EFFECTS OF CONDUCTION AND CONVECTION ON MAGNETOHYDRODYNAMIC FLOW FROM A VERTICAL FLAT PLATE

by

ABDULLAH-AL-MAMUN

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Department of Mathematics

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

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Submitted by

Abdullah-Al-Mamun

Roll No. 040209009F, Registration No. 0402409, Session – April-2002 a full time student of M.Phil. (Mathematics) has been accepted as satisfactory in partial fulfillment for the degree of **MASTER OF PHILOSOPHY** in Mathematics on July 26, 2005.

BOARD OF EXAMINERS

1. **Dr. Md. Abdul Maleque** — 26.07.05 — Chairman & Supervisor
   Associate Professor
   Department of Mathematics
   BUET, Dhaka-1000

2. **Head** — 26-07-05 — Member
   Department of Mathematics
   BUET, Dhaka-1000

3. **Dr. Md. Mustafa Kamal Chowdhury** — 26-07-05 — Member
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   BUET, Dhaka-1000

4. **Dr. Shamsul Alam Sharkar** — 26.07.05 — Member (External)
   Professor
   Department of Mathematics
   University of Rajshahi
   Rajshahi-6205, Bangladesh

To

My Parents.

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I here by declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma.

Date: July 26, 2005

AAm

Abdullah- Al- Mamun

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Abstract

In this dissertation, the effects of conduction and convection on magnetohydrodynamic (MHD) boundary layer flow from a vertical flat plate have been studied numerically. The governing equations are made dimensionless by using suitable transformations. The dimensionless equations are then solved numerically in the entire region starting from the lower part of the plate to the down stream by means of implicit finite difference method known as Keller box scheme.

A steady laminar two dimensional magneto hydrodynamic (MHD) free convection flow from a vertical flat plate of thickness 'b' is investigated. The effects of the magnetic parameter M and the Prandtl number Pr have been examined on the flow field. The analysis has shown that the flow field is appreciably influenced by the effects of conduction and convection. In our study we have also found that the present results are in excellent agreement with Z.I.Khan when $M = 0$

The above effects on the magneto hydrodynamic (MHD) free convection boundary layer flow with viscous dissipation from the vertical flat plate have also been analyzed. Similarity equations of the momentum and energy equations are derived by introducing the same transformations. The dimensionless skin friction co-efficient, the surface temperature distribution, the velocity distribution and the temperature profile over the whole boundary layer are shown graphically for different values of the parameters Pr, the Prandtl number, M, the magnetic parameter and N, the viscous dissipation parameter.

Nomenclature

b	:Plate thickness
c_p	:Specific heat
d	: $(T_h - T_\infty)/T_\infty$
f	:Dimensionless stream function
g	:Acceleration due to gravity
h	: Dimensionless temperature
H_0	:Applied magnetic field
L	:Reference length, $\nu^{2/3}/g^{1/3}$
l	:Length of the plate
M	:Magnetic parameter
N	:Viscous dissipation parameter
p	:Coupling parameter
Pr	:Prandtl number
T	:Temperature of the flow fluid
T_h	:Temperature at outside of the plate
T_{s0}	:Solid temperature
T_∞	:Temperature of the ambient fluid
u	:Velocity component in the x-direction
v	:Velocity component in the y-direction
x	: Stream wise co-ordinate
y	: Transverse co-ordinate

Greek symbols

β	: Co-efficient of thermal expansion
ψ	: Stream function
τ	: Skin friction
η	: Dimensionless similarity variable

ρ	: Density of the fluid inside the boundary layer
ν	: Kinematic viscosity
μ	: Viscosity of the fluid
θ	: Dimensionless temperature
σ	: Electrical conductivity
κ	: Thermal conductivity
κ_f	: Thermal conductivity of the ambient fluid
κ_s	: Thermal conductivity of the ambient solid

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Chapter 1

Introduction

Magnetohydrodynamics (MHD), the appendage of continuum mechanics, contracts with electrically conducting fluids flow in electric and magnetic fields. The field of Astrophysics almost certainly is the main advance towards appreciative of such phenomena. MHD incorporated mainly on the study of severely incompressible fluid, but now a day the terminology is applied also to the studies of ionized gases. Magneto fluid-mechanics or magneto-aerodynamics has been suggested as other names of MHD but innovative taxonomy has persevered.

Various usual phenomenon and engineering problems are vulnerable to MHD analysis. Geophysicists come upon MHD phenomena in the connections of conducting fluids and magnetic fields that are presented in the region of blissful bodies. Engineers make use of MHD ideologies in the devise of heat exchangers, pumps and flow meters, in space vehicle forward motion, control and re-entry, in creating work of fiction power generating systems, and in emergent confinement schemes for proscribed fusion.

The production of electrical power with the flow of an electrical conducting fluid through a transverse magnetic field is the principal application of MHD. In recent times, the experimentations with ionized gases have been carried out with the look forward to of manufacturing power on a hefty scale in stationary plants with large magnetic fields. On a minor scale the generation of MHD power is of concern for space application.

On heat transfer studies, substantial outcome has been intended for towards the convective approach, in which the comparative motion of the fluid supplies an extra mechanism for the transfer of energy and of material, the last one being a more significant contemplation in cases where mass transfer, due to concentration difference, transpires. Only in the presence of a fluid medium the convection is possible. If a fluid flows in the interior of a channel or over a solid body while temperatures of the fluid and the solid surface are different, heat transfer between the fluid and the solid surface takes place as an outcome of the motion of fluid comparative to the surface; this type of mechanism of heat transfer is termed as convection. There are two basic procedures of the convective sort of heat transfer. When with the help of a pump or a fan the fluid motion is unnaturally stimulated that forces the fluid flow over the surface, the heat transfer is tenured as forced convection These types of problems are very recurrently come upon in technology where the heat transfers to or from a body is time and again due to and compelled flow of a fluid at a diverse temperature from that of the body. When on account of buoyancy effects followed by the density diversity caused by temperature variation in the fluid the fluid motion is created, the heat transfer is said to be free or natural convection. The natural convection procedures are governed essentially by three features namely the body force, the temperature difference in the flow field and the fluid density discrepancy with temperature. In the way of life, free convection is the most important style of heat transfer from pipes, transmission lines, refrigerating coils, burning radiators and various realistic situations, though in a lot of cases of sensible attention, both procedures are significant and heat transfer is by mixed convection, in which neither approach is really most important. The manipulation of natural convection of the heat transfer can be deserted in the case of hefty Reynolds number and diminutive Grashof number. Alternatively, the natural convection should be the governing aspect for immense Grashof number and miniature Reynold's number. We countenance in nature this type of situation where forced and free convection is of analogous order, the phenomena may be named as the mixed or combined convection flows.

Since in geophysical, geothermal and nuclear engineering problems free and mixed convection flows are applied, model studies of them have brought in reputations. The foremost researcher Sparrow et al. (1959) contracted with the combined forced and free convective boundary layer flow on a vertical flat plate. Afterwards by means of a perturbation series in terms of the buoyancy parameter Mori (1961) and Sparrow and Minkowycz (1962) investigated mixed convection flow in the boundary layer of a micro polar fluid above a horizontal flat plate. The formulae for calculating the shear stress and heat transfer rate in a mixed flow regime were presented by their investigations. The combined free and forced convection boundary layer flow along an inclined surface entrenched in porous medium was studied by Cheng (1977). The combined forced and free convection in boundary layer flow of a micropolar fluid above a horizontal plate was investigated by Hassanien (1977). Similarity solutions are acquired in his work for the case of wall temperature which is inversely proportional to the square root of the distance from the leading edge.

Free convective steady hydromagnetic flow about a heated vertical flat plate has been considered by Gupta (1961), Poots (1961), Osterle and Yound (1961), Sparrow and Cess (1961), Lykoudis (1962), Cramer (1963) and Riley (1964). The similarity solutions were studied by Gupta (1961) and Lykoudis (1962) considering that the magnetic field differs inversely as the fourth root of the height above the bottom edge of the plate. The solutions of the basic equations were obtained by using the approximate momentum integral technique. As for very low and very high Prandtl number the viscous and thermal boundary layers are unequal,their results are therefore of very limited applications. Afterwards Nanda and Mohanty (1970) made use of the similar technique to solve the hydro magnetic free convection of high and low Prandtl numbers owing to realistic applications, as for liquid metals, the Prandtl number is for all time small. A third degree polynomial for the temperature distribution has been considered by Lykoudis (1962) in the whole boundary layer, whereas Gupta (1961) has considered a third degree velocity profiles which contains a second order zero at the outer edge of the boundary layer. Using the perturbation technique Sparrow and Cess (1961) have considered the case of a constant magnetic field taking the non-magnetic case as the first approximation. In the weak magnetic fields and in the immediate neighborhood of the leading edge their results are applicable. The influence of magnetic field on the laminar free convection flow of liquid metals was investigated by Cramer (1963) above a vertical flat plate and between two parallel plates. For liquid metals he got an analytical solution. Riley (1964) has considered a uniform magnetic field and has integrated the boundary layer equations over a single boundary layer thickness. Effects of transversely applied magnetic field on free convection of an electrically conducting fluid past a semi-infinite plate are studied by Cobble (1979), Wilks (1976) and Wilks and Hunt (1984). MHD mixed convection flow investigated by many researchers such as Yu (1965), Gardner and Lo (1975), Hossain and Ahmed (1990) and Al-Khawaja (1999). Yu (1965) showed the stabilizing effect on combined forced and free convection channel flows similar to the case of horizontal layer heated from below. Grardner and Lo (1975) investigated the laminar problem using a perturbation method, which produced some details of the secondary flow but his result, were limited to small values of the Hartman number. Hossain and Ahmed (1990) studied the combined forced and free convection of an electrically conducting fluid past a vertical flat plate at which the surface heat flux was uniform and magnetic field was applied parallel to the direction normal to plate.

In all the above studies, the conjugate effects of conduction, and convection on magneto hydrodynamic (MHD) fluid flow were neglected. But it is well recognized that when convective heat transfer results are powerfully reliant on thermal boundary conditions. consideration of convective heat transfer problems as conjugated problems is essential to get hold of physically extra severe results. On the conjugate problems of forced convection heat transfer, lots of research labors have been given both experientially and tentatively but a small number of works have been dedicated to the conjugate problem of natural convection. The use of numerical method for solving the preliminary system of governing partial differential equations for instance the finite difference method is unmistakably the most promising in studies of conjugate natural convection was affirmed decisively by Gdalevich and Fertman (1977). In accordance with Kelleher and Yang (1967), the investigative action used comprehensively in conjugate forced convection problems are complicated owing to harmonizing a non-linear solution of free convection in a fluid with linear conduction solution in a solid body at the solid-fluid interface. About narrowed, downstairs projecting fin of a simple power low form, successful analytical solution for the problem of conjugate free convection is obtained by Lock and Gunn (1968). Along this direction with the help of the vertical dimensional analysis Chida and Katto (1976) investigated the conjugate problems. In order to interpret the formal studied conjugate heat transfer problems they applied their method. When convective heat transfer depends stoutly on the thermal boundary conditions, natural convection must be studied as a mixed problem if one need perfect analysis of the thermo-fluid dynamic field that pointed by Miyamoto et. al (1980). With respect to axial heat conduction they analyzed the comparative significance of the parameters of the problem. By expanding the analysis of Gosse (1980), Timma and padet (1985) have built up a technique which looks up the results given by the first term of asymptotic solution of the problem. An innovative correction for the assessment of heat transfer co-efficient had also been presented in the same exertion.

Joined natural convection assessing by the region which the point x_0 falls in, has been made by pozzi and Lupo (1988) humanizing the results regarding the asymptotic expansion by adding terms of higher order with respect to the first order discussed the general form of the asymptotic expansions which is singular for the presence of eigen solutions and determined the expansion holding for small values of x in a exact way. by taking into account many terms of the series using pade approximant techniques. In all the cases mentioned above, the conjugate effect of conduction and convection in presence of transverse magnetic field has not been studied.

Therefore the aim of this dissertation is to study further magnetohydrodynamic heat and mass transfer flow including the effects of conduction of convection by introducing a new class of transformations to the problem posed by Pozzi et at (1988) that the solution for x raising between 0 to ∞ that is solution near the leading edge to down stream region at the vertical surface.

In chapter-1 available information regarding MHD heat and mass transfer flows are discussed from both analytical and numerical point of view. In chapter 2 we have considered a steady laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid with viscosity and constant thermal conductivity past a vertical that plate taking into account the Prandtl number and the magnetic parameter in the presence of uniformly distributed transverse magnetic field. In Chapter-3 a two dimensional laminar MHD free convection flow with viscous dissipation from a vertical flat plate is considered. The above two problems have been solved numerically using a most practical, an efficient and accurate solution technique, known as implicit finite difference method together with Keller-box elimination technique which is well documented and widely used by Keller and Cebeci (1971) and recently by Hossain (1992).

The effects of various parameters entering into the problems are discussed with the help of graphs. We have presented a general conclusion in Chapter-4 of the models studied. At last all references extracted in the book can be come across at the last part of the thesis.

Chapter 2

Effects of conduction and convection on magneto-hydrodynamic flow from a vertical flat plate.

The interaction between an electrically conducting fluid and an applied magnetic field is an important practical problem, which has been studied very often in relation to the magneto-hydrodynamic power generator and boundary layer flow control. The natural convection boundary layer flow of an electrically conducting fluid up a hot vertical wall in the presence of a strong magnetic field has been studied by several authors because of its application in nuclear engineering in connection with the cooling of reactors. Later Crammmer and Pie (1973) presented a similarity solution for the above problem with varying surface temperature. On the other hand Wilks (1976) investigated the problem with uniform heat flux illustrating the problem by formulating in terms of regular and inverse series expansions of characterizing coordinate that provided a link between the similarity states appropriate to the leading edge and down stream. Hossain and Ahmed (1990) have studied a combined effect of forced and free convection with uniform heat flux in the presence of a strong magnetic field. Recently Z. I Khan. (M.phil thesis 2002) investigated the conjugate effect of conduction and convection with natural convection flow from a vertical flat plate. In the present investigation we shall investigate the magneto-hydrodynamic boundary layer flow and heat transfer resulting from the coupling of natural convection along and conduction inside a heated flat plate in an electrically conducting fluid in the presence of the transverse magnetic field.

The transformed non similar boundary layer equations governing the flow together with the boundary conditions based on conduction and convection were solved numerically using the Keller box (implicit finite difference) along with Newton's linearization approximation method in the entire region starting from the lower part of the plate to the down stream for some values of the magnetic parameter M and the Prandtl number Pr. We have studied the effect of the parameters M and Pr on the velocity and temperature fields as well as on the skin friction coefficient and surface temperature coefficient.

In the following sections detailed derivations of the governing equations for the flow and heat transfer and the method of solutions along with the results and discussions are presented. All the investigations for the fluid with low Prandtl number appropriates for the liquid metals are carried out.

2.1. Governing equations of the flow:

The mathematical statement of the basic conservation laws of mass, momentum and energy for the steady viscous incompressible and electrically conducting fluid are given by [Crammer and Pie (1973)]

$$\nabla.\bar{q} = 0 \tag{2.1}$$

$$\rho\left(\vec{q}.\nabla\right)\bar{q} = -\nabla p + \mu\nabla^2\bar{q} + \bar{F} + \bar{J}\times\bar{B} \tag{2.2}$$

$$\rho c_p\left(\bar{q}.\nabla\right)T = k\nabla^2 T \tag{2.3}$$

where $\vec{q} = (u, v)$, u and v are the velocity components along the x and y axes respectively, $\vec{F}$ is the body force per unit volume which is defined as $-\rho g$, the terms $\vec{J}$ and $\bar{B}$ are respectively the current density and magnetic induction vector and the term $\bar{J}\times\bar{B}$ is the force on the fluid per unit volume produced by the interaction of current and magnetic field in the absence of excess charges . T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, k is the thermal conductivity and c_p is the specific heat at constant pressure and μ is the viscosity of the fluid. In the energy equation the viscous dissipation and the Joule heating term are neglected.

Here $\vec{B} = \mu_e H_0$, μ_e being the magnetic permeability of the fluid, H_o is the applied magnetic field and ∇ is the vector differential operator and is defined by $\nabla = \hat{i}_x \frac{\partial}{\partial x} + \hat{i}_y \frac{\partial}{\partial y}$

Where $\hat{i}_x$ and the $\hat{i}_y$ are the unit vector along x and y axes respectively.

When the external electric field is zero and the induced electric field is negligible, the current density is related to the velocity by Ohm's law as follows

$$\vec{J} = \sigma\left(\vec{q} \times \vec{B}\right) \tag{2.4}$$

Where σ denotes the electric conductivity of the fluid. Next under the conduction that the magnetic Renold's number is small, the induced magnetic field is negligible compared with applied field. This condition is usually well satisfied in terrestrial applications, especially so in (low velocity) free convection flows. So we can write

$$\vec{B} = \hat{i}_y H_0 \tag{2.5}$$

Bringing together equations (2.4) and (2.5) the force per unit volume $\vec{J} \times \vec{B}$ acting along the x-axis takes the form:

$$\vec{J} \times \vec{B} = \sigma H_0^2 u \tag{2.6}$$

Under the Boussinesq approximation, the variation of ρ is taken into only in the term $\vec{F}$ in equation (2.2) and the variation of ρ in the inertia term is neglected. We then can write:

$$\rho = \rho_\infty\left[1 - \beta(T - T_\infty)\right] \tag{2.7}$$

where ρ_∞ and T_∞ are the density and temperature respectively out side boundary layer, β is the coefficient of thermal expansion.

We consider the steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a side of a vertical flat plate of thickness 'b' insulated on the edges with temperature T_b maintained on the other side in the presence of a uniformly distributed transverse magnetic field . The flow configuration and the coordinates system are shown in Figure 2.1.

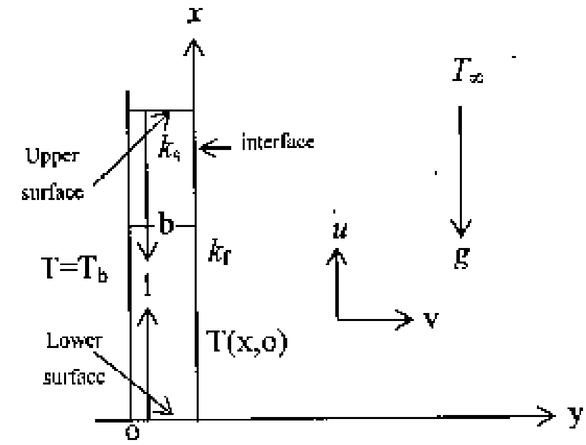

Fig.2.1: Physical model and coordinate systems.

Using the equations (2.4) to (2.6) with respect to our above considerations into the basic equations (2.1) – (2.3), the steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid with viscosity and also constant thermal conductivity past a vertical flat plate in the presence of a uniformly distributed transverse magnetic field take the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\,(T-T_\infty)-\frac{\sigma H_0{}^2 u}{\rho} \tag{2.9}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa_f}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{2.10}$$

The appropriate boundary conditions to be satisfied by the above equations are

$$u=0, v=0 \;\text{ at }\; y=0 \tag{2.11a}$$

$$u\to 0, T\to T_\infty \;\text{ as }\; y\to\infty \tag{2.11b}$$

The temperature and the heat flux are required continuous at the interface for the coupled conditions and at the interface we must have

$$\frac{k_s}{k_f}\frac{\partial T_{so}}{\partial y}=k_f\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{2.12}$$

where k_s and k_f are the thermal conductivity of the solid and the fluid respectively. The temperature T_{so} in the solid as given by A. Pozzi and M. Lupo (1988) is

$$T_{so}=T(x,0)-\{T_b-T(x,0)\}\frac{y}{b} \tag{2.13}$$

where T(x,0) is the unknown temperature at the interface to be determined from the solutions of the equations.

We observe that the equations (2.8) - (2.10) together with the boundary conditions (2.11) - (2.13) are non linear partial differential equations. In the following sections the solution methods of these equations are discussed in details.

2.2. Transformation of the governing equations

Equations (2.8) – (2.13) may now be nondimensionalized by using the following dimensionless dependent and independent variables:

$$\bar{x}=\frac{x}{L},\bar{y}=\frac{y}{L}d^{\frac{1}{4}},u=\frac{\nu}{L}d^{\frac{1}{2}}\bar{u},v=\frac{\nu}{L}d^{\frac{1}{2}}\bar{v},\frac{T-T_\infty}{T_b-T_\infty}=\theta,L=\frac{\nu^{2/3}}{g^{1/3}},d=\beta(T_b-T_\infty) \tag{2.14}$$

As the problem of natural convection, its parabolic character has no characteristic length; L has been defined in terms of ν and g which is the intrinsic properties of the system. The reference length along the 'y' direction has been modified by a factor $d^{-1/4}$ in order to eliminate this quantity from the dimensionless equations and the boundary conditions.

The magnetohydrodynamic field in the fluid is governed by the boundary layer equations, which in the non dimensional form obtained by introducing the dimensionless variables described in (2.4). may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.15}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+Mu=\frac{\partial^2 u}{\partial y^2}+\theta \tag{2.16}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{2.17}$$

where $M=\frac{\sigma H_0{}^2 L^2}{\mu d^{1/2}}$, the dimensionless magnetic parameter which is known as Hartman number (1966) and $\Pr=\frac{\mu C_p}{\kappa_f}$, the Prandtl number.

The corresponding boundary conditions (2.11) - (2.13) take the following form:

$$u=v=0,\ \theta-1=p\frac{\partial \theta}{\partial y}\ \text{ at } y=0 \tag{2.18a}$$

$$u\to 0,\ v\to 0\ \ as\ y\to\infty \tag{2.18b}$$

where p is the conjugate conduction parameter given by $p=\left(\frac{\kappa_f}{\kappa_s}\right)\left(\frac{b}{L}\right)d^{1/4}$

Here the coupling parameter 'p' governs the described problem. The order of magnitude of 'p' depends actnally on $\frac{b}{L}$ and $\frac{\kappa_f}{\kappa_s}$, $d^{1/4}$ being the order of nnity. The term $\frac{b}{L}$ attains values much greater than one because of L being small. In case of air, $\frac{\kappa_f}{\kappa_s}$ becomes very small when the vertical plate is highly conductive i.e. $\kappa_s >> 1$and for materials, $O\left(\frac{\kappa_f}{\kappa_s}\right)$ = 0.1 such as glass. Therefore in different cases 'p' is different but not always a small number. In the present investigation we have considered p = 1 which is accepted for $\frac{b}{L}$ of $O\left(\frac{\kappa_f}{\kappa_s}\right)$.

To solve the equations (2.15) – (2.17) subject to the boundary conditions (2.18), the following transformations are introduced for the flow region starting from up stream to down stream.

$$\psi = x^{4/5}(1+x)^{-1/20} f(\eta,x),\ \eta = yx^{-1/5}(1+x)^{-1/20},\ \theta = x^{1/5}(1+x)^{-1/5} h(\eta,x) \tag{2.19}$$

Here η is the dimensionless similarity variable and ψ is the stream function which satisfies the equation of continuity and $u=\dfrac{\partial\psi}{\partial y}, v=-\dfrac{\partial\psi}{\partial x}$ and $h(\eta,x)$ is the dimensionless temperature.

Substituting (2.19) into equations (2.16) and (2.17) we get the following transformed non dimensional equations.

$$f''' + \frac{16+15x}{20(1+x)} ff'' - \frac{6+5x}{10(1+x)} f'^2 - Mx^{2/5}(1+x)^{1/10} f' + h = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{2.20}$$

$$\frac{1}{P_r}h'' + \frac{16+15x}{20(1+x)} fh' - \frac{1}{5(1+x)} f'h = x\left(f'\frac{\partial h}{\partial x} - h'\frac{\partial f}{\partial x}\right) \tag{2.21}$$

In the above equations the primes denote differentiation with respect to η.

The boundary conditions (2.18) then take the following form

$$\begin{aligned} &f(x,0) = f'(x,0) = 0,\ h'(x,0) = -(1+x)^{1/4} + x^{1/5}(1+x)^{1/20} h(x,0) \\ &f'(x,\infty) = 0,\ h'(x,\infty) = 0\ . \end{aligned} \tag{2.22}$$

2.3. Method of Solution:

To get the solutions of the parabolic differential equations (2.20) and (2.21) along with the boundary condition (2.23) , we shall employ a most practical, an efficient and accurate solution technique . known as implicit finite difference method together with Keller- box elimination technique which is well documented and widely used by Keller (1978) and Cebeci (1984) and recently by Hossain (1992).

To apply the aforementioned method, we first convert the equations (2.20) and (2.21) and their boundary conditions into the system of first order equations. For this purpose we introduce new dependent variables $u\ (\xi,\eta)$, $v\ (\xi,\eta)$ and $p\ (\xi,\eta)$ so that the transformed momentum and energy equations can be written as

$$f' = u \tag{2.23}$$

$$u' = v \tag{2.24}$$

$$g' = p \tag{2.25}$$

$$v' + p_1 fv - p_2 u^2 - p_3 u + g = \xi\left(u\frac{\partial u}{\partial \xi} - v\frac{\partial f}{\partial \xi}\right) \tag{2.26}$$

$$\frac{1}{P_r} p' + p_1 fp - p_4 ug = \xi\left(u\frac{\partial g}{\partial \xi} - p\frac{\partial f}{\partial \xi}\right) \tag{2.27}$$

where $\xi = x$, $h = g$ and

$$\frac{16+5x}{20(1+x)} = p_1, \quad \frac{6+5x}{10(1+x)} = p_2, Mx^{2/5}(1+x)^{1/10} = p_3, \frac{1}{5(1+x)} = p_4$$

and the boundary conditions are

$$\begin{aligned} &f(\xi,0)=0,\ u(\xi,0)=0,\ p(\xi,0)=-(1+\xi)^{1/4}+\xi^{1/5}(1+\xi)^{1/20}g(\xi,0) \\ &u(\xi,\infty)=0, g(\xi,\infty)=0 \end{aligned} \tag{2.28}$$

We now consider the net rectangle on the (ξ, η) plane shown in the figure (2.2) and denote the net points by

$$\begin{aligned} &\xi^0 = 0, \quad \xi^n = \xi^{n-1} + k_n, \quad n = 1,2,\ldots N \\ &\eta_0 = 0, \quad \eta_j = \eta_{j-1} + h_j, \quad j = 1,2,\ldots J \end{aligned} \tag{2.29}$$

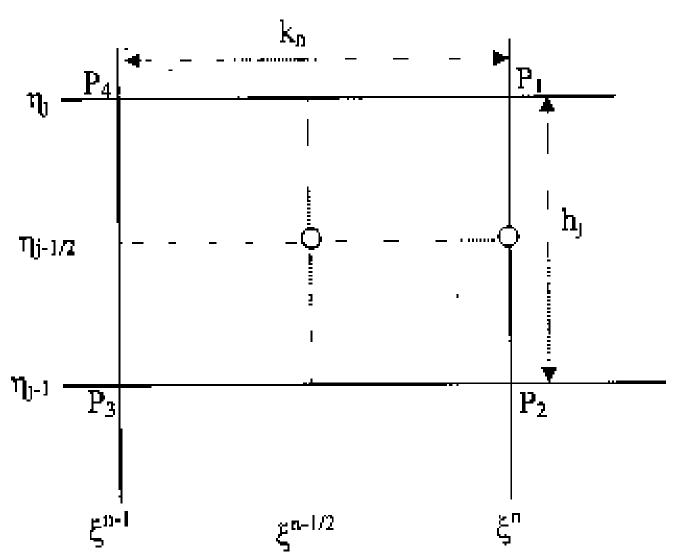

Figure 2.2: Net rectangle for difference approximations for the Box scheme

Here 'n' and 'j' are just sequence of numbers on the (ξ, η) plane. k_n and h_j are the variable mesh widths.

We approximate the quantities f, u, v, p at the points (ξ^n, η_j) of the net by f^n_j, u^n_j, v^n_j, p^n_j which we call net function . We also employ the notation g^n_j for the quantities midway between net points shown in figure (2.2) and for any net function as

$$\xi^{n-1/2} = \frac{1}{2}\left(\xi^n + \xi^{n-1}\right) \tag{2.30}$$

$$\eta_{j-1/2} = \frac{1}{2}\left(\eta_j - \eta_{j-1}\right) \tag{2.31}$$

$$g_j^{n-1/2} = \frac{1}{2}\left(g_j^n + g_j^{n-1}\right) \tag{2.32}$$

$$g^n_{j-1/2} = \frac{1}{2}\left(g_j^n + g^n_{j-1}\right) \tag{2.33}$$

The finite difference approximations according to Box method to the three first order ordinary differential equations (2.23) to (2.25) are written for the mid point $(\xi^n, \eta_{j-1/2})$ of the segment P_1P_2shown in the figure (2.2) and the finite difference approximations to the two first order differential equations (2.26) to (2.27) are written for the mid point $(\xi^{n-1/2}, \eta_{j-1/2})$ of the rectangle $P_1P_2P_3P_4$. This procedure yields.

$$h_j^{-1}\left(f_j^n - f^n_{j-1}\right) = u^n_{j-1/2} = \frac{u^n_{j-1} + u^n_j}{2} \tag{2.34}$$

$$h_j^{-1}\left(u_j^n - u^n_{j-1}\right) = v^n_{j-1/2} = \frac{v^n_{j-1} + v^n_j}{2} \tag{2.35}$$

$$h_j^{-1}\left(g_j^n - g^n_{j-1}\right) = p^n_{j-1/2} = \frac{p^n_{j-1} - p^n_j}{2} \tag{2.36}$$

$$\frac{1}{2}\left(\frac{v_j^n - v_{j-1}^n}{h_j} + \frac{v_j^{n-1} - v_{j-1}^{n-1}}{h_j}\right) + (p_1 fv)_{j-1/2}^{n-1/2} - (p_2 u^2)_{j-1/2}^{n-1/2} - (p_3 u)_{j-1/2}^{n-1/2}$$
$$+ g_{j-1/2}^{n-1/2} = \xi_{j-1/2}^{n-1/2}\left(u_{j-1/2}^{n-1/2}\frac{u_{j-1/2}^n - u_{j-1/2}^{n-1}}{k_n} - v_{j-1/2}^{n-1/2}\frac{f_{j-1/2}^n - f_{j-1/2}^{n-1}}{k_n}\right) \tag{2.37}$$

$$\frac{1}{2P_r}\left(\frac{p_j^n - p_{j-1}^n}{h_j} + \frac{p_j^{n-1} - p_{j-1}^{n-1}}{h_j}\right) + (p_1 fp)_{j-1/2}^{n-1/2} - (p_4 ug)_{j-1/2}^{n-1/2}$$
$$= \xi_{j-1/2}^{n-1/2}\left(u_{j-1/2}^{n-1/2}\frac{g_{j-1/2}^n - g_{j-1/2}^{n-1}}{k_n} - p_{j-1/2}^{n-1/2}\frac{f_{j-1/2}^n - f_{j-1/2}^{n-1}}{k_n}\right) \tag{2.38}$$

Now from the equation (2.36) we get

$$\frac{1}{2}\left(\frac{v_j^n - v_{j-1}^n}{h_j}\right) + \frac{1}{2}\left(\frac{v_j^{n-1} - v_{j-1}^{n-1}}{h_j}\right) + \frac{1}{2}\left\{(p_1 fv)_{j-1/2}^n + (p_1 fv)_{j-1/2}^{n-1}\right\}$$
$$-\frac{1}{2}\left\{(p_2 u^2)_{j-1/2}^n + (p_2 u^2)_{j-1/2}^{n-1}\right\} - \frac{1}{2}\left\{(p_3 u)_{j-1/2}^n + (p_3 u)_{j-1/2}^{n-1}\right\}$$
$$+\frac{1}{2}\left\{g_{j-1/2}^n + g_{j-1/2}^{n-1}\right\} = \frac{1}{2k_n}\xi_{j-1/2}^{n-1/2}\left(u_{j-1/2}^n + u_{j-1/2}^{n-1}\right)\left(u_{j-1/2}^n - u_{j-1/2}^{n-1}\right)$$
$$-\frac{1}{2k_n}\xi_{j-1/2}^{n-1/2}\left(v_{j-1/2}^n + v_{j-1/2}^{n-1}\right)\left(f_{j-1/2}^n - f_{j-1/2}^{n-1}\right)$$

$$\Rightarrow h_j^{-1}\left(v_j^n - v_{j-1}^n\right) + h_j^{-1}\left(v_j^{n-1} - v_{j-1}^{n-1}\right) + (p_1)_{j-1/2}^n (fv)_{j-1/2}^n + (p_1)_{j-1/2}^{n-1}(fv)_{j-1/2}^{n-1}$$
$$-(p_2)_{j-1/2}^n (u^2)_{j-1/2}^n - (p_2)_{j-1/2}^{n-1}(u^2)_{j-1/2}^{n-1} - (p_3)_{j-1/2}^n (u)_{j-1/2}^n$$
$$-(p_3)_{j-1/2}^{n-1}(u)_{j-1/2}^{n-1} + g_{j-1/2}^n + g_{j-1/2}^{n-1} = \alpha_n\left\{(u^2)_{j-1/2}^n - (u)_{j-1/2}^n (u)_{j-1/2}^{n-1}\right\}$$
$$+\alpha_n\left\{(u)_{j-1/2}^n (u)_{j-1/2}^{n-1} - (u^2)_{j-1/2}^{n-1} - (fv)_{j-1/2}^n + v_{j-1/2}^n f_{j-1/2}^{n-1} - v_{j-1/2}^{n-1} f_{j-1/2}^n\right\}$$
$$\alpha_n (fv)_{j-1/2}^{n-1}$$

$$\text{where } \alpha_n = \frac{1}{k_n}\xi_{j-1/2}^{n-1/2} \tag{2.39}$$

$$\Rightarrow h_j^{-1}\left(v_j^n - v_{j-1}^n\right)+\left\{(p_1)_{j-1/2}^n+\alpha_n\right\}(fv)_{j-1/2}^n-\left\{(p_2)_{j-1/2}^n+\alpha_n\right\}\left(u^2\right)_{j-1/2}^n$$
$$-(p_3)_{j-1/2}^n (u)_{j-1/2}^n + g_{j-1/2}^n + \alpha_n\left(v_{j-1/2}^{n-1} f_{j-1/2}^n - v_{j-1/2}^n f_{j-1/2}^{n-1}\right)$$
$$=\alpha_n\left\{-\left(u^2\right)_{j-1/2}^{n-1} + v_{j-1/2}^n f_{j-1/2}^{n-1} - v_{j-1/2}^{n-1} f_{j-1/2}^n + (fv)_{j-1/2}^{n-1}\right\}$$
$$-(p_1)_{j-1/2}^{n-1}(fv)_{j-1/2}^{n-1}+(p_2)_{j-1/2}^{n-1}\left(u^2\right)_{j-1/2}^{n-1}+(p_3)_{j-1/2}^{n-1}(u)_{j-1/2}^{n-1}$$
$$-g_{j-1/2}^{n-1} - h_j^{-1}\left(v_j^{n-1}-v_{j-1}^{n-1}\right)$$
$$\Rightarrow h_j^{-1}\left(v_j^n - v_{j-1}^n\right)+\left\{(p_1)_{j-1/2}^n+\alpha_n\right\}(fv)_{j-1/2}^n-\left\{(p_2)_{j-1/2}^n+\alpha_n\right\}\left(u^2\right)_{j-1/2}^n$$
$$-(p_3)_{j-1/2}^n (u)_{j-1/2}^n + g_{j-1/2}^n + \alpha_n\left(v_{j-1/2}^{n-1} f_{j-1/2}^n - v_{j-1/2}^n f_{j-1/2}^{n-1}\right)$$
$$=-L_{j-1/2}^{n-1}+\alpha_n\left\{(fv)_{j-1/2}^{n-1}-\left(u^2\right)_{j-1/2}^{n-1}\right\}$$
$$L_{j-1/2}^{n-1}=(p_1)_{j-1/2}^{n-1}(fv)_{j-1/2}^{n-1}-(p_2)_{j-1/2}^{n-1}\left(u^2\right)_{j-1/2}^{n-1}$$
$$+h_j^{-1}\left(v_j^{n-1}-v_{j-1}^{n-1}\right)+(p_3)_{j-1/2}^{n-1}(u)_{j-1/2}^{n-1}+g_{j-1/2}^{n-1} \tag{2.40}$$
$$\Rightarrow h_j^{-1}\left(v_j^n - v_{j-1}^n\right)+\left\{(p_1)_{j-1/2}^n+\alpha_n\right\}(fv)_{j-1/2}^n-\left\{(p_2)_{j-1/2}^n+\alpha_n\right\}\left(u^2\right)_{j-1/2}^n$$
$$-(p_3)_{j-1/2}^n (u)_{j-1/2}^n + g_{j-1/2}^n + \alpha_n\left(v_{j-1/2}^{n-1} f_{j-1/2}^n - v_{j-1/2}^n f_{j-1/2}^{n-1}\right)=R_{j-1/2}^{n-1} \tag{2.41}$$
where $R_{j-1/2}^{n-1}=-L_{j-1/2}^{n-1}+\alpha_n\left\{(fv)_{j-1/2}^{n-1}-\left(u^2\right)_{j-1/2}^{n-1}\right\}$ (2.42)

Again from the equation (2.37) we get

$$\frac{1}{P_r}h_j^{-1}\left(p_j^n-p_{j-1}^n\right)+\left\{(p_1)_{j-1/2}^{n-1/2}+\alpha_n\right\}(fp)_{j-1/2}^n-\left\{(p_4)_{j-1/2}^{n-1/2}+\alpha_n\right\}(ug)_{j-1/2}^n$$
$$+\alpha_n\left(u_{j-1/2}^n g_{j-1/2}^{n-1}-u_{j-1/2}^{n-1} g_{j-1/2}^n - p_{j-1/2}^n f_{j-1/2}^{n-1}+p_{j-1/2}^{n-1} f_{j-1/2}^n\right)$$
$$=-M_{j-1/2}^{n-1}+\alpha_n\left\{(fp)_{j-1/2}^{n-1}-(ug)_{j-1/2}^{n-1}\right\}$$
where $M_{j-1/2}^{n-1}=\dfrac{1}{P_r}h_j^{-1}\left(p_j^{n-1}-p_{j-1}^{n-1}\right)+(p_1)_{j-1/2}^{n-1/2}(fp)_{j-1/2}^{n-1}-(p_4)_{j-1/2}^{n-1/2}(ug)_{j-1/2}^n$ (2.43)

$$\frac{1}{P_r}h_j^{-1}\left(p_j^n-p_{j-1}^n\right)+\left\{(p_1)_{j-1/2}^{n-1/2}+\alpha_n\right\}(fp)_{j-1/2}^n-\left\{(p_4)_{j-1/2}^{n-1/2}+\alpha_n\right\}(ug)_{j-1/2}^n$$
$$+\alpha_n\left(u_{j-1/2}^n g_{j-1/2}^{n-1}-u_{j-1/2}^{n-1} g_{j-1/2}^n - p_{j-1/2}^n f_{j-1/2}^{n-1}+p_{j-1/2}^{n-1} f_{j-1/2}^n\right)=T_{j-1/2}^{n-1} \tag{2.44}$$
$$T_{j-1/2}^{n-1}=-M_{j-1/2}^{n-1}+\alpha_n\left\{(fp)_{j-1/2}^{n-1}-(ug)_{j-1/2}^{n-1}\right\} \tag{2.45}$$

The boundary conditions become

$$f_0^n=0\,,\quad u_0^n=0\,,\quad p_0^n=-(1+\xi)^{1/4}+\xi^{1/5}(1+\xi)^{1/20}g_0^n$$
$$u_J^n=0\,,\qquad g_J^n=0 \tag{2.46}$$

If we assume $f_j^{n-1}, u_j^{n-1}, v_j^{n-1}, g_j^{n-1}, p_j^{n-1}$ to be known for $0\le j\le J$, equations (2.33) to (2.35) and (2.40) – (2.45) form a system of 5J + 5 non linear equations for the solutions of the 5J + 5 unknowns $\left(f_j^n, u_j^n, v_j^n, g_j^n, p_j^n\right)$, j = 0,1,2 ...J. These non linear systems of algebric equations are to be linearized by Newton's Quassy linearization method. We define the iterates [$f_j^n, u_j^n, v_j^n, g_j^n, p_j^n$], i = 0, 1, 2 ...N with initial values equal those at the previous x-station (which is usually the best initial available). For the higher iterates we set

$$f_j^{(i+1)}=f_j^{(i)}+\delta f_j^{(i)} \tag{2.47}$$

$$u_j^{(i+1)}=u_j^{(i)}+\delta u_j^{(i)} \tag{2.48}$$

$$v_j^{(i+1)}=v_j^{(i)}+\delta v_j^{(i)} \tag{2.49}$$

$$g_j^{(i+1)}=g_j^{(i)}+\delta g_j^{(i)} \tag{2.50}$$

$$p_j^{(i+1)}=p_j^{(i)}+\delta p_j^{(i)} \tag{2.51}$$

Now by substituting the right hand sides of the above equations in place of f_j^n, u_j^n, v_j^n and g_j^n omitting the terms that are quadratic in δf_j^i, δu_j^i, δv_j^i, δp_j^i we get the equations (2.33) and (2.34) and (2.38) in the following form:

$$f_j^{(i)}+\delta f_j^{(i)}-f_{j-1}^{(i)}-\delta f_{j-1}^{(i)}=\frac{h_j}{2}\left\{u_j^{(i)}+\delta u_j^{(i)}+u_{j-1}^{(i)}+\delta u_{j-1}^{(i)}\right\}$$

$$\delta f_j^{(i)}-\delta f_{j-1}^{(i)}-\frac{h_j}{2}\left(\delta u_j^{(i)}+\delta u_{j-1}^{(i)}\right)=(r_1)_j \tag{2.52}$$

where $(r_1)_j=f_{j-1}^{(i)}-f_j^{(i)}+h_j u_{j-1/2}^{(i)}$ (2.53)

$$\delta u_j^{(i)}-\delta u_{j-1}^{(i)}-\frac{h_j}{2}\left(\delta v_j^{(i)}+\delta v_{j-1}^{(i)}\right)=(r_4)_j \tag{2.54}$$

where $(r_4)_j = u_{j-1}^{(i)} - u_j^{(i)} + h_j v_{j-1/2}^{(i)}$ (2.55)

$$\delta g_j^{(i)} - \delta g_{j-1}^{(i)} - \frac{h_j}{2}(\delta p_j^{(i)} + \delta p_{j-1}^{(i)}) = (r_5)_j \tag{2.56}$$

where $(r_5)_j = g_{j-1}^{(i)} - g_j^{(i)} + h_j p_{j-1/2}^{(i)}$ (2.57)

$$\begin{aligned}
&h_j^{-1}\left(v_j^{(i)} + \delta v_j^{(i)} - v_{j-1}^{(i)} - \delta v_{j-1}^{(i)}\right) + \left\{(p_1)_{j-1/2}^n + \alpha_n\right\}\left\{(fv)_{j-1/2}^{(i)} + \delta (fv)_{j-1/2}^{(i)}\right\} \\
&- \left\{(p_2)_{j-1/2}^n + \alpha_n\right\}\left\{(u^2)_{j-1/2}^{(i)} + \delta (u^2)_{j-1/2}^{(i)}\right\} - (p_3)_{j-1/2}^n \left\{(u)_{j-1/2}^{(i)} + \delta (u)_{j-1/2}^{(i)}\right\} \\
&+ g_{j-1/2}^{(i)} + \delta g_{j-1/2}^{(i)} + \alpha_n \left(f_{j-1/2}^{(i)} + \delta f_{j-1/2}^{(i)}\right) v_{j-1/2}^{n-1} \\
&- \alpha_n \left(v_{j-1/2}^{(i)} + \delta v_{j-1/2}^{(i)}\right) f_{j-1/2}^{n-1} = R_{j-1/2}^{n-1}
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow h_j^{-1}\left(v_j^{(i)} + \delta v_j^{(i)} - v_{j-1}^{(i)} - \delta v_{j-1}^{(i)}\right) + \left\{(p_1)_{j-1/2}^n + \alpha_n\right\} \\
&\left\{(fv)_{j-1/2}^{(i)} + \frac{1}{2}(f_j^{(i)}\delta(v)_j^{(i)} + v_j^{(i)}\delta(f)_j^{(i)} + f_{j-1}^{(i)}\delta(v)_{j-1}^{(i)} + v_{j-1}^{(i)}\delta(f)_{j-1}^{(i)}\right\} \\
&- \left\{(p_2)_{j-1/2}^n + \alpha_n\right\}\left\{(u^2)_{j-1/2}^{(i)} + u_j^{(i)}\,\delta(u)_j^{(i)} + u_{j-1}^{(i)}\,\delta(u)_{j-1}^{(i)}\right\} \\
&- (p_3)_{j-1/2}^n \left\{(u)_{j-1/2}^{(i)} + \frac{1}{2}(\delta(u)_j^{(i)} + \delta(u)_{j-1}^{(i)})\right\} + g_{j-1/2}^{(i)} + \frac{1}{2}(\delta g_j^{(i)} + \delta g_{j-1}^{(i)}) \\
&+ \alpha_n \left\{ v_{j-1/2}^{n-1}(f_{j-1/2}^{(i)} + \frac{1}{2}(\delta f_j^{(i)} + \delta f_{j-1}^{(i)}) - (v_{j-1/2}^{(i)} + \delta v_{j-1/2}^{(i)}) f_{j-1/2}^{n-1} \right\} = R_{j-1/2}^{n-1}
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow (s_1)_j\,\delta v_j^{(i)} + (s_2)_j\,\delta v_{j-1}^{(i)} + (s_3)_j\,\delta f_j^{(i)} + (s_4)_j\,\delta f_{j-1}^{(i)} + (s_5)_j\,\delta u_j^{(i)} \\
&+ (s_6)_j\,\delta u_{j-1}^{(i)} + (s_7)_j\,\delta g_j^{(i)} + (s_8)_j\,\delta g_{j-1}^{(i)} + (s_9)_j\,.0 + (s_{10})_j\,.0 = (r_2)_j
\end{aligned} \tag{2.58}$$

where $(s_1)_j = h_j^{-1} + \dfrac{(p_1)_{j-1/2}^n + \alpha_n}{2} f_j^{(i)} - \dfrac{1}{2}\alpha_n f_{j-1/2}^{n-1}$ (2.59)

$$(s_2)_j = -h_j^{-1} + \frac{(p_1)_{j-1/2}^n + \alpha_n}{2} f_{j-1}^{(i)} - \frac{1}{2}\alpha_n f_{j-1/2}^{n-1} \tag*{2.60)}$$

$$(s_3)_j = \frac{(p_1)_{j-1/2}^n + \alpha_n}{2} v_j^{(i)} + \frac{1}{2}\alpha_n v_{j-1/2}^{n-1} \tag{2.61}$$

$$(s_4)_j = \frac{(p_1)_{j-1/2}^n + \alpha_n}{2} v_{j-1}^{(i)} + \frac{1}{2}\alpha_n v_{j-1/2}^{n-1} \tag{2.62}$$

$$(s_5)_j = -\frac{(p_2)_{j-1/2}^n + \alpha_n}{2} u_j^{(i)} - \frac{(p_3)_{j-1/2}^n}{2} \tag{2.63}$$

$$(s_6)_j = -\frac{(p_2)_{j-1/2}^n + \alpha_n}{2} u_{j-1}^{(i)} - \frac{(p_3)_{j-1/2}^n}{2} \tag{2.64}$$

$$(s_7)_j = 0.5 \tag{2.65}$$

$$(s_8)_j = 0.5 \tag{2.66}$$

$$(s_9)_j = 0 \tag{2.67}$$

$$(s_{10})_j = 0 \tag{2.68}$$

$$\begin{aligned}(r_2)_j &= R_{j-1/2}^{n-1} - \left\{h_j^{-1}\left(v_j^{(i)} - v_{j-1}^{(i)}\right) + \left((p_1)_{j-1/2}^{n} + \alpha_n\right)(fv)_{j-1/2}^{(i)}\right\} \\ &+ \left((p_2)_{j-1/2}^{n} + \alpha_n\right)(u^2)_{j-1/2}^{(i)} - \alpha_n\left(f_{j-1/2}^{(i)} v_{j-1/2}^{n-1} - f_{j-1/2}^{n-1} v_{j-1/2}^{(i)}\right) \\ &+ \alpha_n \left\{(p_3)_{j-1/2}^{n} u_{j-1/2}^{(i)} - g_{j-1/2}^{(i)}\right\}\end{aligned} \tag{2.69}$$

Here the coefficients $(s_9)_j$ and $(s_{10})_j$, which is zero in this case, are included here for the generality.

Similarly by using the equations (2.46) to (2.50) we get the equation (2.43) in the following form:

$$\begin{aligned}&(t_1)_j\,\delta p_j^{(i)} + (t_2)_j\,\delta p_{j-1}^{(i)} + (t_3)_j\,\delta f_j^{(i)} + (t_4)_j\,\delta f_{j-1}^{(i)} + (t_5)_j\,\delta u_j^{(i)} \\ &+ (t_6)_j\,\delta u_{j-1}^{(i)} + (t_7)_j\,\delta g_j^{(i)} + (t_8)_j\,\delta g_{j-1}^{(i)} + (t_9)_j\,\delta v_j^{(i)} + (t_{10})_j\,\delta v_{j-1}^{(i)} = (r_3)_j\end{aligned} \tag{2.70}$$

where $$(t_1)_j = \frac{1}{P_r} h_j^{-1} + \frac{(p_1)_{j-1/2}^{n-1/2} + \alpha_n}{2} f_j^{(i)} - \frac{1}{2}\alpha_n f_{j-1/2}^{n-1} \tag{2.71}$$

$$(t_2)_j = -\frac{1}{P_r} h_j^{-1} + \frac{(p_1)_{j-1/2}^{n-1/2} + \alpha_n}{2} f_j^{(i)} - \frac{1}{2}\alpha_n f_{j-1/2}^{n-1} \tag{2.72}$$

$$(t_3)_j = \frac{(p_1)_{j-1/2}^{n-1/2} + \alpha_n}{2} p_j^{(i)} + \frac{1}{2}\alpha_n p_{j-1/2}^{n-1} \tag{2.73}$$

$$(t_4)_j = \frac{(p_1)_{j-1/2}^{n-1/2} + \alpha_n}{2} p_{j-1}^{(i)} + \frac{1}{2}\alpha_n p_{j-1/2}^{n-1} \tag{2.74}$$

$$(t_5)_j = \frac{(p_4)_{j-1/2}^{n-1/2} + \alpha_n}{2} g_j^{(i)} + \frac{1}{2}\alpha_n g_{j-1/2}^{n-1} \tag{2.75}$$

$$(t_6)_j = -\frac{(p_4)_{j-1/2}^{n-1/2} + \alpha_n}{2} g_{j-1}^{(i)} + \frac{1}{2}\alpha_n g_{j-1/2}^{n-1} \tag{2.76}$$

$$(t_7)_j = -\frac{(p_4)_{j-1/2}^{n-1/2} + \alpha_n}{2} u_j^{(i)} - \frac{1}{2}\alpha_n u_{j-1/2}^{n-1} \tag{2.77}$$

$$(t_8)_j = -\frac{(p_4)_{j-1/2}^{n-1/2} + \alpha_n}{2} u_{j-1}^{(i)} - \frac{1}{2}\alpha_n u_{j-1/2}^{n-1} \tag{2.78}$$

$$(t_9)_j = 0 \tag{2.79}$$

$$(t_{10})_j = 0 \tag{2.80}$$

$$\begin{aligned}(r_3)_j =& T_{j-1/2}^{n-1} - \frac{1}{P_r} h_j^{-1}\left(p_j^{(i)} - p_{j-1}^{(i)}\right) - \frac{(p_1)_{j-1/2}^{n-1/2} + \alpha_n}{2}(fp)_{j-1/2}^{(i)} \\ &+ \frac{(p_4)_{j-1/2}^{n-1/2} + \alpha_n}{2}(ug)_{j-1/2}^{(i)} - \alpha_n\left(u_{j-1/2}^{(i)} g_{j-1/2}^{n-1} - u_{j-1/2}^{n-1} g_{j-1/2}^{(i)}\right) \\ &+ \alpha_n\left(p_{j-1/2}^{(i)} f_{j-1/2}^{n-1} - p_{j-1/2}^{n-1} f_{j-1/2}^{(i)}\right)\end{aligned} \tag{2.81}$$

The boundary conditions (2.45) become

$$\begin{aligned}&\delta f_0^n = 0,\ \delta u_0^n = 0,\ \delta p_0^n = \delta\left[-(1+\xi)^{1/4} + \xi^{1/5}(1+\xi)^{1/20} g_0^n\right] \\ &\delta u_J^n = 0,\ \delta g_J^n = 0\end{aligned} \tag{2.82}$$

which just express the requirement for the boundary conditions to remain during the iteration process.

Now the system of linear equations (2.51) - (2.57) , (2.68) , (2.69) and (2.80) together with the boundary conditions (2.81) can written in a block matrix from a coefficient matrix , which are solved by modified 'Keller Box' methods especially introduced by Keller (1978) . Later, this method has been used most efficiently by Cebeci and Bradshow (1984) and recently by Hossain (1992). Hossain et. all. (1994), taking the initial iteration to be given by convergent solution at $\zeta = \zeta_{j-1}$. Results are shown in graphical form by using the numerical values obtained from the above technique.

The solutions of the above equations (2.20) and (2.21) together with the boundary conditions (2.23) enable us to calculate the skin friction τ and the rate of heat transfer θ at the surface in the boundary layer from the following relations:

$$\tau = x^{2/5}(1+x)^{-3/20} f''(0,x) \tag{2.83}$$

$$\theta = x^{1/5}(1+x)^{-1/5} h(0,x) \tag{2.84}$$

2.4. Results and discussion

If we know the values of the functions $f(\eta, x)$, $h(\eta, x)$ and their derivatives for different values of the Prandtl number Pr and the magnetic parameter M, we may calculate the numerical values of the surface temperature $\theta(0, x)$ and the velocity gradient $f''(0, x)$ at the surface that are important from the physical point of view.

First of all, to verify the proper treatment for the problem, the present solution for $M = 0.0$ has been compared with that of Z. I. Khan (2002). It can be seen from the Fig.2.3 and Fig.2.4 that the present results are in excellent agreement with Z. I. Khan.

Numerical values of the velocity gradient $f''(0, x)$ and the surface temperature $\theta(0, x)$ are depicted graphically in Fig.2.5 and Fig.2.6 respectively against the axial distance x in the interval [0, 30] for different values of the magnetic parameter M (= 0.2, 0.5, 0.8, 1.0) for the fluid haviug Prandtl number $Pr = 0.73$. In Fig.2.7 and Fig2.8, the shear stress coefficient $f''(0,x)$ and the surface temperature $\theta(0, x)$ respectively are shown graphically for different values of the Praudtl number Pr (= 0.05 , 0.73 , 1.0) when the value of the magnetic parameter M is 1. The values of the Prandtl number Pr are taken to 0.05 that corresponds physically the sodium, 0.73 that corresponds the air and 1.0 corresponding to electrolyte solutions such as salt water.

From Fig. 2.5 , it can be observed that increase in the value of the magnetic parameter M leads to decrease the value of the shear stress coefficient $f''(0,x)$. Again Fig.2.6 shows that the increase of the magnetic parameter M leads to increase the surface temperature $\theta(0, x)$. Similar results hold for skin friction $f''(0, x)$ and surface temperature distribution $\theta(0, x)$ shown in Fig 2.9 and Fig.2.10 respectively for different values of the magnetic parameter M when Prandtl number $Pr = 0.05$

From Fig. 2.7, it is shown that the shear stress coefficient $f''(0,x)$ decreases monotonically with the increase of the Prandtl number Pr (= 0.05 , 0.73 , 1.0) and from the Fig. 2.8, the same result is observed on the surface temperature distribution due to increase of the value of the Prandtl number when the value of the magnetic parameter M is 1. Fig 2.11 and Fig.2.12 represent the same result of skin friction and surface temperature distribution respectively for different values of Prandtl number Pr when the value of the magnetic parameter $M = 0.5$

Fig. 2.13 and Fig.2.14 deal with the effect of the magnetic parameter M (= 0.2, 0.5, 0.8, 1.0) for $Pr = 0.73$ on the velocity profile $f'(\eta, x)$ and the temperature profile $\theta\ (\eta, x)$. From Fig. 2.13, it is revealed that the velocity profile $f'(\eta, x)$ decreases with the increase of the magnetic parameter M which indicates that the magnetic parameter retards the fluid motion. Fig.2.14 shows that for increasing values of M, the temperature profile $\theta\ (\eta, x)$ increases . Same results are observed from Fig.2.17 and Fig.2.18 in velocity profile $f'(\eta, x)$ and temperature profile $\theta\ (\eta, x)$ respectively for different values of M while Prandtl $Pr = 0.05$

Fig. 2.15 depicts the velocity profile for different values of the Prandtl number Pr (= 0.05, 0.73, 1.0) while the magnetic parameter M is 1. Corresponding distribution of the temperature profile $\theta\ (\eta, x)$ in the fluids is shown in Fig. 2.16. From Fig. 2.15, it can be seen that if the Prandtl number increases , the velocity of the fluid decreases . On the other hand , from Fig. 2.16 we observe that the temperature profile decreases within the boundary layer due to increase of the Prandtl number Pr. Fig. 2.19 and Fig.2.20 represent the same result occurred in Fig.2.15 and Fig.2.16 in velocity profile $f'(\eta, x)$ and temperature profile $\theta\ (\eta, x)$ respectively for different values of the Prandtl number Pr when the value of the magnetic parameter $M = 0.6$.

2.5. Conclusion

The coupled effect of natural convection and conduction on the magnetohydrodynamic boundary layer flow about a vertical flat has been investigated introducing a new class of transformations. The coupling of conduction required that the temperature and the heat flux be continuous at the interface. Numerical solutions of the equations governing the flow are obtained by using the implicit finite difference method together with the Keller Box scheme. From the present investigation, the following conclusion may be drawn:

1. The skin friction coefficient and the velocity distribution decrease for increasing value of the magnetic parameter M.

2. Increased value of the magnetic parameter M leads to increase the surface temperature distribution as well as the temperature distribution.

3. It has been observed that the skin friction coefficient, the temperamre distribution and the velocity distribution decrease with the increase of the Prandtl number Pr.

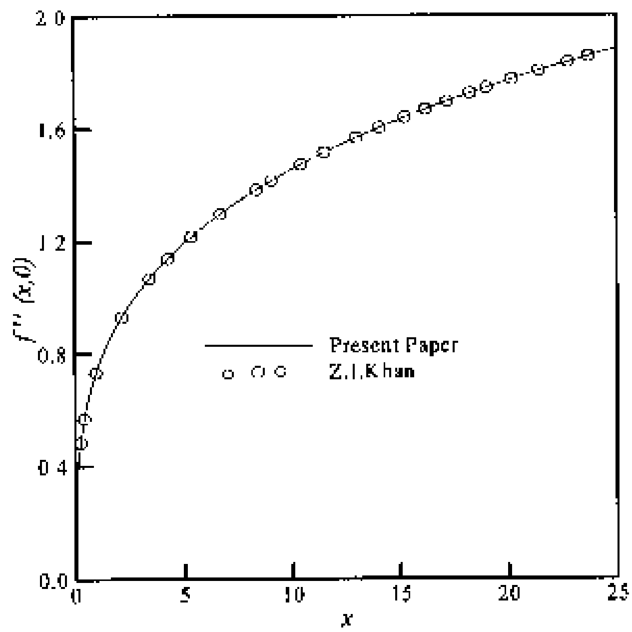

Fig.2.3: Comparison of skin frictions when Prandtl number Pr = 0.73 and M = 0.0

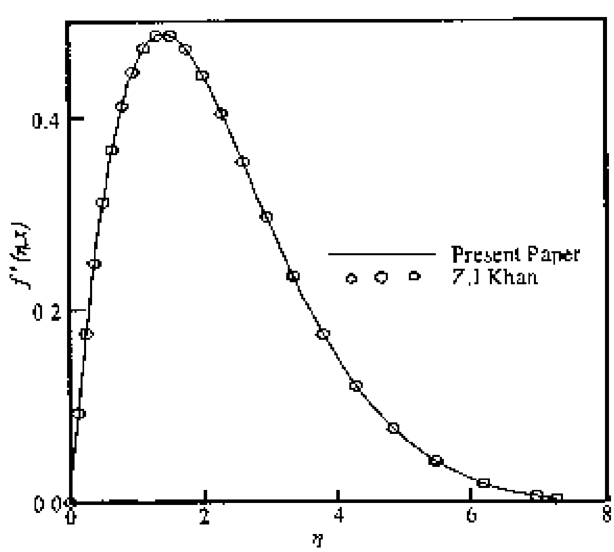

Fig.2.4: Comparison of velocity profiles when Prandtl number Pr =0.73 and M= 0.

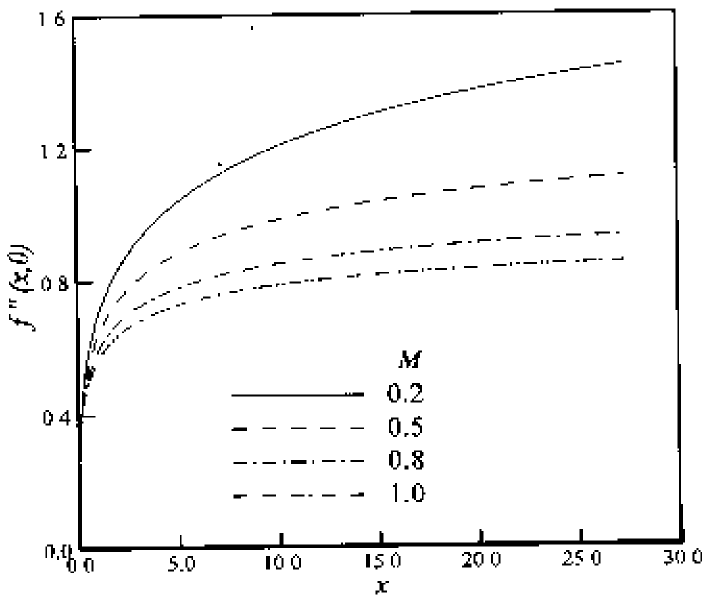

Fig.2.5: Skin frictions for different values of M when number $Pr = 0.73$

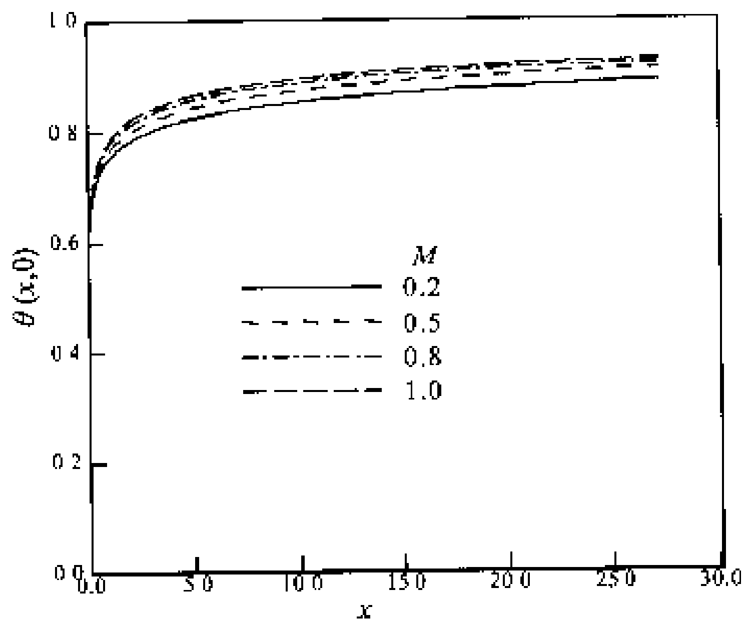

Fig.2.6:Surface temperature distribution for different values of M when $Pr = 0.73$

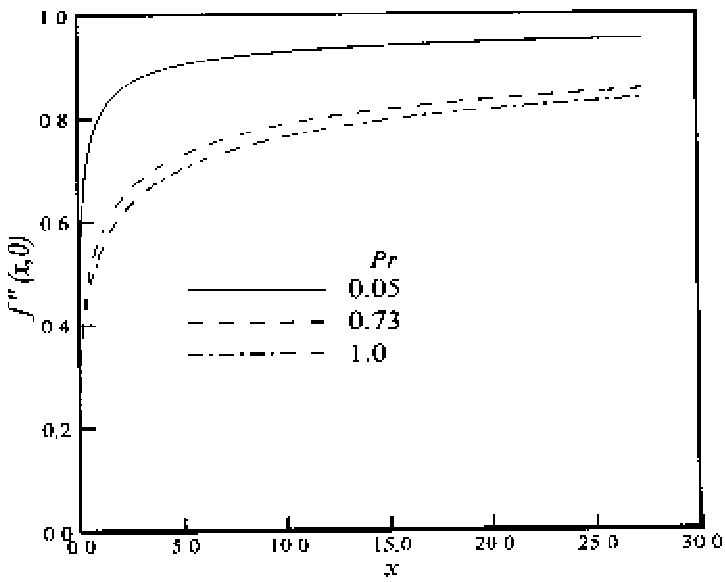

Fig.2.7:Skin frictions for different values of Pr when $M = 1.0$

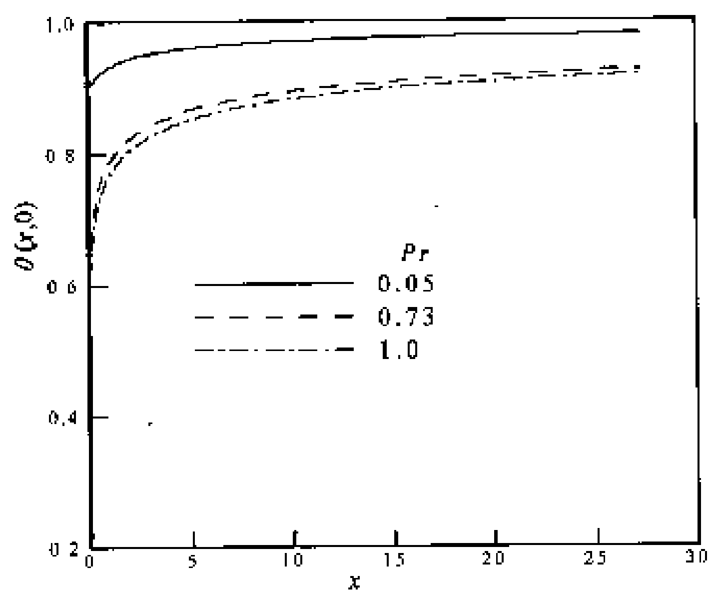

Fig.2.8:Surface temperature distribution for different values of Pr when $M = 1.0$

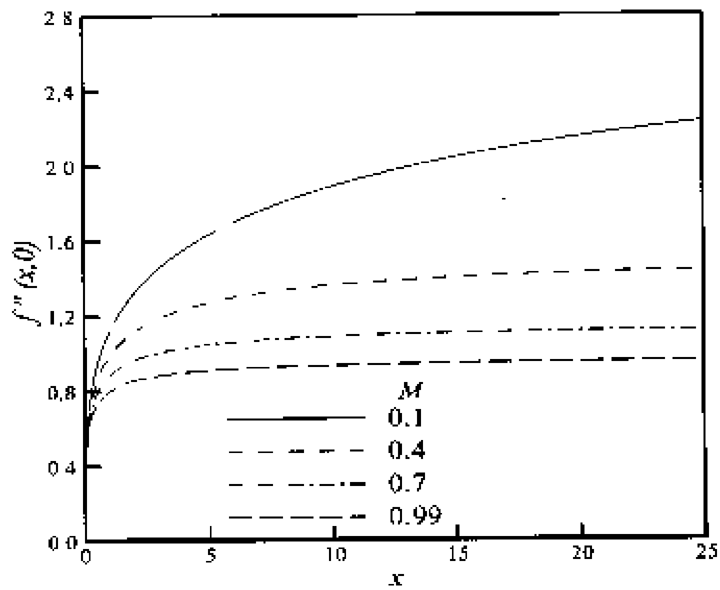

Fig.2.9: Skin frictions for different values of M when number $Pr = 0.05$

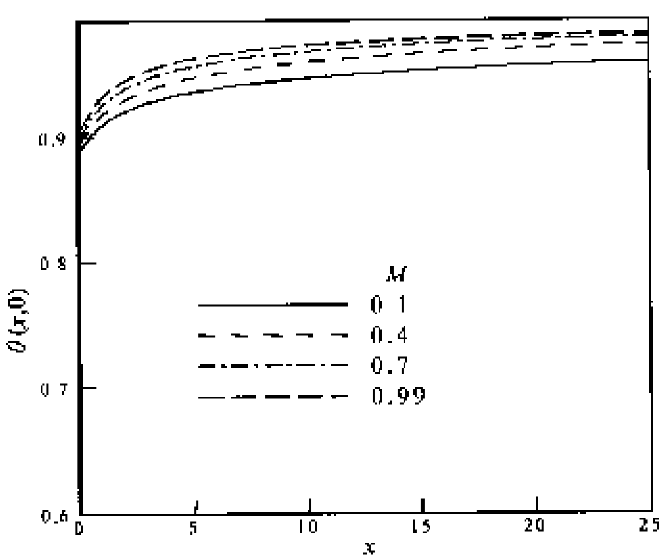

Fig.2.10: Surface temperature distribution for different values of M when Pr =0.05

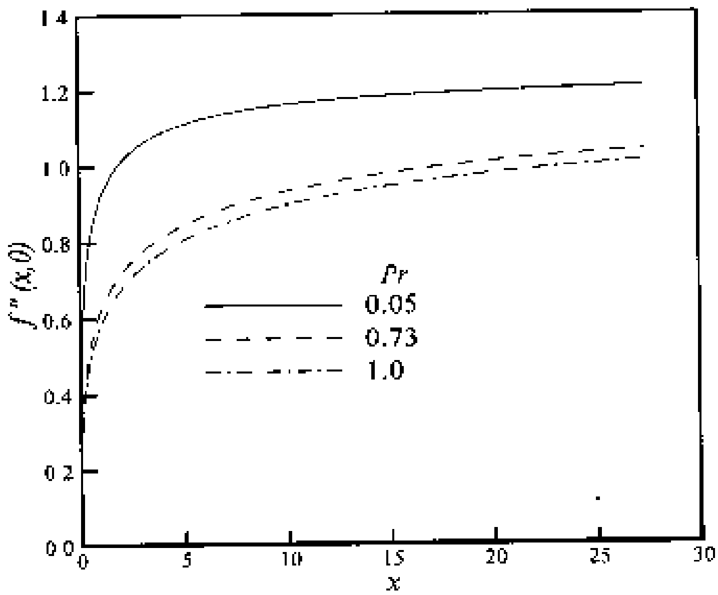

Fig.2.11:Skin frictions for different values of Pr when $M = 0.6$

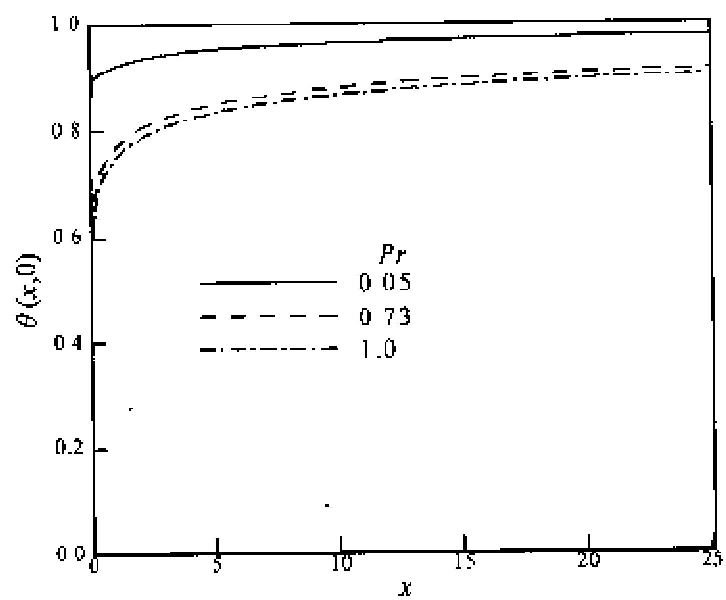

Fig.2.12:Surface temperature distribution for different values of Pr when $M = 0.6$

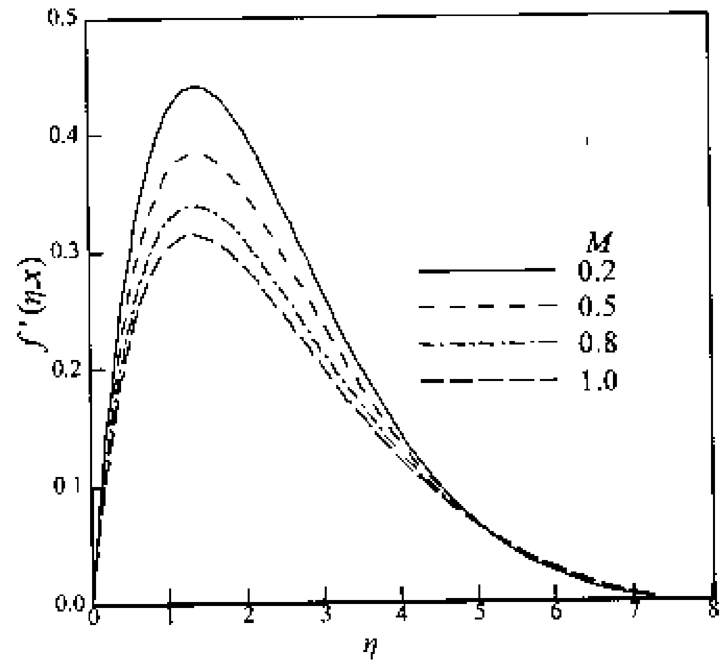

Fig.2.13 :Velocity profile for different values of M when $Pr = 0.73$

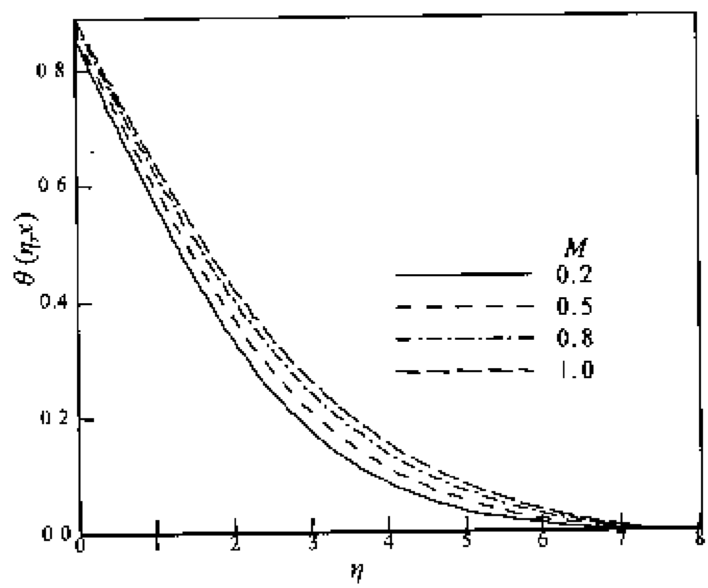

Fig.2.14:Temperature profile for different values of M when $Pr = 0.73$

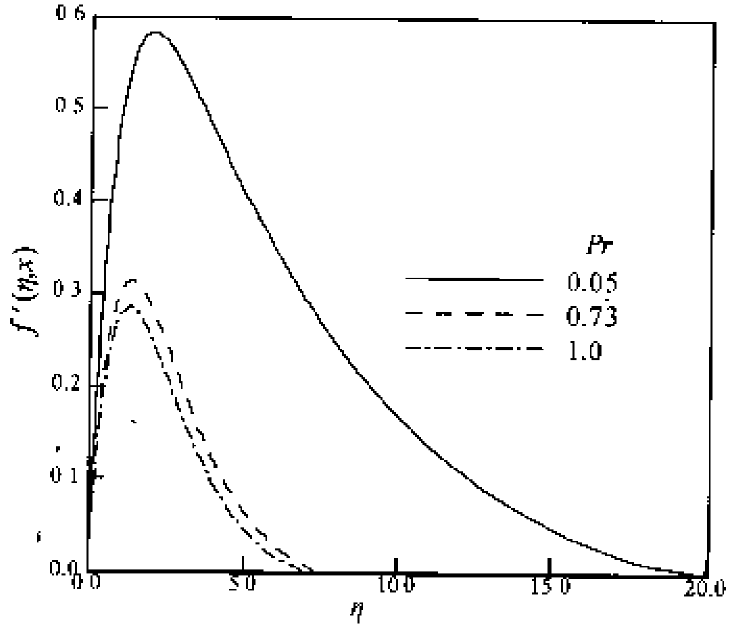

Fig.2.15:Velocity profile for different values of Pr when $M = 1.0$

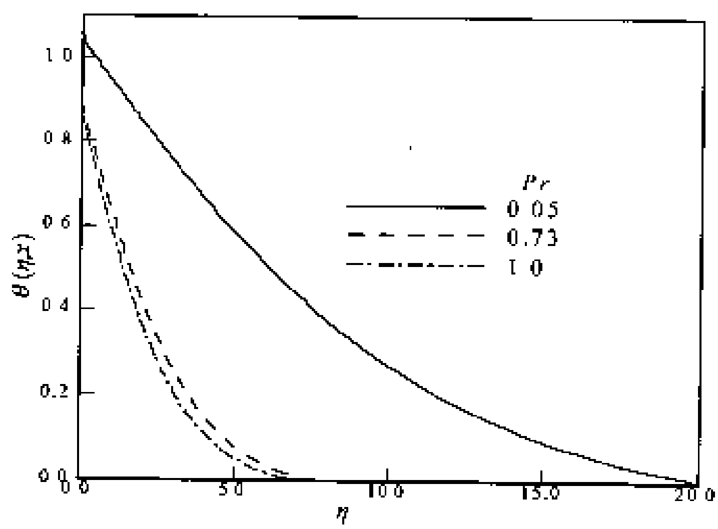

Fig.2.16: Temperature profile for different values of Pr when $M = 1.0$

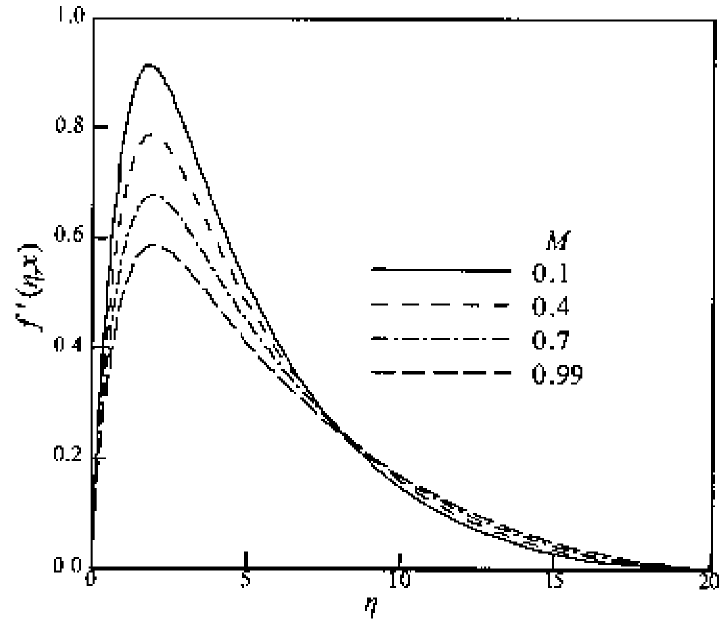

Fig.2.17:Velocity profile for different values of M when $Pr = 0.05$

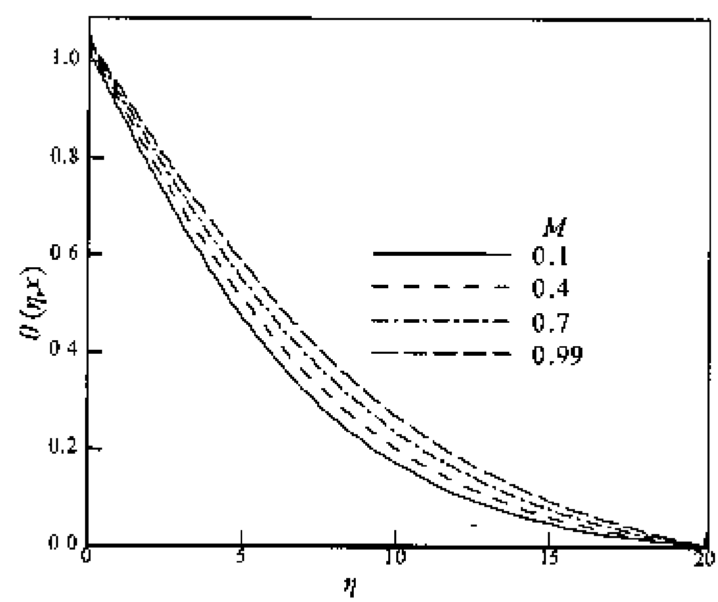

Fig.2.18:Temperature profile for different values of M when $Pr = 0.05$

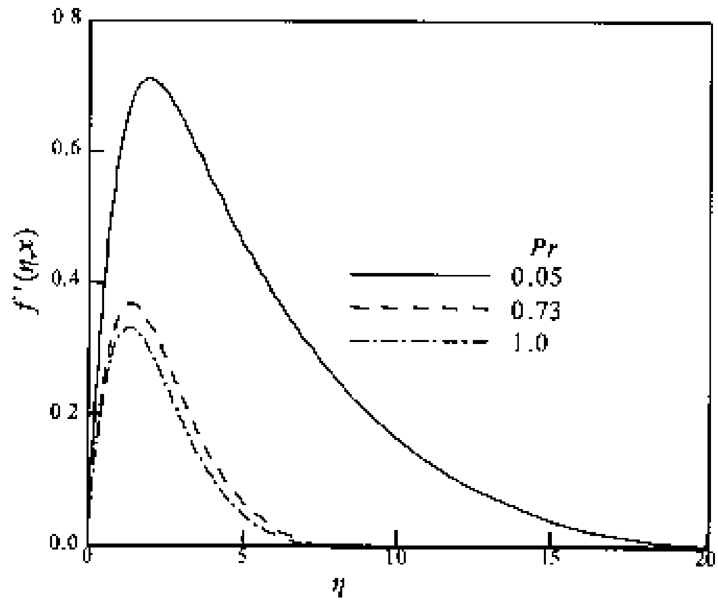

Fig.2.19: Velocity profile for different values of Pr when M=0.6

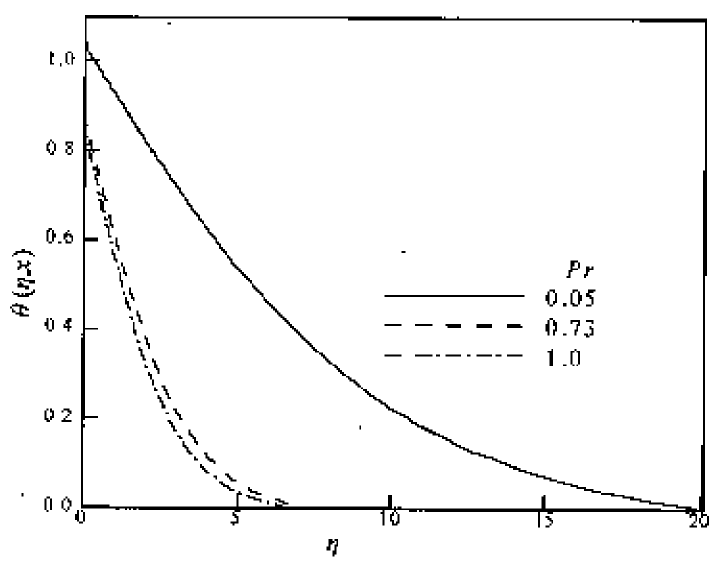

Fig.2.20:Temperature profile for different values of Pr when M= 0.6

Chapter 3

Effects of conduction and convection on magneto hydrodynamic flow with viscous dissipation from a vertical flat plate.

Free convection flow is often encountered in cooling of nuclear reactors or in the study of the structure of stars and planets. Along with the free convection flow the phenomenon of the boundary layer flow of an electrically conducting fluid up a vertical flat plate in the presence of a strong magnetic field is also very common because of its application in nuclear engineering in connection with the cooling of reactors. Gebhart (1962) has shown that the viscous dissipation effect plays an important role in natural convection in various devices which are subjected to large deceleration or which operate at high rotative speeds and also in strong gravitational field processes on large scales (on large planets) and in geological processes. With this understanding Takhar and Soundalgekar (1980) have studied the effects of viscous and Joule heating on the problem posed by Sparrow and Cess (1961), using the series expansion method of Gebhart (1962). Hossain (1992) have studied the effect of viscous and Joule heating on the flow of an electrically conducting and viscous incompressible fluid past a semi infinite plate of which temperature varies linearly with the distance from the leading edge and in the presence of uniform transverse magnetic field. He has solved the equations numerically governing the flow applying the finite difference method along with Newton's linearization approximation. Alam (1995) has investigated the effects of viscous dissipation as well as Joule heating on the unsteady magneto hydrodynamic free convection and mass transfer flow with Hall current of an electrically conducting and viscous incompressible fluid past an accelerated infinite vertical porous plate with time dependent wall temperature and concentration.

In the present chapter we shall investigate the viscous dissipation effect on the skin friction and the rate of heat transfer in the entire region from up stream to down stream of a viscous incompressible and electrically conducting fluid from a vertical flat plate in presence of transverse magnetic field.

In the following sections detailed derivations of the governing equations for the flow and heat transfer and the method of solutions along with the results and discussions are presented. All the investigations for the fluid with low Prandtl number appropriates for the liquid metals are carried out.

3.1 Governing equations of the flow:

We consider the steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a side of a vertical flat plate of thickness 'b' insulated on the edges with temperature T_b maintained on the other side in the presence of a uniformly distributed transverse magnetic field . The flow configuration and the coordinates system are shown in Fig.3.1.

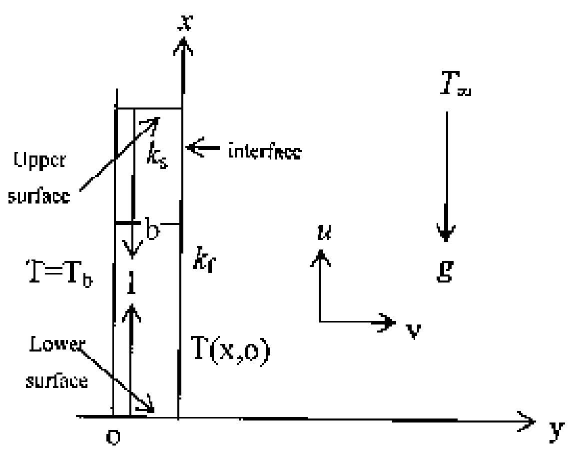

Fig.3.1: Physical configuration and coordinates system.

The equations governing the flow are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma H_0{}^2 u}{\rho} \tag{3.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa_f}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3.3}$$

The appropriate boundary conditions to be satisfied by the above equations are

$$u=0, v=0 \ \text{ at } \ y=0 \tag{3.4}$$

$$u \to 0, T \to T_\infty \ \text{ as } \ y \to \infty$$

The temperature and the heat flux are required continuous at the interface for the coupled conditions and at the interface we must have

$$\frac{k_s}{k_f}\frac{\partial T_{so}}{\partial y}=k_f\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{3.5}$$

where k_s and k_f are the thermal conductivity of the solid and the fluid respectively. The temperature T_{so} in the solid as given by A. Pozzi and M. Lnpo (1988) is

$$T_{so}=T(x,0)-\{T_b-T(x,0)\}\frac{y}{b} \tag{3.6}$$

where $T(x,0)$ is the unknown temperature at the interface to be determined from the solutions of the equations.

We observe that the equations (3.1) - (3.3) together with the boundary conditions (3.4) - (3.5) are non linear partial differential equations. In the following sections the solution methods of these equations are discussed details.

3.2. Transformation of the governing equations

Equations (3.1) – (3.3) may now be nondimensionalized by using the following dimensionless dependent and independent variables:

$$\bar{x} = \frac{x}{L}, \bar{y} = \frac{y}{L} d^{\frac{1}{4}}, u = \frac{\nu}{L} d^{\frac{1}{2}} \bar{u}, v = \frac{\nu}{L} d^{\frac{1}{2}} \bar{v}, \frac{T - T_\infty}{T_b - T_\infty} = \theta, L = \frac{\nu^{2/3}}{g^{1/3}}, d = \beta(T_b - T_\infty) \quad (3.7)$$

As the problem of natural convection, its parabolic character, has no characteristic length, L has been defined in terms of ν and g which are the intrinsic properties of the system. The reference length along the 'y' direction has been modified by a factor $d^{-1/4}$ in order to eliminate this quantity from the dimensionless equations and the boundary conditions.

The magnetohydrodynamic field in the fluid is governed by the boundary layer equations, which in the non dimensional form obtained by introducing the dimensionless variables described in (3.7), may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.8)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + Mu = \frac{\partial^2 u}{\partial y^2} + \theta \quad (3.9)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + N\left(\frac{\partial u}{\partial y}\right)^2 \quad (3.10)$$

where $M = \frac{\sigma H_0{}^2 L^2}{\mu d^{1/2}}$, the dimensionless magnetic parameter, $P_r = \frac{\mu C_p}{\kappa}$, the Prandtl number and $N = \frac{\nu^2 d}{L^2 c_p (T_b - T_\infty)}$, the dimensionless viscous dissipation parameter.

The corresponding boundary conditions (3.4) - (3.6) take the following form:

$$u=v=0,\ \theta-1=p\frac{\partial\theta}{\partial y} \text{ at } y=0 \tag{3.11a}$$

$$u\to 0,\ v\to 0 \text{ as } y\to\infty \tag{3.11b}$$

where p is the conjugate conduction parameter given by $p=\left(\frac{\kappa_f}{\kappa_s}\right)\left(\frac{b}{L}\right)d^{1/4}$

Here the coupling parameter 'p' governs the described problem. The order of magnitude of 'p' depends actually on $\frac{b}{L}$ and $\frac{\kappa_f}{\kappa_s}$. $d^{1/4}$ being the order of unity. The term $\frac{b}{L}$ attains values much greater than one because of L being small. In case of air, $\frac{\kappa_f}{\kappa_s}$ becomes very small when the vertical plate is highly conductive i.e. $\kappa_s >> 1$ and for materials, $O\left(\frac{\kappa_f}{\kappa_s}\right) = 0.1$ such as glass. Therefore in different cases 'p' is different but not always a small number. In the present investigation we have considered $p = 1$ which is accepted for $\frac{b}{L}$ of $O\left(\frac{\kappa_f}{\kappa_s}\right)$.

To solve the equations (3.8) – (3.10) subject to the boundary conditions (3.11), the following transformations were introduced for the flow region starting from up stream to down stream.

$$\psi=x^{4/5}(1+x)^{-1/20}f(\eta,x),\ \eta=yx^{-1/5}(1+x)^{-1/20},\ \theta=x^{1/5}(1+x)^{-1/5}h(\eta,x) \tag{3.12}$$

Here η is the dimensionless similarity variable and ψ is the stream function which satisfies the equation of continuity and $u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}$ and $h(\eta,x)$ is the dimensionless temperature.

Substituting (3.12) into equations (3.9) and (3.10) we get the following transformed non dimensional equations.

$$f''' + \frac{16+15x}{20(1+x)} ff'' - \frac{6+5x}{10(1+x)} f'^2 - Mx^{2/5}(1+x)^{1/10} f' + h = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{3.13}$$

$$\frac{1}{P_r} h'' + \frac{16+15x}{20(1+x)} fh' - \frac{1}{5(1+x)} f'h + Nxf''^2 = x\left(f'\frac{\partial h}{\partial x} - h'\frac{\partial f}{\partial x}\right) \tag{3.14}$$

In the above equations the primes denote differentiation with respect to η.

The boundary conditions (3.11) then take the following form

$$\begin{aligned} &f(x,0) = f'(x,0) = 0,\, h'(x,0) = -(1+x)^{1/4} + x^{1/5}(1+x)^{1/20} h(x,0) \\ &f'(x,\infty) = 0,\, h'(x,\infty) = 0 \end{aligned} \tag{3.15}$$

3.3. Method of Solution:

To get the solutions of the parabolic differential equations (3.13) and (3.14) along with the boundary condition (3.15), we shall employ a most practical, an efficient and accurate solution technique , known as implicit finite difference method together with Keller- box elimination technique which is well documented and widely used by Keller and Cebeci (1971) and recently by Hossain (1992). Since a good description of this method has been discussed in details in Chapter-2, further discussion is disregarded here. The numerical results obtained are presented in the following section

3. 4. Results and discussion

Here we have investigated the problem of the steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid with viscous dissipation along a side of a vertical flat plate of thickness 'b' insulated on the edges with temperature T_b maintained on the other side in the presence of a uniformly distributed transverse magnetic field. Solutions are obtained for the fluid having Prandtl number Pr = (0.05, 0.73, 1.0) and for a wide range of the values of the viscous dissipation parameter N = (0.01, 0.1, 0.3, 0.5, 0.6, 0.8, 1.0) and the magnetic parameter M = (0.1, 0.2, 0.5, 0.8, 1.0). If we know the values of the functions $f(\eta, x)$, $h(\eta, x)$ and their derivatives for different values of the Prandtl number Pr, the magnetic parameter M and the viscous dissipation parameter N, we may calculate the numerical values of the surface temperature $\theta(0, x)$ and the velocity gradient $f''(0, x)$ at the surface that are important from the physical point of view.

Numerical values of the velocity gradient $f''(0, x)$ and the surface temperature $\theta(0, x)$ are depicted graphically in Fig.3.2 and Fig.3.3 respectively against the axial distance x in the interval [0, 30] for different values of the viscous dissipation parameter N (= 0.3, 0.6, 0.8, 1.0) for the fluid having Prandtl number Pr = 0.73 and the magnetic parameter M=1.0. In Fig.3.4 and Fig.3.5, the shear stress coefficient $f''(0, x)$ and the surface temperature $\theta(0, x)$ are shown graphically for different values of the Prandtl number Pr (= 0.05, 0.73, 1.0) when value of the magnetic parameter M is 0.1 and the viscous dissipation parameter N = 0.01. The values of the Prandtl number Pr are taken to 0.05 that corresponds physically the sodium, 0.73 that corresponds the air and 1.0 corresponding to electrolyte solutions such as salt water.

From Fig.3.2, it can be observed that increase in the value of the viscous dissipation parameter N leads to increase of the value of the shear stress coefficient $f''(0,x)$ which is usually expected.

Again Fig.3.3 shows that the increase of the viscons dissipation Parameter N leads to increase of the snrface temperature $\theta(0,x)$. Similar results are observed from Fig.3.6 and

Fig.3.7 for skin friction $f''(0, x)$ and surface temperature distribution $\theta(0, x)$ respectively for different values of the magnetic M when Prandtl number Pr = 0.73 and viscous dissipation parameter N = 0.5

From Fig.3.4, it is shown that the shear stress coefficient $f''(0,x)$ decreases monotonically with the increase of the Prandtl number Pr (= 0.05 , 0.73 , 1.0) and from the Fig.3.5, the same result is observed on the surface temperature distribution due to increase of the value of the Prandtl number when the value of the magnetic parameter M is 0.1 and the value of the viscous dissipation parameter N = 0.01.

Fig.3.8 and Fig.3.9 deal with the effect of the viscous dissipation parameter N (= 0.1, 0.5, 0.8 , 1.0) for Pr = 0.73 and for the magnetic parameter M = 1.0 on the velocity profile $f'(\eta, x)$ and the temperature profile $\theta(\eta, x)$. From Fig. 3.8 , it is revealed that the velocity profile $f'(\eta, x)$ increases slowly with the increase of the viscous dissipation parameter N which indicates that viscous dissipation increases the fluid motion .Small increment is shown from Fig.3.9 on the temperature profile $\theta(\eta, x)$ for increasing values of N . From Fig.3.12 we observe that the velocity profiles decrease monotonically with the increase of the magnetic parameter M when the values of the Prandtl number Pr and the dissipation parameter N are respectively 0.73 and 0.5. Opposite result is shown in Fig.3.13 for temperature distribution for the same values of the magnetic parameter M.

Fig. 3.10 depicts the velocity profile for different values of the Prandtl number Pr (= 0.05, 0.73, 1.0) while the magnetic parameter M is 0.1 and the viscous dissipation parameter N = 0.01. Corresponding distribution of the temperature profile $\theta(\eta, x)$ in the fluids is shown in Fig. 3.11. From Fig.3.10, it can be seen that if the Prandtl number increases, the velocity of the fluid decreases. On the other hand, from Fig.3.11 we observe that the temperature profile decreases within the boundary layer due to increase of the Prandtl number Pr.

3.5. Conclusion

The effect of viscous dissipation and magnetic parameters N and M respectively for small Prandtl number Pr (= 0.05, 0.73, 1.0) on the magneto hydrodynamic(MHD) natural convection boundary layer flow with viscous dissipation from a vertical flat plate has been studied introducing a new class of transformations. The transformed non similar boundary layer equations governing the flow together with the boundary conditions based on conduction and convection were solved numerically using the very efficient implicit finite difference method together with Keller box scheme. The coupled effect of natural convection and condnction required that the temperature and the heat flux be continuous at the interface. From the present investigation, the following conclusions may be drawn:

1. The skin friction coefficient and the velocity distribution increase for increasing value of the viscous dissipation parameter N.

2. Increased value of the viscous dissipation parameter N leads to increase the surface temperature distribution as well as the temperature distribution.

3. It has been observed that the skin friction coefficient, the surface temperature distribution, the temperature distribution over the whole boundary layer and the velocity distribution decrease with the increase of the Prandtl number Pr.

4. The skin friction coefficient, the surface temperature distribution and the velocity profile decrease while the temperature profile increases for the increased values of the magnetic parameter M.

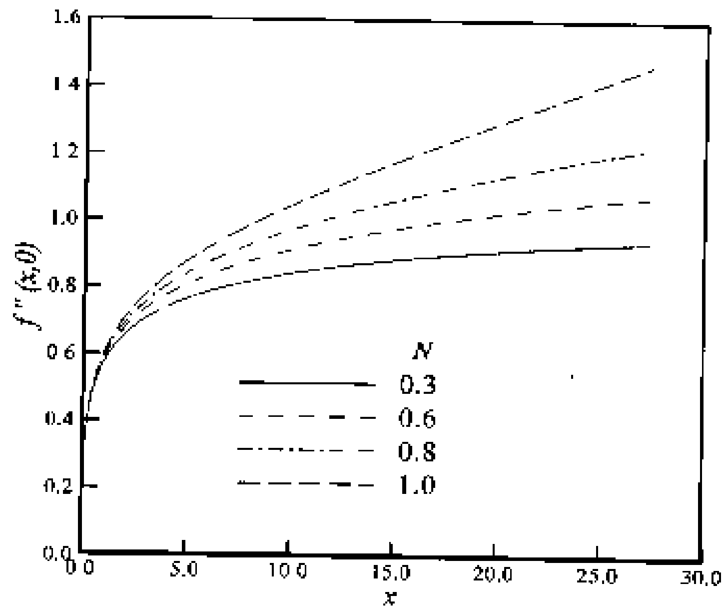

Fig.3.2:Skin frictions for different values of N when number $Pr = 0.73$ and $M = 1.0$

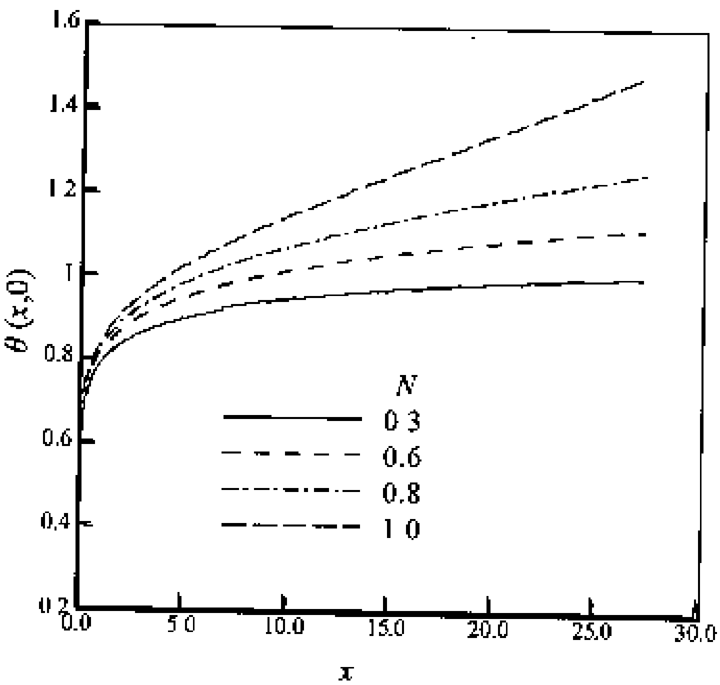

Fig.3.3:Surface temperature distribution for different values of N when Pr =0.73 and M =1.0

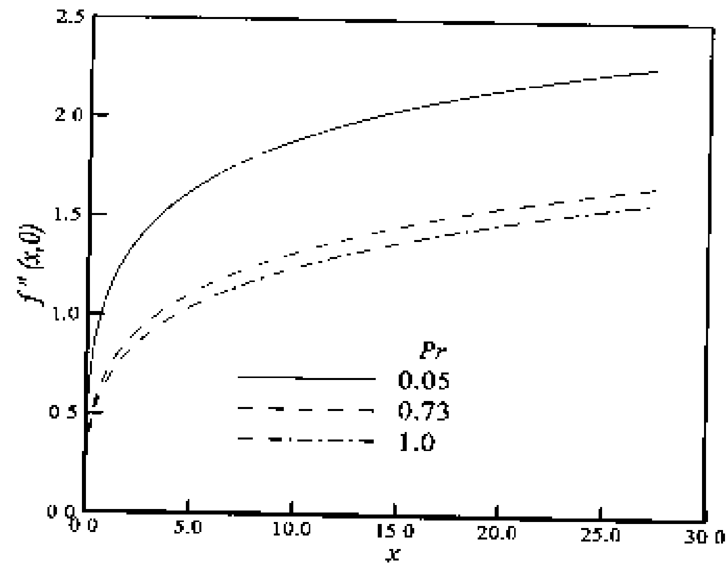

Fig.3.4:Skin frictions for different values of Pr when $M = 0.1$ and $N = 0.01$

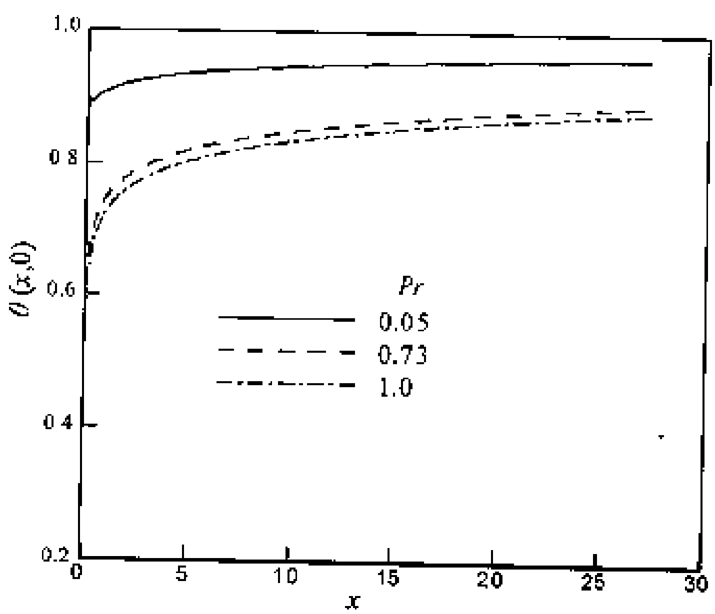

Fig.3.5:Surface temperature distribution for different values of Pr when M=0.1and $N = 0.01$

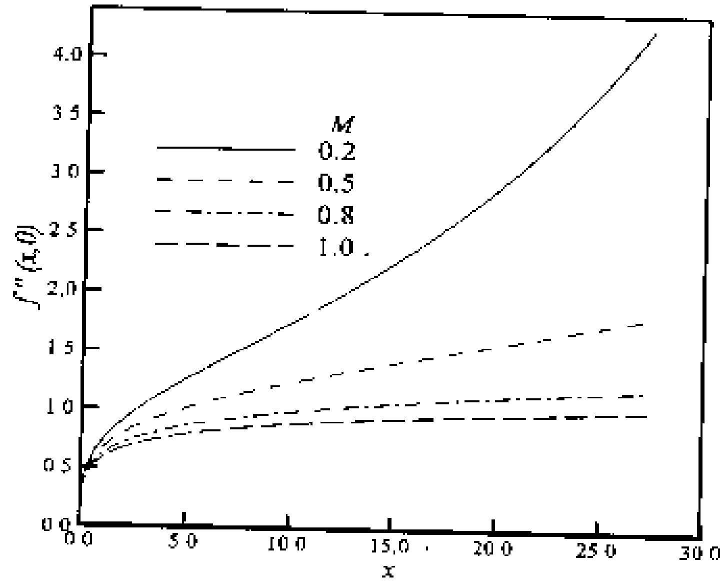

Fig.3.6: Skin frictions for different values of M when $Pr = 0.73$ and $N = 0.5$

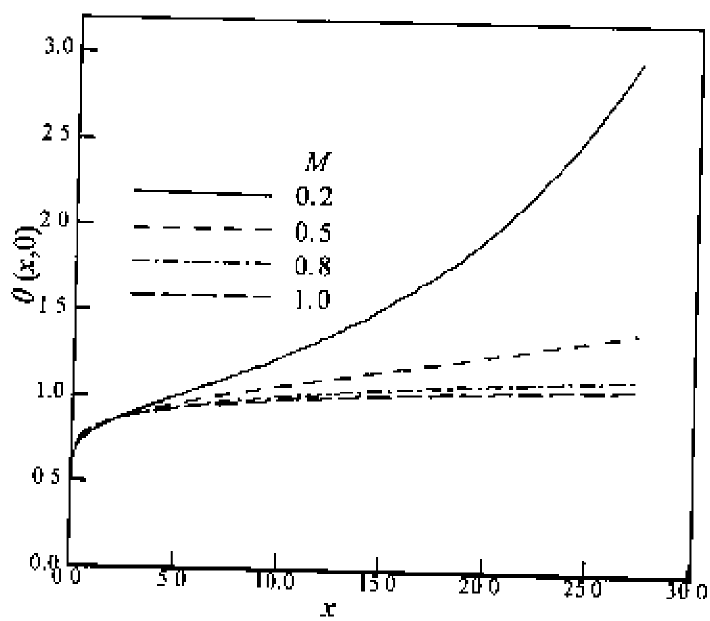

Fig.3.7:Surface temperature distribution for different values of M when Pr =0.73 and N =0.5

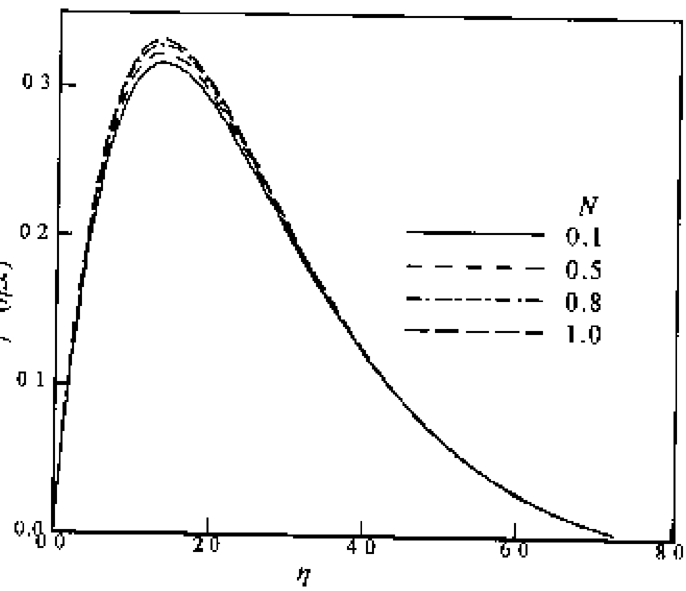

Fig.3.8 :Velocity profile for different values of N when $Pr = 0.73$ and $M = 1.0$

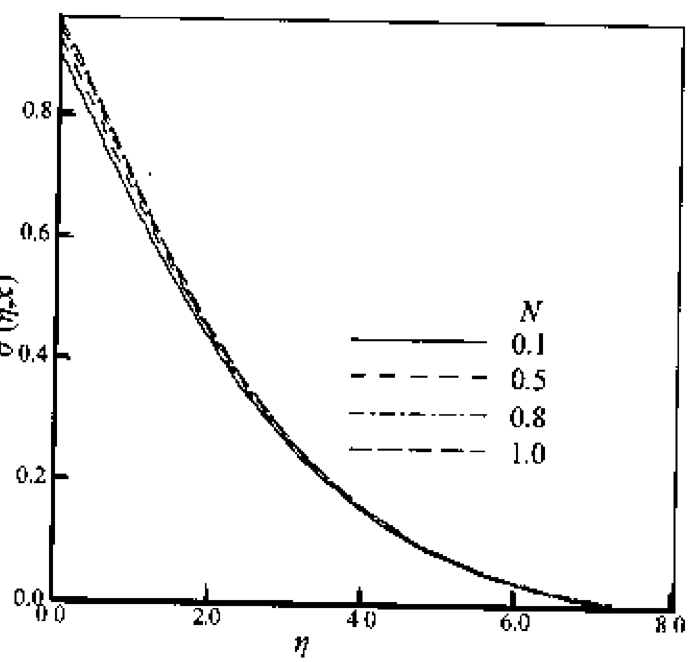

Fig.3.9:Temperature profile for different values of N when $Pr = 0.73$ and $M = 1.0$

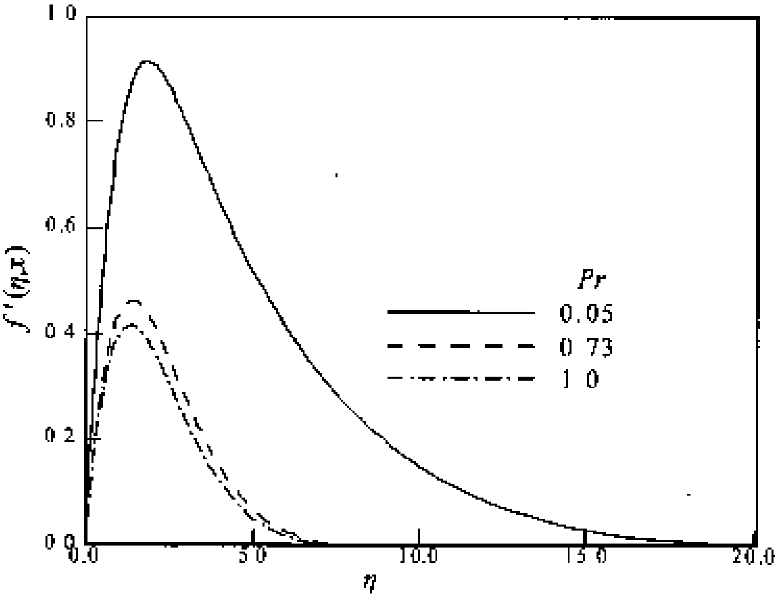

Fig.3.10 :Velocity profile for different values of Pr when $M = 0.1$ and $N = 0.01$

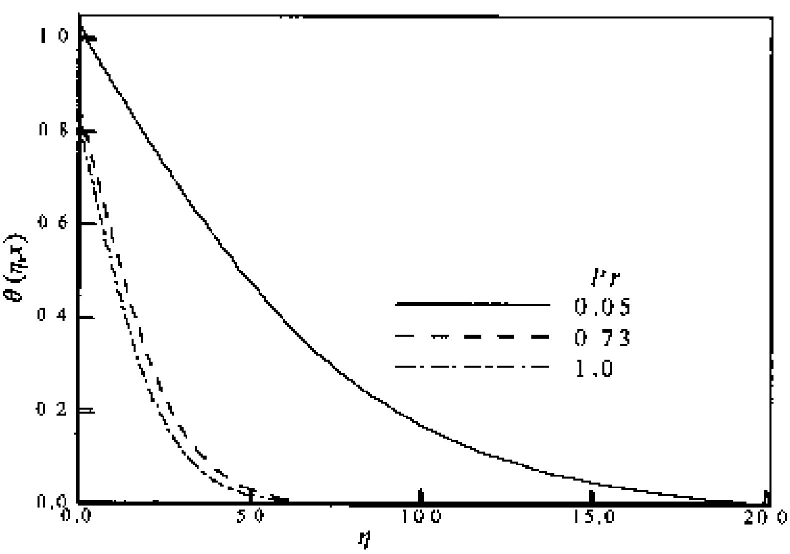

Fig.3.11:Temperature profile for different values of Pr when $M = 0.1$ and $N = 0.01$

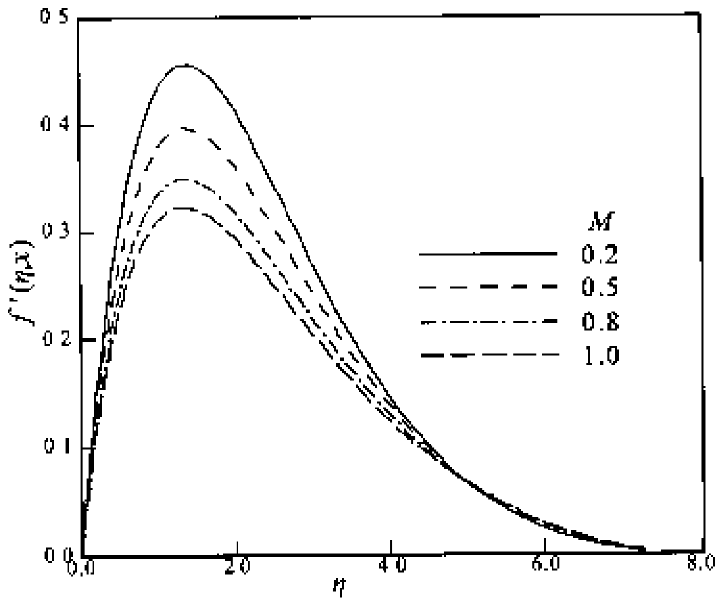

Fig.3.12 :Velocity profile for different values of M when $Pr = 0.73$ and $N = 0.5$

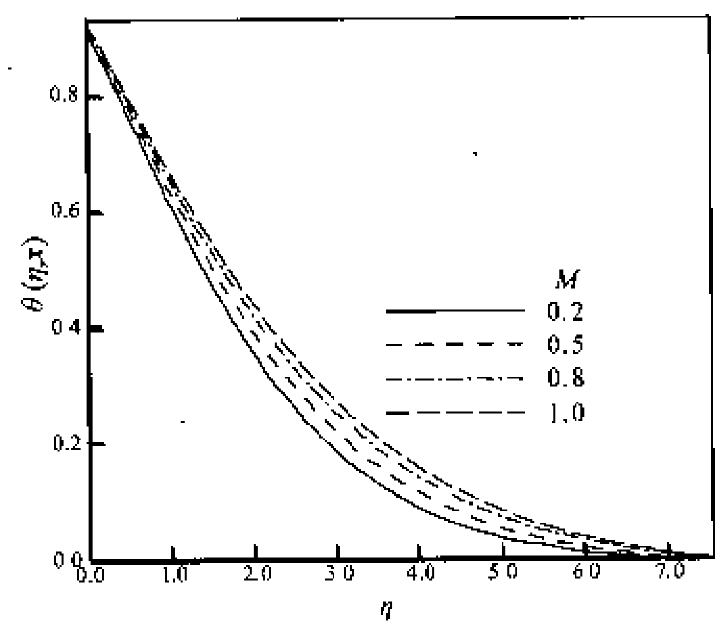

Fig.3.13:Temperature profile for different values of M when $Pr = 0.73$ and $N = 0.5$

Chapter 4

Conclusions

In this dissertation, we have investigated the coupled effect of conduction and convection on steady two dimensional magneto-hydrodynamic flows with viscous dissipation from a vertical flat plate. Using the appropriate transformations the basic equations are transformed to non similar boundary layer equations, which have been solved numerically in the entire region starting from the lower part of the plate to the down stream using a very efficient implicit finite difference method known as Keller box scheme. Here we have focused our attention on the evolution of the skin friction, the surface temperature distribution, velocity distribution, as well as temperature distribution for a selection of parameter sets consisting of the Prandtl number Pr, the magnetic parameter M and the viscous dissipation parameter N.

From chapter 2

The coupled effect of natural convection and conduction on the magneto hydrodynamic boundary layer flow about a vertical flat has been investigated introducing a new class of transformations. From the present investigation, the following conclusions may be drawn:

1. The skin friction coefficient and the velocity distribution decrease for increasing value of the magnetic parameter M.
2. Increased value of the magnetic parameter M leads to increase in the surface temperature distribution as well as the temperature distribution.
3. It has been observed that the skin friction coefficient, the temperature distribution and the velocity distribution decrease with the increase of the Prandtl number Pr.

From chapter 3

The effect of viscous dissipation and magnetic parameters N and M respectively for small Prandtl number Pr (= 0.1, 0.73, 1.0) on the magneto-hydrodynamic(MHD) natural convection boundary layer flow with viscous dissipation from a vertical flat plate has been studied introducing a new class of transformations. From the present investigation, the following conclusions may be drawn:

1. The skin friction coefficient and the velocity distribution increase for increasing value of the viscous dissipation parameter N.
2. Increased value of the viscous dissipation parameter N leads to increase in the surface temperature distribution as well as the temperature distribution.
3. It has been observed that the skin friction coefficient, the surface temperature distribution, the temperature distribution over the whole boundary layer and the velocity distribution decrease with the increase of the Prandtl number Pr.
4. The skin friction coefficient, the surface temperature distribution and the velocity profile decrease while the temperature profile increases for the increased values of the magnetic parameter M.

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