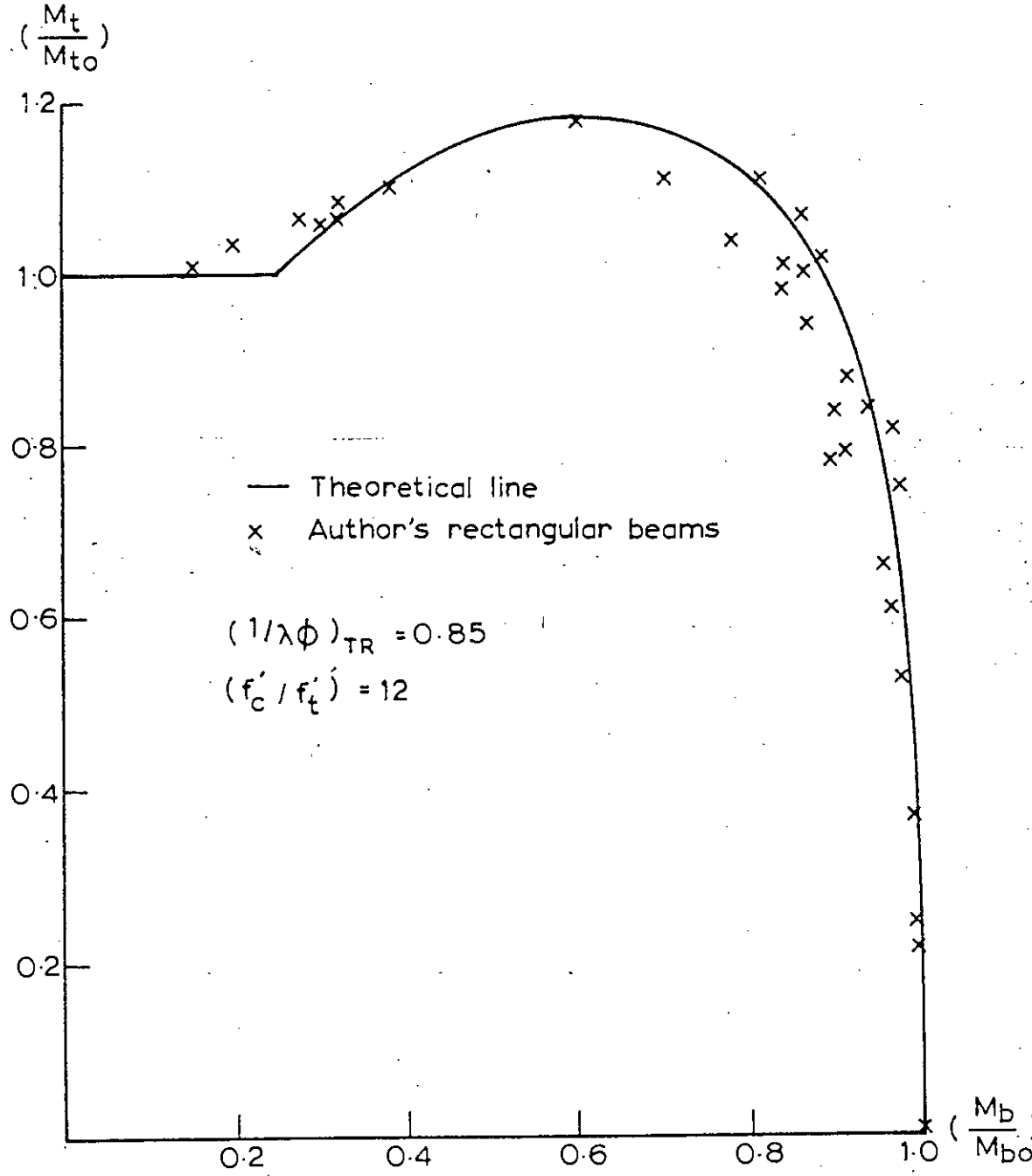


FIG.I.63. INTERACTION DIAGRAM.

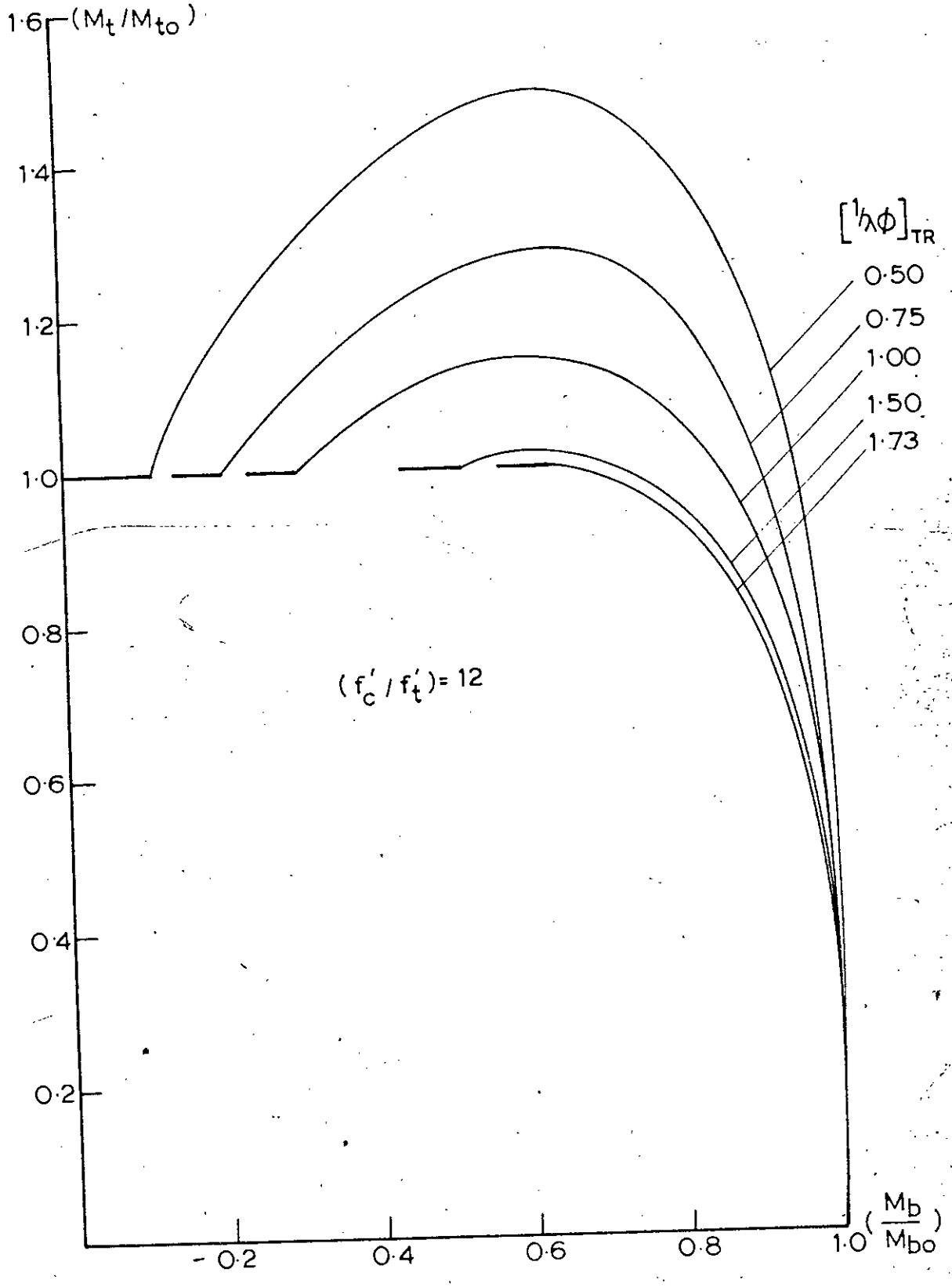


FIG. I. 64. INFLUENCE OF $(\frac{1}{\lambda\phi})_{TR}$ ON INTERACTION DIAGRAMS.

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Using Eqns. I.10.9 and I.10.10, the interaction diagram can be sketched for the given values of $(1/\lambda\phi)_{TR}$ and (f'_c/f'_t). For the rectangular beams tested in the author's investigation the average value of $(1/\lambda\phi)_{TR}$ was approximately equal to 0.85 and the value of f'_c/f'_t was equal to 12. Using these values the interaction diagram shown in Fig. I.63 has been sketched. In the same figure the author's experimental results are also shown. From Fig. I.63, it can be seen that the theoretical curve shows a similar trend as shown by the experimental results. It was not possible for the author to have similar study for other values of $(1/\lambda\phi)_{TR}$ and (f'_c/f'_t) using the test results of other investigations available to him due to the reason that the values of these parameters varied considerably between individual test results. From the comparison shown in Fig. I.63, it can be said that the interaction of moments as indicated by the experiments is also shown by theory. However, more experimentation on these lines is necessary to have further confirmation of the theory.

It may be interesting to study the variations in the Interaction Diagrams given by the theory. In Fig. I.64, Interaction Diagrams as obtained by the theory for different values of $(1/\lambda\phi)_{TR}$ are shown. From Fig. I.64, it is clear that the Interaction Diagram will vary depending on the

physical properties of the beam as the value of $(1/\lambda\phi)_{TR}$ is a direct function of the quantity $M_{to}/\beta eb^2Df'_c$ and the value of f'_c/f'_t. As can be seen from Fig. I.64, the increase in the torque capacity of beams increases with the decrease in the value of $(1/\lambda\phi)_{TR}$ and for values of $(1/\lambda\phi)_{TR}$ greater than 1.50, the increase is very small. For the limiting value of $(1/\lambda\phi)_{TR}$ i.e. 1.73, it is interesting to note that the torque capacity of beam is in no case higher than the pure torsion capacity. These theoretical results can be qualitatively studied using the test results of Gesund et al[52,53]. From the analysis of their test results, it has been found that in beams without transverse steel, the value of M_t/M_{to} is as high as 1.50 (compare the results of beams 1, 2, 5 and 6 Gesund and Boston's[52] investigation) when the values of M_b/M_{bo} varied between 0.50 and 0.60. For the case of beams with a high percentage of transverse steel, their test results indicate the value of M_t/M_{to} is less than 1.0 for all values of M_b/M_{bo} (see the results of beams 2, 4 and 6 of Gesund et al's[53] investigation). These two types of beams had same cross sectional properties. The value of $(1/\lambda\phi)_{TR}$ of beams without transverse steel is equal to 0.50 and for the beams with transverse steel the value of $(1/\lambda\phi)_{TR}$ is greater than the optimum value 1.73. So these test results indicate that the interaction of moment varies with the

value of $(1/\lambda\phi)_{TR}$. The interaction diagrams shown in Fig. I.64 attempt to explain these test observations.

From the above discussion of theoretical results, the following conclusions can be drawn:-

a. As such there is no "universal" interaction diagram for beams subjected to combined bending and torsion.

b. The interaction diagram varies with all the properties of the beams such as cross sectional dimensions, concrete strength, amount and strength of transverse reinforcement, shape of the cross section etc.

As there is no single common interaction curve for beams under combined bending and torsion, it may not be reasonable to base any design criterion on interaction diagrams obtained from a limited number of test results. Hence the interaction diagrams are of restricted utility in the design of beams under combined bending and torsion.

I.10.5 Maximum Torque Capacity of Beams:-

In the flexure failure zone, torque capacity of the beam is given by,

$$\left(\frac{M_t}{\beta_0 b^2 D f'_c}\right) = \frac{1}{2} \cdot \frac{1}{(1/\lambda\phi)} \left(\frac{f'_t}{f'_c}\right)^2 \frac{(\gamma+1)}{(\gamma-1)^2} \left[\sqrt{1 + \left\{\left(\frac{2f'_c}{f'_t}\right)\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^2} - 1\right]$$

... I.10.11

It has been observed from the experimental results (see

Fig. I.40) that the torque capacity of a beam increases over its pure torsion capacity in certain ranges of ϕ. This is also indicated by the theory as can be seen from Fig. I.63. Using Eqn. I.10.11 it is possible to know the maximum torque capacity of beams.

Differentiating $(M_t / \beta_e b^2 D f'_c)$ with respect to $(1/\lambda\phi)$ and equating the resulting equation to zero, the particular value of $(1/\lambda\phi)$ at which torque capacity will be maximum can be known. Performing these operations, we get,

$$\left[1/\lambda\phi\right]_{max} = \frac{\left(\frac{2k'}{\sqrt{3}} - 1\right)}{2\sqrt{\left(\frac{2k'}{\sqrt{3}}\right)}} \qquad \ldots \text{ I.10.12}$$

where $\left[\frac{1}{\lambda\phi}\right]_{max}$ is that particular value of $1/\lambda\phi$ at which the torque capacity is maximum and

$$k' = f'_c/f'_t$$

Substituting this value of $\left[1/\lambda\phi\right]_{max}$ in Eqn. I.10.11, the maximum torque capacity of beams, $(M_t)_{max}$ can be known. The values of $\left[1/\lambda\phi\right]_{max}$ and $(M_t)_{max}$ for three values of k' are given in Table I.10.4.

It may be interesting to mention here that the value of M_b/M_{bo} corresponding to $\left[1/\lambda\phi\right]_{max}$ varied from 0.60 to 0.62 for the three values of k' studied here. Hence it can be

TABLE I.10.4

MAXIMUM TORQUE CAPACITY OF BEAMS

$k' = f'_c/f'_t$	$[1/\lambda\phi]_{max}$	$\frac{(M_t)_{max}}{\beta_e b^2 D f'_c}$
8	1.36	0.214
10	1.55	0.195
12	1.73	0.179

said that the maximum torque capacity of a beam occurs when the bending moment present is about 60 percent of the capacity of the beam under pure flexure.

The value of $(M_t)_{max}$ is useful in fixing-up the dimensions of a beam in a design problem. That is, let M_t be the ultimate design torque for which the dimensions of the beam are to be fixed-up. For any particular dimensions, the design torque M_t should not be greater than $(M_t)_{max}$ for that particular section.

or, $$\frac{M_t}{\beta_e b^2 D f'_c} \leq \frac{(M_t)_{max.}}{\beta_e b^2 D f'_c} \qquad \ldots \text{I.10.13}$$

Designating $\frac{(M_t)_{max.}}{\beta_e\, b^2 D\, f'_c} = K_m$

we have,

$$\frac{M_t}{\beta_e\, b^2 D\, f'_c} \leq K_m \qquad \ldots \text{ I.10.14}$$

The values of K_m are given in Table I.10.4. So for the known values of K_m and M_t the dimensions of the beam can be fixed-up so that the inequality I.10.14 is satisfied. For rectangular beams the in-equality I.10.14 can be put thus:

$$\frac{M_t}{\beta_e K_m f'_c} \leq b^3 \left(\frac{D}{b}\right)$$

$$\text{or} \quad b^3 \geqslant \frac{M_t}{\beta_e \left(\frac{D}{b}\right) K_m f'_c}$$

$$b \geqslant \sqrt[3]{\frac{M_t}{\beta_m f'_c}} \qquad \ldots \text{ I.10.15}$$

where $\beta_m = \beta_e \left(\frac{D}{b}\right) K_m$ and the values of β_m for the three values of k' are given in Table I.10.5.

It may be interesting to compare these results obtained for rectangular beams with those given by previous

TABLE I.10.5

VALUES OF β_m FOR RECTANGULAR BEAMS

D/b	β_m		
	$f'_c / f'_t = 8$	$f'_c/f'_t = 10$	$f'_c/f'_t = 12$
1.0	0.045	0.041	0.037
1.2	0.060	0.054	0.050
1.5	0.087	0.080	0.072
2.0	0.132	0.120	0.110
2.5	0.178	0.163	0.150
3.0	0.227	0.207	0.189
4.0	0.324	0.296	0.272
5.0	0.417	0.380	0.350
10.0	0.960	0.880	0.800

investigators. Based on her own test results, Lessig[17] had given that $M_t/b^2Df'_c \leq 0.07$. Later Collins et al[57] confirmed this relation given by Lessig using their test results and for design purposes they gave the following inequality:

$$b \geq \sqrt[3]{M_t/0.07f'_c \cdot \frac{D}{b}}$$

The beams used in Lessig's as well as Collins' investigations had D/b approximately between 1.50 and 2.00 and the ratio f_c'/f_t' varied between 8 and 10. For these values of D/b ratio and f_c'/f_t', the numerical coefficient predicted by author's theory varies between 0.053 and 0.066 which is close to the safe limit given by Collins et al. It is to be pointed out here that the inequality suggested by Leseig did not account for the D/b ratio.

I.10.6 Limitation on the Amount of Transverse Steel:-

In the preceding article, expressions were derived for the maximum torque capacity of a beam. It has been assumed that its magnitude is independent of the amount of transverse steel. Using this result it is possible to know the limitation of the torsion failure equation (Eqn. I.9.11).

As per the Eqn. I.9.11, the torque capacity of the beam can be increased by increasing the value of M_{ts} for a fixed value of M_{tc}. But indefinite increase in the strength is not possible as the maximum torque capacity of a beam is restricted to $(M_t)_{max}$. It may not be possible to increase the value of M_{to} beyond $(M_t)_{max}$ by increasing the value of M_{ts}. So in all cases,

$$M_{to} \leq (M_t)_{max} \qquad \ldots \text{ I.10.16}$$

i.e. $$\left[\left(\frac{2k'-1}{2k'^2}\right)\beta_s\, b^2 D f'_c + A_w f_{yw}\frac{b'd'}{s}\right] \leq (M_t)_{max.}$$

or $$\frac{A_w}{bs} \leq \frac{bDf'_c}{b'd'f_{yw}}\left[\frac{(M_t)_{max.}}{b^2Df'_c} - \left(\frac{2k'-1}{2k'^2}\right)\beta_s\right] \quad \ldots \text{ I.10.17}$$

For rectangular beams,

$$\frac{A_w}{bs} \leq \frac{bDf'_c}{b'd'f_{yw}}\left[\frac{\beta_m}{(D/b)} - \left(\frac{2k'-1}{2k'^2}\right)\beta_s\right] \quad \ldots \text{ I.10.18}$$

Eqns. I.10.17 and I.10.18 give the limiting values of Aw/bs to be used in any beam.

The results given by Eqn. I.10.18 can be studied by applying it to the test results reported by Ernst[30]. The properties of beams are:

b = 6 in; D = 12 in; b' = 4.5 in; d' = 10 in;

f'_c = 3923 psi; f_{yw} = 55.5 ksi; Aw/bs = 0 to 0.42 %

β_s = 0.332; β_m (from Table I.10.5) = 0.11

k' = 12.

From Eqn. I.10.18,

$$\frac{Aw}{bs} \leq \frac{6 \times 12 \times 3.923}{4.5 \times 10 \times 55.5}\left[\frac{0.11}{2} - 0.08 \times 0.332\right]$$

or $$\frac{Aw}{bs} \leq 0.32\%$$

For values of Aw/bs greater than 0.32%, the increase in the torque capacity of beams may not be proportionate with the increase in the transverse steel and the maximum torque capacity will be equal to $(M_t)_{max.}$. For Ernst's beams $(M_t)_{max.}$ = 93 inch kips. From the experimental results it can be seen that the torque capacities of beams with Aw/bs greater than 0.32% were in no case greater than 93 inch kips (see Table I.10.2). Also from the experimental data reported by Ernst, it has been found that in these cases the strains in the transverse hoops were below yield strain. This indicates that it is uneconomical to use transverse steel beyond the limiting values given by Eqns. I.10.17 and I.10.18. If the value of Aw/bs for a particular beam is greater than the limiting value given by Eqn. 10.18, then it is not possible to determine $(1/\lambda\phi)_{TR}$ using Fig. 10.1. This can be illustrated using the results of some beams reported by Gesund et al[53]:-

Specimen Mark : Beam 2

$b = D = 8$ in; $\frac{Aw}{bs} = 0.625\%$; $f'_c = 5.3$ ksi

$f_{yw} = 50$ ksi; $b' = d' = 5$ in. $f'_c/f'_t = 12$

$\beta_m = 0.037$ (from Table I.10.5); $\beta_s = 0.271$; $\lambda = 1.53$

$M_t = M_b = 102$ inch kips; $M_{bo} = 180$ inch kips.

From Eqn. I.10.18,

$$\frac{Aw}{bs} \leq \frac{8 \times 8 \times 5.3}{5 \times 5 \times 50}\left[\frac{0.037}{1} - 0.08 \times 0.271\right]$$

$$\leq 0.417\ \%$$

As per Eqn. I.9.11,

$$M_{to} = (0.08 \times 0.271 \times 8^3 \times 5.3) + \left(\frac{0.10 \times 50 \times 5 \times 5}{2}\right)$$

$$= \underline{121.3} \text{ inch kips}$$

$$(M_t)_{max.} = 0.037 \times 8^3 \times 5.3 = \underline{100.2} \text{ inch kips}$$

$$\frac{M_{to}}{\beta_e\, b^2 D f'_c} = \frac{121.3}{0.208 \times 8^3 \times 5.3} = \underline{0.216}.$$

For this value of $(M_{to}/\beta_e b^2 D f'_c)$ it is not possible to find the value of $(1/\lambda\phi)_{TR}$ using the curve corresponding to $f'_c/f'_t = 12$ shown in Fig. I.58. This is due to the fact that the value of Aw/bs used in this beam is greater than the limiting value given by the Eqn. I.10.18. In such cases the calculation of M_{to} using Eqn. I.9.11 is no more valid and the value of M_{to} is to be taken equal to $(M_t)_{max.}$. So we have

$$M_{to} = (M_t)_{max.} = \underline{100.2} \text{ in kips}$$

$$\frac{M_{to}}{\beta_e b^2 D\, f'_c} = \frac{100.2}{0.208 \times 8^3 \times 5.3} = 0.179.$$

The value of $(1/\lambda\phi)_{TR}$ using this revised value of $M_{to}/\beta_e b^2 D f'_c$ will be equal to $[1/\lambda\phi]_{max.}$ as can be seen from Fig. I.58.

For this beam, $(1/\lambda\phi) = 0.655 < (1/\lambda\phi)_{TR}$.

So torsion failure equation governs the calculation and

$$M_{to} = (M_t)_{max.} = \underline{100.2} \text{ inch kips}$$

$$(M_{to})_{test} = 102 \text{ inch kips}$$

$$\text{Ratio: Test/Calc.} = \frac{102}{100.2} = \underline{1.02}.$$

Using the method of analysis indicated above, the results of beams with Aw/bs ratio greater than the limiting value given by Eqn. I.10.18 were analysed and the results are summarised in Table I.10.6.

It may be hard to draw any conclusive remarks from Table I.10.6 regarding the method of calculation indicated for the case of beams with Aw/bs ratio higher than the limiting value. Hence it is necessary to restrict the amount of transverse steel to the limiting value so that the theory can be applied without any discrepancy in the determination of $(1/\lambda\phi)_{TR}$. As indicated by the discussion on Ernst's results, this restriction on the amount of transverse steel is necessary from practical view-point so that the transverse steel provided can be used to the fullest extent.

TABLE I.10.6

COMPARISON OF RESULTS : BEAMS WITH A_w/b_s VALUES GREATER THAN THE LIMITING VALUES GIVEN BY EQN. I.10.18

pe-i-en ark	M_{to} (Eqn.I.9.11) in-kips	$(M_t)_{max}$ in-kips	$\left(\frac{A_w}{b_s}\right)$ %	$\left(\frac{A_w}{b_s}\right)$ % Eq. I.10.18	$(1/\lambda\phi)_{TR}$	$1/\lambda\phi$ (test)	M_t (test) in-kips	M_t (calc) in-kips	M_b / M_{bo} Test.	Calc	Rat Te Ca
2	121.3	100.2	0.625	0.417	1.73	0.655	102	100.2	-	-	1.
4	114.5	88.8	0.625	0.360	1.73	1.31	67	88.8	-	-	0.
6	107.5	77.0	0.625	0.312	1.73	1.96	-	-	0.93	0.71	1.
8	126.0	109.0	0.625	0.440	1.73	2.62	-	-	0.98	0.85	1.

::

CHAPTER I.11

DESIGN RECOMMENDATIONS

I.11.1 Since torsion occurs as a secondary effect in many Civil Engineering Structures (for example in circular girders under water tanks, Ell beams in a beam cum slab structure, shell structures etc.) due consideration for design of the same was not given for many years. Only in recent years the codes of various countries have come out with some recommendations for the design of members subjected to torsion. It seems that these recommendations have been proposed after realizing the catastrophic nature of torsional stresses. Mainly all the codes recommend design formulae for working load conditions except the Russian Code whose recommendations are based on ultimate load methods of design.

I.11.2 Code Requirements for Torsion:

The recommendations of various codes for Torsion Design are summarised in Table I.11.1 and the detailed version of them can be seen in the paper by Paul Zia and Fisher[76].

The design formulae of many codes depend basically on two theories: one due to Rausch[1] and the other developed by

TABLE I.11.1

CODE REQUIREMENTS FOR TORSION DESIGN

Code	Area of 45° spiral	Area of one leg of closed hoop.	Area of longitudinal steel.
1	2	3	4
Australia	-	$A_w = \frac{M'_{ts} \cdot s}{1.6 b'd' f_w}$ where $M'_{ts} = (M'_t - \frac{b^2 D \tau}{5})$, $\tau = 80$ psi.	$A_s = \frac{M'_{ts}(b'+d')}{0.8\, b'd' f_s}$
Germany Egypt and Hungary	$A_w = \frac{M'_t \cdot s}{2\sqrt{2}\, b'd' f_w}$	$A_w = \frac{M'_t \cdot s}{2 b'd' f_w}$	$A_s = \frac{M'_t (b'+d')}{b'd' f_s}$ 4 bars to be provided one at each corner.
Poland	$A_w = \frac{M'_t \cdot s}{2\sqrt{2}\, b'd' (\frac{f_w}{m})}$	$A_w = \frac{M'_t \cdot s}{2 b'd' (\frac{f_w}{m})}$	$A_s = \frac{M'_t (b'+d')}{b'd' (\frac{f_s}{m})}$
Russia	$A_w \geqslant \frac{M_t \cdot s}{2\sqrt{2} b'd' mm_s f_{yw}}$	$A_w \geqslant \frac{M_t \cdot s}{2 b'd' mm_s f_{yw}}$ But $A_w \leq (\frac{2A_s}{b'+d'})$	$A_s \geqslant \frac{M_t (b'+d')}{b'd' mm_s f_y}$

(continued)

Table I.11.1 (contd)

1	2	3	4
France	-	$A_w = \frac{0.2bs(b+D)\tau}{D f_w}$ $A_s = \frac{b(b+D)\tau}{3f_s}$ where $\tau = \frac{M'_t}{b^2 D}\left[4.81 - \frac{1.81(D-b)}{\sqrt{2b^2 + D^2}}\right]$	
G S A	-	$A_w = \frac{S_t \cdot s}{f_w}$ where $f_w = 20$ ksi $S_t = \frac{b(\tau - 90)}{4}$ $\tau = \frac{4.5\ M'_t}{b^2 D}$	Nominally use minimum four 5/8" diameter bars one at each corner
India (Ref.No.26)	-	$A_w = \frac{M'_t \cdot s}{1.6\ b'd' f_w}$ If τ exceeds the permissible shear stress, provide transverse steel for the entire torque where $\tau = \frac{M'_t}{b^2 D}\left(3 + \frac{2b}{D}\right)$	Longitudinal steel shall have the same volume per unit length as the volume contained in the corresponding legs of the stirrups ie. $A_s = \frac{M'_t\ (b'+d')}{0.8b'd'\ f_s}$

M'_t - torque at working loads; f_s, f_w - maxm. allowable stresses in longitudinal and transverse steel, respectively; m, m_s - working load coefficients varying between 0.90 and 1.00.

Cowan[2] which differ only in the value of a numerical coefficient, viz. 2 versus 1.60 as readily seen from Table I.11.1. Generally all the codes recommend both transverse and longitudinal reinforcements to resist the torque acting on the structure.

Firstly, it is evident from Table I.11.1., that none of the codes considers the influence of interaction between bending and twisting moments in giving their recommendations. In most of the structures encountered in practice, the twisting moment is accompanied by bending moment, shear force etc. Even though it is generally said that the addition of bending moment increases the torque capacity of members, indefinite increase is not possible. Also increase in the torque capacity of beams varies with the physical properties of the beams and in some cases there may not be any increase in the torque capacity of beams (see Fig. I.64 and Art.I.10.4). On the other hand, the presence of twisting moment causes an adverse effect on the flexural capacity of the member. Experiments of the author as well as those of others available in the literature have shown that the reduction in the flexural capacity of a beam is not small and varies with the amount of torque present. These test observations were obtained from the ultimate tests on beams. Since most of the codes recommend formulae for working load conditions, it may not be strictly possible to interpret these results

obtained from ultimate load tests for working load conditions. But the test results reported by Cowan[23] at "visco-elastic" stage tend to support the conclusions drawn from ultimate load tests. This indicates that the interaction of moments as obtained from ultimate load tests may be true even at working load conditions. In such a case neglecting the interaction of moments may not give a correct design procedure.

Secondly, in all the codes the torsional shear stress is limited to a certain value. If the shear stress exceeds this limiting value, all codes, except the Australian and GSA Codes, recommend that the members are to be designed by neglecting the contribution of concrete and assuming that the entire torque is to be carried by the transverse reinforcements. The test results indicate that even at ultimate stages the contribution of concrete is considerable and the torque capacity of a beam is not very much reduced even in the presence of flexural cracks. This is well brought-out from the test results of B Series beams tested by the author (see Art. I.7.5.5). At working loads the situation may be more favourable and the contribution from concrete may be considerable. So neglecting the contribution from concrete may under-estimate the capacity of beams to a considerable extent and may result in an uneconomical design.

Thirdly, most of the codes recommend that longitudinal steel having the same volume per unit length as the volume contained in the corresponding legs of the stirrups is to be provided. But from the discussion of test results given in a previous article (see Art. I.7.5.2) it can be seen that there is an optimum percentage of longitudinal steel which is necessary along with the transverse reinforcements to resist the torsional stresses. Increasing the longitudinal steel alone beyond this optimum value will not have any significant influence on the capacity of the beam.

Further it is to be pointed out here that it may not be possible if not impossible to have direct check on the accuracy of these recommendations using experimental results. Hence the design procedures based on ultimate load methods of analysis is more preferable than the working stress method of design.

I.11.3 New Design Recommendations:

Using the analytical and experimental results discussed in the preceding chapters, the following recommendations are given for the ultimate strength design of reinforced concrete beams subjected to combined bending and torsion:-

a. For the given working loads, the ultimate moments are to be calculated using specified load factors. In giving these recommendations, it is assumed that the analysis of structures

is known and the maximum moments, forces etc. are known.

b. Choosing a value for D/b ratio, the dimensions of the cross section are to be fixed-up such that $b \geq 3\sqrt{M_t/\beta_e (\frac{D}{b}) K_m f'_c}$. For rectangular beams*, $b \geq 3\sqrt{M_t/\beta_m f'_c}$.

TABLE I.11.2

VALUES OF COEFFICIENTS β_m, β_s, β_e AND λ FOR RECTANGULAR BEAMS

D/b	β_m			β_s	β_e	λ
	$f'_c/f'_t = 8$	$f'_c/f'_t = 10$	$f'_c/f'_t = 12$			
1.0	0.045	0.041	0.037	0.271	0.208	1.528
1.2	0.060	0.054	0.050	0.285	0.234	1.640
1.5	0.087	0.080	0.072	0.310	0.270	1.762
2.0	0.132	0.120	0.110	0.332	0.309	2.060
2.5	0.178	0.163	0.150	0.346	0.334	2.400
3.0	0.227	0.207	0.189	0.356	0.354	2.690
4.0	0.324	0.296	0.272	0.370	0.379	3.355
5.0	0.417	0.380	0.350	0.379	0.390	4.060
10.0	0.960	0.880	0.800	0.400	0.448	7.600

c. The section is to be first designed for the twisting moment using the following equation:

$$M_t = \left[\frac{(2k'-1)}{2k'^2} \cdot \beta_s b^2 D f'_c + A_w f_{yw} \frac{b'd'}{s}\right] \quad \ldots \text{ I.11.1}$$

* The values of the coefficients β_m, β_e, λ and β_s for rectangular beams are given in Table I.11.2.

The spacing of hoops should not be greater than d'.

d. From the known properties of the cross section, the value of $M_{to}/\beta eb^2Df'_c$ is to be calculated where the value of M_{to} can be obtained from Eqn. I.11.1 taking into account the exact quantities of transverse steel provided in the beam.

e. For the calculated value of $M_{to}/\beta eb^2Df'_c$ and known value of f'_c/f'_t, the value of $(1/\lambda\phi)_{TR}$ can be known using Fig.I.58. The value of $1/\lambda\phi$ for the beam can be calculated by knowing the value of λ for the cross section (see Table I.11.2 for rectangular beams) and the ratio of twisting moment to bending moment, ϕ.

f. If the value $1/\lambda\phi$ is greater than $(1/\lambda\phi)_{TR}$, then,

$$\text{Equivalent Design bending moment} = \frac{\text{Bending moment at the section}}{(M_b / M_{bo})} \quad \ldots \text{ I.11.2}$$

where M_b/M_{bo} can be obtained using Fig. I.55 for the known values of $1/\lambda\phi$ and f'_c/f'_t. The beam is to be designed for the Equivalent Design Bending Moment using the requirements of codes for the design of beams under pure bending.

g. If the value of $1/\lambda\phi$ is less than or equal to $(1/\lambda\phi)_{TR}$, the flexural capacity of the beam is not controlling the design. In such cases, the longitudinal steel is to be designed only for the bending moment at the section using

usual methods. The transverse hoops designed in step (c) are sufficient to take up the twisting moment.

h. In the absence of any bending moment, the beam can be directly designed for the twisting moment using Eqn. I.11.1. In such cases a nominal amount of longitudinal steel of 0.5% is to be provided in addition to the transverse hoops. This recommendation is based on Ernst's[30] experimental results.

i. It is assumed that the beams have sufficient bond strength and proper anchorage of reinforcements such that these may not influence the cause of failure.

The above recommendations are mainly given for the case of rectangular beams. It is to be pointed out here that we require two figures (i.e. Figs. I.55 and I.58) and one Table (i.e. Table I.11.2) for the design of rectangular beams under combined bending and torsion.

I.11.4 Illustrative Examples:-

Example 1:

Design a beam which has the following working moments:

Bending Moment = 1000 in kips

Twisting Moment = 350 in kips

Given that f'_c = 3000 psi.; $f_y = f_{yw}$ = 40000 psi; Load factor = 2; effective cover to steel is 2 in. f'_c/f'_t = 10.

Design:-

For the load factor of 2, ultimate bending moment

$$M_b = 2000 \text{ in kips.}$$

Ultimate twisting moment $M_t = 700$ in kips

Choosing a rectangular beam of D/b ratio equal to 1.5, from Table I.11.2,

$$\beta_m = 0.080, \quad \beta_e = 0.274;$$

$$\beta_s = 0.310, \quad \lambda = 1.762$$

$$b \geq \sqrt[3]{\frac{M_t}{\beta_m f'_c}}$$

$$\geq \sqrt[3]{\frac{700}{0.08 \times 3.0}} \geq 14.3 \text{ in.}$$

So select b = 16 in; D = 24 in; d = 22 in;

b' = 12 in; d' = 20 in.

$$\frac{2k'-1}{2k'^2} \cdot \beta_s b^2 D f'_c = 0.095 \times 0.310 \times 16 \times 16 \times 24 \times 3.0$$

$$= \underline{542} \text{ in kips}$$

$$A_w f_{yw} \frac{b'd'}{s} = 700 - 542 = 158 \text{ in kips.}$$

Using 3/8" diameter two-legged stirrups,

$$s = \frac{0.11 \times 40 \times 12 \times 20}{158}$$

$$= \underline{6.68} \text{ in.}$$

Adopt 3/8" diameter two-legged stirrups at 6.5 in centre to centre.

$$M_{to} = (542) + \left(\frac{158}{6.50} \times 6.68\right)$$

$$= (542) + (162) = \underline{704} \text{ in kips}$$

$$\beta_e\, b^2 D f'_c = 0.274 \times 16 \times 16 \times 24 \times 3$$

$$= \underline{5050} \text{ in kips.}$$

$$\frac{M_{to}}{\beta_e b^2 D f'_c} = \frac{704}{5050} = 0.139.$$

From Fig. I.58, $(1/\lambda\phi)_{TR} = 0.45$

$$1/\lambda\phi = \frac{1}{1.762 \times 0.35} = \underline{1.62}.$$

$$(1/\lambda\phi) > (1/\lambda\phi)_{TR}$$

From Fig. I.55, $M_b/M_{bo} = 0.64.$

Equivalent Design Bending Moment $= \frac{2000}{0.64} = \underline{3130}$ in kips

$$\frac{M}{bd^2 f'_c} = \frac{3130}{16 \times 22 \times 22 \times 3} = \underline{0.135}.$$

$$\frac{f_y}{100 f'_c} = \frac{40}{100 \times 3} = \underline{0.133}.$$

Using the Nomogram given by Sundara Raja Iyengar and Author[77],

$$p = 1.20\%$$

$$\text{or} \quad As = \frac{1.2}{100} \times 16 \times 22 = \underline{4.23} \text{ in}^2.$$

Provide 1" diameter bars - 6 Nos.

Example 2:

Design a beam for the following working moments:

Bending Moment = 75 in kips

Twisting Moment = 150 in kips

Given that f'_c = 3000 psi; $f_y = f_{yw}$ = 30000 psi;

Load factor = 2; f'_c/f'_t = 10; cover = 2 in.

Design:

Ultimate bending moment = 150 in kips

Ultimate Twisting Moment = 300 in kips

Choosing a rectangular beam of D/b = 2.5, from Table I.11.2

β_m = 0.163; β_e = 0.334;

β_s = 0.346; λ = 2.400

$$b \geq \sqrt[3]{\frac{300}{0.163 \times 3}}$$

$$\geq \underline{8.5} \text{ in.}$$

Select b = 10 in; D = 25 in; d = 23 in; b' = 7in; d' = 21 i

$$\frac{2k'-1}{2k'^2}\ \beta_s\ b^2 D f'_c = 0.095 \times 0.346 \times 10 \times 10 \times 25 \times 3$$

$$= \underline{246} \text{ in kips}$$

$$A_w\ f_{yw}\ \frac{b'\ d'}{s} = 300 - 246 = 54 \text{ in kips}$$

Using 3/8" diameter two-legged stirrups,

$$s = \frac{0.11 \times 30 \times 7 \times 21}{54} = \underline{9 \text{ in.}}$$

Adopt 3/8" diameter two-legged stirrups at 9 in. c/c.

$$M_{to} = (246) + (54 \times \tfrac{9}{9}) = \underline{300} \text{ in kips}$$

$$\beta_e\ b^2 D f'_c = 0.334 \times 10 \times 10 \times 25 \times 3$$

$$= \underline{2505} \text{ in kips}$$

$$\frac{M_{to}}{\beta_e b^2 D\ f'_c} = \frac{300}{2505} = 0.120$$

From Fig. I.58, $(1/\lambda\phi)_{TR} = 0.250$.

$$1/\lambda\phi = \frac{1}{2.4 \times 2.0} = 0.208$$

$$1/\lambda\phi < (1/\lambda\phi)_{TR}$$

So the flexural capacity of the beam is not controlling the design and longitudinal steel is to be designed for a bending moment of 150 in kips only. Since this bending moment is very small (i.e. it is far less than the flexural

cracking moment) no longitudinal steel is required for the bending moment. But a nominal percentage of longitudinal steel is to be provided. That is we can provide 0.50% of longitudinal steel.

$$\frac{As}{bD} = 0.50\%$$

$$\text{or } As = \frac{0.50}{100} \times 10 \times 25 = \underline{1.25} \text{ in}^2$$

Provide 4 Nos. of 5/8" diameter bars - one at each corner of the beam - as a nominal reinforcement.

I.11.5 Comparison of Results:

Even though a rigorous comparison of results given by various code recommendations with those obtained using the recommendations given by the present author may not be permissible, it may be of interest to do such a comparison to get a qualitative idea about the code recommendations. In Table I.11.3, the results given by the present recommendations for the two examples discussed in the preceding article are compared with those obtained using various code recommendations. The comparison is made with respect to the torsion reinforcements only.

From Table I.11.3 it can be seen that the recommendations by the codes seem to be very conservative. The possible reason for this may be due to the fact that most of the codes ignore

TABLE I.11.3

COMPARISON OF RESULTS : REINFORCEMENTS FOR TORSION BY VARIOUS RECOMMENDATIONS

Code or Recommendation	Example 1		Example 2	
	Longitudinal steel (in^2)	Transverse hoops: 3/8" at (in)	Longitudinal steel (in^2)	Transverse hoops: 3/8" at (in)
Author	-	6.50	Nominal	9.00
Australia	2.10	3.25	1.31	4.50
Germany	2.34	3.00	1.43	4.25
Egypt, Hungary	2.34	3.00	1.43	4.25
Poland	2.34	3.00	1.43	4.25
France	2.00	1.50	0.90	5.00
Russia	2.60	2.75	2.12	3.00
GSA	Nominal	3.25	Nominal	5.00
India	2.92	2.00	1.79	3.50

the contribution from concrete to the torque capacity of the beam and this may result in the under-estimation of the torque capacity of the beam to a great extent. Even in the case of Australian and GSA codes, the results are very conservative as they allow only a small percentage of torque to be resisted by

the concrete. Due consideration by taking into account the contribution of concrete in the calculation of torque capacity of beams may improve the recommendations of codes.

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CHAPTER I.12

CONCLUSIONS

I.12.1 Summary of Conclusions:

The following conclusions can be drawn from the experimental and analytical investigation conducted by the author on reinforced concrete beams under combined bending and torsion:-

1. Strains in beams subjected to combined bending and torsion:

The magnitudes of maximum compressive strains measured on the concrete surface depend on the value of ϕ. For values of ϕ greater than 0.65, the strains are very low ranging from 30 to 50 x 10^{-5} inch/inch. In the case of beams with $\phi < 0.65$, the maximum compressive strains are considerably high ranging from 180 to 220 x 10^{-5} inch/inch. In general, the bending moment-strain diagrams are approximately linear upto 80 to 85% of the collapse moment.

The strains measured in the transverse steel for the case of beams with high values of ϕ ($\phi > 0.75$) indicate that the transverse steel yields in such cases. The hoops

registered considerable strain only after the appearance of diagonal cracks. The strains measured in the longitudinal steel in the case of beams under pure torsion are of small magnitude till failure. At the collapse of the beam there is a sharp increase in the strains; but the maximum strains are far less than the yield strain of steel.

2. Modes of failures:

The observed modes of failures can be classified under the following three categories:

Mode 1: Premature flexure failure:

When $\phi < 0.15$, premature flexure failure of the beam occurs. This type of failure occurs even before the formation of diagonal crack and the crack pattern resembles that of a beam under pure bending showing little influence of torsional stresses on them.

Mode 2: Premature torsion failure:

For values of $\phi > 0.65$, beams prematurally failed in torsion. Beams failed due to a diagonal crack inclined at about 45° to the longitudinal axis of the beam. Failure is very violent and there are no signs of crushing in any part of the failure crack.

Mode 3: Diagonal compression failure:

This type of failure occurs in the case of beams tested

with values of ϕ in between 0.15 and 0.65. The inclination of the failure crack varies with the value of ϕ. The beams failed according to this mode indicate heavy crushing of concrete in the compression zone of the beam. The projecting portions of the flanges of tee beams failed in this manner show a tendency to separate out from the remaining cross section.

3. Factors investigated:

a. Concrete Strength:

Experiments indicate that the degree of influence of concrete strength on capacity of beams varies with the value of ϕ. For beams failed either in premature flexure or in diagonal compression, the influence of concrete strength on the reduction in the flexural capacity of the beams is not very significant. The concrete strength influences the torque capacity of beams failed in premature torsion to a considerable extent. The diagonal cracking torque depends mainly on the strength of concrete and the torque capacity of a beam with transverse hoops is interdependent on the torque capacity of a corresponding beam without transverse reinforcement. Hence it is necessary to consider the torque carried by the concrete in the evaluation of torque capacities of beams.

b. Longitudinal Reinforcements:

In the absence of transverse reinforcement, the increase in the torque capacity of a beam by increasing the longitudinal ste alone is not significant. In the case of beams with transverse reinforcement a minimum amount of longitudinal steel is required. Increase in the longitudinal steel alone beyond this minimum requirement is not having any significant contribution to the torque capacity. The test results reported by Ernst[30] indicate that this minimum requirement of longitudinal steel can be taken as 0.5%. The longitudinal steel influences the flexural capacity of the beam under combined bending and torsion in the same manner as in the case of a beam under pure bending.

c. Transverse Reinforcements:

The effect of transverse hoops on the flexural capacity of beams failed either in premature flexure or in diagonal compression is not very significant. But the torque capacity of beams failed in premature torsion increases with the increase in the amount of transverse reinforcement. However, there seems to be minimum and maximum limiting values of transverse steel ratio (A_w/bs). If the spacings of transverse hoops are greater than the depth of the beam, there is no increase in the torque capacity of the beam. Similarly the increase in the torque capacity beyond certain

values of $M_t/b^2Df'_c$ is not in proportion with the amount of transverse steel. The test results of Ernst[30] indicate that this limiting value of $M_t/b^2Df'_c$ is approximately equal to 0.05; but this value is likely to be influenced by the cross sectional properties of the beams. Hence it is necessary to restrict the transverse steel ratio between these limiting values.

d. Ratio of Twisting Moment to Bending Moment, ϕ:

In general, flexural capacity of a beam subjected to combined bending and torsion is reduced by the presence of twisting moment. But the percentage reduction in the strength varies with the value of ϕ. The flexural capacity of a beam decreases with the increase in the value of ϕ and for $\phi < 0.10$ there is practically no reduction in the strength of the beam. Also for values of $\phi > 0.50$ the reduction in the flexural capacity is approximately constant and its variation with ϕ is not significant.

The presence of bending moment increases the torque capacity of beams only in certain ranges of ϕ. For $0.20 \leq \phi \leq 0.35$, the quantity M_t/M_{to} increases with the increase in the value of ϕ and for values of $\phi > 0.35$ it decreases with the increase in the value of ϕ. The maximum value of M_t/M_{to} occurs when the bending moment present is about 60 percent of the flexural capacity of the beam under pure bending. For values of $\phi > 0.75$, the value of

M_t/M_{to} is nearly equal to unity and hence in such cases the influence of bending moment on torque capacity of beams is negligible and can be ignored for all practical purposes.

e. Presence of Flexural Cracks:

From the test results of a special group of beams tested in the present investigation, it is found that the presence of flexural cracks has no significant influence on the torque capacity of beams. In most of the cases, the torque resisted by a flexurally cracked beam is equal to the torque capacity of an uncracked beam. The test results reported by Ersoy[58] on semi-continuous Ell beams also confirm this conclusion. Hence it seems reasonable to consider the gross section in the calculation of torque capacity of beams under combined bending and torsion.

f. Shape of the Cross Section:

The test results of the present investigation indicate that the shape of the cross section has very significant influence on the capacity of the beam at all ranges of ϕ except in the case of very low values of ϕ. In the case of rectangular and tee beams tested by the author, it has been found that the relative difference between the strength of these two types of beams varied approximately in the same ratio as their torsional rigidities.

4. Analytical Investigation:

a. Torsion of Tee, Ell and I-Sections:

Using a Fourier Series technique, a solution is given to solve the problem of elastic torsion of these sections. The method is simplified by means of charts. Simple expressions are given to calculate the torsional rigidity of these sections under full plastic conditions. These elastic and rigid plastic solutions can be used to calculate torsional rigidities of these sections encountered in reinforced and prestressed concrete construction.

b. Strength of Beams under Combined Bending and Torsion:

Based on certain assumptions and using a failure criterion for concrete under compressive-tensile stresses, equations have been derived to predict the strength of beams under combined bending and torsion. The equations are summarised below:

For Flexure Failure, $(1/\lambda\phi) > (1/\lambda\phi)_{TR}$:

$$\frac{M_b}{M_{bo}} = \left(\frac{f'_t}{f'_c}\right)^2 \frac{(\gamma+1)}{(\gamma-1)^2}\left[\sqrt{1+\left\{\left(\frac{2f'_c}{f'_t}\right)\cdot\frac{\gamma-1}{\gamma+1}\right\}^2}-1\right]$$

For Torsion Failure, $1/\lambda\phi \leq (1/\lambda\phi)_{TR}$:

$$M_t = \left[\left(\frac{2k'-1}{2k'^2}\right)\beta_s\, b^2 D f'_c + A_w f_{yw}\frac{b'd'}{s}\right]$$

The equation for flexural failure is graphically represented

in Fig. I.55. The values of λ are tabulated for rectangular beams (Table I.9.2) and methods are given to evaluate its value for other sections (Art. I.9.4). Using the known values of λ and for given values of ϕ and f'_o/f'_t, the determination of M_b/M_{bo} using Fig. I.55 is simple and straightforward. On comparison with the test results of about three hundred beams in total, it is found that the theoretical predictions are good in the majority of the cases. The theoretical equations are applicable both for beams with and without transverse reinforcements (ie. vertical hoops).

c. Conclusions from Theoretical Results:

i. The interaction diagram for the moments varies with all the properties of the beams such as cross sectional dimensions, strength of concrete, amount and strength of transverse reinforcement, shape of the cross section etc. As there is no single common interaction curve, it may not be possible to base any design criterion purely on the basis of a single interaction diagram.

ii. The maximum torque capacity of a beam occurs when the bending moment present is between 60 and 62 percent of the capacity of the beam under pure bending. This theoretical result is in close agreement with the experimental observation stated in Art. 3(d) of this chapter. The maximum torque capacity of a beam can be known using

Eqn. I.10.14. This provides a basis for restricting the dimensions of the beam and for rectangular beams this limit can be expressed as follows:

$$b \geq \sqrt[3]{\frac{M_t}{\beta_m f'_c}}$$

The values of β_m are given in Table I.10.5 for different D/b ratios. The results given by the above inequality are close to the results given by a similar limitation obtained by Lessig[17] and Collins et al[56] on the basis of their test results.

iii. The theory restricts the amount of transverse steel to a certain limit. For the case of rectangular beams this limit can be expressed as follows:

$$\frac{A_w}{bs} \leq \frac{bDf'_c}{b'd'f_{yw}}\left[\left(\frac{\beta_m}{(D/b)}\right) - \left(\frac{2k'-1}{2k'^2}\right)\beta_s\right]$$

The test results reported by Ernst[30] attempt to confirm the necessity of the above limitation given by the theory.

d. Recommendation for Design:

Using the results of experimental and analytical investigations conducted by the author, a set of recommendations have been given in the preceding chapter for the

design of reinforced concrete beams subjected to combined bending and torsion. Most of the codes have given recommendations for working load conditions only. These code recommendations give very conservative results because they ignore the contribution from concrete to the torque capacity of beams. The recommendations given herein provide a basis for the design of beams using ultimate load methods.

I.12.2 Suggestions for further research:

In the present investigation, the ratio of twisting moment to bending moment was selected as a major factor. The influence of other factors has been studied only to a certain extent. However, because of the immensity of the problem some factors and their interdependence could not be investigated over their complete range. In such circumstances, the need for further work is a necessity.

It was not possible for the author to know fully the influence of concrete strength on the capacity of beams. However the results of the author have indicated that the contribution of concrete to the torque capacity of beams is considerable. Hence further research on these lines is necessary to get detailed information in this respect.

In general, it has been found that the increase in the torque capacity by increasing the longitudinal steel along

is not considerable. But it is necessary to have a minimum amount of longitudinal steel along with transverse hoops. From the available results it has been found that about 0.50% of steel is sufficient for this purpose. A detailed study on this aspect is recommended. More data is also needed to have further study on the maximum amount of transverse steel which we can provide in the light of the theoretical predictions given in the present thesis.

It would be interesting to conduct tests on beams of different cross sectional properties and on frame members. It would be possible, then, to examine the theoretical conclusions drawn with respect to the interaction diagrams in the light of these experimental results. After adequate knowledge on beams with concentrated loads has been obtained, future work can be planned to study the influence of distributed loads on the strength of beams under combined bending and torsion.

Further both the experimental and analytical investigations reported in this part of the thesis have been confined to the case of beams with transverse steel in the form of closed hoops only. But it is well known that the most effective type of transverse reinforcement

for members subjected to torsional stresses is the spiral reinforcement. So a detailed experimental and analytical study on beams with spiral reinforcement is recommended.

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PART II : REINFORCED CONCRETE BEAMS UNDER COMBINED BENDING AND SHEAR

CHAPTER II.1

INTRODUCTION

II.1.1 The need for research in this problem:

The problem of shear strength of beams has attracted many research workers and still remains to do so because of its practical importance. The problem is not fully solved because of the reason that the behaviour and analysis of concrete under combined stresses is very complex. The existing methods recommended by the codes and specifications of various countries for design of members to resist shear stresses are different from one another. Moreover, these design equations are generally of empirical nature and may not be applicable for all cases. In recent years it has been emphasised by some of the investigators that designing members subjected to combined bending and shear on the basis of shear stresses alone cannot represent the true behaviour of the structure. In fact, the shear stresses have less effect on the mechanism of shear failure in certain cases in which the collapse of a beam is not due to tensile stresses, but on the contrary it is due to

compressive stresses. So an inevitable change is necessary for the recommendations by the codes. This is further evident from the report of the ACI-ASCE Committee 426 (1963)[78] which states thus: ' ..... It is again emphasized that the design procedures proposed are empirical because the fundamental nature of shear and diagonal tension strength is not yet clearly understood. Further basic research should be encouraged to determine the mechanism which results in shear failures of reinforced concrete members. With this knowledge it may then become possible to develop fully rational design procedures'.

Attempts have already been made to evolve equations to predict the ultimate strength of beams failing in combined bending and shear. Many of these attempts were semi-rational and some of the latest works have resulted in important new concepts, but again these are accompanied by a new awareness of gaps in our present knowledge. Some of the available solutions result in complicated equations which may not find place in everyday design office use while the others involve certain parameters which are to be determined from large number of experimental results. So a rational solution resulting in simple equations is desirable in improving the present codes and specifications.

Shear failures are sudden and unexpected, and a thorough knowledge of the mechanism involved is essential because of the great danger involved in such failures. So a rational, general and simple method of calculation which can be directly applied to practical cases is essential. It is a necessity and it will afford greater economy whilst ensuring satisfactory safety.

II.1.2 Object and scope of the present investigation:

A fairly large number of experimental results are available in the literature with regard to the beams failing in shear covering both reinforced and prestressed concrete beams. So further experimentation on large number of beams has been avoided in the present investigation. Instead, the experimental observations of various investigators available in the literature have been closely studied with physical interpretations in order to device a mechanism for shear failure. The object of the present investigation is two-fold and it tries to answer the following two questions:

i. What is the so-called mechanism of shear failure in members subjected to combined bending and shear?

ii. What is the strength of this mechanism and how it can be strengthened to avoid the more dangerous shear failures?

In the past, there has been too much emphasis in finding certain parameters like depth of compression block,

dowel shear etc., accurately thus making the analysis and the resulting equations complex. In the analysis presented herein, this type of procedure is not adopted. The effect of certain unknown factors like dowel shear which has been proved by experiments to have negligible influence on the ultimate strength of beams, has been omitted in the analysis. This procedure has resulted in simple equations which are a little bit conservative. It has been felt that conservative solutions are essential to take care of the approximations made in the estimation of loads on the structure and to allow for the variation in physical properties of the materials used in the construction.

The investigation has been conducted in three phases. In the first phase of the work, the test observations available in the literature have been closely examined and the analytical solutions presented by previous investigations have been critically reviewed. In addition, a minor experimental programme consisting of thirty beams has been conducted to clarify some aspects of the problem.

In the second phase of the work, the test observations have been physically interpreted and a general mechanism of shear failure has been formulated. Then using a failure criterion for concrete under compressive-tensile stresses, equations have been derived on the basis of the failure mechanism.

In the last part of the work, the predictions by the theoretical equations have been compared with the test results. The applicability of the equations to different type of structures, both statically determinate and indeterminate with concentrated and uniformly distributed loads has been studied. On comparison with over one thousand test results, it has been found that the proposed theory and equations are fairly good in predicting the test results within an error of $\pm$ 15% in most of the cases. Finally, based on the theoretical results, design recommendations have been suggested and they have been compared with the provisions of some of the recent codes.

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CHAPTER II.2

REVIEW OF EARLIER RESEARCH

This chapter is devoted to the study of the historical development of the problem. The problems relating to the resistance of concrete and reinforced concrete structures to the combined action of normal (or bending) and shearing stresses have received considerable attention from many investigators throughout the globe. A large number of investigations containing over one thousand beam tests have been reported in the literature. Based on their own experimental results, many investigators proposed empirical equations to evaluate the shear strength of reinforced concrete beams. Some of the test observations resulted in new concepts about the problem, but once again accompanied by new awareness of some more " unknown factors ".

A detailed and exhaustive historical and critical review of the subject has been given by Hognestad[79] in 1953 and by Ramakrishnan[80] in 1960. In this chapter the author presents a critical review of some of the investigations which resulted in new concepts or rational methods for calculating the shear strength of beams in some detail. The results are summarised in Table II.2.1.

TABLE II.2.1

SHEAR AND DIAGONAL TENSION - REVIEW OF EARLIER RESEARCH

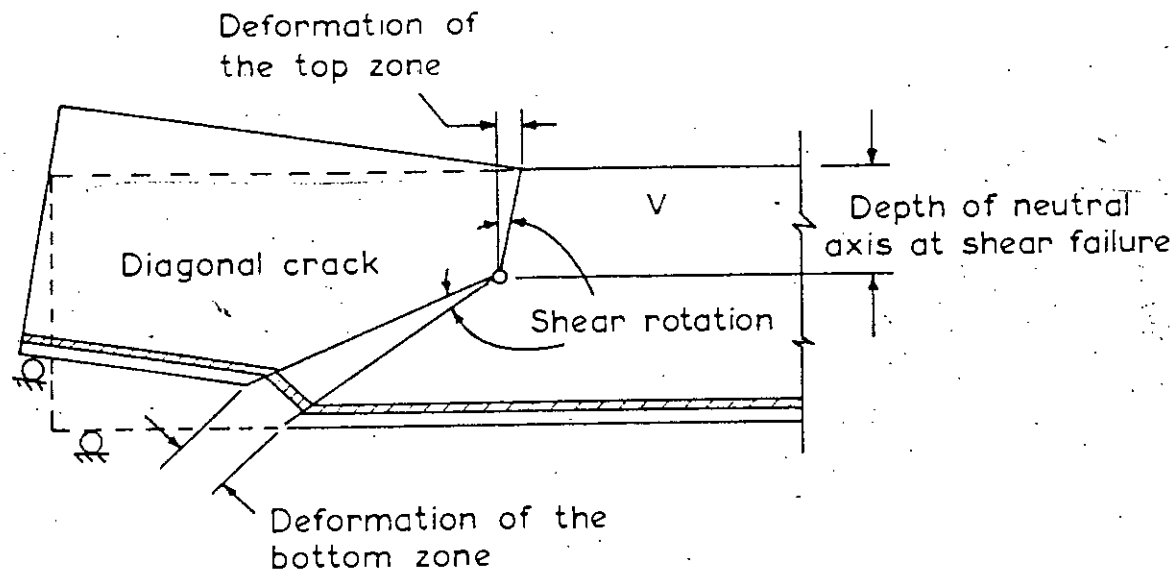
Investigator/s	Important Conclusions	General comments
1	2	3
Ritter[81] (1899)	Introduced the "Truss Analogy" concept neglecting the effect of uncracked concrete area.	Neglecting the effect of concrete in the calculation of shear strength results in very conservative design.
Morsch[82] (1902)	The well known expression for the calculation of nominal shear stress has been suggested.	The nominal shear concept for estimating the shear strength of beam is of limited utility as the failure depends on the interaction of normal and shear stresses.
Talbot[83] (1909)	Influence of concrete strength, span-depth ratio, percentage of tension reinforcement, and amount of shear reinforcement on the shear strength has been indicated.	Qualitative knowledge of the behaviour of beams has been given.
Moretto[84] (1945)	Perhaps the first empirical equation in terms of nominal shear stress has been suggested.	The influence of interaction of normal and shear stresses in terms of the well known parameter a/d has been ignored in the given equation.

(continued)

Table II.2.1 (contd.)

Investigator/s	Important Conclusions	General comments
Clark[85] (1951)	First equation introducing a/d ratio.	It is the first investigation which accounted for the influence of various parameters expressed by Talbot[83] in a quantitative manner.
Ferguson[86] (1951)	It has been pointed out that the shear stress varies with the a/d ratio.	Brought into light the important influence of shear arm-depth ratio on the shear strength of beams which has not been considered till that time.
Laupa, Siess and Newmark[87] (1955)	A new concept using the basic moment equation has been introduced for the so-called "shear-compression failure". The coefficients were evaluated using empirical methods. For Tee beams an empirical shape factor has been suggested.	Ignores the important influence of a/d ratio on shear strength of beams. General behaviour of beams cannot be explained by this concept and is not valid for all the ranges of a/d ratio.
Ferguson[88] (1956)	Distinguished two types of failures: (1) "restrained" and (2) "unrestrained" failures. In the case of low a/d ratios, the vertical	The concept of these two types of failures gave a new line of thought and has been adopted by some of the later investigators like Kani[99] etc. The same type of behaviour of

(continued)



IDEALIZED MECHANISM OF THE RELATIVE DISPLACEMENTS PRIOR TO A SHEAR FAILURE.

(After walther)

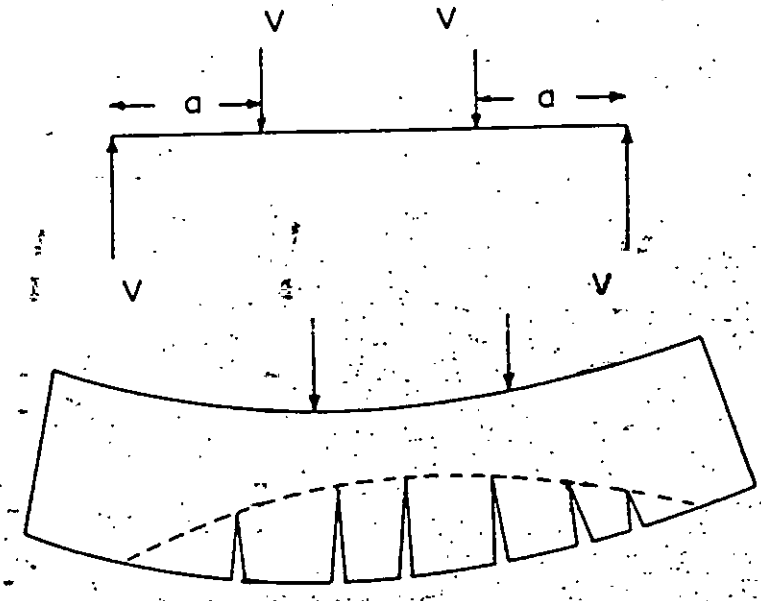
FIG. II. 1.

Table II.2.1 (contd.)

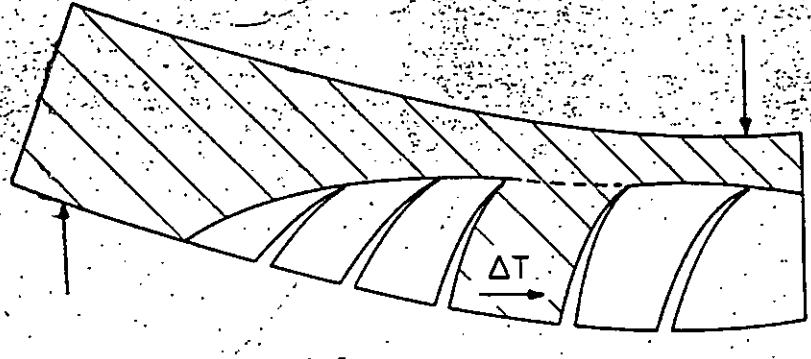
Investigator/s	Important Conclusions	General comments
Ferguson[88] (1956)	compressive stresses set up near the supports reduce the potential diagonal tension resulting in "restrained" failure. In the case of large a/d ratios unrestrained type of failure occurs in such case the diagonal tension resistance is primarily a function of shear force.	beams has been physically interpreted to device the mechanism of shear failure in the present thesis also.
Bresler and Pister[89] (1958)	Using a failure law for concrete under combined stresses, shear strength of certain group of beams has been evaluated. The stress in tension steel has been assumed to be its yield stress.	The basic assumption with respect to stress in tension steel is not in confirmation with the test results. The evaluation of certain quantities has been based on empirical equations. Applicable only for a special group of beams which is not defined by the analytical expressions. Moreover, the suggested failure law does not take into account the ratio f'_c/f'_t.
Walther[90] (1956)	Based on a general concept of shear rotation and a idealized mechanism of the relative displacements prior to a shear failure (Fig. II.1) expressions have been derived. Certain simplifications were suggested to the resulting complicated equations.	The theory resulted in complicated equations. Expressions involved many parameters some of which have been evaluated by empirical methods.

Table II.2.1 (contd.)

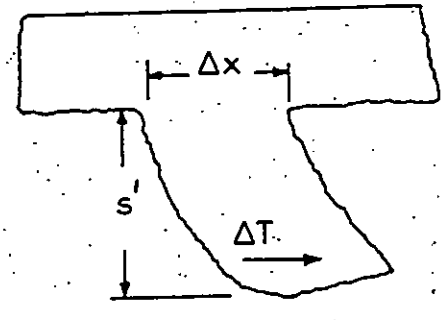
Investigator/s	Important Conclusions	General Comments
Leonhardt and Walther[91] (1953)	Exhaustive experimental work of reinforced and prestressed concrete beams of different cross sections, different types of loading etc. has been reported.	Experimental results are very much useful in understanding the behaviour of beams under combined bending and shear
Guralnick[92] (1960)	Approximating the Mohr's envelope by a straight line and using the stress block suggested by Hognestad et al, analytical expressions have been developed. It is pointed out that the derived expressions are not valid for $a/d < 3.0$.	The derived expressions are applicable for the socalled "shear-compression" type of failure. The limitation of the equations is not derived from the analysis and hence it may be applicable for certain groups of beams only.
Ramakrishnan[80] (1960)	(1)Contribution from dowel action of longitudinal tension steel tc the shear strength is negligible at failure. (2)Compression reinforcement is little effective in increasing the shear strength. (3)The "shear-compression" theory has been proved to be of limited applicability. (4)Deformation characteristics of beams indicated plastic state in shear at failure.	These experimental observations lend support to some of the assumptions made in the theory proposed in the present thesis.



Beam with idealized type of loading.

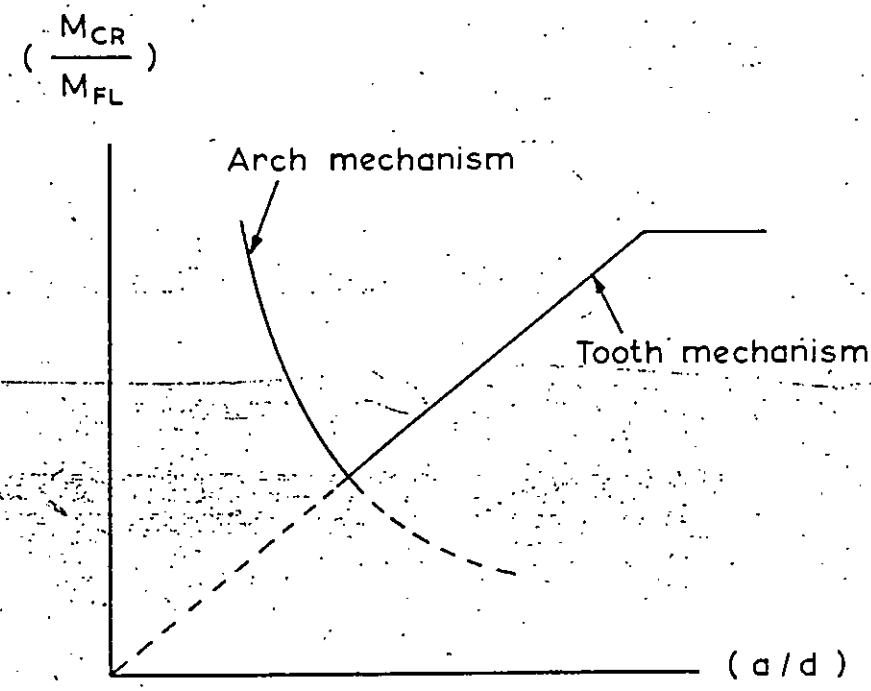


ΔT - Dowel force



KANI'S MECHANISM OF SHEAR FAILURE
ANALOGY OF CONCRETE TOOTH.

FIG. II. 2.



VALLEY OF SHEAR FAILURE AS DEFINED BY KANI'S ANALYSIS.

FIG. II.3.

Table II.2.1 (contd.)

Investigator/s	Important Conclusions	General Comments
Sen et al[93-97]	Using a failure criterion for concrete under compressive tensile stresses, expressions have been derived for the shear strength of beams. A successive approximation procedure has been suggested to evaluate certain parameters.	The solution is somewhat complicated. The influence of physical properties of beams on the ranges of applicability of different equations has been ignored.
Kani[99,100] (1964)	Equations have been developed by considering two types of mechanisms for failure i.e. "tooth" mechanism for large a/d ratios and arch mechanism for "low" a/d ratios. Shear reinforcement provided in the form of bent-up bars have been found to be more effective than the stirrups(Fig.II.2 & 3)	Equations involve crack factor for which solution is still awaited. The same two types of mechanisms have been interpreted in a different way by the present author and the resulting equations are similar to those given by Kani.
Lorentsen[101] (1965)	A theory for the strength of beams in bond and shear has been introduced. According to the theory, the shear is partly carried by concrete by concrete and partly by dowel action. The contribution from concrete was obtained from concrete plate tests and that from dowel action from the tests on dowel beams. Application of the theory to prestressed concrete beams is also indicated.	The expressions are suitable for calculating cracking shear of beams in which both bond and shear have influence.

(continued)

Table II.2.1 (contd.)

Investigator/s	Important Conclusions	General Comments
Acharya and Kemp[102] (1965)	By calculation of principal stresses at the load point and using a failure criterion for concrete under compressive-compressive stresses, it has been shown that the dowel force contributes about 60% of the total shear.	Conclusion seems to be based on erroneous calculation of principal stresses. By correct calculation it has been found that the dowel force contributes negligible amount to the shear strength of beams.
Krefeld and Thurston[103,104] (1966)	Equations were developed for the diagonal cracking shear considering the dowel force to be paramount. An empirical equation arrived from dowel tests is used in conjunction with an idealized failure crack to develop the final equation. The numerical coefficients have been evaluated by empirical methods.	The failure hypothesis given did not have any check with the test results as the final coefficients have been derived from test results themselves. The final equations are very similar to those given in ACI Building Code[98].
Leonhardt[105] (1965)	Stirrups are more preferable than bent-up bars as shear reinforcement.	The relative merits of stirrups and bent-up bars in resisting shear force are not yet fully known.
Amishaw, Bunni and Neville[106,107] (1966)	The dowel force increases to a high value at the time of diagonal cracking, but soon drops down to an insignificant amount after diagonal cracking.	Further confirms the test observations of earlier investigators with respect to the contribution of dowel action of longitudinal steel to shear strength of beams.

(continued)

Table II.2.1 (contd.)

Investigator/s	Important Conclusions	General Comments
Kani[108] (1966)	The influence of concrete strength on the shear strength of beams is insignifioant while the percentage steel and moment/shear ratio has greater influence on the same.	The conclusion regarding the influence of concrete strength requires modification in the light of the experimental results available in the literature.

General Remarks

From the critical review of earlier work discussed in this chapter, the following conclusions can be drawn:

1. The general behaviour of beams under combined bending and shear described by various investigators are similar, but the interpretations given for it differ from person to person.

2. The effect of a/d ratio on the shear strength of beams is well understood and appreciated in recent years.

3. Even though there are theories and equations for predicting the carrying capacity of beams, its application to design office works is limited either by the complex nature of the solutions or lack of knowledge of certain parameters involved in the final expressions.

4. Most of the solutions available in the literature have been checked only with respect to simply supported beams with concentrated loads and their applicability to beams with distributed loads is not fully known.

5. Even though there are large number of test results with respect to rectangular beams with concentrated loads, comparitively very less work has been done on tee beams or beams with uniformly distributed loads.

6. The contribution from the longitudinal steel to the shear strength of the beam by dowel action is not yet completely known. The results of the available investigations fail to explain fully the exact mechanism of dowel action of longitudinal steel while the conclusions of some of the investigations discussing this aspect seem to be unrealistic. But some of the latest investigations tend to support the conclusion that the contribution of dowel action of longitudinal steel to the shear strength of beams is insignificant.

7. So far, many investigators have considered only the strength aspects of members subjected to combined bending and shear and little information is available on the deformation characteristics of the beams. There is no rational method available to determine the deformations such as deflection, curvature etc., of beams under bending and shear.

8. Finally it has to be mentioned here that there is considerable scatter and inconsistency in the test results available in the literature.

The present author has attempted to solve some aspects of this problem in this part of the thesis.

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CHAPTER II.3

FABRICATION AND TESTING OF BEAMS

This chapter briefly describes the details of the experimental investigation conducted by the author on rectangular and tee beams under combined bending and shear. As a large number of test results is already available in the literature, the experimental programme of this part of the thesis is of minor nature. Greater attention was given to the study of effect of shear span-depth ratio on the shear strength of beams keeping other variables constant.

Materials used:-

The cement used was ordinary portland cement conforming to Indian Code requirements. Angular gravel of less than ½" in size was used as the coarse aggregate and the fine aggregate had a fineness modulus of 2.54. Potable water was used for mixing the concrete. Ordinary plain round mild steel bars were used as reinforcements. The mild steel bars had an average yield stress of 50000 psi.

Mix proportion:-

The mix used was of 1:1½:3 proportion by weight with water/cement ratio varying between 0.45 and 0.55. The

average cylinder strength of concrete used was between 3500 and 5000 psi.

Description of specimens:-

In total, thirty beams were cast of which twenty one were rectangular beams and nine were tee beams. The rectangular beams had an overall cross section of 4" x 8", the total length being varied. The beams were reinforced on the tension side only with mild steel round bars. They had an effective depth of 6.78" with a clear cover of 1" and were tested with different shear spans.

The tee beams had a web thickness of 4" and an overall depth of 7". The flange was of 10" wide and 1½" thick. The beams were provided with three mild steel round bars of ½" diameter on the tension side, the percentage of reinforcement being 2.56. The effective depth of beams was 5.75". The overall lengths of specimens were varied depending upon the length of shear span with which they were tested. All the tee beams had a cylinder strength of 4500 psi and they were tested with different a/d ratios varying from 0.96 to 4.52.

With each beam, cubes of 6" size and 6" x 12" cylinders were cast as auxiliary specimens.

Casting of beams:-

All the beams were cast in plywood moulds. The required lengths of reinforcing steel were normally cut from

one lot and the hooks were cold bent using metal jigs. The reinforcements were placed inside the mould on two or three briquettes of 1" thick, spaced at intervals along the length of the beam to ensure proper cover for them.

The exact quantities of materials required for one batch of concrete were calculated and they were mixed together adopting normal procedure. A good control was maintained throughout the work. The beams were cast in pairs by filling concrete in layers and compaction was done in the usual manner. The auxiliary specimens were cast in steel moulds along with each casting.

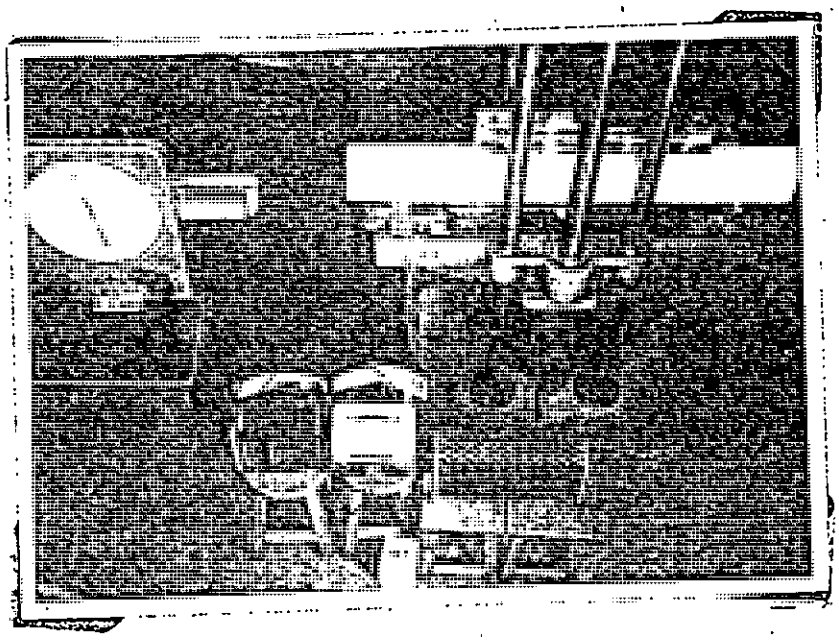
One day after casting, the beams were removed from moulds and were cured under water till the time of testing. The auxiliary specimens were also cured under the same conditions as main beams.

Testing of Beams:-

One day before testing, the specimens were removed from the curing tank and were dried in atmosphere to remove surface moisture. The beams were then white-washed and were kept ready for testing with load positions and support positions marked on it.

All the beams were tested in a 100-ton capacity Universal Testing Machine with simply supported ends. The span of the beams varied according to the length of shear

Fig. II 4 : Beams under combined bending and shear : Details of Test set-up.



span. The testing arrangement used is shown in Fig. II.4 (photograph). The surface strains in concrete were measured using Demec Gauges of 8" gauge length. The dial gauges of 2" travel with a least count of 1 x 10^{-3} inch gave the deflections of the beams at the centre of the span.

Normally, loading was done through one or two point loads. The loads were increased in small increments so as to measure the deformations of beams at various stages and to study the behaviour of beams closely. After each increment of load, deformations were measured and the crack developments were marked. The loading was continued till failure and the ultimate failure load was noted. Then the beams were photographed showing the mode of failure and crack patterns at failure.

The auxiliary specimens were tested in the same testing machine on the same day on which the main beams were tested. The total time of testing for each beam was normally less than two hours and hence they can be classified as short-time tests.

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CHAPTER II.4

GENERAL BEHAVIOUR AND OBSERVED MODES OF FAILURE OF BEAMS

II.4.1 Any investigation on reinforced concrete beams tested upto collapse can be broadly classified under the following two major heads:

1. Study of general behaviour of beams at various stages of loading till failure so as to understand the mechanism of failure, and

2. Interpretation of the physical behaviour of beams and hence to formulate expressions to evaluate its strength at collapse.

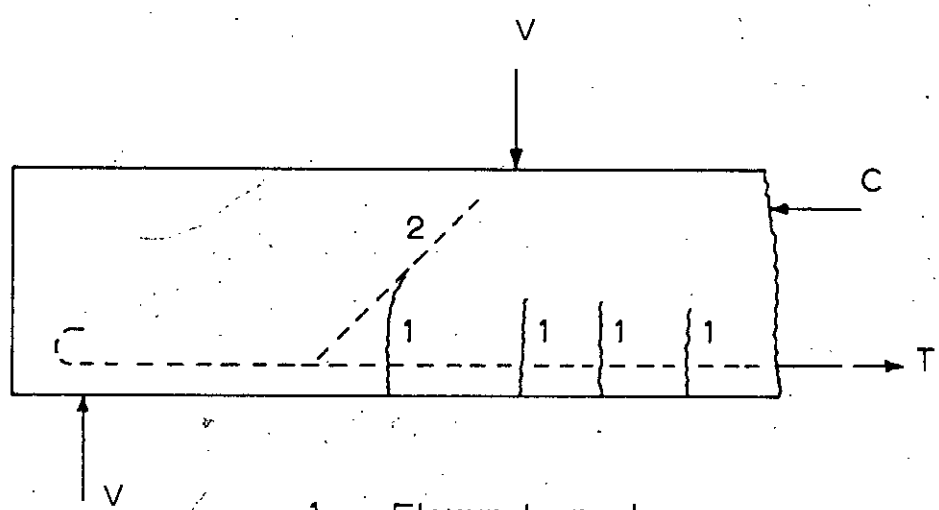
In the above, the second part of the investigation may change from author to author depending upon the methods of approach adopted, assumptions made, etc. But the first part about the behaviour of beams basically remains the same. So following this path of research, the general behaviour of reinforced concrete beams under the combined action of bending and shear was closely observed at various stages of loading. Such observations are reported in the following pages. Then these observations have been physically interpreted to give the mechanism of shear failure and hence to

determine the strength of this mechanism which are given in Chapters II.5 and II.6.

II.4.2 Formation and Development of Cracks:

II.4.2.1 Flexural Cracks:

The initial flexural cracks appeared in the region of maximum bending moment near the load points and at mid-span. The load at which the first flexural cracks formed depended on the shear span and the physical properties of the beam. Normally these cracks were hair cracks visible through magnifying glass extending from bottom of the beam vertically upwards to the level of tension steel. The spacings between these flexural cracks were almost equal and they were evenly distributed along the length of beam. For increments in loads, more flexural cracks appeared in the region of maximum bending moment and new cracks formed in between load points and supports. All these cracks penetrated upwards with the increase in loads. The cracks in the pure bending zone extended vertically whereas the cracks in the shear span curved towards the load points as they extend upwards, showing evidence of the shear effect. Till the formation of diagonal cracks, the movement of flexural cracks was fast and after the appearance of diagonal cracks, flexural cracks remained stationary or moved upwards very slowly. Sometimes a flexural crack formed in the shear span joined with the existing diagonal



1 – Flexural crack
2 – Diagonal crack

DEVELOPMENT OF A FLEXURAL CRACK INTO A DIAGONAL CRACK.

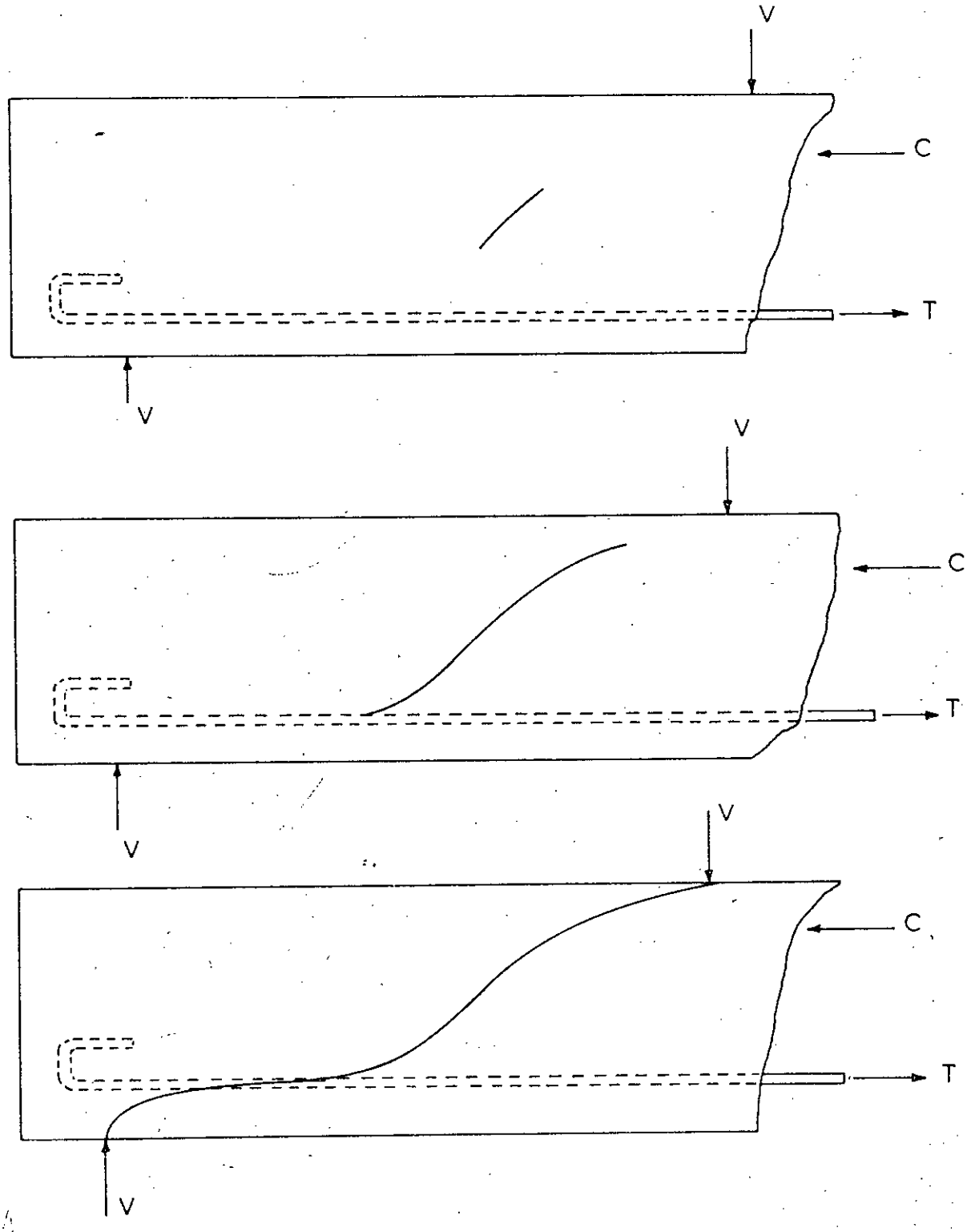
FIG. II. 5

crack at mid-depth of the beam and became a full length diagonal crack for further increments in load. This is illustrated in Fig. II.5. In certain cases this crack itself has become the failure crack before any proper diagonal crack appeared.

II.4.2.2 Diagonal Cracks:

Diagonal cracks mainly originate at about mid-depth of the beam where the influence of normal stresses on shear stresses is least. The paths of diagonal cracks approximately follow the principal compression trajectories.

Usually a short diagonal crack inclined at an angle of 45° to the axis of the beam appeared at mid-depth of the beam. This crack extended downwards and upwards for further increments in loads. The downward extension of the crack was closely a straight line with the same inclination of about 45° till it reached the reinforcement level. The upward extension of the crack deviated from its original inclination becoming more and more flat as it propagated upwards. It became almost parallel to the longitudinal axis of the beam as it reached the top surface of the beam. The lower end of the diagonal crack between steel level and bottom face of the beam was normally flat running along the reinforcement and reached the supports. The failure of the beam occurred



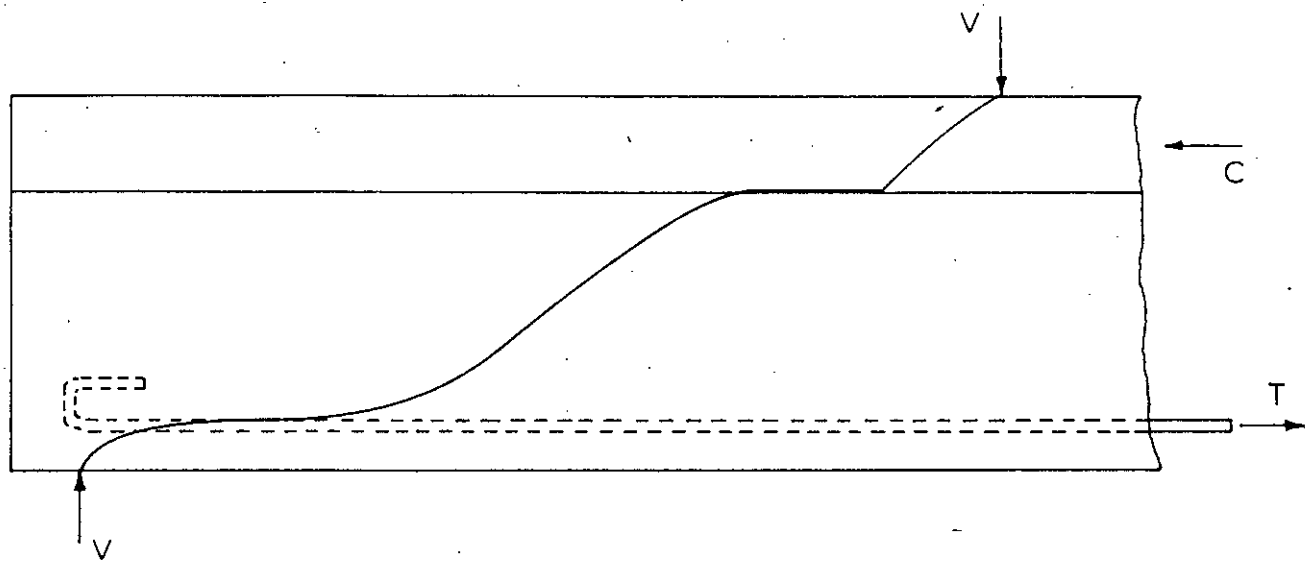
FORMATION AND DEVELOPMENT OF A DIAGONAL CRACK IN A RECTANGULAR BEAM.

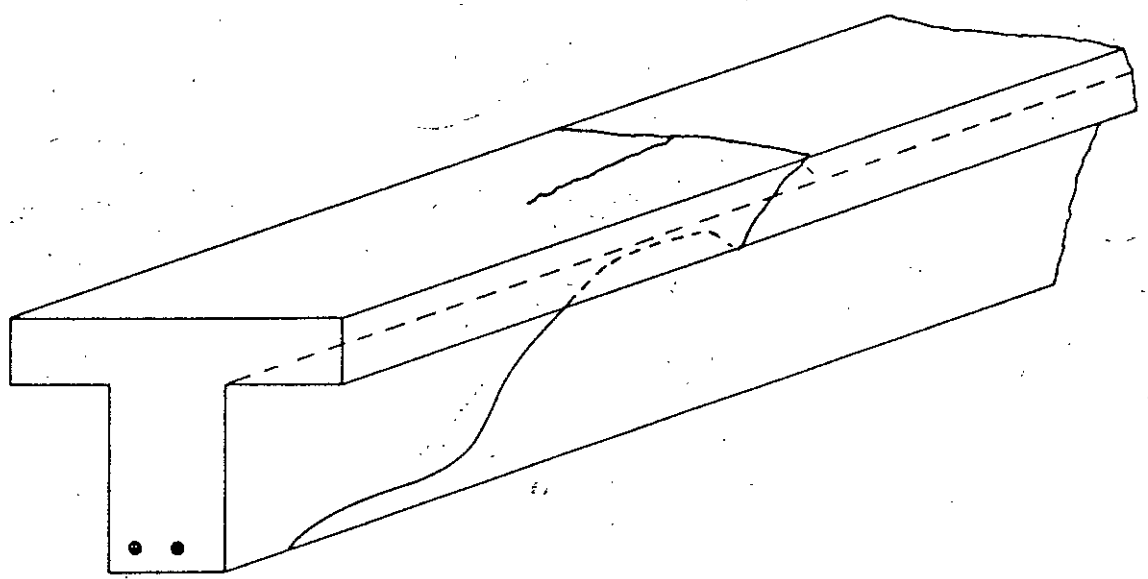
FIG. II.6.

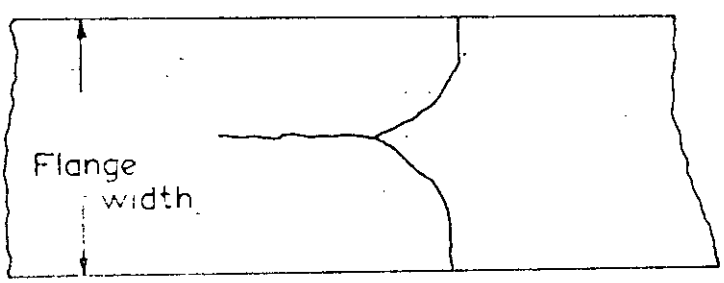
either by wide opening of the diagonal crack or by crushing of concrete at the top of diagonal crack depending on the value of a/d ratio. In some cases the failure was accompanied by splitting of concrete along the reinforcement level. The formation and development of a diagonal crack is illustrated in Fig. II.6.

In the case of high values of a/d ratios, the whole process of development of cracks described above took place instantaneously with the appearance of diagonal crack and the beam collapsed suddenly. In other ranges of a/d ratios the development of cracks was somewhat less rapid.

In tee beams, the diagonal crack, after reaching the junction of flange and web, extended horizontally to some distance along the web, separating the flange from the web along this line. Then this crack crossed the flange thickness and at collapse a tension crack formed at top surface of the flange along its full breadth connecting the diagonal cracks on the sides of the beam. The projecting portions of the flanges broke in the manner described above because, having been separated from the web by the diagonal crack, they acted as a cantilevers with tension crack on top surface and concrete crushing at the junction of flange and web. At the time of failure, a crack appeared at about middle of the top of the flange running parallel to the







CRACK PATTERNS IN A TEE BEAM.

FIG. II.7.

beam axis. This may be due to the transverse bending of the separated flanges from the web. The crack pattern in case of a Tee beam is shown in Fig. II.7. Such crack patterns have also been reported by Ramakrishnan[80] and Taylor[109].

In all the beams the crack patterns were similar in both the shear spans. But failure occurred only in one shear span while the other portion of the beam remained unaffected. Many cracks appeared before failure; but failure of the beam normally occurred by opening of one crack while the others remained as hair cracks. In general all failures occurred suddenly indicating the danger of shear failures.

II.4.2.3 In the past, some investigators[80,87] distinguished two types of diagonal cracks such as "initial diagonal crack" and "major diagonal crack" or "first diagonal crack" and "second diagonal crack". Since the development of the former into the latter occurs simultaneously in many cases, these two cracks are not differentiated from one another in the present investigation. So these two cracks are commonly represented by the term "Diagonal Crack" in the later chapters of this part of the thesis. The load corresponding to the "Diagonal Crack" is termed as "Diagonal Cracking Load".

The diagonal cracking load can be easily known from the changes in the deformations of the beam. In case of

high a/d ratios, the failure of the beam will follow immediately after the formation of "Diagonal Crack". Hence in that case failure load will be equal to the diagonal cracking load itself. It is to be mentioned here that only after the formation of diagonal crack redistribution of stresses takes place in the compression zone of the beam. Hence its appearance changes the behaviour of beams under the combined action of bending and shear.

II.4.3 Observed modes of failure:-

The study of modes of failure is an important aspect of any investigation of concrete beams tested upto collapse. The failure of reinforced concrete beams can occur mainly in three ways:

1. By crushing of concrete due to excessive compressive stresses,
2. By splitting of concrete due to tensile stresses,
3. By yielding or breaking of reinforcement.

These three main causes of failure remain the same for any type of failure. The shear failures can be easily distinguished from the flexure failure which results by crushing of concrete with or without the yielding of tension reinforcement as a result of bending stresses. In the past, many investigators gave different classifications for shear failures. But mainly

two types of failures are discussed here keeping others as secondary effects of the main failures. Obviously a shear failure is not a flexure failure! But the basic causes of failure remain the same. So shear failures are classified under two heads:

1. Diagonal tension failure which results from tensile stresses;
2. Diagonal compression failure which results from compressive stresses.

Depending upon the ultimate cause of failure, these two types of shear failures are described below:

II.4.3.1 Diagonal Tension Failure:-

This type of failure normally occurred in case of beams tested with medium and high a/d ratios (i.e. a/d ratio greater than about 3). The failures were mostly sudden and catastrophic. In case of high values of a/d ratio, the failure followed immediately after the formation of diagonal crack whereas for beams with medium shear spans, the beams sustained some more extra load after the full development of diagonal crack. But if the diagonal cracking load was held for some time, even in case of medium shear spans, the beams failed suddenly developing a wide diagonal crack. The beams failed in this manner showed a few flexural cracks which remained as

Diagonal Tension Failure

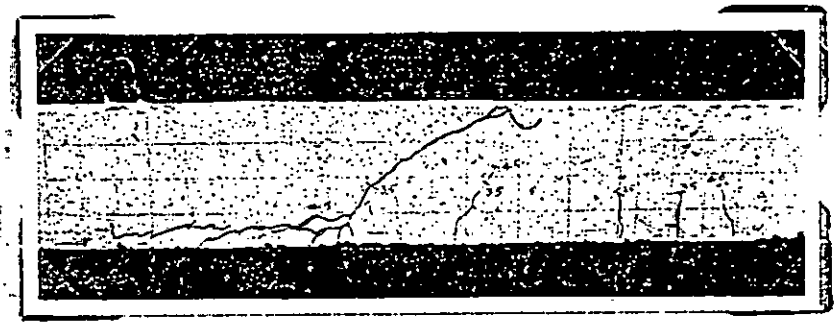
Fig. II.8 : Beam SR 13 (a/d = 6)

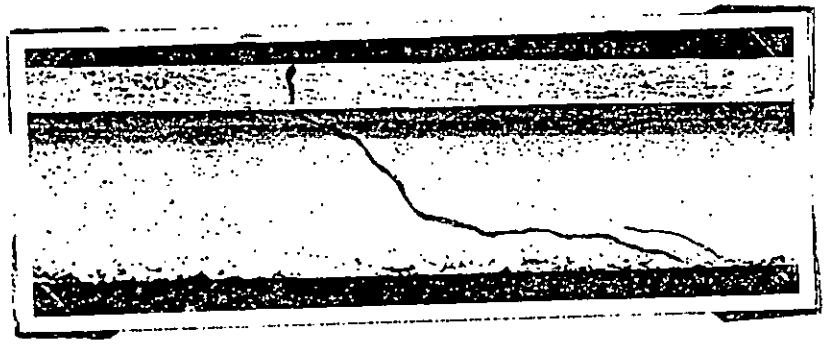
II.9 : Beam ST 29 (a/d = 4.52)

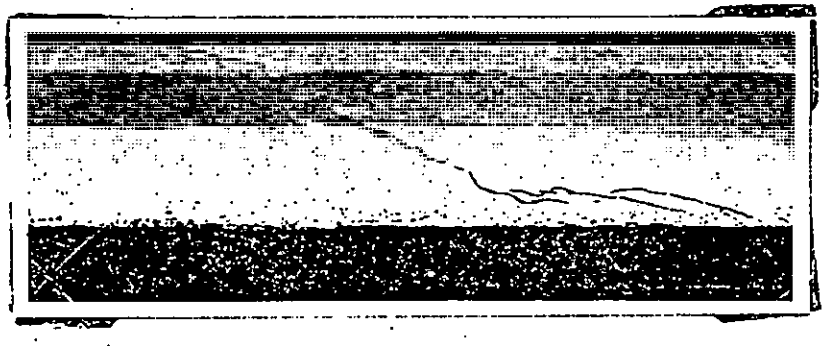
II.10 : Beam ST 30 (a/d = 4.00)

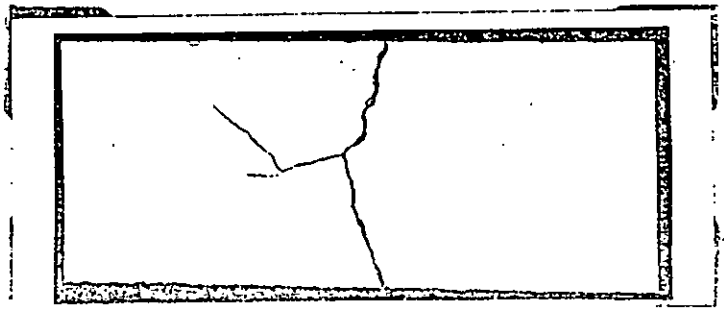
II.11 : Beam SR 17 (a/d = 3.50)

II.12 : Beam ST 30 (a/d = 4.00) - Top view of Tee beam showing yield lines.









hair cracks till the collapse of the beam. There was absolutely no sign of crushing of concrete at any point. Typical views of this type of failure are shown in Figs. II.8-10.

This type of failure was sometimes accompanied by splitting of concrete along the tension steel. A view of this failure is shown in Fig. II.11. Fig. II.12 shows yield pattern of cracks on top surface of a tee beam.

II.4.3.2 Diagonal Compression Failure:-

In contrast with the above type of failure, diagonal compression failure was caused by compressive stresses which resulted in crushing of concrete at the top of diagonal cracks near the load points. This type of failure occurred in case of low values of a/d ratios. Failure was not so sudden as the previous one and was somewhat gradual. Concrete crushing was seen all along the diagonal crack and the process of crushing continued even while releasing the load in many cases.

In case of tee beams, crushing of concrete was also seen at the junction of flange and web followed by a wide cracking on the top surface of the flange along its breadth. A few hair cracks appeared on the top of the flange along the longitudinal axis of the beam at the time of failure.

Figs. II.13 to II.15 show typical examples of this type of failure.

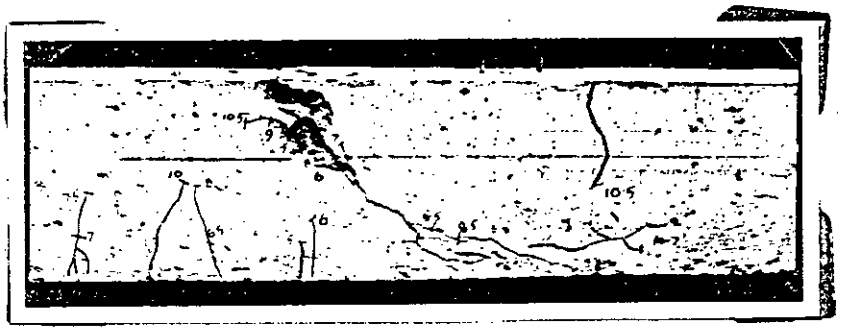
Diagonal Compression Failure

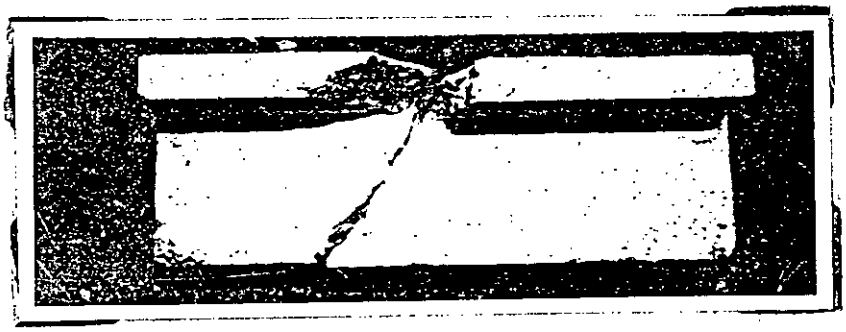
Fig. II.13 : Beam SR 11 (a/d = 2.50)

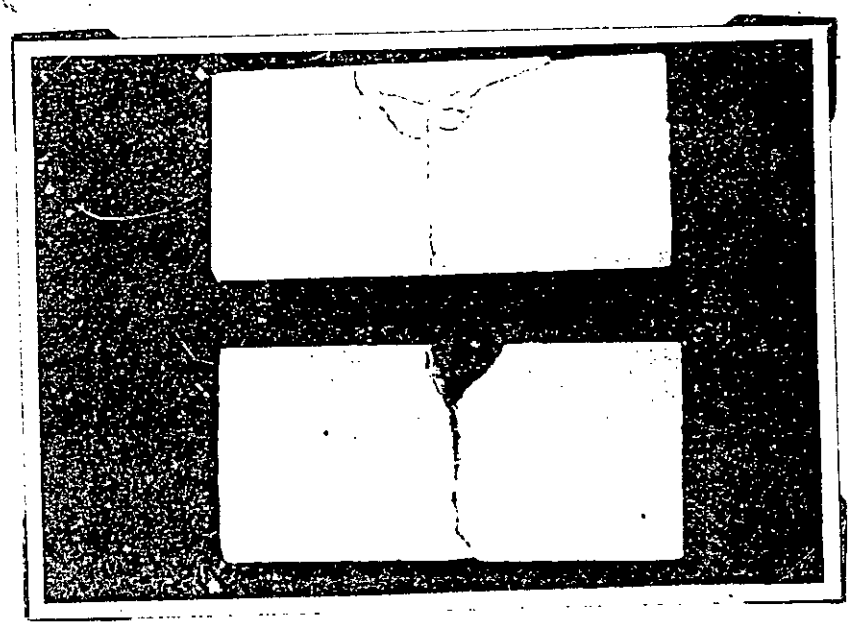
II.14 : Beam ST 27 (a/d = 0.96)

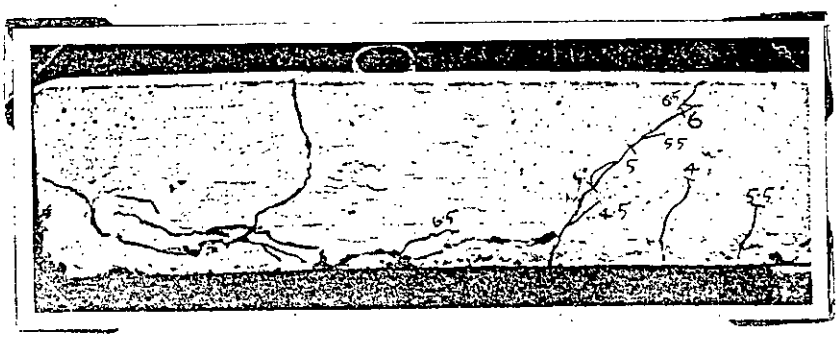
II.15 : Beam ST 27 & 28 (a/d = 0.96 & 1.48) - Top view of beams

II.16 : Beam SR 10 (a/d = 1.56) - Anchorage Failure









II.4.3.3 Secondary Failures:

a. The diagonal tension failures are sometimes accompanied by horizontal splitting of concrete along the tension steel. This failure is only a secondary failure as the splitting of concrete along tension steel will always be preceded by diagonal splitting of compression zone concrete. Certain portion of the vertical shear is transferred to the tension steel as dowel force. At failure, these dowel forces along the tension steel cause horizontal splitting of concrete at this level. Fig. II.11 shows this type of secondary failure.

b. Another type of secondary failure is the bond and anchorage failure. Normally this failure will occur when the bond between steel and concrete is poor. A wide vertical crack appeared at the support suddenly and the concrete sheared off immediately. The failure is shown in Fig. II.16.

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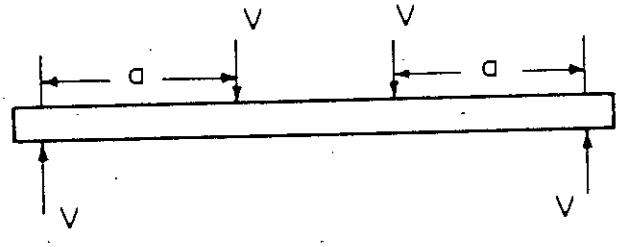
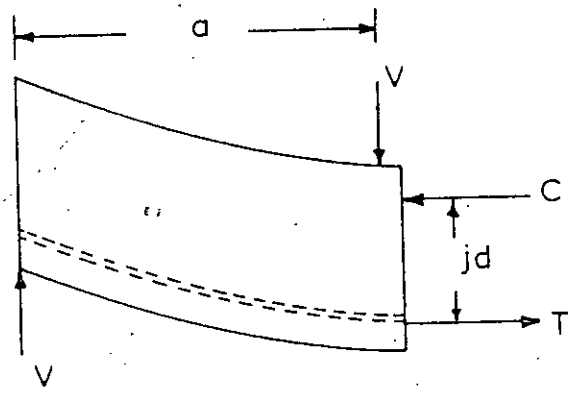


IDEALIZED CASE OF LOADING.



$$C = T$$

$$Va = M = Cjd = Tjd$$

BEAM SECTION IN PURE BENDING.

FIG. II. 17.

CHAPTER II.5

MECHANISM OF SHEAR FAILURE

II.5.1 An important aspect of the present investigation is the understanding of the mechanism of shear failure in reinforced concrete beams. This chapter is devoted to the study of failure mechanisms in beams under combined bending and shear. The observations made during the experiments helped to understand the physical behaviour of beams and hence to establish the mechanisms of failure. The test observations already available in the existing literature were also of much use to the present author in this respect.

Based on the behaviour of beams at different stages of loading, the mechanism of shear failure is discussed briefly in the following pages:

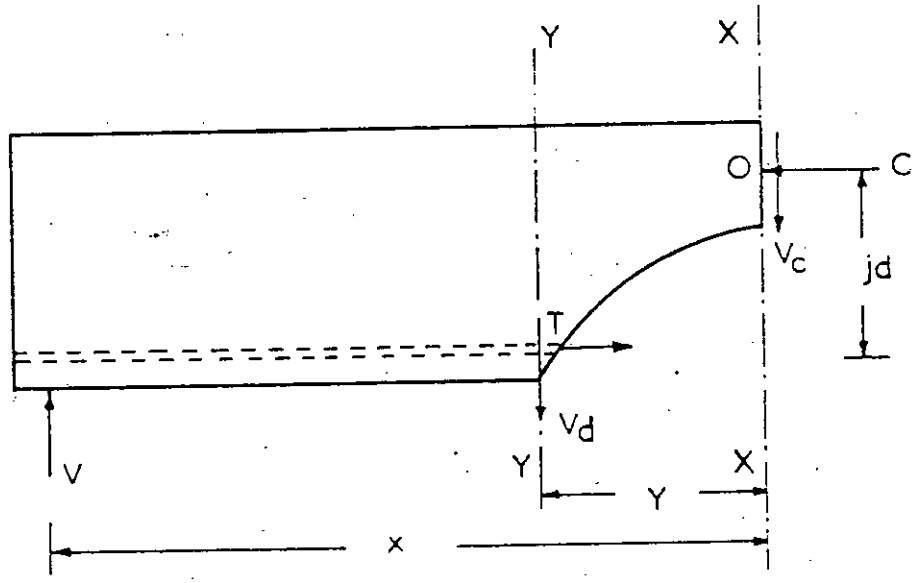
II.5.2 Beams without shear reinforcement:

Consider an idealized case of loading as shown in Fig. II.17. This type of loading is the one normally adopted in laboratories to study the effect of bending moment and shear force on reinforced concrete beams. Two critical combinations of bending moment and shear force are obtained in this idealized loading. The bending moment varies from zero over support to the maximum at load points while the shear force is constant over the entire shear span. So we

have maximum bending moment and maximum shear force at load points, and minimum bending moment and maximum shear force at support points. These are the critical points of failure. But experiments have shown that beams fail near load points than at support points. Hence sections near the load points become critical sections for failure.

In the ideal type of loading shown in Fig. II.17 if the shear span is made stronger and if the central region is weaker, flexure failure will occur. On the other hand if the central region is stronger, shear failure will occur in the shear span. The mechanism of flexure failure is well known and hence it is not discussed here.

In the shear span both bending moment and shear force act and with bending moment, shear force also increases accordingly and a diagonal crack will appear when the shear stress exceeds certain limiting value. As the influence of normal stresses on shearing stresses is to reduce its effect, the diagonal crack will appear usually at the level of neutral axis where the normal stress is zero. In some cases it is possible that this diagonal crack may join the flexural crack which has penetrated already to the level of neutral axis. Such cases have already been discussed in Chapter II.4 (Art. II.4.2.1). After the formation of diagonal crack, redistribution of stresses will take place and the mechanism of failure can be discussed in three stages.



REDISTRIBUTION OF STRESSES AFTER FORMATION OF DIAGONAL CRACK.

FIG. II. 18.

Stage 1

The first stage represents the behaviour of beam before the formation of diagonal crack. At this stage the beam will behave truly like a beam and the Bernoulli's hypothesis remains valid. The deflections will be proportional to the bending moment. The total compressive force in concrete and the total tensile force in tension steel at any cross section will be directly proportional to the bending moment at that section.

Stage 2

This stage corresponds to the behaviour of the beam after the formation of diagonal crack. Immediately after the formation of diagonal crack, redistribution of stresses takes place above the diagonal crack. The external shear force acting at the section is partly distributed to the tension steel and the balance is carried by the concrete in the compression zone as the tension zone concrete has already failed due to the tensile stresses caused by the bending moment. The conditions existing at the diagonal crack are shown in Fig. II.18. Even though the linearity of strain distribution is disturbed to a certain extent by the redistribution of stresses, the laws of statics are still valid. Now consider the beam portion to the left of the external load as a free body. The free body diagram is shown in

Fig. II.18. The free body is kept in equilibrium by the action of following forces:

1. The reaction at support point, V
2. The shear force carried by the concrete, V_c
3. The shear force carried by the longitudinal tension steel, V_d as dowel force.
4. The force in the tension steel, T
5. The compressive force acting on the compression zone concrete, C.

(The force carried by the tension zone concrete is neglected). By taking moments about the point O, (vide Fig. II.18) we have,

$$(T \times jd) = V.x - V_d\, y$$

$$\text{or} \quad T = \frac{V.x - V_d\, y}{jd} \qquad \ldots \text{II.5.1}$$

In the above equation V.x represents the bending moment at the section XX. So the force T in tension steel at the section YY, depends on the bending moment at the section XX and the shear force carried by tension steel. We can fix up two extreme values, one least value and another maximum value, for V_d. That is, if we can assume that total shear force is completely carried by tension steel alone, then $V_c = 0$ or, $V = V_d$.

$$\text{Also,} \quad T = V \frac{(x-y)}{jd} \qquad \ldots \text{II.5.2}$$

The quantity V (x-y) represents the bending moment at section YY thus indicating that the force in tension steel is proportional to the bending moment.

On the other hand, if we assume that the entire shear force is carried by concrete alone we have

$$V = V_c \text{ and } V_d = 0. \quad \text{Then, } T = \frac{Vx}{jd}$$

This indicates that the steel force at the section YY is equal to the steel force at the section XX.

The correctness of these two different extreme cases can be verified by measurement of steel strains. The strain data reported by Ramakrishnan[80] indicates that there is a sharp increase in force T, at the section YY immediately after the formation of diagonal crack, but it does not become equal to the tensile force in steel at the section XX. But for further increments in loads, the tensile force T at the section YY increased rapidly and towards the ultimate load it became nearly equal to the force at the section XX. This clearly indicates that even though certain portion of the shear force has been transferred to tension steel at the formation of diagonal crack, at failure this transferred shear once again redistributes itself to the compression zone concrete. Thus the shear carried by the tension steel is almost negligible and is of least importance at ultimate load.

The above discussion is also in conformation with the observations of Neville et al[106,107]. Recently, they have reported that the shear carried by longitudinal steel as dowel force is negligible before the formation of diagonal crack. It suddenly increases to a high value at the formation of diagonal crack and drops once again to a negligible quantity at the time of failure.

The formation of diagonal crack also reduces the depth of compression block. So the redistributed shear force will produce high shear stresses and hence we can assume uniform shear stress distribution in the compression zone. As the effect of normal stress is to reduce the value of principal tensile stress, the critical point for failure is at the level of neutral axis where the normal stress is zero. At this point only shear stresses exist and hence the resulting principal stresses will be equal in magnitude, to the shear stress itself. So the concrete element at this critical point will fail when the resulting principal stresses satisfy a failure criterion for concrete under compressive-tensile stresses. Since the stresses are at plastic state, even for very small increment in load the position of neutral axis will shift further. So the failure of elements at the level of neutral axis will also extend from one element to the other soon causing failure of entire compression zone concrete. The load sustained by the beam in this "extension

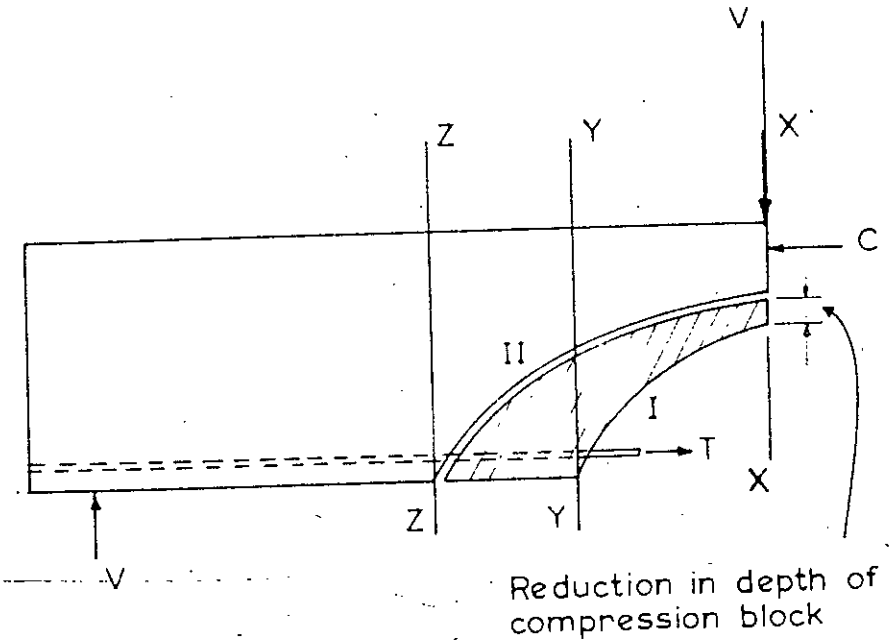
period" will be very small and hence can be ignored. This is evident from the test observation also. That is, immediately after the formation of diagonal crack, if the same load is maintained for some time the beam failed soon without any further increment in load. Thus the strength of the concrete element at the level of neutral axis gives the shear strength of the beam.

But beyond the sections where diagonal crack appears, the concrete under the main steel is subjected to tensile stresses due to the dowel force in addition to other tensile stresses such as shrinkage stresses, anchorage stresses etc. Hence these failures are sometimes accompanied by splitting of concrete along the tension steel.

Stage 3

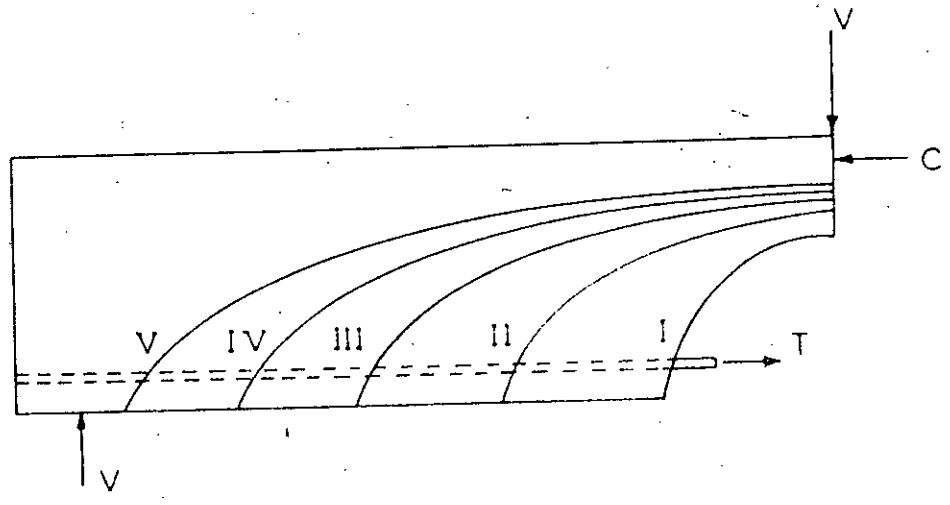
If the beam has not already failed due to the failure of concrete elements at the level of neutral axis, the transformation of the beam into an arch will take place. The transformation is a "step-by-step" process as described by Kani[99].

Consider a beam with a fully developed diagonal crack as shown in Fig. II.18. If the beam does not fail due to this diagonal crack, some more diagonal cracks will appear either parallel to the existing one or branch off from it.



TRANSFORMATION OF BEAM INTO A TIED ARCH.

FIG. II. 19.



TRANSFORMATION OF BEAM INTO A TIED ARCH.

FIG. II. 20

One such diagonal crack is shown in Fig. II.19. The fresh diagonal crack is marked as II in the Fig. II.19. The stress conditions under the presence of first diagonal crack has already been discussed under stage 2. Similarly, when the diagonal crack II crosses the tension steel, the force in the steel will increase suddenly and it will be equal to the force at the section YY or XX. If the tensile forces in longitudinal steel were equal at these sections, then the concrete in between these diagonal cracks (hatched portion in Fig. II.19.) becomes ineffective. Thus the penetration of the diagonal crack II into the compression zone reduces its resisting area further by a small amount as seen in Fig. II.19.

Soon after this, one more diagonal crack will appear and the same process will continue as described above thus reducing the area of compression zone still further. The development of diagonal cracks is shown in Fig. II.20. This process of transformation will continue until a final diagonal crack appears beyond which the support restricts the formation of any fresh diagonal crack. This indicates the limiting case and the area of compression zone will be least at this instant.

Now the beam behaves like a tied-arch with the tension steel as the tie-rod. The force in tension steel will be equal at all points and this fact is in conformity with the strains measured during experiments. The

resultant of the forces near the load point will be inclined to the compressive force, C, thus disturbing the linearity of the strain distribution over the depth of the cross section to a certain extent. Such observations are also given by Kani[99].

The transformed arch has a cross section of varying depth; but the critical section for failure is at the load points where the net area resisting the forces is least. The concrete elements beneath the load points are acted upon by three forces namely,

1. Horizontal compressive force, C
2. Transverse shear force, V
3. Vertical normal load, V.

The compressive force, C, will cause a horizontal normal stress and the shear force, V will produce a state of shear stresses along the sides of the element. In addition to these two stresses, the external load will cause a normal stress in the vertical direction. Thus the critical concrete element at the section is subjected to three stresses.

1. A normal stress in the horizontal direction of the beam.
2. A normal stress in the vertical direction
3. A shear stress due to the applied shear force at the section.

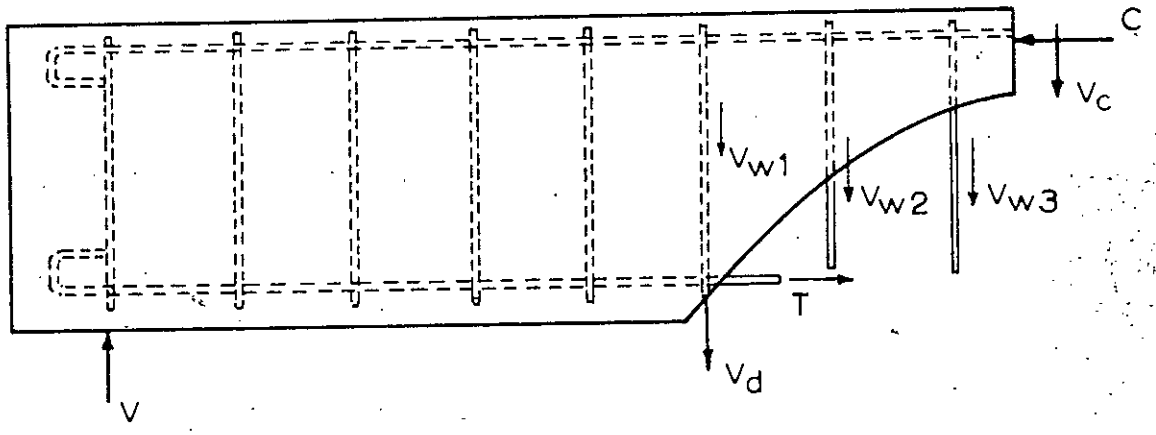
These three stresses will interact with each other and will cause two principal stresses which are compressive. So the failure of the critical concrete element will occur when these compressive-compressive principal stresses satisfy a failure criterion for concrete under biaxial state of stresses.

II.5.3 Beams with shear reinforcement:

The shear reinforcements do not basically change the mechanism of failure of reinforced concrete beams under the combined action of bending moment and shear force. The main functions of shear reinforcement are two-fold: (1) it contributes a certain amount to the strength of the beam and (2) it restricts the widening and development of diagonal cracks thus providing more concrete area to resist the shear force and moment acting at the section.

The shear reinforcement can be provided in different forms such as vertical stirrups, inclined stirrups, bent-up bars, etc. But the present investigation confines itself to the case of beams with vertical stirrups only.

Till the formation of diagonal crack, the behaviour of beams with shear reinforcement is similar to that of beams without shear reinforcement. The strain measurements (made on shear reinforcement) reported in the literature[80,85] clearly indicate that the stirrups did not register any



CONDITIONS AT THE FORMATION OF DIAGONAL CRACK IN A BEAM WITH SHEAR REINFORCEMENT.(Verticle stirrups)

FIG. II. 21.

appreciable strain till the formation of diagonal crack. After the development of diagonal crack, the axial stresses in stirrups will increase rapidly so as to balance the increase in shear force. Similar to the beams without shear reinforcement, the shear force will be shared by concrete, longitudinal steel and shear reinforcement. The stress conditions existing at the formation of initial diagonal crack are shown in Fig. I.21.

For further increments in load, at failure, the shear force carried by the longitudinal steel as dowel force will be once again redistributed to the compression zone concrete. The contribution from the vertical stirrups ceases when once the axial stress in them reaches the yield stress of steel. So for a beam with stirrups, the strength of the beam is given by two parts: the full contribution by stirrups is obtained when the axial stresses in them reach the yield point and the contribution by compression zone concrete is given by the mechanism discussed earlier (stages 2 and 3) under beams without shear reinforcement.

II.5.4 Factors influencing the strength of the mechanisms of shear failure:

The main factor, which influences the strength of the mechanism of shear failure is the shear span-effective depth

ratio, a/d. Experiments show that beams with high and medium a/d ratios, fail in stage 2 of the failure mechanism discussed earlier while the beams with low a/d ratios sustain more load even after the stage 2 and finally collapse in stage 3. In case of beams with high a/d ratios, failure occurs immediately after the formation of diagonal crack and in case of beams with medium values of a/d ratios, failure follows a little later after the formation of diagonal crack. This is in accordance with the failure mechanism discussed under stage 2.

Any change in the physical properties of the beam will alter the strength of the mechanism. By varying the concrete strength, the strength of the mechanism will vary because the strengths of the mechanism of failures discussed earlier are directly interrelated with the concrete strength of the beam.

Obviously any change in the cross section or physical properties of tension reinforcement will alter the strength of the mechanism. The effects of compression reinforcement in a beam or the flange reinforcements in the case of a flanged beam have been ignored in the discussion. In other words, it has been assumed that these reinforcements do not have much influence on the mechanism of failure and shear strength of the beams. Experimental results available in the literature[80,110] confirm this assumption.

CHAPTER II.6

THEORETICAL ANALYSIS OF SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS

II.6.1 Based on the mechanism of shear failure discussed in the previous chapter, analytical equations are derived in this chapter. The analysis is made for rectangular beams in the beginning and then they are extended to the case of tee beams as the basic concepts involved in the mechanism of failure of both the types of beams are same. The analysis can be broadly classified into two heads:

1. Analysis of beams without shear reinforcement
2. Analysis of beams with shear reinforcement

It is to be pointed out here that beams with shear reinforcement in the form of closed vertical stirrups only are considered in the present analysis. The analysis is done for a simply supported beam with idealized case of loading shown in Fig.II.17. The applicability of the resulting equations to other types of structures with different types of loadings are discussed later (Chapter II.7).

II.6.2 Assumptions:

The analysis is based on the following assumptions:

1. The concrete in the tension zone of the beam is neglected. It is well known that the concrete is weak in resisting tensile stresses. So the tension zone concrete is normally neglected in the analysis of beams under pure bending. Hence the same assumption is kept-up in the present analysis of beams under combined bending and shear. This will facilitate the analysis of reinforced concrete beams to have a common hypothesis.

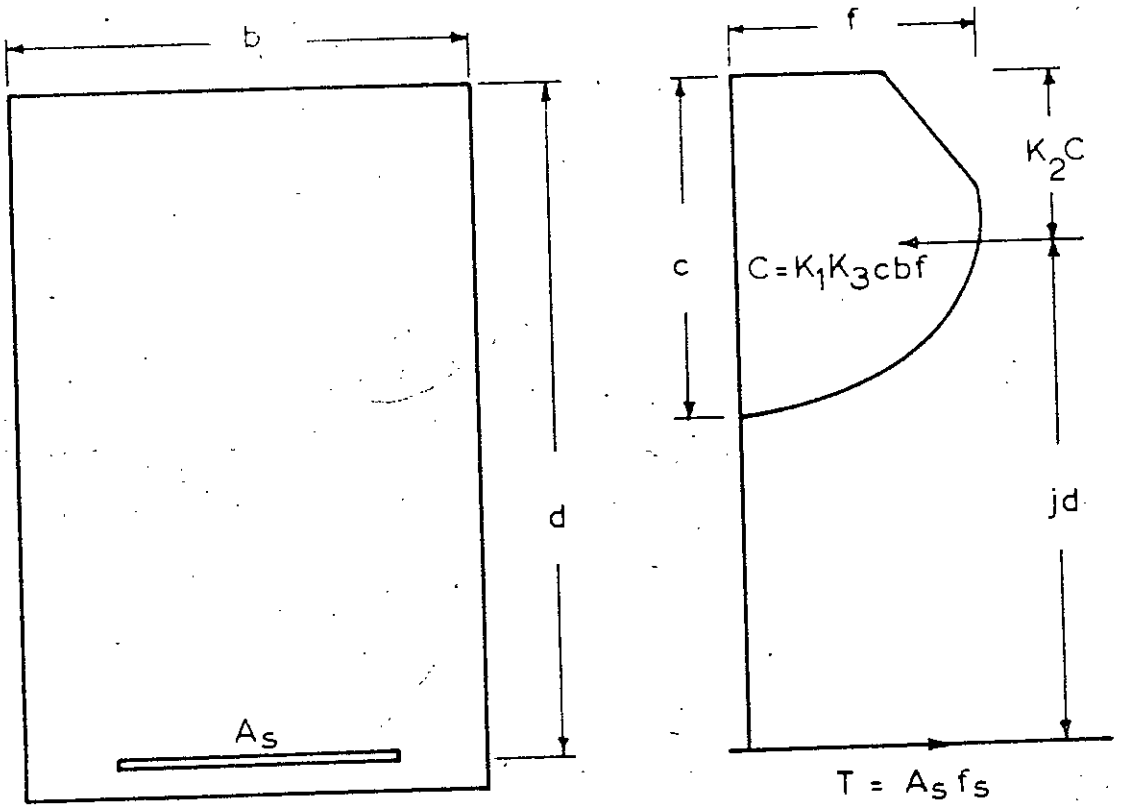
2. The intensity of shear stress is assumed to be constant at all points in the compression zone of the beam; i.e. a rectangular block is assumed for shear stress distribution in the compression zone of the beam. This assumption has also been made by some of the earlier investigators like Walther[90], Bresler and Pister[89], Guralnick[92] and Acharya and Kemp[102].

3. The shear reinforcement (vertical stirrups) yields before the final collapse of the beam. This is in conformation with the test observations available in the literature.

4. The concrete will fail under compression-tension or compression-compression when the stresses satisfy the corresponding failure criterion for concrete.

5. Bond and anchorage failures are excluded from the analysis

6. Contribution from dowel action of longitudinal steel is negligible. This has been shown by the measured strains in longitudinal steel and has already been discussed in detail under



RECTANGULAR BEAM WITHOUT SHEAR REINFORCEMENT.

FIG. II. 22.

the mechanisms of failure. Further this assumption is also in agreement with the discussion of results given by Rusch[111] and Werrille et al[106,107] in recent years.

II.6.3 Theory:

Rectangular Beams

Beams without shear reinforcement

Fig. II.22 shows a rectangular beam with tension steel only. The stress distribution in the compression zone is assumed to be non-linear. Let V be the shear force acting at the section and M_{CR} be the corresponding bending moment. Then, at the load points,

$$M_{CR} = V \times a \qquad \ldots \text{II.6.1}$$

where a is the length of shear span.

From Fig. II.22 by static conditions,

$$C = T$$

$$\text{and } M_{CR} = Va = Tjd = A_s f_s d\left(1 - \frac{k_2 c}{d}\right) \qquad \ldots \text{II.6.2}$$

or from Eqn. II.6.2

$$c = \frac{d}{k_2}\left(1 - \frac{M_{CR}}{A_s f_s d}\right) \qquad \ldots \text{II.6.3}$$

II. 6.3.1 For medium and high a/d ratios:

From the mechanism of failure discussed in the previous chapter, beams with medium and high a/d ratio will fail in stage 2 which corresponds to diagonal tension failure.

Assuming rectangular distribution of shear stress in the compression zone we have,

$$V = bc\tau \qquad \ldots \text{II.6.4}$$

where τ is the intensity of shear stress in the compression block.

Substituting for c from Eqn. II.6.3,

$$V = b.\frac{d}{k_2}\left(1 - \frac{M_{CR}}{A_s f_s d}\right)\tau \qquad \ldots \text{II.6.5}$$

or

$$\tau = \frac{k_2 V}{bd\left(1 - \frac{M_{CR}}{A_s f_s d}\right)} \qquad \ldots \text{II.6.6}$$

The most critical point for failure is at the level of neutral axis where the influence of normal stress (compressive) on shear stress is least. The principal stresses at the level of neutral axis are opposite in sign and each is equal to the shear stress, τ.

Therefore, $\sigma_1 = -\sigma_2 = \tau \qquad \ldots \text{II.6.7}$

The concrete element at the level of neutral axis will fail when these principal stresses satisfy a failure criterion for concrete under compressive-tensile stresses. Now we require the knowledge of a failure criterion for concrete under compressive-tensile stresses. In the first part of this thesis, the failure criterion suggested by Krishnaswamy[69] was used in the analysis. The same is used in this part of the thesis also and it can be put in the following form:

$$\left(\frac{\sigma_1}{f'_c}\right) + \left(\frac{\sigma_2}{f'_t}\right)^2 = 1 \qquad \ldots \text{II.6.8}$$

where

σ_1 - compressive stress
σ_2 - tensile stress
} in a biaxial field

f'_c, f'_t - uniaxial compressive and tensile strengths of concrete respectively.

Substituting for σ_1 and σ_2 from Eqn. II.6.7,

$$\frac{\tau}{f'_c} + \left(\frac{\tau}{f'_t}\right)^2 = 1 \qquad \ldots \text{II.6.9}$$

Substituting for τ from Eqn. II.6.6 and solving for V we get,

$$V = \frac{bd}{k_2}\left(1 - \frac{M_{CR}}{A_s f_s d}\right) f'_t \left[\frac{\sqrt{1 + \left(\frac{2f'_c}{f'_t}\right)^2} - 1}{(2f'_c / f'_t)}\right] \qquad \ldots \text{II.6.10}$$

or

$$M_{CR} = Va = \frac{bda}{k_2}\left(1 - \frac{M_{CR}}{A_s f_s d}\right)\left(\frac{f'_t}{f'_c}\right)\left[\sqrt{1 + \left(\frac{2f'_c}{f'_t}\right)^2} - 1 \over (2f'_c / f'_t)\right] f'_c \quad \ldots \text{II.6.1}$$

Based on their exhaustive experimental data, Krefeld and Thurston[104] approximated the failure surface occurring in the case of beams subjected to combined bending and shear as shown in Fig. II.23.

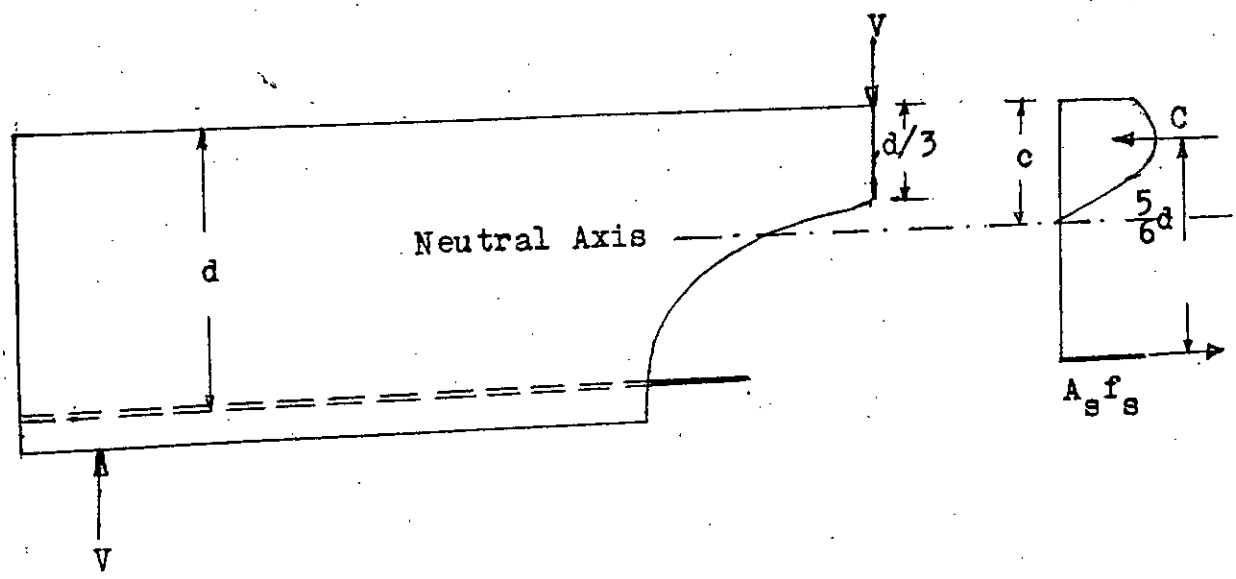


Fig. II.23

From Fig. II.23,

$$M_{CR} = \frac{5}{6} A_s f_s d \quad \ldots \text{II.6.12}$$

Further the strength of an under-reinforced beam failing in flexure, M_{FL} can be put in the following form:

$$M_{FL} = A_s f_y \, jd \quad \ldots \text{II.6.13}$$

where the value of j can be taken equal to $\frac{7}{8}$ as suggested by Kani[99]. The value of f_c'/f_t' will vary between 8 and 10 in the case of normal strengths concrete used in Reinforced Concrete construction. So taking average we get $f_c'/f_t' = 9$.

Using these relations and putting $k_2 = 0.42$ (see Appendix 3 for details) Eqn. II.6.11 reduces to,

$$\left(\frac{M_{CR}}{M_{FL}} \right) = \frac{a/d}{(21\ p\ f_y/f_c')} \qquad \ldots \text{II.6.14}$$

Putting $\left(\frac{M_{CR}}{M_{FL}} \right) = 1.00$ and designating that particular value of a/d ratio as α_{TR} we get,

$$\left(\frac{M_{CR}}{M_{FL}} \right) = \frac{a/d}{\alpha_{TR}} \qquad \ldots \text{II.6.15}$$

where $\alpha_{TR} = \left(\frac{21 p f_y}{f_c'} \right)$.

It is to be pointed out here that Eqn. II.6.15 is of the same form as obtained by Kani[99] in an entirely different way. Kani's expression for α_{TR} involves one more parameter known as crack factor which has to be evaluated as a mean value of large number of experimental results. The present analysis by the author predicts a constant value in the place of the crack factor.

II.6.3.2 For low a/d ratios:

For low a/d ratios, after the formation of diagonal crack, the transformation of the beam into a tied-arch takes place. This corresponds to the stage 3 of the failure mechanism discussed in an earlier chapter. This transformation may occur suddenly or gradually and it can be observed only from the measurement of strains both in steel and concrete. Such test observations were reported by some of the previous investigators[99] and they are in agreement with the observations of the present investigation also.

As discussed earlier, the critical section for failure of this transformed arch is located underneath the load points. The load acting directly on the beam produce vertical compressive stresses. Thus the critical section is acted upon by

i. a horizontal normal stress due to the applied bending moment acting in the longitudinal direction;

ii. a vertical normal stress produced by the loads in the transverse direction and,

iii. a shearing stress caused by the shear force. These three types of stresses at the critical point give rise to a pair of principal stresses thus:

$$\sigma_1 = \frac{f'_x + f'_y}{2} + \sqrt{\left(\frac{f'_x - f'_y}{2}\right)^2 + \tau^2}$$

$$\sigma_2' = \frac{f'_x + f'_y}{2} - \sqrt{\left(\frac{f'_x - f'_y}{2}\right)^2 + \tau^2} \qquad \text{... II.6.16}$$

where f'_x - horizontal normal stress

f'_y - vertical normal stress

τ - shear stress

Of the two principal stresses, σ_1 is compressive and for σ_2' to be compressive,

$$\frac{f'_x + f'_y}{2} > \sqrt{\left(\frac{f'_x - f'_y}{2}\right)^2 + \tau^2} \qquad \text{... II.6.17}$$

$$\text{or} \quad f'_x f'_y > \tau^2$$

This inequality is possible and logical as the magnitudes of the normal stresses f'_x and f'_y will normally be very high when compared to that of the shear stress, τ. So the other principal stress σ_2' will also be compressive. Thus we can deduce that the critical concrete element is under the action of two principal stresses which are compressive. Under these conditions, as shown by experiments[69], the stress at which failure occurs will be more than the uniaxial compressive

stress. But this increase in strength depends on the relative magnitudes of the principal stresses.

Now the problem reduces to the determination of the principal stresses. But it is difficult to determine exactly these principal stresses as the accurate evaluation of the vertical normal stress f'_y may not be possible or sometimes impossible. Hence only the horizontal normal stress and shear stress are considered in the analysis and the influence of vertical normal stress is discussed later.

The resulting principal compressive stress is equated to the uniaxial compressive stress. Thus

$$\frac{f'_x}{2}\left[\sqrt{1 + \left(\frac{2\tau}{f'_x}\right)^2} + 1\right] = f'_c \qquad \text{... II.6.18}$$

or omitting the suffix we can put thus:

$$\frac{f}{2}\left[1 + \sqrt{1 + \left(\frac{2\tau}{f}\right)^2}\right] = f'_c \qquad \text{... II.6.19}$$

where f is the horizontal normal stress (compressive) at the critical point due to M_{CR}.

From Fig. II.22

$$M_{CR} = k_1 k_3 \, f \, cbjd$$

$$\text{or} \quad f = \frac{M_{CR}}{k_1 \, k_3 \, cbjd} \qquad \text{... II.6.20}$$

By Eqn. II.6.4

$$\tau = \frac{M_{CR}}{acb} \qquad \ldots \text{II.6.21}$$

From Eqns. II.6.20 and II.6.21 we get

$$\left(\frac{2\tau}{f}\right) = \frac{2k_1k_3j}{(a/d)} \qquad \ldots \text{II.6.22}$$

Putting $k_1k_3 = 0.80$ (from Appendix 3)

$j = \frac{5}{6}$ and substituting for $(\frac{2\tau}{f})$ from Eqn. II.6.22 in Eqn. II.6.19, we have,

$$\frac{f}{f'_c} = \frac{2}{1+\sqrt{1 + 2 / (a/d)^2}} \qquad \ldots \text{II.6.23}$$

Let

$$\left.\begin{aligned} M_{CR} &= k_1k_3cbjfd \\ M_{FL} &= k_1k_3c_obf'_c jd \end{aligned}\right\} \qquad \ldots \text{II.6.24}$$

where c_o is that particular value of depth of compression block when the beam has to fail in flexure.

Substituting for $(\frac{f}{f'_c})$ from Eqn. II.6.24 in Eqn. II.6.23,

$$\frac{M_{CR}}{M_{FL}} = \frac{c}{c_o}\left[\frac{2}{1+\sqrt{1 + 2/(a/d)^2}}\right] \qquad \ldots \text{II.6.25}$$

Putting $\frac{c}{c_o} = \frac{d}{a}$ (the validity of this relation is discussed in the next chapter) Eqn. II.6.25 reduces to,

$$\frac{M_{CR}}{M_{FL}} = \frac{2}{\frac{a}{d} + \sqrt{(\frac{a}{d})^2 + 2}} \qquad \ldots \text{ II.6.26}$$

To account for the biaxial state of compressive stresses near the load points, we can conservatively increase the above value of (M_{CR}/M_{FL}) by introducing a coefficient k in the denominator which is always less than 1.00 as has been suggested by Kani[99]. The evaluation of k is discussed in the next chapter.

Hence Eqn. II.6.26 reduces to

$$\frac{M_{CR}}{M_{FL}} = \frac{(2/k)}{\frac{a}{d} + \sqrt{(\frac{a}{d})^2 + 2}} \qquad \ldots \text{ II.6.27}$$

For a/d ratios less than 1.00 the quantity given by Eqn. II.6.27 will be greater than unity and at a/d = 0 it reaches a value of $\frac{\sqrt{2}}{k}$. This is not correct because at a/d = 0, (M_{CR}/M_{FL}) should be equal to zero as it is the 'pure shear' case. Hence it is evident that Eqn. II.6.27 is not valid for a/d ratios less than 1.00. For $a/d < 1.00$, the shape of the curve is not known. Hence large number of experiments are necessary to establish the mechanism of failure of such beams.

II.6.3.3 Beams with shear reinforcement:

(Only vertical stirrups are considered in the analysis)

In beams with shear reinforcement, the shear force has to be resisted both by concrete in the compression zone and the shear reinforcing steel.

Therefore total shear force at any section = shear resisted by stirrups + shear resisted by compression zone concrete.

Consider a length equal to d of the beam and assuming the stirrups yield before failure, we have,

$$V = A_w f_{yw} \frac{d}{s} + bc\tau \qquad \text{... II.6.28}$$

where

A_w - total area of cross section of shear reinforcement

f_{yw} - yield stress of stirrups steel

s - spacing of stirrups

Solving for τ in Eqn. II.6.28 we get,

$$\tau = \frac{(V - A_w f_{yw} \frac{d}{s})}{bc}$$

$$\text{or } \tau = \frac{V}{bc}\left(1 - \frac{A_w f_{yw} d}{sV}\right) \qquad \text{... II.6.29}$$

With the value of α as given by Eqn. II.6.29, proceeding on similar lines to the case without shear reinforcement we can deduce the following equations:

For medium and high a/d ratios:

$$\frac{M_{CR}}{M_{FL}} = \frac{(a/d)}{\alpha_{TR}} \qquad \text{... II.6.30}$$

where

$$\alpha_{TR} = \left(1 - \frac{\Lambda_w f_{yw} d}{sV}\right)\left(\frac{21p\, f_y}{f'_c}\right)$$

For low a/d ratios:

$$\frac{M_{CR}}{M_{FL}} = \frac{(2/k)}{\left(\frac{a}{d}\right) + \sqrt{\left(\frac{a}{d}\right)^2 + 2\left(1 - \frac{\Lambda_w f_{yw} d}{sV}\right)^2}} \qquad \text{... II.6.31}$$

II.6.3.4 Flanged beams:

The solutions derived hitherto for rectangular beams can be easily extended to flanged beams i.e. Tee and Ell beams.

The addition of a flange to a rectangular beam without altering any other factor increases the shear resistance of the beam. But as observed from test results available in the literature[80,110] these increases are not uniform for all a/d ratios and are not directly proportional to the increase in the width of the flange. The thickness of the flange is an additional variable further complicating the problem. However,

the basic mechanism of failure is same for both rectangular and flanged beams. The formation of a diagonal crack is similar in both the types of beams; but the development of diagonal crack towards top of the beam is somewhat different in flanged beams. As observed from experiments, the diagonal crack separates the projecting portion of the flanges from the web. In addition, similar to the flanged beams subjected to pure bending, the position of the neutral axis governs the net concrete area resisting the shear force. So two cases are to be discussed:

Case 1 - Neutral axis in the flange:

Normally this case will arise in thick-flanged beams. When the neutral axis falls within the flange, the whole flange width will be effective in resisting the shear force. So the area of stress block will be equal to (B' x c) where 'B' is flange width. Since other concepts of the mechanism of failure are similar to that of rectangular beams, the problem of shear strength flanged beams with the neutral axis in the flange reduces to that of a rectangular beam with breadth of the rectangle equal to the flange width. Hence the equations derived for rectangular beams can be applied to this case with appropriate value for the breadth of the rectangle in the final equations.

Case 2 - Neutral axis in the web:

In the case of flanged beams with normal flange thickness (which are the normal beams encountered in practice), the neutral axis will fall in the web. The effective concrete area resisting the shear force depends on the area available above the neutral axis and its distribution over the depth of the neutral axis. The contribution of the projecting portions of the flanges towards the shear strength of the beam varies with a/d ratios and experiments available in the literature[80] have shown that its contribution for large values of a/d ratios is practically nil. Also near failure, by the propagation of the diagonal crack, the projecting flanges will tend to separate from the web portion of the beam and the separated flanges will act separately. The flanges will be subjected to bending in both lateral and longitudinal directions. The mechanism of failure of these flanges bending in both direction can be thought of analogous to the bending of a slab. This is evident from the mode of crack pattern obtained on the top of flanges. For instance, the crack pattern shown in Fig. II.7 is analogous to the yield line pattern of a continuous slab with free edges. But the analysis of the flange by treating it as a slab will involve too many parameters resulting in complicated solutions. Instead, conservatively, we can assume that the net area resisting the shear force at collapse is (b x c) where b is the

breadth of the web and c is the depth of neutral axis from the top surface of the beam. Thus the contribution from the projecting portions of the flanges is neglected in the analysis. In one respect this analysis is consistent with the flexure analysis of flanged beams. That is, normally one method of analysis of flanged beams under pure bending is to replace the projecting portions of flanges as an equivalent area of steel at mid depth of the flange and to analyze the flanged beam as a rectangular beam with the replaced area as the compression reinforcement of the beam. So if we adopt the same method of analysis here for the flanged beams under combined bending and shear, we can treat the projecting flanges as the compression reinforcement of the modified rectangular beam. But experimental results available in the literature[80] have shown that the contribution by the compression steel towards the shear strength of a beam is practically nil. On the same lines, we can conservatively neglect the effect of projecting portions of the flange which has been replaced by an equivalent amount of compression steel.

Thus neglecting the effect of projecting portions of the flanges, the problem once again reduces to the one similar to rectangular beams and the equations derived for rectangular beams can be applied here also. Similarly, for flanged beams with shear reinforcement the equations derived for rectangular

compression reinforcement and/or with transverse flange reinforcement as the experimental results available in the literature[80,110] have shown that none of these reinforcements influences the shear strength of beams. The method of analysis suggested for flanged beams is illustrated in the following examples:

In general we can put

$$M = A_s f_s d \left(1 - k_2 \frac{c}{d}\right)$$

or taking $M = 0.833 A_s f_s d$, $c = 0.40d$.

With this approximate value of c, the method of calculation is illustrated for two beams of Ramakrishnan's investigation[80].

Beam 3a-S9:-

Flange width = 10.06 in

Wed width = '3 in

Flange thickness, t = 1.25 in

d = 4.8 in

c = 0.40d = 0.4 x 4.8 = 1.92 in > t

Therefore neutral axis falls in the web.

Effective cross section: 3 x 4.8 in^2

M_{CR} = 141 in kips, M_{FL} = 163 in kips; a/d = 8

α_{TR} = 9.70

$$(M_{CR}/M_{FL})_{Test} = \frac{141}{163} = 0.86$$

$$(M_{CR}/M_{FL})_{Calc.} = \frac{8}{9.7} = 0.83$$

$$\frac{Test}{Calc} = 1.04$$

Beam 3b-F5:

Flange thickness, $t = 2.28$ in

Flange breadth $= 6$ in

$d = 4.8$ in; $c = 1.92$ in $< t$

Therefore, neutral axis falls in the flange.

Effective cross section: 6×4.8 in^2

$M_{CR} = 112$ in kips; $M_{FL} = 153$ in kips

$a/d = 3.50$; $\alpha_{TR} = 4.67$

$$(M_{CR}/M_{FL})_{Test} = \frac{112}{153} = 0.73$$

$$(M_{CR}/M_{FL})_{Calc} = \frac{3.50}{4.67} = 0.75$$

Test/Calc $= 0.73/0.75 = 0.97$

II.6.4 The theory and equations developed so far were derived by considering the idealized case of loading on a simply-supported beam. These equations can be easily extended to any other case because the a/d ratio, which is an important parameter in the analysis, represents only the relation between bending moment and shear force. The same equations can be

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applied to any other type of structure statically determinate or indeterminate, with any type of loading by replacing the term a/d by the term M/Vd which is a general parameter giving the ratio between bending moment and shear force in the non-dimensional form.

In the next chapter, the test results of the author as well as those reported in the literature are discussed in the light of the theory and equations derived in this chapter.

CHAPTER II.7

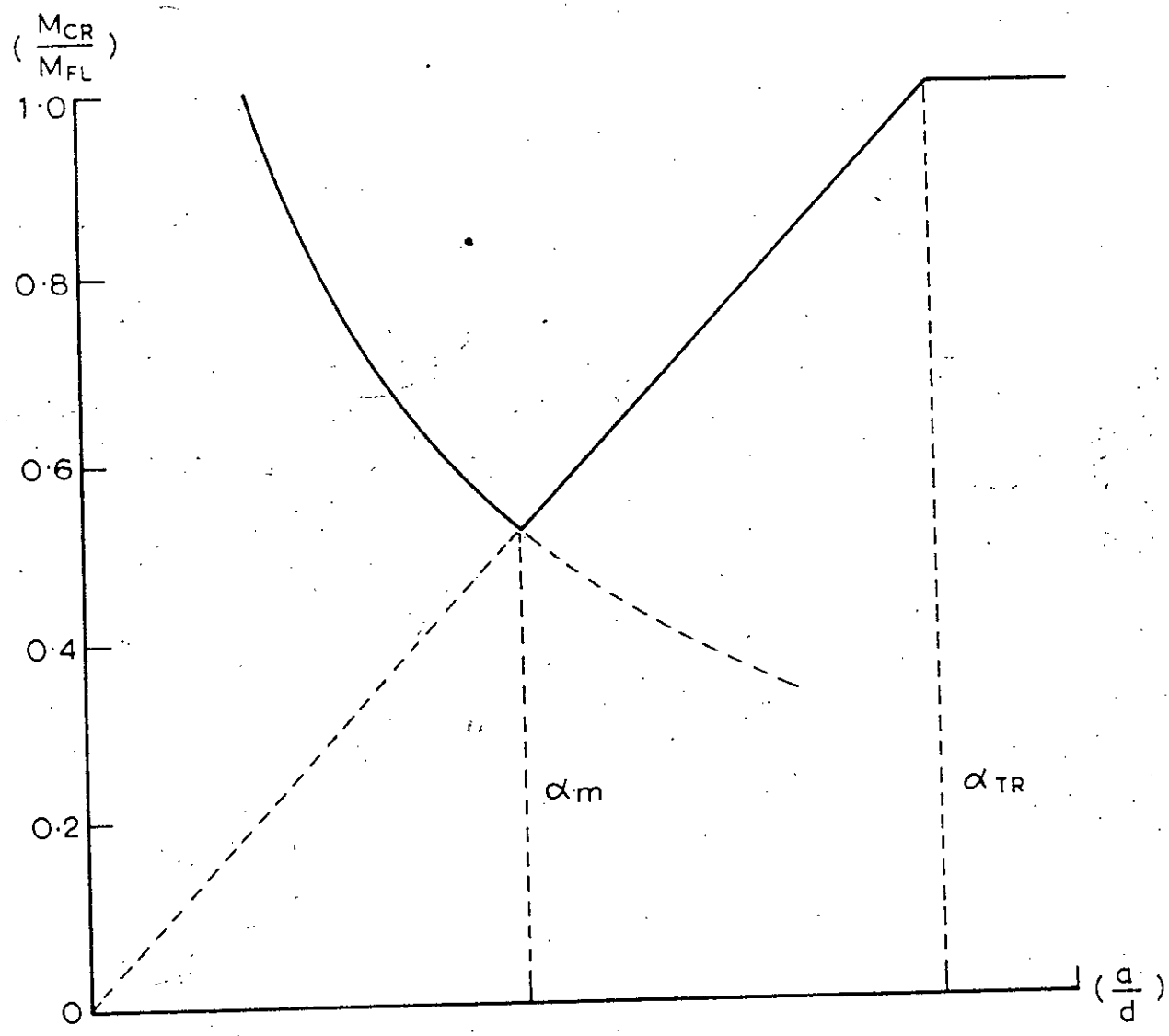
DISCUSSION OF ANALYTICAL AND TEST RESULTS

II.7.1 In the previous chapter, analytical equations have been derived for predicting the shear strength of beams based on certain assumptions and a failure criterion for concrete under compressive-tensile stresses. The validity of some of these assumptions has already been discussed. The justification for other assumptions will be discussed in this chapter in the light of experimental results. Also the effect of certain parameters introduced in the analysis on the final results is also discussed. The analysis is extended to the case of restrained and continuous beams and to the case of beams with uniformly distributed loads. In addition, results of over one thousand tests available in the literature are compared with the theoretical predictions of the proposed theory.

The analytical expressions derived in the previous chapter require the knowledge of ranges of low and high and medium a/d ratios and the value of the biaxial coefficient k. These aspects are discussed in the following two articles.

II.7.2 Ranges of low and high a/d ratios:

It is essential to determine the ranges of low and high a/d ratios theoretically as the theory is based on two different



VALLEY OF SHEAR FAILURE.
FIG. II. 24.

mechanisms of failure of beams:- one applicable to low a/d ratios and the other to high and medium a/d ratios. Considering beams without shear reinforcement, Eqn. II.6.15 is applicable for high and medium values of a/d ratio while Eqn. II.6.27 is suitable for low a/d ratios. On a close observation of these equations, it can be seen that the quantity (M_{CR}/M_{FL}) varies directly with a/d ratio in Eqn. II.6.15 and it varies inverseely with a/d ratio in Eqn. II.6.27. If these two equations are represented in a single plot, as shown in Fig.II.24 a valley is formed in between the plots of these two equations. This has been called by Kani[99] as "valley of shear failure". That is, if the a/d ratios do not fall within the ranges of the valley, there is no danger of shear failure.

The deepest point in the valley of shear failure represents that particular value of a/d ratio at which the quantity (M_{CR}/M_{FL}) is minimum. In other words, at this particular value of a/d ratio, the reduction in the flexural capacity of the beam is maximum. This particular value of a/d ratio is denoted by α_m. The value of α_m is obtained as the intersection of the plots of Eqns. II.6.15 and II.6.27 Thus equating Eqns. II.6.15 and II.6.27 we have,

$$\frac{M_{CR}}{M_{FL}} = \frac{(a/d)}{\alpha_{TR}}$$

$$= \frac{(2/k)}{\frac{a}{d} + \sqrt{(\frac{a}{d})^2 + 2}} \qquad \ldots \text{II.7.1}$$

$$\text{or } (a/d)_m = \alpha_m = \frac{(2/k)\alpha_{TR}}{\sqrt{\frac{4}{k}\alpha_{TR} + 2}} \qquad \ldots \text{II.7.2}$$

Using the value of α_m, the quantity $\left(\frac{M_{CR}}{M_{FL}}\right)_m$ can be easily obtained thus:

$$\left(\frac{M_{CR}}{M_{FL}}\right)_m = \frac{(2/k)}{\sqrt{\frac{4}{k}\alpha_{TR} + 2}} \qquad \ldots \text{II.7.3}$$

Proceeding on similar lines, we can derive similar expressions for the case of beams with shear reinforcement:

$$\alpha_m = \frac{(2/k)\alpha_{TR}}{\sqrt{\frac{4}{k}\alpha_{TR} + 2\left(1 - \frac{A_w f_{yw} d}{sV}\right)^2}} \qquad \ldots \text{II.7.4}$$

$$\text{and } \left(\frac{M_{CR}}{M_{FL}}\right)_m = \frac{(2/k)}{\sqrt{\frac{4}{k}\alpha_{TR} + 2\left(1 - \frac{A_w f_{yw} d}{sV}\right)^2}} \qquad \ldots \text{II.7.5}$$

Then we have,

i. If the a/d ratio is less than or equal to α_m, it represents the lower ranges of a/d ratios and the reduction in the flexural capacity of the beam is known from Eqn. II.6.27 or II.6.31.

ii. If the a/d ratio is greater than or equal to α_m, it represents the higher and medium ranges of a/d ratio and the reduction in the flexural capacity of the beam is determined from Eqn. II.6.15 or II.6.30.

iii. For a/d ratio greater than or equal to α_{TR} the quantity (M_{CR}/M_{FL}) is equal to unity; in other words only flexure failure is possible.

II.7.3 The biaxial coefficient, k:

The biaxial coefficient k has been introduced in the analysis to account for the influence of vertical normal stress on the carrying capacity of beams. This has been done due to the reason that the exact determination of the distribution of vertical normal stress across the depth of the beam is not known. Moreover the behaviour of concrete under biaxial compression is not yet clearly known. Of all the available results, the most extensive one is the results reported by Iyengar et al[112]. According to their test results it has been found that the effect of lateral stress is to increase the major principal stress at failure by four times the minimum principal stress for ratios of minimum principal stress/cube strength upto 0.30. Using the results of this investigation the

failure law can be put in the form thus:

$$\frac{\sigma_1}{f'_c} \leq (1 + 4 \frac{\sigma_2'}{f'_c}) \qquad \ldots \text{II.7.6}$$

where σ_1 - major principal stress (compressive)

σ_2' - minor principal stress (compressive)

From the test results reported by Acharya and Kemp[102] it has been found that the magnitude of (σ_2'/f'_c) varies between 0.050 and 0.125 in most of the cases. Using these values for σ_2'/f'_c and the failure law given by Eqn. II.7.6, the value of the biaxial coefficient k can be obtained as given below:

For example, $\sigma_2'/f'_c = 0.05$

From Eqn. II.7.6, $\frac{\sigma_1}{f'_c} = 1 + 4 \frac{\sigma_2'}{f'_c}$

$= 1 + 4\ (0.05) = 1.20$

The value of 1/k represents nothing but the increase in the value of uniaxial compressive strength of concrete f'_c. Therefore $1/k = \sigma_1/f'_c$, or $k = \frac{1}{1.20} = 0.833$. Similarly for $\sigma_2'/f'_c = 0.125$, $k = 0.667$. From these calculations it is found that the value of k may vary between 0.667 and 0.833. Taking average of these two extreme values, we can find $k = \frac{0.667 + 0.833}{2} = 0.75$. With this value of k, the

solutions derived in the previous chapter are complete and their respective ranges of applicability are well defined by the discussion presented in the previous article.

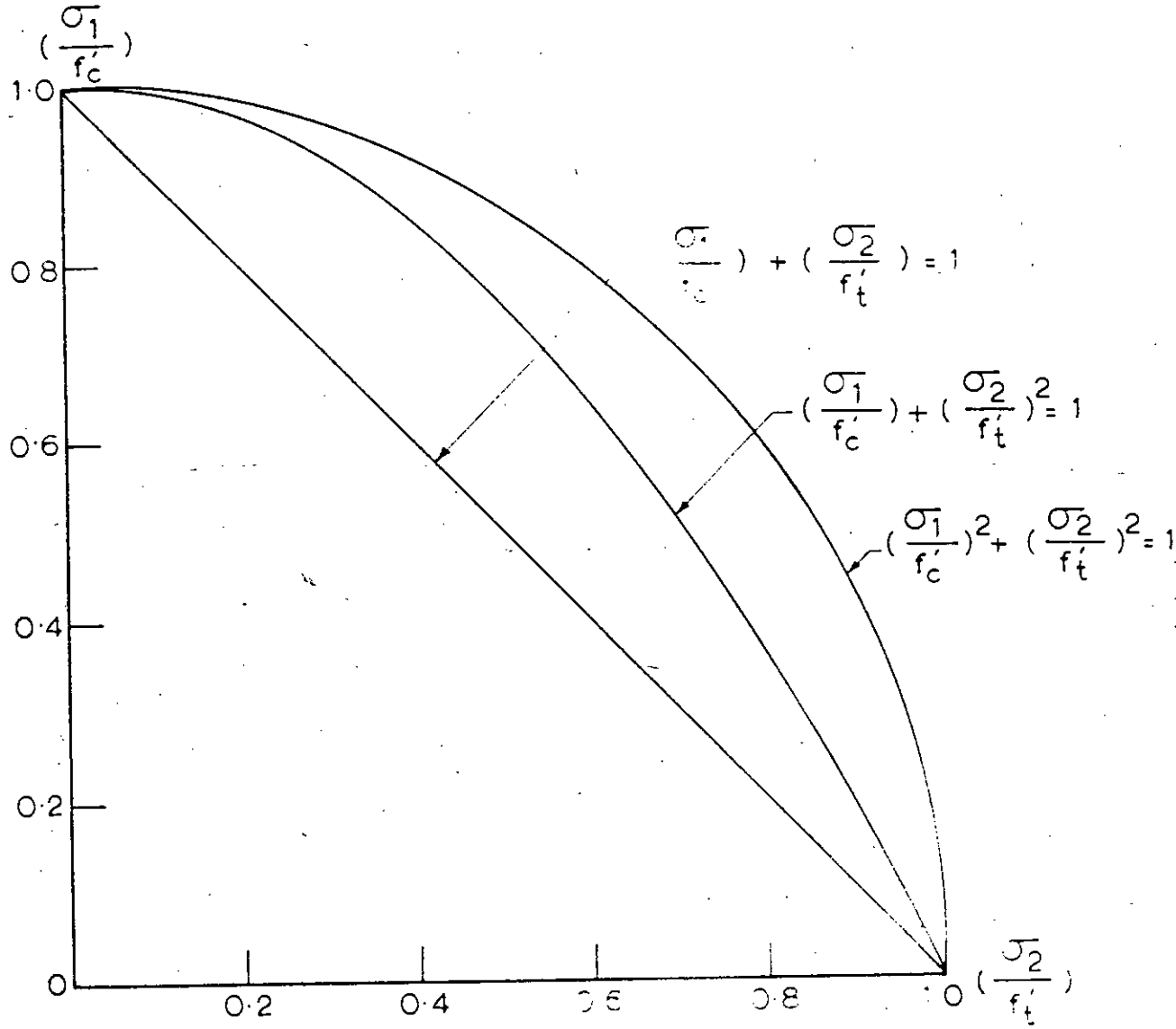
II.7.4.1 Failure criterion for concrete:

In order to determine the strength of the concrete element at the level of neutral axis, the author has used a failure criterion suggested by Krishnaswamy[69] for concrete under compressive-tensile stresses. It is restated here:

$$\left(\frac{\sigma_1}{f'_c}\right) + \left(\frac{\sigma_2}{f'_t}\right)^2 = 1 \qquad \ldots \text{II.7}$$

In the past, some of the investigators like Bresler and Pister[89], Walther[90], Guralnick[92] Sen et al[93-97] have used failure criteria in terms of normal and shearing stresses to evaluate the shear strength of beams. But the author preferred to use a failure criterion in terms of principal stresses rather than in terms of normal and shearing stresses. The main reason which made the author to use the failure criterion given by Eqn. II.7.7 in his analysis is two-fold: firstly it is simple and it is in terms of principal stresses as the author preferred and secondly it has been confirmed by large number of tests on plain concrete specimens subjected to compression, torsion, splitting etc.[69].

In order to examine the sensitivity of the resulting equations with respect to failure criteria, two more failure



σ_1 — Compressive principal stress

σ_2 — Tensile principal stress

COMPARISON OF VARIOUS FAILURE CRITERIA.

FIG. II. 25.

criteria of the same type as given by Eqn. II.7.7 have been considered. They have been assumed of the form:

$$\left(\frac{\sigma_1}{f'_c}\right)^2 + \left(\frac{\sigma_2}{f_t}\right)^2 = 1 \qquad \ldots \text{ II.7.8}$$

$$\left(\frac{\sigma_1}{f'_c}\right) + \left(\frac{\sigma_2}{f'_t}\right) = 1 \qquad \ldots \text{ II.7.9}$$

In Fig. II.25, these three failure criteria are represented in graphical form. From the figure it can be easily seen that there is considerable difference among these three. Using these failure criteria the value of α_{TR} introduced in Eqn. II.6.15 can be calculated. The influence of the various criteria on the value of α_{TR} indirectly reveals the influence of failure criteria on the final expressions for arriving at the shear strength of beams. The results are summarised below:

Failure Criterion			$\alpha_{TR} / (pf_y/f'_c)$
$(\sigma_1/f'_c) + (\sigma_2/f'_t)^2$	=	1.0	21.0
$(\sigma_1/f'_c)^2 + (\sigma_2/f'_t)^2$	=	1.0	19.7
$(\sigma_1/f'_c) + (\sigma_2/f'_t)$	=	1.0	22.4

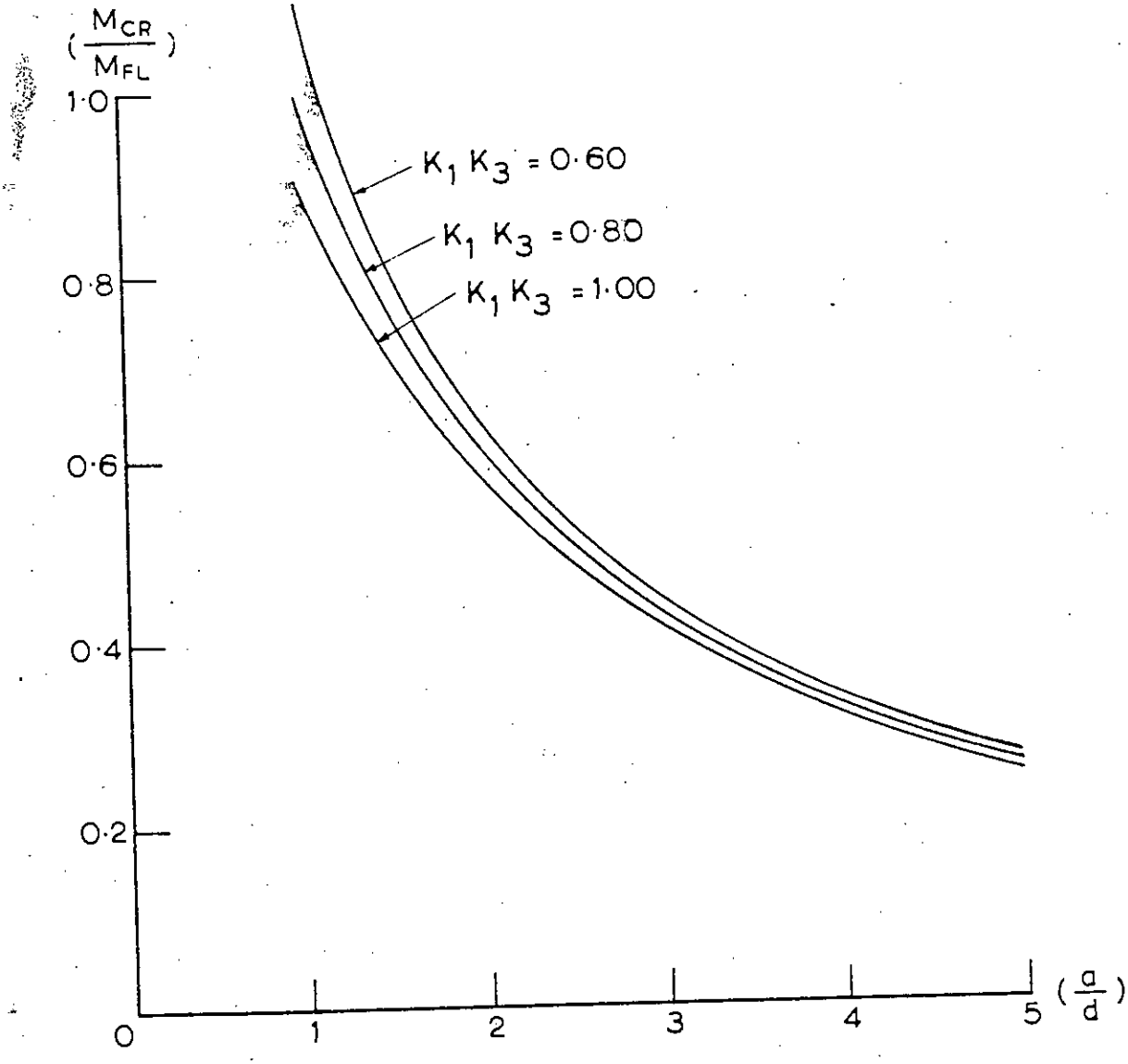
Considering the value of α_{TR} given by Eqn. II.7.7 which has been used in the analysis of the author, as datum it can be easily

seen that the other two values corresponding to the other two failure criteria are erring only by ± 6%. Hence it can be concluded that the final equations are not very sensitive to the variations in the failure criterion. Even though the straight-line failure criterion (Eqn. II.7.9) seems to be much simpler than the one used by the author, obviously it cannot be used in an analysis as its experimental justification is not known.

II. 7.4.2 Coefficients of the compression block:

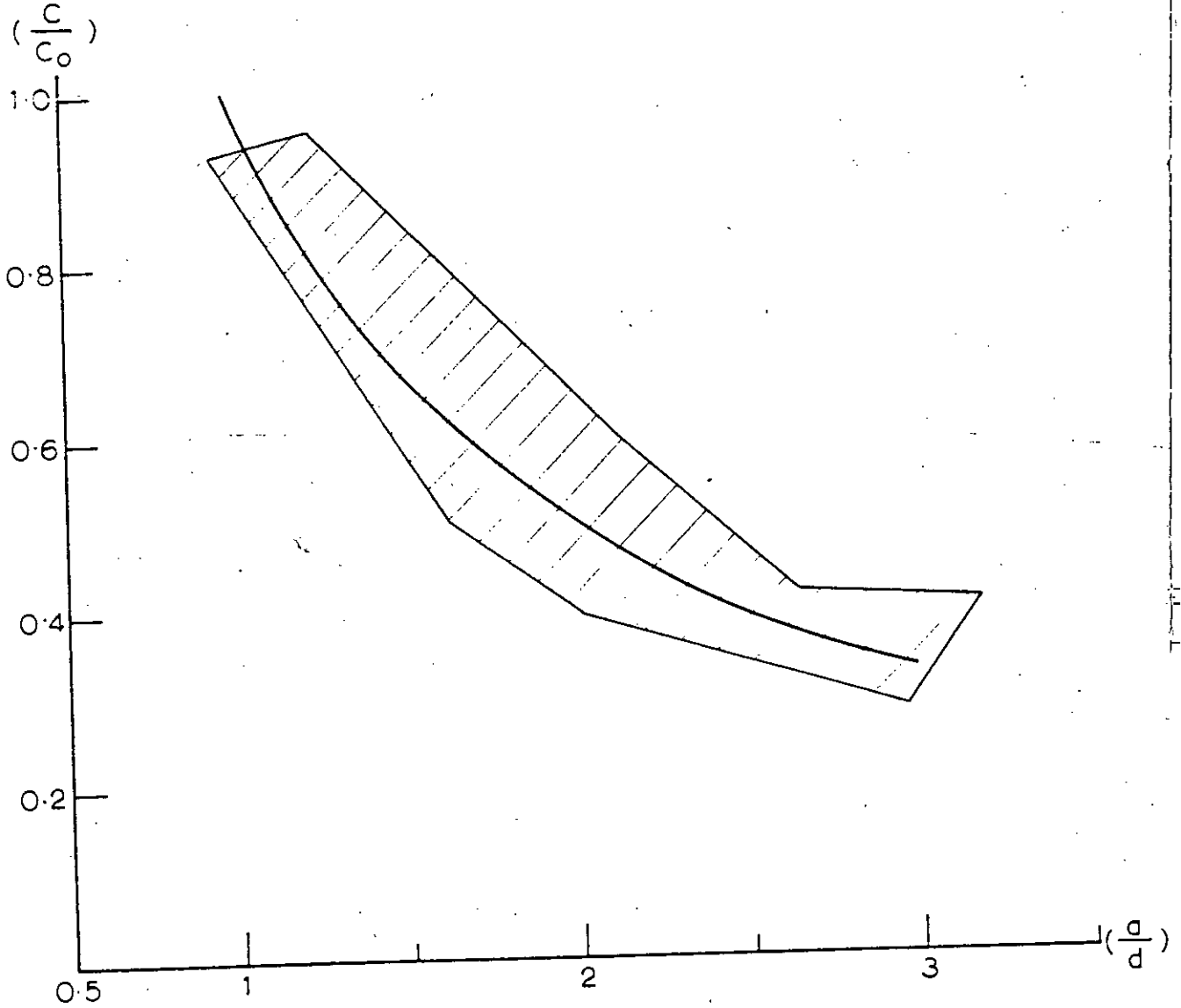
The other important parameters used in the derivation of equations (Eqns. II.6.15 and II.6.23 are the compression block coefficients, k_2 and k_1k_3 (vide Fig. II.22). The respective values used in the analysis are 0.42 and 0.80 (see Appendix 3 for details). This value of the coefficient k_2 (i.e 0.42) has a special advantage as it represents the mean value of two extreme values of k_2 i.e. 0.333 and 0.500 where the former belongs to triangular distribution of stress and the latter refers to the rectangular distribution of stress.

By varying the value of k_2 between 0.400 and 0.460 (based on Hognestad et al[113]) the value of $\frac{\alpha_{TR}}{(pf_y/f'_c)}$ varies between 20 and 23. With $\frac{\alpha_{TR}}{(pf_y/f'_c)} = 21$ as datum, the results are found to vary between + 10% and -5%, the results becoming more conservative when k_2 approaches 0.460.



INFLUENCE OF $K_1 K_3$ ON THE VARIATION OF M_{CR}/M_{FL} WITH $\frac{a}{d}$ RATIO

FIG. II. 26.



▨ — Experimental (Acharya & Kemp)

——— — Theoretical (Author)

VARIATION OF $(\frac{C}{C_o})$ WITH $\frac{a}{d}$ RATIO.

FIG. II. 27.

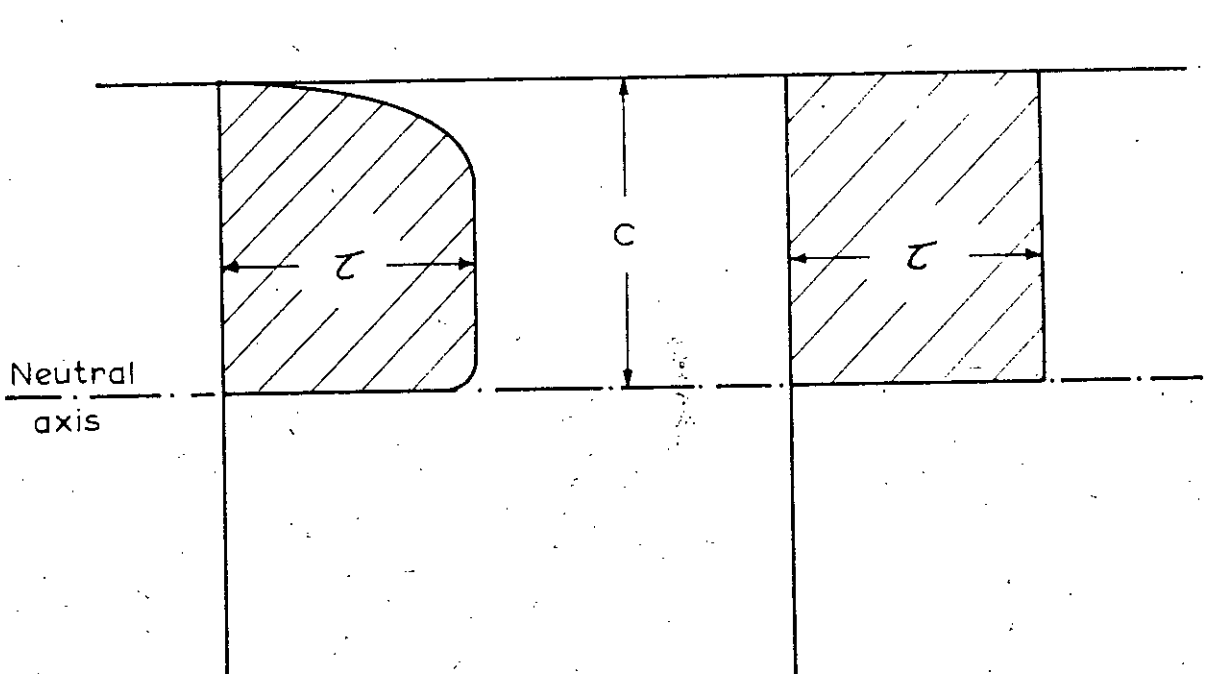
The coefficient k_1k_3 has been used in the derivation of Eqn. II.6.27. Fig. II.26 shows the influence of k_1k_3 on the variation of (M_{CR}/M_{FL}) with a/d ratio. Three values i.e. 0.60, 0.800 and 1.00 (based on Hognestad et al[113]) have been used for k_1k_3 and from the Fig. II.26 it is clear that the value of k_1 k_3 has very little influence on the magnitude of (M_{CR}/M_{FL}).

Thus the above discussion indicates that the resulting equations are not very sensitive to the values of compression block coefficients.

II.7.5.1 Validity of the relation $\frac{c}{c_o} = \frac{d}{a}$

In the development of the Eqn. II.6.27 which is applicable for determining the strength of the transformed arch, at one stage it has been assumed that $c/c_o = d/a$ and the resulting equation has been derived based on this assumption. Now it is necessary to examine the validity of this assumption.

Recently Acharya and Kemp[102] have reported the results of their investigation in which they have measured both c and c_o experimentally for beams failing in shear tested with different a/d ratios. Their experimental results, showing the variation of c/c_o with a/d ratio, are summarised in Fig. II.27. The experimental results are shown by a strip and the theoretical line as given by the equation $c/c_o = d/a$ is



ASSUMED SHEAR STRESS DISTRIBUTION OVER THE DEPTH OF NEUTRAL AXIS.

FIG. II. 28

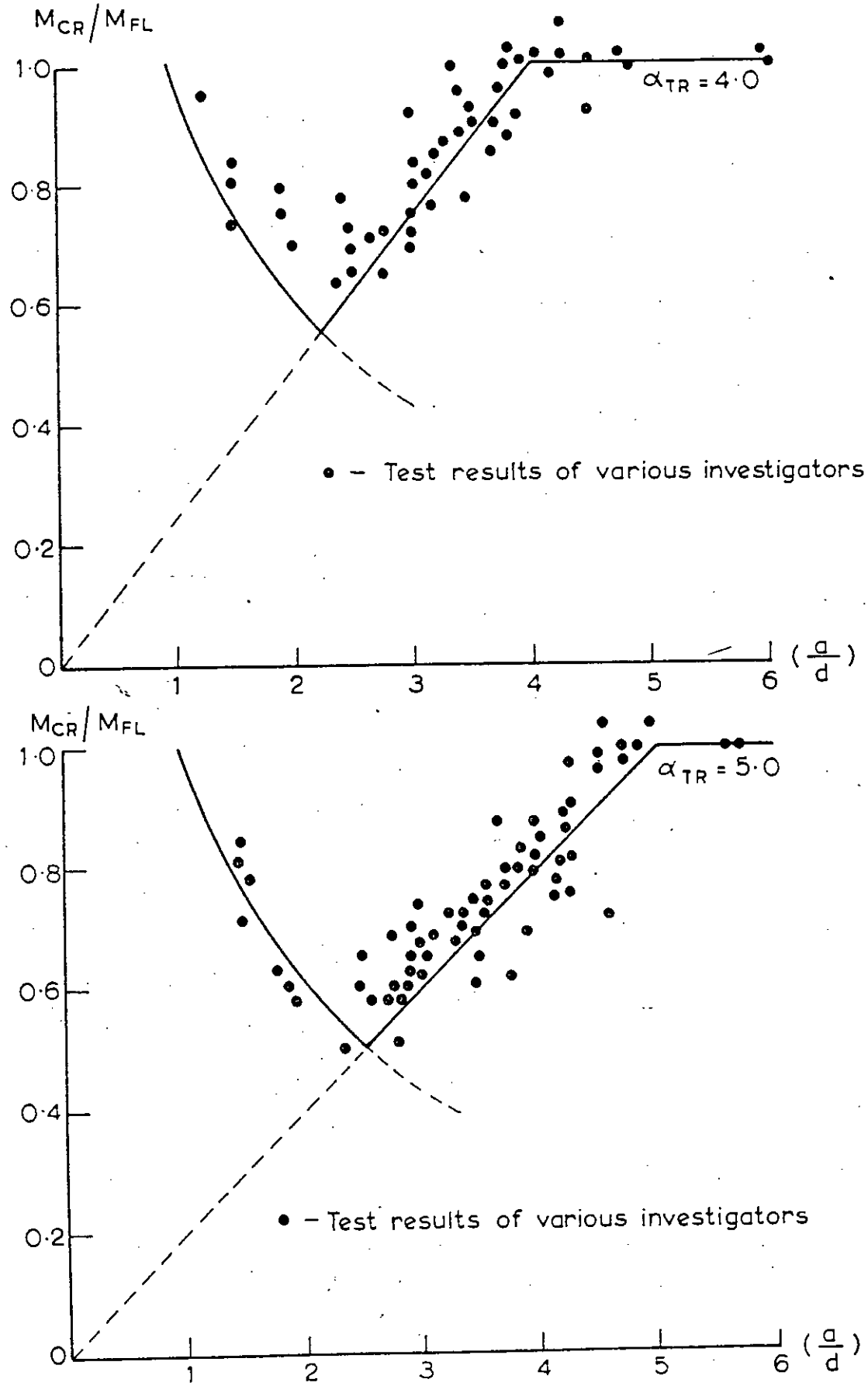
overlapped on the same figure by a thick line. From the Fig. II.27 it can be clearly seen that the experimental and theoretical results agree closely well. This evidently shows that the assumption $c/c_o = d/a$ is justified.

II.7.5.2 Distribution of shear stress

In the present analysis, the author has assumed a rectangular distribution for shear stress in the compression zone of the beam at the time of failure. If the loads are applied vertical and if there are no other shear stresses except the one caused by the shear force, the possibly "exact" distribution of shear stress may be similar to the one as shown in Fig. II.28a. According to this distribution, shear stress is equal to zero at the extreme fibre in the compressic zone (i.e. top of beam). Considering the simplicity of the resulting equations, the possibly "exact" distribution has been approximated by a rectangular distribution (Fig. II.28b) as the difference between the area of distribution diagrams shown in Figs. II.28a and II.28b is quite negligible.

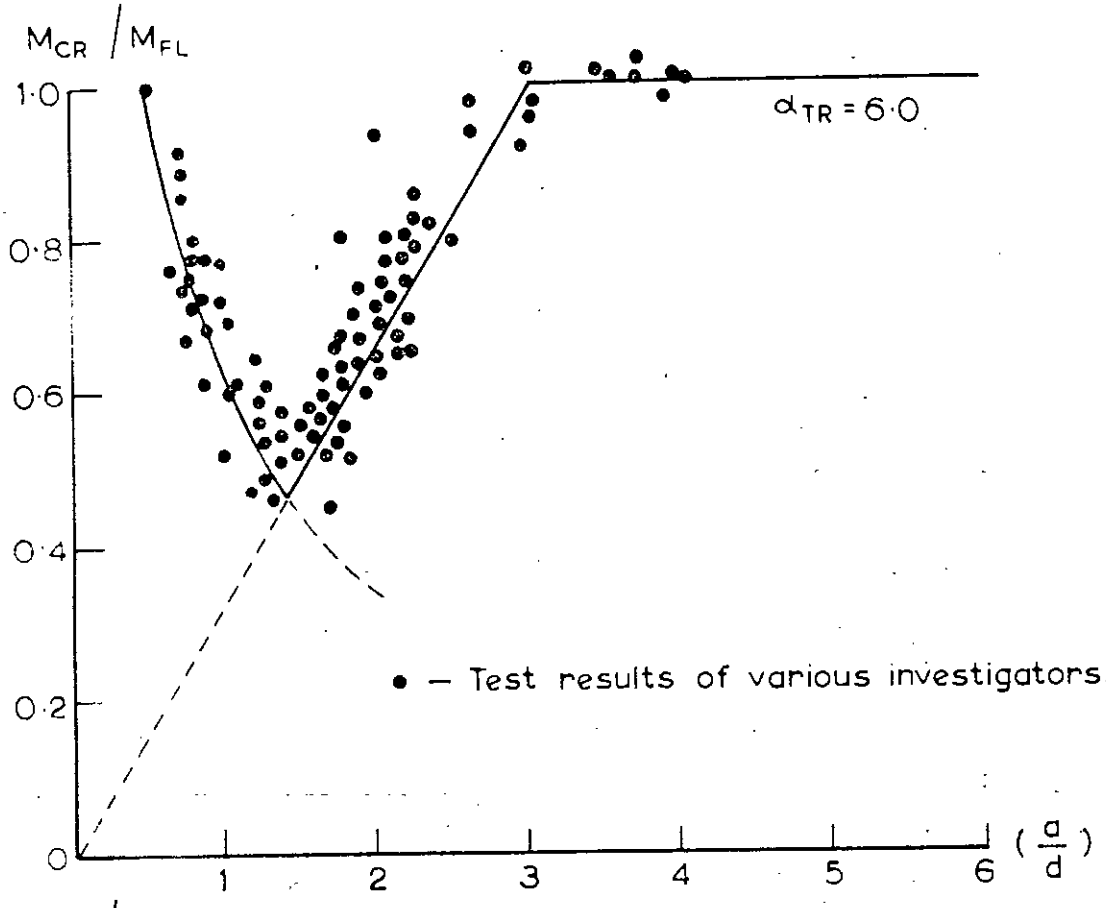
II.7.6 Comparison of calculated and test results:

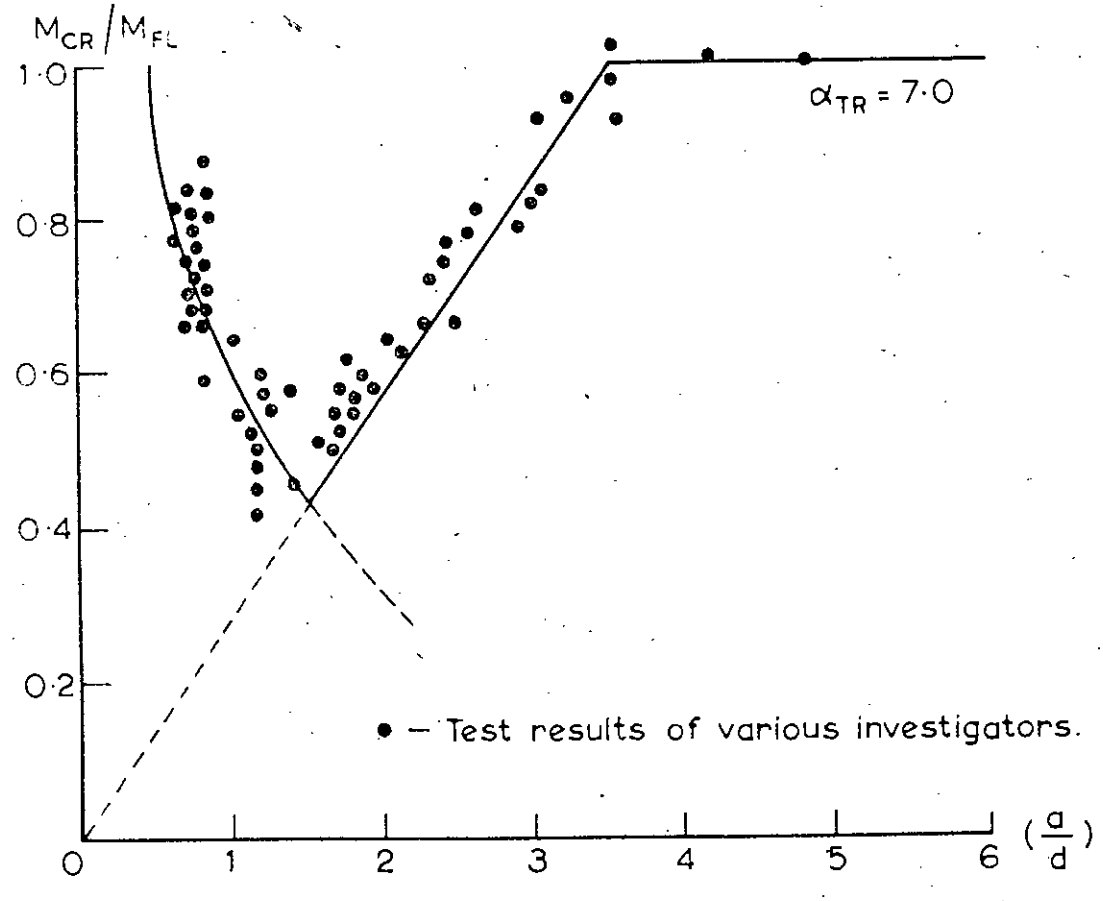
No theory or equation stands valid unless it is confirmed by experimental results. In order to verify the theory and the resulting equations, the author has collected large number of test results available in the literature.



COMPARISON OF AUTHOR'S THEORITICAL LINES WITH THE TEST RESULTS OF VARIOUS INVESTIGATORS.

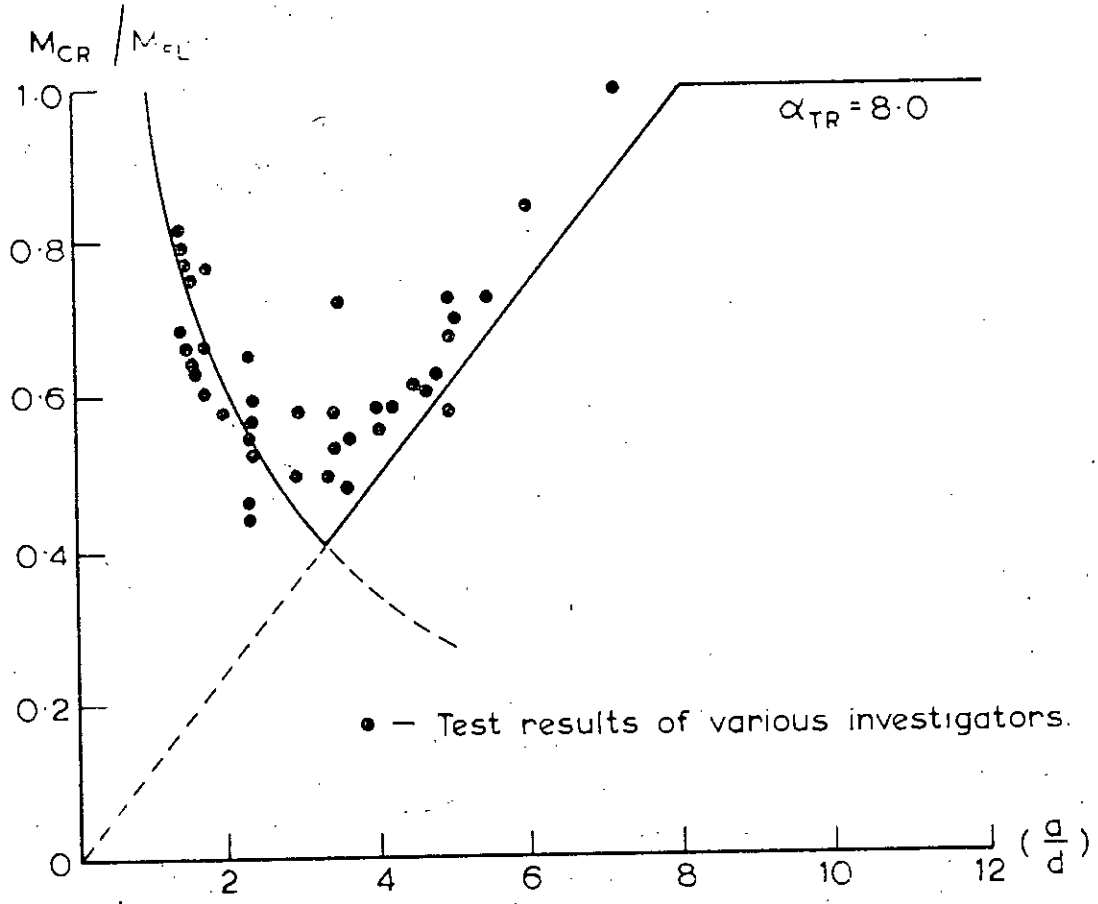
FIG. II. 29.

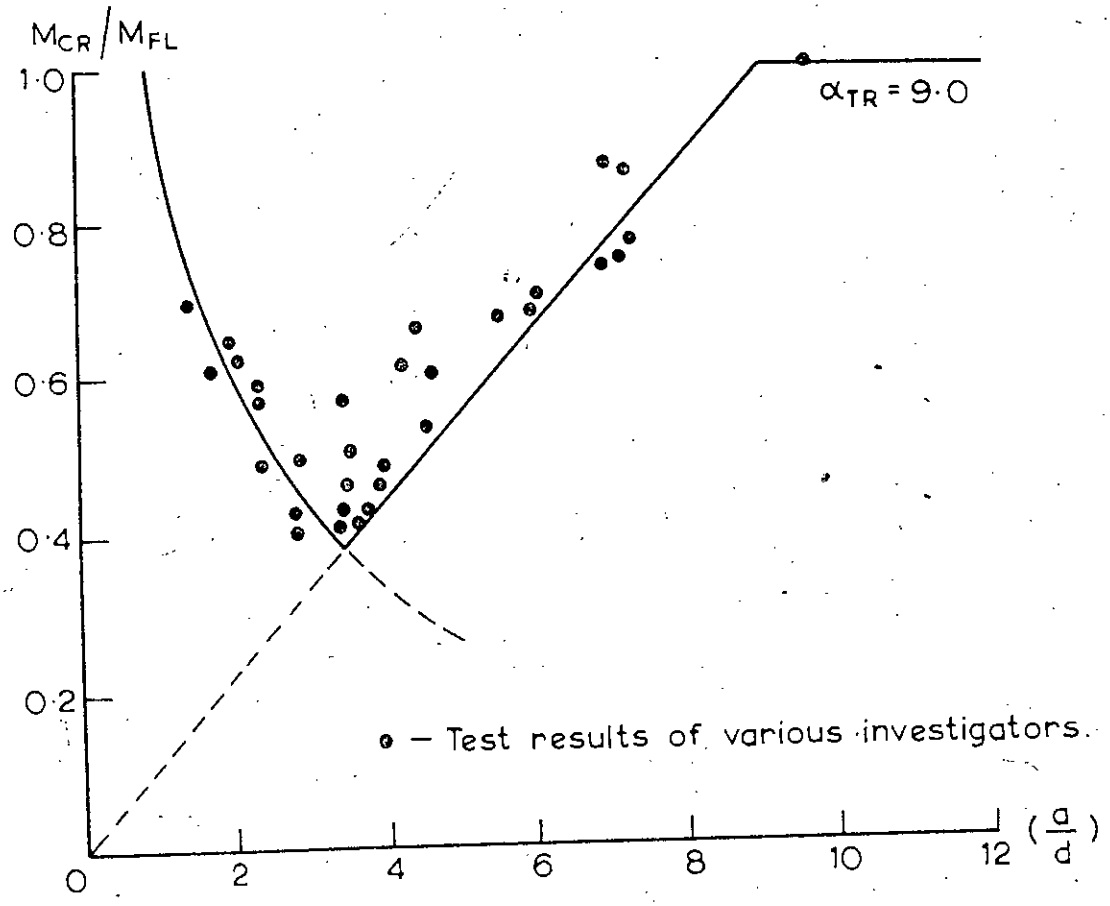




COMPARISON OF AUTHOR'S THEORITICAL LINES WITH THE TEST RESULTS OF VARIOUS INVESTIGATORS.

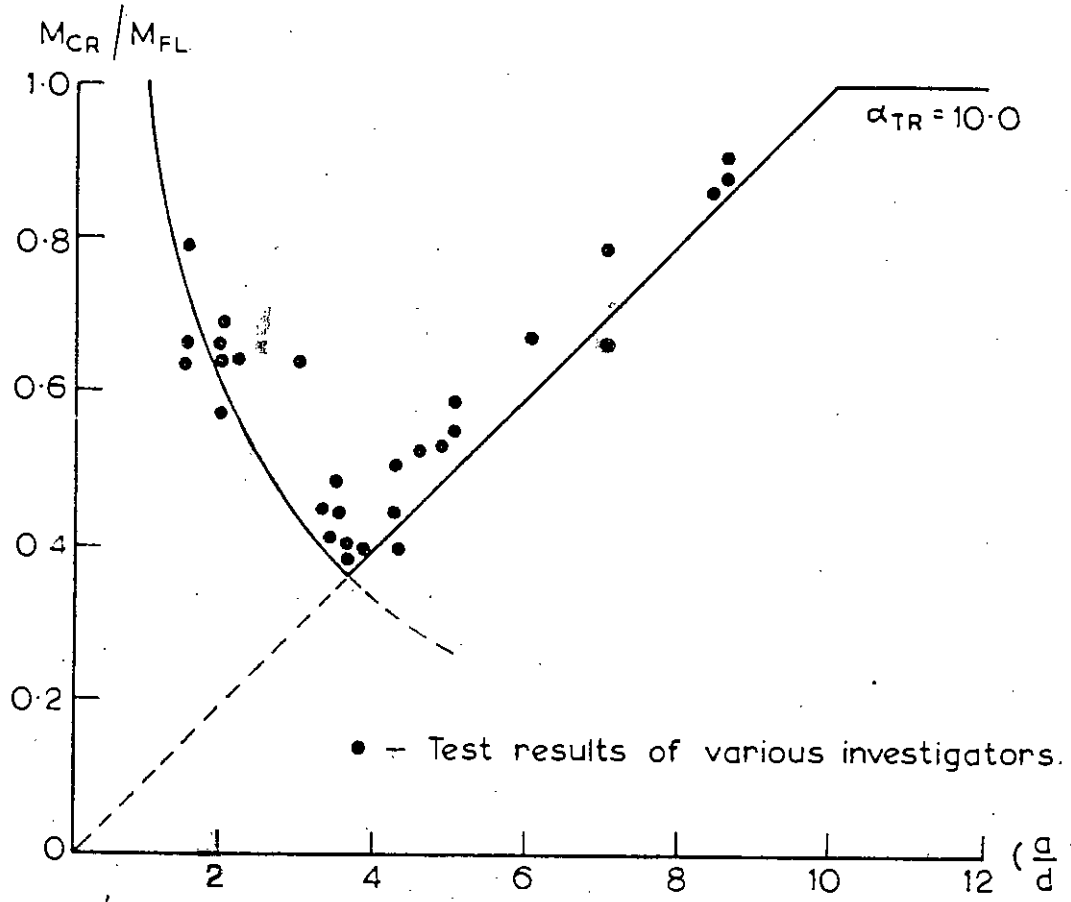
FIG. II.30.

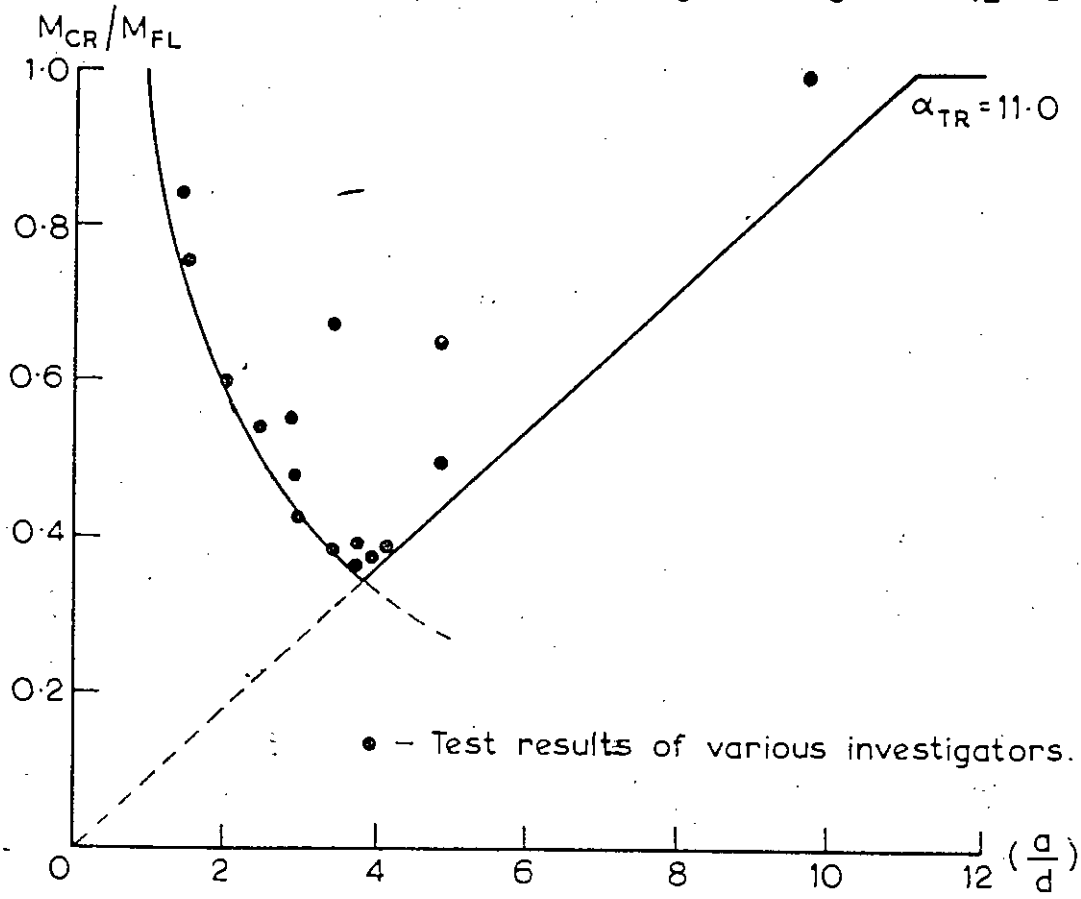




COMPARISON OF AUTHOR'S THEORITICAL LINES WITH THE TEST RESULTS OF VARIOUS INVESTIGATORS.

FIG. II. 31.





COMPARISON OF AUTHOR'S THEORITICAL LINES WITH THE TEST RESULTS OF VARIOUS INVESTIGATORS.

FIG. II. 32.

In addition, as stated earlier, a minor programme of experimental work was conducted. The details of the beams have already been given in an earlier chapter. The experimental work consisted of twenty one rectangular beams and nine tee beams tested with different a/d ratios. A summary of test results is given in Table II.7.1. In Table II.7.2 the test results of the author are compared with the predictions of the theory.

The predictions of the theory and equations are compared with the test results of other investigators[80,84,85,87,91-96, 101-104, 108-110, 114-129] in Figs. II.29 to II.32. The details of comparison are voluminous and hence they are not given in this thesis; but they are being included in a forthcoming research report of the Civil Engineering Department, Indian Institute of Science. From the comparison it can be seen that the agreement between the computed and test results is good and the error made by the theory is normally on the conservative side in most of the cases. Also it is to be pointed out here that there is considerable variation even in the experimental results. This can be clearly seen in Figs. II.29 through II.32.

TABLE II.7.1

SUMMARY OF TEST RESULTS

Specimen Mark	Flange width (in)	Flange thickness (in)	Web thickness (in)	d (in)	p%	f_y (ksi)	f'_c (psi)	a/d	At Failure V (kips)	At Failure M_{CR} (in-kips)	Mode of failure
1	2	3	4	5	6	7	8	9	10	11	
SR 1	-	-	4	6.78	2.26	50.0	5000	3.54	6.00	144	D
2	-	-	4	6.78	2.26	50.0	5000	4.14	6.13	171	D
3	-	-	4	6.78	2.26	50.0	5000	1.33	16.25	146	D
4	-	-	4	6.78	2.26	50.0	5000	4.42	5.85	175	D
5	-	-	4	6.78	2.26	50.0	5000	3.00	8.00	136	D
6	-	-	4	6.78	2.26	50.0	5000	1.99	9.60	130	D
7	-	-	4	6.78	2.26	50.0	5000	2.95	6.40	128	F
8	-	-	4	6.75	2.18	47.5	3820	3.50	6.15	146	
9	-	-	4	6.75	2.18	47.5	3500	7.20	4.00	201	
10	-	-	4	6.75	2.18	47.5	3740	1.56	8.95	94	
11	-	-	4	6.75	2.18	47.5	3050	2.50	6.20	105	
12	-	-	4	6.75	2.18	47.5	4000	3.00	6.95	141	
13	-	-	4	6.75	2.18	47.5	3500	6.00	4.50	182	
14	-	-	4	6.75	2.18	47.5	3050	5.00	5.38	181	
15	-	-	4	6.75	2.18	47.5	3300	5.00	5.13	174	

(continued)

* DT - Diagonal Tension
DC - Diagonal Compression
F - Flexure
A - Anchorage

Table II.7.1 (contd.)

	1	2	3	4	5	6	7	8	9	10	11	12
	16	-	-	4	6.75	2.18	47.5	3050	6.00	4.13	168	DT
	17	-	-	4	6.75	2.18	47.5	3300	3.50	7.40	175	B
	18	-	-	4	6.75	2.26	45.0	3500	4.00	5.68	153	DT
	19	-	-	4	6.75	2.26	45.0	3500	4.00	5.54	150	DT
	20	-	-	4	6.75	2.26	45.0	3500	4.00	3.47	94	A
	21	-	-	4	6.75	2.26	45.0	3500	4.00	3.08	83	A
ST	22	10	1.5	4	5.75	2.56	50.0	4500	2.78	4.88	78	DC
	23	10	1.5	4	5.75	2.56	50.0	4500	2.57	5.85	86	DC
	24	10	1.5	4	5.75	2.56	50.0	4500	2.04	8.46	100	DC
	25	10	1.5	4	5.75	2.56	50.0	4500	1.78	9.86	101	DC
	26	10	1.5	4	5.75	2.56	50.0	4500	2.28	6.85	90	DC
	27	10	1.5	4	5.75	2.56	50.0	4500	0.96	26.80	147	DC
	28	10	1.5	4	5.75	2.56	50.0	4500	1.48	14.50	123	DC
	29	10	1.5	4	5.75	2.56	50.0	4500	4.52	4.96	129	DT
	30	10	1.5	4	5.75	2.56	50.0	4500	4.00	4.92	113	DT

DT - Diagonal Tension F - Flexure A - Anchorage

DC - Diagonal Compression B - Bond

TABLE II.7.2

COMPARISON OF THEORETICAL AND TEST RESULTS

Specimen Mark		a/d	α_{TR}	α_m	M_{FL} (in-kips)	M_{CR}/M_{FL} Test	M_{CR}/M_{FL} Calc.	Ratio: Test/Calc.	Remarks
1		2	3	4	5	6	7	8	9
SR	1	3.54	4.75	2.43	200	0.72	0.74	0.97	
	2	4.14	4.75	2.43	200	0.86	0.87	0.99	
	3	1.33	4.75	2.43	200	0.73	0.82	0.89	
	4	4.42	4.75	2.43	200	0.87	0.93	0.94	
	5	3.00	4.75	2.43	200	0.68	0.63	1.08	
	6	1.99	4.75	2.43	200	0.65	0.60	1.08	
	7	2.95	4.75	2.43	200	0.64	0.62	1.03	
	8	3.50	5.70	2.68	200	0.73	0.62	1.18	
	9	7.20	6.22	-	200	1.00	1.00	1.00	Flexure failure
	10	1.56	5.81	2.70	200	0.47	-	-	Anchorage failure
	11	2.50	7.12	3.02	200	0.53	0.50	1.06	
	12	3.00	5.45	2.60	200	0.70	0.55	1.27	
	13	6.00	6.22	2.80	200	0.91	0.96	0.95	
	14	5.00	7.12	3.02	200	0.90	0.71	1.27	
	15	5.00	6.60	2.89	200	0.87	0.76	1.14	

(continued)

Table II.7.2 (contd.)

1	2	3	4	5	6	7	8	9
16	6.00	7.12	3.02	200	0.84	0.84	1.00	
17	3.50	6.60	2.89	200	0.87	-	-	Bond failure
18	4.00	6.10	2.78	200	0.76	0.66	1.15	
19	4.00	6.10	2.78	200	0.75	0.66	1.14	
20	4.00	6.10	2.78	200	0.47	-	-	
21	4.00	6.10	2.78	200	0.42	-	-	
ST 22	2.78	6.00	2.76	150	0.52	0.47	1.11	
23	2.57	6.00	2.76	150	0.57	0.50	1.14	
24	2.04	6.00	2.76	150	0.67	0.60	1.12	
25	1.78	6.00	2.76	150	0.67	0.67	1.00	
26	2.28	6.00	2.76	150	0.60	0.55	1.09	
27	0.96	6.00	2.76	150	0.98	1.00	0.98	
28	1.48	6.00	2.76	150	0.82	0.76	1.08	
29	4.52	6.00	2.76	150	0.86	0.76	1.13	
30	4.00	6.00	2.76	150	0.75	0.67	1.12	

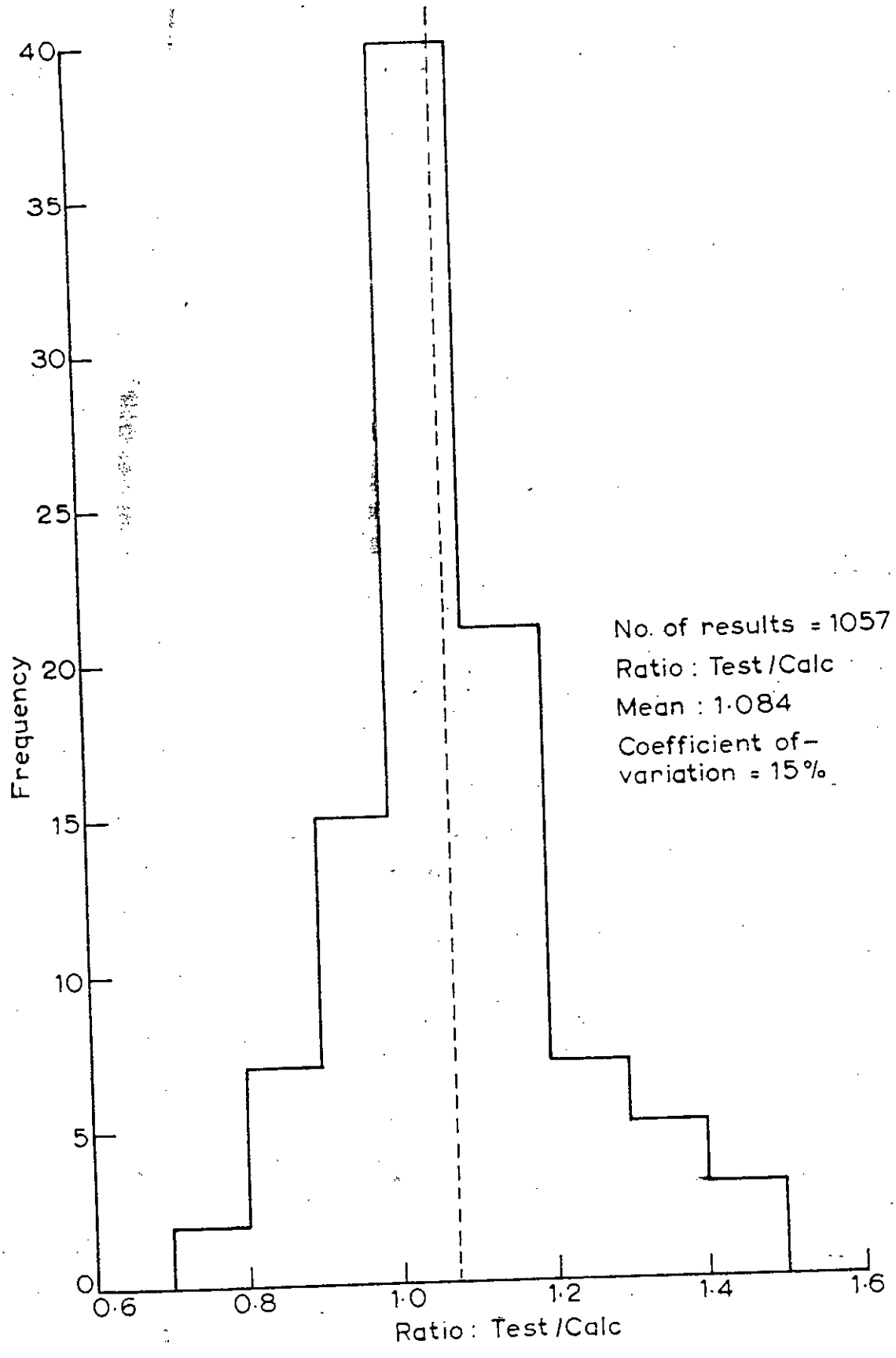
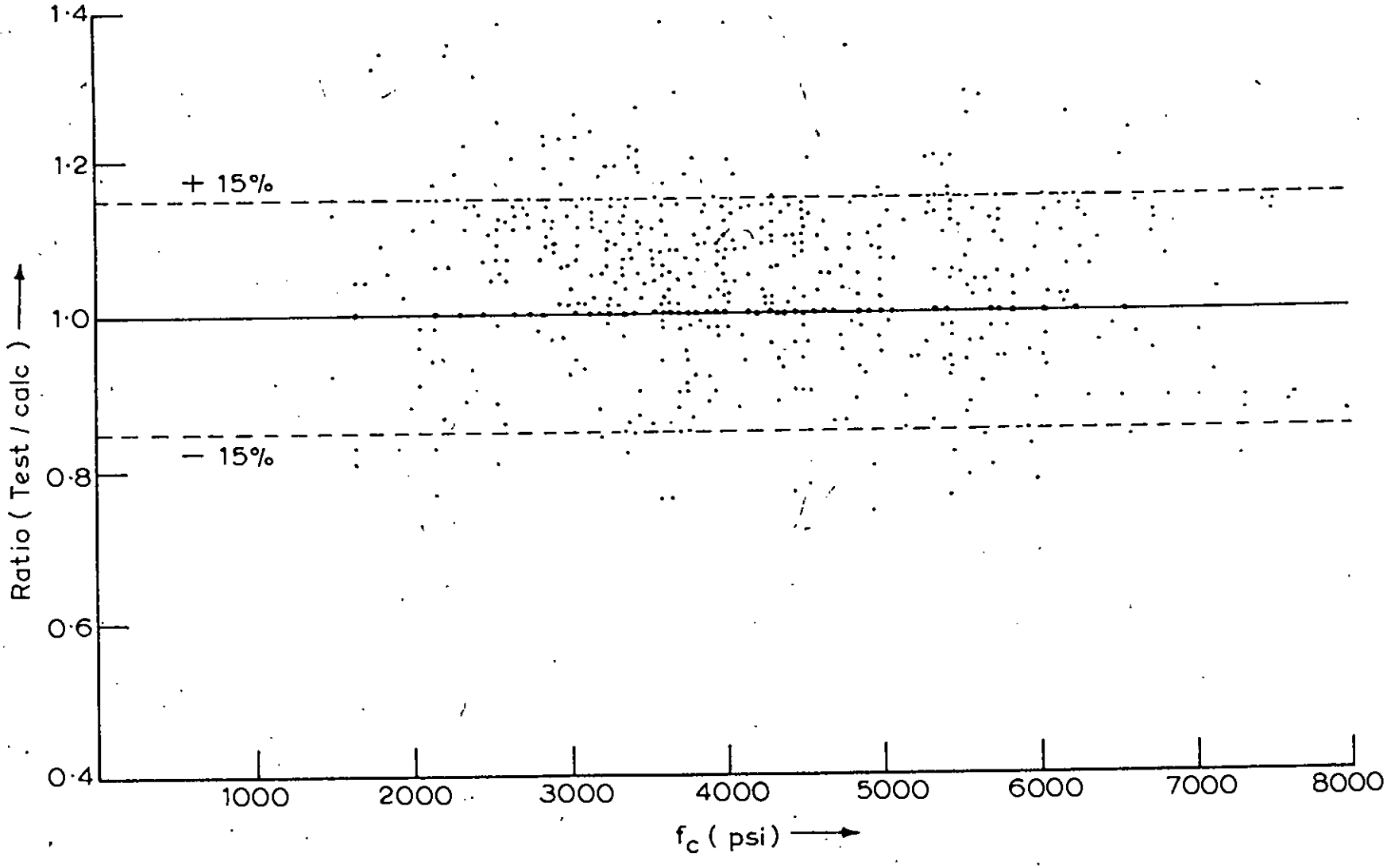


FIG. II. 33. SHEAR STRENGTH OF R.C. BEAMS : COMPARISON OF RESULTS : HISTOGRAM.

The results of comparison are summarised below and Fig. II.33 shows the histogram.

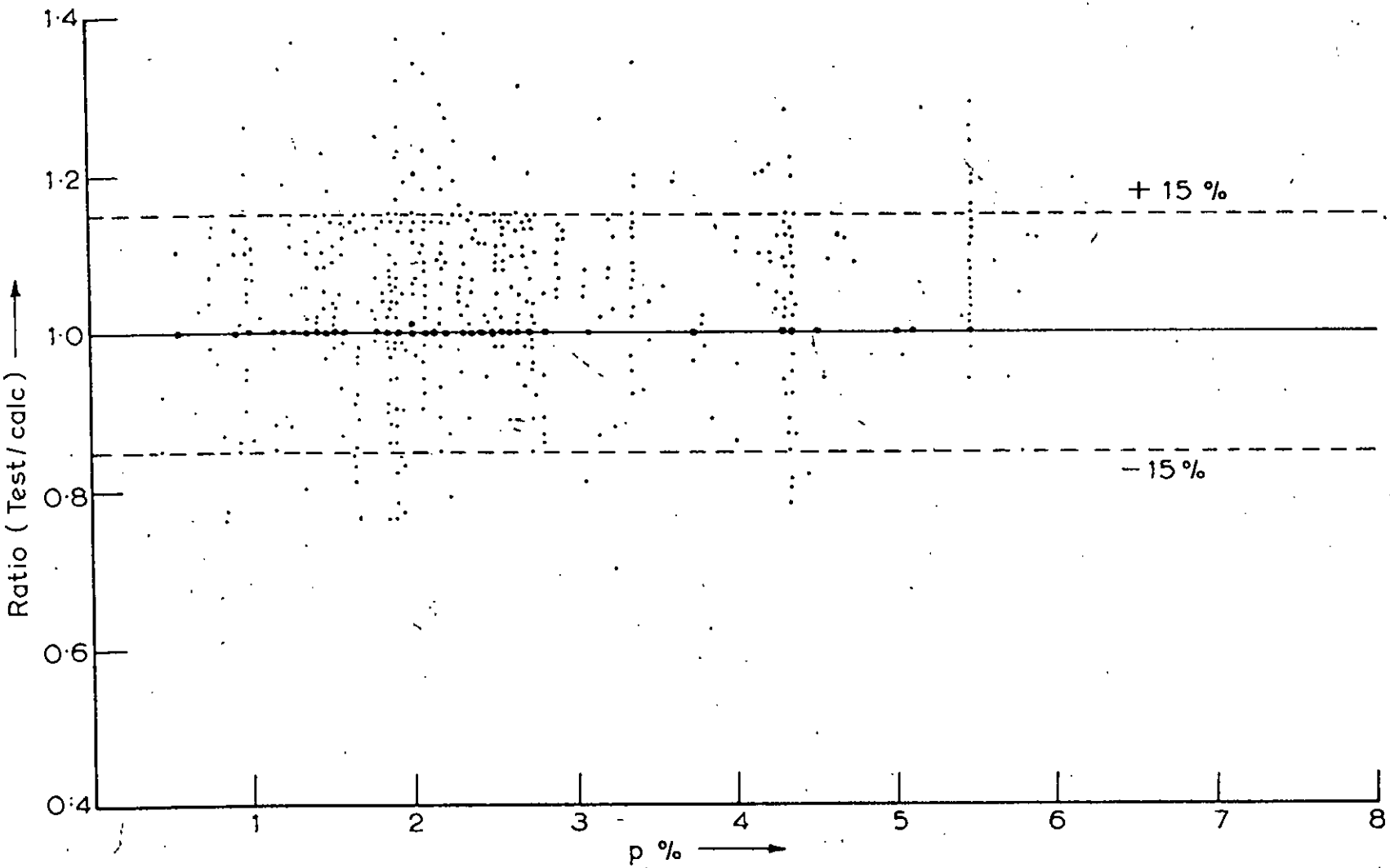
Type of Beams.	No. of test results	Ratio: (Test/Calc)	
		Mean value	Coefficient of variation
Without shear reinforcement			
Rectangular	702	1.065	15%
Tee	146	1.100	9%
With shear reinforcement			
Rectangular	175	1.130	15%
Tee	34	1.095	12%
Total	1057	1.084	15%

From the comparison, it can be seen that the results given by Eqn. II.6.15 are conservative in most of the cases. This can possibly be explained as follows: In evaluating the strength of the beam according to the first mechanism of failure corresponding to the Eqn. II.6.15, it has been said that the strength of the concrete element at the level of neutral axis can be taken as the strength of the beam. As already discussed, the beam is capable of resisting some more load; but this extra load carried by the beam is



ACCURACY OF THEORETICAL RESULTS WITH VARIATION IN STRENGTH OF CONCRETE.

FIG. II. 35.



ACCURACY OF THEORETICAL RESULTS WITH VARIATION IN AMOUNT OF LONGITUDINAL STEEL.

FIG. II. 34.

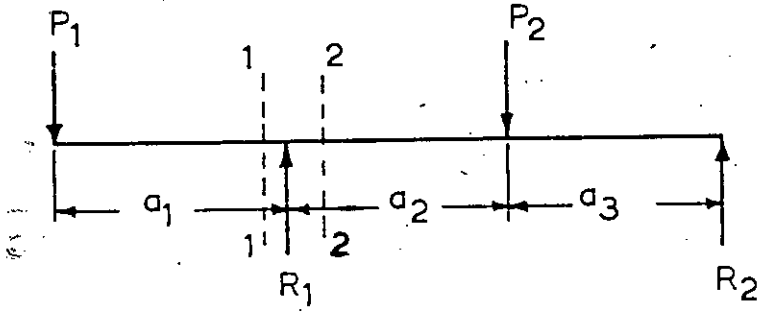
not reliable. This is evident from the sudden failure of beams in diagonal tension in many cases and as observed from experiments the cracking load and the ultimate load are more or less equal with a maximum difference of about 10% in case of beams tested with medium and high a/d ratios[80,91,104]. In addition, the vertical external loads applied on top surface of the beam may have some local effects in reducing the effect of shear stresses. This may further influence the concrete below the loading points to resist more shear stress. But once again the influence of these local effects on the shear strength of the beam is not reliable as the exact influence of them on the concrete element at the level of neutral axis is not known. These reasons may possibly explain the conservative nature of the results given by Eqn. II.6.15.

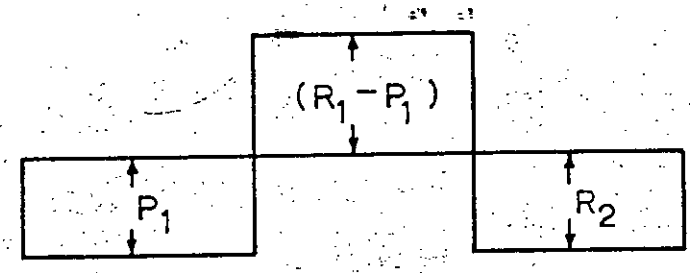
The general applicability of the proposed theory and equations is evident from Figs. II.34 and II.35. In these figures it can be seen that most of the results lie within $\pm$ 15% and the general band of variation remains almost constant for all the ranges of f'_c and p% thus evincing the general applicability of the derived equations for all the ranges of f'_c and p%. It is to be mentioned here that the theory and equations presented herein are not applicable for beams failing in bond and anchorage since these failures do not fall under the category of shear failures. These types of secondary failures may pose an entirely different problem

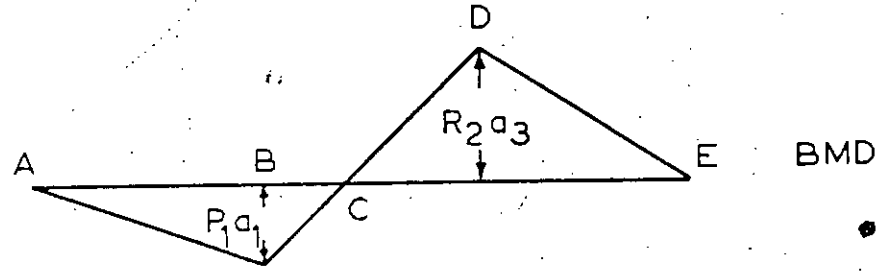
which is beyond the scope of this thesis. Also, the equations derived for beams with shear reinforcement are based on the assumption that proper bond is ensured between the stirrups and the concrete because poor anchorage of stirrups will have adverse effect on the strength of the beam as indicated by Kani[100].

II.7.7 Restrained Beams

So far, the applicability of the theory for simply-supported beams has been discussed. Thetheory and equations can be easily applied to the case of restrained beams also by observing the relation between bending moment and shear force as M/Vd instead of a/d. The restrained beams can be classified under two heads: (i) statically determinate (e.g. overhanging beams) and (ii) statically indeterminate (e.g. continuous beams). The analysis of statically determinate restrained beams is easy; but the analysis of statically indeterminate reinforced concrete beams poses difficulties due to the inelastic behaviour of concrete and non-availability of exact moment-curvature relationship for reinforced concrete beams. However, for symmetrical beams (i.e. with equal spans, uniform cross section and symmetrical loadings), experiments[132] have shown that the elastic analysis provides reasonably accurate results. Assuming the analysis of such beams are known, the method of calculation for the shear strength of such beams is discussed here.







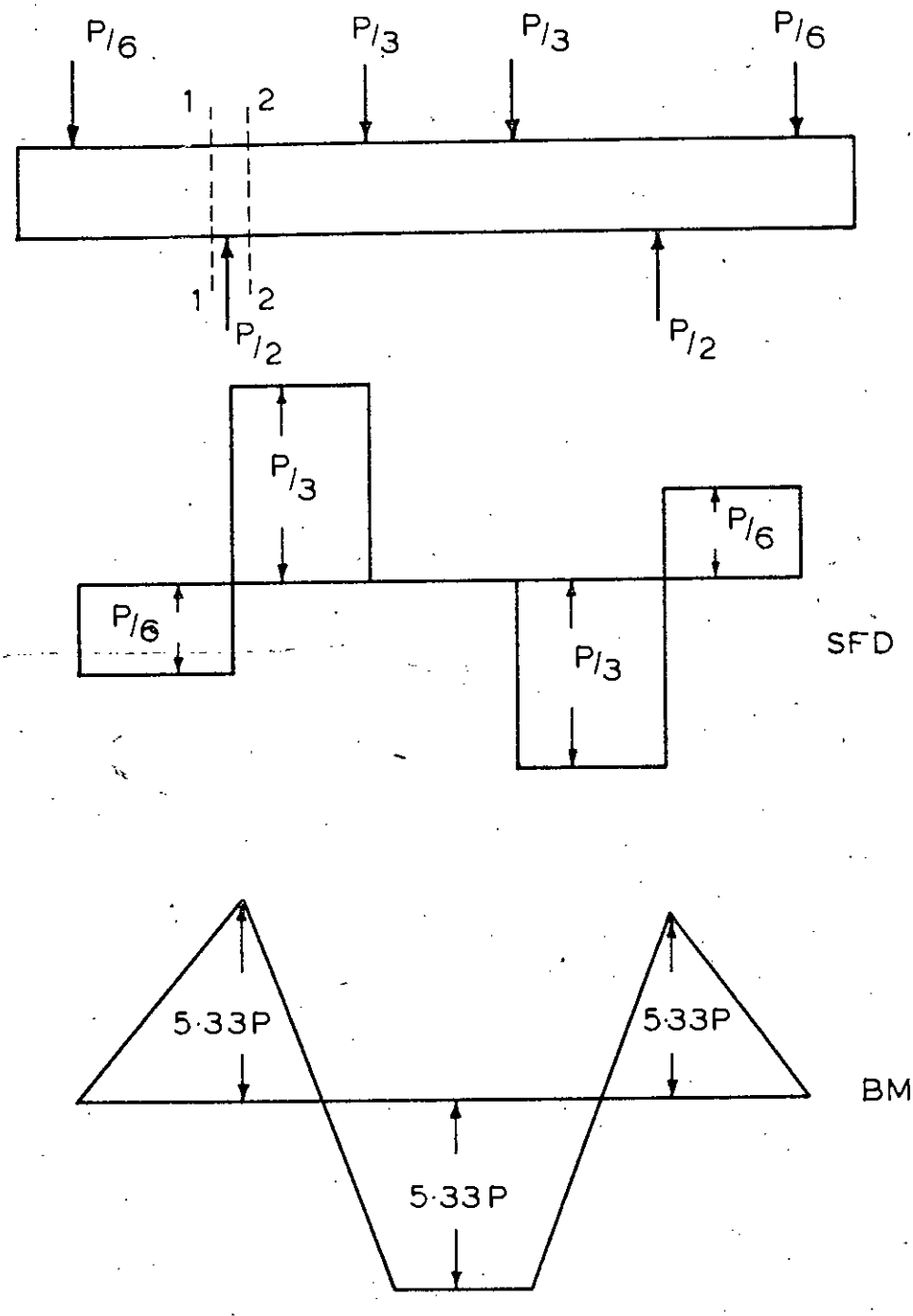
A RESTRAINED BEAM. [STATICALLY DETERMINATE]

FIG. II. 36.

Considering a statically determinate restrained beam with restraint at one of its supports (Fig. II.36), the variations of bending moment and shear force can be easily known as shown in Fig. II.36. Referring to Fig. II.36, the bending moment to shear force ratio M/Vd varies linearly starting from zero at A to a maximum of $\frac{a}{d}_1$ at B. In between the sections B and C the ratio decreases uniformly to zero at C. The ratio once again increases uniformly between C and D and decreases between D and E becoming zero at E. The ratio once again increases uniformly between C and D and decreases between D and E becoming zero at E. At the restrained support, there is a sudden jump in the magnitude of M/Vd ratio. That is, at section 1-1 just left of the restrained support the M/Vd ratio equals a_1/d and at section 2-2 just right of the restrained support $M/Vd = \frac{1}{d}\left(\frac{P_1 a_1}{R_1 - P_1}\right)$. Similar jump in the ratio M/Vd can be observed at the load point P_2 also. Considering the valley of shear failure discussed earlier, the deepest point in the valley was denoted by α_m at which the quantity M_{CR}/M_{FL} is minimum. The value of α_m is given by:

$$\left(\frac{M}{Vd}\right)_m = \alpha_m = \frac{(2/k)\,\alpha_{TR}}{\sqrt{\frac{4}{k}\alpha_{TR} + 2}} \qquad \ldots \text{ II.7.2}$$

where $\alpha_{TR} = 21\, p f_y / f'_c$.



LOADING ARRANGEMENT OF RESTRAINED BEAMS REPORTED BY MOODY ET. AL (REF No. 130)

FIG. II. 37.

Using the above relations, the value of α_m can be easily known. By knowing the value of α_m the weakest section for shear failure can be known by equating α_m to the variation of M/Vd over the span length of the beam. Here two cases may arise:

Case 1: It is possible in certain cases that the both values of M/Vd ratios on either side of a support in a restrained beam may be greater or lesser than α_m. Obviously, in this case the least value of M_{CR}/M_{FL} is obtained by taking the higher value of M/Vd ratio if M/Vd ratios are lesser than α_m or by taking the lower value of M/Vd ratio if M/Vd ratios are higher than α_m.

Case 2: If the value of α_m happens to be an intermediate value in between the jump in M/Vd ratios, then M_{CR}/M_{FL} is to be calculated using both the values of M/Vd ratio and the lesser of these values is to be taken as the shear strength of the beam.

The above discussed method of calculation is illustrated below by applying it to two test specimens reported by Moody et al[130]. The general loading arrangement of the beams is shown in Fig. II.37. The physical properties of the beams are summarised below:

Specimen mark	b x d in^2	p%	f_y ksi	f'_c psi
I-h	7 x 12	1.47	47.6	3540
I-i	7 x 12	2.10	47.5	3320

Beam I-h

$$\alpha_{TR} = \frac{21\, p f_y}{f'_c} = 4.15$$

$$\alpha_m = \frac{(2/k)\alpha_{TR}}{\sqrt{\frac{4}{k}\alpha_{TR} + 2}} = 2.26$$

At section 1-1, $\left(\frac{M}{Vd}\right) = \frac{5.33\ P}{(P/6)12} = 2.67$

At section 2-2, $\left(\frac{M}{Vd}\right) = \frac{5.33\ P}{(P/3)x12} = 1.33$

So in this case, the value of M/Vd jumps from 1.33 to 2.67 at the restrained support and the value of α_m lies in between the jump values. Hence the support section becomes the critical section for failure.

The value (M/Vd) at section 1-1 is greater than α_m and hence the magnitude of (M_{CR}/M_{FL}) can be calculated using Eqn. II.6.15 i.e.

$$\left(\frac{M_{CR}}{M_{FL}}\right) = \frac{\alpha}{\alpha_{TR}} \quad \text{where } \alpha = \left(\frac{M}{Vd}\right)$$

$$= \frac{2.67}{4.15} = \underline{0.645}.$$

Similarly (M/Vd) at section 2-2 is less than α_m and (M_{CR}/M_{FL})

can be obtained using Eqn. II.6.27.

$$(M_{CR}/M_{FL}) = \frac{2/k}{\alpha + \sqrt{\alpha^2 + 2}} \quad \text{where } \alpha = (M/Vd)$$

$$= \frac{(2/0.75)}{1.33 + \sqrt{1.33^2 + 2}} = \underline{0.82}$$

Now the actual (M_{CR}/M_{FL}) resisted by the beam will be least of the two values calculated above. Thus we obtain $\frac{M_{CR}}{M_{FL}}_{calc} = 0.645$. From the experimental result, $(M_{CR}/M_{FL}) = 0.78$ which is as expected a conservative result.

Beam I-i

$$\alpha_{TR} = \frac{21\ pf_y}{f'_c} = 6.32$$

$$\alpha_m = \frac{(2/k)\alpha_{TR}}{\sqrt{(\frac{4}{k}\alpha_{TR} + 2)}} = 2.82$$

The values of (M/Vd) at sections 1-1 and 2-2 remain the same as for Beam I-h. In this case both the values of (M/Vd) at sections 1-1 and 2-2 are less than α_m and hence both of them correspond to the arch mechanism of the beam. Obviously section 1-1 will have lesser strength than the other because the former represents a lower point in the valley of shear failure. Therefore,

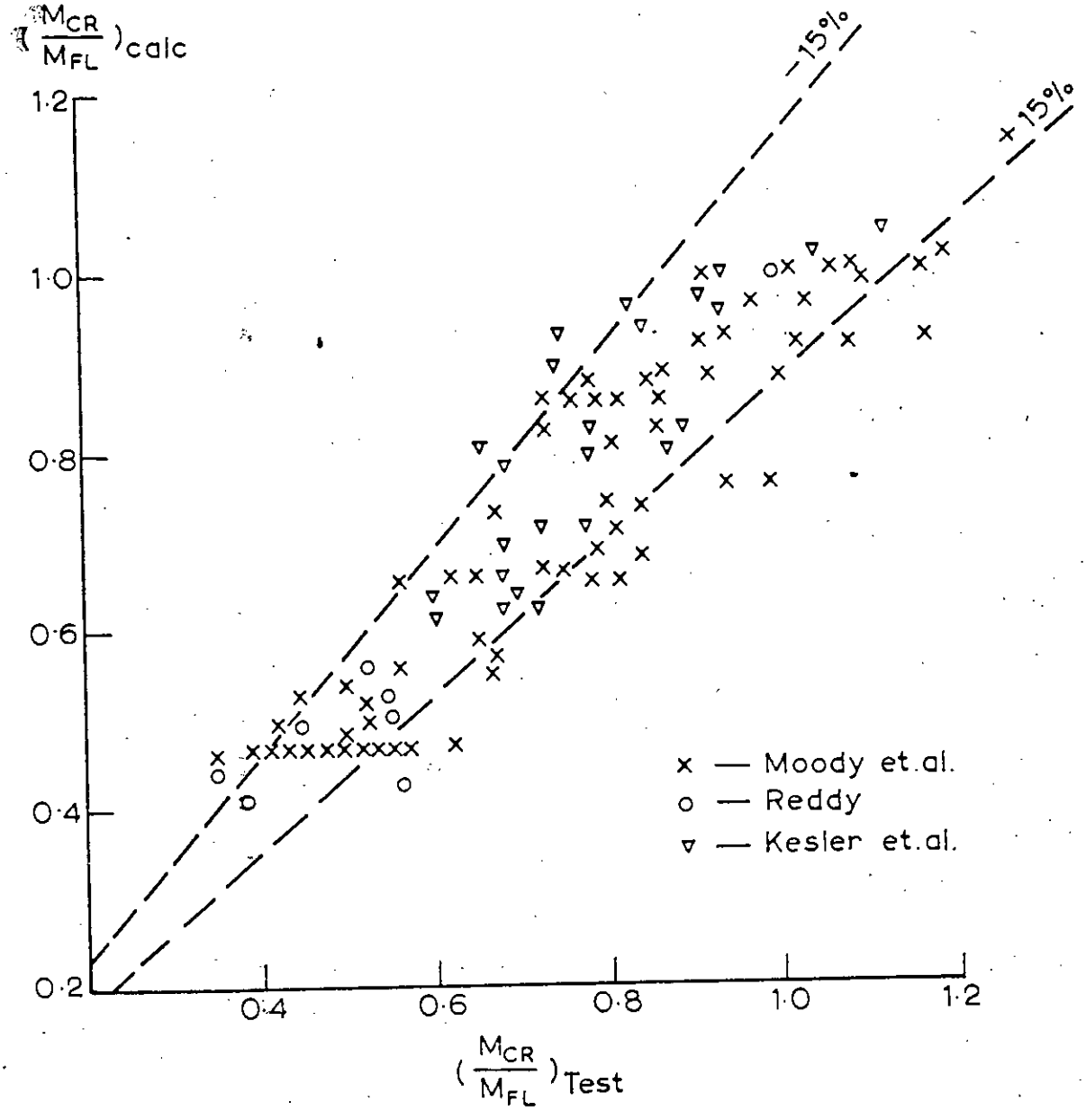


FIG. II.38. COMPARISON OF RESULTS : RESTRAINED BEAMS.

$$(M_{CR}/M_{FL})_{calc.} = \frac{(2/0.75)}{2.67 + \sqrt{2.67^2 + 2}} = \underline{0.47}$$

$$(M_{CR}/M_{FL})_{test} = \underline{0.47}.$$

The above two examples illustrate the procedure to be adopted in calculating the shear strength of restrained beams using the proposed theory and equations.

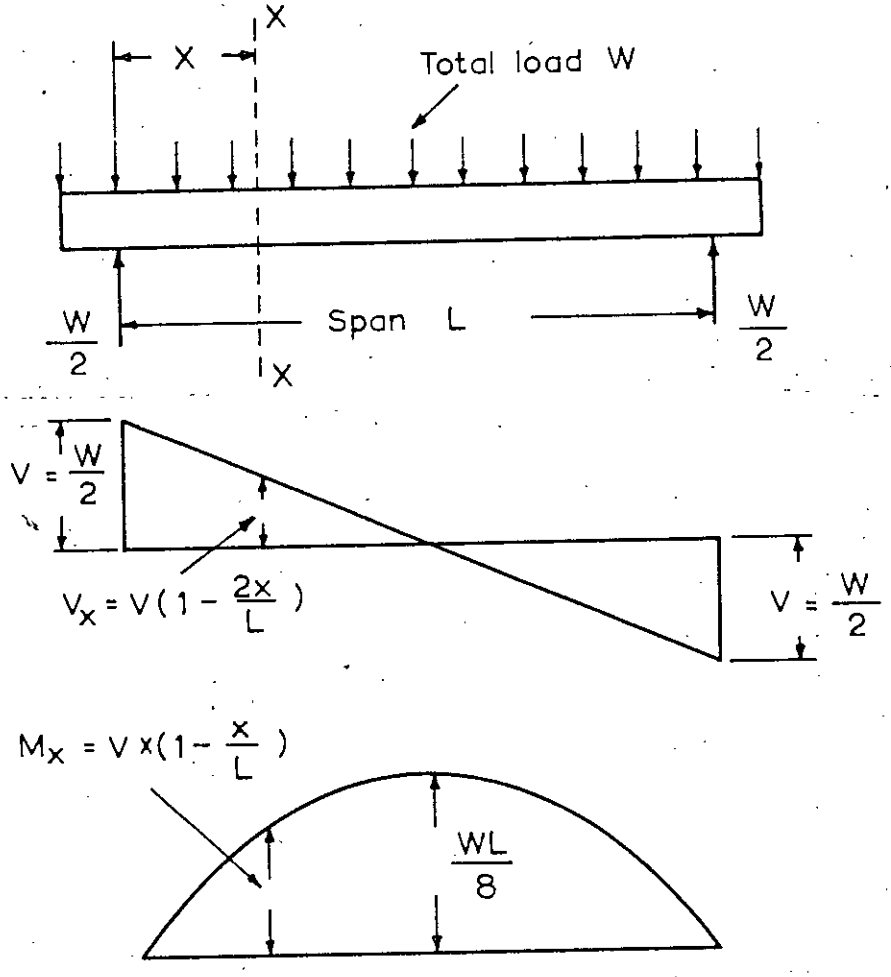
The procedure discussed above is also applicable to restrained beams which are statically indeterminate. The restrained beams with shear reinforcement can be analyzed on similar lines.

Using the method of calculation discussed above, the shear strength of restrained beams, both with and without shear reinforcement, reported in the literature[96,130-132] has been calculated. Fig. II.38 provides a comparison between computed and test results.

From the comparison, it can be concluded that the proposed equations and the method of calculation are fairly good in predicting the test results. The mean value of (Test/Calc) ratio is 1.060, the coefficient of variation being 12%.

II.7.8 Beams with uniformly distributed loads:

So far, we have discussed the calculation of shear



A SIMPLY SUPPORTED BEAM WITH UNIFORMLY DISTRIBUTED LOAD.

FIG. II. 39.

strength of beams with concentrated loads. In practice, we rarely come across structural members with only point loads and normally the members will be subjected to a combination of concentrated and distributed loads. So it is essential to derive methods for calculating the shear strength of beams with uniformly distributed loads using the proposed equations.

For a simply-supported beam with uniformly distributed load as shown in Fig. II.39, the variation of bending moment and shear force along the span length of the beam is well known. The shear force varies linearly from maximum over the support to zero at mid span whereas the bending moment varies parabolically from zero over support to a maximum at mid span. Also the M/Vd ratio varies from zero over support to infinity at mid-span of the beam. Considering a section XX at a distance of x from the support (Fig. II.39) we have,

$$M_x = \frac{W.x}{2}\left(1 - \frac{x}{L}\right) \qquad \ldots \text{II.7.3}$$

and $$V_x = \left(1 - \frac{2x}{L}\right)\frac{W}{2} \qquad \ldots \text{II.7.4}$$

where W is the total load on the beam.

From Eqns. II.7.3 and II.7.4 we get,

$$(M/Vd)_x = \frac{x}{d}\ \frac{(1 - x/L)}{(1 - 2x/L)} \qquad \ldots \text{II.7.5}$$

or designating $(M/Vd)_x$ as α_x and solving for x we get,

$$\frac{x}{L} = \frac{1}{(L/d)}\left[\alpha_x + \frac{L}{2d} - \sqrt{\alpha_x^2 + \left(\frac{L}{2d}\right)^2}\right] \qquad \ldots \text{II.7.6}$$

So using Eqn. II.7.6 the critical section can be known if the critical value of α_x is known.

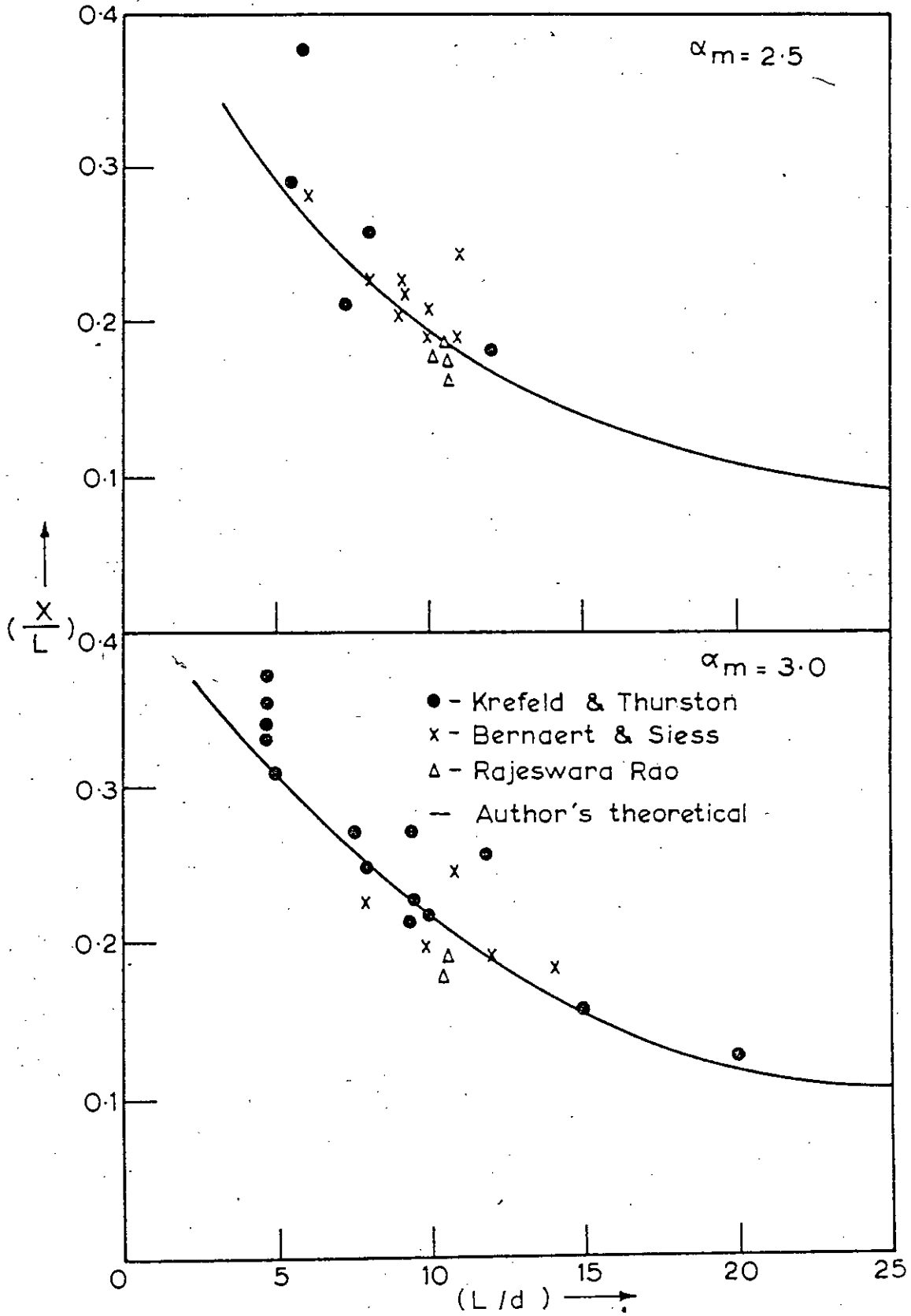
II.7.8.1 To evaluate the critical value of α_x

Since in a beam with distributed loads, the value of (M/Vd) ratio varies from zero to infinity, there will be a particular value of it which will become the critical value. In discussing the valley of shear failure (Fig. II.24), it has been indicated that the deepest point in the valley occurs at $(M/Vd) = \alpha_m$ and at this point the reduction in the flexural capacity of the beam is maximum. As observed from the experimental results available in the literature[103,91], it has been found that the basic mechanism of shear failure is independent of the type of loading. This point has been indicated by Kani also in one of his recent papers[108]. Hence the proposed theory and equations derived earlier are applicable for beams with distributed loads.

Thus accepting the valley of shear failure, derived for the case of beams with point-loaded beams, for the present case also, it can be readily said that the critical section for failure is that section at which (M/Vd) ratio is equal to α_m. So we get for the critical section,

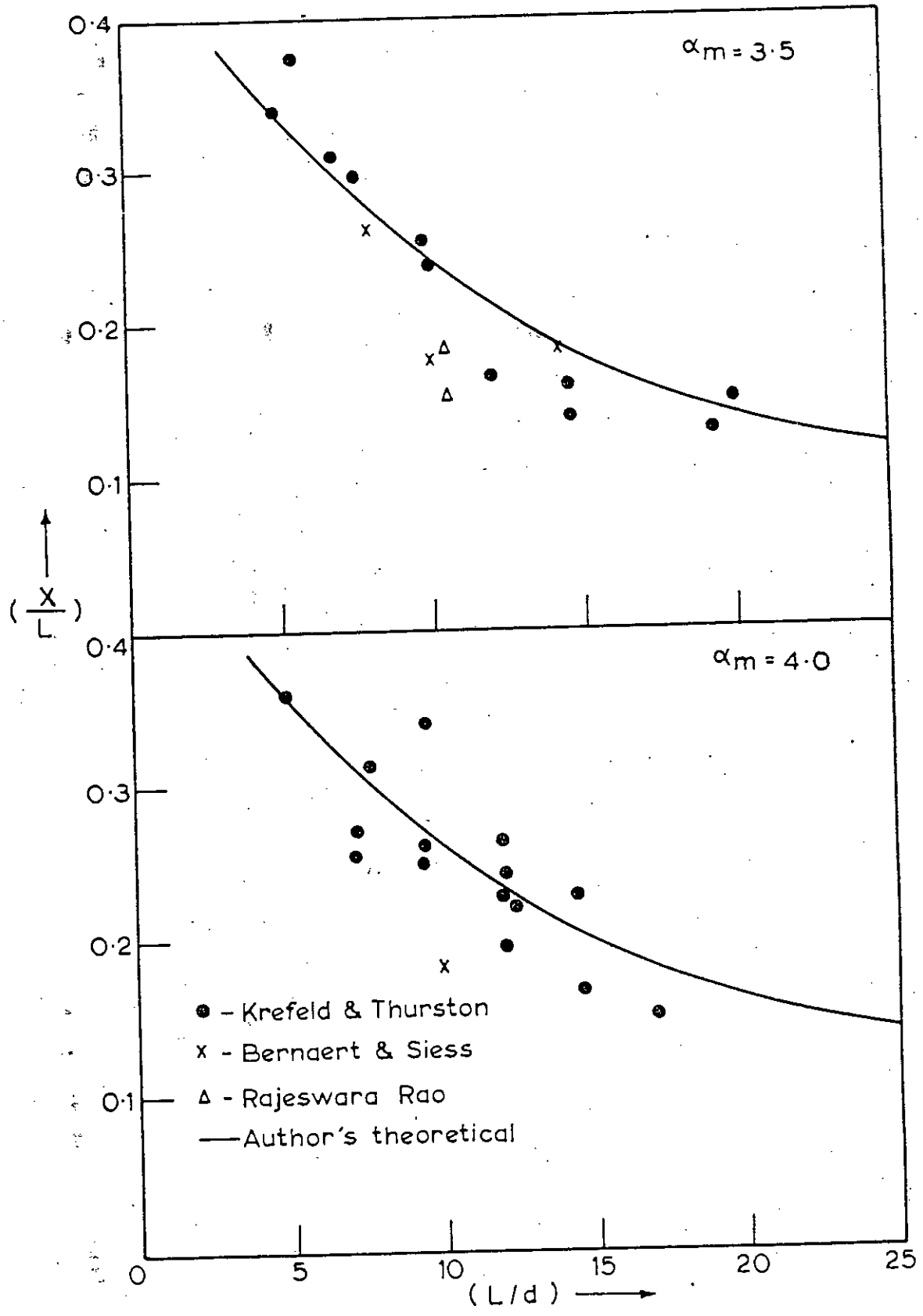
$$\alpha_x = \alpha_m \qquad \ldots \text{II.7.7}$$

and Eqn. II.7.6 can be put thus:



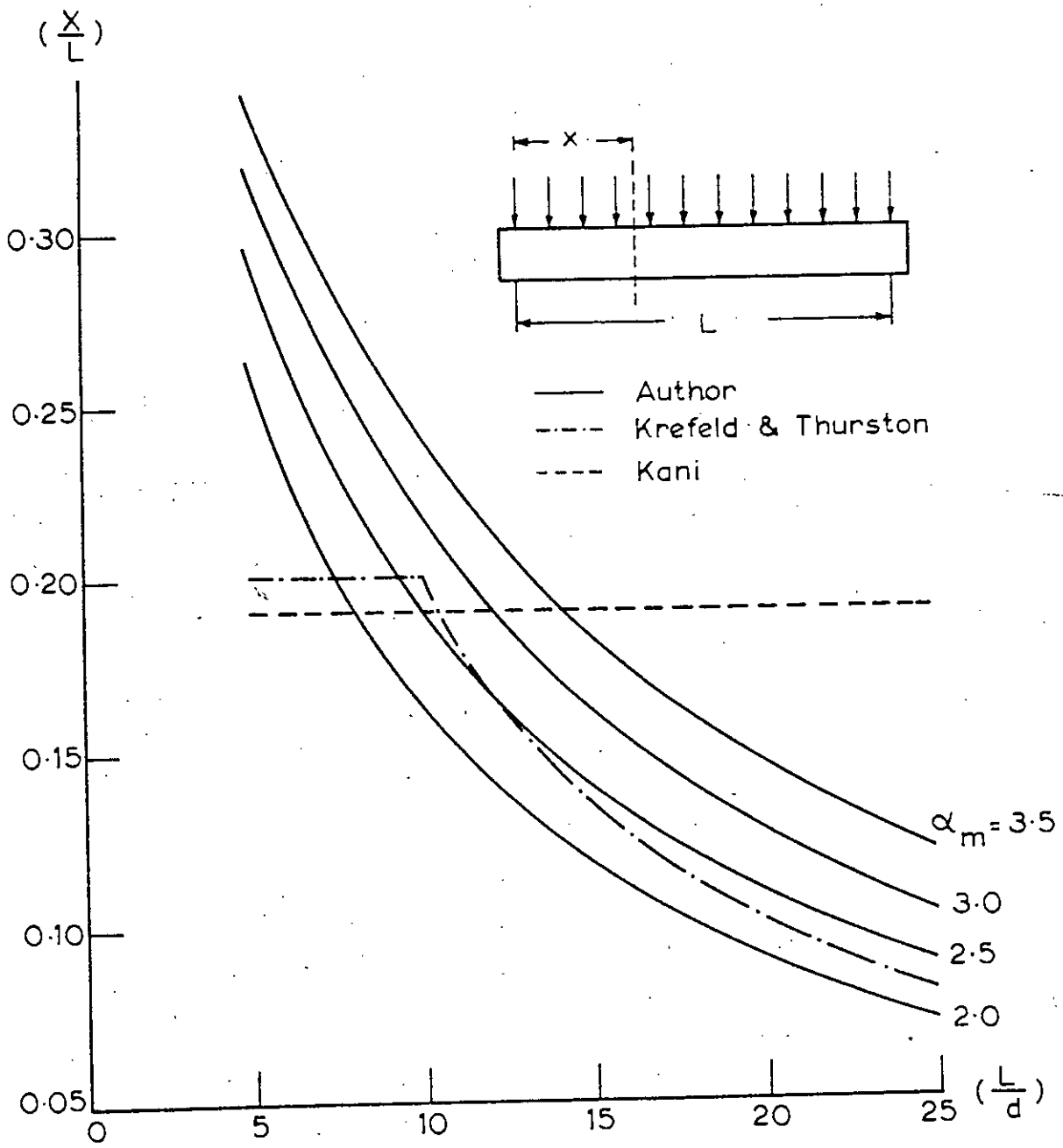
COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS OF CRITICAL SECTION FOR SHEAR FAILURE IN BEAMS WITH UNIFORMLY DISTRIBUTED LOADS.

FIG. II. 40



COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS OF CRITICAL SECTION FOR SHEAR FAILURE IN BEAMS WITH UNIFORMLY DISTRIBUTED LOADS.

FIG. II. 41.



VARIATION OF $\frac{X}{L}$ WITH $\frac{L}{D}$: COMPARISON OF RESULTS.

FIG. II. 42.

$$\frac{x}{L} = \frac{1}{(L/d)}\left[\alpha_m + \frac{L}{2d} - \sqrt{\alpha_m^2 + \left(\frac{L}{2d}\right)^2}\right] \quad \ldots \text{ II.7.8}$$

where $$\alpha_m = \frac{\left(\frac{2}{k}\right)\alpha_{TR}}{\sqrt{\frac{4}{k}\alpha_{TR} + 2}}$$

$$\alpha_{TR} = \frac{21 p f_y}{f'_c} \; ; \quad k = 0.75$$

It may be interesting to compare the test results given by various investigators[91,104,134,135] for the critical section with those given by Eqn. II.7.8. In Figs. II.40 & 41 the results are compared for four different values of α_m. From Figs. II.40 and II.41, it can be seen that the agreement between the results is fairly good.

In Fig. II.42, the results of the Eqn. II.7.8 are compared with those recommended by Krefeld and Thurston[104] and Kani[108]. Krefeld and Thurston suggested a constant value for $L/d \leq 10$ and a drooping curve for $L/d > 10$. As seen from Figs. II.40 and II.41, the value of x/L depends on α_m which in turn depends on the physical properties of the beam. It is to be pointed out here that the x/L values suggested by Krefeld and Thurston did not take into account the influence of physical properties of the beam on the position of critical section for failure. Moreover, the suggested constant value of x/L for $L/d \leq 10$ is not in

support with the test results as can be seen from Figs. II.40 and II.41.

Kani has defined the value of a for beams with distributed loads such that a = 0.25L or M/Vd = 0.25 L/d. Using this value of M/Vd, from Eqn. II.7.8 we can get x/L = 0.19. From the general trend of test results plotted in Figs. II.40 and 41, it is evident that Kani's definition of a has limited applicability.

II.7.8.2 Strength of beams with distributed loads:

Similar to the case of beams with concentrated loads, the strength of beams with distributed loads is to be evaluated considering two aspects. [illegible] beam is made stronger in shear, flexure failure will occur at the position of maximum bending moment. On the other hand if the beam is made stronger so as to avoid flexure failure, a shear failure will occur. As discussed in the previous article at the critical section for shear failure, $\alpha = \alpha_m$ and hence the shear strength of the beam is given by,

$$(M_{CR}/M_{FL})_{udl} = \frac{\alpha_m}{\alpha_{TR}} \qquad \ldots \text{ II.7.9}$$

It may be possible that at one instant both flexure and shear failures may occur simultaneously. This transition point can be obtained as follows:-

$$\text{Let } M_{FL} = W_f \frac{L}{8} \qquad \ldots \text{ II.7.10}$$

where W_f is the total load on the beam at flexure failure.

$$\text{Then } M_x = \frac{W_f x}{2} \left(1 - \frac{x}{L}\right) \qquad \ldots \text{ II.7.11}$$

Substituting the value of W_f from Eqn. II.7.10 in Eqn. II.7.11, we get,

$$M_x = M_{FL} \cdot \frac{4x}{L} \left(1 - \frac{x}{L}\right)$$

or

$$\frac{M_x}{M_{FL}} = \frac{4x}{L} \left(1 - \frac{x}{L}\right) \qquad \ldots \text{ II.7.12}$$

Since at the transition point both flexure and shear failure will occur simultaneously from Eqns. II.7.9. and II.7.12, we can put,

$$\frac{\mathcal{L}_m}{\mathcal{L}_{TR}} = 4 \frac{x}{L} \left(1 - \frac{x}{L}\right) \qquad \ldots \text{ II.7.13}$$

where x/L is given by the Eqn. II.7.8. But we have,

$$\mathcal{L}_m = (x/d) \frac{(1 - x/L)}{(1 - 2x/L)}$$

$$\text{or } x (1 - x/L) = \mathcal{L}_m (1 - 2x/L)d \qquad \ldots \text{ II.7.14}$$

Therefore,

$$\frac{\mathcal{L}_m}{\mathcal{L}_{TR}} = (4/L) \mathcal{L}_m (1 - 2x/L)d$$

$$\text{or } 2\frac{x}{L} = \left[1 - \frac{(L/d)}{4\,\alpha_{TR}}\right] \qquad \text{... II.7.15}$$

Substituting the value of x/L from Eqn. II.7.8

$$\left[\left(\alpha_m + \frac{L}{2d}\right) - \sqrt{\alpha_m^2 + \left(\frac{L}{2d}\right)^2}\right] = \left(1 - \frac{L/d}{4\,\alpha_{TR}}\right)\left(L/2d\right)$$

$$\text{or } \sqrt{\alpha_m^2 + (L/2d)^2} = \frac{(L/d)^2}{8\,\alpha_{TR}} + \alpha_m \qquad \text{... II.7.16}$$

squaring both sides and solving for L/d we get,

$$L/d = 4\alpha_{TR}\sqrt{1 - \left(\frac{\alpha_m}{\alpha_{TR}}\right)} \qquad \text{... II.7.17}$$

Denoting that particular value of L/d ratio which represents the transition point as $(L/d)_{TR}$ we ca Eqn.II.7.17.,

$$(L/d)_{TR} = 4\alpha_{TR}\sqrt{1 - \left(\frac{\alpha_m}{\alpha_{TR}}\right)} \qquad \text{... II.7.18}$$

So the strength of beams under distributed loads can be evaluated thus:

i. For L/d ratios less than or equal to $(L/d)_{TR}$ shear failure will occur and the critical section for shear failure can be calculated using Eqn. II.7.8 and the shear strength of the beam at the critical section is given by the Eqn. II.7.9.

ii. For L/d ratio greater than or equal to $(L/d)_{TR}$, only flexure failure is possible or the beam will develop full flexural capacity even if it fails in shear.

It is to be pointed out here that the expression for $(L/d)_{TR}$ given by Eqn. II.7.18 has been derived for the case of simply supported beam and similar expressions can be obtained for other types of beams adopting the same procedure outlined for the case of simply-supported beams.

II.7.8.3 Comparison of results:-

The method of calculation discussed in the preceding article is illustrated in the following examples:

Investigator: Bernaert and Siess[134]

Beam D12:- b x d = 6" x 9.94"

$p = 1.01\ \%$ L/d = 11.10

$f_y = 67.3$ ksi L = 110"

$f'_c = 4160$ psi W = 32.70 kips

$$\alpha_{TR} = 21\, p f_y / f'_c$$

$$= \frac{21 \times 0.0101 \times 67.3}{4.16} = \underline{3.42}.$$

$$\alpha_m = \frac{(2/0.75)\alpha_{TR}}{\sqrt{\frac{4}{0.75}\alpha_{TR} + 2}} = \frac{(\frac{2}{0.75})\ 3.42}{\sqrt{(\frac{4}{0.75} \times 3.42) + 2}}$$

$$= \underline{2.03}.$$

$$(L/d)_{TR} = 4\alpha_{TR}\sqrt{1 - \frac{\alpha_m}{\alpha_{TR}}}$$

$$= 4 \times 3.42\sqrt{1 - \frac{2.03}{3.42}} = \frac{8.80}{<11.10}$$

$$\therefore \quad L/d > (L/d)_{TR}$$

Therefore only flexure failure is possible.

$$\therefore \quad (M_{CR}/M_{FL})_{Calc} = 1.0$$

From experimental results, it has been found that this beam has failed in flexure. Hence $(M_{CR}/M_{FL})_{test} = 1.0$

Beam D13:-

$b \times d = 6" \times 9.94"$

$p = 1.01\ \%$ $\quad L/d = 11.10$

$f_y = 67.3$ ksi $\quad L = 110"$

$f'_c = 2780$ psi $\quad W = 25.5$ kips

$$\alpha_{TR} = 21\, pf_y/f'_c$$

$$= \frac{21 \times 0.0101 \times 67.3}{2.78} = \underline{5.13}.$$

$$\alpha_m = \underline{2.51}.$$

$$(L/d)_{TR} = 4 \times 5.13\sqrt{1 - \frac{2.51}{5.13}} = 14.65 > 11.10$$

$$\left(\frac{L}{d}\right) < \left(\frac{L}{d}\right)_{TR}$$

Therefore only shear failure will occur.

From Eqn. II.7.8,

$$\frac{x}{L} = 0.0865, \text{ or } x = \underline{9.5"}$$

$$M_x = Wx\,(1 - x/L)$$

$$= 25.5 \times 9.5\,(1 - 0.0865) = 221 \text{ inch kips}$$

M_{FL}(from test) = 450 inch kips

$$(M_{CR}/M_{FL})_{udl} = \frac{221}{450} = \underline{0.49} \text{ (test)}$$

From Eqn. II.7.9,

$$(M_{CR}/M_{FL})_{udl} = \frac{\alpha_m}{\alpha_{TR}} = \frac{2.51}{5.13} = \underline{0.49} \text{ (calc)}$$

Comparing the results of beams D12 and 13, it is interesting to note that both these beams had same physical properties except the strength of concrete. The decrease in the strength of concrete in beam D13 precipitated in a shear failure. Thus the influence of strength of concrete on shear strength of beams is obvious.

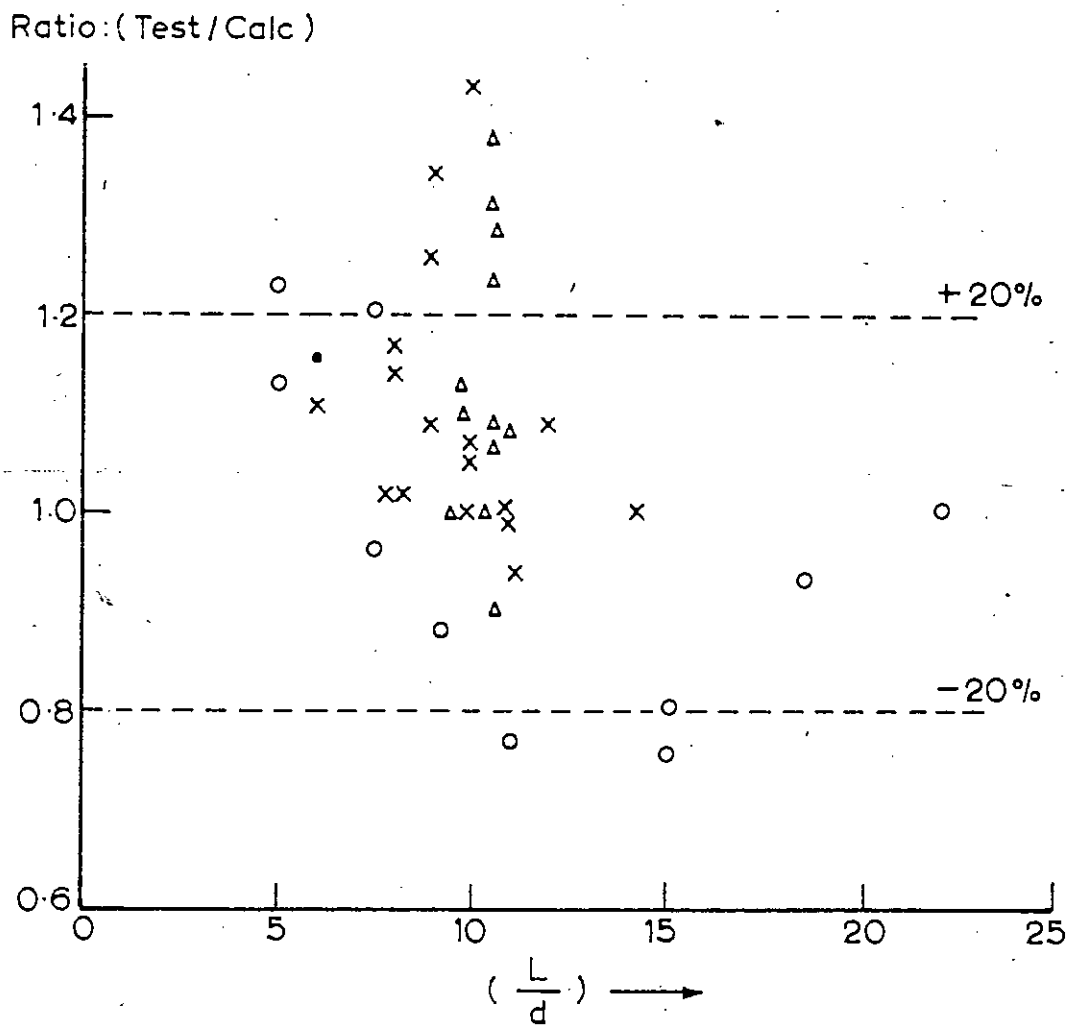


FIG. II. 43. CONTD.

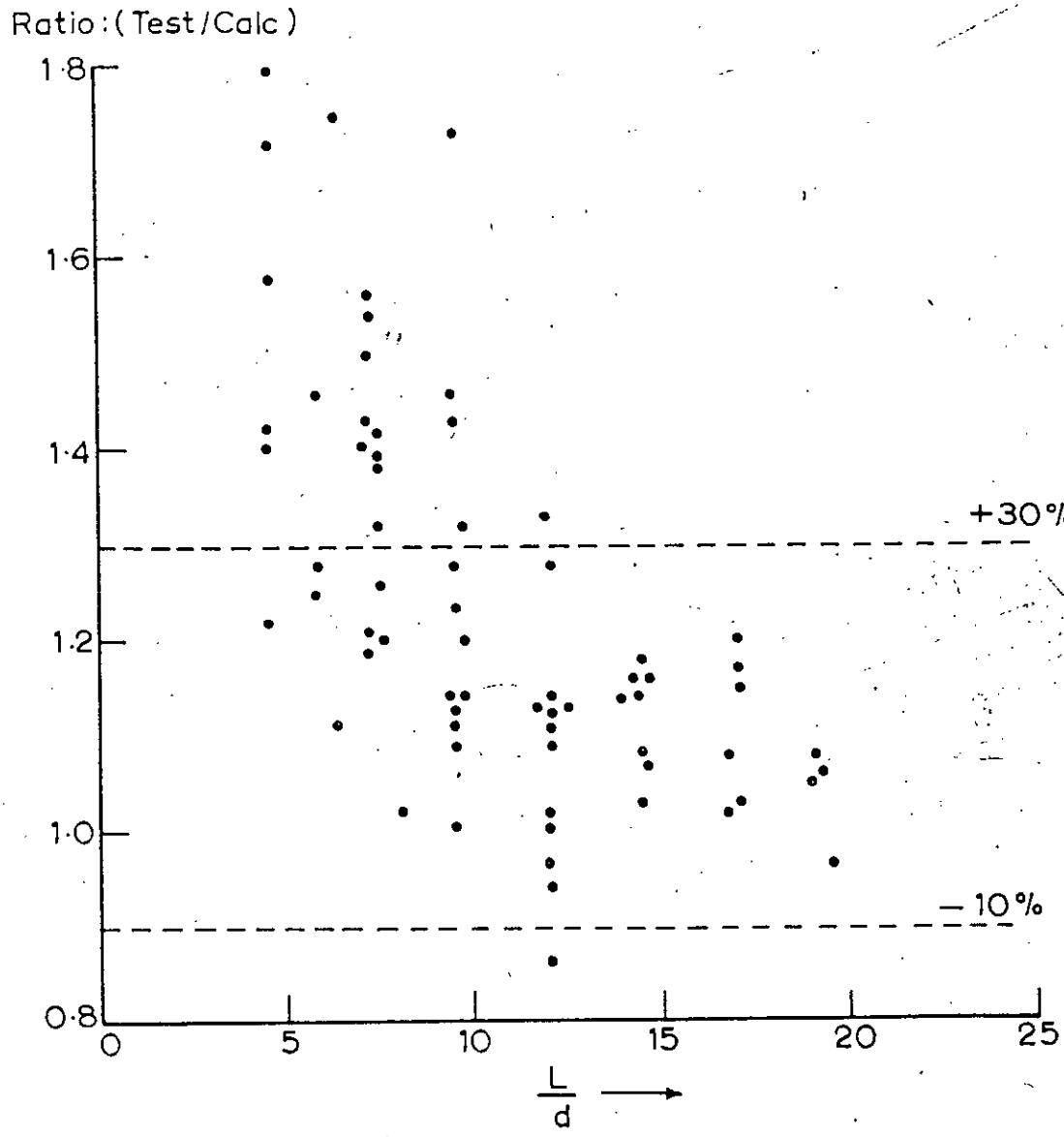


FIG. II. 43. COMPARISON OF RESULTS : BEAMS WITH DISTRIBUTED LOADS.

Using the method of calculation indicated above, the strength of number of beams with distributed loads reported in the literature[91,104,134,135] have been calculated. The results of comparison are shown in Fig. II.43. From the comparison, it can be seen that in some cases (the majority of them have been reported by Krefeld and Thurston[104]) the theoretical results seem to be over conservative. The reason for this discrepancy between the results in such cases can be explained as follows:

In the calculation of shear strength of beams with distributed loads, Eqn. II.7.9 has been used. The quantity α_m in Eqn. II.7.9 represents the deepest point in the valley of shear failure and it is the common point in the two mechanisms of failure discussed earlier for the case of beams with concentrated loads. In discussing the strength of beams with high and medium α values, the strength of the concrete element at the level of neutral axis has been taken as the strength of the beam ignoring the small extra load the beam may carry after the failure of this concrete element. But it is quite possible that in the case of beams with low L/d values under the action of distributed loads, the extra load resisted by the beam after the failure of the concrete element at the level of neutral axis may be considerable. In such cases the failure of the beam will occur when the entire compression zone fails and hence the strength of beams in such cases can be obtained by considering the strength of concrete elements above the position of neutral axis.

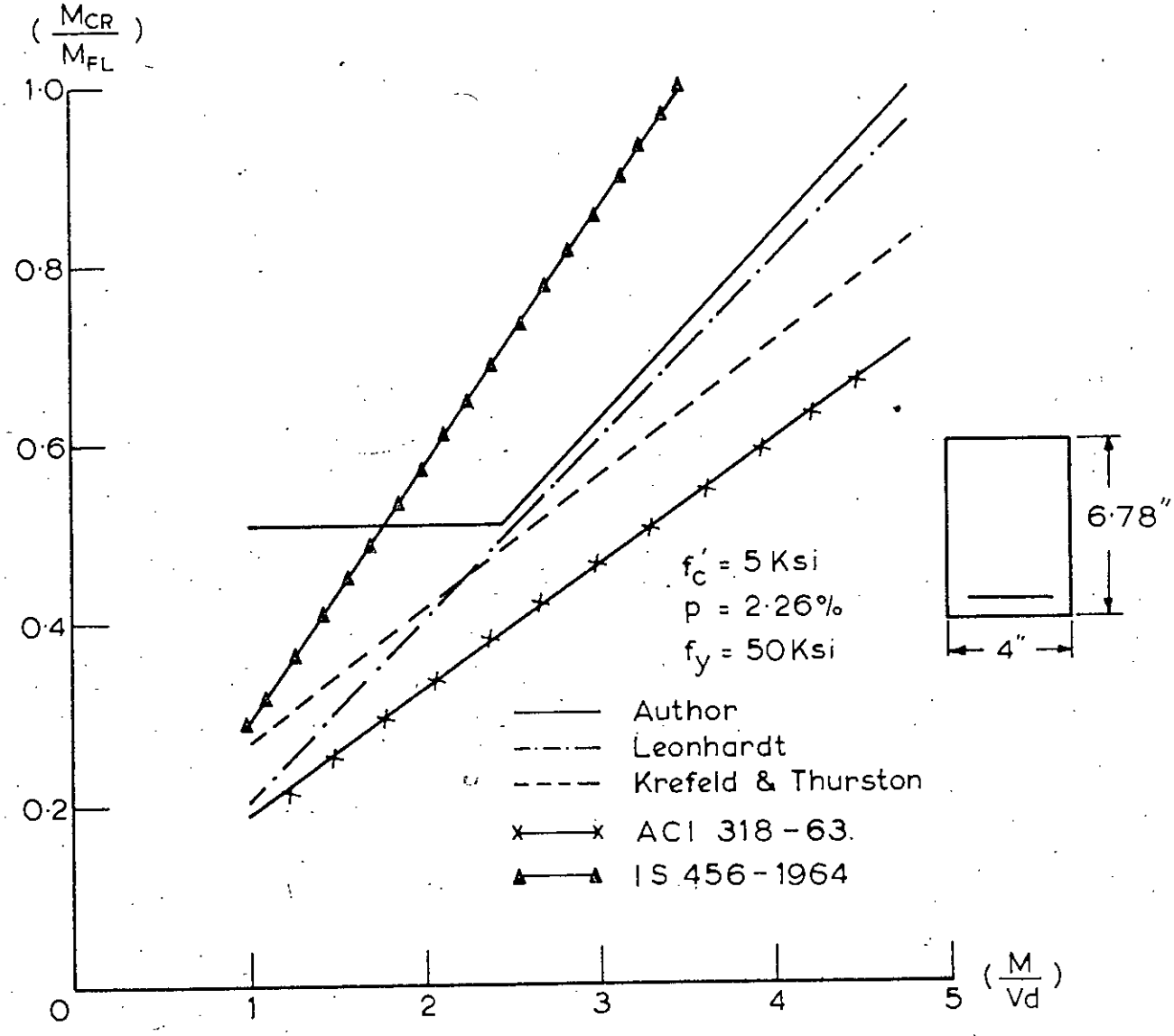
In a few other cases the beams had high percentage of longitudinal steel or low concrete strength or a combination of both. Perhaps these beams might have drawn the extra strength from some other source. A primary source in such cases is normally the poor bond between steel and concrete resulting in an unbonded beam which has been shown by experiments to possess significant amount of reserve strength[91,100].

II.7.9 Useful strength of beams in combined bending and shear:

So far, the proposed theory and the resulting equations have been critically examined in the light of the experimental results and it has been found that the theoretical predictions are generally in agreement with the test observations. In this article, the useful strength of a beam under combined bending and shear is briefly discussed.

The useful strength of a beam can be defined as that strength beyond which the beam looses its servicability due to large deformations, excessive cracking etc. So it may be necessary to restrict the strength given by the theoretical equations to a certain limit from practical view point.

The theoretical equations directly give the reduction in the flexural capacity of beams in combined bending and shear. It is well recognised nowadays that treating moment as a basis



COMPARISON OF RESULTS: USEFUL SHEAR STRENGTH OF BEAMS.

FIG. II. 44.

for the estimation of shear strength of beams is a more logical approach than the method of calculation based on the nominal shear stress. This point has been well emphasized by Kani[108] in one of his recent papers. According to the theory, maximum reduction in the flexural capacity of beams occurs at $M/Vd = \alpha_m$. On either side of α_m, the reduction in the flexural capacity of beams decreases uniformly and becomes zero at M/Vd equal to 1.0 and α_{TR}. Even though the beams with M/Vd ratios less than α_m have enough reserve strength such that their flexural capacity increases with the decrease in the M/Vd ratio, this increase in strength can be obtained only at the cost of servicability of the structure. So from practical view point it may be necessary to ignore this reserve strength of beams. Hence for all M/Vd ratios less than α_m, the reduction in the flexural capacity of the beams can be taken as the one obtained at M/Vd ratio equal to α_m. This may result in somewhat conservative results.

It may be interesting to compare the above suggested method of estimating the useful shear strength of beams with those suggested by other investigators[26,98,104,135]. In Fig. II.44 the results are presented in a graphical form. From Fig. II.44 it is evident that most of the published equations seem to be very conservative in the case of low M/Vd ratios (i.e. $M/Vd < 2.5$). The ACI Building Code Formula

gives somewhat over-conservative results even for the cases with M/Vd > 2.5. The tentative recommendation of Indian Code errs on the unsafe side in the higher ranges of M/Vd ratio. The equation suggested by Krefeld and Thurston gives better results than the ACI Building Code Formula while the line representing the equation given by Leonhardt comes closer to the author's theoretical line for M/Vd > 2.5.

II. 7.10 Factors influencing the shear strength of beams:

In this article the influence of certain factors on the shear strength of beams are discussed in the light of the analytical results. The important factors are:

1. properties of concrete
2. properties of longitudinal steel
3. amount and strength of shear reinforcement
4. M/Vd or a/d ratio
5. bond between concrete and steel and anchorage of reinforcements.

The importance of each factor is discussed in detail in the following pages:

II.7.1.0.1 Properties of Concrete:

As expected, a change in the concrete properties of the beam will influence the shear strength of the beam. An

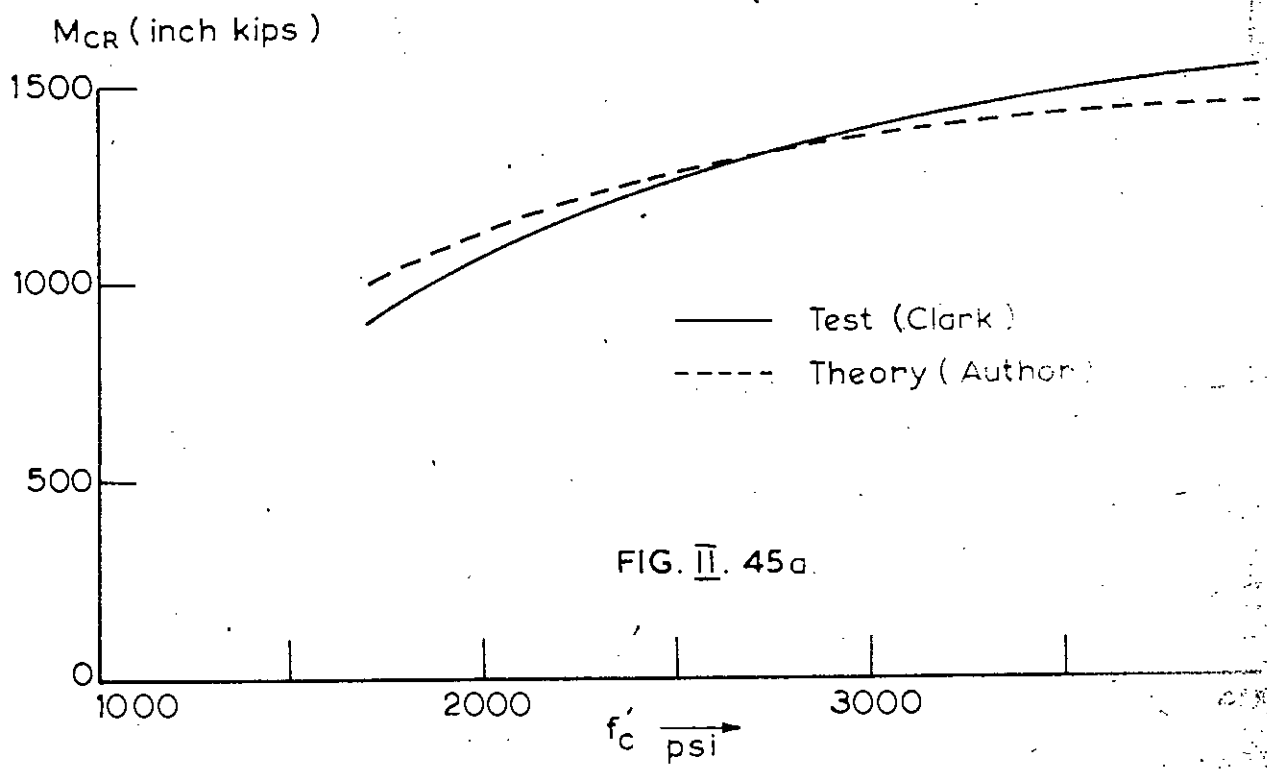
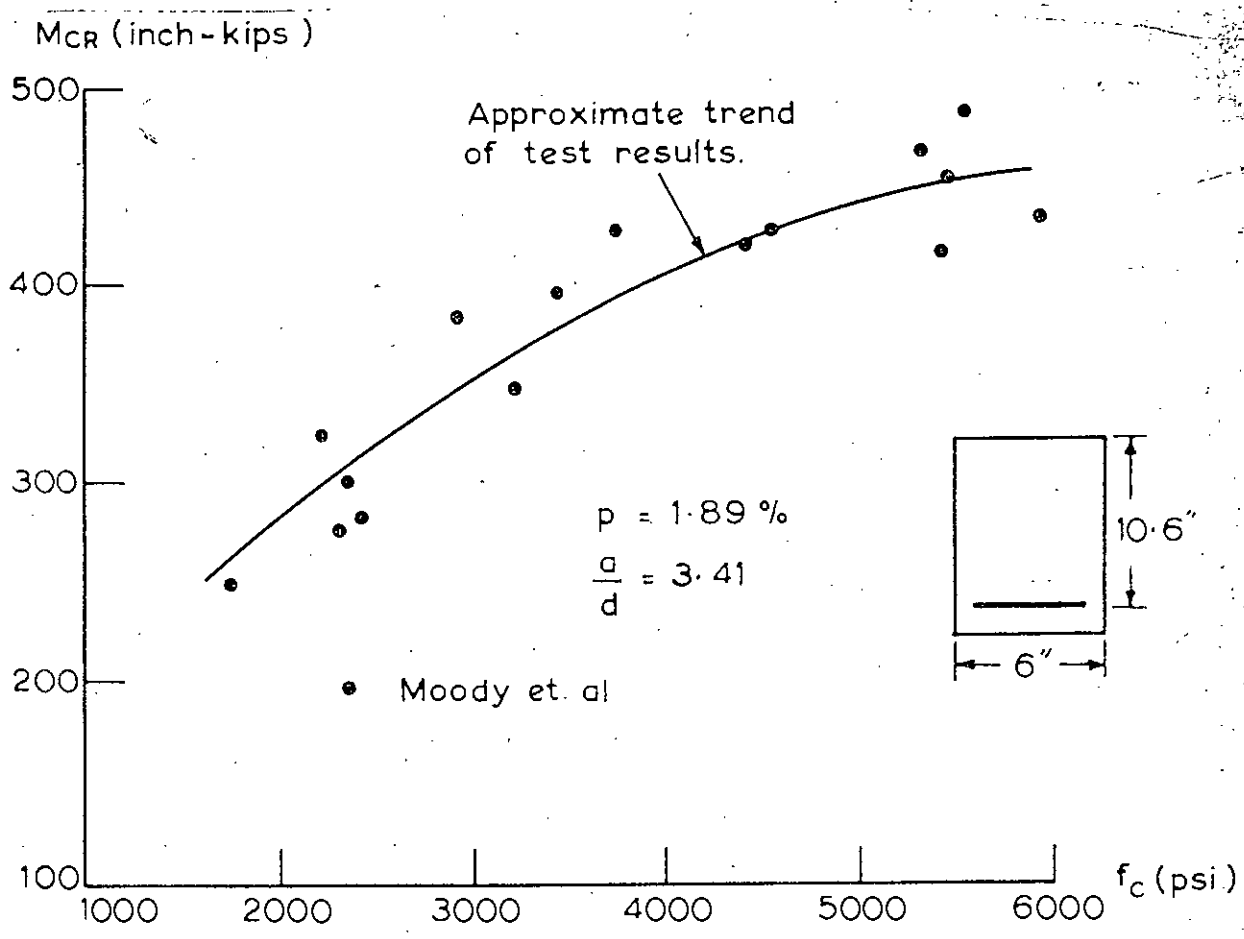


FIG. II. 45a.



INFLUENCE OF CONCRETE STRENGTH ON SHEAR STRENGTH OF BEAMS WITH LOW $\frac{a}{d}$ RATIO. & HIGH $\frac{a}{d}$ RATIO.

FIG. II. 45 b

increase in the compressive strength of concrete is accompanied by a corresponding increase in the tensile strength of concrete resulting in the increase in the shear strength of the beam. Fig. II.45a illustrates the influence of concrete strength on the shear strength of the beam with low a/d ratio. In the figure both theoretical variation as predicted by the theory and equations proposed herein and the variation obtained from tests are shown so as to give a comparative picture. From the figure it can be seen that the results fairly agree well with each other.

Similarly the theoretical and experimental variations of shear strength with concrete strength for beams with high a/d ratio is shown in Fig. II.45b. From Figs. II.45a and b, it is also seen that the influence of concrete strength on the shear strength of beams is significant within certain limits. The increase in the value of f'_c will also result in the increase value of 'modulus of elasticity' of concrete and in turn the stiffness of the beam will increase.

II.7.10.2 Longitudinal steel:

The experiments available in the literature have clearly revealed that the tension steel influences the shear strength of beams considerably. But the results available do not allow a direct comparison to study the influence of tension steel as there are variations in different parameters in a

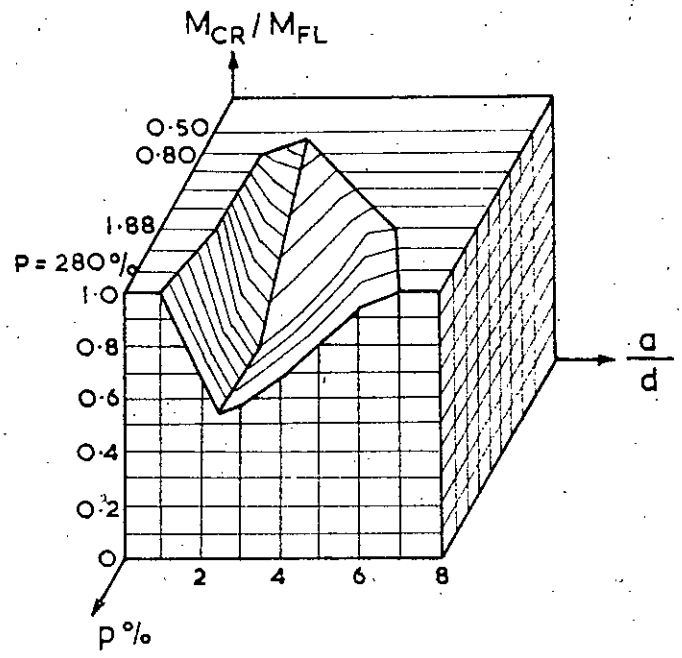


FIG. II. 46a. WEDGE OF SHEAR FAILURE AS OBTAINED FROM TEST RESULTS OF KANI. (REF :- 108)

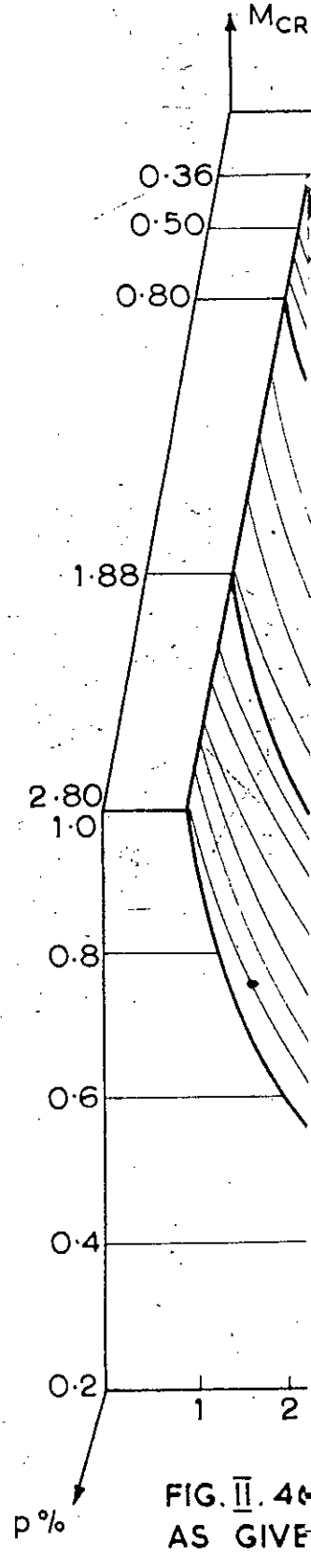
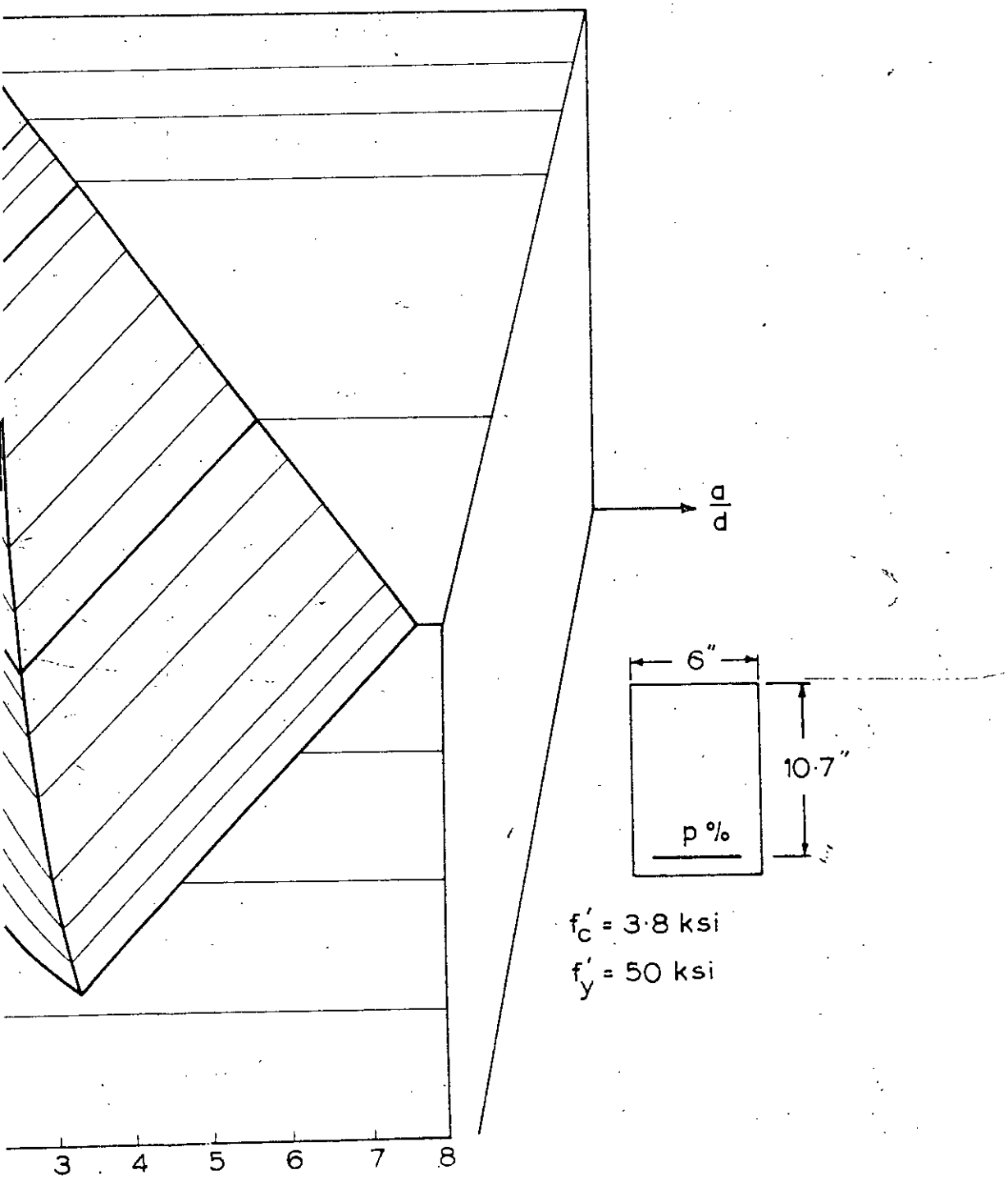
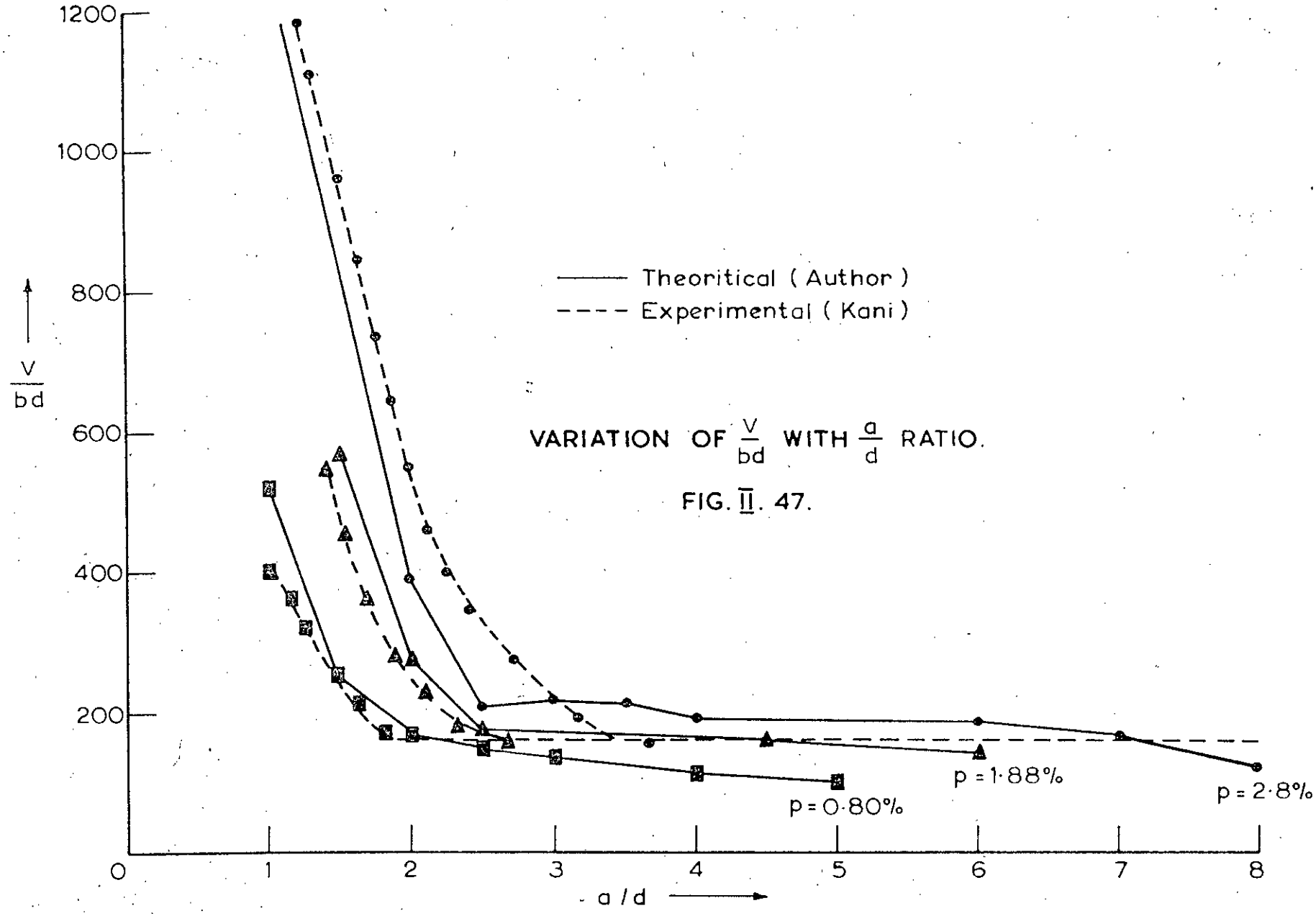


FIG. II. 46
AS GIVE

$/ M_{FL}$



b. 'WEDGE OF SHEAR FAILURE'
N BY AUTHOR'S THEORY.



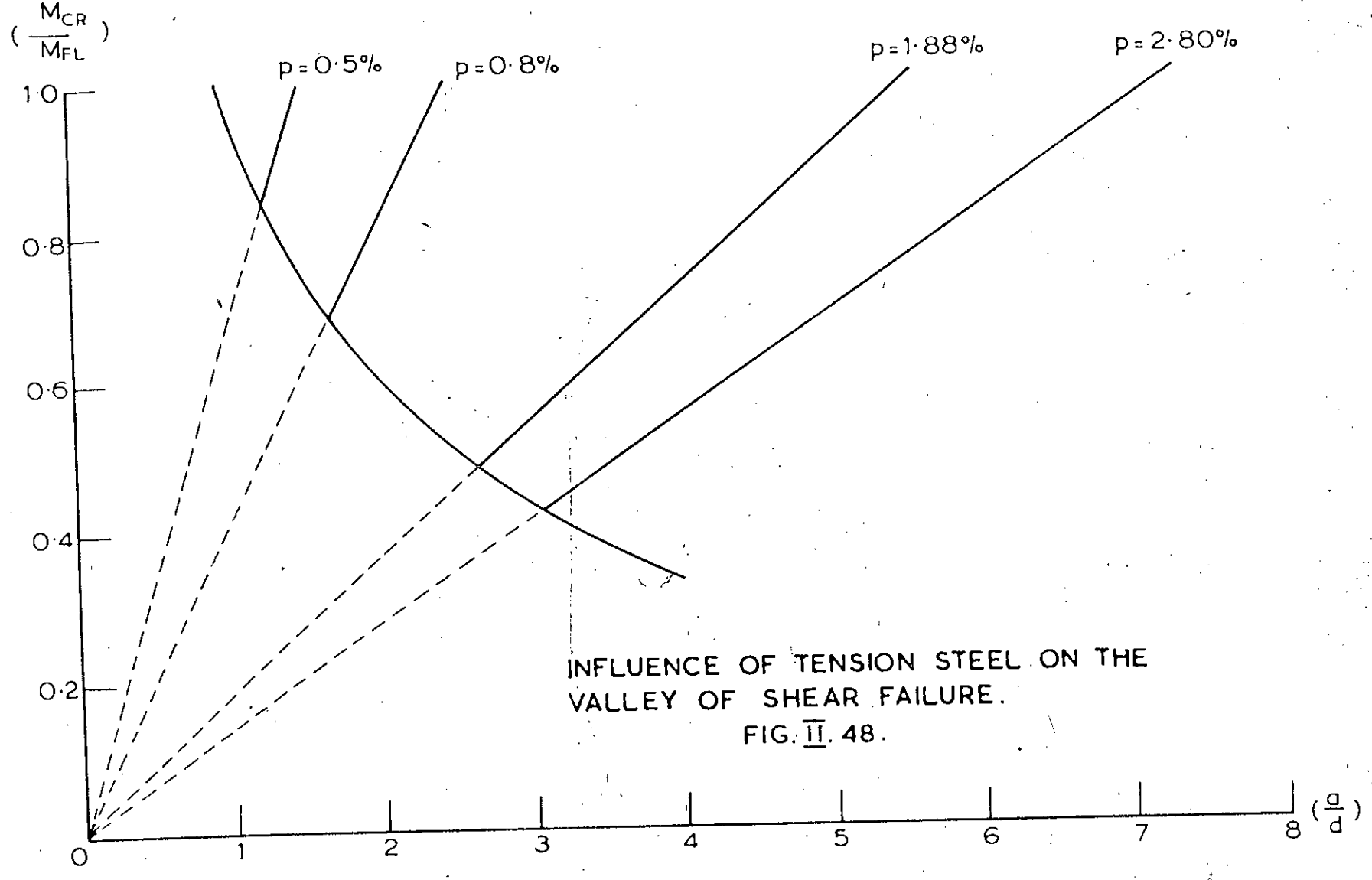
VARIATION OF $\frac{V}{bd}$ WITH $\frac{a}{d}$ RATIO.

FIG. II. 47.

particular group of beams. However, the three-dimensional diagram shown in Fig. II.46a gives a better presentation of the influence of the two parameters a/d and p% on diagonal strength of beems. The figure is a reproduction of the original one reported by Kani in one of his papers[108].

As seen from the Fig. II.46a at (a/d) = 1.00 full flexural strength has been obtained whatever be the value of p%. Also for p = 0.50% the valley of diagonal failure vanishes and hence there is no danger of shear failure. The variation of M_{CR}/M_{FL} with different values of a/d ratio appears to be similar for different values of p%; but the valley of shear failure reduces with the reduction in the value of p%. Also the value of a/d ratio at which the shear failure changes into flexure failure, which is represented as α_{TR} in the analysis discussed in this thesis, varies with percentage of tension steel. For instance from the Fig. II.46a, the value of α_{TR} = 7.00 for p = 2.80% and for p = 0.50%, α_{TR} = 1.00.

In addition, the amount of tensile reinforcement has a decisive influence on the ratio V/bd. Fig. II.47 illustrates the influence of longitudinal tension steel on the value of V/bd. From the figure it can be seen that the variation in V/bd for different values of p is more predominant in case of low a/d ratios than in the case of higher values. On the same figure, the theoretical lines showing the relation between



INFLUENCE OF TENSION STEEL ON THE VALLEY OF SHEAR FAILURE.

FIG. II. 48.

424

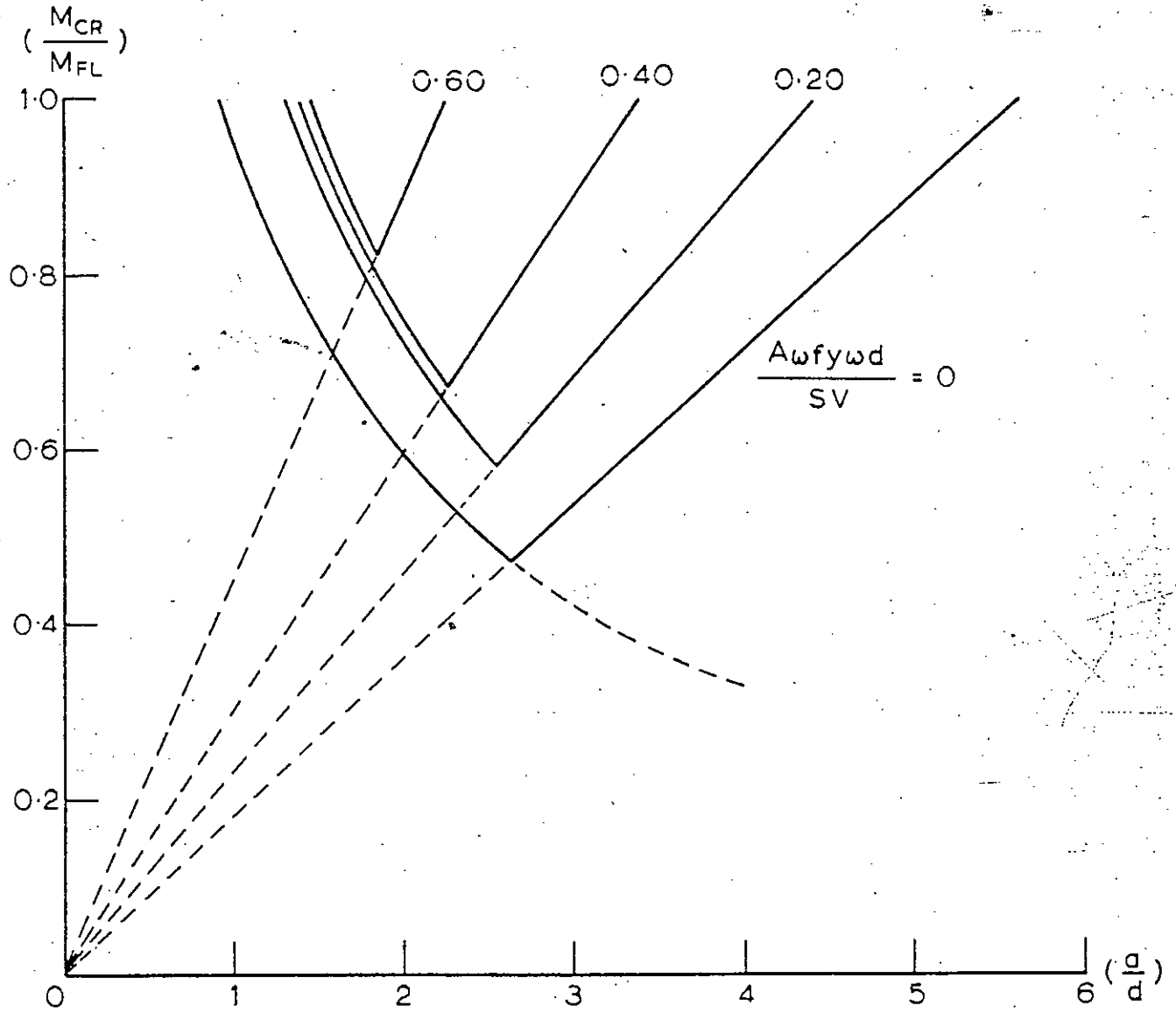
V/bd and percentage of tension steel are also given. From the comparison, it can be seen that the theory predicts a constant value of V/bd for various values of p% in case of high and medium a/d ratios; the theoretical line lies in between the test results.

Fig. II.48 shows the influence of p% on the valley of shear failure as predicted by the theoretical equations. The theoretical variations in the value of α_{TR} and in the valley of shear failure are compatible with the test observations discussed earlier. Using the results of Fig. II.48, a three dimensional diagram showing the influence of p% and a/d ratio on M_{CR}/M_{FL} has been plotted and it is shown in Fig. II.46b. It is interesting to note that the theoretical diagram shown in Fig. II.46b is very similar to the diagram given by Kani[108] (Fig. II.46a) based on his own test results.

II.7.9.3 Shear reinforcement:

As discussed earlier the main functions of shear reinforcement are two fold: (1) it helps to resist certain portion of the shear force; and (2) it restricts the widening of diagonal cracks.

Once again the results available in the literature do not allow a direct comparison of results to study the independent influence of shear reinforcement on the strength of beams. But qualitative study of results has revealed



INFLUENCE OF SHEAR REINFORCEMENT ON THE VALLEY OF SHEAR FAILURE.

FIG. II. 49.

that the shear strength of beams varies linearly with the strength and amount of shear reinforcement upto certain limit. Fig. II.49 illustrates the influence of shear reinforcement on the valley of shear failure as given by the theory and equations proposed here.

Further an important point which need to be stressed is about the anchorage of shear reinforcements in the forms of stirrups because poor anchorage of such stirrups resulted in lesser carrying capacity of beams; favourable conditions exist if such stirrups are omitted[100].

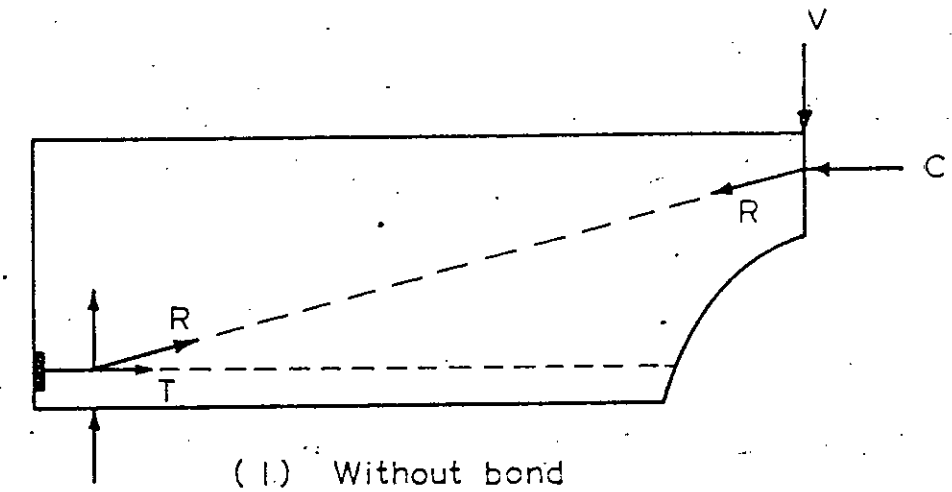
II. 7.10.4 (M/Vd) or a/d ratio:

An important parameter which influences the shear strength of beams to a greater extent than any other one is the ratio between bending moment and shear force M/Vd or a/d ratio. Its influence has been discussed in many contexts of this thesis, and also shown in Figs. II.46 to II.49.

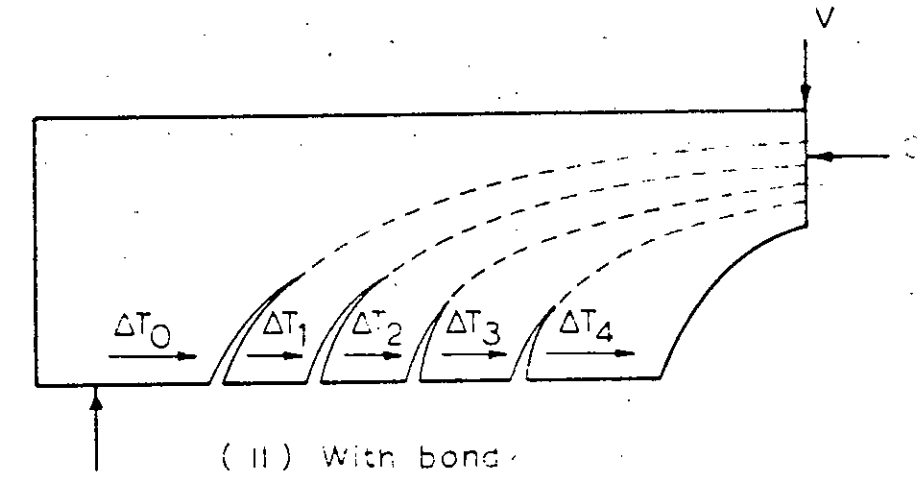
II. 7.10.5 Bond and anchorage:

i. Influence of bond on shear strength of beams:

The bond between the steel and the concrete can be defined as the physical interconnection between these two materials such that concrete has to follow the deformations of steel. The bond between concrete and steel has been proved to have considerable influence on the shear strength



(I.) Without bond



(II) With bond

FREE BODY DIAGRAMS OF THE SHEAR SPAN OF A REINFORCED CONCRETE BEAM. (After Kani)

FIG. II. 50.

of beams. The basic reason for this is due to the fact that the physical behaviour of bonded and unbonded beams are entirely different from one another. The various forces acting in bonded and unbonded beams are shown in Fig. II.50.

In an unbonded beam more favourable conditions exist. Since there is no bond between steel and concrete, ~~there are no bond between steel and concrete~~, there are no forces between steel and concrete except at the anchorage of the bars. The concrete in the shear span remains uncracked and hence providing more area of concrete to resist the resultant force R shown in Fig. II.50. The portion of the beam in shear span is essentially a concrete body under diagonal compression and, the flow of stresses is not obstructed by flexural cracks. Hence beam with no bond will resist more load than a beam with bond between steel and concrete because in case of bonded beams the shear span is affected by the flexural cracks.

The above discussion is well brought out from the test results of a group of beams reported by Leonhardt and Walther[91]. Their results have indicated that beams having poorer bond resisted at least 30% more load than the corresponding beams having better bond. The test results of beams 8 and 17 of author's investigation once again attempt to confirm the above observations of Leonhardt and Walther. The beam 17, even though it had lesser concrete strength than beam 8, had resisted

about 20% more load than the corresponding beam 8. After-examination has revealed that beam 17 had poorer bond than beam 8 resulting in a bond failure.

ii. Influence of anchorage of reinforcements on shear strength of beams:

The beam 10 of author's investigation failed due to the poor anchorage of reinforcing rods. The beam resisted about 30% lesser load than the corresponding beam and failed at the support point before proper opening of the diagonal crack. In order to examine the influence of anchorage on the shear strength of beams, beams 18 and 21 have been tested. In these, beams 18 and 19 were regular beams and beams 20 and 21 were designed to get failure at the anchorage. The beams 20 and 21 failed prematurally at the anchorage accompanied by crushing of concrete at the load point. They resisted about 40% lesser load than their companion beams (beams 18 and 19). The above test observation indicates that poor anchorage of reinforcing rods will result in a premature failure before the beam develops its full shear capacity.

Thus it is to be pointed out here that evaluation of strength of beams failing in bond or anchorage poses entirely different problem and it is beyond the scope of this thesis.

The theory proposed herein is based on the assumption that the beams have proper bond and anchorage and the failure is due to the combined action of bending moment and shear force only.

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CHAPTER II.8

RECOMMENDATIONS FOR DESIGN

II.8.1 So far, in the previous chapters the general behaviour and mechanism of failure of beams subjected to combined bending and shear have been discussed and based on this, theoretical equations have been derived. In the previous chapter, these equations were examined in the light of experimental results and it has been found that the proposed theory and equations are in good agreement with the tests normally erring on the safer side in the majority of the cases.

The fruitfulness of any research can only be obtained by applying its results to practical problems. The applicability of any equation to the design-office use is mainly governed by three aspects:

i. The equations should be of general form and should have been confirmed by large number of experimental results;

ii. They should be simple in form and should not involve too many complicated calculations;

iii. They should be little-bit conservative so as to account for the variations in the quality of the building materials used

and in the nature and magnitude of loads acting on the structure.

The author's equations seem to satisfy these three basic conditions thus suggesting themselves their applicability to design office use.

Further, recommendations of many codes of various countries[26,98,136-138] (vide Table II.8.1) are having provisions restricting the nominal shear stress to a specified limit. It has been already discussed that treating shear stress alone as a measure of shear strength of the beam does not seem to be correct. Also in certain cases, especially in case of low ratios of moment to shear, the shear stresses may not give a true picture of the carrying capacity of the beam. In addition these codes have given equations mainly in terms of shear force and concrete strength for the design of members under combined bending and shear.

All the codes have provisions for designing shear reinforcement if the shear force at the section exceeds the diagonal cracking shear force. According to most of the codes, the shear reinforcements are to be designed to take up the shear force along with the concrete so as to avoid shear failure and thus ensuring only flexure failure in an indirect way. Further, many codes have not considered the influence of M/Vd ratio on the strength of

beams under combined bending and shear. The importance of M/Vd ratio on shear strength of beams has been stressed at number of instances in this thesis and has been well brought out by most of the recent investigators.

Owing to the reasons discussed above, it has been felt that designing shear reinforcements by considering only shear force does not seem to be fully realistic. The proposed methods of calculations discussed here attempt to overcome these inconsistencies in the design procedure.

II.8.2 Discussion of proposed equations:

The main aim of providing shear reinforcement is to make the structure to attain its full flexural capacity. The reason for this is obvious that the sudden and more dangerous type of shear failure is not preferable when compared to the flexure failure.

In the preyious chapter, the useful shear strength of a beam has been taken as follows:

i. For high and medium ranges of M/Vd ratio: $(M/Vd) \geq \alpha_m$

$$\frac{M_{CR}}{M_{FL}} = \frac{(M/Vd)}{\alpha_{TR}}$$

ii. For low values of M/Vd ratio

$(M/Vd) \leq \alpha_m$:-

$$\frac{M_{CR}}{M_{FL}} = \frac{\alpha_m}{\alpha_{TR}}$$

As it has been felt that the above modifications to the proposed theory are necessary from practical view point, the following recommendations are provided for the design of beams under combined bending and shear incorporating the above equations.

II.8.3 Design recommendations:

The members subjected to combined bending and shear are to be designed according to the following recommendations:

a. For the given working loads, the ultimate loads and moments are calculated using the load factors given in the codes.

b. The member is first designed for the ultimate bending moment calculated at the section using the specifications and formulae given in the codes for the design of members subjected to pure bending alone.

c. From the properties of the beam, the quantity α_{TR} is calculated using the relation

$$\alpha_{TR} = \frac{21\ pf_y}{f'_c}$$

d. For sections with M/Vd ratio greater than or equal to α_m the shear-moment capacity of the member is increased to the flexural capacity by providing shear reinforcement using

the relation:

$$(M/Vd) = \frac{\alpha_{TR}}{\phi}\left(1 - \frac{A_w f_{yw} d}{sV}\right)$$

where α_{TR} is calculated from (c) and ϕ is the capacity reduction factor which can be taken equal to 0.85.

e. For sections with M/Vd ratio less than or equal to α_m, the M/Vd ratio for such sections is taken as equal to α_m and the shear reinforcement is calculated using the equation given in (d) where α_m is given by

$$\alpha_m = \frac{\left(\frac{2}{k}\right)\alpha_{TR}}{\sqrt{\frac{4}{k}\alpha_{TR} + 2}}$$

where k = 0.75 and α_{TR} as given in (c).

f. If M/Vd is greater than or equal to $\frac{\alpha_{TR}}{\phi}$ theoretically no shear reinforcement is required; but nominal shear reinforcements can be provided in accordance with the working stress method recommendations of the codes.

g. If M/Vd $\angle$ 1.0, the recommendations given above are not applicable.

h. The spacing of stirrups is not to be more than the effective depth of the beam such that at least one stirrup is traversed by the critical diagonal crack assumed to be inclined at an angle at 45° to the horizontal axis of the member.

i. It is assumed that members have sufficient bond strength and proper anchorage of reinforcements such that these may not influence the cause of failure.

j. The stirrups are to be anchored properly in accordance with the provisions available in the codes.

k. The information available with respect to the efficiency of inclined stirrups and bent-up bars are less conclusive to draft any recommendations.

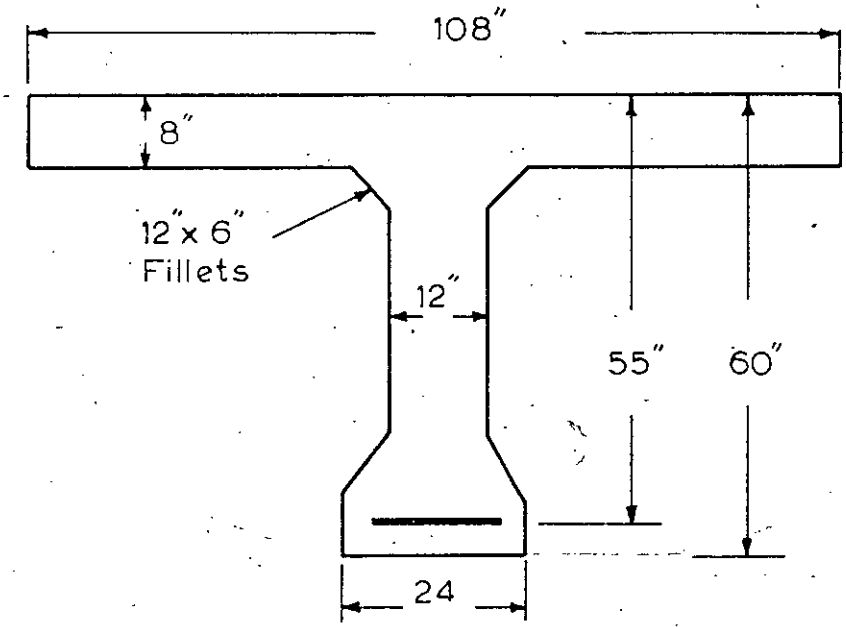
l. The analysis of the author does not provide the basis for limiting the nominal shear stress.

m. For beams with I or T - section, if the position of neutral axis, determined during the flexure analysis falls within the flange then the whole flange width is considered to be effective in resisting the shear force; otherwise the effective width of the member can be taken as equal to the width of rib or web.

n. The frame members and members with uniform loads can be designed using the same recommendations discussed above provided enough reinforcements are given to avoid failures at the junction of frame members.

II.8.4 Numerical Example:

In order to illustrate the application of the design recommendations described in this chapter, the web



MAIN BEAM OF THE BRIDGE.
FIG. II. 51.

reinforcement requirements for a main beam of a Tee Beam-Slab Bridge will be determined. The beam cross section is shown in the Fig. II.51. The other details are:

Span: Clear span = 50 ft.

Effective span = 52.5 ft. (simple supports)

Loading:

IRC Class AA or IRC class A - 2 lanes whichever gives maximum effect, Impact factor 10%.

Cube strength (6" size) at 28 days = 3500 psi

f'_c = 3000 psi; $f_y = f_{yw}$ = 40000 psi

Section	Distance from support (ft)	Under working loads				A_s	p %
		Moments (ft-lb)		Shear (lb)			
		D.L.	L.L	D.L.	L.L	(in^2)	
1	0 (support)	0	0	57,110	114,100	12.5	1.89
2	5.25	283,300	539,900	49,400	101,400	12.5	1.89
3	10.50	518,520	960,100	41,670	88,430	20.0	3.03
4	15.75	672,140	1260,000	24,760	75,540	25.0	3.79
5	21.00	777,600	1440,000	16,960	62,690	30.0	4.55
6	26.25 (mid-span)	801,200	1500,000	0	49,760	34.0	5.15

Design

As per I.S. Code[26]

Ultimate load = 1.8 D.L. + 2.2 L.L

Using the above relation, the ultimate moments and shear forces at various sections and the ratios (M/Vd) are calculated and are given below:

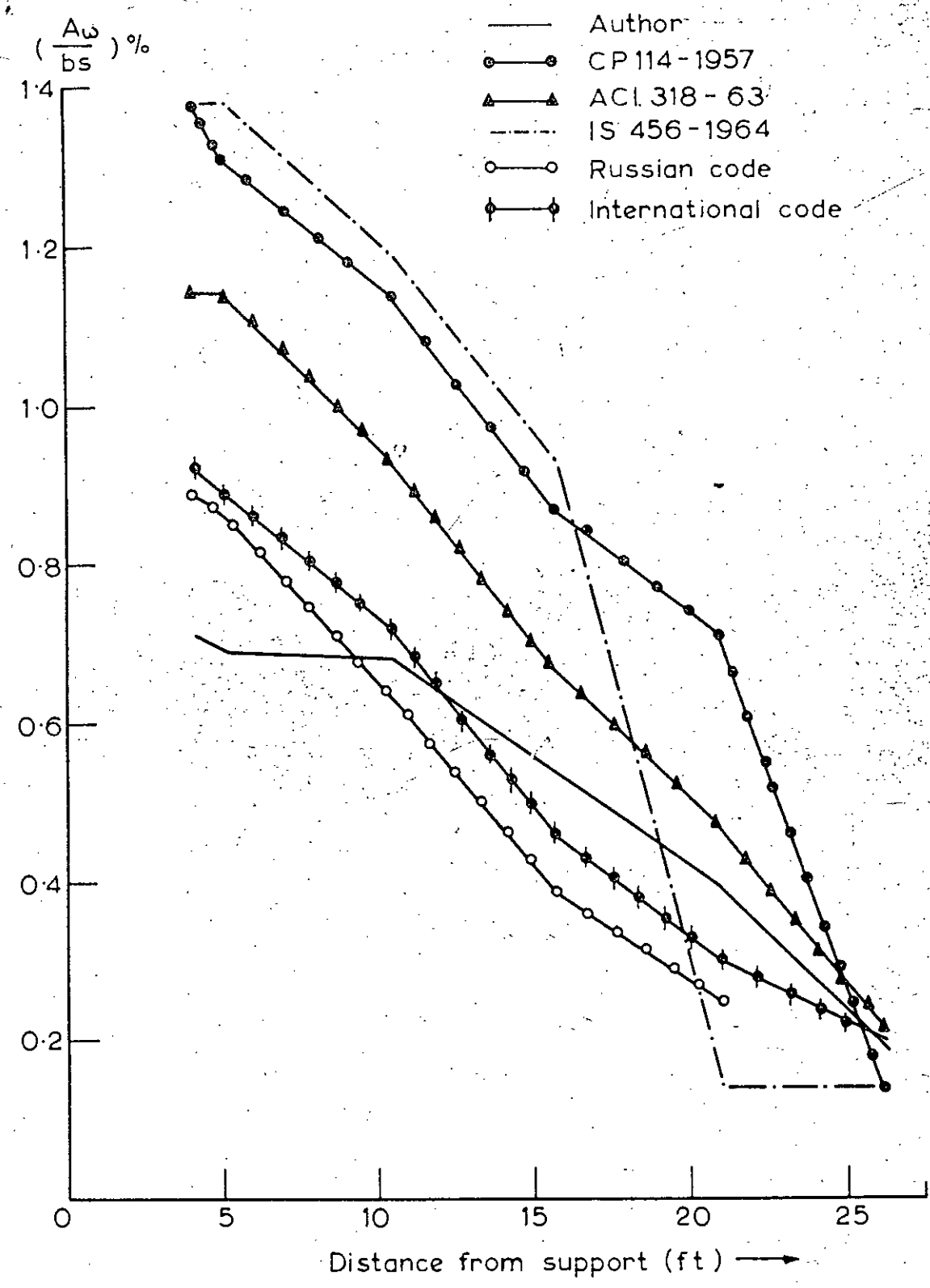
Section	Distance from support (ft)	Ultimate moment (in-kips)	Ultimate shear force (kips)	(M/Vd)
1	0	0	354	0
1'	4.25	17600	320	1.00
2	5.25	20400	313	1.19
3	10.50	36600	269	2.48
4	15.75	47800	211	4.12
5	21.00	55000	169	5.92
6	26.25	57000	110	9.42

Section 6:

$$\alpha_{TR} = \frac{21\, p f_y}{f'_c} =$$

$$\alpha_m = \frac{(2/k)\,\alpha_{TR}}{\sqrt{\frac{4}{k}\alpha_{TR} + 2}} = \frac{21 \times 0.0515 \times 40000}{3000} = \underline{14.40}$$

$$= 4.34.$$



COMPARISON OF RESULTS : DESIGN OF SHEAR REINFORCEMENT BY VARIOUS RECOMMENDATIONS.

FIG. II. 52.

$$\frac{\alpha_{TR}}{\phi} = \frac{14.40}{0.85} = 16.95$$

Since $M/Vd > \alpha_m$ as per the recommendations,

$$\frac{M}{Vd} = \frac{\alpha_{TR}}{\phi}\left(1 - \frac{A_w f_{yw} d}{sV}\right)$$

$$9.42 = 16.95\left(1 - \frac{A_w f_{yw} d}{sV}\right)$$

$$\text{or } \left(1 - \frac{A_w f_{yw} d}{sV}\right) = 0.555$$

$$\text{or } \frac{A_w f_{yw} d}{sV} = 0.445$$

At section 6, V, from table, = 110 kips

$$\text{Therefore, } \frac{A_w f_{yw} d}{s} = 0.445 \times 110$$

$$\text{or } r = \frac{A_w}{bs} = \frac{0.445 \times 110}{55 \times 12} \times 100\%$$

$$= \underline{0.19\%}.$$

Similarly the quantity of transverse steel required can be calculated. The results are given in Table II.8.2

It may be interesting to compare the results obtained using different design recommendations for the numerical example discussed above. The results are summarised in Table II.8.1. Fig. II.52 provides a comparison between results diagramatically. From Fig. II.52 it can be seen that

1. the recommendations of Indian and British Codes err on the unsafe side in certain ranges of (M/Vd) ratio. For low values of M/Vd ratio they are over - conservative.

2. the ACI Building Code seems to be somewhat over-conservative.

3. the Russian Code requires lesser amount of shear reinforcement than that required by the recommendations of other codes. The International Code gives similar results to Russian Code.

In this context, it is hoped that the recommendations proposed herein may be of use in revising the codes as they are based on a general theory which is in agreement with large number of test results.

::

TABLE II.8.1

SUMMARY OF CODE RECOMMENDATIONS FOR SHEAR DESIGN

Code	Design Equations	Maximum spacing of stirrups.	Minimum amount of stirrups	Maximum allowable nominal shear stress
1	2	3	4	5
*British Code: CP114:1957 with Amendment No.1 Feb. 1965.	$\tau = V/bjd$ If τ exceeds the permissible limits, shear reinforcement is to be provided to take up the entire shear force using the equation: $V = \frac{A_w f_w jd}{s}$ where f_w = 20 ksi if dia. of shear reinforcing rod is $\leq 1\frac{1}{4}$ in. = 18 ksi if the same is $> 1\frac{1}{4}$ in. Cube strength C_u (psi) — Permissible shear stress (psi) More than 3000 .. $(\frac{C_u}{50} + 40)$ 3000 ... 100 Less than 3000 ... $(C_u/30)$	jd	-	130 psi

(continued)

Table II.8.1 (contd)

1	2	3	4	5
ACI Building Code: ACI318-63	$V = V_c + V_s$ where $\frac{V_c}{bd} = \phi\left[1.9\sqrt{f'_c} + \frac{2500p}{(M/Vd)}\right]$ $\leq 3.5\,\phi\,f'_c$ $V_s = \phi\,A_w f_{yw}\frac{d}{s}$ $\phi = 0.85$	d	$\frac{A_w}{bs} = 0.15\%$	$10\,\phi\sqrt{f}$
Indian Code: IS456:1964	$\tau = V/bjd$ If τ exceeds two and half times the allowable shear stress, provide shear reinforcement for the entire shear force using the following equation: $V = A_w f_{yw}\frac{jd}{s}$	jd	-	-
Russian Code	$s = \frac{m_s m_{di} A_w f_w}{q_{st}}$ where $q_{st} = \frac{(V/m)^2}{0.6\,fbd^2}$ $f = k_c C_u$ $f_w = k_s f_{yw}$ $k_c = 0.40$ to 0.65 $k_s = 0.70$ to 0.90 $m = 1.0; m_s = 1.0$ $m_{di} = 0.8$ C_u - cube strength	$s \not> \frac{0.1mfbd^2}{V}$ $\not> 8$ in. if $D \leq 16$ in. $\not> \frac{D}{2}$ if $D \not> 16$ in $\not> 20$ in. where D is the total depth of beam and s is the minimum value as given by the above inequalities.		

Table II.8.1 (contd.)

1	2	3	4	5
*International Code	$V = V_{sc} + V_{sw}$ where $V_{sc} = f_t\, bd$ $V_{sw} = 0.9 A_w f_w \frac{d}{s}$ $f_t = \frac{0.5\sqrt{f'_c}}{V_b}$ $V_b = 1.5$ f'_c = compressive strength in kg/cm^2. f_w - allowable stress in shear reinforcing steel.	-	$\frac{A_w}{bs} = 0.2\%$	$\frac{V}{bd} \neq 5 f_t$ (for rectangular beam) $\neq 6\, f_t$ (for beams with a compression flange provided that the width of the flange is greater than or equal to three times the web thickness)

*Recommendations are for working load conditions; ultimate load method of calculations are not given in these codes.

TABLE II.8.2

COMPARISON OF RESULTS : DESIGN OF SHEAR REINFORCEMENT BY VARIOUS RECOMMENDATIONS

Section	M/Vd	α_{TR}	$\alpha_m \left(1 - \frac{A_w f_{yw} d}{sV}\right) \frac{A_w f_{yw} d}{sV}$			$= (A_w/bs)\%$					
						Author	CP114-1957*	ACI318-63	IS:456-1964	Russian Code	International Code*
6	9.42	14.40	4.34	0.555	0.445	0.19	0.14	0.21	0.14	0.11	0.20
5	5.92	12.75	4.06	0.395	0.605	0.39	0.71	0.47	0.14	0.25	0.30
4	4.12	10.60	3.70	0.330	0.670	0.54	0.87	0.67	0.93	0.39	0.46
3	2.48	8.48	3.29	0.330	0.670	0.68	1.14	0.93	1.19	0.63	0.72
2	1.19	5.30	2.58	0.415	0.585	0.69	1.31	1.14	1.38	0.86	0.89
1'	1.00	5.30	2.58	0.415	0.585	0.71	1.38	1.14	1.38	0.89	0.92

*Ultimate load methods of calculations are not given in these codes and hence the results given here are based on working stress methods of calculations.

564

CHAPTER II.9

CONCLUDING REMARKS

II.9.1 The results of the investigation are summarised below:

1. Modes of Failures:

Depending on the ultimate cause of failure, the shear failures are classified under two heads: (a) Diagonal tension failure which occurs in case of high and medium a/d ratios, (b) Diagonal compression failure in case of low values of a/d ratio. The other types of secondary failures, such as bond failure and anchorage failure are not considered as shear failures.

2. Mechanism of Shear Failure:

An hypothesis is presented for the mechanism of shear failure with the help of which the behaviour and modes of failures are explained. Two different types of mechanism of failure are considered. The first mechanism considers the failure of concrete element at the level of neutral axis and it is applicable in the case of high and medium a/d ratios. The second mechanism corresponds to the transformation of the beam into an arch and failure of the arch under biaxial

compression. The second mechanism is applicable for low values of a/d ratio.

In developing the theory it has been assumed that certain amount of shear force is transferred to the longitudinal steel as dowel force at the time of diagonal cracking which rapidly drops to zero at ultimate failure. This is in agreement with the test observations. Also the dowel forces existing at the level of steel beyond the diagonal cracking section sometimes cause splitting along the longitudinal steel. This type of failure is considered as a secondary failure.

3. Shear Strength of Beams:

Based on certain assumptions and using a failure criterion for concrete under compressive-tensile stresses theoretical equations are derived for the two types of mechanism discussed earlier. The equations bring-in the effect of well known " $\frac{\text{Moment}}{\text{shear}}$ or a/d ratio " on the shear strength of beams. The analysis is then extended to flanged beams and beams with shear reinforcement in the form of vertical stirrups only.

Later, the applicability of the proposed equations to the case of restrained beams and beams with distributed loads is discussed. From the study of sensitiveness of results with respect to certain parameters such as compression

block coefficients k_2, k_1 k_3, failure criterion for concrete, etc. it has been found that the resulting equations get altered only to a little extent due to the changes in the above parameters. This clearly indicates the general applicability of the proposed equations.

4. Summary of Equations:

Beams without shear reinforcement

1. For low a/d ratios: $(a/d \leq \alpha_m)$

$$\frac{M_{CR}}{M_{FL}} = \frac{(2/k)}{\frac{a}{d} + \sqrt{(\frac{a}{d})^2 + 2}}$$

2. For high and medium a/d ratios: $(a/d \geq \alpha_m)$

$$\frac{M_{CR}}{M_{FL}} = \frac{(a/d)}{\alpha_{TR}}$$

where

$$\alpha_{TR} = \frac{21\ pf_y}{f'_c} .$$

$$\alpha_m = \frac{(2/k)\alpha_{TR}}{\sqrt{\frac{4}{k}\alpha_{TR} + 2}}$$

Beams with shear reinforcement (vertical stirrups only)

i. For low a/d ratios $(a/d \leq \alpha_m)$

$$\frac{M_{CR}}{M_{FL}} = \frac{2/k}{\frac{a}{d} + \sqrt{(\frac{a}{d})^2 + 2(1 - \frac{A_w f_{yw} d}{sV})^2}}$$

ii. For high and medium a/d ratios $(a/d \geq \alpha_m)$

$$\frac{M_{CR}}{M_{FL}} = \frac{(a/d)}{\alpha_{TR}}$$

where

$$\alpha_{TR} = (\frac{21 p f_y}{f'_c})(1 - \frac{A_w f_{yw} d}{sV})$$

$$\alpha_m = \frac{(2/k)\alpha_{TR}}{\sqrt{\frac{4}{k}\alpha_{TR} + 2(1 - \frac{A_w f_{yw} d}{sV})^2}}$$

Test results of more than one thousand beams have been compared with the theoretical predictions. From the comparison of results it has been found that the theoretical equations give fairly good results normally erring on the safer side in majority of the cases.

The results of comparison are summarised in Table II.9.1 and Fig. II.33 shows the histrogram for the results.

5. Design Recommendations:

Using the results of the investigation, a set of recommendations are given in the previous chapter for the

TABLE II.9.1

Type of beams	No. of test results	Ratio : (Test/Calc) Mean value	Coefficient of variation
1. Beams without shear reinforcement.			
Simple beams:			
Rectangular	702	1.065	15%
Tee	146	1.100	9%
Restrained beams	52	1.070	11%
2. Beams with shear reinforcement.			
Simple beams:			
Rectangular	175	1.130	15%
Tee	34	1.095	12%
Restrained beams	31	1.053	13%
3. Beams with UDL	120	1.150	15%
In General	1260	1.075	14%

design of beams under combined bending and shear. The following formula is suggested for the design of shear reinforcement in the form of vertical stirrups only:

$$\left(\frac{M}{Vd}\right) = \frac{\mathcal{L}_{TR}}{\phi}\left(1 - \frac{A_w f_{yw} d}{sV}\right)$$

where $\mathcal{L}_{TR} = \frac{21\, p f_y}{f'_c}$

ϕ = capacity reduction factor (= 0.85).

It can be added here that the above design formula is simple and conservative and it predicts the strength of beams to a reasonable degree of accuracy.

Hence the equation can be used for the design of members subjected to combined bending and shear.

II.9.2 Suggestions for future research:

From Table II.9.1, it can be seen that most of the results available in the literature are with respect to simple rectangular beams. But in most of the actual structures, the beams have either Tee section or Ell section and are subjected to a combination of distributed and concentrated loads. So more experimentation is necessary on these lines. Further, the tests conducted on continuous beams are very few in number and most of them confine to equal-span symmetrically loaded continuous beams. The influence of redistribution of moments on the shear strength of continuous beams is little known from these test results.

Hence exhaustive investigation on this aspect is suggested.

The analysis presented in this thesis confines itself to the case of beams with shear reinforcement in the form of vertical stirrups only. Most of the codes suggest both inclined stirrups and bent-up bars as shear reinforcement in addition to ordinary vertical stirrups. But the test results available in the literature with regard to the relative merits of bent-up bars and vertical stirrups are less conclusive. Hence a detailed study on these lines is necessary. It would then be possible to extend the equations derived herein to the case of beams with bent-up bars and inclined stirrups.

Another interesting aspect of the problem is the influence of bond on shear strength of beams about which only a very little information is available. It is necessary to conduct a detailed investigation on this aspect so as to know the interaction between bond and shear strengths of beams subjected to combined bending and shear.

PART III : TORSIONAL STRENGTH OF CONCRETE BEAMS UNDER AXIAL COMPRESSION AND OF PRESTRESSED CONCRETE BEAMS

PART III

TORSIONAL STRENGTH OF CONCRETE BEAMS UNDER AXIAL COMPRESSION AND OF PRESTRESSED CONCRETE BEAMS

III.1 Introduction:

The torsional strength of a plain concrete member is low and the modes of failure are very destructive and sudden. Because of this reason, the concrete beams are either reinforced or prestressed. The behaviour and strength of reinforced concrete beams have been discussed in Part I of this thesis. In this Part, the torsional strength of prestressed concrete beams is discussed.

The basic principle of prestressing a concrete member is to induce a certain amount of initial compression in the member. This compressive stress suppresses the effect of shear stress caused by the applied torque thus enhancing the carrying capacity of the member in torsion. So, in this Part, the torque capacity of concrete beams under axial compression are initially analyzed and later expressions are derived for the case of prestressed concrete beams. The derivations are based on the same failure criterion for concrete under compressive-tensile stresses used throughout

this thesis. The theoretical predictions are compared with all the test results available to the author and it has been found that they are good in predicting the test results. It is to be mentioned here that the review of literature on this subject is not given here as it has already been included in Part I of this thesis (see Chapter I.2).

III.2 Beams under axial compression:

In this article, expressions are derived for the torsional strength of concrete beams under axial compression. To the author's knowledge, Nylander[4] was the only investigator who has reported results of tests on beams under axial compression. Even though such investigations may be of academic interest only, they are useful in understanding the behaviour of prestressed concrete beams under torsion.

Let f be the axial compressive stress and τ be the shear stress caused by the applied torque at the critical point for failure. These two stresses will interact with each other and yield two principal stresses which are compressive-tensile in nature. Then,

$$\sigma_1 = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + \tau^2} \qquad \text{(compressive)}$$

$$\sigma_2 = \frac{f}{2} - \sqrt{\left(\frac{f}{2}\right)^2 + \tau^2} \qquad \text{(tensile)}$$

... III(1)

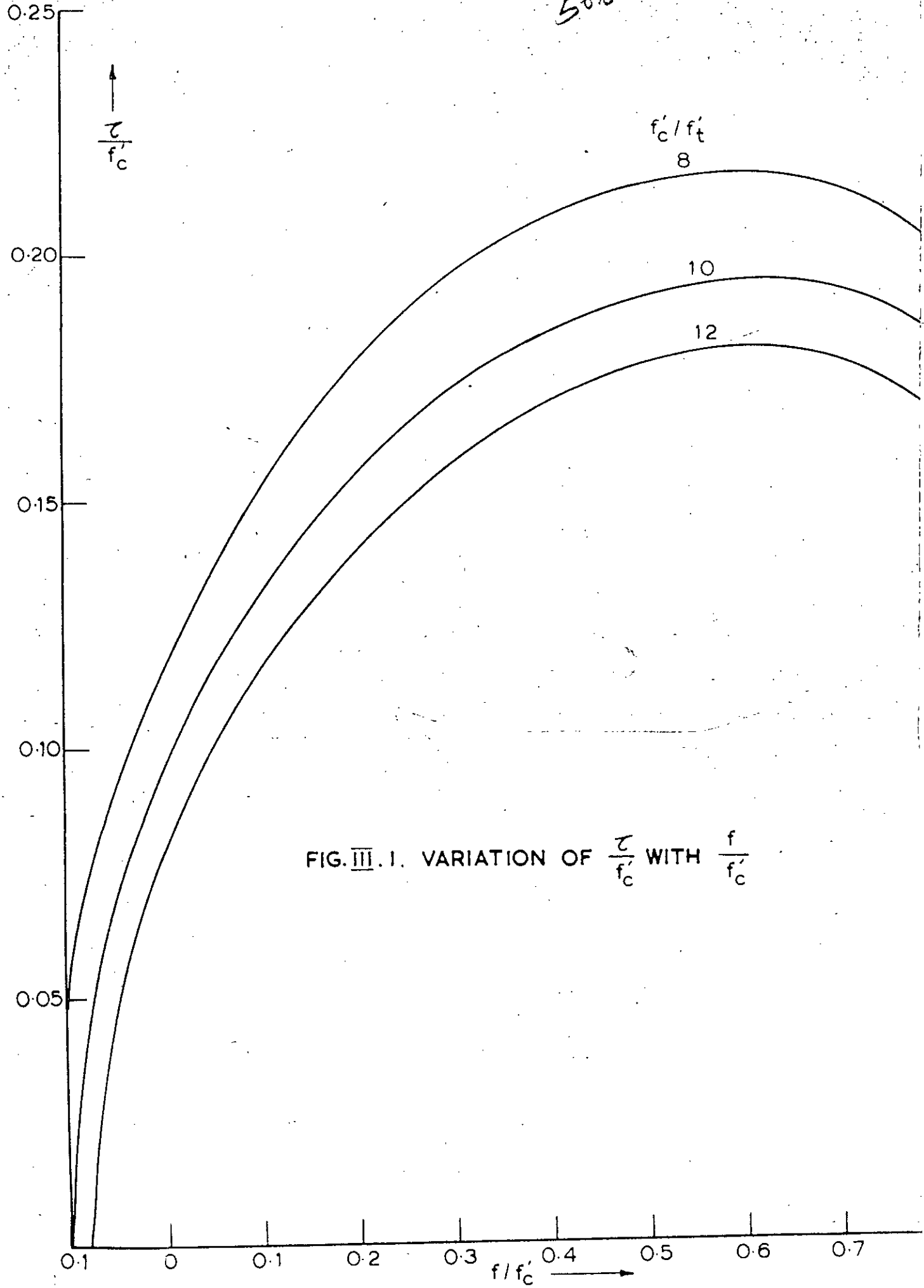


FIG. III.1. VARIATION OF $\frac{\tau}{f_c'}$ WITH $\frac{f}{f_c'}$

For failure, these two principal stresses should satisfy the failure criterion for concrete under compressive-tensile stresses given below:

$$\left(\frac{\sigma_1}{f'_c}\right) + \left(\frac{\sigma_2}{f'_t}\right)^2 = 1 \qquad \ldots \text{ III (2)}$$

Substituting the values of σ_1 and σ_2 from Eqn. III.1 in Eqn. III.2 and solving for τ we get,

$$\left(\frac{\tau}{f'_c}\right) = \varphi\left(\frac{f}{f'_c}\right) \qquad \ldots \text{ III.3}$$

where $\varphi(f/f'_c)$ is a function of normal stress, f. Eqn.III.3 can be represented in the form of an interaction diagram. Such a diagram is shown in Fig. III.1.

$$\text{Let} \qquad M_t = \beta b^2 D\tau \qquad \ldots \text{ III.4}$$

where

M_t - applied torque

β - torsion coefficient

b & D - breadth and depth of the section

The value of β depends on the shape of the cross section and strength of concrete. For normal strength of concrete its value can be taken equal to the semi-plastic torsion

coefficient β_s defined in Part I of this thesis.

The Eqn. III.4 can be put in the following form:

$$M_t = \beta b^2 D f'_c . \left(\frac{\tau}{f'_c}\right) \qquad \ldots \text{ III.5}$$

The value of τ/f'_c can be known from Fig. III.1 for the known values of f/f'_c and f'_c/f'_t. Substituting this value of τ/f'_c in Eqn. III.5, the torque capacity of a concrete beam under axial compression can be known.

Using Eqn. III.5, the torsional strengths of beams reported by Nylander[4] were calculated. The results are given in Table III.1. From Table III.1, it can be seen that the theoretical results agree well with the test results.

III.3 Strength of prestressed concrete beams under pure torsion:

The behaviour and strength of a prestressed concrete beam under pure torsion should be same as those of concrete beams under axial compression. So the Eqn. III.5 derived in the preceding article is also applicable for prestressed concrete beams under pure torsion. But the value of torsion coefficient β to be used in the case of prestressed concrete beams requires certain discussion. Some of the earlier investigators like Turner and Davis[12], Marshall[16] and Nylander[4] have reported that stress redistribution takes place before

TABLE III.1

STRENGTH OF CONCRETE BEAMS UNDER THE COMBINED ACTION OF AXIAL COMPRESSION AND TORSION : COMPARISON OF THEORETICAL RESULTS WITH THE TEST RESULTS OF NYLANDER[4]

Specimen Mark		f'_c psi	$\frac{f'_c}{f'_t}$	β_s	f psi	f/f'_c (test)	τ/f'_c (calc)	M_t (test) inch-kips	M_t (calc) inch-kips	Ratio Test/Calc.
I:	1	4300	10	0.271	0	0	0.095	47.0	56.4	0.84
	2	4300	10	0.271	0	0	0.095	52.1	56.4	0.93
	3	4300	10	0.271	530	0.123	0.138	92.5	80.5	1.15
	4	4300	10	0.271	560	0.130	0.139	83.5	82.0	1.02
	5	4300	10	0.271	1080	0.251	0.265	109.5	98.0	1.12
	6	4300	10	0.271	1070	0.249	0.265	109.5	98.0	1.12
	7	4300	10	0.271	1320	0.307	0.174	115.0	103.5	1.11
	8	4300	10	0.271	1340	0.312	0.175	117.2	104.0	1.13
II:	1	2730	10	0.271	424	0.155	0.146	73.0	55.0	1.33
	3	2730	10	0.271	1000	0.366	0.182	78.2	69.0	1.13
	4	2730	10	0.271	1010	0.370	0.182	78.2	69.0	1.13
	5	2730	10	0.271	0	0	0.095	41.7	36.0	1.16
	6	2730	10	0.271	0	0	0.095	36.5	36.0	1.02
	7	2730	10	0.271	1305	0.477	0.191	93.6	72.0	1.30
	8	2730	10	0.271	1330	0.486	0.192	83.5	72.5	1.15

Cross section of all the beams : 8" x 8".

failure in a beam subjected to pure torsion. But the extent of redistribution depends on the strength of concrete and shape of the cross section. For very low strengths of concrete, the redistribution of stresses will influence the strength of beams to a considerable extent and in such cases the rigid plastic theory is more suitable. As the plasticity ratio of concrete decreases with the increase in the strength of concrete[70,139], the influence of redistribution of stresses on torque capacity of beams is also decreases with the increase in the strength of concrete. Perhaps a very high strength concrete will behave purely as an elastic material as observed by Rausch[139]. Because of this reason only, a semi-plastic theory has been suggested in Part I for the calculation of torsional stresses in the case of reinforced concrete beams. Normally in prestressed concrete construction only high strength concrete is used. For such concrete the plasticity ratio is small and hence the little redistribution of stresses that may occur near the failure may not have any considerable influence on the carrying capacity of beams. So in prestressed concrete beams the torsional stresses can be calculated using the elastic theory. Further it is to be pointed out here that most of the earlier investigators like Cowan[22], Humphrey[35] and Zia[36] preferred elastic theory in the case of prestressed concrete beams than the rigid plastic theory. This might be due to the reasons discussed above. So the value of β in Eqn. III.5 is to be obtained from the results of

elastic torsion theory. Since most of the sections encountered in prestressed concrete construction are flanged beams (either Tee or I), the solutions obtained in Part I of this thesis for the elastic torsion of these sections are very useful in these cases. However it is to be pointed out here that prestressed concrete beams with spiral reinforcement may pose an entirely different problem. It is quite possible in such cases that considerable redistribution of stresses may occur near failure. The test results reported by Ramakanth[140] show this type of behaviour. Hence the analysis presented in this Part of the thesis considers beams with transverse steel in the form of vertical hoops only.

In general prestress can be applied concentrically or eccentrically. So two cases are to be discussed:

III.3.1 Concentrically Prestressed Beams:

In a concentrically prestressed beams, the normal stress is constant at all points in the cross section. So the critical point for failure is that point where the shear stress is maximum as per the elastic theory. So in this case,

$$M_t = \beta_e^{\max.} b^2 D f'_c \left(\frac{\tau}{f'_c}\right) \qquad \ldots \text{III.6}$$

where $\beta_e^{\max.}$ - elastic torsion coefficient corresponding to maximum shear stress.

$(\frac{\tau}{f'_c})$ - as given by Fig. III.1 for the known values of f/f'_c and f'_c/f'_t and f is the concentric prestress in the section.

III.3.2 Eccentrically Prestressed Beams:

In an eccentrically prestressed beam, the normal stress varies across the depth of the section. In certain cases, if the eccentricity is large, there may be even tensile stresses at certain points in the section. In such cases, the critical point for failure may not necessarily be the one at which the shear stress is maximum. So the torque capacity of an eccentrically prestressed concrete beam is to be calculated (i) at the point where the normal stress is minimum and (ii) at the point where shear stress is maximum. The torque capacity of the beam is the lower of these two values.

The method of calculation indicated above is applicable for rectangular and tee beams. As observed from the test results reported by Zia[37], the I-beams present a somewhat different behaviour. Due to the eccentricity of prestress, the normal stress in the top flange of an I-beam will be usually less and cracking will be initiated at that point. With the propagation of the crack, the I section will transform itself into an inverted Tee section. This transformation will result in the reduction of eccentricity of the prestressing force to a considerable extent resulting in a more uniform

prestress with respect to the uncracked inverted Tee section. The torque capacity of the transformed tee section with more uniform stress is considerably increased and it is nearly equivalent to a tee beam with concentric prestress. The transformed tee section will fail when the shear stress at mid depth of the web, which is the point at which maximum shear stress occurs, reaches the critical value as given by the interaction diagram shown in Fig. III.1. So in the case of I beams the mid depth of the web is to be considered as the critical point for failure irrespective of the value of normal stress present at other points. This seems reasonable due to the reasoning discussed above. So we have,

For rectangular and tee sections:-

$$M_t = \beta_e^c \; b^2 \, D f'_c \left(\frac{\tau}{f'_c}\right) \qquad \ldots \text{ III.7}$$

For I-sections:-

$$M_t = \beta_e^{max.} \; b^2 D f'_c \left(\frac{\tau}{f'_c}\right) \qquad \ldots \text{ III.8}$$

where

β_e^c - elastic torsion coefficient corresponding to shear stress at the point where normal stress is minimum.

β_e^{max} - elastic torsion coefficient corresponding to

shear stress at mid depth of web in the case of I - sections and the values of $(\frac{\sigma}{f'_c})$ in Eqns. III.7 and III.8 are to be obtained from Fig. III.1 and the values of f/f'_c correspond to minimum normal stress in the case of rectangular and tee sections and to the normal stress at mid depth of web in the case of I - sections.

III.3.3 Beams with transverse reinforcement:

(Only vertical hoops are considered in the analysis)

Experiments[37] have shown that the transverse hoops increase the torque capacity of prestressed concrete beams. The torque capacity of beams with shear reinforcement is inter-dependent on the torsional strength of corresponding beams without transverse hoops. Hence, for simplicity the following relationship is assumed:

Torque capacity of a beam with transverse hoops = Torque resisted by concrete + Torque resisted by transverse hoops.

... III.9

The torque resisted by transverse hoops can be calculated in a similar manner as is done in the case of reinforced concrete beams (vide Ch.I.9). The axial stress in the transverse hoops can be taken equal to the yield stress of steel as the experiments of Paul Zia[37] have shown that the transverse steel

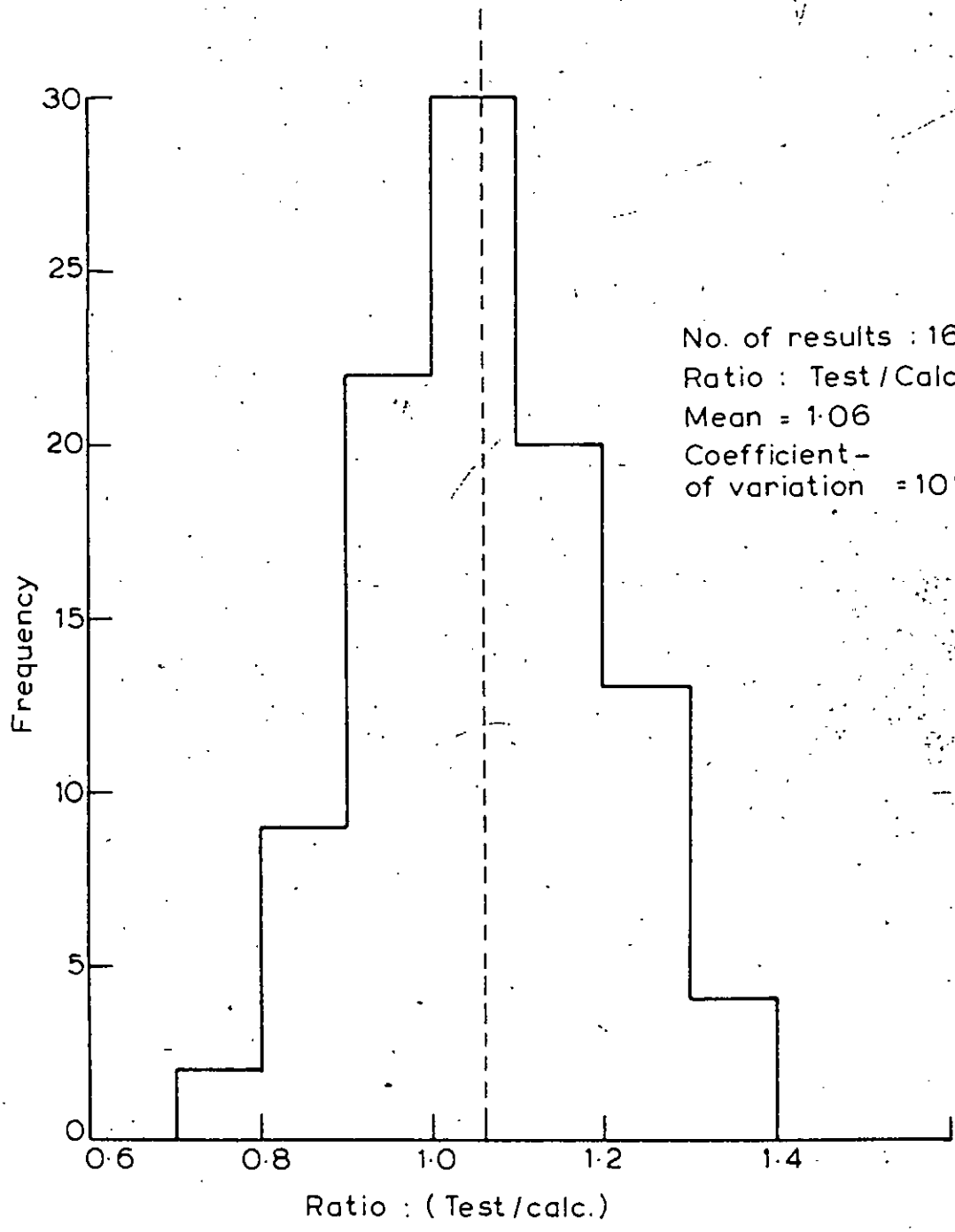


FIG. III. 2. PRESTRESSED CONCRETE BEAMS UNDER PURE TORSION : COMPARISION OF RESULTS : HISTOGRAM.

yields at failure of the beam. So, torque resisted by transverse hoops,

$$M_{ts} = A_w f_{yw} \frac{b'd'}{s} \quad \ldots \text{III.10}$$

The torque resisted by concrete, M_{tc} can be known from Eqn.III.6 or III.7 or III.8.

III.3.4 Comparison of Results:

In order to check the accuracy of the equations derived above, the torque capacities of over one hundred fifty prestressed concrete beams[35,36] tested in pure torsion were calculated using the derived equations. The results are given in Tables III.2 and III.3. Since the test results reported by Humphrey[35] are in terms of shear stress calculated as per the elastic theory, the comparison is made in terms of shear stresses for these results in Table III.2. In Table III.3 the results are compared in terms of torque capacities of beams. From Tables III.2 and III.3 it can be seen that the theoretical predictions are fairly good when compared with the test results. The mean value of test/calc. ratio is equal to 1.06, the coefficient of variation being 10%. The histogram is shown in Fig. III.2.

III.4 Strength of prestressed concrete beams under combined bending and torsion:

As in the case of reinforced concrete beams, there are two possible causes for failure in a prestressed concrete beam

TABLE III.2

TORSIONAL STRENGTH OF PRESTRESSED CONCRETE BEAMS : COMPARISON OF THEORETICAL RESULTS WITH THE TEST RESULTS OF HUMPHREYS[35]

Specimen Mark	f (test) psi	τ (test) psi	f/f'_c	τ/f'_c Test	τ/f'_c Calc.	Ratio: Test / Calc.
1	2	3	4	5	6	7
			Size : 5" x 5"			
P0 A	0	778	0	0.099	0.095	1.04
B	0	745	0	0.095	0.095	1.00
C	0	774	0	0.098	0.095	1.03
D	0	758	0	0.096	0.095	1.01
E	0	765	0	0.097	0.095	1.02
P1 A	0	760	0	0.096	0.095	1.01
B	0	828	0	0.105	0.095	1.11
C	0	855	0	0.109	0.095	1.15
D	0	814	0	0.103	0.095	1.08
E	0	742	0	0.094	0.095	0.99
P2 A	1340	1160	0.17	0.171	0.150	1.14
B	1340	1130	0.17	0.144	0.150	0.96
C	1340	1046	0.17	0.133	0.150	0.89
D	1025	935	0.13	0.119	0.140	0.85
P3 A	1775	1390	0.23	0.177	0.163	1.09
B	1775	1190	0.23	0.151	0.163	0.93
C	1820	1230	0.23	0.157	0.163	0.96
P4 A	2720	1740	0.35	0.222	0.180	1.23
B	3000	1815	0.38	0.231	0.184	1.25
C	2620	1695	0.33	0.216	0.178	1.21

(contd)

Table III.2 (contd.)

1		2	3	4	5	6	7
P5	A	3810	2070	0.48	0.264	0.190	1.39
	B	3810	1985	0.48	0.252	0.190	1.32
	C	3560	2083	0.45	0.265	0.190	1.39
	X	3270	1870	0.42	0.238	0.189	1.26
P6	A	4380	2205	0.56	0.280	0.196	1.43
	B	4380	2220	0.56	0.282	0.196	1.44
	C	4510	2270	0.57	0.289	0.196	1.47
P7	A	6250	1620	0.80	0.206	0.186	1.11
	B	5800	2075	0.74	0.264	0.190	1.39
	C	6250	2020	0.80	0.257	0.196	1.31
				Size : 10" x 5"			
PR0	A	0	784	0	0.100	0.095	1.05
	B	0	680	0	0.087	0.095	0.92
	C	0	690	0	0.088	0.095	0.93
PR1	A	0	728	0	0.093	0.095	0.98
	B	0	750	0	0.095	0.095	1.00
	C	0	745	0	0.095	0.095	1.00
PR2	A	821	934	0.10	0.118	0.132	0.90
	B	875	934	0.11	0.118	0.135	0.88
	C	920	970	0.12	0.123	0.138	0.89
PR 3A		2050	1290	0.26	0.164	0.168	0.98
PR 4A		2520	1475	0.32	0.187	0.176	1.06
	B	2610	1320	0.33	0.168	0.178	0.95
	C	2750	1542	0.35	0.196	0.180	1.09
PR 6A		4280	1670	0.55	0.212	0.196	1.08
	B	4090	1820	0.52	0.231	0.194	1.19
	C	4350	1765	0.55	0.224	0.196	1.14

(continued)

Table III.2(contd.)

1	2	3	4	5	6	7
			Size: 15" x 5"			
PS 0A	0	689	0	0.088	0.095	0.93
B	0	683	0	0.087	0.095	0.92
C	0	689	0	0.088	0.095	0.93
PS 1A	0	740	0	0.094	0.095	0.99
B	0	735	0	0.093	0.095	0.98
C	0	720	0	0.092	0.095	0.97
PS 2A	570	920	0.07	0.117	0.122	0.96
B	550	835	0.07	0.116	0.122	0.87
C	685	860	0.09	0.109	0.126	0.87
PS 4A	2270	1260	0.29	0.160	0.172	0.93
B	2200	1502	0.28	0.191	0.171	1.12
C	2200	1355	0.28	0.172	0.171	1.00
PS 6A	3810	1940	0.48	0.246	0.194	1.27
B	3890	1830	0.50	0.232	0.195	1.19
C	3770	1775	0.48	0.225	0.194	1.16
			Size : 9" x 3"			
PT 0A	0	767	0	0.098	0.095	1.03
B	0	735	0	0.093	0.095	0.98
C	0	680	0	0.086	0.095	0.91
PT 1A	0	760	0	0.097	0.095	1.02
B	0	714	0	0.091	0.095	0.96
C	0	690	0	0.088	0.095	0.93

(Contd.)

Table III.2(contd.)

1	2	3	4	5	6	7
			Size : 9" x 3"			
PT 2A	610	853	0.08	0.108	0.125	0.87
B	620	897	0.08	0.114	0.125	0.91
C	620	921	0.08	0.117	0.125	0.94
PT 4A	1740	1410	0.22	0.179	0.162	1.11
B	1670	1550	0.20	0.197	0.158	1.25
C	1740	1270	0.16	0.161	0.148	1.09
PT 6A	2540	1800	0.32	0.229	0.177	1.29
B	2470	1800	0.31	0.229	0.175	1.31
C	3000	1800	0.38	0.229	0.184	1.24
			Size : 12" x 3"			
PU 0A	0	728	0	0.093	0.095	0.98
B	0	740	0	0.091	0.095	0.96
C	0	740	0	0.094	0.095	0.99
PU 1A	0	808	0	0.103	0.095	1.08
B	0	694	0	0.088	0.095	0.93
C	0	726	0	0.092	0.095	0.97
PU 2A	630	1056	0.08	0.134	0.126	1.06
B	660	1150	0.09	0.146	0.127	1.15
C	640	1056	0.08	0.134	0.126	1.06
PU 4A	1480	1421	0.19	0.180	0.154	1.17
B	1770	1565	0.22	0.199	0.161	1.23
C	1690	1472	0.21	0.187	0.159	1.18

(contd.)

Table III.2(contd.)

1	2	3	4	5	6	7
			Size: 12" x 3"			
PU 6A	3420	1681	0.43	0.213	0.188	1.13
B	3180	1785	0.41	0.227	0.186	1.22
C	3480	1856	0.44	0.236	0.189	1.25
			Size : 10" x 5"			
PRE A*	0	890	0	0.113	0.095	1.19
B*	0	795	0	0.101	0.095	1.06
			Size : 5" x 5"			
PE A*	0	885	0	0.112	0.095	1.18
B*	0	930	0	0.118	0.095	1.24

* Eccentrically Prestressed Beams.

Note:- For all the beams of Humphrey's investigation,

Cube strength = 8730 psi;

f'_c = 7870 psi; f'_t = 740 psi

TABLE III.3

TORSIONAL STRENGTH OF PRESTRESSED CONCRETE BEAMS : COMPARISON OF THEORETICAL RESULTS WITH THE TEST RESULTS OF ZIA[36].

Specimen Mark	f'_c psi	$\frac{f'_c}{f'_t}$	f_p (top) psi	f_p (bottom) psi	$\frac{f}{f'_c}$	$\frac{\tau}{f'_c}$	M_{tc} inch-kips	M_{ts} inch-kips	M_t inch-kips Test	M_t inch-kips Calc	Ratio Test/Calc
1	2	3	4	5	6	7	8	9	10	11	12
)R 1	8050	12	1020	1020	0.13	0.124	49.8	0	44.2	49.8	0.89
2	8050	12	1020	1020	0.13	0.124	49.8	0	46.5	49.8	0.93
3	8200	12	1010	1010	0.12	0.122	50.8	0	46.7	50.8	0.92
?R 1	8500	12	0	2080	0	0.080	46.2	0	47.9	46.2	1.04
2	8540	12	0	2080	0	0.080	46.4	0	51.0	46.4	1.10
3	8540	12	0	2080	0	0.080	46.4	0	49.2	46.4	1.06
?.5R1	8540	12	-250	2370	-0.03	0.067	40.3	0	45.6	40.3	1.13
2	8540	12	-250	2370	-0.03	0.067	40.3	0	47.2	40.3	1.17
3	8540	12	-250	2370	-0.03	0.067	40.3	0	43.0	40.3	1.07
RP 1	6000	12	0	0	0	0.080	24.6	0	28.6	24.6	1.16
2	6200	12	0	0	0	0.080	25.4	0	26.4	25.4	1.04
3	6350	12	0	0	0	0.080	26.0	0	24.2	26.0	0.93
4	6350	12	0	0	0	0.080	26.0	0	27.4	26.0	1.06
5	6400	12	0	0	0	0.080	26.2	0	28.0	26.2	1.07
6	6400	12	0	0	0	0.080	26.2	0	32.0	26.2	1.22
).25T1	6590	12	855	1130	0.13	0.124	48.5	0	51.1	48.5	1.05
2	6590	12	855	1130	0.13	0.124	48.5	0	52.0	48.5	1.07
?.25T1	6785	12	- 20	2310	-0.003	0.079	32.4	0	35.2	32.4	1.09
2	6785	12	- 20	2310	-0.003	0.079	32.4	0	32.9	32.4	1.02
?.75T1	6785	12	-225	2575	-0.03	0.067	32.4	0	34.0	32.4	1.05
2	6880	12	-225	2575	-0.03	0.067	33.0	0	33.2	33.0	1.00
TP 1	6590	12	0	0	0	0.080	31.6	0	33.7	31.6	1.06
2	6785	12	0	0	0	0.080	32.4	0	32.1	32.4	0.99
3	6880	12	0	0	0	0.080	33.0	0	29.2	33.0	0.89

(continued)

Table III.3 (contd.)

1	2	3	4	5	6	7	8	9	10	11	12
0.75I1	4750	12	620	1104	0.18	0.135	38.4	0	46.6	38.4	1.21
2	4750	12	620	1104	0.18	0.135	38.4	0	46.1	38.4	1.20
3I1	5270	12	-100	1766	0.16	0.131	41.0	0	45.2	41.0	1.10
2	5270	12	-100	1766	0.16	0.131	41.0	0	50.8	41.0	1.24
3.5 I1	6210	12	-255	1925	0.14	0.126	48.2	0	54.1	48.2	1.12
2	6210	12	-255	1925	0.14	0.126	48.2	0	53.3	48.2	1.11
IP 1	6210	12	0	0	0	0.080	27.5	0	33.0	27.5	1.20
2	6700	12	0	0	0	0.080	29.6	0	30.0	29.6	1.01
3	6700	12	0	0	0	0.080	29.6	0	31.0	29.6	1.05
4	6700	12	0	0	0	0.080	29.6	0	34.0	29.6	1.15
5	6700	12	0	0	0	0.080	29.6	0	27.5	29.6	0.93
OR W 1	7180	12	1050	1050	0.14	0.126	45.8	8.2	50.6	54.0	0.94
2	7180	12	1050	1050	0.14	0.126	45.8	8.2	57.4	54.0	1.06
2RW 1	6500	12	0	2120	0	0.080	35.0	8.2	48.4	43.2	1.12
2	6950	12	0	2120	0	0.080	38.0	8.2	52.3	46.2	1.13
2.5RW1	6950	12	-260	2300	-0.03	0.067	33.0	8.2	53.4	41.2	1.29
2	6950	12	-260	2300	-0.03	0.067	33.0	8.2	52.4	41.2	1.27
RW1	7180	12	0	0	0	0.080	29.4	8.2	28.6	37.6	0.76
2	7180	12	0	0	0	0.080	29.4	8.2	30.1	37.6	0.83
3	6950	12	0	0	0	0.080	28.6	8.2	33.2	36.8	0.90
4	6950	12	0	0	0	0.080	28.6	8.2	29.2	36.8	0.79
7	7180	12	0	0	0	0.080	29.4	8.2	28.2	37.6	0.75
8	6500	12	0	0	0	0.080	26.6	8.2	29.8	34.8	0.86
9	6950	12	0	0	0	0.080	28.6	8.2	31.2	36.8	0.85
0.25TW1	6590	12	855	1130	0.13	0.124	48.5	6.5	50.2	55.0	0.91
2	6590	12	855	1130	0.13	0.124	48.5	6.5	54.4	55.0	0.99
2.25TW1	6785	12	-20	2310	-0.003	0.079	32.4	6.5	41.0	38.9	1.05
2	6785	12	-20	2310	-0.003	0.079	32.4	6.5	46.7	38.9	1.20

(continued)

Table III.3 (contd.)

1	2	3	4	5	6	7	8	9	10	11	12
2.75TW1	6880	12	-225	2575	-0.03	0.067	33.0	6.5	46.8	39.5	1.18
2	6880	12	-225	2575	-0.03	0.067	33.0	6.5	50.6	39.5	1.28
TW 1	6590	12	0	0	0	0.080	31.6	6.5	30.8	38.1	0.81
2	6785	12	0	0	0	0.080	32.4	6.5	29.3	38.9	0.76
3	6880	12	0	0	0	0.080	33.0	6.5	35.3	39.5	0.89
0.75IW1	4750	12	620	1104	0.18	0.135	38.4	7.2	54.2	45.6	1.19
2	4750	12	620	1104	0.18	0.135	38.4	7.2	50.2	45.6	1.10
3IW1	5270	12	-100	1766	0.16	0.131	41.0	7.2	59.7	48.2	1.24
2	5270	12	-100	1766	0.16	0.131	41.0	7.2	57.8	48.2	1.20
3.5IW1	5270	12	-250	1920	0.16	0.131	41.0	7.2	59.0	48.2	1.22
2	5270	12	-250	1920	0.16	0.131	41.0	7.2	61.2	48.2	1.27
IW 1	5270	12	0	0	0	0.080	25.4	7.2	36.8	32.6	1.13
2	5270	12	0	0	0	0.080	25.4	7.2	36.0	32.6	1.11
3	5270	12	0	0	0	0.080	25.4	7.2	36.9	32.6	1.13
IP 6	4750	12	0	0	0	0.080	23.0	7.2	34.9	30.2	1.16
7	5270	12	0	0	0	0.080	25.4	7.2	35.2	32.6	1.08

Negative sign indicates tensile stresses.

under combined bending and torsion:

i. Torsion failure by diagonal splitting of concrete;

ii. Flexure failure by diagonal compression of compression zone concrete after yielding of longitudinal steel.

III.4.1 Torsion Failure:

The analysis of a prestressed concrete under combined bending and torsion for torsion failure is similar to that of a eccentrically prestressed beam under pure torsion. The normal stress caused by the bending moment and prestress will vary along the depth of the beam. So the critical point for failure is that point where normal stress is minimum. Hence the equations derived for the case of eccentrically prestressed concrete beams under pure torsion, ie. Eqns. III.7 and III.8, are valid for this case also. For beams with transverse reinforcement, the torque contributed by transverse hoops can be calculated using Eqn. III.10 and hence the torque capacity of such beams can be known using Eqn. III.9 along with Eqns. III.7, ~~and~~ III.8 and III.10.

III.4.2 Flexure Failure:

The behaviour and mechanism of failure of a prestressed concrete beams after flexural cracking is similar and same as that of a reinforced concrete beam. Hence the strength of a prestressed concrete beam under combined bending and torsion

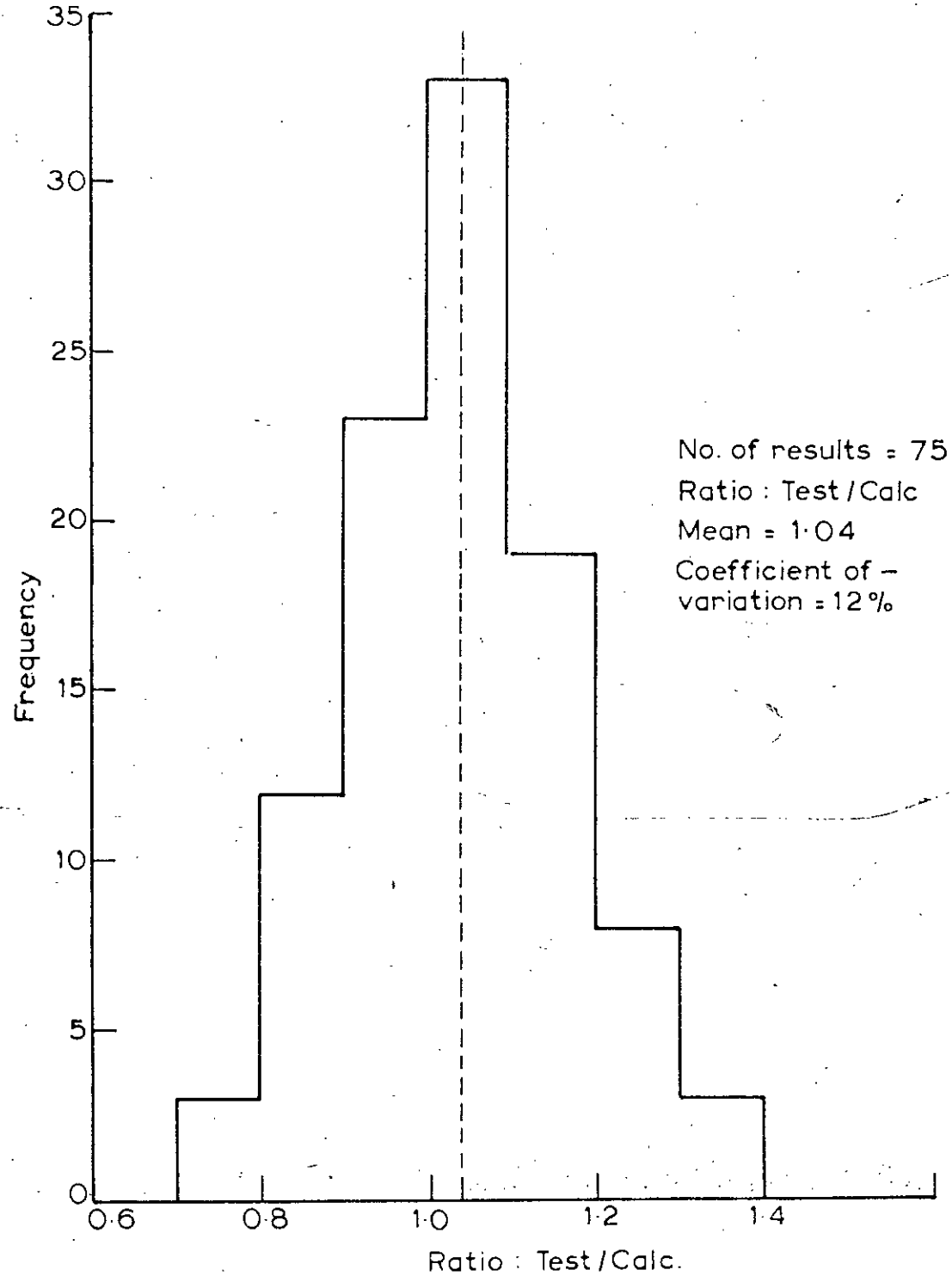


FIG. III.3. PRESTRESSED CONCRETE BEAMS UNDER COMBINED BENDING AND TORSION : COMPARISON OF RESULTS : HISTOGRAM.

failing in flexure can be calculated using the equations derived in the case of reinforced concrete beams for such type of failure. The detailed derivation of these equations have already been given in Part I of this thesis (see Ch. I.9) and hence it is not repeated here.

III.4.3 Comparison of Results:

In Table III.4 and III.5, the experimental results of various investigators[23,40,41,43] are compared with the theoretical results. Table III.4 gives a comparison for torsion failure and Table III.5 provides the same for flexure failure. From Tables III.4 and III.5 it can be seen that the comparison is fairly good. The histogram is shown in Fig.III.3

III.5 Concluding Remarks:

Using a failure criterion for concrete under compressive-tensile stresses, equations have been derived for the case of concrete beams under axial compression. Later the derivations have been extended to the case of prestressed concrete beams under pure torsion and under combined bending and torsion. On comparison with the test results of over two hundred beams in total, it has been found that the theoretical equations are good in predicting the test results.

TABLE III.4

PRESTRESSED CONCRETE BEAMS UNDER COMBINED BENDING AND TORSION : COMPARISON OF RESULTS (TORSION FAILURE)

Investigator	Specimen mark	f'_c psi	$\frac{f'_c}{f'_t}$	f (top)	f (bottom)	$\frac{f}{f'_c}$	M_{tc} inch-kips	M_{ts} inch-kips	M_t inch-kips Test	M_t inch-kips Calc.	Ratio Test/Calc.
1	2	3	4	5	6	7	8	9	10	11	12
Gardner[40]	1	5660	12	290	1860	0.19	18.5	0	21.2	18.5	1.15
	2	5160	12	500	1640	0.21	17.1	0	18.6	17.1	1.09
	3	5650	12	510	1640	0.19	18.4	0	22.2	18.4	1.21
	4	6530	12	950	1190	0.16	20.6	0	20.9	20.6	1.02
	5	6580	12	950	1190	0.16	20.8	0	21.0	20.8	1.01
	6	6400	12	1360	780	0.17	20.2	0	20.7	20.2	1.03
	7	6480	12	1770	370	0.17	20.4	0	19.1	20.4	0.94
	8	6020	12	1770	370	0.18	19.3	0	22.0	19.3	1.14
	9	6100	12	2240	-100	0.18	19.6	0	20.4	19.6	1.04
	10	5800	12	2660	-520	0.18	18.7	0	20.1	18.7	1.07
	11	5960	12	2690	-550	0.18	19.2	0	22.7	19.2	1.18
Reeves[41]	A4	7660	12	470	1035	0.09	23.2	0	23.0	23.2	0.99
	5	8210	12	-25	2080	-0.003	15.0	0	17.0	15.0	1.13
	6	8400	12	320	1315	0.11	26.4	0	23.0	26.4	0.87
	7	6650	12	410	1200	0.14	22.1	0	25.0	22.1	1.13
	8	7600	12	655	465	0.06	30.0	0	26.0	30.0	0.87
	9	6500	12	-150	2335	-0.02	12.0	0	13.9	12.0	1.16
	B3	6550	12	30	2120	0.20	21.0	0	18.5	21.0	0.88
	4	9000	12	-15	1945	-0.002	13.5	0	20.0	13.5	1.48
	7	5250	10	370	1140	0.16	18.5	0	21.2	18.5	1.1
	9	5550	10	815	515	0.09	25.5	0	26.1	25.5	1.0
	10	4900	10	755	490	0.10	24.1	0	24.3	24.1	1.0

(Continued)

Table III.4 (contd.)

1	2	3	4	5	6	7	8	9	10	11	12
Reeves[41]	C3	6180	12	65	1700	0.17	16.5	0	13.9	16.5	0.85
	4	6200	12	25	2115	0.21	18.0	0	13.9	18.0	0.78
	5	5130	10	500	1240	0.18	15.9	0	14.6	15.9	0.92
	6	5130	10	915	565	0.11	20.0	0	19.5	20.0	0.98
	7	4750	10	1245	-95	0.09	14.0	0	19.8	14.0	1.41
	10	4330	10	225	1650	0.25	14.3	0	15.6	14.3	1.09
	11	5280	10	820	660	0.12	20.9	0	18.3	20.9	0.88
	12	5320	10	1210	30	0.01	16.0	0	19.2	16.0	1.20
Cowan and Armstrong[23]	M1	5850	10	1300	1300	0.22	70.0	0	104.2	70.0	1.49
	J4	8460	12	- 5	2700	0	56.0	0	49.6	56.0	0.89
	J5	8300	12	2355	340	0.16	69.0	0	75.4	69.0	1.09
	J3	8650	12	2915	-220	-0.02	57.5	0	62.2	57.5	1.08
Narayan-swamy[43]	C14	8250	13	2612	32	0.004	125.0	0	140.2	125.0	1.14
	15	8250	13	2012	632	0.077	162.0	0	151.0	162.0	0.93

TABLE III.5

PRESTRESSED CONCRETE BEAMS UNDER COMBINED BENDING AND TORSION :

COMPARISON OF RESULTS (FLEXURE FAILURE)

Investi-gator	Speci-men mark	$\frac{f'_c}{f_t}$	λ	ϕ	M_b (test) inch-kips	M_{bo} (test) inch-kips	M_b/M_{bo} Test	M_b/M_{bo} Calc.	Ratio Test/Calc.
1	2	3	4	5	6	7	8	9	10
Hard-er[40]	12	12	3.82	0.122	177	218	0.81	0.75	1.07
	13	12	3.82	0.103	177	218	0.81	0.83	0.98
	14	12	3.82	0.108	177	218	0.81	0.81	1.00
Reeves[41]	A10	12	3.36	0.378	76	200	0.38	0.25	1.52
	11	12	3.36	0.270	95	200	0.48	0.38	1.26
	12	12	3.36	0.200	105	200	0.52	0.54	0.96
	13	12	3.36	0.248	113	200	0.56	0.40	1.40
	14	12	3.36	0.200	114	200	0.57	0.54	1.06
	15	12	3.36	0.161	128	200	0.64	0.66	0.97
	B 5	12	3.36	0.211	105	200	0.53	0.49	1.08
	6	12	3.36	0.359	74	200	0.37	0.26	1.42
	11	12	3.36	0.204	90	200	0.45	0.52	0.87
	12	12	3.36	0.171	114	200	0.57	0.63	0.91
	13	12	3.36	0.124	126	200	0.63	0.80	0.79
	14	12	3.36	0.215	90	200	0.45	0.48	0.94
	C 8	12	3.36	0.232	83	160	0.52	0.44	1.18
	9	12	3.36	0.151	102	160	0.64	0.70	0.92
	13	12	3.36	0.295	73	160	0.46	0.32	1.43
	14	12	3.36	0.182	92	160	0.58	0.58	1.00
	15	12	3.36	0.113	111	160	0.70	0.84	0.83

(Continued)

Table III.5 (contd.)

1	2	3	4	5	6	7	8	9	10
Cowan and Armstrong[23]	M 2	12	1.76	0.500	186	350	0.53	0.40	1.32
Narayana-swamy[43]	C 1	13	1.25	0.167	362	376	0.96	0.97	0.99
	3	13	1.25	0.167	370	376	0.98	0.97	1.01
	5	13	1.25	0.167	359	376	0.95	0.97	1.02
	7	13	1.25	0.167	360	376	0.96	0.97	0.99
	9	13	1.25	0.167	371	376	0.98	0.97	1.01
	16	13	1.25	0.100	373	376	0.99	0.99	1.00
	17	13	1.25	0.167	360	376	0.96	0.97	0.99
	13	13	1.25	0.200	350	376	0.93	0.94	0.99
	12	13	1.25	0.250	302	376	0.80	0.89	0.90
	S 1	13	1.53	0.167	478	500	0.96	0.94	1.02
	C11	13	1.25	0.333	237	376	0.63	0.76	0.83
	C 2	13	1.25	0.500	248	376	0.66	0.53	1.24
	4	13	1.25	0.500	223	376	0.59	0.53	1.11
	6	13	1.25	0.500	208	376	0.55	0.53	1.04
	8	13	1.25	0.500	256	376	0.68	0.53	1.28
	10	13	1.25	0.500	230	376	0.61	0.53	1.15
	18	13	1.25	0.500	243	376	0.64	0.53	1.21
	S 2	13	1.53	0.500	248	500	0.50	0.42	1.19

PART IV : SHEAR STRENGTH OF PRESTRESSED CONCRETE BEAMS

PART IV

CHAPTER IV.1

INTRODUCTION

IV.1.1 The need for research in this problem

From the existing literature one can easily understand how little the strength and behaviour of prestressed concrete beams under combined bending and shear has been understood. Unlike reinforced concrete structures, the knowledge of prestressed concrete structures under ultimate load conditions is essential because of the basic difference in the initial conditions of these two types of structures. But the behaviour of these two structures near ultimate load conditions are similar and same, although there are advantages from the points of view of cracking and stiffness in prestressed concrete structures. The recommendations by the codes of various countries for calculating the shear strength of prestressed concrete structures are basically different from those for reinforced concrete structures. In the case of prestressed concrete the shear stress at the working load as such is not considered critical. Instead, the greatest principal tensile stress is calculated in the uncracked part

of the beam and the shear strength is considered to be satisfactory if the principal tensile stress does not exceed a limiting value. Some of the codes also recommend that the principal tensile stress is to be checked under both working load and ultimate load conditions. In both the cases the checks are made on the uncracked part of the beam.

The design of structures by these calculations offers low factor of safety against failure and there are many flaws as indicated by Lorensten[141]. Also shear failure seldom occurs in uncracked part of the beam. As indicated by tests, failure generally arises in the cracked part or on the boundary between the cracked and uncracked parts. While equating the principal tensile stress to a limiting value, the present-day codes ignore the effect of principal compressive stress perpendicular to the principal tensile stress.

The number of investigations conducted so far to evaluate the shear strength of prestressed concrete beams are small - when compared to that of reinforced concrete beams. Attempts have already been made to formulate empirical equations to predict the shear strength of prestressed concrete beams. The moment-shear equations given by some of the authorities seem to be of the same form and differ only in the evaluation of certain parameters. These equations are of limited applicability within the ranges of their formulation.

As the behaviour of prestressed concrete beams under ultimate load conditions has been found to be similar to that of reinforced concrete there should be a common basis of calculation for both of these structures. This will bridge the gap between prestressed concrete and reinforced concrete. This is necessary if a fair comparison between these two types of structures is to be made.

Shear failures in prestressed concrete beams are proved to be more violent than those of reinforced concrete beams. In addition, as discussed earlier, prestressed concrete structures require inevitable check at ultimate load conditions. So a rational method of approach similar to the analysis of reinforced concrete beams is necessary to make the design methods more rational.

IV.1.2. Object and scope of the present investigation

The necessity of establishing a rational solution having a common basis of calculation both for reinforced and prestressed concrete beams has been stressed in the preceding article. The main object of this investigation is to establish such a solution. As this investigation was a minor part of the major investigation reported in this thesis, the author did not conduct any test on prestressed concrete beams. On the other hand, the experimental results reported by previous investigators were collected and they have been used in

explaining the mechanism of failure and to verify the theoretical predictions of the proposed theory and equations.

In the first phase of the work, the details of the other investigations have been collected. They have been reviewed critically so as to help the formulation of a new theory which, it is hoped will overcome the limitations of the existing formulae.

In the second phase, test observations of reinforced as well as prestressed concrete beamsreported in the existing literature have been compared and a common basis of calculating the shear strength of these beams has been established. Then the theory and equations proposed for reinforced concrete beams have been extended to cover prestressed concrete beams.

In the last phase of the work, the predictions by the theory have been compared with the test results available in the literature. On confirmation of the theory and equations, new design recommendations have been suggested for the design of prestressed concrete beams under combined bending and shear.

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CHAPTER IV.2

REVIEW OF EARLIER WORK

The problem of shear strength of prestressed concrete beams has been engaging the attention of many investigators. The existing works are critically reviewed in Table IV.2.1.

TABLE IV.2.1

SHEAR AND DIAGONAL TENSION IN PRESTRESSED CONCRETE BEAMS : REVIEW OF EARLIER WORK

Investigator/s. 1	Important Conclusions 2	General Comments 3
Sozen, Zwoyer and Siess[142,143] (1954)	Experimental work consisted of testing of 43 rectangular and 56 I-beams, primary variables being prestress, shear span, concrete strength and amount of longitudinal steel. An empirical equation has been suggested to calculate the shear strength of beams.	Experimental results indica that the behaviour of prestressed concrete beams in bending and shear is simila to that of reinforced concr beams. The influence of M/Vd ratio has not been included in the suggested equation.
Evans and Hosny[144] (1958)	In total 22 beams were tested and the influence of concrete strength, a/d ratio, amount of prestress and amount of longitudinal steel on shear strength of beams has been studied. Based on experimental results, equations have been framed to predict the shear strength of beams.	Test observations are in conformity with those of Sozen et al[142]. The parameter M/Vd ratio has not been included in the final equations.

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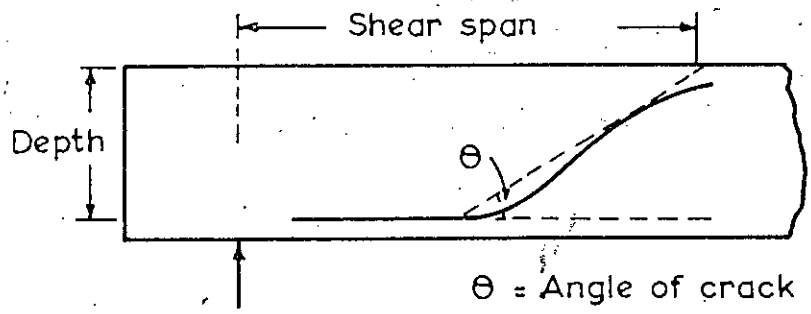
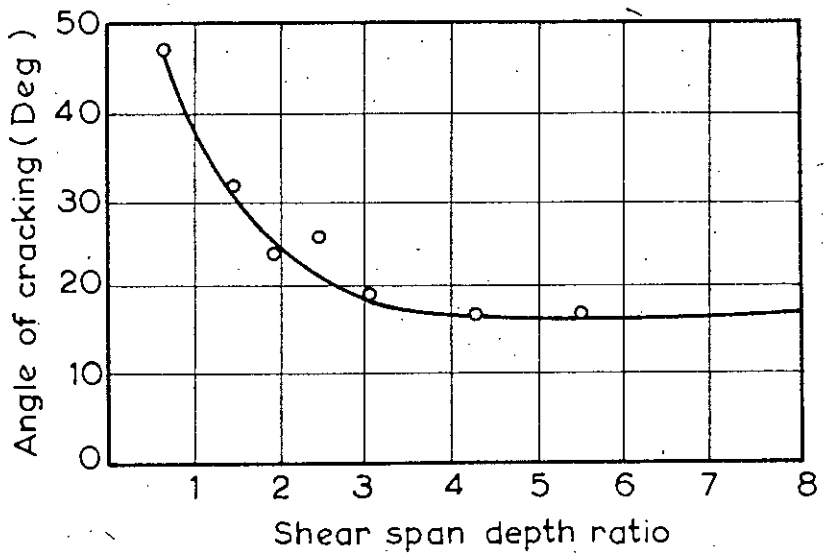
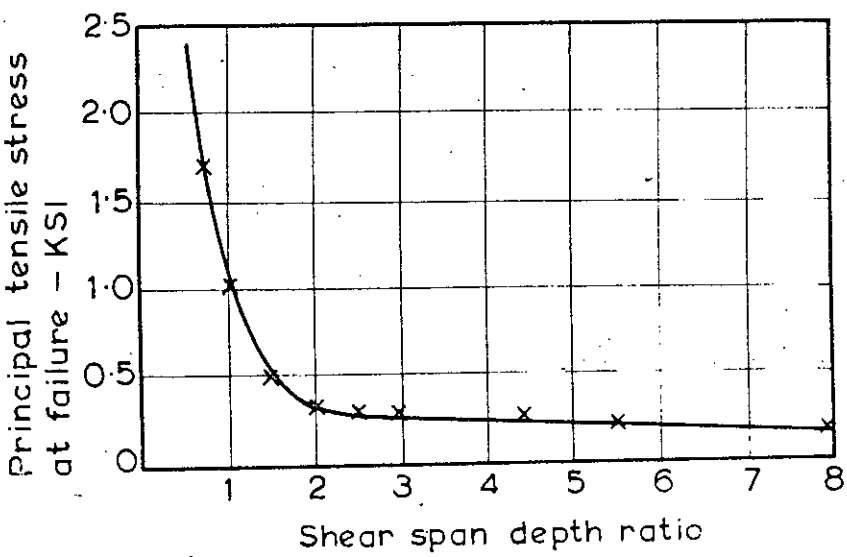




RELATION BETWEEN ANGLE OF CRACKING AND SHEAR SPAN TO DEPTH RATIO (After Hicks)

FIG. IV. 1.



RELATION BETWEEN PRINCIPAL TENSILE STRESS AT FAILURE AND SHEAR SPAN DEPTH RATIO.(After Hicks)

FIG. IV. 2

Table IV.2.1 (contd.)

1	2	3
Warner and Hall[145] (1958)	Test results describing the behaviour of simply supported beams under two-point loading have been reported. By considering the principal tensile stress, an equation has been derived to predict the diagonal tension strength of beams.	The effect of principal compressive stress acting perpendicular to the principal tensile stress has not been considered i the derivation of propose equation.
Hicks[146] (1958)	The effect of a/d ratio and concrete strength on shear resistance has been studied. The variations of the angle between the diagonal crack and the longitudinal axis of the beam and of principal tensile stress at failure with respect to a/d ratio have also been studied (See Figs. IV. 1 and 2)	From Fig. IV.2, it is clear that the principal tensile stress is not the which causes shear failur at all ranges of a/d rati
Sethu-narayanan[147] (1960)	Using the concept of failure of plate girders, an equation has been derived for calculating the shear strength of thin-webbed I-beams. In the analysis the shear span has been considered as a pin-jointed panel and the structure, as a whole has been analyzed for failure by equating the total normal stress to the limiting value of compressive strength of concrete.	In the analysis the effec of shear stress and that of other principal stress has been omitted. The limitation of the derived equation is not given.
Rusch and Vigerust[148] (1960)	Experimental work included testing of 25 I-beams with varying degree of prestress. Based on test results, equations have been derived for predicting the shear strength of beams.	The influence of M/Vd ratio has not considered in the derived equation.

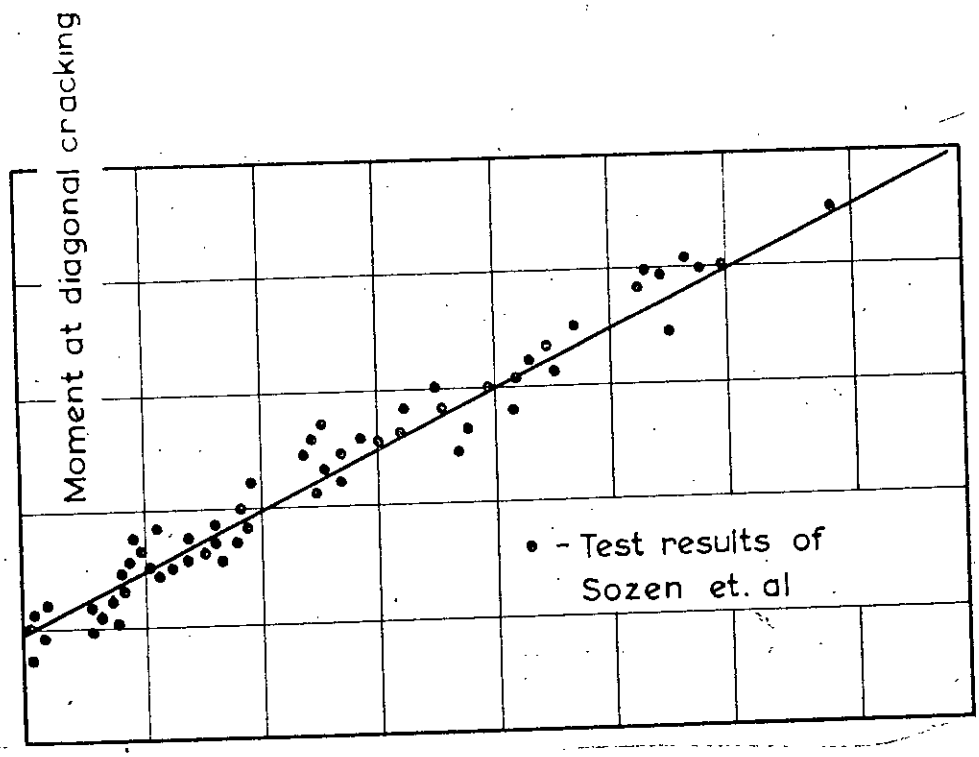
Table IV.2.1 (contd.)

1	2	3
Evans and Schumacher[149] (1963)	Experimental studies included testing of 54 beams, the major variables being amount of longitudinal steel, length of shear span, shape of the beam cross section etc. Based on experimental data, equations have been derived for predicting diagonal cracking strength as well as ultimate strength of beams.	Experimental results are very useful in understanding the behaviour of beams at different stages of loading.
Wilby and Nazir[150] (1964)	Test results on I beams with distributed loads have been reported. It has been found that the failure occurs at a distance of about 0.22 x span from the support. Based on test results, an equation has been derived to calculate the diagonal cracking strength of beams.	Test results indicate that the moment of failure is about 10% higher for beams with distributed loads than for beams with concentrated loads.
Hanson and Hulsbos[151] (1964)	Experimental studies included testing of 20 prestressed concrete beams with web reinforcement. Then the test results have been compared with the provisions of ACI Building Code.	Test results show that the ACI Building Code gives very conservative results in the case of low M/Vd ratios.
Kar[97] (1964)	The results of tests conducted on 56 beams have been reported. Based on a failure criterion for concrete under combined stresses and the results of tests, equations have been derived to calculate the shear strength of beams.	The analysis is the extensic of the theory proposed by Sen[93] for shear strength of reinforced concrete beams.

(continued)

Table IV.2.1 (contd.)

1	2	3
Arthur[152] (1965)	The results of two-point loaded shear tests on 50 I-beams without web steel have been presented. Based on experimental data, an equation has been suggested for predicting the diagonal cracking strength of beams.	The test results show that the design rules of ACI Code and British Code are rather very conservative for values of a/d less than 2.
Olsen et al[153,154] (1965)	Exhaustive study on beams with web steel has been reported. Based on test observations, two types of diagonal cracks (i.e. flexure-shear crack and web-shear crack) were distinguished and equations have been derived for these two cases.	The results of this investigation formed basis of the recommendations of Canadian Code.



INFLUENCE OF PRESTRESS ON DIAGONAL CRACKING MOMENT.

FIG. IV. 3

CHAPTER IV. 3

THEORETICAL ANALYSIS OF PRESTRESSED CONCRETE BEAMS UNDER COMBINED BENDING AND SHEAR

IV. 3.1 From the test observations reported in the literature, it is clear that the general behaviour of prestressed concrete beams under combined bending and shear is similar to that of reinforced concrete beams. The so-called various types of failures are also same for both prestressed and reinforced concrete beams. So it is obvious that the mechanism of failure is to be same for both these structures. However, the prestress increases the shear strength of beams (vide Fig. IV.3). Hence the basic principles of analysis of prestressed concrete beams are same as that of reinforced concrete beams discussed earlier. Similar to reinforced concrete beams, the analysis and mechanism of failure of prestressed concrete beams are discussed in the following pages under two heads, i.e.

i. Beams without shear reinforcement

ii. Beams with shear reinforcement

The analysis of beams with shear reinforcement confines itself to those with shear reinforcement in the form of vertical hoops.

IV. 3.2 Mechanism of Shear Failure:

In most of the laboratory investigations the idealized type of loading (vide Fig. IV.4) has been adopted even though this may

544

not represent the types of loading encountered in practice. This type of loading has two advantages: - firstly it is an easy type of loading and the ratio of moment and shear can be directly known from the value of shear span, a. Secondly, it simulates the effects of both bending moment and shear force. So the mechanism of failure will be discussed for this idealized type of loading and the theory and equations will be developed in terms of the shear span, a. Since the value of shear span represents the ratio between applied bending moment and shear force, say M/V, we can easily generalize the proposed theory and the resulting equations to any other types of loadings encountered in practice replacing the parameter a/d by M/Vd.

Referring to Fig. IV.4, the central region of this beam between the load points is free from shear force. Outside this range (i.e. between load points and support reactions) the beam is subjected to a constant shear force and the bending moment varies from zero over support reaction to the maximum at the load point. If the shear span is made stronger to avoid diagonal tension failure and if the central region is weak, flexure failure occurs finally by crushing of concrete in the compression zone or by failure of prestressing tendons. Otherwise, if the central region is made stronger, shear failure occurs. The mechanism of flexure failure of pre-stressed concrete beams is same as that of reinforced concrete

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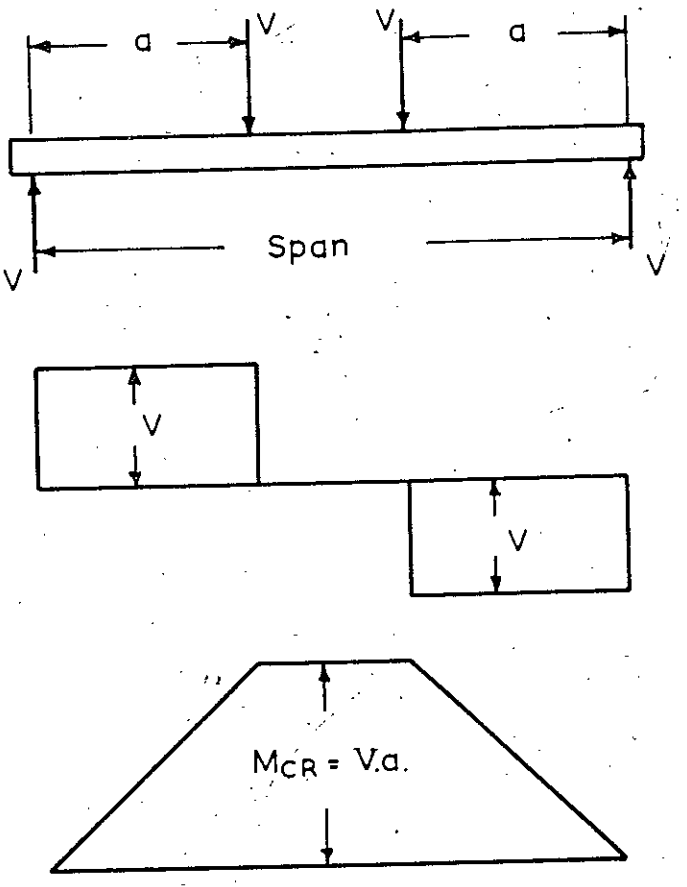
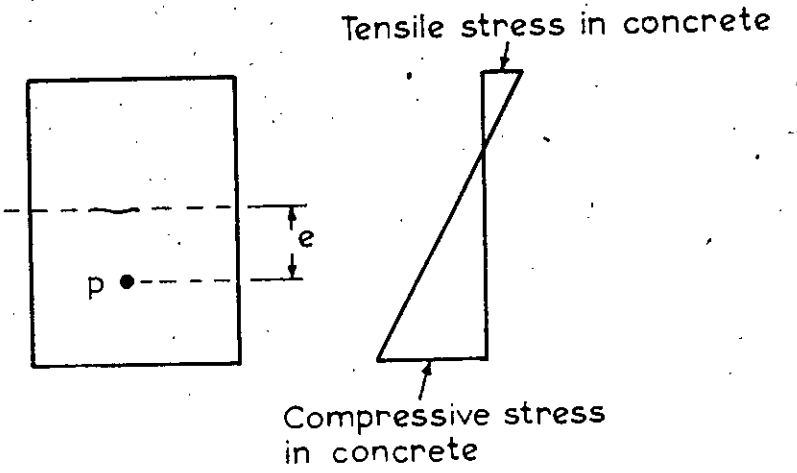


FIG. IV. 4. IDEALIZED TYPE OF LOADING.



p = Prestressing force

e = Eccentricity of prestressing force

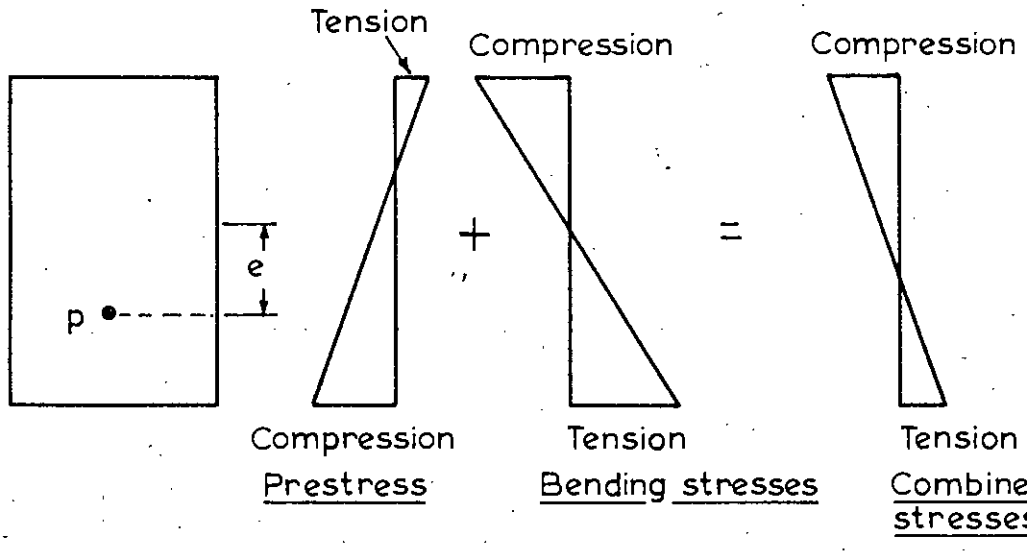
INITIAL STRESS CONDITIONS IN A PRESTRESSED CONCRETE BEAM.

FIG. IV. 5

beams. Since the mechanism of flexure failure is well known it need not be repeated here. So only the mechanism[8] of shear failure are discussed.

IV. 3.2.1 Beams without shear reinforcement:

As shown in Fig. IV.3, the prestress increases the shear strength of concrete beams. This is due to the initial stress conditions in concrete in a prestressed concrete beam before the application of loads. This initial stage of stresses in concrete is shown in Fig. IV.5. The eccentricity of prestressing force creates compressive stress near bottom fibres and tensile stresses near the top fibres. When the bending moment is applied, normal stresses which are parallel to the longitudinal axis of the beam, are produced to balance the applied bending moment. As the moment increases, the concrete in the tension zone of the beam is stressed more and more and a stage will come when the compressive stress due to prestress in the tension zone of the beam will be completely nullified. For further increments in moments the tension zone concrete is subjected to tensile stresses and when the tensile stress exceeds the limiting value, the first flexural crack will occur. Till the formation of flexural crack, the stress in the prestressing steel will be approximately equal to the prestress only. After the development of first flexural crack, the tension zone concrete fails and further resistance to the

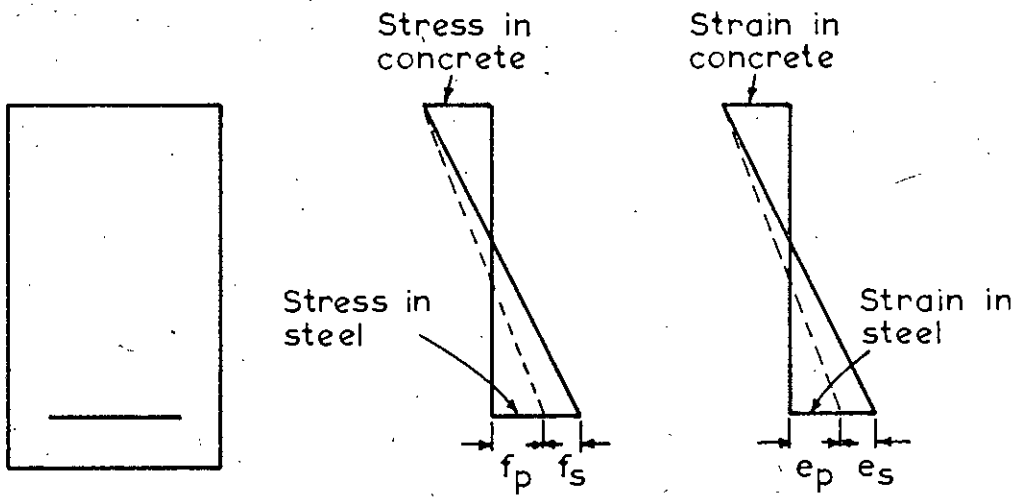


p - Prestressing force
e - Eccentricity

(When tensile stress equals tensile strength of concrete first flexural crack occurs)

STRESS CONDITIONS IN A PRESTRESSED CONCRETE BEAM BEFORE FLEXURAL CRACKING.

FIG. IV. 6.



f_p – Prestress in steel

f_s – Stress in steel due to external moment

e_p – Strain in steel due to prestress

e_s – Strain in steel due to applied bending moment.

CONDITIONS IN A PRESTRESSED CONCRETE BEAM JUST AFTER FLEXURAL CRACKING.

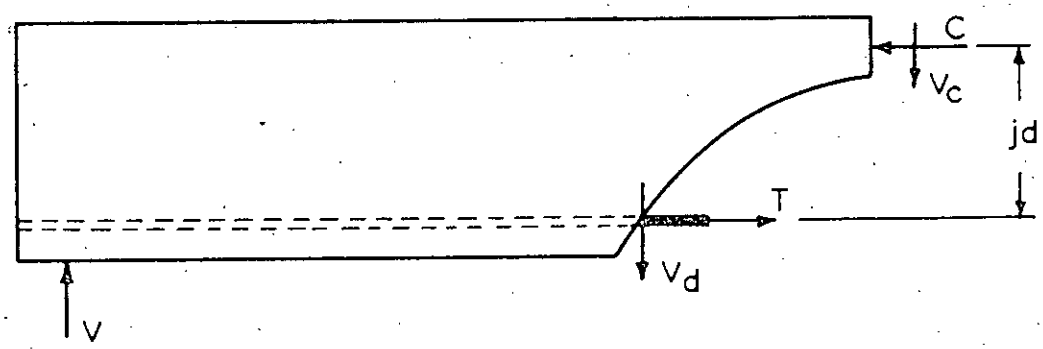
FIG. IV. 7.

stresses is given by the prestressing wires. This stage is shown in Figs. IV.6 and IV.7.

After the formation of first flexure crack, the behaviour of prestressed concrete beams is similar to that of reinforced concrete beams. For further increments in the external moment, the stress in steel and compressive stress in concrete will increase to balance the external moment. The flexure crack will move upwards and its penetration is limited by the relative load-carrying capacities of the steel and concrete. This crack almost extends to the neutral axis and stops there or any further penetration is very much limited.

In the shear span, along with bending moment, shear force also increases accordingly. An inclined tension crack (diagonal crack) appears at a point in the shear span where the shear stresses exceed their limiting value. In some cases, a flexural crack may transfer itself into a diagonal crack due to the action of shear stresses by deviating from its vertical direction to an inclined direction. The formation of this diagonal crack is accompanied by an increase in the inclined tensile stresses and hence the crack penetrates upwards for further increment in shear stresses. At this stage a redistribution of stresses takes place.

The conditions existing immediately after the redistribution of stresses are illustrated in Fig. IV.8. The part of the beam



REDISTRIBUTION OF STRESSES AFTER THE FORMATION OF DIAGONAL CRACK. (Beams without stirrups).

FIG. IV. 8.

to the left of the load is considered as a free body and the free body is kept in equilibrium by the following forces:

i. The reaction V

ii. Shear transferred by concrete, V_c

iii. Shear transferred by main steel as a dowel force, V_d

iv. The force in the tensile steel, T.

v. The compressive force acting on the concrete, C.

By static conditions we have

$$C = T$$

$$V_c + V_d = V$$

$$T\ jd = C\ jd = \text{External moment, } M_{CR}$$

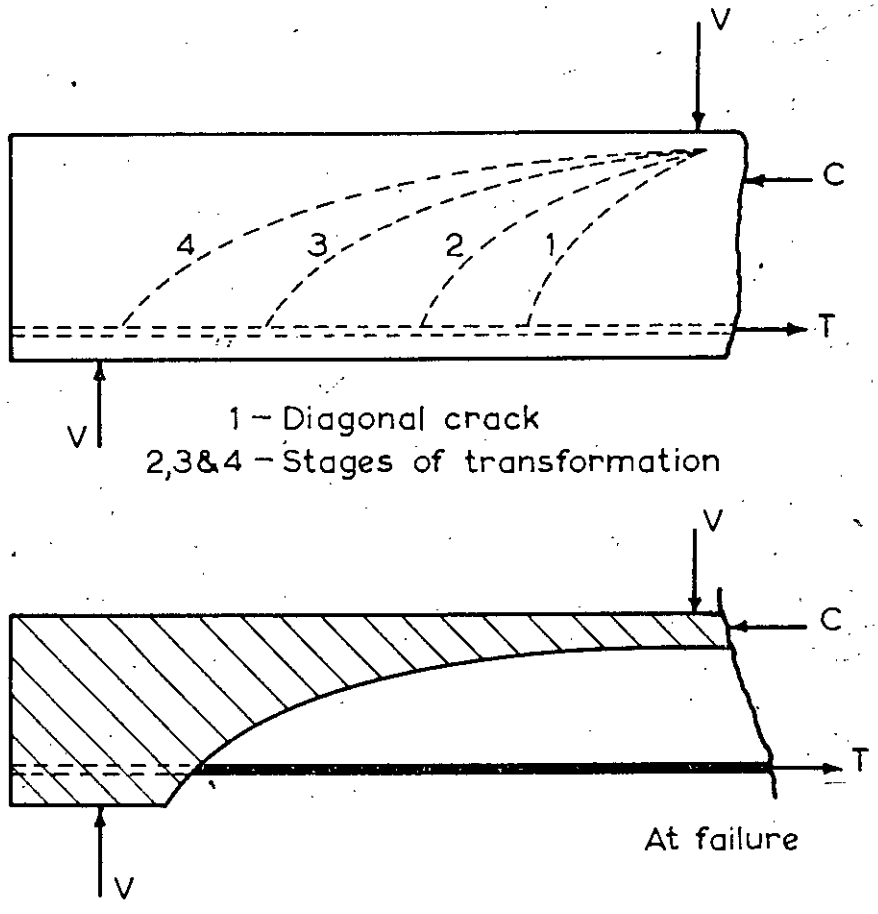
For further increments in loads, the shear stress in the compression block increases rapidly to balance the increasing shear force. After a certain stage, the distribution of shear force between compression zone concrete and the longitudinal prestressing steel changes. That is, near ultimate loads, the transfer of shear to steel ceases and the major portion of the shear is taken by the compression block concrete itself.

The effect of normal stress is to reduce the effect of shear stress. So the most critical point of failure is that point where the influence of normal stress is least. This

critical point is at the level of neutral axis where normal stress due to bending moment is equal to zero. So the concrete element at this critical point is subjected to a state of simple shear. The shear stress will cause two principal stresses which are of opposite nature and equal in magnitude and also equal to the shear stress. Therefore, failure of this element will occur when these compressive-tensile principal stresses satisfy failure criterion for concrete.

The diagonal crack also produces an increase in the compressive stress in concrete since the concrete area under compression is reduced due to extension of the diagonal crack. Since the diagonal crack penetrates upwards very rapidly, even for small increments in load, the other points in the compression zone will attain the critical stage soon causing failure of the entire compression block. As the extra load resisted by the beam after the failure of the concrete element at the neutral axis is small, conservatively we can take the strength of the beam as equal to the strength of the concrete element at the level of neutral axis. This mechanism of failure is valid for 'medium' and 'high' values of a/d ratios.

For 'low' a/d ratios, concentrated loads and the reaction from the supports have some local modifying influence on the principal stresses. As a result, in short shear spans, the



TRANSFORMATION OF BEAM INTO A TIED ARCH.

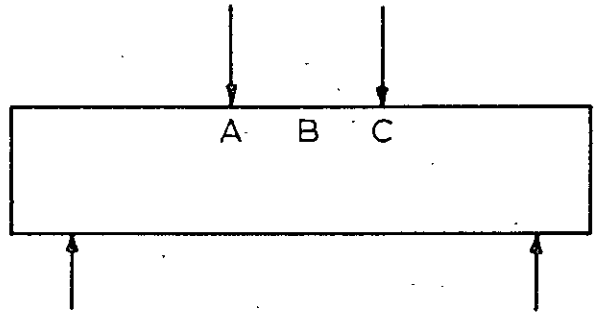
FIG. IV. 9.

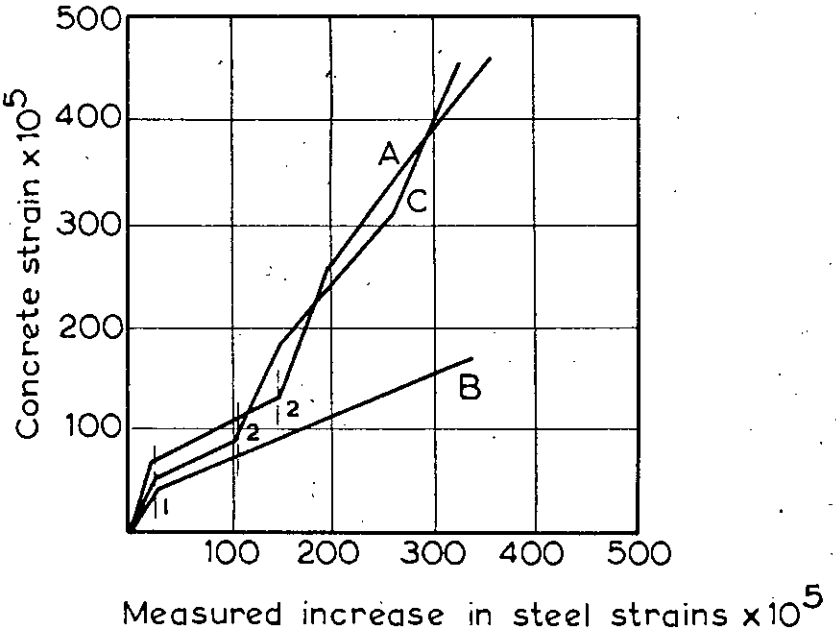
diagonal crack appears at a higher load. After the failure of concrete below neutral axis, the beam no more behaves truly like a beam. The remaining concrete area of varying depth along the length of beam transforms itself into a tied-arch with the prestressing wires as the tie-rod connecting both ends. This arch will find its support near support reactions of the original beam. This transformation of the beam into an arch is shown in Fig. IV.9.

The critical point for failure in the transformed arch is near the load points where the stresses are critical. The compressive stresses will increase enormously due to the reduction in the area resisting the compressive stresses near load points. The concrete element under the load, which is the critical element for failure, is subjected to three types of stresses:

a. the applied bending moment will produce compressive horizontal normal stress;
b. the concentrated load will produce a normal stress in the vertical direction;
c. the applied shear force will cause a shear stress.

These three types of stresses will combine together and will yield two principal stresses which are compression in nature. The major principal stress will be greater in magnitude than the





1 – Flexural crack
2 – Diagonal crack

RELATION BETWEEN CRITICAL STRAINS IN CONCRETE AND STEEL (After Sozen et. al[142]).

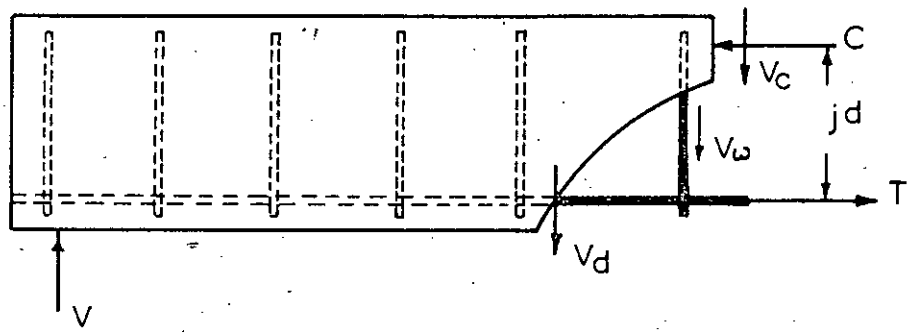
FIG. IV. 10.

normal stress caused by the applied bending moment. So the compressive strains in the concrete will be more than the corresponding strains in prestressing steel. The increase in compressive strains will be more rapid than that of steel strains showing the influence of the principal stresses. This argument is in agreement with test observations of Sozen, Zwoyer and Siess[142] shown in Fig. IV.10. From the above discussion, it is clear that only the compressive stresses cause failure. The failure of the critical element near the load point will occur when the principal stresses satisfy a failure law for concrete under compressive-compressive stresses.

IV.3.2.2 Beams with shear reinforcement:

As stated earlier, the shear reinforcement in the form of vertical stirrups only are considered in the analysis of present investigation. The main function of shear reinforcement is to delay the extension and widening of diagonal cracks. As discussed earlier, the formation of diagonal crack depends mainly on the properties of concrete. So the behaviour and mechanism of cracking of beams with shear reinforcement is similar to that of beams without shear reinforcement till the formation of diagonal crack. The shear reinforcement will be less effective till the diagonal cracking moment.

After the formation of diagonal crack, redistribution of stresses takes. The condition of stresses existing



REDISTRIBUTION OF STRESSES AFTER THE FORMATION OF DIAGONAL CRACK (Beams with stirrups)

FIG. IV. 11.

immediately after redistribution is similar to that of beams without shear reinforcement except that the total shear force will be shared now by the shear reinforcement also. Therefore $V = V_c + V_d + V_w$ where V_w is the shear taken by the stirrups. The state of stresses are shown in Fig. IV.11. For further increments in load, the axial stresses in the stirrups will rapidly increase to balance the increasing shear force and the quantity V_d will drop down. The transfer of shear force to the stirrups will continue until the shear reinforcement yields. Thus the shear strength of a beam with vertical hoops can be calculated by adding the contribution from stirrups to the strength of a beam without stirrups given by the mechanisms of failure discussed in the preceding article.

IV.3.3 Assumptions:

Since the analysis of prestressed concrete beams is similar to that of reinforced concrete beams, the assumptions made in the latter case (See Ch.II.6) are kept the same for the present case also.

IV.3.4 Theory:

RECTANGULAR BEAMS

IV.3.4.1 Beams without shear reinforcement:

Let V be the shear force acting at the section and M_{CR} be the corresponding bending moment at that section. The ratio between bending moment and shear force i.e. (M_{CR}/V) be a.

In the idealized case of point loading in a simply supported beam (Fig. IV.4), the ratio a is known as shear span of the beam. So we have $M_{CR} = Va$. As discussed in the article IV.3.2, we have to determine the strength of two mechanisms to know the shear strength of prestressed concrete beam i.e:

i. Strength of first mechanism which considers the strength of the concrete element at the level of neutral axis.

ii. Strength of the transformed arch which is valid for 'low' a/d ratios.

i. <u>For 'medium' and 'high' a/d ratios:</u>

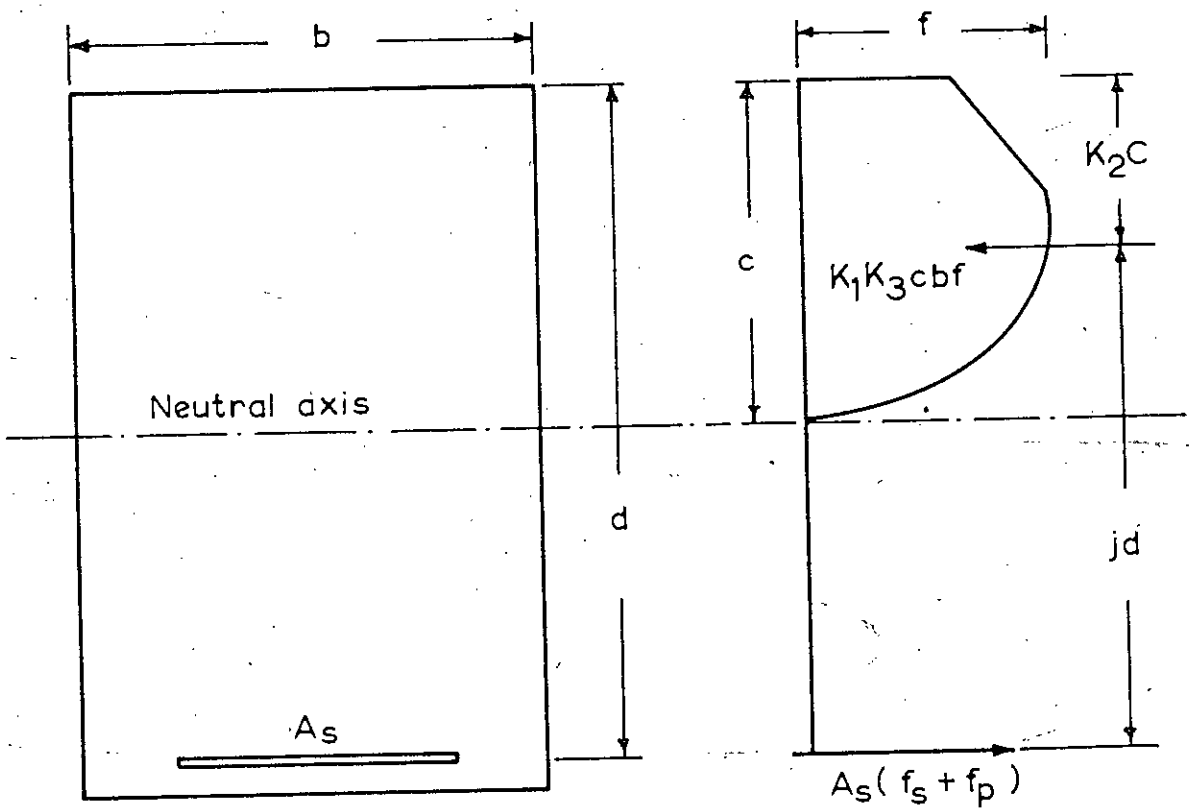
From the mechanism of failure, in beams without shear reinforcement only concrete in the compression zone of the beam resists the complete shear force coming at the section as the contribution from the dowel action of longitudinal steel ceases near collapse of the beam. Assuming rectangular distribution for the shear stress in the compression zone we have,

$$V = bc\tau \qquad \ldots \text{IV.3.1}$$

where τ is the shear stress in the compression zone of the beam,

$$\text{or } M_{CR} = Va = abc\tau \qquad \ldots \text{IV.3.2}$$

$$\text{or } \tau = \frac{M_{CR}}{abc} \qquad \ldots \text{IV.3.3}$$



BEAM WITHOUT SHEAR REINFORCEMENT.

FIG. IV. 12.

To determine the value of α from Eqn. IV.3.3 we require the value of depth of compression block c as other quantities are known.

The compression zone of a concrete beam is normally represented by the stress-strain curve for concrete which is non-linear in nature. Referring to Fig. IV.12 by static conditions we have,

$$C = T$$

$$\text{or } k_1 k_3 \, bcf = A_s (f_s + f_p) \qquad \ldots \text{ IV.3.4}$$

where f_p - prestress in steel

f_s - stress in steel due to the applied moment M_{CR}.

f - stress in concrete

or

$$\frac{c}{d} = \frac{p (f_s + f_p)}{k_1 k_3 f}$$

$$= \frac{1}{k_1 k_3} (q_s + q_p) = \frac{q}{k_1 k_3} \qquad \ldots \text{ IV.3.5}$$

where

$$q_s = \frac{A_s}{bd} \frac{f_s}{f} = \left(\frac{pf_s}{f} \right)$$

$$q_p = \frac{A_s}{bd} \frac{f_p}{f} = \left(\frac{pf_p}{f} \right)$$

and $q = (q_s + q_p)$.

In the above equation, the parameter q contains two parts: one due to prestress in steel and the other due to the applied moment, M_{CR}. These two values q_s and q_p can be approximately evaluated by the following method:

Adopting the same non-linear stress block we can write,

$$M'_{CR} = A_s f_s d \left(1 - \frac{k_2}{k_1 k_3} q_s \right) \qquad \ldots \text{IV.3.6}$$

where M'_{CR} is the extra moment added to the beam after the formation of first flexural crack.

$$\text{or } q_s = \frac{\left(1 - \frac{M'_{CR}}{A_s f_s d}\right)}{(k_2 / k_1 k_3)} \qquad \ldots \text{IV.3.7}$$

Putting $M'_{CR} \simeq 0.875\ A_s f_s d$, we can deduce that

$$q_s = \frac{0.125}{(k_2/k_1 k_3)} \qquad \ldots \text{IV.3.8}$$

From Eqn. IV.3.5, it is clear that the value of c is directly proportional to q and q_p. The term q_p is indirectly proportional to the stress in concrete f. So the minimum value q_p is obtained when $f = f'_c$. As the exact determination of f may not be possible, we can conservatively assume $f = f'_c$ so as to yield the least value of q_p.

Hence we can write now,

$$q = q_s + q_p = \frac{0.125}{(k_2 / k_1 k_3)} + \left(\frac{pf_p}{f'_c}\right) \qquad \ldots \text{IV.3.9}$$

Substituting the value of c from Eqn. IV.3.5 in Eqn. IV.3.3,

$$\tau = \frac{M_{CR}}{abd\left(\frac{q}{k_1 k_3}\right)} \qquad \ldots \text{IV.3.10}$$

where q is given by the Eqn. IV.3.9.

By the mechanism of failure, the critical point for failure is at the level of neutral axis where only shear stress exists. The two unlike principal stresses produced will be equal in magnitude to the shear stress, τ. So we have,

$$\sigma_1 = -\sigma_2 = \tau \qquad \ldots \text{IV.3.11}$$

The failure criterion for concrete under compressive-tensile stresses is after Krishnaswamy[69] and of the form:

$$\left(\frac{\sigma_1}{f'_c}\right) + \left(\frac{\sigma_2}{f'_t}\right)^2 = 1 \qquad \ldots \text{IV.3.12}$$

Substituting for σ_1 and σ_2 from Eqn. IV.3.11 in Eqn. IV.3.12 we get,

$$\left(\frac{\tau}{f'_c}\right) + \left(\frac{\tau}{f'_t}\right)^2 = 1 \qquad \ldots \text{IV.3.13}$$

Solving for τ we have,

$$\tau = f''_t \qquad \ldots \text{IV.3.}$$

where

$$f''_t = f'_t \left[\frac{\left\{\frac{2f'_c}{f'_t} - 1\right\}}{(2f'_c / f'_t)} \right]$$

Substituting for τ in Eqn. IV.3.10

$$M_{CR} = abd \left(\frac{q}{k_1 k_3} \right) f''_t \qquad \ldots \text{IV.3.}$$

The flexural capacity of a prestressed concrete beam can be put in the following form:

$$M_{FL} = A_s f_y jd \qquad \ldots \text{IV.3.}$$

where f_y is the proof stress of the prestressing steel.

Experimental results available in the literature[155-158] have shown that the Eqn. IV.3.16 is a fairly good approximation for under-reinforced beams as the influence of prestress on the ultimate moment in such cases is insignificant. After analyzing the test results reported in the literature, (details not given here) it has been found that the value 0.90 for j

in the Eqn. IV.3.16 is a good approximation. It is to be pointed out here that Cowan[158] has suggested that by taking $j = 0.90$ the strength of I and hollow sections can be calculated to a sufficient degree of accuracy. So we can put:

$$M_{FL} = 0.90\, A_s f_y d \qquad \ldots \text{IV.3.17}$$

Using Eqn. IV.3.17 and putting $k_1 k_3 = 0.800$ (see Appendix 3) the Eqn. IV.3.15 reduces to

$$\frac{M_{CR}}{M_{FL}} = \frac{(a/d)}{\alpha_{TR}} \qquad \ldots \text{IV.3.18}$$

where $$\alpha_{TR} = \left(\frac{0.72\, pf_y}{f''_t \cdot q}\right)$$

Assuming $f'_t = 8\sqrt{f'_c}$ and putting $k_2/k_1 k_3 = 0.525$ (see Appendix 3) the value α_{TR} can be calculated.

<u>For 'low' a/d ratios:</u>

As already discussed, for 'low' a/d ratios, the beam transforms itself into a tied-arch. Failure of the arch depends on the values of two principal stresses (both are compressive) caused by the interaction of two normal stresses and one shear stress. But as indicated earlier the determination of vertical normal stress due to the local effects of the

applied loads and reactions may not be possible. So only the horizontal normal stress and shear stress are considered in the present analysis and the effect of other normal stress is obtained by introducing a coefficient in the final equation of the analysis. The resulting principal stress is equated to the uniaxial compressive strength of concrete. Thus

$$\frac{f}{2}\left[1 + \sqrt{1 + \left(\frac{2\tau}{f}\right)^2}\right] = f'_c \qquad \ldots \text{IV.3.19}$$

where f is the horizontal normal stress due to the applied bending moment, M_{CR}.

Let

$$\left.\begin{aligned} M_{CR} &= k_1 k_3\, bcfjd \\ M_{FL} &= k_1 k_3\, bc_o f'_c\, jd \\ M_{CR} &= V \cdot a = abc\tau \end{aligned}\right\} \qquad \ldots \text{IV.3.20}$$

Substituting for f and τ from Eqn. IV.3.20 in Eqn. IV.3.19 and observing $k_1 k_3 = 0.800$ (see Appendix 3) and $j = 0.90$ we get,

$$\left(\frac{M_{CR}}{M_{FL}}\right) = \frac{c}{c_o}\left(\frac{2}{1+\sqrt{1 + 2/\,(a/d)^2}}\right) \qquad \ldots \text{IV.3.21}$$

where c_o is that particular value of depth of compression block when the beam has to fail in flexure.

Putting $\frac{c}{c_o} = \frac{d}{a}$, the validity of which has already been discussed in the case of reinforced concrete beams, the Eqn. IV.3.21 reduces to

$$\frac{M_{CR}}{M_{FL}} = \frac{2}{\left(\frac{a}{d}\right) + \sqrt{\left(\frac{a}{d}\right)^2 + 2}} \qquad \ldots \text{ IV.3.22}$$

As stated earlier, we can conservatively increase the above value of (M_{CR}/M_{FL}) by introducing a numerical coefficient k in the denominator, which is always less than 1.0, to account for the biaxial state of stress i.e. to care of the local effects of point loads.

Thus,

$$\frac{M_{CR}}{M_{FL}} = \frac{(2/k)}{\left(\frac{a}{d}\right) + \sqrt{\left(\frac{a}{d}\right)^2 + 2}} \qquad \ldots \text{ IV.3.23}$$

IV.3.4.2 <u>Beams with shear reinforcement</u>:

In beams with shear reinforcement, the total shear force acting at the section will be shared both by compression zone concrete and shear reinforcement in the form of vertical stirrups. Therefore,

total shear force = shear resisted by concrete
+ shear resisted by shear reinforcing hoops. ... IV.3.24

Shear resisted by vertical stirrups in a length of d of the beam $= A_w f_{yw} \frac{d}{s}$ ... IV.3.25

where A_w - total area of shear reinforcement

f_{yw} - yield stress of shear reinforcement

and s - spacing of hoops

Therefore total shear force $V = \left(A_w f_{yw} \frac{d}{s} \right) + bc\tau$ ... IV.3.26

or

$$\tau = \frac{V\left(1 - \frac{A_w f_{yw} d}{sV}\right)}{bc} \quad \text{... IV.3.27}$$

or

$$\tau = \frac{M_{CR}\left(1 - \frac{A_w f_{yw} d}{sV}\right)}{abc} \quad \text{... IV.3.28}$$

Proceeding similar to the derivations of beams without shear reinforcement, once again we can deduce two equations for the two mechanism of failures discussed earlier. Thus we get,

i. <u>For 'medium' and 'high' a/d ratios</u>:

$$\left(\frac{M_{CR}}{M_{FL}}\right) = \frac{a/d}{\alpha_{TR}\left(1 - \frac{A_w f_{yw} d}{sV}\right)} \quad \text{... IV.3.29}$$

ii. For 'low' a/d ratios:

$$\left(\frac{M_{CR}}{M_{FL}}\right) = \frac{(2/k)}{\left(\frac{a}{d}\right) + \sqrt{\left(\frac{a}{d}\right)^2 + 2\left(1 - \frac{A_w f_{yw} d}{sV}\right)^2}} \quad \ldots \text{IV.3.30}$$

IV.3.5 Flanged Beams:

The equations derived hitherto can also be applied to flanged beams. Even though the addition of flange to a rectangular beam increases its carrying capacity, it has been found that the increase is not uniform and depends on a number of factors. Some of the known factors which may affect this increase in strength are Moment/Shear ratio, prestressing force, cross sectional dimensions etc. But evaluation of this increase in strength by considering the influence of these parameters will lead to complicated solutions. Basically the strength of a flanged beam depends on the effective concrete area resisting the shear force. So two cases are to be discussed:

Case 1: Neutral axis in the flange

For the case when the neutral axis falls within the flange, the complete flange width will be effective in resisting the shear force. This case will arise only in the

case of beams with thick flanges. So the problem reduces to that of a rectangular beam with breadth equal to the flange width of the beam and the equations derived for rectangular beams are valid for this case.

Case 2: Neutral axis is in the web

If the neutral axis falls in the web, the effective concrete area resisting the shear depends on the effective width of the flange resisting the shear. As evinced by some of the experimental observations available in the literature near the final collapse, the projecting portions of the flanges will have a tendency to separate from the remaining section thus reducing the effective width of the cross section to the thickness of web. So, for this case also we can conservatively calculate the shear strength using the equations derived for rectangular beams taking the breadth of the rectangle as equal to the thickness of web.

But the exact determination of the position of neutral axis poses another difficulty because the number of parameters involved are too many. So an indirect approach has been adopted. After analysing all the test results reported in the literature, it has found that if flange thickness/ total depth $\geqslant$ 0.25 and if flange width / web thickness $\leqslant$ 3.0, the whole flange is active in resisting the shear force. Hence it can be assumed that for such cases, the shear

strength can be calculated as discussed in Case 1. Otherwise, the shear strength can be calculated as per the Case 2. It may be interesting to mention here that with the above limiting values, the moment of inertia of a symmetrical I - section is approximately equal to that of a rectangular section of same overall dimensions of the I-section.

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CHAPTER IV. 4

DISCUSSION OF RESULTS

IV.4.1 This chapter is devoted to the study of experimental results in the light of the theory and equations proposed in the previous chapter. The experimental results are also used to verify the theory and equations developed by the author. In addition, the influence of certain parameters on the shear strength of beams is studied. The experimental observations showing these influences are compared with the variations in shear strength of beams predicted by the theory. Prior to the discussion of analytical and experimental results, two more points are to be discussed regarding the equations derived in the previous chapter, i.e.

i. value of the biaxial coefficient k and
ii. demarcation between two different mechanism of failures discussed earlier.

IV.4.2 Value of biaxial coefficient, k.

In order to know the strength of the transformed arch and to use Eqn. IV.3.23 and IV.3.30, we require the value of the biaxial coefficient k introduced in these equations. As discussed earlier, the value of k depends on the relative

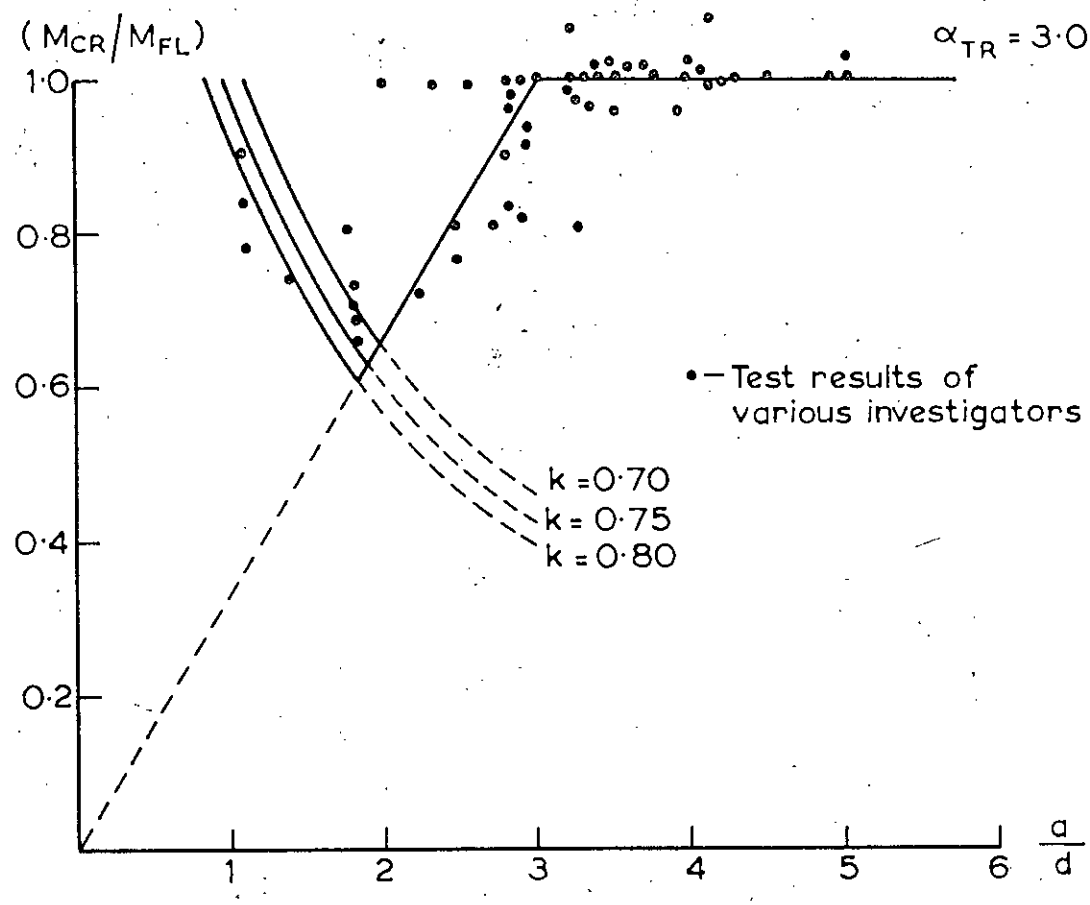
magnitude of the two principal stresses (both are compressive) developed. These principal stresses are caused due to interaction of two normal stresses and one shear stress. Of these stresses, the exact determination of vertical normal stress is not possible as the exact determination of local effects of vertical loads is not possible. Based on the test results of Acharya and Kemp[102] and the failure law given by Iyengar et al[112], the value of k has been taken equal to 0.75 for the case of reinforced concrete beams. The same value can be assumed in the case of prestressed concrete beams also.

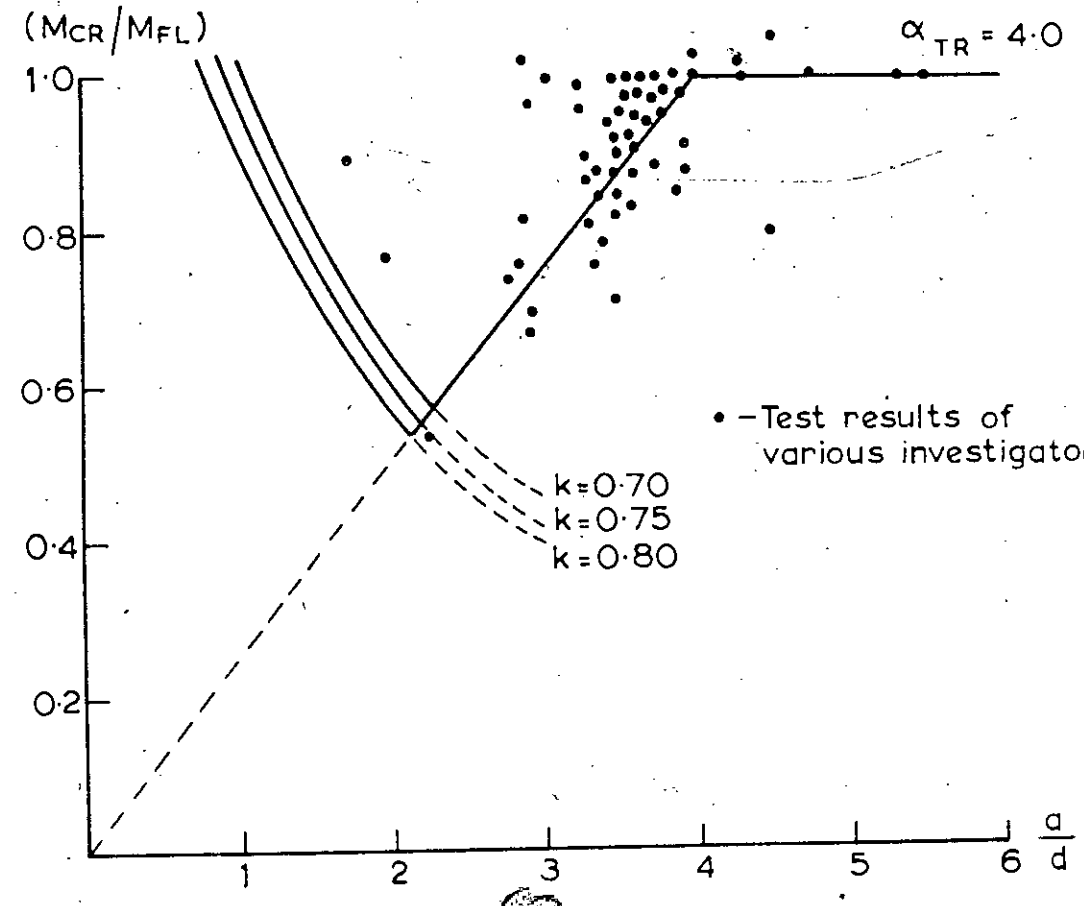
IV.4.3 Demarcation between two different mechanism of failure:

The theory and equations reported in this thesis were developed using two mechanisms of failures. The corresponding equations are applicable to two different ranges of a/d ratios defined earlier as 'medium and high a/d ratios' and 'low a/d ratios'. Using the different equations derived for the two types of mechanisms of failure and proceeding on similar lines with the derivations for the case of reinforced concrete beams, we can put thus:

i. If a/d is less than or equal to α_m, it is termed as 'low a/d ratio';

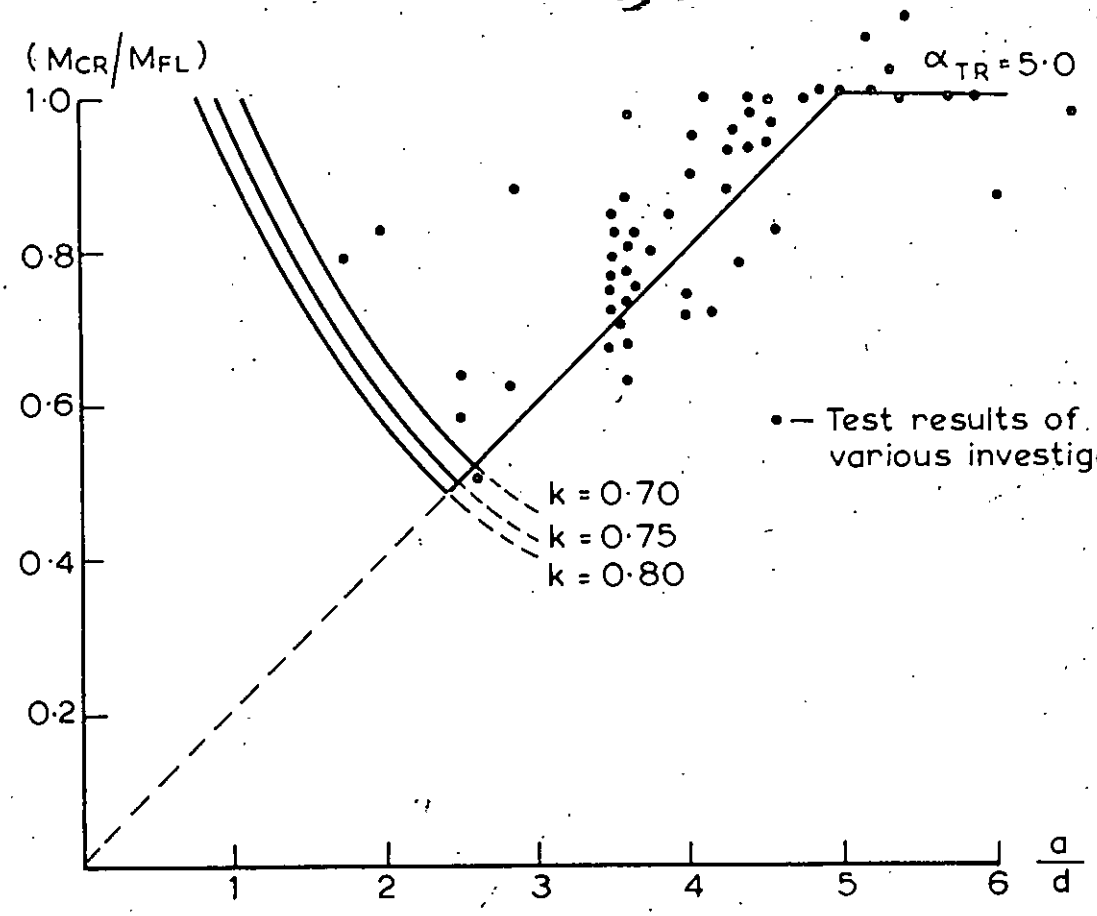
ii. If a/d is greater than or equal to α_m, it is termed as 'high or medium a/d ratio'

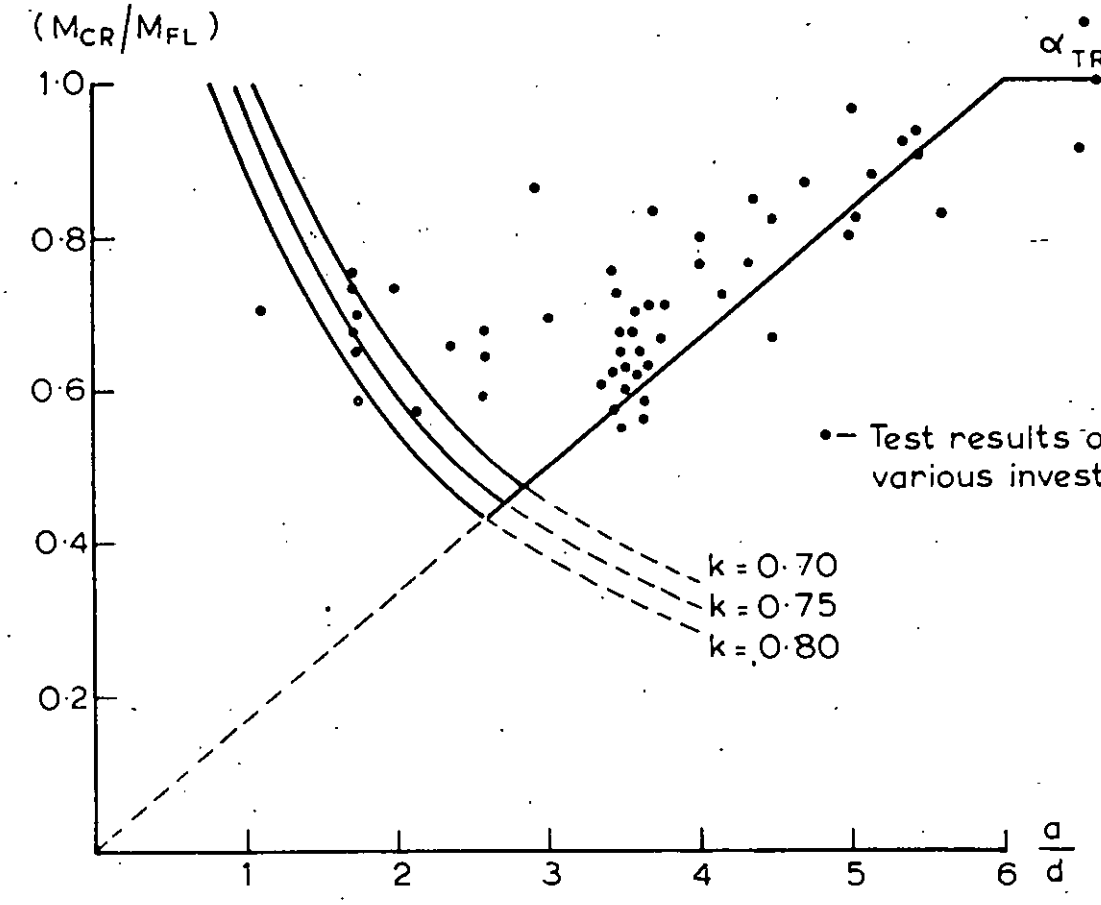




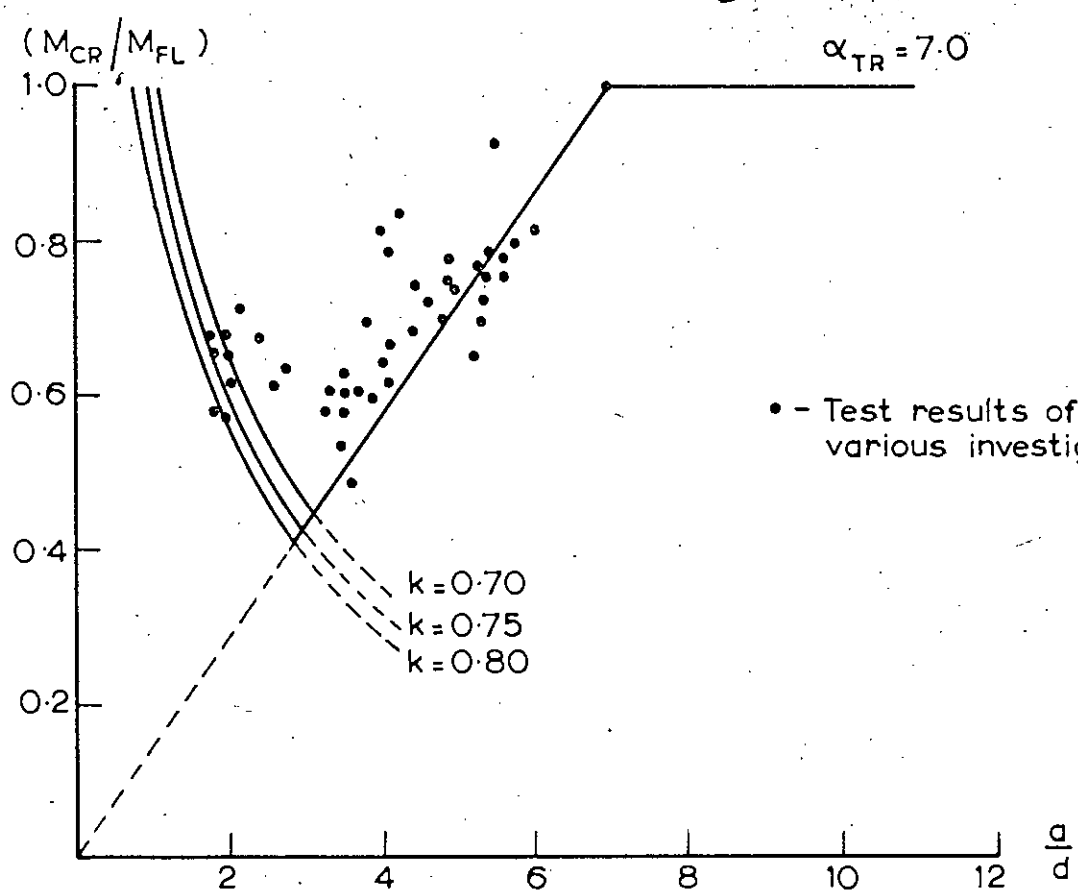
COMPARISON OF AUTHOR'S THEORETICAL LINES WITH THE TEST RESULTS OF VARIOUS INVESTIGATORS.

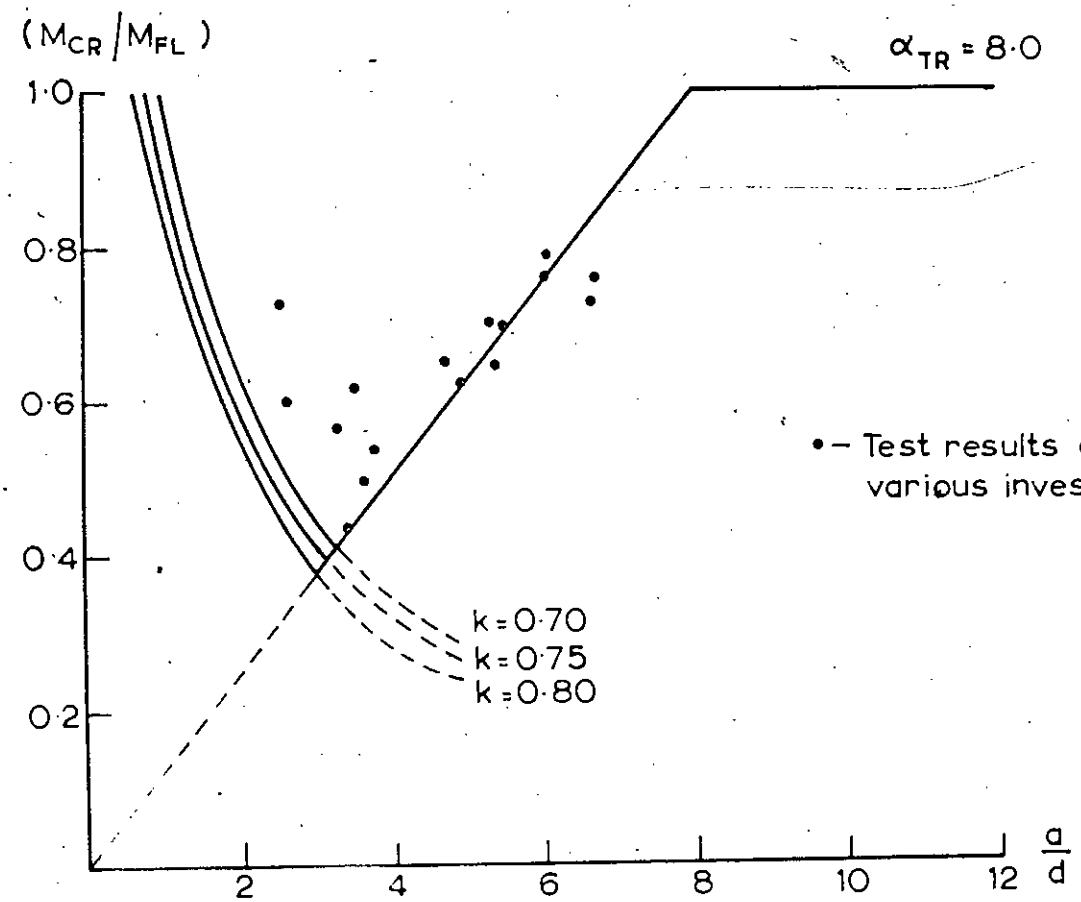
FIG. 3





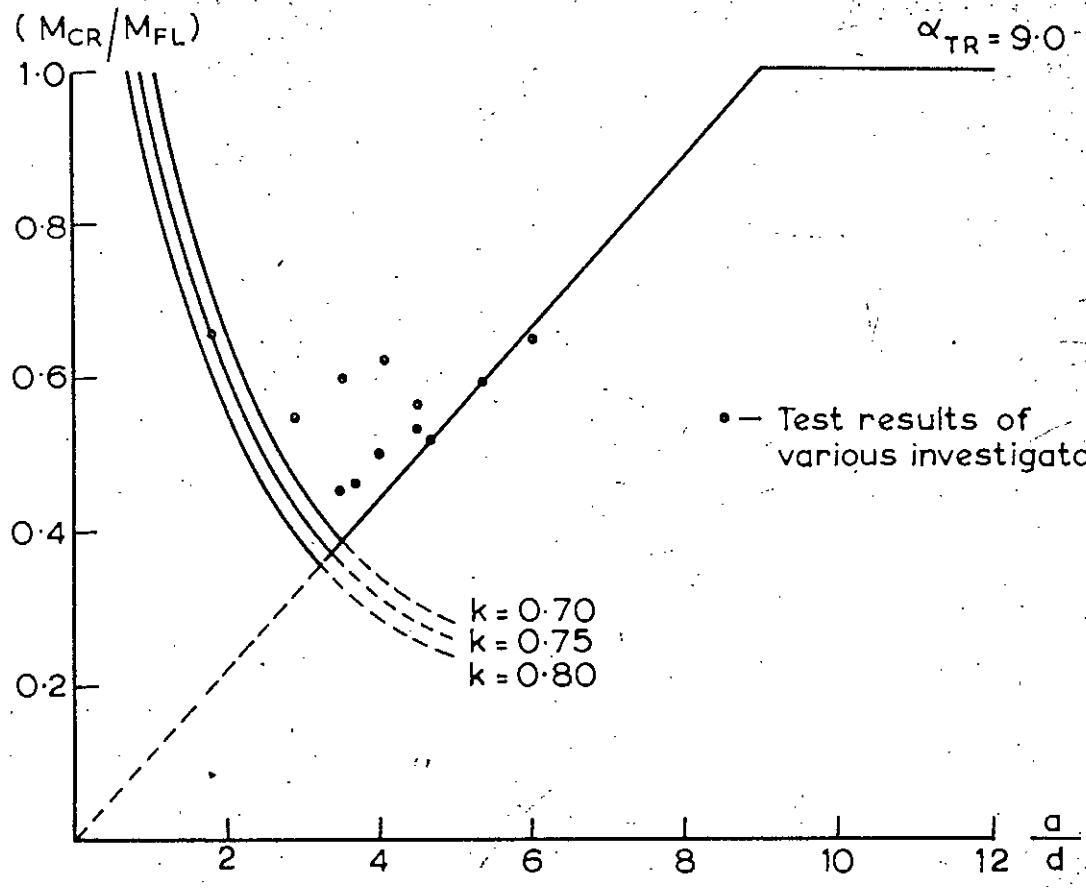
COMPARISON OF AUTHOR'S THEORETICAL LINES WITH THE TEST RESULTS OF VARIOUS INVESTIGATORS.

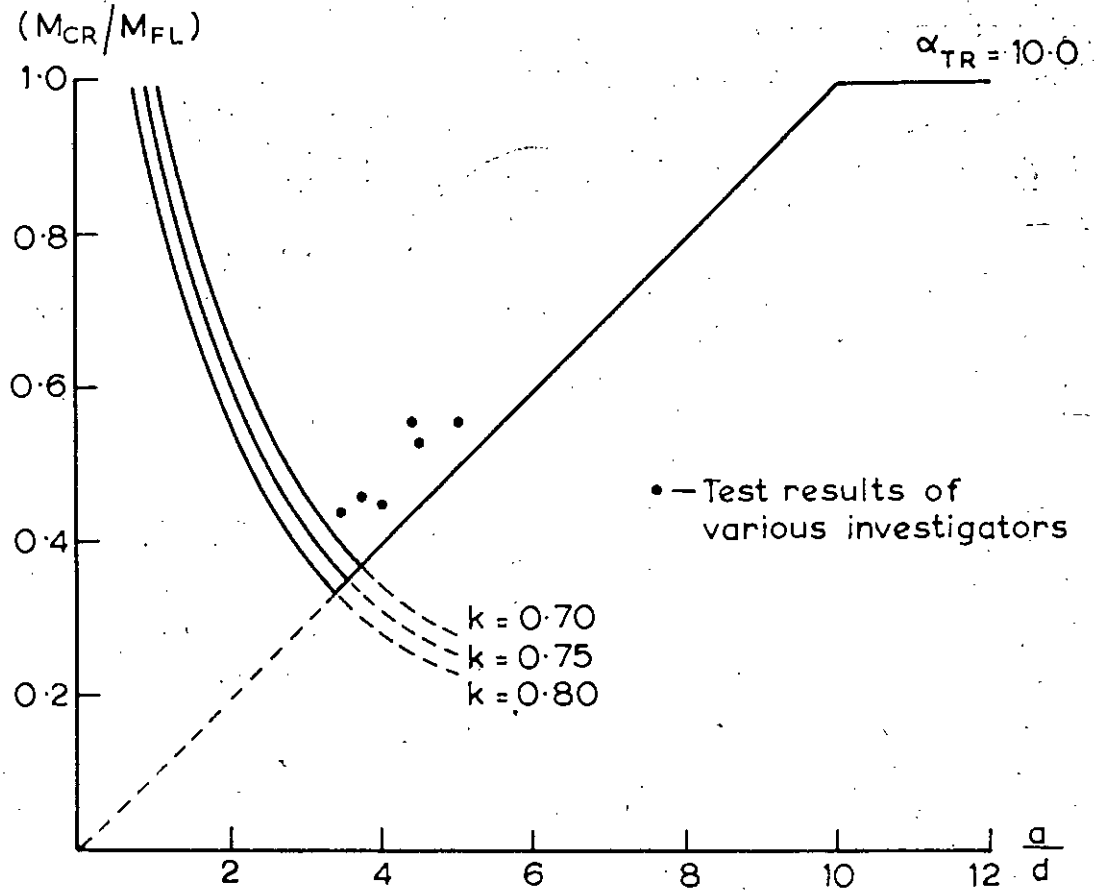




COMPARISON OF AUTHOR'S THEORETICAL LINES WITH THE TEST RESULTS OF VARIOUS INVESTIGATORS.

FIG IV 15





COMPARISON OF AUTHOR'S THEORETICAL LINES WITH THE TEST RESULTS OF VARIOUS INVESTIGATORS.

where

$$\alpha_m = \frac{(2/k)\alpha_{TR}}{\sqrt{\frac{4}{k}\alpha_{TR} + 2}}$$

for the case of beams without shear reinforcement and

$$\alpha_m = \frac{(2/k)\alpha'_{TR}}{\sqrt{\frac{4}{k}\alpha'_{TR} + 2\left(1 - \frac{A_w f_{yw} d}{sV}\right)^2}}$$

for the case of beams with shear reinforcement.

IV.4.4.1 Comparison of test results with calculated values

In order to verify the theory and the resulting equations proposed in the previous chapter, the author has collected large number of test results available in the literature[142,143,146-153]. A qualitative picture of the comparison is provided in Figs. IV.13 to IV.16. (The details of comparison are being included in a forthcoming publication of the department). From the comparison it can be seen that the predictions by the theory are fairly good. The results of comparison are summarised below:

Type of beam	No. of results.	Ratio: Tests/Calc	
		Mean	Coefficient of variation
Beams without shear reinforcement	300	1.00	15%
Beams with shear reinforcement.	140	1.09	12%
In total	440	1.04	14%

It is to be pointed out here that there is wide variation even in the experimental results as seen from the Figs. IV. 13 to 16.

IV.4.4.2 Limitation of the theory:

Consider the Eqn. IV.3.23 applicable for low a/d ratios. At a/d = 1.0, $(M_{CR}/M_{FL}) \simeq 1.00$ and for values of a/d less than 1.00, (M_{CR}/M_{FL}) is greater than unity and reaches a value of $(\sqrt{2}/k)$ at a/d = 0 which corresponds to 'pure' shear case. Obviously this is not correct as at a/d = 0, (M_{CR}/M_{FL}) should be equal to zero. So for a/d ratios less than 1.00, Eqn. IV.3.23 is not applicable and the exact variation of (M_{CR}/M_{FL}) for a/d ratios between 0 and 1.0 is not known as there are no sufficient test results to define this part of the curve.

IV.4.5 Effect of compression block coefficients on the theory:

As discussed in the case of reinforced concrete beams, it is necessary to study the influence of compression block coefficients k_2, k_1, k_3 on the theoretical results. The ratio k_2/k_1k_3 has been used in the derivation of the Eqn. IV.3.16. It has been shown by Hognestad et al[113] that the ratio (k_2/k_1k_3) may vary between 0.45 to 0.60 in most of the cases. The author has used a value equal to 0.525 for k_2/k_1k_3 (see Appendix 3). Using the result given for this value of

k_2/k_1k_3 as basis, the results obtained using 0.45 and 0.60 have been found to vary by ± 12%.

Similarly the coefficient k_1 k_3 has been used in the derivation of the Eqn. IV.3.23. In the reinforced concrete section, it has been shown (vide Fig. II.26) that the variation in k_1 k_3 has negligible influence on the final results. Since the Eqn. IV. 3.23 is same as the companion equation derived for the case of reinforced concrete beams, it can be said that the coefficient k_1 k_3 has no significant influence on the final results obtained from Eqn. IV.3.23.

IV.4.6 Diagonal cracking strength of beams

The prediction of diagonal cracking strength is an important aspect of the present investigation. It has high practical value as most of the codes and recommendations restrict the shear strength of beams to its diagonal cracking strength. After close observation of experimental results available in the literature, the author considers two aspects of the problem:

i. Diagonal cracking strength before the formation of flexural crack;

ii. Diagonal cracking strength after the formation of flexural crack.

In the case of low values of M/Vd ratio a diagonal crack may appear even before the formation of flexural cracks.

On the other hand in the case of high values of (M/Vd) ratio, the beam may diagonally crack only after the formation of flexural cracks. In between these two extreme cases, there is a transition stage at which both the cracks may form simultaneously or the flexural crack may transform itself into a diagonal crack. Hence the diagonal cracking strength of beams is to be evaluated for these three cases.

IV.4.6.1 Diagonal cracking strength in case of low values of M/Vd ratio : (Flexural crack is absent)

Let f and τ be the normal and shearing stresses, respectively at the critical point at which the diagonal crack may originate. Then the principal stresses are:

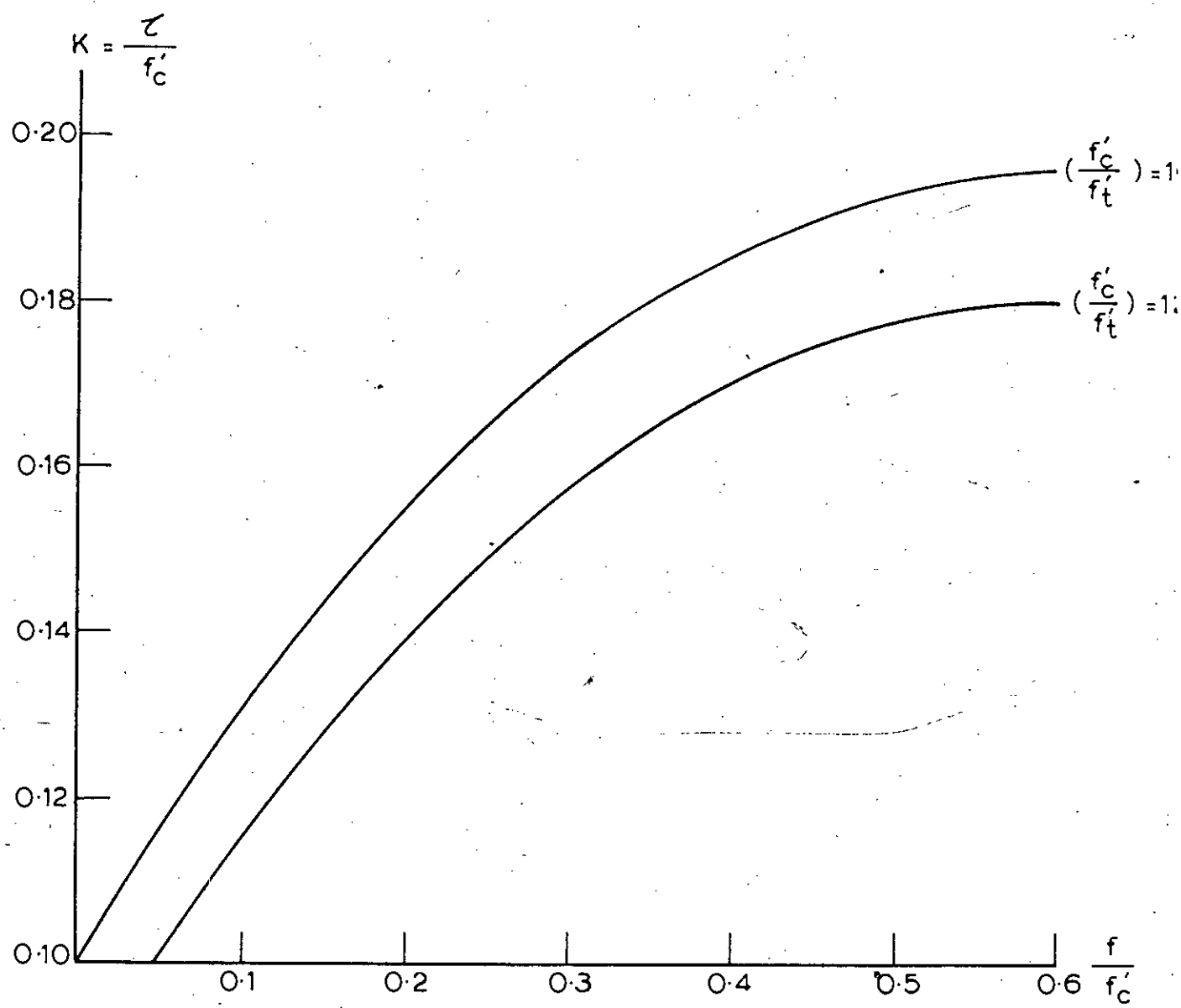
$$\sigma_1 = \frac{f}{2}\left[\sqrt{1 + \left(\frac{2\tau}{f}\right)^2} + 1\right] \text{ (compressive)}$$

$$\sigma_2 = \frac{f}{2}\left[\sqrt{1 + \left(\frac{2\tau}{f}\right)^2} - 1\right] \text{ (tensile)} \qquad \text{... IV.4.1}$$

For diagonal crack to form these two principal stresses should reach the limiting state satisfying the failure criterion discussed earlier (Eqn. IV.3.12). So substituting for σ_1 and σ_2 in Eqn. IV.3.12, we have

$$\frac{f}{2f'_c}\left[\sqrt{1 + \left(\frac{2\tau}{f}\right)^2} + 1\right] + \left[\frac{f}{2f'_t}\left\{\sqrt{1 + \left(\frac{2\tau}{f}\right)^2} - 1\right\}\right]^2 = 1$$

... IV.4.2



VARIATION OF K WITH f / f_c'.

FIG. IV. 17.

Solving the Eqn. IV.4.2 for $\frac{\tau}{f'_c}$, we can write,

$$\left(\frac{\tau}{f'_c}\right) = \phi \left(\frac{f}{f'_c}\right) \qquad \ldots \text{IV.4.3}$$

where the parameter ϕ accounts for the interaction between the stresses. The Eqn. IV.4.3 can be diagramatically represented for different values of f'_c/f'_t. Fig. IV.17 shows such an interaction diagram. Using Fig. IV.17 the value of τ/f'_c for any particular value of f/f'_c can be easily known.

After establishing the relationship between the stresses, it is now necessary to know the magnitudes of stresses. Since the section is uncracked the stresses can be evaluated using the elementary strength of materials formulae. According to the elementary theory, the shear stress varies parabolically with its maximum value at the centroid of the section. The normal stress varies linearly across the depth of the cross section and its magnitude at various depths is the algebraic sum of the normal stresses caused by the external moment and the prestressing force.

Experimental results available in the literature[142,153] have shown that the critical point for diagonal cracking in an uncracked beam is the centroid of the section. The magnitude of f can be easily calculated by knowing the stresses at the centroid caused by prestressing force, external moment and

dead load of the structure. For ordinary beams $f = \frac{P}{A}$ where P is the prestressing force and A is the gross area of cross section. Similar expressions can be written for other cases such as composite section, etc. The shear stress at the centroid of the section can be calculated using the expression:

$$\tau = \frac{V_{cw}}{Ib} \cdot Q \qquad \ldots \text{ IV.4.4}$$

where V_{cw} is the shear force causing diagonal cracking

I is moment of inertia of the cross section about centroid.

b is width of web at centroid and

Q is the first moment of the area below or above centroid taken about the centroid of the section.

Let $\frac{\tau}{f'_c} = K$

From the given properties of a beam the value of K can be known with the help of Fig. IV.17.

So we get, from Eqn. IV.4.4

$$V_{cw} = K \left(\frac{Ib}{Q} \right) f'_c \qquad \ldots \text{ IV.4.5}$$

The above Eqn. IV.4.5 can be suitably modified for the case of composite sections.

It is to be pointed out here that in the calculation of principal stresses the effect of vertical normal stress caused by the local effects of reactions and loadings has been ignored assuming its influence on the final results as negligible.

IV.4.6.2 Diagonal cracking strength in the case of high values of M/Vd ratio (Flexural crack is present)

In discussing the mechanism of failure for high and medium M/Vd ratios, the concrete element at the level of neutral axis has been considered as the critical one for failure and its strength has been taken as the shear strength of the beam. Since the difference between ultimate load and diagonal cracking load in the case of high M/Vd ratios is insignificant and negligible, the Eqn. IV.3.18 can be used to predict the diagonal cracking load. The Eqn.IV.3.18 can be put in the following form:

$$V_{cd} = bdf''_t \frac{q}{k_1 k_3} \qquad \ldots \text{IV.4.6}$$

or substituting for $k_1 k_3 = 0.800$ we get,

$$V_{cd} = 1.25\, bdf''_t\, q \qquad \ldots \text{IV.4.7}$$

where V_{cd} is the diagonal cracking shear force in the case of high values of M/Vd ratio.

IV.4.6.3 Diagonal cracking strength in the transition stage:

As discussed earlier, in certain cases a flexural crack may transform itself into a diagonal crack. These cases form a transition between the diagonal cracking strength of cracked and uncracked beams. The exact determination of the shear force at the transformation of flexural crack into a diagonal crack may not be possible as the stress field above the crack is a complicated one. However, as shown by experiments, the shear carried by the beam after flexural cracking in such cases is small. So we can conservatively take the shear force at flexural cracking as the diagonal cracking shear force for this type of diagonal cracking. So we have,

$$V_{cf} = \frac{M_{cf}}{a} = \left[(f_r + f_{pc}) \frac{I}{yd}\right] \frac{1}{\alpha} \qquad \ldots \text{ IV.4.8}$$

where V_{cf} = diagonal cracking shear force in the transition stage

M_{cf} = bending moment at flexural cracking

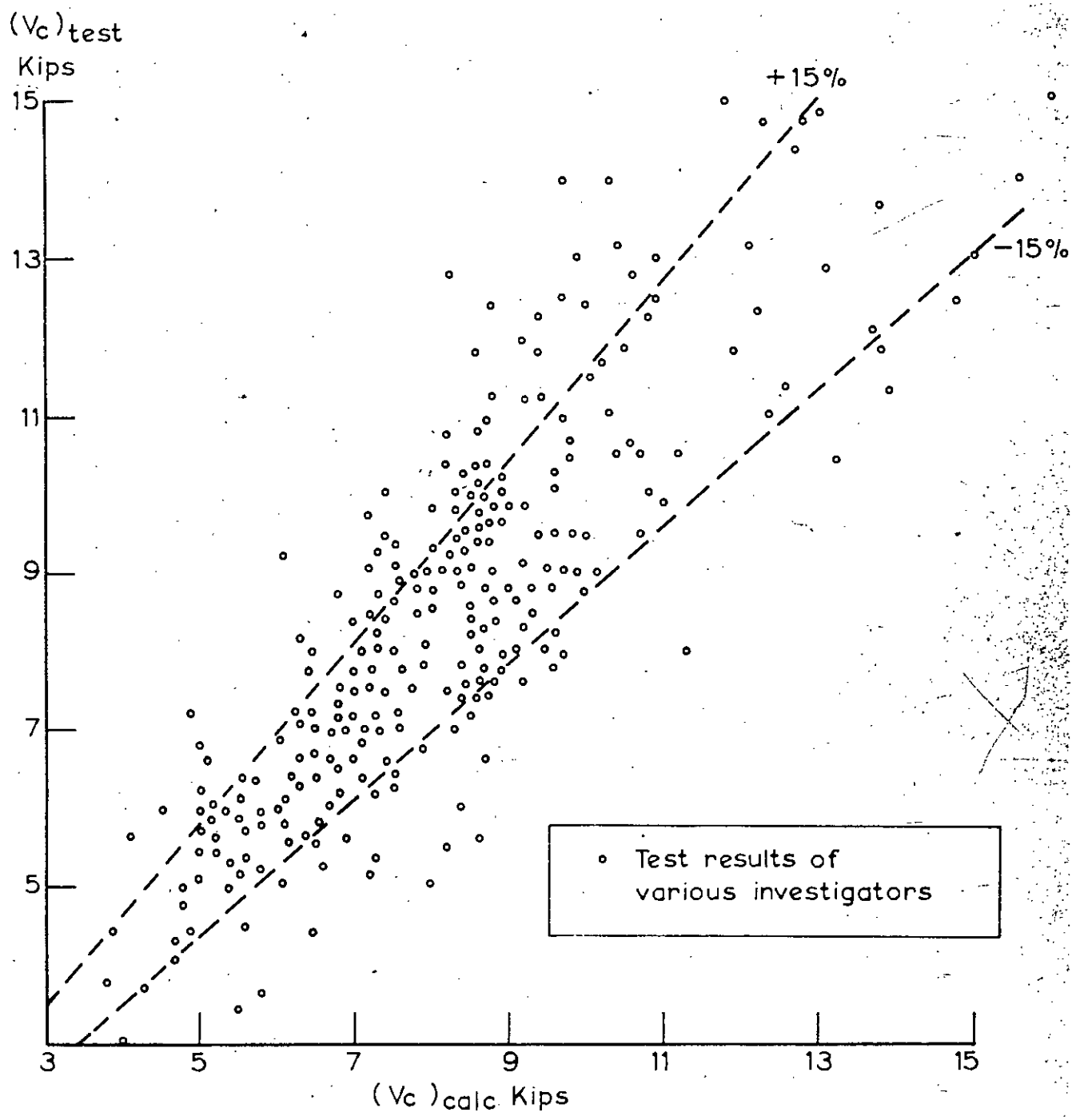
f_r = modulus of rupture of concrete

f_{pc} = prestress in the concrete at the extreme fibre in the tension zone

I = moment of inertia of the cross section about centroid

y = distance of extreme fibre in the tension zone from centroid

and α = M/Vd.



DIAGONAL CRACKING STRENGTH OF BEAMS : COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS.

FIG. IV. 18.

Thus summarising the results we can write that the diagonal cracking strength of a prestressed concrete beam V_c' is given thus:

$$\begin{aligned} &\text{i. For } (M/Vd) \leq \alpha_{cw},\ V_c' = V_{cw} \\ &\text{ii. For } (M/Vd) \geq \alpha_{cd},\ V_c' = V_{cd} \\ &\text{iii. For } \alpha_{cw} \leq \frac{M}{Vd} \leq \alpha_{cd}\ V_c' = V_{cf} \end{aligned} \qquad \ldots \text{ IV.4.9}$$

where

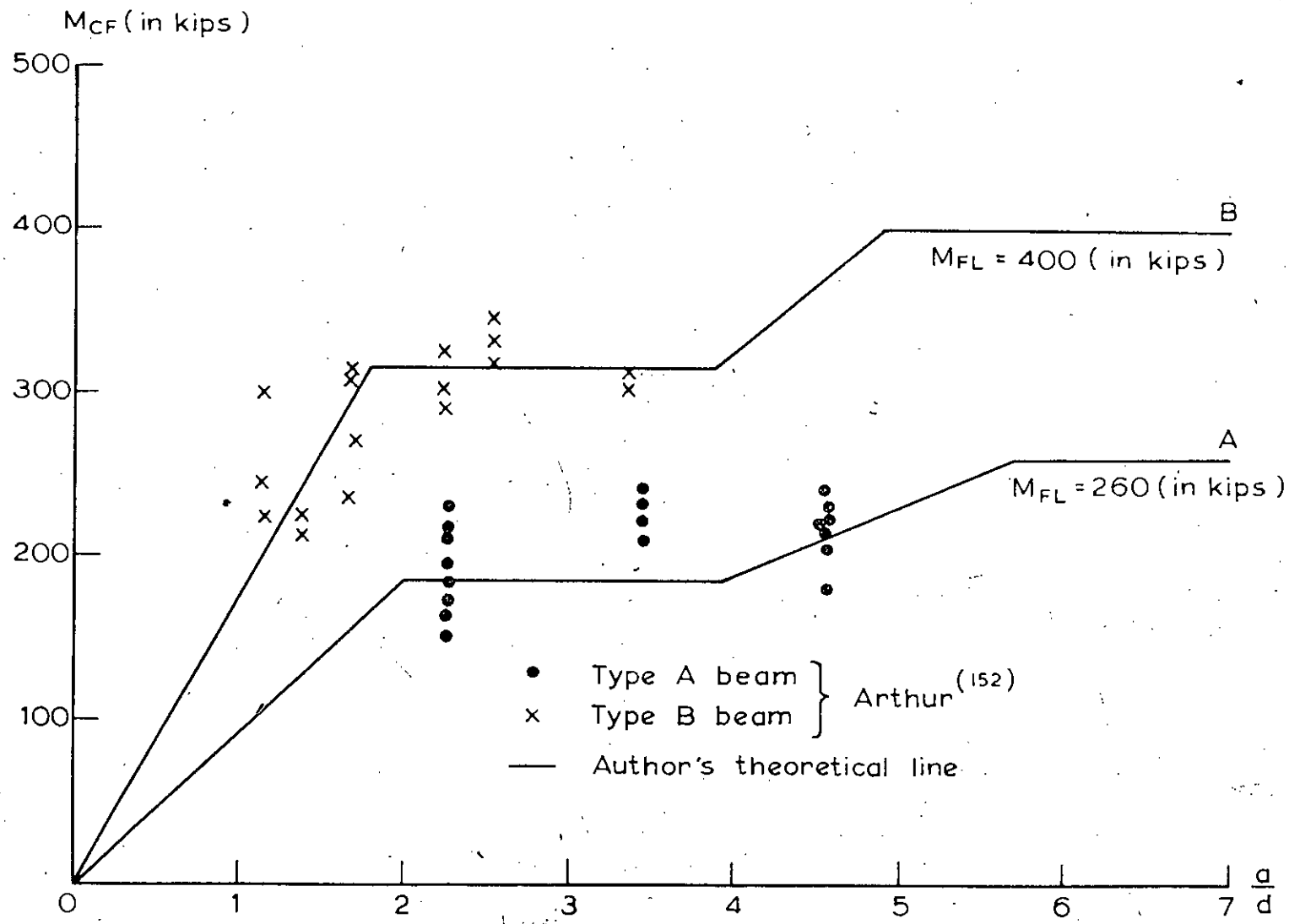
$$\alpha_{cw} = \frac{(f_r + f_{pc})\ \frac{I}{yd}}{V_{cw}}$$

$$\alpha_{cd} = \frac{(f_r + f_{pc})\ \frac{I}{yd}}{V_{cd}} \qquad \ldots \text{ IV.4.10}$$

where the values of V_{cf}, V_{cd} and V_{cw} can be known using Eqns. IV.4.8, IV.4.7 and IV.4.5 respectively.

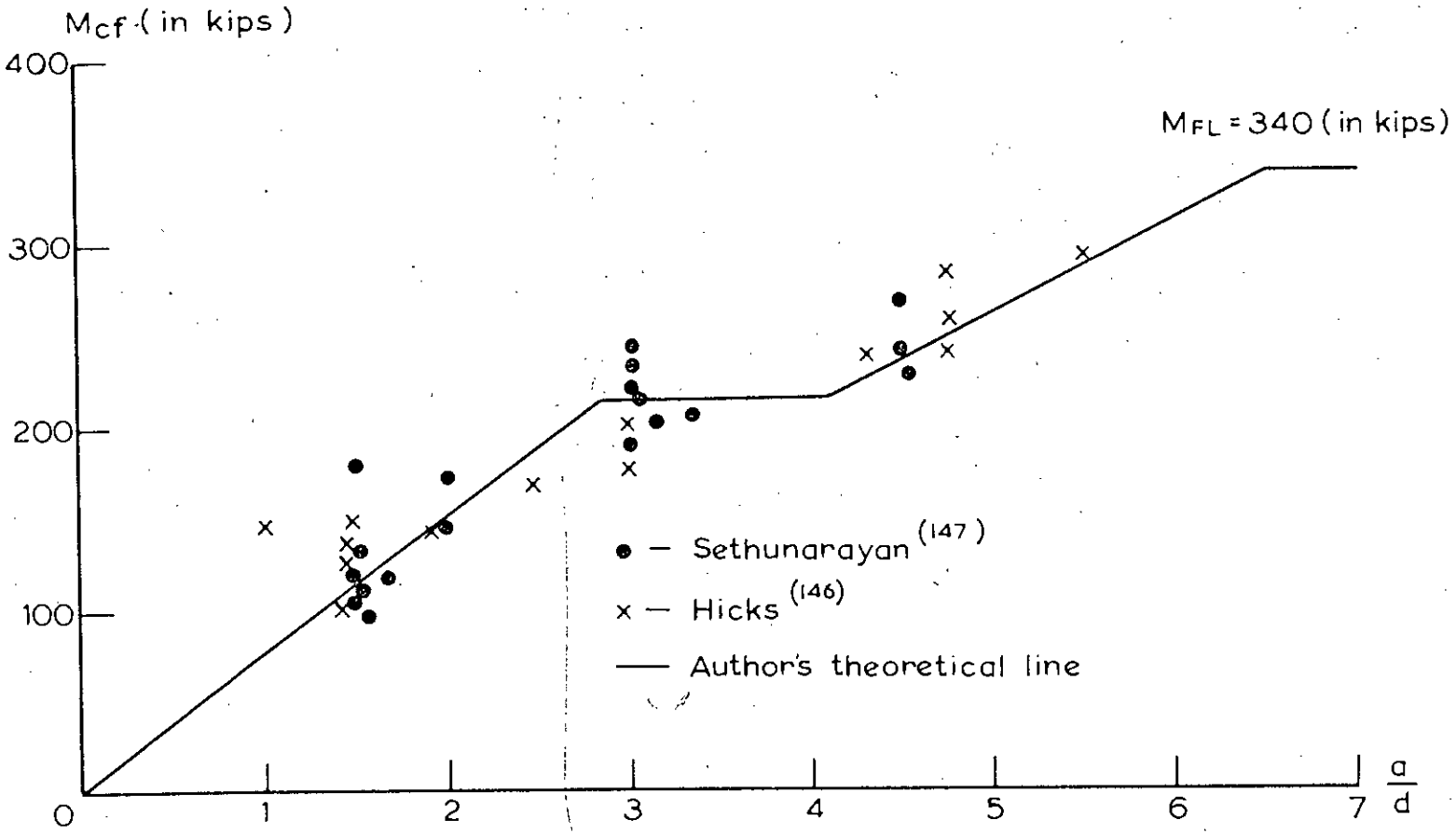
IV.4.6.4 Comparison of Results:

Using the method of calculation indicated above, the diagonal cracking strength has been calculated for number of beams reported in the literature[142,143,146-153]. Fig. IV.18 provides a general comparison between test and computed results. From Fig. IV.18 it can be seen that



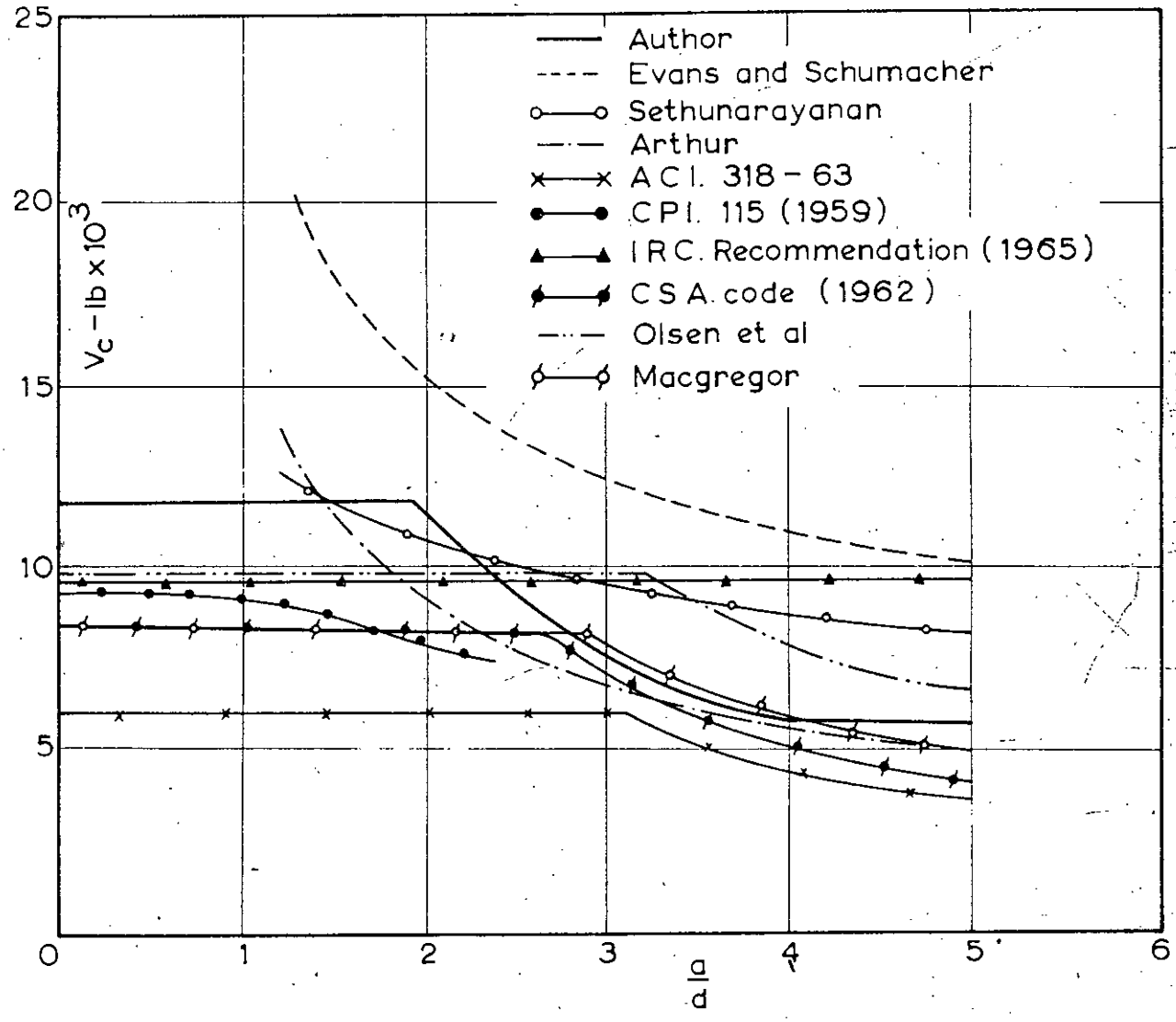
VARIATION OF DIAGONAL CRACKING MOMENT WITH $\frac{a}{d}$ RATIO.

FIG. IV. 20.



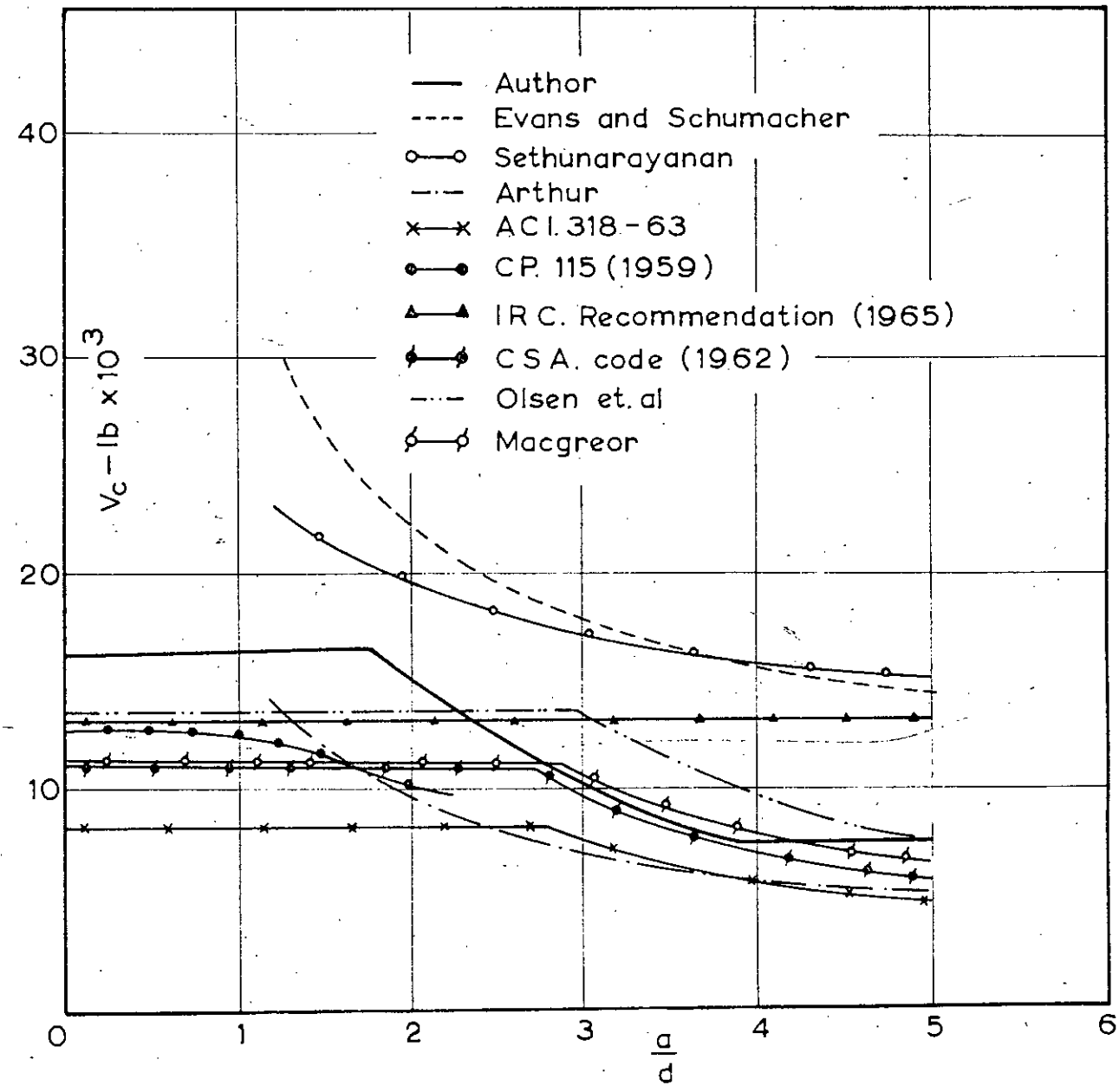
VARIATION OF DIAGONAL CRACKING MOMENT WITH $\frac{a}{d}$ RATIO.

FIG. IV. 19.



RELATIONSHIP BY VARIOUS THEORIES BETWEEN CRACKING SHEAR FORCE AND $\frac{a}{d}$ FOR ARTHUR'S TYPE A BEAM.

FIG. IV. 21.



RELATIONSHIP BY VARIOUS THEORIES BETWEEN CRACKING SHEAR FORCE AND $\frac{a}{d}$ FOR ARTHUR'S TYPE B BEAM.

FIG. IV. 22.

most of the test results lie within ± 15% of the computed results.

Figs. IV.19 and IV.20 present the results in terms of moments for the beams certain investigations[146,147,152]. In the same figures the test results of these investigations are also shown. From these figures it can be seen that the proposed equations agree fairly well with the experimental results.

It may be interesting to compare the results of proposed equations with those of other equations given by previous investigators and Code recommendations. Recently Arthur[152] has given such a comparison for two cases. The same two cases are studied here incorporating the results of the present investigation and those of some recent investigations. The results are shown in Figs. IV.21 and IV.22. From the comparison it can be seen that most of the published equations seem to be somewhat over conservative for $a/d < 2.0$. The ACI Building Code[98] equations give very conservative results when compared to those recommended in CSA Code[159]. The British Code[160] equations seem to have limited applicability and give conservative results within this limitation. The recommendations of Indian Road Congress (IRC)[161] are fairly good for $a/d < 2.0$ and in the

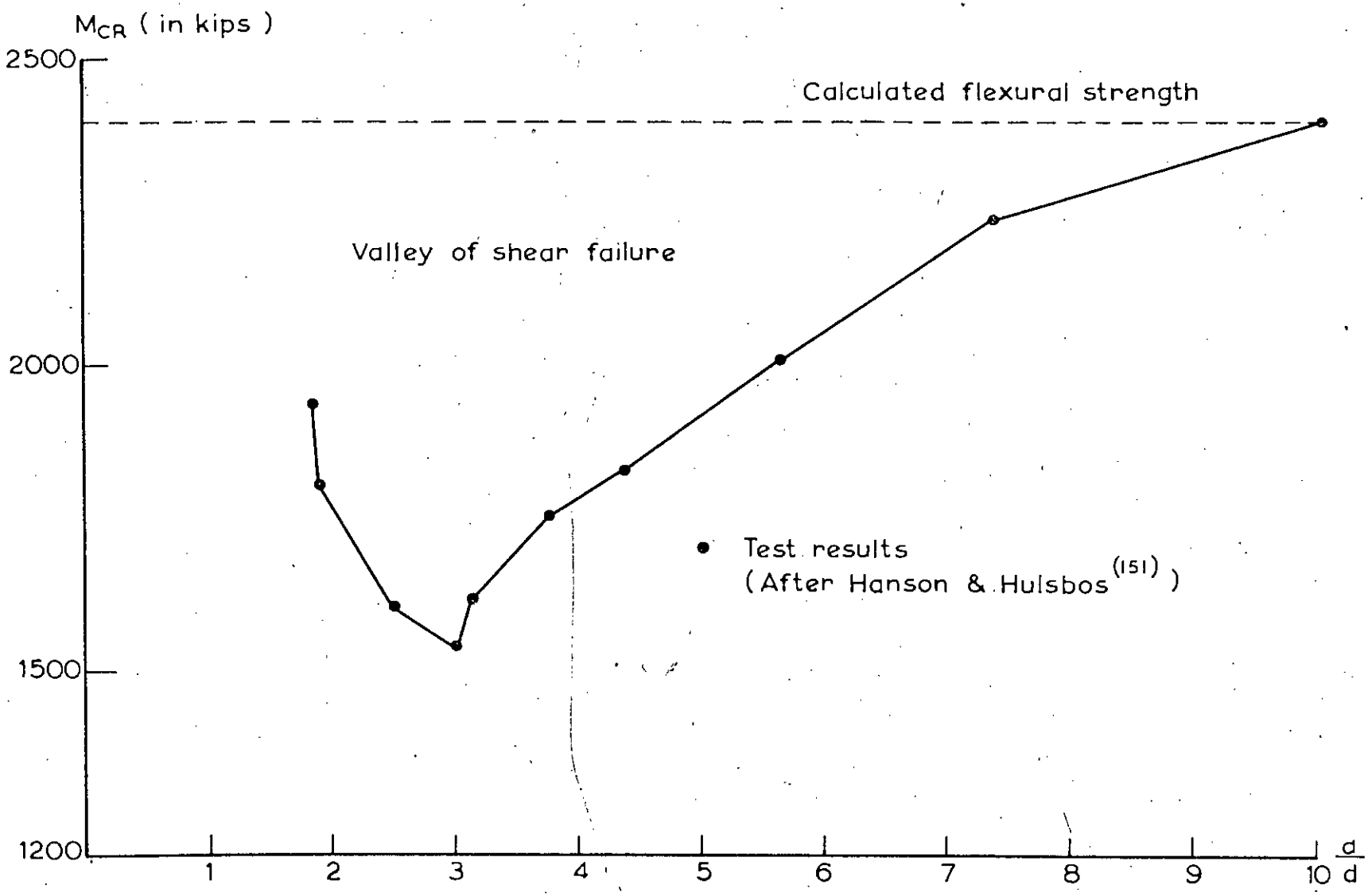


FIG. IV.23. VARIATION OF SHEAR STRENGTH WITH $\frac{a}{d}$ RATIO.

other ranges of a/d ratio they seem to over-estimate the capacity to a very great extent.

IV.4.7 Factors which affect shear strength of prestressed concrete beams:

The main factors which influence the shear strength of prestressed concrete beams are:

1. shear span-depth ratio a/d
2. prestressing force
3. concrete strength of the beam
4. shear reinforcement

The influence of these factors are discussed in some detail below:

IV.4.7.1 Shear span-depth ratio, a/d:

Similar to reinforced concrete beams, the shear strength of prestressed concrete beams varies with a/d ratio. Fig. IV.23 shows such a variation indicating the 'valley of shear failure'. The two different variation of shear strength with a/d on both the sides of a/d = 3.0 corresponds to the two different mechanism of failure of beams discussed earlier. It may be recalled here that the variation of M_{CR} with a/d ratio shown in IV.23 is similar to the variation predicted by the theory proposed in this thesis. The comparison between experimental and theoretical results has already been discussed and diagramatically demonstrated in Figs. IV.13 to IV.16. The variation of $(\frac{V}{bdf''_t})$ with a/d ratio

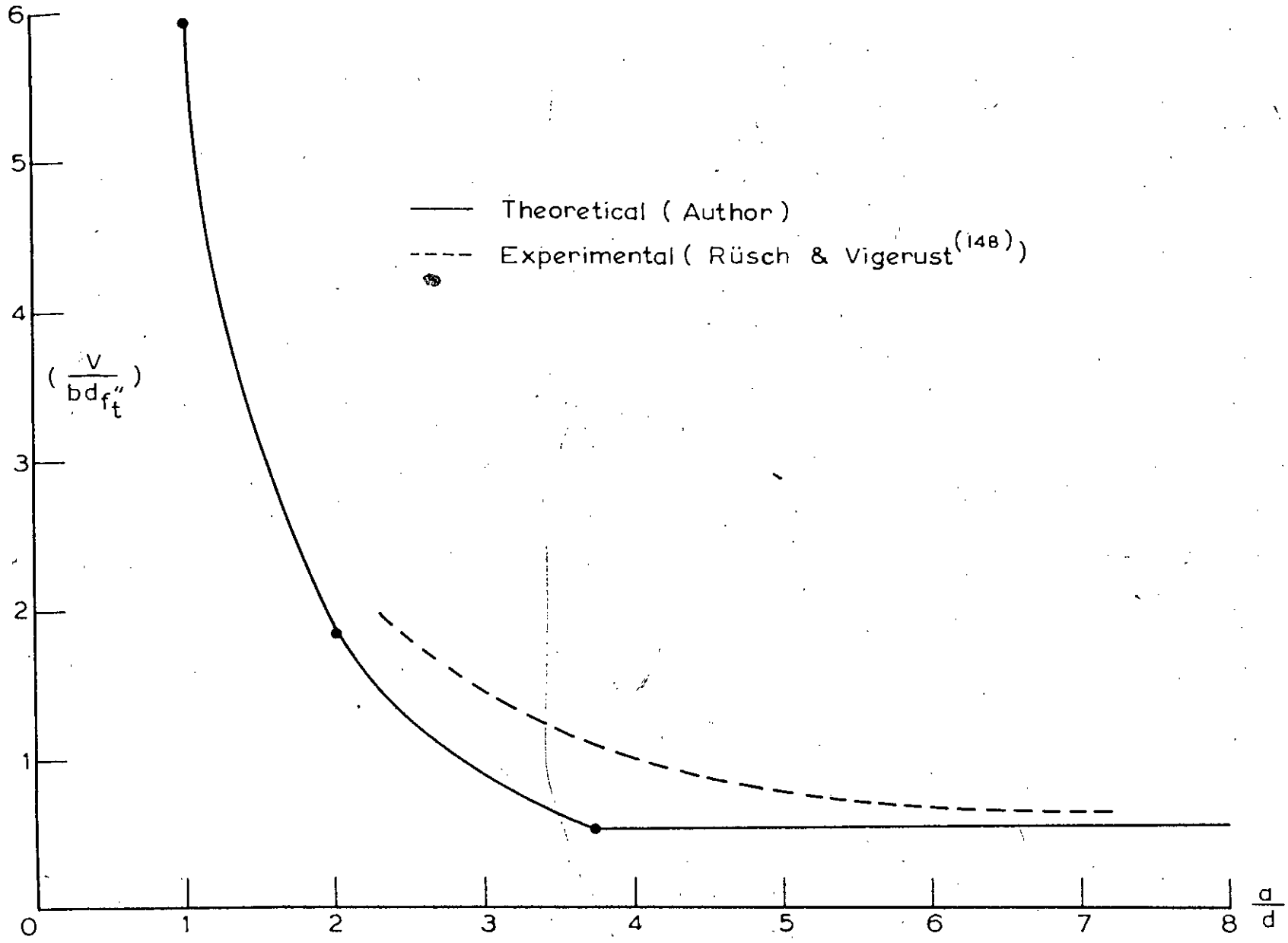
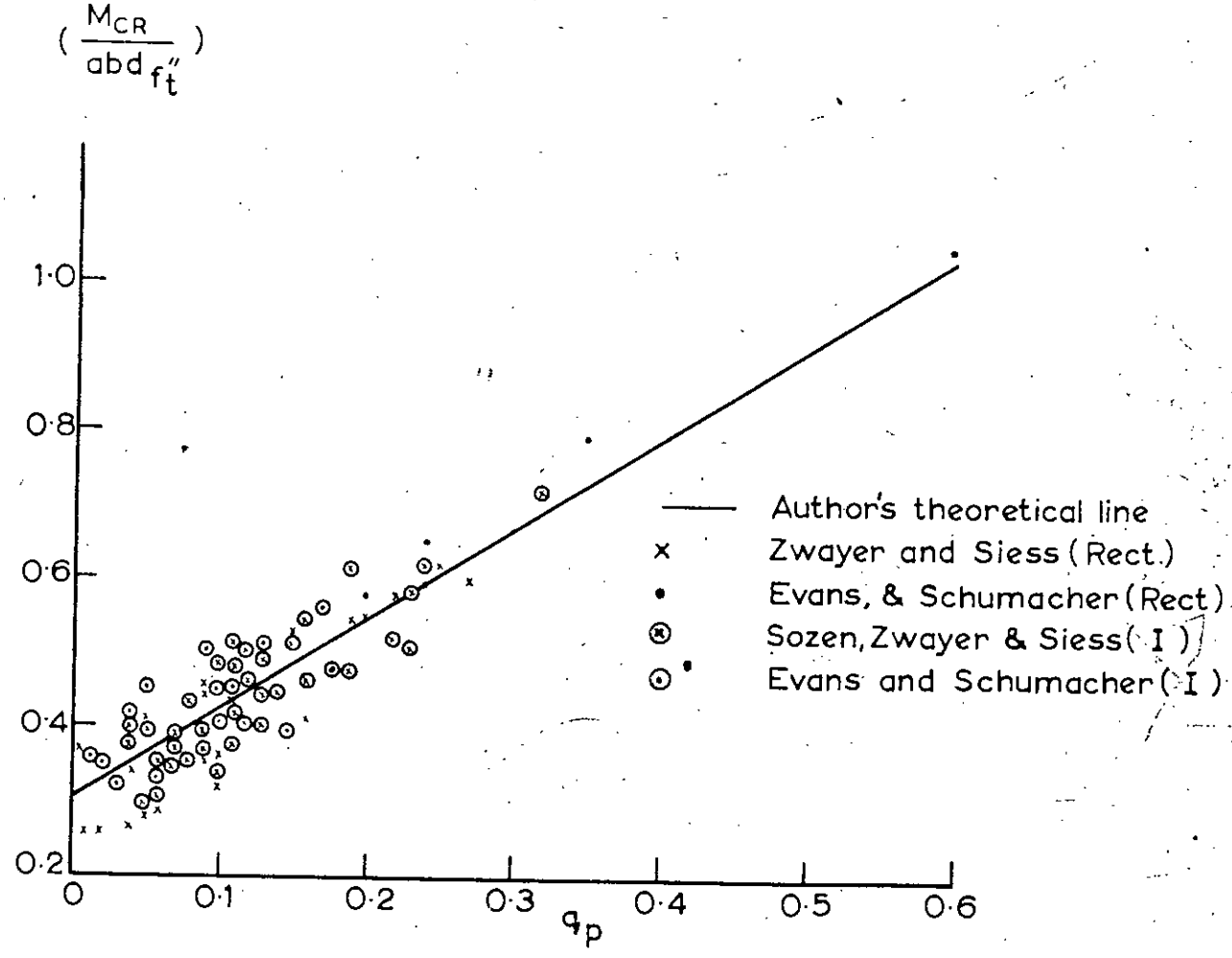


FIG. IV. 24. RELATION BETWEEN SHEAR FORCE AT FAILURE AND SHEAR SPAN-DEPTH RATIO.



INFLUENCE OF PRESTRESS ON THE SHEAR STRENGTH OF BEAM.

FIG. IV. 25.

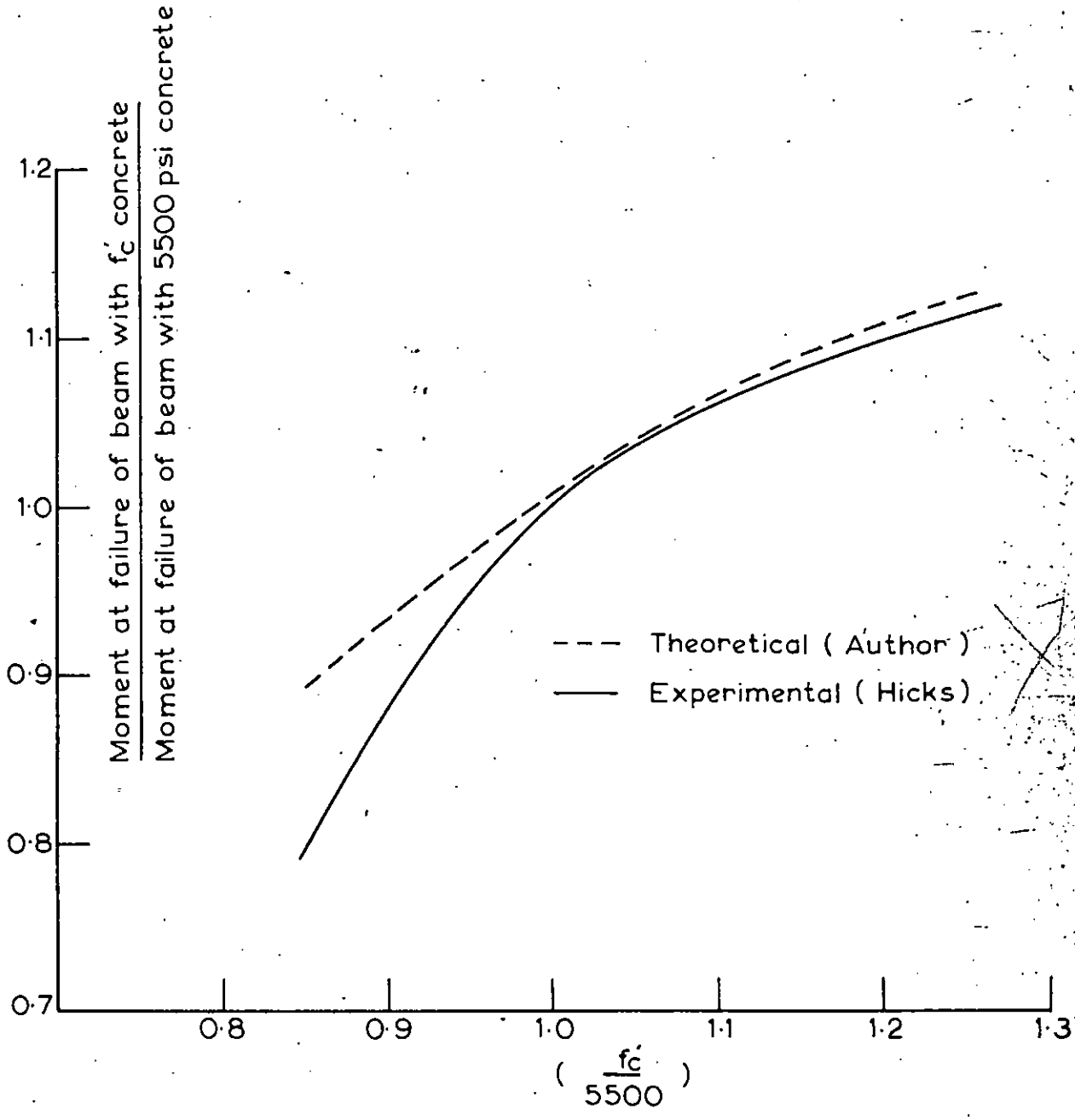
is shown in Fig. IV.24. The experimental curve agrees fairly with the theoretical one. In addition it can be seen that the trend of the theoretical curve is similar to the one reported by Hicks[146] shown in Fig. IV.2.

IV.4.7.2 Prestressing Force:

An important physical parameter which influences the strength of a prestressed concrete beam is the prestressing force. As already shown in Fig. IV.3 the shear strength of the beam increases with the increase in prestress. Experiments[142] have shown that this variation is almost linear, increase in shear strength being directly proportional to the increase in the prestress. In order to show the comparison between the variation as predicted by the theory and the variation as given by experiments, Fig. IV.25 has been plotted. The figure shows a linear variation between prestress and shear strength of the beam. In the figure prestress has been expressed in terms of the non-dimensional parameter q_p ($= \frac{pf_p}{f'_c}$) and the shear strength in the form of another non-dimensional parameter ($M_{CR}/abdf''_t$). From Fig. IV.25 it can be seen that the agreement between the experimental variation and theoretical variation is fairly good.

IV.4.7.3 Concrete Strength

As expected, shear strength of beams will increase



EFFECT OF CONCRETE STRENGTH ON THE SHEAR STRENGTH OF THE BEAM.

FIG. IV. 26.

with the increase in concrete strength. Such a variation is shown in Fig. IV.26. From the figure, it is evident that the variation in shear strength is not linear and also not directly proportional to the increase in concrete strength, f'_c. The increase in shear strength with the increase in f'_c is more in the lower ranges of f'_c than in higher ranges. This is due to the fact that the shear strength of concrete varies directly with the square root of its compressive strength. The variation in the shear strength with the concrete strength as predicted by the theory is also compared with the experimental variation in Fig. IV.26. The agreement between them is found to be good.

IV.4.7.4 Shear Reinforcement:

Normally, prestressed concrete beams are provided with shear reinforcement in the form of vertical stirrups. Other types of shear reinforcement, such as inclined stirrups, bent-up cables, vertical prestressing etc. are also occasionally provided in practice. The functions of vertical stirrups are three fold:

i. it increases the shear strength of the beam by sharing certain portion of the shear force acting at the section;

ii. it restricts the widening and development of diagonal cracks thus providing more concrete area to resist the shear force;

iii. it stiffens the core concrete thus offering more stiffness to the beam to resist deformations.

However, proper anchorage between vertical stirrups and concrete in compression zone is to be ensured so as to derive the three functions of stirrups stated above. Poor anchorage between stirrups and concrete will have adverse effects, on the shear strength of the beams. As in reinforced concrete beams, shear reinforcement becomes effective only after the formation of diagonal crack. Large variation in the individual parameters do not allow the author to have a direct study of the influence of shear reinforcement on shear strength of beams. However from the general trend of experimental results it has been found that the shear strength increases in direct proportion with the quantity $(A_w f_{yw}/bs)$ within certain limiting values of the same.

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CHAPTER IV.5

RECOMMENDATIONS FOR DESIGN AND CONCLUDING REMARKS

IV.5.1 In the previous chapter, the proposed theory and equations derived for predicting the ultimate as well as diagonal cracking strength of prestressed concrete beams have been examined in the light of experimental results. On examination, it has been found that the derived equations are fairly good in predicting the test results. This chapter is devoted to the discussion of a design procedure. The recommendations proposed herein are based on the analytical results discussed in the earlier chapters. The chapter is concluded with an illustrative example using the proposed design procedure followed by a set of conclusions.

IV.5.2 Useful shear strength of beams

As shown by experiments, the load resisted by a prestressed concrete beam after the formation of diagonal crack varies with M/Vd ratio. In the case of low M/Vd ratios, the beam resists considerable amount of load even after diagonal cracking while in the case of medium and high M/Vd ratios failure follows soon after the formation of diagonal crack. Since the shear failures are catastrophic and the formation of a diagonal crack affects the serviceability of a structure to a considerable

extent it is necessary to restrict the useful shear strength of a beam to its diagonal cracking strength.

In the previous chapter, equations have been developed to calculate the diagonal cracking strength considering three types of diagonal cracking. On comparison with the experimental results, it has been found that the predictions of the equations are within error of ± 15%. However, it has been felt that the design equation should always be conservative. Hence the diagonal cracking load can be taken as 0.85 times as given by calculations, such that the calculated results are always erring on the safer side.

IV. 5.3 Discussion of current specifications:

In this article the recommendations of some of the present-day codes are briefly reviewed and their relative merits are discussed. Only the following four documents are considered:

i. 'Design criteria for prestressed concrete road bridges' recommendations by Bridges Committee, Indian Road Congress, April 1965[161].

ii. 'The structural use of prestressed concrete in buildings', British Standard Code No. 115 (1959)[160].

iii. 'A 135 Code for prestressed concrete construction' Canadian Standards Association, 1962[159].

iv. 'ACI Standard Building Code requirements for Reinforced Concrete (ACI 318-63', American Concrete Institute, 1963[98].

The recommendations of the Indian Road Congress are based on the principal tensile stress criterion.

It considers the shear strength of beams both under service loads and under ultimate loads. In the case of service loads the Congress recommends that the principal tensile stresses should not exceed $1.5\sqrt{F_c}$ psi where F_c is the cube crushing strength in psi. When this limit is exceeded, mild steel shear reinforcement calculated as in the case of reinforced concrete is to be provided to take up fully all the principal tensile stresses. Under ultimate loads, if the value of the principal tensile stress is greater than $6\sqrt{F_c}$ psi the section is to be redesigned. If its value is less than $6\sqrt{F_c}$ psi but greater than $4.5\sqrt{F_c}$ psi, adequate mild steel shear reinforcement is to be provided to resist the whole of the effective shear at a stress equal to 0.8 of the yield stress of mild steel.

Under working loads, the British Code restricts the principal tensile stress to $(50 + 0.02f'_c)$* psi and under

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* modified expressions for the tabulated values of British Code, as given by MacGregor[162].

ultimate loads this value corresponds to $(150 + 0.039f'_c)^*$ psi, where f'_c is the cylinder strength in psi. If these limits are exceeded, shear reinforcement is to be provided to take up the complete shear force and the stress in shear reinforcement under ultimate load conditions has been restricted to 80% of the yield stress (or 0.2% proof stress, where appropriate). Thus it can be seen that the recent recommendations by the Indian Road Congress are in similar lines with those of British Code.

The recommendations of the CSA Code are basically an outgrowth of the tests of MacGregor[162]. The diagonal cracking shear is taken as the smaller of the following:

a. The shear which produces a principal tensile stress of $4\sqrt{f'_c}$ in the web for sections not closer than a distance equal to the depth of the member.

b. The shear which produces a net tensile stress of $6\sqrt{f'_c}$ in the extreme fibre in the tension zone of the beam.

Where the value of V_c exceeds the above limit, shear reinforcement is to be designed using the formula:

$$A_w = \frac{(V_u - V_c)\, s}{f_{yw}\, d}$$ and should not exceed

7bs $\sqrt{f'_c}$ / f_{yw}. The code recommends a minimum quantity of web steel and restricts the spacing of stirrups to three-quarters the depth of the member.

The ACI Building Code recommendations are partly based on the results of tests carried out at University of Illinois[153]. According to these recommendations, the shear reinforcement is to be designed using the formula:

$$A_w = \frac{(V_u - \phi V_c)s}{\phi d f_{yw}}$$

where ϕ is the capacity reduction factor, taken equal to 0.85 and V_c is the smaller of V_{ci} and V_{cw} defined as given below:

$$V_{ci} = 0.6bd\sqrt{f'_c} + \frac{M_c}{\left(\frac{M}{V} - \frac{d}{2}\right)}$$

but not less than 1.7 bd $\sqrt{f'_c}$

$$V_{cw} = bd\,(3.5\sqrt{f'_c} + 0.3\,f)$$

where f is the net compressive stress in concrete at centroid of the section or at the junction of web and flange if the centroid lies in the flange.

The code restricts the spacing of stirrups to three-quarters the depth of the member or 24 inches whichever

is smaller and also recommends a minimum quantity of web steel in similar-lines with that of CSA Code.

From the above discussion, it is clear that the recommendations of Indian Road Congress and British Code belong to one group and those of CSA and ACI Codes form another group. Both Indian Road Congress and British Code account for only one type of diagonal cracking which is applicable for low values of M/Vd ratio when the section is uncracked. The British Code does not recommend any design criteria if the section is cracked. But the recommendation of Indian Road Congress does not seem to have considered the effect of cracking in the section. From Figs. IV.21 and IV.22, in which the results of various investigations and code recommendation have been compared, it is evident that the recommendations of both Indian Road Congress and British Code are somewhat conservative for low values of M/Vd ratio. The Indian Road Congress errs on the unsafe side for M/Vd ratio greater than 2.0. Its over-estimation of the diagonal cracking strength of the beam is so high that the factor of safety may be reduced to one at higher ranges of M/Vd ratio.

Further both the recommendations suggest design of shear reinforcement to take up the whole shear. Present-day test results have clearly indicated that the concrete in the compression zone of the beam carries considerable amount of

shear and the shear reinforcement included in the structure helps to increase the capacity of the uncracked concrete by restricting the development and widening of cracks and sharing certain portion of the shear by itself. So ignoring the effect of concrete does not seem to be correct and may affect the economy of the structure to a certain extent.

The recommendations of CSA and ACI Codes are the results of the latest investigations. The ACI Code seems to be somewhat over-conservative at lower values of M/Vd ratio.

IV.5.4 Design Recommendations:

Based on the results of present investigation the following recommendations are given for the design of prestressed concrete members under the combined action of bending and shear. These recommendations may be useful in improving the recommendations of Indian Road Congress.

a. The useful strength of a prestressed concrete member under bending and shear may be restricted to its diagonal cracking strength.

b. The diagonal cracking strength of a prestressed concrete member, V_c may be taken equal to 0.85 V_c' where V_c' is given by

i. For $\frac{M}{Vd} \leq \alpha_{cw}$, $V'_c = V_{cw}$

ii. For $\frac{M}{Vd} \geq \alpha_{cd}$, $V'_c = V_{cd}$

iii. For $\alpha_{cw} \leq \frac{M}{Vd} \leq \alpha_{cd}$, $V'_c = V_{cf}$

where

$$\alpha_{cw} = \frac{(f_r + f_{pc}) \frac{I}{yd}}{V_{cw}}$$

$$\alpha_{cd} = \frac{(f_r + f_{pc}) \frac{I}{yd}}{V_{cd}}$$

$$V_{cf} = \left[(f_r + f_{pc})\left(\frac{I}{yd}\right)\right]\left(\frac{1}{\alpha}\right)$$

$$V_{cw} = K \left(\frac{Ib}{Q}\right) f'_c$$

$$V_{cd} = 1.25\ bdf''_t.q.$$

Similar expressions may be derived for composite sections and to account for dead loads and draping in prestressing wires.

c. If the ultimate shear V_u exceeds V_c, shear reinforcement is to be provided using the formula

$$\frac{A_w}{bs} = r = \frac{(V_u - V_c)}{bdf_{yw}}$$

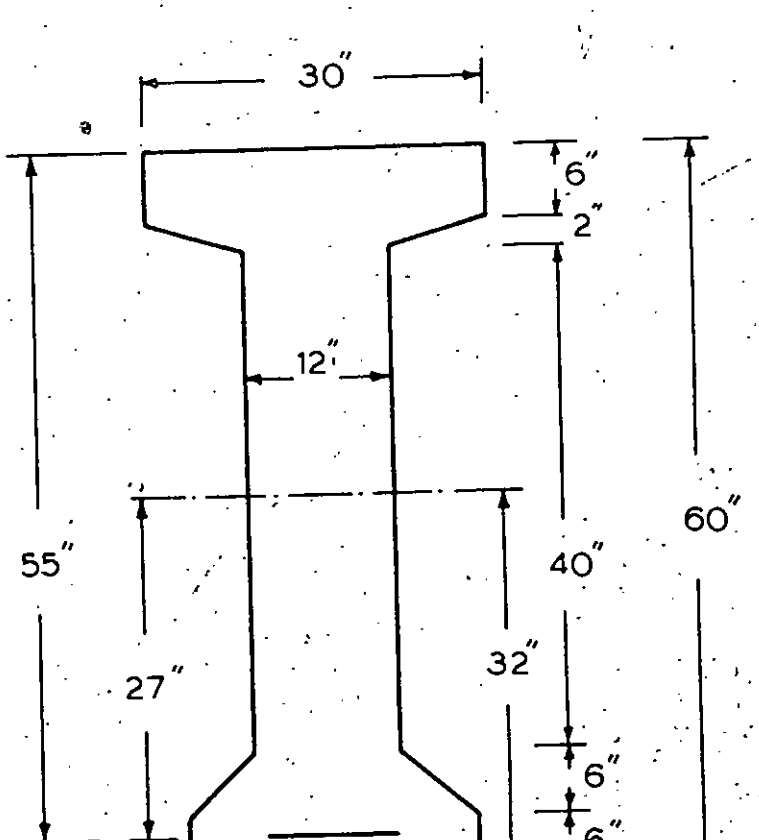
d. If V_u is less than V_c, theoretically no shear reinforcements is required. However nominal reinforcements can be provided to increase the stiffness of the structure by an amount equal to the minimum shear reinforcement provided in the length of the member or as specified by the Code for working load conditions.

e. The spacing of shear reinforcement as calculated from (c) is not to be greater than the clear depth of web in an I-beam or the effective depth of the member in the case of rectangular sections. This clause accounts for the effectiveness of stirrups such that at least one stirrup will be crossed by the potential diagonal crack developing at about mid-depth of the member.

f. The shear reinforcement so designed is to be anchored properly as per the recommendations of code such that bond and anchorage failures are avoided.

g. Experiments have indicated that it may not be necessary to restrict the nominal shearing stress or the value of principal tensile stress.

h. It can be assumed that for sections with (M/Vd) ratios greater than (α_{TR} / ϕ), there is no danger of



CROSS SECTION OF THE BRIDGE GIRDER.

FIG. IV. 27.

612

shear failure and the flexure failure is ensured in such sections. So in these sections, only nominal shear reinforcement may be provided as per the clause (d). The value of α_{TR} is given by:

$$\alpha_{TR} = \frac{0.72\phi pf_y}{qf''_t} ; \quad \phi = 0.85$$

IV.5.5 Numerical Example:

The suggested design procedure is illustrated in the following example:

Fig. IV.27 shows the cross section of a prestressed concrete girder of span 60 ft. It is subjected to combined bending and shear and it is required to design the shear reinforcement for this beam given that:

A	= 858 in^2	A_s	= 3.5 in^2
I	= 327,500 in^4	f_y	= 250 ksi
Q	= 7524 in^3	f_{yw}	= 40 ksi
e	= 27 in	P	= 500 kips
y	= 32 in	f'_c	= 5.5 kips
d	= 55 in		

Distance from support (ft)	At ultimate stage	
	Shear force (kips)	(M/Vd)
30	100	6.00
25	125	4.75
20	150	3.80
15	174	2.68
10	200	1.68
5	224	0.80

Design:

Prestress in concrete at the extreme fibre in tension zone, $f_{pc} = \frac{P}{A} + \frac{Pey}{I}$

$$= \frac{500}{858} + \frac{500 \times 27 \times 32}{327500}$$

$$= \underline{1.902} \text{ ksi}$$

$$f_r = 9\sqrt{f'_c} = 9\sqrt{5500} = 0.667 \text{ ksi}$$

$$f_{pc} + f_r = 1.902 + 0.667 = \underline{2.569} \text{ ksi}$$

$$V_{cf} = (f_r + f_{pc}) \frac{I}{yd} \frac{1}{(M/Vd)}$$

$$= 2.569 \times \frac{327500}{32 \times 55} \frac{1}{(M/Vd)}$$

$$= \frac{478}{(M/Vd)}$$

Net compressive stress at centroid of the section

$$f = \frac{P}{A} = 500/858 = 0.583 \text{ ksi}$$

$$\frac{f}{f'_c} = \frac{0.583}{5.5} = 0.106$$

From Fig. IV.17, corresponding to $f/f'_c = 0.106$ and $f'_c/f'_t = 10$, the value of K = 0.131

$$V_{cw} = K \left(\frac{Ib}{Q} \right) f'_c$$

$$= 0.131 \times \frac{327500 \times 12}{7524} \times 5.5$$

$$= \underline{376} \text{ kips}$$

$$q_p = \frac{p_{fp}}{f'_c} = \frac{3.5}{12 \times 55} \times \frac{f_p}{5.5} = \frac{P}{bdf'_c}$$

$$= \frac{500}{12 \times 55 \times 5.5} = 0.138$$

$$q = 0.238 + q_p = 0.238 + 0.138 = 0.376$$

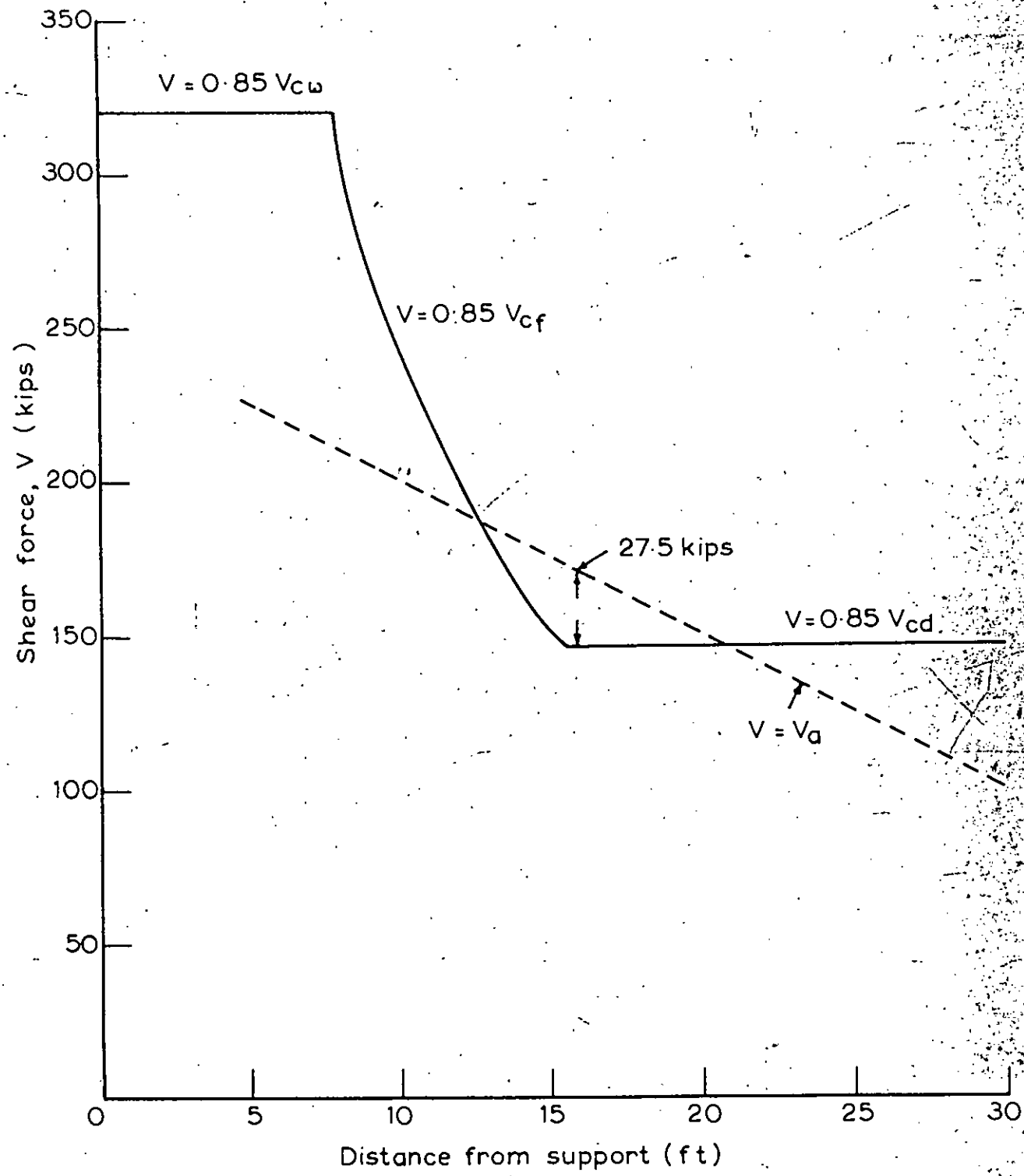
$$f''_t = \frac{f'_t \left[\frac{2f'_c}{f'_t} - 1 \right]}{(2f'_c / f'_t)} \text{ where } f'_t = 8\sqrt{f'_c}$$

$$f''_t = 0.562 \text{ ksi.}$$

$$V_{cd} = 1.25 \, bdf''_t \cdot q$$

$$= 1.25 \times 12 \times 55 \times 0.562 \times 0.376$$

$$= \underline{179 \text{ kips}}.$$



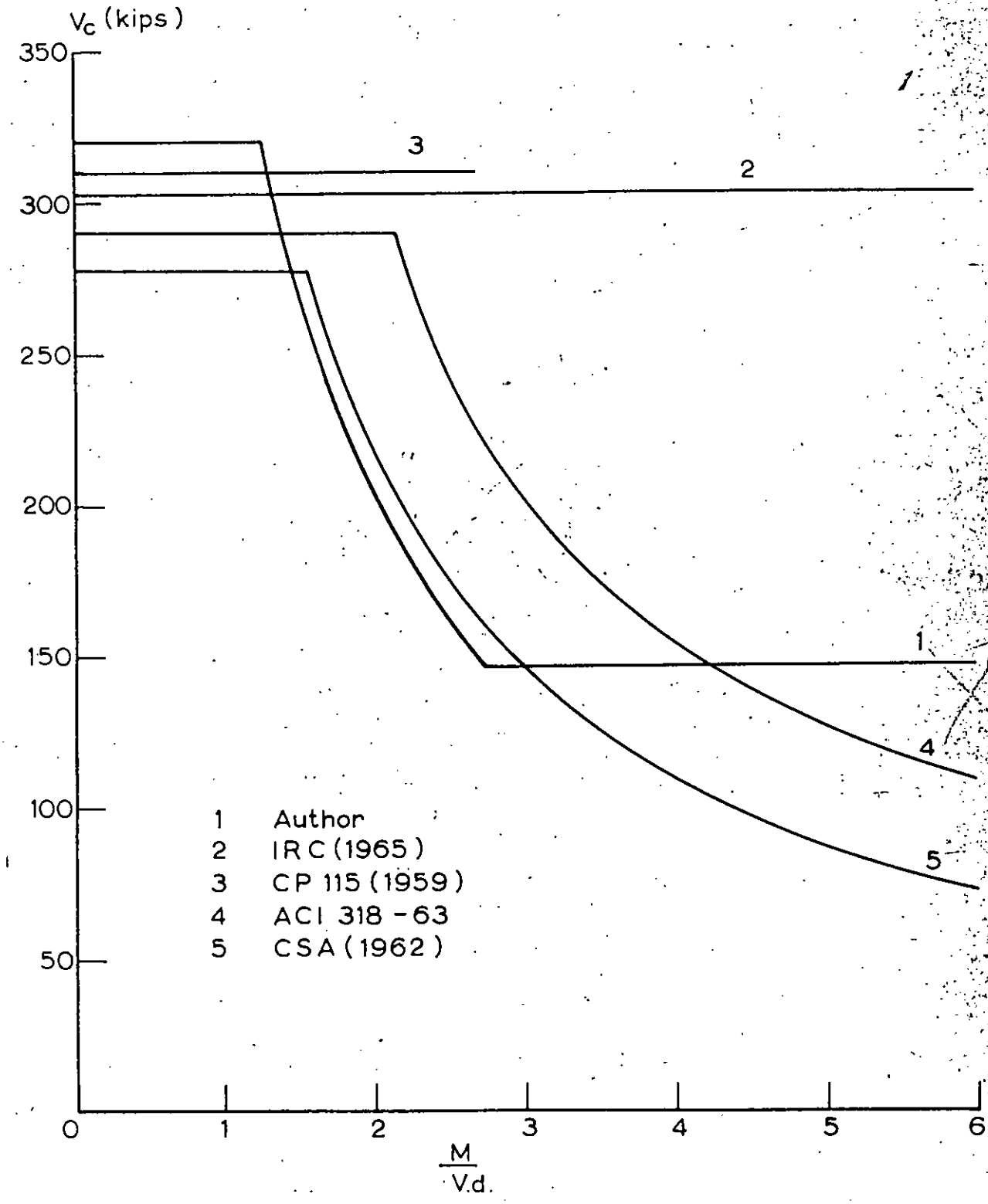
VARIATION OF SHEAR FORCE AT DIAGONAL CRACKING ALONG THE SPAN OF THE BRIDGE GIRDER.

FIG. IV. 28.

Having evaluated different quantities, the diagonal cracking shear V_c at various sections can be known using the clause (b). The results are presented in the following table:

Distance from support (ft)	V_{cf} kips	V_{cd} kips	V_{cw} kips	V'_c kips	V_c kips	V_u kips
30	80	179	376	179	152	100
25	101	179	376	179	152	125
20	126	179	376	179	152	150
15	178	179	376	179	152	174
10	284	179	376	284	242	200
5	597	179	376	376	320	224

A quantitative picture of the results can be obtained by representing the results in a diagram. Such a diagram is given in Fig. IV.28. In this both the diagonal cracking strength and the external shear at various sections have been plotted. From Fig. IV.28, it can be found that the maximum difference between diagonal cracking strength and external shear force is 27.5 kips. The major portion of the beam length is safe against diagonal cracking and theoretically no shear reinforcement is necessary for those portions. However, assuming that



COMPARISON OF RESULTS: SHEAR FORCE AT DIAGONAL CRACKING.

FIG. IV. 29.

reinforcement can be provided at constant spacing throughout the span length, we get,

$$r = \frac{(V_u - V_c)}{bdf_{yw}}$$

$$= \frac{27.5}{12 \times 55 \times 40} = \underline{0.104\ \%}$$

Assuming $\frac{1}{2}$ in. diameter single legged stirrups we have,

$$r = \frac{A_w}{bs} = 0.104\%$$

$$\text{or } s = \frac{A_w}{br} = \frac{0.196}{12 \times 0.00104} = 15.7 \text{ in.}$$

$$= \underline{\text{say 15 in.}}$$

Hence provide 1/2 in diameter single legged stirrups at 15 in. spacing throughout the length of the beam.

In Fig. IV.29 the values of V_c given by various Codes are compared with that calculated value using the recommendations given herein. The results of ACI Code and CSA Code are in general agreement with those of proposed recommendations for lower ranges of M/Vd ratio; but these codes seem to be somewhat over conservative at higher values of M/Vd ratio. Within its limitation, the British Code agrees fairly well with other recommendations. The recommendations

of the Indian Road Congress give fairly good results for $M/Vd < 1.5$ and for $M/Vd > 1.5$, it seems to over-estimate the capacity of the beams to a great extent owing to the reasons discussed earlier. Hence the recommendations of the Indian Congress need certain revisions.

IV.5.6 CONCLUDING REMARKS

1. The mechanisms of shear failure of bonded prestressed concrete beams have been classified under two heads and they have been explained using a failure criteria for concrete under combined stresses.

2. Based on certain assumptions, a theory has been developed to evaluate the strength of the above mechanisms. The theory resulted in simple equations which can be used to calculate the shear strength of beams for all ranges of $M/Vd \geq 1.0$.

3. The resulting equations bring-in the influence of moment/shear ratio and other important parameters such as prestress, concrete strength, amount and strength of longitudinal steel etc., on the shear strength of beams. The theoretical predictions have been compared with the results of over four hundred tests and it has been found that they are fairly good with an error $\pm$ 15% in most of the cases. The mean value and coefficient of variation of (Test/Calc.) ratio is 1.04 and 14% respectively.

4. It has been indicated that the useful strength of a prestressed concrete beam in shear can be taken as its diagonal cracking strength. From the analysis of test results, it has been found that diagonal crack can occur in three different ways. Using the results of the proposed theory and failure criterion for concrete under compressive-tensile stresses, equations have been derived for these three types of diagonal cracking. On comparison with test results, it has been found that the predictions of the equations agree fairly well with the test results within an error of ± 15% in majority of the cases.

5. Using the results of the present investigation, a set of recommendations has been proposed for the design of prestressed concrete beams under combined bending and shear. The results of the proposed recommendations have been compared with those of other codes and it has been indicated that ACI and CSA codes are rather very conservative at higher ranges of moment/shear ratio. The limitations of the recommendations of Indian Road Congress have been indicated and revision of these recommendations has been stressed.

PART V : COMBINED TORSION, BENDING AND SHEAR IN REINFORCED CONCRETE BEAMS

COMBINED TORSION, BENDING AND SHEAR IN REINFORCED CONCRETE BEAMS

V.1 Introduction:

The importance of the problem of torsion combined with bending and shear has been discussed in the beginning of this thesis. In Parts I and II, the individual influences of torsion and shear on the flexural strength of reinforced concrete beams have been discussed. In this part the more general problem of combined torsion, bending and shear is discussed in the light of the experimental and analytical results given in the preceding sections.

The purpose of this investigation has been primarily analytical. However, a minor experimental programme was also conducted to study the behaviour of beams under combined torsion, bending and shear. The main variables were torque/moment ratio and moment/shear ratio. All the beams were of rectangular cross section and were reinforced on the tension side only.

Based on certain failure mechanisms and using a failure criterion for concrete under compressive-tensile stresses,

equations have been derived for three modes of failure. The theoretical predictions have been compared with all the test results available to the author and it has been found that the agreement is good.

V.2 Review of previous research:

From the literature it can be seen that not much work has been done in this field of research. Probably Nylander[4] was the first investigator who reported tests on rectangular beams under combined bending, torsion and shear in the year 1945. Later, in 1955, Brown[27] reported test results obtained from rectangular, Ell and Tee beams without transverse reinforcement. In 1961, Lessig[17] reported test results on rectangular beams reinforced both longitudinally and transversely. Since the results of these three investigations have already been discussed in detail in Part I of this thesis, they are not repeated here.

In 1956, Farmer[163*] tested twenty-seven semi-continuous reinforced concrete tee beams subjected to torsion combined with shear and bending moment. He proposed a new method, which he called as 'Core-shear Theory', to predict the ultimate strength of beams under combined bending, shear and torsion. In this theory he has assumed that a part of the cross section

*Details of Farmer's investigation have been obtained from Ersoy's thesis[58] as a cross reference.

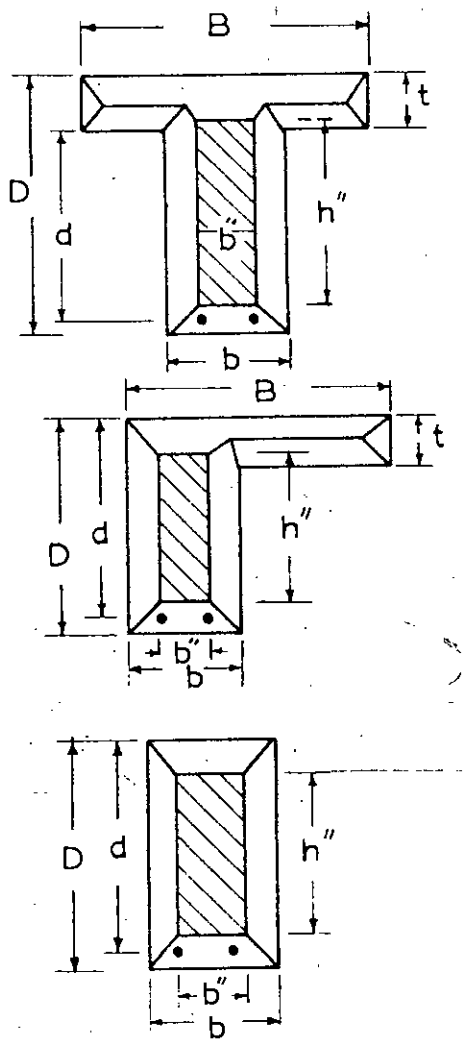


FIG. V. 1. "CORE-SHEAR" THEORY (After Farmer).

625

of the beam resists shear force and the remaining portion resists torsion. The shear stress caused by transverse shear force is calculated assuming a rectangular distribution and the torsional stress is calculated assuming full plastic conditions. He further assumed that core of the web resists the direct shear and the remaining portion of the web resists the torsion. Based on these assumptions, Farmer derived the following equations for predicting the strength of beams of various cross sections:

For rectangular beams:

$$V = b'' h'' \tau_s \quad \text{for } h'' \leq 2d-D$$
$$= b'' h''' \tau_s \quad \text{for } h'' > 2d-D \qquad \ldots \text{V.1}$$

where

$$h'' = D-b+b''$$
$$h''' = d-\tfrac{1}{2}(D-h'')$$

$$M_t = \tfrac{1}{2} \tau_t \left[b^2 (D-\tfrac{b}{3}) - (b'')^2 (h''-\tfrac{b''}{3}) \right] \qquad \ldots \text{V.2}$$

For Tee and Ell beams:

$$V = b''h'' \tau_s \quad \text{for } h'' \leq 2d-D$$
$$= b''h''' \tau_s \quad \text{for } h'' > 2d-D \qquad \ldots \text{V.3}$$

$$M_t = \tfrac{1}{2} \tau_t \left[b^2 (D-\tfrac{b}{3}) + t^2 (B-b) - (b'')^2 (h''-\tfrac{b''}{3}) \right] \qquad \ldots \text{V.4}$$

where $h'' = D-b+b''$

$h''' = d-\tfrac{1}{2}(D-h'')$.

The other notations are explained in Fig. V.1. By properly selecting the values for τ_s and τ_t, Farmer found very good agreement between the theory and the test results.

In 1965, Ersoy[58] tested twenty-five semi-continuous reinforced concrete Ell beams without stirrups in combined bending, shear and torsion. The investigation has been programmed to study the effect of percentage of longitudinal steel, flange width and the eccentricity of loading on both the behaviour and the strength of beams. Some of the important conclusions of this investigation are summarised below:

a. Test results indicated that the simple addition of direct shear stresses to torsional stresses would yield non-uniform safety factors in design.

b. If plastic theory is used to compute torsional stresses, the total shear stress is given by $\tau = (2.55+0.82\ M_t/V)\sqrt{f'_c}$, but not greater than $5.0\sqrt{f'_c}$.

c. A lower bound solution has been suggested to predict the ultimate strength of reinforced concrete beams subjected to torsion combined with shear and flexure. The suggested equation is of the following form, representing a circle.

$$\left(\frac{M_t}{M_{to}}\right)^2 + \left(\frac{V}{V_o}\right)^2 = 1.0 \qquad \ldots \text{V.5.}$$

It has been found that when $\tau_s = 2.68\sqrt{f'_c}$ and $\tau_t = 4.80\sqrt{f'_c}$ were used to compute V_o and M_{to} the Eqn.V.5 formed a safe lower limit for the test results. The value of M_{to} is calculated using plastic theory and $V_o = Bd\tau_s$, where B is the width of flange.

d. The ultimate shear capacity of test specimens decreased with increasing "eccentricity"*.

e. The influence of longitudinal steel on shear capacity of beams decreased with increasing eccentricity. At very high eccentricities it had no significant effect on the strength.

f. Previous tension cracking had very little influence on the cracking pattern of beams which were tested under large eccentricities. The ultimate strength of such beams was 6 to 7 percent lower than their companion specimens with no previous cracking.

Very recently, Johnson Victor has conducted tests on semi-continuous tee beams without stirrups in continuation of Ersoy's investigation. It is understood that Eqn. V.5 proposed by Ersoy has been modified to the following form

---

* In Ersoy's investigation, the torque was caused by applying the transverse load eccentrically with respect to shear centre of the cross section. Thus the term "eccentricity" means only the eccentricity of the load which is nothing but the quantity torque/shear force, M_t/V.

by Johnson Victor:

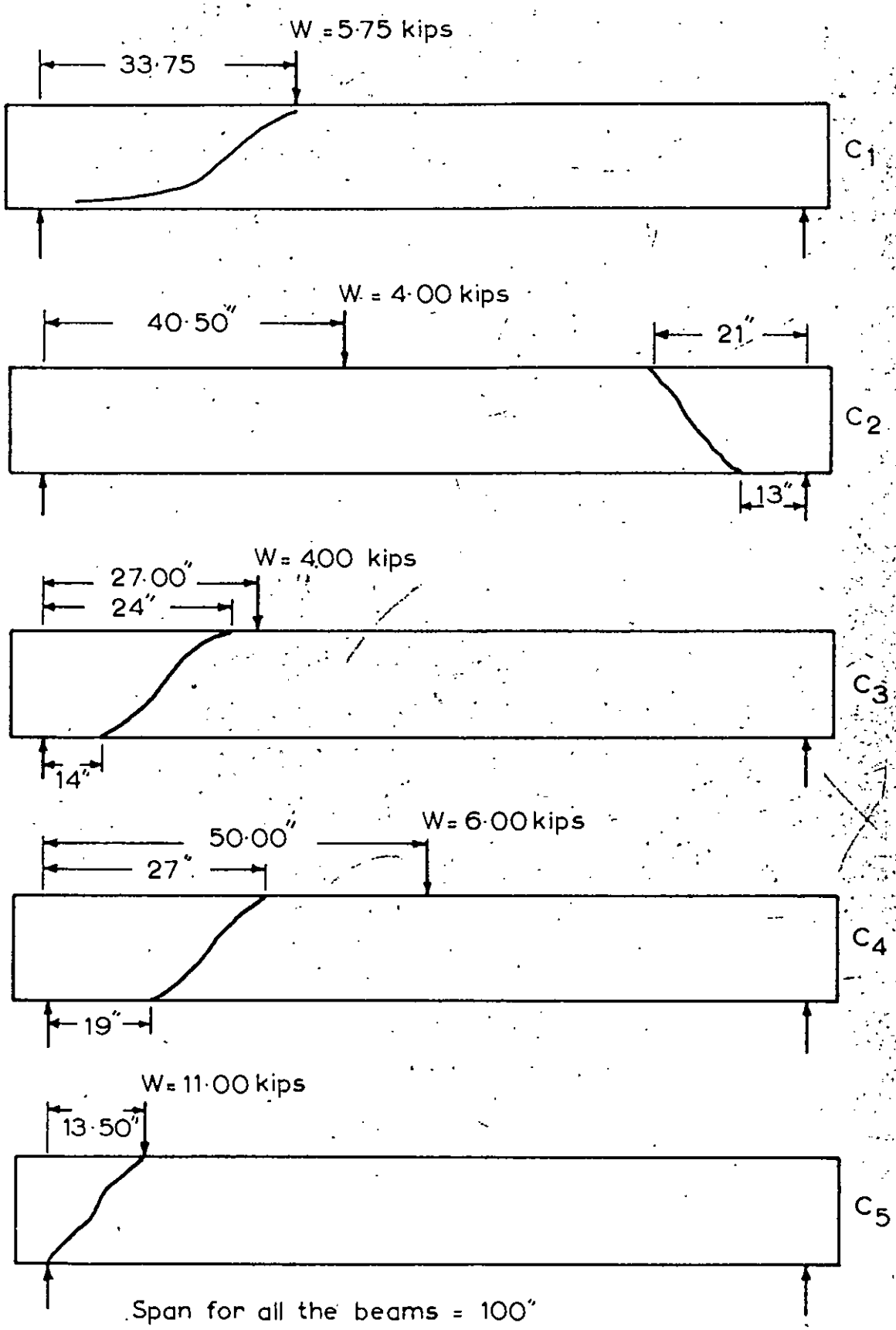
$$\left(\frac{M_t}{\bar{a}\, M_{to}}\right)^2 + \left(\frac{V}{\bar{b}\, V_o}\right)^2 = 1.0 \qquad \ldots \text{ V.5(a)}$$

where $\bar{a}$ and $\bar{b}$ are numerical constants obtained from experiments. The details of his investigation are not yet available to the present author.

V.3 Experimental Investigation:

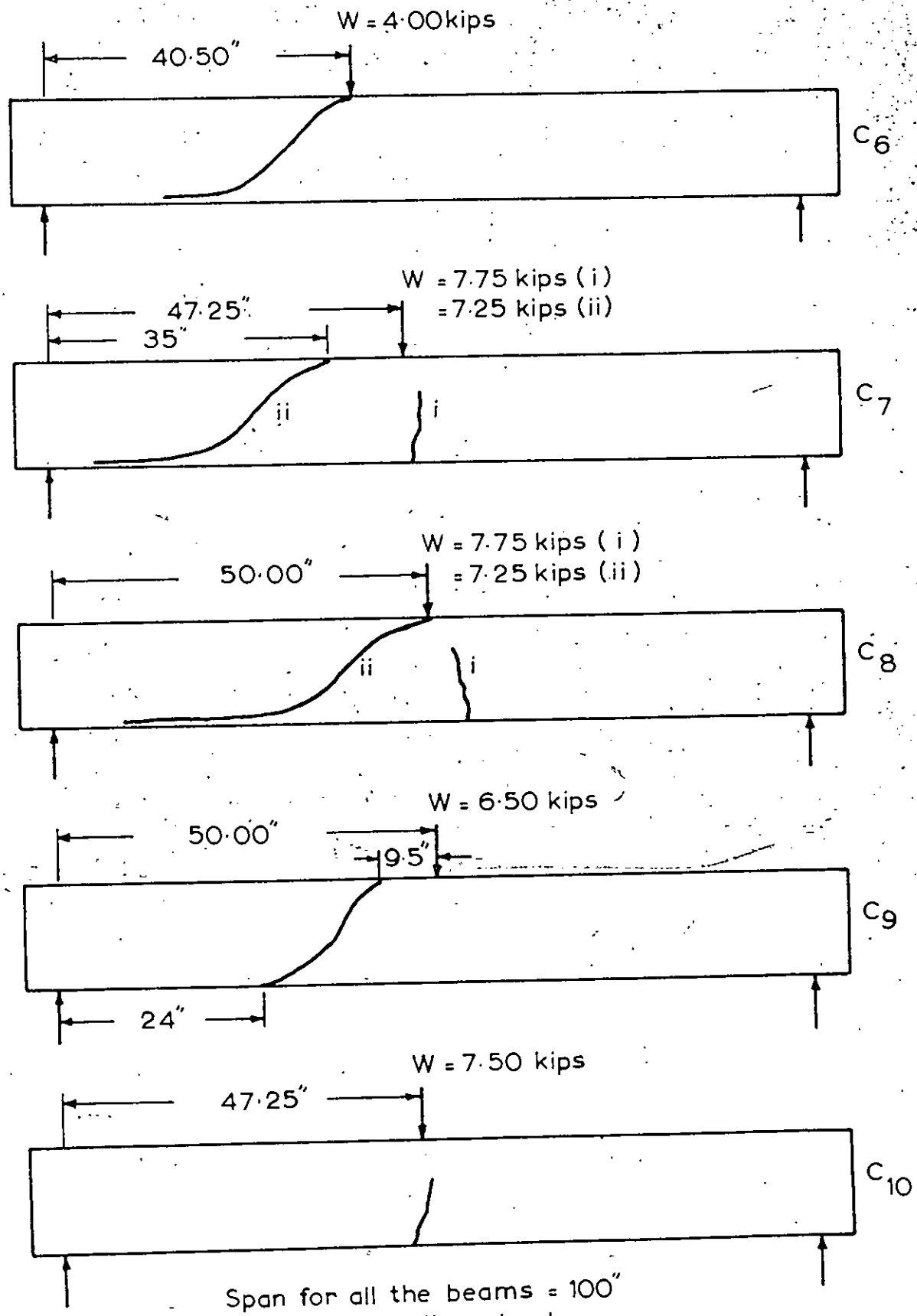
V.3.1 Description of specimens and conduct of tests:

In total, ten beams were tested. All the beams were of rectangular section of 5 in. x 8 in. reinforced with 3 numbers of 1/2 in. diameter round bars on the tension side only. The beams had an effective depth of 6.75 in. The materials used for casting the beams were of same standard as described in Part I of this thesis. The reinforcing rods had a yield stress of 45 ksi. The beams were cast in pairs using steel and plywood moulds and were cured till the date of testing using the methods described in Part I. With each pair of beams, 6" cubes, 6" x 12" prisms and 6" x 12" cylinders were cast. The cubes and prisms were used to know the compressive strength of concrete and the cylinders were used to determine the split-strength of concrete.



Span for all the beams = 100"
W - indicates failure load.

FIG. V. 2. SCHEMATIC ARRANGEMENTS OF LOAD POSITIONS.



Span for all the beams = 100"
W - indicates failure load.
(i) First test ; (ii) Retest.

FIG. V.3. SCHEMATIC ARRANGEMENTS OF LOAD POSITIONS.

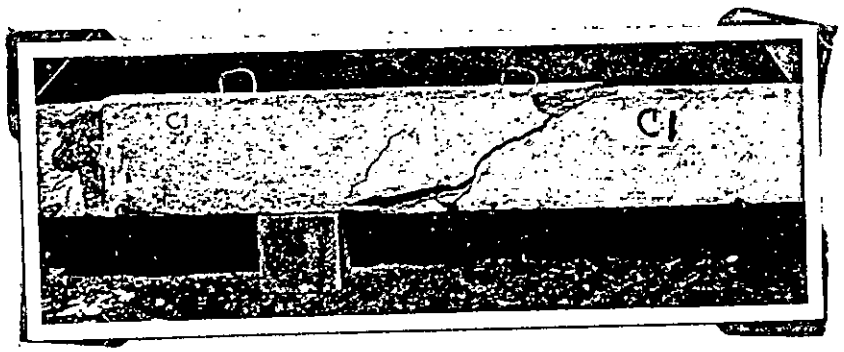
Fig. V.4 : Beams under combined torsion, bending and shear : Modes of Failure

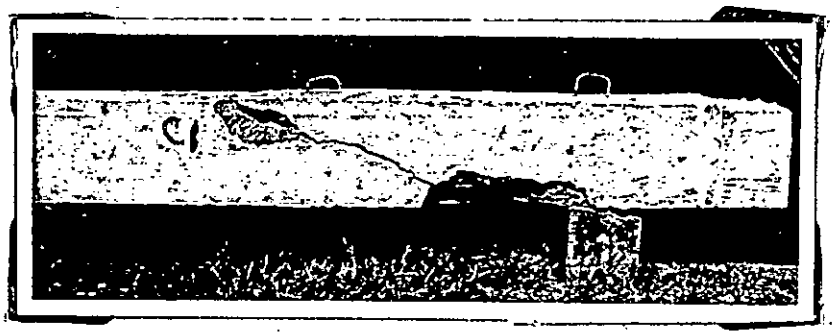
Beam C1 ($\alpha = 5.0$, $\phi = 0.116$)

Beam C1 (-do-)

Beam C2 ($\alpha = 3.12$, $\phi = 0.406$)

Beam C2 (-do-)


C1
C1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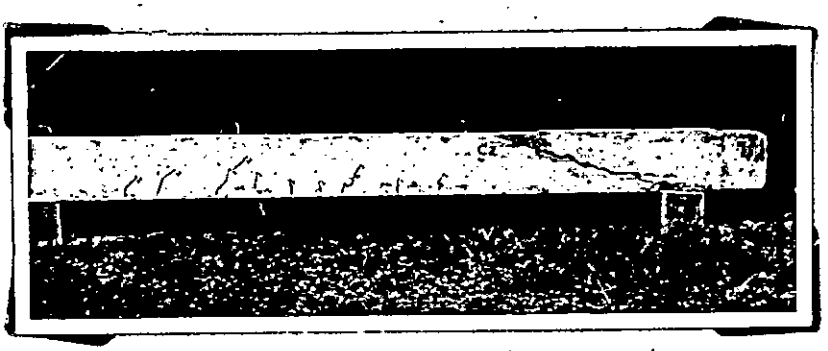

C2

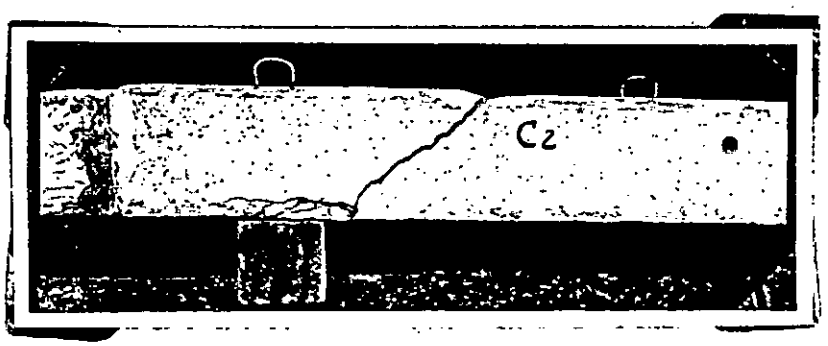

C2

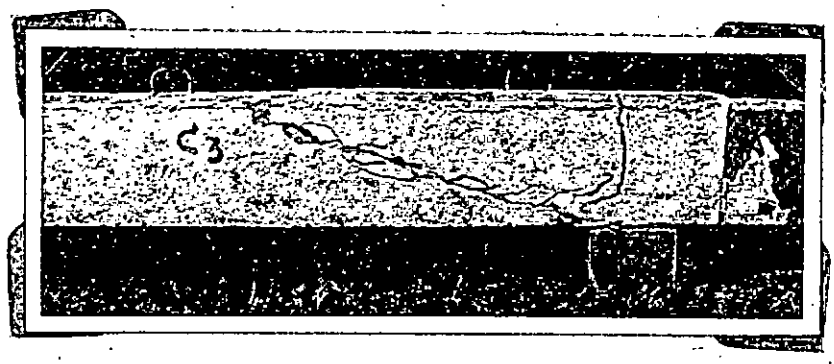
Fig. V.5 : Beams under combined torsion, bending and shear :

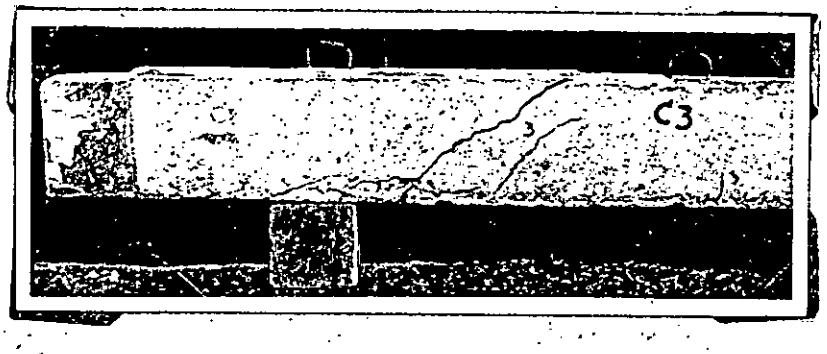
Modes of Failure

Beam C3 (α= 3.55, ϕ = 0.164)

Beam C3 (-do-)

Beam C4 (α= 4.00, ϕ = 0.163)


C3


C3

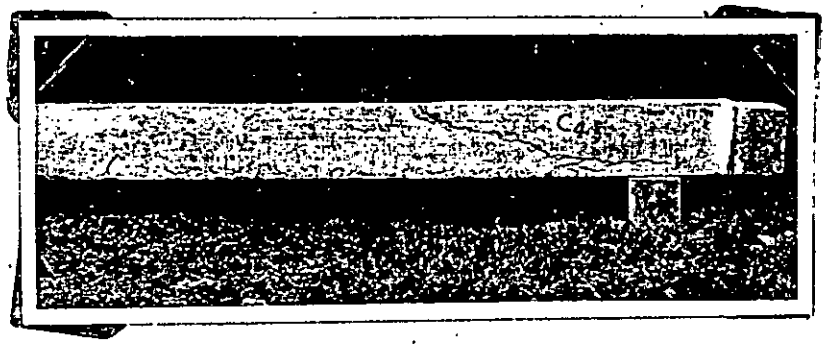
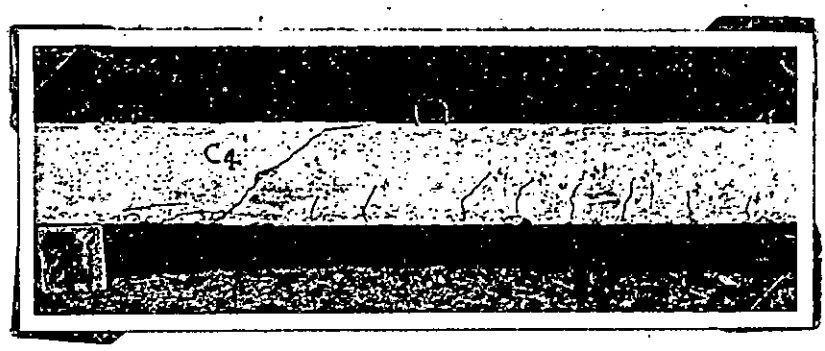

C4

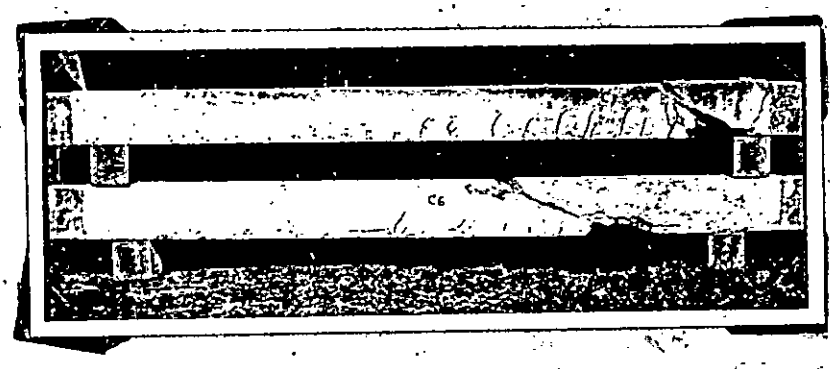
Fig. V.6 : Beams under combined torsion, bending and shear : Modes of Failure

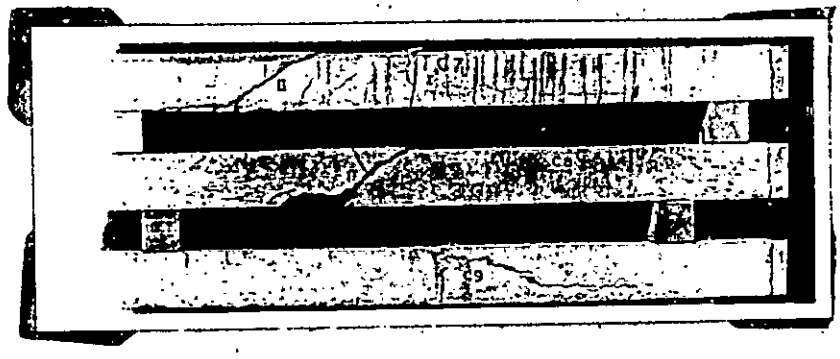
Beam C4 (α = 4.00, ϕ = 0.163)

Beam C5 (α = 2.00, ϕ = 0.090) and
Beam C6 (α = 6.00, ϕ = 0.102)

Beam C7 (α = 7.00, ϕ = 0.050),
Beam C8 (α = 7.40, ϕ = 0.060) and
Beam C9 (α = 6.00, ϕ = 0.130).


C4


C6



All the beams were tested after 28 days of curing in a reinforced concrete loading frame. They were simply-supported over a span of 100 in. on special type of saddles which allowed free rotation. Two cantilever lattice brackets attached to the beam applied the torque and the bending moment and shear force were induced by a point load applied on the top of the beam. The details of the loading system were same as discussed in Part I and hence they are not given here in detail. The deflections of the beams under the load point were measured using dial gauges of least count 1×10^{-3} inch.

The position of load point on the span was varied for each beam so as to get a predetermined value for the moment/shear ratio. The loads were applied in small increments so as to study the behaviour of beams closely. After each increment of load, the deflection readings were taken and the cracks were traced. The loading was continued till the final collapse of the beam. After failure, the beams were photographed. In Fig. V.2 and V.3, the schematic arrangements of load positions for various beams are given. In the same figures, the position and mode of failure cracks are also shown. Figs. V.4 to V.6 show the crack patterns of beams. In Table V.1, a summary of test results is given.

TABLE V.1

SUMMARY OF TEST RESULTS

Specimen Mark	Cube strength psi	Cylinder split strength psi	f'_c+ psi	At Failure Section					*Mode of failure
				α	ϕ	M_b in-kips	M_t in kips	V kips	
C 1	2500	200	2100	5.00	0.116	128	14.84	3.81	3
C 2	2200	200	1900	3.12	0.406	34	13.17	1.62	1
C 3	2200	200	1900	3.55	0.164	70	11.50	2.92	3
C 4	2500	200	2100	4.00	0.163	81	13.17	3.00	3
C 5	2500	200	2100	2.00	0.090	128	11.50	9.51	2
C 6	2500	200	2100	6.00	0.102	96	9.84	2.38	3
C 7 I	2600	220	2250	7.00	0.051	193	9.84	4.09	F
II	2600	220	2250	5.20	0.098	134	13.17	3.82	3
C 8 I	2600	220	2250	7.40	0.060	193	11.50	3.86	F
II	2600	220	2250	7.40	0.082	181	14.84	3.62	3
C 9	2600	220	2250	6.00	0.130	132	17.10	3.25	3
C 10	2500	200	2100	7.00	0	185	0	3.92	F

* Mode 1 : Predominent shear and torsion failure
Mode 2 : Predominent moment and torsion failure
Mode 3 : Predominent moment and shear failure
F : Flexure Failure

See Art. V.3.3

\+ f'_c as given by prism test.

V.3.2 Behaviour of Beems:

Since the beams were tested with different moment/shear ratio, the behaviour of each beam is described briefly in the following pages:

Beam C 1 was tested with a/d ratio equal to 5.0. The torque and bending moment were increased simultaneously till failure. Initially flexural cracks appeared near the loads and for further increments in loads they propagated upwards. At about 80 percent of the ultimate torque, a diagonal crack appeared. The beam failed along the diagonal crack with signs of crushing of concrete in the compression zone of the beam. The failure was very sudden and the deflections increased enormously at failure. The face on which the torsional and direct shear stresses have mutually opposite effects experienced a more flat type of diagonal crack. The flexural cracks were very few in number on this face.

In Beam C 2, the value of a/d ratio was equal to 6.0 in the shorter shear span. Initially flexural cracks were seen near the load point. But this beam suddenly failed in the longer shear span at about 38.5 ins. from the load point. There was no flexural crack in the failure region. The opening of the diagonal crack was very wide and there were certain secondary cracks along the tension steel. The inclination of the crack was approximately equal to 45°.

The shorter shear span of beam C3 had a length equal to 4d. In the initial stages, flexural cracks appeared near the load point end for increments in load, more flexural cracks appeared. At about 75 percent of the bending load a diagonal crack appeared in the shorter shear span and after this stage the deflection of the beam increased considerably. The beam failed suddenly along the diagonal crack and this crack was about 3 in from the load point. The bottom end of the crack propagated along the tension steel and there was a vertical hair crack near the support. The other face of the beam experienced a more flat type of diagonal crack similar to Beam C1.

Beam C4 was tested with a concentrated load at the centre of the span. The behaviour of this beam was similar to that of beam C3 and the diagonal crack was about 23 in. from the load point.

In beam C5, the value of a/d ratio was equal to 2 for the shorter shear span. Flexural cracks appeared near the load point and there were only very few flexural cracks in the longer shear span. A diagonal crack appeared at about 70 percent of the ultimate bending load. After the formation of diagonal crack, the beam sustained considerable amount of extra bending moment. In the meantime some more diagonal cracks formed in parallel to the existing one. The beam

failed by progressive opening of the diagonal crack and large crushing of concrete was seen in the compression zone of the beam. In contrast to the sudden failures of other beams, the failure of this beam was gradual and the deflection of the beam was comparitively small. The failure crack was accompanied by a vertical crack at the support point.

The shorter shear span of Beam C6 had a length equal to 6d. A full-length diagonal crack appeared at about 80 percent of the ultimate bending load. In the next stage of loading this crack considerably widened and precipitated in the failure of the beam for further increments in bending moment. Immediately after the formation of diagonal crack, the beam experienced large deflections.

The length of shorter shear span of Beam C7 was equal to 7d. The behaviour of this beam was similar to that of a beam under bending and shear. Large number of flexural cracks were seen along the span length and those which formed away from the load point curved towards the load point when the bending load was increased. A diagonal crack formed at about 85 percent of the ultimate bending moment. But finally the beam failed due to the wide opening of the flexural crack just near the load point even though the existing diagonal crack was considerably wide. On this beam a retest was conducted by maintaining approximately the flexure failure

load and increasing the twisting moment. The flexure failure load dropped by 0.5 kip and maintaining this load, the torque was increased. The beam failed by opening of the existing diagonal crack. This diagonal tension failure pattern of the beam was similar to that of a beam under bending and shear. There were secondary cracks along the tension steel on both the faces of the beam.

Beam C8 was tested with a point load at the centre of the span. This beam also first failed in flexure and a diagonal tension failure was obtained by conducting a retest as in the case of Beam C7. The behaviour and crack patterns of these two beams were similar.

In Beam C9 also, the point load was placed at the centre of the span. The diagonal crack occurred at about 80 percent of the ultimate bending moment. The behaviour of this beam was similar to those of Beams C3 and C4. The failure crack was at a distance of 9.5 in. from the load point.

The shorter shear span of Beam C10 had a length equal to 7d. This beam was tested in bending and shear. The beam finally collapsed by wide opening of the flexural crack under the load point. Large deflections were noticed near the failure of the beam.

V.3.3 Modes of Failures:

From the general behaviour of beams discussed in the

preceding article, it can be said that there are three possible modes of failure in reinforced concrete beams under combined torsion, bending and shear.

Mode 1: Predominent shear and torsion failure:

This type of failure occurs when the bending moment is small. Mostly the failure is brought out by the combined action of torsion and shear force. The failure crack is approximately 45° to the longitudinal axis of the beam. The beam C2 of the present investigation failed as per this mode*

Mode 2: Predominent moment and torsion failure:

This type of failure occurs for low values of $\alpha\phi$ (say $\alpha\phi$ less than 0.20). The influence of shear force on the mode of failure is secondary and the failure crack resembles that of diagonal compression type of failure discussed in the case of beams under combined bending and torsion (vide Part I, Ch.I.6). The beam C5 failed in this manner.

Mode 3 : Predominent moment and shear failure:

This type of failure occurs for values of $\alpha\phi$ between 0.20 and 1.00. The failure modes are very similar to those discussed in the case of beams under combined bending and shear (see Chapter II.5). Beams C1, C3, C4, C6, C7II, C8 II and C9 failed as per this mode.

* This failure is similar to the predominent torsion failure discussed in the case of beams under combined bending and torsion (See Ch.I.6)

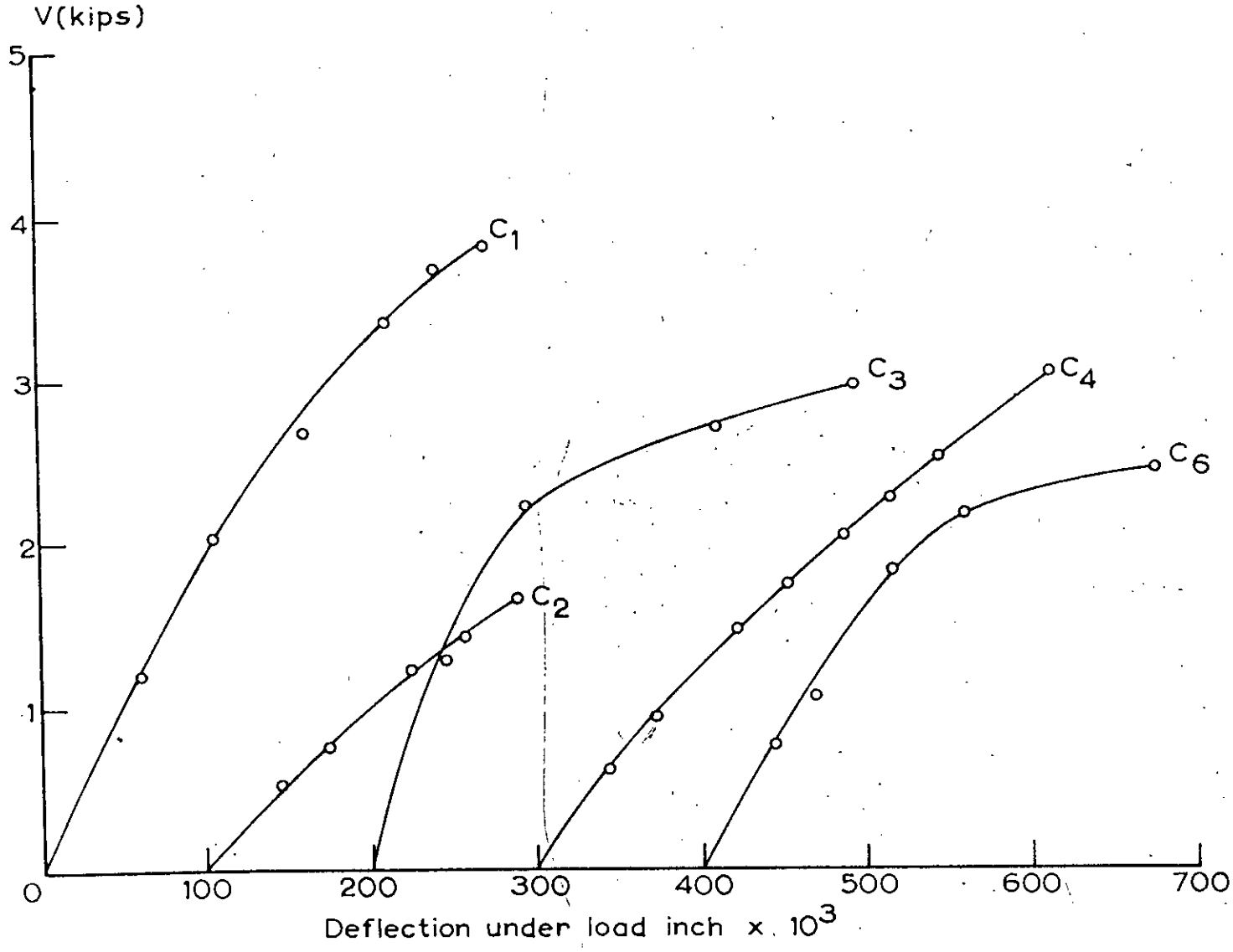


FIG. V. 7.A. CONTD.

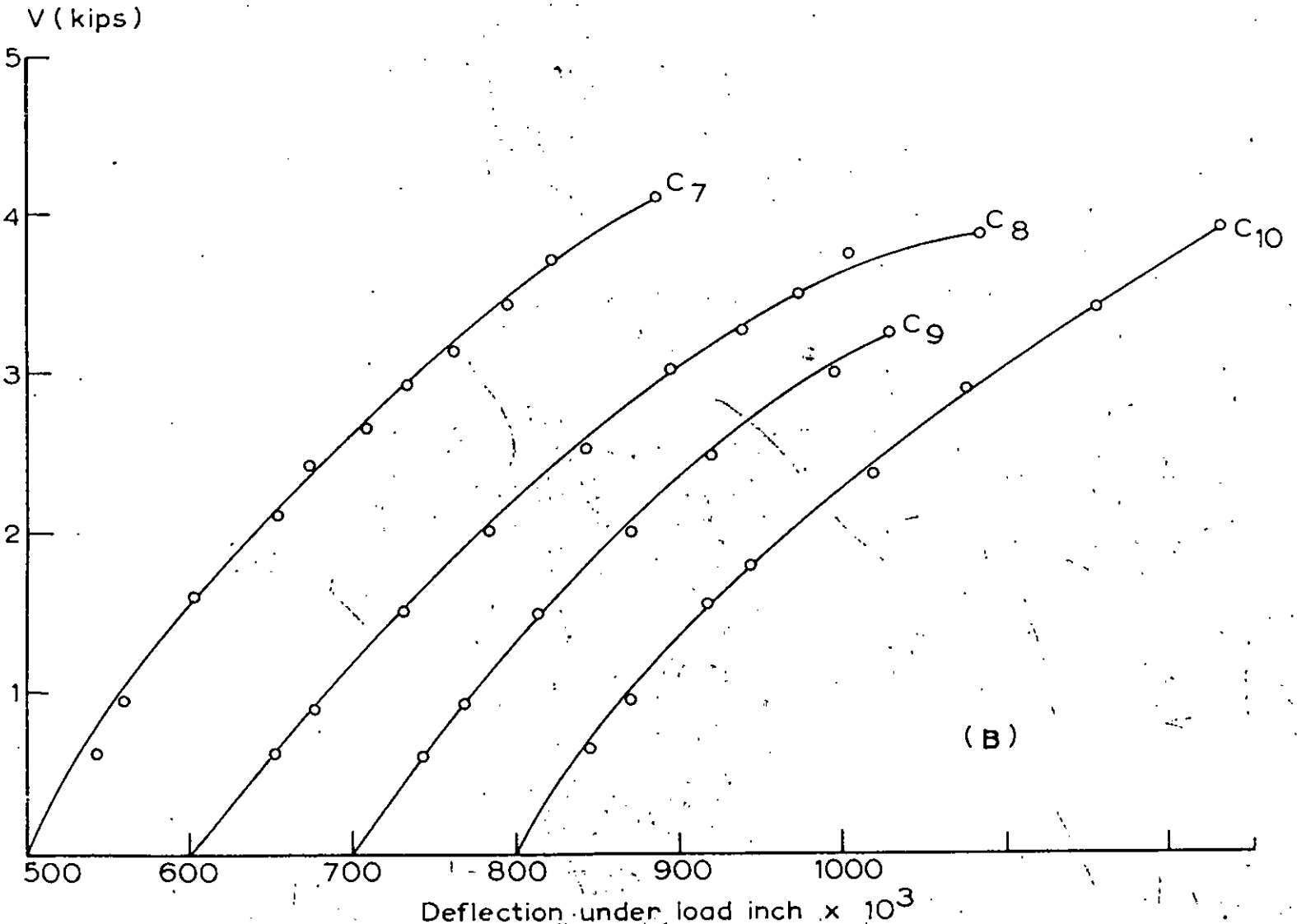


FIG. V. 7 A & B. SHEAR FORCE Vs DEFLECTION CURVES.

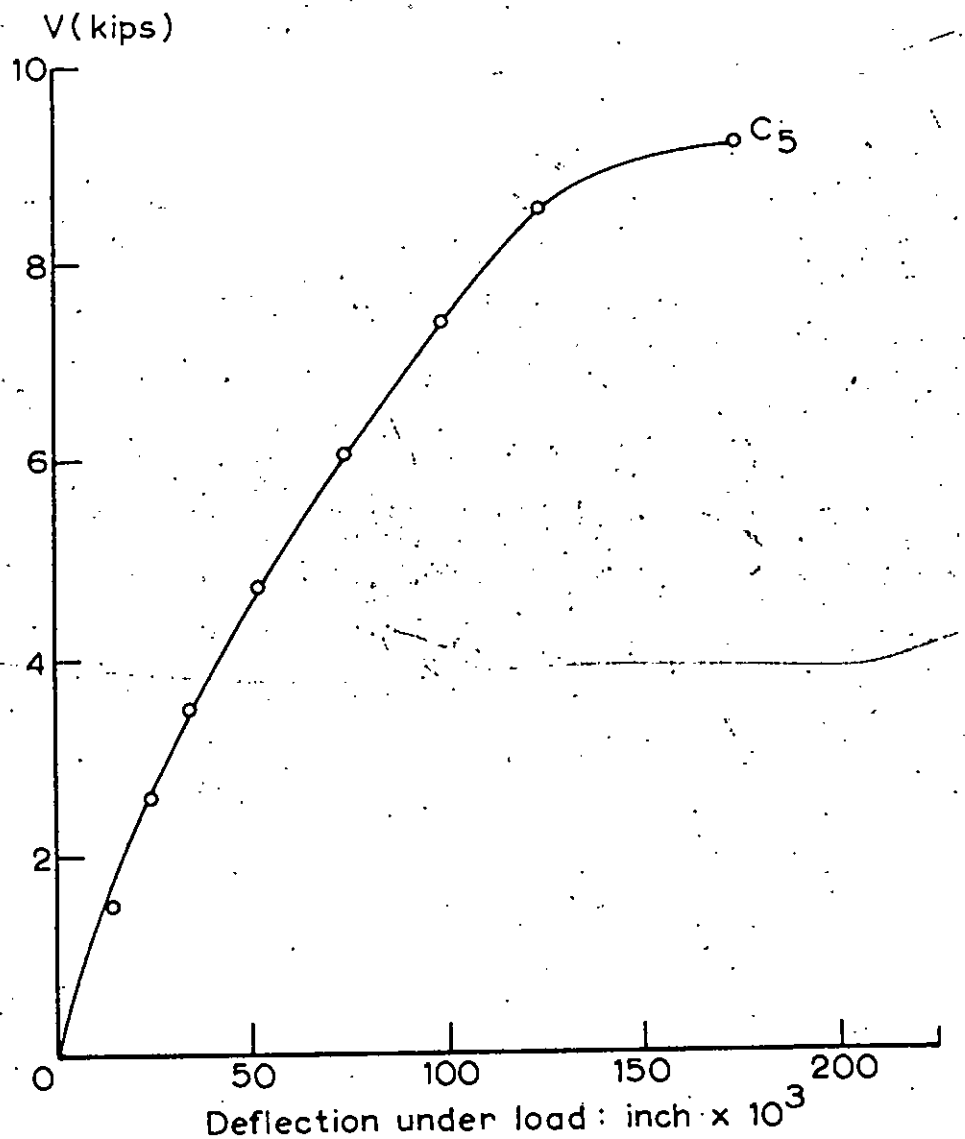


FIG. V. 8. SHEAR FORCE Vs DEFLECTION CURVES.

V.3.4 Deflections of Beams:

For all the beams, deflections were measured under the load points. The shear-deflection curves are shown in Figs. V.7 and V.8. From these curves, the following conclusions can be drawn:

a. The shear-deflection curves are initially linear and become non-linear after the formation of cracks.

b. The formation of diagonal cracks is indicated by a sudden change in the shear-deflection curves. The curves of beams C3, C5 and C6 are typical examples.

c. The deflection curves of beams with low values of $\alpha\phi$ ratio are similar to that of a beam failing in flexure. The similarity of curves of beams C7, C8 and C10 is a typical example of this. This indicates that the behaviour of such beams are similar.

d. The increase in the value of $\alpha\phi$ decreased the deflections of beams under ultimate load. For instance, the ultimate deflection of beam C7 ($\alpha\phi$ = 0.36) is greater than that of beam C6 ($\alpha\phi$ = 0.61). Obviously this is due to the decrease in the ultimate moments for increase in the value of $\alpha\phi$ (compare the ultimate moments of beams C6 and C7).

V.4 Analytical Investigation:

V.4.1 In this article, analytical equations are derived for the

determination of ultimate strength of reinforced concrete beams subjected to combined torsion, bending and shear. The derivations are done at the first instance for the case of rectangular beams and then they are extended to the case of flanged beams.

In beams under combined bending, torsion and shear, there are three possible modes of failures. When the bending moment is small, the failure will be caused by the combined action of torsion and shear. This type of failure is called "Mode 1" failure. The "Mode 2" failure will occur when the failure is predominently caused by bending moment and torque with the secondary influence of shear force on the resulting stresses. In "Mode 3" failure, the failure will be brought out predominently by bending moment and shear force in the presence of secondary influence of torque on the resulting stresses. Equations are derived for these three modes of failures.

V.4.2 Assumptions:

The derivations are based on the following assumptions:

1. Concrete in the tension zone of the beam is neglected in the calculation of stresses caused by bending moment for Mode 2 and Mode 3 types of failures.

2. The torsional shear stress is calculated using a semi-plastic concept and a rectangular distribution is assumed for the shear stress caused by the shear force.

3. The transverse steel yields before failure of the beam and hence the axial stress in it is assumed to be equal to its yield stress.

4. It is assumed that for Mode 2 and Mode 3 types of failures, the transverse hoops completely resist the shear force and they do not contribute to the torque.

5. Contribution from dowel action of longitudinal steel is neglected.

6. The beam will fail in one of the three modes of failure when the concrete element at the critical point for failure fails under the combined action of compressive-tensile stresses.

7. Bond and anchorage failures are excluded from the analysis.

8. It is assumed that the influence of bending moment on the failure mechenism of beams failing as per Mode 1 is negligible and that the failure is primarily due to shear force and twisting moment.

V.4.3 Rectangular Beams

Beams without transverse hoops

Mode 1:

In the case of Mode 1 type of failure bending moment will be very small and hence its influence on torsion and shear can be ignored. At any point in the cross section, there are

two shear stress components, say τ_{xz} and τ_{yz}, caused by the twisting moment. The resultant of these two components is denoted by τ_t and by the semi-plastic condition it is assumed that the magnitude of τ_t is constant at all points in the cross section. Let τ_s be the shear stress caused by the shear force. The state of stresses existing in a beam is shown in Fig. V.9

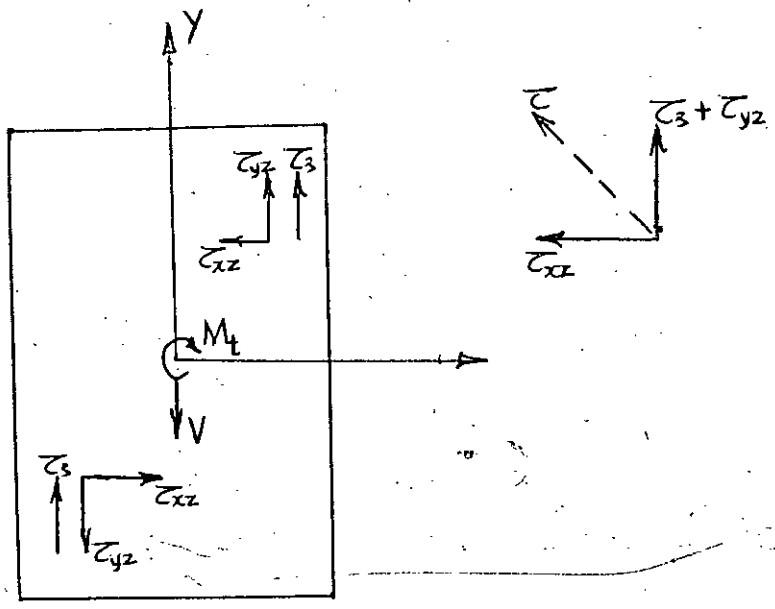


Fig. V.9

From Fig. V.9, it can be seen that the shear stress τ_s will have additive effect with τ_{yz} in one half of the cross section and in the other half these two shear stresses will have subtractive effect. Obviously that portion of the cross section in which τ_s and τ_{yz} have additive effect will contain the critical point for failure. So considering any

point in this part of the cross section, the net resultant shear stress τ is given by,

$$\tau^2 = (\tau_{yz} + \tau_s)^2 + \tau_{xz}^2 \qquad \ldots \text{V.6}$$

$$= \tau_{yz}^2 + \tau_s^2 + 2\tau_{yz}\tau_s + \tau_{xz}^2$$

$$= \tau_s^2 + (\tau_{yz}^2 + \tau_{xz}^2) + 2\tau_{yz}\tau_s \qquad \ldots \text{V.7}$$

The experimental results of author as well as those of Nylander[4] have indicated that in this mode failure was always initiated at the mid-depth of the beam. Hence at the critical point for failure, $\tau_{yz} = \tau_t$, $\tau_{xz} = 0$ and thus we get,

$$\tau^2 = \tau_s^2 + \tau_t^2 + 2\tau_t \cdot \tau_s \qquad \ldots \text{V.8}$$

$$= (\tau_s + \tau_t)^2$$

$$\text{or } \tau = \tau_s + \tau_t \qquad \ldots \text{V.9}$$

The shear stress τ will cause two unlike principal stresses which are equal in magnitude to the shear stress, τ. Failure is caused by the destruction of concrete when these principal stresses satisfy the failure criterion for concrete under compressive-tensile stresses.

Since the bending moment present is small, τ_s can be calculated using the relation,

$$V = bd\tau_s \qquad \ldots V.10$$

where V is the shear force. A similar relation has also been used by some of the previous investigators like Brown[27], Ersoy[58], etc.

The value of τ_t can be calculated using the semi-plastic concept introduced in Part I of this thesis. Then,

$$M_t = \beta_s b^2 D \tau_t \qquad \ldots V.11$$

The values of β_s for rectangular beams are given in Table V.2

TABLE V.2

VALUES OF COEFFICIENTS β_s AND β_c FOR RECTANGULAR BEAMS

D/b	1.0	1.2	1.5	2.0	2.5	3.0	4.0	5.0	10.0
β_s	0.271	0.285	0.310	0.332	0.346	0.356	0.370	0.379	0.400
β_c	1.07	1.22	1.40	1.75	2.10	2.46	3.14	3.82	7.25

From Eqns. V.10 and V.11,

$$\tau_s = \frac{V}{bd}$$

$$\tau_t = M_t / \beta_s b^2 D \qquad \ldots V.12$$

$$\text{or } \tau = \tau_s + \tau_t = \frac{V}{bd} + \frac{M_t}{\beta_s b^2 D} \qquad \ldots V.13$$

In the absence of normal stresses,

$$\sigma_1 = -\sigma_2 = \tau \qquad \ldots V.14$$

Substituting for σ_1 and σ_2 in the failure criterion given by Krishnaswamy and solving for τ, we get,

$$\tau \simeq \left(\frac{2k'-1}{2k'^2}\right) f'_c \qquad \ldots V.15$$

Substituting the value of τ from Eqn. V.15 in Eqn. V.13,

$$\left(\frac{V}{bd} + \frac{M_t}{\beta_s b^2 D}\right) = \left(\frac{2k'-1}{2k'^2}\right) f'_c \qquad \ldots V.16$$

Designating

$$V_o = bd \left(\frac{2k'-1}{2k'^2}\right) f'_c$$

and

$$M_{to} = \beta_s b^2 D \left(\frac{2k'-1}{2k'^2}\right) f'_c \qquad \ldots V.17$$

Eqn. V.16 becomes,

$$\left(\frac{V}{V_o}\right) + \left(\frac{M_t}{M_{to}}\right) = 1.0 \qquad \ldots \text{V.18}$$

It is to be pointed out here that Eqn. V.18 is of the same form as given by Sozen and Kemp[164] and Collins et al[56] based on the test results of Nylander[4].

Mode 2:-

As discussed earlier, Mode 2 failure occurs predominently by the action of bending moment and torque with the secondary effects of shear force. The critical point for failure is that point where the normal stress caused by the bending moment is maximum. The normal stress and the resultant shear stress at this point will interact with each other and will yield a set of compressive-tensile principal stresses and failure will occur when these principal stresses satisfy a failure criterion for concrete under compressive-tensile stresses.

Let f be the normal compressive stress and τ be the resultant shear stress. Then,

$$\sigma_1 = f/2\,(\gamma + 1)$$
$$\sigma_2 = f/2\,(\gamma - 1) \qquad \ldots \text{V.19}$$

where $\gamma = \sqrt{1 + \beta^2 \phi^2}$ and $\phi = \frac{2\tau}{f}$

Substituting for σ_1 and σ_2 in the failure law given by

Krishnaswamy and solving for f we get,

$$\frac{f}{f'_c} = \left(\frac{f'_t}{f'_c}\right)^2 \frac{\gamma+1}{(\gamma-1)^2}\left[\sqrt{1+\left\{\frac{2f'_c}{f'_t}\cdot\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^2}-1\right] \quad \ldots \text{V.20}$$

Putting $M_b/M_{bo} \simeq f/f'_c$

We get,

$$\frac{M_b}{M_{bo}} = \left(\frac{f'_t}{f'_c}\right)^2 \frac{\gamma+1}{(\gamma-1)^2}\left[\sqrt{1+\left\{\frac{2f'_c}{f'_t}\cdot\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^2}-1\right] \quad \ldots \text{V.21}$$

where M_b is the ultimate flexural capacity of a beam under combined bending, torsion and shear failing as per Mode 2.

M_{bo} is the flexural capacity of the corresponding beam under pure flexure.

The Eqn. V.21 is of the same form as derived for the case of flexure failure in beams under bending and torsion. The value of $\lambda\phi$ can be determined as follows:

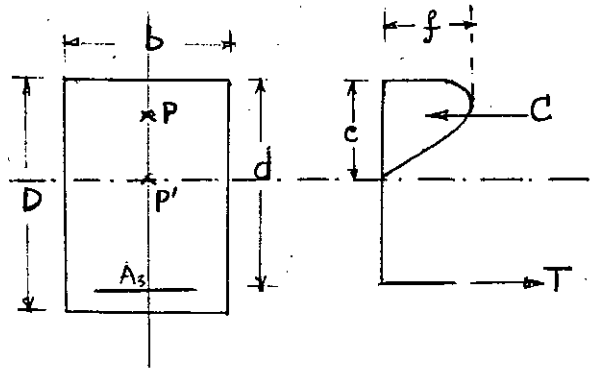


Fig. V.10

Referring to Fig. V.10,

$$M_b = k_1 k_3 \, bcf \, jd$$

$$\text{or} \quad f = \frac{M_b}{k_1 k_3 bcjd} \qquad \ldots \text{V.22}$$

where f is the maximum normal compressive stress (contribution from the tension zone concrete is conservatively neglected)

$$M_t = \beta_s b^2 D \, \tau_t$$

$$V = bc \, \tau_s \qquad \ldots \text{V.23}$$

$$\text{or} \quad \tau_t = M_t / \beta_s b^2 D$$

$$\tau_s = V/bc = \frac{M_b}{\alpha bcd} \qquad \ldots \text{V.24}$$

where $\alpha = M_b/Vd$.

Assuming point P in Fig. V.10 as the critical point for failure, we have,

Resultant shear

$$\text{Stress } \tau = \sqrt{\tau_t^2 + \tau_s^2}$$

$$= \sqrt{\left(\frac{M_t}{\beta_s b^2 D}\right)^2 + \left(\frac{M_b}{\alpha bcd}\right)^2} \qquad \ldots \text{V.25}$$

$$= \frac{M_t}{\beta_s b^2 D} \sqrt{1 + \left(\frac{\beta_s b^2 D}{bcd\alpha\phi}\right)^2}$$

$$= \frac{M_t}{\beta_s b^2 D} \cdot N. \qquad \ldots \text{V.26}$$

where

$$N = \sqrt{1 + \left(\frac{\beta_s b^2 D}{bcd\alpha\phi}\right)^2}$$

$$\phi = M_t/M_b$$

$$\lambda\phi = 2\tau/f$$

Substituting the values of τ and f from Eqns. V.22 and V.26,

$$\lambda\phi = \frac{2 M_t}{\beta_s b^2 D} \cdot N. \frac{k_1 k_3 bcjd}{M_b}$$

$$\text{or } \lambda = \frac{2N k_1 k_3 bcjd}{\beta_s b^2 D} \qquad \ldots \text{V.27}$$

Observing $c = (1-j)\frac{d}{k_2}$

$j = 7/8$

$D/d \simeq 1.2$ and $k_2/k_1 k_3 = 0.525$

we get

$$\lambda = \frac{N. (D/b)}{3.456 \beta_s} \qquad \ldots \text{V.28}$$

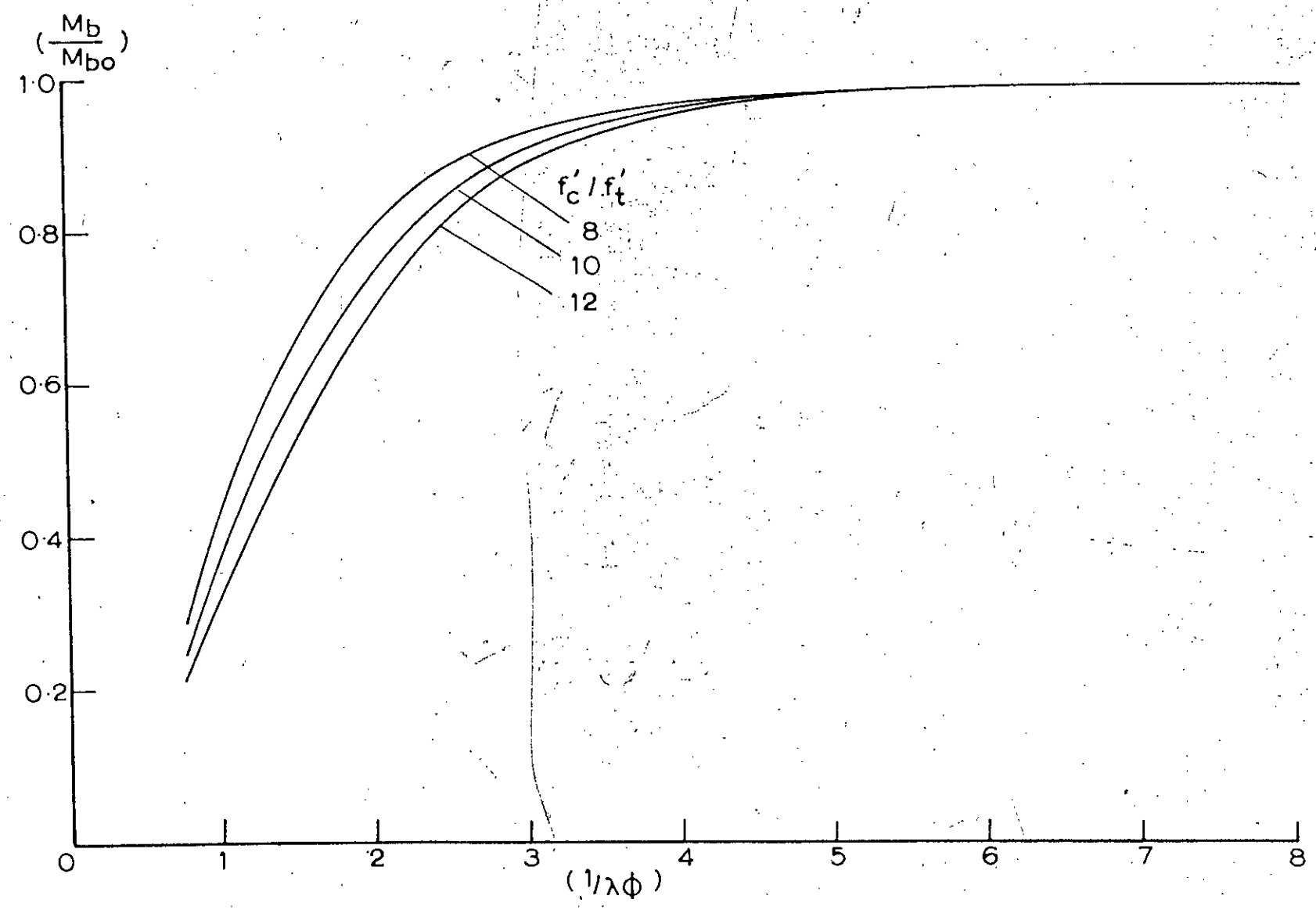


FIG. V. 11. VARIATION OF $\frac{M_b}{M_{bo}}$ WITH $\frac{1}{\lambda\Phi}$.

where

$$N = \sqrt{1 + \left\{\frac{4.84\ (b/D)\beta_s}{\alpha\phi}\right\}^2}$$

or designating $\beta_c = \dfrac{(D/b)}{3.456\ \beta_s}$

We get

$$\lambda = N.\beta_c$$

$$N = \sqrt{1 + \left(\frac{1.4}{\alpha\phi\beta_c}\right)^2} \qquad \ldots \text{ V.29}$$

The values of β_c for rectangular section are given in Table V.2. Thus for the known values of λ, ϕ and f'_c/f'_t, the strength of beams for Mode 2 failure can be determined using Fig. V.11 which is a graphical representation of Eqn.V.21.

Mode 3:-

Since Mode 3 failure will be caused predominently by bending moment and shear force, we have to consider two mechanisms of failure similar to the case of beams under combined bending and shear. That is, for high and medium values of M/Vd ratio, the strength of the concrete element at the level of neutral axis gives the strength of the beam. For low values of M/Vd, the beam transforms itself into an arch and the strength of the arch is to be evaluated.

These two types of mechanisms of failure have been discussed in detail in Part II of this thesis and hence they need not be repeated here. So equations are derived for these two cases:

For high and medium values of M/Vd ratio :

Let V and M_t be the shear force and torque at the failure section. Then,

$$V = bc\,\tau_s$$
$$M_t = \beta_s b^2 D\,\tau_t \qquad \text{... V.30}$$

Assuming the point P' shown in Fig. V.10 as the critical point for failure, Resultant shear stress

$$\tau = \sqrt{\tau_s^2 + \tau_t^2}$$
$$= \sqrt{\left(\frac{V}{bc}\right)^2 + \left(\frac{M_t}{\beta_s b^2 D}\right)^2} \qquad \text{... V.31}$$

$$\text{or } \tau = \frac{V}{bc} \cdot \bar{N} \qquad \text{... V.32}$$

where $$\bar{N} = \sqrt{1 + \left\{\frac{M_t\ bc}{\beta_s b^2 DV}\right\}^2}$$

For the case of beams with high and medium values of M/Vd ratio, the critical point for failure is at the neutral axis. The resultant shear stress τ at this point will yield two unlike principal stresses of magnitude equal to τ. So we have,

$$\sigma_1 = -\sigma_2 = \tau \qquad \text{... V.33}$$

From Eqn. V.32

$$\tau = \bar{N} \cdot \frac{V}{bc}$$

Observing $c = \frac{(1-i)}{k_2} \cdot d$, $\alpha = M_b/Vd$

$$\tau = \bar{N} \cdot \frac{M_b}{bd^2 \alpha \left(\frac{1-i}{k_2}\right)} \qquad \ldots \text{ V.34}$$

Using the value of τ as given by Eqn. V.34 and the failure criterion given by Krishnaswamy and proceeding on similar lines to the derivations made in Part II, finally we get,

$$\frac{M_b}{M_{bo}} = \frac{\alpha}{\bar{N} \alpha_{TR}} \qquad \ldots \text{ V.35}$$

where $\alpha_{TR} = \frac{21\ p f_y}{f'_c}$.

<u>For low values of M/Vd ratio:-</u>

Let $M_b = k_1 k_3\ bc\ f\ jd$

$$\text{or } f = \frac{M_b}{k_1 k_3 bcjd} \qquad \ldots \text{ V.36}$$

In the case of low values of M/Vd ratio, the interaction of the horizontal normal stress due to bending moment, the vertical normal stress due to the applied loads and the

resultant shear stress will result in two principal stresses which are compressive-compressive in nature. Since it may not be possible to evaluate the vertical normal stress, the major principal stress can be obtained using the other two stresses and the influence of vertical normal stress can be introduced in a similar way as done in Part II.

That is,

$$\frac{f}{2}\left[\sqrt{1+\left(\frac{2\tau}{f}\right)^2}+1\right]=f'_c$$

$$\text{or}\quad \frac{f}{f'_c}=\frac{2}{1+\sqrt{1+\left(\frac{2\tau}{f}\right)^2}} \qquad \ldots \text{ V.37}$$

From Eqns. V.32 and V.36

$$\frac{2\tau}{f}=\frac{2V}{bc}\cdot\frac{k_1k_3bcjd}{M_b}\cdot\bar{N} \qquad \ldots \text{ V.38}$$

$$=\frac{2k_1k_3j}{\alpha}\cdot\bar{N}$$

or observing $k_1k_3 = 0.80$ and $j = 0.875$,

$$\left(\frac{2\tau}{f}\right)^2\simeq\frac{2}{\alpha^2}\bar{N}^2 \qquad \ldots \text{ V.39}$$

Then,

$$\frac{f}{f'_c}=\frac{2}{1+\sqrt{1+\frac{2\bar{N}^2}{\alpha^2}}} \qquad \ldots \text{ V.40}$$

Let $M_b = k_1 k_3 b c f jd$

$M_{bo} = k_1 k_3 b c_o f'_c jd$ V.41

Substituting the value of f/f'_c from Eqn. V.41 in Eqn.V.40,

$$\frac{M_b}{M_{bo}} = \frac{c}{c_o}\left[\frac{2}{1+\sqrt{1+\frac{2\bar{N}^2}{\alpha^2}}}\right] \quad \ldots \text{ V.42}$$

Similar to the derivations of Part II putting $c/c_o = 1/\alpha$ and introducing a coefficient k to account for the influence of vertical normal stress, Eqn. V.42 becomes,

$$\frac{M_b}{M_{bo}} = \frac{(2/k)}{\alpha + \sqrt{\alpha^2 + 2\bar{N}^2}} \quad \ldots \text{ V.43}$$

The ranges of applicability of Eqns. V.35 and V.43 can be obtained by similar derivations given in Part II. Thus we get,

i. for $\alpha \geq \alpha_m$, M_b/M_{bo} can be obtained using Eqn. V.35 and

ii. for $\alpha \leq \alpha_m$, M_b/M_{bo} is given by Eqn. V.43

where

$$\alpha_m = \frac{(2/k)\,\bar{N}\,\alpha_{TR}}{\sqrt{\frac{4}{k}\bar{N}\alpha_{TR} + 2\bar{N}^2}}$$

$$\bar{N} = \sqrt{1+\left\{\frac{M_t \cdot bc}{\beta_s b^2 DV}\right\}^2}$$

$$\text{or } \bar{N} = \sqrt{1 + \left(\frac{\alpha_\phi \beta_c}{1.4}\right)^2}$$

Beams with transverse hoops

The equations derived above can be easily extended to the case of beams with transverse hoops. For Mode 2 and Mode3 types of failures, assuming the transverse steel yields at failure and resist the shear force only, we have, for two-legged hoops,

$$V = bc\,\tau_s + 2\,A_w f_{yw}\,\frac{d}{s}$$

$$\text{or } \tau_s = \frac{V}{bc}\left[1 - 2A_w f_{yw}\,\frac{d}{sV}\right]$$

$$= \frac{V}{bc}\,\bar{K}. \qquad \ldots \text{ V.44}$$

where A_w is the area of one leg of hoop and

$$\bar{K} = (1 - 2A_w f_{yw}\,\frac{d}{sV}) \geqslant 0.$$

Using the value of τ_s as given by Eqn. V.44, the following equations can be obtained for Mode 2 and Mode 3 failures.

Mode 2:

For Mode 2 failure only the value of N will change and other parameters remain as for the case of beams without

transverse hoops. The value of N can be obtained from the following equation:

$$N_w = \sqrt{1 + \left\{\frac{1.4\,\bar{K}}{\alpha\phi\,\beta_c}\right\}^2} \qquad \text{... V.45}$$

Mode 3:

For $\alpha \geq \alpha_m$:

$$\frac{M_b}{M_{bo}} = \frac{\alpha}{\bar{N}_w\,\bar{K}\,\alpha_{TR}} \qquad \text{... V.46}$$

For $\alpha \leq \alpha_m$:

$$\frac{M_b}{M_{bo}} = \frac{(2/k)}{\alpha + \sqrt{\alpha^2 + 2\bar{K}^2\,\bar{N}_w^2}} \qquad \text{... V.47}$$

where

$$\bar{N}_w = \sqrt{\left[1 + \left\{\frac{\alpha\phi\,\beta_c}{1.4\,\bar{K}}\right\}^2\right]}$$

$$\alpha_m = \frac{(2/k)\,\bar{N}_w\bar{K}\alpha_{TR}}{\sqrt{\frac{4}{k}\,\bar{N}_w\bar{K}\alpha_{TR} + 2\bar{K}^2\,\bar{N}_w^2}}$$

Mode 1:

Since there are no test results available to the author to know the behaviour of beams with transverse hoops failed as per Mode 1, the author has not tried extention of Eqn.V.18 for such cases. However, he believes that Eqn. V.18 can

as well be used to know the strength of beams with transverse hoops when the following values for M_{to} and V_o are assumed:-

$$M_{to} = \left[\frac{2k'-1}{2k'^2}\beta_s\, b^2 D f'_c + A_w f_{yw}\frac{b'd'}{s}\right]$$

and $$V_o = \left[\frac{2k'-1}{2k'^2}\, bdf'_c + 2A_w f_{yw}\frac{d'}{s}\right] \qquad \ldots \text{ V.48}$$

A detailed experimentation on this line is needed to know the applicability of Eqn. V.18 with values of M_{to} and V_o as given by Eqn. V.48

V.4.4 FLANGED BEAMS

Since the mechanisms of failure of beams are independent of the cross section, the equations derived for rectangular beams can be modified to the case of flanged beams by changing certain parameters introduced in the theory. Similar procedure has been adopted in the analysis presented in Parts I and II.

The equation derived for Mode 1 failure (Eqn. V.18) can be used in the same form for flanged beams also with the following values for V_o and M_{to}.

Referring to Fig. V.12,

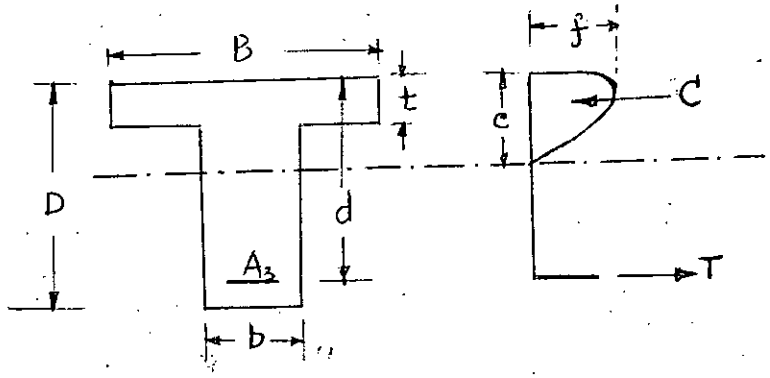


Fig. V.12

$$M_{to} = \beta_s^{f1} b^2 D \left(\frac{2k'-1}{2k'^2}\right) f_c' \qquad \ldots \text{ V.49}$$

where $\beta_s^{f1} = \dfrac{\beta_e^{f1} + \beta_p^{f1}}{2}$

β_e^{f1} - torsion coefficient corresponding to maximum shear stress as per elastic theory.

β_p^{f1} - torsion coefficient corresponding to the shear stress as per rigid plastic theory.

The values of β_e^{f1} and β_p^{f1} for flanged sections (say tee and ell sections) can be obtained using the solutions derived in Chapter I.8 of Part I of this thesis.

$$V_o = \left[\{b.d + (B-b)t\} \left(\frac{2k'-1}{2k'^2}\right) f'_c\right] \quad \ldots \text{V.50}$$

For Mode 2 and Mode 3 failures,

$$\left.\begin{aligned} M_b &= k_1 k_3 \left[bc + (B-b)t\right] fjd \\ V &= \left[bc + (B-b)t\right] \tau_s \end{aligned}\right| \quad \text{for } c > t$$

$$M_t = \beta_s^{fl}\, b^2 D\, \tau_t \quad \ldots \text{V.51}$$

Designating $\left[1 + \frac{(B-b)t}{bc}\right] = \bar{A}$

we get,

$$\left.\begin{aligned} M_b &= k_1 k_3\, bc\, \bar{A}.jd.f. \\ V &= bc.\,\bar{A}\,\tau_s \end{aligned}\right| \quad \ldots \text{V.52}$$

Therefore,

$$f = \frac{M_b}{k_1 k_3 bc\, \bar{A}.jd}$$

$$\left.\begin{aligned} \tau_s &= \frac{V}{bc.\bar{A}} \\ \tau_t &= \frac{M_t}{\beta_s^{fl}\, b^2 D} \end{aligned}\right| \quad \ldots \text{V.53}$$

Using these values for stresses and proceeding on similar lines to the derivations of rectangular beams, we can get the following relation for various parameters introduced in the theory.

Mode 2:

$$\lambda = N.\bar{A}.\beta_c \qquad \ldots \text{V.54}$$

$$N = \sqrt{1 + \left(\frac{1.4}{\alpha\phi\beta_c.\bar{A}.}\right)^2}$$

Mode 3:

$$\bar{N} = \sqrt{1 + \left(\frac{\alpha\phi\beta_c.\bar{A}.}{1.4}\right)^2} \qquad \ldots \text{V.55}$$

$$\alpha_{TR} = \frac{1}{\bar{A}}.\frac{21pf_y}{f'_c}$$

$$\beta_c = \frac{(D/b)}{3.456\ \beta_s^{fl}} \qquad \ldots \text{V.56}$$

The value of $\bar{A}$ can be simplified further,

$$\bar{A} = \left[1 + \frac{(B-b)t}{bc}\right] \qquad \ldots \text{V.57}$$

Observing $c = (1-j)\frac{d}{k_2}$; $j = \frac{7}{8}$; $k_2 = 0.42$

and $D \simeq 1.2d$,

$$\bar{A} = \left[1 + 4\,\frac{t}{D}\left(\frac{B}{b} - 1\right)\right] . \text{ (for } c > t\text{)} \qquad \ldots \text{V.58}$$

Thus using Eqns. V.54, 55, 56 and 57 the various parameters can be evaluated. If $c \leq t$, the value of $\bar{A}$ will be equal to

(B/b). With these modified values for different parameters, the strength of flanged beams can be known using the equations derived for rectangular beams.

V.5 COMPARISON OF RESULTS:

In the preceding article, equations were derived for the three modes of failures. The equation derived for Mode 1 failure is applicable only if bending moment is small. When bending moment is considerable either Mode 2 or Mode 3 failure may occur. In such cases it may be very difficult to find out whether the failure will be caused by a predominent combination of shear force and bending moment or torque and bending moment. So in these cases, the values of M_u/M_{uo} is to be calculated using the equations derived for Mode 2 and Mode 3 failures and the least of these two values is to be taken as the strength of the beam.

A coefficient k was introduced in the derivation of Eqn. V.43 to account for the influence of vertical normal stress caused by the applied loads on the strength of beam. The value of k has been taken equal to 0.75 in the case of beams under combined bending and shear (see Part II, Chapter II.7). The same value is assumed in the present analysis of beams under combined bending, shear and torsion. Further, the Eqn. V.43, similar to its companion equation derived for the case of beams under bending and shear, has

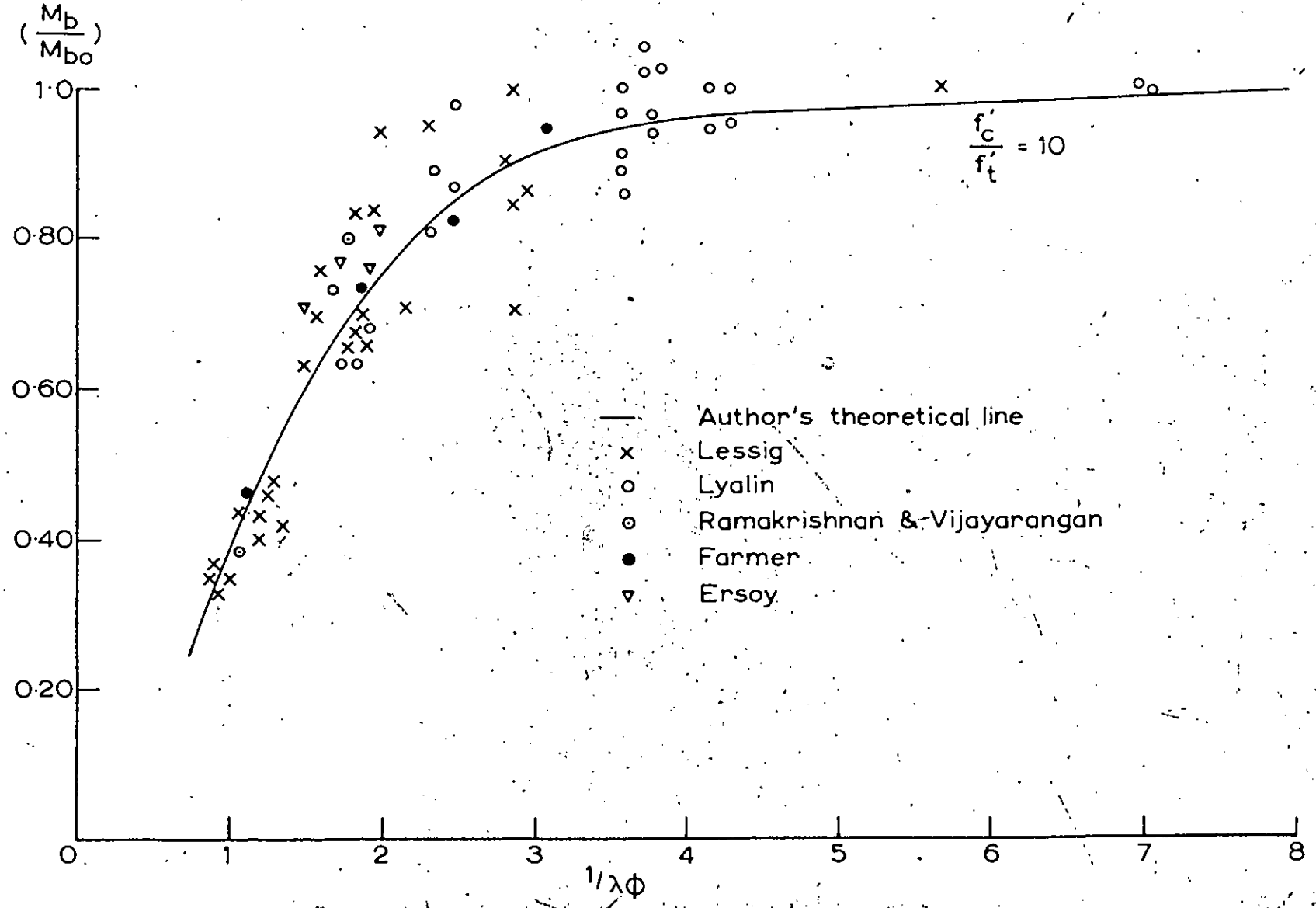


FIG. V.13a. COMPARISON OF RESULTS : MODE 2 FAILURE.

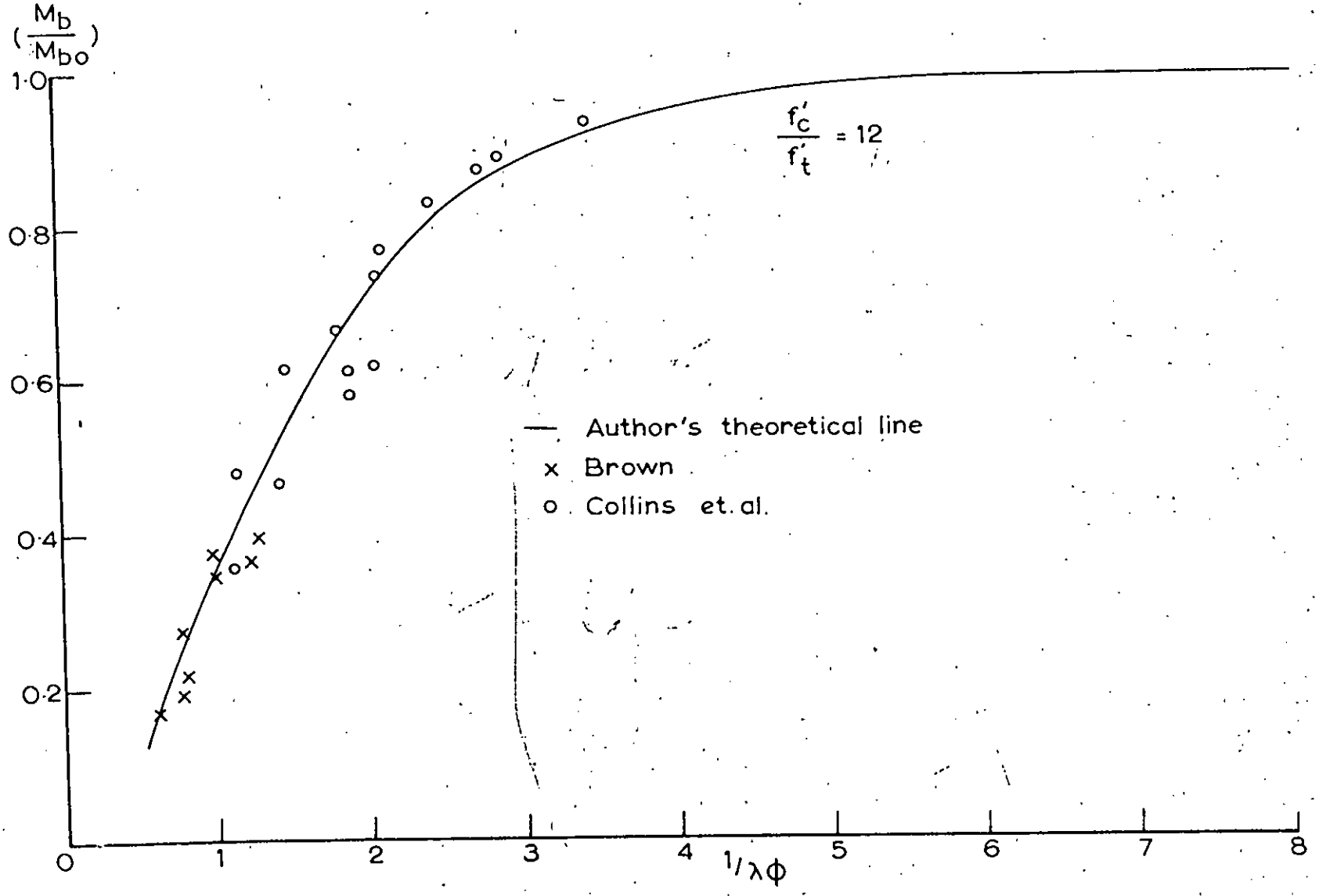


FIG. V. 13b. COMPARISON OF RESULTS : MODE 2 FAILURE.

certain limitation. The value of M_b/M_{bo} given by Eqn.V.43 should not be greater than 1.0. So this equation is valid only upto that particular value of α beyond which M_b/M_{bo} given by the equation is not greater than 1.0. This particular value of α can be obtained by equating the right hand side of Eqn. V.43 equal to 1.0. Thus we get,

$$\alpha = \left[\frac{1}{k} - \frac{N^2}{(2/k)}\right] \qquad \ldots \text{V.59}$$

where k = 0.75.

The Eqn. V.43 is not valid if the given value of α is less than $\left[\frac{1}{k} - \frac{N^2}{(2/k)}\right]$.

In order to check the theoretical equations, the test results reported by Nylander[4], Lessig[17], Lyalin[34], Brown[27], Ramakrishnan and Vijaya Rangan[6], Farmer[163], Collins et al[56] and Ersoy[58] have been collected. The predictions of the theory are compared with the test results in Tables V.3, V.4 and V.5. Figs. V.13(a) and (b) give a graphical picture for the comparison. In these figures the results are compared for Mode 2 failure with $f'_c/f'_t = 10$ (Fig.V.13a) and $f'_c/f'_t = 12$ (Fig. V.13 b). These values for f'_c/f'_t have been used by the author based on the strength of concrete used in the investigation of these research workers. That is

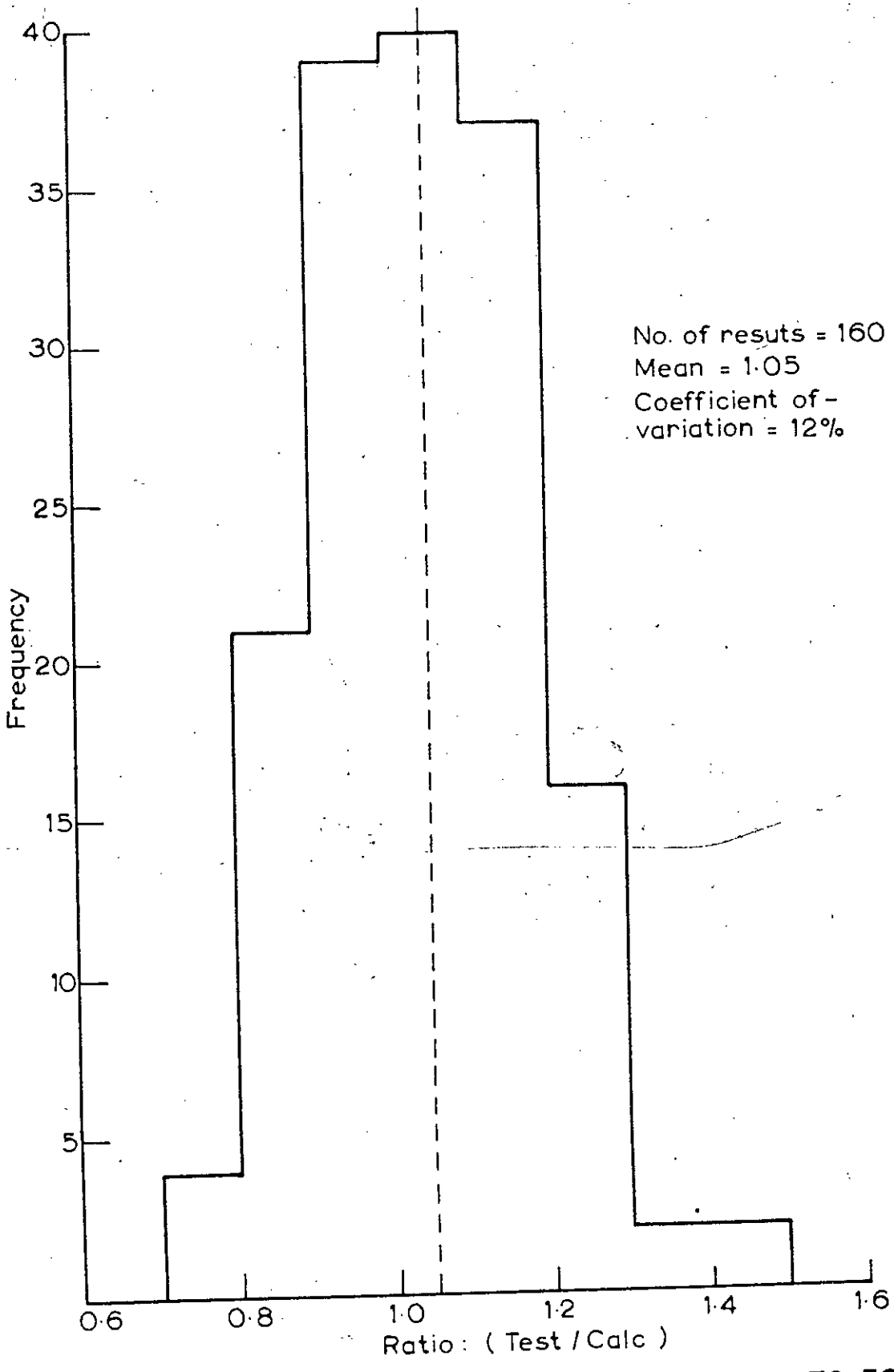


FIG. V. 14. HISTOGRAM : COMPARISON OF RESULTS FOR BEAMS UNDER COMBINED TORSION, BENDING AND SHEAR.

for the results shown in Fig. V.13a, the concrete strength of the beams varied between 2500 psi and 3500 psi and for these values it is reasonable to assume a value of 10 for the ratio f'_c/f'_t . Similarly for the results shown in Fig. V.13b, the strength of concrete varied between 4500 psi and 5000 psi and for these values it is reasonable to assume $f'_c/f'_t = 12$. However, it is to be pointed out here, that the influence of variation in the values of f'_c/f'_t on the theoretical results is not very significant as can be seen from Fig. V.11. Since most of these test results check the accuracy of equations derived for Mode 1 and Mode 2 failures, the author programmed his experimental investigation so as to get results for Mode 3 failure. From the comparison of results, it can be seen that the predictions of the theory are fairly good. The mean value of test/calc. ratio is 1.05, the coefficient of variation being 12%. The histogram is shown in Fig. V.14. However, comparisons with more results, especially for Mode 3 type of failure, are necessary to have further check on the theoretical results.

TABLE V.3

COMPARISON OF RESULTS : MODE 1

Failure (Failure predominently by torque and shear)

Investigator/s	Specimen Mark	V_o (calc) kips	M_{to} (calc) inch-kips	V (test) kips	$(\frac{V}{V_o})$	M_t inch kips Test	M_t inch kips Calc	Ratio: Test/Calc
1	2	3	4	5	6	7	8	9
Nylander	IV 2a	8.60	13.50	4.60	0.535	6.94	6.28	1.10
	2b	8.60	13.50	4.40	0.510	6.76	6.60	1.02
	3a	8.00	13.20	2.70	0.340	10.31	8.70	1.18
	3b	8.00	13.20	2.17	0.270	12.74	9.65	1.32
	4a	9.15	14.10	1.78	0.195	13.36	11.40	1.17
	4b	9.15	14.10	1.64	0.180	13.60	11.60	1.17
Farmer	TR 1	38.00	58.50	7.60	0.200	52.70	47.00	1.12
	2	38.00	58.50	7.06	0.186	54.20	47.80	1.13
	3	38.00	58.50	5.34	0.141	57.20	50.30	1.14
	4	10.20	9.50	1.34	0.137	8.30	8.20	1.01
	7	10.20	9.50	2.49	0.254	6.62	7.10	0.93
	10	10.20	9.50	1.55	0.158	8.80	8.00	1.10
	13	10.20	9.50	1.54	0.157	9.28	8.00	1.16
	14	10.20	9.50	1.50	0.153	9.10	8.05	1.13
	17	10.20	9.50	0.99	0.101	10.10	8.55	1.17
	TW 1	38.00	58.50	6.84	0.181	47.10	48.00	0.98
	2	38.00	58.50	6.84	0.181	52.60	48.00	1.10
	3	38.00	58.50	5.17	0.136	57.90	50.80	1.14
	4	10.20	9.50	1.45	0.148	9.05	8.05	1.13
	6	10.20	9.50	2.50	0.256	8.93	7.05	1.26
	7	10.20	9.50	1.74	0.178	9.98	7.82	1.28
	9	10.20	9.50	2.82	0.288	5.46	6.80	0.80
	10	10.20	9.50	0.99	0.101	9.90	8.55	1.16

(Contd.)

Table V.3 (contd.)

1	2	3	4	5	6	7	8	9
Ersoy	LI 1	9.15	8.00	1.22	0.133	7.50	6.95	1.08
	1a	9.15	8.00	1.29	0.141	8.00	6.90	1.16
	2	9.15	8.00	1.25	0.136	7.75	6.92	1.12
	5	9.15	8.00	0.83	0.091	8.57	7.30	1.17
	5a	9.15	8.00	0.76	0.083	7.92	7.35	1.08
	6	9.15	8.00	0.75	0.082	7.64	7.35	1.04
	6a	9.15	8.00	0.83	0.091	8.58	7.30	1.17
	7	9.15	8.00	2.09	0.228	5.72	6.18	0.93
	8	9.15	8.00	1.91	0.208	5.21	6.30	0.83
	LII 1	8.00	7.60	1.18	0.147	7.26	6.45	1.13
	2	8.00	7.60	0.76	0.095	7.81	6.86	1.14
	3	8.00	7.60	1.92	0.240	5.20	5.78	0.90
	LIII 1	6.85	7.50	0.67	0.098	6.85	6.75	1.02
	3	6.85	7.50	1.06	0.155	6.52	6.35	1.03
	4	6.85	7.50	0.64	0.093	6.59	6.80	0.97
	5	6.85	7.50	1.82	0.266	4.92	5.50	0.90
Collins et al	SL 1	15.00	31.50	2.25	0.150	46.20	26.80	1.72
	LS 3	17.60	38.60	2.18	0.124	39.60	33.80	1.17
	S 5	23.00	53.60	2.84	0.123	47.70	47.00	1.01
	L 6	28.60	54.50	2.50	0.088	54.20	49.60	1.09
Author	C 2	7.22	13.50	1.62	0.224	13.17	10.50	1.25

TABLE V.4

COMPARISON OF RESULTS : MODE 2

Failure (Failure predominently by moment and torque)

Investigator	Specimen Mark	$\frac{f'_c}{f'_t}$	ϕ	λ	M_b in-kips	M_{bo} in-kips	M_b/M_{bo} Test	M_b/M_{bo} Calc	Ratio: Test/Calc
1	2	3	4	5	6	7	8	9	10
Lessig	BII 7	10	0.100	1.75	625	625	1.00	0.99	1.01
	7a	10	0.100	1.75	625	625	1.00	0.99	1.01
	8	10	0.190	1.75	382	450	0.85	0.91	0.93
	8a	10	0.190	1.75	382	450	0.85	0.91	0.93
	9	10	0.570	1.75	157	450	0.35	0.39	0.90
	9a	10	0.600	1.75	156	450	0.35	0.36	0.97
	10	10	0.400	1.40	348	450	0.77	0.69	1.12
	10a	10	0.400	1.40	313	450	0.70	0.69	1.01
	11	10	0.780	1.40	156	450	0.35	0.32	1.09
	11a	10	0.730	1.40	156	450	0.35	0.36	0.97
	12	10	0.600	1.40	243	600	0.40	0.46	0.87
	12a	10	0.580	1.40	261	600	0.43	0.47	0.92
	13	10	0.180	1.75	520	625	0.83	0.92	0.90
	13a	10	0.200	1.75	625	625	1.00	0.90	1.11
	14	10	0.200	1.75	416	625	0.67	0.90	0.75
	14a	10	0.200	1.75	572	625	0.91	0.90	1.01
	15	10	0.370	1.40	416	475	0.88	0.74	1.19
	15a	10	0.360	1.40	416	475	0.88	0.76	1.16
	16	10	0.590	1.75	156	225	0.69	0.37	1.86[x]
	16a	10	0.530	1.75	156	225	0.69	0.41	1.68[x]
	17	10	0.300	1.75	278	320	0.87	0.73	1.19
	17a	10	0.290	1.75	313	320	0.98	0.75	1.30[x]

(Continued)

Table V.4 (contd.)

1	2	3	4	5	6	7	8	9	10
Lessig	18	10	0.600	1.75	191	450	0.43	0.44	0.98
	18a	10	0.580	1.75	191	450	0.43	0.46	0.94
	19	10	0.250	1.75	313	320	0.98	0.82	1.19
	19a	10	0.250	1.75	313	320	0.98	0.82	1.19
	20	10	0.400	1.40	313	475	0.66	0.69	0.96
	20a	10	0.380	1.40	339	500	0.68	0.72	0.95
	21	10	0.380	1.40	313	475	0.66	0.72	0.92
	21a	10	0.320	1.40	313	475	0.66	0.80	0.83
II	B	10	0.200	2.77	207	325	0.64	0.70	0.92
	B'	10	0.280	2.33	207	325	0.64	0.60	1.07
W	B	10	0.400	1.88	132	325	0.41	0.52	0.79*
	B'	10	0.400	1.90	143	325	0.44	0.50	0.88
Lyalin	B8-0.1	10	0.100	1.40	520	520	1.00	0.99	1.01
	0.1a	10	0.100	1.40	555	550	1.00	0.99	1.01
	0.2	10	0.200	1.40	451	520	0.87	0.95	0.92
	0.2a	10	0.200	1.40	486	520	0.96	0.95	1.01
	0.4	10	0.380	1.40	347	550	0.63	0.72	0.88
	0.4a	10	0.400	1.40	347	550	0.63	0.69	0.91
	B7-0.2	10	0.200	1.40	468	550	0.85	0.95	0.90
	0.2a	10	0.200	1.40	451	520	0.87	0.95	0.92
	B10-0.2	10	0.200	1.40	521	520	1.00	0.95	1.05
	0.2a	10	0.200	1.40	521	520	1.00	0.95	1.05
	B 1	10	0.200	1.07	452	450	1.00	0.98	1.02
	1a	10	0.200	1.07	452	475	0.94	0.98	0.96
	2	10	0.200	1.34	694	725	0.95	0.96	0.99
	2a	10	0.200	1.34	694	725	0.95	0.96	0.99
	3	10	0.400	1.34	486	725	0.67	0.72	0.93
	3a	10	0.400	1.34	486	725	0.67	0.72	0.93

(contd.)

Table V.4 (continued)

1	2	3	4	5	6	7	8	9	10
Lyalin	4	10	0.470	1.34	556	725	0.77	0.64	1.20
	4a	10	0.500	1.34	556	725	0.77	0.59	1.30[x]
	5	10	0.200	1.61	972	1025	0.95	0.91	1.04
	5a	10	0.200	1.61	972	1025	0.95	0.91	1.04
	6	10	0.200	2.00	833	900	0.93	0.86	1.08
	6a	10	0.200	2.00	903	900	1.00	0.86	1.16
	9	10	0.190	1.41	208	200	1.04	0.96	1.08
	9a	10	0.200	1.41	235	215	1.09	0.95	1.15
	10	10	0.200	1.33	182	190	0.96	0.96	1.00
	10a	10	0.200	1.41	209	200	1.05	0.95	1.11
	11	10	0.170	1.41	209	210	1.00	0.97	1.03
	11a	10	0.170	1.41	209	220	0.95	0.97	0.98
	12	10	0.310	1.41	78	98	0.80	0.82	0.98
	12a	10	0.300	1.41	104	112	0.93	0.83	1.12
Brown	IIa	12	0.092	8.00	33	85	0.39	0.47	0.83
	IIIa	12	0.154	6.30	32	85	0.38	0.34	1.12
	c	12	0.153	6.30	30	85	0.35	0.34	1.03
	IVa	12	0.228	5.54	18	85	0.21	0.25	0.84
	c	12	0.232	5.52	23	85	0.27	0.25	1.08
	Va	12	0.304	5.15	18	85	0.21	0.17	1.23[x]
	VIa	12	0.378	4.91	15	85	0.17	0.15	1.13
	VIIa	12	0.092	8.30	31	85	0.36	0.45	0.80
	VIIIa	12	0.183	7.32	18	85	0.21	0.23	0.91
Rama-krishnan and Vijaya Rangan (Author)	B4	10	0.173	3.37	99	120	0.82	0.67	1.22[x]
	B5	10	0.550	1.71	45	120	0.38	0.40	0.95

(contd.)

Table V.4 (continued)

1	2	3	4	5	6	7	8	9	10
Farmer	TR 5	10	0.074	11.40	47	65	0.72	0.47	1.53[x]
	8	10	0.056	9.65	80	110	0.73	0.72	1.01
	9	10	0.036	11.40	90	110	0.82	0.85	0.96
	15	10	0.074	11.40	49	110	0.45	0.47	0.96
	TW 5	10	0.074	11.40	50	65	0.77	0.47	1.64[x]
Ersoy	LI10	10	0.059	9.62	37	50	0.74	0.70	1.06
	10a	10	0.058	9.70	41	50	0.82	0.70	1.17
	11	10	0.086	7.20	39	50	0.78	0.65	1.20
Collins	36T4	12	0.260	1.41	240	276	0.87	0.86	1.01
et al	77T5	12	0.350	1.41	262	360	0.73	0.72	1.01
	7705	12	0.350	1.41	278	360	0.77	0.72	1.07
	77T4	12	0.480	1.41	223	360	0.62	0.52	1.19
	7703	12	0.340	1.41	219	360	0.61	0.73	0.84
	38T5	12	0.370	1.41	216	360	0.60	0.68	0.88
	RE 5	12	0.610	1.41	108	300	0.36	0.38	0.95
	RU 4	12	0.590	1.41	145	300	0.48	0.40	1.20
	RU 5	12	0.300	1.41	250	300	0.83	0.80	1.04
	RU5A	12	0.250	1.41	266	300	0.89	0.87	1.02
	RU6	12	0.210	1.41	281	300	0.93	0.92	1.01
	L 5	12	0.300	1.74	144	250	0.58	0.68	0.85
	S 1	12	0.232	2.41	181	274	0.66	0.65	1.02
	S 4	12	0.370	1.87	127	274	0.46	0.51	0.90
Author	C 5	8	0.090	7.90	128	185	0.69	0.62	1.11

TABLE V.5

COMPARISON OF RESULTS : MODE 3

Failure (Failure predominently by moment and shear)

Investigator	Specimen Mark	α	α_{TR}	$\bar{N}\alpha_{TR}$	α_m	M_b in-kips	M_{bo} in-kips	M_b/M_{bo} Test	M_b/M_{bo} Calc	Ratio Test/Calc.
1	2	3	4	5	6	7	8	9	10	11
Collins et al	LS 4	3.71	4.50	6.76	2.84	118	240	0.49	0.55	0.89
	LS 6	3.76	4.50	7.70	3.00	102	240	0.43	0.49	0.88
Rama Krishnan and Vijaya Rangan (Author)	C1 II	2.67	5.00	5.93	2.68	101	185	0.54	0.46	1.17
	C3	2.67	5.10	5.80	2.67	111	185	0.60	0.46	1.30[x]
	C4	2.67	5.55	6.50	2.83	90	185	0.49	0.46	1.07
	C5	2.67	5.00	5.85	2.67	105	185	0.56	0.46	1.22[x]
	C6 II	2.67	5.45	6.55	2.84	90	185	0.49	0.46	1.07
Author	C1	5.00	7.85	9.20	3.42	128	185	0.69	0.55	1.25[x]
	C3	3.55	8.68	10.15	3.58	70	185	0.38	0.36	1.06
	C4	4.00	7.85	9.45	3.47	81	185	0.44	0.42	1.05
	C6	6.00	7.85	9.35	3.45	96	185	0.52	0.64	0.82
	* C7 I	7.00	7.35	7.76	3.25	193	193	1.00	0.90	1.11
	C7 II	5.20	7.35	8.30	3.32	134	193	0.69	0.63	1.10
	* C8 I	7.40	7.35	8.00	3.28	193	193	1.00	0.93	1.08
	C8 II	7.40	7.35	8.60	3.34	181	193	0.94	0.86	1.09
	C9	6.00	7.35	9.45	3.47	132	193	0.68	0.64	1.06

* Flexure Failure

V.6 DESIGN METHOD:

Using the theoretical equations, the following procedure is suggested for the design of beams under combined bending, shear and torsion:

a. Choosing a value for D/b ratio and for given values of α and ϕ, the value of λ can be determined using Eqn. 25.

b. For this value of λ, the value of M_b/M_{bo} can be known using Fig.V.11. Then,

$$\text{Equivalent Design Bending Moment} = \frac{\text{(Bending Moment at the Section)}}{(M_b / M_{bo})} \qquad \ldots \text{V.60}$$

Longitudinal steel is to be designed for the Equivalent Design Bending Moment using the provisions available in the codes for the design of beams under pure bending.

c. The step (b) avoids Mode 2 type of failure. In order to avoid Mode 3 failure, transverse hoops are to be designed using the following equation:

$$\frac{A_w}{bs} = \frac{V}{bd\, f_{yw}} \left(1 - \frac{\alpha}{N\, \alpha_{TR}}\right) \qquad \ldots \text{V.61}$$

where $\alpha \geq \alpha_m$

$$\alpha_{TR} = 21\, p f_y / f'_c$$

$$\bar{N} = \sqrt{1 + \left(\frac{\alpha_\phi \beta_c}{1.4}\right)^2}$$

$$\alpha_m = \frac{(2/k)\,\bar{N}\,\alpha_{TR}}{\sqrt{\frac{4}{k}\bar{N}\,\alpha_{TR} + 2\,\bar{N}^2}} \quad ; \quad k = 0.75$$

Similar to the case of beams under bending and shear, the increase in the strength of beams for $\alpha < \alpha_m$ has been ignored in Eqn. V.61 and the strengths of beams in such cases have been taken equal to the strength of beam with $\alpha = \alpha_m$. This has been done considering the serviceability of the structure.

d. The designed beam is to be checked for Mode 1 failure such that

$$\left(\frac{M_t}{M_{to}}\right) + \left(\frac{V}{V_o}\right) \leq 1.0 \qquad \ldots \text{ V.62}$$

where M_{to} and V_o are to be calculated using Eqn. V.48. Otherwise increase the amount of transverse steel until Eqn. V.62 is satisfied.

The suggested design procedure is illustrated in the following example.

Design Example:

Design a beam for the following ultimate loads:

Bending moment = 2000 in kips

Torque = 700 in kips

Shear Force = 35 kips

Given that: $f'_c = 3000$ psi; $f_y = f_{yw} = 40$ ksi;

$f'_c/f'_t = 10.$

Design:

Choosing a rectangular beam of D/b ratio equal to 1.50, from Table V.2,

$$\beta_c = 1.40$$
$$\beta_s = 0.310$$

The dimensions of the beam can be fixed-up using the limitation on breadth suggested for the case of beams under bending and torsion (see Part I, Chapter I.11, Art. I.11.3)

$$\text{i.e.} \quad b \geqslant \sqrt[3]{M_t/\beta_m f'_c}$$

$$\text{or} \quad b \geqslant \sqrt[3]{700/.08 \times 3.0} \geqslant \underline{14.3} \text{ in}$$

Select $b = 16$ in; $D = 24$ in; $d = 22$ in;

$b' = 12$ in; $d' = 20$ in.

$$\alpha = \frac{M_b}{Vd} = \frac{2000}{35 \times 22} = 2.60$$

$$\phi = \frac{M_t}{M_b} = \frac{700}{2000} = 0.35$$

$$N = \sqrt{1 + \left(\frac{1.4}{\alpha \phi \beta_c}\right)^2}$$

$$= \sqrt{1 + \left(\frac{1.4}{2.6 \times 0.35 \times 1.40}\right)^2} = \underline{1.49}$$

$$\lambda = N.\beta_c = 1.49 \times 1.40 = \underline{2.08}.$$

$$(1/\lambda\phi) = \frac{1}{2.08 \times 0.35} = 1.37.$$

From Fig. V.11, $(M_b/M_{bo}) = 0.54$.

Equivalent design Bending moment $= \frac{2000}{0.54} = \underline{3700}$ in kips

$$\frac{M}{bd^2 f'_c} = \frac{2700}{16 \times 22 \times 22 \times 3} = \underline{0.160}$$

$$\frac{f_y}{100\, f'_c} = \frac{40}{100 \times 3} = 0.133$$

From the Nomogram given by Sundara Raja Iyengar and Author[77],

$$p = 1.45\%$$

$$\text{or } A_s = \frac{1.45}{100} \times 16 \times 22 = \underline{5.10}\ \text{in}^2.$$

Provide 1 in diameter rods - 7 Nos.

$$\alpha_{TR} = \frac{21 p f_y}{f'_c} = \frac{21 \times 0.0145 \times 40}{3} = 4.06$$

$$\bar{N} = \sqrt{1 + \left(\frac{\alpha \phi \beta_c}{1.4}\right)^2}$$

$$= \sqrt{1 + \left(\frac{2.60 \times 0.35 \times 1.4}{1.4}\right)^2} = \underline{1.35}.$$

$$\alpha_m = \frac{2.67 \times 1.35 \times 4.06}{\sqrt{(5.33 \times 1.35 \times 4.06) + 2 \times 1.35^2}}$$

$$= \underline{2.55}.$$

$\underline{\alpha > \alpha_m.}$

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$$\frac{A_w}{bs} = \frac{35}{16 \times 22 \times 40}\left(1 - \frac{2.60}{1.35 \times 4.06}\right)$$

$$= \underline{0.131}\ \%$$

Provide 3/8 in. diameter two legged hoops at 10.5 in c/c.

$$M_{to} = (0.095 \times 0.310 \times 16 \times 16 \times 24 \times 3) + \left(\frac{0.11 \times 40 \times 12 \times 20}{10.5}\right)$$

$$= (543) + (101) = \underline{644} \text{ in kips.}$$

$$V_o = (0.095 \times 16 \times 22 \times 3) + \left(\frac{2 \times 0.11 \times 40 \times 20}{10.5}\right)$$

$$= (100.0) + (16.7) = \underline{116.7} \text{ kips}$$

$$\frac{V}{V_o} = \frac{35}{116.7} = 0.30$$

$$\frac{M_t}{M_{to}} = 1 - 0.30 = 0.70$$

$$\therefore M_{to} = \frac{M_t}{0.70} = \frac{700}{0.70} = \underline{1000} \text{ in kips}$$

$> \underline{644}$ in-kips

So increase the amount of transverse steel.

Adopting a spacing of 3 in;

$$M_{to} = (543) + (357) = 900 \text{ in kips.}$$

$$V_o = (100.0) + (59.0) = 159 \text{ kips}$$

$$\frac{V}{V_o} = 0.22$$

$$\therefore M_{to} = \frac{700}{0.78} = 897 \text{ in kips} < \underline{900} \text{ in kips}$$

Summary of Design:

Section: 24 in. x 16 in. Rectangular beam

Longitudinal Steel : 1 in diameter rods - 7 Nos.

Transverse steel : 3/8 in diameter two - legged hoops at 3 in. c/c.

V.7 CONCLUDING REMARKS:

a. It has been assumed that beams subjected to combined bending, torsion and shear will fail in any one of the following modes:

Mode 1 : Failure predominently by shear and torsion, when bending moment is small.

Mode 2 : Failure predominently by bending moment and torsion with the secondary influence of shear force.

Mode 3 : Failure predominently by bending moment and shear force with the secondary influence of torque.

b. Based on a failure criterion for concrete under compressive-tensile stresses, equations have been derived for these three modes of failures. The predictions of the theoretical equations were compared with the test results of over one hundred fifty beams available in the literature. From the comparison it has been found that the predictions of the equations are fairly good within an error $\pm$ 20% in most of the cases.

c. The theoretical investigation was supplemented by a minor experimental programme. Ten reinforced concrete rectangular beams without transverse reinforcement were tested to destruction in combined torsion, bending and shear. Primarily, the influence of length of shear span on the strength of beams was studied. Measured deflections provided qualitative information regarding the behaviour of beams.

d. Based on the results of analytical investigation, a method has been suggested for the design of beams under the combined action of bending, torsion and shear. The suggested design method has been illustrated by a numerical example.

More work is needed in this field of research. The results available for the Mode 3 failure are very few in number. Further experimentation is necessary to yield more results in this mode of failure by varying the lengths of shear spans and amount of torque. It would be desirable to have detailed study on the influence of various physical properties of beams (such as amount of longitudinal steel, transverse steel, concrete strength, shape of the cross section etc.) on behaviour and strength

of beams. Since this combined loading mostly occurs in monolithic grid systems, it would be interesting to study the behaviour of such members to know the influence of monolithic action on the strength of such members.

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PART VI : STIFFNESS OF REINFORCED CONCRETE BEAMS UNDER COMBINED BENDING AND TORSION AND BENDING AND SHEAR

PART VI

STIFFNESS OF REINFORCED CONCRETE BEAMS UNDER COMBINED BENDING AND TORSION AND BENDING AND SHEAR

VI.1 INTRODUCTION

The stiffness of a structural member is as important as its strength. One of the objects of the present investigation was to study the load-deformation characteristics of reinforced concrete beams under combined loadings. In this part, the behaviour of beams is studied in the light of the moment-deflection and torque-twist curves obtained from the experiments. The influence of various parameters on the stiffness of beams is also examined. Further, based on experimental data, an approximate method is suggested for the calculation of stiffness of beams at working loads.

VI.2.1 Moment-Deflection Curves:

1. Beams under bending andtorsion:-

The moment-deflection curves obtained from tests on beams under combined bending and torsion resembled those of beams under pure bending; but the deflections at corresponding moments are different. The curves obtained

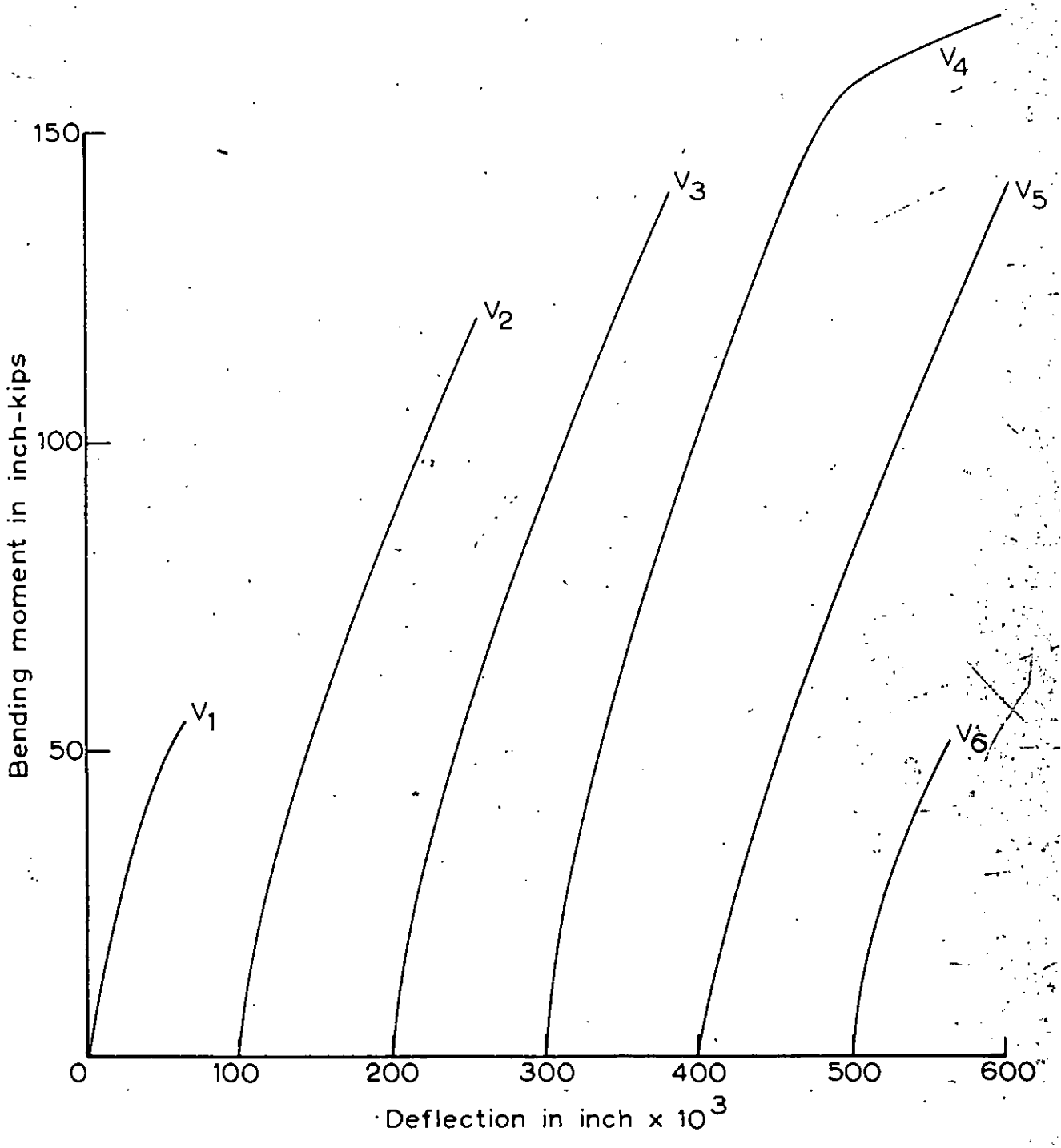


FIG. VI. 1. MOMENT - DEFLECTION CURVES - V SERIES.

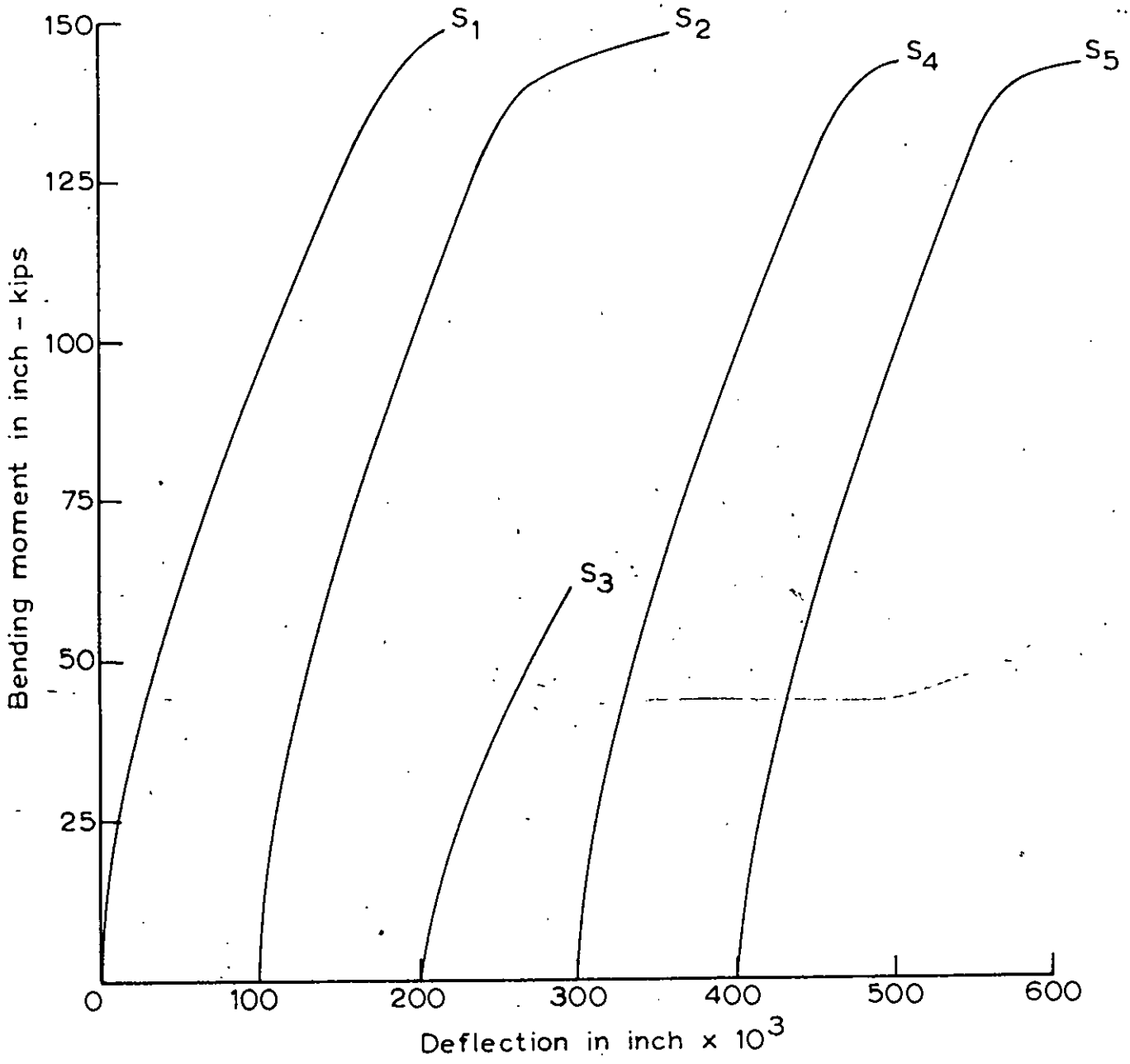


FIG. VI. 2. MOMENT - DEFLECTION CURVES - S SERIES.

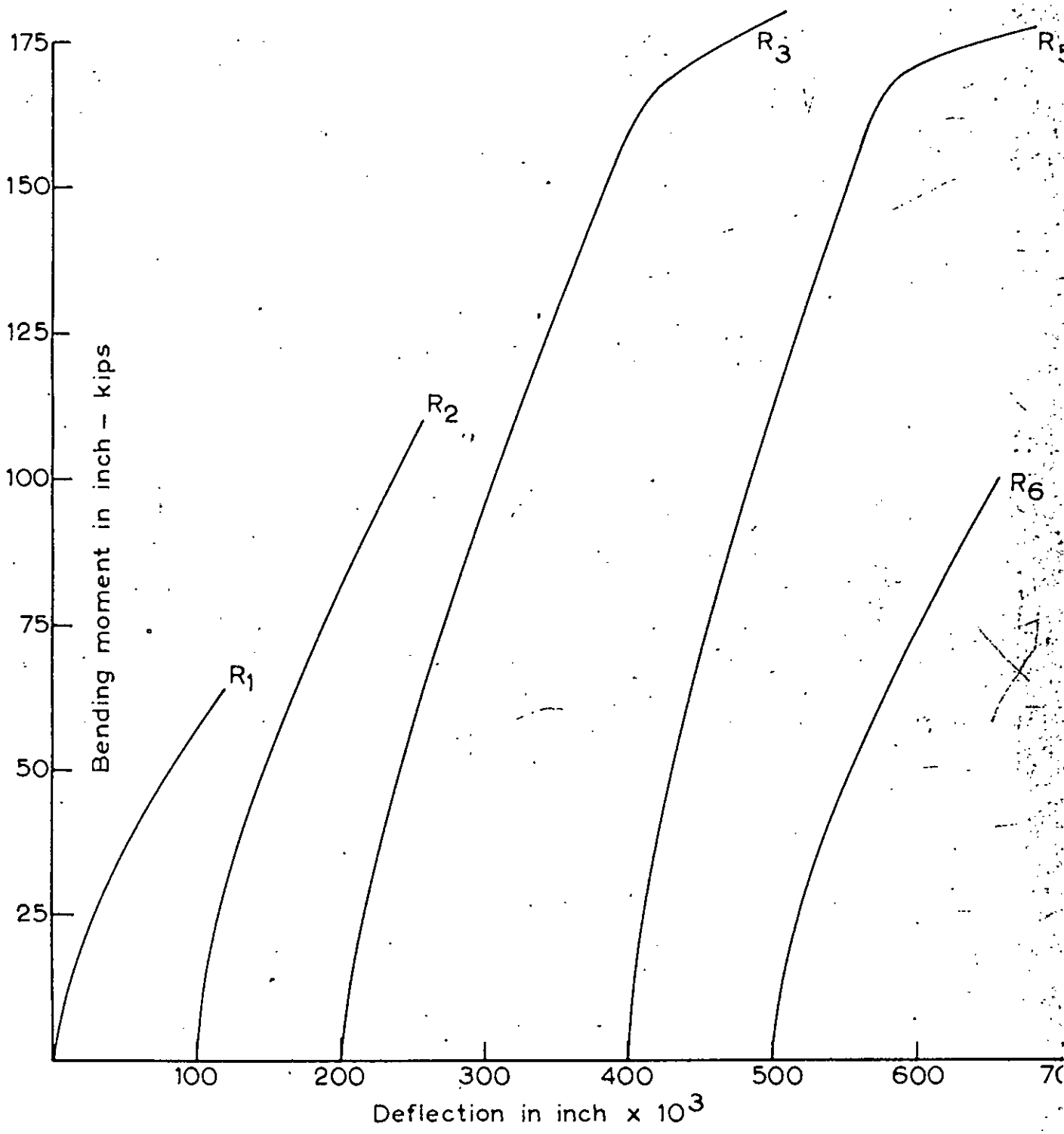
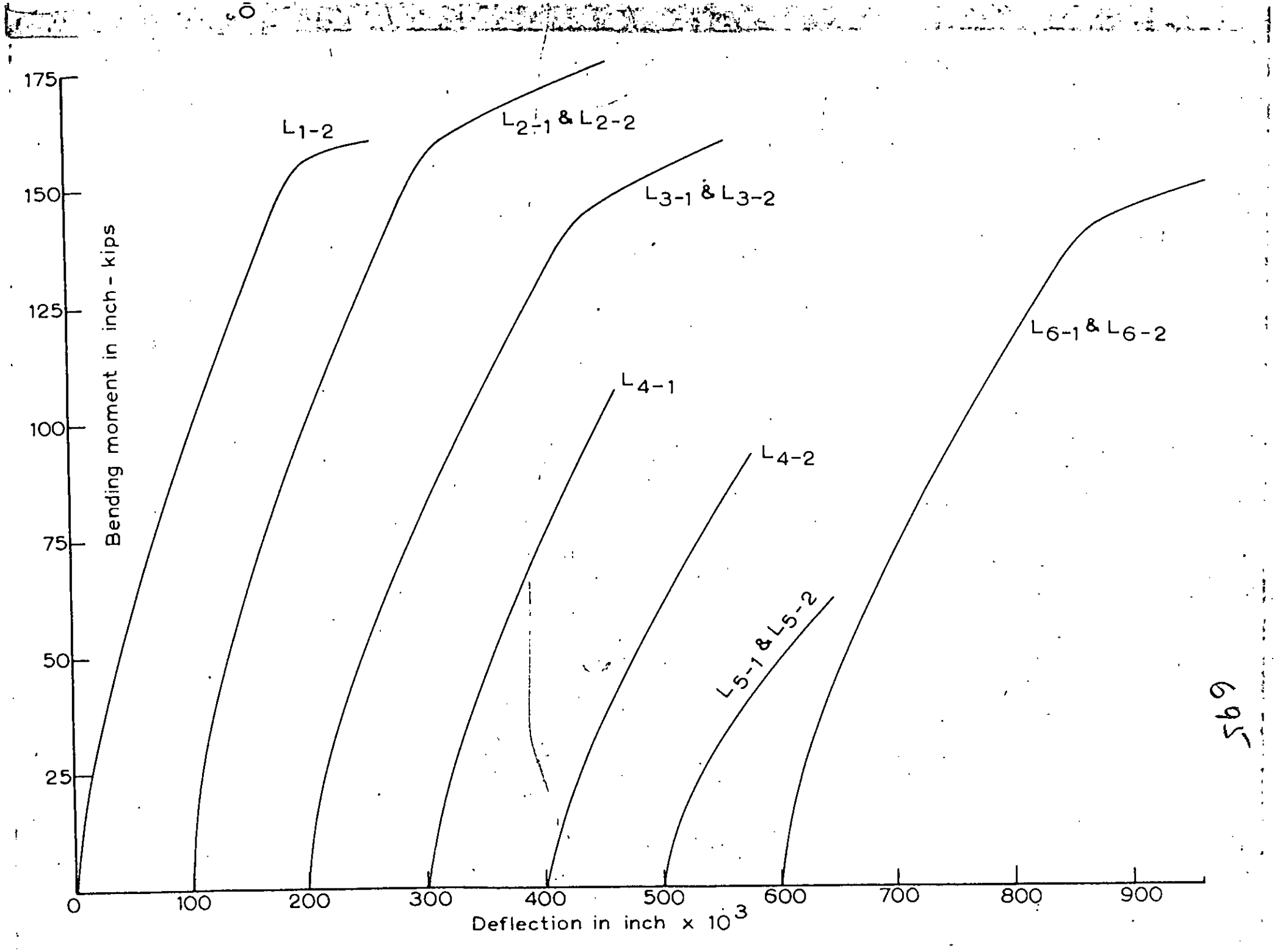


FIG. VI.3. MOMENT - DEFLECTION CURVES - R SERIES.


175
150
125
100
75
50
25
0
100
200
300
400
500
600
700
800
900
Bending moment in inch - kips
Deflection in inch x 10^3
L_{1-2}
L_{2-1} & L_{2-2}
L_{3-1} & L_{3-2}
L_{4-1}
L_{4-2}
L_{5-1} & L_{5-2}
L_{6-1} & L_{6-2}

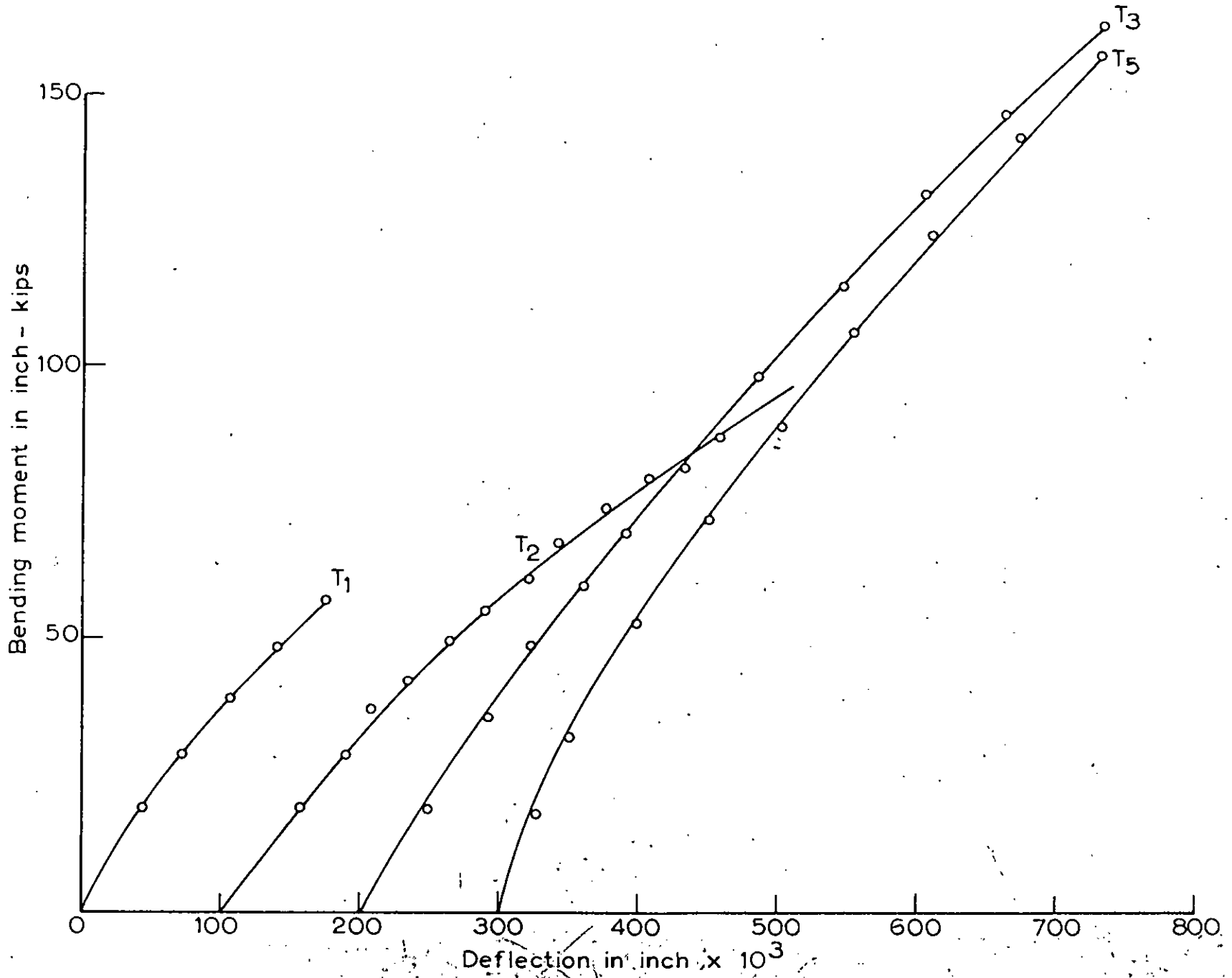


FIG. VI. 5. MOMENT - DEFLECTION CURVES - T SERIES.

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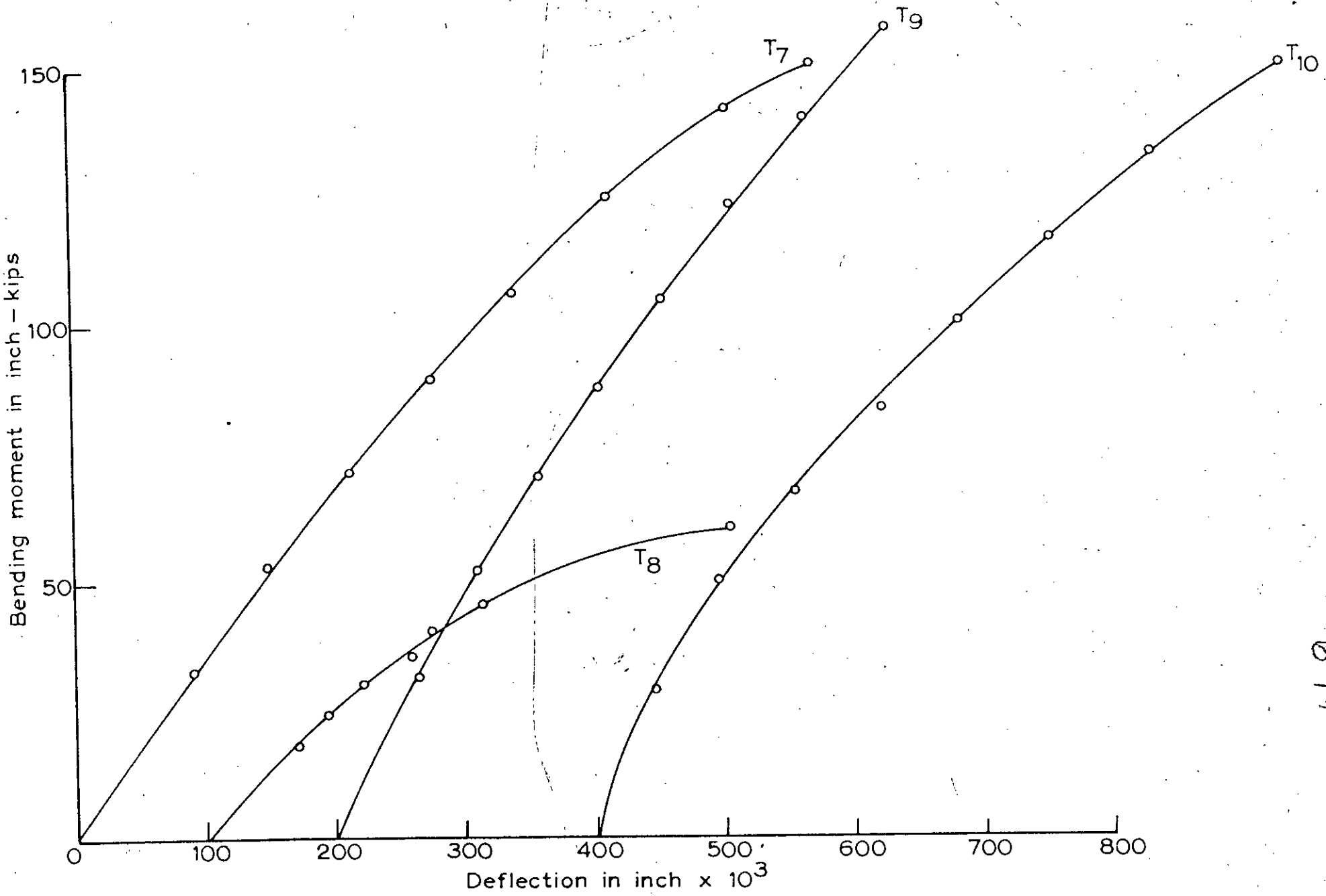


FIG. VI. 6. MOMENT – DEFLECTION CURVES – T SERIES.

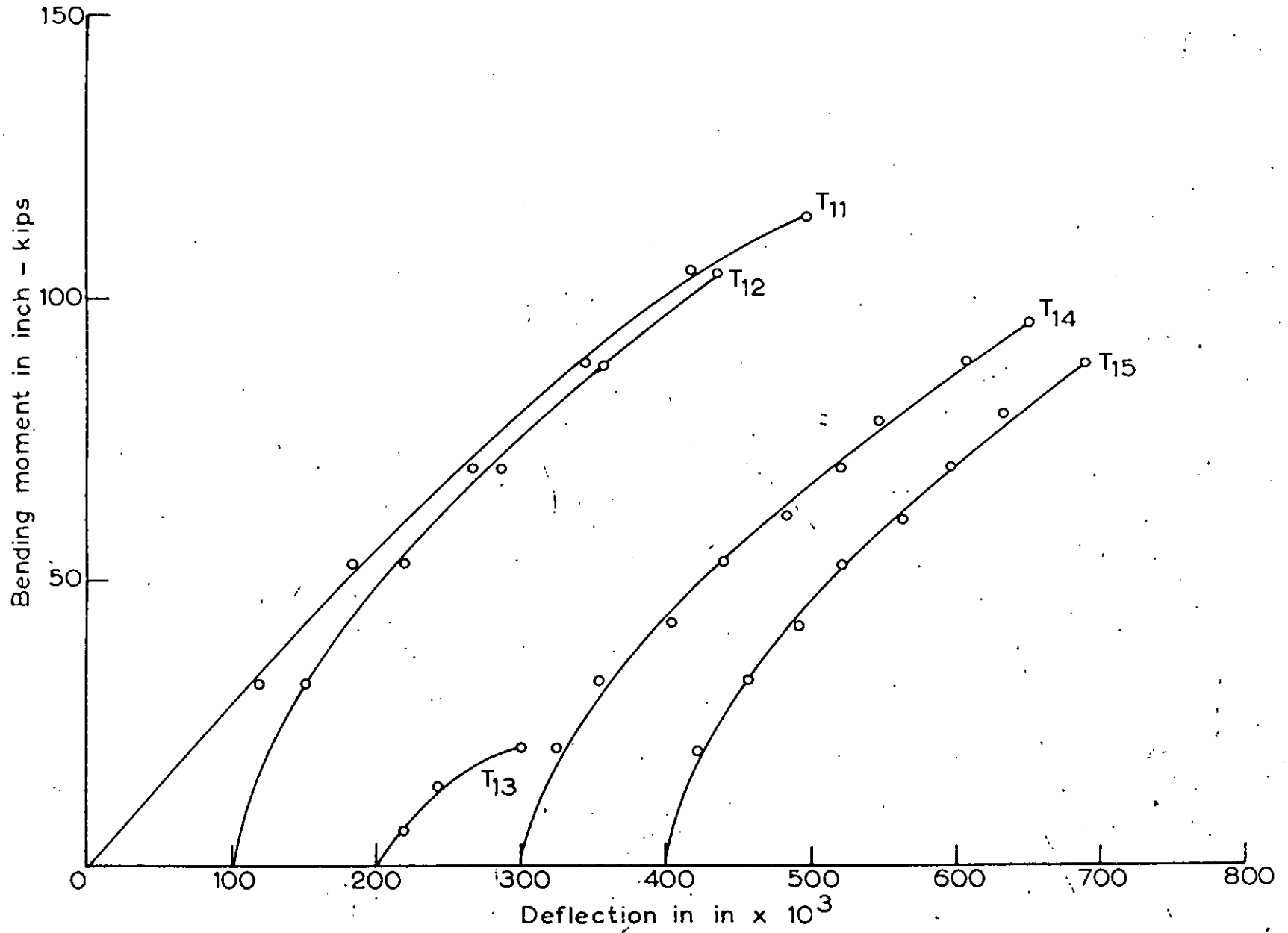


FIG. VI. 7. MOMENT - DEFLECTION CURVES - T SERIES.

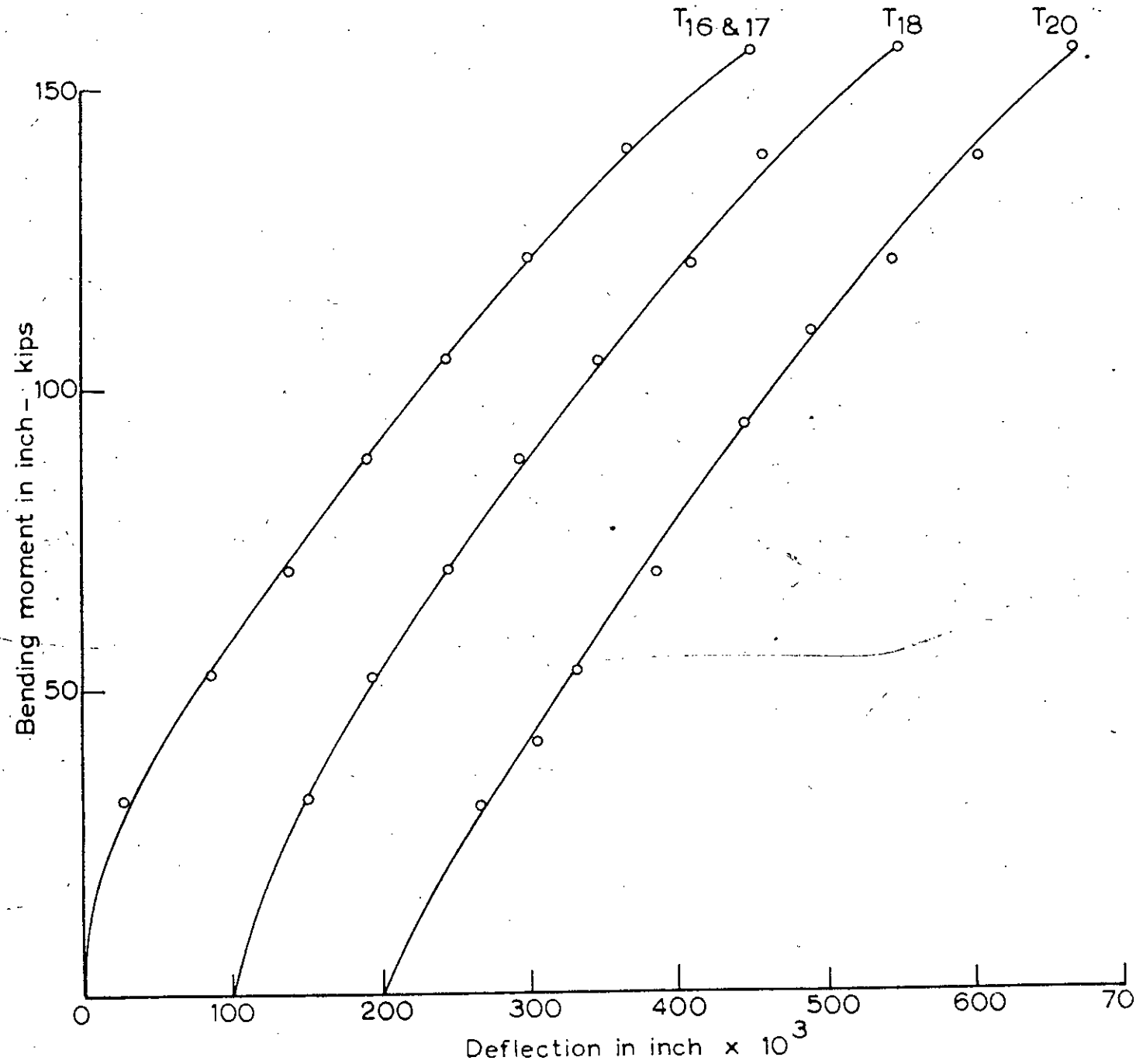


FIG. VI. 8. MOMENT - DEFLECTION CURVES - T SERIES.

from the experiments are shown in Figs. VI.1 to VI.8. According to the shape of the curves, three different stages can be defined.

The first stage corresponds to that part of the curve before the appearance of first flexural crack. The curve is a straight line in this range and the deflections were proportional to the applied moment. The extent of this stage depended mainly on the concrete strength of the beam. It may be possible to know the flexural cracking moment using moment-deflection curves.

The second stage represents that part of the curve between the beginning of flexural cracking and before the final collapse of the beam. After the formation of flexural cracks, the deflection curve takes the first change of slope and follows more or less a straight line. This stage continues upto 85 to 90 percent of ultimate moment.

The third stage of the curve represents that part before the final collapse of the beam. In this stage, the moment-deflection curve undergoes a second change of slope and the deflections are greater even for small increments of moments. Almost for all beams nearly a flat region exists in this range. Moreover, the curve becomes more erratic in this stage and the prediction of

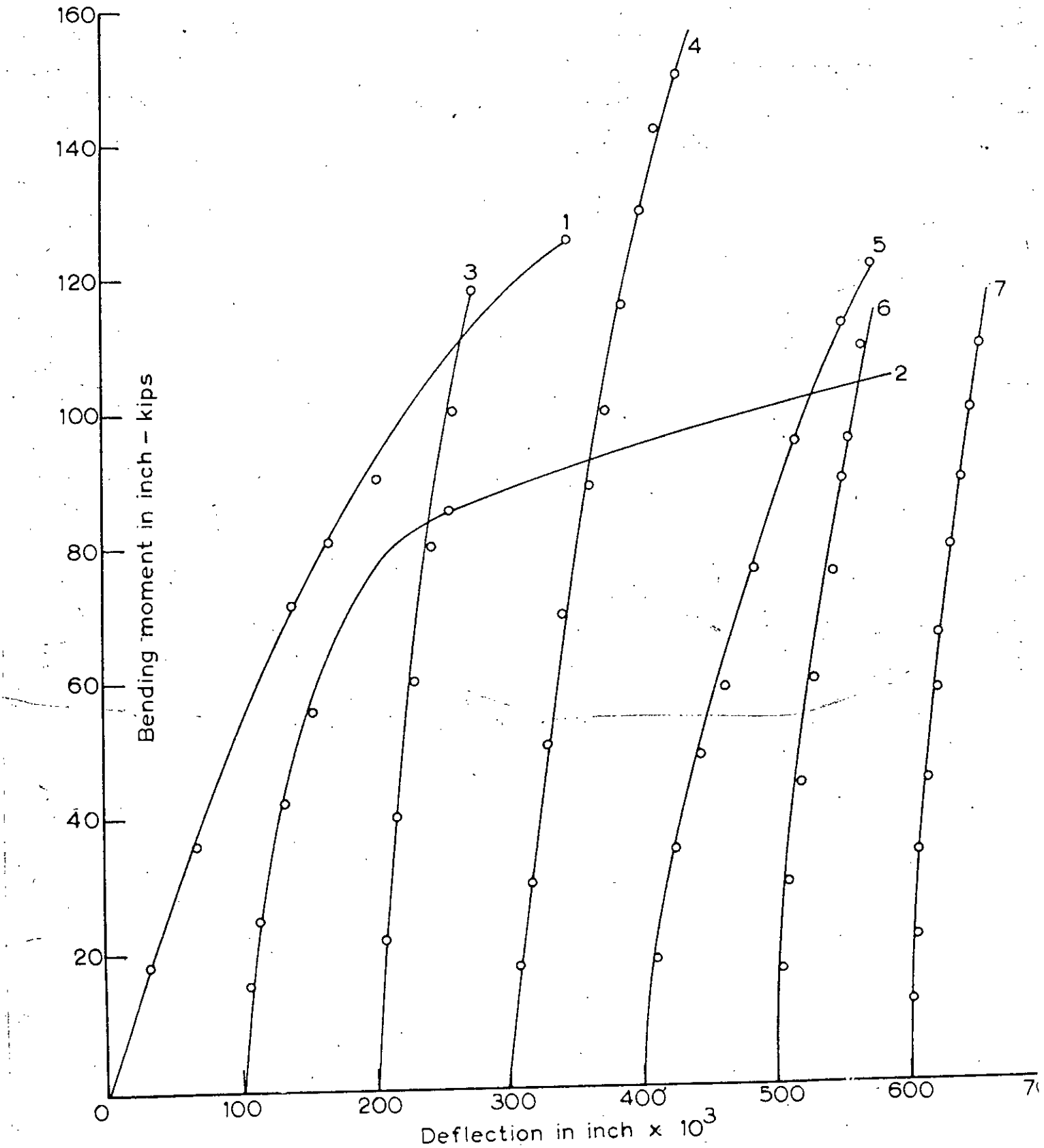


FIG. VI. 9. MOMENT - DEFLECTION CURVES SR SERIES.

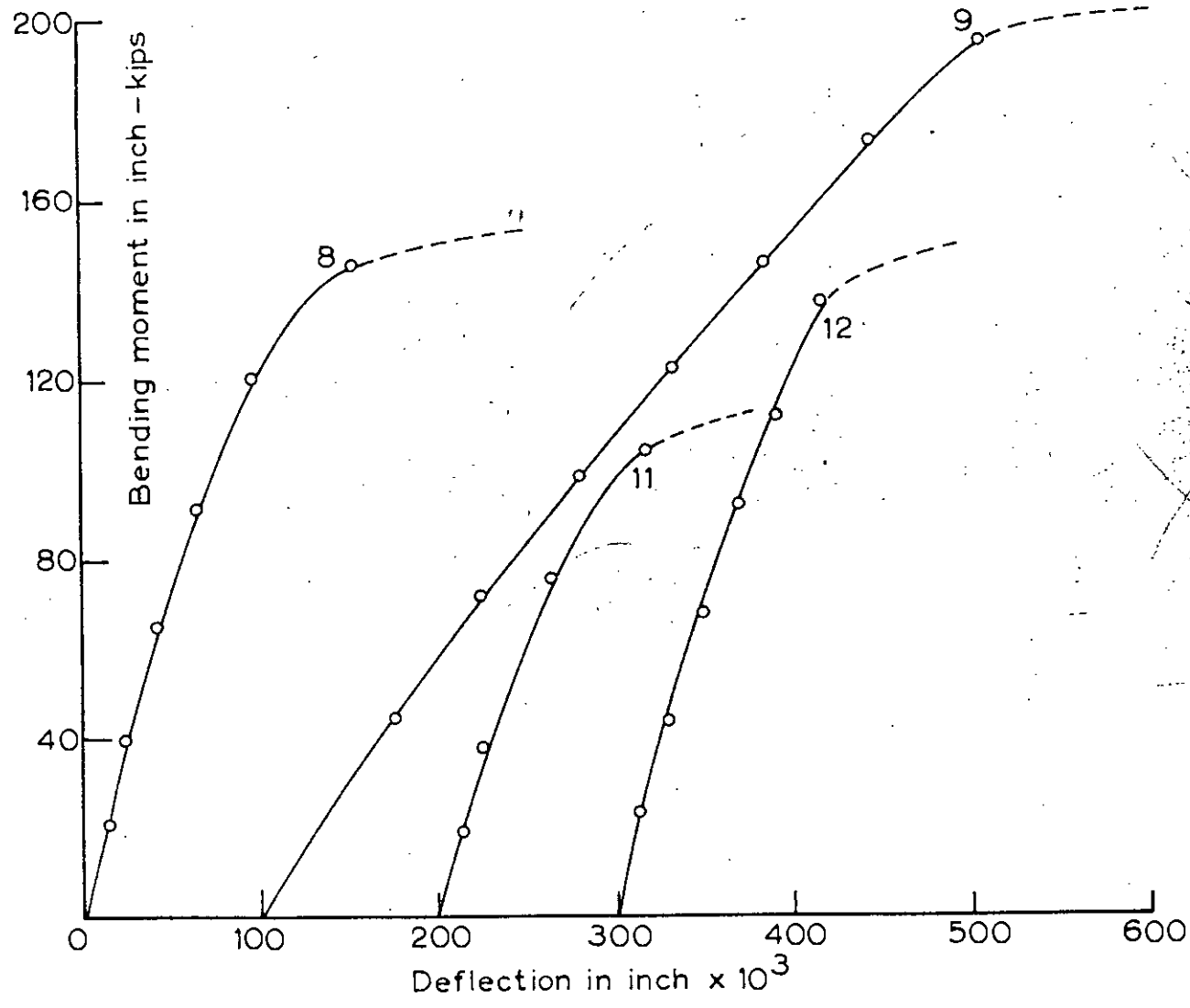


FIG. VI. 10 a. MOMENT - DEFLECTION CURVES - SR SERIES.

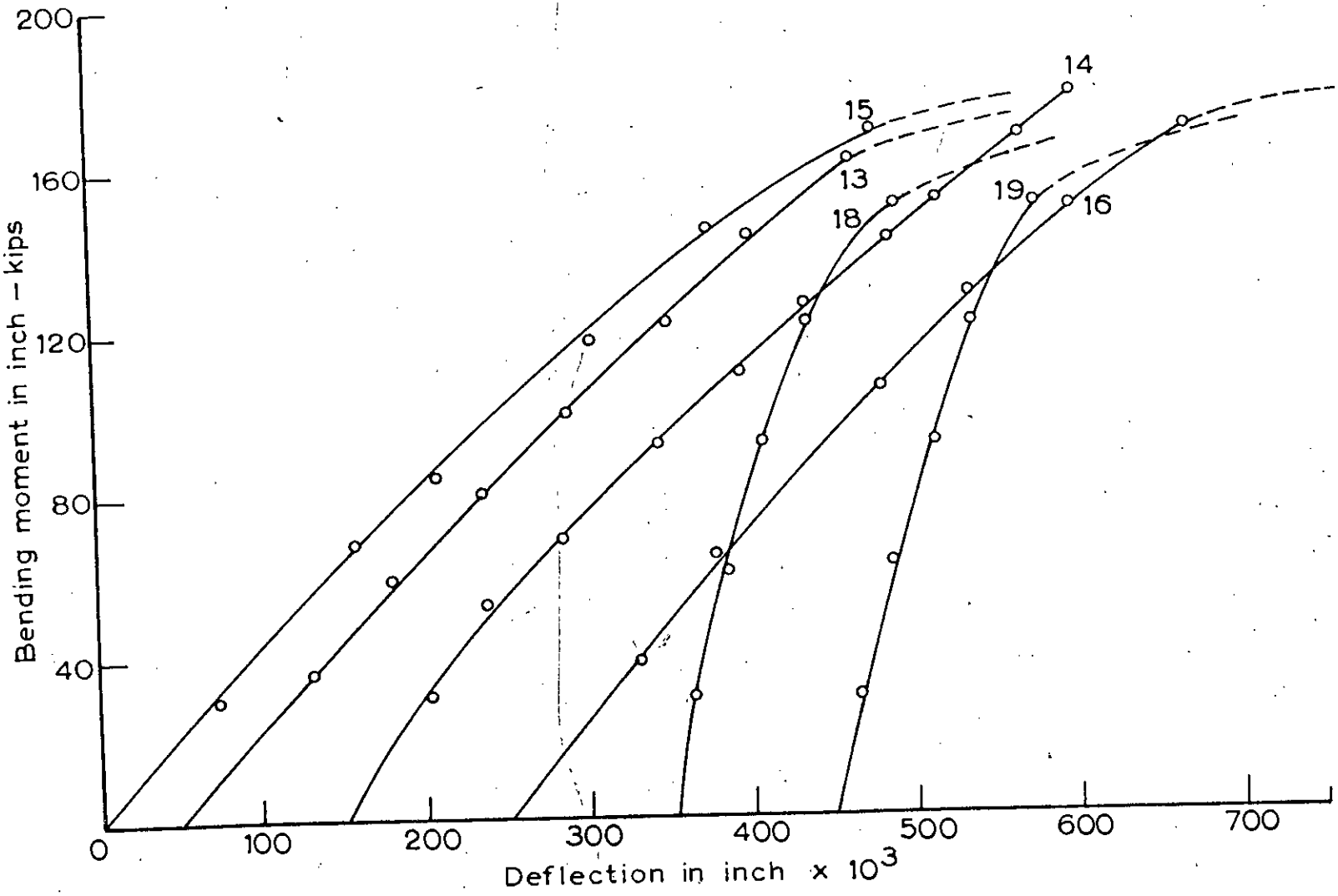


FIG. VI.10b. MOMENT – DEFLECTION CURVES – SR SERIES.

it is more difficult. So in some of the curves shown in Figs. VI.1 to VI.8, this stage of the curve is shown in dotted lines indicating that the deflections increased enormously at failure of the beam. Always failure occurred only in this stage of the curve.

In most of the beams tested, the stages described above are sharply defined by sudden changes in the slope of the moment-deflection curves. For beams prematurally failed in torsion, the third stage of the curve is absent. The moment deflection curves for beams V3, V4, S1, S2, S3, L2 etc. provide a good illustration of the three stages of behaviour of beams. When two companion beams were tested, the deflections agreed remarkably well (vide curves for Beams L2-1 and L2-2, L3-1 and L3-2 and L6-1 and L6-2).

ii. Beams under bending and shear:

The moment-deflection curves obtained from tests on beams under combined bending and shear are given in Figs.VI.9 and VI.10. The first two stages are not well defined in many of these curves; but the third stage of the curve is indicated in most of the cases. The moment-deflection curve of Beam S2 provides a good illustration of the rapid increase in the deflection of the beam near failure.

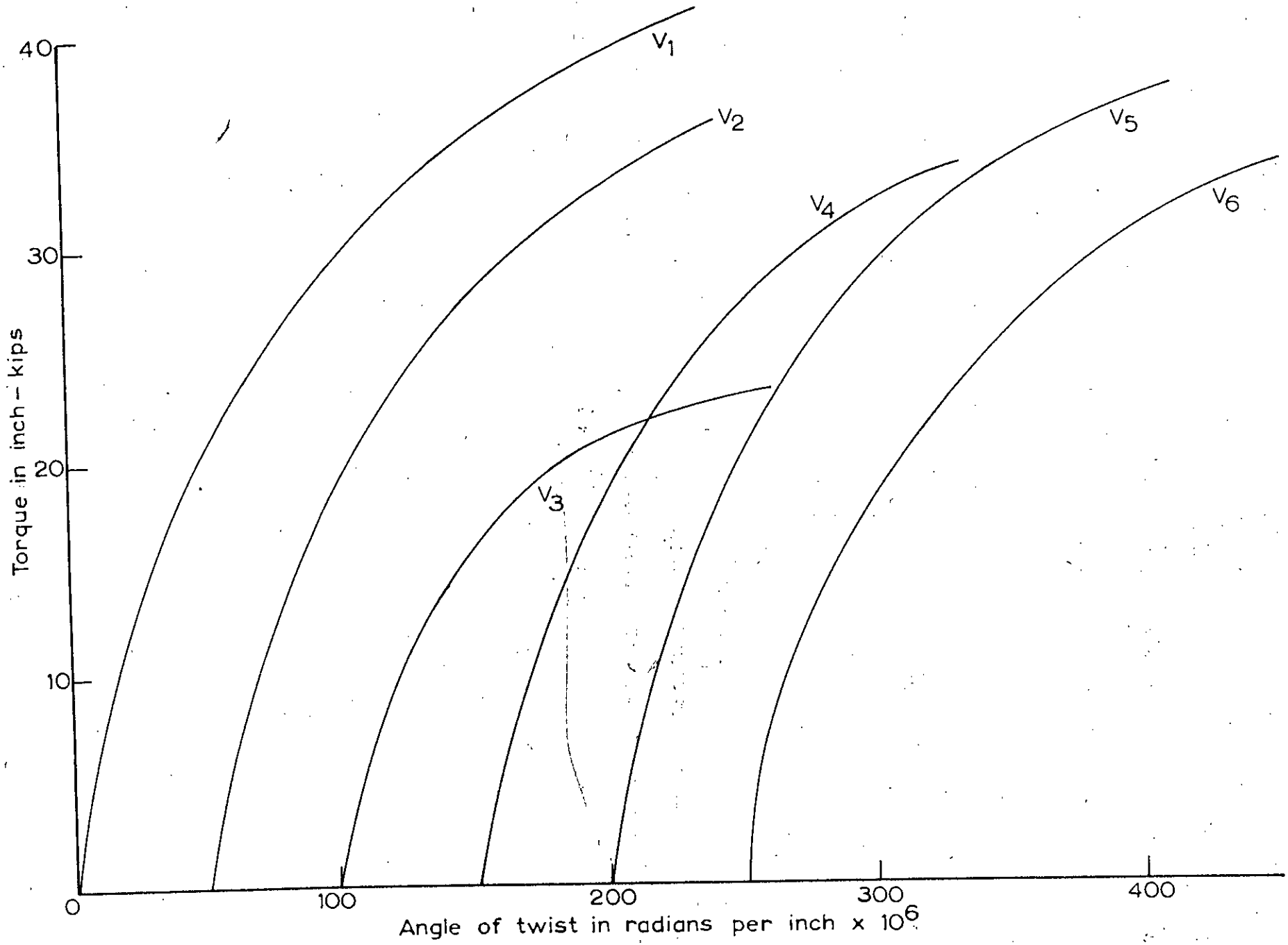


FIG. VI. 11. TORQUE - TWIST CURVES - V SERIES.

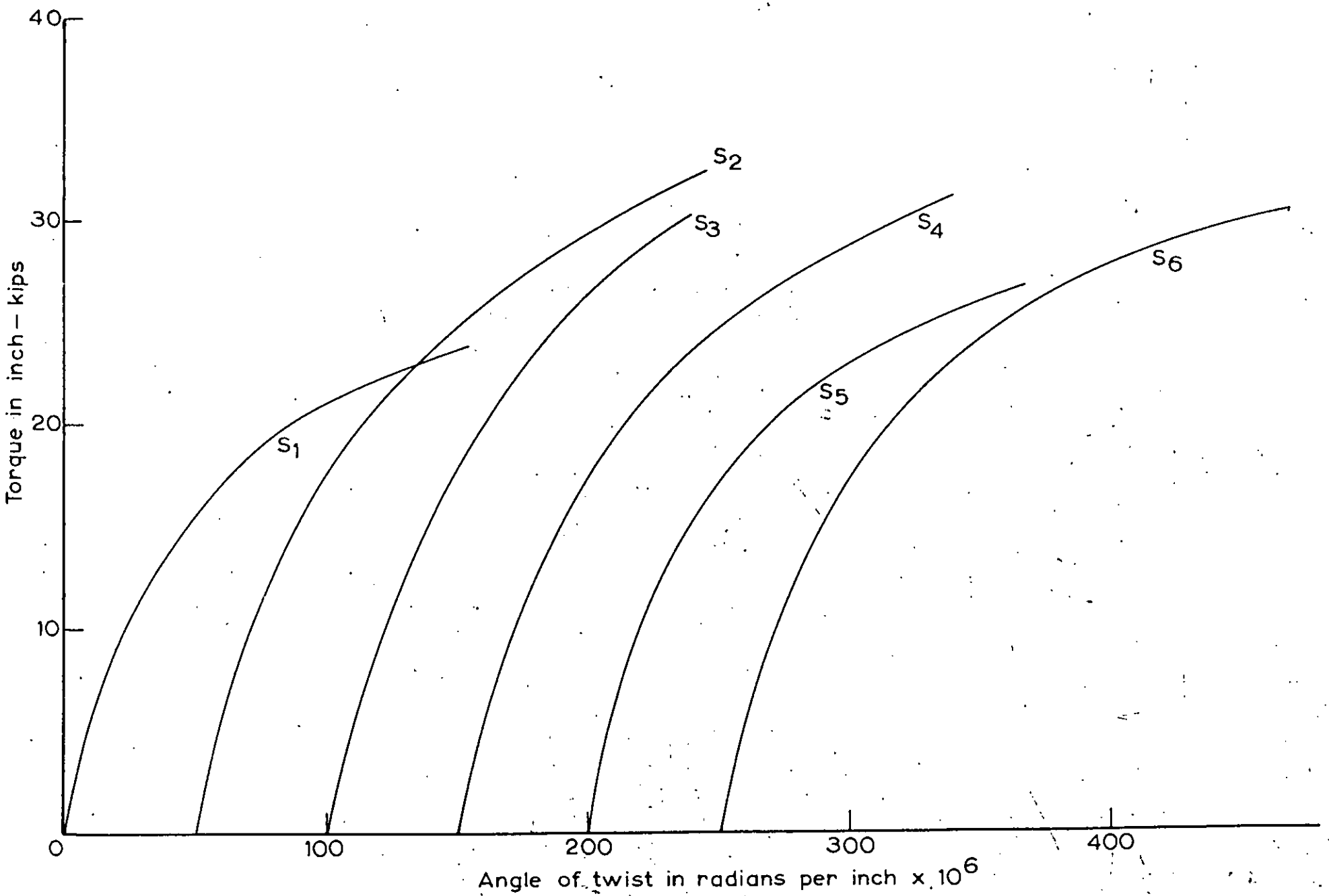


FIG. VI. 12. TORQUE – TWIST CURVES – S SERIES.

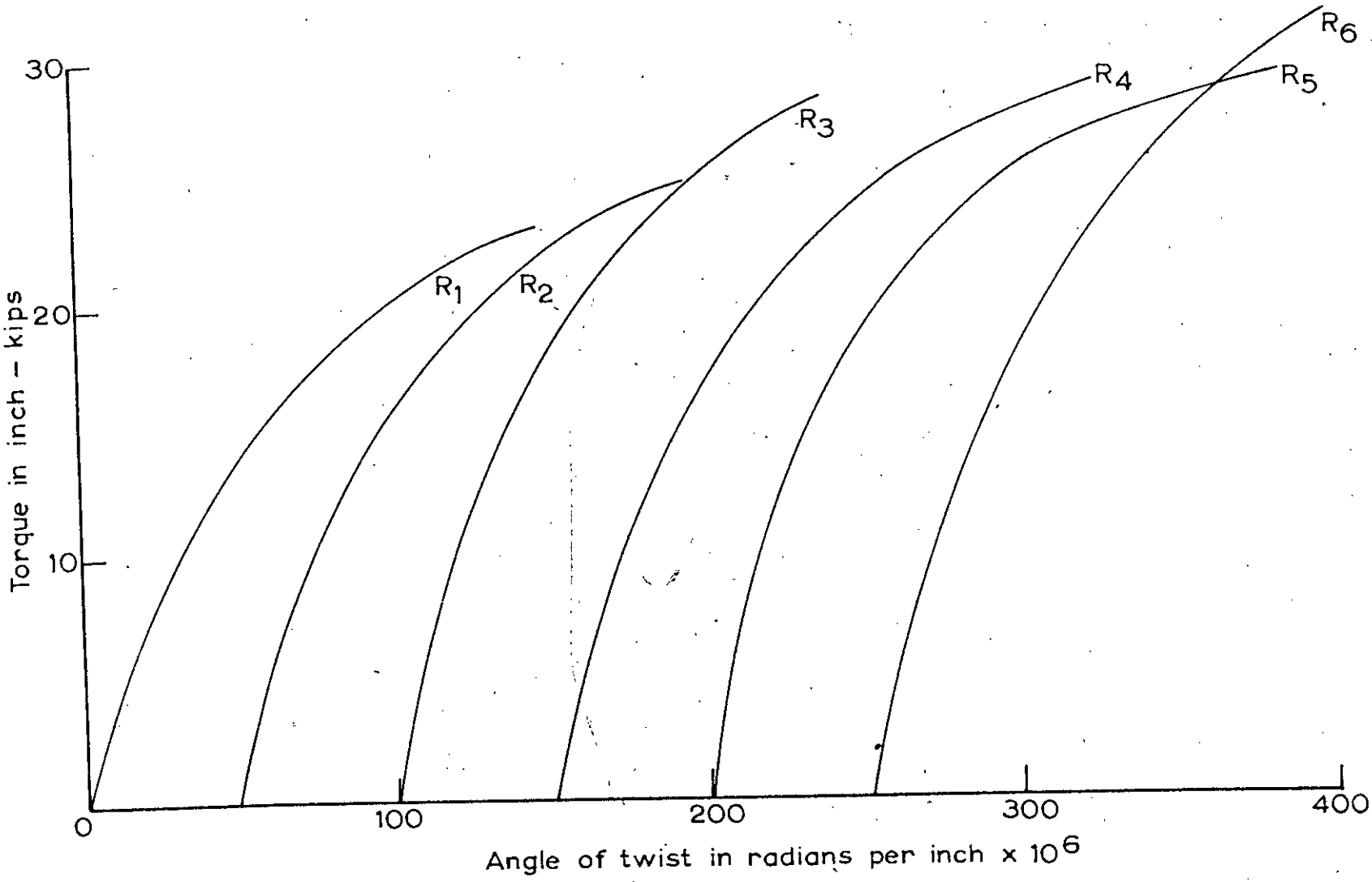


FIG. VI. 13. TORQUE— TWIST CURVES - R SERIES.

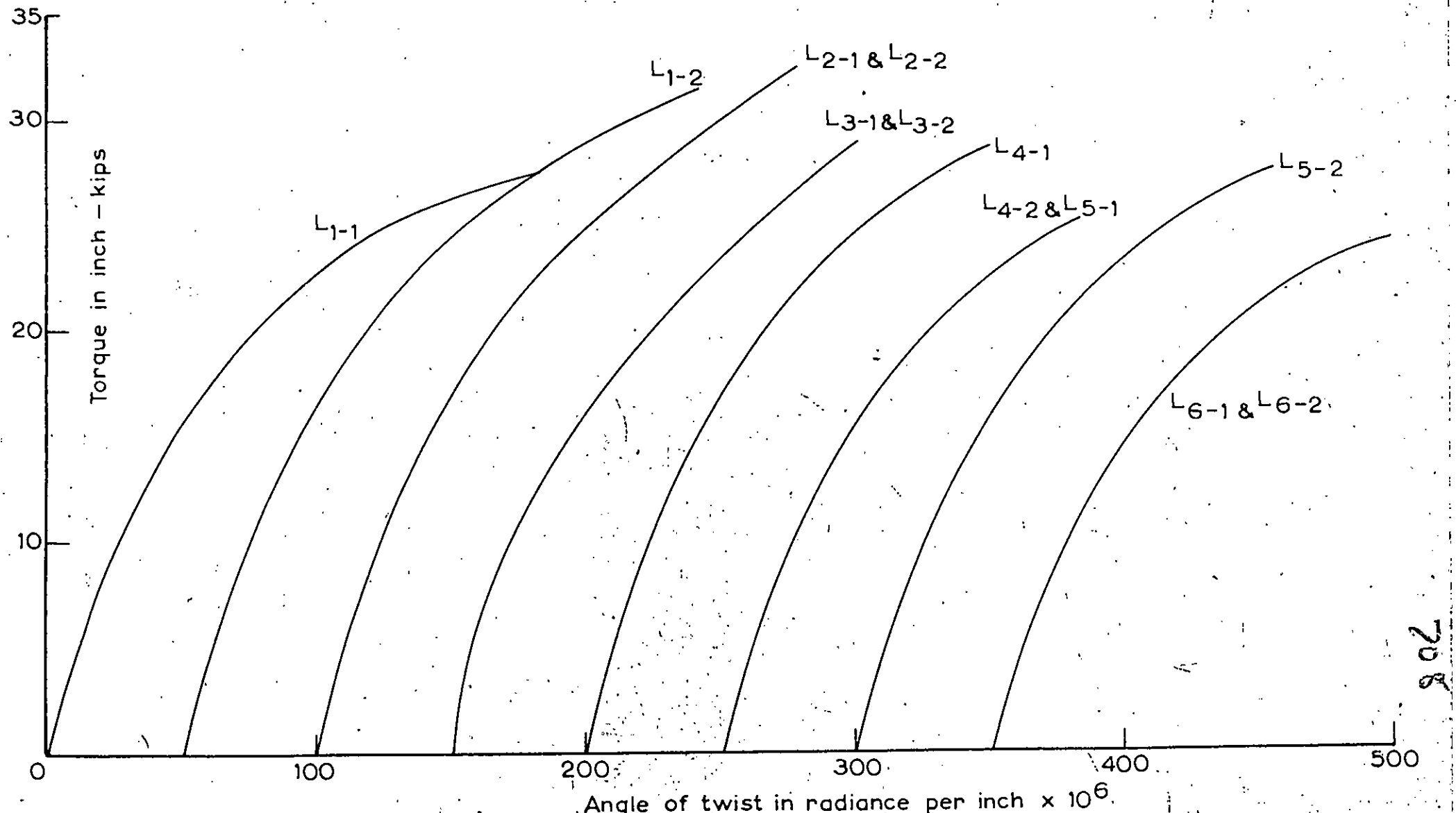


FIG. VI. 14. TORQUE - TWIST CURVES - L SERIES.

VI.2.2 Torque-Twist Curves:

Figs. VI.11 to VI.14 show the torque-twist curves of beams tested in combined bending and torsion. From the shape of the curves, it can be seen that there are mainly two stages. In the first stage, the curve is somewhat linear in the initial stages; but becomes non-linear for greater torques. The end of this stage was usually marked by the appearance of diagonal cracks. After the formation of diagonal cracks, the angle of twist of beams increased considerably. As in the case of moment-deflection curves, the torque-twist curves of identical beams coincided with one another (vide torque-twist curves of beams L2-1 and L2-2, L3-1 and L3-2 and L6-1 and L6-2.)

VI.3 Influence of various factors on the stiffness of beams:

a. Strength of Concrete:

As it is normally expected, the stiffness of beams decreased with the reduction in the strength of concrete. This can be noticed when the curves of same group of beams having different concrete strength are compared together. For instance, comparing the moment-deflection curves of beams L6-1 and L3-1, the beam L6-1 (C_u = 2600 psi) has suffered about 20% more deflection than the beam L3-1 (C_u = 4230 psi) for corresponding moments. Similarly the beam

R3 (C_u = 5600 psi) had a lesser concrete strength than that of beam R5 (C_u = 6350 psi) and hence it has suffered about 10% more twist than the later beam. These examples show the influence of strength of concrete on the stiffness of beams; but the variation in stiffness is not in direct proportion with the variation in concrete strength.

b. Longitudinal Reinforcement:

In general, the stiffness of a beam increased by the presence of longitudinal reinforcements. But increase in the stiffness by the increase in the longitudinal steel alone is not very significant. For instance, the angle of twists of beams S6 and R4 are nearly equal (within a variation of 5 percent) at working torques even though the beam R4 had about 50 percent more of longitudinal steel than the other beam. Similarly, the deflection of beams R3 and L2-1 are nearly equal at corresponding moments. These experimental observations indicate that increase in the stiffness by increasing the longitudinal steel alone beyond certain limit may not be considerable. It may be interesting to note here that these observations are in similar line with those discussed under strength aspects of the beams in Part I.

c. Transverse Reinforcements:

The transverse steel does not contribute by itself to the stiffness of the beam. That is, the area of transverse

steel may be so negligible such that its contribution to the rigidity, either torsional or flexural is insignificant. But the transverse steel restricts the widening of cracks.

d. Ratio of twisting moment to bending moment, $\emptyset$:

From experiments, it has been found that the angle of twist decreases with the increase in the value of $\emptyset$ at corresponding torques. The beams S3 and L4-1 were similar beams and were tested under similar conditions. The angle of twist of the beam L4-1 ($\emptyset = 0.353$) is about 20 percent more than that of the beam S3 ($\emptyset = 0.935$) at corresponding torques. This observation can be explained as follows: With the increase in the value of $\emptyset$, the amount of bending moment decreases and the penetration of flexural cracks is limited. So the net area of the section is more thus offering more stiffness to the beam.

The deflections of beams tested in combined bending and torsion slightly increased with the increase in the values of $\emptyset$. But the variation of deflections of beams with that of $\emptyset$ is not appreciable in many cases. Further, for low values of $\emptyset$, say $\emptyset < 0.10$, the deflection of a beam is nearly equal to that of a corresponding beam in pure bending. The deflection curves of Beams S1, S2 and S4 shown in Fig. VI.2 provide a good illustration of this.

For beams under combined bending and shear, the moment-deflection curves vary with the value of a/d ratio. The deflections of beams decrease with the increase in the value of a/d ratio. For high values of a/d ratio, the curves are similar to those obtained from beams failed in flexure. In the case of low values of a/d ratio, the stiffness of beams reduces considerably due to the presence of more diagonal cracks.

VI.4 Computation of Maximum Deflection and Angle of Twist of Beams at Working Loads:-

In general, the deflection δ of a beam is a function of bending moment M_b, span L, Modulus of Elasticity of the material E and Moment of Inertia of the section, I. Expressing it analytically,

$$\delta = F(M_b, L, E, I) \qquad \ldots \text{VI.1}$$

$$\text{or } F'(M_b, L, E, I, \delta) = 0 \qquad \ldots \text{VI.2}$$

By dimensional analysis we can easily show that

$$\delta = \beta_\delta \cdot \frac{M_b \cdot L^2}{EI} \qquad \ldots \text{VI.3}$$

where the parameter β_δ depends on the position and nature of loadings and the nature of end conditions of the beam. For example, for a simply supported beam with a uniformly

distributed load the value of β_δ is equal to 5/48 corresponding to the deflection at the centre of the span. Similarly, the values of β_δ for any other case can be obtained using the principles of elementary strength of materials. The International Code[138] recommends these values of β_δ for reinforced concrete beams also.

So the computation of deflections mainly depends on the value of the flexural rigidity, EI. If EI is known, for any given moment and other physical quantities, the deflection δ can be calculated assuming that the value of β_δ given by the elementary principles of strength of materials are sufficiently accurate for all practical purposes. The exact computation of EI at any instant for a reinforced concrete beam is rather complex or sometimes impossible. In this respect the reinforced concrete beams differ from the steel beams. The quantity EI is <u>not</u> a constant for reinforced concrete beams at all stages of loading, but it goes on decreasing as the applied bending moment increases and reaches a minimum value when the final collapse of the beam occurs. This is due to the formation of cracks in the tension zone of the beam in the initial stages and penetration of them into the compression zone at later stages of loading. Moreover, the so-called modulus of elasticity for concrete is not a constant quantity. This is due to the non-linear nature of the stress-strain

curve of concrete in compression. These two facts discussed above introduce difficulty in calculating the exact value of EI.

Having in view the difficulty in the calculation of EI, the following 'semi-inverse' method has been adopted to find the value of EI at working moments.

All the beams tested in the present investigation were simply supported at the ends with two concentrated loads placed symmetrically on the span. For such cases, deflection at the centre of the span can be calculated using the equation

$$\delta = \frac{M}{24\ EI}\left(3L^2 - 4a^2\right) \qquad \ldots \text{VI.4}$$

where a is the distance between load points and supports.

Or from Eqn. VI.3

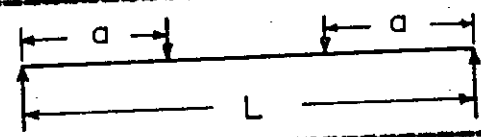
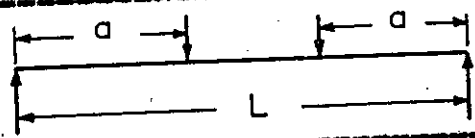
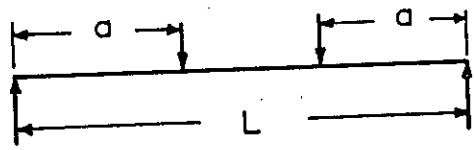
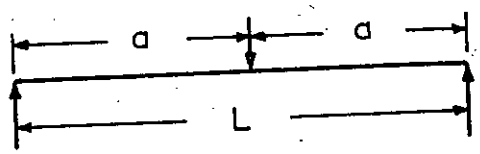
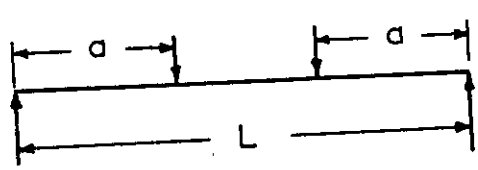
$$\beta_\delta = \left[\frac{3 - 4\ (a/L)^2}{24}\right] \qquad \ldots \text{VI.5}$$

The values of a and L for various beams are summarised in Table VI.1.

The working moments were obtained using a load factor of 2 for the ultimate moments. The deflections of beams corresponding to working moments, δ_ω were got from the moment-deflection curves obtained from the experiments.

TABLE VI.1

LOADING ARRANGEMENTS OF BEAMS

Specimen Mark	Type of Loading	a (in)	L (in)
V1 to L6-2		25	75
T1 to T20		35	100
SR 1		24	84
SR 2		28	84
SR 3		9	40
SR 5		17	60
SR 6		13.5	60
SR 4		30	60
SR 7		20	40
SR 9		49	98
SR 18		27	54
SR 19		27	54
SR 8		23.63	54
SR 11		16.87	54
SR 12		20.25	54
SR 13		40.50	98
SR 14		33.75	98
SR 15		33.75	98
SR 16		40.50	98

From Eqn. VI.3,

$$EI = \beta_\delta \frac{M_{bw} L^2}{\delta} \qquad \ldots \text{VI.6}$$

Putting $M_{bw} = M_b/2$ and

$\delta = \delta_w$ in Eqn. VI.6

the values of EI at working moments, say $(EI)_w$ were calculated. The values of EI before loading, $E_o I_o$, were calculated as follows. The value of E_o was taken equal to the value of modulus of elasticity of concrete as given in the International Code[138], i.e.

$$E_o = 19000 \sqrt{C_u} = 21000 \sqrt{f'_c}$$

when C_u and f'_c are expressed in kg/cm^2

$$\text{or } E_o = 71500 \sqrt{C_u} = 79000 \sqrt{f'_c}$$

when C_u and f'_c are expressed in lb/in^2.

where C_u - cube strength of concrete and

f'_c - cylinder strength of concrete

The value of I_o was taken equal to the moment of inertia of the uncracked concrete section neglecting the effect of reinforcements. Using these calculated values of $(EI)_w$ and $E_o I_o$, the ratios of $(EI)_w / E_o I_o$ were computed for all the beams. The results are given in Tables VI.2 to VI.4.

TABLE VI.2

STIFFNESS PROPERTIES OF BEAMS UNDER COMBINED BENDING AND TORSION

Specimen Mark	Cube strength psi	E_o: psi x 10^6	μ	G_o: psi x 10^6	$E_o I_o$ lbin^2 x 10^6	$G_o J_o$ lbin^2 x 10^6
1	2	3	4	5	6	7
V1	6650	5.84	0.200	2.02	1242	600
V2	5750	5.42	0.200	2.26	1156	670
V3	4920	5.02	0.200	2.09	1070	620
V4	5650	5.38	0.200	2.24	1150	665
V5	5400	5.25	0.200	2.19	1120	650
V6	5040	5.08	0.200	2.12	1085	630
S1	4900	5.00	0.200	2.08	1068	617
S2	5400	5.25	0.200	2.19	1120	650
S3	4800	4.95	0.200	2.06	1058	612
S4	5600	5.35	0.200	2.23	1140	662
S5	5250	5.18	0.200	2.16	1105	640
S6	5350	5.24	0.200	2.18	1118	646
R1	3550	4.26	0.200	1.78	910	528
R2	6000	5.55	0.200	2.32	1182	690
R3	5600	5.35	0.200	2.23	1140	662
R4	5160	5.14	0.200	2.14	1095	635
R5	6350	5.70	0.200	2.38	1215	706
R6	5160	5.14	0.200	2.14	1095	635
L1-1	4100	4.58	0.200	1.91	978	566
L1-2	5160	5.14	0.200	2.14	1095	635
L2-1	5725	5.42	0.200	2.26	1158	670
L2-2	5725	5.42	0.200	2.26	1158	670
L3-1	4230	4.65	0.200	1.94	990	575
L3-2	4230	4.65	0.200	1.94	990	575
L4-1	4660	4.88	0.200	2.04	1040	605
L4-2	3800	4.40	0.200	1.83	938	544

Table VI.2 (continued)

1	2	3	4	5	6	7
L5-1	3600	4.28	0.200	1.78	915	528
L5-2	3650	4.32	0.200	1.80	920	535
L6-1	2600	3.65	0.200	1.52	778	452
L6-2	2600	3.65	0.200	1.52	778	452
T1	3420	4.18	0.200	1.74	705	562
T2	3450	4.20	0.200	1.75	706	565
T3	3500	4.23	0.200	1.76	714	567
T4	3860	4.45	0.200	1.85	750	597
T5	4800	4.95	0.200	2.06	834	665
T6	4200	4.64	0.200	1.93	780	623
T7	5160	5.14	0.200	2.14	865	690
T8	3500	4.23	0.200	1.76	714	567
T9	3900	4.46	0.200	1.86	785	600
T10	2210	3.36	0.200	1.40	566	452
T11	3500	4.23	0.200	1.76	714	567
T12	3500	4.23	0.200	1.76	714	567
T13	3700	4.35	0.200	1.86	732	600
T14	5800	5.45	0.200	2.27	918	732
T15	4200	4.64	0.200	1.93	780	624
T16	4650	4.87	0.200	2.03	820	655
T17	3750	4.37	0.200	1.82	735	588
T18	5200	5.15	0.200	2.15	866	695
T19	5300	5.20	0.200	2.17	875	700
T20	5200	5.15	0.200	2.15	866	695

TABLE VI.3

SUMMARY OF CALCULATIONS: BEAMS UNDER COMBINED BENDING AND TORSION

(LOAD FACTOR = 2)

pe-men mark	M_{bu} ($\frac{}{2}$) in-kips	M_{tu} ($\frac{}{2}$) in-kips	ϕ	δ inch $\times 10^3$ (test)	θ radians/inch $\times 10^6$ (test)	$(EI)_w$: lbin^2 $\times 10^6$	$(GJ)_w$: lbin^2 $\times 10^6$	$\frac{(EI)_w}{Eo\ Io}$	$\frac{(GJ)_w}{Go\ Jo}$
1	2	3	4	5	6	7	8	9	10
1	27.5	20.5	0.745	28	52	572	580	0.46	0.97
2	60.5	18.0	0.298	64	50	552	530	0.48	0.79
3	70.0	12.5	0.179	80	32	510	575	0.48	0.93
4	86.5	16.5	0/∞	88	42	574	580	0.50	0.87
5	71.0	19.5	0.275	92	52	450	550	0.40	0.85
5	26.0	17.0	0.655	24	46	632	545	0.58	0.87
6	77.0	12.0	0/0.156	84	34	535	520	0.50	0.84
2	77.0	15.5	0.104/0.238	72	44	625	518	0.56	0.80
3	16.0	14.5	0.935	16	38	584	564	0.55	0.92
4	74.0	15.5	0.081/0.235	76	42	568	540	0.50	0.82
5	74.0	14.0	0.135/0.212	80	38	540	542	0.49	0.85
6	0	15.0	∞	-	42	-	525	-	0.81
1	17.5	12.5	0.715	18	40	566	460	0.62	0.87
2	55.5	14.0	0.252	60	40	540	515	0.46	0.75
3	92.5	15.0	0/0.162	108	40	498	550	0.44	0.83
	0	15.0	∞	-	42	-	525	-	0.83
	100.0	15.0	0.045/0.162	96	35	608	630	0.50	0.89
	32.0	16.5	0.515	28	46	665	526	0.61	0.83
-1	0	14.5	∞	-	48	-	445	-	0.79
-2	84.5	16.0	0/0.190	96	50	514	470	0.47	0.74
-1	90.5	16.5	0.055/0.180	96	48	550	505	0.48	0.75
-2	90.5	16.5	0.055/0.180	96	48	550	505	0.48	0.75
-1	81.5	15.0	0.123/0.286	112	48	425	460	0.43	0.80

(contd.)

Table VI.3 (contd.)

1	2	3	4	5	6	7	8	9	10
L3-2	81.5	15.0	0.123/0.294	112	48	425	460	0.43	0.80
L4-1	42.5	15.0	0.353	52	46	476	480	0.46	0.79
L4-2	11.0	13.5	0.795	16	42	400	472	0.43	0.87
L5-1	10.0	13.5	0.820	12	42	485	472	0.53	0.89
L5-2	29.0	14.5	0.500	48	48	353	445	0.39	0.83
L6-1	81.5	12.5	0.123/∞	132	45	360	410	0.46	0.91
L6-2	78.0	12.5	0.141/∞	124	45	367	410	0.47	0.91
T1	28.5	10.5	0.370	70	-	426	-	0.60	-
T2	49.0	10.5	0.216	164	-	312	-	0.44	-
T3	81.5	0	0	236	-	361	-	0.51	-
T4	0	8.0	∞	-	-	-	-	-	-
T5	81.5	10.0	0.098/.127	175	-	486	-	0.58	-
T6	0	8.5	∞	-	-	-	-	-	-
T7	79.0	11.0	0/0.157	242	-	341	-	0.40	-
T8	30.0	9.0	0.300	123	-	256	-	0.36	-
T9	87.5	9.0	0.103	205	-	446	-	0.57	-
T10	79.0	6.5	0.083	200	-	414	-	0.74	-
T11	57.0	8.5	0/∞	210	-	284	-	0.40	-
T12	57.0	7.5	0.105/.143	135	-	441	-	0.62	-
T13	10.0	10.0	1.000	30	-	350	-	0.48	
T14	48.0	10.0	0.208	125	-	402	-	0.44	-
T15	52.5	11.5	0.219	125	-	438	-	0.56	-
T16	87.5	0	0	244	-	376	-	0.46	-
T17	87.5	9.0	0.097/.114	244	-	376	-	0.51	-
T18	79.0	11.5	0.146	170	-	485	-	0.56	-
T19	0	11.5	∞	-	-	-	-	-	-
T20	87.5	11.5	0.114/∞	239	-	383	-	0.44	-

519

TABLE VI.4

SUMMARY OF CALCULATIONS: BEAMS UNDER BENDING AND SHEAR

(Load Factor = 2)

Specimen mark	f'_c psi	Eo : psi$\times 10^6$	Eo Io : lbin$^2\times 10^6$	M_{bu} / 2 in-kips	α	δ inch $\times 10^3$	$(EI)_w$ lbin$^2\times 10^6$	$\frac{(EI)_w}{Eo\ Io}$
SR 1	5000	5.60	955	72.0	3.54	140	406	0.43
2	5000	5.60	955	85.5	4.14	160	403	0.42
3	5000	5.60	955	73.0	1.33	40	340	0.36
4	5000	5.60	955	87.5	4.42	60	438	0.46
5	5000	5.60	955	68.0	3.00	75	364	0.38
6	5000	5.60	955	65.0	2.00	50	546	0.57
7	5000	5.60	955	64.0	2.95	23	370	0.39
8	3820	4.88	834	73.0	3.50	50	398	0.48
9	3500	4.67	796	100.5	7.20	185	435	0.54
11	3050	4.36	745	52.5	2.50	40	417	0.56
12	4000	5.00	854	70.5	3.00	50	420	0.49
13	3500	4.67	796	91.0	6.00	220	384	0.48
14	3050	4.36	745	90.5	5.00	195	470	0.63
15	3300	4.54	774	87.0	5.00	220	402	0.52
16	3050	4.36	745	84.0	6.00	180	433	0.58
18	3500	4.67	796	76.5	4.00	45	413	0.52
19	3500	4.67	796	75.0	4.00	48	380	0.48

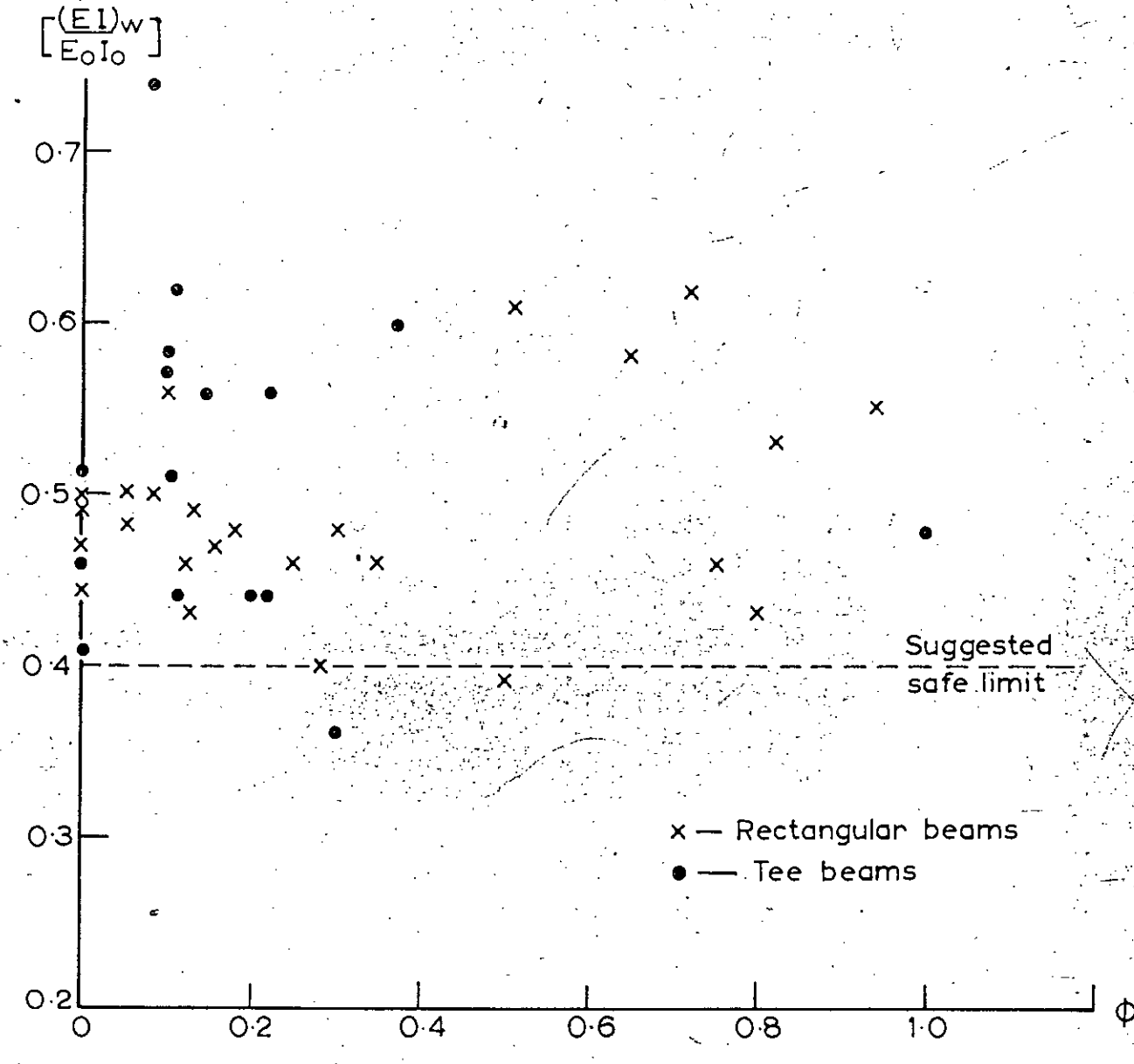


FIG. VI. 15. VARIATION OF $(EI)_w$ FOR BEAMS UNDER COMBINED BENDING AND TORSION.

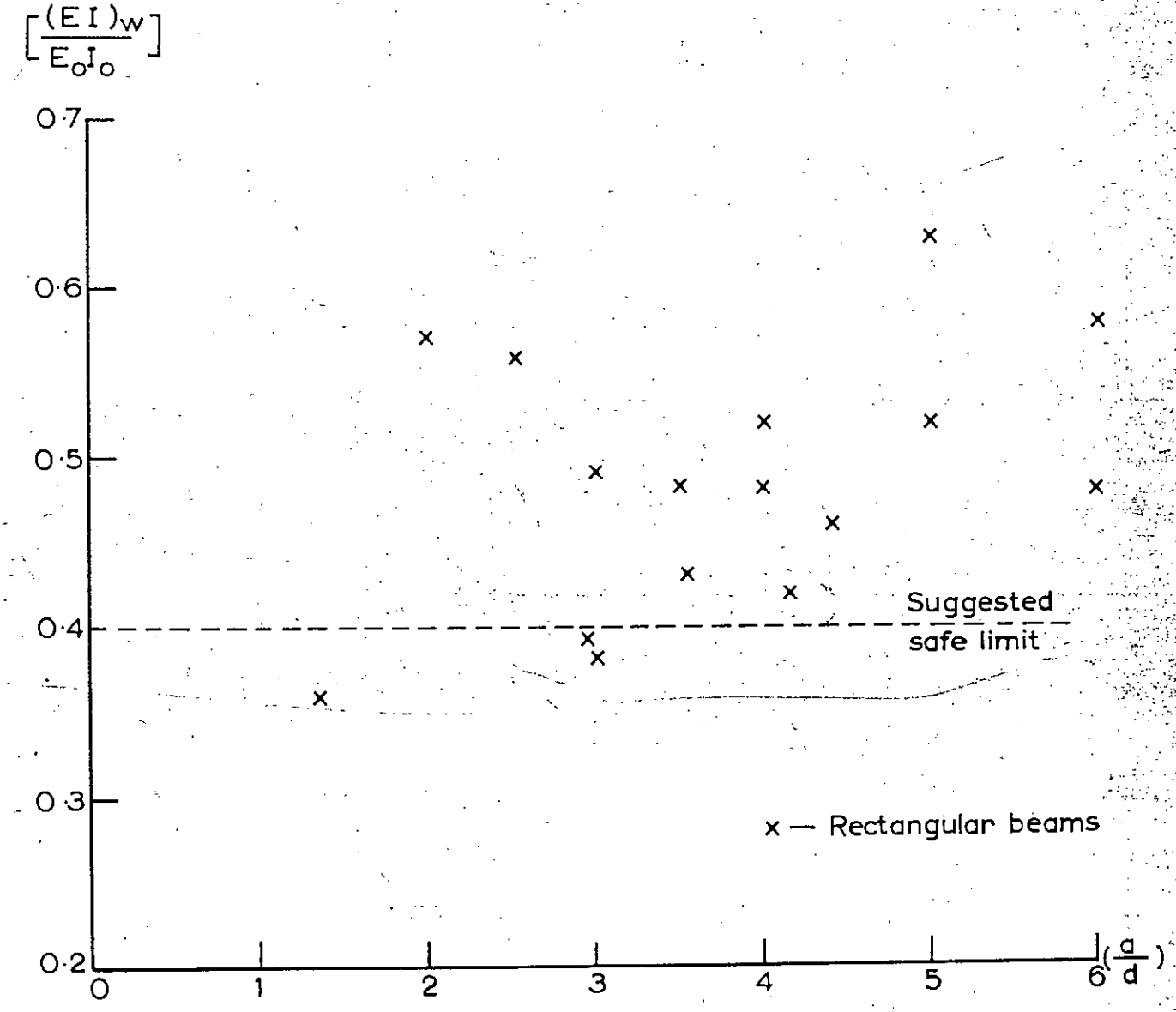


FIG. VI. 16. VARIATION OF $(EI)_w$ FOR BEAMS UNDER COMBINED BENDING AND SHEAR.

The computed values of $(EI)_w/Eo\ Io$ were plotted against ϕ in Fig. VI.15 for the beams tested under combined bending and torsion and against a/d in Fig. VI.16 for the case of beams subjected to combined bending and shear.

Keeping in view the scatter in the results, from Figs. VI.15 and 16, it can be seen that the ratio $(EI)_w/EoIo$ lies within a band of 0.40 and 0.55 in most of the cases. A safe lower bound can be obtained using a value of 0.40 for $(EI)_w/EoIo$. So the deflections under working loads of beams under bending and torsion or bending and shear can be calculated using the following equation:

$$\delta_w = \beta_\delta \frac{M_{bw} L^2}{0.40\ EoIo} \qquad \ldots \text{VI.7}$$

where M_{bw} is the working moment and EoIo is to be calculated as indicated earlier.

b. Angle of twist of beams:

In general, the angle of twist per unit length, θ of a prismatic member is a function of the applied torque M_t, the centroidal or polar moment of inertia J of the cross section and the shear modulus G of the material. It can be expressed as follows:

$$\theta = F(M_t, J, G)$$

$$\text{or} \quad F'(M_t, J, G, \theta) = 0 \qquad \ldots \text{VI.8}$$

By dimensional analysis we can easily show that

$$\theta = \beta_\theta \frac{M_t}{GJ} \qquad \ldots \text{VI.9}$$

where β_θ depends on the dimensions and shape of the cross section.

For rectangular beams,

$$J = \frac{1}{12} b^3 D \left[1 + (D/b)^2\right] \qquad \ldots \text{VI.10}$$

As per Elastic Torsion theory,

$$M_t = K_1 G\theta\, b^3 D \qquad \ldots \text{VI.11}$$

where b and D denote the breadth and overall depth of the rectangular beam and K_1 is St. Venant's torsion coefficient depending on D/b ratios.

Substituting for b^3D from Eqn. VI.10 in Eqn.VI.11 we have,

$$\theta = \left(\frac{M_t}{GJ}\right) \frac{\left[1 + (D/b)^2\right]}{12\, K_1} \qquad \ldots \text{VI.12}$$

Comparing Eqns. VI.9 and VI.12, the value of β_θ for rectangular beams can be written as

$$\beta_\theta = \left[\frac{1 + (D/b)^2}{12\, K_1}\right] \qquad \ldots \text{VI.13}$$

The values of β_θ for various D/b ratios are given in Table VI.5. The values of β_θ for other sections can be obtained similarly.

TABLE VI.5

VALUES OF β_θ FOR RECTANGULAR BEAMS

D/b	1.0	1.2	1.5	2.0	2.5	3.0	4.0	5.0	10.0
β_θ	1.186	1.225	1.381	1.820	2.429	3.170	5.049	7.436	26.967

The calculation of angle of twist of beams reduces to the determination of torsional rigidity GJ. Similar to the flexural rigidity of beams, the value of GJ for a reinforced concrete beam decreases with the increase in the torque reaching a minimum value at failure. The exact determination of GJ at any instant poses the same difficulty as in the case of calculation of EI. So the semi-inverse method used in the case of deflection of beams is adopted here also. Using the torque-twist curves obtained from experiments and a load factor of 2 for the ultimate torques, the angle of twists at working torques were obtained. Using these values the values of torsional rigidities at working torques $(GJ)_w$ were computed using Eqn. VI.9. The values of torsional rigidities before loading Go Jo were obtained in a similar manner as those of Eo Io. That is, Jo represents the centroidal or polar moment of inertia of the uncracked concrete section ignoring the effect of reinforcements. The value of Go was calculated using the equation $Go = Eo/2(1+\mu)$ where μ is the Poisson's ratio and it is taken equal to 0.20 as recommended in the International Code[138]. From these computed

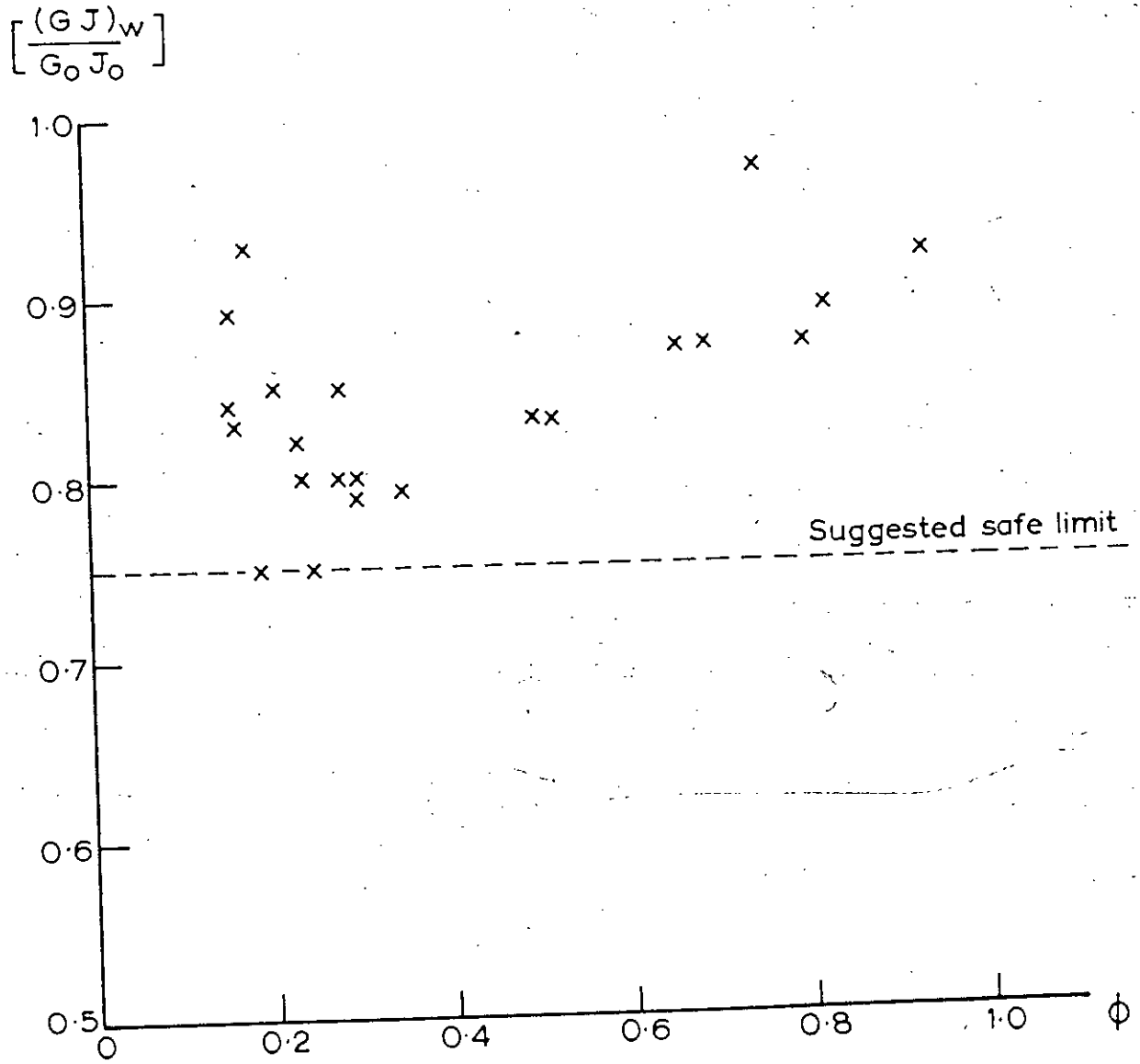


FIG. VI.17. VARIATION OF $(GJ)_w$ FOR BEAMS UNDER COMBINED BENDING AND TORSION.

results the ratios of $(GJ)_w/Go\ Jo$ were obtained. The results are given in Tables VI.2 and 3.

In Fig. VI.17, the values of the ratio $(GJ)_w / Go\ Jo$ were plotted against ϕ. From Fig. VI.17 it can be seen that the ratio $(GJ)_w/Go\ Io$ increases from 0.75 to 0.90 with the increase in the value of ϕ. Since this variation in the values of $(GJ)_w/Go\ Jo$ with ϕ is not considerable, the lower value of 0.75 is suggested as a safe limit. So the angle of twist per unit length under working loads of beams under combined bending and torsion can be calculated using the following equation:

$$\theta_w = \beta_\theta \frac{M_{tw}}{0.75\ Go\ Io} \qquad \ldots\ VI.14$$

where

θ_w is the angle of twist per unit length at working loads,

M_{tw} is the torque at working loads and

$Go\ Io$ is to be calculated as indicated earlier.

c. The semi-inverse method discussed above is carried out using different load factors (ie. 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75, 3.00, 3.50 and 4.00). In each case the safe limit was obtained by plotting the results in the form of graphs

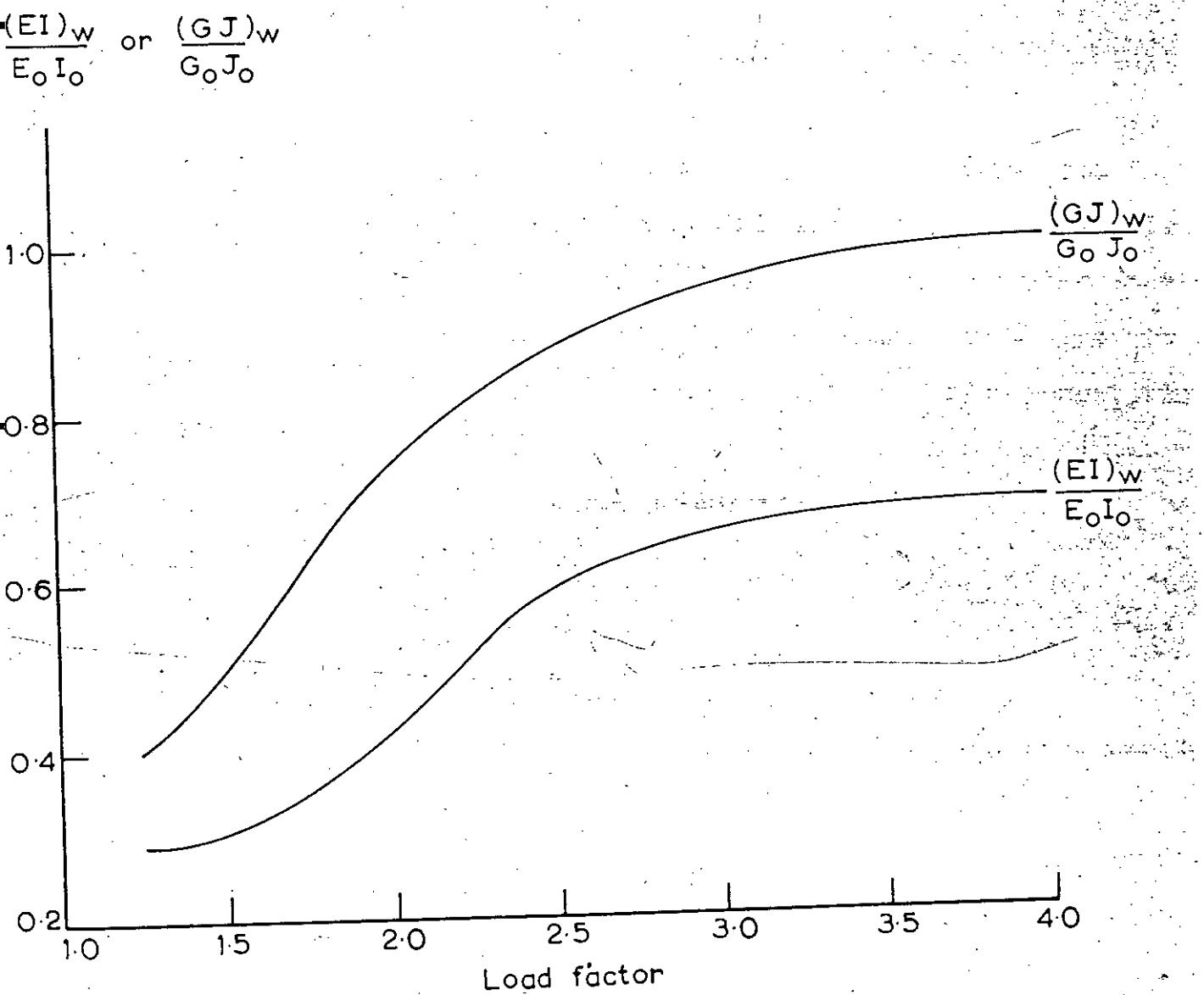


FIG. VI. 18. VARIATION OF $\frac{(EI)_w}{E_o I_o}$ AND $\frac{(GJ)_w}{G_o J_o}$ WITH LOAD FACTOR.

similar to those shown in Figs. VI.15 to VI.17. Since the detailed calculations are voluminious they are not given here. The final results of the analysis are summarised below:

Load Factor	$(EI)_w$ / EoIo	$(GJ)_w$ /GoIo
1.25	0.29	0.40
1.50	0.30	0.50
1.75	0.34	0.62
2.00	0.41	0.75
2.25	0.52	0.82
2.50	0.60	0.87
2.75	0.63	0.92
3.00	0.65	0.95
3.50	0.67	0.99
4.00	0.69	1.00

In Fig. VI.18, these results are shown in a graphical form. Thus for a given load factor the quantities $(EI)_w$ / Eo Io and $(GJ)_w$ / Go Io can be known using Fig. VI.18 and hence the deflection and angle of twist of beams can be calculated.

VI.5 Illustrative Example:

The suggested procedure for the calculation of deflection and angle of twist of beams under combined loading

is illustrated in the following numerical example:

To determine deflection and angle of twist of a rectangular beam having the following working moments:

Bending moment = 1000 in kips

Torque = 350 K in kips

Given that f'_o = 3000 psi; b = 16 in; D = 24 in;
L = 40 ft; β_δ = 5/48; Load Factor = 2.

Computation of Deflection:

$$E_o = 79000\sqrt{f'_o} = 79000\sqrt{3000} = \underline{4.325} \times 10^6 \text{ psi}$$

$$I_o = 1/12\ bD^3 = \frac{1}{12} \times 16 \times 24^3 = \underline{18432}\ \text{in}^4.$$

$$\delta_w = \beta_\delta \frac{M_{bw} L^2}{0.40\ E_o\ I_o}$$

$$= \frac{5}{48} \times \frac{1000 \times 10^3 \times (40 \times 12)^2}{0.4 \times 4.325 \times 10^6 \times 18432}$$

$$= \underline{0.752}\ \text{in.}$$

$$\text{or } \frac{\delta_w}{L} = \frac{0.752}{40 \times 12} = \frac{1}{640}.$$

$$\text{or } \delta_w = \underline{L/640}.$$

Computation of angle of twist:

From Table VI.5, $\beta_\theta = 1.381$

$$J_o = \frac{1}{12} b^3 D \left[1 + (D/b)^2\right]$$

$$= \frac{1}{12} \times 16 \times 16 \times 16 \times 24 \left(1 + 1.5^2\right)$$

$$= \underline{26,600} \text{ in}^4$$

$$G_o = E_o/2(1+\mu)$$

$$= \frac{4,325}{2(1+0.2)} \times 10^6 = \underline{1.80 \times 10^6} \text{ psi}$$

$$\theta_w = \beta_\theta \frac{M_{tw}}{0.75\, G_o\, J_o}$$

$$= 1.381 \times \frac{350 \times 10^3}{0.75 \times 1.80 \times 10^6 \times 26600}$$

$$= \underline{13.45} \times 10^{-6} \text{ radians per inch}$$

6. CONCLUDING REMARKS:

The behaviour of reinforced concrete beams under combined bending and torsion and under combined bending and shear has been studied in the light of moment-deflection and torque-twist curves. The influence of various parameters on the stiffness of beams is examined. Based on experimental

data, an approximate method has been suggested to obtain deflection and angle of twist of beams at working loads. The suggested method of calculation is illustrated by a numerical example.

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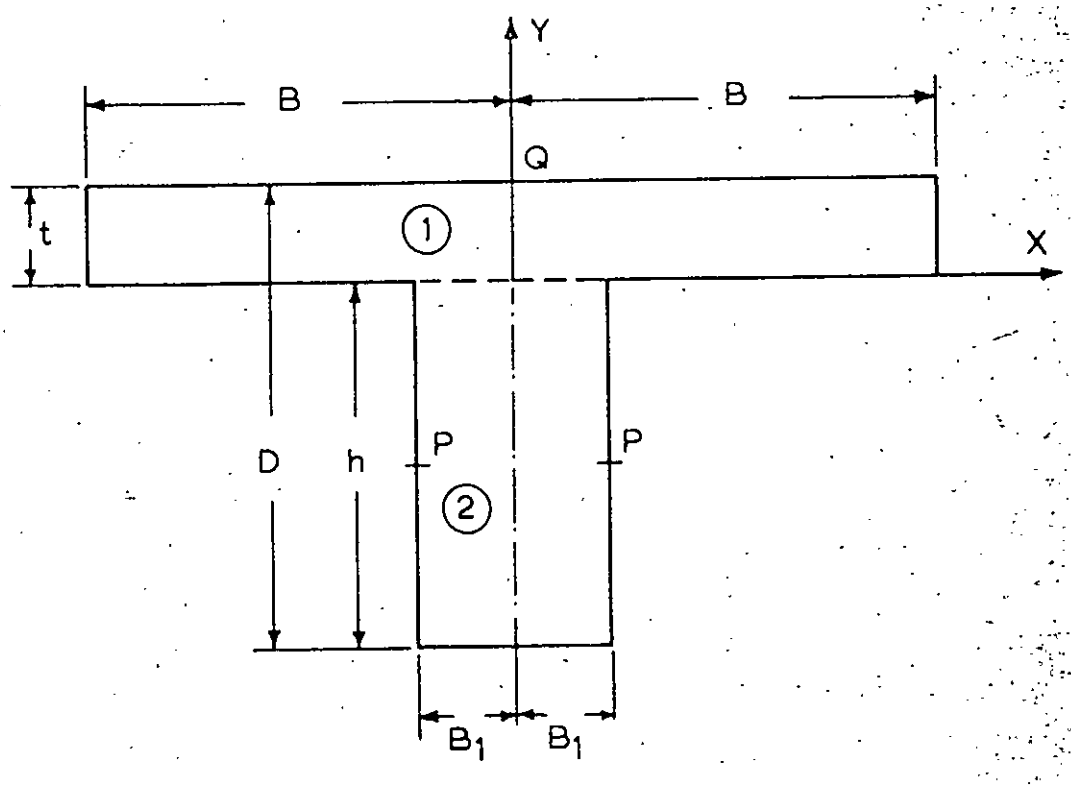
APPENDIX 1 : CHARTS FOR THE CALCULATION OF ELASTIC TORSIONAL PROPERTIES OF TEE SECTION.



$$C' = \frac{M_t}{G\theta t^4} \quad , \quad \tau'_m = \tau_m / G\theta t \quad , \quad \tau'_{xz} = \tau_{xz} / G\theta t$$

Where M_t = Twisting moment

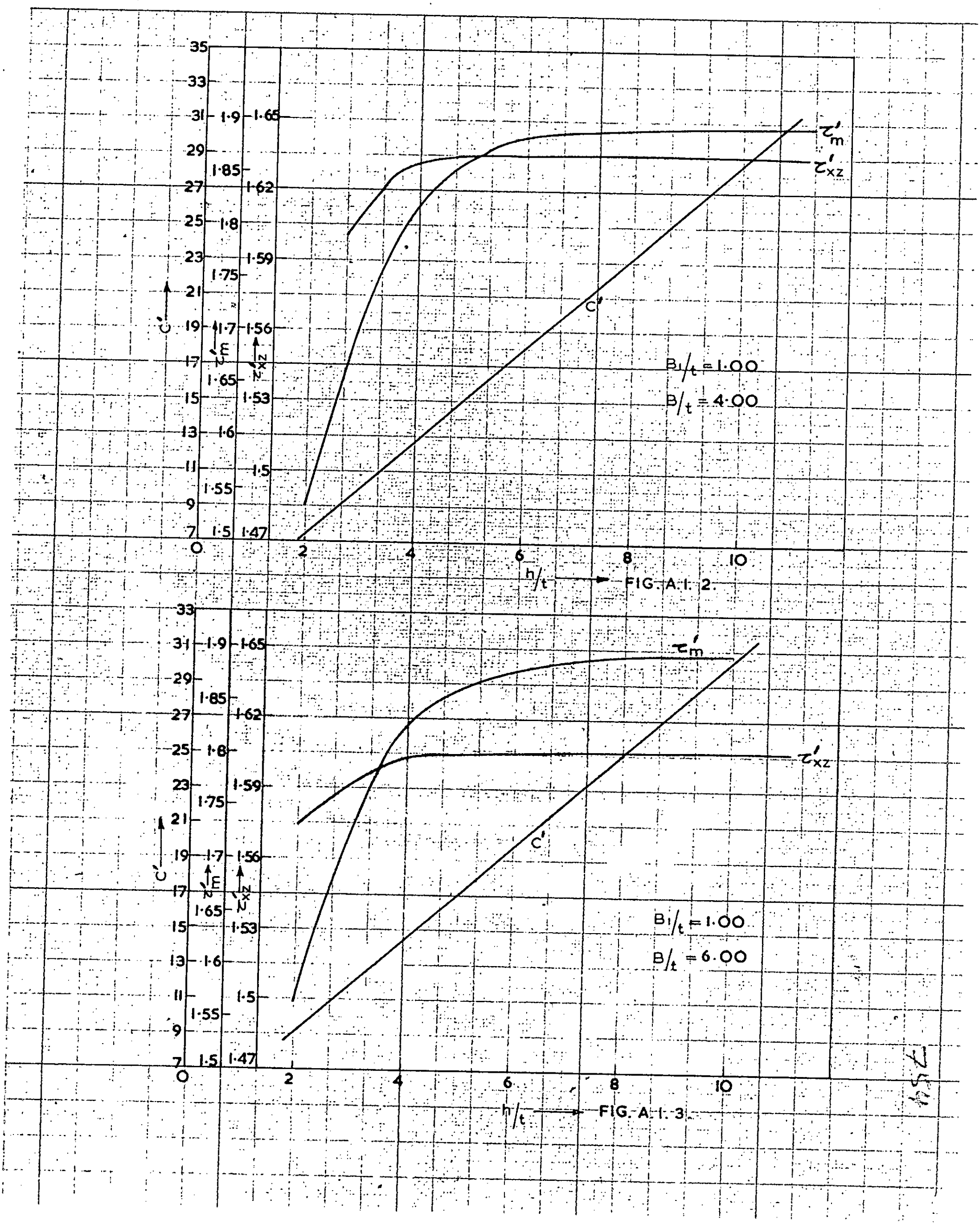
τ_m = Shear stress at mid point P of web

τ_{xz} = Shear stress at mid point Q of flange

G = Shear modulus

θ = Angle of twist per unit length

FIG. A.1.1. TORSION OF TEE SECTION.


35
33
31
29
27
25
23
21
19
17
15
13
11
9
7
1·9
1·85
1·8
1·75
1·7
1·65
1·6
1·55
1·5
1·65
1·62
1·59
1·56
1·53
1·5
1·47
C'
E
N
N
x
z
O
2
4
6
8
10
h/t
τ'_m
τ'_{xz}
C'
$B_1/t = 1.00$
$B/t = 4.00$
FIG. A.I. 2.
$B_1/t = 1.00$
$B/t = 6.00$
FIG. A.I. 3.

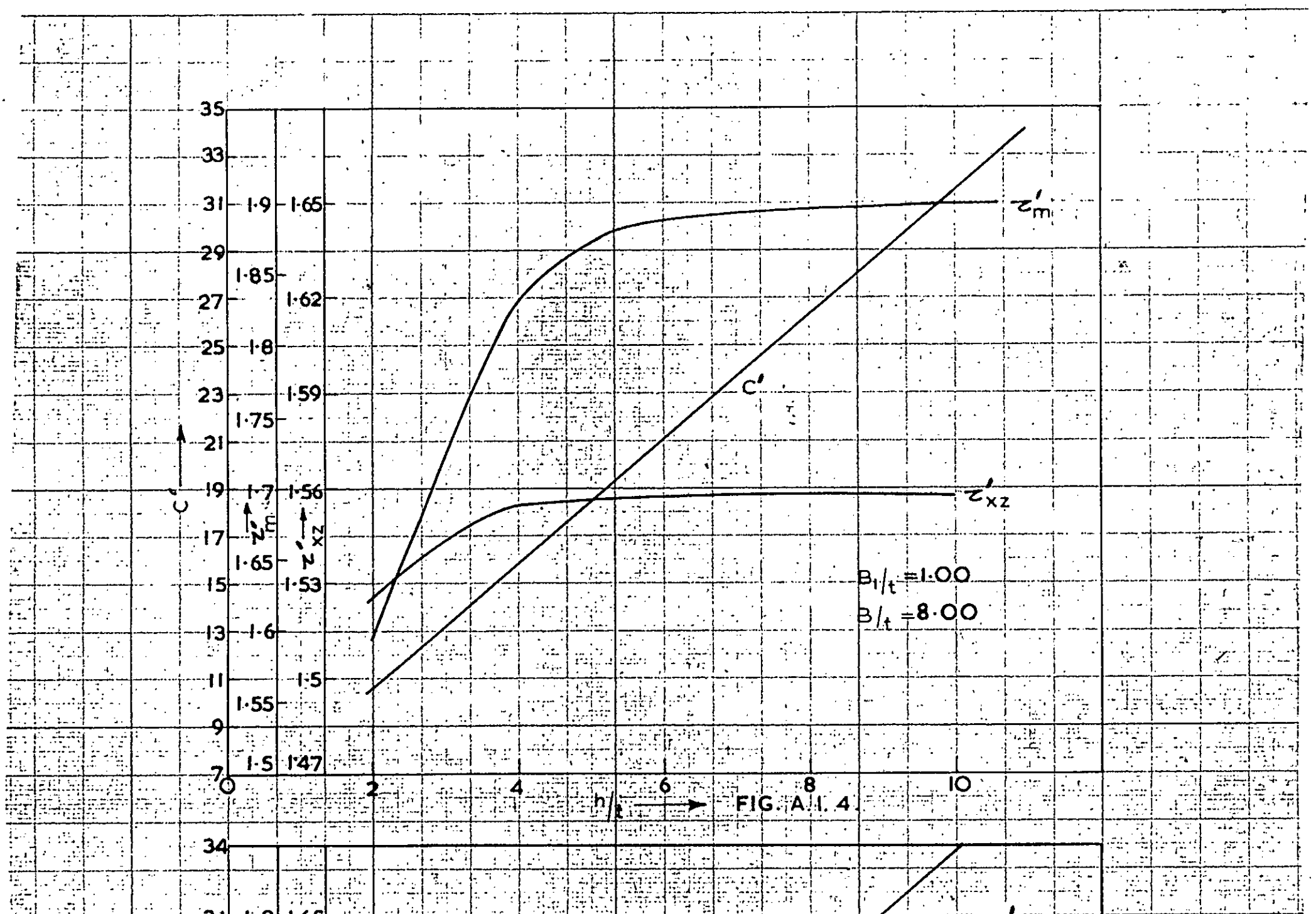


FIG. A I. 4

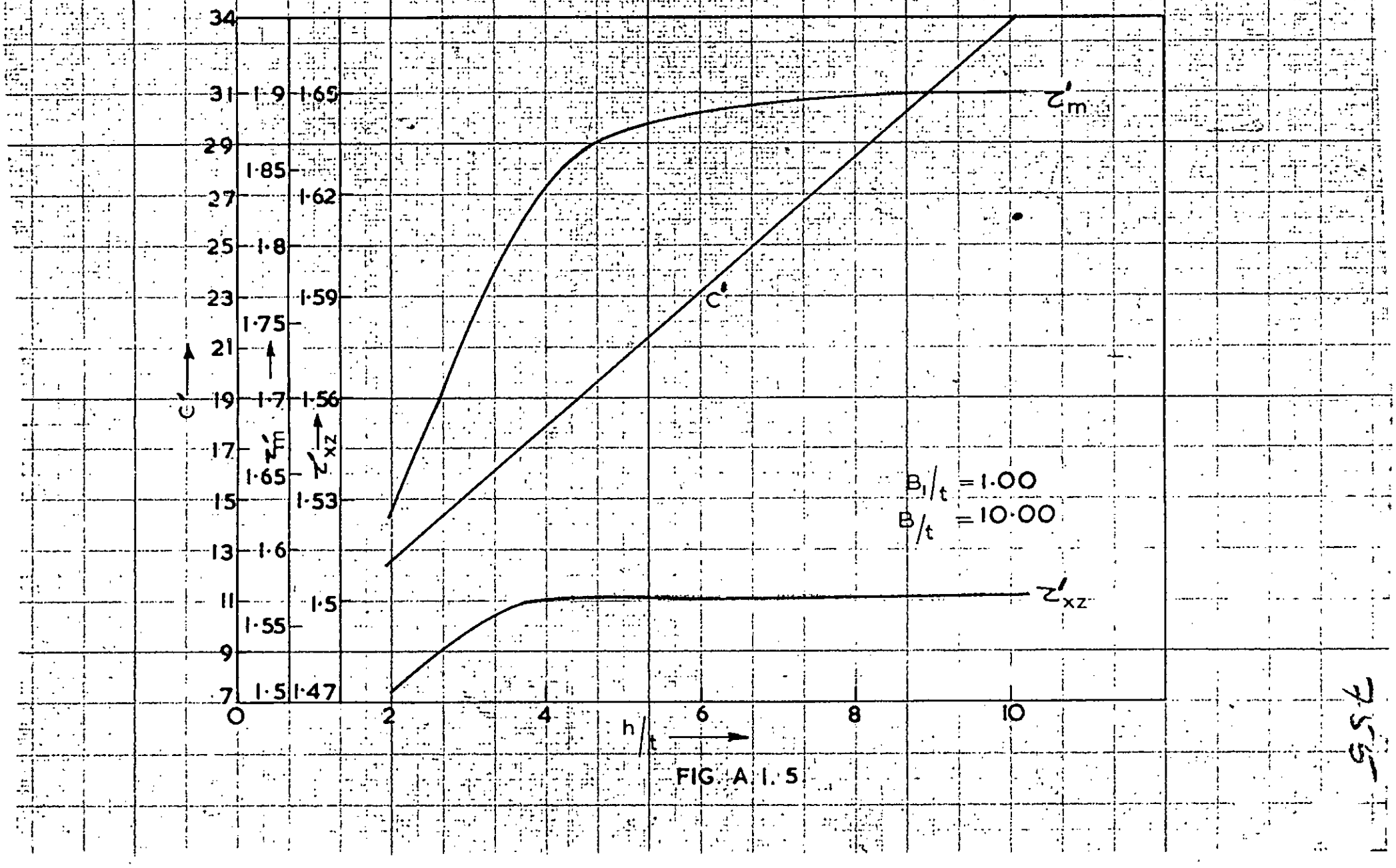


FIG. A I. 5

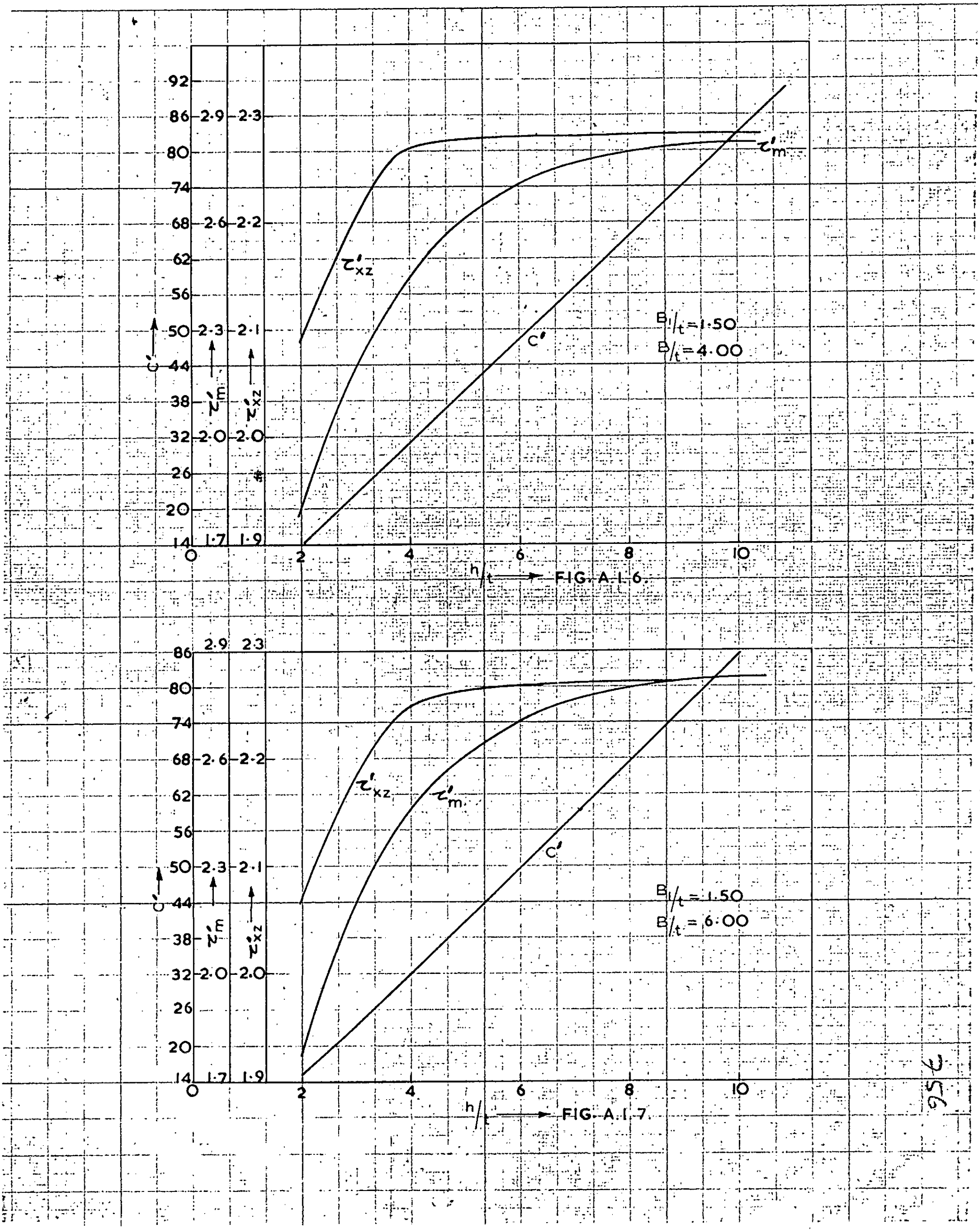


FIG. A.I.6.

FIG. A.I.7.

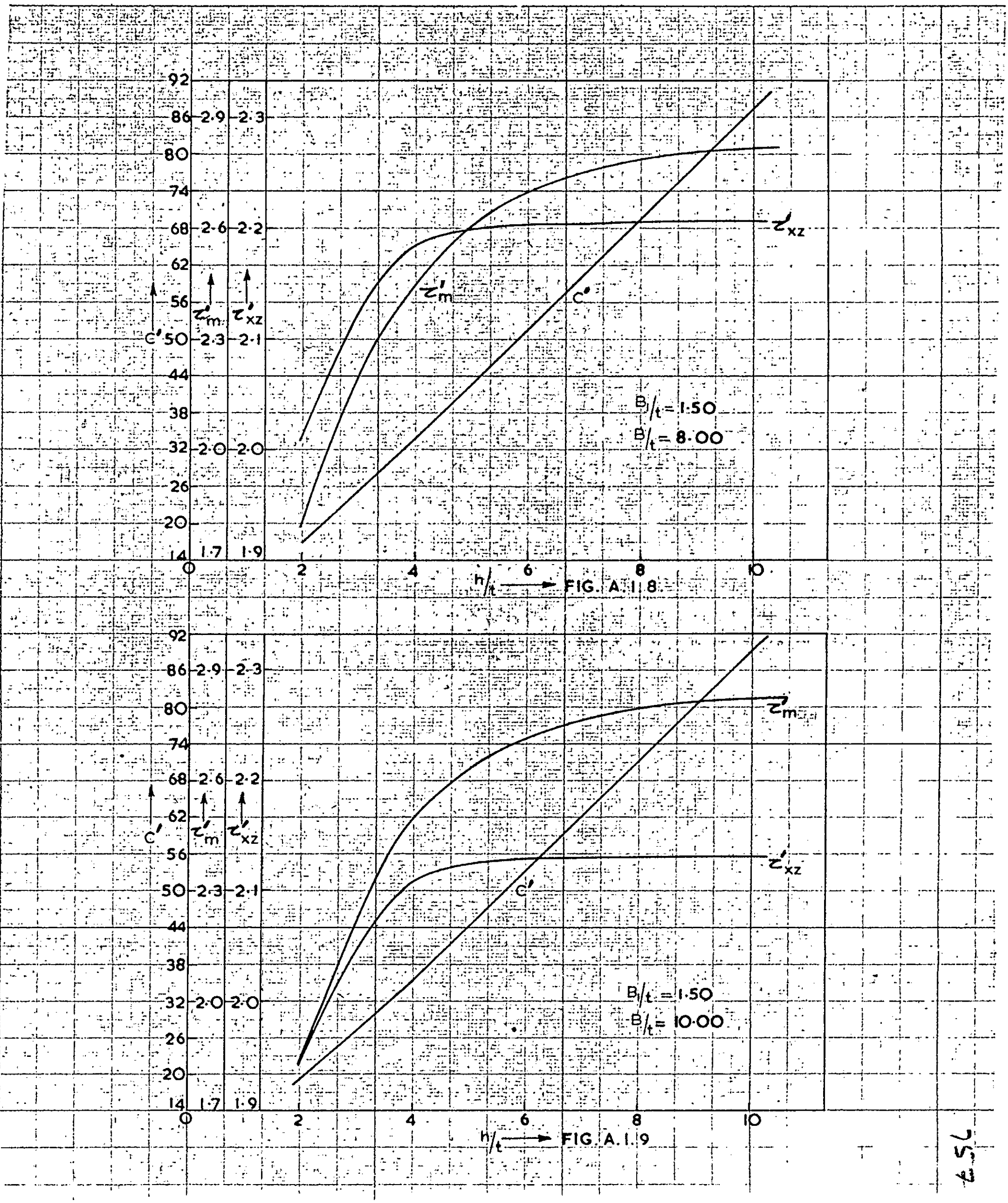


FIG. A.1.8

FIG. A.1.9

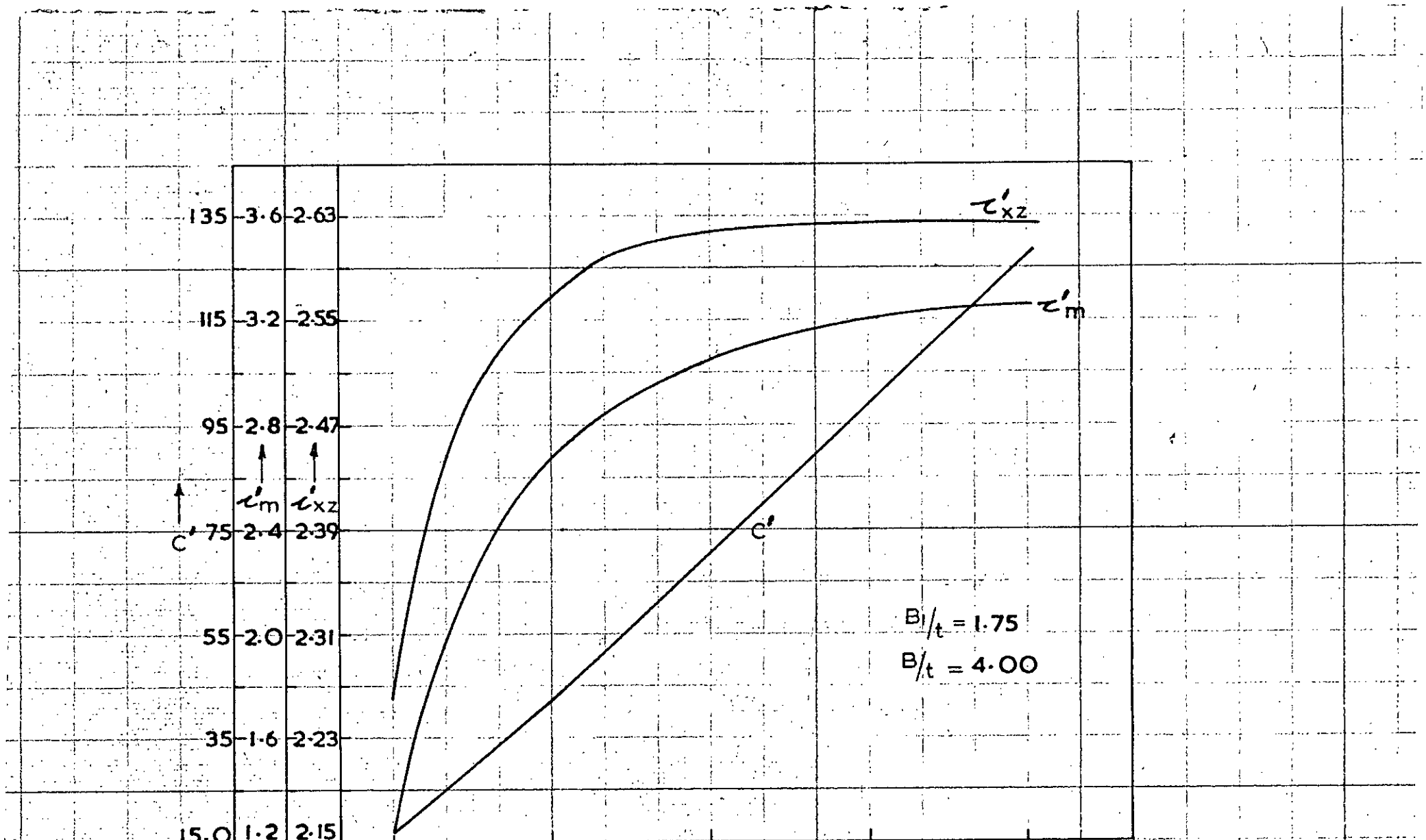


FIG. A.I.10.

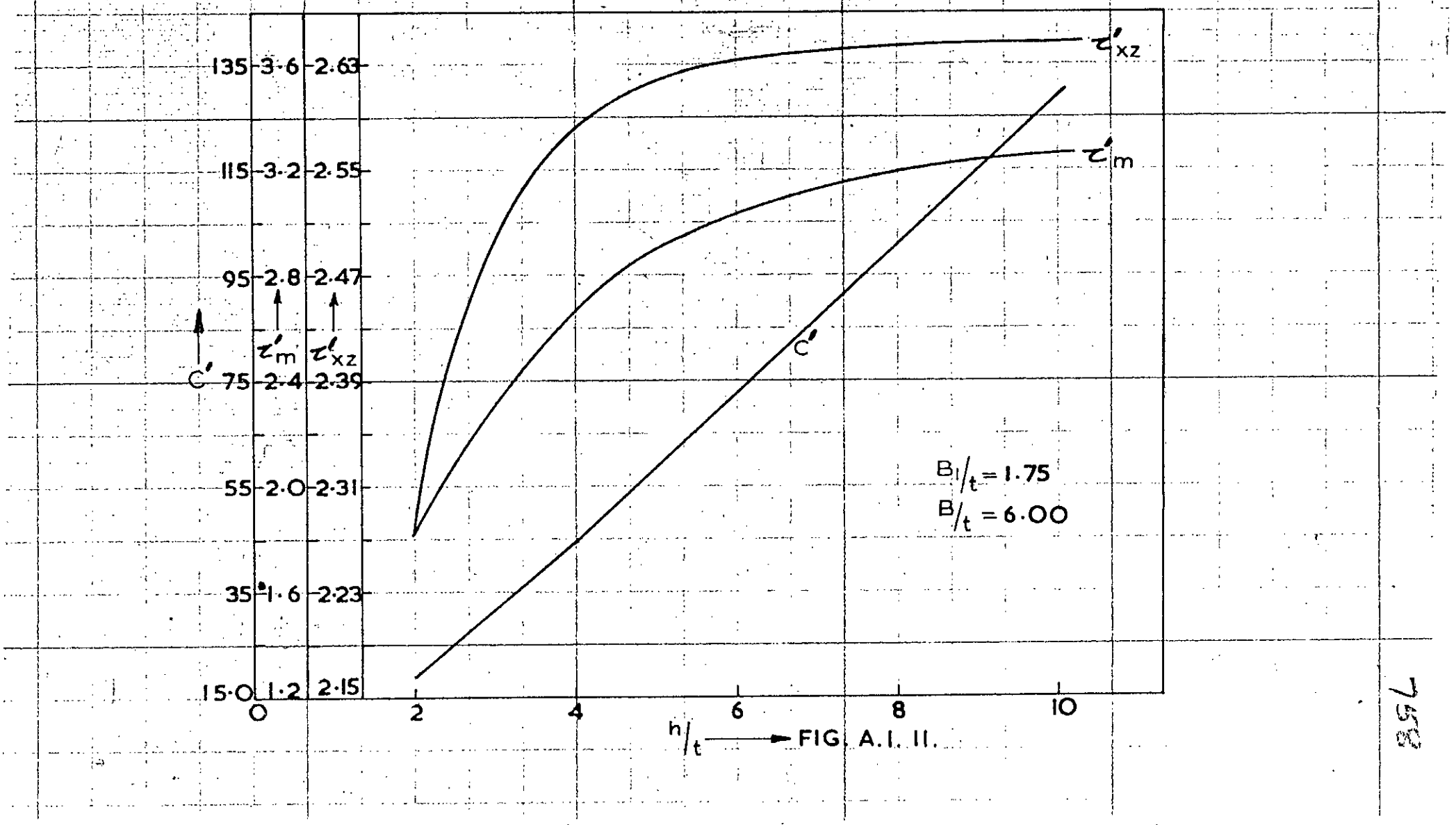


FIG. A.I. 11.

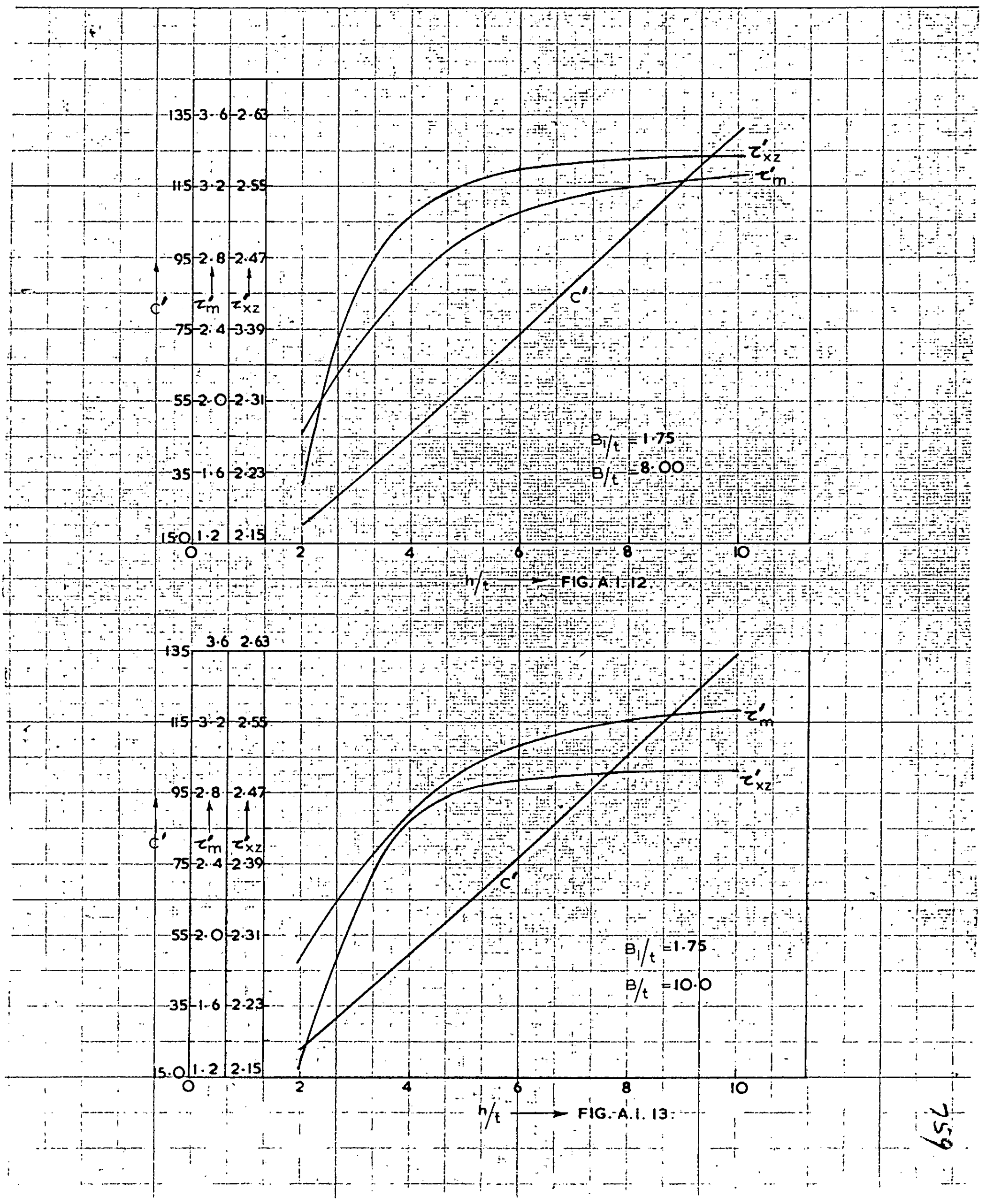


FIG. A.I. 12.

FIG. A.I. 13.

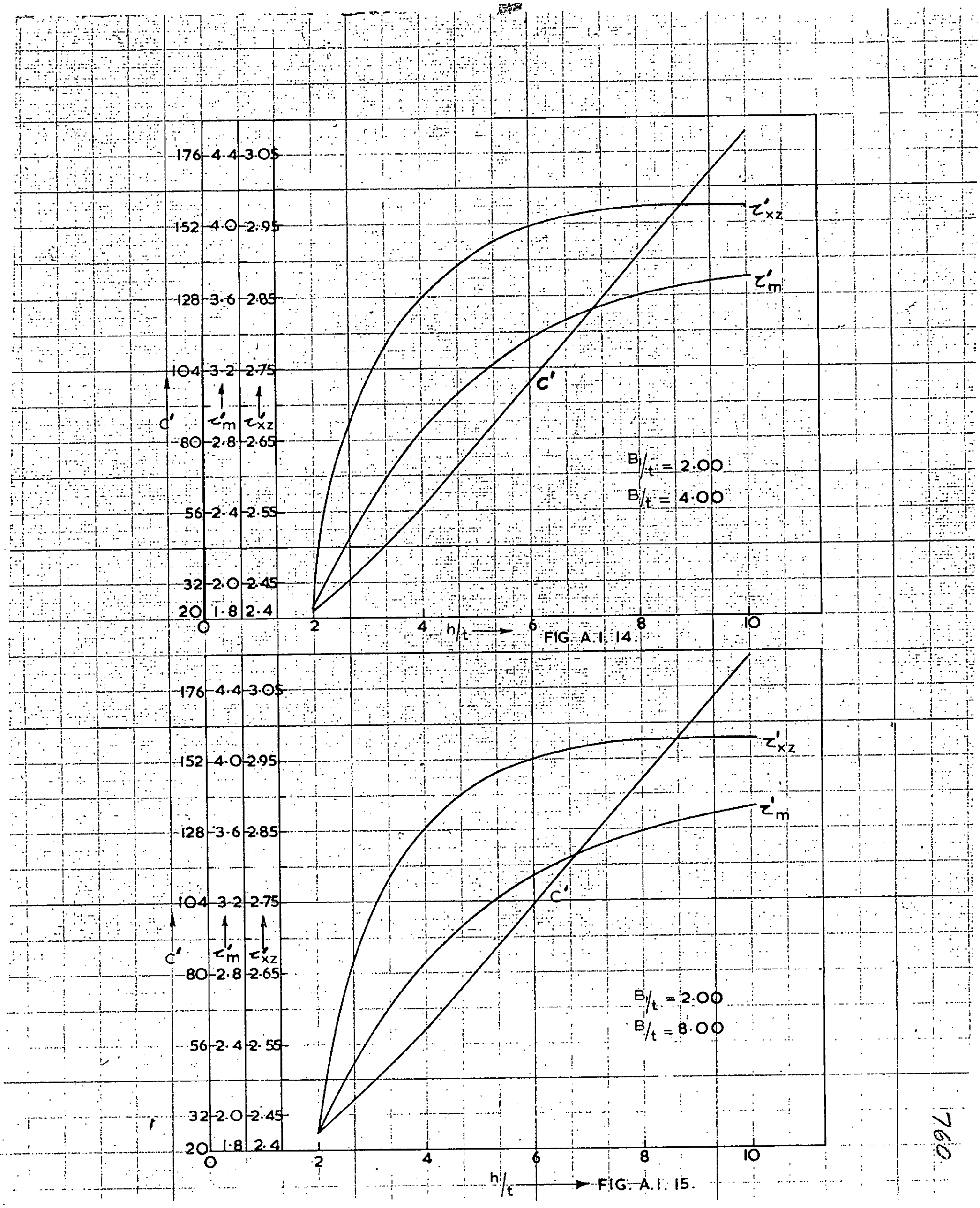
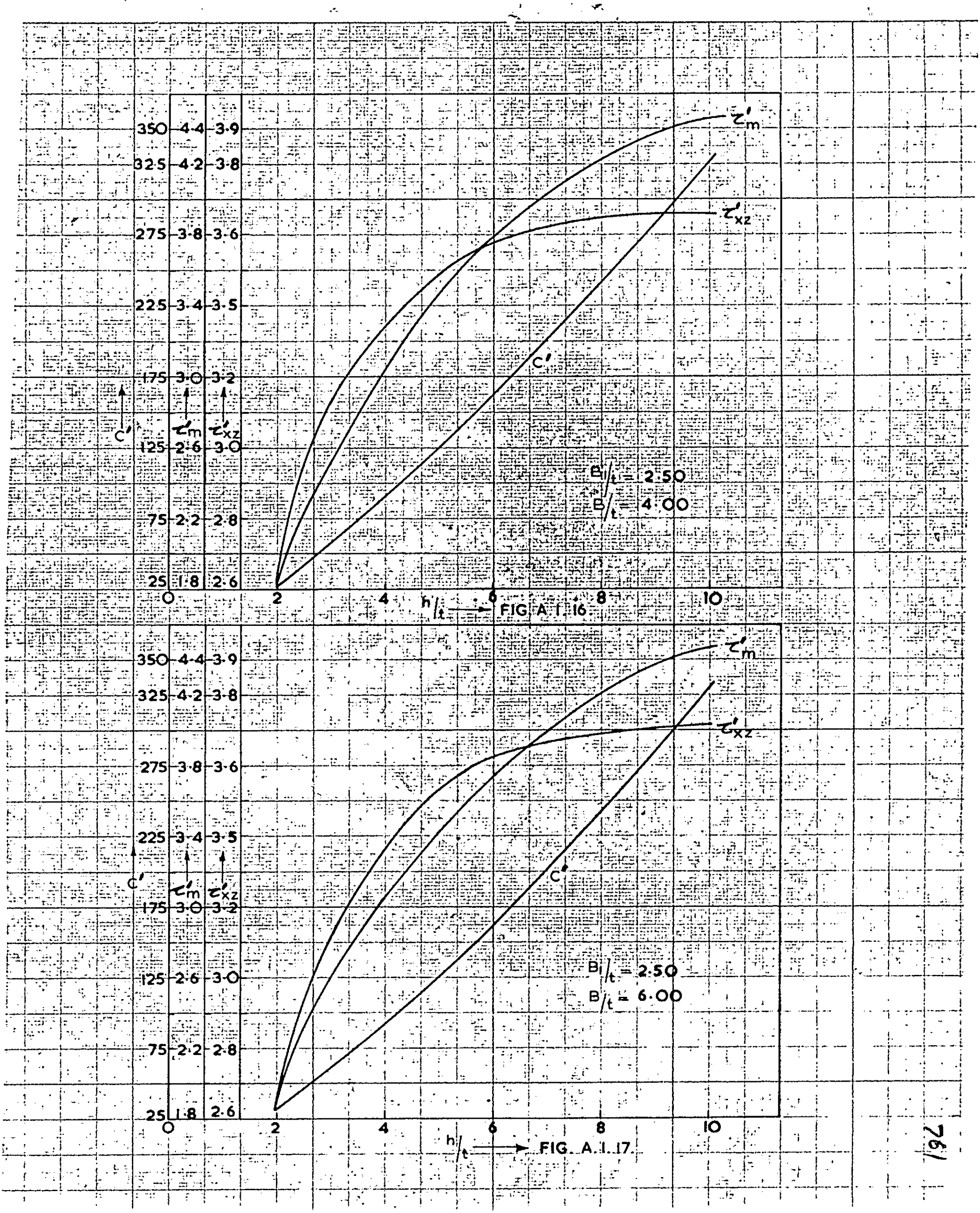
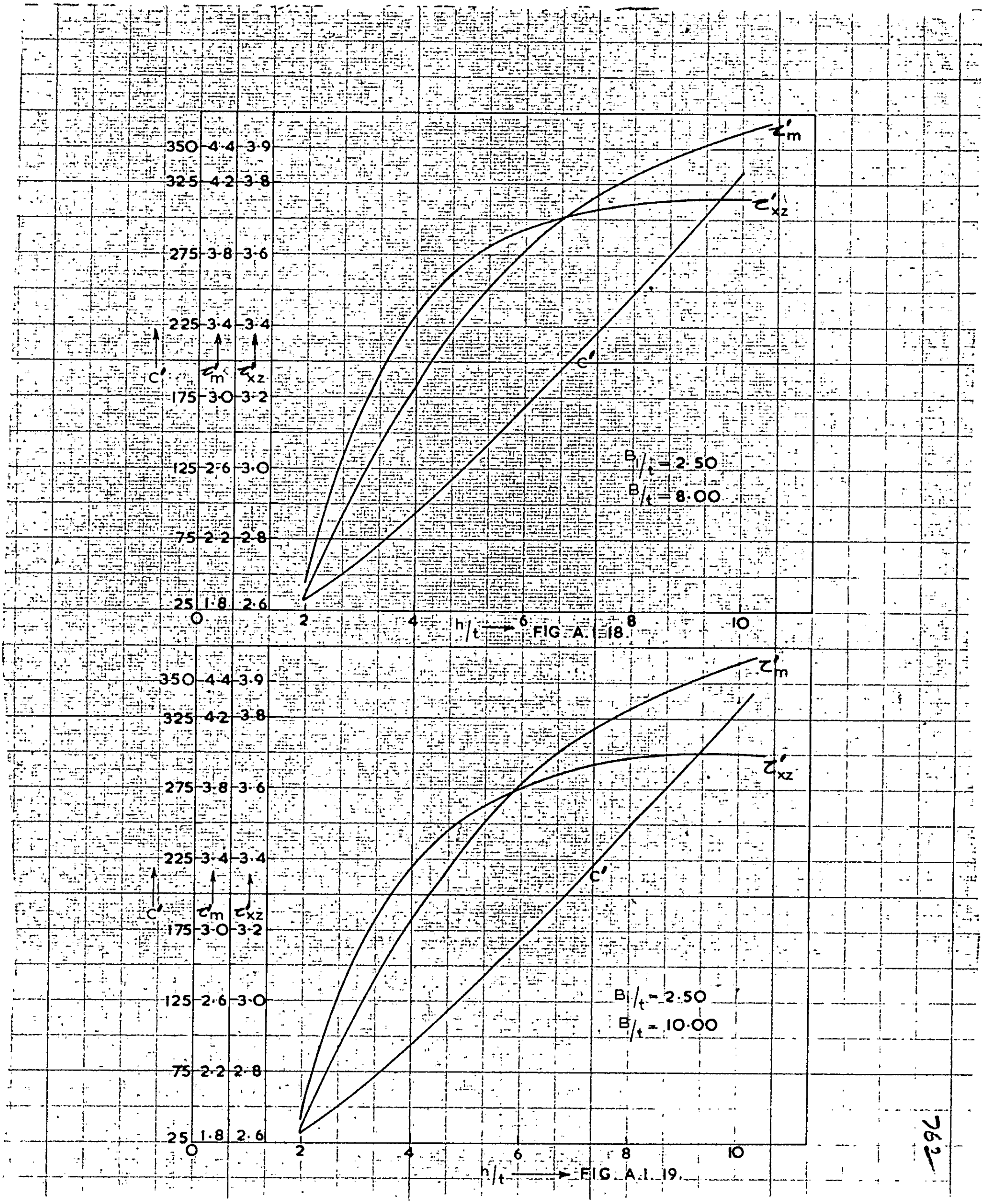


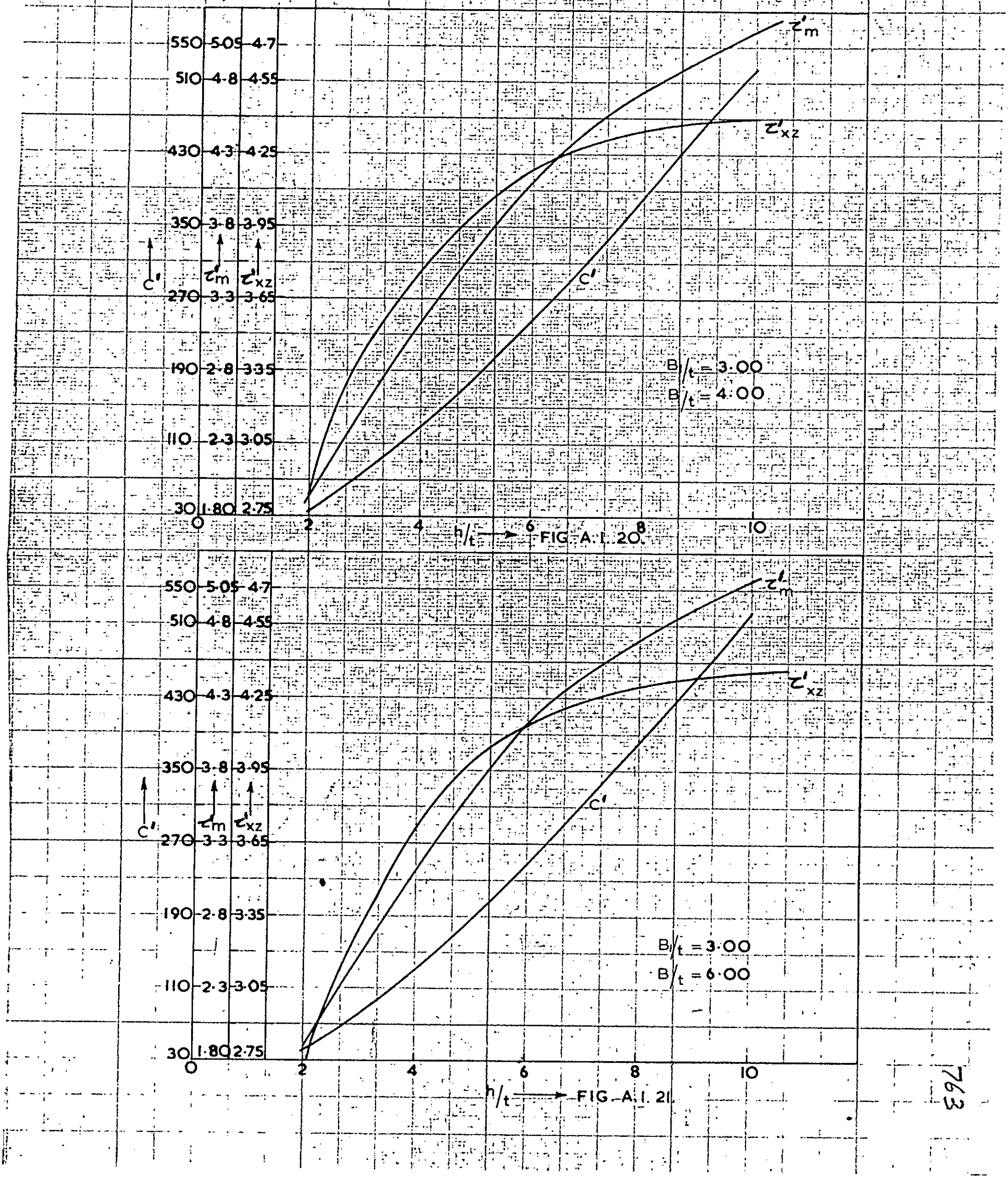
FIG. A.I. 14.

FIG. A.I. 15.

760


C'
350 4·4 3·9
325 4·2 3·8
275 3·8 3·6
225 3·4 3·5
175 3·0 3·2
125 2·6 3·0
75 2·2 2·8
25 1·8 2·6
τ'm
τ'xz
O
2
4
6
8
10
h/t
τ'm
τ'xz
C'
B/t = 2·50
B/t = 4·00
FIG. A.I.16.
C'
B/t = 2·50
B/t = 6·00
FIG. A.I.17.


350 4·4 3·9
325 4·2 3·8
275 3·8 3·6
225 3·4 3·4
175 3·0 3·2
125 2·6 3·0
75 2·2 2·8
25 1·8 2·6
C'
τ'_m
τ'_{xz}
τ'_m
τ'_{xz}
C'
$B_1/_t = 2.50$
$B/_t = 8.00$
0
2
4
6
8
10
h/t
FIG. A. I. 18.
350 4·4 3·9
325 4·2 3·8
275 3·8 3·6
225 3·4 3·4
175 3·0 3·2
125 2·6 3·0
75 2·2 2·8
25 1·8 2·6
C'
τ'_m
τ'_{xz}
τ'_m
τ'_{xz}
C'
$B_1/_t = 2.50$
$B/_t = 10.00$
0
2
4
6
8
10
h/t
FIG. A.I. 19.


550 5·05 4·7
510 4·8 4·55
430 4·3 4·25
350 3·8 3·95
270 3·3 3·65
190 2·8 3·35
110 2·3 3·05
30 1·80 2·75
C'
τ'm
τ'xz
0
2
4
6
8
10
h/t
B/t = 3·00
B/t = 4·00
FIG A.I. 20
B/t = 3·00
B/t = 6·00
FIG A.I. 21

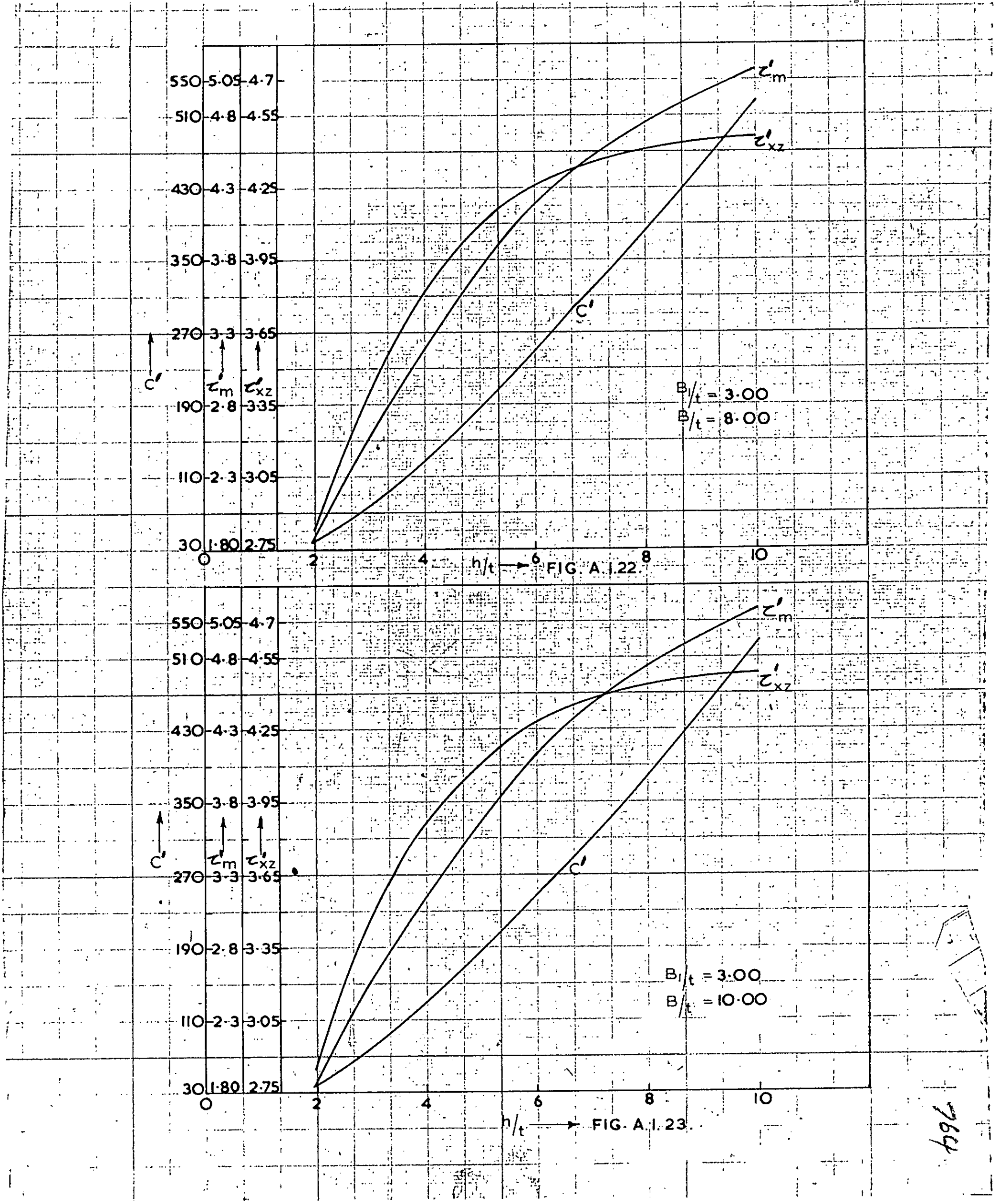


FIG. A.1.22.

FIG. A.1.23.

766 765

APPENDIX 2

THEORIES OF FAILURE OF CONCRETE UNDER COMPRESSIVE-TENSILE STRESSES

It is well known that when a material is subjected to two or more stresses, its strength under combined stresses is not entirely determined by its uniaxial strength under any one of the stresses. Concrete is a very peculiar material showing different behaviour under the action of compressive and tensile stresses. In all the problems discussed in this thesis, the concrete element at the critical point for failure is subjected to a combination of compressive normal stress and shear stress. The interaction of these stresses will yield two principal stresses which are compressive-tensile in nature. The case of simple shear is a special case of this general combination in which normal compressive stress is zero. So in order to determine the strength of the critical concrete element, it is essential to know the law or criterion governing the failure of concrete under compressive-tensile stresses. A brief review of different failure criteria suggested by various investigators is given below. A detailed version of them can be seen elsewhere[33,34].

1. Maximum Stress Theory[62]

The oldest and simplest of the principal macroscopic theories of failure is the maximum stress theory which is

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1. Maximum Stress Theory[62]

The oldest and simplest of the principal macroscopic theories of failure is the maximum stress theory which is

so known as Rankine's Theory. This theory assumes that the terial will fail under any condition of loading if one of the incipal stresses at any point reaches the value of the limiting ress as determined by the simple tension or compression test the material, regardless of what the stresses may be on other anes through the point. That is,

$$\sigma_2 = f'_t \quad \text{or} \quad \sigma_1 = f'_c \qquad \ldots \text{A2.1}$$

ichever causes failure earlier, where σ_1 and σ_2 are compressive d tensile principal stresses, respectively. In the presence of normal compressive stress f and a shear stress τ we have

$$\sigma_1 = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + \tau^2} \qquad \text{(compressive)}$$

and

$$\sigma_2 = \frac{f}{2} - \sqrt{\left(\frac{f}{2}\right)^2 + \tau^2} \qquad \text{(tensile)} \qquad \ldots \text{A2.2}$$

bstituting the values of σ_1 and σ_2 from Eqn. A2.1 in Eqn. A2.2 can get the equations for the interaction curve for f/f'_c and $/f'_t$ as given below:

$$\frac{\tau}{f'_t} = \sqrt{1 + f/f'_t}$$

$$\frac{\tau}{f'_t} = \frac{f'_c}{f'_t}\sqrt{1 - f/f'_c} \qquad \ldots \text{A2.3}$$

where

τ - shear stress

f - compressive normal stress

As per Rankine's Theory, the value of optimum compression (i.e. peak of the interaction curve) varies from 88 percent to 92 percent of f'_c when the ratio f'_c/f'_t varies between 8 and 12. On comparison with test results, Zia[37] found that this theory forms an upper bound to the test results. The over-estimation of capacity of beams by Rankine's Theory is due to the fact that the influence of one principal stress on the other is ignored in the failure condition.

2. Internal Friction Theory[63]

This theory, formulated by Coulomb, assumes that failure of a material is caused by a sliding action along a certain plane within the material. The resistance of this sliding is composed of a constant shearing strength, similar to cohesion, and an internal friction which is proportional to the normal pressure on the sliding plane. When the shearing stress overcomes this combined resistance, failure occurs. That is,

$$\tau = f \tan\theta + \text{constant} \qquad \ldots A2.4$$

where θ is the angle of internal friction. The equation for the interaction curve can be written as follows: