ACOUSTICS OF RECTANGULAR FLAT PLATES OF MIXED BOUNDARY CONDITIONS

BY

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RECOMMENDATION OF THE BOARD OF EXAMINERS

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ABSTRACT

The total acoustic power radiation from one side of a rectangular flat plate, vibrating in one of its natural mode under different mixed boundary conditions has been investigated. The conditions at the edges of plates which have been investigated here include (i) two opposing ends fixed and other two ends free (ii) one end fixed and the other three ends free. The radiation efficiency of a rectangular flat plate, with two opposing ends fixed and other two ends free, has also been found to check the soundness of the method employed by comparing the efficiency with the results available in literature. Numerical results on the acoustic power radiation by the plate with different sets of values of thickness ratio, aspect ratio and mode orders have been obtained.

The characteristic beam functions developed by Warburton have been used to define the deformation mode of vibrating plates.

When a plate with two opposing ends fixed and other two ends free vibrates in one of its natural modes, acoustic power radiation increases in a regular fashion. But at higher mode orders it fluctuates maintaining nearly the same average magnitude and the waviness is gradually reduced as the mode order increases.

In the case of a plate with one end fixed and other three ends free, power radiation is more than that of a plate with two opposing ends fixed and other two ends free. But the variation of power radiation with the change of mode orders, thickness ratio and aspect ratio is similar.

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CONTENTS

N O T A T I O N S

a	Half of the length of the plate
b	Half of the width of the plate
c	Velocity of sound in air
E	Modulus of elasticity of the plate material
f	Frequency of vibration of the plate
G_x	A function of m, factor of the dimensionless frequency factor
G_y	A function of n, factor of the dimensionless frequency factor
h	Thickness of the plate
H_x	A function of m, factor of the dimensionless frequency factor
H_y	A function of n, factor of the dimensionless frequency factor
J_x	A function of m, factor of the dimensionless frequency factor
J_y	A function of n, factor of the dimensionless frequency factor
k	Acoustic wave number
k_p	Plate wave number
m	Number of nodal lines in x-direction
n	Number of nodal lines in y-direction
P_w	Total acoustic power radiation
P	Acoustic pressure distribution
R	Distance of the receiving point from the centre of the plate
R_a	Aspect ratio of the plate, b/a
R_t	Thickness ratio of the plate, h/a
r	Distance of the receiving point from the elemental area

S_{mn} Radiation efficiency

T Kinetic energy

t Time

U Surface velocity distribution, potential energy

U_o Amplitude of velocity distribution

W Amplitude of vibration

w Instantaneous displacement of the plate surface

(x,y,z) Cartesian coordinate system

γ Roots of the equation $\tan\gamma/2 + \tanh\gamma/2 = 0$ and $\cos\gamma\cosh\gamma = -1$

$\theta(x)$ Displacement function in x-direction

$\theta(y)$ Displacement function in y-direction

λ Wave length of sound

λ_f Dimensionless frequency factor

ρ Density of the surrounding medium

ρ_m Density of the plate material

σ Poission's ratio of the plate material

Φ Velocity potential

Ψ Wave number ratio, k/k_p

ω Circular frequency of plate vibration

(R,θ,α) Spherical coordinate system

CHAPTER-1

INTRODUCTION

1.1 GENERAL :

The mechanical and electrical equipments which are an integral part of modern living are also the source of many acoustical problems. Intentionally or otherwise, the choice of heating, ventilating or air conditioning systems, for example, is an acoustical decision. Nearly every piece of mechanical equipment in a building is a source of sound and vibration; the piping and the duct systems are superb paths for both air-borne and structure-borne sound.

The subject of acoustics is broad enough to challenge specialists in many branches of the field. Engineers and scientists may concentrate on the acoustic design of concert halls and theaters, design of quicker mechanical equipment, reduction of aircraft noise or such specialised fields as underwater sound and medical applications of ultrasonic. One of the most important and extensive application of acoustics and noise control relates to mechanical systems of building. Here concern must be focused on ensuring such things as adequate speech privacy, provision for acceptable sleeping habits and little or no annoyance from equipment noise. Sound and acoustics are appropriate for the designer of the thermal system to consider for two reasons.

1. A good designer thinks in terms of providing a favorable environment extending beyond the realm of temperature, humidity, radiation and air motion.

2. The compressors, fans, pumps and flow of air are some times the major generators of noise in buildings, but same equipment may offer the means of controlling other more objectionable noise.

1.2 <u>MECHANISM OF PRODUCTION AND PROPAGATION OF SOUND</u> :

Knowledge of acoustics is very essential to identify the source of sound and to realise the mechanism by which it is produced. Noise is caused by the vibration of solid, liquid or gaseous medium. When this vibration is within the range of audible frequency of the human being, a human ear can perceive the noise; otherwise, they go unnoticed. When a solid body vibrates at a frequency within the audible frequency range, a part of the energy dissipated is transmitted to the surrounding environment as perceivable sound. Energy is transferred from one vibrating particle through the next and the acoustic energy travels through the surrounding medium as longitudinal waves. In this study, we are interested to find the amount of acoustic energy dissipated in the form of noise due to the natural vibration of rectangular plates with mixed boundary conditions.

1.3 MOTIVATION TO SELECT THE PROBLEM :

Due to the mechanical vibration of solid structures sound is produced. It is then transferred to the surrounding medium in the form of acoustic energy which often causes discomfort to human life. Everyday, many problems are encountered with unwanted sound. Many of these noise sources are in the form of flat plates. In our civilization, the familiar flat plate noise source are walls, windows, floors and roofs of the buildings, many enclosed surfaces and integrated parts of machine, wall of air-conditioning ducts etc. Attempts were made in the past to evaluate the noise characteristic of different vibrating sources by measurements. Engineers always endevour to eliminate acoustical noise from the mechanical systems and buildings. Therefore, understanding of the acoustics of rectangular plates is an absolute necessity in order to determine noise dissipation under various mixed boundary conditions when these plates vibrate in one of their natural mode of vibration.

1.4 OBJECTIVES OF THE STUDY :

In this analysis, attempt is made to study the noise generating characteristic of a rectangular flat plate in an infinite baffle. The plate is subjected to two mixed boundary conditions. These are : (i) Two opposing ends fixed and other two ends free,(ii) One end fixed and other three ends free. The objectives of the study are as follows :

(i) The development of appropriate displacement function that satisfy the required boundary conditions of the rectangular flat plate vibrating in flexure.

(ii) Derivation of expressions for the natural frequency of the vibrating plate for different mode shapes.

(iii) Evaluation of the frequency of vibration for a particular case and study its variation with the mode shape.

(iv) Derivation of an expression for the power radiation due to natural vibration of the plates under consideration.

(v) Evaluation of the magnitude of the average power radiated from one side of the baffled plates by the method of numerical integration of the expression for the radiation of power. For this purpose, a computer program is to be developed.

(vi) To study the variation in the radiated power from the plate with the different mode numbers, aspect ratio, and thickness ratio under different boundary conditions.

(vii) Determination of the radiation efficiency of the flat plate with two opposing ends fixed and other two ends free and plotting the same against the wave number ratio for comparing the result of the current investigation with those of the previous works [1,46] in order to verify the soundness of the present analysis and to ascertain the absence of any error in the numerical procedure employed.

1.5 DEFINITION OF TERMS :

Some terms used in this thesis are defined here in order to remove ambiguity in their use and to attach precise meaning to them.

(a) Acoustic Pressure :

Sound is a disturbance that propagates through an elastic material at a speed characteristic of that medium. Sound travels as a wave of altarnate compression and rarefaction with an associated pressure variation. The acoustic pressure at any point is the difference between the actual pressure at that point in the presence of the sound and the pressure that would exist at that point under identical conditions in the absence of sound. The sound pressure is the incremental variation in pressure above atmospheric pressure which varies sinusoidally with time.

(b) R M S Sound Pressure :

Sound consists of a rapid irregular series of positive pressure disturbance (compression) and negative pressure disturbance (rarefaction) measured from the equilibrium pressure value. If the mean value of the sound pressure disturbance is measured, it would be zero, because there are as many positive compression as negative rarefaction. Thus, the mean value of sound pressure is not a useful measure. Care must be taken to look for a measure that permits the effects of rarefactions to be added to the effects of compressions. One such measure is the root mean square (rms) sound pressure P_{rms}. P_{rms} value of sound pressure is obtained by square root of the effects produced by adding the squares of compression and rarefaction effects of sound pressure. Mathematically,

$P_{rms} = P_o/\sqrt{2}$, where P_o is peak value of pressure wave.

(c) <u>Acoustic Intensity</u> :

It is the most basic quantity in understanding the sound power. Due to the vibration of a body, acoustic power is transferred to the surrounding in the form of noise from that body. For defining the acoustic intensity, a point at some distance from a source of sound and a small area perpendicular to the line joining the point to the source has to be considered. Some of the power being generated by the source will be transmitted through the area; the exact amount depends not only on the strength of the source but also on the directional property, the distance of the area from the source and the presence of sound absorbing and sound reflecting material. If the power passing through the area A is W, then the acoustic intensity I is given by

$$I = W/A.$$

CHAPTER - 2

Literature Review

2.1 Introduction :

The works on the vibration of rectangular plates started in the last part of the 19th century. Rayleigh was the first person who came forward to begin the study of the dynamical behaviour of structures and developed fundamental governing equations of vibrating plates. With the passage of time, research in this field got momentum with the recognition of the fact that environmental pollution, caused by noise, is an important factor affecting human comfort. Yet, the acoustic radiation characteristics of flat plates is still awaiting its exhausting study.

2.2 Vibration of plates:

Early part of the study on dynamical structures were mostly on vibrational analysis of plates. Most of the initiators confined their study to the determination of the frequency of vibrating plates.

Warburton [47] derived approximate expression for the frequencies of all the modes of vibration of isotropic plates subjected to any combination of free, simply supported or clamped edges. He applied the Rayleigh method, assuming that the deflections of the plates could be represented by suitable characteristic functions satisfying the boundary conditions [48].

In his analysis, the author first developed a set of beam functions satisfying the end conditions of vibrating beams and applied those results to the vibration of plates. He expressed the frequency in terms of a dimensionless frequency factor which in its turm is a function of mode shapes.

In 1959, Hearmon [18] extended the treatment presented by Warburton [47] to the orthotropic plates of material having three mutually perpendicular axis of symmetry with any of its edges either clamped or simply supported.

In 1960, Lin [29] studied the free vibration of plates stiffened with many stringers. The differential equation was solved for an individual plate and the solutions for an individual plate were coupled with the boundary conditions.

For the problem of solid circular plate, an analytical solution was presented by Wah [44] in 1962. Approximate values of the natural frequency of an isotropic rectangular plate with thickness varying in one direction were reported by Gontkevich [11] and Apple and Byers [2]. Apple and Byers calculated the upper and lower bounds for the fundamental natural frequency of simply supported plates. Gontkevich derived approximate expression for calculating the natural frequency for the plate with various boundary conditions by the use of finite difference method.

In 1969, Dickinson [4] extended the sine series solution, previously used for the study of flexural vibration of rectangular isotropic plate, to freely vibrating orthotropic plates. The author used the sine series solution developed by Dill and Pister [5] and applied the method to three particular plates with different support conditions.

In 1972, Jain [23] analyzed circular plates of variable thickness by a Frobenius method to calculate fundamental frequency and buckling load. Pardoen [34] dealt with the problem by finite element approach Ritz and Galekin solutions have been obtained by Laura and co-workers [25,26] to calculate frequency. Saito and Suzuki [36] performed an analytical evaluation on the viscoelastic beam effect on the plate vibration characteristics in 1977.

Gorman [12] conducted free vibration analysis of plates with all combinations of clamped-simply supported edges, excepting those of two opposite edges simply supported. He discussed the vibration of plates with two opposite edges simply supported in a separate paper [13]. The author also introduced the analysis of free vibration of rectangular plate in his earlier work [14].

In 1978, Gupta and Lal [16] analyzed the ciruclar plates of variable thickness by a Frobenies method to have fundamental frequency.Celep [3] studied dynamic stability of free circular plate under non conservative load. The problem of variable thickness was solved by Laura and Valergrade [43] by Ritz method. K. Ohtomi [33] studied analytically the free vibration of a simply supported rectangular plate, stiffened with viscoelastic beam. The effects of the volume and number of stiffening beam on the plate were clarified. In 1986, Laura and co-workers [27] studied the transverse vibration of rectangular plate to calculate fundamental frequency of vibration. The authors used the Rayleigh-Schmidt methodology coupled with the use of polynomial coordinate function. The frequency values obtained by these authors were very close to those obtained by Leissa [28] in 1973.

2.3 Acoustic Radiation from plates:

Extensive research works have been carried out to study the vibration of solid structures like rectangular plates, but the study of acoustic radiation due to the vibration of solid structures like flat plates is yet very inadequate and unsatisfactory.

The first solution to the radiation of power from an elastic plate, modeled by the classical plate theory, was obtained by Skudrzyk [37,38] and Heckl [19,20,21] for a time harmonic point force and by Thompson and Rattya [42] for a time harmonic point moment. The solution for the acoustic pressure radiated from a point excited plate using the classical theory was given by Gutin [17].

The influence of fluid loading on the radiation from the elastic plate was investigated by Maindanik and Kerwin [31]. To improve the high frequency prediction of elastic plate, in 1966, Feit [8] employed the Timshenko-Miudlin theory for such a prediction. This theory adds shear deformation and rotary inertia to the classical flexural theory. There are two dispersion curves for this plate. The flexural branch (acoustic) has phase and group velocities approaching the Rayleigh velocity of the plate material as the frequency increases. Stuart [39] explored the solution for the same plate with new insights into the leaky wave emanating from the plate and obtained a more accurate solution when the angle of observation approached the coincidence angle. This new solution would be accurate at closer observation distance than that of Feit's [8].A solution for the acoustic pressure radiation from a point excited plate was also given by Feit [6] in 1970. All these investigations employed the classical plate theory. Such theory fails at high frequency where the phase and group velocities become infinite as the frequency become unbounded. Such a plate theory is useful only for frequencies where the ratio of the wave length to the plate thickness exceeds eight.

Sound radiation from the beam-reinforced plate, excited by point or line forces, has been investigated by a few authors. Ramanov [35] obtained the solution of the radiated pressure from a plate reinforced with beams and excited by a line force. Feit and Saurenman [7] analyzed the acoustic radiation of a point excited plate reinforced with a beam, but confined their interests to high frequencies.

In 1972, Wallace [45] found radiation resistance from the farfield pressure distribution produced by a baffled beam, vibrating with a simple harmonic motion in one of its natural modes. He considered beams hinged at one end and clamped at the other. The author derived expression for the radiation resistance which are asymptotic to the exact solution as the frequency approaches zero. In addition numerical integration of the farfield acoustic intensity was used to obtain graphs covering the entire frequency range for the first ten modes of the beam. In another paper [46] the author determined the radiation resistance corresponding to the natural mode of a finite rectangular panel supported in an infinite baffle. He used the approximate beam functions developed by Warburton [47]. But his analysis was confined to simply supported panel only. In 1974, Gorman [15] obtained the solution for a plate reinforced by many beams and excited by a line force parallel to the beam. His solution thus makes the beams reaction on the plate purely as rotary and transverse impedences without flexural wave travelling in the beam. Garrelick and Lin [9] analyzed the radiation from a beam reinforced plate and confined their attention to on-axis response. Lin and Hayck [30] obtained the exact solution for the radiation from a point excited plate reinforced by one beam, which is valid in the entire acoustic space.

Ahmed [1] studied the acoustic power radiation from one side of a baffled rectangular isotropic plate for different thickness ratio, aspect ratio and mode orders. The author also found radiation efficiency of the plate with all edges simply supported. The author used Warburtons [47] displacement functions to define the deformation mode of the surface of the plate.

It is seen that much research works have been done on natural frequency of rectangular plates but in the case of acoustic radiation they are still less researched. In Bangladesh,works on natural frequency and acoustic radiation of rectangular plates are in a state of infancy.

CHAPTER - 3

FORMULATION OF THE PROBLEM

3.1 Introduction:

Before considering the various boundary conditions, the basic equations and methods of solution are described. In this analysis a rectangular plate of uniform thickness is assumed to be confined in an infinite baffle. The baffle prevents the movement of air arround the edges of the plate and permits radiation into the spaces in front of either of the surfaces of the plate. Sound waves are generated by the vibration of the plate in contact withthe fluid medium and energy is transferred from the source to the fluid. The characteristics of the source determine the frequency and directional properties of the generated sound field. In this paper we shall devote our attention to the generated sound energy and radiation efficiency. We shall assume that the fluid medium out side the source region is initially uniform and at rest and the acoustic pressure generated outside the source is small enough so that the first order equations of sound are valid in the region out side the source. The plate may have any combination of simply supported clamped or freely suspended condition at the edges. But the current analysis is confined to two mixed boundary conditions namely (1) two opposing ends are clamped and the other two opposing ends freely suspended ,(2) one end clamped and the three other ends freely suspended. The plate, along with the coordinate system used in the analysis is shown in fig-1. Different boundary conditions are dealt with in separate sections. But only a general mathematical model is presented in this chapter.

3.2 Assumptions:

The governing equations are based on the following assumptions :

(1) There is no deformation in the middle plane of the plate. This plane remain neutral during bending.

(2) The plate is isotropic, elastic and of uniform thichness.

(3) The plate is thin and free from applied load.

(4) Points of the plate lying initially on a normal to the middle surface of the plate remain on the normal to the middle surface of the plate after bending.

(5) The normal stresses in the direction transverse to the plate can be disregarded. These stresses must be zero at the free surfaces and, provided that the plate is thin, it is reasonable to assume that it is zero at any section.

(6) Only transverse displacement is considered and this is very small in comparison with the thickness of the plate.

3.3 Mathematical Modeling

(A) Determination of power radiation

For a rectangular plate of sides 2a and 2b, the vibration form must satisfy the boundary conditions at the edges and also satisfy the plate equation of dynamic equilibrium. When excited, the plate vibrates in simple harmonic motion in one of its natural modes. The transverse displacement at a point (x,y) on the surface of the plate at any time t corrosponding to the (m,n)th mode of vibration is given by [1]

$$w_{mn} = W_{mn} \quad \Theta(x)\ \Theta(y) \quad e^{i\omega_{mn} t} \quad \ldots\ldots\ldots (3.1)$$

where,

W_{mn} = amplitude of transverse displacement

ω = natural angular frequency of vibration of the plate corrosponding to the (m,n)th mode

t = time

$\Theta(x)$, $\Theta(y)$ = displacement function describing the wave form of the vibrating plate and satisfying the conditions at the edges.

The motion of the plate surface which generates the acoustic radiation is given by the normal velocity distribution,

$$U = \frac{d\, w_{mn}}{dt} = i\omega_{mn} \quad W_{mn}\ \Theta(x)\ \Theta(y)\ e^{i\omega_{mn} t}$$

$$= U_0 e^{i\omega_{mn} t} \quad \ldots\ldots\ldots\ldots (3.2)$$

Whenever a distarbance is created at any point in a compressible fluid there is a propagation of that disturbance through out the fluid. It is desired to study the characteristics of this propagation. For this purpose the following symbols are introduced.

x,y,z coordinates of a particle of the medium.

u,v,w component velocities of a particle of the medium.

ρ' Instantaneous density of the medium

P pressure in the medium

c velocity of propagation of the disturbance,

S' the condensation defined by the relation

$$S' = (\rho' - \rho)/\rho = (\delta\rho')/\rho \quad , \text{ where}$$

ρ is the constant mean density at any point.

ϕ the velocity potential.

All the above quantities except c are functions of x,y,z & t. Now it is necessary to define the velocity potential - the most important single quantity in the study of the irrotational motion of fluids.

Velocity potential is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velicity in that direction. Mathematically, velocity potential ϕ is given by

$$u = -\frac{\partial \varphi}{\partial x}$$
$$v = -\frac{\partial \varphi}{\partial y}$$
$$w = -\frac{\partial \varphi}{\partial z}$$

Let us consider a small volume element of the fluid. The difference between the efflux and influx of the medium in this element is equal, by the so-called principle of continuity, to the time rate of growth of mass in the element.

The most simple method of obtaining the mathematical expression of this principle is by the consideration of an element parallelopiped of dimensions Δx, Δy, Δz. By considering the influx and efflux through each pair of faces respectively, the difference between the later and the former for the whole cube is found to be :

$$-\left\{ \frac{\partial(\rho' u)}{\partial x} + \frac{\partial(\rho' v)}{\partial y} + \frac{\partial(\rho' w)}{\partial z} \right\} \Delta x \, \Delta y \, \Delta z$$

Moreover, the rate of growth of mass in the cube is clearly $\frac{\partial \rho'}{\partial t} \Delta x \, \Delta y \, \Delta z$. Equating these two quantities we get the following:

$$\frac{\partial \rho'}{\partial t} + \frac{\partial(\rho' u)}{\partial x} + \frac{\partial(\rho' v)}{\partial y} + \frac{\partial(\rho' w)}{\partial z} = 0 \quad \ldots\ldots\ldots \quad (3.3)$$

If in this equation we make the substitution, $\rho' = \rho(1+S')$, we have,

$$\rho \frac{S'}{t} + \rho(1+S')\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$+ \rho\left(u \frac{\partial S'}{\partial x} + v \frac{\partial S'}{\partial y} + w \frac{\partial S'}{\partial z}\right) = 0 \quad \ldots\ldots\ldots\ldots (3.4)$$

In acoustics the condensation is a very small quantity compared with unity. Movereover, acoustical wave lengths are so long that u,v,w, and S' change very little with x, y, z. Hence $\frac{\partial u}{\partial x}$, $\frac{\partial S'}{\partial x}$, etc. are very small quantities. We can therefore neglect terms like $S' \frac{\partial u}{\partial x}$ and $u \left(\frac{\partial S'}{\partial x}\right)$ in comparison to $\frac{\partial u}{\partial x}$ and to this approximation the continuity equation becomes

$$\frac{\partial S'}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \ldots\ldots\ldots\ldots\ldots (3.5)$$

Substituting the values of u,v,w in terms of the potential function, we have,

$$\frac{\partial S'}{\partial t} = \nabla^2 \varphi \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad \ldots\ldots \quad (3.6)$$

To replace (3.3) by an equation in which φ is the only independent variable we must have recourse to the hydrodynamical equations of motion [24] these are,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho'} \frac{\partial P}{\partial x},$$

and two similar equation for v and w, where in, the left-hand side represents the acceleration of the medium in the x-direction and the right-hand side, the force per unit mass, and there are no external forces acting. From what has been said above, we can neglect $u \left(\frac{\partial u}{\partial x}\right)$, etc. compared with $\frac{\partial u}{\partial t}$, etc. Equations of motion then take the simpler form

$$\frac{\partial u}{\partial t} = - \frac{1}{\rho'} \frac{\partial P}{\partial x},$$

$$\frac{\partial v}{\partial t} = - \frac{1}{\rho'} \frac{\partial P}{\partial y},$$

$$\frac{\partial w}{\partial t} = - \frac{1}{\rho'} \frac{\partial P}{\partial z},$$

If we substitute into these equations $u = - \frac{\partial \varphi}{\partial x}$, etc, multiply the three equations by dx, dy, dz respectively and add, we have

$$\frac{\partial}{\partial t} d\varphi = \frac{1}{\rho'} dP$$

or integrating :

$$\frac{\partial \varphi}{\partial t} = \int \frac{dP}{\rho'} \qquad \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad \ldots\ldots \quad (3.7)$$

Since ρ' changes but little it will be approximately correct to remove it from under the integral sign, calling it ρ , the mean density. Then, $\int dP$ reduces simply to the excess pressure, so

$$P = \rho \frac{\partial \phi}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.8)$$

On the other hand, if we write it as δP,we have the approximate relation,

$$\frac{\partial \phi}{\partial t} = \frac{\delta P}{\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.9)$$

Newton assumed that the passage of train of longitudinal wave through air is so slow that heat generated during compression or cold produced during rarefaction is dissipated or absorbed to or from the body of the gas respectively, so the process is isothermal.

Hence, Boyles law, PV = constant, holds
So isothermal elasticity of the gas is given by the equation

$$PdV + VdP = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3,10)$$

or, $P = - V \frac{dP}{dV} = E$, bulk modulas (volume stress/volume strain)

Therefore, velocity, $c = \sqrt{\frac{E}{\rho'}} = \sqrt{\frac{P}{\rho'}} \quad \ldots\ldots\ldots\ldots \quad (3.11)$

or, it can be written in the form,

$\delta P = c^2 \delta \rho'$. But $\delta \rho' = \rho S'$ by the definition of condensation and hence there follows the important relationship.

$$\delta P = c^2 \rho S' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.12)$$

Moreover, (3.9) now becomes

$$\frac{\partial \varphi}{\partial t} = c^2 S' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.13)$$

which is a general equation connecting velocity potential and condensation. If we substitute S' from (3.13) into (3.6) we finally have,

$$\frac{\partial^2 \varphi}{\partial t^2} = c^2 \nabla^2 \varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.14)$$

Which is the familiar equation of wave motion. The general solution of (3.14) shows that the influence of any value of φ is propagated with velocity c.

In the case of spherical wave, diverging from a point, let us assume spherical symmetry about the point. That is, φ will be assumed to depend only on t and the distance R from that point. In polar coordinates, it may be shown [22] that $\nabla^2 \varphi$ takes the form,

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial R^2} + \frac{2}{R} \frac{\partial \varphi}{\partial R} + \frac{1}{R^2 \operatorname{Sin}\theta} \frac{\partial}{\partial \theta}\left(\operatorname{Sin}\theta \frac{\partial \varphi}{\partial \theta}\right) +$$

$$\frac{1}{R^2 \operatorname{Sin}\theta} \frac{\partial^2 \varphi}{\partial^2 \alpha} \quad \ldots\ldots\ldots\ldots\ldots \quad (3.15)$$

Where $x = R \sin\theta \cos\alpha$

$y = R \sin\theta \sin\alpha$

$z = R \cos\theta$

In the case of assumed symmetry, the above equation reduces to

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial R^2} + \frac{2}{R}\frac{\partial\varphi}{\partial R}$$

$$\text{or} \quad \nabla^2\varphi = \frac{1}{R}\frac{\partial^2}{\partial R^2}(R\varphi) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.16)$$

Therefore, the general wave equation (3.14) for spherical wave propagation becomes

$$\frac{\partial^2(R\varphi)}{\partial t^2} = c^2\frac{\partial^2(R\varphi)}{\partial R^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.17)$$

Let $\varphi = (1/R)F\,(ct + R)$, where F is any arbitrary function.

It is clear that φ satisfies the general wave equation (3.17). So is the solution of the above equation.

As before we shall postulate a harmonic wave and for convenience, shall employ the complex notation for φ of the plate problem,as in fig. 1

$$\varphi = \frac{\alpha' A}{r}\, e^{ik(ct + r)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.18)$$

where,

r is the distance from the source dA to the receiving point.

α' is the amplitude of velocity potential per unit area.

A is the area of the rectangular plate on which disturbances are created and propagated in to the surrounding medium.

$k = 2\pi/\lambda = \omega/\lambda f = \omega/c$, acoustic wave number.

ω is angular frequecy of vibration.

c is velocity of sound in air.

f is frequency of harmonic vibration.

λ is wave length of sound wave.

The differential form of the equation (3.18) is

$$d\varphi = \frac{\alpha' dA}{r} e^{ik(ct + r)}, \text{ where } dA = dxdy.$$

The total velocity potential at the receiving point (R,θ,α) will be obtained by the integration of the contributions from over the whole plane surface of the rectangular flat plate and so

$$\varphi_r = \int \frac{\alpha'}{r} e^{ik(ct + r)} dA$$

In order to evaluate α', let the source of hemispherical waves at dA to be a pulsating hemisphere of radius r_o, which is finally made vanishingly small. Hence area $= 2\pi r_0^{\,2}$, and the velocity potential at the receiving point is

$$\varphi_r = \frac{2\pi r_0^{\,2}\alpha'}{r} e^{i(\omega t + kr)} \quad \ldots\ldots\ldots\ldots \quad (3.19)$$

The particle velocity at the surface of the hemisphere will be given by

$$U_{r = r_0} = -\left(\frac{\partial\varphi}{\partial r}\right)_{r = r_0}$$

$$= -2\pi r_0^{\,2}\alpha' e^{i(\omega t + kr)}\left\{\frac{k}{r_0} - \frac{1}{r_0^{\,2}}\right\} \quad \ldots (3.20)$$

where U is the linear surface velocity which follows the harmonic variation. According to (3.2)

$$U = U_0 e^{i\omega t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.21)$$

where U_0 is the amplitude of velocity distribution.

From equation (3.20) and (3.21) it is evident that if $r_0 \to 0$

$$U_0 e^{i\omega t} = -\left(\frac{\partial\varphi}{\partial r}\right)_{r \quad 0} = 2\pi\alpha' e^{i\omega t}$$

or $\alpha' = \dfrac{U_0}{2\pi}$

Therefore $d\varphi = \dfrac{U_0}{2\pi r} e^{\,i(\omega t + kr)}\, dA$

$$= \frac{U}{2\pi r} e^{\,ikr}\, dA \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.22)$$

From the expression (3.8) we have for area dA (= dx dy)

$$P = \rho \frac{\partial \varphi}{\partial t} \text{ , then}$$

$$dP = \frac{i\rho\omega U_0}{2\pi r} e^{i(\omega t + kr)} dA$$

$$= ik\rho c \quad d\varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.23)$$

Substituting $d\varphi$ from the equation (3.22) into equation (3.23) we have,

$$dP = \frac{ik\rho cU}{2\pi r} c^{ikr} \quad dA \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.24)$$

If the receiving point (R,θ,α) is located in the farfield, then the distance r of the receiving point from the elemental area dA can be expressed with the first order approximation as [32]

$$r \simeq R - (x \operatorname{Cos}\alpha + y \operatorname{Sin}\alpha)\operatorname{Sin}\theta \ldots\ldots\ldots\ldots\ldots\ldots (3.25)$$

The second term $[x \operatorname{Cos}\alpha + y \operatorname{Sin}\alpha]\operatorname{Sin}\theta$ in the equation (3.25) is very small compared to the first and its effect can be neglected in the amplitude factor of the equation (3.24). However, both the terms are equally significant as far as the the phase factor of the equation (3.24) is concerned.

Subtituting equation (3.2) and (3.25) in equation (3.24) and neglecting the second term in the expression of r in the amplitude factor of equation (3.24), it can be shown that,

$$dP = -\frac{k\rho c\,\omega_{mn}W_{mn}}{2\pi R}\,\theta(x)\,\theta(y)\,e^{i(\omega t+kr)}\,dA$$

$$= -\frac{k\rho c\,\omega_{mn}W_{mn}}{2\pi R}\,\theta(x)\,\theta(y)\,e^{i\{\omega t+kR-k(x\,\cos\alpha+y\,\sin\alpha)\sin\theta\}}\,dA$$

Thus the net acoustic pressure at the point (R,θ,α) due to (m,n)th mode of vibration of the plate is given by

$$p = -\frac{\rho ck\omega_{mn}W_{mn}}{2\pi R}\,e^{i(\omega t+kr)}\int_{-a}^{a}\int_{-b}^{b}\theta(x)\,\theta(y)\,e^{-ik(x\,\cos\alpha+y\,\sin\alpha)\sin\theta}\,dxdy$$

$$= -\frac{\rho ck\omega_{mn}W_{mn}}{2\pi R}\,e^{i(\omega t+kr)}\int_{-a}^{a}\int_{-b}^{b}\theta(x)\,\theta(y)\,e^{-i\left(\frac{sx}{2a}+\frac{qy}{2b}\right)}\,dxdy \qquad \ldots\ldots\ldots\ldots\ (3.26)$$

where $s = 2ak\,\cos\alpha\,\sin\theta$

and $q = 2bk\,\sin\alpha\,\sin\theta$

Acoustic intensity of a sound wave is defined as the average power transmitted per unit area in the direction of propagation of the wave. Instantaneous power per unit area is equal to the product of instantaneous pressure (P) and instantaneous particle velocity (v). Average power per unit area measures the acoustic intensity. Therefore,acoustic intensity,

$$I = 1/T \cdot \int_0^T Pv\,dt.$$

Here $P = -\rho c^2\,(dw/dx)$

If $w = W \cos(\omega t - kx)$, Where W is the amplitude of displacement w,

$v = dw/dt = -\omega W \sin(\omega t - kx)$

$dw/dx = -Wk \sin(\omega t - kx)$

$kc = \omega$, and T is time period, then

$P = -\rho c W \omega \sin(\omega t - kx)$

Here c is the velocity of sound and ρ is the density of the medium.

Therefore,

$$I = 1/T \cdot \int_0^T [-\rho c W \omega \sin(\omega t - kx)][-\omega W \sin(\omega t - kx)]dt.$$

$$= 1/2 . \rho c \omega^2 W^2 \quad \text{------------------------------} \quad (3.27)$$

Since, $P = -\rho c W \omega \sin(\omega t - kx)$,

$$P_{max} = -\rho c W \omega .$$

The root mean sqare value of pressure,

$$P_{r.m.s} = P_{max} / \sqrt{2}$$

From equation (3.27), $I = \dfrac{\rho^2 c^2 W^2 \omega^2}{2 \rho c} = \dfrac{(P_{max}{}^2)}{2 \rho c}$

$$= \frac{|P|^2}{c} \quad .... \quad (3.28)$$

where, $|P|^2$ is the square of the root mean square value of the net acoustic pressure.

The total average power radiated from one side of the baffled plate, found by integrating the farfield acoustic intensity over the hemispherical surface is

$$P_w = \int_0^{2\pi}\int_0^{\pi/2} \frac{|p|^2}{c} R^2 \sin\theta \, d\theta \, d\alpha \quad \ldots . (3.28a)$$

(B) Determination of Natural frequency :

The natural frequecny of the plate ω_{mn} corresponding to the transverse mode of vibration (m,n) has been determined by a number of authors in terms of boundary conditions, the nodal patterns, the dimensions of the plate and the constants of the plate material. Warburton [47] derived the expression for the natural frequecny from energy equation. He studied a rectangular plate of sides of length a and b where the vibration form must satisfy the boundary conditions at the edges; also it must satisfy the governing equation of plate (Love 1927)

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{12\rho(1-\sigma^2)}{Egh^2}\frac{\partial^2 w}{\partial t^2} = 0 \quad ..(3.29)$$

Here w, the displacement at any point (x,y) at time t, is given by

$$w = W_{xy} \sin\omega t = A\,\Theta(x)\,\Theta(y) \sin\omega t \quad \ldots\ldots\ldots\ldots(3.30)$$

where W_{xy} is amplitude of the wave form.

$\Theta(x)$, $\Theta(y)$ is space functions defined by the author [47]and ω is natural frequency.

In general, it was not possible to find a form for ω to satisfy equation (3.29) togather with the boundary conditions. For these cases, an infinite series was assumed for W_{xy} ; each term of the series satisfied equation (3.29) and some of the boundary conditions, and by considering suitable values of the coefficients A, the remaining conditions were satisfied.

Accurate values of frequency may also be obtained by the considerations of energy. For a rectangualr plate, the potential energy of bending U (Timoshenko 1937) is

$$U = \int_0^a\int_0^b (1/2)\frac{Eh^3}{12(1-\sigma^2)}\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\sigma\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2(1-\sigma)\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]dxdy \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.31)$$

and the kinetic energy is

$$T = \int_0^a\int_0^b (1/2)\cdot(\rho_m h/g)\left(\frac{\partial w}{\partial t}\right)^2 dxdy \quad \ldots\ldots\ldots (3.32)$$

From maximum values of potential and kinetic energy, natural frequency may be obtained as

$$\omega^2_{m,n} = \frac{U_{max}}{(\rho_m h/2g)\int_0^a\int_0^b W^2\, dxdy} \quad \ldots\ldots\ldots\ldots (3.33)$$

For any boudnary condition, expressions for space function $\Theta(x)$, $\Theta(y)$ were substituted in equation (3.30) and (3.33) to give an expression for frequency. Warburton expressed the natural frequency in terms of a dimensionless frequency factor λf, given by

$$\lambda_f^2 = Gx^4 + Gy^4 (a/b)^4 + 2(a/b)^2 [\sigma HxHy+(1-\sigma)JxJy] \quad ..(3.34)$$

and the relation between ω_{mn} and λ_f^2 was obtained as

$$\lambda_f^2 = \frac{\rho_m a^4 \omega^2 12(1-\sigma^2)}{\pi^4 Eh^2 g} \quad .... (3.35)$$

Thus the frequency factor λ_f was obtained for any ratio a/b; the frequency was given by

$$f = \frac{\lambda_f h \pi}{a^2} \left[\frac{Eg}{48 \rho_m (1 - \sigma^2)} \right]^{1/2} \quad .... (3.36)$$

Where,

ρ_m	=	density of the plate material,
σ	=	poissions ratio,
E	=	modulus of elasticity of the plate material
h	=	thickness of the plate
g	=	acceleration due to gravity

Gx,Gy,Hx,Hy, Jx,Jy = functions of boundary conditions and mode shapes. The values of these quantities were given by Warburton [47] in the form of a table presented here for ready reference.

Table giving the values of the parameters Hx,Hy,Gx,Gy,Jx,Jy of dimensionless frequency factor.

Boundary condition	m	Gx	Hx	Jx
Two end fixed and two ends free	2	1.506	1.248	1.248
	3,4,5	m-1/2	(m-1/2)[1- 2/(m-1/2)]	(m-1/2)[1- 2/(m-1/2)]
One end fixed and three ends free	2	1.499		
	3,4,5	m-1/2	(m-1/2)[1- 2/(m-1/2)]	(m-1/2)[1+ 2/(m-1/2)]
	n	Gy	Hy	Jy
Two end fixed and two ends free	2	1.506	1.248	5.017
	3,4,5	n-1/2	(n-1/2)[1- 2/(n-1/2)]	(n-1/2)[1+ 6/(n-1/2)]
One end fixed and three ends free	2	1.506	1.248	5.017
	3,4,5	n-1/2	(n-1/2)[1- 2/(n-1/2)]	(n-1/2)[1+ 6/(n-1/2)]

(C) Determination of radiation efficiency :

The radiation resistance corresponding to the natural modes of a finite ractangular panel is theoritically determined from the total energy radiated to the farfield. Wallace, in his papers[45,46], introduced the concept of radiation efficiency with a view to simplify the matters. The main idea behind the introduction of the concept of radiation effiicency was to eliminate the dependence on the impedance of the acoustic medium and the plate size while calculating the acoustic radiation from vibrating plates. The radiation efficiency is defined by Wallace [45,46] as

$$S_{mm} = \frac{P_w}{4 \rho cab \langle |u_w|^2 \rangle} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.37)$$

where P_w is the total average power radiation from one side of the baffled plate and $\langle |u_w|^2 \rangle$ is the average of the temporal and spatial factor of the square of the surface velocity, given by

$$\langle |u_w|^2 \rangle = (1/4ab) \int_{-a}^{a} \int_{-b}^{b} (1/2) \, . \, u_w^2 \, dxdy \qquad \ldots\ldots\ldots\ldots\ldots \quad (3.38)$$

u_w is obtained from the equation (3.2) as

$$u_w = i \omega_{mn} W_{mn} \Theta(x) \Theta(y).$$

3.4 Displacement Functions :

The characteristic beam functions are usually adopted to represent the deformation of plates in vibration in both the x & y directions. By this method the approximate freequency expressions are obtained. Warburton [47] found a number of beam functions for describing the displacement of a vibrating beam under different combination of end conditions. He also used these functions to represent the vibration of rectangular flat plate. The slightly modified beam functions, which are hereby referred to as displacement function, for different boundary conditions are presented here.

(i) Two opposite ends fixed, other two ends free :

If the two opposing edges parpendicular to x-direction are fixed, the corresponding beam function is given by Warburton [47] as

$\Theta(x) = \operatorname{Cos}\gamma_m x/2a + R_m \operatorname{Cosh}\gamma_m x/2a$ for $m = 2,4,6 \ldots.$

$\Theta(x) = \operatorname{Sin}\gamma_m' x/2a + R_m' \operatorname{Sinh}\gamma_m' x/2a$ for $m = 3,5,7 \ldots..$

where, $R_m = \dfrac{\operatorname{Sin}\gamma_m/2}{\operatorname{Sinh}\gamma_m/2}$ and γ_m are the roots of the equation $\tan\gamma/2 + \tanh\gamma/2 = 0$, and

$R_m' = -\dfrac{\operatorname{Sin}\gamma_m'/2}{\operatorname{Sinh}\gamma_m'/2}$ and γ_m' are the roots of the equation $\tan\gamma/2 - \tanh\gamma/2 = 0$.

When the opposing ends perpendicular to y-direction are free, the corresponding beam functions are

$$\theta(y) = \cos \gamma_n y/2b + R_n \cosh \gamma_n y/2b \quad \text{for} \quad n = 2,4,6 \ldots.$$

$$\theta(y) = \sin \gamma_n' y/2b + R_n' \sinh \gamma_n' y/2b \quad \text{for} \quad n = 3,5,7 \ldots.$$

where, $R_n = - \dfrac{\sin \gamma_n/2}{\sinh \gamma_n/2}$ and γ_n are the roots of the equation $\tan \gamma/2 + \tanh \gamma/2 = 0$, and

$R_n' = \dfrac{\sin \gamma_n'/2}{\sinh \gamma_n'/2}$, and γ_n' are the roots of the equation $\tan \gamma/2 - \tanh \gamma/2 = 0.$

(ii) <u>One end fixed, other three ends free plate</u> :

If one of the edges perpendicular to x-direction is fixed while the others are free, then the corresponding beam functions is

$$\theta(x) = \cos \gamma_m \left(\frac{x+a}{2a}\right) - \cosh \gamma_m \left(\frac{x+a}{2a}\right) -$$

$$R_m \left\{\sin \gamma_m \left(\frac{x+a}{2a}\right) - \sinh \gamma_m \left(\frac{x+a}{2a}\right)\right\}$$

$$\text{for} \quad m = 1,2,3 \ldots\ldots.$$

Where $R_m = \dfrac{\cos \gamma_m + \cosh \gamma_m}{\sin \gamma_m + \sinh \gamma_m}$ and γ_m are the roots of the euation $\cos \gamma \cdot \cosh \gamma = -1$

If both the edges of the plate perpendicular to y-axis are free, the corresponding beam function is

$$\theta(y) = \text{Cos}\ \gamma_n y/2b + Rn\ \text{Cosh}\ \gamma_n y/2b \quad \text{for} \quad n = 2,4,6 \ldots.$$

$$\theta(y) = \text{Sin}\ \gamma_n' y/2b + Rn'\ \text{Sinh}\ \gamma_n' y/2b \quad \text{for} \quad n = 3,5,7 \ldots.$$

where, $Rn = -\dfrac{\text{Sin}\ \gamma_n/2}{\text{Sinh}\ \gamma_n/2}$ and γ_n are the roots of the equation $\tan\gamma/2 + \tanh\gamma/2 = 0$, and

$$Rn' = \frac{\text{Sin}\ \gamma_n'/2}{\text{Sinh}\ \gamma_n'/2},$$ and γ_n' are the roots of the equation $\tan\gamma/2 - \tanh\gamma/2 = 0.$

CHAPTER- 4

Solution of the problem

4.1 Introduction :

In this chapter expression for analytical determination of total average acoustic power from one side of the baffled plate for various boundary conditions are presented. For the plate with two oppsite ends fixed and the other two ends free, the expression for radiation efficiency is also derived and presented here. In the subsequent sections, the method of integration of the expression for power and radiation efficiency for various boundary conditions are treated.

4.2 Plate with two opposing ends fixed and other two ends free,

(a) Power radiation :

From equation (3.26), the acoustic pressure distribution at a point in the farfield is given by

$$P = \frac{-k \rho c \omega_{mn} W_{mn}}{2 \pi R} e^{i(\omega_{mn} t + kR)} \times \int_{-a}^{a}\int_{-b}^{b} \theta(x).\theta(y) e^{-i(sx/2a + qy/2b)} \, dxdy$$

The displacement functions $\theta(x)$ and $\theta(y)$ for this plate with the fixed edges parallel to y-axis and the free edges parallel to x-axis are :

$$\theta(x) = \cos\left(\gamma_m \frac{x}{2a}\right) + R_m \cosh\left(\gamma_m \frac{x}{2a}\right) \quad \text{for} \quad m = 2,4,6 \ldots$$

$$\theta(x) = \sin\left(\gamma_m' \frac{x}{2a}\right) + R_m' \sinh\left(\gamma_m' \frac{x}{2a}\right) \quad \text{for} \quad m = 3,5,7 \ldots.$$

where $R_m = \dfrac{\sin(\gamma_m/2)}{\sinh(\gamma_m/2)}$ and γ_m are the roots of the equation, $\tan(\gamma/2) + \tanh(\gamma/2) = 0$

$R_m' = -\dfrac{\sin(\gamma_m'/2)}{\sinh(\gamma_m'/2)}$ and γ_m' are the roots of the equation, $\tan(\gamma/2) - \tanh(\gamma/2)) = 0$

$$\theta(y) = \cos\left(\gamma_n \frac{y}{2b}\right) + R_n \cosh\left(\gamma_n \frac{y}{2b}\right) \quad \text{for} \quad n = 2,4,6 \ldots$$

$$\theta(y) = \sin\left(\gamma_n' \frac{y}{2b}\right) + R_n' \sinh\left(\gamma_n' \frac{y}{2b}\right) \quad \text{for} \quad n = 3,5,7 \ldots.$$

where $R_n = -\dfrac{\sin(\gamma_n/2)}{\sinh(\gamma_n/2)}$ and γ_n are the roots of the equation, $\tan(\gamma/2) + \tanh(\gamma/2) = 0$

$R_n' = \dfrac{\sin(\gamma_n'/2)}{\sinh(\gamma_n'/2)}$ and γ_n' are the roots of the equation, $\tan(\gamma/2) - \tanh(\gamma/2)) = 0$

It is now required to consider each of the combination of odd and even values of m and n separately.

Case 1. <u>Even values of m and n</u>

Substituting the displacement functions $\Theta(x)$ and $\Theta(y)$, the acoustic pressure distribution P becomes,

$$P = \frac{-\rho c k \omega W}{2\pi R} e^{i(\omega t + kR)} \int_{-a}^{a}\int_{-b}^{b} [\{Cos(\gamma_m x/2a) +$$

$$Rm\ Cosh(\gamma_m x/2a)\}e^{-isx/2a}\ dx\ ] \times [\{\ Cos(\gamma_n y/2b) +$$

$$Rn\ Cosh\ (\gamma_n y/2b)\}\ e^{-iqy/2b}\ dy\ ] \quad \ldots\ldots\ldots\ldots\ldots \quad (4.1)$$

where $S = 2ak\ Cos\alpha\ \ Sin\Theta$

$q = 2bk\ Sin\alpha\ \ Sin\Theta$

Integration of equation (4.1) gives,

$$P= - \frac{\rho k\ c\omega W}{2\pi R} e^{\ i(\omega t + kR)} [\ \frac{4a}{\gamma_m^2\{(s/\gamma_m)^2-1\}}\{S\ Cos(\gamma_m/2)Sin(s/2)-$$

$$\gamma_m Sin(\gamma_m/2).\ Cos\ (s/2)\} + \frac{Rm.\ 4a}{\gamma_m^2\{(s/\gamma_m)^2+\ 1\}}.\{S\ Cosh(\gamma_m/2)Sin\ (s/2) +$$

$$\gamma_m Sinh(\gamma_m/2)\ Cos(s/2)\}] \times [\ \frac{4b}{\gamma_n^2\{(q/\gamma_n)^2 - 1\}}.\ \{q\ Cos(\gamma_n/2)Sin(q/2) -$$

$$\gamma_n Sin(\gamma_n/2)cos(q/2)\} + \frac{4b\ Rn}{\gamma_n^2\{(q/\gamma_n)^2+\ 1\}}\ \{q\ Cosh(\gamma_n/2)Sin(q/2) +$$

$$\gamma_n Sinh(\gamma_n/2)Cos(q/2)\}] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.2)$$

The farfield acoustic intensity is given by

$$I = |P|^2/\rho c$$

Substituting P from equation (4.2), it is found that,

$$I = \frac{32k^2 \rho_c\, \omega^2 W^2 \alpha^2 b^2}{\pi^2 R^2 \gamma_m^4 \gamma_n^4} \Big[\frac{1}{\{(s/\gamma_m)^2 - 1\}} \{S \operatorname{Cos}(\gamma_m/2) \operatorname{Sin}(s/2) -$$

$$\gamma_m \operatorname{Sin}(\gamma_m/2) . \operatorname{Cos}(s/2)\} + \frac{Rm}{\{(s/\gamma_m)^2 + 1\}} .\{S \operatorname{Cosh}(\gamma_m/2) \operatorname{Sin}(s/2) +$$

$$\gamma_m \operatorname{Sinh}(\gamma_m/2) \operatorname{Cos}(s/2)\}\Big]^2 \times \Big[\frac{1}{\{(q/\gamma_n)^2 - 1\}} . \{q \operatorname{Cos}(\gamma_n/2) \operatorname{Sin}(q/2) -$$

$$\gamma_n \operatorname{Sin}(\gamma_n/2) \cos(q/2)\} + \frac{Rn}{\{(q/\gamma_n)^2 + 1\}} \{q \operatorname{Cosh}(\gamma_n/2) \operatorname{Sin}(q/2) +$$

$$\gamma_n \operatorname{Sinh}(\gamma_n/2) \operatorname{Cos}(q/2)\}\Big]^2$$

From equation (3.35) the dimensionless frequency factor λ_f is given by,

$$\lambda_f^2 = \frac{\rho_m a^4 \omega^2\, 12(1-\sigma^2)}{\pi^4 E h^2 g}, \quad \text{So } \omega^2 = \frac{\lambda_f^2 \pi^4 E h^2 g}{a^4 \rho_m\, 12(1-\sigma^2)}$$

where $\lambda_f^2 = Gx^4 + (a/b)^4 Gy^4 + 2(a/b)^2 [\sigma HxHy + (1-\sigma) JxJy]$

and $S = 2ak \operatorname{Cos}\alpha \operatorname{Sin}\theta = \dfrac{2a}{c} \operatorname{Cos}\alpha \operatorname{Sin}\theta$

$$= \frac{R_t \lambda_f \pi^2 \sqrt{Eg}}{c\sqrt{3 \rho_m (1-\sigma^2)}} \operatorname{Cos}\alpha \operatorname{Sin}\theta$$

$$q = 2bk \sin\alpha \sin\theta = \frac{2b}{c} \sin\alpha \sin\theta$$

$$= \frac{R_t\, R_a\, \lambda_f\, \pi^2 \sqrt{Eg}}{c\sqrt{3\,\rho_m\,(1-\sigma^2)}} \sin\alpha \sin\theta$$

Here R_t = Thickness ratio = h/a

R_a = Aspect ratio = b/a

The total average power radiated from one side of the baffled plate is then given by

$$P_w = \int_0^{2\pi}\int_0^{\pi/2} |P|^2/\rho c \,.\, R^2 \sin\theta \, d\theta d\alpha = \int_0^{2\pi}\int_0^{\pi/2} I R^2 \sin\theta d\theta d\alpha$$

$$= \frac{8\rho\, W^2 \lambda_f^4 \pi^6 E^2 g^2}{9c\, \gamma_m^4\, \gamma_n^4\, \rho_m^2 (1-\sigma^2)^2}\, Ra^2\, Rt^4 \int_0^{\pi/2}\int_0^{\pi/2} [\frac{1}{\{(s/\gamma_m)^2-1\}}\{S \cos(\gamma_m/2)\sin(s/2)-$$

$$\gamma_m \sin(\gamma_m/2).\cos(s/2)\} + \frac{Rm}{\{(s/\gamma_m)^2+1\}}.\{S \cosh(\gamma_m/2)\sin(s/2) +$$

$$\gamma_m \sinh(\gamma_m/2)\cos(s/2)\}]^2 \times [\frac{1}{\{(q/\gamma_n)^2-1\}}.\{q \cos(\gamma_n/2)\sin(q/2) -$$

$$\gamma_n \sin(\gamma_n/2)\cos(q/2)\} + \frac{Rn}{\{(q/\gamma_n)^2+1\}}\{q \cosh(\gamma_n/2)\sin(q/2) +$$

$$\gamma_n \sinh(\gamma_n/2)\cos(q/2)\}]^2 \sin\theta\, d\theta d\alpha \quad \ldots\ldots\ldots\ldots (4.3)$$

It should be noted that the total average power radiated from one side of the plate is not a function of R. Only the acoustic intensity varies inversely as the square of R as seen in equation (4.3).

(b) Radiation efficiency :

The expression for the radiation efficiency of the plate with two opposing ends fixed and other two ends free, needed for comparing the result of this study with those by the previous works [1,46], are derived here. The radiation efficiency of a vibrating plate is defined by the equation (3.37) as

$$S_{mn} = \frac{P_w}{4 \rho c \, ab \, \langle |u_w|^2 \rangle}$$

where the expression of total acoustic power P_w, without substituting the value of k, is

$$P_w = \frac{128 \, k^2 \rho c \, \omega^2 W^2 \, a^2 \, b^2}{\pi^2 \gamma_m^4 \, \gamma_n^4} \int_0^{\pi/2} \int_0^{\pi/2} \Big[\frac{1}{\{(s/\gamma_m)^2 - 1\}} \{ S \, \mathrm{Cos}(\gamma_m/2) \mathrm{Sin}(s/2) -$$

$$\gamma_m \mathrm{Sin}(\gamma_m/2) . \ \mathrm{Cos}\ (s/2)\} + \frac{Rm}{\{(s/\gamma_m)^2 + 1\}} . \{ S \, \mathrm{Cosh}(\gamma_m/2) \mathrm{Sin}\ (s/2) +$$

$$\gamma_m \mathrm{Sinh}(\gamma_m/2) \ \mathrm{Cos}(s/2)\} \Big]^2 \times \Big[\frac{1}{\{(q/\gamma_n)^2 - 1\}} . \{ q \, \mathrm{Cos}(\gamma_n/2) \mathrm{Sin}(q/2) -$$

$$\gamma_n \mathrm{Sin}(\gamma_n/2) \mathrm{cos}(q/2)\} + \frac{Rn}{\{(q/\gamma_n)^2 + 1\}} \{ q \, \mathrm{Cosh}(\gamma_n/2) \mathrm{Sin}(q/2) +$$

$$\gamma_n \mathrm{Sinh}(\gamma_n/2) \mathrm{Cos}(q/2)\} \Big]^2 \ \mathrm{Sin}\theta \ d\theta d\alpha$$

Letting $A_1 = [\frac{1}{\{(s/\gamma_m)^2-1\}}\{S\ \mathrm{Cos}(\gamma_m/2)\mathrm{Sin}(s/2) -$

$\gamma_m \mathrm{Sin}(\gamma_m/2).\ \mathrm{Cos}\ (s/2)\} + \frac{Rm}{\{(s/\gamma_m)^2+1\}}.\{S\ \mathrm{Cosh}(\gamma_m/2)\mathrm{Sin}\ (s/2) +$

$\gamma_m \mathrm{Sinh}(\gamma_m/2)\ \mathrm{Cos}(s/2)\}]^2 \times [\frac{1}{\{(q/\gamma_n)^2-1\}}.\ \{q\ \mathrm{Cos}(\gamma_n/2)\mathrm{Sin}(q/2) -$

$\gamma_n \mathrm{Sin}(\gamma_n/2)\cos(q/2)\} + \frac{Rn}{\{(q/\gamma_n)^2+1\}}\{q\ \mathrm{Cosh}(\gamma_n/2)\mathrm{Sin}(q/2) +$

$\gamma_n \mathrm{Sinh}(\gamma_n/2)\mathrm{Cos}(q/2)\}]^2,$

total power is expressed as

$$P_w = \frac{128\ k^2 \rho c \omega^2 W^2\ a^2\ b^2}{\pi^2\ \gamma_m^4\ \gamma_n^4} \int_0^{\pi/2} \int_0^{\pi/2} A_1\ \mathrm{Sin}\theta\ d\theta\ d\alpha$$

The average of the temporal and spatial factor of the square of the surface velocity, $< |u_w|^2 >$, is given by

$$< |u_w|^2 > = 1/4ab. \int_{-a}^{a} \int_{-b}^{b} u_w^2/2.dx\ dy \ldots\ldots\ldots\ldots (4.4)$$

But $u_w = i\omega W\ \Theta(x)\ \Theta(y)$

$$= i\omega W\ \{\mathrm{Cos}(\gamma_m x/2a) + Rm\ \mathrm{Cosh}(\gamma_m x/2a)\} \times$$

$$\{\mathrm{Cos}(\gamma_n y/2b) + Rn\ \mathrm{Cosh}(\gamma_n y/2b)\}$$

Substituting in equation (4.4) and carrying out the integration

$$\langle |u_w|^2 \rangle = \frac{\omega^2 W^2}{8} \Big[\Big\{ \Big(1 + \frac{\sin\gamma_m}{\gamma_m}\Big) + R_m^2 \Big(1 + \frac{\sinh\gamma_m}{\gamma_m}\Big) +$$

$$\frac{4R_m}{\gamma_m}(\sin\gamma_m/2 \cosh\gamma_m/2 + \cos\gamma_m/2 \sinh\gamma_m/2)\Big\} \times$$

$$\Big\{\Big(1 + \frac{\sin\gamma_n}{\gamma_n}\Big) + R_n^2\Big(1 + \frac{\sinh\gamma_n}{\gamma_n}\Big) +$$

$$\frac{4R_n}{\gamma_n}(\sin\gamma_n/2 \cosh\gamma_n/2 + \cos\gamma_n/2 \sin\gamma_n/2)\Big\}\Big]$$

$$= \frac{\omega^2 W^2}{8} B_1$$

where,

$$B_1 \quad \Big[\Big\{ \Big(1 + \frac{\sin\gamma_m}{\gamma_m}\Big) + R_m^2 \Big(1 + \frac{\sinh\gamma_m}{\gamma_m}\Big) +$$

$$\frac{4R_m}{\gamma_m}(\sin\gamma_m/2 \cosh\gamma_m/2 + \cos\gamma_m/2 \sinh\gamma_m/2)\Big\} \times$$

$$\Big\{\Big(1 + \frac{\sin\gamma_n}{\gamma_n}\Big) + R_n^2\Big(1 + \frac{\sinh\gamma_n}{\gamma_n}\Big) +$$

$$\frac{4R_n}{\gamma_n}(\sin\gamma_n/2 \cosh\gamma_n/2 + \cos\gamma_n/2 \sin\gamma_n/2)\Big\}\Big]$$

Substituting $\langle |u_w|^2 \rangle$ in the expression of radiation efficiency

$$S_{mn} = \frac{256\, k^2 ab}{\pi^2 \gamma_m^4 \gamma_n^4 B_1} \int_0^{\pi/2}\int_0^{\pi/2} A_1 \sin\theta \, d\theta \, d\alpha \quad \ldots\ldots\ldots\ldots (4.5)$$

The plate wave number k_p is now defined as,

$k_p = [(m\pi/2a)^2 + (n\pi/2b)^2]^{0.5}$ and the wave number ratio,

$\Psi = k/k_p$

Thus, $k = \Psi k_p$ or $k^2 = \Psi^2 k_p^2$.

Substituting k in equation (4.5), the expression of the radiation efficiency is obtained as

$$S_{mn} = \frac{256\ ab\ [(m\pi/2a)^2 + (n\pi/2b)^2]}{\pi^2 \gamma_m^4 \gamma_n^4 B_1} \Psi^2 \int_0^{\pi/2}\int_0^{\pi/2} A_1 \ \mathrm{Sin}\theta \ d\theta \ d\alpha$$

$$= \frac{64\ (m^2 R_a + n^2/R_a)}{\gamma_m^4 \gamma_n^4 B_1} \Psi^2 \int_0^{\pi/2}\int_0^{\pi/2} A_1 \ \mathrm{Sin}\theta \ d\theta \ d\alpha \qquad \ldots\ldots\ldots\ldots\ldots \quad (4.6)$$

Making the same substitution, the expressions of s and q become

$$s = (m^2 + n^2/R_a^2)^{0.5} \Psi\pi \ \mathrm{Sin}\theta \ \mathrm{Cos}\alpha$$

$$q = (R_a^2 m^2 + n^2)^{0.5} \Psi\pi \ \mathrm{Sin}\theta \ \mathrm{Sin}\alpha$$

Case II. <u>Odd values of m and n</u>

(a) Power radiation :

For this case, the expression for the farfield pressure, after substitution of the appropriate displacement function, becomes,

$$P = \frac{-k\rho c \omega W}{2\pi R} e^{i(\omega t + kR)} \int_{-a}^{a}\int_{-b}^{b} [\mathrm{Sin}(\gamma_m' x/2a) +$$

$$Rm' \ \mathrm{Sinh}(\gamma_m' x/2a)] e^{-isx/2a} \ dx \ x \ [\mathrm{Sin}\ \gamma_n' y/2b +$$

$$Rn' \ \mathrm{Sinh}(\gamma_n' y/2b)] e^{-iqy/2b} \ dy$$

After proper integration of the above expression of P,

$$P = \frac{-k\rho c\omega W\, e^{i(\omega t + kR)}.16\, ab}{2\pi R\, \gamma_m'^2\, \gamma_n'^2}\left[\frac{1}{(s/\gamma_m')^2 - 1}.\{S\, \mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(s/2) - \right.$$

$$\gamma_m'\mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(s/2)\} + \frac{Rm'}{(s/\gamma_m')^2+1}.\{S\, \mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(s/2) -$$

$$\left.\gamma_m'\mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(s/2)\}\right] \times \left[\frac{q\, \mathrm{Sin}(\gamma_n'/2)\mathrm{Cos}(q/2) - \gamma_n'\mathrm{Cos}(\gamma_n'/2)\mathrm{Sin}(q/2)}{(q/\gamma_n')^2 - 1}\right.$$

$$\left. + Rn'\, \frac{q\, \mathrm{Sinh}(\gamma_n'/2)\mathrm{Cos}(q/2) - \gamma_n'\mathrm{Cosh}(\gamma_n'/2)\mathrm{Sin}(q/2)}{(q/\gamma_n')^2 + 1}\right]$$

Following the same procedure as in the case of even values of m and n, the total average power radiated from one side of clamped plate vibrating with odd values of m and n comes out to be

$$P_w = \frac{8\rho W^2 \lambda_f^4 \pi^6 E^2 g^2\, R_t^4 R_a^2}{9c\, \rho_m^2\, \gamma_m'^4 \gamma_n'^4 (1 - \sigma^2)^2}\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{1}{(s/\gamma_m')^2 - 1}.\{S\, \mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(s/2) - \right.$$

$$\gamma_m'\mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(s/2)\} + \frac{Rm'}{(s/\gamma_m')^2+1}.\{S\, \mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(s/2) -$$

$$\left.\gamma_m'\mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(s/2)\}\right]^2 \times \left[\frac{q\, \mathrm{Sin}(\gamma_n'/2)\mathrm{Cos}(q/2) - \gamma_n'\mathrm{Cos}(\gamma_n'/2)\mathrm{Sin}(q/2)}{(q/\gamma_n')^2 - 1}\right.$$

$$\left. + Rn'\, \frac{q\, \mathrm{Sinh}(\gamma_n'/2)\mathrm{Cos}(q/2) - \gamma_n'\mathrm{Cosh}(\gamma_n'/2)\mathrm{Sin}(q/2)}{(q/\gamma_n')^2 + 1}\right]^2 \mathrm{Sin}\theta\, d\theta\, d\alpha$$

where, s and q are the same as in the case of I.

(b) <u>Radiation efficiency</u> :

The power has to be expressed interms of plate parameters in order to evaluate radiation efficiency. Now,

$$P_w = \frac{128\, k^2 \rho c \omega^2 W^2 a^2 b^2}{\pi^2 \gamma_m'^4 \gamma_n'^4} \int_0^{\pi/2}\int_0^{\pi/2} \Big[\frac{1}{(s/\gamma_m')^2 - 1} .\{S \operatorname{Sin}(\gamma_m'/2)\operatorname{Cos}(s/2) - \gamma_m' \operatorname{Cos}(\gamma_m'/2)\operatorname{Sin}(s/2)\} + \frac{Rm'}{(s/\gamma_m')^2+1} .\{S \operatorname{Sinh}(\gamma_m'/2)\operatorname{Cos}(s/2) - \gamma_m' \operatorname{Cosh}(\gamma_m'/2)\operatorname{Sin}(s/2)\}\Big]^2 \times \Big[\frac{q \operatorname{Sin}(\gamma_n'/2)\operatorname{Cos}(q/2) - \gamma_n' \operatorname{Cos}(\gamma_n'/2)\operatorname{Sin}(q/2)}{(q/\gamma_n')^2 - 1} + Rn' \frac{q \operatorname{Sinh}(\gamma_n'/2)\operatorname{Cos}(q/2) - \gamma_n' \operatorname{Cosh}(\gamma_n'/2)\operatorname{Sin}(q/2)}{(q/\gamma_n')^2 + 1}\Big]^2 \operatorname{Sin}\theta \, d\theta \, d\alpha$$

$$= \frac{128\, k^2 \rho c \omega^2 W^2 a^2 b^2}{\pi^2 \gamma_m'^4 \gamma_n'^4} \int_0^{\pi/2}\int_0^{\pi/2} A_2 \operatorname{Sin}\theta \, d\theta \, d\alpha$$

where, $A_2 =$

$$\Big[\frac{1}{(s/\gamma_m')^2 - 1} .\{S \operatorname{Sin}(\gamma_m'/2)\operatorname{Cos}(s/2) - \gamma_m' \operatorname{Cos}(\gamma_m'/2)\operatorname{Sin}(s/2)\} + \frac{Rm'}{(s/\gamma_m')^2+1} .\{S \operatorname{Sinh}(\gamma_m'/2)\operatorname{Cos}(s/2) - \gamma_m' \operatorname{Cosh}(\gamma_m'/2)\operatorname{Sin}(s/2)\}\Big]^2 \times \Big[\frac{q \operatorname{Sin}(\gamma_n'/2)\operatorname{Cos}(q/2) - \gamma_n' \operatorname{Cos}(\gamma_n'/2)\operatorname{Sin}(q/2)}{(q/\gamma_n')^2 - 1} + Rn' \frac{q \operatorname{Sinh}(\gamma_n'/2)\operatorname{Cos}(q/2) - \gamma_n' \operatorname{Cosh}(\gamma_n'/2)\operatorname{Sin}(q/2)}{(q/\gamma_n')^2 + 1}\Big]^2$$

The average of the temporal and spatial factor of the square of the surface velocity $<|u_w|^2>$ is now obtained as

$$<|u_w|^2> \;=\; (1/4ab)\int_{-a}^{a}\int_{-b}^{b}(u_w^2/2)\,dx\,dy.$$

Where, $\quad u_w \;=\; i\omega\, W\, \Theta(x)\,\Theta(y)$

$$= i\omega\, W\,[\mathrm{Sin}(\gamma_m' x/2a) + R_m'\,\mathrm{Sinh}(\gamma_m' x/2a)] \times [\mathrm{Sin}(\gamma_n' y/2b) + R_n'\,\mathrm{Sinh}(\gamma_n' y/2b)]$$

Substituting u_w in the expression of $<|u_w|^2>$ and integrating, we have,

$$<|u_w|^2> \;=\; \frac{\omega^2 W^2}{8}\Big[\Big(1-\frac{\mathrm{Sin}\gamma_m'}{\gamma_m'}\Big) + R_m'^2\Big(\frac{\mathrm{Sinh}\gamma_m'}{\gamma_m'} - 1\Big) + \frac{4R_m'}{\gamma_m'}\big(\mathrm{Sin}\,\gamma_m'/2\;\mathrm{Cosh}\,\gamma_m'/2 - \mathrm{Cos}\,\gamma_m'/2\;\mathrm{Sinh}\,\gamma_m'/2\big)\Big] \times$$

$$\Big[\Big(1-\frac{\mathrm{Sin}\gamma_n'}{\gamma_n'}\Big) + R_n'^2\Big(\frac{\mathrm{Sinh}\gamma_n'}{\gamma_n'} - 1\Big) + \frac{4R_n'}{\gamma_n'}\big(\mathrm{Sin}\,\gamma_n'/2\;\mathrm{Cosh}\,\gamma_n'/2 - \mathrm{Cos}\,\gamma_n'/2\;\mathrm{Sinh}\,\gamma_n'/2\big)\Big]$$

$$= \frac{\omega^2 W^2}{8}\,.\,B_2$$

where $B_2 = \Big[\Big(1-\dfrac{\mathrm{Sin}\gamma_n'}{\gamma_m'}\Big) + R_m'^2\Big(\dfrac{\mathrm{Sinh}\gamma_m'}{\gamma_m'} - 1\Big) +$

$$\frac{4R_m'}{\gamma_m'}\big(\mathrm{Sin}\,\gamma_m'/2\;\mathrm{Cosh}\,\gamma_m'/2 - \mathrm{Cos}\,\gamma_m'/2\;\mathrm{Sinh}\,\gamma_m'/2\big)\Big] \times$$

$$\Big[\Big(1-\frac{\mathrm{Sin}\gamma_n'}{\gamma_n'}\Big) + R_n'^2\Big(\frac{\mathrm{Sinh}\gamma_n'}{\gamma_n'} - 1\Big) +$$

$$\frac{4R_n'}{\gamma_n'}\big(\mathrm{Sin}\,\gamma_n'/2\;\mathrm{Cosh}\,\gamma_n'/2 - \mathrm{Cos}\,\gamma_n'/2\;\mathrm{Sinh}\,\gamma_n'/2\big)\Big]$$

Substitution of $<|u_w|^2>$ in the expression of radiation efficiency gives

$$S_{mn} = \frac{256\ k^2\ ab}{\pi^2\ \gamma_m'^4\ \gamma_n'^4\ B_2} \int_0^{\pi/2}\int_0^{\pi/2} A_2\ \text{Sin}\theta\ d\theta\ d\alpha$$

Substitution of acoustic wave number k, in terms of plate wave number k_p and wave number ratio Ψ, gives

$$S_{mn} = \frac{256\ ab[(m\pi/2a)^2+(n\pi/2b)^2]_2}{\pi^2\ \gamma_m'^4\ \gamma_n'^4\ B_2}\Psi^2 \int_0^{\pi/2}\int_0^{\pi/2} A_2\ \text{Sin}\theta\ d\theta\ d\alpha$$

$$= \frac{64(m^2\ R_\alpha + n^2/R_\alpha\ )}{\gamma_m'^4\ \gamma_n'^4\ B_2}\Psi^2 \int_0^{\pi/2}\int_0^{\pi/2} A_2\ \text{Sin}\theta\ d\theta\ d\alpha$$

Now after the same substitution of s and q as in the case of radiation efficiency of case I, the radiation efficiency may be evaluated through numerical integration.

Case III : <u>Even values of m and odd values of n</u> :

(a) <u>Power radiation</u> :

Making proper substitution of displacement function for the case of a plate vibrating in even modes in the x-direction and odd modes in the y-direction the pressure distribution is obtained as

$$P = \frac{-k\rho c\,\omega W}{2\pi R}\ e^{i(\omega t+kR)} \int_{-a}^{a}\int_{-b}^{b} [\text{Cos}(\ \gamma_m x/2a) +$$

$$R_m\ \text{Cosh}\ (\gamma_m x/2a)]\ e^{-isx/2a}dx\ x\ [\text{Sin}(\gamma_n' y/2b) +$$

$$R_n'\ \text{Sinh}(\gamma_n' y/2b))]\ e^{-iqy/2b}\ dy$$

After integration, pressure distribution becomes

$$P = \frac{-k\rho c\omega W\, e^{i(\omega t + kR)}\, 16\, abi}{2\pi R\, \gamma_m^2 \gamma_n'^2} \left[\frac{S \cos\gamma_m/2 \sin S/2 - \gamma_m \sin\gamma_m/2 \cos s/2}{(s/\gamma_m)^2 - 1} + \right.$$

$$\left. R_m \frac{S\cosh\gamma_m/2 \sin s/2 + \gamma_m \sinh\gamma_m/2 \cos s/2}{(s/\gamma_m)^2 + 1} \right] x$$

$$\left[\frac{q \sin\gamma_n'/2 \cos q/2 - \gamma_n' \cos\gamma_n'/2 \sin q/2}{(q/\gamma_n')^2 - 1} + \right.$$

$$\left. R_n' \frac{q \sinh\gamma_n'/2 \cos q/2 - \gamma_n' \cosh\gamma_n'/2 \sin q/2}{(q/\gamma_n')^2 + 1} \right]$$

Then, the total average power radiated from one side of the plate is

$$P_w = \frac{8\rho W^2 \lambda_f^4 \pi^6 E^2 g^2}{9c \rho_m^2 \gamma_m^4 \gamma_n'^4 \; (1-\sigma^2)^2} R_a^2 R_f^4 \int_0^{\pi/2}\!\!\int_0^{\pi/2} \left[\frac{S \cos\gamma_m/2 \sin S/2 - \gamma_m \sin\gamma_m/2 \cos s/2}{(s/\gamma_m)^2 - 1} + \right.$$

$$\left. R_m \frac{S\cosh\gamma_m/2 \sin s/2 + \gamma_m \sinh\gamma_m/2 \cos s/2}{(s/\gamma_m)^2 + 1} \right]^2 x$$

$$\left[\frac{q \sin\gamma_n'/2 \cos q/2 - \gamma_n' \cos\gamma_n'/2 \sin q/2}{(q/\gamma_n')^2 - 1} + \right.$$

$$\left. R_n' \frac{q \sinh\gamma_n'/2 \cos q/2 - \gamma_n' \cosh\gamma_n'/2 \sin q/2}{(q/\gamma_n')^2 + 1} \right]^2 \sin\theta \, d\theta d\alpha$$

where s and q are the same as in case I.

(b) <u>Radiation efficiency</u> :

Radiation efficiency of a vibrating plate is given by the equation (3.37).

$$S_{mn} = \frac{P_w}{4\, c\, \rho ab < |u_w|^2 >}$$

where, P_w, the total average power radiated from one side of the plate in terms of plate parameter is

$$P_w = \frac{128\, k^2 \rho c \omega^2 W^2 a^2 b^2}{\pi^2 \gamma_m^4 \gamma_n'^4} \int_0^{\pi/2}\int_0^{\pi/2} \Big[\frac{S \cos\gamma_m/2 \,\sin S/2 - \gamma_m \sin\gamma_m/2 \cos s/2}{(s/\gamma_m)^2 - 1} +$$

$$R_m \frac{S \cosh\gamma_m/2 \,\sin s/2 + \gamma_m \sinh\gamma_m/2 \cos s/2}{(s/\gamma_m)^2 + 1} \Big]^2 x$$

$$\Big[\frac{q \sin\gamma_n'/2 \cos q/2 - \gamma_n' \cos\gamma_n'/2 \sin q/2}{(q/\gamma_n')^2 - 1} +$$

$$R_n' \frac{q \sinh\gamma_n'/2 \cos q/2 - \gamma_n' \cosh\gamma_n'/2 \sin q/2}{(q/\gamma_n')^2 + 1} \Big]^2 \sin\theta \, d\theta d\alpha$$

$$= \frac{128\, k^2 \rho c \omega^2 W^2 a^2 b^2}{\pi^2 \gamma_m^4 \gamma_n'^4} \int_0^{\pi/2}\int_0^{\pi/2} A_3 \sin\theta \, d\theta \, d\alpha$$

where A_3 is given by

$$A_3 = \left[\frac{S \cos \gamma_m/2 \sin S/2 - \gamma_m \sin \gamma_m/2 \cos s/2}{(s/\gamma_m)^2 - 1} + R_m \frac{S \cosh \gamma_m/2 \sin s/2 + \gamma_m \sinh \gamma_m/2 \cos s/2}{(s/\gamma_m)^2 + 1} \right]^2 \times$$

$$\left[\frac{q \sin \gamma_n'/2 \cos q/2 - \gamma_n' \cos \gamma_n'/2 \sin q/2}{(q/\gamma_n')^2 - 1} + R_n' \frac{q \sinh \gamma_n'/2 \cos q/2 - \gamma_n' \cosh \gamma_n'/2 \sin q/2}{(q/\gamma_n')^2 + 1} \right]^2$$

The average of the temporal and spatial factor of the square of the surface velocity $\langle |u_w|^2 \rangle$ is given by

$$\langle |u_w|^2 \rangle = (1/4ab) \int_{-a}^{a}\int_{-b}^{b} (u_w^2/2)\, dx\, dy.$$

where $u_w = i\omega W\, \theta(x)\, \theta(y)$

$$= i\omega W\, [\cos(\gamma_m x/2a) + R_m \cosh(\gamma_m x/2)] \times [\sin \gamma_n' y/2b + R_n' \sinh \gamma_n' y/2b]$$

Substituting u_w in the above equation and carrying out the integration,

$$<|u_w|^2> = \frac{\omega^2 W^2}{8} \Big[\Big(1+\frac{\sin\gamma_m}{\gamma_m}\Big) + R_m^2\Big(1+\frac{\sin\gamma_m}{\gamma_m}\Big) +$$

$$(4R_m/\gamma_m)\ (\sin\gamma_m/2\ \cosh\gamma_m/2 + \cos\gamma_m/2\ \sinh\gamma_m/2)\Big]\ x$$

$$\Big[\Big(1-\frac{\sin\gamma_n'}{\gamma_n'}\Big) + R_n'^2\Big(\frac{\sinh\gamma_n'}{\gamma_n'} - 1\Big) +$$

$$(4R_n'/\gamma_n')\ (\sin\gamma_n'/2\ \cosh\gamma_n'/2 - \cos\gamma_n'/2\ \sinh\gamma_n'/2)\Big]$$

$$= \frac{\omega^2 W^2}{8} B_3$$

where $B_3 = \Big[\Big(1+\frac{\sin\gamma_m}{\gamma_m}\Big) + R_m^2\Big(1+\frac{\sinh\gamma_m}{\gamma_m}\Big) +$

$$(4R_m/\gamma_m)\ (\sin\gamma_m/2\ \cosh\gamma_m/2 + \cos\gamma_m/2\ \sinh\gamma_m/2)\Big]\ x$$

$$\Big[\Big(1-\frac{\sin\gamma_n'}{\gamma_n'}\Big) + R_n'^2\Big(\frac{\sinh\gamma_n'}{\gamma_n'} - 1\Big) +$$

$$(4R_n'/\gamma_n')\ (\sin\gamma_n'/2\ \cosh\gamma_n'/2 - \cos\gamma_n'/2\ \sinh\gamma_n'/2)\Big]$$

Substituting $<|u_w|^2>$ in the expression of radiation efficiency,

$$S_{mn} = \frac{256\,k^2\,ab}{\pi^2 \gamma_m^4 \gamma_n'^4 B_3} \int_0^{\pi/2}\int_0^{\pi/2} A_3 \sin\theta\ d\theta\ d\alpha$$

In the same manner, as in case of I,

$$S_{mn} = \frac{256\ ab[(m\pi/2a)^2 + (n\pi/2b)^2]}{\pi^2 \gamma_m^4 \gamma_n'^4 B_3}.\Psi^2 \int_0^{\pi/2}\int_0^{\pi/2} A_3 \sin\theta\ d\theta\ d\alpha$$

$$= \frac{64(m^2 R_a + n^2/R_a)}{\gamma_m^4 \gamma_n'^4 B_3}\Psi^2 \int_0^{\pi/2}\int_0^{\pi/2} A_3 \sin\theta\ d\theta\ d\alpha$$

Case IV : <u>Odd values of m and even values of n</u> :

(a) <u>Power radiation</u> :

Substitution of proper displacement function in the expression of pressure distribution, gives

$$P = \frac{-k\rho c\omega W}{2\pi R} e^{i(\omega t + kR)} \int_{-a}^{a}\int_{-b}^{b} [\mathrm{Sin}(\gamma_m' x/2a) +$$

$$R_m' \,\mathrm{Sinh}(\gamma_m' x/2a)] e^{-isx/2a}\, dx \times [\{\mathrm{Cos}(\gamma_n y/2b) +$$

$$R_n \,\mathrm{Cosh}\,(\gamma_n y/2b)\}\, e^{-iqy/2b}\, dy\,]$$

After integration, the above expression of P becomes,

$$P = \frac{-k\rho c\omega W\, e^{i(\omega t + kR)}.16\, abi}{2\pi R\, \gamma_m'^2 \gamma_n^2} \left[\frac{1}{(s/\gamma_m')^2 - 1} .\{S\,\mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(s/2) - \right.$$

$$\gamma_m' \mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(s/2)\} + \frac{R_m'}{(s/\gamma_m')^2 + 1} .\{S\,\mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(s/2) -$$

$$\left.\gamma_m' \mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(s/2)\}\right] \times \left[\frac{1}{\{(q/\gamma_n)^2 - 1\}} . \{q\,\mathrm{Cos}(\gamma_n/2)\mathrm{Sin}(q/2) - \right.$$

$$\gamma_n \mathrm{Sin}(\gamma_n/2)\mathrm{cos}(q/2)\} + \frac{R_n}{\{(q/\gamma_n)^2 + 1\}} \{q\,\mathrm{Cosh}(\gamma_n/2)\mathrm{Sin}(q/2) +$$

$$\left.\gamma_n \mathrm{Sinh}(\gamma_n/2)\mathrm{Cos}(q/2)\}\right]$$

As in case I, the total power radiation from one side of the plate is

$$P_w = \frac{8\rho W^2 \lambda_f^4 \pi^6 E^2 g^2 R_t^4 R_a^2}{9c\,\rho_m^2\,\gamma_m'^4 \gamma_n^4 (1-\sigma^2)^2} \int_0^{\pi/2}\int_0^{\pi/2} \Big[\frac{1}{(s/\gamma_m')^2 - 1} .\{S\ \mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(s/2) -$$

$$\gamma_m'\mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(s/2)\} + \frac{Rm'}{(s/\gamma_m')^2+1} .\{S\ \mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(s/2) -$$

$$\gamma_m'\mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(s/2)\}\Big]^2 x\ \Big[\frac{1}{\{(q/\gamma_n)^2 - 1\}} . \{q\ \mathrm{Cos}(\gamma_n/2)\mathrm{Sin}(q/2) -$$

$$\gamma_n \mathrm{Sin}(\gamma_n/2)\mathrm{cos}(q/2)\} + \frac{Rn}{\{(q/\gamma_n)^2 + 1\}} \{q\ \mathrm{Cosh}(\gamma_n/2)\mathrm{Sin}(q/2) +$$

$$\gamma_n \mathrm{Sinh}(\gamma_n/2)\mathrm{Cos}(q/2)\}\Big]^2 \mathrm{Sin}\theta\ d\theta d\alpha$$

(b) **Radiation efficiency :**

From the equation (3.37), radiation efficiency is

$$S_{mn} = \frac{P_w}{4\,c\,\rho\,ab < |u_w|^2 >}$$

where P_w is the same way as in the case I,

$$P_w = \frac{128\ k^2 \rho c \omega^2 W^2 a^2 b^2}{\pi^2 \gamma_m'^4 \gamma_n^4} \int_0^{\pi/2}\int_0^{\pi/2} \Big[\frac{1}{(s/\gamma_m')^2 - 1} .\{S\ \mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(s/2) -$$

$$\gamma_m'\mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(s/2)\} + \frac{Rm'}{(s/\gamma_m')^2+1} .\{S\ \mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(s/2) -$$

$$\gamma_m'\mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(q/2)\}\Big]^2 x\ \Big[\frac{1}{\{(q/\gamma_n)^2 - 1\}} . \{q\ \mathrm{Cos}(\gamma_n/2)\mathrm{Sin}(q/2) -$$

$$\gamma_n \mathrm{Sin}(\gamma_n/2)\mathrm{cos}(q/2)\} + \frac{Rn}{\{(q/\gamma_n)^2 + 1\}} \{q\ \mathrm{Cosh}(\gamma_n/2)\mathrm{Sin}(q/2) +$$

$$\gamma_n\ \mathrm{Sinh}(\gamma_n/2)\mathrm{Cos}(q/2)\}\Big]^2 \mathrm{Sin}\theta\ d\theta d\alpha$$

or $$P_w = \frac{128\, k^2 \rho c\, \omega^2 W^2 a^2 b^2}{\pi^2\, \gamma_m'^4\, \gamma_n^4} \int_0^{\pi/2}\!\!\int_0^{\pi/2} A_4 \, \mathrm{Sin}\theta \, d\theta \, d\alpha$$

where $$A_4 = \Big[\frac{1}{(s/\gamma_m')^2 - 1} . \{S\, \mathrm{Sin}(\gamma_m'/2)\mathrm{Cos}(s/2) -$$

$$\gamma_m' \mathrm{Cos}(\gamma_m'/2)\mathrm{Sin}(s/2)\} + \frac{R_m'}{(s/\gamma_m')^2 + 1} . \{S\, \mathrm{Sinh}(\gamma_m'/2)\mathrm{Cos}(s/2) -$$

$$\gamma_m' \mathrm{Cosh}(\gamma_m'/2)\mathrm{Sin}(s/2)\}\Big]^2 \times \Big[\frac{1}{\{(q/\gamma_n)^2 - 1\}} . \{q\, \mathrm{Cos}(\gamma_n/2)\mathrm{Sin}(q/2) -$$

$$\gamma_n \mathrm{Sin}(\gamma_n/2)\mathrm{cos}(q/2)\} + \frac{R_n}{\{(q/\gamma_n)^2 + 1\}} \{q\, \mathrm{Cosh}(\gamma_n/2)\mathrm{Sin}(q/2) +$$

$$\mathrm{Sinh}(\gamma_n/2)\mathrm{Cos}(q/2)\}\Big]^2$$

Now, in the same way as earlier,

$$\langle |u_w|^2 \rangle = (1/4ab) \int_{-a}^{a}\!\!\int_{-b}^{b} (u_w^2/2)\, dx\, dy.$$

where, $$u_w = i\omega W\, \theta(x)\, \theta(y)$$

$$= i\omega W\, [\mathrm{Sin}(\gamma_m' x/2a) + R_m' \mathrm{Sinh}(\gamma_m' x/2a)] \times [\mathrm{Cos}(\gamma_n y/2b) + R_n \mathrm{Cosh}(\gamma_n y/2b)]$$

With the substitution of u_w and integration,

$$\langle |u_w|^2 \rangle = \frac{\omega^2 W^2}{8} \Big[(1 - \frac{\mathrm{Sin}\gamma_m'}{\gamma_m'}) + R_m'^2 (\frac{\mathrm{Sinh}\gamma_m'}{\gamma_m'} - 1) +$$

$$\frac{4R_m'}{\gamma_m'} (\mathrm{Sin}\, \gamma_m'/2\ \mathrm{Cosh}\, \gamma_m'/2 - \mathrm{Cos}\gamma_m'/2\ \mathrm{Sinh}\, \gamma_m'/2)\Big] \times$$

$$\{(1 + \frac{\mathrm{Sin}\gamma_n}{\gamma_n}) + R_n^2 (1 + \frac{\mathrm{Sinh}\gamma_n}{\gamma_n}) +$$

$$\frac{4R_n}{\gamma_n} (\mathrm{Sin}\gamma_n/2\ \mathrm{Cosh}\gamma_n/2 + \mathrm{Cos}\gamma_n/2\ \mathrm{Sin}\gamma_n/2)\}\Big]$$

$$= \frac{W^2 \omega^2}{8} B_4$$

where $B_4 = [(1 - \frac{\sin \gamma_m'}{\gamma_m'}) + R_m'^2 (\frac{\sinh \gamma_m'}{\gamma_m'} - 1) +$

$\frac{4R_m'}{\gamma_m'} (\sin \gamma_m'/2 \cosh \gamma_m'/2 - \cos \gamma_m'/2 \sinh \gamma_m'/2)] \times$

$\{(1 + \frac{\sin \gamma_n}{\gamma_n}) + Rn^2 (1 + \frac{\sinh \gamma_n}{\gamma_n}) +$

$\frac{4Rn}{\gamma_n} (\sin \gamma_n/2 \cosh \gamma_n/2 + \cos \gamma_n/2 \sin \gamma_n/2)\}]$

Therefore, S_{mn} is obtained as

$$S_{mn} = \frac{256\, ab[(m\pi/2a)^2 + (n\pi/2b)^2]}{\pi^2 \gamma_m'^4 \gamma_n^4 B_4} \Psi^2 \int_0^{\pi/2}\int_0^{\pi/2} A_4 \sin\theta \, d\theta \, d\alpha$$

$$= \frac{64(m^2 R_a + n^2/R_a)}{\gamma_m'^4 \gamma_n^4 B_4} \Psi^2 \int_0^{\pi/2}\int_0^{\pi/2} A_4 \sin\theta \, d\theta \, d\alpha$$

where the values of s and q are the same as in the expression of radiation efficiency in case I.

4.3 <u>One end fixed and other three ends free plate</u>:

Total acoustic power radiated from one side of the plate is required to be calculated for the condition of the plate with one end fixed and other three ends free. The total average power radiation is given by the equation (3.28A) which has to be integrated after substituting the expression of acoustic pressure distribution, P. But the acoustic pressure distribution, given by the equation (3.26), has to be analytically integrated first with proper displacement functions as given by Warburton [47].

The displacement function satisfying the conditions at the edges of the plate in this case are :

$$\theta(x) = \cos(\gamma_m \frac{x+a}{2a}) - \cosh(\gamma_m \frac{x+a}{2a}) - R_m \left[\sin(\gamma_m \frac{x+a}{2a}) - \sinh(\gamma_m \frac{x+a}{2a})\right] \quad \text{for } m = 1,2,3, \ldots\ldots\ldots$$

Where, $R_m = \dfrac{\cos\gamma_m + \cosh\gamma_m}{\sin\gamma_m + \sinh\gamma_m}$ and γ_m are the roots of the equation, $\cos\gamma \cosh\gamma = -1$.

$\theta(y) = \cos\gamma_n y/2b + R_n \cosh\gamma_n y/2b$ for $n = 2,4,6,\ldots\ldots\ldots$

$\theta(y) = \sin\gamma_n' y/2b + R_n' \sinh\gamma_n' y/2b$ for $n = 3,5,7,\ldots\ldots\ldots$

Where $R_n = -\dfrac{\sin\gamma_n/2}{\sinh\gamma_n/2}$ and γ_n are the roots of the equation $\tan\gamma/2 + \tanh\gamma/2 = 0$.

and $R_n' = \dfrac{\sin\gamma_n'/2}{\sinh\gamma_n'/2}$ and γ_n' are the roots of the equation $\tan\gamma/2 - \tanh\gamma/2 = 0$.

Now the different combination of nodal vibration of the plate are treated separately.

Case I : Even values of n and both odd-even values m :

(a) Power radiation :

The farfield acoustic pressure distribution for the vibrating plate with even values of n and both odd-even values m after substituting the appropriate displacement function is obtained as

$$P = \frac{-k\rho c\omega W}{2\pi R} e^{i(\omega t+kR)} \int_{-a}^{a}\int_{-b}^{b} [\cos(\gamma_m \frac{x+a}{2a}) - \text{Cosh}(\gamma_m \frac{x+a}{2a}) -$$

$$R_m \{\text{Sin}(\gamma_m \frac{x+a'}{2a}) - \text{Sinh}(\gamma_m \frac{x+a}{2a})\} e^{-isx/2a}]dx \times [\text{Cos}(\gamma_n y/2b)+$$

$$R_n \text{Cosh}(\gamma_n y/2b)] e^{-iqy/2b} dy.$$

where $s = 2ak \text{ Cos}\alpha \text{ Sin}\Theta$

$q = 2bk \text{ Sin}\alpha \text{ Sin}\Theta$

Integration of the above equation of pressure distribution gives

$$P = \frac{-k\rho c\omega W e^{i(\ t+kR)} 8ab}{2\pi R \gamma_m^2 \gamma_n^2} [\frac{-1}{(s/\gamma_m)^2-1} \{(\gamma_m \text{Sin}\gamma_m \text{Cos } s/2 -$$

$$s \text{ Cos}\gamma_m \text{Sin } s/2 - s \text{ Sin } s/2) + i(s \text{ Cos } s/2 - s \text{ Cos}\gamma_m \text{Cos } s/2 -$$

$$\gamma_m \text{Sin}\gamma_m \text{ Sin } s/2)\} - \{1/(s/\gamma_m)^2 + 1\}\{(\gamma_m \text{Sinh}\gamma_m \text{Cos } s/2 +$$

$$s \text{ Sin } s/2 + s \text{ Cosh}\gamma_m \text{ Sin } s/2) + i(s \text{ Cosh}\gamma_m \text{Cos } s/2 - s \text{ Cos } s/2 -$$

$$\gamma_m \text{Sinh}\gamma_m \text{Sin } s/2)\} - R_m\{\frac{-1}{(s/\gamma_m)^2-1}\}\{\gamma_m \text{Cos } s/2 - \gamma_m \text{Cos}\gamma_m \text{ Cos } s/2 -$$

$$s \text{ Sin}\gamma_m \text{Sin } s/2) + i(\gamma_m \text{Sin } s/2 + \gamma_m \text{Cos}\gamma_m \text{Sin } s/2 -$$

$$s \operatorname{Sin}\gamma_m \operatorname{Cos} s/2)\} + \frac{R_m(-1)}{(s/\gamma_m)^2+1} \{(\gamma_m \operatorname{Cos} s/2 - \gamma_m \operatorname{Cosh}\gamma_m \operatorname{Cos} s/2 -$$

$$s \operatorname{Sinh}\gamma_m \operatorname{Sin} s/2) + i(\gamma_m \operatorname{Sin} s/2 + \gamma_m \operatorname{Cosh}\gamma_m \operatorname{Sin} s/2 - s \operatorname{Sinh}\gamma_m \operatorname{Cos} s/2)\}]x$$

$$[\frac{q \operatorname{Cos}\gamma_n/2 \operatorname{Sin} q/2 - \gamma_n \operatorname{Sin}\gamma_n/2 \operatorname{Cos} q/2}{(q/\gamma_n)^2 - 1} + \frac{R_n}{(q/\gamma_n)^2+1} x$$

$$(q \operatorname{Cosh}\gamma_n/2 \operatorname{Sin} q/2 + \gamma_n \operatorname{Sinh}\gamma_n/2 \operatorname{Cos} q/2)] \quad .... \quad (4.7)$$

The farfield acoustic intensity is given by

$$I = \frac{|P|^2}{\rho c}$$

After substituting P from (4.7),

$$I = \frac{8k^2\rho c\omega^2 W^2 a^2 b^2}{\pi^2 R^2 \gamma_m^4 \gamma_n^4} [\frac{1}{(s/\gamma_m)^2-1} \sqrt{\{(\gamma_m \operatorname{Sin}\gamma_m \operatorname{Cos} s/2 -}$$

$$s \operatorname{Cos}\gamma_m \operatorname{Sin} s/2 - s \operatorname{Sin} s/2)^2 + (s \operatorname{Cos} s/2 - s \operatorname{Cos}\gamma_m \operatorname{Cos} s/2 -$$

$$\gamma_m \operatorname{Sin}\gamma_m \operatorname{Sin} s/2)^2\}} - \{1/(s/\gamma_m)^2 + 1\}\sqrt{\{(\gamma_m \operatorname{Sinh}\gamma_m \operatorname{Cos} s/2 +}$$

$$s \operatorname{Sin} s/2 + s \operatorname{Cosh}\gamma_m \operatorname{Sin} s/2)^2 + (s \operatorname{Cosh}\gamma_m \operatorname{Cos} s/2 - s \operatorname{Cos} s/2 -$$

$$\gamma_m \operatorname{Sinh}\gamma_m \operatorname{Sin} s/2)^2\}} - R_m\{\frac{1}{(s/\gamma_m)^2-1}\}\sqrt{\{(\gamma_m \operatorname{Cos} s/2 - \gamma_m \operatorname{Cos}\gamma_m \operatorname{Cos} s/2 -}$$

$$s \operatorname{Sin}\gamma_m \operatorname{Sin} s/2)^2 + (\gamma_m \operatorname{Sin} s/2 + \gamma_m \operatorname{Cos}\gamma_m \operatorname{Sin} s/2 -$$

$$s \operatorname{Sin}\gamma_m \operatorname{Cos} s/2)^2\}} + \frac{R_m}{(s/\gamma_m)^2+1}\sqrt{\{(\gamma_m \operatorname{Cos} s/2 - \gamma_m \operatorname{Cosh}\gamma_m \operatorname{Cos} s/2 -}$$

$$s \text{ Sinh}\gamma_m \text{Sin } s/2)^2 + (\gamma_m \text{Sin } s/2 + \gamma_m \text{Cosh}\gamma_m \text{Sin } s/2 - s \text{ Sinh}\gamma_m \text{Cos } s/2)^2\}]^2 x$$

$$\left[\frac{q \text{ Cos}\gamma_n/2 \text{ Sin } q/2 - \gamma_n \text{Sin } \gamma_n/2 \text{ Cos } q/2}{(q/\gamma_n)^2 - 1} + \frac{R_n}{(q/\gamma_n)^2 + 1} x \right.$$

$$\left. (q \text{ Cosh}\gamma_n/2 \text{ Sin } q/2 + \gamma_n \text{Sinh}\gamma_n/2 \text{ Cos } q/2)\right]^2$$

Total average power radiated from one side of the baffled plate is now given by

$$P_w = \int_0^{2\pi}\int_0^{\pi/2} \frac{|P|^2}{\rho c} R^2 \text{ Sin}\theta \, d\theta \, d\alpha$$

$$= \int_0^{2\pi}\int_0^{\pi/2} IR^2 \text{ Sin}\theta \, d\theta \, d\alpha$$

Therefore, acoustic power radiation for both odd-even values of m and even values of n becomes,

$$P_w = \frac{2\rho W^2 \lambda_f^4 \pi^6 E^2 g^2}{9 c \rho_m^2 \gamma_m^4 \gamma_n^4 (1-\sigma^2)^2} R_a^2 \cdot R_t^4 \int_0^{\pi/2}\int_0^{\pi/2} \left[\frac{1}{(s/\gamma_m)^2 - 1} \sqrt{(\gamma_m^2 \text{Sin}^2\gamma_m + s^2 \text{ Cos}^2\gamma_m + s^2} \right.$$

$$- 2s^2 \text{ Cos}\gamma_m \text{Cos } s - 2\gamma_m s \text{ Sin}\gamma_m \text{Sin } s) - \frac{1}{(s/\gamma_m)^2 + 1} \sqrt{(\gamma_m^2 \text{ Sinh}^2\gamma_m + s^2 +}$$

$$s^2 \text{ Cosh}^2\gamma_m - 2s^2 \text{Cosh}\gamma_m \text{Cos } s + 2\gamma_m s \text{ Sinh}\gamma_m \text{Sin } s) - \frac{R_m}{(s/\gamma_m)^2 - 1} \sqrt{(\gamma_m^2 +}$$

$$\gamma_m^2 \text{Cos}^2\gamma_m + s^2 \text{ Sin}^2\gamma_m - 2\gamma_m^2 \text{ Cos}\gamma_m \text{Cos } s - 2\gamma_m s \text{ Sin}\gamma_m \text{Sin } s) +$$

$$\frac{R_m}{(s/\gamma_m)^2 + 1} \sqrt{(\gamma_m^2 + \gamma_m^2 \text{Cosh}^2\gamma_m + s^2 \text{ Sinh}^2\gamma_m - 2\gamma_m^2 \text{Cosh}\gamma_m \text{Cos } s -}$$

$$\left. 2\gamma_m s \text{ Sinh}\gamma_m \text{Sin } s)\right]^2 x$$

$$\left[\frac{q \text{ Cos}\gamma_n/2 \text{ Sin } q/2 - \gamma_n \text{Sin } \gamma_n/2 \text{ Cos } q/2}{(q/\gamma_n)^2 - 1} + \frac{R_n}{(q/\gamma_n)^2 + 1} x \right.$$

$$\left. (q \text{ Cosh}\gamma_n/2 \text{ Sin } q/2 + \gamma_n \text{Sinh}\gamma_n/2 \text{ Cos } q/2)\right]^2 \text{Sin}\theta \, d\theta \, d\alpha$$

Here, $$\lambda_f^2 = G_X^4 + (a/b)^4 G_Y^4 + 2(a/b)^2 \left[\sigma H_X H_Y + (1-\sigma) J_X J_Y \right]$$

$$\omega^2 = \frac{\lambda_f^2 \pi^4 E h^2 g}{\rho_m a^4 12(1-\sigma^2)}$$

$$s = \frac{R_t \lambda_f \pi^2 \sqrt{Eg}}{c\sqrt{3 \rho_m (1-\sigma^2)}} \quad \text{Cos}\alpha \ \text{Sin}\theta$$

$$q = \frac{R_\alpha R_t \lambda_f \pi^2 \sqrt{Eg}}{c\sqrt{3 \rho_m (1-\sigma^2)}} \quad \text{Sin}\alpha \ \text{Sin}\theta$$

Case II : Even-odd values of m and odd values n :

(a) Power radiation :

For this case, when the displacement functions are substituted into the expression for the farfield pressure distribution, the expression takes the form

$$P = \frac{-k\rho c \omega W}{2\pi R} e^{i(\omega t + kR)} \int_{-a}^{a}\int_{-b}^{b} \left[\cos\left(\gamma_m \frac{x+a}{2a}\right) - \text{Cosh}\left(\gamma_m \frac{x+a}{2a}\right) - R_m \left\{ \text{Sin}\left(\gamma_m \frac{x+a}{2a}\right) - \text{Sinh}\left(\gamma_m \frac{x+a}{2a}\right) \right\} e^{-isx/2a} \right] dx \times \left[\text{Sin}(\gamma_n' y/2b) + R_n' \, \text{Sinh}(\gamma_n' y/2b) \right] e^{-iqy/2b} \, dy,$$

where s and q are the same as in case I for even values of m and n. After integration it becomes

$$P = \frac{-k\rho c\omega W\, e^{i(\omega t+kR)}\; 8\,abi}{2\pi R\, \gamma_m^2\, \gamma_n'^2} \Big[\frac{-1}{(s/\gamma_m)^2-1}\{(\gamma_m \mathrm{Sin}\,\gamma_m \mathrm{Cos}\, s/2 -$$

$$s\,\mathrm{Cos}\,\gamma_m\, \mathrm{Sin}\, s/2 - s\,\mathrm{Sin}\, s/2) + i(s\,\mathrm{Cos}\, s/2 - s\,\mathrm{Cos}\,\gamma_m \mathrm{Cos}\, s/2 -$$

$$\gamma_m \mathrm{Sin}\,\gamma_m\; \mathrm{Sin}\, s/2\,)\} - \{1/(s/\gamma_m)^2+1\}\{(\gamma_m \mathrm{Sinh}\,\gamma_m \mathrm{Cos}\, s/2 +$$

$$s\,\mathrm{Sin}\, s/2 + s\,\mathrm{Cosh}\,\gamma_m\, \mathrm{Sin}\, s/2) + i\,(s\,\mathrm{Cosh}\,\gamma_m \mathrm{Cos}\, s/2 - s\,\mathrm{Cos}\, s/2 -$$

$$\gamma_m \mathrm{Sinh}\,\gamma_m \mathrm{Sin}\, s/2)\} - R_m'\{\frac{-1}{(s/\gamma_m)^2-1}\}\{\gamma_m \mathrm{Cos}\; s/2 - \gamma_m \mathrm{Cos}\,\gamma_m\, \mathrm{Cos}\; s/2 -$$

$$s\;\; \mathrm{Sin}\,\gamma_m\, \mathrm{Sin}\, s/2) + i\,(\gamma_m \mathrm{Sin}\, s/2 + \gamma_m \mathrm{Cos}\,\gamma_m \mathrm{Sin}\, s/2 -$$

$$s\,\mathrm{Sin}\,\gamma_m \mathrm{Cos}\, s/2)\} + \frac{R_m'(-1)}{(s/\gamma_m)^2+1}\{(\gamma_m \mathrm{Cos}\; s/2 - \gamma_m \mathrm{Cosh}\,\gamma_m \mathrm{Cos}\; s/2 -$$

$$s\,\mathrm{Sinh}\,\gamma_m \mathrm{Sin}\; s/2) + i(\gamma_m \mathrm{Sin}\; s/2 + \gamma_m \mathrm{Cosh}\,\gamma_m \mathrm{Sin}\; s/2 \;- s\,\mathrm{Sinh}\,\gamma_m \mathrm{Cos}\; s/2)\}\Big]\,x$$

$$\Big[\frac{q\,\mathrm{Sin}\,\gamma_n'/2\;\mathrm{Cos}\; q/2 - \gamma_n'\mathrm{Cos}\;\gamma_n'/2\;\mathrm{Sin}\; q/2}{(q/\gamma_n')^2-1} + \frac{R_n'}{(q/\gamma_n')^2+1}\,x$$

$$(q\,\mathrm{Sinh}\,\gamma_n'/2\;\mathrm{Cos}\; q/2 - \gamma_n'\mathrm{Cosh}\;\gamma_n'/2\;\mathrm{Sin}\; q/2)\Big]$$

Following the same procedure as in case I, the final form of acoustic power radiation becomes

$$P_w = \frac{2\rho W^2 \lambda_f^4 \pi^6\; E^2 g^2}{9\; c\, \rho_m^2 \gamma_m^4 \gamma_n'^4 (1-\sigma^2)^2} R_a^2\; R_t^4 \int_0^{\pi/2}\!\!\int_0^{\pi/2} \Big[\frac{1}{(s/\gamma_m)^2-1}\sqrt{(\gamma_m^2 \mathrm{Sin}^2\gamma_m + s^2\,\mathrm{Cos}^2\gamma_m + s^2}$$

$$- 2s^2\;\mathrm{Cos}\,\gamma_m \mathrm{Cos}\; s - 2\gamma_m s\;\mathrm{Sin}\,\gamma_m \mathrm{Sin}\; s) - \frac{1}{(s/\gamma_m)^2+1}\sqrt{(\gamma_m^2 \mathrm{Sinh}^2\gamma_m + s^2 +}$$

$$s^2\;\mathrm{Cosh}^2\gamma_m - 2s^2\,\mathrm{Cosh}\,\gamma_m \mathrm{Cos}\; s + 2\gamma_m s\;\mathrm{Sinh}\,\gamma_m \mathrm{Sin}\; s) - \frac{R_m'}{(s/\gamma_m)^2-1}\sqrt{(\gamma_m^2 +}$$

$$\gamma_m^2 Cos^2\gamma_m + s^2 Sin^2\gamma_m - 2\gamma_m^2 Cos\,\gamma_m Cos\, s - 2\gamma_m s\, Sin\gamma_m Sin\, s) +$$

$$\frac{R_m'}{(s/\gamma_m)^2+1}\sqrt{}(\gamma_m^2 + \gamma_m^2 Cosh^2\gamma_m + s^2\, Sinh^2\gamma_m - 2\gamma_m^2 Cosh\,\gamma_m Cos\, s -$$

$$2\gamma_m s\, Sinh\gamma_m Sin\, s)]^2 \; x$$

$$[\frac{q\, Sin\gamma_n'/2\, Cos\, q/2 - \gamma_n Cos\,\gamma_n'/2\, Sin\, q/2}{(q/\gamma_n')^2 - 1} + \frac{R_n'}{(q/\gamma_n')^2 + 1} \; x$$

$$(q\, Sinh\gamma_n'/2\, Cos\, q/2 - \gamma_n' Cosh\gamma_n'/2\, Sin\, q/2)]^2 Sin\theta\, d\theta\, d\alpha$$

Above equation gives the total average power radiated from one side of the baffled plate with one edge fixed and the other three edge free, vibrating in its (m,n)th mode with even-odd values of m and odd values n.

4.4 Numerical Solution :

The analytical solutions of the equations for calculating the total average acoustic power radiation could not be obtained Numerical method of solution has thus been developed to solve equation (3.26) with the help of a computer. The equations, $\tan(\gamma/2) \pm \tanh(\gamma/2) = 0$ and $Cos(\gamma)Cosh(\gamma) + 1 = 0$, are solved by the method of bisection and the integrations of equation (3.26) are performed by applying Simpson Rule. The result of integration are presented in tabular as well as in graphical forms.

The method of bisection :

The method of bisection is very efficient for finding the roots of continuous nonlinear as well as complicated linear equations. In this method, the equation to be solved is expressed in the form, $f(x) = 0$. Then an approximate root is determined either by observation or by graphical method. The values of the function $f(x)$ are then determined for successive regular intervals. If x_o be the approximate value of a root of the equation $f(x) = 0$, then, $f(x_o)$, $f(x_o + h)$, $f(x_o + 2h)$, $\cdots$, are determined, where h is the successive increment of x. If the product of any two consecutive values of the function becomes negative, it can be concluded that at least one root of the equation lies in that interval. Then the average (arithmatic mean) of these two successive values of x gives a better approximation of the root of the euqation. The accuracy of the result can be increased to any degree by taking smaller value of h and repeating the process stated above.

The Simpson Formula :

The trapezoidal rule was based on passing straight lines through pairs of consecutive points on the graph of $y = f(x)$. Since the equation of a straight line is of the form, $y = mx+c$, which is the first degree or linear in x, this is equivalent to fitting each pair of points with a first degree polynomial. In

Simpson's rule, a parabola or second degree polynomial is passed through each set of three consecutive points. The formula is obtained by summing the areas under the parabolic arcs. If the range (b-a) of the integration, $I = \int_a^b f(x)dx$, is divided into n even equal divisions, so that $h = (b-a)/n$, then, by Simpson's formula, the result of integration is given by

$$I = (h/3) [f(a) + 4 f(a+h) + 2 f(a+2h) + \dots\dots\dots\dots + 4f\{a + (n-1)h\} + f(b)]$$

In the present solution, the range, 0 to $\pi /2$, was divided into 34 equal intervals for applying the Simpson formula. The number 34 was taken because of the fact that the accuracy of the result with further increase in the number of divisions increases only slightly, but the computer time required increases proportionately. If highly accurate result is desired, the number of divisions may be increased or the method of Romberg integration can be used for improving the accuracy of the results.

CHAPTER - 5

Reliability and Limitations of the Method

5.1 Reliability of the method of analysis :

When a new technique is employed, it should be verified that the method is reliable and efficient. This is achieved usually by comparing the results of the new mathod with the results that are already available in the literature. This would also help to ascertain that no error due to logic is committed in formulating the problem and no mistake is made in the computer programming. Keeping all these in mind, a number of standard problems are solved with the present method of solution and the results are compared with those of others,obtained analytically or by some other technique. On the basis of this comparison, the reliability and efficiency of the present method are determined.

Wallace worked on the vibration characteristics of a simply supported rectangular panel of uniform thickness [46]. In his work, the author used the beam functions developed by Warburton [47], for defining the deformation mode of rectangular panels. The radiation resistance and radiation efficiency corresponding to the natural modes of a finite rectangular panel were theoretically determined from the total energy radiated to the farfield. The panel was assumed to be simply supported in an infinite baffle. Asymptotic solutions for low frequency region were derived, and curves covering the entire frequency range for various mode shapes and aspect ratios were obtained through numerical integration. Wallace [45] in his study, found the radiation resistance and radiation efficiency of a baffled beam from the total power radiated into the farfield, vibrating in simple harmonic motion in one of its natural bending modes. The beam was supported in an infinite baffle with both hinged and clamped supports. Asymptotic solution were derived for frequencies well below the critical frequency. In addition, he found the radiation efficiency by numerical integration of the farfield acoustic intensity produced by the vibrating beam, covering the entire frequency range for the first ten modes of vibration of the beam. As the nature of radiation efficiency is of similar and independent of end-fixity, comparison of the result of the present analysis with that of Wallace provides a reasonable opportunity to prove its soundness and credibility.

When compared, it is observed that the graphs of Wallace [46], presented in Fig.19, are identical to the radiation efficiency graphs, presented in Fig.14 to 17, obtained in the present study. Ahmed [1] also obtained the radiation efficiency of a simply supported rectangular flat plate for various combinations of mode orders (Fig.18). Although the graphs of radiation efficiency of a plate at different combinition of mode orders and of different plates (different end-fixity) at the same mode order differ from one another at the lower range of mode orders, they reach the same maximum value and the same asymptotic value at increasing wave number ratio, ψ . This is verified here by comparing the present results (Fig.14 to 17) with those of

Ahmed (Fig.18) and of Wallace (Fig.19). In addition, the power readiations from different plates of the present analysis are compared with that obtained by Ahmed [1] for a simply support plate over the identical range of mode orders. As expected, the graphs differ from another, but they maintain similar nature. The above verification proves that the methods of solution employed here is accurate and no error is committed either in formulating the problem or in the computer programming.

5.2 <u>Validity of beam functions</u> :

Warburton [47] developed a number of beam functions to satisfy the conditions at the end of a vibrating beam. The function were designed to represent accurately the wave form of a beam vibrating in its natural modes. Wallace [45] used Warburton's beam functions in the case of beam with hinged and clamped ends. The results of the investigations by Wallace proved to be satisfactory as the expression for the radiation resistance asymptotically approached the exact solution as frequency decreased. Again Wallace [46] studied the vibrating plate with simply supported end conditions using Warburton's beam function. The expression for the radiation resistance was asymptotic to the exact solution at low frequency.

In the present study the beam functions of Warburton representing deformation modes under various end-fixity have been used to represent the motion of points on the surfaces of vibrating plates. The functions are combined to meet the requirements of the different boundary conditions of vibrating plates like two opposite ends fixed while the other two ends free and one end fixed while the other three ends free. The exact representations of these beam functions as used in the present analysis are shown in figures 2,3,4 and 5.

The above discussions prove that the beam functions developed by Warburton are sufficiently valid and that those functions can be applied to the vibrating rectangular plate with different pure or mixed boundary conditions.

5.3 <u>Limitations of the method</u>:

The end results of the solution of any complex engineering problem are always obtained under certain simplifying assumptions and approximations.The limitations thus impossed on the results must be stated clearly and should be justified.

The present analysis of the acoustics of vibrating rectangular plates with mixed boundary conditions have the following limitations :

1. The displacement functions used in defining the deformation modes of the vibrating plates satisfy the governing equation only approximately [47]. Thus, the actual deformation modes of the vibrating plates may differ from what have been used in the analysis.

2. The method of finding the fundamental frequency of the vibrating plates includes certain approximations [47] and thus the present analysis is also limited by these approximations.

3. The governing equation used in finding the fundamental frequency of the plates are based on certain assumptions as stated on page 15 and hence this analysis is limited by those assumptions.

4. The expressions for acoustic power radiation used in this analysis include certain assumption as stated in Ref.[40,41] and hence this analysis is limited by those assumptions.

But the nature of approximations and assumptions included in this analysis had been tested in different contexts by different authors and was found to be of not much significance. It can thus be concluded that the end results of this analysis would be good enough for engineering uses.

CHAPTER - 6

Results and Discussion :

6.1 Results and Discussion:

The appropriate beam functions satisfying the condition of the edges of the vibrating plate to be solved has to be combined. In this analysis, plate with two mixed boundary conditions are considered. These are:(1) two opposite ends fixed while the other two ends free,(2) one end fixed while the other three ends free. The beam functions in fact satisfy the boundary conditions for plates with fixed or freely- supported edges, but are only approximate for free edges. The input variables for any rectangular flat plate of uniform thickness are aspect ratio,(b/a), and the thickness ratio, (h/a). The results are found for three values of the aspect ratio and three values of the thickness ratio in each case of the boundary conditions. The results are presented in both tabular and graphical forms. Three values of each of the two plate parameters considered are 0.5, 1.0 and 2.0 for aspect ratio and 0.001,0.002,0.004 for thickness ratio.The values of other parameters as required in numerical evaluations are : density of the surrounding medium, ρ= 1.21 kg/m^3, velocity of sound in surrounding medium, c = 341 m/sec, density of the plate material, ρ_m = 7700.0 kg/m^3, modulus of elasticity of the plate material, E=206 x 10^9 N/m^2,amplitude of vibration W = .0001m and Poission's ratio,σ= .4

In the following section, the results are discussed for each case separately.

(i) Plates with two opposite ends fixed and the other two ends free :

For these edge conditions of vibrating rectangular flat plates the values of acoustic power radiation are obtained by changing the plate parameters and mode shapes both in x and y directions. Figures 6 to 11 show the total average acoustic power radiated from one side of the baffled plate, plotted against the number of nodal lines in the y-direction. In these figures, power radiations at different mode orders and for different thickness ratio and aspect ratios are compared.

In figures 6 and 7, the values of the aspect ratio and thickness ratio are kept constant and the variation of radiation of power are plotted against the variation of nodal lines in the x-direction. In order to offer a plausible explanation of the nature of curves in Figs. 6 & 7, it is necessary to refer to Fig. 12 & 13. In Figs. 12 & 13, it is seen that, with the increase of n, frequency factor λ_f increases. This is also true for increasing m. But from the definition of frequency factor λ_f by Warburton [47], it is observed that the frequency of vibration is directly proportional to the frequency factor λ_f . So it can be concluded that the nature of variation of frequency of the plates will be the same as the frequency factor because they have the same trend. Hence it can be concluded that, with the increase of the values of m and n, the frequency of vibration increases.

In figure 6, it is seen that the acoustic power radiation increases somewhat regularly at the lower range of n and fluctuates at the higher values of n. This is more obivious from figure 7. These can be explained by the fact that the power radiation should increase with the increase of the frequency of vibration which, in its turn, increase with the increase of the value of m and n. But the rate of increase of frequecny is higher for the lower values of n and rises slightly over the higher values of n. Therefore, power radiation increases sharply at the lower values of n and only moderately at the higher values. At the high range of the values of mode orders, the values of m and n have less impact over the acoustic power increase. The waviness of the graph after some value of n are due to the interference of the waves from x and y directions. At the high mode orders, the wave form from two directions interfere frequently. This interference makes the acoustic power to fluctuate. When two crests from two directions superimpose,the total average acoustic power radiation increases. On the otherhand, when the crest from one direction superimposes on the trough from another, the total average acoustic power radiation decreases as depicted by the roots of the graphs. The frequency of waviness depends upon the interference of the waves. This frequency of waviness has a lower value at the low range of mode orders and increase with the rise of mode orders. This is why the waviness of the graph exist at the higher values of m and n. It is also noted that the amplitude of waviness decreases asymptotically to zero as mode orders increase indefinitely. The variation of average power radiation can be explained with the help of acoustic pressure in surrounding medium due to the vibration of the plate. There are three major factors affecting the variation of acoustic power. These are frequency of vibration, absolute acoustic pressure and effective radiating surface of the plate. Due to the vibration of the plate, wave forms are generated in the surrounding medium, resulting in its compression,i.e. positive pressure and rarefaction,i.e. negative pressure. At very high mode orders, the plate wave numbers become very high and therefore, alternate crests and roots of the plate wave come closer to one another. Eventually, compressions, made by the crests and rarefactions made by the roots come also closer. Hence rarefactions and compressions are neutrallized. The effective pressure variation decreases. Thus, the intensity of absolute pressure variation is high at the lower mode orders and low at the higher mode order. It is already known that the frequency of vibration increases significantly with the increase of mode orders at lower range, but for higher range, rate of increase of frequency of vibration falls. Again, effective radiating surfaces of the plate are larger in the low range of mode shape because of less points of zero displacement in the plate. With the increase of mode orders, this points of zero displacement rises so that effective radiating surfaces decreases. Hence in the low range of mode orders, power radiation sharply rises due to the positive effects of the frequency of vibration, effective pressure variation and effective radiating surface of the plate. But at the higher mode orders, frequency of vibration of the plate produces insignificant positive effect on power variation. In this range effective surface of the plate implies negative effect on power variation. And effective acoustic pressure in fact has no role on power change at this high values of m and n. Therefore, the absolute value of average power radiation is more or less constant. Though the power radiation displays a fluctuating characteristic at high modes, the radiation efficiency converges to unity and does not show any change with further increase in the mode number as in the figures 14,15,16 and 17. This is because of the fact that, radiation efficiency presents the power radiated from a plate due to its vibration at certain mode orders in comparison to the power radiated by the same plate vibrating as a solid without forming waves, with a velocity equal to the root mean square value of the surface velocity distribution of the plate under consideration. At high mode orders, the inteferences of the compressions and rarefaction produced by alternate crests and roots of the plate waves tends to reduce the rate of increase in the total acoustic power radiation. Above the critical frequency of vibration of the plate, the acoustic radiation from the plate becomes stable and does not increase with increasing mode orders. This stable acoustic radiation is equivalent to the acoustic radiation from a plate vibrating as a solid body without forming waves, with a velocity equal to the root mean square value of the surface velocity distribution of the plate. The effect of this stable magnitude of the acoustic radiation from a plate converges the radiation efficiency to unity above the critical frequency the frequency at whcih the wave length of the standing wave of the plate is equal to the wave length of the radiated acoustic wave of the plate.

The radiation efficiency of the vibrating plate with two opposing ends fixed and other two ends free increases sharply with the increase of wave number ratio. When the wave number ratio is unity, radiation efficiency is the maximum in magnitude which is about 1.2. In the papers [1,46], maximum value of radiation efficiency is nearly 1.2 when the wave number ratio reaches unity. With further increase of wave number ratio, radiation efficency converges to unity and remains constant. It is also true in [1,46].

In figures 8 and 9, it is shown that the variations of average acoustic power radiation depends substantially on the variation of thickness ratio. Naturally, more power will be needed to excite a plate if its thickness is increased. Therefore, a plate of higher thickness radiates larger acoustic powers. This is witnessed in the figures 8 & 9. In figure 8, the value of nodal line in x direction is low and it is high in the fig. 9.

Figure 10 and 11 show the effect of aspect ratio over the power radiation. Figure 11 shows the variation of average power with aspect ratio at a lower value of m and fig. 10 for higher value of m. From both these figures it is seen that the total acoustic power radiation increases with the increase of aspect ratio. It can be explained with the help of figures 12 and 13. These figures present the variation of dimensionless frequency factor λ_f with the variation of aspect ratio for different values of n. There are two sets of graph for two values of m of 6 and 30 (Fig. 12). It shows that the dimensionless frequency factor λ_f increases with the increase of aspect ratio. As dimensionless frequency factor increases, frequency of vibration also increases, which in tern increase the total acoustic power. Therefore, average acoustic power radiation increases as the aspect ratio is raised. But at the lower mode order this rise is steeper than that at the higher mode orders. From figures 12 and 13, it is seen that at high mode orders the variation of dimensionless frequency factor is almost insignificant.

(ii) <u>One end fixed and other three ends free plate</u> :

Figures 18 and 19 show the variation of total average acoustic power radiation from one side of this plate. The variation of power is plotted against the number of nodal lines in y-driection at a particular value of nodal line in x-direction. In figure 18, acoustic power radiation is observed to increase sharply with the increase of the values of n for the lower values of m. But in figure 19, with the increase in the values of n for higher values of m, power radiation increases slowly with moderate fluctuation. The power radiation due to the vibration of the plate is largely dependent upon the frequency of vibration and the effective radiating surface.From figures 24 and 25 it is evident that dimensionless frequency factor increases with the increase of mode orders. At the lower range of the mode orders, increasing tendency is higher than that at the higher mode orders. Again the variation of frequency of vibration follows the same trend as the dimensionless frequency factor. Therefore, with the increase of dimensionless frequency factor, frequency of vibration increases in the same manner as there exist a linear relation between frequency of vibration of the plate and dimensionless frequency factor. But acoustic power radiation increases if frequency of vibration increases. Hence, from the figures 24 and 25 it may be concluded that the radiation of power should increase with increasing n at any particular value of m. Now effective radiating surface may be defined as the surface of the plate from which acoustic radiation takes place. With the increase of mode orders effective radiating surface decreases but frequency of vibration increases. That is why for lower mode order both frequency of vibration of the plate and effective radiating surface plays a positive effect on acoustic power variation. At the higher mode orders frequency of vibration increases at a decreasing rate with the increase of mode orders. On the other hand effective radiating surface decrease significantly. Hence acoustic power will be in a stable state with some waviness of decreasing magnitude. With further increase in mode orders, waviness will be asymptotically zero. For the higher value of the mode orders, waviness of acoustic power radiated is due to the interference of the standing wave coming from x-and y-directions. If two crests coming from two directions superimpose, net result will produce a sharp crest. On the other hand, if two crests oppose one another, a root is produced. The frequency of interference increases with the increase of mode orders. In the higher mode orders, neighbouring compression and rarefaction made by the crest and root of standing waves in the plate neutralize each other. Waviness then approaches asymptotically to zero. It should also be noted that acoustic power radiation in this case is much greater than the previous case. There is also a significant rise in the values of total acoustic power between the consecutive values of nodal lines in x-direction.

Figures 20 and 21 give the nature of variation of power radiation with the variation of thickness ratio. Fig. 20 is for the lower value of m and figure 21 for the higher value of m. It is evident that with the increase of thickness, total acoustic power radiation also increases. These two figures also give the same information. More power is thus needed to excite a plate with higher thickness as the plate radiates more energy.

Figures 22 and 23 shows the effect of aspect ratio on the variation of total acoustic power radiation. Fig.22 presents the power variation with aspect ratio for low value of m and fig. 23 for higher value of m. From both the figures, it is seen that total acoustic power radiation increases with the increase of aspect ratio. This may be explained by referring to figs. 24 and 25. In figures 24 and 25, dimensionless frequency factor is plotted taking aspect ratio as a parameter. It is clear that as aspect ratio increases, dimensionless frequency factor λ_f also increases. Frequency of vibration increases with the increases of dimensionless frequency factor as they are linearly related. But the frequency of vibrations have significant positive effect on acoustic power. Then it may be concluded that if dimensionless frequency factor increases, power radiation also increases. Hence increasing aspect ratio is responsible for increase in the total acoustic power radiation from a vibrating plate.

6.2 Comparison of power radiation from plates of different boundary conditions :

In the present analysis, the power radiation from a rectangular plate with the following two boundry conditions are studied:(i) two opposing end fixed and other two ends free (ii) one end fixed and other three ends free. It is observed that total acoustic power radiation from a plate with one end fixed and other ends free is higher than that of the case with two opposing ends fixed and other two ends free. This is due to the following reasons :

a. The absolute value of dimensionless frequency factor for a particular mode shape is higher in the case of a plate with one end fixed and other three ends free in comprison to the plate with two opposing ends fixed and other two ends free.

b. Effective radiating surface plays an important role in the power radiation. Power radiation will be higher for larger effective radiating surface. Plate with one end fixed and other three ends free boundary has more radiating surface due to less point of zero displacement in comparison to a plate with two opposing end fixed and other two ends free boundary.

Therefore, plate with one end fixed and other three ends free boundary radiates more power during vibration than the other case.

CHAPTER-7

Conclusions and Recommendations :

7.1 Conclusion :

The following conclusions are made from the investigation of the results and graphs for different boundary conditions and for different values various parameters.

i. The average acoustic power radiated from one side of a baffled plate vibrating in its natural mode increases with the rise of mode numbers, thickness ratio and also aspect ratio. But the mode of increasing tendency of acoustic power radiation is high for increasing values of mode orders in the low range. At the high range of mode orders, the acoustic power radiation is infact in a stable state with the increase of mode number but waviness exists with a tendency of decreasing magnitude.

ii. The average power radiated from a baffled plate increases with the increase of thickness ratio and aspect ratio.

iii. The radiation effiicency of two opposing ends fixed and the other two ends free plate sharply increases with the increase of wave number ratio ψ upto the value of unity. With further increases in values of wave number ratio, radiation efficiency converges to unity and remains constant.

iv. The total average acoustic power radiation from a baffled plate with one end fixed and the other three ends free is high in comparison to the power radiated from the plate with two opposite ends fixed and other two ends free.

7.2 Recommendation :

From the knowledge and experience of the present study, the following fields on plate vibration and acoustics are recommended as the scope of future research.

i. Accuracy in this study may be enhanced by adopting more appropriate displacement functions to define correctly the motion of the surface of the vibrating plate.

ii. Plates with variable thickness and other irregularities may also be studied.

iii. Displacement functions may be evolved for the study of the effect of holes and cracks in vibrating plates.

iv. The effects of various other mixed boundary conditions may be studied by using Warburton's displacement functions for dealing with practical problems.

v. Plates, reinforced with beams, may be studied and the effects of reinforcement can be investigated.

vi. Methods should be evolved to investigate these problems experimentally.

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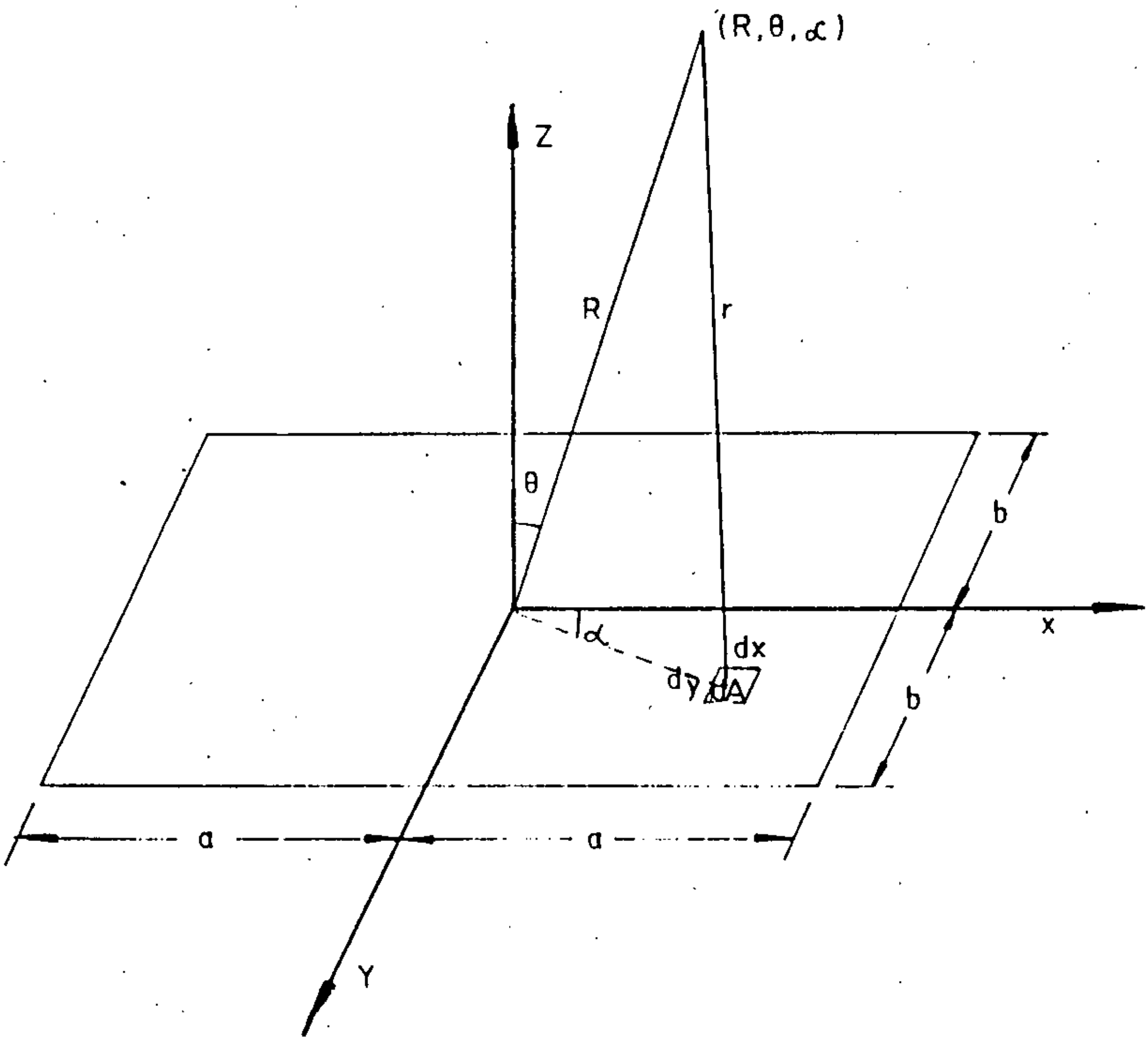

Fig. 1 : Rectangular flat plate with coordinate systems.

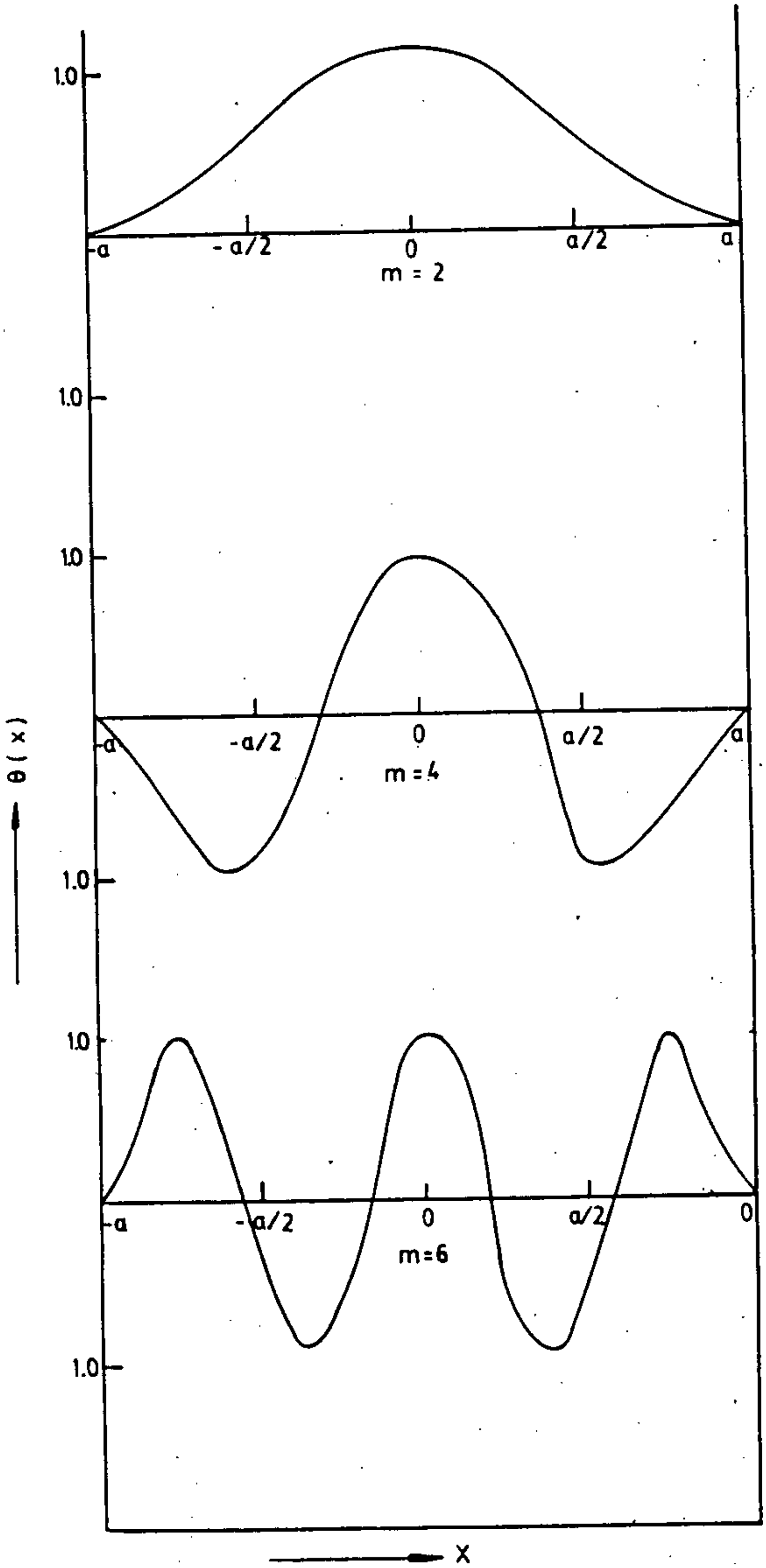

Fig. 2 : Beam function θ(x) for the plate with two opposing ends fixed for even values of m.

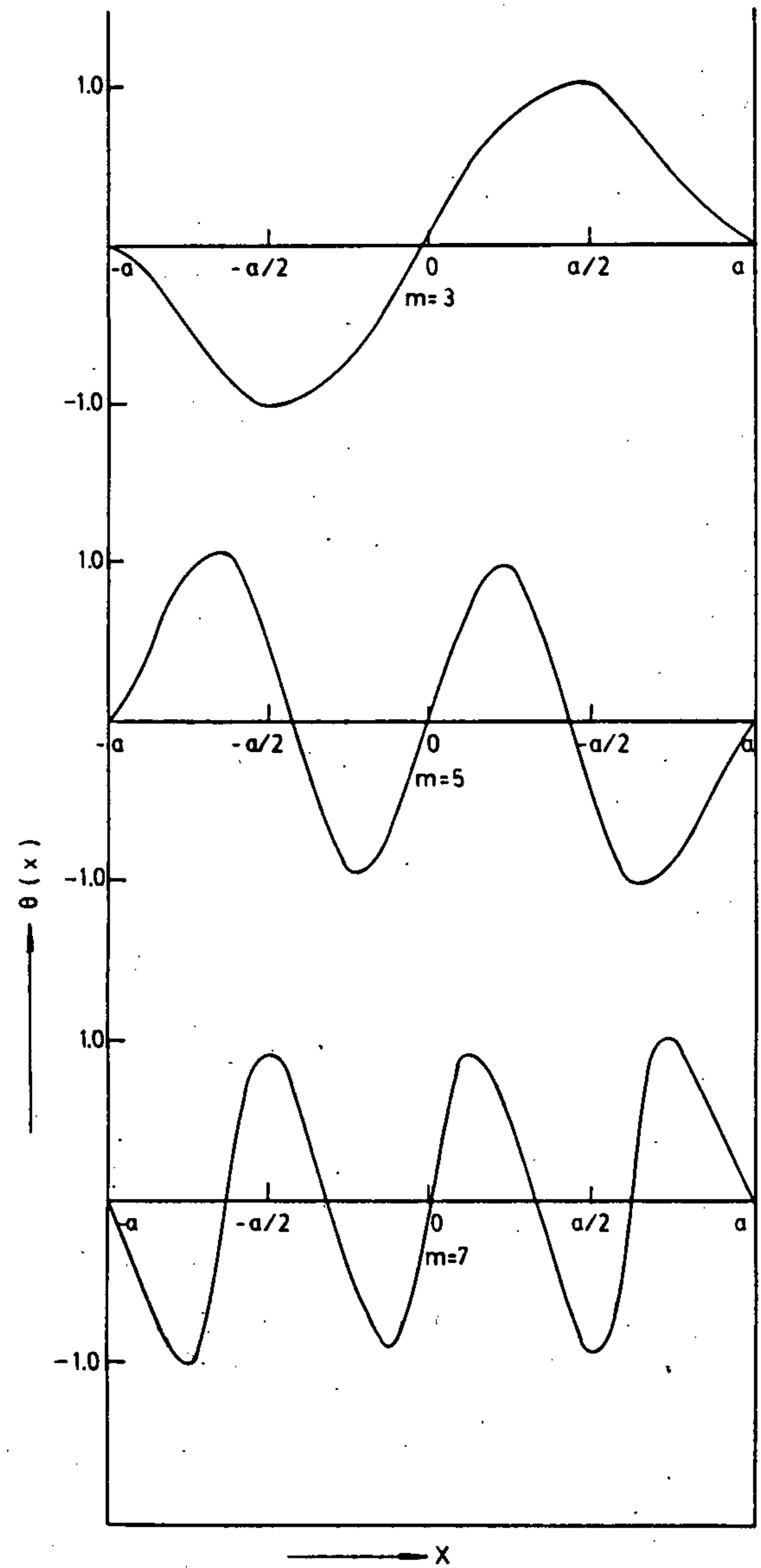

Fig. 3 : Beam function θ(x) for the plate with two opposing ends fixed for odd values of m.

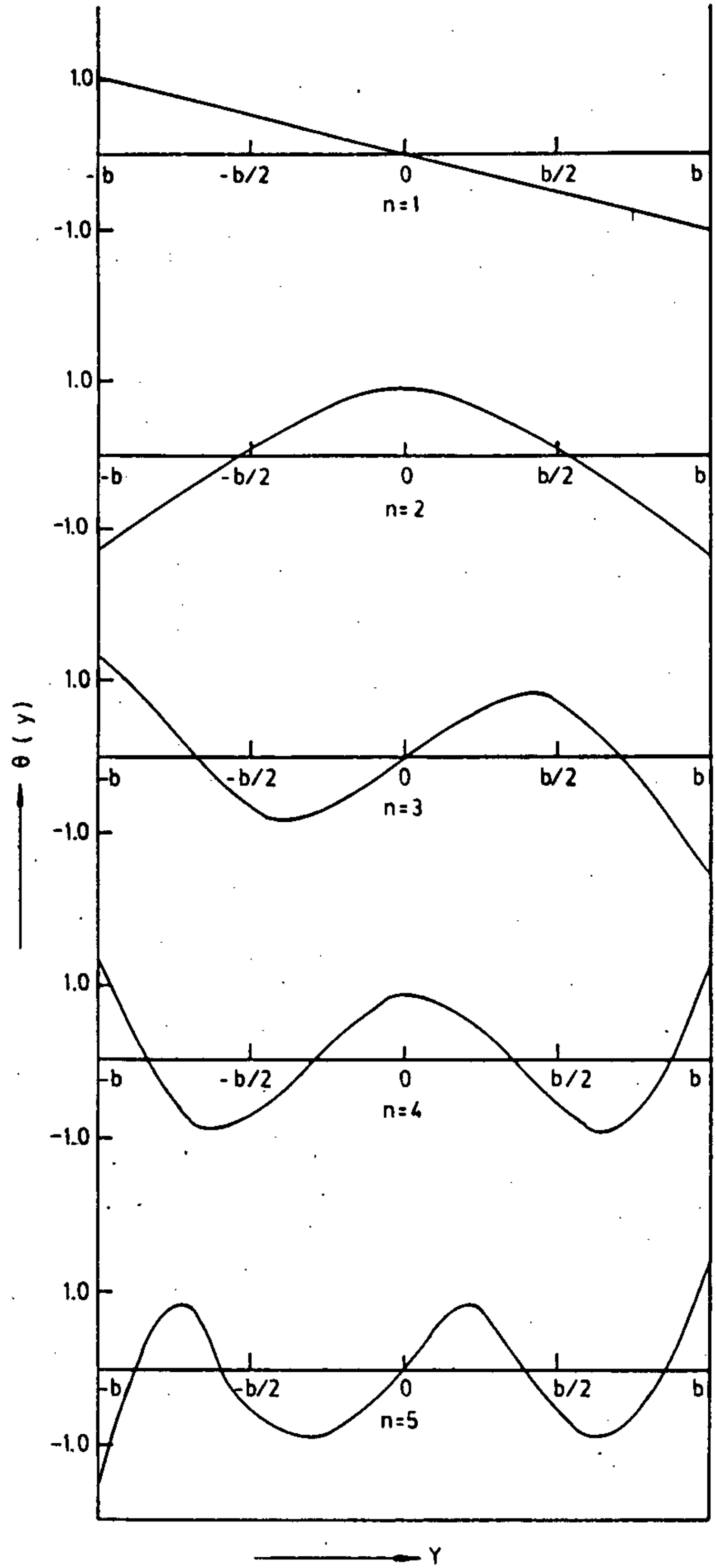

Fig. 4 : Beam function $\theta(y)$ for the plate with two opposing ends free for even & odd values of n.

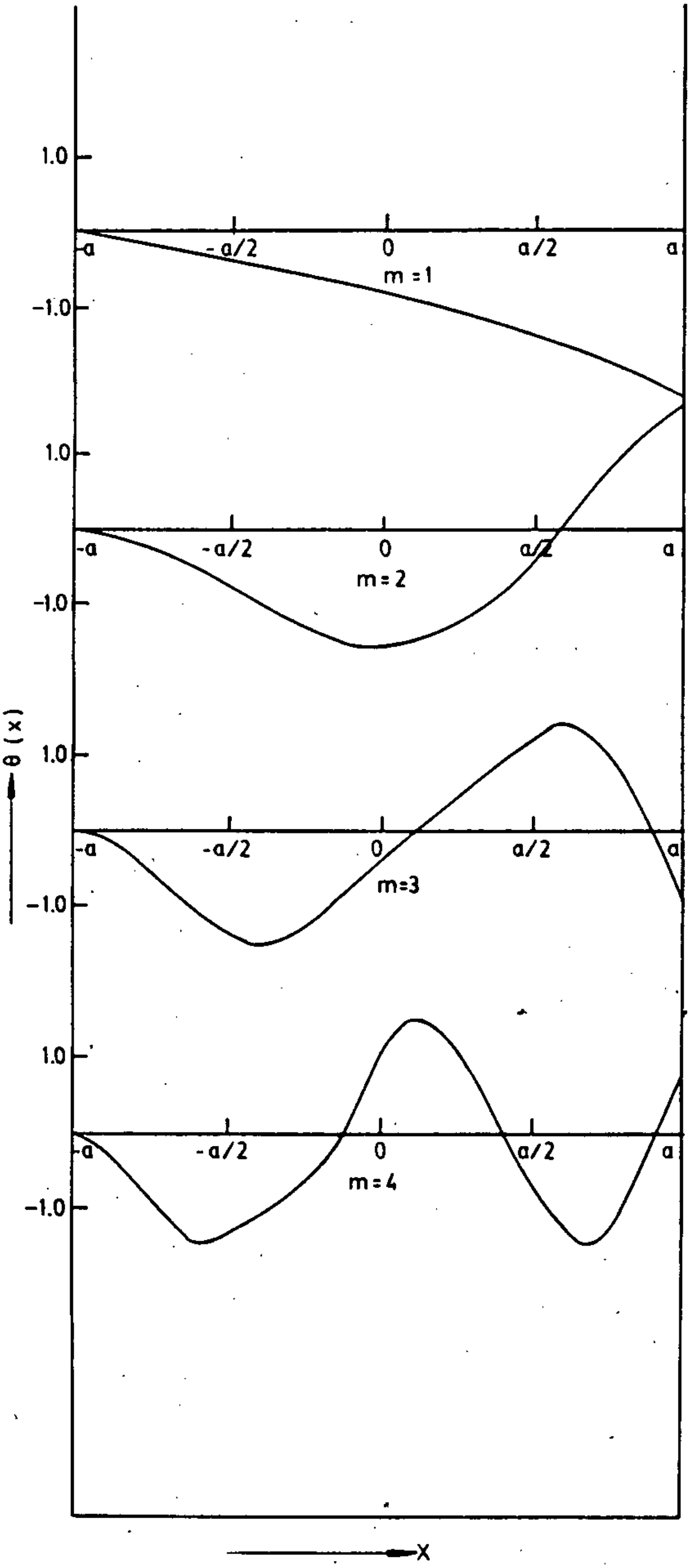

Fig. 5 : Beam function θ(x) for the plate with opposing ends fixed and free for even & odd values of m.

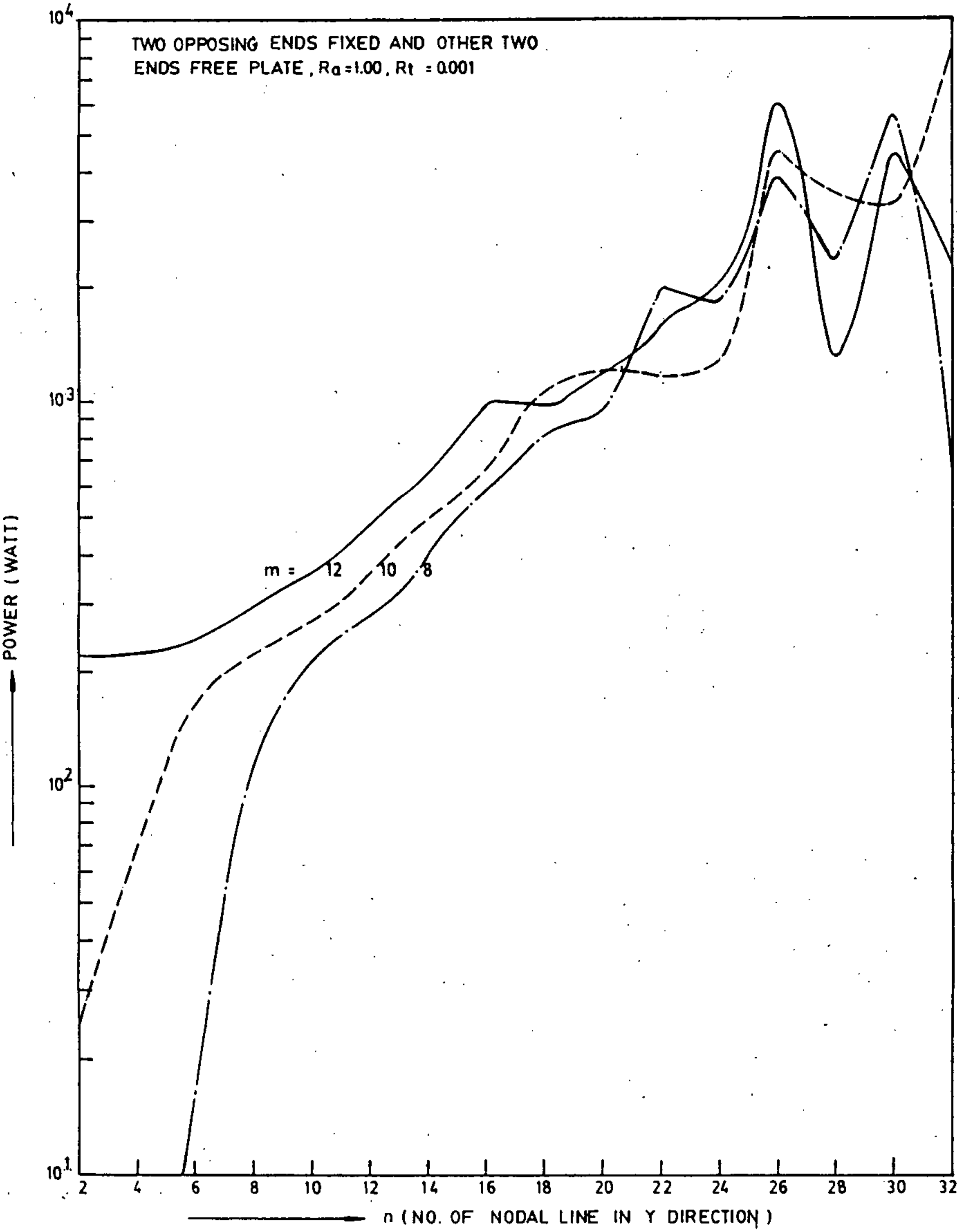

Fig. 6 : Power radiation from two opposing ends fixed and other two ends free plate at low values of m.

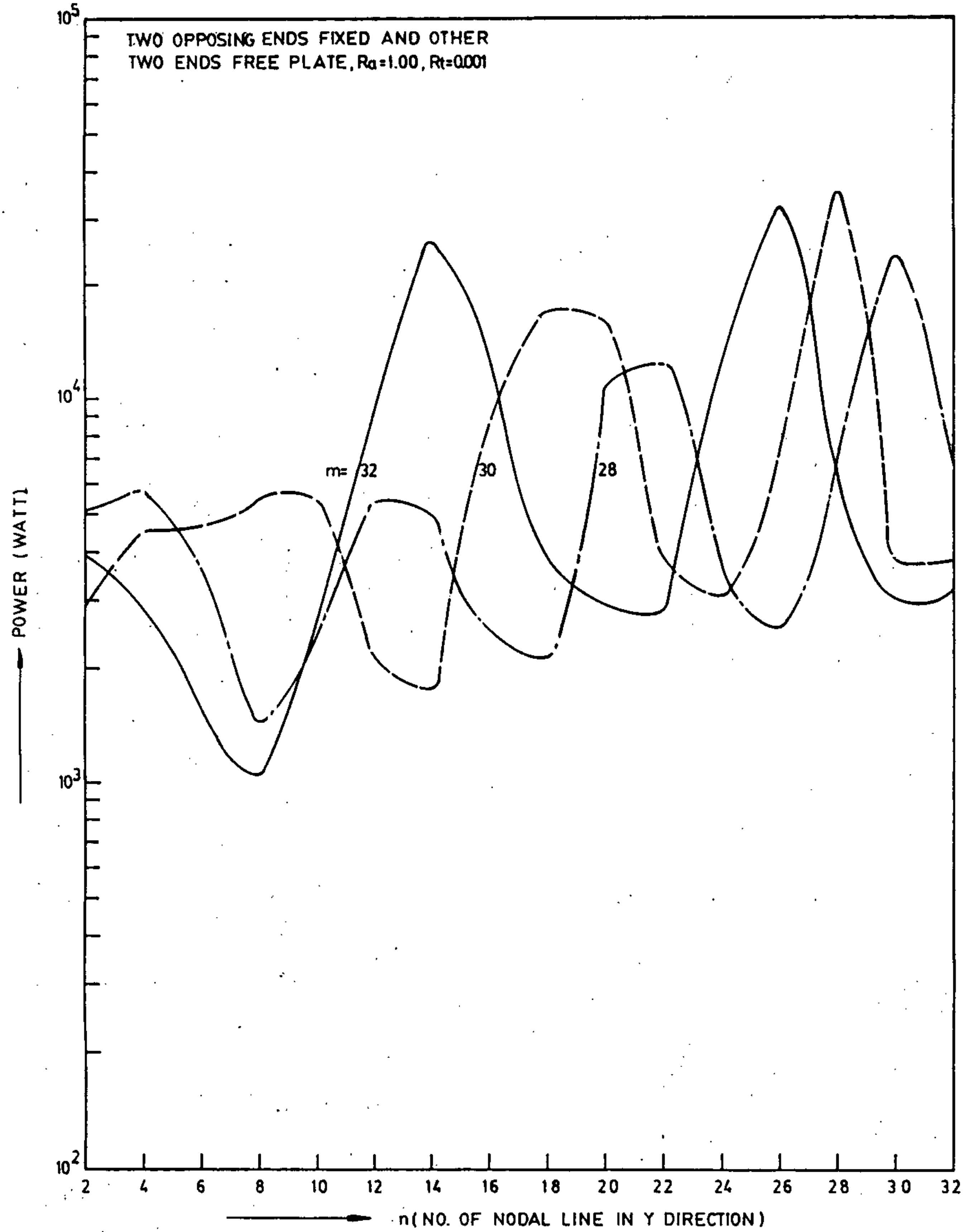

Fig. 7 : Power radiation from two opposing ends fixed and other two ends free plate at high values of m.

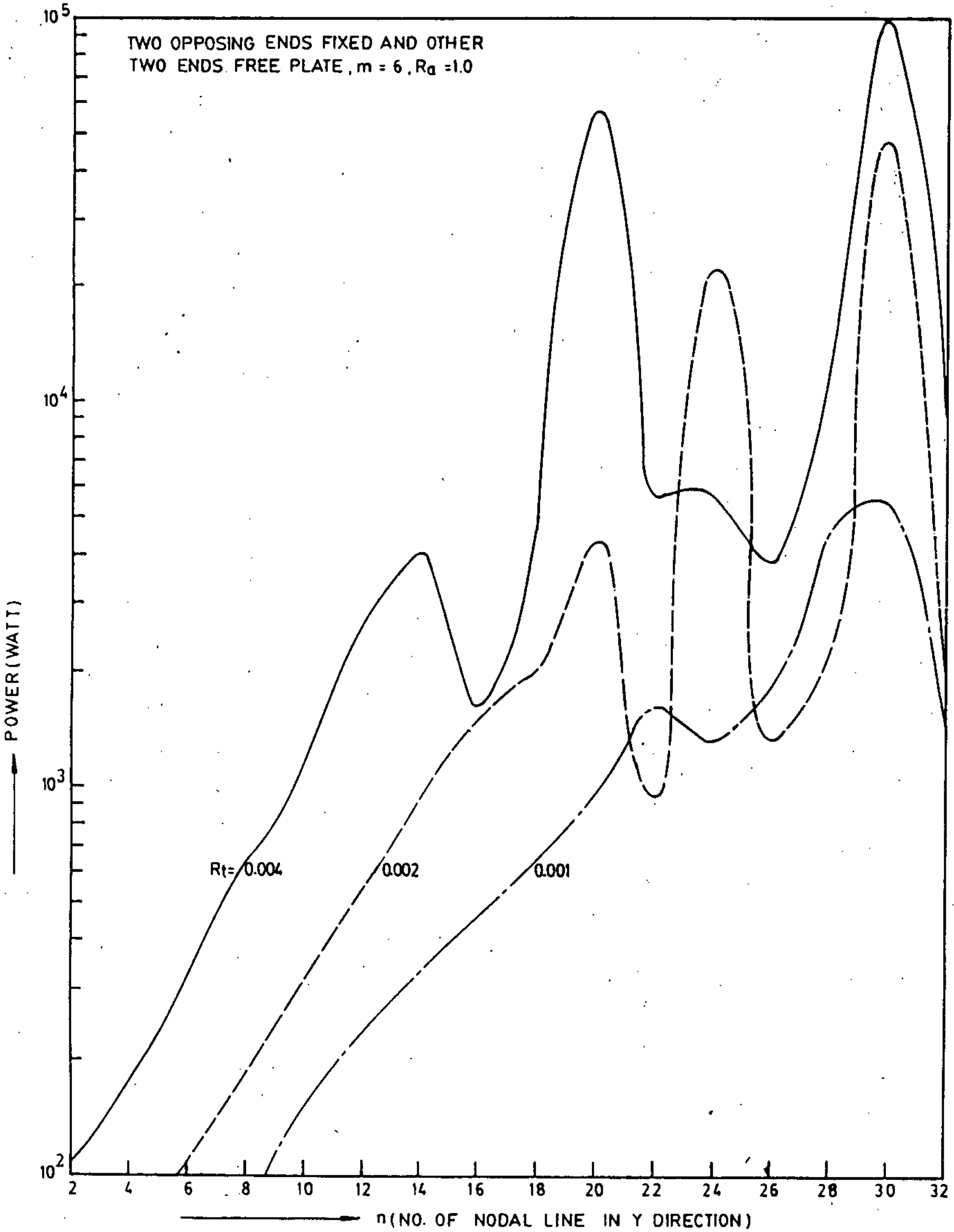

Fig. 8 : Power radiation from two opposing ends fixed and other two ends free plate for different thickness ratio at low values of m.

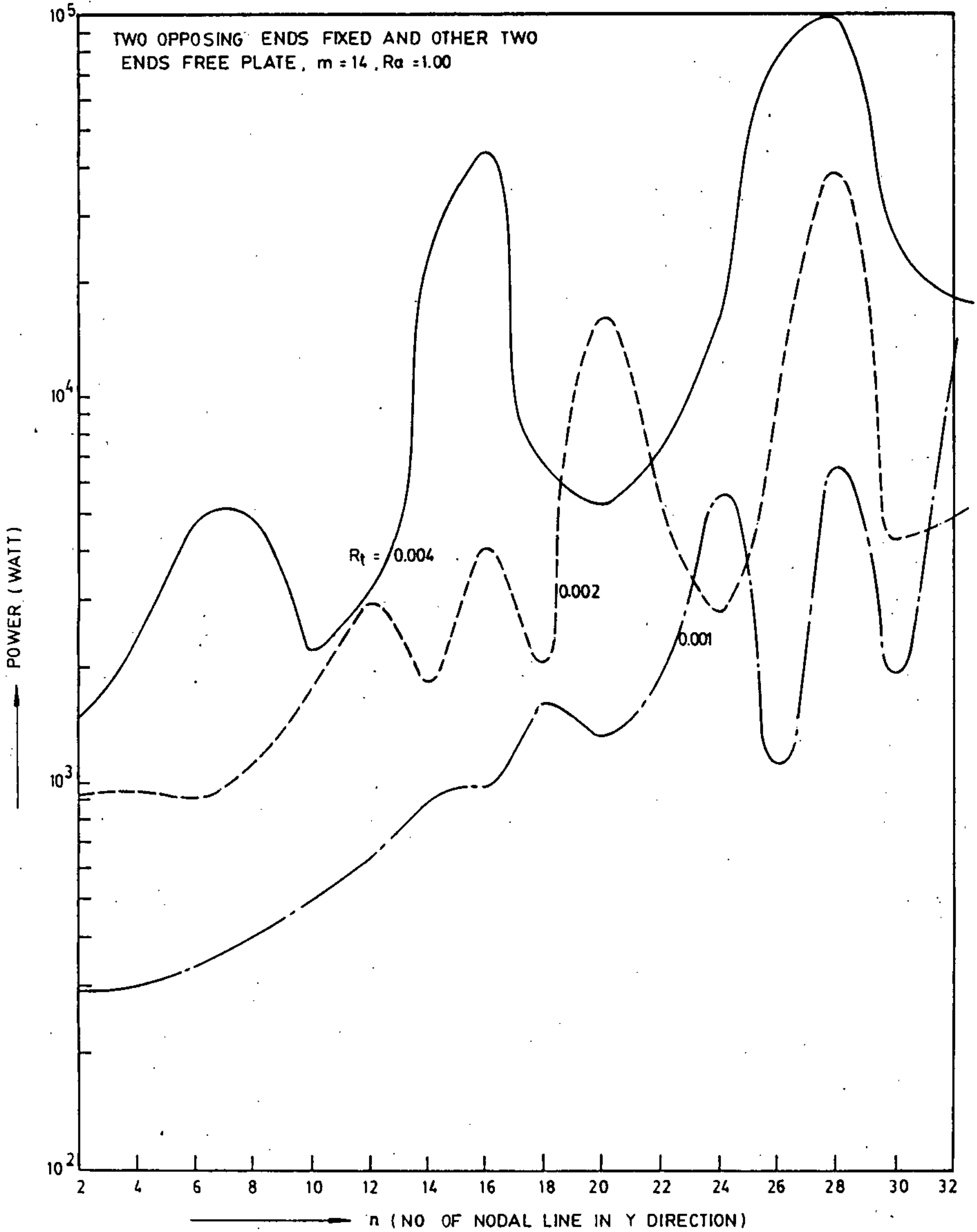

Fig. 9 : Power radiation from two opposing ends fixed and other two ends free plate for different thickness ratio at high values of m.

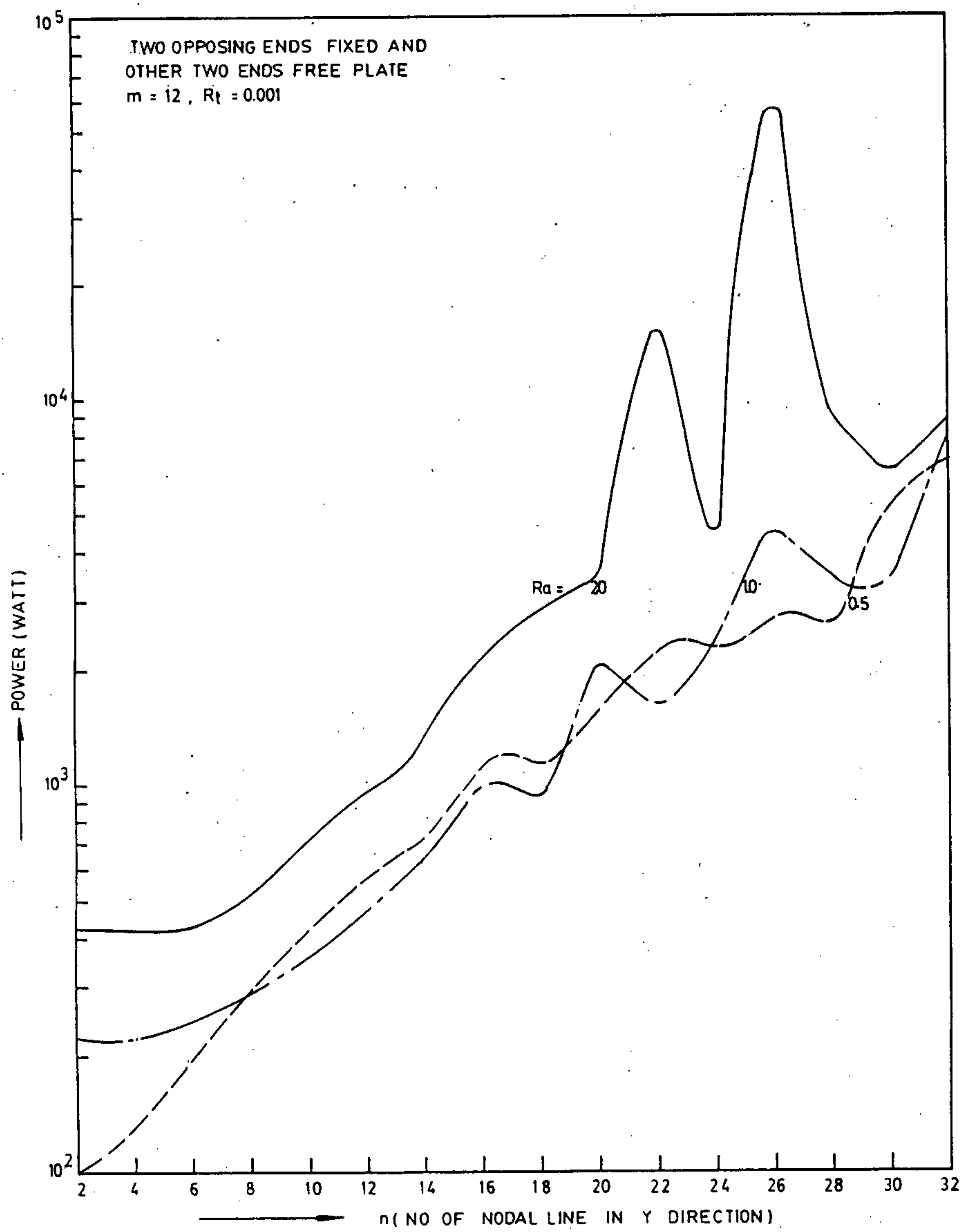

Fig. 10 : Power radiation from two opposing ends fixed and other two ends free plate for different aspect ratio at high values of m.

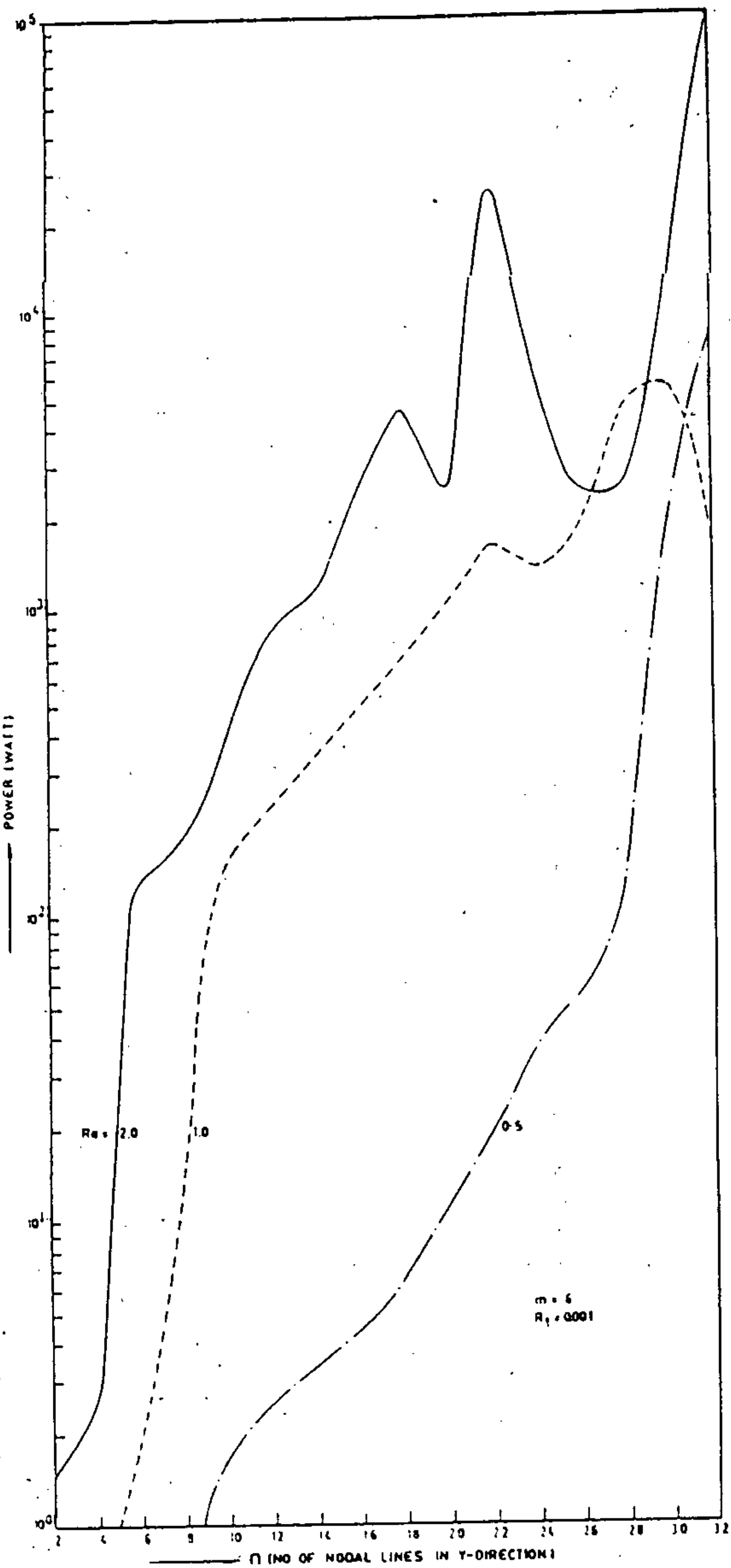

Fig. 11 : Power radiation from two opposing ends fixed and other two ends free plate for different aspect ratio at low values of m.

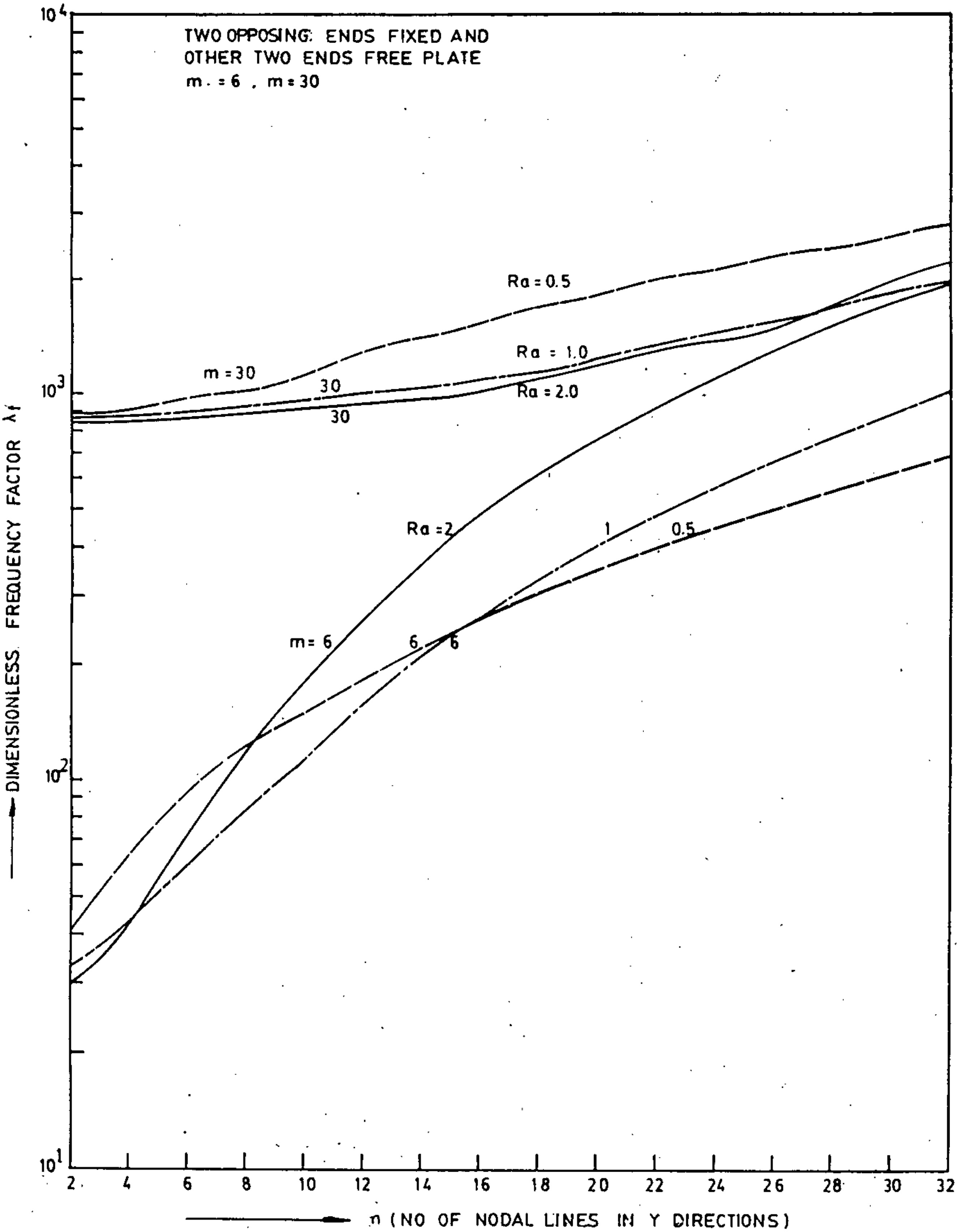

Fig.12 : Dimensionless frequency factor for different aspect ratio at high and low values of m.

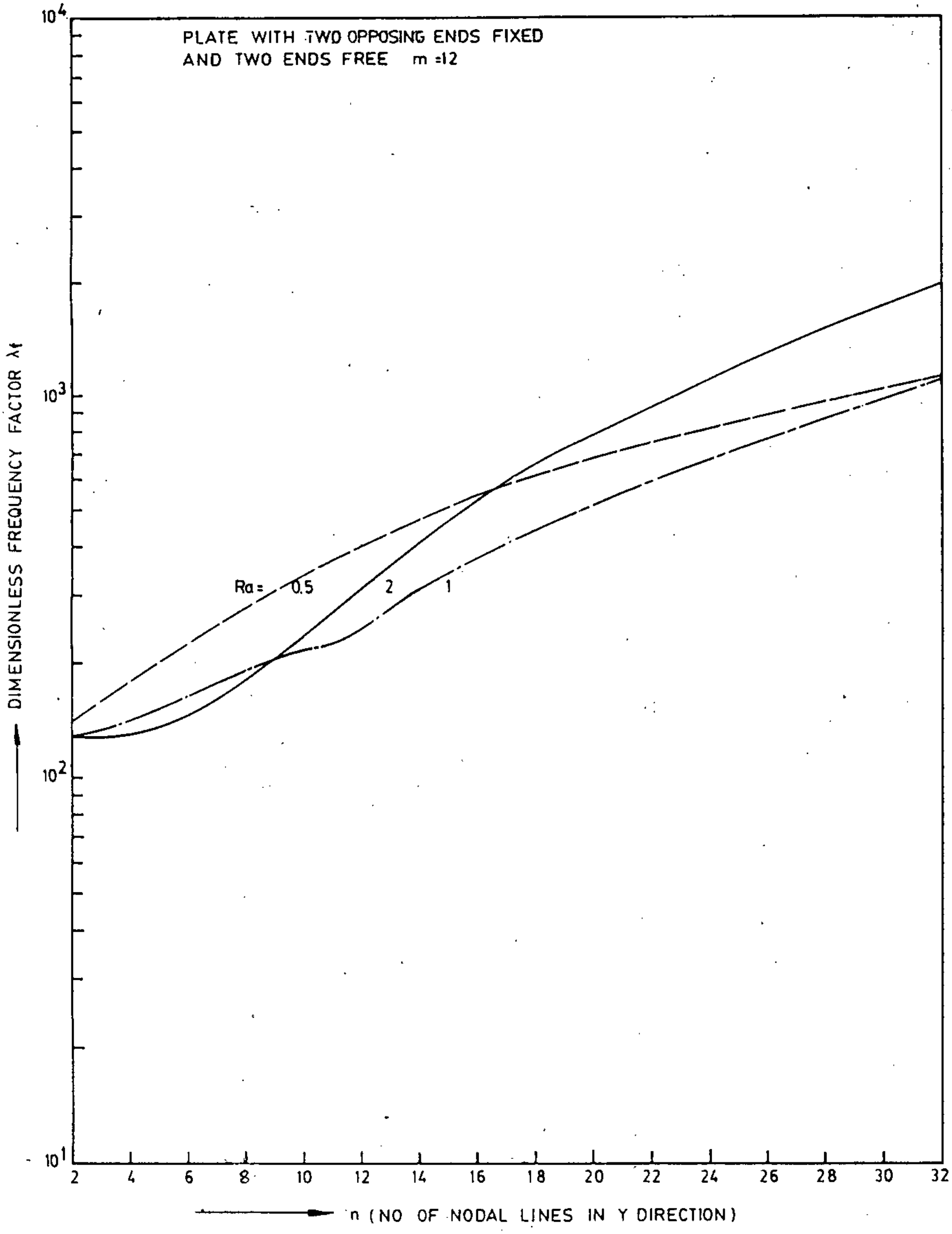

Fig.13 : Dimensionless frequency factor for different aspect ratio at high values of m.

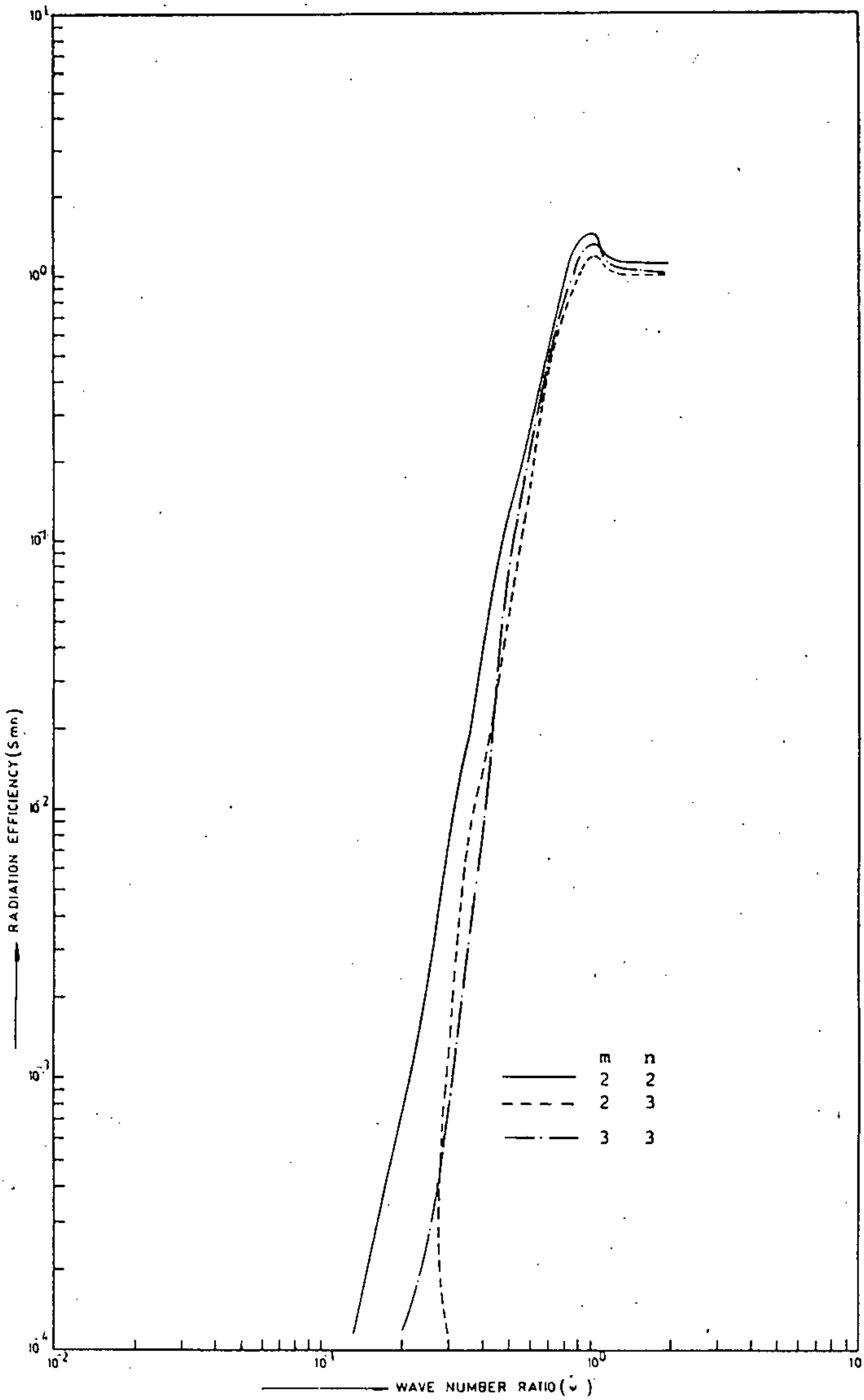

Fig. 14 : Radiation efficiency for two opposing ends fixed & other two ends free plate at low-low mode orders.

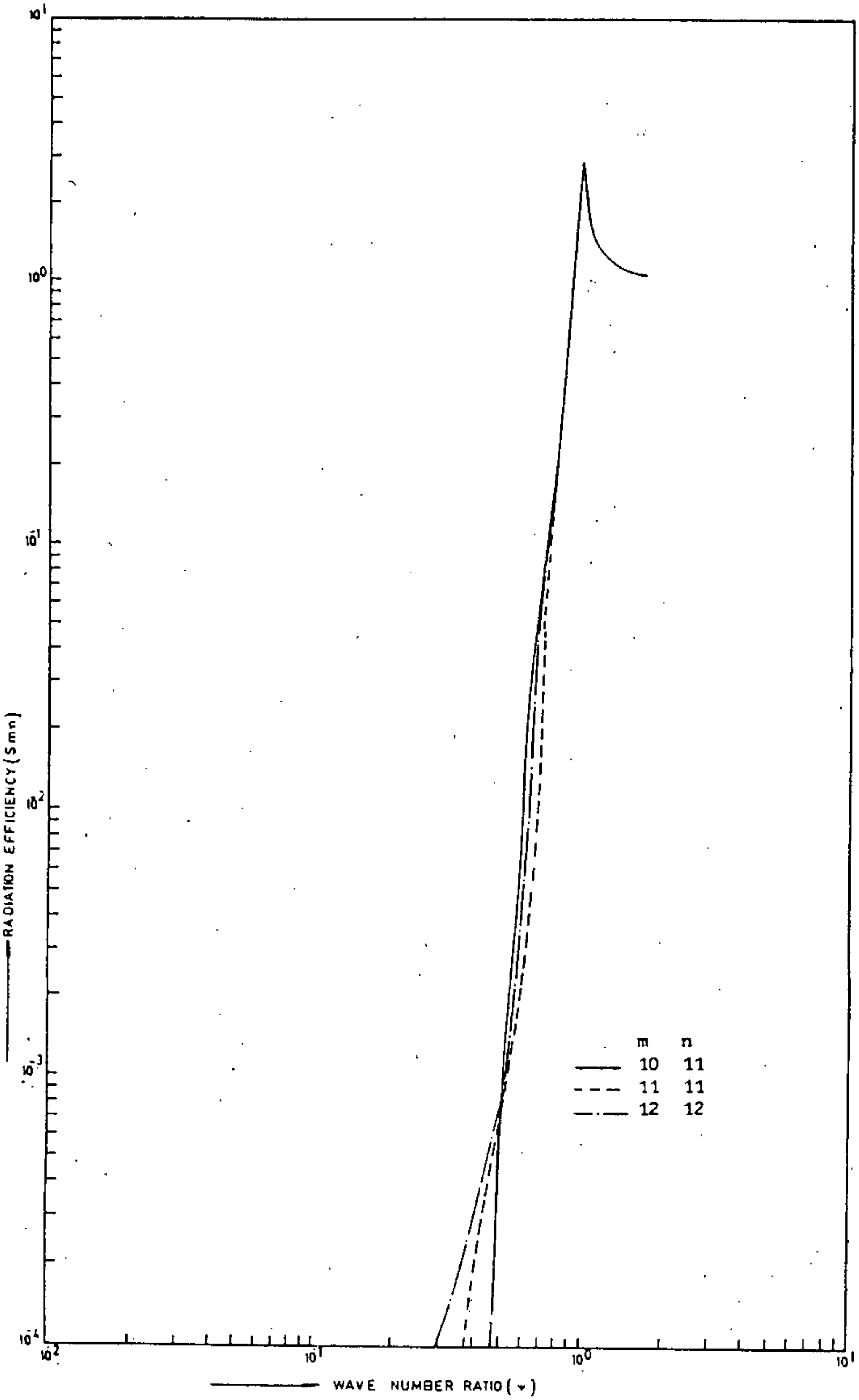

Fig. 15 : Radiation efficiency for two opposing ends fixed & other two ends free plate at high-high mode orders.

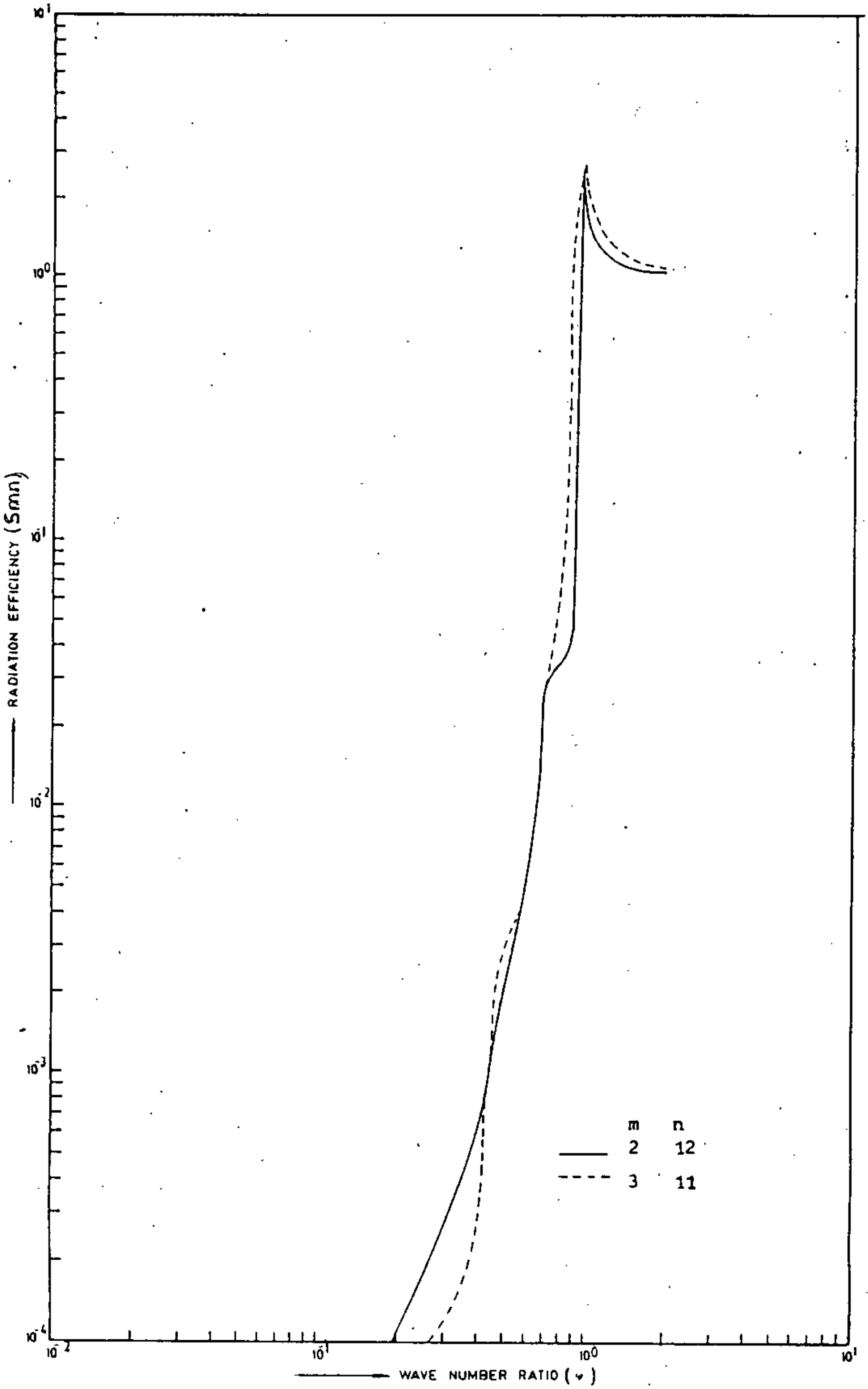

Fig. 16 : Radiation efficiency for two opposing ends fixed and other two ends free plate at low-high mode orders.

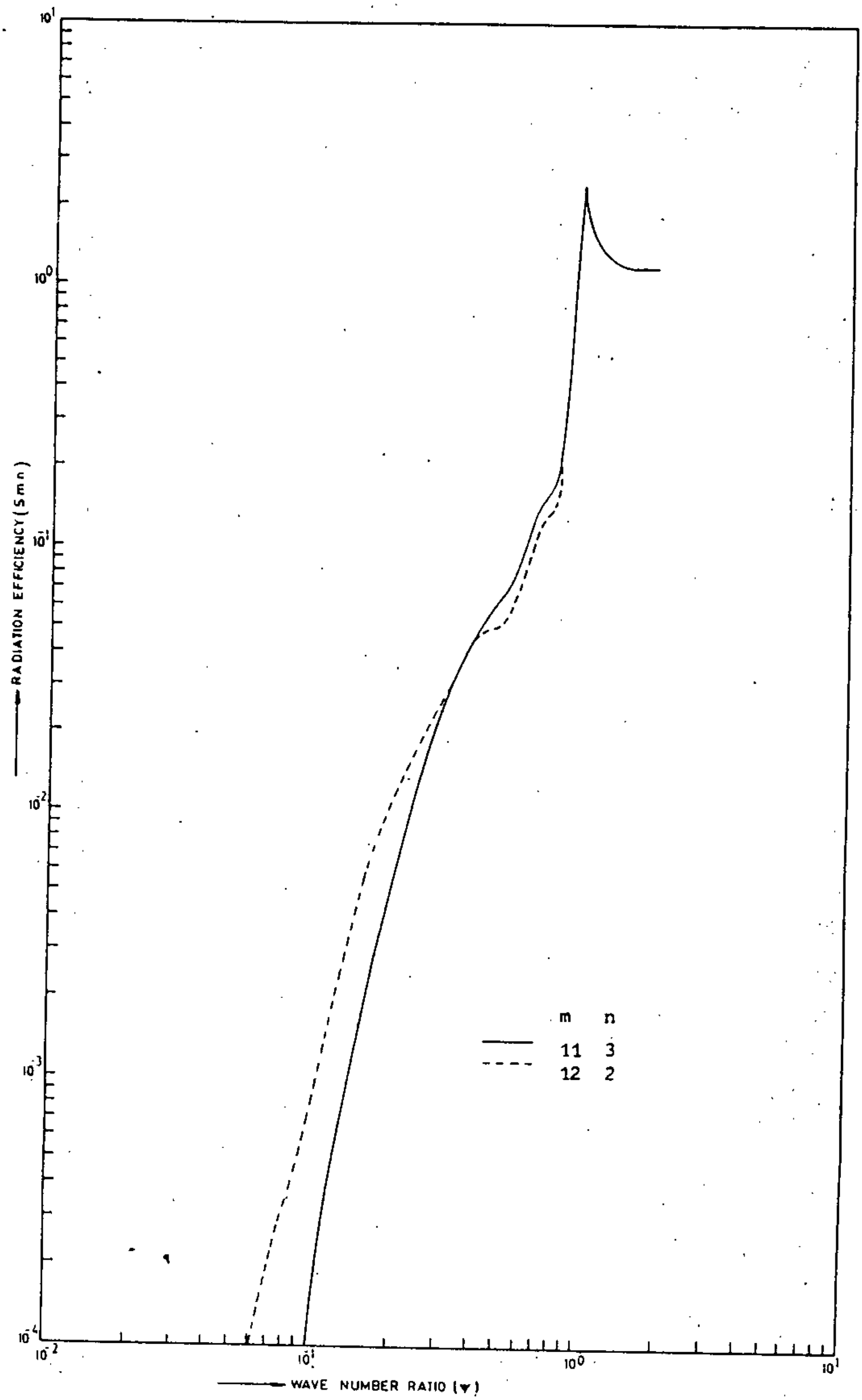

Fig. 17 : Radiation efficiency for two opposing ends fixed and other two ends free plate at high-low mode orders.

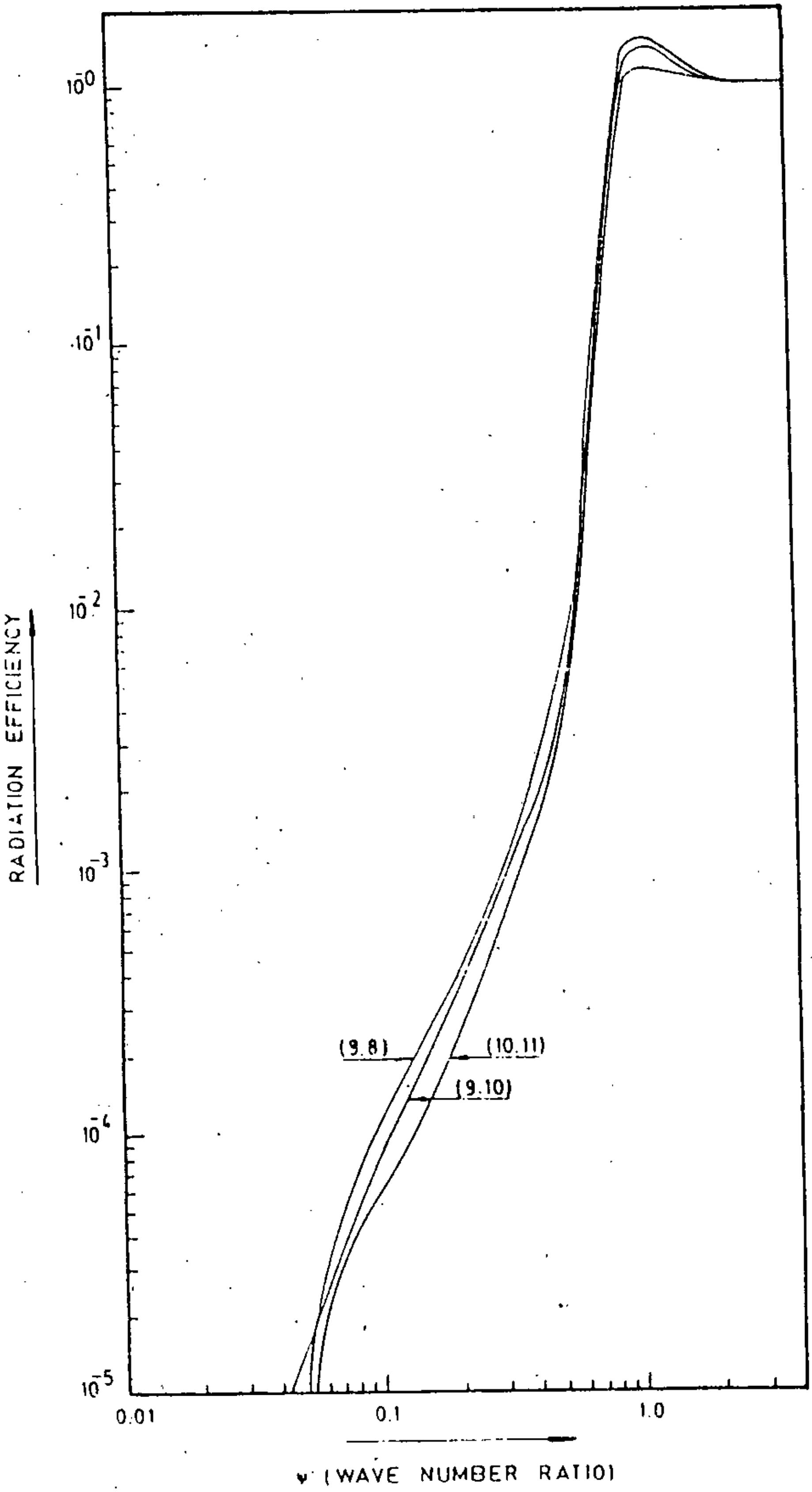

Fig. 18 : Radiati.n efficiency of a simpiy supported plate for high-high mode orders. Ref. [1].

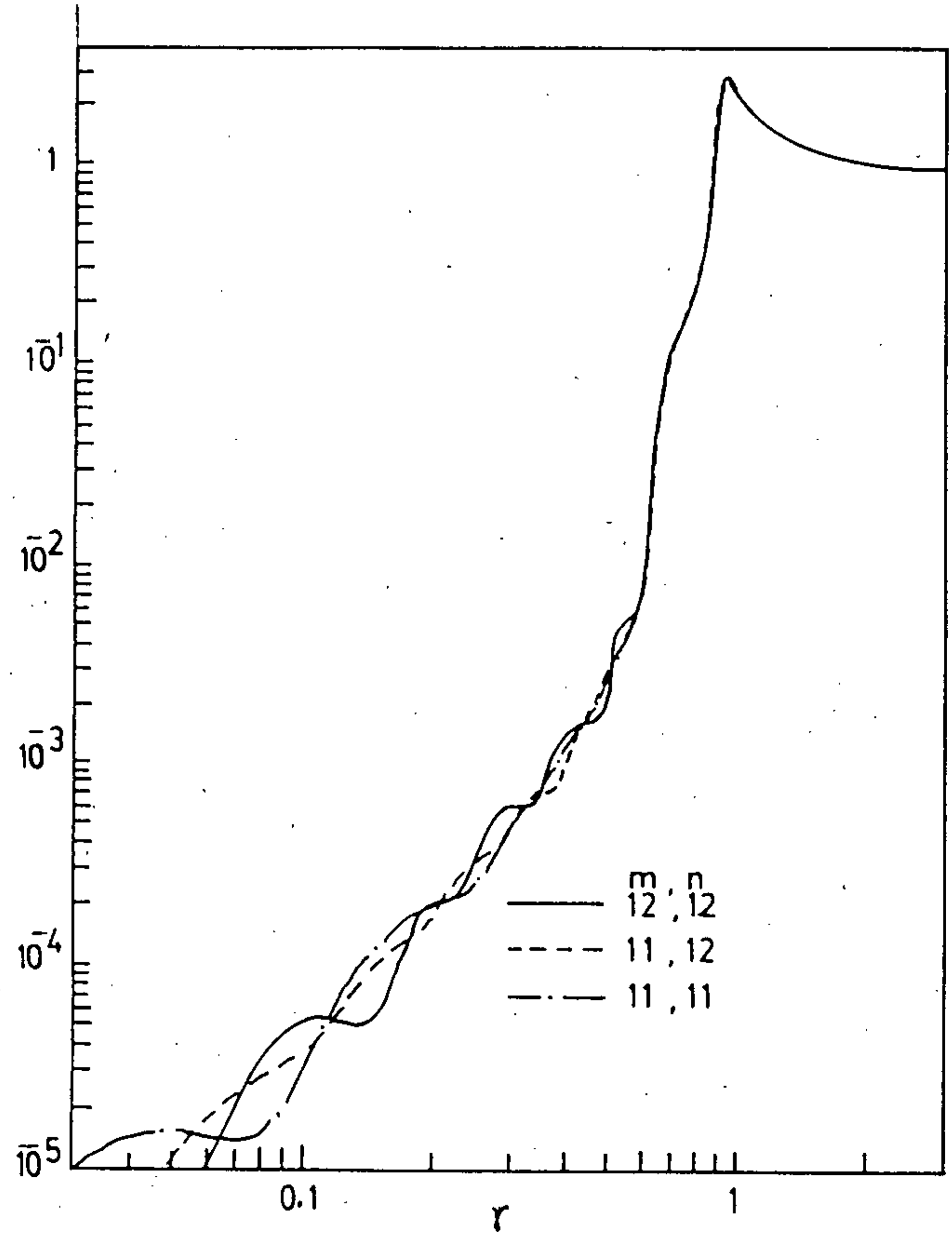

Fig 19 Radiation efficiency for typical high-numbered modes of a square panel ref. [46]

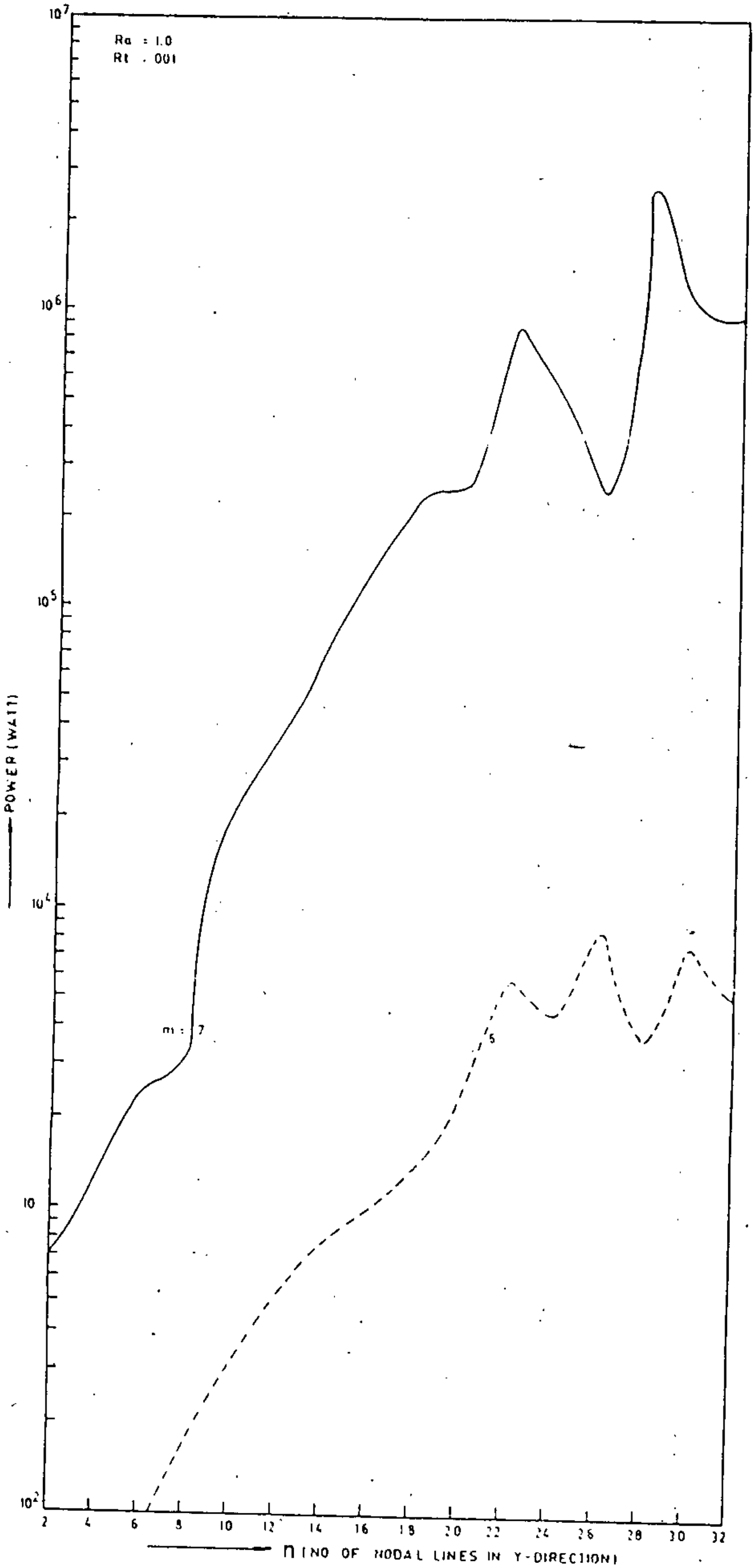

Fig.20 : Power radiation from one end fixed and other three ends free plate at low values of m.

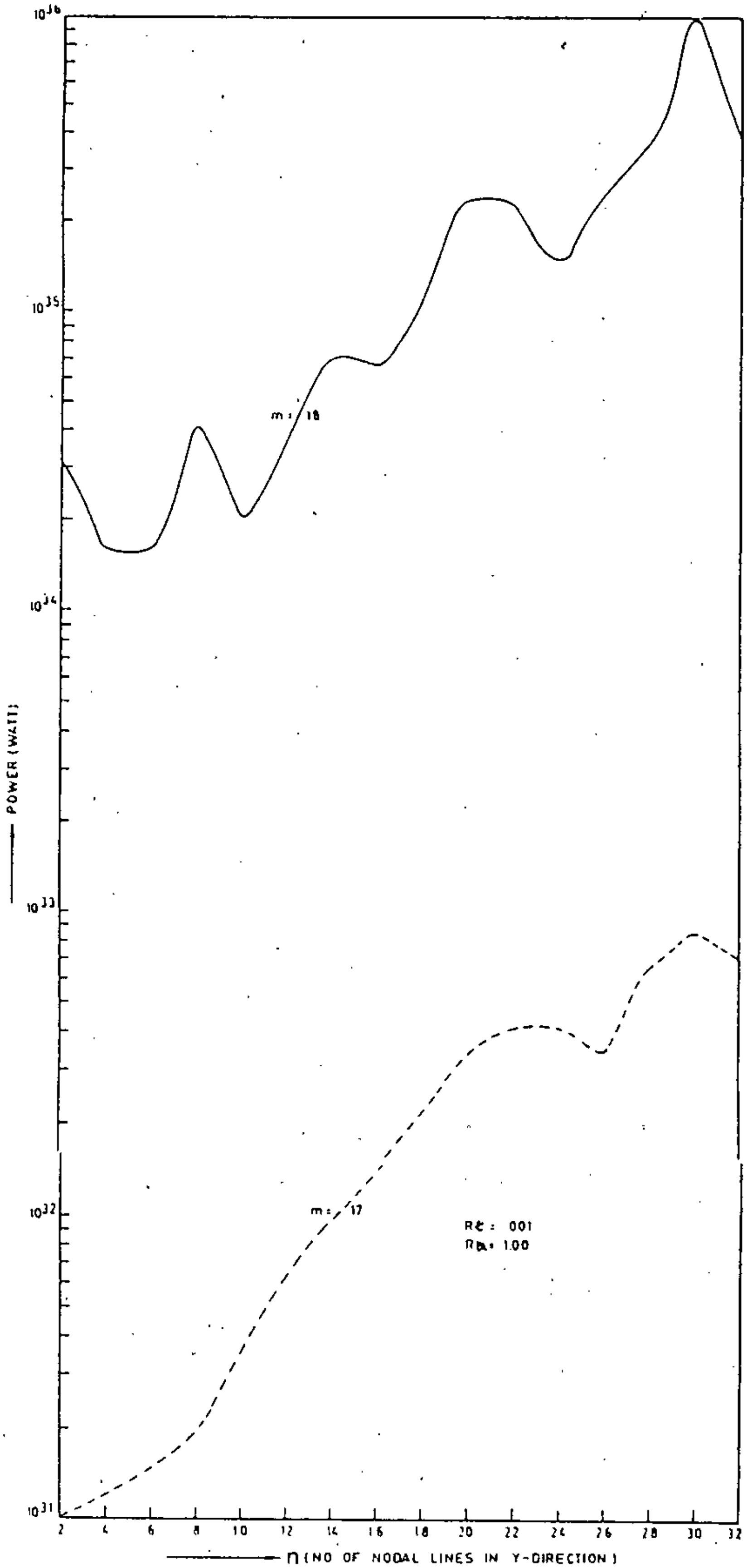

Fig.21 : Power radiation from one end fixed and other three ends free plate at high values of m.

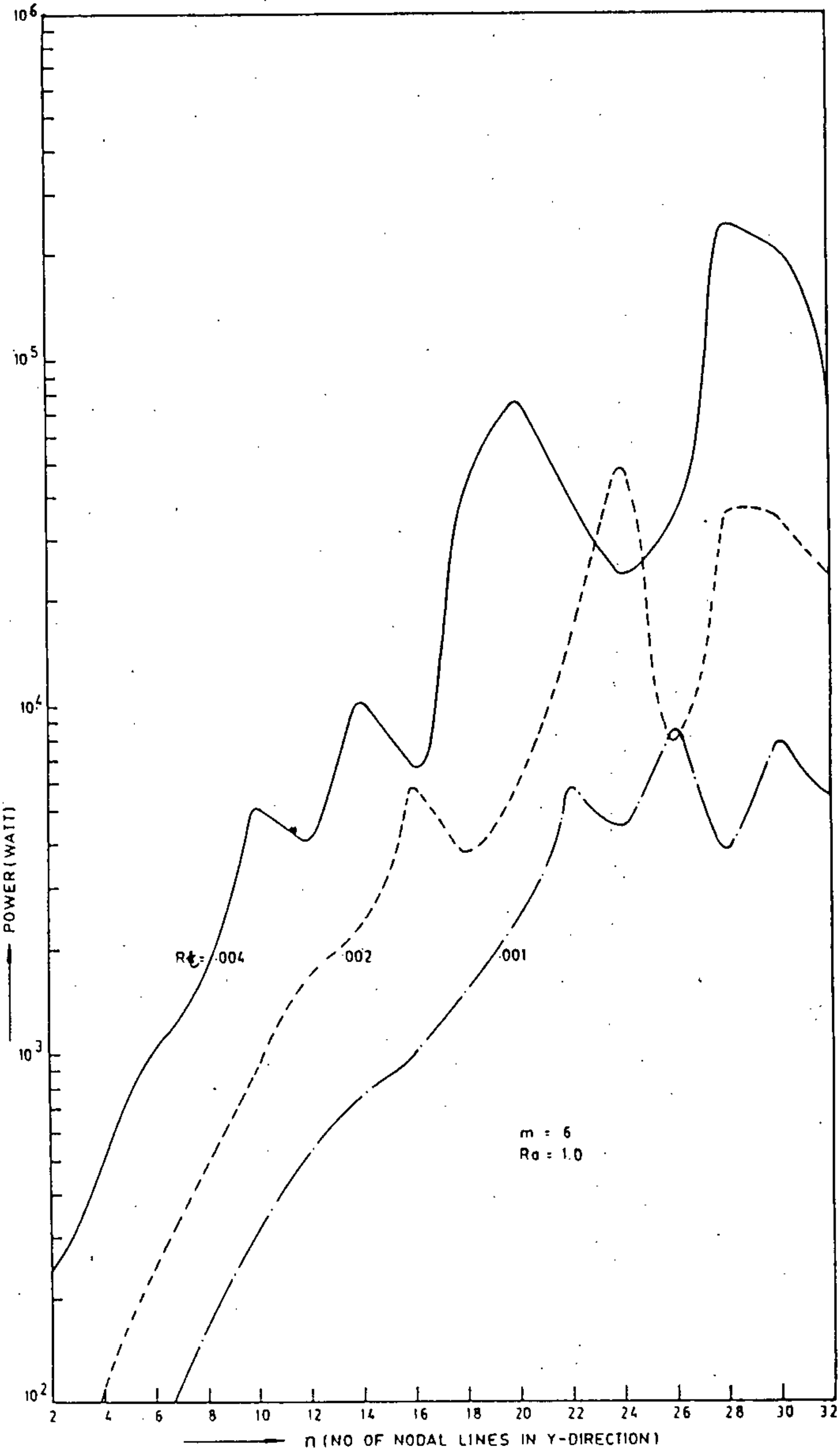

Fig. 22 : Power radiation from one end fixed and other three ends free plate for different thickness ratio at low values of m.

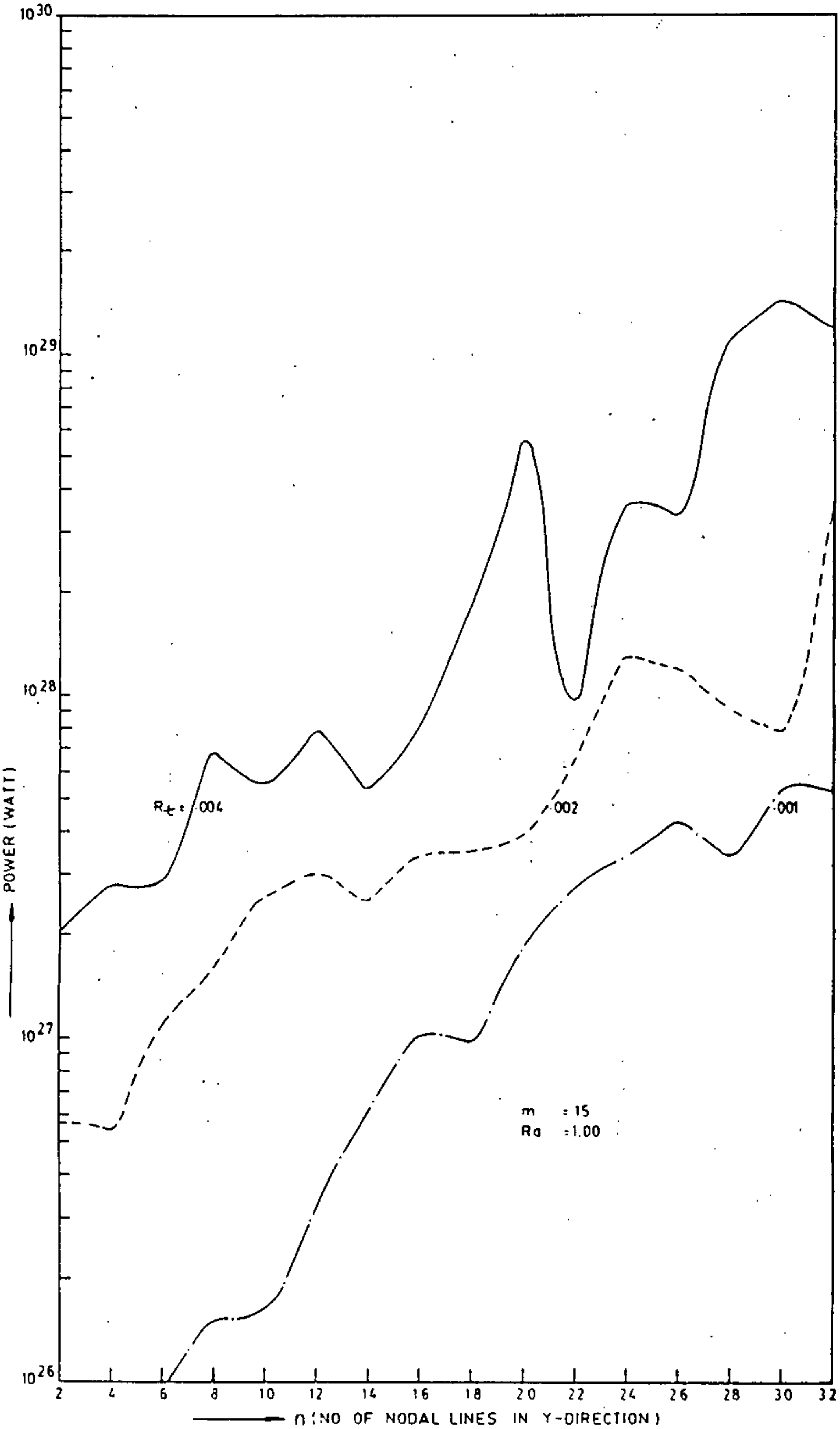

Fig. 23 : Power radiation from one end fixed and other three ends free plate for different thickness ratio at high values of m.

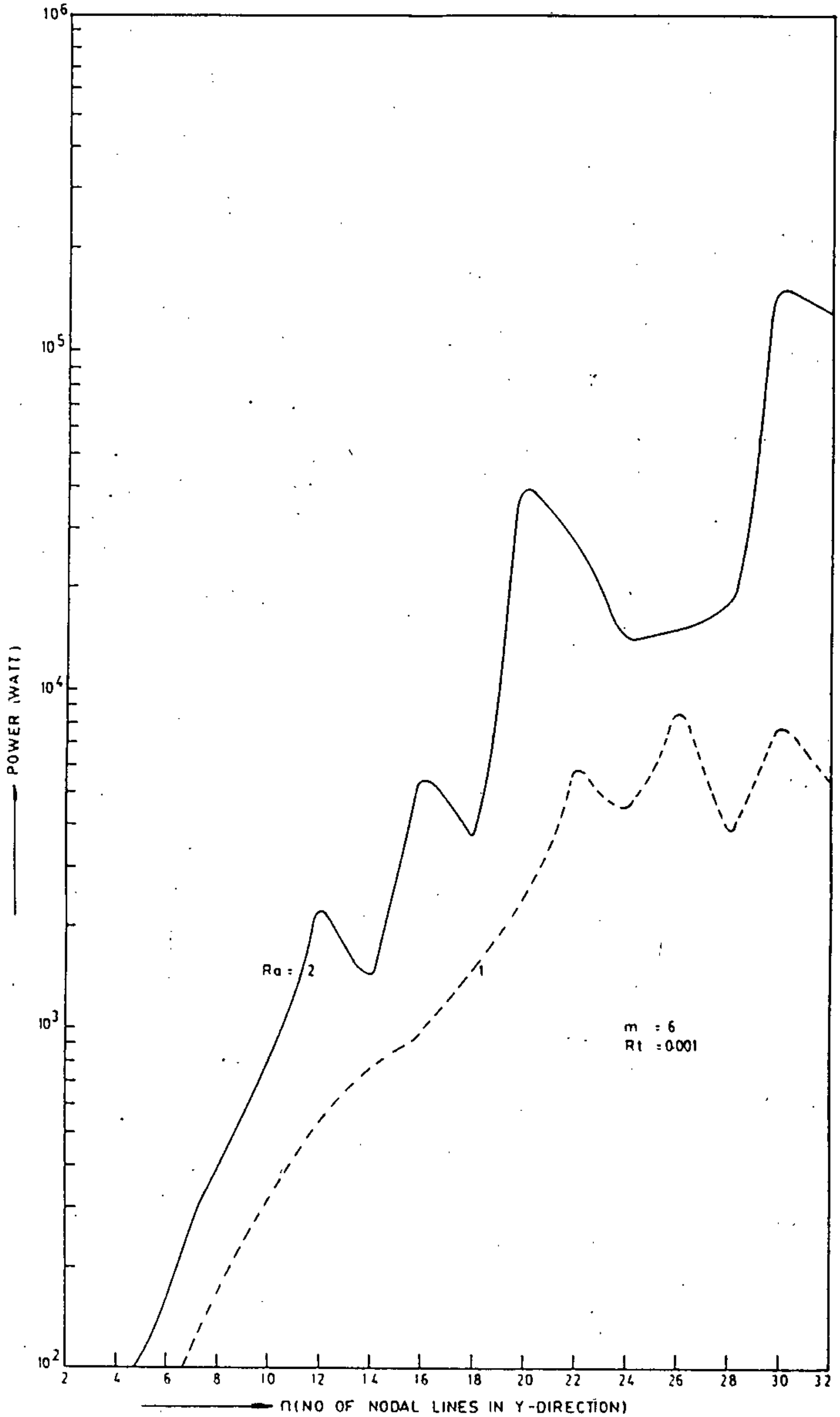

Fig. 24 : Power radiation from one end fixed and other three ends free plate for different aspect ratio at low values of m.

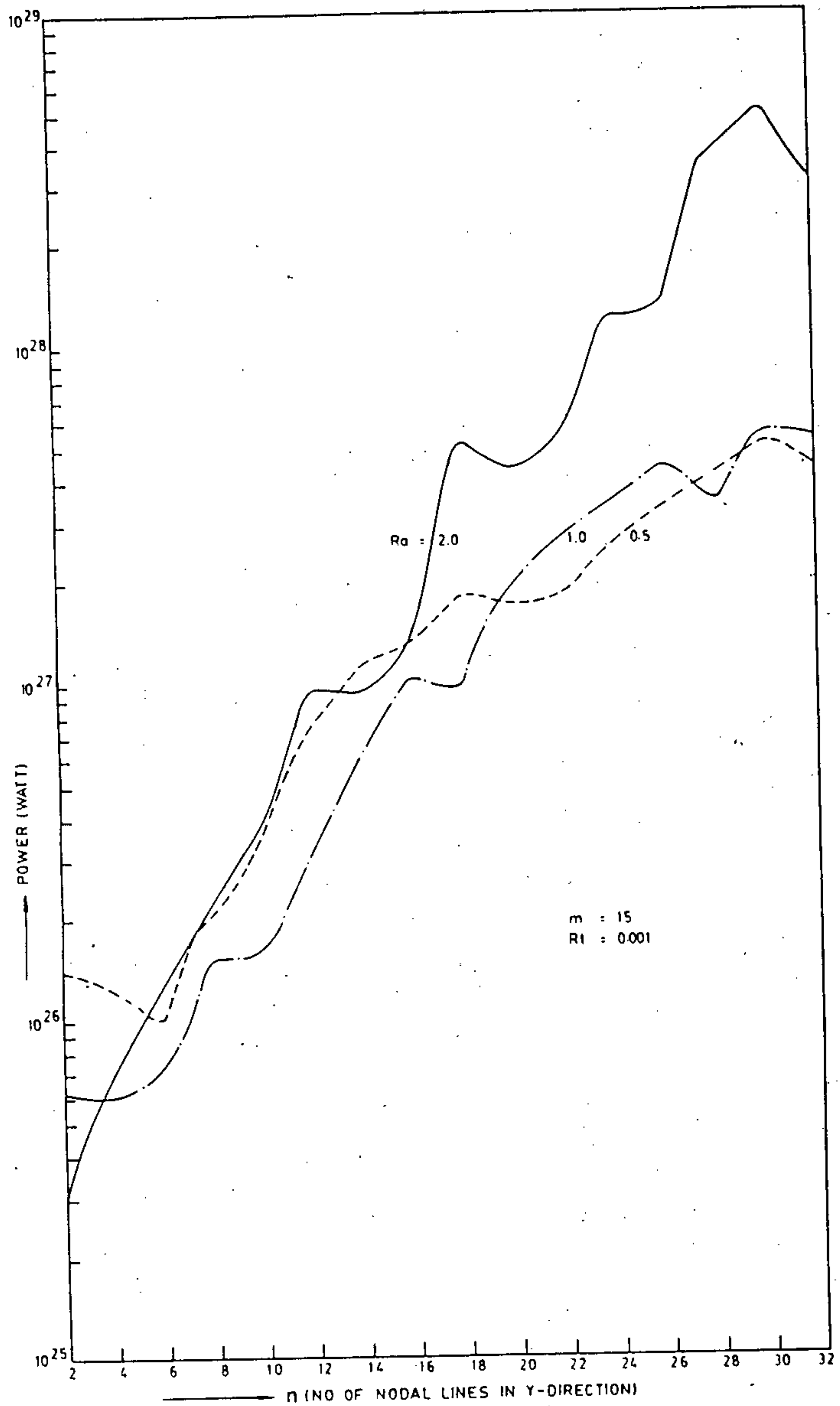

Fig. 25 : Power radiation from one end fixed and other three ends free plate for different aspect ratio at high values of m.

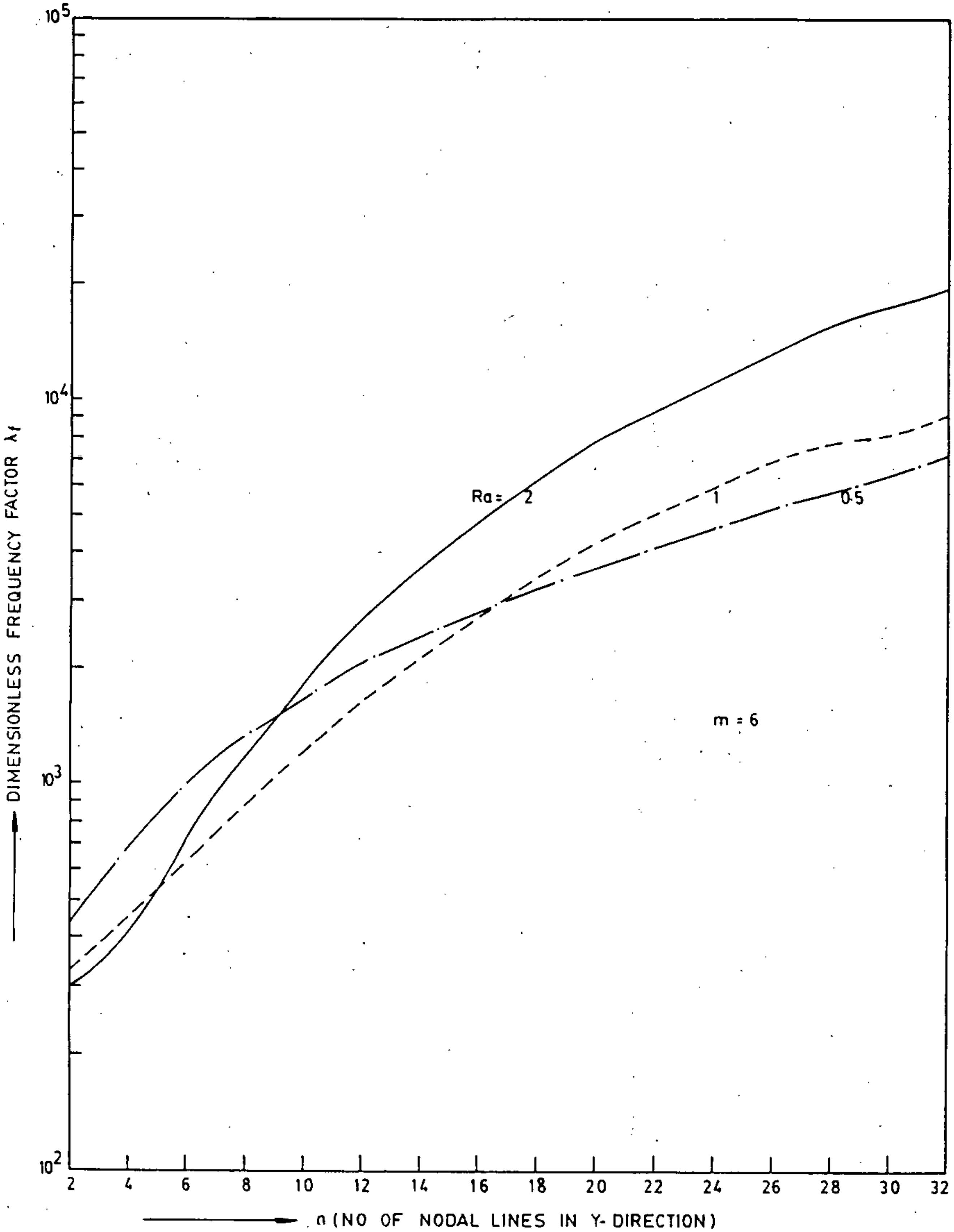

FIG. 26. DIMENTIONLESS FREQUENCY FACTOR FOR DIFFERENT VALUES OF ASPECT RATIO AT LOW VALUES OF M OF ONE END FIXED AND OTHER THREE ENDS FREE PLATE

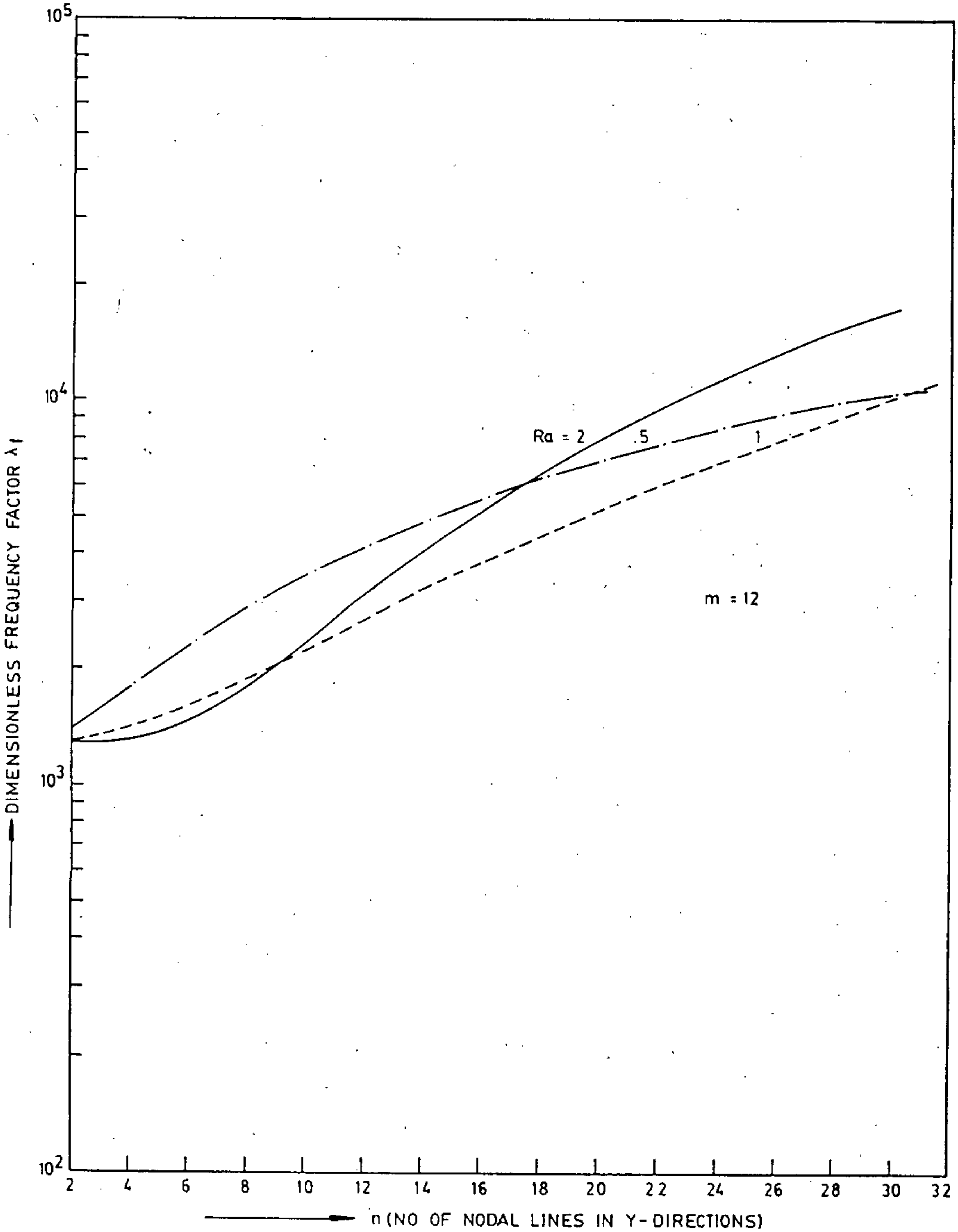

FIG. 27 DIMENSIONLESS FREQUENCY FACTOR FOR DIFFERENT ASPECT RATIO AT HIGHER VALUE OF M OF ONE END FIXED AND THREE ENDS FREE PLATE

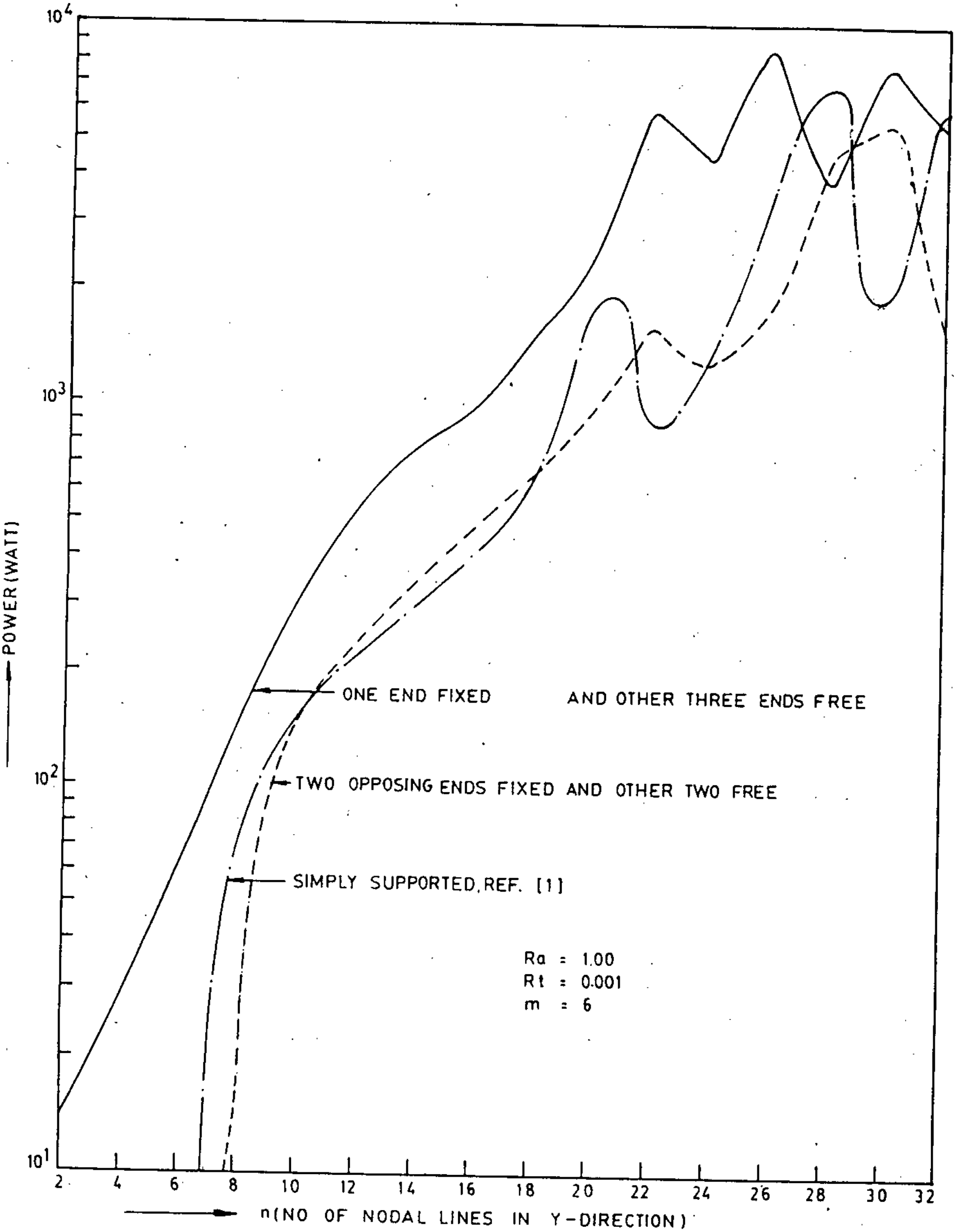

FIG. 28 POWER RADIATED FROM PLATES WITH DIFFERENT BOUNDARY CONDITIONS

Table-1 : Power radiation by two oppsing ends fixed and other two free plate. $R_a = 1$, $R_t = .001$

m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
8	2.0E0	4.0E0	1.7E1	1.2E2	2.2E2	2.8E2	4.0E2	5.8E2	8.4E2	9.2E2	2.0E3	1.8E3	3.9E3	2.3E3	5.6E3	6.9E2
9	4.8E0	1.3E1	7.0E1	2.1E2	2.4E3	3.2E2	4.4E2	6.4E2	1.0E3	1.0E3	2.0E3	1.9E3	4.3E3	1.2E3	6.7E3	1.2E3
10	2.5E1	7.0E1	1.7E2	2.2E2	2.7E2	3.6E2	5.0E2	6.5E2	1.0E3	1.2E3	1.6E3	2.0E3	6.023	1.3E3	4.6E3	2.4E3
28	5.1E3	5.8E3	3.6E3	1.4E3	2.3E3	5.4E3	5.0E3	2.5E3	2.1E3	1.0E4	2.5E4	3.9E4	2.5E3	7.0E3	2.4E4	6.6E3
30	2.8E3	4.5E3	4.6E3	5.4E3	5.5E3	2.1E3	1.7E3	8.5E3	1.7E4	1.6E4	3.8E3	3.0E3	7.0E3	3.5E4	3.7E3	3.7E3
32	3.9E3	3.1E3	1.6E3	1.0E3	2.9E3	8.4E3	2.6E4	1.4E4	3.8E3	3.1E3	2.8E3	1.3E4	3.2E4	6.8E3	3.0E3	3.1E3

Table-2 : Power radiation by a two oppsing ends fixed and other two free plate. $R_a = 1.0$ & different values of R_t.

Rt	m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
.001	6	7.0E-1	9.6E-1	2.1E0	1.4E1	1.6E2	2.4E2	3.4E2	4.9E2	6.7E2	9.8E2	1.6E3	1.3E3	1.8E3	4.7E3	5.4E3	1.5E3
.002	6	3.7E1	6.2E1	1.0E2	1.8E2	3.3E32	5.5E2	9.3E2	1.5E3	2.0E3	4.2E3	9.3E2	2.2E4	1.3E3	2.2E2	4.7E4	1.8E3
.004	6	1.0E2	3.1E2	3.1E2	5.6E2	1.1E3	2.6E3	4.0E3	1.6E3	4.6E3	5.7E4	5.6E3	5.7E3	3.8E3	1.3E4	3.7E5	9.7E3
.001	14	2.9E2	3.0E2	3.4E2	4.1E2	5.1E2	6.2E2	9.1E2	9.6E2	1.6E3	1.3E3	1.8E3	5.5E3	1.1E3	6.7E3	1.9E3	1.4E4
.002	14	9.1E2	9.6E2	9.1E2	1.0E3	1.8E3	2.9E3	1.8E3	4.0E3	2.0E3	1.6E4	5.4E3	2.7E3	9.6E3	3.8E4	4.1E3	4.9E3
.004	14	1.5E3	2.3E3	4.6E3	4.8E3	2.2E3	3.1E3	2.4E3	4.3E4	6.7E3	5.1E3	6.9E3	1.7E4	8.8E4	2.5E5	2.6E4	1.8E4

Table-3 : Power radiation by two oppsing ends fixed and other two free plate. R_t = .001 & different value of R_a.

Ra	m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
.5	6	2.E-1	4.2E-1	3.6E-1	7.1E-1	1.8E0	2.8E0	3.4E0	4.3E0	6.8E0	1.1E1	2.0E1	3.8E1	5.6E1	1.4E2	2.2E3	8.1E3
1.0	6	7.0E1	9.6E1	2.1E0	1.4E1	1.6E2	2.4E2	3.4E2	4.9E2	6.7E2	9.8E2	1.6E3	1.3E3	1.8E3	4.7E3	5.4E3	1.5E3
2.0	6	1.5E0	2.6W0	1.0E2	1.8E2	4.0E2	8.9E2	1.1E3	2.4E3	4.8E3	2.5E3	2.5E4	6.7E3	2.4E3	2.6E3	1.5E4	1.8E5
.5	12	1.0E2	1.3E2	2.0E2	2.9E2	4.2E2	5.9E2	7.2E2	1.0E3	1.0E3	1.6E3	2.2E3	2.2E3	2.6E3	2.5E3	5.6E3	6.9E3
1.0	12	2.2E2	2.2E2	2.4E2	2.9E2	3.6E2	4.7E2	6.4E2	9.8E2	9.2E2	2.0E3	1.6E3	2.3E3	4.5E3	3.5E3	3.3E3	8.2E3
2.0	12	4.2E2	4.3E2	4.2E2	5.0E2	7.2E2	9.9E2	1.3E3	2.2E3	2.8E3	3.4E3	1.5E4	4.2E3	5.7E4	9.4E3	6.4E3	8.7E3

Table-4 : Dimensionless frequency factor for two opposing ends fixed and other two ends free plate for different values of R_a.

Ra	m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
.5	6	4.1E1	6.5E1	9.3E1	1.2E2	1.5E2	1.8E2	2.2E2	2.6E2	3.0E2	3.4E2	3.9E2	4.4E2	5.0E2	5.5E2	6.1E2	6.8E2
1.0	6	1.3E2	1.4E2	1.6E2	1.9E2	2.2E2	2.6E2	3.1E2	3.7E2	4.3E2	5.1E2	5.9E2	6.8E2	7.8E2	8.8E2	9.9E2	1.1E3
2.0	6	3.1E1	4.1E1	7.1E1	1.2E2	1.8E2	2.6E2	3.6E2	4.8E2	6.1E2	7.6E2	9.2E2	1.1E3	1.3E3	1.5E3	1.7E3	1.9E3
.5	12	1.4E2	1.8E2	2.2E2	2.8E2	3.4E2	4.0E2	4.6E2	5.3E2	6.0E2	6.7E2	7.4E2	8.1E2	8.9E2	9.6E2	1.0E3	1.1E3
1.0	12	1.3E2	1.4E2	1.6E2	1.9E2	2.2E2	2.6E2	3.1E2	3.7E2	4.3E2	5.1E2	5.9E2	6.8E2	7.8E2	8.8E2	9.9E2	1.1E3
2.0	12	1.3E2	1.3E2	1.5E2	1.8E2	2.3E2	3.1E2	4.0E2	5.1E2	6.4E2	7.8E2	9.4E2	1.1E3	1.3E3	1.5E3	1.7E3	2.0E3
.5	30	8.8E2	9.2E2	9.9E2	1.0E3	1.1E3	1.3E3	1.4E3	1.5E3	1.7E3	1.8E3	2.0E3	2.1E3	2.3E3	2.4E3	2.6E3	2.8E3
1.0	30	8.7E2	8.8E2	9.0E2	9.3E2	9.6E2	1.0E3	1.0E3	1.1E3	1.1E3	1.2E3	1.3E3	1.4E3	1.5E3	1.6E3	1.7E3	1.8E3
2.0	30	8.7E2	8.7E2	8.8E2	8.9E2	9.1E2	9.4E2	9.8E2	1.0E3	1.1E3	1.2E3	1.3E3	1.4E3	1.6E3	1.8E3	2.0E3	2.2E3

Table-5 : Radiation efficiency of two oppsing ends fixed and other two free plate for different mode orders.

m	n\Ψ	.01	.11	.21	.31	.41	.51	.61	.71	.81	.91	1.01	1.11	1.21	1.31	1.41	1.51
2	2	1.4E-11	2.4E-5	1.1E-3	9.9E-3	4.5E-2	1.3E-1	2.9E-1	5.3E-1	8.1E-1	1.1E0	1.2E0	1.3E0	1.2E0	1.1E0	1.0E0	9.9E-1
2	3	7.6E-14	8.2E-7	1.3E-4	2.2E-3	1.8E-2	7.6E-2	2.2E-1	4.7E-1	8.2E-1	1.2E0	1.4E0	1.4E0	1.3E0	1.1E0	1.1E0	1.1E0
3	3	2.8E-17	3.8E-8	2.1E-5	8.1E-4	9.6E-3	5.7E-2	2.1E-1	5.1E-1	9.3E-1	1.3E0	1.5E0	1.4E0	1.3E0	1.2E0	1.2E0	1.2E0
3	11	1.8E-17	8.7E-8	3.5E-6	1.3E-4	3.6E-4	3.4E-3	4.8E-3	3.5E-2	8.9E-2	1.3E0	2.5E0	1.8E0	1.5E0	1.4E0	1.3E0	1.3E0
2	12	4.4E-13	5.9E-8	2.6E-5	8.3E-5	7.3E-4	2.6E-3	4.6E-3	3.2E-2	4.1E-2	9.8E-1	2.3E0	1.6E0	1.4E0	1.2E0	1.2E0	1.1E0
11	3	5.3E-15	3.8E-5	5.2E-3	2.6E-2	4.8E-2	6.5E-2	8.2E-2	1.6E-1	1.8E-1	1.2E0	2.2E0	1.8E0	1.6E0	1.4E0	1.3E0	1.3E0
12	2	1.6E-9	1.1E-3	1.3E-2	2.6E-2	4.9E-2	5.3E-2	8.3E-2	1.4E-1	1.5E-1	1.0E0	2.3E0	1.9E0	1.6E0	1.4E0	1.3E0	1.2E0
10	11	3.9E-16	4.5E-8	1.7E-6	1.2E-5	1.4E-4	7.6E-4	2.9E-3	6.5E-2	2.3E-1	1.1E0	2.7E0	1.9E0	1.6E0	1.4E0	1.3E0	1.3E0
11	11	5.8E-18	4.8E-8	2.5E-6	1.6E-5	1.4E-4	1.1E-3	7.2E-3	9.1E-2	2.1E-1	1.1E0	2.7E0	1.9E0	1.6E0	1.4E0	1.3E0	1.3E0
12	12	5.6E-14	3.3E-8	8.8E-7	1.6E-5	1.3E-4	7.2E-4	4.3E-3	7.8E-2	1.9E-1	9.1E-1	2.8E0	1.9E0	1.6E0	1.4E0	1.3E0	1.3E0

Table-6 : Power radiation by one end fixed and other three ends free plate, $R_a = 1$, $R_t = .001$

m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
6	1.4E1	2.8E1	6.3E1	1.6E2	3.0E2	5.4E2	7.8E2	9.7E2	1.5E3	2.4E3	5.9E3	4.4E3	8.5E3	3.1E3	7.8E3	5.3E3
7	7.0E2	1.2E3	2.5E3	8.2E3	2.5E4	3.7E4	7.7E4	1.4E5	2.4E5	2.6E5	8.6E5	5.6E5	2.5E5	2.9E6	1.0E6	9.4E5
17	1.0E31	1.2E31	1.5E31	1.9E31	3.5E31	6.4E31	9.7E31	1.4E32	2.1E32	3.4E32	4.1E32	4.1E32	8.5E32	6.6E32	8.7E32	7.0E32
18	3.0E35	1.6E35	1.6E35	4.1E35	2.0E35	3.7E35	7.2E35	6.9E35	1.1E35	2.4E35	2.2E35	1.5E35	2.6E35	3.9E35	1.5E36	3.6E35

Table-7 : Power radiation by one end fixed and other three ends free plate. R_a = 1.0.

Rt	m\n	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
.001	**6**	**1.4E1**	**2.4E1**	**3.6E1**	**1.6E2**	**3.0E2**	**5.4E2**	**7.8E2**	**9.7E2**	**1.5E3**	**2.4E3**	**5.9E3**	**4.4E3**	**8.5E3**	**3.8E3**	**7.8E3**	**5.3E3**
.002	6	5.9E1	1.1E2	2.5E2	4.9E2	8.7E2	1.7E3	2.3E3	5.8E3	3.7E3	5.3E3	1.4E4	4.9E4	7.7E3	3.5E4	3.4E4	2.2E4
.004	**6**	**2.2E2**	**4.6E2**	**1.0E3**	**1.7E3**	**5.1E3**	**4.1E3**	**1.0E4**	**6.6E3**	**4.3E4**	**7.8E4**	**4.0E4**	**2.3E4**	**3.4E4**	**2.4E5**	**2.0E5**	**5.8E4**
.001	15	6.2E25	5.8E25	7.2E25	1.5E26	1.6E26	3.1E26	6.1E26	1.0E27	9.8E26	1.9E27	2.8E27	3.4E27	4.6E27	3.4E27	5.4E27	5.3E27
.002	**15**	**5.8E26**	**5.4E26**	**1.1E27**	**1.6E27**	**2.6E27**	**3.0E27**	**2.5E27**	**3.4E27**	**3.5E27**	**3.9E27**	**6.5E27**	**1.3E28**	**1.2E28**	**9.0E27**	**7.9E27**	**3.6E28**
.004	15	2.0E27	2.8E27	2.8E27	6.9E27	5.4E27	7.8E27	5.3E27	8.0E27	1.8E28	5.9E28	9.5E27	3.6E28	3.4E28	1.1E29	1.4E29	1.2E29