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# Proposal for a numerical definition of standard depths

Coloration Technology

Society of Dyers and Colourists

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Surfaces of uniform visual depth were mapped in $L^*a^*b^*$ colour space, at six levels of depth, from visual assessments made by a panel of professional colourists employed in the dye manufacturing industry. By interpolation, the surfaces of uniform visual depth at depths corresponding to the average of the ISO standard depths were determined. The algorithms developed enable the determination of the $L^*$ value that a colour of any hue or chroma should have in order for it to correspond to one of the standard depths.

## Introduction

The standard depths that are used for assessing colour fastness properties were originally developed in the 1920s by German and Swiss dye manufacturers and are known as the Hilfstypen samples. Standard depths (SDs) are used principally by dye manufacturers and sellers for assessing all types of fastness in order to ensure that, as far as possible, all dyes are tested at the same depth regardless of shade. In this way, a meaningful comparison of candidate dyes in a formulation can be made. What has become known as 1/1 SD was defined by a series of 18 shades dyed on wool fabric. The shades represented various hues and were selected as being of the same visual depth. Other series of samples were produced at the ratios, *inter alia*, 2/1 (more dye) and 1/3, 1/6, 1/12 and 1/25 (less dye). In ISO 105 (Part A01: General Principles of Fastness Testing) [1], it is stipulated that dye manufacturers should publish fastness data obtained by testing 1/1 SD specimens wherever possible, supplemented by data for one or more of the other ratios as required.

These SD samples are not perfect. Most of the formulae that have been developed to quantify depth show wide variations in value for the samples of any given series [1]. For example, for the 1/1 SD samples, the values for depth computed using the Integ formula [2] vary between 14.86 and 27.87 [3], a factor of almost 2. Values for depth computed using the formula developed by Sato *et al.* [4] are generally more consistent, and with the exception of samples 1 and 2 ranging from 69.9 to 79.1, varying by a factor of only 1.1. However, samples 1 and 2 exhibit a considerably greater deviation from the average (Figure 1). It should be noted that the variations in the depth values will be caused by inaccuracies in the formulae, and additionally by inconsistencies in the samples themselves.

The visual process of deciding whether a given colour is of a particular SD involves comparing it with the sample that is nearest in hue to it. Given the error that such a visual decision is likely to have, an instrumental method of deciding whether a sample corresponds to an SD is required. In 1976, the ISO requested the development of an algorithm that would indicate the $L^*$ value that a colour of any $a^*b^*$ coordinates should possess if it is at 1/1 SD. The condition was applied that the algorithm should give an error in $L^*$ of not greater than 10% of the $L^*$ value of the Hilfstypen samples. Such a formula was developed by Christ [6]. This formula has been incorporated into ISO 105-A06, a method 'for the instrumental determination of 1/1 SD,' but its error is significantly more than 10%. Hawkyard and Kelly [7] have proposed a method for defining SD that is based on an equation that relates the lightness ($L^*$) of dyeings, applied at increasing depths of shade, to maximum chroma ($C^*$) reached. However, this method has not been substantiated by visual assessment and does not allow for

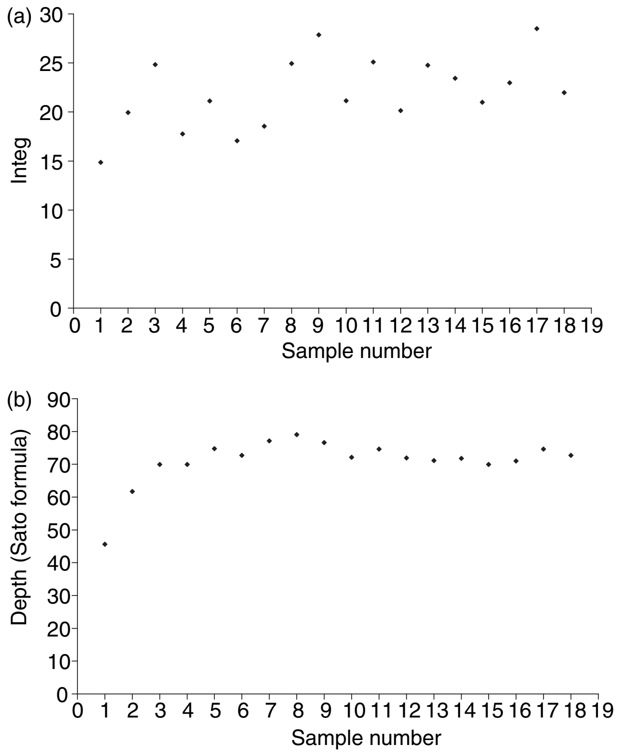


**Figure 1** Depth values for the 1/1 standard depth samples given by the (a) Integ formula and (b) Sato formula

  

the established variation of $L^*$ with $C^*$ at a given hue angle ($h°$), for colours of constant depth.

Although the Christ [6] formula predicts the $L^*$ values of the 1/1 SD series as well as any useful formula is likely to do so, the fact remains that the actual 1/1 SD samples are widely recognised as not being of uniform visual depth. In this study, surfaces of constant visual depth have been mapped throughout the $L^*a^*b^*$ colour space at six different depth levels. By interpolation, it is possible then to predict the surface for any other depth. Using this approach, surfaces of uniform depth have been predicted for the averages of the 2/1, 1/1, 1/3, 1/6, 1/12 and 1/25 SDs. It is suggested that the algorithms developed be used to define new sets of SDs. To allow the continued practice of visual assessment it will be necessary to generate corresponding physical standards.

## Experimental

### Methods

Surfaces of constant visual depth were mapped at six different depth levels having $L^*$ axis intercepts ($a^* = b^* = 0$) at $L^* = 79.23$, 68.32, 50.82, 34.14, 24.89 and 20.23, respectively. The method employed to map the surfaces through the $L^*a^*b^*$ space has been previously reported [8], where the precise details of the determination of the surface of intercept $L^* = 34.14$, the first surface to be defined, are described. As before, for each depth level, the lines of constant visual depth along each of eight hue directions, spaced at 45° intervals round the hue circle (starting at 0°) were determined.

Some minor variations were made to the procedure for the new depth surfaces. First, it was found that steps of 5 CIELAB units of chroma along any one hue direction were large and gave the assessors some difficulty in determining which samples of a depth range had the same visual depth as the standard, which was at a point 5 CIELAB units lower in chroma. This situation was the most acute at the two palest depths. Therefore, for these two depths, the procedure was modified so that the mapping of samples of constant visual depth along each hue direction was made at increments of 2.5 CIELAB units in chroma, moving outwards from the neutral point. The actual number of chroma points at which assessments were made varied with the depth and with the hue angle, but are shown by the number of datum points in Figure 2. Secondly, for the first surface to be mapped [8], dyeings were made round the hue circle, between the points of maximum chroma with the dyes used, to check for the consistency of the assessments. This exercise was repeated for just one of the new depths ($L^* = 50.82$ at $a^* = b^* = 0$), but not for the remaining ones, due to the massive extra amount of work required. Thirdly, the dyeings for the two heaviest depths were produced on woven wool fabric, using acid levelling dyes. The reason for changing to wool was to aid in the production of dark, saturated shades that would be difficult to obtain with reactive dyes on cotton. Wool was not used for paler depths because its natural colour would restrict the range of shades that could be produced. It was found that at heavy depths, the depth ranges of samples at 90, 95, 100, 105 and 110% of the recipe gave only very small visual differences. Accordingly, these depth ranges were prepared at 80, 90, 100, 110 and 120% of the recipe.

Of the four assessors who carried out the assessments at the original depth studied [8], one had to withdraw, but was replaced by a colleague from the same company. The other three assessors were the same people who participated previously. As before, the samples were either sent to the observers by post or delivered in person by the authors and assessments conducted at the time of the visit. In all cases, none of the observers had any knowledge of the results of any of the others of the panel.

## Results and Discussion

The colorimetric coordinates of the samples, determined from the visual assessments as having the same visual depth, for each hue direction, for the different depth levels studied are represented graphically in Figure 2. As explained previously [8], it was desirable to ensure a smooth transition across opposite hue angles, so the graphs in Figure 2 were plotted in such a way that, for the curve fitting process, the $C^*$ values of one of the hue angles of each pair, in each case, were assigned negative values. The experimental points illustrated in Figure 2 show clear trends of (generally increasing) lightness value with increase in chroma, along a given hue direction. The rate of change of $L^*$ was different for the different hue angles and also at the different depth levels. At the very high levels of depth, there was very little change in lightness with chroma, whereas at low depths, lightness varied considerably with chroma, especially in the red–orange–yellow hue directions.

### Mapping the lines of constant visual depth at each hue direction and depth level

As with the analyses carried out for the original surface, the relationships between $L^*$ and $C^*$ along each hue direction were established first. As it was desirable to maintain a continuity of the profile of the curves across opposite hue directions (0–180°, 45–225°, 90–270° and 135–315°), the general conic formula (Eqn 1) was used as previously described [8] to define the curves.

$$k_1L^{*2} + k_2C^{*2} + k_3L^* + k_4L^*C^* + k_5C^* + k_6 = 0 \qquad (1)$$

This formula can be solved using Eqns 2 and 3:

$$L^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (2)$$

where

$$\begin{aligned} a &= k_1 \\ b &= k_3 + k_4C^* \\ c &= k_2C^{*2} + k_5C^* + k_6 \end{aligned} \qquad (3)$$

Only the positive value within the square-root term is required. In fitting the conic formula to the experimental data, the relationship was kept as simple as possible. First, the value of $k_3$ was set to −1.0 for all surfaces and all hue angles. Secondly, it was found that, for any given surface, Eqn 1 could be made to fit the experimental results satisfactorily for all eight hue directions by

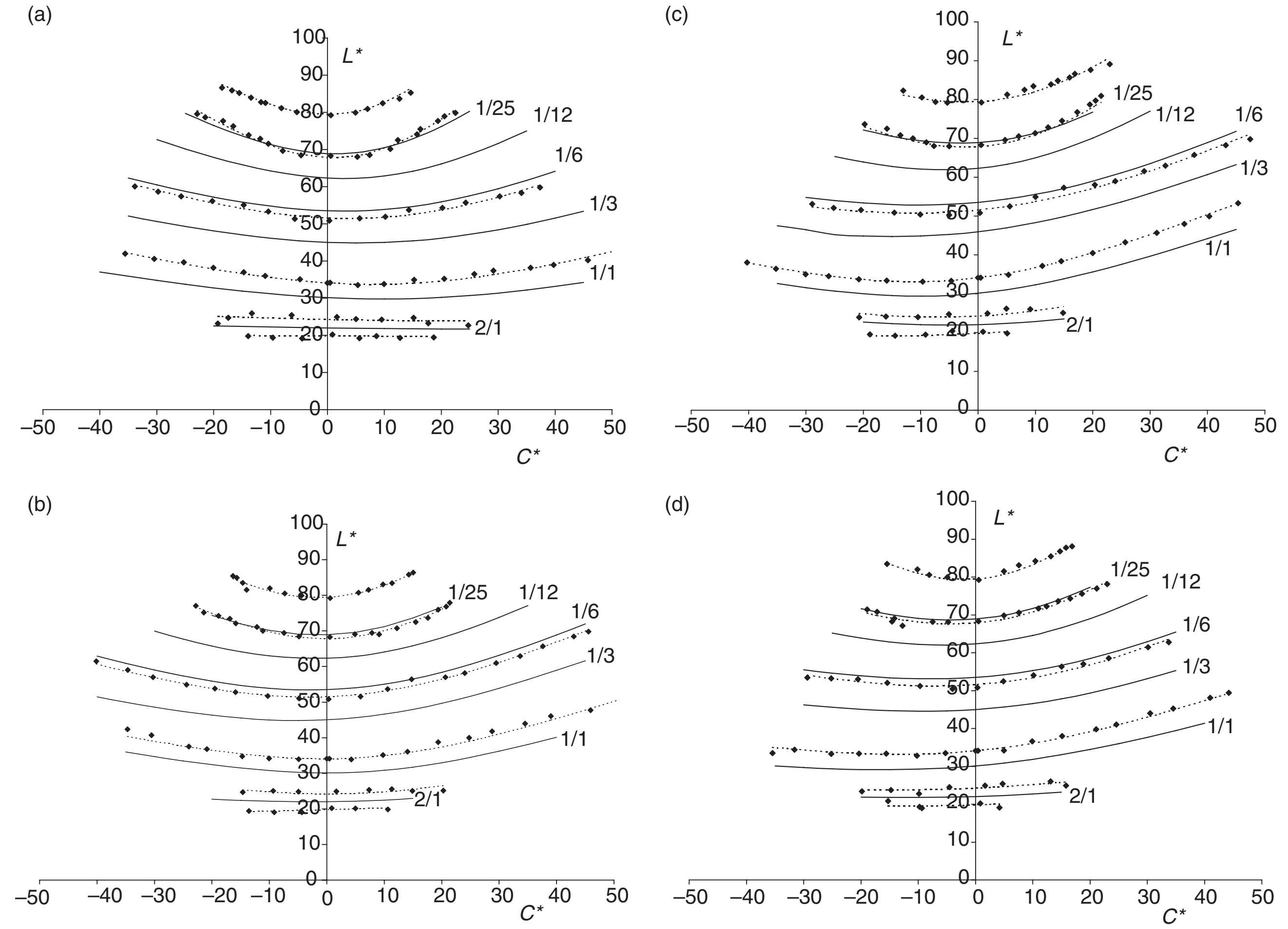


**Figure 2** Values of lightness ($L^*$) plotted against chroma ($C^*$) across opposite hue directions, for samples of constant visual depth: (a) 0–180°, (b) 45–225°, (c) 90–270° and (d) 135–315°; also shown are the proposed new standard depth lines

keeping the values of the constants $k_1$ and $k_6$ invariant, and varying only the values of the constants $k_2$, $k_4$ and $k_5$ individually. In doing so, care was taken to ensure that, for a given depth, at $C^* = 0$, all predicted $L^*$ values of the hue directions were equal. In meeting these criteria, the values of the constants did not always give the minimum least squares fit to the experimental data; a small degree of inaccuracy had to be accepted.

In fitting the conic formula across the opposite hue directions (to ensure continuity of curve profile), it was necessary to make the $C^*$ values of one of the hue directions of a pair negative. This is of course meaningless in practical terms, but by reversing the sign of the values of the constants $k_4$ and $k_5$ the curves are made to fit in the opposite direction (i.e. as a mirror image). In this way, the values of the six constants for the individual hue directions can be obtained, and the final values are presented in Table 1. The profiles of the curves relating $L^*$ to $C^*$ are illustrated in Figure 2.

**Determining the lines of constant visual depth at the Integ levels corresponding to the standard depths**

Having determined the lines of constant visual depth at the eight hue angles at each of the various levels of depth, it was necessary to determine the lines corresponding to the several SD samples. For this purpose, two sets of Hilfstypen SD samples, defined by DIN 54000, were sourced. The first set was provided by the Society of Dyers and Colourists [9] and the second was purchased from DEK in Germany [10]. Each of these sets probably dates back to the 1950s. The reflectance values of the samples were measured, using a Datacolor Spectraflash SF600 spectrophotometer (Datacolor International, Switzerland), set with the large area aperture, specular reflection and the ultraviolet component of irradiation included. However, it should be noted that the samples are of wool serge, approximately $5 \times 4$ cm, just one layer of fabric mounted on the card of the booklet. This means that the reflectance measurements obtained will not be true measurements of the fabric alone and that the colour of the cardboard background (an off-white colour) will influence the readings obtained. It is likely that this influence will be the greatest on the heavier depths, especially the 2/1 and 1/1 samples. Nevertheless, the same situation applies to visual assessment of the samples.

In the absence of any other information about the samples, the reflectance data obtained were used to determine the Integ [2] and CIE $L^*a^*b^*C^*h$ values of the samples. All colorimetric data used in this study were derived using the D65, 10° standard observer data. The colour-difference values determined from the $L^*a^*b^*$ values (CIEDE2000) between the corresponding samples of each set were found to be very small (Table 2).

**Table 1** Optimised constants $k_1$–$k_6$ for the conic formula fitted to experimental data

| $L^*$ at $a^* = b^* = 0$ | Hue direction | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ |
|---|---|---|---|---|---|---|---|
| 79.23 | 0 | 0.0110 | −0.0230 | −1 | −0.0078 | 0.6360 | 10.00 |
| | 45 | 0.0110 | −0.0200 | −1 | −0.0018 | 0.0085 | 10.00 |
| | 90 | 0.0110 | −0.0128 | −1 | −0.0070 | 0.5000 | 10.00 |
| | 135 | 0.0110 | −0.0200 | −1 | −0.0050 | 0.3200 | 10.00 |
| | 180 | 0.0110 | −0.0230 | −1 | 0.0078 | −0.6360 | 10.00 |
| | 225 | 0.0110 | −0.0200 | −1 | 0.0018 | −0.0085 | 10.00 |
| | 270 | 0.0110 | −0.0128 | −1 | 0.0070 | −0.5000 | 10.00 |
| | 315 | 0.0110 | −0.0200 | −1 | 0.0050 | −0.3200 | 10.00 |
| 68.32 | 0 | 0.0130 | −0.0230 | −1 | −0.0057 | 0.4500 | 8.00 |
| | 45 | 0.0130 | −0.0165 | −1 | −0.0023 | 0.1470 | 8.00 |
| | 90 | 0.0130 | −0.0189 | −1 | −0.0066 | 0.3800 | 8.00 |
| | 135 | 0.0130 | −0.0125 | −1 | −0.0060 | 0.3100 | 8.00 |
| | 180 | 0.0130 | −0.0117 | −1 | 0.0057 | −0.4500 | 8.00 |
| | 225 | 0.0130 | −0.0165 | −1 | 0.0023 | −0.1470 | 8.00 |
| | 270 | 0.0130 | −0.0189 | −1 | 0.0066 | −0.3800 | 8.00 |
| | 315 | 0.0130 | −0.0117 | −1 | 0.0060 | −0.3100 | 8.00 |
| 50.82 | 0 | 0.0174 | −0.0067 | −1 | −0.0029 | 0.2000 | 5.30 |
| | 45 | 0.0174 | −0.0075 | −1 | −0.0039 | 0.1550 | 5.30 |
| | 90 | 0.0174 | −0.0049 | −1 | −0.0061 | 0.1990 | 5.30 |
| | 135 | 0.0174 | −0.0058 | −1 | −0.0068 | 0.2700 | 5.30 |
| | 180 | 0.0174 | −0.0067 | −1 | 0.0029 | −0.2000 | 5.30 |
| | 225 | 0.0174 | −0.0075 | −1 | 0.0039 | −0.1550 | 5.30 |
| | 270 | 0.0174 | −0.0049 | −1 | 0.0061 | −0.1990 | 5.30 |
| | 315 | 0.0174 | −0.0058 | −1 | 0.0068 | −0.2700 | 5.30 |
| 34.14 | 0 | 0.0260 | −0.0046 | −1 | −0.0011 | 0.1130 | 3.87 |
| | 45 | 0.0260 | −0.0060 | −1 | −0.0049 | 0.1600 | 3.87 |
| | 90 | 0.0260 | −0.0062 | −1 | −0.0059 | 0.0590 | 3.87 |
| | 135 | 0.0260 | −0.0039 | −1 | −0.0078 | 0.1560 | 3.87 |
| | 180 | 0.0260 | −0.0046 | −1 | 0.0011 | −0.1130 | 3.87 |
| | 225 | 0.0260 | −0.0060 | −1 | 0.0059 | −0.1600 | 3.87 |
| | 270 | 0.0260 | −0.0062 | −1 | 0.0065 | −0.0590 | 3.87 |
| | 315 | 0.0260 | −0.0039 | −1 | 0.0078 | −0.1560 | 3.87 |
| 24.89 | 0 | 0.0390 | −0.0005 | −1 | −0.0009 | 0.0500 | 1.30 |
| | 45 | 0.0390 | −0.0050 | −1 | −0.0064 | 0.1600 | 1.30 |
| | 90 | 0.0390 | −0.0050 | −1 | −0.0050 | 0.0500 | 1.30 |
| | 135 | 0.0390 | −0.0016 | −1 | −0.0078 | 0.1300 | 1.30 |
| | 180 | 0.0390 | −0.0005 | −1 | 0.0009 | −0.0500 | 1.30 |
| | 225 | 0.0390 | −0.0050 | −1 | 0.0064 | −0.1600 | 1.30 |
| | 270 | 0.0390 | −0.0050 | −1 | 0.0050 | −0.0500 | 1.30 |
| | 315 | 0.0390 | −0.0016 | −1 | 0.0078 | −0.1300 | 1.30 |
| 20.23 | 0 | 0.0480 | −0.0001 | −1 | −0.0007 | 0.0200 | 0.98 |
| | 45 | 0.0480 | −0.0001 | −1 | −0.0100 | 0.1600 | 0.98 |
| | 90 | 0.0480 | −0.0050 | −1 | −0.0050 | 0.0100 | 0.98 |
| | 135 | 0.0480 | −0.0016 | −1 | −0.0078 | 0.1100 | 0.98 |
| | 180 | 0.0480 | −0.0001 | −1 | 0.0007 | −0.0200 | 0.98 |
| | 225 | 0.0480 | −0.0001 | −1 | 0.0100 | −0.1600 | 0.98 |
| | 270 | 0.0480 | −0.0050 | −1 | 0.0050 | −0.0100 | 0.98 |
| | 315 | 0.0480 | −0.0016 | −1 | 0.0078 | −0.1100 | 0.98 |

**Table 2** Colour-difference values between the two sets of Hilfstypen samples (CIEDE2000 formula)

| Standard depth | $\Delta E_{av}$ | $\Delta E$ (range) |
|---|---|---|
| 2/1 | 0.48 | 0.06–1.11 |
| 1/1 | 0.55 | 0.13–0.96 |
| 1/3 | 0.50 | 0.21–0.87 |
| 1/6 | 0.44 | 0.13–1.05 |
| 1/12 | 0.42 | 0.17–0.71 |
| 1/25 | 0.56 | 0.29–1.06 |

**Table 3** Average Integ values of two sets of Hilfstypen samples

| Colour number | 2/1 SD | 1/1 SD | 1/3 SD | 1/6 SD | 1/12 SD | 1/25 SD |
|---|---|---|---|---|---|---|
| 1 | 21.23 | 14.46 | 4.16 | 1.99 | 1.06 | 0.80 |
| 2 | 34.14 | 20.17 | 6.78 | 3.17 | 1.83 | 1.12 |
| 3 | 42.67 | 25.09 | 7.88 | 4.54 | 2.88 | 1.52 |
| 4 | 37.31 | 17.81 | 7.88 | 4.59 | 2.58 | 1.34 |
| 5 | 37.51 | 21.46 | 8.02 | 4.71 | 2.56 | |
| 6 | 44.58 | 17.62 | 6.71 | 3.65 | 2.12 | |
| 7 | 33.82 | 19.22 | 7.00 | 4.26 | 2.48 | |
| 8 | 43.99 | 25.63 | 9.66 | 5.92 | 2.92 | 1.88 |
| 9 | 46.55 | 28.74 | 9.90 | 5.48 | 2.88 | 2.04 |
| 10 | 37.45 | 21.78 | 7.86 | 4.18 | 2.25 | 1.50 |
| 11 | 43.02 | 25.55 | 7.92 | 3.88 | 2.26 | 1.43 |
| 12 | 36.52 | 21.18 | 6.64 | 3.52 | 1.80 | |
| 13 | 39.89 | 24.81 | 7.75 | 4.02 | 2.03 | |
| 14 | 44.18 | 23.52 | 8.74 | 5.17 | 2.89 | |
| 15 | 40.61 | 20.96 | 8.84 | 5.30 | 2.55 | 1.59 |
| 16 | 41.49 | 23.16 | 8.03 | 4.77 | 2.37 | 1.22 |
| 17 | 53.87 | 29.44 | 11.12 | 5.92 | 3.08 | 1.39 |
| 18 | 39.49 | 22.79 | 10.52 | 5.89 | 3.39 | 2.25 |
| Average | 39.90 | 22.41 | 8.08 | 4.50 | 2.44 | 1.51 |

The Hilfstypen SD samples are known to be visually inconsistent and some measure of depth was required on which to base the comparison. It was therefore felt appropriate to use the average Integ values of the corresponding samples of each of the two sets for the development of the SD lines, and these values are shown in Table 3. The values of the conic constants for the average Integ value for each SD series (2/1, 1/1, etc.) were determined by interpolation between those obtained experimentally at the arbitrary depths. The values of the smoothed conic constants obtained for each of the six SD levels are given in Table 4.

**Determining the surfaces of constant visual depth at each depth level corresponding to the standard depths**

In order to define completely the surfaces of constant visual depth for each SD level, it was necessary to establish the relationships between the conic constants, $k_1$–$k_6$, of the conic formula and hue angle. The general formula is shown in Eqn 4 where $i$ = 1–6.

$$k_i = l_0 + l_1\cos(h + m_1) + l_2\cos(2h + m_2) + l_3\cos(3h + m_3) + l_4\cos(4h + m_4) + l_5\cos(5h + m_5) \quad (4)$$

The values of the constants $l_0$–$l_5$ and $m_1$–$m_5$ were determined for each of the six SD surfaces according to the procedure previously explained [8]. Inspection of the values of the conic constants along the eight hue directions studied (Table 4) shows that only the constants $k_2$, $k_4$ and $k_5$ vary with hue angle. For the determination of the values of the $k_2$, $k_4$ and $k_5$ constants at a particular hue angle, not all terms of Eqn 4 were required in every case and the final formulae for their determination are given in Table 5 for each of the SDs. The constants of the cosine formula for a given SD effectively define the surface for that SD, because they define the way in which the values of the conic constants vary with hue angle.

**Table 4** Values of the conic constants for the various standard depths at the hue angles shown

| Standard depth | Hue direction | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ |
|---|---|---|---|---|---|---|---|
| 2/1 | 0 | 0.0430 | −0.0003 | −1 | −0.0008 | 0.036 | 1.151 |
| | 45 | 0.0430 | −0.0027 | −1 | −0.0078 | 0.160 | 1.151 |
| | 90 | 0.0430 | −0.0035 | −1 | −0.0050 | 0.075 | 1.151 |
| | 135 | 0.0430 | −0.0016 | −1 | −0.0078 | 0.132 | 1.151 |
| | 180 | 0.0430 | −0.0003 | −1 | 0.0008 | −0.036 | 1.151 |
| | 225 | 0.0430 | −0.0027 | −1 | 0.0078 | −0.160 | 1.151 |
| | 270 | 0.0430 | −0.0035 | −1 | 0.0050 | −0.075 | 1.151 |
| | 315 | 0.0430 | −0.0016 | −1 | 0.0078 | −0.132 | 1.151 |
| 1/1 | 0 | 0.03 | −0.0031 | −1 | −0.0010 | 0.089 | 2.907 |
| | 45 | 0.03 | −0.0056 | −1 | −0.0053 | 0.160 | 2.907 |
| | 90 | 0.03 | −0.0057 | −1 | −0.0060 | 0.060 | 2.907 |
| | 135 | 0.03 | −0.0030 | −1 | −0.0086 | 0.154 | 2.907 |
| | 180 | 0.03 | −0.0031 | −1 | 0.0010 | −0.089 | 2.907 |
| | 225 | 0.03 | −0.0056 | −1 | 0.0053 | −0.160 | 2.907 |
| | 270 | 0.03 | −0.0057 | −1 | 0.0060 | −0.060 | 2.907 |
| | 315 | 0.03 | −0.0030 | −1 | 0.0086 | −0.154 | 2.907 |
| 1/3 | 0 | 0.0198 | −0.0045 | −1 | −0.0024 | 0.155 | 4.892 |
| | 45 | 0.0198 | −0.0060 | −1 | −0.0048 | 0.155 | 4.892 |
| | 90 | 0.0198 | −0.0053 | −1 | −0.0060 | 0.159 | 4.892 |
| | 135 | 0.0198 | −0.0040 | −1 | −0.0070 | 0.237 | 4.892 |
| | 180 | 0.0198 | −0.0045 | −1 | 0.0024 | −0.155 | 4.892 |
| | 225 | 0.0198 | −0.0060 | −1 | 0.0048 | −0.155 | 4.892 |
| | 270 | 0.0198 | −0.0053 | −1 | 0.0060 | −0.159 | 4.892 |
| | 315 | 0.0198 | −0.0040 | −1 | 0.0070 | −0.237 | 4.892 |
| 1/6 | 0 | 0.0167 | −0.0065 | −1 | −0.0035 | 0.227 | 5.691 |
| | 45 | 0.0167 | −0.0075 | −1 | −0.0037 | 0.154 | 5.691 |
| | 90 | 0.0167 | −0.0050 | −1 | −0.0062 | 0.225 | 5.691 |
| | 135 | 0.0167 | −0.0050 | −1 | −0.0067 | 0.275 | 5.691 |
| | 180 | 0.0167 | −0.0065 | −1 | 0.0035 | −0.227 | 5.691 |
| | 225 | 0.0167 | −0.0075 | −1 | 0.0037 | −0.154 | 5.691 |
| | 270 | 0.0167 | −0.0050 | −1 | 0.0062 | −0.225 | 5.691 |
| | 315 | 0.0167 | −0.0050 | −1 | 0.0067 | −0.275 | 5.691 |
| 1/12 | 0 | 0.0142 | −0.0100 | −1 | −0.0044 | 0.335 | 7.166 |
| | 45 | 0.0142 | −0.0095 | −1 | −0.0030 | 0.149 | 7.166 |
| | 90 | 0.0142 | −0.0090 | −1 | −0.0070 | 0.329 | 7.166 |
| | 135 | 0.0142 | −0.0080 | −1 | −0.0062 | 0.298 | 7.166 |
| | 180 | 0.0142 | −0.0100 | −1 | 0.0044 | −0.335 | 7.166 |
| | 225 | 0.0142 | −0.0095 | −1 | 0.0030 | −0.149 | 7.166 |
| | 270 | 0.0142 | −0.0090 | −1 | 0.0070 | −0.329 | 7.166 |
| | 315 | 0.0142 | −0.0080 | −1 | 0.0062 | −0.298 | 7.166 |
| 1/25 | 0 | 0.0128 | −0.0160 | −1 | −0.0052 | 0.404 | 8.136 |
| | 45 | 0.0128 | −0.0140 | −1 | −0.0023 | 0.120 | 8.136 |
| | 90 | 0.0128 | −0.0110 | −1 | −0.0066 | 0.388 | 8.136 |
| | 135 | 0.0128 | −0.0110 | −1 | −0.0059 | 0.311 | 8.136 |
| | 180 | 0.0128 | −0.0160 | −1 | 0.0052 | −0.404 | 8.136 |
| | 225 | 0.0128 | −0.0140 | −1 | 0.0023 | −0.120 | 8.136 |
| | 270 | 0.0128 | −0.0110 | −1 | 0.0066 | −0.388 | 8.136 |
| | 315 | 0.0128 | −0.0110 | −1 | 0.0059 | −0.311 | 8.136 |

Figure 3 illustrates the variation of $L^*$ with hue angle, at a constant chroma ($C^* = 20$), for each of the six SDs.

### Determining whether a particular colour is of a selected standard depth

The process of deciding whether a colour is of a specific SD involves first measuring the $L^*$, $C^*$ and $h$ values of the colour. The value of the hue angle $h$ is entered into the relevant formulae shown in Table 5a–f for the SD selected. For example, if it is to be established whether the colour corresponds to 1/1 SD, the formulae shown in Table 5b are used, and the values of the six conic constants are calculated.

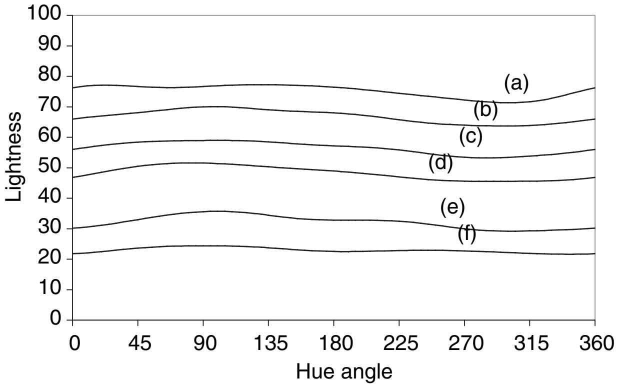


**Figure 3** Mapping of $L^*$ values round the hue circle at $C^* = 20$, for colours at the standard depths: (a) 1/25, (b) 1/12, (c) 1/6, (d) 1/3, (e) 1/1 and (f) 2/1

**Table 5** Formulae used to compute values of conic constants at standard depths indicated

| Conic constant | Formula |
|---|---|
| (a) 2/1 | |
| $k_1$ | 0.0431 |
| $k_2$ | $-0.002 + 0.0017 \cos(2h + 19) + 0.0001 \cos(4h)$ |
| $k_3$ | −1.0 |
| $k_4$ | $-0.008 \cos(h - 87) - 0.003 \cos(3h - 82)$ |
| $k_5$ | $0.1435 \cos(h - 79) + 0.0662 \cos(3h - 83)$ |
| $k_6$ | 1.151 |
| (b) 1/1 | |
| $k_1$ | 0.03 |
| $k_2$ | $-0.0044 + 0.0018 \cos(2h + 45)$ |
| $k_3$ | −1.0 |
| $k_4$ | $-0.0079 \cos(h - 95) - 0.0025 \cos(3h - 49)$ |
| $k_5$ | $0.1485 \cos(h - 72) + 0.0914 \cos(3h - 62)$ |
| $k_6$ | 2.907 |
| (c) 1/3 | |
| $k_1$ | 0.0195 |
| $k_2$ | $-0.0049 + 0.0011 \cos(2h + 68)$ |
| $k_3$ | −1.0 |
| $k_4$ | $-0.0072 \cos(h - 87) - 0.0023 \cos(3h - 31)$ |
| $k_5$ | $0.2234 \cos(h - 77) + 0.1218 \cos(3h - 29)$ |
| $k_6$ | 4.892 |
| (d) 1/6 | |
| $k_1$ | 0.0167 |
| $k_2$ | $-0.006 + 0.0015 \cos(2h + 121) + 0.0002 \cos(4h)$ |
| $k_3$ | −1.0 |
| $k_4$ | $-0.0068 \cos(h - 84) - 0.0029 \cos(3h - 12)$ |
| $k_5$ | $0.2735 \cos(h - 75) + 0.1611 \cos(3h - 14)$ |
| $k_6$ | 5.691 |
| (e) 1/12 | |
| $k_1$ | 0.0142 |
| $k_2$ | $-0.0091 + 0.0009 \cos(2h + 124) - 0.0004 \cos(4h)$ |
| $k_3$ | −1.0 |
| $k_4$ | $-0.0068 \cos(h - 81) - 0.0033 \cos(3h + 4)$ |
| $k_5$ | $0.3424 \cos(h - 70) + 0.2203 \cos(3h + 2)$ |
| $k_6$ | 7.166 |
| (f) 1/25 | |
| $k_1$ | 0.0128 |
| $k_2$ | $-0.013 + 0.0029 \cos(2h + 149) - 0.0005 \cos(4h)$ |
| $k_3$ | −1.0 |
| $k_4$ | $-0.0063 \cos(h - 78) - 0.0039 \cos(3h + 6)$ |
| $k_5$ | $0.3716 \cos(h - 69) + 0.2727 \cos(3h + 9)$ |
| $k_6$ | 8.136 |

The next stage is to apply the values of the conic constants determined, together with the value of $C^*$ for the colour, to Eqn 3 to determine the values of the

**Table 6** Maximum values of $C^*$ to which the algorithm may be applied, at each of the standard depths

| | Maximum chroma, at hue angle shown | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Standard depth | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° |
| 2/1 | 25 | 15 | 15 | 15 | 20 | 20 | 20 | 20 |
| 1/1 | 45 | 40 | 45 | 40 | 40 | 35 | 35 | 35 |
| 1/3 | 45 | 45 | 45 | 40 | 35 | 40 | 35 | 35 |
| 1/6 | 40 | 45 | 45 | 35 | 35 | 40 | 30 | 30 |
| 1/12 | 35 | 35 | 30 | 30 | 30 | 30 | 25 | 25 |
| 1/25 | 25 | 20 | 25 | 25 | 25 | 25 | 20 | 20 |

**Table 7** Differences between the $L^*$ values of the Hilfstypen samples and the $L^*$ values computed for colours of the same $a^*b^*$ values using the proposed new algorithm, at standard depths shown

| | | $\Delta L^*$ | | | | | |
|---|---|---|---|---|---|---|---|
| Sample number | Hue angle (average) | 2/1 | 1/1 | 1/3 | 1/6 | 1/12 | 1/25 |
| 1 | 91 | −33.67 | −13.37 | −9.06 | −10.17 | −3.32 | −1.27 |
| 2 | 53 | −14.69 | −0.46 | 4.86 | 2.81 | 4.47 | 5.02 |
| 3 | 33 | −8.61 | 3.51 | 9.18 | 13.45 | 16.42 | 15.59 |
| 4 | 21 | −9.54 | −1.07 | 9.07 | 14.59 | 15.4 | 4.45 |
| 5 | 12 | −6.68 | 0.53 | 6.40 | 11.57 | 11.88 | |
| 6 | 337 | −1.29 | −2.23 | 1.78 | 3.48 | 5.97 | |
| 7 | 322 | −2.40 | 0.04 | 2.77 | 5.29 | 6.65 | |
| 8 | 301 | 2.43 | 3.29 | 5.57 | 6.78 | 5.22 | 7.23 |
| 9 | 248 | 2.67 | 3.95 | 3.94 | 4.36 | 3.92 | 5.85 |
| 10 | 250 | −1.15 | 0.31 | 1.03 | 1.74 | 1.99 | 3.95 |
| 11 | 189 | −2.52 | 1.89 | 0.77 | 0.22 | 2.08 | 3.58 |
| 12 | 172 | −6.29 | −1.01 | −1.56 | −1.31 | −0.62 | |
| 13 | 92 | −4.32 | 0.96 | −0.28 | −0.99 | 1.92 | |
| 14 | 148 | 0.67 | 0.89 | 1.15 | 2.03 | 2.57 | |
| 15 | 96 | −1.52 | −0.57 | 1.74 | 2.34 | 1.28 | 2.46 |
| 16 | 77 | −2.03 | 0.57 | 0.53 | 1.23 | 1.06 | 0.26 |
| 17 | 52 | −1.99 | 3.40 | 5.71 | 4.42 | 4.35 | 2.82 |
| 18 | 223 | 0.42 | 1.85 | 4.99 | 4.91 | 5.11 | 6.78 |

constants $a$, $b$ and $c$, and then to solve the quadratic formula (Eqn 2) for $L^*_{SD}$. The difference between the values of $L^*$ and $L^*_{SD}$ is calculated and if the two are sensibly close ($\Delta L^* \approx 0$), then the colour can be said to be of 1/1 SD. If $L^* < L^*_{SD}$, the sample is greater than 1/1 SD and vice versa.

It is recommended that the use of the algorithm is restricted to colours whose $C^*$ values are less than the limits shown in Table 6. At hue angles in between those shown in Table 6, the limit can be judged by interpolation. This caveat is placed upon the method because of the limiting values of $C^*$ to which experimental data for constant depth surfaces were obtained. It is acknowledged that more experimental data are required at greater values of chroma than those shown in Table 6 to prove the validity of the method for very bright colours.

### Comparison between the $L^*$ values of the Hilfstypen samples and the predicted ($L^*_{SD}$) values

The $L^*$ values for each of the Hilfstypen colours were determined using the procedure described above. The differences between the $L^*$ values of the samples of each of the SD series and the $L^*$ values calculated, using the procedure described above ($\Delta L^*$), are given in Table 7. The largest differences occur for the yellow, orange and red colours (samples 1, 2 and 3, respectively).

## Conclusions

The outcome of this study is a new, consistent algorithm for the determination of the $L^*$ value that a colour of any given hue and chroma should possess for it to correspond to one of the SDs. A comparison of the proposed algorithm to define the SDs with the Hilfstypen samples confirms a significant inconsistency in the Hilfstypen samples. The proposed method has a significant advantage for the colour manufacturing and using industries in establishing exactly whether a particular shade corresponds to an SD.

It is recommended that the appropriate international bodies (such as ISO/TC38/SC1 for textiles) investigate its suitability for adoption as the basis for the colorimetric definition of revised series of SDs and control of the manufacture of corresponding physical samples.

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