MODIFIED FAST FOURIER TRANSFORM TECHNIQUE IN POWER SYSTEM TRANSIENT ANALYSIS

BY

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A THESIS

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DECLARATION

I do hereby declare that neither this thesis nor any part thereof has been submitted or is being concurrently submitted in candidature for any degree at any other University.

Candidate.

CERTIFICATE OF RESEARCH

Certified that the work presented in this Thesis is the result of the investigation carried out by the candidate under the Supervision of Dr. Md. Mujibur Rahman, Professor, Department of Electrical and Electronic Engineering, BUET, Dhaka.

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ABSTRACT

The modified Fast Fourier Transform (FFT) Technique has been used to calculate switching transient over-voltages in a complicated power system network. The beauty of the FFT algorithm lies in the fact that it automatically provides, in one pass, results for a number of time-domain points equal to the number of frequency samples used in the synthesis process. This is a tremendous advantage over conventional Fourier Transform methods where response at each instant of time has to be calculated separately. The FFT algorithm thus achieves a considerable savings in terms of computer time, especially for large complicated power networks.

As usual, the frequency response of the system is calculated from a transformation of the system equations and a numerical inversion of this frequency response is then attempted, analytical inversion being extremely difficult, if not impossible. The inversion integral is evaluated using the inverse Discrete Fourier Transform (IDFT) technique, implemented by Fast Fourier Transform algorithm.

The work starts with a critical appreciation of the available methods of analysis and goes on to a description of the IDFT technique and FFT algorithm. Numerical inversion gives rise to problems peculiar to itself which are discussed and the interrelationships between various integration parameters are investigated in an attempt to establish psuedo-optimum values.

The National Grid of Bangladesh Power Development Board has been selected for validation of the method. Using a network reduction technique, the voltages at different buses for switching at a particular bus, under varying system conditions, has been investigated. Results are compared with a previous work and it is found that at least a similar order of accuracy, if not better, can be achieved, while, at the same time, obtaining a substantial saving in computer time.

LIST OF PRINCIPAL SYMBOLS AND ABBREVIATIONS:

DFT	Discrete Fourier Transform
IDFT	Inverse Discrete Fourier Transform.
FFT	Fast Fourier Transform.
IFFT	Inverse Fast Fourier Transform.
KV	Kilo-volt, 1.0E03 volt.
HZ	Hertz.
rad/sec	Radian per second.
C/S	Cycles per second
p.u	Per unit.
μsec	Micro-second, 1.0E-06 second
μF	Micro-Farad, 1.0E-06 Farad
mH	Milli- Henry, 1.0E-03 Henry
π	pi, 3.1415927
ν	Nu, an integer
γ	Gamma, The propagation constant.
τ	Tau, The travel time
σ	Sigma, Sigma factor
ζ	Zeta
η	Eta
Ω	OMEGA, Maximum frequency limit in rad/sec.
DW	Increment in Angular frequency, rad/sec.
DF	Increment in frequency, C/s.
T	Time domain sampling interval
A	Smoothing parameter.
N	Number of frequency or time domain samples.
l	Length of the transmission line.
BPDB	Bangladesh Power Development Board.

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CHAPTER 1

INTRODUCTION

1. INTRODUCTION

1.1 GENERAL

The power-system operating voltages are rising continually. Owing to the rise in system operating voltage and to a desire to reduce capital costs by a reduction in system insulation level, the transient overvoltages arising when long transmission lines are energized and reenergized and the means for reducing these transient overvoltages have become of considerable importance. Besides this , for the reliable and uninterrupted operation of power system networks it is important to know the conditions under which transient overvoltages may be developed within the system and to have the means for their calculation. If this can be done at the design stage, precautions can be taken either to avoid the overvoltages completely or at least to minimise their effects. Therefore, a detailed analysis of the conditions under which transient overvoltages may develop within the system and a thorough understanding of the methods for the calculation of these overvoltages are of vital importance in ensuring smooth and continuous operation of power system networks. A number of methods for calculating the transient response of a complex power system network are available, (i) Analog method (ii) Bewley - Lattice Method (iii) Schnyder- Bergeron's Method and iv) Fourier Transform Method. All the methods have their relative advantages and disadvantages. The use of the most accurate methods, however, may not always be justified, owing to system data limitations and the larger computation time and hence costs involved[15].

The analog method, using the transient analyzer, has the shortcomings that distributed single and double circuit lines, considering the self and mutual impedances of the conductors, cannot be realistically represented and introduces a frequency band width limitations[15]. The elements of a power system, under switching conditions, are subjected to voltages and current having a wide frequency range varying from 50 c/s to arround 100 k c/s and the system parameters do vary with the frequency. The method is also unable to take account of this frequency dependance of the system parameters[8].

The travelling wave approach using Bewley-Lattice or Schnyder-Bergerons method is attractive for the simpler types of system but is rather cumbersome when lumped parameter or an extensive network are involved. The greatest restriction, possibly, is that the computation process must be carried through stage by stage from the instant of switching, thus errors inevitably accumulate. It also proves difficult to include the frequency dependance of the line parameters which arises due to skin effect and earth penetration,[1,7]. The Fourier Transform Method as used, is very suitable in situations it has been applied. Its biggest limitation is the large computation time required. Despite its own limitations, the method seems to be the best in that it overcomes most of the problems encountered with the previous methods. Further, since calculations are performed in the frequency domain, the frequency dependance of the system parameters are autometically incorporated in the analysis.

To reduce the computation time without sacrificing accuracy, modified Fast Fourier Transform (FFT) method can be resorted to. In the Fourier Transform Method, the response at a particular instant of time can only be obtained from a large number of data points in the frequency plane, thereby requiring a very substantial amount of computation time for finding the response over an interval of time. On the other hand, using FFT technique a number of time domain responses can be obtained from an equal number of data points in the frequency plane. Hence the time required for the calculation of the transient over a large interval of time will greatly be reduced. In effect, it is enough to consider a number of freequency domain data points equal to half the time domain data points at which response is to be evaluated (see Appendix - A).

As is well-known, high voltage transmission is necessary to reduce the long distance transmission loss. At present within our country or elsewhere there are many systems in use or coming into use at 132 kv, 230 kv, 400 kv, 500 kv, 600 kv, 735 kv or even 1000kv [1,4,15]. So it is evident that still higher voltages would be used in the future for long distance transmission of electrical energy. At these high operating voltages, internally developed overvoltages such as those due to switching operations within the system can exceed that due to lightning surges [4,15]. Switching overvoltages of as much as four times the phase to neutral voltage of the system are theoretically possible [15]. Therefore in determining the insulation level of the overall power system network, switching surges are fast becoming the major criteria to investigate. Our study therefore, is limited only to the switching transinet behaviour of power systems, specially the national grid of Bangladesh Power Development Board (BPDB).

1.2 DIFFERENT TYPES OF TRANSIENTS:

The insulation requirements of a power system are determined by transient overvoltages. Overvoltages, surges or transients are voltage waves of magnitude higher than desirable value and these persist usually for a short duration. These overvoltages travel along the transmission lines at arround the velocity of light and act as a source of serious potential danger even to physically remote terminal equipment.

Transient overvoltages occur due to several causes; among them, important causes are (1) Lightning surges (2) Sudden changes in circuit conditions (3) Switching (4) Resonance (5) Arching grounds (6) Different kinds of faults (7) Travelling waves etc[4]. The wave shape, magnitude of voltages, current, time and effects of the overvoltages caused by the above phenomena are different and likewise , the approach for protection of the various electrical system components against overvoltages differs[4,7]. The large spark accompanied by light produced by an abroupt, discontinous discharge of electricity through the air, from the clouds generally under turbulent conditions of atmosphare is called lighting. The representative values for an average lighting stroke are: 2.0E08 volts (peak), 4.0E04 Amps and a duration of arround 1.0E-05 seconds.

The basic phenomenon involved in a lightning stroke is that the clouds get charged during thunderstorms and the resulting high potential gradient causes break down of insulation of air. The stroke usually tries to hit the earth but if may also be attracted by HV overhead transmission lines. Due to such direct lightning strokes, or even indirect strokes, the line insulators may flash over or puncture, and the travelling wave may also reach substations and generating stations, thus overstressing the insulation of equipment and machines. Lightning surges influence the line design and insulation requirements upto system voltages of about 275 kv[4]. For system voltages above 360 kv, the switching overvoltages become more important in deciding the insulation requirements of system equipment and in the insulation coordination of EHV lines[4,15].

Switching surges may be caused due to one or more of the following reasons:

- Closing of uncharged lines
- Closing of charged lines
- Autoreclosure of circuit breakers (CB)
- Interruption of short circuits and line faults
- Load shedding at receiving ends of lines.
- Switching of magnetizing currents.
- Switching of high voltage reactors.
- Single line to ground fault without switching operation etc.

The waveforms and magnitudes of overvoltages caused by the above mentioned causes are different, hence they are not equally hazardous. Switching overvoltages are proportional to the system voltages and hence they can reach very high values in EHV and UHV systems (above 400 kv and upto 1000kv). Out of the various reasons mentioned above, the most severe are the following two[4]:

- Closing of charged lines
- Autoreclosure of CB.

The other reasons are relatively less severe. Switching surges due to closing of unloaded lines arise due to successive reflections of travelling waves. When the CB is closed on an unloaded transmission line, voltage is suddenly applied to the transmission line. This gives rise to a travelling wave, which gets reflected and the successive reflected waves give rise to voltages even as high as six times the normal system voltage. The overvoltages resulting on the closing of a pre-charged line are higher, because the applied voltage gets superimposed on the original voltage due to pre-charge. The worst case is the three phase high speed autoreclosure, due to closure on the trapped charge of line. The overvoltages are influenced by non-simultenious closure and /or non-simultenious reclosure of the three poles.

In an ungrounded transmission line due to Arching ground, the line to ground capacitance discharged through an earth fault. The process of charging and discharging gives rise to intermittant arcs. Thus the temporary fault grows into permanent fault and a voltage oscillation of 3 to 4 times of normal system voltage may be seen. Very high voltage surges occur due to a fault causing resonance between the inductance and capacitance in a part of the circuit. Insulation failure is likely to occur for such high voltage rises.

When a travelling wave reaches a junction or an end of the line the high voltage waves get reflected at the impedance mismatch and this reflected wave gets superimposed on the initial wave. The magnitude of voltage may rise to several times the normal voltage.

Considering all types of transient overvoltages it is found that internally developed overvoltages due to switching operations within the system, can exceed that due to other types of transients, specially at the higher operating voltages. Switching surges, therefore, are fast becoming the major criterion of investiagtion in designing the insulation level and in dealing with the insulation coordination of a power system. From here we will use the term transient in place of switching transient and all of our discussions will be limited for switching transient overvoltages only.

1.3 METHODS FOR CALCULATING POWER SYSTEM TRANSIENTS:

Some of the methods commonly in use for analysis and determination of switching surges for various switching-conditions and parameters are discussed here. In the Transient Network Analizer Method the system is represented by a scaled-down model, CB closings are simulated by synchronously driven switches and the resulting transients at different points on the system are recorded on an oscilloscope. Field tests involve tests on actual circuit breaker's and artificial transmission lines in a short-circuit testing laboratory or in actual field type testing stations, to determine the switching overvoltages. However, with the advent of high speed digital computer and as a result of its ready availability, the design engineers need no longer rely on the traditional methods, nor be forced to avoid problems which appear to be insolvable by these methods. Various programs have been developed to analyze the switching surges using digital computers. A comparative study of the different methods available is given in this section .

1.3.1 ANALOG METHOD USING TRANSIENT NETWORK ANALYZER[1,4,15]:

An A.C. Network Analyzer consists of a number of independent single phase units such as generator units, variable resistors, reactors and capacitors, autotransformers etc. The units are arrenged, adjusted and connected to have a circuit which represents the system under study. Sensitive electrical measuring instruments are provided for making electrical measurements at points in the system. The nominal voltage or base voltage for a typical network analyzer is 50 volts, base current 50 m A and the operating frequency is 400 or 480 HZ. Some of them are also operated at 60 HZ or even 10,000 HZ. Higher frequency permits smaller size of reactors. Transient investigations involve the opening or closing of switches placed at appropriate positions on the representative network. The resultant voltage and current transients appearing at different parts of the system are usually measured on an ocilloscope. Multiplying factors are then used to get the values corresponding to the actual system.

Although this method is accurate when applied to elements having lumped constants, error is introduced by the representation used for lines and cables. The artificial line used behaves in exactly the same way as the actual line for a particular frequency, but it has a bandwidth which is given by the natural frequency of a π'-section. If this bandwidth is exceeded by a particular transient the high frequency components are attenuated, introducing error into the solution. When energizing transmission lines, the most severe conditions occur when the line is energized at a peak of the supply voltage wave, and therefore this is the condition most frequently studied. This is equivalent to applying a step voltage to the line, and the response obtained from the artificial line when subject to such a discontinuous voltage function may, under certain conditions, contain errors which are not acceptable.

1.3.2 BEWLEY - LATTICE METHOD[1,15]:

In this method, transmission lines and cables are specified by their surge impedances and surge travel times, and the reflected and refracted voltages and currents at junctions and terminations are calculated by the use of reflection and refraction coefficients, defined for a single-phase system as:

$$K_R = (R_e - Z_c) / (R_e + Z_c) \qquad \ldots \qquad (1.1)$$

$$K_T = 2R_e / (R_e + Z_c) \qquad \ldots \qquad (1.2)$$

Where Z_c is the surge impedance of the line or cable on which the wave is travelling and R_e is the effective surge impedances seen by the wave when it reaches the termination or junction. K_R is the reflection coefficients and K_T is the refraction coefficents. When a 3-phase system is considered, although the basic travelling-wave equations remain unaltered, it is necessary to replace the individual surge impedances by surge-impedance matrices. Computer programs may be developed incorporating various facilities including the provision for taking into account the lumped parameter elements, non-linear resistors and earth return path losses etc. This Method is alttractive for simpler types of problems but is rather combersome when extensive network is required to be investigated.

1.3.3 SCHNYDER BERGERONS METHOD [1,6,7,8]:

Basically, the method assumes a fictitious observer moving along the line with a velocity equal to the phase velocity of the travelling wave. If the observer moves along the line in the forward direction at velocity v, then, since (x-vt) remains constant, the expression $V(x,t) + Zi(x,t)$ encountered by the observer when he leaves node m at time (t-τ) must be same when he arrives at node k at time t. Using the travelling wave equations and this above argument an expression for current and voltage equation is set up. (Details are given in Appendix- B). The method implies that to calculate the state at any time 't', the previous state at t-Δt, t-2Δt..... etc. over a time span of 'τ' the travel time is required to know and a step by step procedure is to be followed.

In this method it is difficult to include the frequency dependance of line parameters which arises due to skin effects and earth penitration [8]. Furthermore, to deduce the state of the system at any time, the computation must be carried through step-by-step from the instant of switching , thus errors inevitibly accumulate. The node admittance matrices formed in this method use first order approximation in evaluating trapizoidal rule of integration [1].

1.3.4 FOURIER TRANSFORM METHOD [1,9]:

At first this method was developed for examining in detail the initial transient response of a system, but more recently, however, attempts have been made to extend. The method to determine the global response of the system [1]. In this method the frequency response of the system is calculated first, then by the help of transform calculus the time domain response is calculated from an inversion of the frequency response. The frequency dependance of the parameters of the system can be incorporated using methods based on the fourier transform. For an extensive power system network, due to the complexity of the system, the integrals forming the inverse transforms have to be evaluated numerically using digital computer. The Fourier Transform of a continuous signal x(t) is given by:

$$x\ (jw) = \int_{-\alpha}^{\alpha} x(t)\ \exp\ (-jwt)\ dt \qquad \ldots \qquad (1.3)$$

Where the integral is assumed to converge in order for the definition to be valied. The Fourier Transform is an operation that creates from the time function x(t) a function x (jw) in the frequency domain. Then x(jw) displays the frequency content of the signal x(t) as a function of the continuous frequency variable w. To use numerical methods to evaluate the Fourier transform, the continuous signal x(t) has to be approximated by a discrete set of values. The zero order approximation is to assume x(t) to be constant at x(KT) for $KT \leqslant t < (K+1)\ T$, and the integral over this time interval is then approximated by the area of the rectangle, which is equal to Tx (TK). Hence the Fourier Transform in Eq. (1.3) becomes under this numerical approximation:

$$X(jw) \simeq \sum_{k=-\alpha}^{\alpha} Tx(t_k)\ \exp\ (-jwt_k) \qquad \ldots \qquad (1.4)$$

Where t_k= KT and the summation is over an infinite range in general. Of course, any actual numerical computation of the Fourier transfrom must involve only a finite number of terms, instead of the infinite summation in Eq. (1.4). In practice the signal is of finite duration of, say, NT and can be assumed to start at t = 0. Then Eq. (1.4) becomes:

$$X(jw) \simeq \sum_{k=0}^{N-1} Tx(t_k) \exp(-jwt_k) \qquad \ldots \qquad (1.5)$$

This represents the approximated Fourier Transform of a finite duration signal. The accuracy of this apprximation depends, of course on the time step size T selected. The smaller T is, the better the approximation becomes.

In a similar manner we can approximate the inverse Fourier transform integral defined by:

$$x(t) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} X(jw) \exp(jwt)\, dw \qquad \ldots \qquad (1.6)$$

and by the summation:

$$x(t) \simeq \frac{Dw}{2\pi} \sum_{n=-M}^{M-1} X(jw_n) \exp(jw_n t) \qquad \ldots \qquad (1.7)$$

Where the signal is assumed to be band limited in frequency to MDw, with Dw being the step size in frequency used in the approximation of the frequency function X(jw), and

w_n=nDw. Again x(t) is still a continuous function of the time variable t and is periodic in t with period $2\pi/Dw$. The accuracy of the approximation in Eq. (1.7) depends on the frequency step size Dw.

For numerical integration, the range of integration must be finite. But if the infinite integral is not reasonably convergent, Gibb's oscillation may arise in a way comparable to their occurances in approximating an infinite Fourier series by a trigonometric series. However, these unwanted oscillations, resulting from the trancation of the infinite range of integration, can be reduced by incorporating the standard sigma factor. It is also shown later that the modified Fourier transform can be used successfully to move the path of integration away from the poles of the integraud, thus reducing the peaks of the spectrum function, smoothing its shape and enabling the use of a larger step length or sampling interval in performing the integration. The modification is done in order to carry out the integration on the complex plance. The modified Fourier Transform pair consists of:

$$X(A+jw) = \int_{-\alpha}^{\alpha} x(t)\ \exp(-(A+jw)t)\ dt \qquad \ldots \quad (1.8)$$

together with its inverse integral:

$$x(t) = \frac{1}{2\pi}\int_{-\alpha}^{\alpha} X(A+jw)\ \exp((A+jw)t)\ dw \qquad \ldots \quad (1.9)$$

Where 'A' is taken sufficiently large to ensure that the integrals converge. In real stable system, this is ensured if $A > 0$; apart from this the value of 'A' is at one's disposal and considered for a particular problem being investigated. On the other hand the presence of exp(At) in the integral in Eq. (1.9) imposes a practical restriction on the choice of the value of 'A'.

The Fourier Transform method is the least restrictive in the assumptions that have to be made and consequently yields high accuracy in any particular calculation. The effect of trancating the infinite range of integration, as necessitated in the numerical inversion restrict the maximum rise time obtainable for the calculated system response. The method is capable, by its very nature, of handling frequency dependant parameters but is also inherently limited to linear or psuedo-linear systems. Despite its own limitations the Fourier transform method seems to be the best in that it overcomes most of the problems encountered with the other methods.

However, as has been pointed out in sec. 1.1 the method involves relatively large computational time, this problem can be overcome by evaluating the inverse Fourier transform integral using (DFT) discrete Fourier transform, instead of continuous Fourier transform, implimented by Fast Fourier Transform (FFT) algorithm.

1.4 FAST FOURIER TRANSFORM METHOD

Discrete Fourier transform is a special case of continuous Fourier transform and is easily amenable to machine computation. The Fast Fourier Transform (FFT) is simply an algorithm (i.e. a particular method of performing a series of computations) that can compute the discrete Fourier transform much more rapidly than other available algorithms, thus saving substantial amount of computation time.

In 1965 cooley and Tukey published a famous paper showing that DFT can be efficiently computed by using approximately $N \log_2 N$ operations (an operation being defined to be a complex multiplication and addition) where N is a power of 2. Whereas the direct DFT requires N^2 operations (see in ch-2). Hence, the Fast Fourier transform is sometimes reffered to as the cooley-Tukey algorithm. Certain special algorithms for computing the discrete Fourier transform (DFT) with considerable savings in computation time are discussed in chapter-2 with applications. Later we will discuss how this FFT algorithm can be applied to the efficient evaluation of time response from frequency domain data points.

1.5 LITERATURE SURVEY:

The field of power system transients has been the subject of a great deal of analysis, investigations and research, and a large number of references to such work are available in the literature. Some of the more important ones, which are relevant to and associated with the present study, are discussed in this section.

Bickford and Doepel have presented a paper[15] in which a comparative study of different methods for the calcualtion of transient overvoltages have been discussed. The paper outlines first a method based on a lumped parameter representation of the system, and secondly a method based on a lattice-diagram solution of the transmission line wave equations. The results obtained using these methods are compared. But they did not comment on the relative accuracy of calculation using the two methods. The transient overvoltages resulting mainly from line energization are discussed and the factors affecting the results are ivestigated in this paper.

A Budner in his paper[17] on the 'introduction of frequency-dependant line parameters into an electromagnetic transients program' describes a new method of digitally modelling the frequency dependance of the line parameters for an overhead transmission line. The method uses Fourier Transforms and presents models only the line characteristics, leaving the terminations to be chosen by the programer at will. The method of analysis is applied on a two-phase line. This paper seems to be among the first to present a practical method of including the effects of frequency upon the ground-mode propagation, in a transient analysis program. All the discussers of this paper have commented on the difficulties of the numerical inversion of the response transform . In this connection, D.E. Hedman and W.F. B'Rells of General Electric Company, N.Y. have suggested the recently developed First Fourier Transform (FFT) method of evaluating the inverse Fourier transform. The direct method of calculating the inverse Fourier transform, as opposed to the FFT method is prohibitively time consuming, specially when a large number of points in the time domain have to evaluated. Discusser Professor K.R. Shah of Verginia Polytechnic Institute has advocated in favour of the method of Fourier transform specially for this type of problems.

N. Mullineux et al, in a paper [9] present examples of the application of modified Fourier transform to single-phase and three-phase power system networks. The problems encountered in source side representation and the simulation of line energization and three phase switching are discussed. Investigation results on short transmission line fault and switching overvoltages in cable networks are presented. However, the paper did not focus on the problems encountered in the choice of optimum parameters in performing the numerical inversion of the response transform No particular suggestions have been put forward for the accurate calculation of global responses.

In another paper [18] the same authors have described how the problems of sequential pole closure of three-phase network or CB may be attempted using the same method of transformation for the calculation of switching transients. The analysis was performed using a very simple source-side representation and therefore, the suitability of its application to sources feeding through a complex network of lines and other elements has not been clearly established.

Four papers [11,12,13,14] by S.J. Day et al on "Developments in Obtaining Transient Response using Fourier Transforms" discusses some of the problems associated with the direct computations of inverse Fourier transform integral for the calculation of switching transients of a complicated power systems network. Part I [11] deals with the problems which arise because of the necessity of trancating the infinite range of integration. It has been shown that the oscillatory effects caused by the trancation of the infinite range of integration, as necessitated due to numerical inversion could be reduced by the use of sigma factors. But the incorporation of sigma factors results in a still lower rate of rise at the points of discontinuty. Part II [12] deals with the problems arising from the fact that poles of the integrand of the inverse integral may lie close to the path of integration. This causes the integrand to peak at intervals along the path of integration and necessitates using a very small step length in the numerical integration process. It is shown in the paper that the modified Fourier transform can be used to move the path of integration away from the poles and hence smooth the integrand thus enabling the use of a greater step length. The methods are illustrated firstly by considering the receiving end voltages of an open-circuit transmission line and secondly by considering the voltage appearing at a switch clearing a short line fault. Part III [13] describes the extension of the methods to determine the response over several time periods. The computer time necessary for calculating the response over such a long time interval is obviously large but no attention to this aspect has been given in this paper. Finally in part IV [14] the authors present a physical interpretation of the theory involved in the method and the aliasing error, sequential switching and some other non-linear effects are discussed.

S.M. Lutful Kabir, BUET,Dhaka, in his M.Sc. Engg. (E.E) thesis [1] presents a very interesting and detailed analysis and results using modified Fourier transform method for the calculation of power system transients. In this work the time response of the system was obtained from a numerical inversion (direct computation of inverse transform) of its frequency response using a digital computer. The numerical iversion process have been investigated in detail in an attempt to understand the inter-relationships between the various parameters associated with the process and hence to select the pseudo-optimum values. A network reduction technique, for application of the method developed to complicated power system networks, has been presented. The developed method and network reduction technique was applied to the Eastern Grid of Bangladesh Power Development Board (BPDB) and switching surges at different busbars, for switching at a particular bus, under varying system conditions, were calculated. The results obtained are quite realistic but the numerical inversion process seems to be prohibitively time consuming and hence costly.

1.6 SCOPE OF THE PRESENT WORK:

Objectives of the research is to calculate the switching transient response of a complicated power system networks using the Fast Fourier Transform Algorithm on a digital computer. The calculation may be done using the modified Fourier Transform method, but this method is time consuming and hence costly. It has been mentioned earlier that, for high voltage transmission design of system insulation level is being governed more by the behaviour of the system under switching conditions than by its response to lightning surges. For this reason we devoted our time to investigate the switching performance of the systems. A comparative study of different methods and their limitations, associated problems, advantages and disadvantages are mentioned earlier. Now it is established that Fourier transform method is very suitable in the situations it has been applied, but its biggest limitation is the large computational time. For this we have resorted to the Fast Fourier Transform technique in which discrete Fourier transformation is performed instead of continuous Fourier transformation. However, numerical evaluation of the inverse transform poses certain problems that need thorough investigation in relation to the particular system being considered.

The work starts with the Fourier transform method as the basis of investiagtions reported here, giving explanation of the transient phenomena, and the processes involved in the calculation of switching transient. Detailed investigations into the Fast Fourier transform technique with application and its associated problems are performed. An attempt to establish the method as an accurate, reliable and faster tool for application to the transient analysis of a generalized power system is also presented. The various problems associated with the numerical inversion of the response transform are studied with a view to establish pseudo-optimum integration parameters and for this number of computer programs has been written & presented. Finally the method is applied to the national interconnected Grid of Bangladesh Power Development Board. Various switching conditions are simulated and the results are presented in graphical form.

CHAPTER 2

METHOD OF ANALYSIS

2.1 INTRODUCTION

In this chapter the analysis of sampling theorem, discrete and Fast Fourier transform with their application and analysis of long transmission lines are presented. The discrete evaluation of continuous Fourier transform integral is also presented with applications. The DFT evaluation implimented by FFT and its associated problems are discussed. Finally an algorithm for computer program is written to calculate the time domain response from spectrum function for an open-ended 10 mile long transmission line considering a unit step input at the sending end. The results are presented in graphical form.

2.2 SAMPLING THEOREM

In most of the signal processing applications, the DFT is used to find the sampled frequency spectrum of a continuous signal as an approximation to the continuous Fourier Integral. Because of the relationships among the time domain and frequency domain parameters it is necessary to investigate the constraints placed on some of the parameters in order to use the FFT algorithm in a minigful fashion. The parameters involved in the time and frequency-domain sampling of a signal are as follows:

T = Sampling time interval in seconds.

N = Number of signal samples in time or frequency domain

$f_s = \frac{1}{T}$ = Sampling frequency in HZ.

$td = NT$ = duration of time signal being sampled.

DF = frequency step size in HZ.

Dw = frequency step size in rad/sec.

f_h = highest frequency content of signal in HZ.

OMEGA = highest frequency content of signal in rad/sec.

According to the sampling theorem, in order for a band-limited signal to be recoverable from the samples of the signal, the sampling frequency must be equal to at least twice the highest frequency component in the signal [2]. That is:

$$f_s = \frac{1}{T} > 2f_h \tag{2.1}$$

Therefore, $$T < \frac{1}{2f_h} \tag{2.2}$$

This relation places an upper limit on the sampling interval in the time - domain. The requirement that $T = \frac{1}{2f_h}$ is simply the maximum spacing between samples for which the theorem holds. Frequency $\frac{1}{T} = 2fh$ is known as the Nyquist sampling rate. If $T > \frac{1}{2f_h}$, then aliasing will result [2,3]. In practice the sampling interval is chosen to be much lower than the limiting value of $\frac{1}{2f_h}$ in order to reduce the aliasing error.

Now let us look at the number of signal samples that must be taken in order to meet a specified frequency resolution. To have the relationship between the time interval T and the frequency interval Dw let us assume that discrete signal x(kT) is obtained from a continuous signal x(t) by sampling it at a uniform rate of T seconds. Then from the principles of sampling, the Fourier transform of the sampled signal is periodic with period $2\pi/T$. Therefore, for the N frequency points to cover the entire frequency range, we must have:

$$DW = 2\pi/(NT) \tag{2.3}$$

Hence $td = NT = 2\pi/DW = 1/DF$ (2.4)

From Eq. (2.2) and Eq. (2.4) we have the constraint on the number of signal samples:

$$N > 2fh/DF = 2*OMEGA/DW \tag{2.5}$$

Eq. (2.5) illustrates the lower limit on the number of samples required to compute the DFT. In practice, more samples are taken in order to reduce the aliasing error. Eq. (2.2) and Eq. (2.5) are used to select the sampling interval and the number of samples respectively to get better reproduction of the original signal i.e. without lossing any information as well as to maintain considerable accuracy.

2.3 DISCRETE FOURIER TRANSFORM

Considering the discrete signal x(kT), which can be the result of sampling a continuous signal x(t) or can occur inherently as a sequence of numbers. If the signal has a finite number of points and its values are used in the approximate Fourier transform derived in sec.-1.3,4, then Eq. (1.5) and Eq. (1.7) yields the approximate frequency and time information about the signal at the N discrete frequency and time values respectively. Eq. (1.5) & Eq. (1.7) can be rewritten as:

$$X(jw_n) = T \sum_{k=0}^{N-1} x(t_k) \exp(-jw_n t_k) \quad \text{for } n=0,1,2,\ldots N-1 \tag{2.6}$$

and
$$x(t_k) = \frac{Dw}{2\pi} \sum_{n=-N/2}^{N/2-1} X(jw_n) \exp(jw_n t_k) \quad \text{for } k=0,1,2 \ldots N-1 \tag{2.7a}$$

Where for convenience and without loss of generality, M in Eq. (1.7) is replaced by N/2, so that the same number of data points in both the time and frequency domain is obtained. Since the frequency function $X(jw_n)$ in Eq. (2.6) is periodic with period N, the summation in Eq. (2.7a) can be written as:

$$x(t_k) = \frac{Dw}{2\pi} \sum_{n=0}^{N-1} X(jw_n) \exp(jw_n t_k) \quad \text{for } k=0,1,2, \ldots N-1 \tag{2.7b}$$

From Eq. (2.3) $Dw = 2\pi/(NT)$

Then $\frac{Dw}{2\pi} = \frac{1}{NT}$ and $w_n t_k = nk2\pi/N$ (2.8)

Now let us consider:

$$x_k = Tx\ (t_k) \quad ; \quad k=0,1\ldots\ldots N-1 \tag{2.9}$$

and

$$X_n = X(jw_n) \qquad n=0,1,\ldots\ldots N-1 \tag{2.10}$$

Then Eq. (2.6) and (2.7) becomes:

$$X_n = \sum_{k=0}^{N-1} x_k \exp\left(-(j2\pi/N)nk\right) \quad n=0,1\ldots\ldots N-1 \tag{2.11}$$

and

$$x_k = 1/N \sum_{n=0}^{N-1} X_n \exp\left((j2\pi/N)\ kn\right) \quad k=0,1\ \ldots\ N-1 \tag{2.12}$$

The two expressions in Eq. (2-11) and Eq. (2.12) are called the Discrete Fourier Transform (DFT) pair. Notation X_n = DFT (x_k) is used to denote the direct discrete Fourier transform in Eq. (2.11) and x_k = IDFT(X_n) is used to denote the inverse discrete Fourier transform in Eq. (2.12).

2.4 FAST FOURIER TRANSFORM:

It has been mentioned earlier that the FFT algorithm is frequently used to compute the discrete Fourier transform

DFT and its inverse (IDFT), with considerable savings in computational time. In this section we shall see, how FFT algorithm can be evaluated efficiently and how it saves the arithmatic operation and time of computation and hence cost of computation.

2.4.1 MATRIX FORMULATION AND INTUITIVE DEVELOPMENT [2,3]:

Before deriving the FFT algorithm it is necessary to look at the number of operations involved in the direct computation of the DFT and that of involved in the FFT computation of DFT. Rewriting the DFT Eq. (2.11):

$$X(n) = \sum_{k=0}^{N-1} x_k \exp\,(-(j2\pi/N)\; nk), \quad n = 0,1.....\; N-1$$

If we let N=4 and $W = \exp(-j2\pi/N)$, then the above equation can be written as:

$$\begin{aligned}
X(0) &= x_o(0)\; w^o + x_o(1)\; w^o + x_o(2)\; w^o + x_o(3)\; w^o \\
X(1) &= x_o(0)\; w^o + x_o(1)\; w' + x_o(2)\; w^2 + x_o(3)\; w^3 \\
X(2) &= x_o(0)\; w^o + x_o(1)\; w^2 + x_o(2)\; w^4 + x_o(3)\; w^6 \\
X(3) &= x_o(0)\; w^o + x_o(1)\; w^3 + x_o(2)\; w^6 + x_o(3)\; w^9
\end{aligned} \qquad (2.13$$

Eq. (2.13) can more easily be represented in matrix form:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W^0 & W^0 & W^0 & W^0 \\ W^0 & W^1 & W^2 & W^3 \\ W^0 & W^2 & W^4 & W^6 \\ W^0 & W^3 & W^6 & W^9 \end{bmatrix} \begin{bmatrix} x_o(0) \\ x_o(1) \\ x_o(2) \\ x_o(3) \end{bmatrix} \tag{2.14}$$

Or more compactly as:

$$\bar{X}(n) = \bar{W}^{nk}\,\bar{x}_o(k) \tag{2.15}$$

Here we use the vector notation to denote a matrix. Examination of Eq. (2.15) reveals that since $\bar{W}$ and $\bar{x}_o(k)$ are complex, then n^2 complex multiplications and $N(N-1)$ complex additions are necessary to perform the required matrix computation. The FFT owes its success to the fact that the algorithm reduces the number of multiplications and additions required in the computation of Eq. (2.15). We will now discuss, on an intuitive level, how this reduction is accomplished.

To illustrate the FFT algorithm, it is convenient to choose the number of samples N of $x_o(k)$ according to the relation $N = 2^{\nu}$, where ν is an integer, Recall that Eq. (2.14) results from the choice of $N = 4 = 2^{\nu} = 2^2$; Therefore, we can apply the FFT to the computation of Eq. (2.14)

The first step in developing the FFT algorithm for this particular example is to rewrite Eq. (2.14) as:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & w^1 & w^2 & w^3 \\ 1 & w^2 & w^o & w^2 \\ 1 & w^3 & w^2 & w^1 \end{bmatrix} \begin{bmatrix} x_o(0) \\ x_o(1) \\ x_o(2) \\ x_o(3) \end{bmatrix} \tag{2.16}$$

Eq. (2.16) was derived from Eq. (2.14) by using the relationship $w^{nk} = w^{nk \bmod (N)}$. Recall that nk mod(N) is the remainer upon division of nk by N; Hence, if N=4, n=2 & k=3 then

$$w^4 = w^o, \quad w^6 = w^2 \quad \text{and} \quad w^9 = w^1 \tag{2.17}$$

[w^6 is the element in 3rd row (n=2), 4rth column (k=3) of the $\bar{w}$ matrix in Eq. (2.14).]

$$\text{Since} \quad w^{nk} = w^6 = \exp \frac{j2\pi}{4} .6 = \exp(-j3\pi) = \exp(-j\pi)$$

$$= \exp \frac{-j2\pi}{4} . 2 = w^2 = w^{nk \bmod (N)} \tag{2.18}$$

The other elements in the $\bar{w}$ - matrix in Eq. (2.16) can be derived as Eq. (2.17) in a similar fashion. The second step in the development is to factor the square matrix in Eq. (2.16) as follows:

$$\bar{X}(n) = \begin{bmatrix} X(0) \\ X(2) \\ X(1) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & w^0 & 0 & 0 \\ 1 & w^2 & 0 & 0 \\ 0 & 0 & 1 & w^1 \\ 0 & 0 & 1 & w^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & w^0 & 0 \\ 0 & 1 & 0 & w^0 \\ 1 & 0 & w^2 & 0 \\ 0 & 1 & 0 & w^2 \end{bmatrix} \begin{bmatrix} x_o(0) \\ x_o(1) \\ x_o(2) \\ x_o(3) \end{bmatrix} \qquad (2.19)$$

The method of factoriization is based on the theory of the FFT algorithm[3,12] provided in the literature. For the present it suffices to show that multiplication of the two square matrices of Eq. (2.19) yields the square matrix of Eq. (2.16) with the exception that rows 1 and 2 have been interchanged (The rows are numbered 0,1,2 & 3). It is noted that this interchange has been taken into account in Eq. (2.19) by rewriting the column vector X(n) as:

$$\bar{X}(n) = \begin{bmatrix} X(0) \\ X(2) \\ X(1) \\ X(3) \end{bmatrix} \qquad (2.20)$$

This rearrangement is inherent in the matrix factoring process and is a minor problem because it is straight forward to generalize a technique for unscrambling $\bar{X}(n)$ to obtain $X(n)$, as shown below:

Rewriting $\bar{X}(n)$ of Eq. (2.20) by replacing argument n with its binary equivalent:

$$\bar{X}(n) = \begin{bmatrix} X(0) \\ X(2) \\ X(1) \\ X(3) \end{bmatrix} \quad \text{becomes} \quad \begin{bmatrix} X(00) \\ X(10) \\ X(01) \\ X(11) \end{bmatrix} \tag{2.21}$$

It is observed that if the binary arguments of Eq. (2.21) are flipped or bit reversed then:

$$\bar{X}(n) = \begin{bmatrix} X(00) \\ X(10) \\ X(01) \\ X(11) \end{bmatrix} \quad \text{flips to} \quad \begin{bmatrix} X(00) \\ X(01) \\ X(10) \\ X(11) \end{bmatrix} = X(n) \tag{2.22}$$

It is straightforward to develop a generalized result for unscrambing the FFT result, following the above argument.

Now we want to calculate the number of operations requied to compute Eq. (2.19).

First assuming:

$$\begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & w^0 & 0 \\ 0 & 1 & 0 & w^0 \\ 1 & 0 & w^2 & 0 \\ 0 & 1 & 0 & w^2 \end{bmatrix} \begin{bmatrix} x_0(0) \\ x_0(1) \\ x_0(2) \\ x_0(3) \end{bmatrix} \tag{2.23}$$

or,

$$\begin{aligned} x_1(0) &= x_0(0) + w^0 \; x_0(2) \\ x_1(1) &= x_0(1) + w^0 \; x_0(3) \\ x_1(2) &= x_0(0) + w^2 \; x_0(2) \\ x_1(3) &= x_0(1) + w^2 \; x_0(3) \end{aligned} \tag{2.24}$$

or,

$$\begin{aligned} x_1(0) &= x_0(0) + w^0 \; x_0(2) \\ x_1(2) &= x_0(0) - w^0 \; x_0(2) \\ x_1(1) &= x_0(1) + w^0 \; x_0(3) \\ x_1(3) &= x_0(1) - w^0 \; x_0(3) \end{aligned} \tag{2.25}$$

From the above Eq. (2.25) it is clear that to calculate the intermediate vector $\bar{X}_1(k)$, four complex additions and two complex multiplications are required.

which for $N = 1024 = 2^{10}$ is a computational reduction of more than 200 to 1. A comparison between the number of operations required for direct computation and computation using FFT is shown in Table 2.1.

For $N > 4$ it is combersome to describe the matrix factorization process analogous to Eq. (2.19). In literature[2(pp249-263),3] the FFT algorithms, both diamation-in-time and decimation in-frequency are provided to interpret Eq. (2.19) in a graphical manner using flow-graphs. On the basis of those algorithms a FORTRAN subroutine FFT() and a discussion on the FORTRAN subroutine FFT is presented in Appendix-C.

2.4.2 APPLICATION OF FAST FOURIER TRANSFORM

2.4.2.1 DFT & IDFT

To illustrate the application of the DFT to the computation of Fourier transform we consider a test signal exp (-t) and we wish to compute the frequency spectrum, by means of DFT an approximation to the Fourier transform, of this function. The computer program is given in Appendix-D1.

The first step in applying DFT is to choose the number of samples N and the sampling interval T. Let us assume $T = 0.25$ and $N=32$. The value of the function at a discontinuity ($t = 0$) must be defined to be the mid-value if the inverse Fourier transform is to hold. Recalling Eq. (2.11):

$$X_n = T \sum_{k=0}^{N-1} x_k \exp(-(j2\pi/N)\, nk), \quad n= 0,1,2 \ldots. N-1 \qquad (2.28)$$

Now
$$\begin{pmatrix} X(0) \\ X(2) \\ X(1) \\ X(3) \end{pmatrix} = \begin{pmatrix} x_2(0) \\ x_2(1) \\ x_2(2) \\ x_2(3) \end{pmatrix} = \begin{pmatrix} 1 & W^0 & 0 & 0 \\ 1 & W^2 & 0 & 0 \\ 0 & 0 & 1 & W^1 \\ 0 & 0 & 1 & W^3 \end{pmatrix} \begin{pmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{pmatrix} \qquad (2.26)$$

Therefore, to determine $x_2(k)$, two complex multiplications and four complex additions are required. The computation of X(n) by means of Eq. (2.19) thus requires a total of four complex multiplications and eight complex additions, whereas the computation of X(n) by Eq. (2.15) requires 16, or 4^2, complex multiplications and twelve complex additions. Since the computation time is governed by the required number of operations, we see the reason for the efficiency of the FFT algorithm.

For $N = 2^\nu$ the FFT is simply a procedure for factoring an $N \times N$ matrix into ν matrices ($N \times N$) such that each of the factored matrices has the special property of minimizing the number of complex operations. It is noted that the FFT requires $N\nu/2$ complex multiplications and $N\nu$ complex additions, whereas the direct method (DFT) requires N^2 complex multiplications and $N(N-1)$ complex additions. If we assume that computation time is proportional to the number of multiplications, then the approximate ratio of direct DFT to FFT computing time is given by:

$$\frac{N^2}{N\nu/2} = \frac{2N}{\nu} \qquad (2.27)$$

TABLE 2.1: COMPARISON OF NUMBER OF COMPLEX ARITHMATIC OPERATIONS REQUIRED FOR DFT AND FFT.

	MULTIPLICATIONS		ADDITIONS	
N	DFT	FFT	DFT	FFT
4	16	4	12	8
8	64	12	56	24
16	256	32	240	64
32	1024	80	992	160
64	4098	192	4032	384
128	16384	448	16256	896
256	65536	1024	65280	2048
512	262144	2304	261632	4608
1024	1048576	5120	1047552	10240
2048	4194304	11264	4192256	22528
4096	16777216	24576	16773120	49152

Where T is introduced to produce equivalence between the continuous and discrete transforms. From the result (See Fig. 2.1) it is shown that the discrete transform is symmetrical about $n = N/2$. This follows from the fact that the real part of the transform is even and that the result for $n > N/2$ are values for hypothetical negative frequencies. The discrete transform approximates the continous transform rather poorly at the higher frequencies. To reduce this error it is necessary to decrease the sampling interval T and increase N.

Now assuming that the frequency function are given and it is necessary to determine the corresponding time function by means of the IDFT. Recalling Eq. (2.12) :

$$x_k = DF/N \sum_{n=0}^{N-1} X_n \exp\left((j2\pi/N)\, nk\right) \quad k= 0,1.\ldots\ldots N-1 \tag{2.29}$$

Where DF is the frequency step size in Hertz, $N = 32$, $DF = 1/8$. Since $R(f)$, the real part of the complex frequency function, must be an even function then we fold $R(f)$ about the frequency $f = 2.0$, which corresponds to the sample point $n= N/2$. From Fig. 2.2 it is clear that the imaginary part of the frequency function is odd, so we must not only fold the sample value about $n = N/2$ but also flip the results to preserve symmetry The illustrations are given in Appendix-A. A program has been

64 POINT FREQUENCY SPECTRUM OF EXPONENTIAL
FUNCTION COMPUTED FROM 32 SIGNAL SAMPLES

	FREQ(HZ)	REAL PART	IMAGINARY PART	MAGNITU
I	0.0	1.008	0.0	1.008
2	0.156	0.583	-0.519	0.781
3	0.313	0.246	-0.386	0.458
4	0.469	0.160	-0.314	0.353
5	0.625	0.107	-0.226	0.250
6	0.781	0.094	-0.199	0.221
7	0.938	0.076	-0.154	0.171
8	1.094	0.075	-0.142	0.161
9	1.250	0.064	-0.114	0.131
10	1.406	0.066	-0.109	0.127
11	1.563	0.059	-0.089	0.106
12	1.719	0.062	-0.086	0.106
13	1.875	0.056	-0.071	0.090
14	2.031	0.059	-0.070	0.092
15	2.188	0.054	-0.058	0.079
16	2.344	0.058	-0.057	0.081
17	2.500	0.053	-0.048	0.071
18	2.656	0.057	-0.047	0.074
19	2.813	0.052	-0.039	0.065
20	2.969	0.056	-0.038	0.068
21	3.125	0.051	-0.032	0.061
22	3.281	0.056	-0.031	0.064
23	3.438	0.051	-0.026	0.057
24	3.594	0.055	-0.025	0.060
25	3.750	0.051	-0.020	0.054
26	3.906	0.055	-0.019	0.058
27	4.063	0.051	-0.015	0.053
28	4.219	0.055	-0.013	0.056
29	4.375	0.050	-0.010	0.051
30	4.531	0.055	-0.008	0.055
31	4.688	0.050	-0.005	0.051
32	4.844	0.055	-0.003	0.055
33	5.000	0.050	0.0	0.050
34	5.156	0.055	0.003	0.055
35	5.313	0.050	0.005	0.051
36	5.469	0.055	0.008	0.055
37	5.625	0.050	0.010	0.051
38	5.781	0.055	0.013	0.056
39	5.938	0.051	0.015	0.053
40	6.094	0.055	0.019	0.058
41	6.250	0.051	0.020	0.054
42	6.406	0.055	0.025	0.060
43	6.563	0.051	0.026	0.057
44	6.719	0.056	0.031	0.064
45	6.875	0.051	0.032	0.061
46	7.031	0.056	0.038	0.068
47	7.188	0.052	0.039	0.065
48	7.344	0.057	0.047	0.074
49	7.500	0.053	0.048	0.071
50	7.656	0.058	0.057	0.081
51	7.813	0.054	0.058	0.079
52	7.969	0.059	0.070	0.092
53	8.125	0.056	0.071	0.090
54	8.281	0.062	0.086	0.106
55	8.438	0.059	0.089	0.106
56	8.594	0.066	0.109	0.127
57	8.750	0.064	0.114	0.131
58	8.906	0.075	0.142	0.161
59	9.063	0.076	0.154	0.171
60	9.219	0.094	0.199	0.221
61	9.375	0.107	0.226	0.250
62	9.531	0.160	0.314	0.353
63	9.688	0.246	0.386	0.458
64	9.844	0.583	0.519	0.781

FIG.2.1 : 64-POINT FREQUENCY SPECTRUM OF EXP(-T)
COMPUTED BY DFT

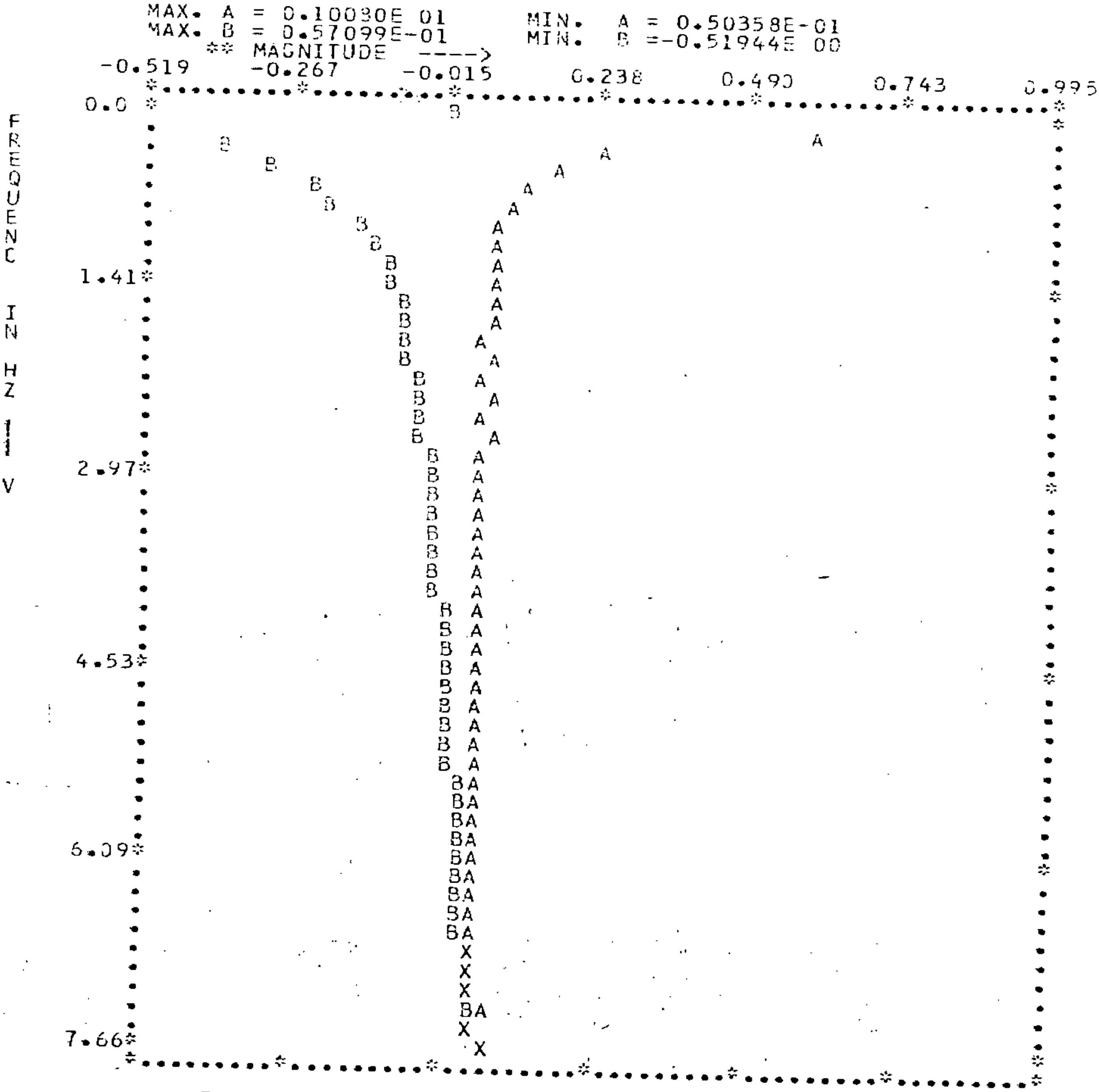

FIG. 2.2 : 64-POINT DFT OF EXP. FUNCTION
CURVE-A FOR REAL PART & B FOR IMAGINARY PART

EOJ EF53TEST

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written (See Appendix - D2) for the computation of Eq. (2.29) using IDFT, the results of which are presented in Fig. 2.3. The result is a complex function whose imaginary part is approximately zero and real part is apprximately the absolute value. It is noted that at k=0 the result is approximately the correct mid value and reasonable agreement is obtained for all except the large - k results. Improvement can be obtained by reducing DF and increasing N.

2.4.2.2 FFT AND IFFT

A sample program calling the FFT subroutine to compute the frequency spectrum of a square wave test signal is given in Appendix - D3. The square wave is sampled for 64 points at a sampling interval of T = 0.1. The computed frequency spectrum is shown in Fig. 2.4. If we compare this spectrum with the one obtained from the thoretical Fourier series:

$$X_n = 1/(NT) \int_0^{NT} x(t) \exp(-jnwt)\, dt \qquad (2.30)$$

or

$$X_n = \begin{cases} -j2/(n\pi) & ; \ n = \text{odd} \\ 0 & ; \ n = \text{even} \end{cases} \qquad (2.31)$$

Where NT = 6.4, the period of the square wave, and $w = 2\pi/(NT)$. It is observed that the FFT implements only the integral portion

// EXEC

THE VALUE OF DW IS= 0.9812
& THE VALUE OF N IS = 64
& THE VALUE OF DT IS= 0.10E 00
& THE VALUE OF OMEGA IS = 0.63E 02

* MAX. = 0.997P.U.
* MIN. = 0.005P.U.

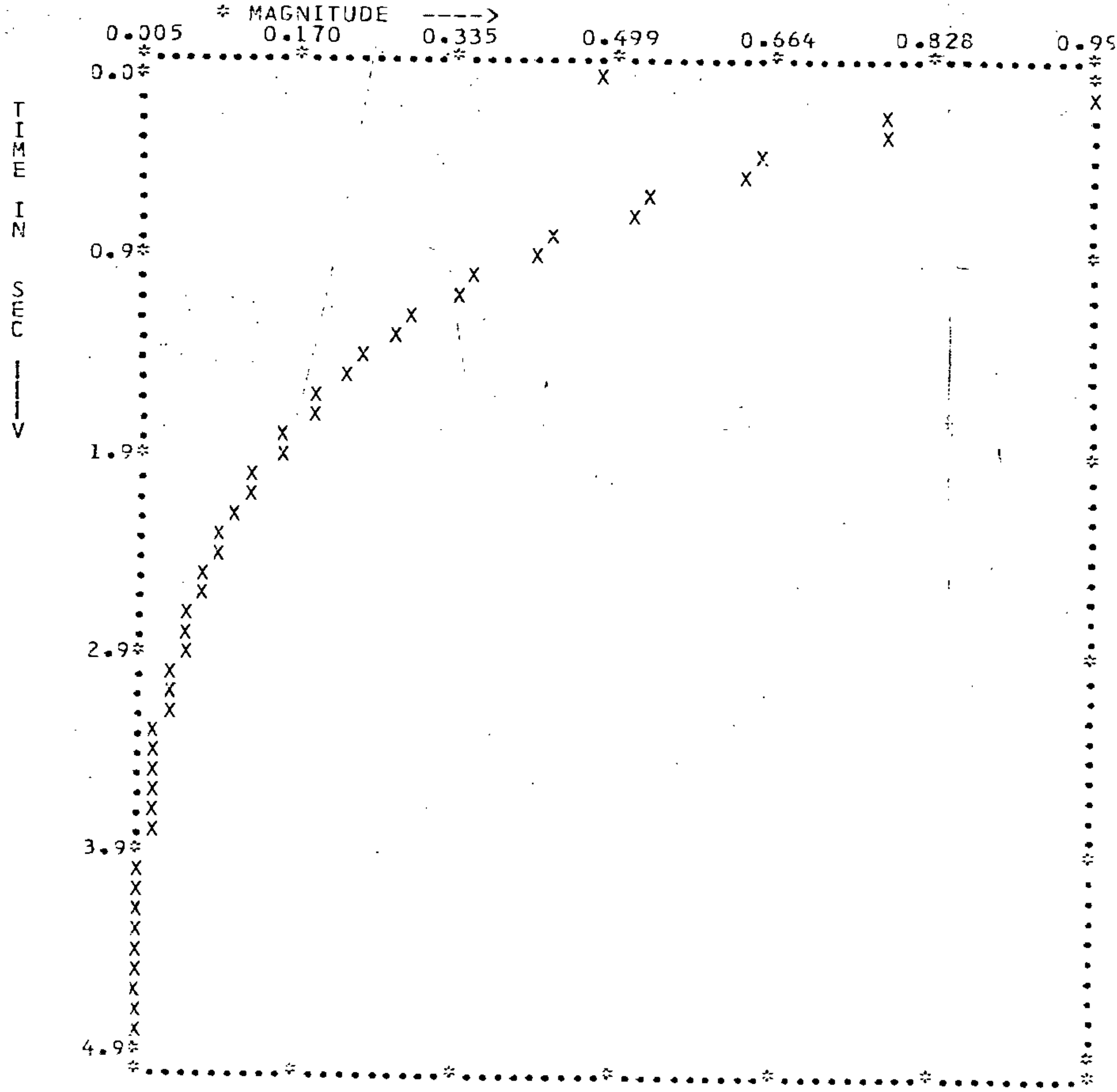

FIG. 2.3 : TIME DOMAIN RESPONSE OF EXP. FUNCTION USING IDFT

// EXEC

FREQUENCY SPECTRUM OF A SQUARE WAVE AS COMPUTED FROM FFT

	FREQUENCY(HZ)	REAL PART	IMAGINARY PART	MAGNITUDE
1	0.0	0.0	0.0	0.0
2	0.156	0.313E-01	-0.636E 00	0.637E 00
3	0.313	0.0	0.0	0.0
4	0.469	0.313E-01	-0.211E 00	0.213E 00
5	0.625	0.0	0.0	0.0
6	0.781	0.313E-01	-0.125E 00	0.129E 00
7	0.938	0.0	0.0	0.0
8	1.094	0.313E-01	-0.873E-01	0.928E-01
9	1.250	0.0	0.0	0.0
10	1.406	0.312E-01	-0.661E-01	0.731E-01
11	1.563	0.0	0.0	0.0
12	1.719	0.313E-01	-0.521E-01	0.608E-01
13	1.875	0.0	0.0	0.0
14	2.031	0.313E-01	-0.421E-01	0.525E-01
15	2.188	0.0	0.0	0.0
16	2.344	0.313E-01	-0.345E-01	0.465E-01
17	2.500	0.0	0.0	0.0
18	2.656	0.312E-01	-0.283E-01	0.422E-01
19	2.813	0.0	0.0	0.0
20	2.969	0.313E-01	-0.232E-01	0.389E-01
21	3.125	0.0	0.0	0.0
22	3.281	0.312E-01	-0.187E-01	0.364E-01
23	3.438	0.0	0.0	0.0
24	3.594	0.313E-01	-0.148E-01	0.346E-01
25	3.750	0.0	0.0	0.0
26	3.906	0.312E-01	-0.112E-01	0.332E-01
27	4.063	0.0	0.0	0.0
28	4.219	0.313E-01	-0.783E-02	0.322E-01
29	4.375	0.0	0.0	0.0
30	4.531	0.313E-01	-0.464E-02	0.316E-01
31	4.688	0.0	0.0	0.0
32	4.844	0.313E-01	-0.153E-02	0.313E-01
33	5.000	0.0	0.0	0.0
34	5.156	0.312E-01	0.153E-02	0.313E-01
35	5.313	0.0	0.0	0.0
36	5.469	0.312E-01	0.464E-02	0.316E-01
37	5.625	0.0	0.0	0.0
38	5.781	0.312E-01	0.783E-02	0.322E-01
39	5.938	0.0	0.0	0.0
40	6.094	0.313E-01	0.112E-01	0.332E-01
41	6.250	0.0	0.0	0.0
42	6.406	0.312E-01	0.148E-01	0.346E-01
43	6.563	0.0	0.0	0.0
44	6.719	0.312E-01	0.187E-01	0.364E-01
45	6.875	0.0	0.0	0.0
46	7.031	0.312E-01	0.232E-01	0.389E-01
47	7.188	0.0	0.0	0.0
48	7.344	0.313E-01	0.283E-01	0.422E-01
49	7.500	0.0	0.0	0.0
50	7.656	0.312E-01	0.345E-01	0.465E-01
51	7.813	0.0	0.0	0.0
52	7.969	0.312E-01	0.421E-01	0.525E-01
53	8.125	0.0	0.0	0.0
54	8.281	0.312E-01	0.521E-01	0.608E-01
55	8.438	0.0	0.0	0.0
56	8.594	0.312E-01	0.661E-01	0.731E-01
57	8.750	0.0	0.0	0.0
58	8.906	0.312E-01	0.873E-01	0.928E-01
59	9.063	0.0	0.0	0.0
60	9.219	0.312E-01	0.125E 00	0.129E 00
61	9.375	0.0	0.0	0.0
62	9.531	0.312E-01	0.211E 00	0.213E 00
63	9.688	0.0	0.0	0.0
64	9.844	0.312E-01	0.636E 00	0.637E 00

FIG. 2.4 : 64-POINT FREQUENCY SPECTRUM OF SQUARE WAVE COMPUTED USING FFT

of the above Eq. (2.30). Therefore the results should all be multiplied by $1/(NT) = 0.1563$ HZ, which is implemented in line 15 of the sample program. The theoretical value for $n=1$ is $X_1 = -j2/\pi = -j0.6366$, while the FFT output is $X_1 = 0.6368 \angle -87.18$. A small error is introduced in the FFT result, as expected in any numerical approximation scheme.

Another sample program given in Appendix - D4 illustrates the application of the DFT to generate a time sequence from a given set of frequency samples through IDFT, which can be implemented by IFFT. The test signal is a periodic square wave. It is known from Fourier series analysis that a square wave contains only odd harmonic in addition to its fundamental frequency. Here we use first four frequency components as fundamental, 3rd, 5th, and 7th harmonic. The fundamental amplitude is set to 1, 3rd as ⅓, 5th as 1/5 and 7th as 1/7. The result is shown in Fig. (2.5), the imaginary part of which is in the order of 1.0E-07. If more frequency components were added in the synthesis procedure, the output would be closer to a square wave.

2.4.2.3 ZERO PADDING:

Recalling the sampling theorem and Fig. (2.2) & Fig. (2.5) it was established that, to use DFT to evaluate samples of the frequency spectrum to a given frequency resolution, first

// EXEC

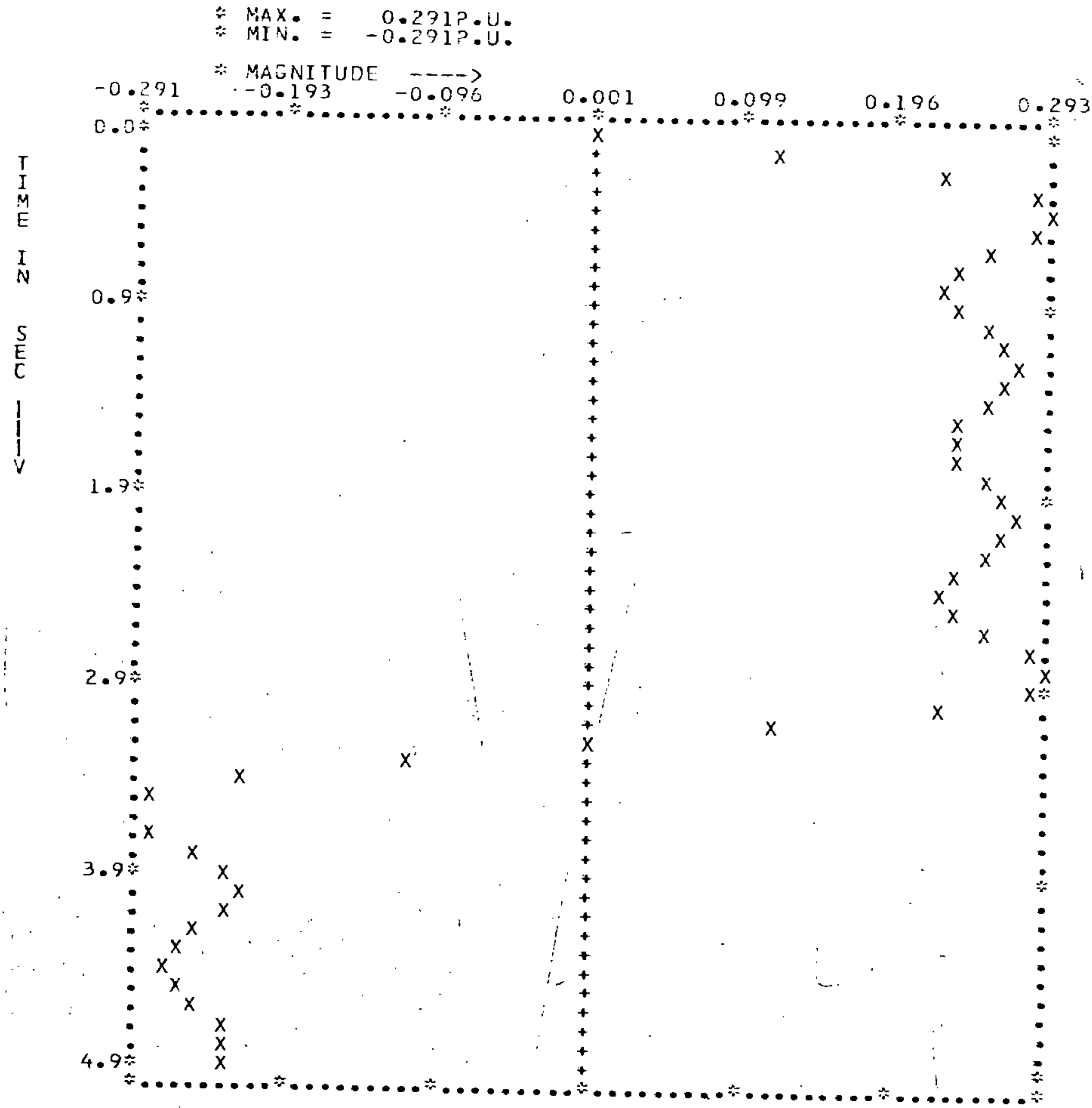

FIG. 2.5 : SYNTHESIZED SQUARE WAVE USING IFFT

the sampling interval and the number of signal samples N must be selected properly. Now if M samples of a continuous signal is given and if FFT at N frequency points are required to be found, where $N > M$ and the parameter M and T satisfy the constraints Eq. (2.2) and Eq. (2.5) , then the N-point DFT or FFT is still meaningful and it is possible to find out N-point FFT. In this case, (N-M) zeroes are appended to the end of the signal sequence to get an N-point sequence. It can be formed the N-point DFT as:

$$X_n = \sum_{k=0}^{N-1} x_k \exp(-j(2\pi/N)\,nk)$$

$$= \sum_{k=0}^{M-1} x_k \exp(-j(2\pi/N)\,nk) + \sum_{k=m}^{N-1} x_k \exp(-j(2\pi/N)\,nk)$$

$$= \sum_{k=0}^{M-1} x_k \exp(-j(2\pi/N)\,nk) \; ; \quad n=0,1,2 \ldots\ldots N-1$$

Since $x_k = 0$ for $k= M, M+1 \ldots\ldots N-1$. This zero-padding technique can be used to improve the frequency resolution of the DFT.

A sample program given in Appendix - D5 illustrates this zero-padding technique where the frequency function of Exp(-t) it taken and using IFFT the time response function is calculated. The results are shown in Fig. 2.6.

THE VALUE OF DW IS= 0.9812
& THE VALUE OF N IS = 64
& THE VALUE OF DT IS= 0.100E 00
& THE VALUE OF OMEGA IS = 0.100E 05

* MAX. = 0.992P.U.
* MIN. = 0.002P.U.

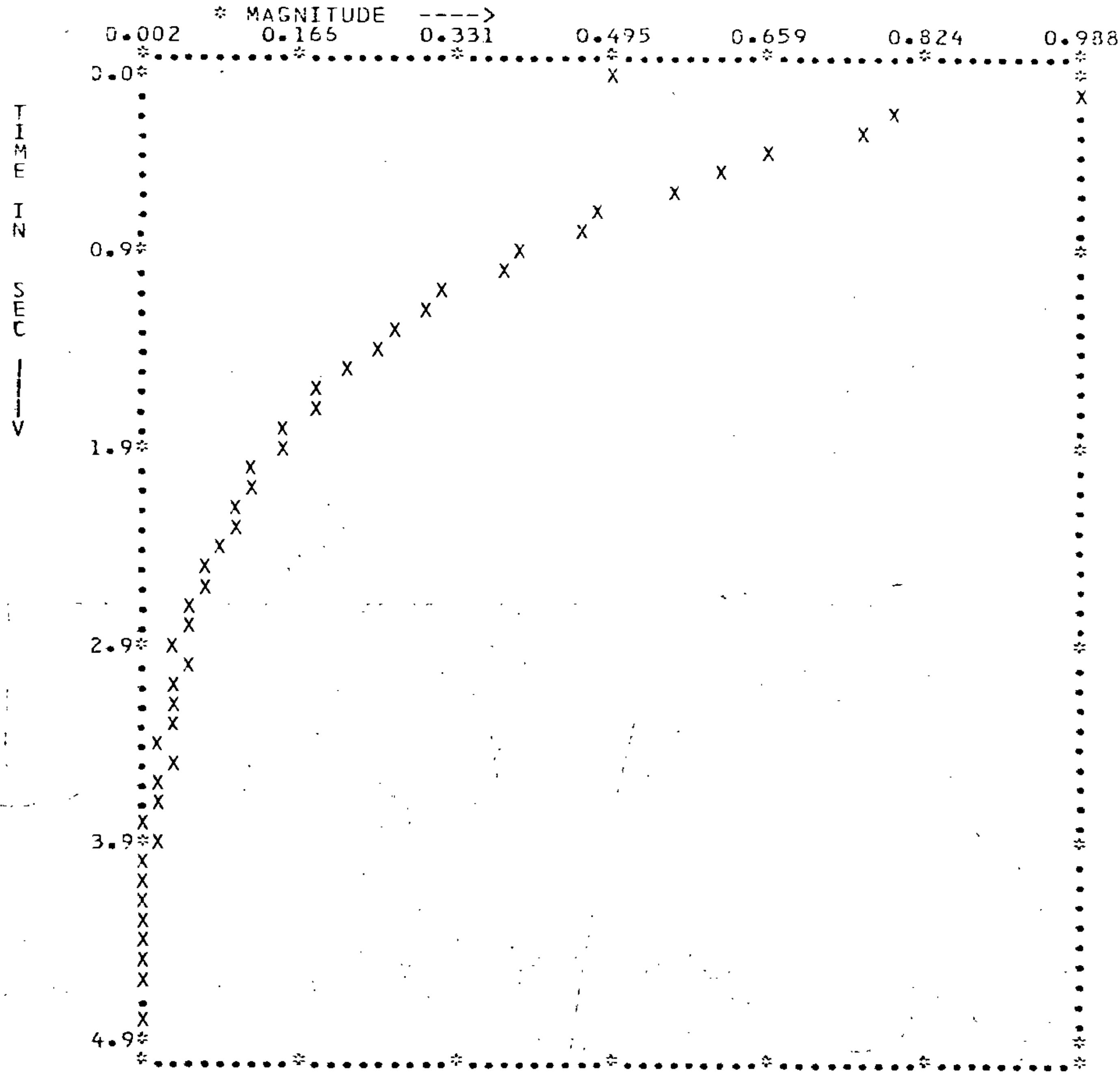

FIG. 2.6 : ZERO PADDING TECHNIQUE FOR
EXP(-T) TEST SIGNAL

2.5 ANALYSIS OF LONG TRANSMISSION LINE:

2.5.1 INTRODUCTION

In this section of voltage appearing at the open-end of a long transmission line due to the application of a step input voltage at the sending end is investigated, as a simple illustration of the method of analysis. For an elemental section of the line differential equations relating voltage and currents are written first. By modified Fourier transform, these equations are converted to algebric equations and the expression for receiving end voltage transform is written. Section 2.5.2.1 and 2.5.2.2 present mathematical analysis considering infinite source and source having finite impedances respectively. The receiving end continuous voltage transform is then sampled and by using inverse discrete Fourier transform (IDFT) the time-domain response is found out. The iversge DFT is implemented by FFT algorithm. A computer program has been written and the results are presented in sec.-2.5.5.

2.5.2 MATHEMATICAL ANALYSIS

2.5.2.1 LINE ENERGIZATION FROM A ZERO-IMPEDANCE SOURCE:

The defferential equations for the line with series inductance L H/mile, series resistance R ohms/mile and shunt capacitance C F/mile are:

$$- \frac{\partial}{\partial x} v(x,t) = L \frac{\partial i}{\partial t} (x,t) + Ri(x,t) \tag{2.32}$$

$$\frac{\partial}{\partial x} i(x,t) = C \frac{\partial}{\partial t} v(x,t) \tag{2.33}$$

The boundary conditions being:

$$\begin{aligned} &\text{at} \quad x=0 \quad v=0 \quad \text{for} \quad t<0 \\ &\qquad\qquad\quad = 1 \ \text{for} \quad t>0 \\ &\text{at} \quad x=1 \quad i=0 \quad \text{for all} \quad t \\ &\qquad\qquad\quad v = \nu_R \end{aligned} \tag{2.34}$$

Applying modified Fourier transform, i.e. multiplying throughout by exp (- (A+jw) t) and integrating over (0,α) w.r. to t, and rearranging terms we have :[12,1]

$$\frac{d2}{dx^2} V(x,s) = (R + A+jwL) (A+jw) CV (x,s)$$

$$= (R+Ls) \ CSV (x,s)$$

$$= \gamma^2 V(x,s) \tag{2.35}$$

and $\frac{d}{dx}$ $1(x,s) = -CSV (x,s)$ (2.36)

Where $\gamma = (R+Ls)\ Cs$; $S = A+jw$ and

$$V(x,s) = \int_0^{\alpha} v(x,t) \exp(-(A+jw))t \; dt$$

If the line is energized from a source of voltage v at the sending end (at x=0) and is terminated with a load impedance Z_L at x= 1, the expression for voltage transform at any point on the transmission line making use of the transformed boundary condition is:

$$V(x,s) = \frac{V_s(s) \quad \gamma \quad \sinh \gamma(1-x) + CSZ_L \cosh \gamma(1-x)}{\gamma \sinh 1\gamma + CSZ_L \cosh \gamma 1} \qquad (2.37)$$

Where $V_s(s)$ is the transform of v_s, the sending end voltage. Similarly the expression for voltage transform at any point on the transmission line V(x,s) can be written interms of sending end current transform $I_s(s)$ as :

$$V(x,s) = \frac{\gamma I_s(s)}{CS} \left[\frac{\gamma \sinh \gamma(1-x) + CSZ_L \cosh \gamma(1-x)}{\gamma \cosh \gamma 1 + CSZ_L \sinh \gamma 1} \right] \qquad (2.38)$$

Where $I_s(s)$ is the sending end current transform. Fig. 2.7 shows the line energized from an infinite source (having zero impedance source).

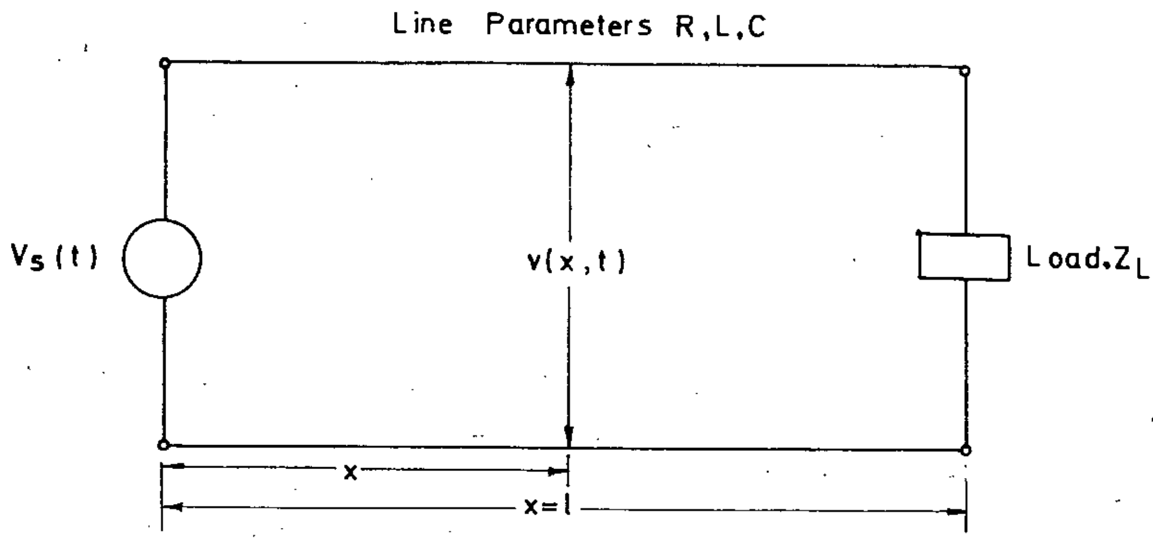

Fig. 2.7: Line Energized from Infinite Source.

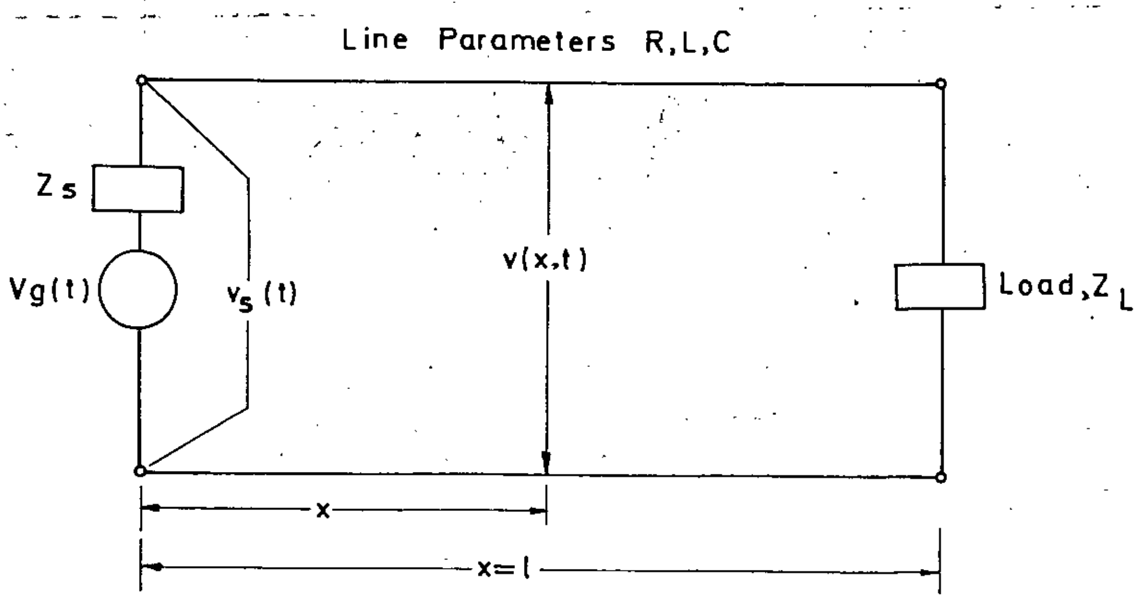

Fig. 2.8: Line Energized from Finite Source.

2.5.2.2 LINE ENERGIZATION FROM A SOURCE HAVING FINITE IMPEDANCE:

Fig. 2.8 show the line energized from a finite source. Here the parameters are:

C = line shunt capacitance F/mile

R, L = line series resistance ohms/mile and inductance H/mile.

R_s = internal source resistance

L_s = source inductance

$V_g(s)$ = generator voltage transform

and $V_s(s)$ = sending end voltage transform.

These parameters are related by:

$$V_g(s) = R_s + L_s\ S)\ I_s(S) + V_s(S) \tag{2.39}$$

Using Eq. (2.39) the equation of voltage transform at any point on the transmission line becomes:(1)

$$V(x,s) = \frac{V_g(S)\gamma\{\gamma \sinh \gamma(1-x) + CSZ_L \sinh \gamma(1-x)\}^2}{\left[Z_s CS\{\gamma \cosh \gamma l + CSZ_L \sinh \gamma l\} + \gamma\{\gamma \sinh \gamma(1-x) + CSZ_L \sinh \gamma(1-x)\}\right]\{\gamma \sinh \gamma l + CSZ_L \cosh \gamma l\}} \tag{2.40}$$

2.5.3 TIME DOMAIN RESPONSE

The time domain response $\nu(x,t)$ is obtained by taking the inverse modified Fourier transform of response transform $V(x,s)$ and is given by:

$$v\ (x,t) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} V(x,s)\ \exp\ (\ (\ A + Jw)\ t)\ dw \tag{2.41}$$

If the above integral is required to be evaluated numerically, it is necessary to trancate the range of integration from $(-\alpha,\alpha)$ to $(-\Omega,\Omega)$. The resulting Gibb's Oscillations can be reduced by incorporating the standard sigma factor (see chapter-3):

$$\sigma = \frac{\mathrm{Sin}\ (\pi w/\Omega)}{\pi w/\Omega} \tag{2.42}$$

So the integral becomes:

$$V_{\sigma}\ (\ x,t) = \frac{\exp(At)}{2\pi} \int_{-\Omega}^{\Omega} V\ (x,s)\ \exp\ (jwt)\ \frac{\sin(\pi w/\Omega)}{\pi w/\Omega}\ dw \tag{2.43}$$

For the time being if we neglect the constant term exp(At) from the integral Eq. (2.41), we have a similar equation like Eq.(1.6), and we can approximate this integral by the summation:

$$V(x,t) \simeq \frac{Dw}{2\pi} \sum_{n=-M}^{M-1} V(x,s) \exp(jw_n t) \qquad (2.44)$$

where $W_n = n * Dw$

If we evaluate the Eq. (2.44) at N discrete points in time, the the above equation will take the form:

$$V(x,t_k) = \frac{Dw}{2\pi} \sum_{n=-N/2}^{N/2-1} V(x,s) \exp(jw_n t_k) \quad \text{for } k= 0,1, .. N-1 \qquad (2.45)$$

By the help of Eq. (2.7a) and Eq. (2,7b) , Eq. (2.45) can be rewritten as:

$$V(x,t_k) = \frac{Dw}{2\pi} \sum_{n=0}^{N-1} V(x,s) \exp(jw_n t_k) \quad \text{for } k=0,1,2,...N-1 \qquad (2.46)$$

In article 2.3 it was shown that this equation can be expressed as Eq. (2.12) which is called the inverse discrete Fourier transform IDFT.

We evaluated the integral Eq. (2.43) by IDFT implemented by IFFT algorithm and then the time response is multiplied by the time function exp (At) to have the actual time domain response. The perfection and validation of the method of analysis has been established by varying the different parameters involved in Eq. (2.43) by a trail and error method and a pseudo-optimum value of each parameter is found out (See chapter-3 for details). The algorithm for computer program is given in the next section.

2.5.4 ALGORITHM FOR COMPUTER PROGRAM:

Step-1: Input the data for the value of π , maximum frequency limit (OMEGA), frequency step size DF in Hertz or DW in rad/sec, number of signal samples N, the value of smoothing parameter A, line parameter R,L,C and line length LL.

Step-2: Set w(1) = 0.0, I=1, calcualte N1 = 2* OMEGA/DW, N2= N1/2+1 and N3= N2 + 1, M = N/2+1, M1 = M+1 : If N > N1, zero padding is necessary.

Step-3: Define S=A+jw(I) and calculate the value of V(x,s) from Eq. (2.37) for the current value of S.

Step-4: Store the value of V(x,s) in F1 and F2 and if w(I) is not equal to zero, multiply F2 by sin(π*w/OMEGA)/(π*w/OMEGA) and store it in F2.

Step-5: Increase the value of w by Dw and I by one.

Step-6: Repeat steps 3,4 and 5 for N2 times.

Step-7: Set F1 and F2 to zero for N3 to M number of data.

Step-8: Fold and flip the data from M1 to N to preserve symmetry of DFT, take complex conjugate of M to 1 data of F1 and F2 to have M1 to N number of data.

Step-9: Call IFFT for F1 and F2.

Step-10: Set T=0.0

Step-11: Multiply time response F1 and F2 by Exp (A*T)

Step-12: Increase T by DT = π/OMEGA.

Step-13: Repeat steps 11 and 12 for N times and take the absolute value of F1 and F2.

Step-14: Write necessary output data & plot T VS FM1 and FM2 on the same scale simulteneously for first 100 time samples.

2.5.5 RESULTS FOR AN OPEN-ENDED TRANSMISSION LINE

The voltage transform at the open end of a long transmission line can be calculated from Eq. (2.37). At x=1, the value of the load impedance $Z_L = \alpha$ for an open line, so we have from Eq. (2.37):

$$V(x,1) = V_s(S)/\cosh \gamma l \tag{2.47}$$

The input voltage transform $V_s(S)$ for a step input is 1/s. Therefore, Eq. (2.47) becomes:

$$V(x,l) = 1/(S^*\cosh \gamma l) \qquad (2.48)$$

Results of the computer program (See Appendix-D6) written on the basis of the algorithm described in Sec-2.5.4 for Eq. (2.48) are presented in Fig. (2.9). The line parameters were chosen as:

Series resistance R= 2.75 E-03 ohm/mile

Series inductance L = 1.386 E-03 Henry/mile

Shunt capacitance C = 2.09 E-08 Farad/mile

and the length of the line LL = 10.0 miles.

The number of samples for IDFT and the IFFT evaluation were taken as N = 1024. The sampling frequency DF = 318 HZ, Dw = 2000 rad/sec. The other parameters, such as maximum frequency limit OMEGA = 1.0 E06, the smoothing parameter A = 1.0 E04, were chosen so that optimum output with greater accuracy can be obtained. Since the travel time τ = 53.8 μsec for a lossless transmission lime of length 10 miles, the number of samples at which the step rise would occur, for a time step size of 3.14 μ sec is 17.

From the plotted resut it is observed that first rise to about 2.0 p.u occur at 53.8 μ sec, and the output fall off to

OBSERVATION

THE VALUE OF N = 1024
THE VALUE OF LMT = 0.10000E 07
THE VALUE OF DW= 0.19531E 04
THE VALU E OF DT= 0.31415E-05
THE VALUE OF DF= 0.31086E 03

THE VALUE OF A IS= 0.100E 05

* MAX. = 2.017P.U.
* MIN. = 0.000P.U.

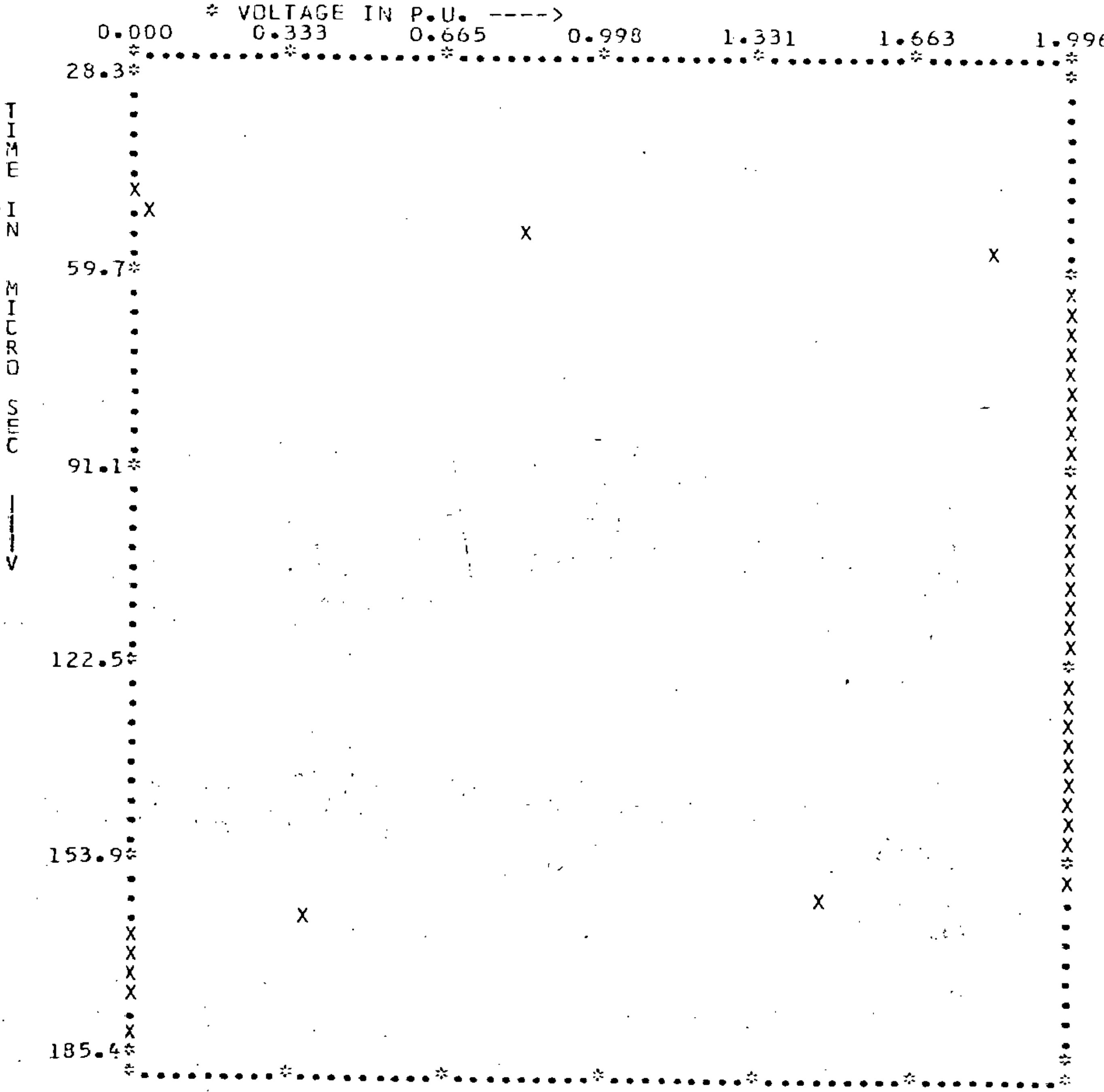

FIG. 2.9 : RESULTS FOR A 10-MILE LONG
OPEN-ENDED TRANSMISSION LINE

zero at 162 μ sec which is equal to 3τ. The output then rises again at 269 μ, sec, which is equal to about 5τ. This situation can be explained by the backward and forward propagation and 100% reflection at the open end of the traveling wave. The result are plotted for 100 time samples i.e. upto 314 μ sec which can be extended further if necessary.

A number of factors contribute to the derivation and the interrelationships among the parameters, and the outputs are investigated thoroughly (See chapter-3) in attempts to establish the method for application to an actual power system networks like the National Grid of Bangladesh Power Development Board.

CHAPTER - 3

DEVELOPMENT OF THE METHOD

3.1 INTRODUCTION

In this chapter details investigations are performed as an attempt to establish the psuedo-optimum integration parameters taking the same 10 mile long open-ended transmission line of chapter-2 as an example. Section 3.2 and sec-3.3 describes the effect of maximum frequency limit on the FFT solution, its associated problems and how to tackle those problems. In sec-3.4 the use of modified FFT is described. Section 3.4.2 describes the selection of smoothing parameter and the frequency step size for numerical evaluation of the time domain response from frequency spectrum, over first cycle. In sec- 3.5 the whole thing is considered for global response i.e. the time domain response over larger time duration.

3.2 THE EFFECT OF MAXIMUM FREQUENCY LIMIT (OMEGA) ON THE FFT SOLUTION.

Theoretically the frequency spectrum contains an infinite number of frequencies. To perform the integration of Eq. (2.41) numerically the infinite range ($-\infty,\infty$) of frequency must be truncated to a finite value, say, ($-\Omega,\Omega$). This truncation of the range of integration results in excluding the contributions from the higher frequencies thereby introducing an error called 'Truncation error'. As the value of maximum frequency limit is increased, the accuracy of the results would increase but if we are not ready to sacrifice the required frequency resolution nor the contribution due to the higher frequencies, then the number of samples (N) considered in the frequency domain must be of a higher value. If N increases, the number of complex arithmatic operations required to perform IFFT will increases (see Table 2.1) and thus the time required to perform Eq. (2.41) will increase. So a compromise must be set up in choosing the maximum frequency limit and the number of samples. Besides this, the rate of rise of the transient wave front depends on the maximum frequency limit. The transient response, upto a few cycles after the closing of the switch at sending end should be calculated as accurately as possible-so far as the rate of rise in concerned. This is because circuit breaker operations could very well be taking place during the initial few cycles. The rate of rise of the transient wave front has an important role on circuit breaker operation. Hence, due attention must be given in determining the range of frequency and to the front rise time.

A computer program (See Appendix-E1) has been developed to study the effect of the maximum frequency limit on the rate of rise. A 10 mile long open-ended transmission line is taken as an example. The results are shown in Fig. 3.1 and Fig. 3.2. Two different value of maximum frequency limit, OMEGA: 0.5E06 and OMEGA = 1.0E06 were considered and the outputs for this two values are compared in Fig. 3.1. The rate of rise obtained was

OBSERVATION 2

N = 1024
DT = 0.307E-05
A = 0.100E 05

DW =2000.000
DF = 318.316

CURVE-A FOR OMEGA= 0.100E 07
CURVE-B FOR OMEGA= 0.500E 06
MAX. A = 0.20587E 01 MIN. A = 0.95284E-02
MAX. B = 0.21481E 01 MIN. B = 0.23370E-01

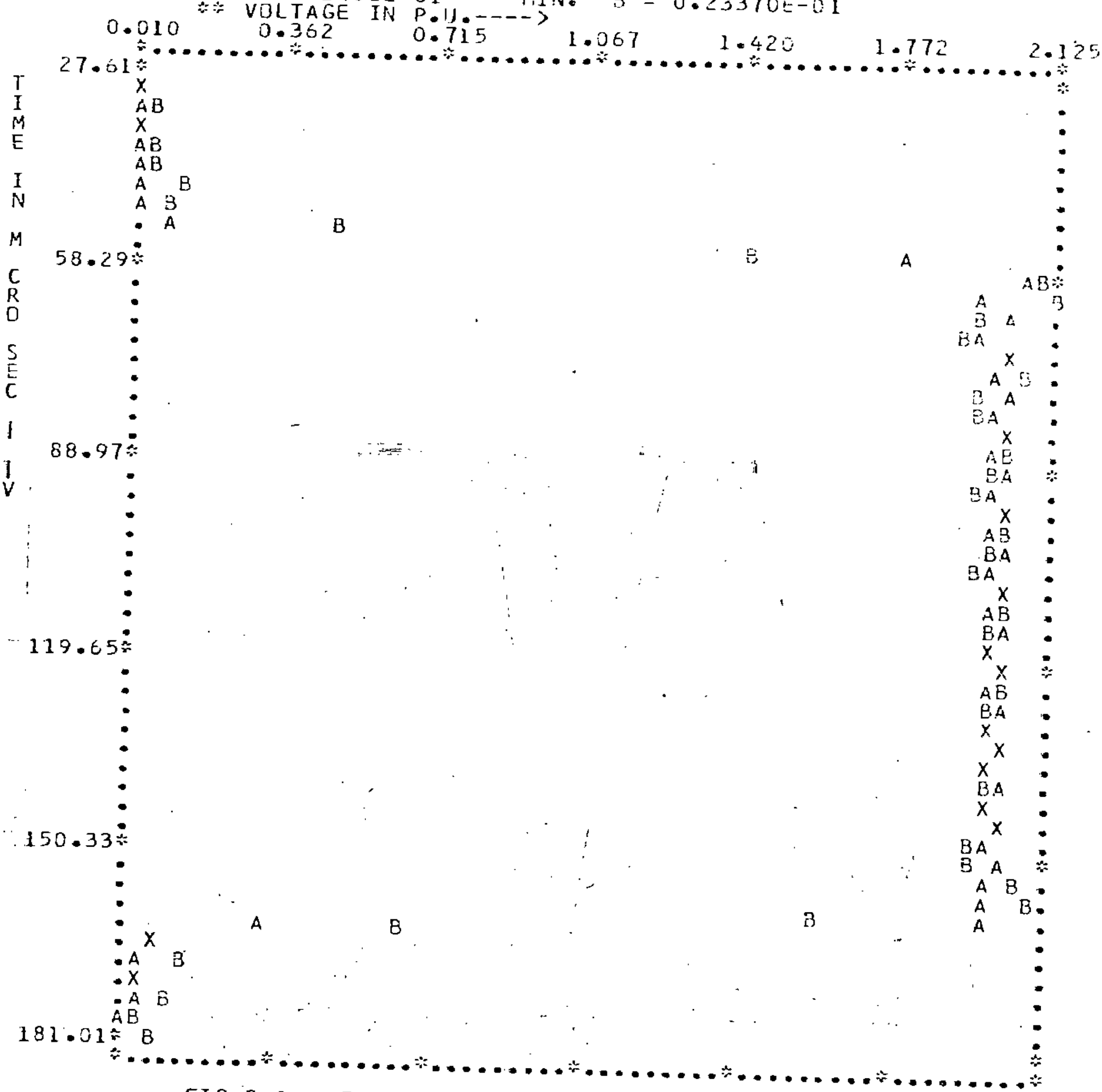

FIG.3.1 : EFFECT OF MAX. FREQUENCY LIMIT (OMEGA) ON THE FFT SOLUTION

OBSERVATION 1

```
N = 1024                  DW =2000.000
DT = 0.307E-05            DF =    318.316
A = 0.100E 05

CURVE-A FOR OMEGA=  0.200E 07
CURVE-B FOR OMEGA=  0.100E 07
MAX. A = 0.20410E 01       MIN.  A = 0.14314E-02
MAX. B = 0.20587E 01       MIN.  B = 0.95284E-02
```

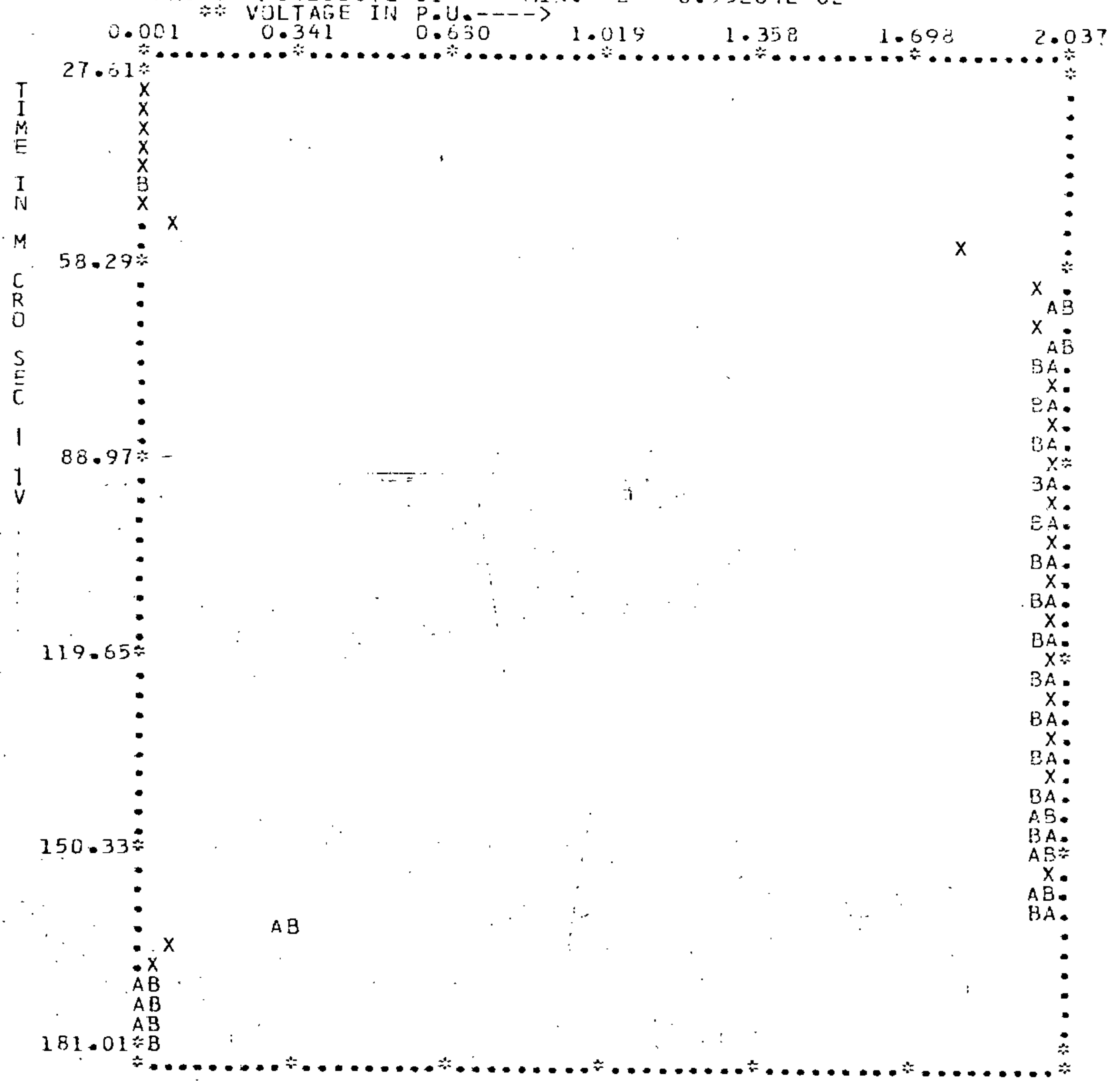

FIG.3.2 : EFFECT OF MAX. FREQUENCY LIMIT (OMEGA) ON THE FFT SOLUTION

slower in the case of OMEGA = 0.5E06 than that with OMEGA = 1.0E06. Hence OMEGA should be 1.0E06 or higher. A second comparison was made for OMEGA = 1.0E06 and 2.0E06 as shown in Fig. 3.2. But no improvement in the rate of rise was noted. Therefore the maximum frequency limit OMEGA = 1.0E06, seems to be suitable for the purpose.

3.3 GIBB'S OSCILLATION: [1,11]

The Gibb's oscillation which arise, when only a finite number of terms of an infinite Fourier series are used for synthesis are pronounced (See Fig. 3.1 & Fig. 3.2) and slow to die away. In fact the Fourier series, inherently, does not converge at points of discontinuity. So, the amplitude of these oscillations are the most marked at the points of discontinuity. The magnitude of the oscillations is largely independent of the number of terms considered, as long as sufficient number of terms are summed up. Lanczos[11] showed a method of handling this problem.

3.3.1 SIGMA FACTOR FOR SERIES:

When a trigonometric series in used to approximate a given function f(t), the partial sum may be written as:

$$f_m(t) = \sum_{k=-(m-1)}^{m-1} C_k \exp\,(jk\omega t) \tag{3.1}$$

Where $$C_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \exp(-jkx)\, dx \qquad (3.2)$$

and $$f(t) = \sum_{k=-\alpha}^{\alpha} C_k \exp(jk\omega t) \qquad (3.3)$$

Gibb's oscillations have peak and crest arround the true value. The local average of this function $f_m(t)$ over the range $\pm \pi/m$ should lie close to the actual value. The local average of $f_m(t)$ is:

$$f\ (t) = \frac{m}{2\pi} \int_{-\pi/m}^{\pi/m} f_m(t+\tau)\, d\tau \qquad (3.4)$$

It is readily shown by substituting for f_m in Eq. (3.4) from Eq. (3.1) and interchanging the order of integration and summation that:

$$f_\sigma(t) = \sum_{k=-(m-1)}^{m-1} \sigma_k C_k \exp(\ jk\omega t) \qquad (3.5)$$

Where $\sigma_k = \frac{\sin(\pi k/m)}{\pi k/m}$ is Lanczos sigma factor (3.6)

For most practical purposes $f_\sigma(t)$ is a better representation of $f(t)$ than $f_m(t)$.[11]

3.3.2 SIGMA FACTOR FOR INTEGRAL:[1,11]

The inverse Fourier integral is defined as:

$$f(t) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} f(\omega) \exp(j\omega t) \, d\omega \qquad (3.7)$$

This development directly follows from the Fourier series as the time period of the function is allowed to approach infinity.

By truncating the range of integration from $(-\alpha, \alpha)$ to $(-\Omega, \Omega)$ in the above integral, a function similar to $f_m(t)$ may be expressed as:

$$f_\Omega(t) = \frac{1}{2\pi} \int_{-\Omega}^{\Omega} F(\omega) \exp(j\omega t) \, d\omega \qquad (3.8)$$

If we take the local average of $f_\Omega(t)$ over the range $\pm \pi/\Omega$, we obtain:

$$f_\sigma(t) = \frac{\Omega}{2\pi} \int_{t-\pi/\Omega}^{t+\pi/\Omega} f_\Omega(\tau) \, d\tau \qquad (3.9)$$

or $$f_\sigma(t) = \frac{1}{2\pi} \int_{-\Omega}^{\Omega} \sigma F(\omega) \exp(j\omega t) \, d\omega \qquad (3.10)$$

Where $$\sigma = \frac{\sin(\pi\omega/\Omega)}{\pi\omega/\Omega} \qquad (3.11)$$

σ is reffered to as the standard sigma factor.

3.3.3 INCORPORATION OF STANDARD SIGMA FACTOR TO REDUCE GIBB'S OSCILLATIONS:

Fig. 3.1 & Fig. 3.2 represent the voltage appearing at the receiving end of a 10 mile long open ended transmission line, for a unit step input at the sending end. In order to reduce Gibb's oscillations in the results for our 10-mile long line, a program incorporating the standard sigma factor has been developed (See Appendix-E2) keeping the value of $\Omega = 10^6$. The output of the program is compared in Fig. 3.3, against that for the case of no sigma factor. It is observed that incorporation of the standard sigma factor gives much more improved results so far as the unwanted Gibb's oscillations are concerned.

3.4 THE MODIFICATION OF FOURIER INTEGRAL:

The inverse Fourier transform is:

$$f(t) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} F(\omega) \exp(j\omega t) \, d\omega \qquad (3.12)$$

Where $F(\omega)$ is the frequency response and the integration is carried out along the real axis.

For numerical evaluation the range of the integral must be made finite and any resulting Gibb's oscillations can be

OBSERVATION = 1

THE VALUE OF N = 1024
THE VALUE OF OMEGA = 0.10000E 07
THE VALUE OF DW= 0.20000E 04
THE VALUE OF DT= 0.30679E-05
& THE VALUE OF A = 0.10000E 05

MAX. A = 0.20587E 01　　MIN. A = 0.95284E-02
MAX. B = 0.20211E 01　　MIN. B = 0.55808E-03

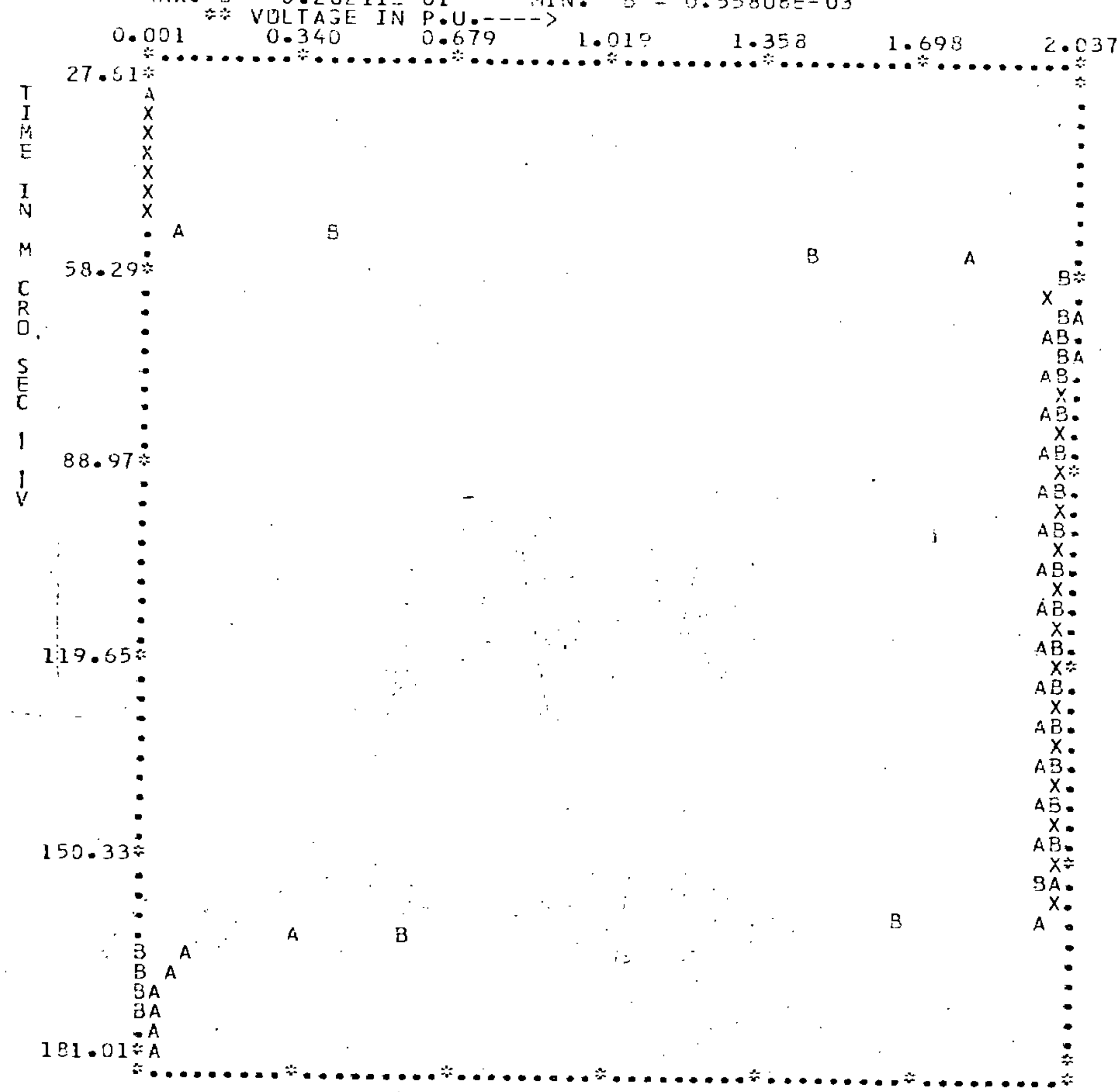

FIG.3.3: EFFECT OF SIGMA FACTOR ,CURVE -A WITHOUT SIGMA & CURVE-B WITH SIGMA FACTOR　　X FOR DOUBLE PL

reduced by incorporating the sigma factor as described in the previous article. However the poles of the frequency response F(w), of a transmission line or a power system, may, and in many cases do, lie very close to the real axis. This causes the integrand to peak, over a series of small intervals lying in the neighbourhood of the poles and hence for accurate numerical integration, it is necessary to take a very small step length, which also makes the computation time larger. This limitations can be overcome by using the Modified Fourier Transform [12] and at the same time this secures the additional advantage of avoiding problems associated with the convergence of the integrals involved, at the points of discontinuity in the time domain. The modification is done by incorporating an artificial forcing function U(A,t), where 'A' is the smoothing parameter- so named because it smoothens out the peaks of the integrand.

Let the differential equation of a system be:

$$F(D)\ f(t) = g(t), \quad D = d/dt \tag{3.13}$$

Where f(t) is the output, g(t) the input, both of which are zero for t<o.

In taking Modified Fourier Transform, Eq. (3.13) is multiplied throughout by Exp $(-(A+j\omega)t)$ and integrated ω, r, to 't' over the range $(-\infty,\infty)$, or, since, $f(t)$ and $g(t)$ are zero for $t<0$, over the range $(0,\infty)$; Eq. (3.13) then becomes:

$$F\,(A+j\omega)\ \bar{f}_{+}\,(A+j\omega) = \bar{g}_{+}\,(A+j\omega) \tag{3.14}$$

Where $$\bar{g}_{+}\,(A+j\omega) = \int_0^{\infty} g(t)\ \exp(-(A+j\omega)\ t)\ dt \tag{3.15}$$

and A can be chosen large enough to ensure that the integral converges. The expression for $\bar{f}_{+}\,(A+j\omega)$ can then be found and $f(t)$ is determined from:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{f}_{+}(A+j\omega)\ \exp\,((A+j\omega)\ t)\ d\omega, \quad t>0$$

$$= \frac{\exp\,(At)}{2\pi}\int_{-\infty}^{\infty} \bar{f}_{+}\,(A+j\omega)\ \exp\,(j\omega t)\ d\omega, \ t>0 \tag{3.16}$$

and $$0 = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{f}_{+}\,(A+j\omega)\ \exp\,((A+j\omega)\ t\ )\ d\omega, \quad t<0 \tag{3.17}$$

For real, stable systems all the roots of $F(D)$ have negative real parts and consequently the zeroes of $F(A+j\omega)$ in the $\zeta = \omega + j\eta$ plane (say) lie above the line $I_m(\zeta) = A$. Similarly the poles of $\bar{g}\,(A+j\omega)$ lie either above this line or in it.

Hence all the poles of $\bar{f}_+ (A+j\omega)$ lie on or above the line $I_m(\zeta) = A$. Since the path of integrations in Eq. (3.16) and Eq. (3.17) is the real axis the process has the effect of moving the poles away from it by a distance 'A' which we are relatively free to choose. The effect of this choice on the integrand is illustrated in Fig. 3.4.

Limiting the range of integration to Ω and incorporating the standard sigma factor, the inverse integral $f_\sigma(t)$ can be found also from the Eq. (3.16) as described below:

$$f_\sigma(t) = \frac{\exp(At)}{2\pi} \int_{-\Omega}^{\Omega} \bar{f}_+ (A+j\omega) \exp(j\omega t) \frac{\sin(\pi\omega/\Omega)}{\pi\omega/\Omega} d\omega \tag{3.18}$$

Inverse Discrete Fourier Transform (IDFT) of this equation can then be implemented using the FFT algorithm.

3.4.1 THE EFFECT OF SMOOTHING PARAMETER ON THE SHAPE OF INTEGRAND:

The Modified Fourier Transform is used to overcome the effects, on the integrand, of resonances of the system. This method has the effect of moving the integrand away from the real axis and thus resulting in a relatively smoother integrand. This will now be seen for the same transmission line of length 10 miles, described in chapter-2.

OBSERVATION 1

THE VALUE OF N= 1024
& THE VALUE OF OMEGA = 0.10000E 07
& THE VALUE OF DW = 0.20000E 04
& THE VALUE OF DT = 0.30679E-05

CURVE -A FOR A = 0.100E 04
CURVE -B FOR A = 0.100E 05
CURVE -C FOR A = 0.200E 05

+ FOR DOUBLE PLOT
* FOR TRIPPLE PLOT

MAX. A= 0.999E-03 MIN. A= 0.162E-04
MAX. B= 0.871E-04 MIN. B= 0.129E-04
MAX. C= 0.305E-04 MIN. C= 0.702E-05

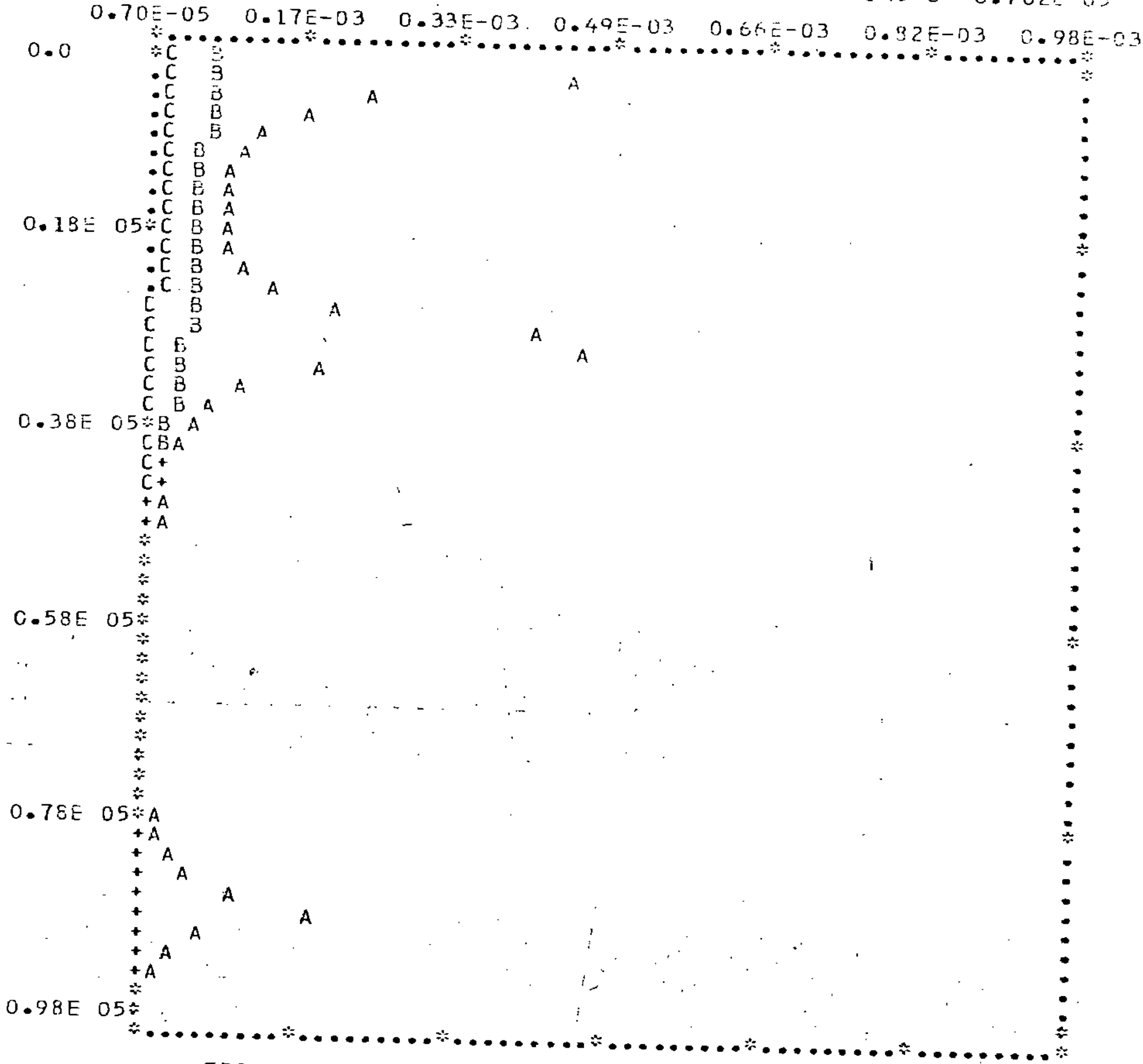

FIG. 3.4 : EFFECT OF A ON THE SHAPE OF INTEGRAL

The inverse Fourier integral, including the standard sigma factor can be written as:

$$f_\sigma(t) = \frac{\exp(At)}{2\pi} \int_{-\Omega}^{\Omega} F(A+j\omega) \exp(j\omega t) \frac{\sin(\pi\omega/\Omega)}{(\omega\pi/\Omega)} d\omega \tag{3.19}$$

Expression for $F(A+j\omega)$ in Eq. (3.19) is computed for three different values of 'A' and the results are plotted in Fig. 3.4 (See Appendix-E3. for computer program). It is evident that the response transform $F(A+j\omega)$ for $A = 1.0E04$ is smoother than that for $A = 1.0E03$, but the shape for $A = 1.0E05$ is the smoothest. Hence, a larger step length can be chosen, for the same degree of accuracy, for the integrand with a larger value of 'A' thus requiring a lesser computation time.

3.4.2 SELECTION OF STEP LENGTH & SMOOTHING PARAMETER:

It was observed that the voltage transform looses its peaky nature with the increase in the value of smoothing parameter A. The voltage wave shape (time-domain) at the receiving end of a ten mile long open-ended line is calculated for A=1.0E03 and 1.0E04, for the step length and maximum frequency limit being fixed at 2.0E03 and 1.0E06 respectively. The results are plotted in Fig. 3.5. It is observed that for A= 1.0E04 a much better representation of the true output is obtained than with A=1.0E03.

// EXEC

OBSERVATION 1

THE VALUE OF N = 1024
THE VALUE OF OMEGA = 0.10000E 07
THE VALUE OF DW= 0.19531E 04
THE VALUE OF DT= 0.31415E-05
THE VALUE OF DF= 0.31026E 03
CURVE -A FOR A = 0.100E 04
CURVE -B FOR A = 0.100E 05
MAX. A = 0.20920E 01 MIN. A = 0.95307E-03
MAX. B = 0.20171E 01 MIN. B = 0.49173E-04
** VOLTAGE IN P.U.---->

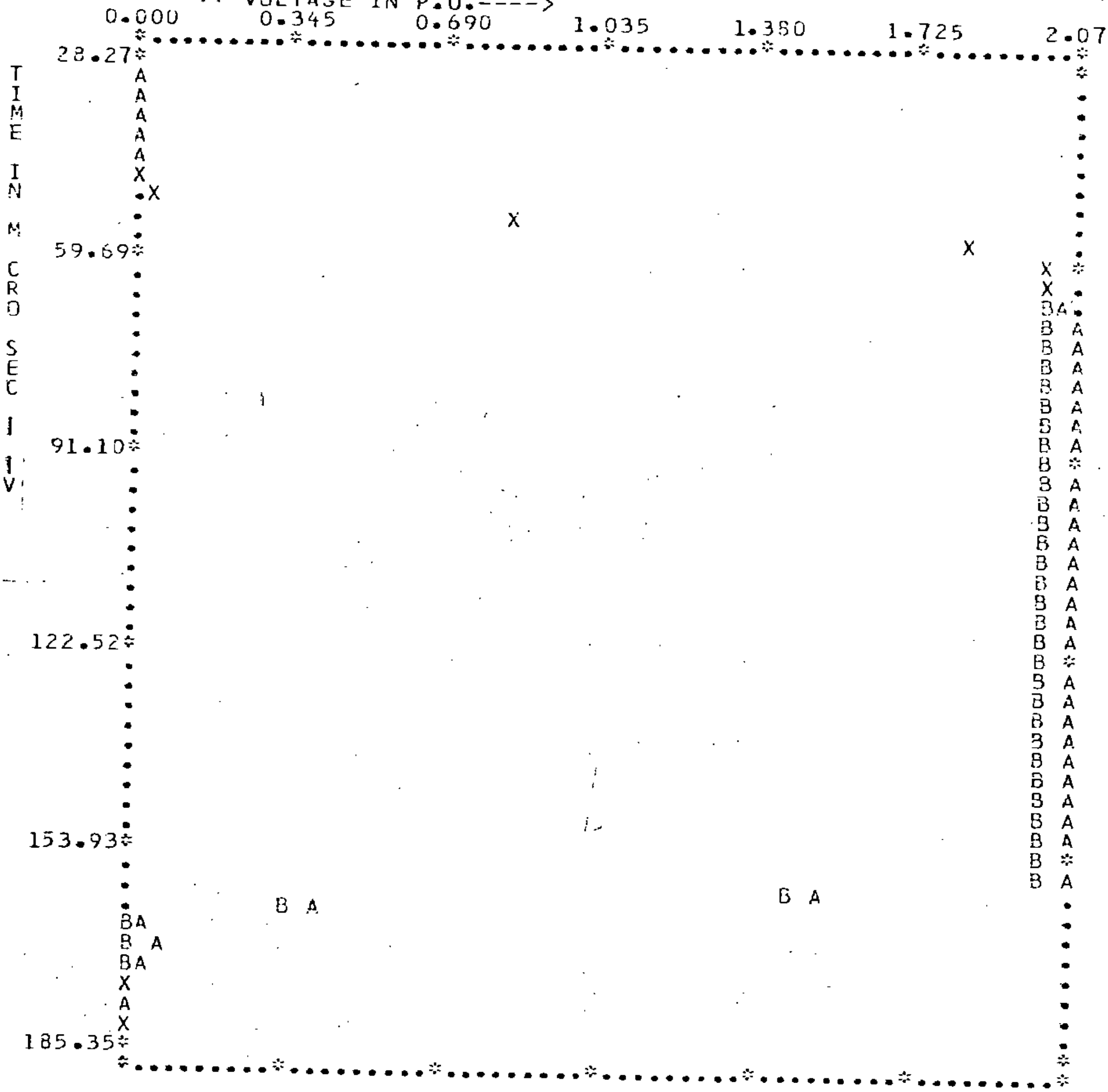

FIG.3.5 : EFFECT OF SMOOTHING PARAMETER A
ON THE FFT SOLUTION

This situation can be explained as follows. For A = 1.0E04, the integrand i.e. voltage transform is sufficiently smooth for a step length 2000 to be chosen, whereas the same step length of 2000 cannot cater for the relatively peaky nature of the integrand at A = 1.0E03 and introduces noticeable errors in the calculated time domain response. To obtain a good result with A = 1.0E03, the step length must therefore be chosen at a smaller value than 2000, but this would increase the computational time. So there is a role of 'A' in the choice of step length and on the computation time involved. The natural tendency would be to choose as high a value of 'A' as possible, so that computation time could be reduced as a result of the larger step length that can be chosen. To verify this, the step length DW was kept constant at the value of 2000, while a still higher value of 'A', A = 1.0E05, was chosen and the result was compared with the result obtained for A = 1.0E04 (See Fig. 3.6). But unexpectedly, the output for A = 1.0E05 is a worse representation of the true wave shape. There may be at least two reasons for this apparently anomalous result. In the process of IDFT evaluation of the integral, errors inevitably occur due to truncation and rounding etc. These errors are also multiplied by the factor exp (At), the value of which is increases exponentially with the value of 'A'. Secondly with the increase of the value of 'A' the integrand (i.e. voltage transform) is not only smoothened out, but the spectrum also extends in frequency. That is, relative contribution due to the higher frequency terms increase. Therefore to obtain the same

OBSERVATION 2

THE VALUE OF N = 1024
THE VALUE OF OMEGA = 0.10000E 07
THE VALUE OF DW= 0.19531E 04
THE VALUE OF DT= 0.31415E-05
THE VALUE OF DF= 0.31086E 03
CURVE -A FOR A = 0.100E 05
CURVE -B FOR A = 0.500E 05
MAX. A = 0.20171E 01 MIN. A = 0.49173E-04
MAX. B = 0.20293E 01 MIN. B = 0.53781E-04

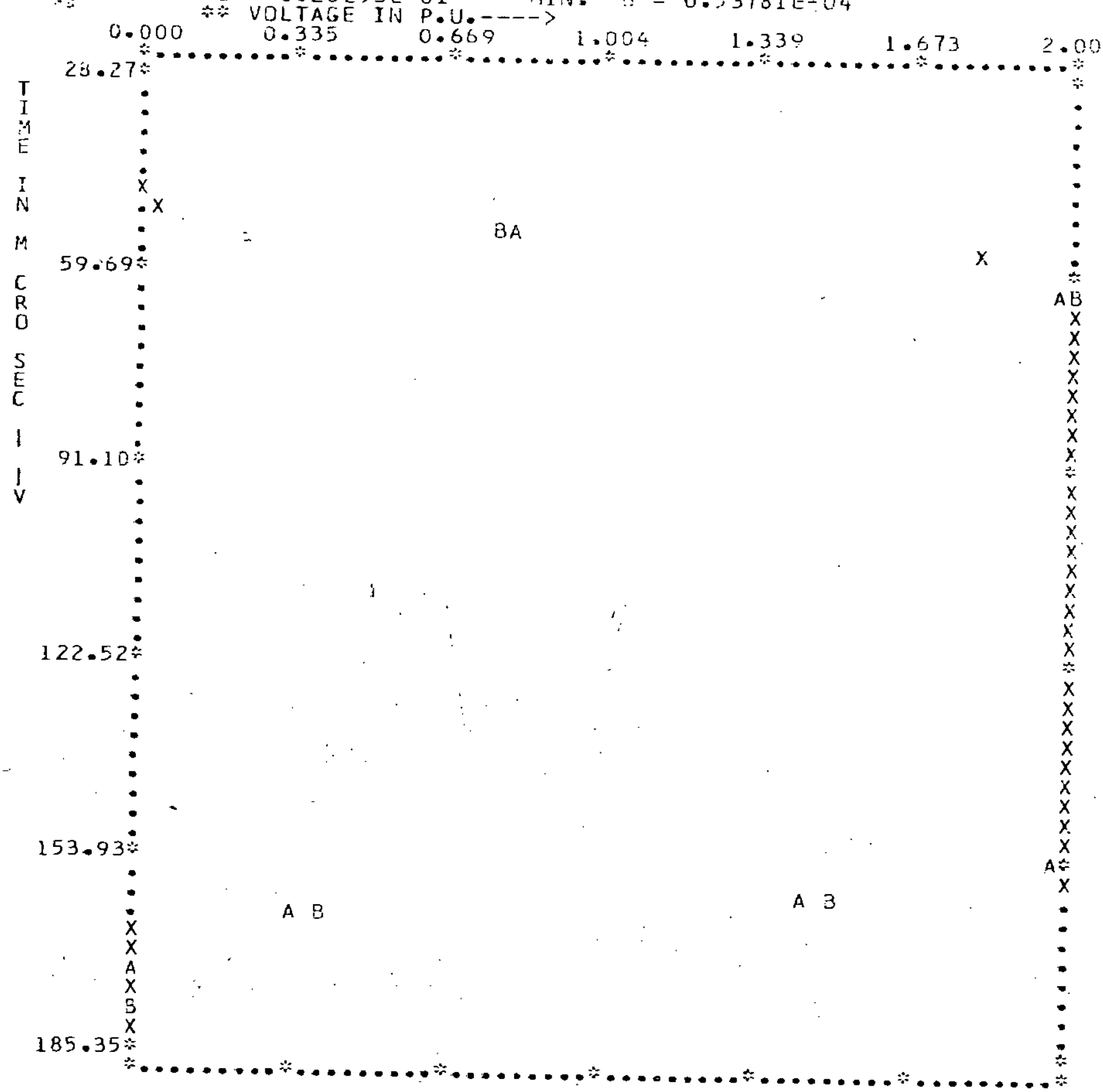

FIG.3.6 : EFFECT OF SMOOTHING PARAMETER A
ON THE FFT SOLUTION

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result the value of maximum frequency chosen must be increased at the same time, but that would involve an increase of the computation time. Thus, the truncation error increases with increasing value of 'A'.

Therefore, it is important to realise that the value of 'A', for a certain value of DW (step length) and OMEGA (the maximum frequency limit), cannot be chosen at will. In our calculations, we limit the value of 'At' to less than 2 (See section 3.5.2 for details).

To investigate the effect of step length DW on the result the same 10 mile long transmission line (see ch. 2) is taken. The computer program is given in Appendix E-5. Fig. 3.7 a,b, shows the output for DW = 1000, 2000 and 2000, 4000 respectively. It is observed that for DW = 2000 the outputs are close to the expected result. This value of DW = 2000 also satisfies the constraint set in the sampling theorem, as stated below. Choosing OMEGA = 1.0E06 in the relation N>2*OMEGA/DW, it is seen that DW must be greater than 1954 (which is true since DW = 2000) for N = 1024.

3.5 GLOBAL RESPONSE:

3.5.1 INTRODUCTION

Section 3.2 to sec. 3.4 describe methods of overcoming the two basic problems associated with the numerical inversion of

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THE VALUE OF OMEGA =  0.100E 07
& THE VALUE OF A IS=  0.200E 04

CURVE-A FOR DW = 1000.000        N = 2048
CURVE-B FOR DW = 2000.000        N = 1024
MAX. A = 0.20219E 01       MIN.  A = 0.68729E-03
MAX. B = 0.20256E 01       MIN.  B = 0.15330E-02
```

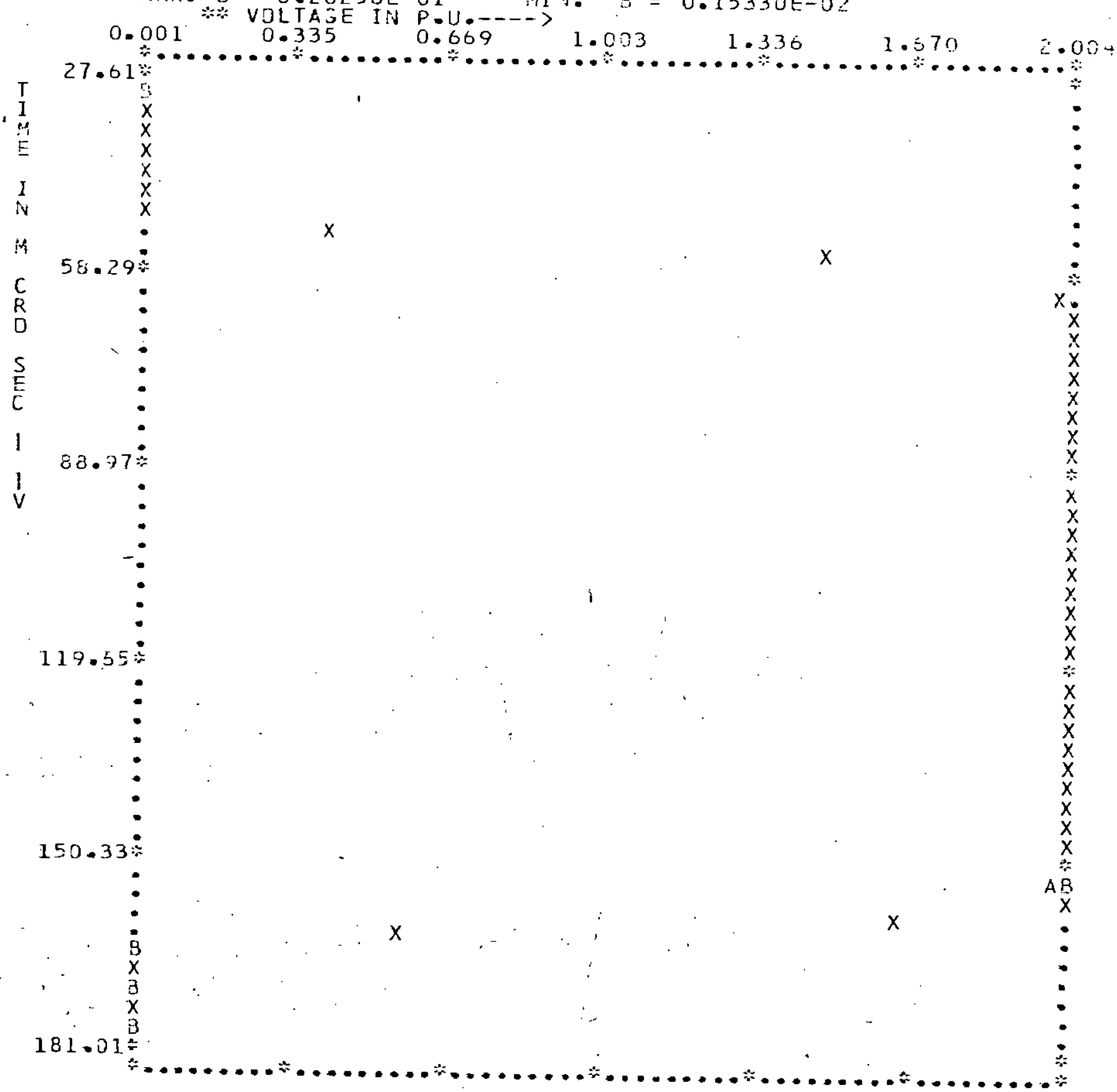

FIG.3.7A : EFFECT OF FREQ. STEP DW
ON THE FFT SOLUTION

THE VALUE OF OMEGA = 0.10E 07
& THE VALUE OF A IS= 0.200E 04

CURVE-A FOR DW = 2000.000 N = 1024
CURVE-B FOR DW = 4000.000 N = 512
MAX. A = 0.20256E 01 MIN. A = 0.15330E-02
MAX. B = 0.20987E 01 MIN. B = 0.36777E-02

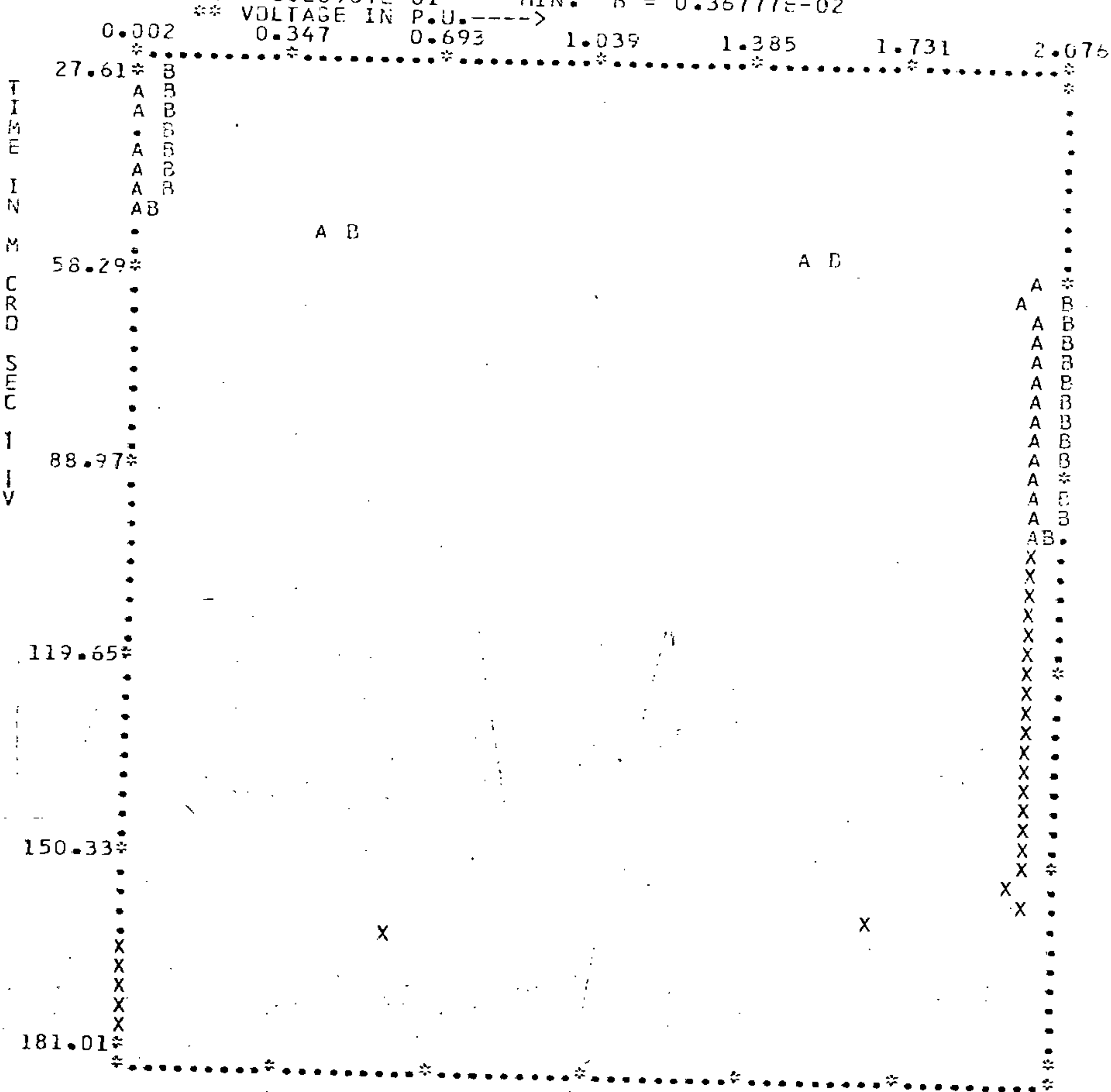

FIG.3.7B : EFFECT OF FREQ. STEP DW
ON THE FFT SOLUTION

the complex integrals Eq. (2.43) encountered when using transform techniques to derive the transient response of any power system network. In sec- 3.2 and sec- 3.3 it was shown that the effects caused by the necessary truncation of the infinite range of integration could be minimized by the use of standard sigma factors, and in sec- 3.4 the modified FFT was used to overcome the effects, on the integrand, of resonances of the system.

In using the method for calculation of transients in real power system network, it soon becomes desirable to use it over a longer time range than that considered so far, In the preceding sections, the method was used for examining in detail the initial transient response over arround the first cycle. If the same optimum values of the different integration parameters are used in calculating the output over a longer period of time, i.e. over several cycles of the transient response, accuracy of computation may decreases. At the same time, the computation time involve would be longer and thus expensive. Besides this, if the length of the transmission line is greater than (say) 100 miles, the travel time would be longer and for this longer time the methods of selecting the integration parameters described so far could be useless. Now it is seldom necessary that great accuracy needs to be combined with such an extended time interval. Generally it is desired to obtaine an overall picture of the phenomena which is reffered to as the 'Global Response'.

Of course, if any point of particular interest appears in the global response than that particular time interval can be investigated in finer detail, using optimum parameter values corresponding to that particular region of interest. The choice of optimum parameter values appropriate when the global response is required over several periods is investigated in the next section.

3.5.2 GLOBAL RESPONSE OVER SEVERAL PERIODS:

When it is desired to obtain the global response for a time interval extending over a number of cycles, many points require serious consideration.

First, the sampling interval and the number of signal samples must be selected properly, to use the IDFT (IFFT) for evaluating time domain response from the response transform. Here, care must be excercised to maintain the periodic property of the DFT and the symmetry property for a real time signal (See Appendix-A).

Recalling from the sampling theorem that, in order for a band-limited signal to be recoverable from the samples of the signal, the sampling frequency must be equal to at least twice the highest frequency component in the signal, i.e.

$$T < \pi / \text{ OMEGA} \tag{3.26}$$

Where T is the time domain sampling period. Assuming the maximum frequency content, OMEGA for a band-limited spectrum function to be 1.0E06, the maximum value of time step length comes out from the above inequality to be π μsec. In order to reduce aliasing error, the time step chosen should be much lower than its limiting value, say, 2.0E06 sec. If the time domain response is desired for a time duration of 2048 μsec, then the value of N would be 1024 for the above sampling rate. From Eq. (2.5) we have the constraint on the number of signal samples as:

$$N > 2*OMEGA/DW \qquad (3.27)$$

Where $$DW = 2\pi/(N*T) \qquad (3.28)$$

It is observed that for a value of DW = 3068, the minimum number of signal sample (N) should be 652 which is less than 1024 thus satisfying inequality (3.27). If we increase the sampling rate to T = 3.0E-06 sec, the value of DW comes out to be 2045 for the same value of N (i.e. increasing the time interval under investigation). In both the cases the value of A is 1.0E04. Results obtained using the two sets of parameter values are compared in Fig. 3.8 (See Appendix-E6 for computer program). The latter set of parameter value produces a better representation of the output. The value of frequency step size DW is less for latter set of parameter values than that of former. For this accurate sample

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THE VALUE OF OMEGA = 0.100E 07
& THE VALUE OF A IS= 0.200E 04

CURVE -A FOR DW = 3067.896 & DT = 0.200E-05
CURVE -B FOR DW = 2045.264 & DT = 0.300E-05
MAX A = 2.025 MIN. A = 0.008
MAX B = 2.021 MIN. B = 0.000

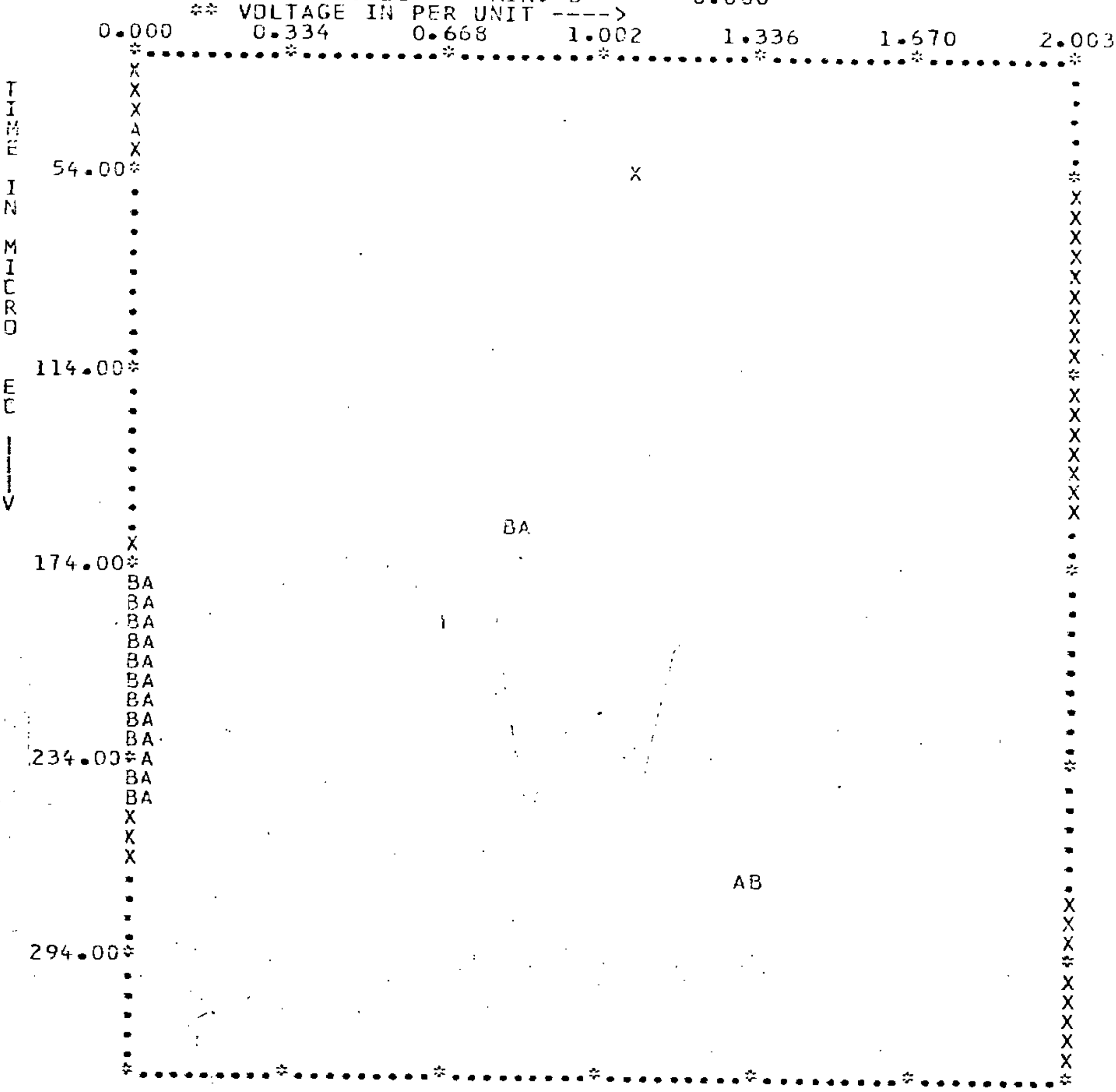


FIG.3.8 : EFFECT OF SAMPLING INTERVAL T
ON THE FFT SOLUTION

value is obtained using smaller frequency step size & hence better representation of the output.

If it is necessary to have a global representation for a time range of 4096 μsec, then for a 2 μsec time step, the corresponding frequency step chosen has to be longer than 1533 rad/sec, so that inequality (3.27) is satisfied. It would appear to be necessary to reduce the step length DW when the response over a larger time interval is desired.

Secondly, in the inverse transforms, the integral Eq. (3.19) is multiplied by a term exp (A*T). It is observed that if A = 1.0E04 is used for a time step T = 3.14 μsec then only 64 samples of the time response make the value of NAT = 2.0, which can be taken as permissible error. For this case a global respose over 200 μsec could be obtained. But in many cases this time is much less than the travel time of the transient wave. After 64 samples as the time increases the error will increase exponentially. Therefore, the selection of A should be revised. It should be noted that if, to reduce the factor exp (A*T), A is decreased below the value chosen for representation of the first cycle, the spectrum function will be less smooth, and then for a given step length the integration errors would increase. On the other hand, if A is increased without increasing the range of integration, the truncation error would increase. Integration error could be minimized by properly selecting DW and using discrete method of integration instead of classical simposon's rule or Trapezoidal rule of integration.

For a long transmission line specially when the switching and surge buses are located at large distances (>100 miles) from each other, we propose the following equation to select the value of the smoothing parameter A, for calculating the global response:

$$(N/2) * A * T = 1$$

or

$$A = 2/(N*T) = 2*DF = DW/\pi \tag{3.29}$$

If it is desired that any specific sub-domain of the global response be investigated in detail, so as, for example, to obtain a more accurate representation of the rate of rise, peak value of the response etc. the choice of integration parameters would be different from those used for determining the global response. This is not a restriction of the method but rather an advantage. As has been explained earlier, one is not usually interested in the detailed response over the entire period of investigation but rather in obtaining a general picture of the response- regions of interest can then be picked up from this general picture and undertaken for detailed investigation.

For the detailed investigation, all parameter values except A could have the same value as for global response calculation. The following equation for the value of A is suggested:

$$A * t = 2$$

$$\text{or} \qquad A = 2/t \quad ; \qquad 0 < t < (N*T) \qquad (3.30)$$

Where t is the elapsed time upto the time of interest, starting from zero. Eq. (3.30) produces a value of A = 1.0E04 for t = 200 μsec (i.e. when the first cycle is of special interest, for a 10 mile long line). This is the same as that assumed for a 10 mile long line (see ch.-2) earlier.

To investigate the justification of Eq. (3.29) and Eq. (3.30) the same 10 mile long open-ended transmission line was studed using different value of A. If the values of OMEGA = 1.0E06, T = 3.14E-06 (i.e. DW = 1953 rad/sec) and N = 1024 are assumed the value of A for global response should be 621.0 according to Eq. (3.29). At the same time if a detailed picture for the second cycle is desired (i.e. t = 430 μsec), the value of A chosen should be 4647.0 according to Eq. (3.30). And the value of A = 1.0E04 was assumed for the first cycle. Using three values of A the output responses are shown in Fig. 3.9. Relevant computer program is given in Appendix-E7. It is observed that the results for A = 1.0E04 and A = 4647.0 are almost coincide for the second cycle and very much accurate as compared to the expected

// EXEC

THE VALUE OF N = 1024
THE VALUE OF OMEGA = 0.10000E 07
THE VALUE OF DW= 0.19531E 04
THE VALUE OF DT = 0.31415E-05
THE VALUE OF DF= 0.31086E 03

CURVE -A FOR A = 0.100E 05
CURVE -B FOR A = 0.465E 04
CURVE -C FOR A = 0.620E 03

\+ FOR DOUBLE PLOT
* FOR TRIPPLE PLOT

MAX. A= 0.201E 01 MIN. A= 0.551E-03
MAX. B= 0.201E 01 MIN. B= 0.335E-03
MAX. C= 0.232E 01 MIN. C= 0.305E-02

VOLTAGE IN P.U.--->

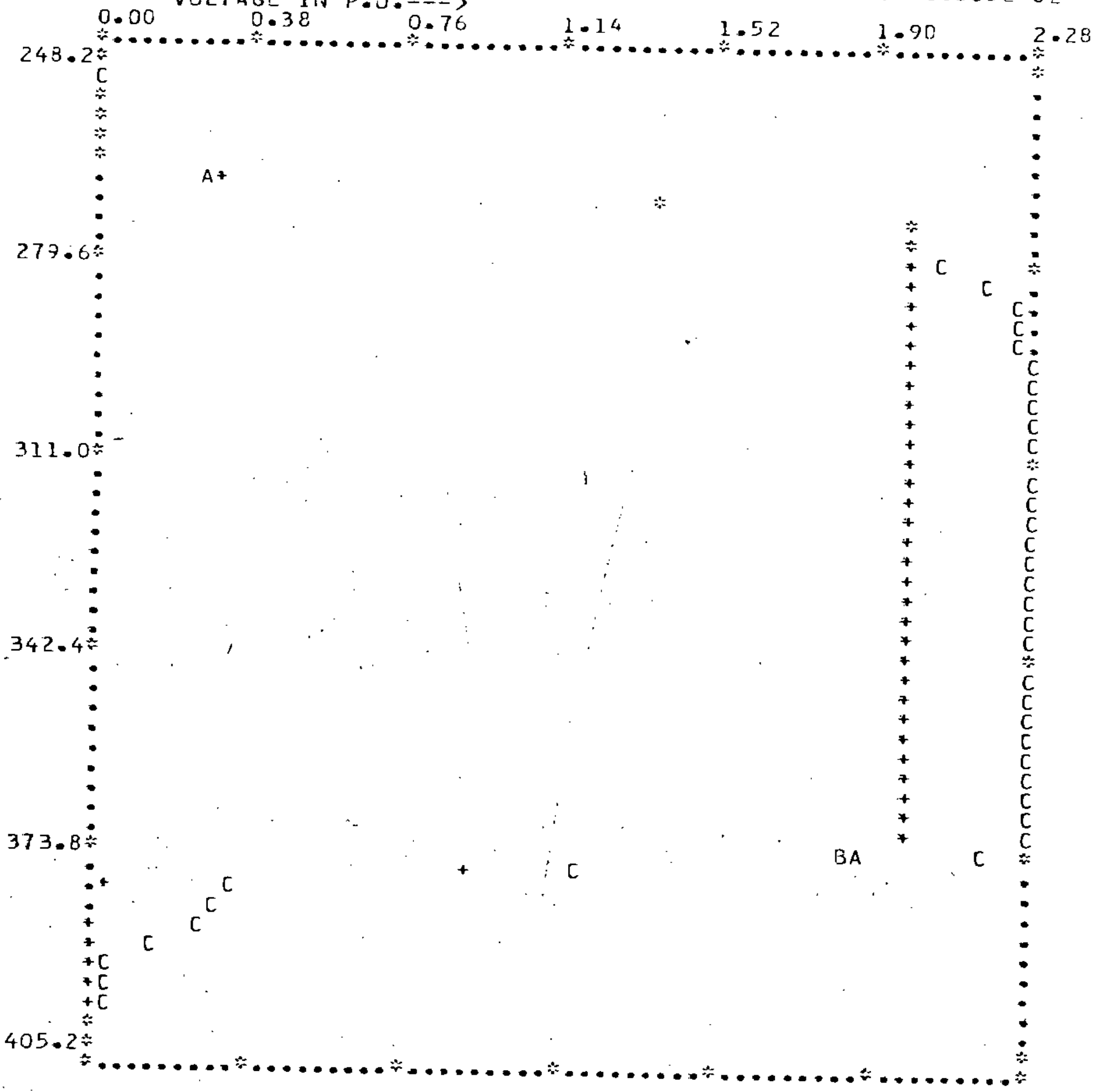


FIG. 3.9 : GLOBAL RESPONSE CALCULATION
2ND CYCLE OF TRASIENT OF 10-MILE LONG OPEN-ENDED L

result. The peak and rise time of the global response (for A = 621.0) are increased. This is due to the selection of sufficiently small value of A, for entire 1024 time samples & for which the peaky nature of the spectrum function does not smooth sufficiently. In the first cycle of Fig. 3.9, it is observed that the parameter value corresponding to second cycle gives better result than that of first cycle. Reasons have already been discussed in sec-3.4.

Similar investigation is performed to have the detailed response for the 5th cycle. Here the value of A = 1858.7 for 5th cycle (t = 1076 μsec) according to Eq. (3.30). All other parameter values are same as above. The results are shown in Fig. 3.10. It will specially be noted that even for the 5th cycle where t is large and the value of A is sufficiently low, the detailed response of the 5th cycle is sufficiently accurate as compared to the expected result.

```
THE VALUE OF N = 1024
THE VALUE OF OMEGA = 0.10000E 07
THE VALUE OF DW= 0.19531E 04
THE VALUE OF DT = 0.31415E-05
THE VALUE OF DF= 0.31086E 03

CURVE -A FOR A = 0.100E 05
CURVE -B FOR A = 0.186E 04        + FOR DOUBLE PLOT
CURVE -C FOR A = 0.620E 03        * FOR TRIPPLE PLOT

MAX. A= 0.201E 01       MAX. B= 0.202E 01       MAX. C= 0.232E 01
 MIN. A= 0.453E-02       MIN. B= 0.236E-02       MIN. C= 0.124E-01
```

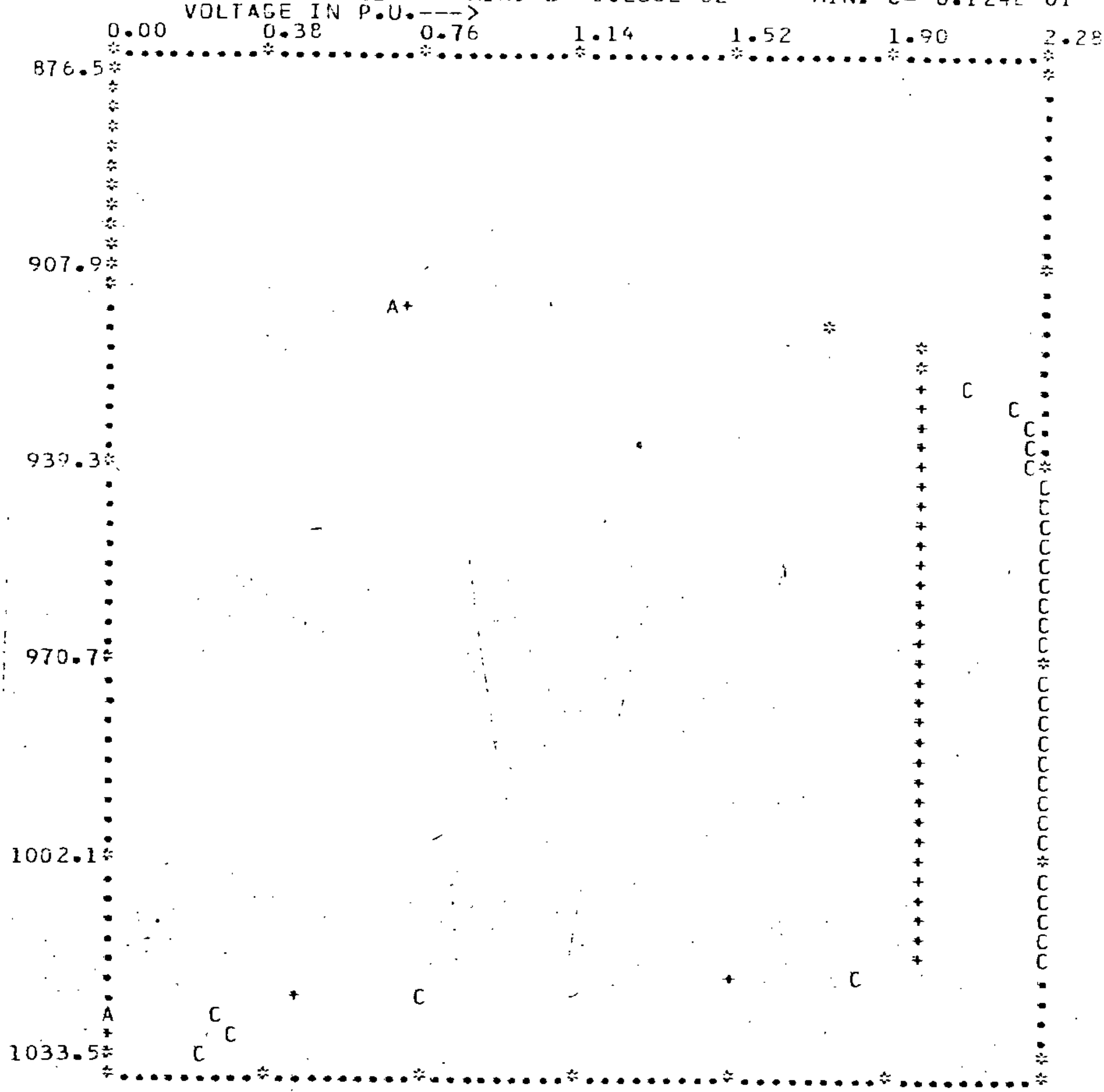


FIG. 3.10 : GLOBAL RESPONSE CALCULATION
5TH CYCLE OF TRASIENT OF 10-MILE LONG OPEN-ENDED L

CHAPTER 4

PRESENT ANALYSIS APPLIED TO THE NATIONAL GRID OF BPDB.

4.1 NETWORK REDUCTION TECHNIQUE:

In chapter-2 a method for calculating the switching transient overvoltages at the receiving end of a long uniform transmission line, due to the application of step voltage at the sending end, is described. Now we will describe, how the basic method of chapter-2 can be applied to a complicated power system network.

From Eq.(2.8) the expression for voltage and current transforms $V(l,s)$ and $I(l,s)$ at the load end of a long transmission line having load impedance Z_L can be written as:[1]

$$V(l,s) = \frac{CSZ_L\ V_s(S)}{\gamma \sinh \gamma l + CSZ_L \cosh \gamma l} \tag{4.1}$$

and $$V(l,s) = \frac{\gamma Z_L I_s(S)}{\cosh \gamma l + CSZ_L \sinh \gamma l} \tag{4.2}$$

$$I(l,s) = \frac{CS\ V_s(S)}{CSZ_L \cosh \gamma l + \sinh \gamma l} \tag{4.3}$$

Where $V_s(S)$ and $I_s(S)$ are the/ transformed input voltage and current respectively. γ is/ the propagation constant and l is/ the length of the line, $S = A + jw$, w is the angular frequency in rad/sec.

Let the line of length '1' be considered as a cascaded serries of two equal line sections, each of length 1/2. Line section-1 includes the source voltage $V_s(S)$ and source impedance Z_s, Using Thevenins theorem the line section-1 can be reduced to an equivalent source in series with an impedance.

The open circuit voltage of line section-1 at the remote end $(Z_L = \alpha)$ is obtained from Eq.(2.8) as:

$$V_{oc}\ (1/2,s) = \frac{V_s(S)}{\cosh(\gamma 1/2)} \tag{4.4}$$

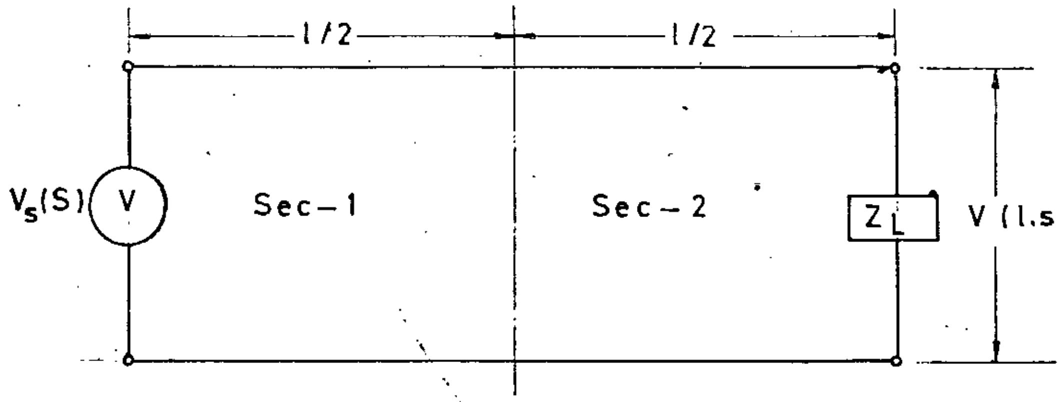
The short circuit current $(Z_L = 0)$ of the same line would be:

$$I_{sc}(1/2,s)\Big|_{Z_L=0} = \frac{CSV_s(S)}{\sinh(\gamma 1/2)} \tag{4.5}$$

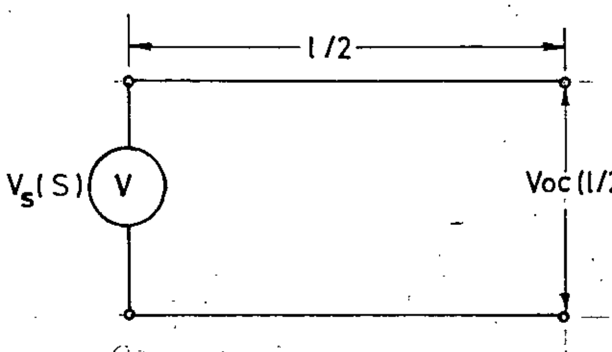
Therefore, the Thevenins equivalent impedance of line sec-1 will be:

$$Z_s(1/2,s) = \frac{V_{oc}(1/2,s)}{I_{sc}(1/2,s)} = \frac{1}{CS}\ \tanh(\gamma 1/2) \tag{4.6}$$

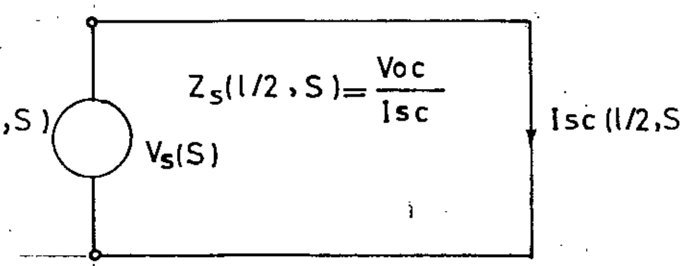
The network reduction technique in figure is given in the next page (See Fig. 4.1).



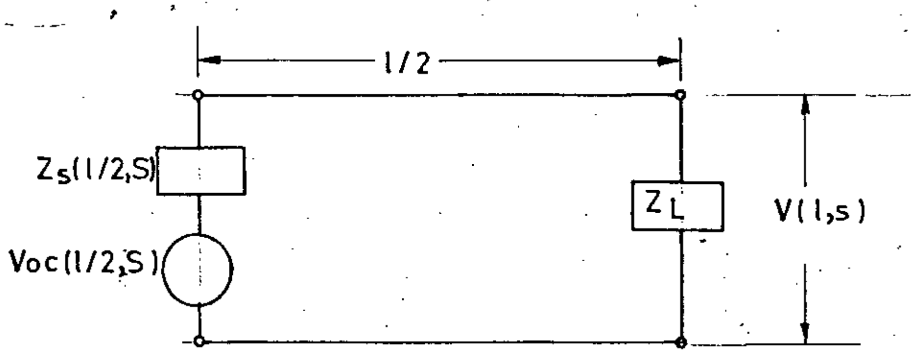
a) Line of length l devided into two equal line sections.



b) Determination of thevenin equivalent voltage.



c) Determination of thevenin equivalent impedance.



d) Equivalent ckt. of Fig.(a) with line section-1 replaced by its thevenins equivalent.

Fig. 4.1: Network Reduction Technique.

The voltage transform at the load end of line sec-2 is obtained as:[1]

$$V(l,s) = \frac{CSZ_L\ V_s(S)}{\sinh \gamma l + CSZ_L \cosh \gamma l} \tag{4.7}$$

Which is exactly the same expression as in Eq.(4.1), derived by assuming the line of Fig. 4.1(a), as one single line section of length l.

The voltage at the junction of the two line sections in Fig. 4.1(a) can be calculated by replacing line sec-2 with input impedance Z_s and considering this impedance as the terminal impedance of line sec-1. This technique of network reduction can be extended to a complicated power system network having interconnections of various line sections of different line parameters, so that the voltage at any junction (BUS) can be found out. The lumped impedances i.e. the generator impedances and loads at different buses can be considered as coming into parallel with the derived Thevenins equivalent input impedances of other sections connected to that bus.

A computer program has been written (See Appendix-F1) to calculate the receiving end voltage of a long transmission line considering the line as consisting of cascaded line sections. In this program the basic technique for network reduction is used.

4.2 NATIONAL GRID OF BPDB

The method developed so far has been applied to the existing National Grid (1984-85) of Bangladesh Power Development Board (including the East-west Interconnector), in an attempt to investigate the overvoltages that may develop at different buses due to various switching conditions. There are 47 buses, 12 of them have generators. In our analysis we have taken all 47 buses including all generators. All the buses have been designated by distinctive numbers, so that the main line consists of maximum number of generator buses. According to the numbering the generators are connected to bus numbers 1,8,11, 19, 22, 23,30,33, 37, 38, 39 and 40. The single line diagram of the network is given in Fig. 4.2, showing in detail the generators and loads. The line data i.e. the bus numbers and names connected by the relevant lines, resistance, inductance and charging capacitance per unit length are given in Table 4.1. A simplified version of the single line diagram, as shown in Fig. 4.3, is considered for details investigation. The West Grid consist of the bus number 1(Goalpara) to 11(Veramara) and 27(Panchagarh) to 34(Ishurdi) and the East Grid consist of 8 branch sections and a main section consist of bus number 35 (Tongi) to 44 (Chatak). Bus 34 to 35 is connected by the Interconnector, so the main section (sec-1) is from bus 27 (Panchagarh) to 44 (chatak). The branch sections of West Grid are: Bus number 1 (Goalpara) to 5 (Botail) and then to 11 (Veramara) consists

SINGLE LINE DIAGRAM OF BPDB NETWORK SHOWING LOADS AND GENERATORS.

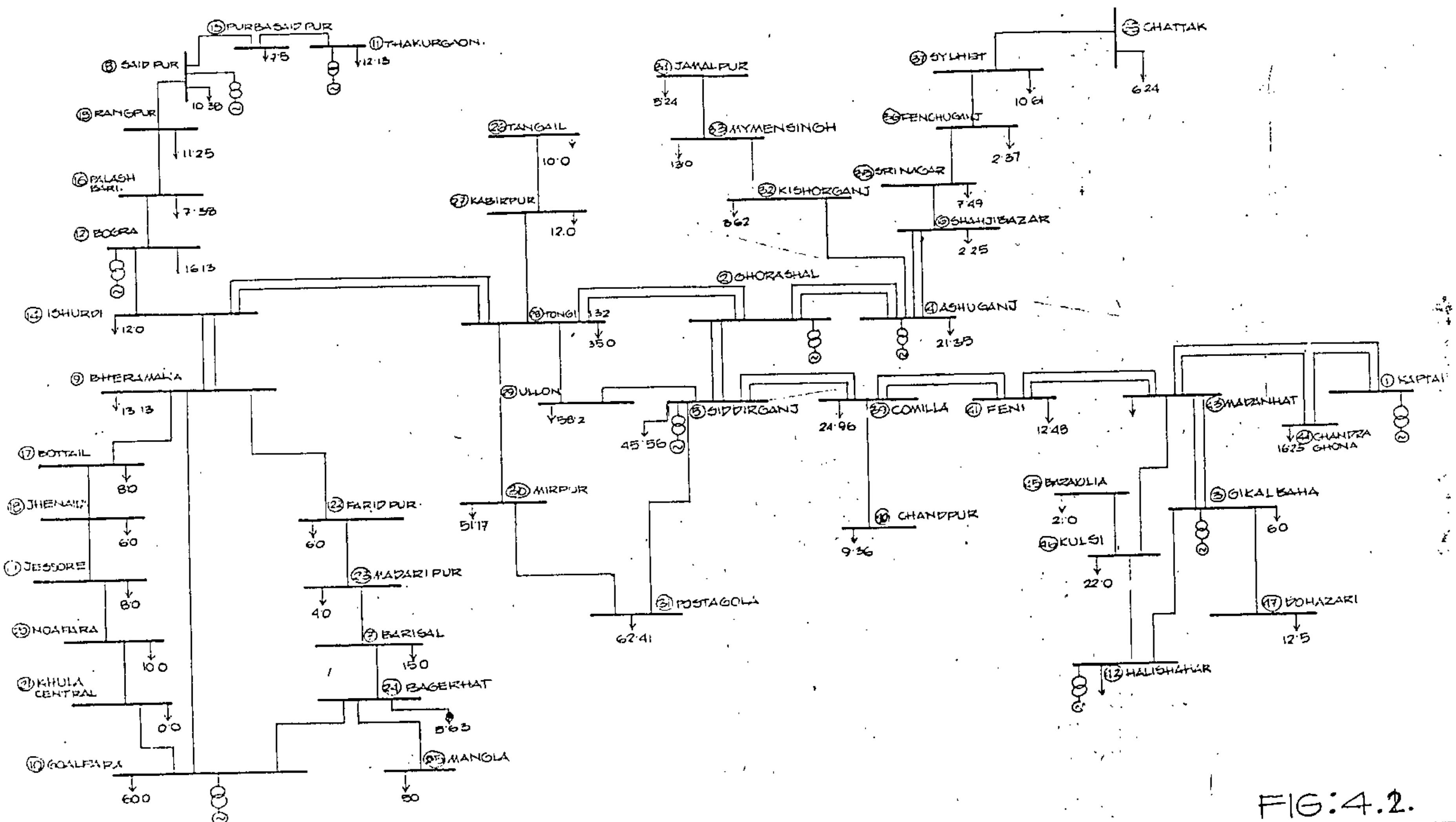


FIG:4.2.

// EXEC TABLE 4.1: LINE DATA

BUS NO. FROM	BUS NO. TO	NAME OF BUS FROM	NAME OF BUS TO	LENGTH IN MILES	R OHM/MILE	L H/MILE	C F/MILE
1	2	GOALPARA	-NOAPARA	14.00	0.216	0.20308E-02	0.147E-07
2	3	NOAPARA	-JESSORE	16.00	0.216	0.20308E-02	0.147E-07
3	4	JESSORE	-JHENIDAH	28.00	0.216	0.20308E-02	0.147E-07
4	5	JHENAIDAH	-BOTAIL	27.00	0.216	0.20308E-02	0.147E-07
5	11	BOTAIL	-BHERAMARA	14.00	0.216	0.20308E-02	0.147E-07
6	7	MONGLA	-BAGERHAT	19.50	0.216	0.20308E-02	0.147E-07
7	8	BAGERHAT	-BARISAL	41.30	0.216	0.20308E-02	0.147E-07
8	9	BARISAL	-MADARIPUR	35.00	0.216	0.20308E-02	0.147E-07
9	10	MADARIPUR	-FARIDPUR	40.00	0.216	0.20308E-02	0.147E-07
10	11	FARIDPUR	-BHERAMARA	65.00	0.216	0.20308E-02	0.147E-07
11	34	BHERAMARA	-ISHURDI	6.00	0.216	0.20308E-02	0.135E-07
12	13	TANGAIL	-KABIRPUR	25.00	0.162	0.19671E-02	0.152E-07
13	35	KABIRPUR	-TANGI	20.00	0.162	0.19671E-02	0.152E-07
14	15	MIRPUR	-POSTAGOLA	18.00	0.162	0.21072E-02	0.141E-07
15	37	POSTAGOLA	-SIDDIRGANG	14.50	0.162	0.21072E-02	0.141E-07
16	17	JAMALPUR	-MYMENSING	34.10	0.162	0.19671E-02	0.152E-07
17	18	MYMENSING	-KISHORGANG	36.60	0.162	0.19671E-02	0.152E-07
18	39	KISHORGANG	-ASHUGANG	32.20	0.162	0.19671E-02	0.152E-07
19	24	KAPTAI	-MADANHAT	24.10	0.162	0.19671E-02	0.152E-07
20	21	BAROULIA	-KULSHI	7.00	0.162	0.19671E-02	0.152E-07
21	22	KULSHI	-HALISHAHAR	12.00	0.162	0.19671E-02	0.152E-07
22	23	HALISHAHAR	-SIKALBAHA	9.50	0.162	0.19671E-02	0.152E-07
23	24	SIKALBAHA	-MADANHAT	10.00	0.162	0.19671E-02	0.152E-07
24	25	MADANHAT	-FENI	60.70	0.162	0.19671E-02	0.152E-07
25	26	FENI	-COMILLA	30.20	0.162	0.19671E-02	0.152E-07
26	37	COMILLA	-SIDDIRGANG	53.70	0.162	0.19671E-02	0.152E-07
27	28	PANCHAGAR	-THAKURGAON	22.00	0.316	0.20531E-02	0.145E-07
28	29	THAKURGAON-	PURBASAIDPUR	27.00	0.316	0.20531E-02	0.145E-07
29	30	PURBASAIDPUR	-SAIDPUR	13.00	0.316	0.20531E-02	0.145E-07
30	31	SAIDPUR	-RANGPUR	25.50	0.216	0.20531E-02	0.150E-07
31	32	RANGPUR	-PALASHBARI	32.60	0.216	0.20531E-02	0.150E-07
32	33	PALASHBARI	-BAGRA	31.20	0.216	0.20531E-02	0.150E-07
33	34	BAGRA	-ISHURDI	63.60	0.216	0.20531E-02	0.150E-07
34	35	ISHURDI	-TANGI	86.10	0.216	0.20531E-02	0.150E-07
35	36	TONGI	-ULLON	12.00	0.162	0.21687E-02	0.140E-07
36	37	ULLON	-SHIDDIRGANG	9.50	0.162	0.21687E-02	0.140E-07
37	38	SHIDDIRGANG	-GHORASAL	27.40	0.162	0.21687E-02	0.140E-07
38	39	GHORASAL	-ASHUGANG	27.00	0.162	0.21687E-02	0.140E-07
39	40	ASHUGANG	-SHAHJIBAZAR	32.20	0.162	0.19671E-02	0.152E-07
40	41	SHAHJIBAZAR	-SRIMANGAL	22.60	0.162	0.19671E-02	0.152E-07
41	42	SRIMANGAL	-FENCHUGANG	30.50	0.162	0.19671E-02	0.152E-07
42	43	FENCHUGANG	-SILHET	19.60	0.162	0.19671E-02	0.152E-07
43	44	SILHET	-CHATAK	20.20	0.162	0.19671E-02	0.152E-07
23	45	SIKALBAHA	-DOHAZARI	20.00	0.162	0.19671E-02	0.152E-07
24	46	MADANHAT-	CHANDRAGHONA	19.10	0.162	0.19671E-02	0.152E-07
26	47	COMILLA	-CHANDPUR	47.10	0.162	0.19671E-02	0.152E-07

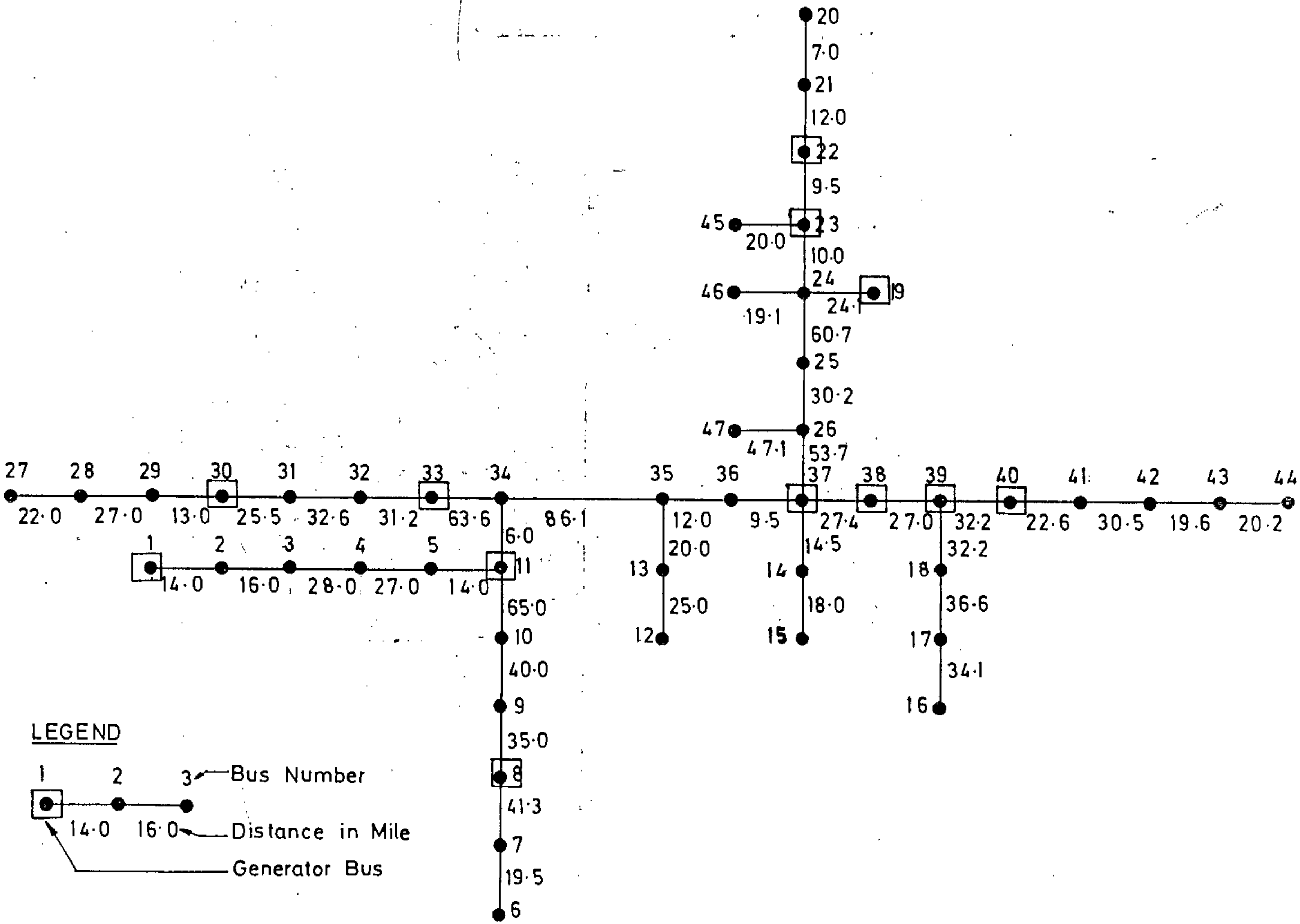


Fig. 4.3: Simplified One-line Diagram of National Grid of BPDB.

section-2; Bus number 6 (Mongla) to 11 and then to 34 consists section-3. The branch sections of East Grid as numbered in our analysis and computer program are: sec-4, bus number 12(Tangail) to 13(Kabirpur) and then to 35 (Tongi); sec-5, 14 (mirpur) to 15 (postagola) and then to 37 (Shiddirganj); sec-6, 16(Jamalpur) to 18(Mymensingh)and then to 39 (Ashuganj); sec-7, 19(Kaptai) to 24 (Madanhat); sec-8, 45 (Dohazari) to 23(Sikalbaha); sec-9, 46 (Chandraghona) to 24 (Madanhat); sec-10, 47(Chandpur) to 26 (Comilla) and sec-11, 20 (Baroulia) to 26 (Comilla)and then to 37 (Shiddirganj).

4.3 DEVELOPMENT OF THE COMPUTER PROGRAM:

The numbering of the buses are given in such a way that the maximum number of generating stations are located on the main section of the network. This is because that switching may usually be assumed to occur at the generator buses. Depending on the location of the surge bus (ND) at which surge voltages need to be calculated, in different sections, the whole program is divided into three distinct parts. Similarly depending on the location of switching bus (NS) for which transient overvoltages are developed at different sections of the line, each part of the program again divided into three distinct subsections. Therefore, the main program is divided into nine distinct parts depending on ND and NS.

The main program (See Appendix-F2) for the calculation of surge voltages due to switching at different sections of the network contains six subroutines, given in Appendix-G.

Subroutine FIRST is used to calculate the input admittance of a line terminated in a load admittance. The repetition of this procedure for cascaded line sections is performed as necessary.

Subroutine SECOND reduces a line, excited from a voltage source through an impedance into its Thevenins equivalent. The Thevenis equivalent of a cascaded line sections along with its excitation source and impedances, can be found out by repetition of this subroutine. When subroutine FIRST and SECOND are repeated for cascaded line sections, the generator impedances and loads at intervening line sections are autometically taken into account.

Subroutine FFT is used to calculate the time domain response of the surge voltages from the frequency domain data of voltage transform. The inverse continuous Fourier transformation integral is done here by inverse Discrete Fourier Transformation (DFT), which is implimented using the inverse Fast Fourier Transform algorithm (FFT). The whole frequency set of data is fed to the FFT subroutine and within very short time the time domain response function is obtained with better accuracy than any other conventional method. This is the novelty of our present analysis, subroutine FFT is given in Appendix-C.

Subroutine STL is used to calculate the line length between switching (NS) surge (ND) bus so that the travel time required to reach the transient wave to ND form NS can be calculated.

Subroutine IN is used to calculate the load admittances, load resistances and load inductances of different buses from the voltage and power (Real and reactive) information of buses at normal operating frequency of 50 C/sec.

Subroutine PLT1 or PLT2 is used to have a plot of the output voltage versus time for required number of data points.

The program is capable of calculating surge voltages at any bus due to both line energization and load rejection at any bus. The system initial conditions are discussed in the next article. The system Flow chart is given in Fig. 4.4, which is self explanetory. The detailed Flow chart is given in Fig. 4.5. The computer program developed on the basis of these Flow chart are given in Appendix -F2.

4.4. INITIAL CONDITIONS:

Switching does not always take place on an initially related system. Initially the system is in a steady state condition

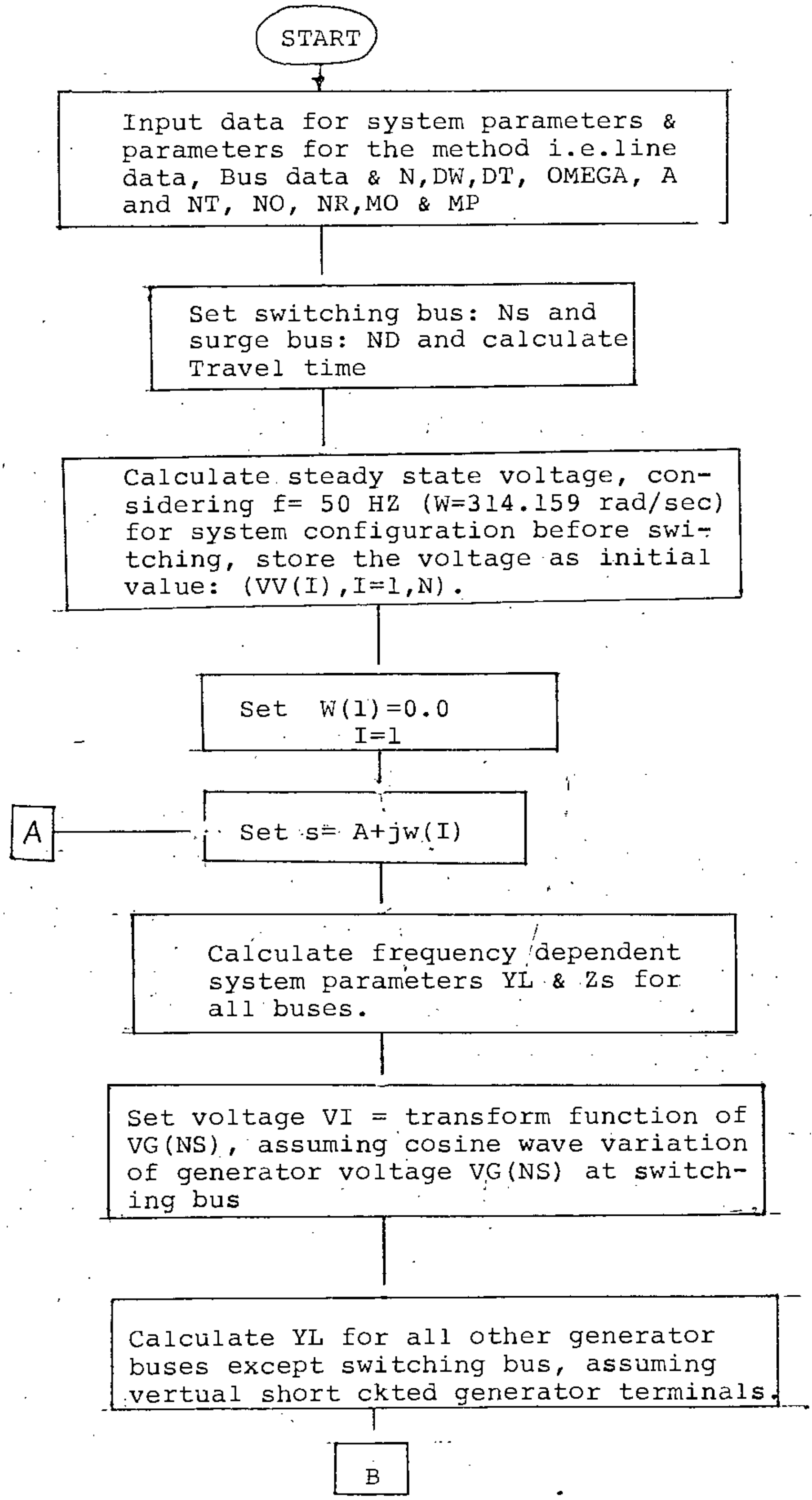


Fig. 4.4 Flow chart for main program.

Fig. 4.4. (Contd.)

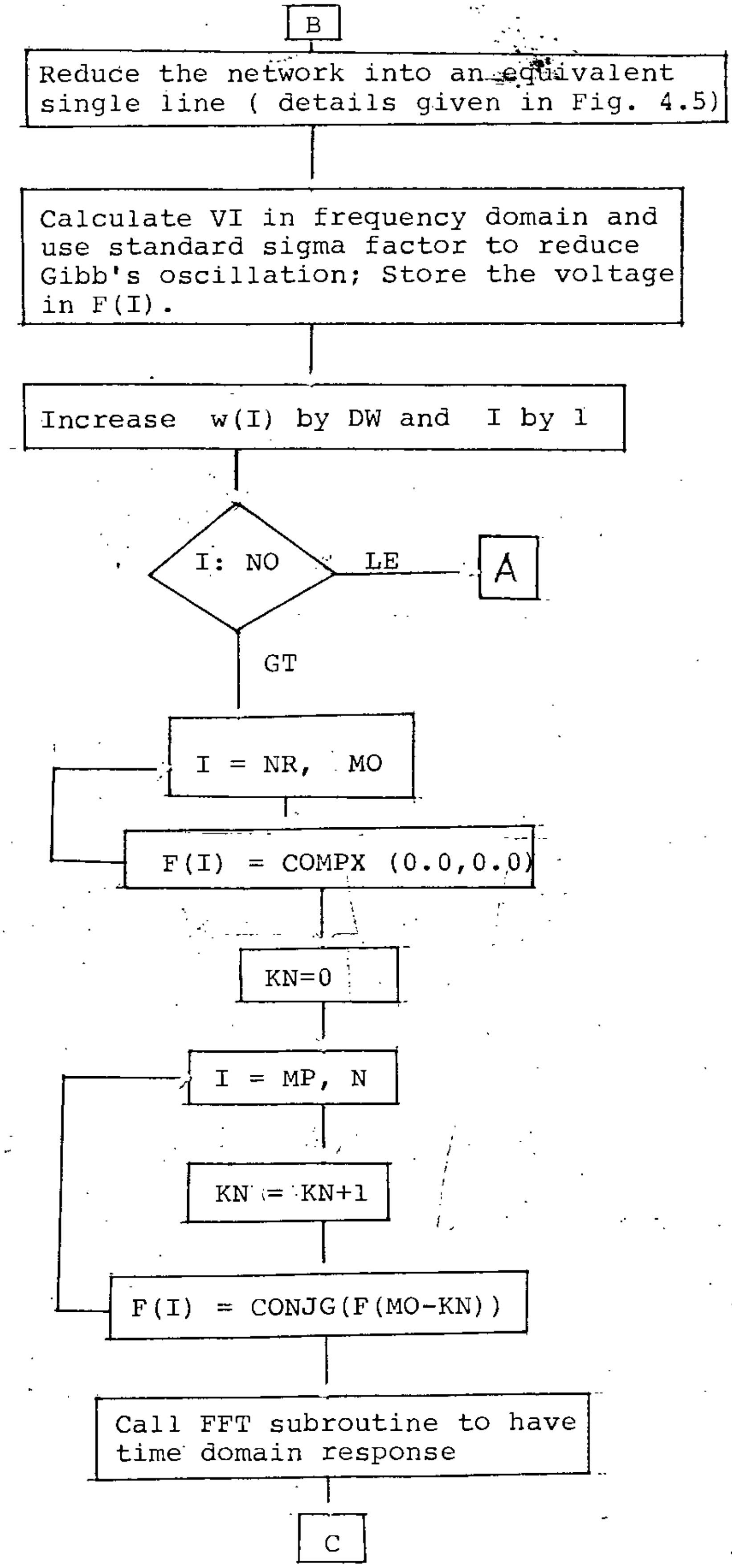

B
Reduce the network into an equivalent single line (details given in Fig. 4.5)
Calculate VI in frequency domain and use standard sigma factor to reduce Gibb's oscillation; Store the voltage in F(I).
Increase w(I) by DW and I by 1
I: NO
LE
A
GT
I = NR, MO
F(I) = COMPX (0.0,0.0)
KN=0
I = MP, N
KN = KN+1
F(I) = CONJG(F(MO-KN))
Call FFT subroutine to have time domain response
C

Fig. 4.4.Contd.

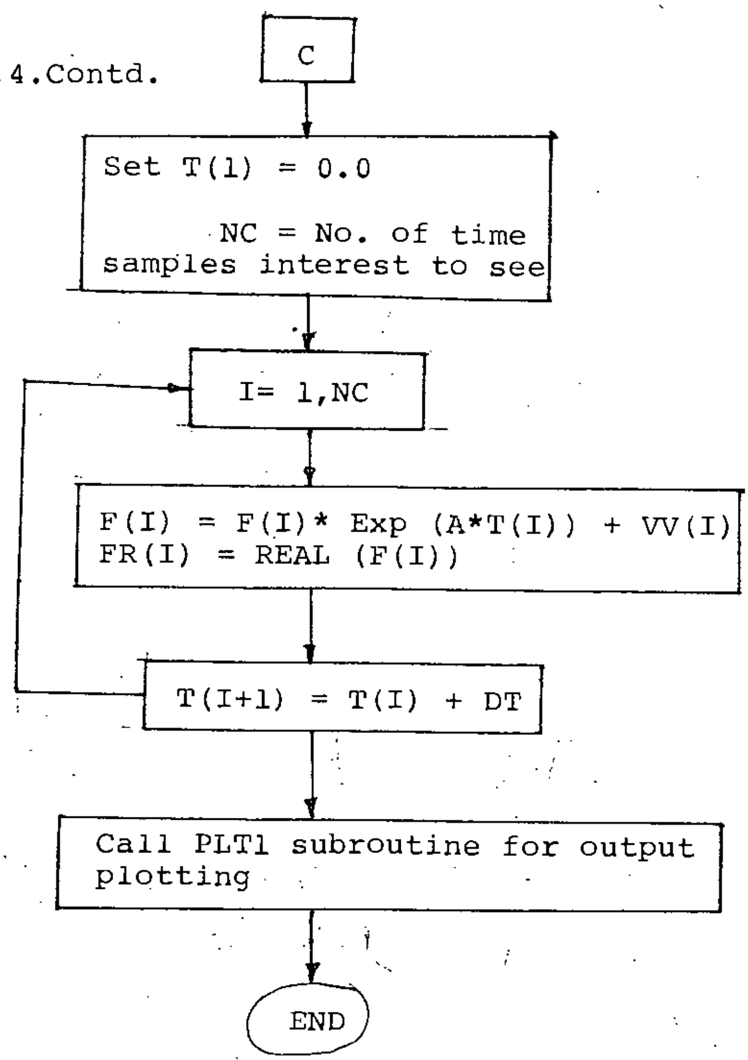

C
Set T(1) = 0.0
NC = No. of time samples interest to see
I= 1,NC
F(I) = F(I)* Exp (A*T(I)) + VV(I)
FR(I) = REAL (F(I))
T(I+1) = T(I) + DT
Call PLT1 subroutine for output plotting
END

Fig. 4.5: Details of Flow-chart/Box-9 for network Reduction.

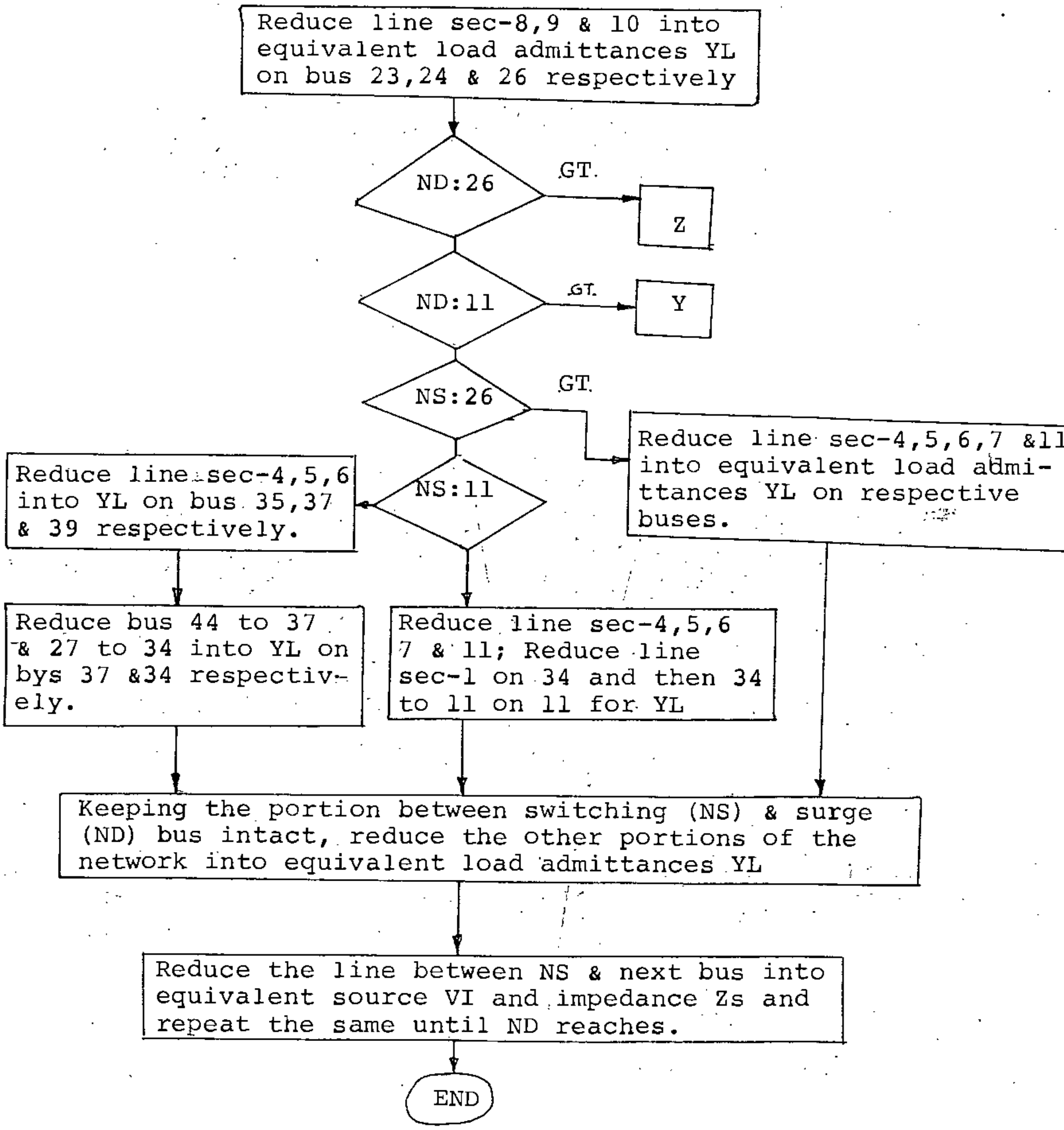
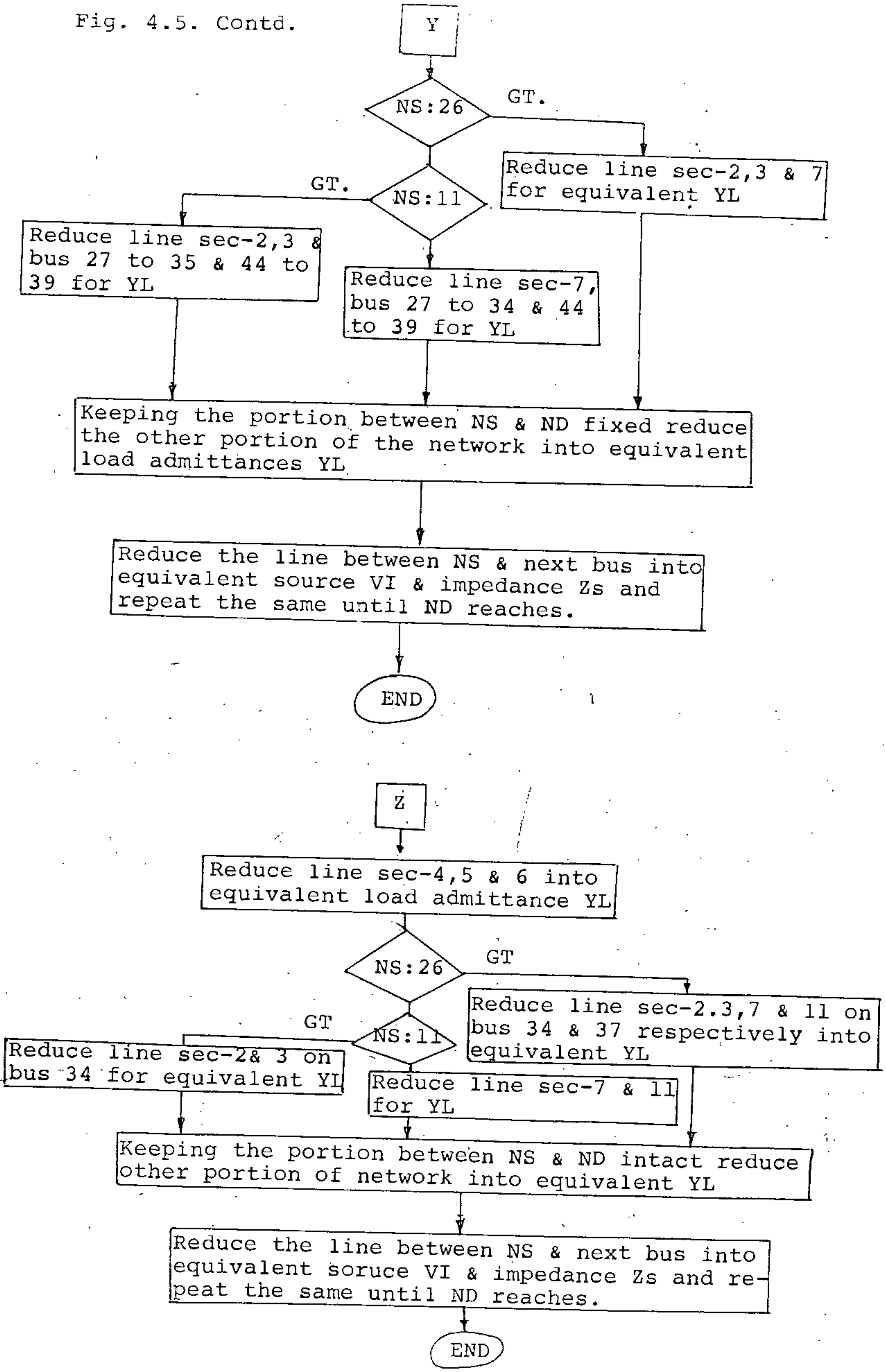


Fig. 4.5. Contd.


Y
NS:26
GT.
Reduce line sec-2,3 & 7 for equivalent YL
GT.
NS:11
Reduce line sec-2,3 & bus 27 to 35 & 44 to 39 for YL
Reduce line sec-7, bus 27 to 34 & 44 to 39 for YL
Keeping the portion between NS & ND fixed reduce the other portion of the network into equivalent load admittances YL
Reduce the line between NS & next bus into equivalent source VI & impedance Zs and repeat the same until ND reaches.
END
Z
Reduce line sec-4,5 & 6 into equivalent load admittance YL
NS:26
GT
Reduce line sec-2.3,7 & 11 on bus 34 & 37 respectively into equivalent YL
GT
NS:11
Reduce line sec-2& 3 on bus 34 for equivalent YL
Reduce line sec-7 & 11 for YL
Keeping the portion between NS & ND intact reduce other portion of network into equivalent YL
Reduce the line between NS & next bus into equivalent soruce VI & impedance Zs and repeat the same until ND reaches.
END

with the transmission lines in a charged state. The transient overvoltages caused by different switching operations is a superimposition of the transient behaviour of the system on this initial steady state condition. Therefore, the steady-state system behaviour before the instant of switching has to be evaluated first.

In our program a provision has been made to accomodate and to calculate the steady state voltage level at the bus where the surge is to be calculated before the instant of switching. Considering the network configuration before switching i.e. at $t \underline{<} 0$ the voltage transform at $w = 2\pi * 50 = 314$ rad/sec (f = 50 C/sec) is calculated and this calculated value is then superimposed on the voltage transform obtained from the transient behaviour of the system network to get the actual response transform.

CHAPTER - 5

RESULTS AND DISCUSSIONS

5.1 INTRODUCTION:

The method of analysis developed in chapter-3 and the basic network reduction technique discussed in chapter-4 is applied to investigate the transient response at different buses of the National Interconnected Grid of BPDB, due to switching operations at specified generating buses. The method presented is very much faster without sacrificing accuracy of calculation. The transient response over any desired time duration can be accurately calculated by the present method. The investigations performed on the BPDB National Grid System have been classified into three distinct categories depending on various system conditions prior to switching:

1. Initially relaxed system- no loads connected.
2. Initially relaxed system- all normal loads connected &
3. System under normal load condition.

The results for different switching conditions investigated are presented in graphical form and discussed in the following sections. In sec-5.2, investigations are performed on a part of the whole system, in which it is assumed that all the intermediate buses are open to form a single transmission line open at the remote end (surge bus) and excited from switching bus by an infinite source (i.e. source having zero source impedance). This assumption will serve as a validity check on the method of analysis presented, the network reduction technique and on the computer program developed. In sec- 5.3 the switching surges under loaded and initially relaxed system are investigated for switching at bus 39- Ashuganj and surge at bus 36-Ullon. In sec- 5.4 the switching surges under normal load and initially energized system are investigated for different switching and surge buses.

5.2 SWITCHING SURGE UNDER NO LOAD CONDITIONS CONSIDERING AN INITIALLY RELAXED SYSTEM:

The switching operation is assumed to take place at bus 39- Ashuganj and surge voltages for different instants of time are observed at bus 43- Sylhet. The result is shown in Fig. 5.1. Initially the system is considered as relaxed and the switching bus is considered to be infinite. In this case the investigations are done on the part of the whole system which, although in reality consists of a series of cascaded line sections, is in effect a single transmission line open at the Sylhet end and excited from an infinite source located at Ashuganj bus. The excitation voltage is 1.0 p.u. The surge starts to travel in forword and backword directions from/to Ashuganj as soon as the generator is switched on to the source bus.It would reach Sylhet bus at a time equal to the travel

LINE LENGTH = 104.900
TRAVEL TIME = 563.978

OMEGA = 0.100E 07 N = 1024
DW = 1954.107 DT = 0.314E-05
DF = 311.007 A = 1640.000

SWITCHING BUS = 39 ASHUGANJ
SURGE BUS = 43 SYLHET

PEAK = 1.966P.U.
* CREST = -0.014P.U.

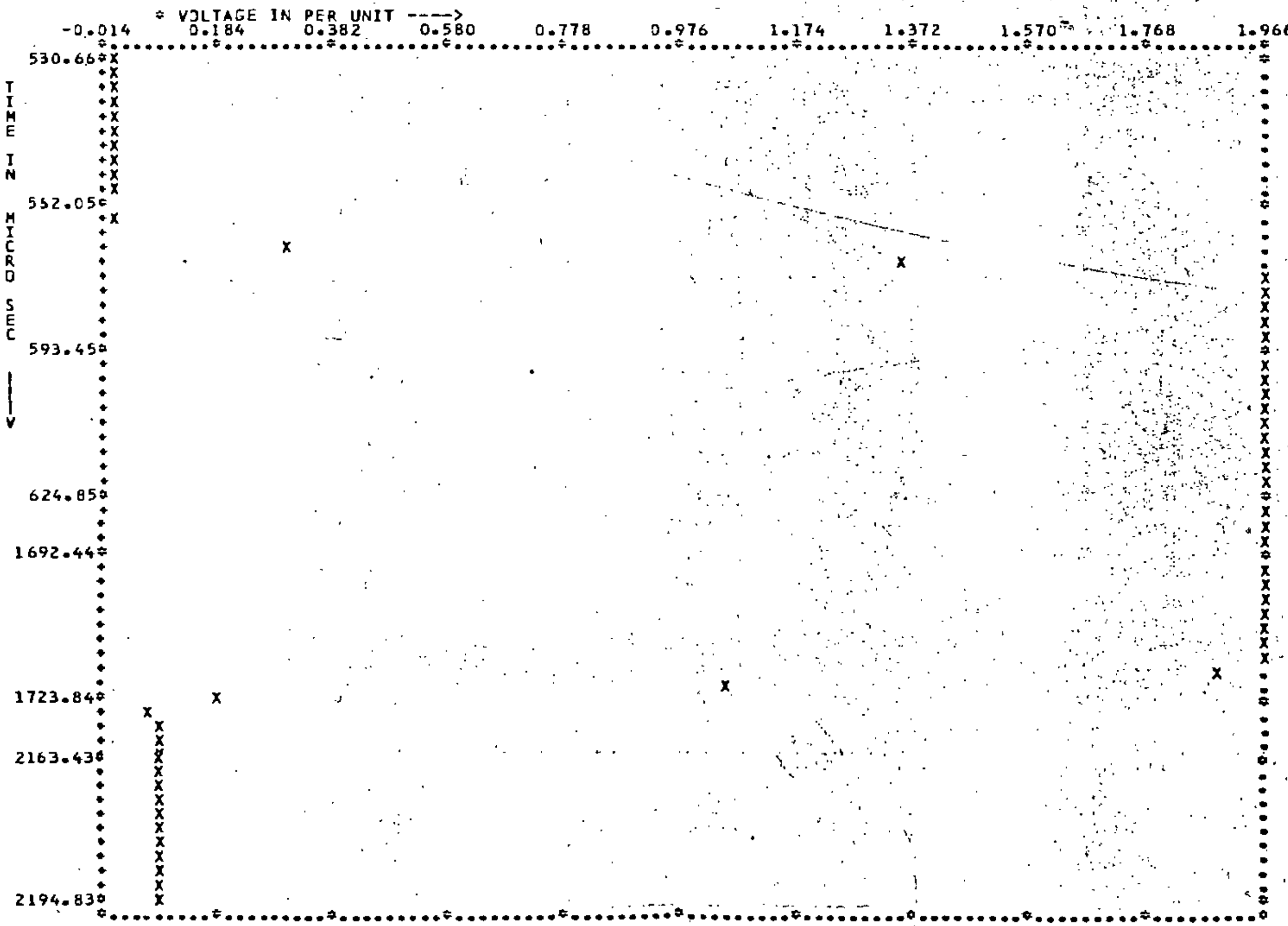


FIG 5.1

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time between Ashuganj and Sylhet i.e. at around 564.0 μsec if the transient wave propagates with a velocity equal to the velocity of light. Since Sylhet bus is open a 100% reflection would occur, resulting in a doubling of the incoming surge voltage. But in practice, losses are present and it will tend to modify the transient voltages to some extent. From Fig. 5.1 it is observed that the calculated voltages at the Sylhet bus at 565.19 μsec and at 577.7 μsec are 0.0058 p.u. and 1.96 p.u. respectively. The expected results and observed results of the calculated voltage differ to some extent; This is because the conductor resistance losses have a two-fold effect:[15] first they cause attenuation of the voltage wave on the line owing to an energy loss, and secondly they cause a distortion of these waves, owing to differential reduction in the wave velocity of the different frequency components. In general, their net result is to cause a reduction in the transient voltage at any point on the line, whereas the reduction in wave velocity can cause a change in the phase relationship between the travelling waves on the line, so that some portion of the transient voltage are increased and the net effect reaches the open end later than the actual travel time. After a time interval, from the moment of switching, equal to three travel times, a 1.0 p.u. negative voltage, due to the virtual short circuit at Ashuganj bus, arrives at Sylhet, and is doubled by the open termination.

This nullifies the former 2.0 p.u. voltage at the Sylhet bus and brings it down to zero. The calculated values of 1.966 p.u. and 0.0652 p.u. at 1715.0 μsec and at 1727.0 μsec respectively (see Fig. 5.1), Once again agree reasonably with the expected results.

LY

5.3 SWITCHING SURGE ON INITIAL/ RELAXED SYSTEM (LOADS CONNECTED):

Before the instant of switching at a particular bus, the system is considered initially relaxed with typical load admittances connected at all buses. Switching is performed at bus 39-Ashuganj and the surge voltage is calculated at bus 36-Ullon. The travel time between switching and surge buses is 343.5 μsec. The first peak is seen to occur (see Fig. 5.2) at arround 352.0 μsec, the value of which is about 2.35 p.u. The peaks & crests observed are created due to multiple reflection from different buses within the system.

5.4 SWITCHING SURGE UNDER LOADED & INITIALLY ENEGIZED SYSTEM:

At the instant of switching, the system is considered to be under normal operating condition i.e. all generators, except the generator being switched, and all normal loads are connected to the system. Surge voltages at different buses are calculated for switching at different generator buses, considering the source impedance.

OMEGA = 0.100E 07
DW = 1953.123
DF = 310.850

N = 1024
DT = 0.314E-05
A = 205.000

SWITCHING BUS = 39 ASHUGANJ
SURGE BUS = 36 ULLON
TRAVEL TIME = 343.5 MICRO SEC

PEAK = 2.358P.U.
CREST = -0.286P.U.

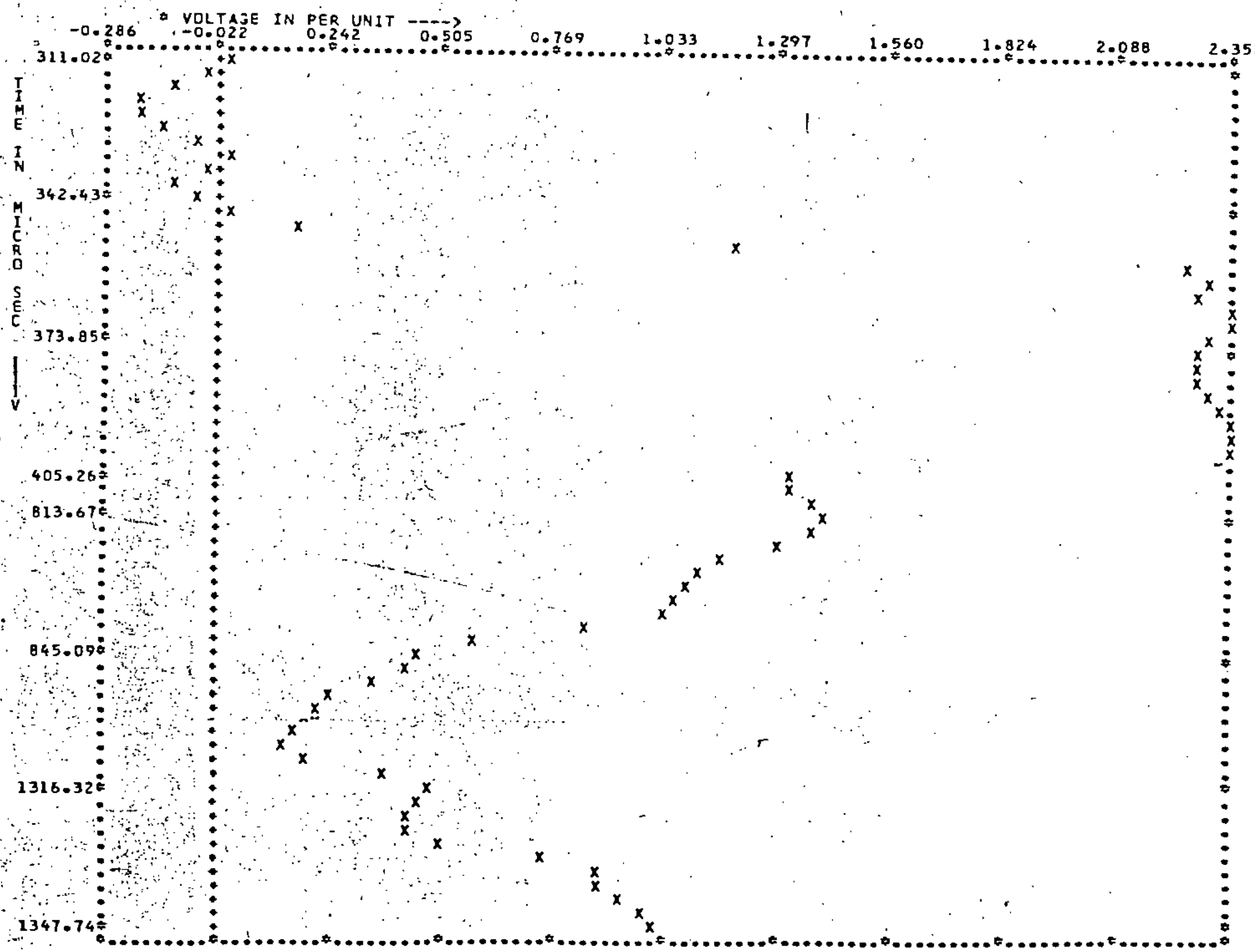

FIG. 5.2

5.4.1 SWITCHING SURGE AT BUS 35-TANGI DUE TO SWITCHING AT BUS 1-GOALPARA:

The length of the line between Goalpara and Tangi is about 191.1.1 miles and the corresponding travel time is arround 1027.0 μsec. From steady-state solution, it is found that the voltage at Tongi, prior to the switching operation, is 0.91 p.u. At the end of the time of interest (i.e. the time upto which voltage transient is calculated), the steady-state component of this voltage has come down to 0.686 p.u. Superimposing this steady-state voltage on the transient response it is observed (see Fig. 5.3) that peaks having magnitudes of 3.21 p.u., 2.4 p.u. and a negative peak of 1.02 p.u. voltages occur at Tangi at 1054.68 μsec, 1547.82 μsec and 1792.0 μsec respectively. Among these, the peak of 3.21 p.u. occuring at 1054.68 μsec, needs to be given serious consideration in determining the insulation level. The first peak of 3.21 p.u. starts to rise at 1044.92 μsec which is slightly greater than the travel time of 1027.0 μsec. Reasons of this late rise has already been discussed in sec-5.2. Besides, the successive peaks and crests are observed due to the multiple reflections at the points of discontinuity within the system.

5.4.2 SWITCHING SURGE AT BUS 35-TANGI DUE TO SWITCHING AT BUS 19-KAPTAI:

Considering switching at bus 19-Kaptai, the surge voltages are calculated at bus 35-Tangi. The distance between these two

// EXEC

* PEAK= 0.91058E 00P.U.
* CREST = 0.64392E 00P.U.
FOR STEADY-STATE CALCULATION

OMEGA = 0.600E 06 N = 4096
DW = 314.159 DT = 0.488E-05
DF = 50.000 A = 600.000

SWITCHING BUS = 1 GOALPARA
SURGE BUS= 35 TANGI
ALL GENERATORS ARE CONNECTED
TRAVEL TIME = 1027.0 MICRO SEC

PEAK = 3.217P.U.
* CREST = -1.027P.U.

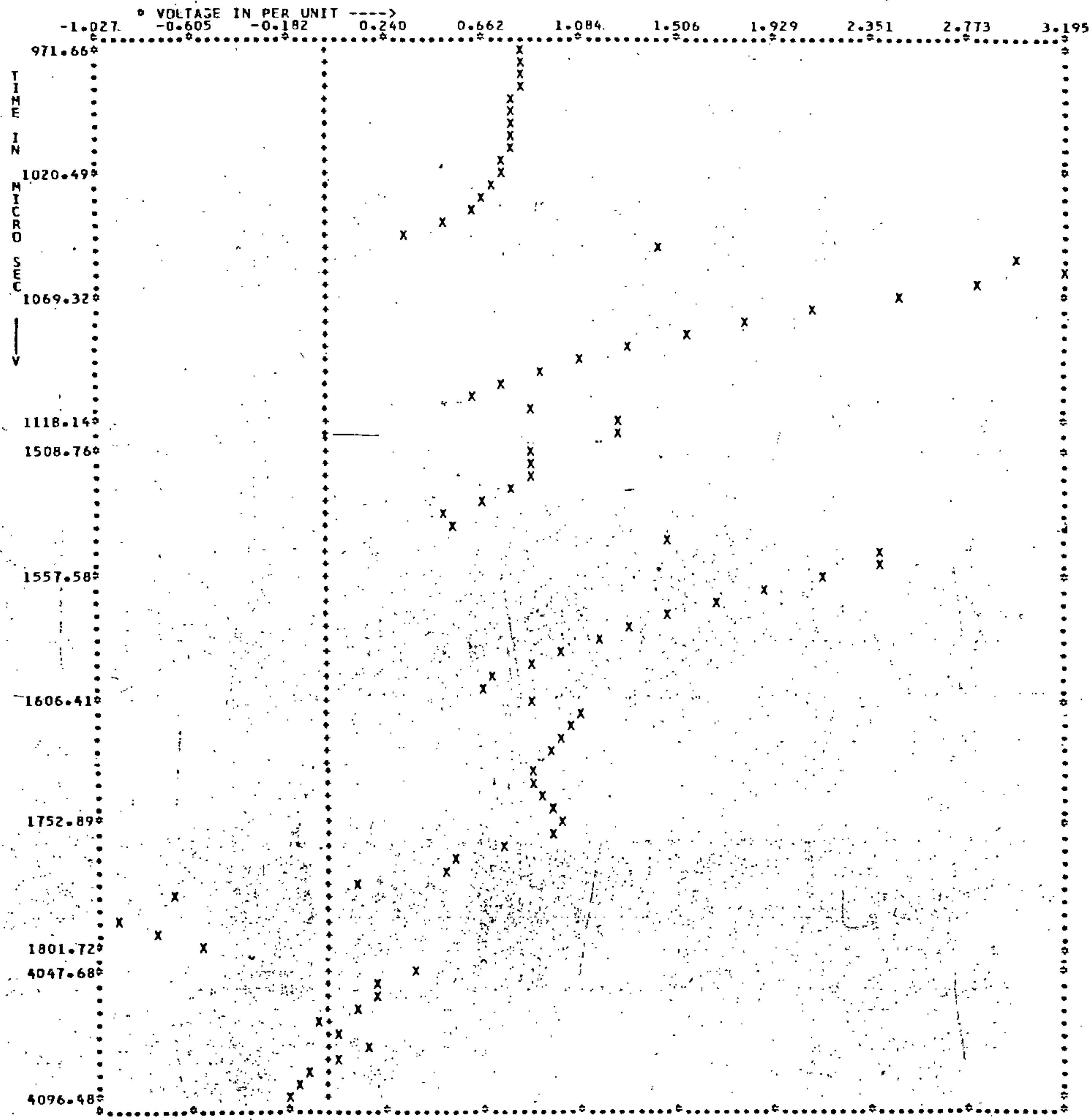

FIG 5.3

buses is about 190.0 miles and the coresponding travel time is arrount 1022.0 μsec. The first peak is seen to arrive at Tangi at arround 1050.0 μsec which is slightly greater than the travel time and it attains a value of 2.66 p.u. Other three peaks are seen (see Fig. 5.4) to reach voltage levels of 2.93 p.u., 2.66 p.u. and 2.66 p.u. at 1562.46 μsec, 1860.14 μsec and at 2432.0 μsec respectively. Besides these, two negative peaks of -0.54 p.u. and -0.88 p.u. are seen to occur at 1713.74 μsec and 2255.42 μsec respectively. The highest positive peak of 2.93 p.u. and the highest negative peak -0.88 p.u. could be potentially dangerous for equipment connected to the surge bus Tangi.

5.4.3 SWITCHING SURGE AT BUS 11-VERAMARA DUE TO SWITCHING AT BUS 38-GHORASHAL:

The travel time required to travel the transient wave from bus 38-Ghorashal to bus 11-Veramara is about 758.0 μsec. Superimposing the steady-state solution on the transient response due to switching at bus 38-Ghorashal it is observed that (See Fig. 5.) the first peak arrive at veramara at arround 786.1 μsec which is slightly greater than the actual travel time and it attains a value of 2.87 p.u. The reason behind this late rise has already been discussed in sec-5.2. Other two peaks are seen to reach voltage levels of 2.19 p.u and 2.25 p.u. at 1137.66 μsec and at 1630.8 μsec respectively. No negative peaks are observed, but the minimum value of voltage is 0.045 p.u. The first peak of 2.87 p.u. voltage requires serious consideration in determining the insulation level of the system.

* PEAK= 0.90850E 00P.U.
* CREST = 0.64182E 00P.U.
FOR STEADY-STATE CALCULATION

OMEGA = 0.600E 06 N = 4096
DW = 314.159 DT = 0.488E-05
DF = 50.000 A = 410.000

SWITCHING BUS = 19 KAPTAI
SURGE BUS= 35 TANGI
ALL GENERATORS ARE CONNECTED
TRAVEL TIME = 1022.0 MICRO SEC

PEAK = 2.932P.U.
* CREST = -1.007P.U.

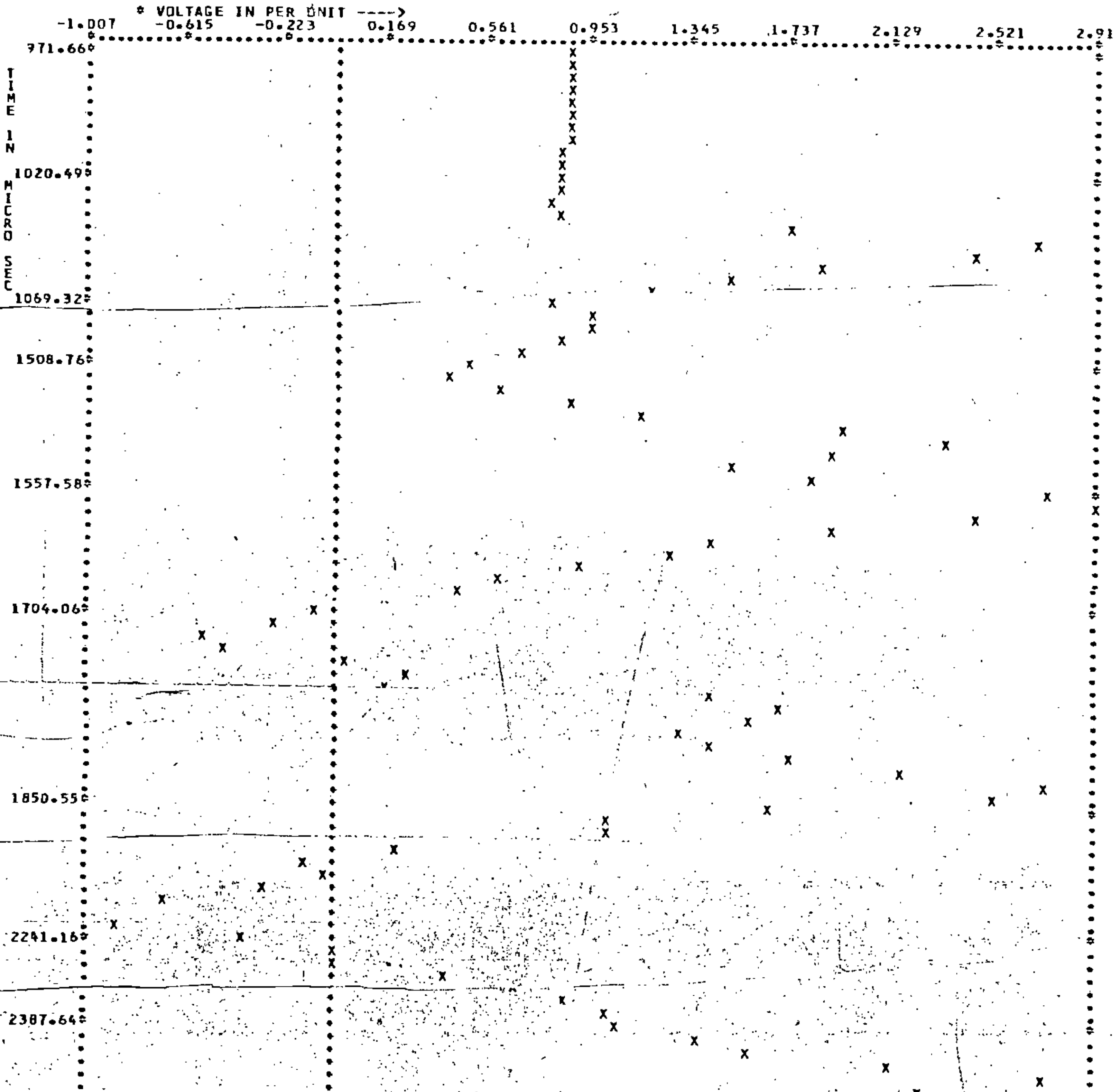

FIG 5.4

* PEAK= 0.11211E 01P.U.
* CREST = 0.10408E 01P.U.
FOR STEADY-STATE CALCULATION

OMEGA = 0.600E 06 N = 4096
DW = 314.159 DT = 0.488E-05
DF = 50.000 A = 205.000

SWITCHING BUS = 38 GHORASHAL
SURGE BUS= 11 VERAMARA
ALL GENERATORS ARE CONNECTED
TRAVEL TIME = 758.0 MICRO SEC

PEAK = 2.880P.U.
* CREST = 0.045P.U.

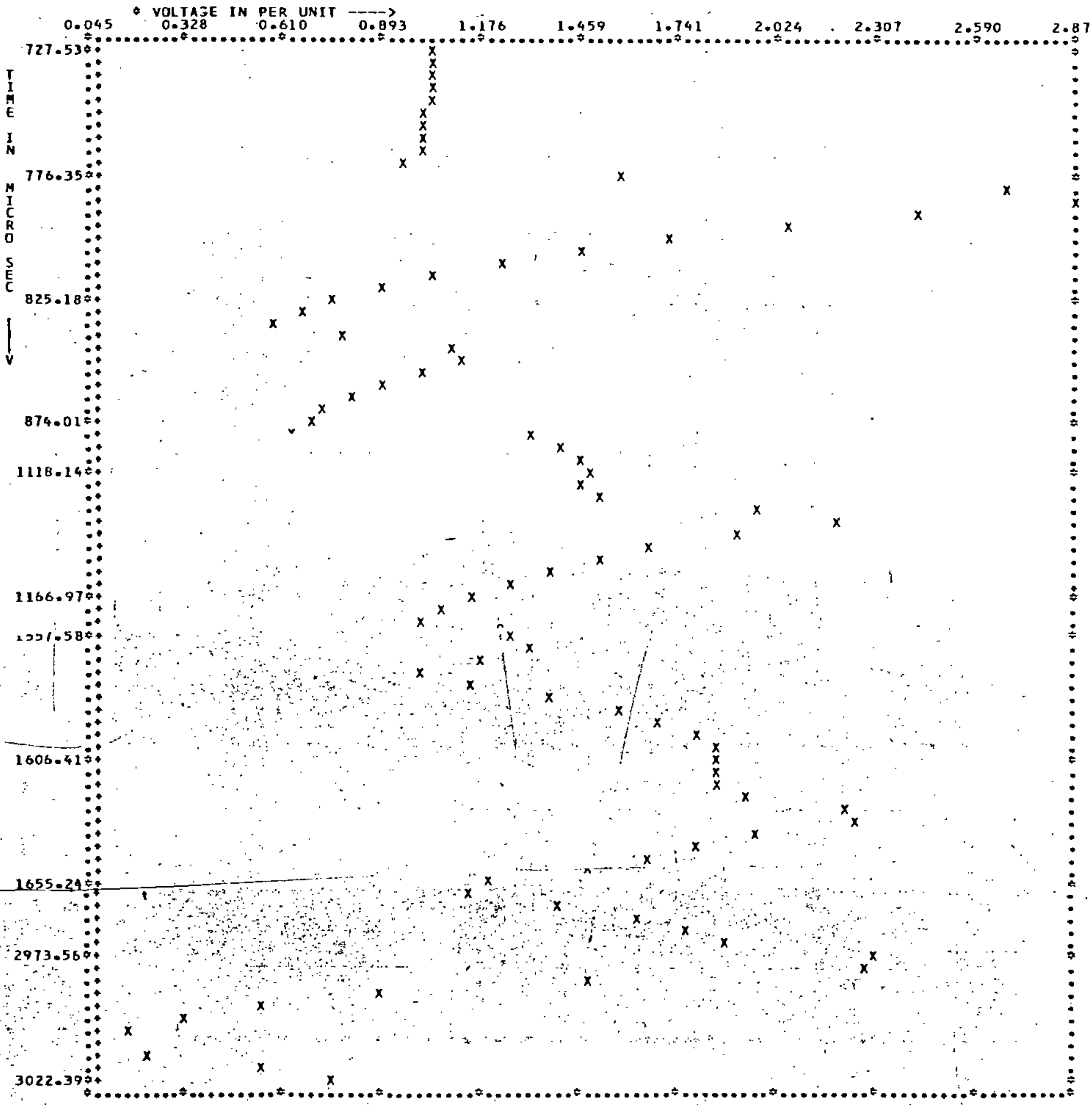

FIG 55

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CHAPTER - 6

CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

6.1 CONCLUSIONS:

A modified FFT method is presented which is capable of yielding solutions to power system problems of any degree of complexity. The superiority of the method lies in the greatly reduced computation time, as compared to conventional Fourier analysis or even other travelling wave techniques. The difficulties which arise from the necessity of numerical inversion are outlined and methods of overcoming them are described. The modified Fourier transform method has many advantages over classical methods as has already been set out in sec. 1.1. It's biggest limitation is the large computation time required. Besides this, the attraction and power of the modified Fourier transform lies in the facility with which frequency dependence of the system parameters can be taken into account if necessary. The method is well suited to analysis involving both lumped and distributed parameters and is also capable of faithfully (accurately) reproducing their variations with frequency. This method can afford a very accurate solution, since it can represent the attenuation and distortion of the line due to resistance of the line conductors and the earth return path loss.

To reduce the computation time without sacrificing accuracy modified Fast Fourier transform method is investigated. In conventional Fourier transform analysis, the response at a particular instant of time can be obtained from a large number of frequency data points i.e. here, the inverse integration is done by simpson's rule or trapizoidal rule of integration. For this a very substantial amount of computation time is required for obtaining the response over an interval of time. On the other hand using discrete Fourier transform (implemented by Fast Fourier Transform (FFT) algorithm) technique, a number of time domain responses can be obtained from an equal number of frequency data points. Therefore, it is evident that the time required for the calculation of transient over a large interval of time will greatly be reduced. However, the symmetry property of DFT (see Appendix-A) makes it possible that only half of the total frequency domain samples are enough to calculate the total time domain response. For faster and accurate calculation procedure the global response over any desired time duration can be obtained.

The value of smoothing parameter A can be chosen at much lower value by properly assigning the sampling interval T or DW, without affecting the accuracy significantly. Here the limitation for the term exp (A*T) on the global response is not applicable.

From the engineering point of view the modified Fourier transform method is less restrictive in the assumptions and very suitable for calculating the transient over voltage of a complicated power system, than any other conventional methods.

But the method is not justified economically for all cases because of its large computation time involved and hence the cost. The present analysis seems to be very effective in calculating the transient over voltages due to its faster and accurate calculation. The author believes that the method,he has adopted and which are described, are quite accurate, well suited to the system data which are normally available and very much faster than any conventional methods.

The power system operating voltage levels are reaching such magnitudes that ransient over voltages due to switching operations within the system can exceed [4,15] those due to lighting surges. Therefore, in EHV and UHV system the overall insulation level and the insulation coordination of the system are being determined more by the switching behaviour of the system. Switching transients can contain frequencies as high as 100 kc/s [16,4] and the line parameter under switching condition are frequency dependant. In the present analysis the maximum frequency chosen is about 160 KHZ. So the present method is very suitable to the problem at hand.

The particular method selected for the analysis is a variation of the familiar Fourier transform method. In this method, the frequency response of the system is calculated after a regular interval of DW, for N/2 sample values i.e. half of the total time domain samples to be evaluated. After folding and flipping the sampled frequency response the inverse discrete Fourier transformation is performed which is implemented by Fast Fourier transform algorithm for faster calculation of DFT.

The IDFT implemented by IFFT gives rise to a number of associated computational problems. After performing a detailed investigations over DFT (FFT) (Ch-2 and 3), a set of optimum parameter values is found out considering sampling theorem and other practical constraints. The effect of maximum frequency limit OMEGA, sampling interval T or DW, the smoothing parameter-'A' and the number of time or frequency domain samples N On the accuracy of calculation and computation time is studied in detail. A network reduction technique[1] is used for application of the basic method to the complicated power system network. The analysis of transmission line is based on distributed parameters and the single phase single line representation of the transmission line is considered. Finally the method has been developed and the network reduction technique has been applied to the National Grid of BPDB for calculating the switching transient response at different busber considering switching at a particular bus. The computer program developed for such calculation is capable of considering different switching conditions especially the line energization and load rejection operation.

In the present analysis, it is observed that the initial conditions of the system can be included easily. Difficulties arise to represent the effect of nonlinearties such as those due to surge diverters, magnetic saturation, corona and circuit-Breaker arc on the transient response. A technique [17] for inclusion of such nonlinear elements by the method of Fourier transform has been discussed in the literature.

6.2. SCOPE OF FURTHER RESEARCH

Further research work in the field of transient analysis of power system networks, under different switching conditions, using modified Fast Fourier Transform Technique could be extended on the basis of following considerations:

1. Considering the three-phase representation of ~~transposed~~ or non-transposed transmission line instead of single phase representation and transforming the line equations into its modal form. For multiphase system the effects of mutual coupling have to be taken into account.

2. Considering the effect of non linearities such as surge diverters, magnetic saturation, circuit-breaker arc, corona etc.

3. Considering the frequency dependancies arising from the Earth-return path.

4. The present analysis could be extended on underground cables instead of over head transmission lines and the line consisting of cables and over head transmisssion lines could be investigated.

5. The analysis could be extended to calculate the electromagnetic transients, considering frequency dependant system parameters for aerial and ground modes of transmission.

6. The factors affecting the transient over voltages such as: i) the trapped charge voltage (sometimes referred to as the residual voltage), ii) the nature and impedence of the source, iii) non-simulteneous closure of the three phases of the circuit breaker energising the line, iv) the amount of reactive compensation (shunt) connected to the line and losses (conductor resistive loss and corona) could be incorporated with the program and their effects on the transient voltages could be investigated.

7. Our National Interconnected Grid consists of three phase single and double circuit transmission lines. So the effect of using double circuit transmission line, on same tower, on the transient overvoltages could be investigated.

APPENDIX - A

PROPERTIES OF THE DISCRETE FOURIER TRANSFORM (DFT)

In this section, some properties of the DFT are studied. It is observed that the DFT has properties similar to those of the continuous Fourier Transfor, But the DFT also possesses some properties not shared by the continuous Fourier Transform.

1. Periodicity:

Both X_n and x_k are periodic with period N, i.e.

$$X_{N+n} = X_n \quad \text{for all } n \qquad \text{(A-1)}$$

and

$$x_{N+k} = x_k \quad \text{for all } k \qquad \text{(A-2)}$$

This property is the result of the periodic property of the complex exponential function exp $(-j2\pi/N)$ used in the defining equations. This property can be displayed by using a circle. If the samples in the time or frequency domain are laid equally spaced around the circle as shown in Fig. (A-1), they repeat themselves with a period of N.

From the circular arrangement of Fig. (A-1), A discrete signal x_k is an even function if $x_{N-k} = x_k$, and a signal x_k is an odd function if $x_{N-k} = -x_k$, this two types of functions are defined.

This property says that as a result of the DFT operation a finite duration time sequence becomes a periodic sequence, called the periodic extension of the original finite duration sequence. Similarly the frequency domain sequence also becomes periodic. This periodic extension is illustrated in Fig. A-2.

As a result of the sampling in the time domain, the frequency spectrum becomes periodic as shown in Fig. A2(d). When the frequency spectrum is sampled as shown in Fig. (f), the time domain sequence becomes periodic, as shown in (e). The sequences in Fig. (g) & (h) represent the periodic extensions of the finite frequency and time sequences in (f) & (c), respectively. The two periodic sequences in Fig. (h) & (g) are related by the discrete Fourier Transform.

2. Time Reversal

If the data points are reversed in the time domain the DFT of the reversed signal is given by:

$$\text{DFT}\ (x_{-k}) = X_{-n} = X_{N-n} \tag{A-3}$$

To prove this property, we use, according to property 1, $x_{-k} = x_{N-k}$ in the original equation defining DFT.

$$\text{DFT}\ (x_{-k}) = \sum_{k=0}^{N-1} x_{N-k} e^{-(j2\pi/N)\ nk}$$

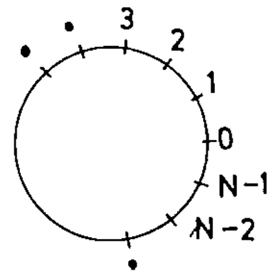

Fig. A.1: A set of samples arranged arround a circle.

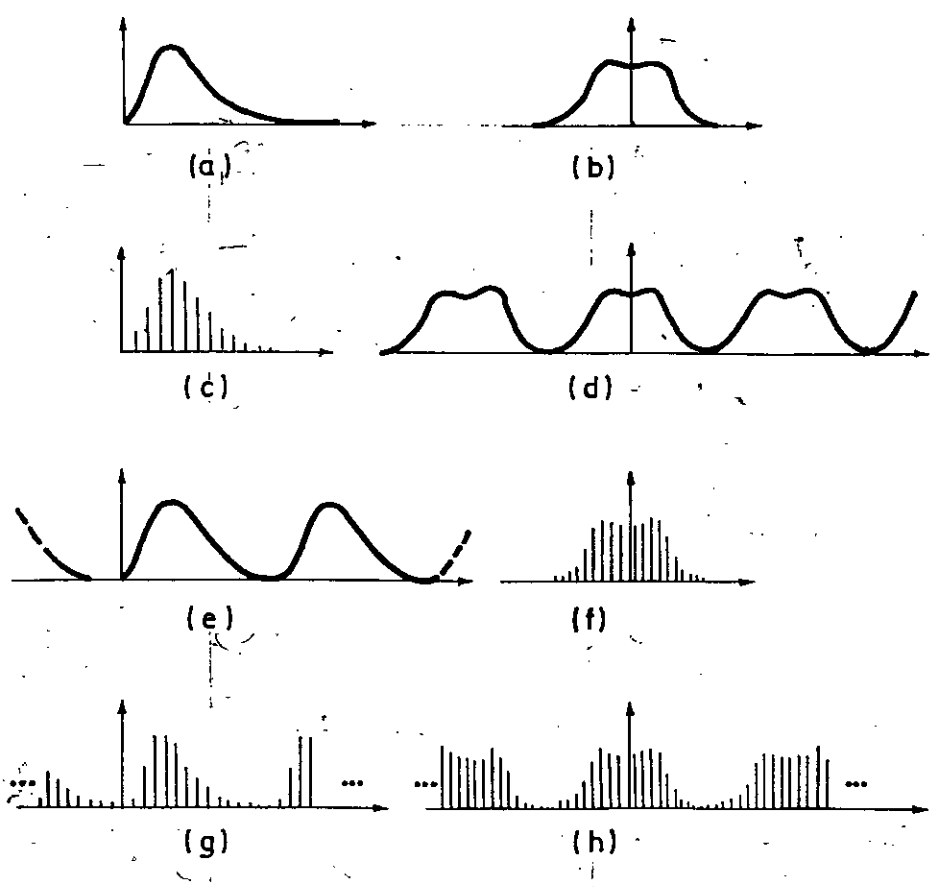

Fig. A.2: a) A continuous signal & (b) its amplitude spectrum
(c) A sample discrete signal and (d) its amplitude spectrum
(f) A sampled discrete spectrum and (e) its corresponding
signal (g) periodic extention of finite duration sequence
in f. (h) periodic extension of finite duration sequence
in C.

Changing the variables N-k = m

$$\text{DFT}\ (x_{-k}) = \sum_{m=1}^{N} x_m e^{-(j2\pi/N)\ n(N-m)}$$

$$= \sum_{m=1}^{N} x_m e^{-(j2\pi/N)\ (-n)m}$$

$$= X\text{-}n = X_{N-n}$$

Where use is made of the fact that $X_N = x_o$ and $e^{-j2\pi}=1$, and the last part is the result of property 1. This property says that reversing the time samples is equvalent to reversing the frequency samples.

3. Complex Conjugate of DFT

The conjugate of any complex discrete signal x_k is given by:

$$\bar{x}_k = \text{DFT}\ (\bar{X}_{n/N}) \qquad \text{(A-4)}$$

Where $\bar{x}_k$ denotes the complex conjugate of x_k. To see this, we take the complex conjugate of both sides of Eq. (2.12)

$$\bar{x}_k = \frac{1}{N}\sum_{n=0}^{N-1} X_n e^{(j2\pi/N)kn} = \sum_{n=0}^{N-1} \bar{X}_n/N\ e^{-j(2\pi/N)kn}$$

$$= \text{DFT}\ (\bar{X}_n/N)$$

The significance of this property is that the direct DFT algorithm (implemented by FFT) can be used to obtain the time domain sequence (i.e. the IDFT) from the frequency domain sequence by adding the scale factor 1/N and taking the indicating conjugates.

4. DFT of Complex-Conjugate Sequence

For any complex discrete signal x_k,

$$\mathrm{DFT}\ (\bar{x}_k) = \bar{X}_{-n} = \bar{X}_{N-n} \tag{A-5}$$

To prove this relationship, we take the conjugate of both sides of DFT equations.

$$\bar{X}_n = \sum_{k=0}^{N-1} \bar{x}_k\ e^{(j2\pi/N)nk} = \sum_{k=0}^{N-1} \bar{x}_k e^{-(j2\pi/N)\ (-n)k}$$

Replacing n by -n in the above expression and by property 1, we have:

$$\bar{X}_{N-n} = \bar{X}_{-n} = \sum_{k=0}^{N-1} \bar{x}_k\ e^{-(j2\pi/N)nk} = \mathrm{DFT}\ (\bar{x}_k)$$

According to this property, taking the complex conjugate of a discrete signal in the time domain is equivalent to taking the conjugate of the frequency spectrum reversed in order in the frequency domain.

5. Real Signal

For a real signal x_k, $\bar{x}_k = x_k$. Therefore, according to property 4.

$$\bar{X}_{N-n} = X_n \qquad \text{for } n = 0,1,2, \ldots\ldots\ldots N-1 \qquad \text{(A.6)}$$

To interpret this property for a real signal, we use the circular display of the frequency samples. The real part (and the magnitude) of the frequency samples is an even function and the imaginary part (and the phase) is an odd function for a real signal. Therefore, it requires only N/2+1 (N-even) or (N+1)/2 (N-odd) samples in the frequency domain to completely characterize a real signal. This property is displayed in Fig. A.2 for the magnitude of the frequency samples of a real signal.

APPENDIX - B

SCHNYDER BERGERONS METHOD [1,6,7,8]

The differential equations of transmission line are:

$$- \frac{\partial v}{\partial x} = L \frac{\partial i}{\partial t} + Ri \qquad \text{(B-1)}$$

$$- \frac{\partial i}{\partial x} = C \frac{\partial v}{\partial t} + Gv \qquad \text{(B-2)}$$

Where v & i are voltage and current in the line at a distance x and L,C,R and G are respectively the line series inductance, shunt capacitance, series resistance and shunt conductance per unit length. The method applies to lossless lines where R and G are zero and L and C are independent of frequency. Therefore the equations become:

$$- \frac{\partial v}{\partial x} = L \frac{\partial i}{\partial t} \qquad \text{(B-3)}$$

$$- \frac{\partial i}{\partial x} = C \frac{\partial v}{\partial t} \qquad \text{(B-4)}$$

The general solutions of the above equations are:

$$i\ (x,t) = f_1(x-vt) + f_2(x+vt) \qquad \text{(B-5)}$$

and $$v\ (x,t) = Z \left[f_1(x-vt) - f_2\ (x+vt) \right] \qquad \text{(B-6)}$$

Where $Z = \sqrt{L/C}$ and $v = 1/\sqrt{LC}$, are the surge impedance and phase velocity respectively. From Eq.(B.5) & (B.6) :

$$v(x,t) + Zi\,(x,t) = 2zf_1\,(x-vt) \qquad (B-7)$$

$$v(x,t) - zi(x,t) = -\,2zf_2(x+vt) \qquad (B-8)$$

Now subject to the limitations stated before, those are relationships between the conditions at each end of the line at time t and at time $t-\tau$ which exist independent of the terminating networks.This fact can be explained assuming a fictitious observer moving along the line in the forword direction at velocity v. Since $(x-vt)$ remains constant, the expression $[v(x,t) + zi(x,t)]$ encountered by the observer when he leaves node m at time $(t-\tau)$ must be the same when he arrives at node k at time t. For the line of figure (B.1) these relationships are-

$$v_k(t) - z.i_k(t) = v_m(t-\tau) + z.i_m(t-\tau) \qquad (B.9)$$

$$v_m(t) - z.i_m(t) = v_k(t-\tau) + z.i_k\,(t-\tau) \qquad (B.10)$$

These equations give relationships between v & i at both ends of the transmission line which, provided the conditions a travel time earlier are known, enable the transmission line to be replaced by a current source in parallel with a resistance z. This allows a solution to be obtained for the voltage and currents at time t in the network consisting of the ends of the transmission lines and the components connected to them.

The quantities $v \pm zi$ are known as characteristics and are directly related to the forward and backword travelling waves. Eq. (B.9) & Eq. (B.10) can be rewritten:

$$B_k(t) = F_m(t-\tau) \qquad (B.11)$$

$$B_m(t) = F_k(t-\tau) \qquad (B.12)$$

Where $F_k = v_k + zi_k$ = forword travelling wave at end k.

$B_k = v_k - zi_k$ = Backword travelling wave at end k.

Eqs. (B.11) & (B.12) are the basic equations for this method. Dommel[6] extends the basic Eq. (B.11) & Eq. (B.12) to include an approximation for series losses. The model used for the transmission lines is shown in Fig. B.2. This results in equations[7]:

$$B_k(t) = \frac{z}{z+R/4} F_m(t-\tau) + \frac{R/4}{z+R/4} F_k(t-\tau) \qquad (B.13)$$

$$B_m(t) = \frac{z}{z+R/4} F_k(t-\tau) + \frac{R/4}{z+R/4} F_m(t-\tau) \qquad (B.14)$$

In these equations the impedance used in defining the characteristics is modified to $(z+R/4)$. Eq. (B.11) & Eq. (B.12) are the particular case of Eq. (B.13) & Eq. (B.14) respectively, where ere R is zero.

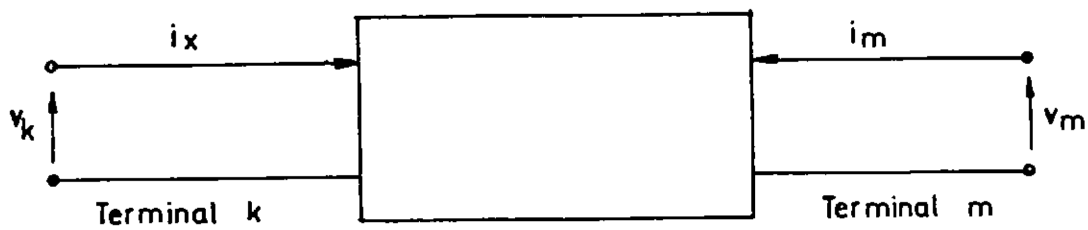

Fig. B.1: Single-phase lossless transmission line.

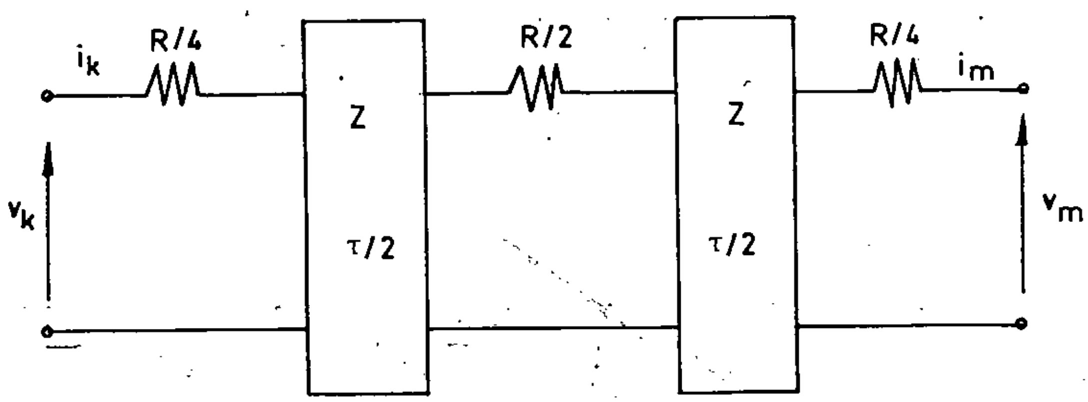

Fig. B.2: Approximate Model for line with series resistance.

APPENDIX-C
DISCUSSION ON THE FFT FORTRAN SUBROUTINE

Based on the Flow-chart given in literature[2,3] a FORTRAN subroutine program implementing a diamation- in-time FFT algorithm is presented in Fig. C-1. The same subroutine FFT can be used to compute both the direct FFT and the inverse FFT, provided the sign in the complex exponential constant W is properly assigned as:

$$W = \exp\ (-j2\pi/N) \tag{C.1}$$

and $$W = \exp\ (j2\pi/N) \tag{C.2}$$

for the direct FFT and inverse FFT respectively (See Eq. (2.11) & Eq. (2.12)).

This is accomplished in the FORTRAN program through an integer code KODE, as implemented in line 24 of the subroutine program. This KODE is to be entered as 1 for direct FFT and -1 for the inverse FFT. Since the algorithm is based on N being a power of 2, the program has to make sure that N is passed to the subroutine as a power of 2. This is achieved in lines 3 through 9. If N is not a power of 2 an error message will be printed out and the FFT run will be aborted. After N is found to be a power of 2 the DO 30 loop then performs the in-place computation through IR stages $(N=2^{IR})$. The in-place computations are implemented in lines 26 to 28. $W^{O}=1$ requires no complex operations which is implemented in lines 19 to 21. It is observed that the power m in w^m in each stage has a bit- reversal relationship to the position number, as shown in line 18 of the program. The bit reversal FUNCTION subprogram KBIRT is used here for this purpose and later in line 36 to bit-reverse the output samples. It accepts an IR bit integer k and returns a bit-reversed integer KBIRT (IR bits). The parameter DELTA is used to multiply the samples so that the FFT algorithm can be used as an approximation to the continuous Fourier transform. According to Eq. (2.9) this factor should be T, the time step size, used in the zero-order approximation of the direct Fourier Integral. For inverse Fourier transform, this factor should be the frequency step size DF in HZ used in the zero-order approximation (see Eq. (2.7b)) of the complex inverse Fourier transform integral. For the implementation of strict DFT, DELTA should be set equal to 1 (see Eq. (2.11)) for direct DFT and equal to 1/N for the inverse DFT (see Eq. (2.12)).

```
      SUBROUTINE FFT(KODE,N,DELTA,X)
      COMPLEX X(N),W,X1
C * *-------------------------------------------------------------------------------
C     KODE =1 FOR DIRECT FFT
C          =-1 FOR INVERSE FFT
C        N = NO. OF TIME OR FREQUENCY DOMAIN DATA POINTS
C     DELTA=DT FOR DIRECT FFT
C          =DF FOR INVERSE FFT
C        X = FREQUENCY OR TIME DOMAIN DATA SAMPLES
C * *------------------------------------------------------------------------------*
      IR=0
      N1=N
    5 N2=N1/2
      IF(N2*2.NE.N1) GO TO 100
      IR=IR+1
      N1=N2
      IF(N1.GT.1)GO TO 5
      PN=6.283185/N
      L=N/2
      IR1=IR-1
      K1=0
      DO 30 IS=1,IR
   15 DO 20 I=1,L
      K=K1+1
      KPL=K+L
      AM=KBIRT(K1/2**IR1,IR)
      IF(AM.NE.0.0) GO TO 18
      X1=X(KPL)
      GO TO 19
   18 ARG=AM*PN
      C=COS(ARG)
      S=SIN(ARG)*(-1)*KODE
      W=CMPLX(C,S)
      X1=W*X(KPL)
   19 X(KPL)=X(K)-X1
      X(K)=X(K)+X1
   20 K1=K1+1
      K1=K1+L
      IF(K1.LT.N) GO TO 15
      K1=0
      IR1=IR1-1
   30 L=L/2
      DO 40 K=1,N
      K1=KBIRT(K-1,IR)+1
      IF(K1.LE.K) GO TO 40
      X1=X(K)
      X(K)=X(K1)
      X(K1)=X1
   40 CONTINUE
      IF(DELTA.EQ.1)GO TO 100
      DO 50 K=1,N
   50 X(K)=DELTA*X(K)
      RETURN
  100 PRINT 101, N
  101 FORMAT(//5X,'*** N= ',I6,'IS NOT A POWER OF TWO,FFT RUN ABORTED*')
      RETURN
      END
      FUNCTION KBIRT(K,IR)
      KBIRT=0
      K1=K
      DO 1 I=1,IR
      K2=K1/2
      KBIRT=2*KBIRT+K1-2*K2
    1 K1=K2
      RETURN
      END
C
C
C
C
```

```
C                    APPENDIX-D1
C                    ===========
C  * *-----------------------------------
C   NON PERIODIC TEST SIGNAL EXP(-T)
C   THE USE OF DFT.
C  * *-----------------------------------
      COMPLEX F(64)
      DIMENSION T(64),DFN(64),FMAG(64),FR(64),FI(54)
      DT=0.10
      N=64
      DF=1.0/(N*DT)
      T(1)=0.0
      DO 5 I=1,32
      T(I)=T(I-1)+DT
5     F(I)=CMPLX(EXP(-T(I)),0.0)
      DO 10 I= 33,64
10    F(I)=CMPLX(0.0,0.0)
      CALL FFT(1,N,DT,F)
      PRINT 11
11    FORMAT('1',25X,'64 POINT FREQUENCY SPECTRUM OF EXPONENTIAL'/25X,
     +'FUNCTION COMPUTED FROM 32 SIGNAL SAMPLES'//19X,'FREQ(HZ)',5X,'RE
     +AL PART'7X,'IMAGINARY PART',4X,'MAGNITUDE'//)
      DO 20 I= 1 ,N
      DFN(I)=(I-1)*DF
      FMAG(I)=CABS(F(I))
      FR(I)=REAL(F(I))
      FI(I)=AIMAG(F(I))
20    PRINT 21, I,DFN(I),F(I),FMAG(I)
21    FORMAT(5X,I5,5X,E12.4,5X,E12.4,5X,E12.4,5X,E12.4/)
C     CALL PLT1(N,1,54,DFN,FMAG)
      CALL PLT2(N,1,64,DFN,FR,FI)
      STOP
      END
C
C
C
C
C
C                    APPENDIX-D2
C                    ===========
C  * *--------------------------------------
C   NON PERIODIC TEST SIGNAL EXP(-T)
C   THE USE OF IDFT.
C  * *--------------------------------------
      COMPLEX F(128)
      DIMENSION FM(128),T(128)
      DT=0.10
      DO 200 JJ=1,2
      N=16*2**JJ
      DF=1.0/(N*DT)
      DW=2.0*3.14*DF
      W=0.0
      M=N/2+1
      M1=M+1
      DO 10 I= 1,M
      F(I)=1.0/CMPLX(1.0,W)
      W=W+DW
   10 CONTINUE
      K=0
      DO 20 I= M1,N
      K=K+1
   20 F(I)=CONJG(F(M-K))
      OMEGA=N*DW
      WRITE(3,175) DW,N,DT,OMEGA
 175  FORMAT(////10X,'THE VALUE OF DW IS= ',F8.4/10X,'& THE VALUE OF N
     +IS = ',I3/10X,'& THE VALUE OF DT IS= ',E9.2/10X,'& THE VALUE OF OM
     +EGA IS = ',E9.2////)
      CALL FFT(-1,N,DF,F)
      T(1)=0.0
      NN=N-1
      DO 22 I=1,NN
      T(I+1)=T(I)+DT
 22   FM(I)=CABS(F(I))
      FM(N)=CABS(F(N))
C      PRINT 129
C129   FORMAT(5X,' I ',5X,' T(I) ',5X,' F(I)',10X,'FM(I)'//)
C      DO 220 I=1,N
C 220  WRITE(3,21) I,T(I),F(I),FM(I)
   21 FORMAT(' ',I5,3X,E14.6,8X,E13.6,5X,E13.6,10X,E13.6)
      CALL PLT1(N,1,N,T,FM)
```

```
200     CONTINUE
        STOP
        END
C
C
C
C
C
C                    APPENDIX-D3
C                    ===========
C  * *-------------------------------------------------------------------
C       SAMPLE PROGRAM TO USE INVERSE FFT TO GENERATE TIME SAMPLES
C       FROM FREQUENCY SAMPLES OF A RECTANGULAR TEST SIGNAL.
C  * *-------------------------------------------------------------------
        DIMENSION XM(64),T(64),XMA(64)
        COMPLEX X(64)
        DATA N,DT/64,0.1/
        DF=1.0/(N*DT)
        DO 5 I=1,64
5       X(I)=CMPLX(0.0,0.0)
        X(2)=CMPLX(0.0,-1.0)
        X(64)=CONJG(X(2))
        X(4)=CMPLX(0.0,-.333333)
        X(62)=CONJG(X(4))
        X(6)=CMPLX(0.0,-0.2)
        X(60)=CONJG(X(6))
        X(8)=CMPLX(0.0,-.142855)
        X(58)=CONJG(X(8))
        CALL FFT(-1,N,DF,X)
        PRINT 6
6       FORMAT('1',10X,'THE RESULT OF FFT'/10X,'TIME(SECONDS)',6X,'REAL PA
       +RT',7X,'IMAGINARY PART',5X,'MAGNITUDE'/)
        DO 11 I=1,64
        XMA(I)=CABS(X(I))
11      XM(I)=REAL(X(I))
        T(1)=0.0
        DO 10 I=2,64
        T(I)=T(I-1)+DT
10      PRINT 7,I,T(I),X(I),XMA(I)
7       FORMAT(8X,I2,5X,E12.5,5X,2E13.5,5X,E13.5)
        CALL PLT1(N,1,N,T,XM)
        STOP
        END
C
C
C
C
C
C
C                    APPENDIX-D4
C                    ===========
C  * *-------------------------------
C       SAMPLE PROGRAM TO USE FFT
C       SQUARE WAVE TEST SIGNAL.
C  * *-------------------------------
        DIMENSION FMAG(64),FR(64),FI(64),DFN(64)
        COMPLEX F(64)
        N=64
        DT=0.1
        DO 5 I=1,32
5       F(I)=CMPLX(1.0,0.0)
        DO 10 I=33,64
10      F(I)=CMPLX(-1.0,0.0)
        CALL FFT(1,N,DT,F)
        DF=1.0/(N*DT)
        PRINT 6
6       FORMAT(10X,'FREQUENCY SPECTRUM OF A SQUARE WAVE AS COMPUTED FROM
       +FFT'//10X,'FREQUENCY(HZ)',10X,'REAL PART',5X,'IMAGINARY PART',
       +'MAGNITUDE'/)
        DFN(1)=0.0
        DO 11 I=1,N
        F(I)=F(I)*DF
        DFN(I+1)=DFN(I)+DF
        FMAG(I)=CABS(F(I))
        FI(I)=AIMAG(F(I))
        FR(I)=REAL(F(I))
11      PRINT 7,I,DFN(I),F(I),FMAG(I)
7       FORMAT(8X,I5,5X,E12.5,5X,2E14.4,5X,E13.5)
        CALL PLT2(N,1,N,DFN,FI,FR)
        STOP
        END
```

```
C
C
C
C
C
C                         APPENDIX-D5
C                         ===========
C * *---------------------------------
C   NON PERIODIC TEST SIGNAL EXP(-T)
C   THE USE OF IFFT.
C   ZERO PADDING TECHNIQUE.
C * *---------------------------------
      COMPLEX F(64)
      DIMENSION FM(64),T(64)
C   N>M *--------------------*
      N=64
      N1=56
      NPAD=N-M
      N2=N1/2+1
      N3=N2+1
      M=N/2+1
      M1=M+1
      DT=250.0E-06
      OMEGA=1.0E04
      DF=1./(N*DT)
      DW=2.0*3.14*DF
      W=0.0
      DO 10 I= 1,N2
      F(I)=1.0/CMPLX(1.0,W)
      W=W+DW
   10 CONTINUE
C * ******************************
C   ZERO PADDING
      DO 11 I=N3,M
   11 F(I)=CMPLX(0.0,0.0)
C * ******************************
C   FOLDING & FLIPPING
      K=0
      DO 20 I=M1,N
      K=K+1
   20 F(I)=CONJG(F(M-K))
C * ******************************
      PRINT 175, DW,N,DT,OMEGA
  175 FORMAT(////10X,'THE VALUE OF DW IS= ',F8.4/10X,'& THE VALUE OF N
     +IS = ',I3/10X,'& THE VALUE OF DT IS=',E10.3/10X,'& THE VALUE OF OM
     +EGA IS = ',E10.3////)
C     DO 44 I= 1,N
C 44  PRINT 33, I,F(I)
   33 FORMAT(' ',I5,7X,E14.5,5X,E14.5)
      CALL FFT(-1,N,DF,F)
      T(1)=0.0
      NN=N-1
      DO 22 I=1,NN
      T(I+1)=T(I)+DT
22    FM(I)=CABS(F(I))
      FM(N)=CABS(F(N))
      DO 220 I=1,N
220   PRINT 21, I,T(I),F(I),FM(I)
  21  FORMAT(' ',I5,3X,E14.6,8X,E13.6,5X,E13.6,10X,E13.6)
      CALL PLT1(N,1,N,T,FM)
200   CONTINUE
      STOP
      END
C
C
C
C
C
C                         APPENDIX-D6
C                         ===========
C * *------------------------------------
C     TO CALCULATE  V(W) FOR  AN OPEN ENDED LOSSY TRANSMISSION LINE OF
C     LENGTH 10 MILES.
C     & TO CALCULATE  V(T)  BY USING  MODIFIED FFT.
C * *------------------------------------
      REAL L,LNG,LMT
      COMPLEX F(1024),ALP,S,XA,COSH
      DIMENSION FM(1024),W(1024),T(1024)
      DATA  PI,R,L,C,LNG/3.141527,2.75E-2,1.386E-3,2.09E-8,10./
      DATA  N,LMT,A/1024,1.0E6.1.0E04/
```

```
      DT=PI/LMT
      DF=1.0/(N*DT)
      DW=2.*PI*DF
      N1=(2.0*LMT)/DW
      IF( MOD(N1,2).EQ.0 ) GO TO 55
      N2=(N1+1)/2
      GO TO 56
 55   N2=N1/2+1
 56   N3=N2+1
      M=N/2+1
      M1=M+1
      PRINT 401,N,LMT,DW,DT,DF
  401 FORMAT(///10X,'OBSERVATION'//10X,'THE VALUE OF N =',I5/10X,'THE
     +VALUE OF LMT =',E12.5/10X,'THE VALUE OF DW=',E12.5/10X,'THE VAL
     +E OF DT=',E12.5/10X,'THE VALUE OF DF=',E12.5/)
       W(1)=0.0
      DO 10 I= 1,N2
      S=CMPLX(A,W(I))
      ALP=CSQRT((R+S*L)*S*C)
      XA=CEXP(ALP*LNG)
      COSH=(XA+1.0/XA)/2.0
      F(I)=1.0/(S*COSH)
      IF(W(I).NE.0.0) F(I)=F(I)*SIN(DT*W(I))/(DT*W(I))
      W(I+1)=W(I)+DW
   10 CONTINUE
C * *------------------
      DO 11 I=N3,M
   11 F(I)=CMPLX(0.0,0.0)
C * *------------------
      K=0
      DO 20 I= M1,N
      W(I)= W(I-1)+DW
      K=K+1
   20 F(I)=CONJG(F(M-K))
C * *-----------------
      WRITE(3,175) A
  175 FORMAT(/10X,'THE VALUE OF A IS= ',E13.6/)
      CALL FFT(-1,N,DF,F)
  402 T(1)=0.0
      DO 22 I=1,100
      F(I)=F(I)*EXP(A*T(I))
      FM(I)=CABS(F(I))
      T(I+1)=T(I)+DT
   22 CONTINUE
C      PRINT 21,(I,T(I),FM(I),I=1,100)
C  21  FORMAT(4X,I4,8X,E12.5,8X,E13.6/)
      CALL PLT1(100,10,60 ,T,FM)
      STOP
      END
```

```
C                       APPENDIX-E1
C                       ===========
C * *------------------------------------------------------------
C         EFFECT OF MAXIMUM FREQUENCY LIMIT ON THE SOLUTION
C                   OMEGA VARIES FROM 1.0E6 TO 4.0E6
C * *------------------------------------------------------------
      REAL L,LL
      COMPLEX F(4096),GAMA,S,XY,COSH,F1(4096),F2(4096)
      DIMENSION FM1(4096),W(4096),T(4096),FM2(4096)
      DATA  PI,R,L,C,LL/3.141527,2.75E-2,1.386E-3,2.09E-8,10.0/
      DATA  N,DW,A/1024,2000.0,1.0E4/
       DF=DW/(2.0*PI)
       DT=1.0/(N*DF)
      DO 400 IT=1,2
      PRINT 401,IT,N,DW,DT,DF,A
  401 FORMAT(///10X,'OBSERVATION',I2//10X,'N =',I5,11X,'DW =',F8.3/
     +10X,'DT =',E10.3,5X,'DF =',F10.3/10X,'A =',E10.3/)
      DO  200 II= 1,2
      IF(IT .EQ.1 .AND. II .EQ.1) OMEGA=2.0E06
      IF(IT .EQ.1 .AND. II .EQ.2) OMEGA=1.0E06
      IF(IT .EQ.2 .AND. II .EQ.1) OMEGA=1.0E06
```

```
      IF(IT .EQ.2 .AND. II .EQ.2) G
      NT=(2.0*OMEGA)/DW
       IF(MOD(NT,2) .EQ. 0) GO TO 456
       NO=(NT+1)/2
       GO TO 457
456   NO=NT/2+1
457   NR=NO+1
      MO=N/2+1
      MP=MO+I
C * *---------------------
      W(1)=0.0
      DO 10 I= 1,NO
      S=CMPLX(A,W(I))
      GAMA=CSQRT((R+S*L)*S*C)
      XY=CEXP(GAMA*LL)
      COSH=(XY+1.0/XY)/2.0
      F(I)=1.0/(S*COSH)
      W(I+1)=W(I)+DW
   10 CONTINUE
C * *--------------------
      DO  11 I=NR,MO
   11 F(I)=CMPLX(0.0,0.0)
C * *--------------------
      K=0
      DO 20 I= MP,N
      W(I)= W(I-1)+DW
      K=K+1
   20 F(I)=CONJG(F(MO-K))
C * *--------------------*
      DO 13 I=1,N
      IF(II .EQ. 1) GO TO 39
      F2(I)=F(I)
      GO TO 13
   39 F1(I)=F(I)
   13 CONTINUE
C * *--------------------*
       IF(II .EQ. 1) PRINT 175 ,OMEGA
       IF(II .EQ. 2) PRINT 176 ,OMEGA
 175  FORMAT(10X,'CURVE-A FOR OMEGA= ',E10.3)
 176  FORMAT(10X,'CURVE-B FOR OMEGA= ',E10.3)
C * *----------------------**
      IF (II .EQ. 1) GO TO 51
      CALL FFT(-1,N,DF,F2)
      GO TO 402
  51  CALL FFT(-1,N,DF,F1)
  402 T(1)=0.0
      NC=100
      DO 22 I=1,NC
      IF (II.EQ.1) GO TO 52
      F2(I)=F2(I)*EXP(A*T(I))
      FM2(I)=CABS(F2(I))
      GO TO 22
52    F1(I)=F1(I)*EXP(A*T(I))
      FM1(I)=CABS(F1(I))
  22  T(I+1)=T(I)+DT
 200  CONTINUE
      CALL PLT2(100,10,60,T,FM1,FM2)
 400  CONTINUE
      STOP
      END
C
C
C
C
C
C
C                  APPENDIX-E2
C                  ===========
C * *-------------------------------------
C            THE VALUE OF OMEGA=1.0E6,N=1024
C                           DW= 2000.0,A=1.0E4
C            INCORPORATION OF SIGMA FACTOR
C            TO REDUCE GIBB'S OSCILLATION
C * *-------------------------------------
      REAL L,LL
      COMPLEX F(1024),GAMA,S,XY,COSH,F1(1024)
      DIMENSION FM(1024),W(1024),T(1024),FM1(1024)
      DATA  PI,R,L,C,LL/3.141527,2.75E-2,1.386E-3,2.09E-8,10./
      DATA  N,OMEGA,DW,A/1024,1.0E6,2000.,1.0E4/
      DS=PI/OMEGA
      NT=(2.0*OMEGA)/DW
```

```
      DF=DW/(2.0*PI)
      DT=1.0/(N*DF)
      N2=NT/2+1
      N3=N2+1
      M=N/2+1
      M1=M+1
C   * ---------------------------
      PRINT 400,N,OMEGA,DW,DT,A
  400 FORMAT(///10X,'OBSERVATION = 1'//10X,'THE VALUE OF N =',I5/10X,'TH
     +E VALUE OF OMEGA =',E12.5/10X,'THE VALUE OF DW=',E12.5/10X,'THE VA
     +LUE OF DT=',E12.5/10X,'& THE VALUE OF A =',E12.5/)
C * *---------------------------
      W(1)=0.0
      DO 10 I= 1,N2
      S=CMPLX(A,W(I))
      GAMA=CSQRT((R+S*L)*S*C)
      XY=CEXP(GAMA*LL)
      COSH=(XY+1.0/XY)/2.0
      F(I)=(1.0/(S*COSH))
       F1(I)=F(I)
      IF(W(I).NE.0.0) F1(I)=F1(I)*SIN(DS*W(I))/(DS*W(I))
      W(I+1)=W(I)+DW
   10 CONTINUE
C * *---------------------------
      DO 12 I=N3,M
       F(I)=CMPLX(0.0,0.0)
   12 F1(I)=CMPLX(0.0,0.0)
C * *---------------------------
      K=0
      DO 20 I= M1,N
      K=K+1
      F(I)=CONJG(F(M-K))
      F1(I)=CONJG     (M-K))
     *---------------------------
      CALL FFT(-1,   DF,F)
      CALL FFT(-1,N,DF,F1)
      T(1)=0.0
      DO 22 I=1,100
      F(I)=F(I)*EXP(A*T(I))
      FM(I)=CABS(F(I))
      F1(I)=F1(I)*EXP(A*T(I))
      FM1(I)=CABS(F1(I))
      T(I+1)=T(I)+DT
    2 CONTINUE
C    *---------------------------
      PRINT 175
  175 FORMAT(10X,'CURVE -A WITHOUT SIGMA FACTOR'/
     +10X,'CURVE -B WITH SIGMA FACTOR',5X,'X FOR DOUBLE PLOT')
      CALL PLT2(100,10,60,T,FM,FM1)
      STOP
      END
C
C
C
C
C
C
C
C                      PENDIX-E3
C                      ==========
C * *----------------------------------------------------------
C   THE SHAPE OF SPECTRUM FUNCTION WITH THE VARIATION OF A.
C * *----------------------------------------------------------
      REAL L,LL
      COMPLEX F(1024),GAMA,S,XY,COSH,F1(1024),F2(1024),F3(1024)
      DIMENSION FM1(1024),W(1024),T(1024),FM2(1024),FM3(1024)
      DATA  PI,R,L,C,LL/3.141527,2.75E-2,1.386E-3,2.09E-8,10./
      DATA  N ,OMEGA,DW/1024,1.0E6,2000.0/
C    *---------------------------------------
      DF=DW/(2.0*PI)
      DS=PI/OMEGA
      DT=1.0/(N*DF)
      NT=(2.0*OMEGA)/DW
      IF(MOD(NT,2) .EQ. 0) GO TO 456
      NO=(NT+1)/2
      GO TO 457
      NO=NT/2+1
      NR=NO+1
      MO=N/2+1
      MP=MO+1
      ---------------------
      PRINT 401,N,OMEGA,DW,DT
```

```
  401 FORMAT(//10X,'OBSERVATION I' //10X,'THE VALUE OF N=',I6   /10X,'&
     +THE VALUE OF OMEGA = ',E12.5/10X,'S THE VALUE OF DW =',E12.5/10X,'
     +S THE VALUE OF DT =',E12.5/)
C * *---------------------------
      DO 200 II=1,3
      IF( II.EQ. 1) A=1.0E03
      IF( II.EQ. 2) A=1.0E04
      IF( II.EQ. 3) A=2.0E04
C * *-----------------------
      W(1)=0.0
      DO 10 I= 1,ND
      S=CMPLX(A,W(I))
      GAMA=CSQRT((R+S*L)*S*C)
      XY=CEXP(GAMA*LL)
      COSH=(XY+1.0/XY)/2.0
      F(I)=1.0/(S*COSH)
      IF(W(I) .NE. 0.0 ) F(I)=F(I)*SIN (DS*W(I))/(DS*W(I))
      W(I+1)=W(I)+DW
   10 CONTINUE
C * *-----------------------
      DO 11 I=NR,MD
   11 F(I) = CMPLX(0.0,0.0)
C * *-----------------------
      K=0
      DO 20 I= MP,N
      K=K+1
   20 F(I)=CONJG(F(MD-K))
C * *-----------------------
      DO 13 I=1,ND
      IF(II.EQ.1) GO TO 39
      IF(II.EQ.2) GO TO 40
      F3(I)=F(I)
      FM3(I)=CABS(F3(I))
      GO TO 13
   40 F2(I)=F(I)
      FM2(I)=CABS(F2(I))
      GO TO 13
   39 F1(I)=F(I)
      FM1(I)=CABS(F1(I))
   13 CONTINUE
C * *------------------------------
      IF(II .EQ. 1)PRINT 175,A
      IF(II .EQ. 2)PRINT 176,A
      IF(II .EQ. 3)PRINT 177,A
  175 FORMAT(10X,'CURVE -A FOR A =',E10.3)
  176 FORMAT(10X,'CURVE -B FOR A =',E10.3,5X,'+ FOR DOUBLE PLOT')
  177 FORMAT(10X,'CURVE -C FOR A =',E10.3,5X,'* FOR TRIPPLE PLOT')
  200 CONTINUE
      CALL PLT3(70,1,60,W,FM1,FM2,FM3)
  400 CONTINUE
      STOP
      END
C
C
C
C
C
C                       APPENDIX-E4
C                       ===========
C * *-------------------------------------------
C     THE EFFECT OF SMOOTHING PARAMETER
C     A=1.0E03 TO 0.5E05
C * *-------------------------------------------
      REAL L,LL
      COMPLEX F(1024),GAMA,S,XY,COSH,F1(1024),F2(1024)
      DIMENSION FM1(1024),RE(1024),AIM(1024),W(1024),T(1024),FM2(1024)
      DATA  PI,R,L,C,LL/3.141527,2.75E-2,1.386E-3,2.09E-8,10.0/
      DATA  N,OMEGA/1024,1.0E6/
      DT=PI/OMEGA
      DF=1.0/(N*DT)
      DW=2.*PI*DF
      MD=N/2+1
      MP=MD+1
C * *-------------------------------------------
      DO 400 IT=1,2
      PRINT 401,IT,N,OMEGA,DW,DT,DF
  401 FORMAT(//10X,'OBSERVATION',I2//10X,'THE VALUE OF N =',I5/10X,'THE
     +VALUE OF OMEGA =',E12.5/10X,'THE VALUE OF DW=',E12.5/10X,'THE VALU
     +E OF DT=',E12.5/10X,'THE VALUE OF DF=',E12.5/)
C * *-------------------------------------------
```

```
      DO 200 II=1,2
      IF(IT .EQ. 1 .AND. II .EQ.1) A=1.0E03
      IF(IT .EQ. 1 .AND. II .EQ.2) A=1.0E04
      IF(IT .EQ. 2 .AND. II .EQ.1) A=1.0E04
      IF(IT .EQ. 2 .AND. II .EQ.2) A=0.5E05
C * *------------------------------------------
      W(1)=0.0
      DO 10 I= 1,MO
      S=CMPLX(A,W(I))
      GAMA=CSQRT((R+S*L)*S*C)
      XY=CEXP(GAMA*LL)
      COSH=(XY+1.0/XY)/2.0
      F(I)=1.0/(S*COSH)
      IF(W(I) .NE.0.0) F(I)=F(I)*SIN(DT*W(I))/(DT*W(I))
      W(I+1)=W(I)+DW
   10 CONTINUE
C * *---------------------------------------------
      K=0
      DO 20 I= MP,N
      W(I)= W(I-1)+DW
      K=K+1
   20 F(I)=CONJG(F(MO-K))
C * *-----------------------*
      DO 13 I=1,N
      IF(II .EQ. 1) GO TO 39
      F2(I)=F(I)
      GO TO 13
   39 F1(I)=F(I)
   13 CONTINUE
C * *-----------------------*
      IF(II.EQ.1) GO TO 51
      CALL FFT(-1,N,DF,F2)
      GO TO 402
   51 CALL FFT(-1,N,DF,F1)
  402 T(1)=0.0
C * *-----------------------*
      NC=100
      DO 22 I=1,NC
      T(I+1)=T(I)+DT
      IF(II.EQ.1) GO TO 52
      F2(I)=F2(I)*EXP(A*T(I))
      FM2(I)=CABS(F2(I))
      GO TO 22
   52 F1(I)=F1(I)*EXP(A*T(I))
      FM1(I)=CABS(F1(I))
   22 CONTINUE
C * *------------------------------
      IF(II .EQ. 1) PRINT 175,A
      IF(II .EQ. 2) PRINT 176,A
  175 FORMAT(10X,'CURVE -A FOR A =',E10.3/)
  176 FORMAT(10X,'CURVE -B FOR A =',E10.3/)
  200 CONTINUE
      CALL PLT2(100,10,60,T,FM1,FM2)
  400 CONTINUE
      STOP
      END
C
C
C
C
C
C
C                   APPENDIX-E5
C                   ===========
C * *-------------------------------------------------
C         THE EFFECT OF SAMPLING INTERVAL DW ON THE SOLUTION
C         OMEGA=1.0E6, A=1.0E4,DW (1000,2000,4000)
C * *-------------------------------------------------
      REAL L,LL
      COMPLEX F(2048),GAMA,S,XY,COSH,F1(2048),F2(2048)
      DIMENSION FM1(500),W(1048),T(500),FM2(500)
      DATA PI,R,L,C,LL/3.141527,2.75E-2,1.386E-3,2.09E-8,10./
      DATA N ,OMEGA,DW,A/2048,1.0E6,1000.,2.0E3/
      DS=PI/OMEGA
C * *-----------------------*
      DO 400 IT=1,2
      PRINT 175, OMEGA,A
  175 FORMAT(/10X,'THE VALUE OF OMEGA = ',E10.3
     +/10X,'& THE VALUE OF A IS= ',E10.3/)
C * *-----------------------*
      DO 200  II=1,2
```

```
      N=N/II
      DW=DW*II
C * *---------------------------*
      DF=DW/(2.*PI)
      DT=1.0/(N*DF)
      NT=(2.0*OMEGA)/DW
      IF(MOD(NT,2) .EQ.0) GO TO 456
      NO=(NT+1)/2
      GO TO 457
456   NO=NT/2+1
457   NR=NO+1
      MO=N/2+1
      MP=MO+1
C * *---------------------------*
      W(1)=0.0
      DO 10 I= 1,NO
      S=CMPLX(A,W(I))
      GAMA=CSQRT((R+S*L)*S*C)
      XY=CEXP(GAMA*LL)
      COSH=(XY+1.0/XY)/2.0
      F(I)=1.0/(S*COSH)
      IF(W(I) .NE. 0.0) F(I)=F(I)*SIN(DS*W(I))/(DS*W(I))
      W(I+1)=W(I)+DW
   10 CONTINUE
C * *---------------------------*
      DO 11 I=NR,MO
   11 F(I) = CMPLX(0.0,0.0)
C * *---------------------------*
      K=0
      DO 20 I= MP,N
      K=K+1
   20 F(I)=CONJG(F(MO-K))
C * *---------------------------*
      DO 13 I=1,N
      IF(II.EQ.1) GO TO 39
      F2(I)=F(I)
      GO TO 13
   39 F1(I)=F(I)
   13 CONTINUE
C * *---------------------------*
      IF (II.EQ.1) GO TO 51
      CALL FFT(-1,N,DF,F2)
      GO TO 402
   51 CALL FFT(-1,N,DF,F1)
C * *---------------------------*
 402  T(1)=0.0
      NC=100
      DO 22 I=1,NC
      T(I+1)=T(I)+DT
      IF(II.EQ.1) GO TO 52
      F2(I)=F2(I)*EXP(A*T(I))
      FM2(I)=CABS(F2(I))
      GO TO 22
52    F1(I)=F1(I)*EXP(A*T(I))
      FM1(I)=CABS(F1(I))
   22 CONTINUE
C * *-----------------------------------*
      IF(II .EQ. 1) PRINT 176,DW,N
      IF(II .EQ. 2) PRINT 177,DW,N
176   FORMAT(10X,'CURVE-A FOR DW =',F9.3,5X,'N =',I5)
177   FORMAT(10X,'CURVE-B FOR DW =',F9.3,5X,'N =',I5)
  200 CONTINUE
      CALL PLT2(100,10,60,T,FM1,FM2)
  400 CONTINUE
      STOP
      END
C
C
C
C
C
C
C                   APPENDIX-E6
C                   ===========
C     ---------------------------------------------------------
C         THE EFFECT OF SAMPLING INTERVAL T ON THE SOLUTION
C         OMEGA=1.0E6, A=1.0E4,DT=2.0E-06,& 3.0E-06
C     ---------------------------------------------------------
      REAL L,LL
      COMPLEX F(1024),GAMA,S,XY,COSH,F1(1024),F2(1024)
      DIMENSION FM1(1024),W(1024),T(1024),FM2(1024)
```

```
      DATA  PI,R,L,C,LL/3.141527,2.75E-2,1.386E-3,2.09E-8,10./
      DATA  OMEGA,A,N/1.0E6,2.0E3,1024/
      DS=PI/OMEGA
C * *----------------------------------------------------
      PRINT 175, OMEGA,A
 175  FORMAT(/10X,'THE VALUE OF OMEGA = ',E10.3/
     +10X,'& THE VALUE OF A IS= ',E10.3/)
C * *----------------------------------------------------
      DO 200  II=1,2
      IF (II .EQ. 1) DT=2.0E-06
      IF (II .EQ. 2) DT=3.0E-06
C * *------------------------------------------------------
      DF=1.0/(N*DT)
      DW=2.0*PI*DF
      NT=(2.0*OMEGA)/DW
      IF(MOD(NT,2) .EQ. 0 ) GO TO 456
      NO=(NT+1)/2
      GO TO 457
456   NO=NT/2+1
457   NR=NO+1
      MD=N/2+1
      MP=MD+1
C * *------------------------------------------------------
      W(1)=0.0
      DO 10 I= 1,NO
      S=CMPLX(A,W(I))
      GAMA=CSQRT((R+S*L)*S*C)
      XY=CEXP(GAMA*LL)
      COSH=(XY+1.0/XY)/2.0
      F(I)=1.0/(S*COSH)
      IF(W(I) .NE. 0.0) F(I)=F(I)*SIN(DS*W(I))/(DS*W(I))
      W(I+1)=W(I)+DW
  10  CONTINUE
C * *-----------------------------------------------------
      DO 11  I=NR,MD
  11  F(I) = CMPLX(0.0,0.0)
C * *-----------------------------------------------------
      K=0
      DO 20  I= MP,N
      K=K+1
  20  F(I)=CONJG(F(MD-K))
C * *-----------------------------------------------------
      DO 13 I=1,N
      IF(II.EQ.1) GO TO 39
      F2(I)=F(I)
      GO TO 13
  39  F1(I)=F(I)
  13  CONTINUE
C * *-----------------------------------------------------
      IF (II.EQ.1) GO TO 51
      CALL FFT(-1,N,DF,F2)
      GO TO 402
  51  CALL FFT(-1,N,DF,F1)
C * *-----------------------------------------------------
 402  T(1)=0.0
      NC=100
      DO 22 I=1,NC
      T(I+1)=T(I)+DT
      IF(II.EQ.1) GO TO 52
      F2(I)=F2(I)*EXP(A*T(I))
      FM2(I)=CABS(F2(I))
      GO TO 22
52    F1(I)=F1(I)*EXP(A*T(I))
      FM1(I)=CABS(F1(I))
  22  CONTINUE
C * *-----------------------------------------------------
      IF(II .EQ. 1) PRINT 176,DW,DT
      IF(II .EQ. 2) PRINT 177,DW,DT
176   FORMAT(10X,'CURVE -A FOR DW =',F10.3,5X,'& DT =',E10.3)
177   FORMAT(10X,'CURVE -B FOR DW =',F10.3,5X,'& DT =',E10.3)
 200  CONTINUE
      CALL PLT2(100,10,60,T,FM1,FM2)
C 400 CONTINUE
      STOP
      END
C
C
C
C
C
```

```
C                     APPENDIX-E7
C                     ===========
C  *  *---------------------------------------------------*
C       THE EFFECT OF SMOOTHING PARAMETER
C       ON GLOBAL RESPONSE.
C  *  *---------------------------------------------------*
        REAL L,LL
        COMPLEX F(1024),GAMA,S,XY,COSH,F1(1024),F2(1024),F3(1024)
        DIMENSION FM1(1024),FM2(1024),FM3(1024),W(1024),T(1024),SM(3)
        DATA  PI,R,L,C,LL/3.141527,2.75E-2,1.386E-3,2.09E-8,10./
        DATA  N,OMEGA,SM/1024,1.0E6,1.0E04,4647.0,620.0/
        DT=PI/OMEGA
        DF=1.0/(N*DT)
        DW= 2.*PI*DF
C  *  *---------------------------------------------------*
        NT=(2.0*OMEGA)/DW
        IF(MOD(NT,2) .EQ. 0) GO TO 456
        NO=(NT+1)/2
        GO TO 457
456     NO=NT/2+1
457     NR=NO+1
        W(1)=0.0
        MO=N/2+1
        MP=MO+1
C  *  *---------------------------------------------------*
        PRINT 401,N,OMEGA,DW,DT,DF
  401   FORMAT(//10X,'THE VALUE OF N =',I5/10X,'THE VALUE OF OMEGA =',E12
       +5/10X,'THE VALUE OF DW=',E12.5/10X,'THE VALUE OF DT =',E12.5
       +/10X,'THE VALUE OF DF=',E12.5/)
        DO 400 IT=1,3
        A=SM(IT)
C  *  *---------------------------------------------------*
        DO 10 I= 1,NO
        S=CMPLX(A,W(I))
        GAMA=CSQRT((R+S*L)*S*C)
        XY=CEXP(GAMA*LL)
        COSH=(XY+1.0/XY)/2.0
        F(I)=1.0/(S*COSH)
        IF(W(I) .NE.0.0) F(I)=F(I)*SIN(W(I)*DT)/(W(I)*DT)
        W(I+1)=W(I)+DW
   10   CONTINUE
C  *  *---------------------------------------------------*
        DO 24 I=NR,MO
24      F(I)=CMPLX(0.0,0.0)
        K=0
        DO 20 I= MP,N
        K=K+1
   20   F(I)=CONJG(F(MO-K))
C  *  *---------------------------------------------------*
        DO 13 I=1,N
        IF(IT .EQ. 1) GO TO 39
        IF(IT .EQ. 2) GO TO 40
        F3(I)=F(I)
        GO TO 13
40      F2(I)=F(I)
        GO TO 13
 39     F1(I)=F(I)
 13     CONTINUE
C  *  *---------------------------------------------------*
        IF(IT.EQ.1) GO TO 51
        IF(IT.EQ.2) GO TO 52
        CALL FFT(-1,N,DF,F3)
        GO TO 402
52      CALL FFT(-1,N,DF,F2)
        GO TO 402
   51   CALL FFT(-1,N,DF,F1)
C  *  *---------------------------------------------------*
 402    T(1)=0.0
        NC=340
        DO 22 I=1,NC
        IF(IT.EQ.1) GO TO 53
        IF(IT.EQ.2) GO TO 54
        F3(I)=F3(I)*EXP(A*T(I))
        FM3(I)=CABS(F3(I))
        GO TO 22
54      F2(I)=F2(I)*EXP(A*T(I))
        FM2(I)=CABS(F2(I))
        GO TO 22
53      F1(I)=F1(I)*EXP(A*T(I))
        FM1(I)=CABS(F1(I))
```

```
 22     T(I+1)=T(I)+DT
C * *-----------------------------------------------------------*
        IF(IT .EQ. 1) PRINT 174,SM(1)
        IF(IT .EQ. 2) PRINT 175,SM(2)
        IF(IT .EQ. 3) PRINT 176,SM(3)
174     FORMAT(10X,'CURVE -A FOR A =',E10.3)
175     FORMAT(10X,'CURVE -B FOR A =',E10.3,5X,'+ FOR DOUBLE PLOT')
176     FORMAT(10X,'CURVE -C FOR A =',E10.3,5X,'* FOR TRIPPLE PLOT')
  400 CONTINUE
      CALL PLT3(340,80,130,T,FM1,FM2,FM3)
C * *-----------------------------------------------------------*
      STOP
      END
```

APPENDIX-F1

```
C                            APPENDIX-F1
C                            ===========
C * *------------------------------------------------------------------
C              PROGRAM DEVELOPED BY MD. JALAL UDDIN
C                     M. SC. ENGG.(EEE) THESIS
C                SUPERVISOR > DR. M. MUJIBUR RAHMAN
C              NETWARK REDUCTION BY THEVNINS THEOREM
C * *------------------------------------------------------------------
      COMPLEX F1(1024),F2(1024)
      DIMENSION W(1024),T(1024),FM1(1024),FM2(1024)
      DIMENSION R(11),L(11),C(11),X(11)
      REAL L,LS
      COMPLEX S,V1,V(1024),ZS,YL,GM,AG,CH,SH,DN
      DATA NN,PI/10,3.1415927/
      DATA RS,LS,GL,BL,LIMIT/4*0.0,1000000/
      DATA N,DW,A/1024,2000.,1.0E4/
C   << ND IS IN DO LOOP >>
      DT=PI/LIMIT
      N1=(2.0*LIMIT)/DW
      DF=1.0/(N*DT)
      N2=N1/2+1
      N3=N2+1
      M=N/2+1
      M1=M+1
C * *-----------------------------------------------------------*
      PRINT 400,N,LIMIT,DW,DT,A
  400 FORMAT(///10X,'OBSERVATION = 1'//10X,'THE VALUE OF N=',I5/10X,'T
     +E VALUE OF LIMIT =',I10/10X,'THE VALUE OF DW=',F10.3/10X,'THE VA
     +LUE OF DT=',E12.5/10X,'& THE VALUE OF A =',E12.5//)
C * *-----------------------------------------------------------
      READ(1,11) (R(I),L(I),C(I),X(I),I=1,10)
   11 FORMAT(4F10.2)
C     PRINT 12, (R(I),L(I),C(I),X(I),I=1,10)
   12 FORMAT(10X,4E18.7)
C * *HEADING-------------------------------------------------
      DO 1111 ND=10,11
C     WRITE(3,20) ND
   20 FORMAT(//5X,'SWITCHING SURGE FOR A SECTIONALLY UNIFORM' /11X,
     +'POWER SYSTEM AT NO LOAD'///20X,'TIME',19X,'VOLTAGE AT BUS',
     +I4/19X,'IN SECOND',10X,'WITHOUT SIGMA',5X,'WITH SIGMA FACTOR'/)
C * *-----------------------------------------------------------*
      W(1)=0.0
      DO 10 I=1,N2
  200 S=CMPLX(A,W(I))
      V1=1.0/S
      ZS=RS+S*LS
      YL=GL-S*BL
C * *-----------------------------------------------------------
      IF( ND .EQ. 2) GO TO 100
C
C     CALCULATION OF ZS & V1 FOR BUS UNDER CONSIDERATION
C * *-----------------------------------------------------------
      IK=ND-2
      K=1
  700 GM=CSQRT((R(K)+S*L(K))*C(K)*S)
      AG=CEXP(GM*X(K))
      CH=(AG+1.0/AG)/2.0
```

```
      SH=(AG-1.0/AG)/2.0
      DN=S*C(K)*(GM*CH+S*C(K)*ZS*SH)
      ZS=GM*(S*C(K)*ZS*CH+GM*SH)/DN
      V1=V1*GM*S*C(K)/DN
C * *-------------------------------------------
      K=K+1
      IF( K.GT.IK) GO TO 100
      GO TO 700
C
C     CALCULATION OF YL FOR BUS UNDER CONSIDERATION
C
  100 IF(ND.EQ.(NN+1)) GO TO 600
      IM=(NN+1)-ND
      K1=1
  800 K=NN+1-K1
      GM=CSQRT((R(K)+S*L(K))*C(K)*S)
      AG=CEXP(GM*X(K))
      CH=(AG+1.0/AG)/2.0
      SH=(AG-1.0/AG)/2.0
      YL=S*C(K)*(GM*YL*CH+S*C(K)*SH)/(GM**2*YL*SH+GM*S*C(K)*CH)
      K1=K1+1
      IF(K1.GT.IM) GO TO 600
      GO TO 800
C
C     CALCULATION OF V(L,S) FOR BUS UNDER CONSIDERATION
C
  600 K=ND-1
      GM=CSQRT((R(K)+S*L(K))*C(K)*S)
      AG=CEXP(GM*X(K))
      CH=(AG+1.0/AG)/2.0
      SH=(AG-1.0/AG)/2.0
      V(I)=V1*GM/((ZS*GM*YL*CH+ZS*S*C(K)*SH)+((GM**2*YL*SH+GM*S*C(K)*CH)
     +/(C(K)*S)))
      F1(I)=V(I)
      F2(I)=V(I)
      IF(W(I).NE.0.0)F2(I)=V(I)*SIN(DT*W(I))/(DT*W(I))
      W(I+1)=W(I)+DW
   10 CONTINUE
C * *-------------------------------------------
      DO 13 I=N3,M
      F2(I)=CMPLX(0.0,0.0)
   13 F1(I)=CMPLX(0.0,0.0)
C * *-------------------------------------------
      K=0
      DO 21 I= M1,N
      W(I)= W(I-1)+DW
      K=K+1
      F1(I)=CONJG(F1(M-K))
   21 F2(I)=CONJG(F2(M-K))
C * *-------------------------------------------
      DO 101 IT=1,2
      IF(IT.EQ.1) GO TO 51
      CALL FFT(-1,N,DF,F2)
      GO TO 402
   51 CALL FFT(-1,N,DF,F1)
  402 T(1)=0.0
C * *-------------------------------------------
      DO 22 I=1,100
      IF(IT.EQ.1) GO TO 52
      F2(I)=F2(I)*EXP(A*T(I))
      FM2(I)=CABS(F2(I))
      GO TO 22
   52 F1(I)=F1(I)*EXP(A*T(I))
      FM1(I)=CABS(F1(I))
   22 T(I+1)=T(I)+DT
C * *-------------------------------------------
  101 CONTINUE
C     DO 23 I=1,100
C  23 PRINT    29, I,T(I),FM1(I),FM2(I)
   29 FORMAT(4X,I4,8X,E12.5,8X,E13.6,8X,E13.6/)
      PRINT 30,ND
   30 FORMAT(10X,'SWITCHING VOLTAGE =1 P.U. AT BUS 1'/10X,'SURGE VOLTAGE
     +AT BUS =',I3/10X,'CURVE-A WITHOUT SIGMA',5X,'& CURVE-B WITH SIGMA
     +FACTOR'/)
      CALL PLT2(100,10, 60,T,FM1,FM2)
 1111 CONTINUE
      STOP
      END
```

APPENDIX-F2
=============

```
C ---------------------------------------------------------------------
C       PROGRAM NO.#8, DEVELOPED BY ENGR. MD.JALAL UDDIN .
C       AN M.SC.ENGG. THESIS WORK UNDER THE SUPERVISION OF
C       DR.MD.MUJIBUR RAHMAN
C       MODIFIED FFT TECHNIQUE IN POWER SYSTEM TRANSIENT ANALYSIS.
C ---------------------------------------------------------------------
      DIMENSION RS(47),LS(47),RL(47),LL(47),W(2000),T(1001)
      DIMENSION XL(47,47),IG(12),FR(1000)
      DIMENSION R(47,47),L(47,47),C(47,47),BF(46,3),BT(46,3)
      DIMENSION NST(47),NEND(47),DXL(47),DR(47),DL(47),DC(47)
      COMPLEX S,VI,AG,CH,SH,GM,YL(47),ZS(47),DN,VG(40)
      COMPLEX F(4096),VV(4096)
          REAL LL,LS,L
          INTEGER BF,BT
          COMMON /EA1/ XL
          COMMON /EA2/ YL,ZS
          COMMON /EA3/ R,L,C
      DATA NB,NB1,PI/47,46,3.14159/
      DATA  N ,OMEGA/4096,0.6E06/
C * *---------------------------------------------------------**
      DO 1 I=1,NB1
      DO 1 J=1,NB1
      XL(I,J)=0.0
      R(I,J)=0.0
      L(I,J)=0.0
      C(I,J)=0.0
1     CONTINUE
      READ(1,2) (NST(I),NEND(I),DXL(I),DR(I),DL(I),DC(I),(BF(I,J),BT(I,J
     +),J=1,3),I=1,NB1)
2     FORMAT(I2,I2,F5.1,1X,F5.3,E12.5,1X,E8.1,1X,3A4,1X,3A4)
      DO 3 I=1,NB1
      NI=NST(I)
      NJ=NEND(I)
      XL(NI,NJ)=DXL(I)
      R(NI,NJ)=DR(I)
      L(NI,NJ)=DL(I)
      C(NI,NJ)=DC(I)
      XL(NJ,NI)=XL(NI,NJ)
      R(NJ,NI)=R(NI,NJ)
      L(NJ,NI)=L(NI,NJ)
      C(NJ,NI)=C(NI,NJ)
3     CONTINUE
C      PRINT 46
46    FORMAT (4X,'BUS NO.',10X,'NAME OF BUS',10X,'LENGTH IN',10X,' R ',
     +10X,' L ',10X,' C '/4X,8('-'),10X,11('-'),10X,9('-'),10X,3('-'),
     +10X,3('-'),10X,3('-')/3X,'FROM',2X,'TO',8X,'FROM',9X,'TO',10X,
     +'MILES',12X,'OHM/MILE',6X,'H/MILE',6X,'F/MILE'/3X,100('-')//)
C      DO 4 I=1,NB1
C      NI=NST(I)
C      NJ=NEND(I)
C 4    PRINT 5,NI,NJ,(BF(I,J),BT(I,J),J=1,3),XL(NI,NJ),R(NI,NJ),
C     +L(NI,NJ),C(NI,NJ)
 5    FORMAT(4X,I2,3X,I2,5X,3A4,3X,3A4,F10.2,5X,F5.3,5X,E12.5,5X,E10.3)
C * *---------------------------------------------------------**
          DS=PI/OMEGA
          DW=314.159
          DF=DW/(2.0*PI)
          DT=1.0/(N*DF)
          NT=(2.0*OMEGA)/DW
          IF( MOD(NT,2) .EQ. 0 ) GO TO 12
          NO=(NT+1)/2
          GO TO 13
   12     NO=NT/2+1
   13     NR=NO+1
          MO=N/2+1
          MP=MO+1
CC * *---------------------------------------------------------**
      READ(1,100) (I,RL(I),LL(I),RS(I),LS(I),I=1,47)
100   FORMAT(I2,3X,F10.6,E12.5,3X,F5.3,E10.4)
110   FORMAT(10X,I5,5X,F10.3,5X,E12.5,5X,F5.3,5X,E12.4/)
      DO 72 I=1,47
      RL(I)=174.24*RL(I)
      LL(I)=174.24*LL(I)
      RS(I)=174.24*RS(I)
 72   LS(I)=174.24*LS(I)
C73   PRINT 110, I,RL(I),LL(I),RS(I),LS(I)
C * *---------------------------------------------------------**
      VG(1)=CMPLX(0.931360,-0.484836)
      VG(8)=CMPLX(0.859046,-0.472264)
      VG(11)=CMPLX(0.9796,-0.37799)
```

```
      VG(19)=CMPLX(1.069557,-0.14651)
      VG(22)=CMPLX(1.026770,-0.1663)
      VG(23)=CMPLX(1.026770,-0.1663)
      VG(30)=CMPLX(0.88590,-0.526)
      VG(33)=CMPLX(0.953933,-0.43876)
      VG(37)=CMPLX(0.987959,-0.15471)
      VG(38)=CMPLX(1.035870,-0.09427)
      VG(39)=CMPLX(1.067814,-0.06344)
      VG(40)=CMPLX(1.058297,-0.057315)
C *  -------------------------------------------------------
C * *-------------------------------------------------------* *
      DATA IG/1,8,11,22,23,30,33,37,38,39,40,19/
C     DATA IG/8,11,30,33,1/
C     DATA IG/22,23,37,38,39,40,19/
      DATA  KK, ND / 12, 35  /
C * *-------------------------------------------------------* *
      DO 445 I=1,N
445   VV(I)=CMPLX(0.0,0.0)
      DO 444 IL=1,KK
      NS=IG(IL)
      II=1
      W(1)=0.0
      A=410.0
      IF(IL.EQ.KK) GO TO 90
      DO 303 I=1,ND
303   F(I)=CMPLX(0.0,0.0)
      II=2
      W(II)=314.159
      A=52.00
90    S=CMPLX(A,W(II))
      DO 221 KI=1,NB
      IF(KI.EQ.19) GO TO 888
      YL(KI)=1.0/(RL(KI)+S*LL(KI))
      GO TO 221
888   YL(KI)=CMPLX(0.0,0.0)
221   ZS(KI)=RS(KI)+S*LS(KI)
C   * *--------------------------------------------------*
      VI=(S*REAL(VG(IG(IL)))-W(II)*AIMAG(VG(IG(IL))))/(S*S+W(II)**2)
      KKK=KK-1
      DO 31 IM=1,KKK
      IF(IM .EQ. IL) GO TO 31
      YL(IG(IM))=YL(IG(IM))+1.0/ZS(IG(IM))
 31   CONTINUE
C * *--------------------------------------------------*
CINCLUDE XRED
CINCLUDE YRED
C ZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZ
33333 K1=12
      K3=1
      K4=13
      CALL FIRST(K1,K3,K4,S)
      K1=13
      K3=22
      K4=35
      CALL FIRST(K1,K3,K4,S)
      K1=14
      K3=1
      K4=15
      CALL FIRST(K1,K3,K4,S)
      K1=15
      K3=22
      K4=37
      CALL FIRST(K1,K3,K4,S)
      K1=16
      K3=1
      K4=18
      CALL FIRST(K1,K3,K4,S)
      K1=18
      K3=21
      K4=39
      CALL FIRST(K1,K3,K4,S)
      IF (NS .GT. 26) GO TO 30003
      IF (NS .GT. 11) GO TO 30002
C
C Z1Z1Z1Z1Z1 BEGINS HERE >6*22+9>141
C
30001 K1=19
      K3=5
      K4=24
      CALL FIRST(K1,K3,K4,S)
```

```
      K1=20
      K3=1
      K4=26
      CALL FIRST(K1,K3,K4,S)
      K1=26
      K3=11
      K4=37
      CALL FIRST(K1,K3,K4,S)
C   ****************************
      IF(NS .GT. 5) GO TO 700
      IF(ND -34) 701,701,702
701   K1=44
      K3=-1
      K4=34
      CALL FIRST(K1,K3,K4,S)
      K1=27
      K3=1
      K4=ND
      CALL FIRST(K1,K3,K4,S)
703   K1=1
      K3=1
      K4=NS
      CALL FIRST(K1,K3,K4,S)
      K1=5
      K3=1
      K4=11
      CALL FIRST(K1,K3,K4,S)
      LI=NS
      L3=1
      L4=5
      CALL SECOND(L1,L3,L4,VI,S)
      L1=5
      L3=6
      L4=11
      CALL SECOND(L1,L3,L4,VI,S)
      L1=11
      L3=23
      L4=34
      CALL SECOND(L1,L3,L4,VI,S)
C   ****************************
      IF(ND -34) 704,704,705
704   L3=-1
706   L1=34
      L4=ND
      CALL SECOND(L1,L3,L4,VI,S)
      GO TO 777
705   L3=1
      GO TO 706
702   K1=27
      K3=1
      K4=34
      CALL FIRST(K1,K3,K4,S)
      K1=44
      K3=-1
      K4=ND
      CALL FIRST(K1,K3,K4,S)
      GO TO 703
700   IF(ND - 34) 707,707,708
707   K1=44
      K3=-1
      K4=34
      CALL FIRST(K1,K3,K4,S)
      K1=27
      K3=1
      K4=ND
      CALL FIRST(K1,K3,K4,S)
709   K1=1
      K3=1
      K4=5
      CALL FIRST(K1,K3,K4,S)
      K1=5
      K3=5
      K4=11
      CALL FIRST(K1,K3,K4,S)
      K1=6
      K3=1
      K4=NS
      CALL FIRST(K1,K3,K4,S)
      LI=NS
      L3=1
```

```
      L4=11
      CALL SECOND(L1,L3,L4,VI,S)
      L1=11
      L3=23
      L4=34
      CALL SECOND(L1,L3,L4,VI,S)
      IF(ND - 34) 710,710,711
710   L3=-1
712   L1=34
      L4=ND
      CALL SECOND(L1,L3,L4,VI,S)
      GO TO 777
711   L3=1
      GO TO 712
708   K1=27
      K3=1
      K4=34
      CALL FIRST(K1,K3,K4,S)
      K1=44
      K3=-1
      K4=ND
      CALL FIRST(K1,K3,K4,S)
      GO TO 709
C
C Z1Z1Z1Z1Z1Z1Z1 FINISH.
C Z2Z2Z2Z2Z2Z2Z2 BEGINS HERE >>130=6*22-2.
C
30002 K1=1
      K3=1
      K4=5
      CALL FIRST(K1,K3,K4,S)
      K1=5
      K3=6
      K4=11
      CALL FIRST(K1,K3,K4,S)
      K1=6
      K3=1
      K4=11
      CALL FIRST(K1,K3,K4,S)
      K1=11
      K3=23
      K4=34
      CALL FIRST(K1,K3,K4,S)
      IF (NS .GT. 19 ) GO TO 800
      IF (ND - 37 ) 801,801,802
801   K1=44
      K3=-1
      K4=37
      CALL FIRST(K1,K3,K4,S)
      K1=27
      K3=1
      K4=ND
      CALL FIRST(K1,K3,K4,S)
806   K1=20
      K3=1
      K4=24
      CALL FIRST(K1,K3,K4,S)
      L1=NS
      L3=5
      L4=24
      CALL SECOND(L1,L3,L4,VI,S)
      L1=24
      L3=1
      L4=26
      CALL SECOND(L1,L3,L4,VI,S)
      L1=26
      L3=11
      L4=37
      CALL SECOND(L1,L3,L4,VI,S)
      IF (ND -37 ) 803,803,804
803   L3=-1
805   L1=37
      L4=ND
      CALL SECOND(L1,L3,L4,VI,S)
      GO TO 777
804   L3=1
      GO TO 805
802   K1=27
      K3=1
      K4=37
```

```
      CALL FIRST(K1,K3,K4,S)
      K1=44
      K3=-1
      K4=ND
      CALL FIRST(K1,K3,K4,S)
      GO TO 806
800   IF(ND-37) 807,807,808
807   K1=44
      K3=-1
      K4=37
      CALL FIRST(K1,K3,K4,S)
      K1=27
      K3=1
      K4=ND
      CALL FIRST(K1,K3,K4,S)
C  ***********************
812   K1=19
      K3=5
      K4=24
      CALL FIRST(K1,K3,K4,S)
      K1=20
      K3=1
      K4=NS
      CALL FIRST(K1,K3,K4,S)
      L1=NS
      L3=1
      L4=26
      CALL SECOND(L1,L3,L4,VI,S)
      L1=26
      L3=11
      L4=37
      CALL SECOND(L1,L3,L4,VI,S)
      IF(ND-37) 809,809,810
809   L3=-1
811   L1=37
      L4=ND
      CALL SECOND(L1,L3,L4,VI,S)
      GO TO 777
810   L3=1
      GO TO 811
808   K1=27
      K3=1
      K4=37
      CALL FIRST(K1,K3,K4,S)
      K1=44
      K3=-1
      K4=ND
      CALL FIRST(K1,K3,K4,S)
      GO TO 812
C Z2Z2Z2Z2Z2Z2 FINISH.
C Z3Z3Z3Z3 BEGINS HERE >>>3*22+14>80
C
30003 K1=1
      K3=1
      K4=5
      CALL FIRST(K1,K3,K4,S)
      K1=5
      K3=6
      K4=11
      CALL FIRST(K1,K3,K4,S)
      K1=6
      K3=1
      K4=11
      CALL FIRST(K1,K3,K4,S)
      K1=11
      K3=23
      K4=34
      CALL FIRST(K1,K3,K4,S)
      K1=19
      K3=5
      K4=24
      CALL FIRST(K1,K3,K4,S)
      K1=20
      K3=1
      K4=26
      CALL FIRST(K1,K3,K4,S)
      K1=26
      K3=11
      K4=37
      CALL FIRST(K1,K3,K4,S)
```

```
C   **************************
      IF (NS-ND) 900,900,901
900   NSURGE=ND
      NSWITC=NS
903   K1=27
      K3=1
      K4=NSWITC
      CALL FIRST(K1,K3,K4,S)
      K1=44
      K3=-1
      K4=NSURGE
      CALL FIRST(K1,K3,K4,S)
      GO TO 902
901   NSWITC=ND
      NSURGE=NS
      GO TO 903
902   IF(NS-ND) 904,904,905
904   L3=1
906   L1=NS
      L4=ND
      CALL SECOND(L1,L3,L4,VI,S)
      GO TO 777
905   L3=-1
      GO TO 906
C * ***   ----ZZZZZZZZZ FINISH -------- *** *
C * *---------------------------------------------*
777   IF(W(II) .NE. 0.0) VI=VI*SIN(DS*W(II))/(DS*W(II))
      F(II)=VI
      W(II+1)=W(II)+DW
      II=II+1
      IF(IL .NE. KK) GO TO 399
      IF(II .GT. NO) GO TO 399
      GO TO 90
399   DO 45 II=NR,MO
45    F(II)=CMPLX(0.0,0.0)
      KN=0
      DO 86 II= MP,N
      KN=KN+1
      F(II)=CONJG(F(MO-KN))
86    CONTINUE
      CALL FFT(-1,N,DF,F)
C * *------------------------------------------*
      T(1)=0.0
      DO 446 I=1,840
      VV(I)=VV(I)+F(I)*EXP(A*T(I))
      FR(I)=REAL(VV(I))
446   T(I+1)=T(I)+DT
      IF(IL .EQ. KKK) GO TO 201
      GO TO 444
201   CALL PEAK1(840,200,840,FR)
444   CONTINUE
C *  *----------------------------------------*
C
      PRINT 74
 74   FORMAT(/10X,'NOTE ***'/10X,'*** ALL GENERATORS  ARE CONNECTED
     +'/10X,'SURGE AT EAST GRID'/)
      PRINT 75,OMEGA,N,DW,DT,DF,A
75    FORMAT(//15X,'OMEGA =',E10.3,5X,'N =',I5/15X,'DW =',F9.3,9X,'DT =
     +',E10.3/15X,'DF =',F9.3,9X,'A =',F9.3/)
      PRINT 77,NS,ND
77    FORMAT(15X,'SWITCHING BUS =',I3,2X,' KAPTAI     '/15X,'SURGE BUS=
     +',I3,2X,' TANGI   '/15X,'ALL GENERATORS ARE CONNECTED
     + '/15X, 'TRAVEL TIME = 1022.0 MICRO SEC'/)
      CALL PLT1(840,200,840,T,FR)
C * *-------------------------------------------* *
22222 STOP
      END
```

```
      COMMON /EA1/ XL
      COMMON /EA3/ R,L,C
      COMMON /EA2/ YL,ZS
125   IF(K1 .EQ. K4) GO TO 124
      K2=K1+K3
      GM=CSQRT((R(K1,K2)+S*L(K1,K2))*C(K1,K2)*S)
      AGG=GM*XL(K1,K2)
      IF(REAL(AGG) .LE.174.0) GO TO 512
      PRINT 68,GM ,AGG
68    FORMAT(10X,4E10.3)
      GO TO 124
512   AG=CEXP(AGG)
      CH=(AG+1.0/AG)/2.0
      SH=(AG-1.0/AG)/2.0
      YL(K2)=S*C(K1,K2)*(GM*YL(K1)*CH+S*C(K1,K2)*SH)/(GM**2*YL(K1)*SH+G*
     +*S*C(K1,K2)*CH)+YL(K2)
      K1=K2
      GO TO 125
124   RETURN
      END
C
C
C
C
C
      SUBROUTINE SECOND(M1,M3,M4,VI,S)
      DIMENSION XL(47,47),R(47,47),L(47,47),C(47,47)
      REAL L
      COMPLEX YL(47),ZS(47),VI,GM,AG,CH,SH,DN,S
      COMMON /EA1/ XL
      COMMON /EA3/ R,L,C
      COMMON /EA2/ YL,ZS
63    VI=VI/(CMPLX(1.0,0.0)+ZS(M1)*YL(M1))
      ZS(M1)=ZS(M1)/(CMPLX(1.0,0.0)+ZS(M1)*YL(M1))
      IF(M1 .EQ. M4) GO TO 64
      M2=M1+M3
      GM=CSQRT((R(M1,M2)+S*L(M1,M2))*C(M1,M2)*S)
      AG=CEXP(GM*XL(M1,M2))
      CH=(AG+1.0/AG)/2.0
      SH=(AG-1.0/AG)/2.0
      DN=GM*CH+S*C(M1,M2)*SH*ZS(M1)
      VI=VI*GM/DN
      ZS(M2)=GM*(S*C(M1,M2)*CH*ZS(M1)+GM*SH)/(DN*S*C(M1,M2))
      M1=M2
      GO TO 63
64    RETURN
      END
C
C
C
C
C
      SUBROUTINE PLT1(N,IST,LST,X,Y)
      DIMENSION X(N),Y(N),BB(50)
      INTEGER BLANK,STAR,LINE(101),DOT,RE,TIME(24),ZVA
      DATA BLANK,STAR,DOT,RE,ZVA/' ','*','.','X','+'/
      DATA TIME/' ','T','I','M','E',' ','I','N',' ','M','I','C','R','O',
     +' ','S','E','C',' ',']','|','|','V',' '/
      DO 204 J=1,N
 204  X(J)=X(J)*1.0E06
      NP=101
      IST1=IST+1
      BY=Y(IST)
      SY=Y(IST)
      DO 1 I=IST1,LST
      IF(Y(I).GT.BY) GO TO 3
      GO TO 24
 3    BY=Y(I)
 24   IF(Y(I)-SY) 7,1,1
 7    SY=Y(I)
 1    CONTINUE
      PRINT 135,BY,SY
135   FORMAT(/15X,' PEAK =',F8.3,'P.U.'/15X,'* CREST =',F8.3,'P.U.'/)
      DO 9 I=1,NP
 9    LINE(I)=DOT
      NPP=NP/10
      SCALE=(BY-SY)/NP
      BB(1)=SY
      J=1
      DO 38 I=11,NP,10
      J=J+1
      BB(J)=BB(J-1)+SCALE*10.0+0.002
      ...NT 201
      ...5X,'* VOLTAGE IN PER UNIT ---->')
      ...),I=1,J)
```

```
37     FORMAT(1X,T6,[illegible],T26,[illegible],T36,F8.[illegible],T46 F8.3,T56,
      +T66,F8.3,T76,F8.[illegible]3,[illegible]3,T106,F8.3,T116,F8.3,T12[illegible]
       DO 39 I=1,NP,10
   39 LINE(I)=STAR
       PRINT 22,LINE
 22    FORMAT(10X,121A1)
       DO 40 I=1,NP
 40    LINE(I)=BLANK
       IMC=0
       ASML=ABS(SY)/SCALE
       DO 10 I=IST,LST
       YC=( Y(I)-SY)/SCALE+0.99
       II=YC
       LINE(1)=DOT
       IF(SY) 101,102,102
101    LINE(1+INT(ASML))=ZVA
102    LINE(NP)=DOT
       IF(II.LT.1.OR.II.GT.NP)GO TO 50
       LINE(II)=RE
 50    IF(I.EQ.1) GO TO 44
       IF((I/10)*10.EQ.I)GO TO 44
       IMC=IMC+1
       IF(IMC .GT. 24) GO TO 203
       PRINT 56,TIME(IMC),LINE
 56    FORMAT(2X,A1,7X,121A1)
       GO TO 43
203    PRINT 22,LINE
       GO TO 43
  44   LINE(1)=STAR
       LINE(NP)=STAR
       PRINT 41,X(I),LINE
 41    FORMAT(3X,F7.2,121A1)
  43   LINE(II)=BLANK
 10    CONTINUE
       DO 36 I=1,NP
 36    LINE(I)=DOT
       DO 35 I=1,NP,10
 35    LINE(I)=STAR
       PRINT 22,LINE
       RETURN
       END
C
       SUBROUTINE STL(MI,M3,M4,TL)
       DIMENSION XL(47,47)
       COMMON /EA1/ XL
 20    IF(M1 .EQ. M4) GO TO 10
       M2=M1+M3
       TL=TL+XL(M1,M2)
       M1=M2
       GO TO 20
 10    RETURN
       END
C
C
C
C
C
       SUBROUTINE YZ(NB,YLL,RL,LL)
       DIMENSION RL(47),NBT(47)
       COMPLEX PL(47),BV(47),ZL(47),YLL(47)
       REAL LL(47)
       COMMON /EA4/PL,BV,NBT
       DO 30 I=1,NB
       XA=AIMAG(BV(I))
       XB=REAL(BV(I))
       XC=XB*COS(XA/57.3)
       XD=XB*SIN(XA/57.3)
       BV(I)=CMPLX(XC,XD)
       YLL(I)=CONJG(PL(I))/(XB**2)
       XE=REAL(YLL(I))
       XF=AIMAG(YLL(I))
       IF(XE.EQ. 0.0 .AND. XF .EQ. 0.0) GO TO 40
       ZL(I)=1/YLL(I)
       RL(I)=REAL(ZL(I))
       LL(I)=AIMAG(ZL(I))/314.15
       GO TO 30
 40    RL(I)=0.0
       LL(I)=0.0
       ZL(I)=(0.0,0.0)
 30    CONTINUE
       PRINT 50,(NBT(I),YLL(I),RL(I),LL(I),I=1,47)
 50    FORMAT(/10X,I5,5X,2E12.4,5X,2E12.3)
       RETURN
       END
C
C
```

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