ANALYSIS OF TALL BUILDINGS WITH INTERCONNECTED SHEAR WALLS AND FRAMES

A Thesis

by

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ABSTRACT

This thesis deals with the methods of analysing tall buildings with frames and shear walls subjected to static loads, considering only the linear elastic behaviour.

The design procedures for frame structure and shear wall interconnected with frame structure are discussed. Exact analysis as well as approximate analysis by different authors are presented.

Analysis of structures with or without shear walls is performed by stiffness matrix method and the solution is made by recursion procedure. Computer programs using the above method are developed and written in Fortran II and IV. A program is also developed and written in Fortran II and IV for the approximate analysis of shear wall structures by using the continuous connection method suggested by Heidebrecht and Smith.

The values of lateral stiffness of frames, as expressed by the GA parameter are tabulated both by the 'exact' method and Heidbrecht and Smith method and compared for different structural properties of frame expressed by a non-dimensional parameter.

Frame programs are run for lateral loads and the results are analyzed to find out the validity of assumptions of portal and cantilever methods. The results are presented in Tabular and graphical form. These graphs may be used to find out the point of inflection at columns at different stories.

A comparison of "interactions" of shear walls and frames by Heidebrecht and Smith method is made with that of 'exact' method.

An attempt is also made to evaluate the role of heavier crown coupling beam in reducing sway of one bay frames.

The problem of load transfer from a vertically loaded column to another column depending on the structural properties of different elements of one bay frames is also discussed.

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NOTATION

A = Cross-sectional area

b = length of girder in frame

E = modulus of el sticity

G = shear modulus

H = height of building

h = storey height

I, I_h, I_b = moment of inertia of shear wall, frame column and frame girder respectively

M_o = total bending moment at base of the sttructure

M_B, M_S = bending moment in flexural and shear components respectively

Q = transverse interactive force at the top of the structure

q = transverse interactive force distribution

S_o = total shearing force at base of the structure

S_B, S_S = shearing force in the flexural and shear components respectively

w = lateral load distribution

x = position of ordinate along the height of the structure, relative to the base

y = lateral deflection

α = $\sqrt{GA/EI}$

P = force or load

B = Width of frame

C = Width of column or shear wall

C_1, C_2 = distances from centerline of be ms to centerline of columns of shear wall.

D = depth of beam

N = number of stories

Δ = deflection

Δ_A = top deflection due to column axial deformation

Δ_B = top deflection due to bending deformation

λ = ratio $\frac{E_c I_c/h}{E_b I_b/\ell}$ or $\frac{\sum(E_c I_c/h)}{2\sum(E_b I_b/\ell}$)

K_w = stiffness of a shear wall. In general K_w = lateral point load applied at top of a shear wall to cause unit deflection in its line of action

K_f = lateral point load applied at top of frame to cause unit deflection in its line of action

W = total lateral loading

LIST OF FIGURES

CHAPTER 1

GENERAL INTRODUCTION

1.1 Introduction

After the Independence of Bangladesh, the social and economic aspects of the urban areas have changed quite considerably, causing the tremendous influx of population to these areas from all over the country. This increase in population in one hand and the limited space in the urban areas on the other hand have necessitated the need for constructing tall office and residential buildings in the urban areas of Bangladesh. As a result several buildings from 10 to 20 stories high are being built in concrete in the major cities; and it is becoming apparent that in the near future a large number will be required.

1.1.1 The Criteria for Tall Buildings

Arbitrary definitions of tall buildings have in some cases included, even a two storey building. From the structural engineer's point of view, a tall building is one whose structural analysis and design is in some way affected by one or more of the following factors:

(a) Height to width ratio.

(b) Forces and moments in the structural members due to lateral loads.

(c) Sway caused by lateral loads.

(d) Perception of motion under strong wind gusts.

1.1.2 Optimum Building Cost

The structural design of buildings of smaller height are governed by the gravity loads, viz. the dead and live load. The stresses due to lateral loads viz. wind and seismic in low height buildings remain within the allowable overstress. But for taller buildings the lateral sway as well as lateral load stresses begin to control the design. As a result, the structural elements designed only for gravity loads need to be increased in order to increase the stiffness and strength of the building.

In the past, it was generally considered by the engineers that concrete buildings more than about 15 stories high are neither economic nor feasible. Such opinions were based on the use of traditional beam-column type frame construction.

In fact, if some preliminary design of taller buildings, using only the frame action, are made, it would be found that the estimated cost per sq. ft. of floor would be too high to be considered competitive. On the other hand, if there was no lateral load such as wind or earthquake any tall building could be primarily designed only for gravity loads in which case there would be no premium for height. This is qualitatively shown by the two curves in Fig. 1. Since there is no way of avoiding the gravity loads, it can be concluded that the minimum possible cost (or material) for a building of any number of stories cannot be less than that shown by the lower curve in Fig. 1. Therefore, from the structural point of view the most efficient or optimum building cost should qualitatively correspond to this curve.

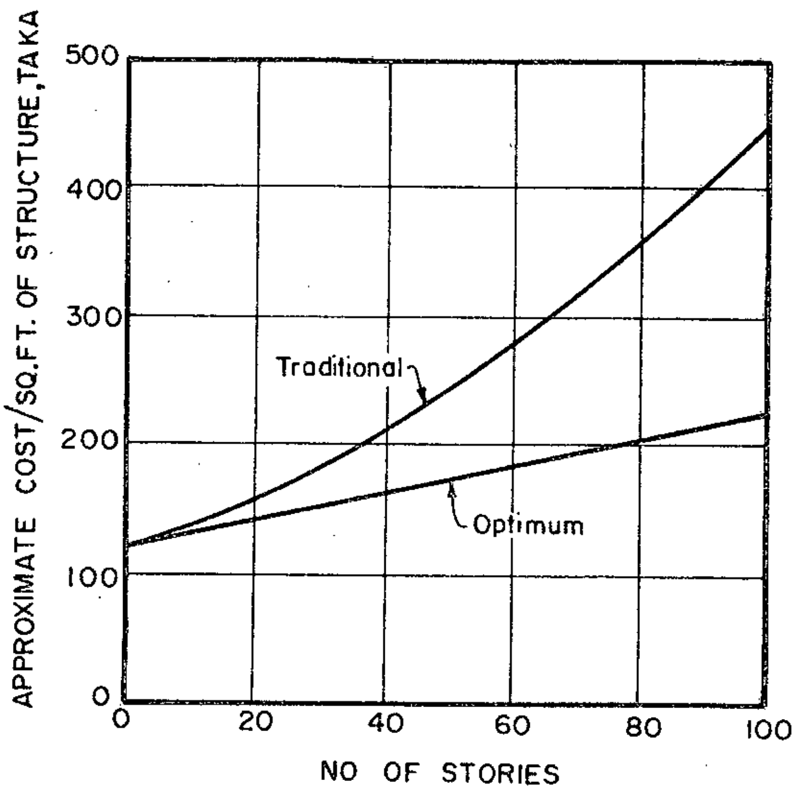


FIG. 1 QUALITATIVE COST PER SQ.FT. VERSUS NUMBER OF STORIES

The search for a system of tall buildings should therefore, be aimed at creating "optimum" structures that need be designed for the gravity loads only while the stresses due to lateral loads will automatically remain within the normally allowable overstress $33\frac{1}{3}\%$ in U.S. code and 25% in British code.

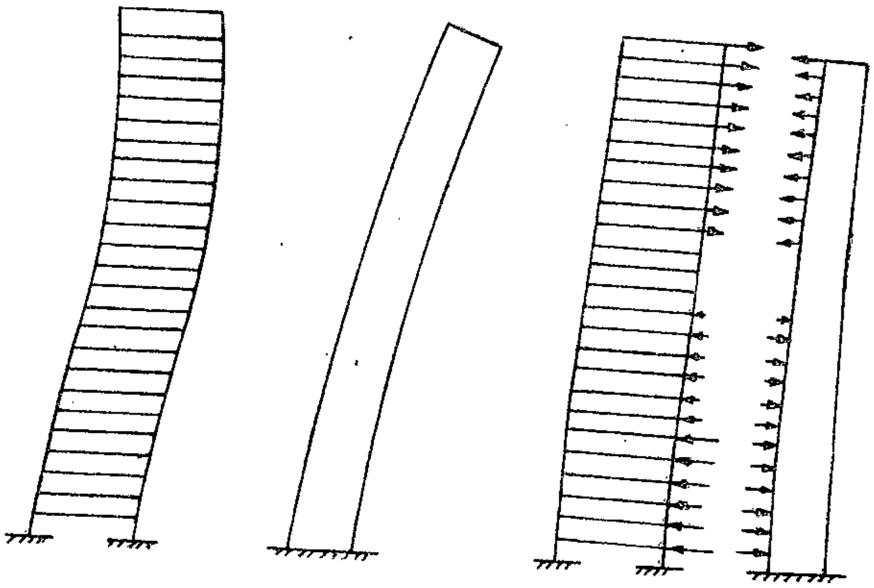
1.1.3 Shear Wall Construction

In the past and in the early post war period the systems providing lateral stability to tall buildings were essentially the rigid structural frames. The stability of such frames depends largely on the rigid connections between the columns and beams. The fully rigid connection, in practice, for a reinforced concrete frame results in a complicated arrangement of reinforcements. As shown by the upper curve of Fig. 1, the traditional rigid frame construction results in excessive structural cost as the building grows taller, thereby making the construction of tall reinforced concrete framed buildings uneconomical. To overcome the problem of lateral stability of tall buildings, the structural engineers have introduced concrete shear walls at suitable locations inside the building.

Shear walls are reinforced concrete walls providing stability to structure against lateral loads. As the wall can take care of most of the lateral shear coming to the structure, that is why it is called shear wall. The primary behavior of a shear wall of normal proportions is however governed by flexural deformations.

The moment resistance capacity of the frame in bending fails to achieve a structural action as efficient as that of a shear wall. The lateral stability of tall building is solved by constructing the walls around the stair and elevator shafts in reinforced concrete. These would act as shafts cantilevering out of the ground and resisting the entire lateral force. By incorporating shear walls in tall buildings, flat-plate type slab construction could be used typically for all floors without any regard to lateral load.

However, to assume that a shear wall resists the entire wind load is an obvious oversimplification. In reality, a shear wall can never act as a truly independent unit but must interact



(a) Free Frame (b) Free Wall (c) Combined Frame & Wall

FIG 2 INTERACTION BEHAVIOR OF SHEAR WALL AND FRAME

with the remaining frame elements of the building. Even though the frame part of the structure may be a rather flexible combination of flat plate and columns, as the number of stories increases its interaction with the shear wall becomes more significant which may actually contribute greatly to the lateral load resistance of the building. Therefore, when the frame portion of the building is fairly rigid by itself the interaction between the shear wall and frame can result in a considerably more rigid and efficient design. The interaction behaviour of a shear wall and frame structure can be schematically shown in Fig. 2. Because of the different lateral deflection characteristics of the frame and shear wall, the frame tends to pull back the shear wall in the upper portion of the building and push if forward in the lower portion. As a result, the frame participates more effectively in the upper portion of the building where the lateral load shears are relatively less and the shear wall carries most of the shear in the lower portion of the building where the frame generally cannot afford to carry high lateral load.

1.1.4 Choice for Shear Walls

The concrete shear wall has come to be accepted as a natural component of a multistorey concrete building. Judicious use of shear walls in a tall building has resulted in their optimum use both as a column element as well as the wind bracing.

Depending on height to width ratio, the use of extra shear wall, other than the walls around the stair and elevator shafts, is a matter of choice for buildings say upto 15 stories. For buildings taller than 15 stories and particularly in the range of 20 - 60 stories, the use of shear walls other than walls around stair and elevator shafts, in one form or other becomes imperative from the point of view of economy. The use of shear walls in several already constructed tall buildings are shown in Fig. 3.

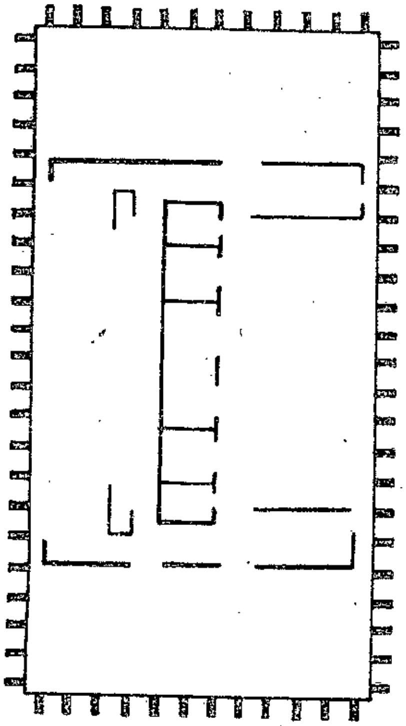


FIG. 3(a) INDEPENDENCE HOUSE LAGOS, 25 STOREY

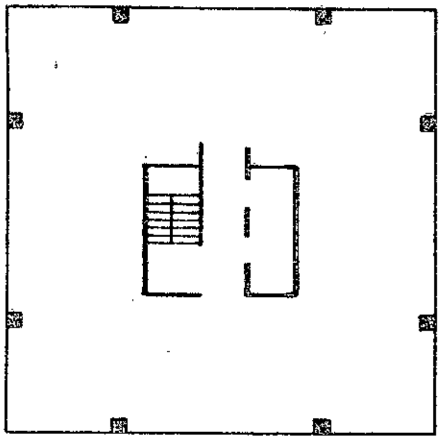


FIG. 3(b) RADIATION LTD. OFFICE NEASDEN, 13 STOREY

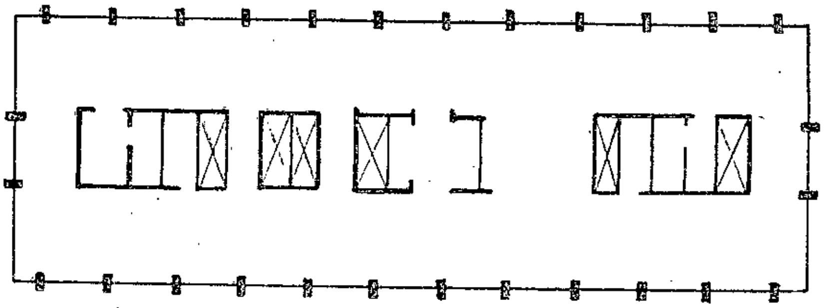


FIG. 3(c) MOORFIELDS DEVELÓPMENT, 36 STOREY

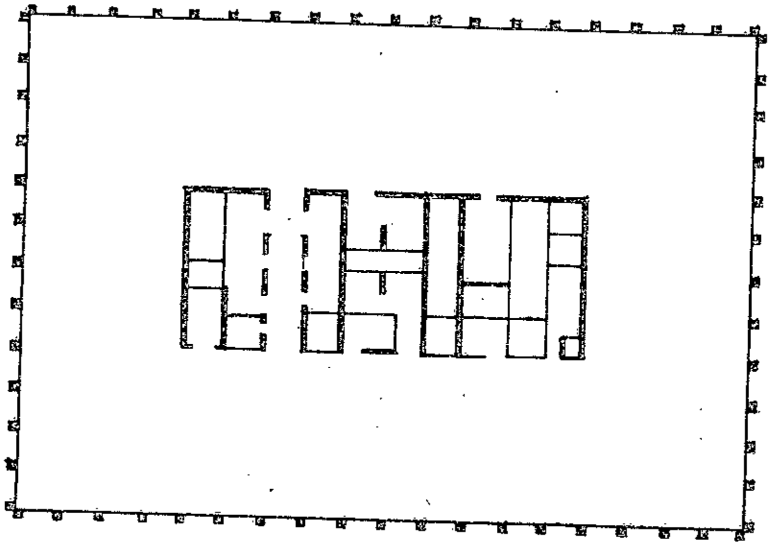


FIG. 3(d) TYPICAL FLOOR PLAN OF BRUNSWICK BUILDING,U.S.A.-38 STOREY

1.2 Assumptions

The following assumptions are made in the analysis of tall buildings:-

1) Shear wall and frames are fixed at its base.
2) All the joints are assumed to be rigid.
3) The floor diaphragms are rigid in their own plane (but have no stiffness normal to this plane) and there produces no rotations of the diaphragms in their plane. Thus, each floor level is constrained to translate without rotation, and each parallel frame is subjected to the same displacement at any given floor level.

(4) Axial deformations of the columns and shear walls are considered in the analysis, but girder axial deformations are neglected. Shear deformations in the columns, walls and girders are neglected.

(5) Plane sections of the wall before bending remain plane after bending so that the moment-curvature relations based on the simple "engineer's theory of bending" (ETB) may be used.

The "Exact" analysis as used in this thesis is based on the above assumptions.

1.3 Scope of the Thesis

The purpose of this work is to investigate methods of analysis of tall buildings with frames and interconnected shear walls. Only the linear elastic behaviour under static loads is considered. The exact analysis of Tall structures with or without shear walls are made and computer programms are developed and written in Fortran II and IV.

Analysis by Heidebracht and Smith method are compared with exact method in relation to GA parameter of frames and the interaction of shear-wall frame structure.

The validity of portal and cantilever methods are studied, and the transfer of vertical concentrated load from one vertical member to others are examined.

The role of heavier crown coupling beam at roof level in reducing the sway of frame is investigated.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In recent years the number of tall buildings, for both commercial and residential purposes, have increased rapidly throughout the world. This increase has necessiated the need for a greater knowledge of the behavior of these structures, and, in particular, the necessity for producing methods of analysis capable of giving rapid and accurate assessments of their overall strength and stiffness, as well as detailed information about any local stress concentration.

Although normal assumptions of lateral and vertical load distribution is adequate for smaller heights, these assumptions result in serious error in analysis and design for taller buildings. As buildings increase in height, it becomes more important to ensure adequate lateral stiffness to resist loads which may arise due to wind, seismic or even blast effects. In framed structures this stiffness may be obtained by bracing members, by the rigidity of the joints, by complete shear truss assemblies acting in conjunction with the frame, or by infilling the frame with shear-resistant panels. A simplification of the latter is shear wall construction, in which the high in-plane stiffness of the walls, is employed to resist the lateral force. The object of this literature review is to attempt to provide a summary of the current knowledge relating to shear wall structures, by reviewing briefly the relevant papers. The detail solution of some of the papers along with relevant figures are given in Chapter 3.

2.2 Review of Different Papers

A simplified method of determining the interaction of shear wall and frames was given by Khan and Sbarounis[1]. The method presented can be solved on a slide rule or a calculator. A method of converging approximations was proposed to provide a solution that can be carried to any degree of refinement. The material presented is applicable to analyses for lateral forces due to wind or earthquake. Useful graphs were presented to enable interaction of frame and shear walls to be determined rapidly. The detail solution of this paper along with figures are given in Chapter 3.

McLeod[2] in his paper on "Shear Wall-Frame Interaction" introduced a so called "component stiffness method" for determining the interaction of frames and shear walls. He used an equation to find the interaction force at top due to point load, or uniformly distributed load or triangular load (due to earthquake). The main assumption is that the frame takes constant shear, that is, the interaction force between the frame and wall can be represented by a concetrated force at the top. If the frame shear is assumed to be constant, the system can be treated as a wall supported at the top by a spring. This method lacks accuracy if the wall is more flexible than the frame. The detail solution of this paper along with figures are presented in Chapter 3.

Another simple approach to the design of shear wall with frames was published by Gould[3] in 1965. The simplest approach to the design problem is to consider the shear wall as an independent member and thus design it as a vertical cantilever allowing all the lateral loads to be taken by this member. It is very simple but quite conservative in a structure where substantial frame is provided along with shear wall system.

Cardan[4] has given a method of analyzing the shear wall with frame. He shows that it is possible to express the angle of deflection of wall at all points with a second degree differential equation taking the effect of bending and shear. By solving the equation the deflection of the system as well as internal actions can be evaluated. The assumptions and the governing equations of this method are given in Chapter 3.

Heidebrecht and Smith[5] has given an approximate analysis of Tall Wall-Frame building structures. The method is relatively simple and can be applied to both static and dynamic analyses of uniform and non-uniform structures. The structure is assumed to consist of a combination of flexural and shear vertical cantilever beams, deforming either in bending or shear configuration, respectively. The detail solution of this paper along with figures are presented in Chapter 3.

Rosman[6] has presented a paper on "Interaction between shear walls and frames" in the Tall building symposium held at the University of Southampton, U.K. In this paper he derived a governing equation by the energy method to deal with laterally triangular or uniformly distributed loaded systems consisting of walls and frames.

Parme[7] in his paper on "Design of Combined frame and Shear Walls" has presented a procedure which consists of relating the total load at each floor level to the displacements of that floor and the two floors above and below. At each level an equation is written in terms of the relative stiffness of the columns, girders, shear wall and the applied loading. This leads to a series of simultaneous equations equal to the number of floors.

Murachev, Sigalov, and Baikov[8] in their analysis of frame wall systems, neglected the total flexural stiffness of columns where the wall is solid.

CHAPTER 3

APPROXIMATE ANALYSIS OF SHEARWALL-FRAME STRUCTURE

3.1 Introduction

With the increasing availability of large scale digital computers, it is now-a-days possible to obtain exact analysis of tall buildings. However, to have the application of analysis af in a design office without the help of computers or to have a preliminary design in rapid hand calculations before sending it to the computer for more exact analysis, numerous methods have came upduring the past decade.

In Chapter 2 a brief review of different papers dealing with analysis of tall shear wall frame structure was given. In this Chapter solution of some of the methods are presented. These methods deals on approximate analysis of tall shear wall-frame structures.

3.2 Heidebrecht and Smith Method[5]

3.2.1 Basic Mathematic Model

The structure is considered to consist of a combination of flexural and shear vertical cantilever beams i.e. deforming either in bending or shear configurations, respectively.

The equations governing the behaviour of the two types of beam are, referring to Figures 4a and 4b, are

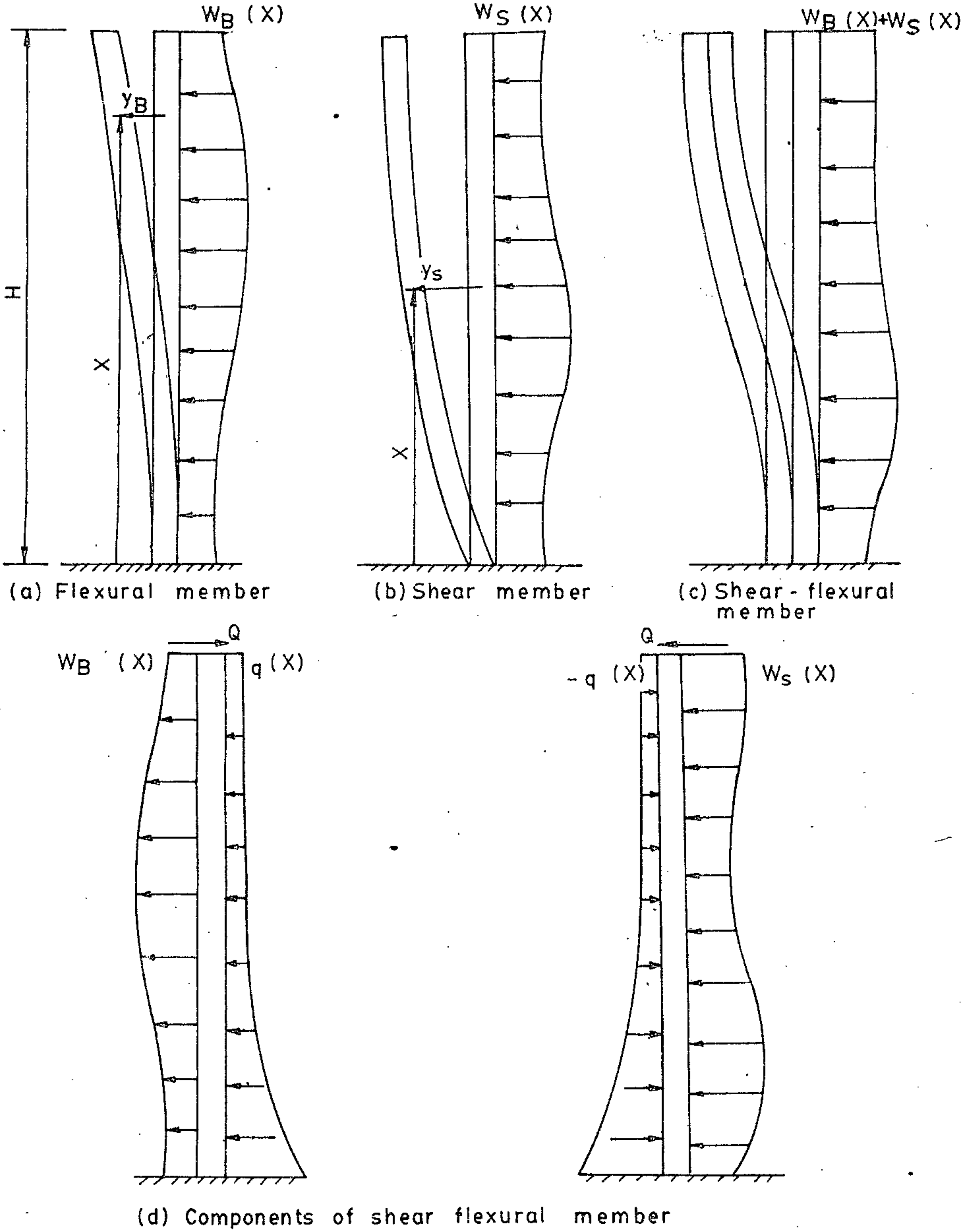


FIG.4 MATHEMATICAL MODEL FOR SHEAR-FLEXURAL MEMBER

$$\text{for flexure,} \quad EI \frac{d^4 y_B}{dx4} = W_B(X) \tag{3.2.1}$$

$$\text{and} \quad \text{for shear,} \quad GA \frac{d^2 y_s}{dx2} = W_s(X) \tag{3.2.2}$$

in which the subscripts B and s refer to the flexural and shear beams respectively; $W(X)$ is the distributed horizontal load; $y(X)$ is the horizontal deflection of the beam; E and I are the modulus of elasticity and the second moment of area, respectively, of the flexural beam; G and A are the shear modulus and cross-sectional area; respectively; of the shear beam.

In addition to the above governing differential equations, the equations for the stress resultants in each case are

$$S_B(X) = -\frac{dM_B(X)}{dX} \tag{3.2.3}$$

$$M_B(X) = EI \frac{d^2 y_B}{dx2} \tag{3.2.4}$$

$$\text{and} \quad S_s(X) = \frac{-dM_s(X)}{dX} = GA \frac{dy_s}{dx} \tag{3.2.5}$$

in which S and M are the distributed shearing force and bending moments, respectively.

Now if the shear beam and flexural beam are linked together so that they have identical horizontal deflection at all positions throughout their height, as shown in Fig. 4c. In this case, a distributed horizontal interaction force of magnitude $q(x)$ and a concentrated top interaction force of magnitude Q are required to maintain this compatibility, as shown in Fig. 4d. The concentrated for Q is required so that both components are in equilibrium at the top of the structure.

The governing equations of the components of this combination member, referred to as a shear-flexure beam or member, are given by

$$EI \frac{d^4 y}{dx^4} = W_B(X) + q(x) \qquad (3.2.6)$$

$$- GA \frac{d^2 y}{dx^2} = W_s(x) - q(x) \qquad (3.2.7)$$

Adding Eqs. (3.2.6) and (3.2.7), and dividing through by EI, yields

$$\frac{d^4 y}{dx^4} - \alpha^2 \frac{d^2 y}{dx^2} = \frac{W(x)}{EI} \qquad (3.2.8)$$

in which $W(x) = W_B(x) + W_s(x)$ (3.2.9)

and $\alpha^2 = \frac{GA}{EI}$ (3.2.10)

The contribution of a single column to the GA parameter of the equivalent shear-flexure beam is given by

$$GA = \frac{12EI_h}{h^2} \quad \frac{1}{1 + \dfrac{2\ I_h}{h\left(\dfrac{Ib_1}{b_1} + \dfrac{Ib_2}{b_2}\right)}}$$

in which I_h = Second moment of area of the column

h = Storey height

b_1, b_2 = total length of adjacent beams

Ib_1, Ib_2 = Second moments of area of corresponding beams

E = Young's modulus of Elasticity.

The total GA contribution of a planar frame is the arithmetic sum of the GA terms for each of the columns in a typical storey of the frame.

The total EI and GA terms of the respective flexure and shear components, can then be substituted in equation (3.2.10) to determine the parameter α^2 for the structure or for the particular segment of the structure.

For the case of a uniformly distributed load of magnitude W_o the solution to equation (3.2.8) can be written in the form

$$y(x) = C_1 + C_2X + C_3 \text{ Cosh } \alpha X + C_4 \text{ Sin } \alpha X - \frac{W_o X^2}{2EI\alpha^2} \quad (3.2.12)$$

in which C_1, C_2, C_3 and C_4 are constants in the homogenous part of the solution, to be determined by the boundary conditions.

A tall building can be considered as a vertical cantilever beam, with zero deflection and rotation at the base and free at the top, the corresponding boundary conditions for the shear-flexure cantilever of height H are:

$$Y(0) = 0 \quad (3.2.13a)$$

$$\frac{dy}{dx}(0) = 0 \quad (3.2.13b)$$

$$M(H) = M_B(H) + M_S(H) = 0 \quad (3.2.13c)$$

$$S(H) = S_B(H) + S_S(H) = 0 \quad (3.2.13d)$$

Since equilibrium requires that the moment is zero at the free and of each of the components of the shear-flexure cantilever, the condition expressed by eqn. (3.2.13c) can be replaced by

$$M_B(H) = EI\,\frac{d^2y}{dx^2}(H) = 0 \qquad (3.2.14)$$

Substituting equations (3.2.3) and (3.2.5) in Eq. (3.2.13d) yields

$$S(H) = GA\,\frac{dy}{dx}(H) - EI\,\frac{d^3y}{dx^3}(H) = 0 \qquad (3.2.15)$$

The constants of equation (3.2.12) can be obtained from the above boundary conditions, leading the following expression for deflexion:-

$$Y(X) = \frac{W_o H^4}{EI(\alpha H)^4}\left\{\left(\frac{\alpha H \,\mathrm{Sin}\,\alpha H + 1}{\mathrm{Cos}\,\alpha H}\right)(\mathrm{Cos}\,\alpha x - 1) - \alpha H\,\mathrm{Sin}\,\alpha x + (\alpha H)^2\left[\frac{x}{H} - \frac{1}{2}\left(\frac{x}{H}\right)^2\right]\right\} \qquad (3.2.16)$$

The expressions for stress resultants and the interacting force are given by

$$\frac{M_B(x)}{M_o} = \frac{2}{(\alpha H)^2}\left[\left(\frac{\alpha H\,\mathrm{Sin}\,\alpha H + 1}{\mathrm{Cos}\,\alpha H}\right)\mathrm{Cos}\,\alpha x - \alpha H\,\mathrm{Sin}\,\alpha x - 1)\right] \qquad (3.2.17)$$

$$\frac{M_s(x)}{M_o} = \left(1 - \frac{x}{H}\right)^2 - \frac{M_B(x)}{M_o} \qquad (3.2.18)$$

$$\frac{S_B(x)}{S_o} = \frac{1}{\alpha H}\left[\alpha H\,\mathrm{Cos}\,\alpha X - \left(\frac{\alpha H\,\mathrm{Sin}\,\alpha H + 1}{\mathrm{Cos}\,\alpha H}\right)\mathrm{Sin}\,\alpha X\right] \qquad (3.2.19)$$

$$\frac{S_S(S)}{S_o} = (1 - \frac{x}{H}) - \frac{S_B(x)}{S_o} \tag{3.2.20}$$

$$\frac{q(x)}{W_o} = \frac{W_s}{W_o} + \frac{(\alpha H)^2}{2} \frac{M_B(x)}{M_o} \tag{3.2.21}$$

Similar expressions for the case of a horizontal concentrated top load and a triangularly distributed load are given in section 3.2.3. These three types of loading can be appropriately combined to simulate the static lateral loadings specified by most of the building codes for wind or earthquake effect.

3.2.2 Design Curves

Design curves for deflection, moment, shearing force and horizontal interacting force constructed from equations (3.2.16) (3.2.17), (3.2.19) and (3.2.21) are constructed and presented by Heidebrecht and Smith[5]. The curves can be used for a rapid estimate of the total deflection and the variation with height of the load distribution within the structure, for a range of the parameter αH.

3.2.3 Design Equations for Concentrated Load and Triangular Load Distribution, Uniform Structure

A. CONCENTRATED LOAD P AT TOP OF STRUCTURE

$$y(x) = \frac{PH^3}{EI}\left\{\frac{\sinh \alpha H}{(\alpha H)^3 \cosh \alpha H}(\cosh \alpha x - 1) - \frac{\sinh \alpha x}{(\alpha H)^3} + \frac{1}{(\alpha H)^2}\,\frac{x}{H}\right\} \qquad (3.2.22)$$

$$\frac{M_B(x)}{M_o} = \frac{1}{\alpha H}\left(\tanh \alpha H \cosh \alpha x - \sinh \alpha x\right) \qquad (3.2.23)$$

$$\frac{M_S(x)}{M_o} = \left(1 - \frac{x}{H}\right) - \frac{M_B(x)}{M_o} \qquad (3.2.24)$$

$$M_o = PH \qquad (3.2.25)$$

$$\frac{S_B(x)}{P} = \left(\cosh \alpha x - \tanh \alpha H \sinh \alpha x\right) \qquad (3.2.26)$$

$$\frac{S_S(x)}{P} = 1 - \frac{S_B(x)}{P} \qquad (3.2.27)$$

$$\frac{Hq(x)}{P} = (\alpha H)^2\,\frac{M_B(x)}{M_o} \qquad (3.2.28)$$

B. TRIANGULAR DISTRIBUTION

$$w(x) = \frac{w_1 x}{H}$$

$$y(x) = \frac{w_1 H^4}{EI(\alpha H)^2} \left\{ \left(\frac{\sinh \alpha H}{2\alpha H} - \frac{\sinh \alpha H}{(\alpha H)^3} + \frac{1}{(\alpha H)^2}\right)\left(\frac{\cosh \alpha X - 1}{\cosh \alpha H}\right) + \left(\frac{x}{H} - \frac{\sinh \alpha x}{\alpha H}\right)\left(\frac{1}{2} - \frac{1}{(\alpha H)^2}\right) - \frac{1}{6}\left(\frac{x}{H}\right)^3 \right\} \qquad (3.2.29)$$

$$\frac{M_B(x)}{M_o} = \frac{3}{(\alpha H)^3} \left\{ \left(\frac{(\alpha H)^2 \sinh \alpha H}{2} - \sinh \alpha H + \alpha H\right) \frac{\cosh \alpha x}{\cosh \alpha H} - \left(\frac{(\alpha H)^2}{2} - 1\right) \sinh \alpha x - \alpha H \left(\frac{x}{H}\right) \right\} \qquad (3.2.30)$$

$$\frac{M_S(x)}{M_o} = \left(1 - \frac{3}{2}\left(\frac{x}{H}\right) + \frac{1}{2}\left(\frac{x}{H}\right)^3 - \frac{M_B(x)}{M_o}\right. \qquad (3.2.31)$$

$$M_o = \frac{w_1 H^2}{3} \qquad (3.2.32)$$

$$\frac{S_B(x)}{S_o} = -\frac{2}{(\alpha H)^2} \left\{ \left(\frac{(\alpha H)^2 \sinh \alpha H}{2} - \sinh \alpha H + \alpha H\right) \frac{\sinh \alpha x}{\cosh \alpha H} - \left(\frac{(\alpha H)^2}{2} - 1\right) \cosh \alpha x - 1 \right\} \qquad (3.2.33)$$

$$\frac{S_S(x)}{S_o} = 1 - \left(\frac{x}{H}\right)^2 - \frac{S_B(x)}{S_o} \qquad (3.2.34)$$

$$S_o = \frac{w_1 H}{2} \qquad (3.2.35)$$

$$\frac{q(x)}{w_1} = \frac{x}{H} + \frac{(\alpha H)^2}{3} \frac{M_B(x)}{M_o} \qquad (3.2.36)$$

3.3 Khan and Sbarounis Method [1]

3.3.1 Analysis

The interaction of a shear wall and frame is a special case of indeterminacy in which two basically different components are tied together to produce one structure. If the frame alone is considered to take the full lateral load, it would develop moments in columns and beams to resist the total shear at each storey while the effects of overturning would normally be considered secandary, and in most cases, negligible. In resisting all lateral loads, a frame would deflect as in Fig. 2(a). If a shear wall on the other hand, is considered to resist all the lateral loads it would develop moments at each floor equal to the overturning moment at that level and the deflected shape Fig. 2(b), would be that of a cantilever.

If a shear wall and a frame exist in a building, each one will try to obstruct the other from taking its natural free deflected shape, and as a result a redistribution of forces between the two would be expected. As shown in Fig. 2(c), the frame will restrain or pull the shear wall back in the upper stories, while in the lower regions the opposite will occur.

The conflicting physical characteristics of the two systems can be considered if the structure is first divided into two parts, a frame, and a shear wall, and then the two parts are brought together so that all structural laws are fully satisfied.

3.3.2 Concept and Method of Analysis

The analysis is performed in two stages. In the first stage of analysis of a structure, shown in plan in Fig. 5(a) it is necessary to determine the deflected shape and the amount of lateral load distributed to the walls and frame, respectively at each story. For this purpose, the structure is separated into two distinct systems Fig. 5(b) as follows:-

3.3.3 System "W"

This system consists shear wall or combination of shear walls. The moment of inertia of this system at any story equals the sum of the moments of inertia of all the shear walls regardless of their shape and size. Shape and size are considered in computing an average L_s, the distance from the N.A. of system "W" to its extreme fibre. Coupled shear walls can often be represented in high multi-storey buildings as a single wall with an equivalent stiffness.

3.3.4 System "F"

This system is a one bay frame connected with the system "W" by means of link beams and consists of all other framing elements outside of system "W". This includes all columns, beams, spandrels, and slabs contributing to the lateral stiffness. Member linking the frames with the shear walls ("Link Beams") are also included in system "F". The stiffnesses of the columns, beams, and "link" beams (S_c, S'_b, S''_b) are simply the sum of the stiffnesses of all such members in the structure. The "link" beam span, L_b of system "F", is an average of the "link" beam spans of the structure when these spans are within the same range of magnitude.

In the first stage of analysis, average rotation and vertical displacement of the frame joints will yield acceptable values of the distribution of lateral forces between the two systems and the computed deflection at each story will be quite satisfactory. The computed distribution of lateral forces in the first stage of analysis will inform the designer of the effectiveness of the shear walls in resisting the lateral forces so that he may adjust the size and stiffness of the shear wall to obtain a more economical structure. The deflected shape is needed in the second stage of computation.

In most cases, a further simplification can be made (Fig. 5(c)) by adding the stiffness of "link" beams to the stiffness of the other flexural members. The two systems are then tied together with members that can transmit only lateral force. The quantities L_b and L_s are no longer needed.

The analysis of the system is performed by the iterative solution. At the completion of the iterative solution the deflections of the combined system comprising the structure are known. It is then possible to analyze each column line as an isolated resisting system. The resisting elements at any column line may form a rigid frame or a combination of rigid frame and shear wall.

The second stage of analysis may be performed by subjecting these isolated bents to the deflection pattern that was derived for the entire structure from the ~~interactive~~ iterative solution. Fixed end moments imposed on the columns and connecting links by this known set of deflections can be balanced rapidly by a moment distribution solution. In this manner, local effects on moments and shears resulting from localized stiffness variations are fully accounted for.

3.3.5 First Stage of Analysis

The conditions required to be satisfied for the equilibrium of total structure are:-

a) Deflection in system "W" and system "F" must be the same at corresponding levels.

b) "Link" members connecting system "F" to system "W" must undergo the same rotations and vertical translations as those of system "W" at their points of connection.

c) Horizontal shear V_w, developed in system W plus the horizontal shear, V_f, developed in system F must be equal to the external shear V_t, at every story.

The steps adopted to achieve the above conditions are as follows:-

1. The total computed external loads (wind or siesmic) on the idealized structure are applied to system W at each floor

level and the slopes and deflections of system W at each floor level are determined. The vertical movements of the connecting points with system W are computed by multiplying the slope at each level by the distance from the neutral axis of the wall to the connecting point.

2. This is the first cycle of iteration. For quick convergence, a final deflected shape could be assumed or approximated from the figures as given by Khan and Sbarounis. In the absence of good guess, the final deflected shape is assumed to be the same as the free deflected shape of system W.

System F is forced to undergo the assumed deflections at each floor. This also requires that the connecting members at each floor must have the same rotations and vertical translations as system W at their points of connection with system W. However, if there is a deliberate hinge at these points, only the vertical translation must be considered.

3. Moments induced by "force fitting" can be determined directly by using moment distribution. The force fitted frame has no external forces but only known story deflections and rotations at the connecting points.

4. After "force-fitting" system F to system W, the total shears in each story of system F as well as moments and reactions applied on system W by the connecting links are computed. The shears generated by force fitting can be used directly in the next step.

5. All shears, forces, and moments generated by force-fitting system F are applied to the isolated free system W. Negative deflections and rotations of the system W are then calculated. The net deflection with respect to the original unloaded shape of system W would be the algebraic sum of step 1 and step 5. This is the end of one cycle of iteration. For the stable condition the assumed initial deflection at any floor at the beginning of the nth cycle must be the same as the end deflections, at the completion of the nth cycle.

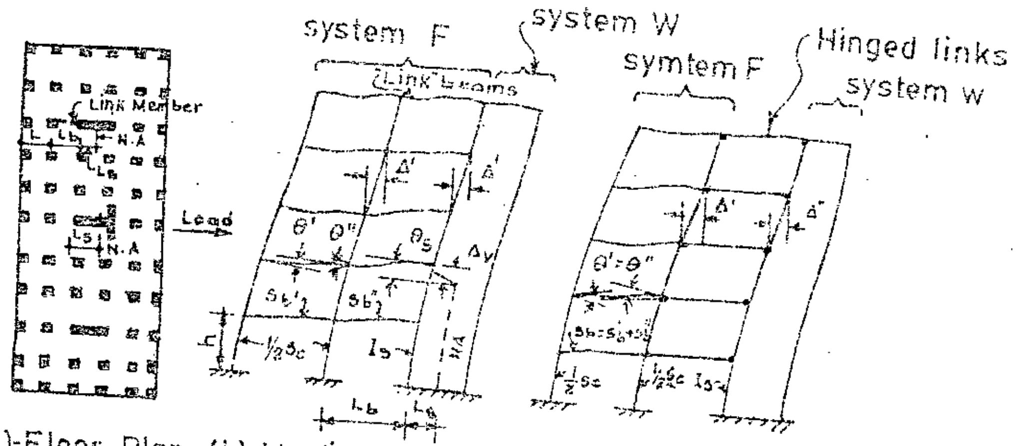
3.3.6 Second Stage of Analysis

After convergence of the iteration solution has been achieved, the final deflected shape of the structure is used to distribute moments and shears to every member in each bent of the structure. At a column line that contains no shear walls, a set of fixed end column moments obtained from the difference in story deflection can be apportioned to all the members by moment distribution. No sidesway correction is needed because the bent is in its final deflected shape.

If a shear wall is contained in a bent, it can be treated separately from the frame segment. With a known deflected shape and EI, the moment at any floor i, can be obtained from

$$M_i = \left(\frac{EI_{si}}{h_i^2}\right)\left(\Delta_{i+1} - 2\,\Delta_i + \Delta_{i-1}\right) \qquad (3.3.1)$$

in which M_i denotes the moment at floor, i, I_{si} refers to the moment of inertia of wall at floor, i, h_i represents the storey height; Δ_i is the deflection at floor, i, Δ_{i+1} describes the deflection at floor, i+1, Δ_{i-1} is the deflection at floor, i-1.



(a)-Floor Plan (b)-Idealized structure (c)-Simplified Idealized structure

FIG. 5 TYPICAL IDEALIZED STRUCTURE

3.4 Component Stiffness Method[2]

The main assumption in component stiffness method by McLeod[2] is that the frame takes constant shear, that is the interaction force between the frame and wall can be represented by a concentrated force at the top. The method is simple but lacks accuracy if the wall is more flexible than the frame ($K_w/K_f < 1$, see below). Consider the single-bay frame and shear wall loaded in-plane by the uniformly distributed load shown in Fig. 6(a). If the frame shear is assumed to be constant, the system can be treated as a wall supported at the top by a spring (Fig. 6(c). The spring stiffness K_f is defined as the lateral point load applied at the top of the frame to cause unit deflection in its line of action. K_f can be calculated using equations A and B (Table 2). K_w is defined as the lateral point load required to cause unit deflection at the top of the wall (similarly to K_f). Normally

$$K_w = \frac{3\ EI_w}{H^3} \tag{3.4.1}$$

where E is the Young's modulus of elasticity, I_w the moment of inertia of the wall, and H the total height. This equation can be modified to take account of variations in properties with height and the effect of openings in the walls.

The relationships between P/W, γ_w, and K_w/K_f for different loading cases can be evaluated by using equation C (Table 1). P is the interaction load at the top of the frame, the constant shear; W is the total applied lateral load; γ_w is a dimensionless parameter which relates the rotational stiffness of the wall to that of the foundation, that is

$$\gamma_w = K_B \; H/4E_w \; I_w \qquad (3.4.2)$$

where K_B is the rotational stiffness of the shear wall support. If the rotation at the base of the shear wall is to be neglected, the term γ_w in Equation C should be omitted. However, the effect of shear wall base rotation can significantly affect the distribution of load between shear walls and frames, and Equation C can be used as a simple method of assessing this factor.

Problems involving several frames and walls may be reduced to that of a single wall and frame. Alternatively, K_f and K_w for each vertical unit may be calculated separately and the results summed. $\sum K_f$ and $\sum K_w$ are then used instead of K_f and K_w in Equation C.

Studies on shear wall-frame interaction normally use three parameters to define behaviour, namely, λ, i_w, and I_c, where

$$\lambda = \frac{E_c \; I_c/h}{E_b \; I_b/l} \qquad (3.4.3)$$

with h as the column height. By using K_f, which is a function of λ and $\sum I_c$, behavior can be discussed in terms of only two variables, K_f and K_w. This simplifies the physical interpretation of the behavior. Also, the parameter P/W is useful for estimating the effectiveness of the frame (or frames) in comparison with the shear wall(s) in resisting lateral load and for assessing the effect of various assumptions in analysis.

When a frame and wall are interconnected as shown in Fig. 6(a), maximum shear on the frame tends to occur towards midheight and in this area Equation C can underestimate maximum frame shear by as much as 30 percent. Therefore, when calculating moments in the frame, it is worth while to increase the calculated value of P by 30%.

If K_w/K_f is less than 1, the use of Equation C is not recommended and the use of charts as given by Khan and Sbarounis[1] produces more accurate results.

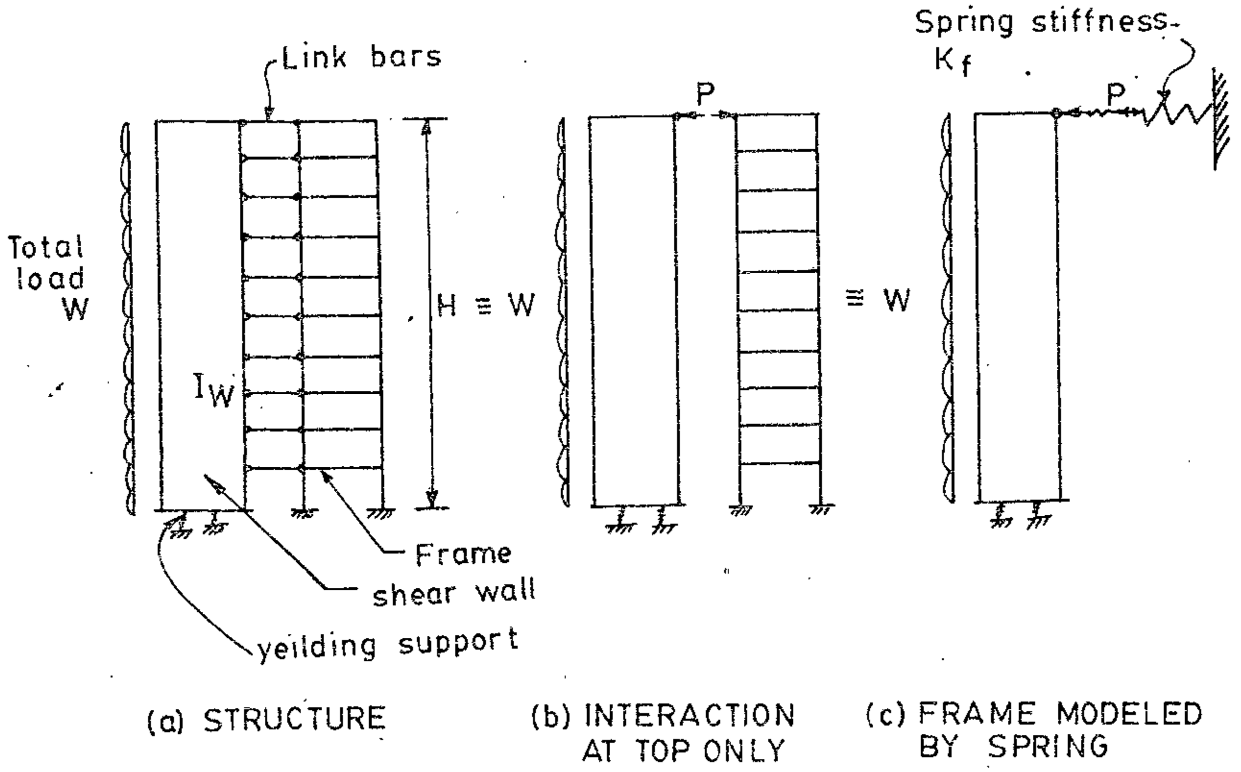


FIG. 6 IDEALIZATION OF STRUCTURE FOR EQUATION C

Table 1. Equation C

Load condition	Equation C
Point load at top	$\frac{P}{W} = \dfrac{1 + \frac{3}{4\gamma_w}}{1 + \frac{3}{4\gamma_w} + \frac{K_w}{K_f}}$
Uniformly distributed	$\frac{P}{W} = \dfrac{\frac{3}{8}(1 + \frac{1}{\gamma_w})}{1 + \frac{3}{4\gamma_w} + \frac{K_w}{K_f}}$
Triangular (earthquake)	$\frac{P}{W} = \dfrac{\frac{11}{20} + \frac{1}{2\gamma_w}}{1 + \frac{3}{4\gamma_w} + \frac{K_w}{K_f}}$

NOTATION

W = total applied load

P = interaction force at top

I_w = moment of inertia of wall

H = total height

K_f = point load at top of frame to cause unit deflection in its line of action

$K_w = \frac{3EI_w}{H^3}$ (with constant I_w)

K_B = rotational stiffness of shear wall support

$\gamma_w = \frac{K_B H}{4E_w I_w}$

top deflection $\Delta = \frac{P}{K_f}$

Table 2. Equation A for Top Deflection of Rigid Regular Frames

EQUATION A- for Axial deformation:

$$A = \frac{wH^3F_n}{E_cA_cB^2}$$

here Δ_A= deflection at top of frame due to axial deformation of exterior columns

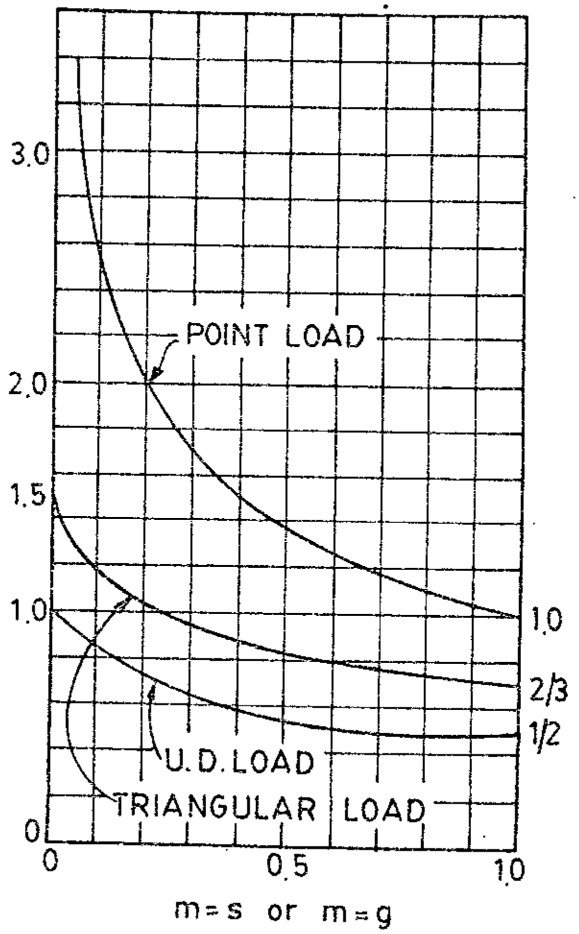
F_n= function of n, dependent on the type of loading

n = ratio $\dfrac{\text{Area of exterior column at top of frame}}{\text{Area of exterior column at bottom of frame}}$

(linear variation of A_c with height)

A_c = area of exterior columns at first-story level

B = total width of frame



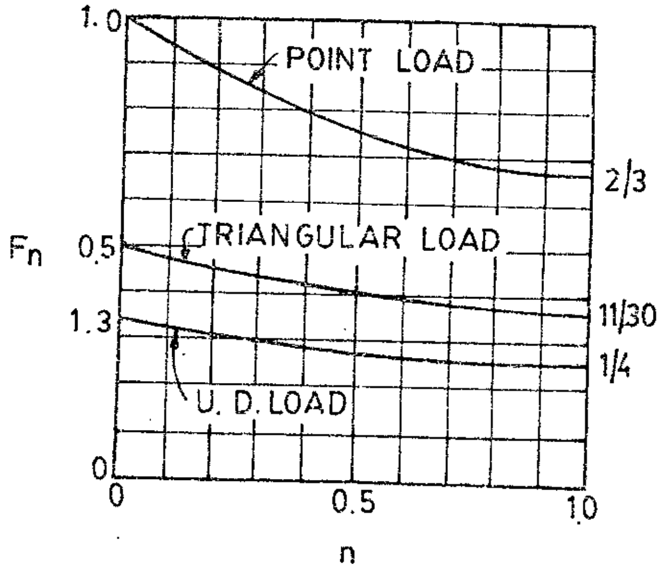


Table 2. Equation B for Top Deflection of Rigid Regular Frames

EQUATION B– for Bending deformation:

$$\Delta_B = \frac{wh^2H}{12\sum(E_cI_c)}\left[F_s(1-\beta_D)^3 + F_g(I-\beta_c)^3 2\lambda\right]$$

where Δ_B = deflection at top of frame due to bending of members

W = total lateral load

h = story height

H = total height

E = Young's modulus (subscript denotes structural system)

I_c = sum of moments of inertia of columns at first-story level

F_s, F_g = functions of s and g, dependent on the type of loading

s = ratio $\dfrac{I_c \text{ at top of frame}}{I_c \text{ at bottom of frame}}$

g = ratio $\dfrac{I_b \text{ at top of frame}}{I_b \text{ at bottom of frame}}$

Linear variation of I_c and I with height. If E varies, use EI instead of I.

$\beta_D = \dfrac{D}{h}$, where D is beam depth

$\beta_c = \dfrac{C}{\ell}$, where C is column width and ℓ is distance between column center lines

$\lambda = \dfrac{\sum(E_cI_c/h)}{2\sum(E_bI_b/\ell)}$, i.e. summation over width of structure at first story level

I_b = moment of inertia of beam at bottom of structure

TOTAL DEFLECTION $\Delta = \Delta_B + \Delta_A$

3.5 Cardan Method[4]

Cardan expressed the angle of deflection of the wall at all points with a second degree differential equation, taking the effect of bending and shear.

3.5.1 Assumption

The assumptions made in Cardan method is as follows:-

a) Shear wall and frames are fixed at its base

b) Columns are monolithic with their bases fixed

c) The joints of the frames are rigid

d) Buildings are symmetrical

e) Girders are infinitely rigid as compared to the walls and frames. All points on the same floor will then have same horizontal deflection.

3.5.2 Governing Equations

The equation governing the effect is

$$\emptyset = \emptyset_B + \emptyset_S \tag{3.5.1}$$

and by differentiating

$$\frac{d^2\emptyset}{dx2} = \frac{d^2\emptyset_B}{dx2} + \frac{d^2\emptyset s}{dx2} \tag{3.5.2}$$

where $\emptyset_B$ = Slope of deflected wall due to bending only

$\emptyset_S$ = Slope of deflected wall due to shear only

$\emptyset$ = Slope of the deflected wall due to all forces.

By solving the above equation the value of $\emptyset$ may easily be determined. Once the value of $\emptyset_B$ and $\emptyset_S$ are known, deflection

$$Y = \int_0^x \emptyset \, dx \tag{3.5.3}$$

For uniform load, Y/H for shear wall coupled with frame is

$$Y/H = \frac{C}{B} H \left(\xi - \frac{1}{2} \xi^2 \right) + \left(\frac{C}{B} - D \right) H/\alpha$$

$$\times \frac{\sin \alpha (1 - \xi) - \sin \alpha - \frac{1}{\alpha} (1 - \cos \alpha)}{\cos \alpha} \tag{3.5.4}$$

Where, $C = \left[1 + 3 (K_1 - K_2)/AE \right] / \beta EI$ (3.5.5)

$D = 3/\beta AE$ (3.5.6)

$B = \left[\beta (K_1 - K_2) + K_2 + K_3 \right] / \beta EI$ (3.5.7)

$\alpha = H\sqrt{B}$ (3.5.8)

$\beta = 1 + 3K_3/AE$ (3.5.9)

$\xi = X/H$ (3.5.10)

K_1 and K_2 = Constants, defined as

$$m_s = K_1 \emptyset_B + K_2 \emptyset_S \tag{3.5.11}$$

K_3 = Constant, defined as $V_f = - K_3$

A = Cross-sectional area of shear wall

m_s = Elastic moment reaction per inch of wall height

Y = Horizontal deflection of wall

CHAPTER 4

"EXACT" ANALYSIS OF PLANE FRAME BY STIFFNESS METHOD AND RECURSION PROCEDURE

4.1 Introduction

In Chapter 3 different methods for approximate analysis of shear-wall frame structure was given. In this chapter "exact" analysis of plane frame with or without shear wall is presented. The analysis of a structure is a process in which information given in one form is transformed into another form and the common objective in the analysis is finding the internal forces which result from the application of external forces. The forces and displacements in a structure can be related by using its flexibility and stiffness co-efficients. These co-efficients are characteristics of a structure and its coordinate system.

The flexibility coefficients characterize the behaviour of the structure by specifying its displacement response to applied forces at the coordinates, and the stiffness coefficients by specifying the forces required to produce given displacements at the coordinates. The stiffness and flexibility coefficients depend on the force displacement properties of the structure and the coordinate system.

A tall structure may be analysed either by flexibility or by stiffness method. But it has been found that the time required for the analysis by stiffness method is much less than the time required for flexibility method. As such the tall plane frames are generally analysed by stiffness method.

The solution of large building frames by stiffness method uses a special technique by taking advantage of the sparse nature of the stiffness matrices of the frame. This technique was first introduced by Clough, Wilson and King(9). To avoid the inversion of large matrices, the system coordinates of the structure are selected in such a way that the resulting matrix which must be inverted will be tri-diagonal band matrix. Using the band matrix, the analysis is conducted by recursion procedure which requires the inversion of low order matrices.

4.2 Types of Frames

At first a one bay frame without shear wall is analyzed. Storey heights, cross-sections, moments of inertia and Youngs modulus of elasticity are assumed to be constant for both columns and beams throughout the full height of the system. The analysis is modified to take care of wall system.

The one bay frame analysis is further modified to take care of variable storey heights, moments of inertia, cross-sections etc. The name of this analysis is given as general one bay frame analysis.

Subsequently a two bay frame with and without shear wall is analyzed. The matrices are formulated in a general form so that they can be used either for frame with wall or without wall. Two Bay frame analysis deals with constant storey heights, cross-sections, moments of inertia etc.

A "general two bay frame" with variable storey heights, cross-sections moments of inertia of vertical and horizontal members throughout the full height is also analyzed.

Computer programs for all of the above analyses are developed and written in Fortran II and Fortran IV.

4.2A ADDITIONAL NOTATION

A_1, A_L = cross-sectional area of left column or shear wall

A_2, A_R = cross-sectional area of right column or shear wall

I_1, C_L = moment of inertia of left column or left

I_2, C_R = moment of inertia of right column or right shear wall

I_3, B_L = moment of inertia of beam

H = storey height

L, X, Y = Beam lengths

BO, B, C, A = stiffness matrices of the frame

$\{F\}_j$ = forces at the coordinate level j

$\{U\}_j$ = displacements at the coordinate level j

S_{ij} = elements of frame stiffness matrix

θ_A = rotation at end A of member AB

θ_B = rotation at end B of member AB

Δ = displacement of one end of a member width respect to other end

M_{FAB} = fixed end moment at end A of member AB

M_{FBA} = fixed end moment at end B of member AB

M_{AB} = moment at end A of member AB

M_{BA} = moment at end B of member AB

L_1, L_2, L_3, L_4 / S_1, S_2, S_3, S_4 = distance from c.g. of wall to end of wall or from c.g. of wall to beam end

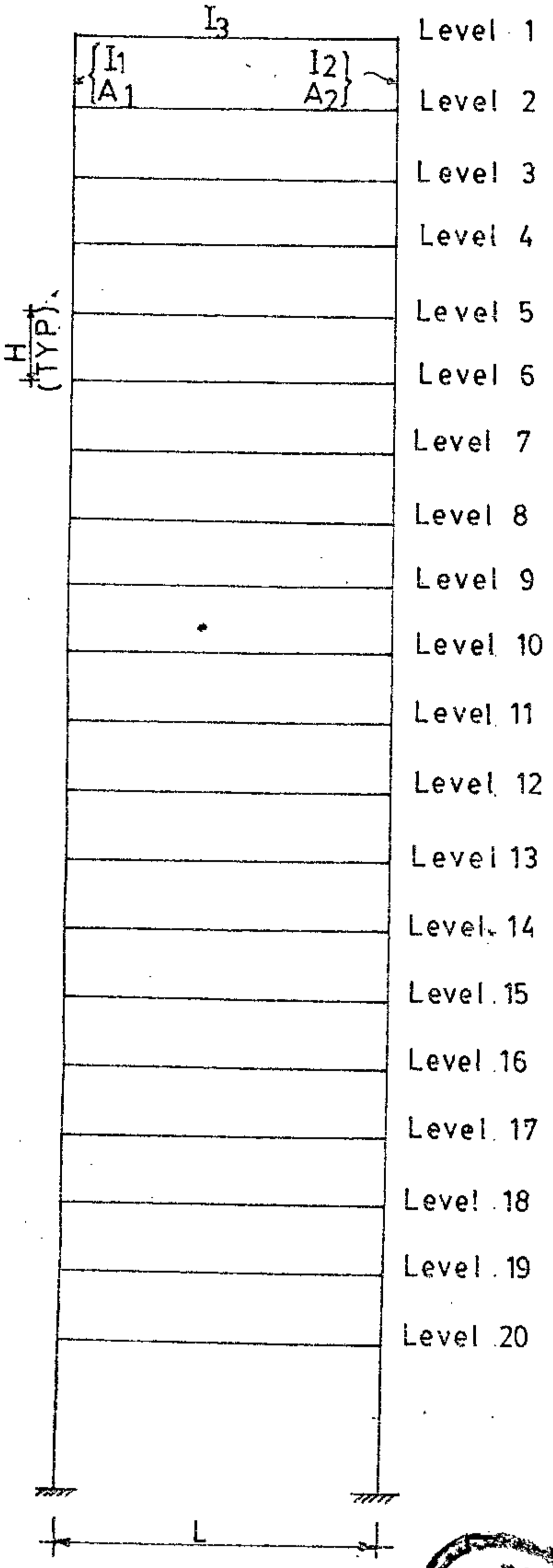
A_M = Gross-sectional area of center column or wall

C_M = Moment of inertia of center column or wall

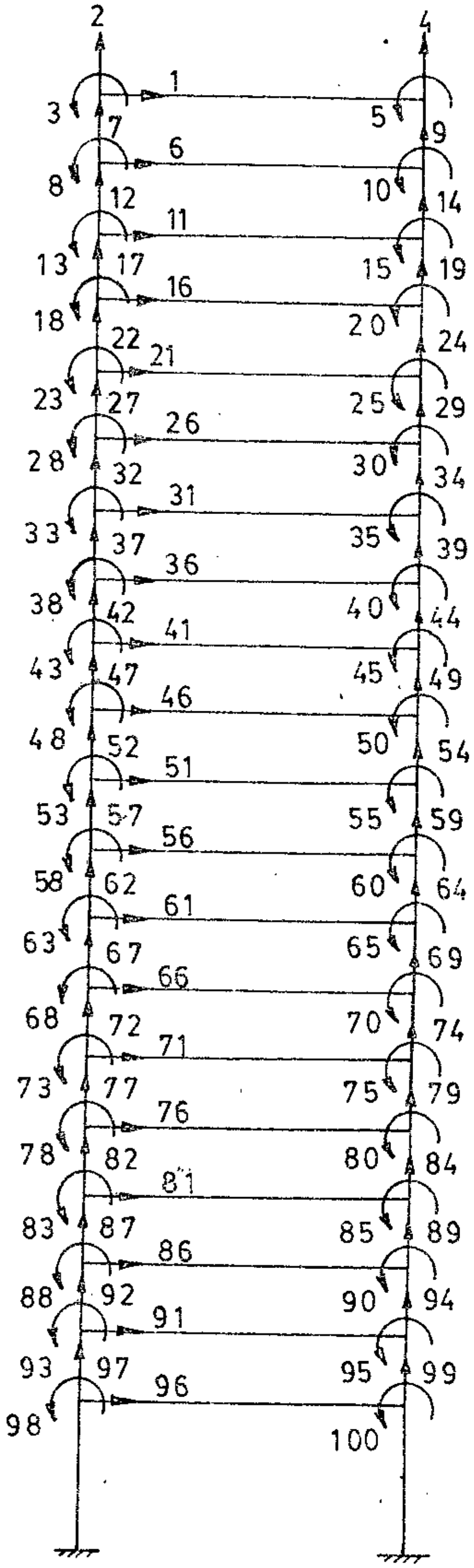
XL = Length of left beam

XR = Length of right beam

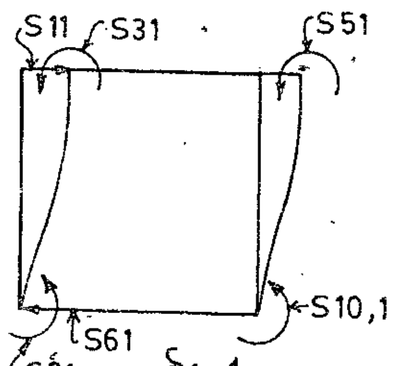
4.3 ONE BAY FRAME
(WITH COLUMN AND BEAM)



FRAME



COORDINATES



$\delta_1 = 1$

$S10,1 = 6EI2/H^2$

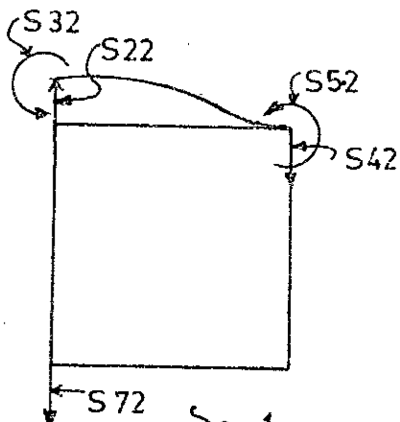
$S11 = 12E/H^3 (I1+I2)$

$S31 = 6EI1/H^2$

$S51 = 6EI2/H^2$

$S61 = -12E/H^3 (I1+I2)$

$S81 = \frac{6EI1}{H^2}$



$\delta_2 = 1$

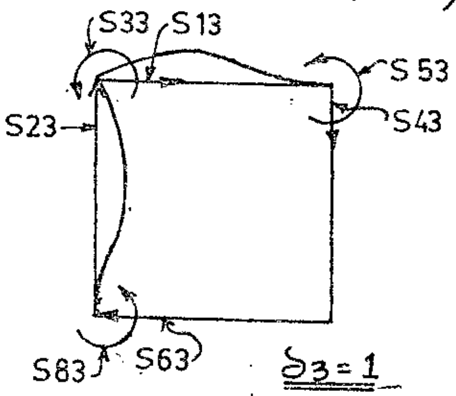
$S22 = EA1/H + 12EI3/L^3$

$S32 = 6EI3/L^2$

$S42 = -12EI3/L^3$

$S52 = 6EI3/L^2$

$S72 = -\frac{EA1}{H}$



$\delta_3 = 1$

$S13 = 6EI1/H^2$

$S23 = 6EI3/L^2$

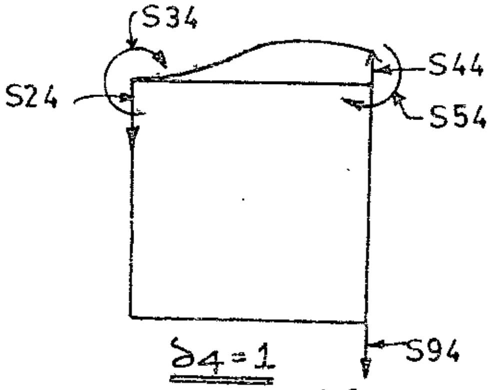
$S33 = 4E(I1/H + I3/L)$

$S43 = -6EI3/L^2$

$S53 = 2EI3/L$

$S63 = -6EI1/H^2$

$S83 = 2EI1/H$



$\delta_4 = 1$

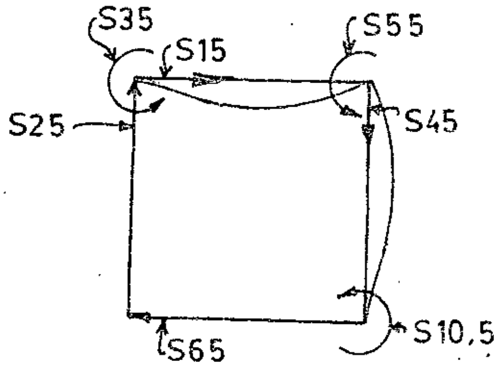
$S24 = -12EI3/L^3$

$S34 = -6EI3/L^2$

$S44 = EA2/H + 12EI3/L^3$

$S54 = -6EI3/L^2$

$S94 = -EA2/H$



$S10,5 = 2EI2/H$

$S15 = 6EI2/H^2$

$S25 = 6EI3/L^2$

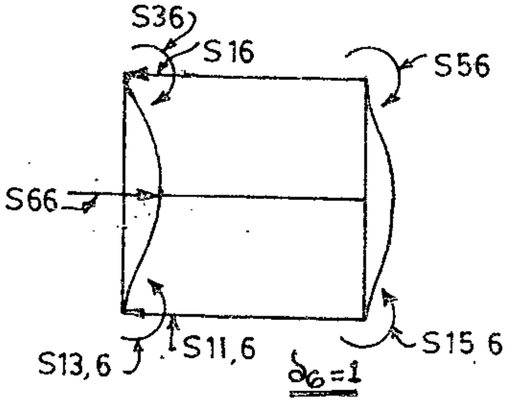
$S35 = 2EI3/L$

$S45 = -6EI3/L^2$

$S55 = 4E\left(\frac{I2}{H} + \frac{I3}{L}\right)$

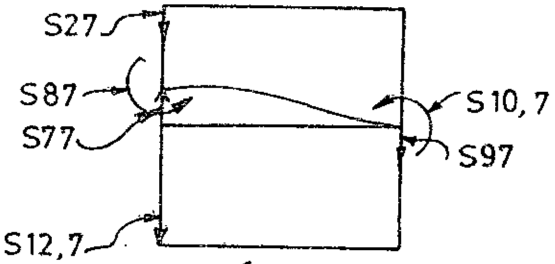
$S65 = -6EI2/H^2$

$\delta_5 = 1$



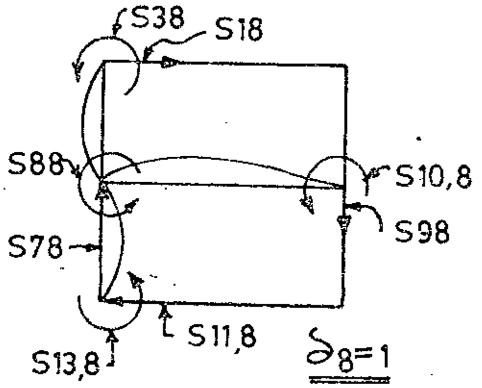
$\delta_6 = 1$

$S_{11,6} = -12E/H^3\,(I_1 + I_2)$
$S_{13,6} = 6EI_1/H^2$
$S_{15,6} = 6EI_2/H^2$ $\quad S_{16} = -12E/H^3\,(I_1 + I_2)$
$S_{36} = -6EI_1/H^2$ $\quad S_{56} = -6EI_2/H^2$
$S_{66} = 24E/H^3\,(I_1 + I_2)$



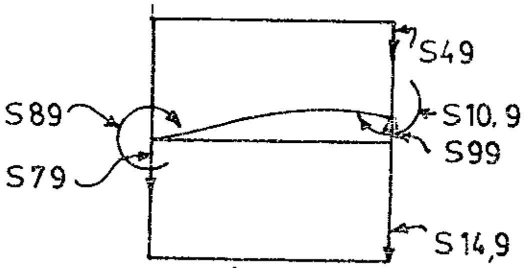
$\delta_7 = 1$

$S_{10,7} = 6EI_3/L^2$
$S_{12,7} = -EA_1/H$
$S_{27} = -EA_1/H$
$S_{77} = 2EA_1/H + 12EI_3/L^3$
$S_{87} = 6EI_3/L^2$
$S_{97} = -12EI_3/L^3$



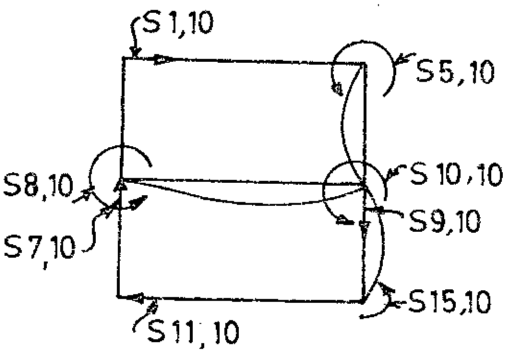
$\delta_8 = 1$

$S_{10,8} = 2EI_3/L$ $\quad S_{11,8} = -6EI_1/H^2$
$S_{13,8} = 2EI_1/H$ $\quad S_{38} = 2EI_1/H$
$S_{78} = 6EI_3/L^2$ $\quad S_{88} = 4E(2I_1/H + I_3/L)$
$S_{98} = -6EI_3/L^2$
$S_{18} = 6EI_1/H^2$



$\delta_9 = 1$

$S_{10,9} = -6EI_3/L^2$
$S_{14,9} = -EA_2/H$
$S_{49} = -EA_2/H$
$S_{79} = -12EI_3/L^2$
$S_{89} = -6EI_3/L^3$
$S_{99} = 2EA_2/H + 12EI_3/L^3$



$S_{1,10} = 6EI_2/H^2$
$S_{5,10} = 2EI_2/H$
$S_{7,10} = 6EI_3/L^2$
$S_{8,10} = 2EI_3/L$
$S_{9,10} = -6EI_3/L^2$
$S_{10,10} = 4E(2I_2/H + I_3/L)$
$S_{11,10} = -6EI_2/H^2$
$S_{15,10} = 2EI_2/H$

$\delta_{10} = 1$

4.3.1 STIFFNESS MATRIX OF THE FRAME

$$
[K] = \left[\begin{array}{cccccccccccccccccccc}
B_0 & C \\
A & B & C \\
& A & B & C \\
& & A & B & C \\
& & & A & B & C \\
& & & & A & B & C \\
& & & & & A & B & C \\
& & & & & & A & B & C \\
& & & & & & & A & B & C \\
& & & & & & & & A & B & C \\
& & & & & & & & & A & B & C \\
& & & & & & & & & & A & B & C \\
& & & & & & & & & & & A & B & C \\
& & & & & & & & & & & & A & B & C \\
& & & & & & & & & & & & & A & B & C \\
& & & & & & & & & & & & & & A & B & C \\
& & & & & & & & & & & & & & & A & B & C \\
& & & & & & & & & & & & & & & & A & B & C \\
& & & & & & & & & & & & & & & & & A & B & C \\
& & & & & & & & & & & & & & & & & & A & B
\end{array}\right]
$$

$$B0 = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{bmatrix}$$

$$B0 = \begin{bmatrix} \frac{12E}{H^3}(I_1+I_2) & 0 & 6EI_1/H^2 & 0 & 6EI_2/H^2 \\ 0 & \frac{EA_1}{H}+\frac{12EI_3}{L^3} & 6EI_3/L^2 & -12EI_3/L^3 & 6EI_3/L^2 \\ 6EI_1/H^2 & 6EI_3/L^2 & 4E(\frac{I_1}{H}+\frac{I_3}{L}) & -6EI_3/L^2 & 2EI_3/L \\ 0 & -12EI_3/L^3 & -6EI_3/L^2 & \frac{EA_2}{H}+\frac{12EI_3}{L^3} & -6EI_3/L^2 \\ 6EI_2/H^2 & 6EI_3/L^2 & 2EI_3/L & -6EI_3/L^2 & 4E(\frac{I_2}{H}+\frac{I_3}{L}) \end{bmatrix}$$

$$B = \begin{bmatrix} S_{66} & S_{67} & S_{68} & S_{69} & S_{6,10} \\ S_{76} & S_{77} & S_{78} & S_{79} & S_{7,10} \\ S_{86} & S_{87} & S_{88} & S_{89} & S_{8,10} \\ S_{96} & S_{97} & S_{98} & S_{99} & S_{9,10} \\ S_{10,6} & S_{10,7} & S_{10,8} & S_{10,9} & S_{10,10} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{24E}{H^3}(I_1+I_2) & 0 & 0 & 0 & 0 \\ 0 & \frac{2EA_1}{H}+\frac{12EI_3}{L^3} & 6EI_3/L^2 & -12EI_3/L^3 & 6EI_3/L^2 \\ 0 & 6EI_3/L^2 & 4E(\frac{2I_1}{H}+\frac{I_3}{L}) & -6EI_3/L^2 & 2EI_3/L \\ 0 & -12EI_3/L^3 & -6EI_3/L^2 & \frac{2EA_2}{H}+\frac{12EI_3}{L^3} & -6EI_3/L^2 \\ 0 & 6EI_3/L^2 & 2EI_3/L & -6EI_3/L^2 & 4E(\frac{2I_2}{H}+\frac{I_3}{L}) \end{bmatrix}$$

$$A = \begin{bmatrix} S_{61} & S_{62} & S_{63} & S_{64} & S_{65} \\ S_{71} & S_{72} & S_{73} & S_{74} & S_{75} \\ S_{81} & S_{82} & S_{83} & S_{84} & S_{85} \\ S_{91} & S_{92} & S_{93} & S_{94} & S_{95} \\ S_{10,1} & S_{10,2} & S_{10,3} & S_{10,4} & S_{10,5} \end{bmatrix}$$

$$A = \begin{bmatrix} -\frac{12E}{H^3}(I_1+I_2) & 0 & -6EI_1/H^2 & 0 & -6EI_2/H^2 \\ 0 & -EA_1/H & 0 & 0 & 0 \\ 6EI_1/H^2 & 0 & 2EI_1/H & 0 & 0 \\ 0 & 0 & 0 & -EA_2/H & 0 \\ 6EI_2/H & 0 & 0 & 0 & 2EI_2/H \end{bmatrix}$$

$$C = \begin{bmatrix} S_{16} & S_{17} & S_{18} & S_{19} & S_{1,10} \\ S_{26} & S_{27} & S_{28} & S_{29} & S_{2,10} \\ S_{36} & S_{37} & S_{38} & S_{39} & S_{3,10} \\ S_{46} & S_{47} & S_{48} & S_{49} & S_{4,10} \\ S_{56} & S_{57} & S_{58} & S_{59} & S_{5,10} \end{bmatrix}$$

$$C = \begin{bmatrix} -\frac{12E}{H^3}(I_1+I_2) & 0 & 6EI_1/H^2 & 0 & 6EI_2/H^2 \\ 0 & -EA_1/H & 0 & 0 & 0 \\ -6EI_1/H & 0 & 2EI_1/H & 0 & 0 \\ 0 & 0 & 0 & -EA_2/H & 0 \\ -6EI_2/H & 0 & 0 & 0 & 2EI_2/H \end{bmatrix}$$

4.3.2 SOLUTION BY RECURSION PROCEDURE

By using the stiffness matrix of the frame the force displacement equation of the frame may be written as

$$\begin{Bmatrix} \{F\}_1 \\ \{F\}_2 \\ \{F\}_3 \\ \vdots \\ \{F\}_j \\ \vdots \\ \{F\}_n \end{Bmatrix} = \begin{bmatrix} B0 & C & & & & & \\ A & B & C & & & & \\ & A & B & C & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & A & B & C & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & & & A & B \end{bmatrix} \begin{Bmatrix} \{U\}_1 \\ \{U\}_2 \\ \{U\}_3 \\ \vdots \\ \{U\}_j \\ \vdots \\ \{U\}_n \end{Bmatrix} \quad \text{---} \quad (A)$$

Where $\{F\}_1$, $\{F\}_2$, $\{F\}_3$, ... $\{F\}_n$ & $\{U\}_1$, $\{U\}_2$, $\{U\}_3$ ... $\{U\}_n$ etc. are all 5X1 column matrix.

Equation (A) can be written as n separate matrix equations as follows :

$$\{F\}_1 = Bo\,\{U\}_1 + C\,\{U\}_2 \quad (1)$$

$$\{F\}_2 = A\,\{U\}_1 + B\,\{U\}_2 + C\,\{U\}_3 \quad (2)$$

$$\{F\}_3 = A\,\{U\}_2 + B\,\{U\}_3 + C\,\{U\}_4 \quad (3)$$

$$\{F\}_4 = A\,\{U\}_3 + B\,\{U\}_4 + C\,\{U\}_5 \quad (4)$$

$$\{F\}_j = A\,\{U\}_{j-1} + B\,\{U\}_j + C\,\{U\}_{j+1} \quad (J)$$

$$\{F\}_{n-1} = A\,\{U\}_{n-2} + B\,\{U\}_{n-1} + C\,\{U\}_n \quad (n\ 1)$$

$$\{F\}_n = A\,\{U\}_{n-1} + B\,\{U\}_n \quad (n)$$

From eq^n (1) Solve for $\{U\}_1$

$$\{U\}_1 = Bo^{-1}\{F\}_1 - Bo^{-1}C\,\{U\}_2 \quad (21)$$

Substituting eq^n (21) into (2) and rearranging yields

$$\{F\}_2^* = B_1\{U\}_2 + C\,\{U\}_3 \quad (22a)$$

in which $\{F\}_2^* = \{F\}_2 - ABo^{-1}\,\{F\}_1$ (22b)

$$B_1 = B - ABo^{-1}C \quad (22c)$$

From eq^n.(22a) $\{U\}_2 = B_1^{-1}\,\{F\}_2^* - B_1^{-1}C\,\{U\}_3$ (22d)

Substituting eq^n (22d) in (3) and rearranging yields

$$\{F\}_3^* = B_2\,\{U\}_3 + C\,\{U\}_4 \quad (23a)$$

in which $\{F\}_3^* = \{F\}_3 - AB_1^{-1}\,\{F\}_2^*$ (23b)

$$B_2 = B - AB_1^{-1}C \quad (23c)$$

From eq^n (23a), $\{U\}_3 = B_2^{-1}\{F\}_3^* - B_2^{-1} C \{U\}_4$ (23d)

Proceeding in this manner the general equations in the recursion process have the following form

$$\{F\}_j^* = B_{j-1}\{U\}_j + C\{U\}_{j+1} \quad (24a)$$

in which
$$\{F\}_j^* = \{F\}_j - AB_{j-2}^{-1}\{F\}_{j-1}^* \quad (24b)$$

$$B_{j-1} = B - AB_{j-2}^{-1} C \quad (24c)$$

From eq^n (24a) $\{U\}_j = \{B\}_{j-1}^{-1}\{F\}_j^* - \{B\}_{j-1}^{-1} C \{U\}_{j+1}$ (24d)

The last of the matrix eq^n shall be

$$\{F\}_n^* = B_{n-1}\{U\}_n \quad (25a)$$

in which
$$\{F\}_n^* = \{F\}_n - AB_{n-2}^{-1}\{F\}_{n-1}^* \quad (25b)$$

$$B_{n-1} = B - AB_{n-2}^{-1} C \quad (25c)$$

From eq^n (25a) $\{U\}_n = B_{n-1}^{-1}\{F\}_n^*$ (25d)

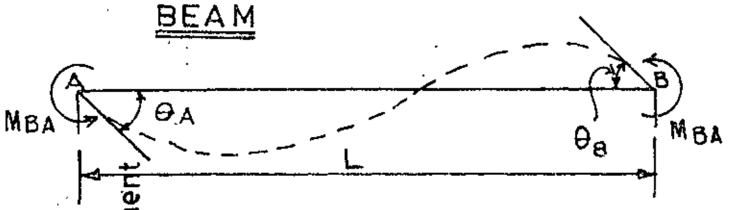
By back substitution we solve for $\{U\}_j$

The general equation for $\{U\}_j$ shall be

$$\{U\}_j = B_{j-1}^{-1}\left[\{F\}_j^* - C\{U\}_{j+1}\right]$$

4.3.3 SLOPE DEFLECTION EQUATIONS

BEAM

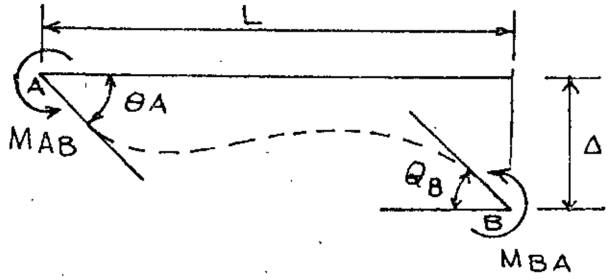


Anticlockwise moment = +ve
Clockwise rotation = +ve

with no vertical movement

$$M_{AB} = M_{FAB} + \frac{2EI}{L}(-2\theta_A - \theta_B)$$
$$M_{BA} = M_{FBA} + \frac{2EI}{L}(-\theta_A - 2\theta_B)$$

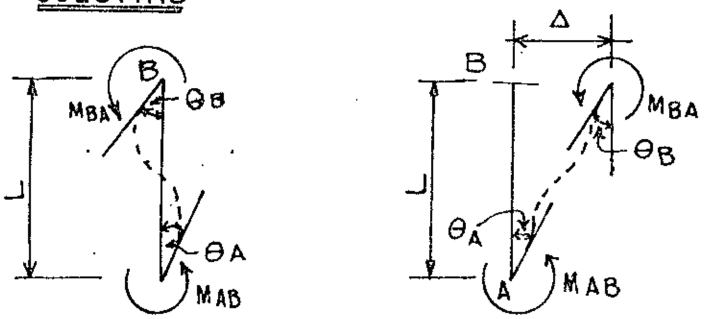
[M_{FAB} & M_{FBA} are Fixed end moments]



with vertical movement = Δ

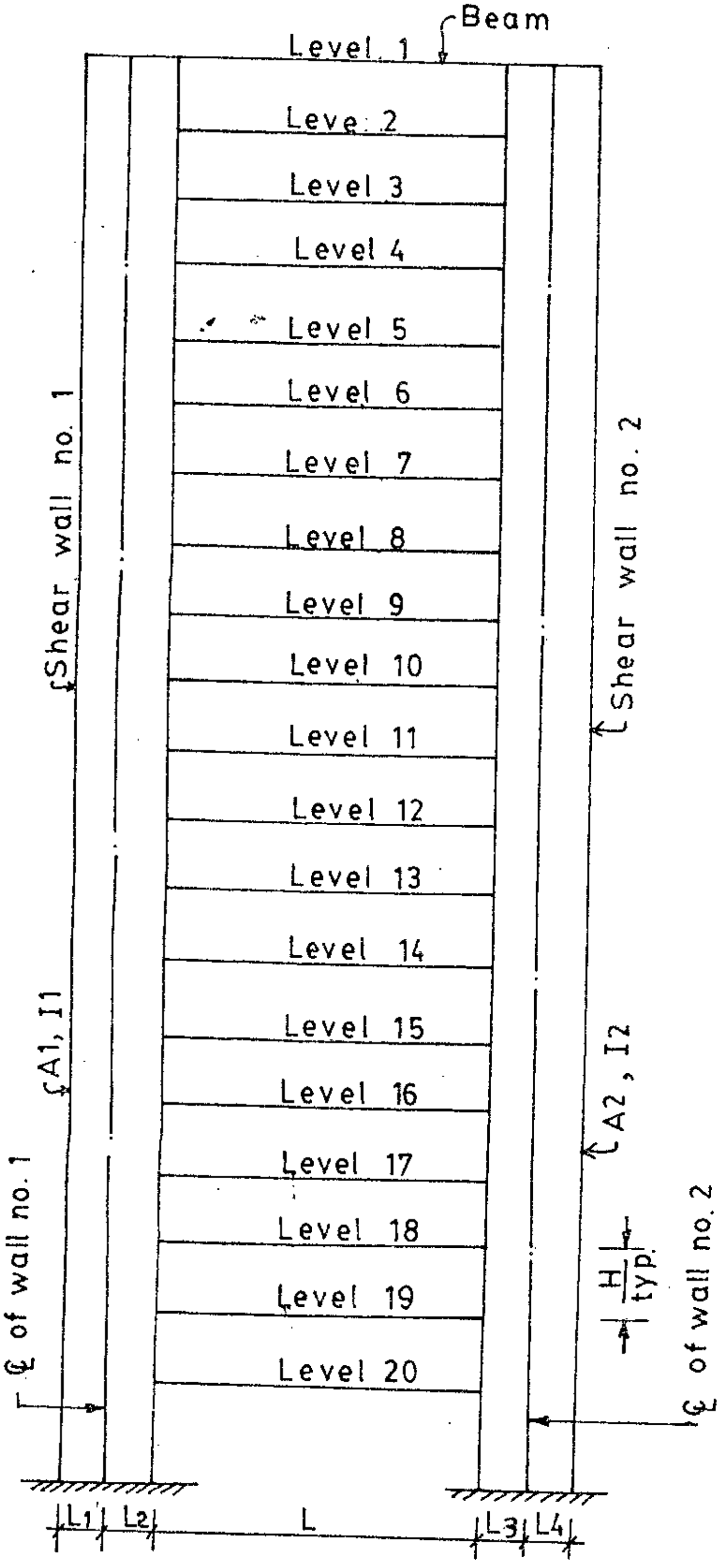
$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(-2\theta_A - \theta_B + \frac{3\Delta}{L}\right)$$
$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(-\theta_A - 2\theta_B + \frac{3\Delta}{L}\right)$$

COLUMNS

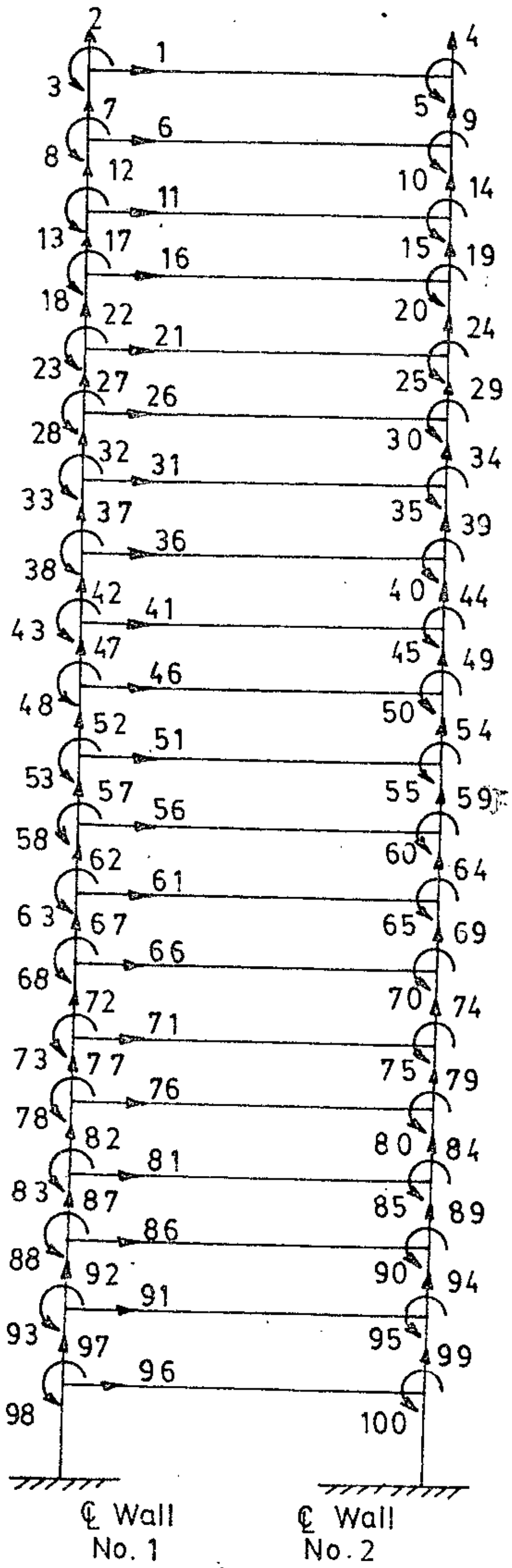


APPLY SAME FORMULAS AS THOSE FOR BEAM

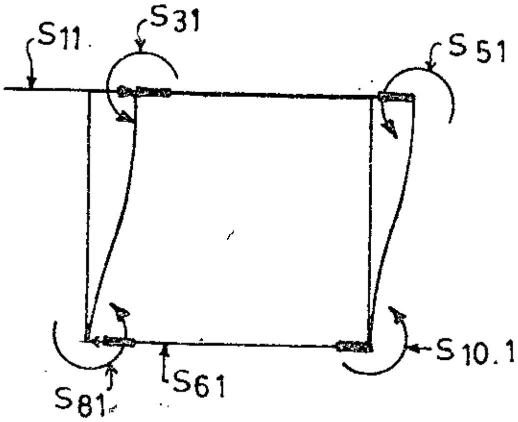
4.4. FRAME ANALYSIS (ONE BAY) (SHEAR WALL AND BEAM)

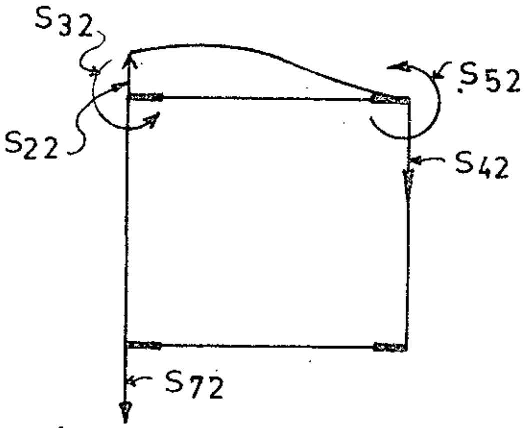


SHEAR WALL FRAME



COORDINATES





$S10.1 = 6EI2/H^2$ $\quad S11 = 12E/H^3\ (I1+I2)$

$S31 = 6EI1/H^2$ $\quad S51 = 6EI2/H^2$

$S61 = -12E/H^3\ (I1+I2)$

$S81 = 6EI1/H^2$

$\underline{\delta_1 = 1}$

$S22 = EA1/H + 12EI3/L^3$

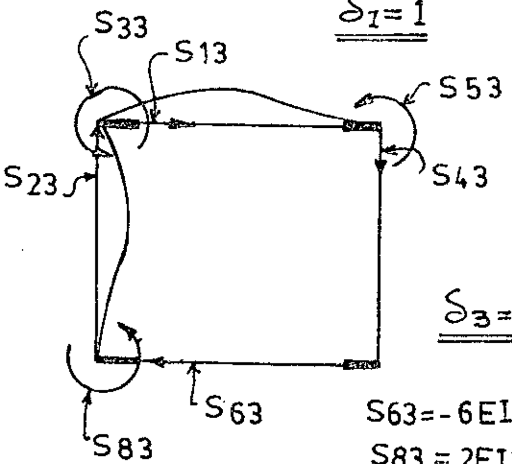
$S32 = 6EI3/L^2\ (1+2L2/L)$

$S42 = -12EI3/L^3$

$S52 = 6EI3/L^2\ (1+2L3/L)$

$S72 = -EA1/H$

$\underline{\delta_2 = 1}$



$\underline{\delta_3 = 1}$

$S63 = -6EI1/H^2$

$S83 = 2EI1/H$

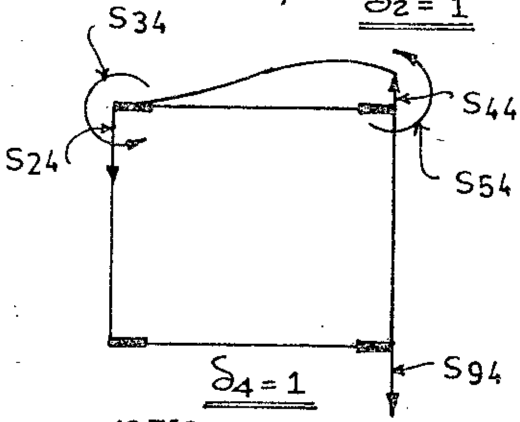
$S13 = 6EI1/H^2$

$S23 = 6EI3/L^2\left[1+\frac{2L2}{L}\right]$

$S33 = \frac{4EI1}{H} + \frac{4EI3}{L}\left[1+\frac{3L2}{L}\left(1+\frac{L2}{L}\right)\right]$

$S43 = -6EI3/L^2\ [1+2L2/L]$

$S53 = \frac{2EI3}{L}\left[1+\frac{3}{L}\left(L2+L3+2L2L3/L\right)\right]$



$\underline{\delta_4 = 1}$

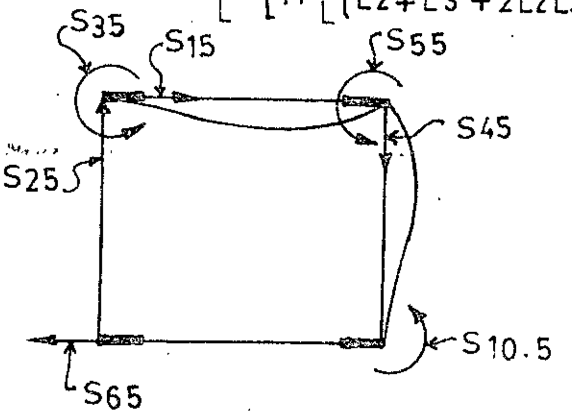
$S24 = -\frac{12EI3}{L^3}$

$S34 = -6EI3/L^2\ (1+2L2/L)$

$S44 = \frac{EA2}{H} + \frac{12EI3}{L^3}$

$S54 = -\frac{6EI3}{L^2}\ (1+2L3/L)$

$S94 = -\frac{EA2}{H}$



$\underline{\delta_5 = 1}$

$S10.5 = \frac{2EI2}{H}$

$S15 = \frac{6EI2}{H^2}$

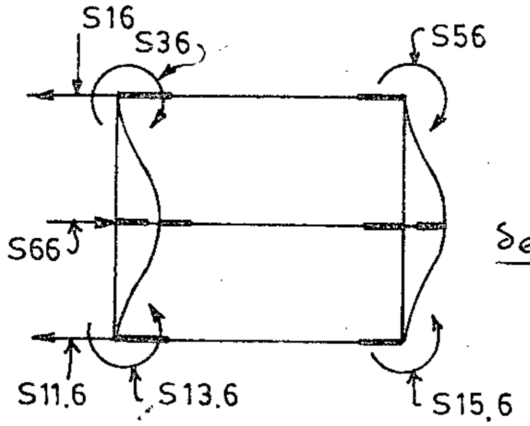
$S25 = \frac{6EI3}{L^2}\left[1+\frac{2L3}{L}\right]$

$S35 = \frac{2EI3}{L}\left[1+\frac{3}{L}\left(L2+L3+2L2L3/L\right)\right]$

$S45 = -\frac{6EI3}{L^2}\left[1+\frac{2L3}{L}\right]$

$S55 = \frac{4EI2}{H} + \frac{4EI3}{L}\left[1+\frac{3L3}{L}\left(1+\frac{L3}{L}\right)\right]$

$S65 = -6EI2/H^2$



$\underline{\delta_6 = 1}$

$S_{11.6} = -12E/H^3\,(I1 + I2)$

$S_{13.6} = 6EI1/H^2$

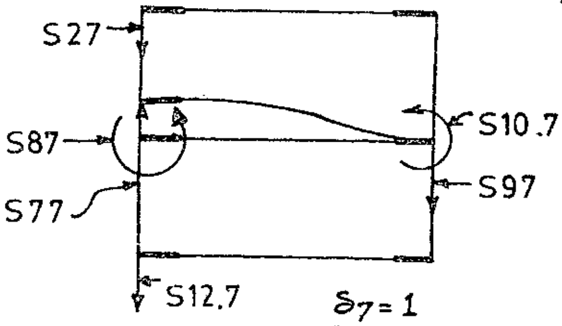
$S_{15.6} = 6EI2/H^2$

$S_{16} = -12E/H^3\,(I1 + I2)$

$S_{36} = -6EI1/H^2$

$S_{56} = -6EI2/H^2$

$S_{66} = \dfrac{24E}{H^3}\,(I1 + I2)$



$\underline{\delta_7 = 1}$

$S_{12.7} = -\dfrac{EA1}{H}$

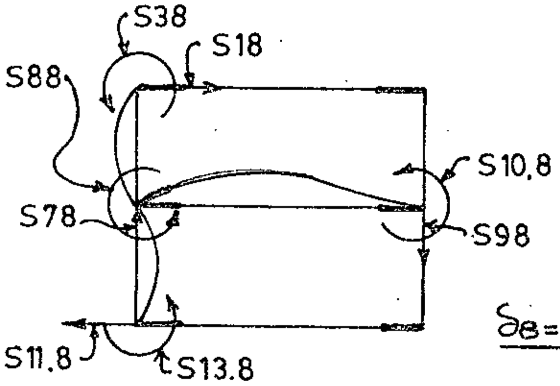
$S_{27} = -\dfrac{EA1}{H}$

$S_{10.7} = 6EI3/L^2\,[1 + 2L3/L]$

$S_{77} = 2EA1/H + 12EI3/L^3$

$S_{87} = 6EI3/L^2\,[1 + 2L2/L]$

$S_{97} = -12EI3/L^3$



$\underline{\delta_8 = 1}$

$S_{10.8} = 2EI3/L\,[1 + 3/L\,(L2 + L3 + 2L2\,L3/L)]$

$S_{11.8} = -6EI1/H^2$

$S_{13.8} = 2EI1/H$

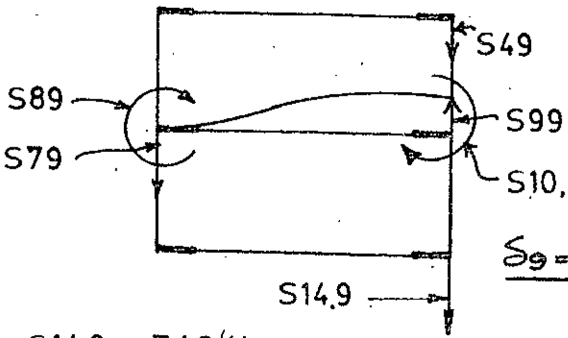
$S_{18} = 6EI1/H2$

$S_{38} = 2EI1/H$

$S_{78} = 6EI3/L^2\,[1 + 2L2/L]$

$S_{88} = 8EI1/H + 4\dfrac{EI3}{L}\,[1 + 3L2/L\,(1 + L2/L)]$

$S_{98} = -6EI3/L^2\,[1 + 2L2/L]$



$\underline{\delta_9 = 1}$

$S_{14,9} = -EA2/H$

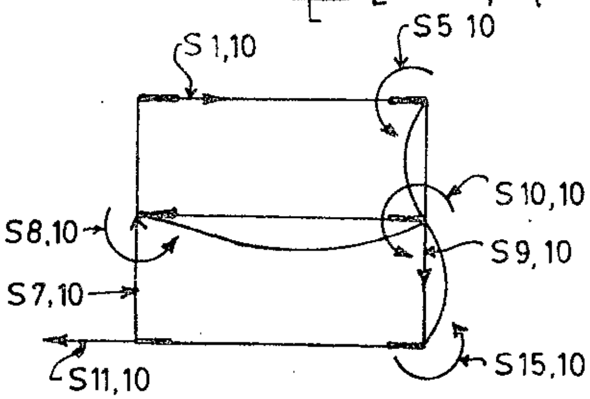
$S_{10,9} = -6EI3/L^2\,[1 + 2L3/L]$

$S_{49} = -EA2/H$

$S_{79} = -12EI3/L^3$

$S_{89} = -6EI3/L^2\,[1 + 2L2/L]$

$S_{99} = 2EA2/H + 12EI3/L^3$



$\underline{\delta_{10} = 1}$

$S_{1,10} = 6EI2/H^2$

$S_{5,10} = 2EI2/H$

$S_{7,10} = 6EI3/L^2\,[1 + 2L3/L]$

$S_{8,10} = 2EI3/L\,[1 + 3/L\,(L2 + L3 + 2L2L3/L)]$

$S_{9,10} = -6EI3/L^2\,[1 + 2L3/L]$

$S_{10,10} = 8EI2/H + 4EI3/L\,[1 + 3L3/L\,(1 + L3/L)]$

$S_{11,10} = -6EI2/H^2$

$S_{15,10} = 2EI2/H$

C =

S_{16}	S_{17}	S_{18}	S_{19}	$S_{1\,10}$
S_{26}	S_{27}	S_{28}	S_{29}	$S_{2\,10}$
S_{36}	S_{37}	S_{38}	S_{39}	$S_{3\,10}$
S_{46}	S_{47}	S_{48}	S_{49}	$S_{4\,10}$
S_{56}	S_{57}	S_{58}	S_{59}	$S_{5\,10}$

or C =

$-\frac{12E}{H^3}(I_1+I_2)$	0	$6EI1/H^2$	0	$6EI2/H^2$
0	$-EA1/H$	0	0	0
$-6EI1/H^2$	0	$2EI1/H$	0	0
0	0	0	$-EA2/H$	0
$-6EI2/H^2$	0	0	0	$2EI2/H$

Note All the elements of matrix "C" is same as frame stiffness matrix

MATRIX A = TRANSPOSE OF MATRIX C OR, $A = C^T$

B =

S_{66}	S_{67}	S_{68}	S_{69}	$S_{6,10}$
S_{76}	S_{77}	S_{78}	S_{79}	$S_{7,10}$
S_{86}	S_{87}	S_{88}	S_{89}	$S_{8,10}$
S_{96}	S_{97}	S_{98}	S_{99}	$S_{9,10}$
$S_{10,6}$	$S_{10,7}$	$S_{10,8}$	$S_{10,9}$	$S_{10,10}$

$$\text{or, } B = E^* \begin{bmatrix} \frac{24}{H^3}(I1+I2) & 0 & 0 & 0 & 0 \\ 0 & 2A1/H + 12I3/L^3 & 6I3/L^2 + 12I3L2/L^3 & -12I3/L^3 & 6I3/L^2 + 12I3L3/L^3 \\ 0 & 6I3/L^2 + 12I3L2/L^3 & \frac{8I1}{H} + \frac{4I3}{L} + \frac{12I3L2}{L^2}\left(1+\frac{L2}{L}\right) & -6I3/L^2 - 12I3L2/L^3 & \frac{2I3}{L} + \frac{6I3}{L^2}(L2 + L3 + 2L2L3/L) \\ 0 & -12I3/L^3 & -6I3/L^2 - 12I3L2/L^3 & 2A2/H + 12I3/L^3 & -6I3/L^2 - 12I3L3/L^3 \\ 0 & 6I3/L^2 + 12I3L3/L^3 & \frac{2I3}{L} + \frac{6I3}{L^2}(L2 + L3 + 2L2L3/L) & -6I3/L^2 - 12I3L3/L^3 & \frac{8I2}{H} + \frac{4I3}{L} \; \frac{12I3L3}{L^2}\left(1+\frac{L3}{L}\right) \end{bmatrix}$$

$$BO = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{bmatrix}$$

$$\text{or, } BO = E^* \begin{bmatrix} \frac{12}{H^3}(I1+I2) & 0 & 6I1/H^2 & 0 & 6I2/H^2 \\ 0 & \frac{A1}{H} + \frac{12I3}{L^3} & \frac{6I3}{L^2} + \frac{12I3L2}{L^3} & -12I3/L^3 & 6I3/L^2 + 12I3L3/L^3 \\ 6I1/H^2 & \frac{6I3}{L^2} + \frac{12I3L2}{L^3} & 4I1/H + 4I3/L + \frac{12I3L2}{L^2}\left(1+\frac{L2}{L}\right) & -6I3/L^2 - 12I3L2/L^3 & 2I3/L + 6I3/L^2(L2 + L3 + 2L2L3/L) \\ 0 & -12I3/L^3 & -6I3/L^2 - 12I3L2/L^3 & \frac{A2}{H} + \frac{12I3}{L^3} & -6I3/L^2 - 12I3L3/L^3 \\ 6I2/H^2 & 6I3/L^2 + 12I3L3/L^3 & \frac{2I3}{L} + \frac{6I3}{L^2}(L2 + L3 + 2L2L3/L) & -6I3/L^2 - 12I3L3/L^3 & 8I2/H + 4I3/L + \frac{12I3L3}{L^2}(1 + L3/L) \end{bmatrix}$$

4.5. GENERAL ONE BAY FRAME ANALYSIS

MOment of Inertia of Left Column or Wall = CL

Moment of Inertia of Right Column or Wall = CR

Area of Left Column or Wall = AL

Area of Right Column or Wall = AR

Moment of Inertia of Beam = BL

Beam Length = X

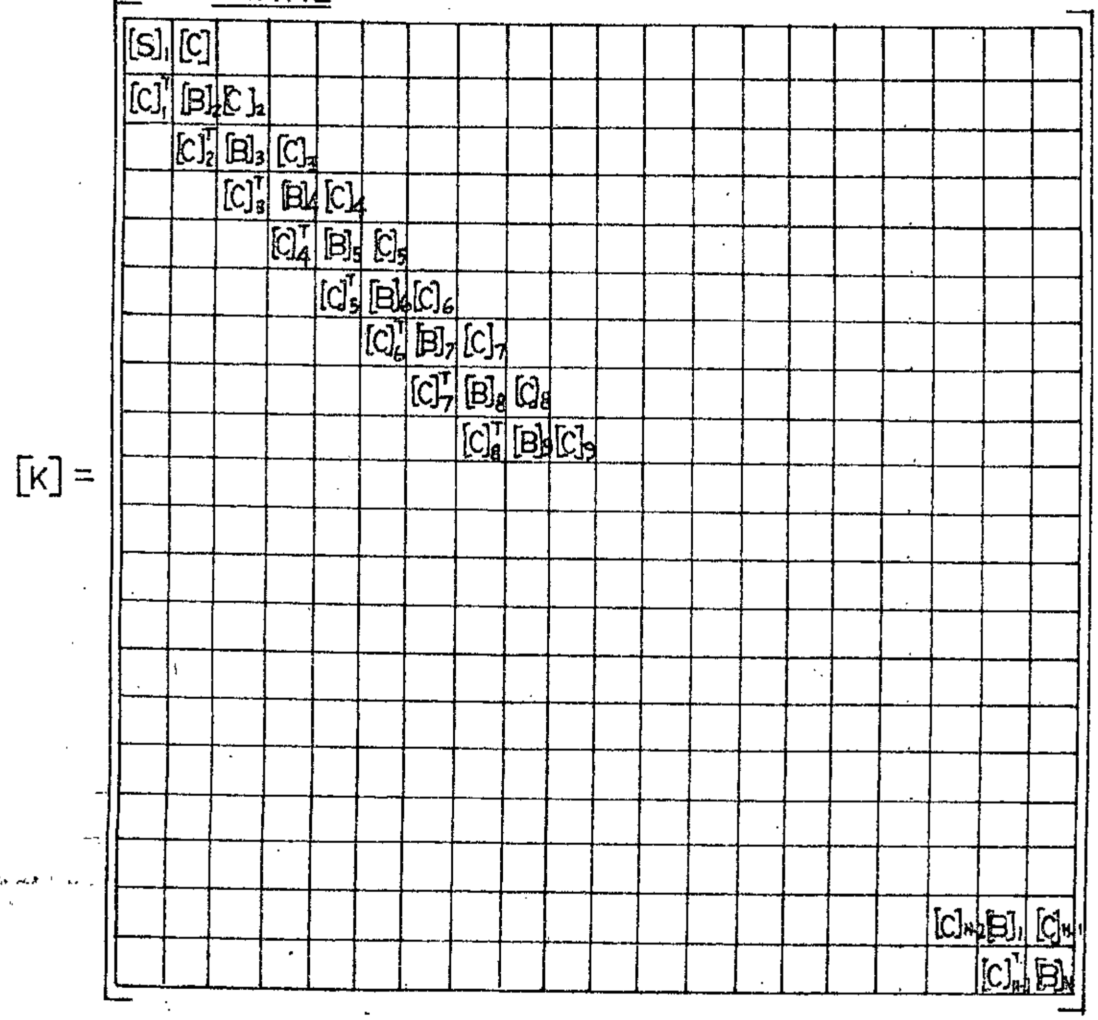
Story Height = H

Distance from c.g. of left wall to beam end = S_2

Distance from c.g. of right wall to beam end = S_3

All the above quantities may vary
from story to story

4.5.1. STIFFNESS MATRIX FOR GENERAL ONE BAY FRAME



$$[S]_1 = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{bmatrix}$$

$$S_{11} = \frac{12E}{H(1)^3}(CL(1) + CR(1)); \quad S_{21} = 0; \quad S_{31} = \frac{6E\,CL(1)}{H(1)^2}$$

$$S_{41} = 0; \quad S_{51} = \frac{6ECR(1)}{H(1)^2};$$

$$S_{22} = \frac{EAL(1)}{H(1)} + \frac{12EBL(1)}{X(1)^3}; \quad S_{32} = \frac{6EBL(1)}{X(1)^2} * \left(1 + \frac{2\,S2(1)}{X(1)}\right)$$

$$S_{42} = -\frac{12EBL(1)}{X(1)^3}; \quad S_{52} = \frac{6EBL(1)}{X(1)^2}\left(1 + \frac{2S3(1)}{X(1)}\right);$$

$$S_{33} = \frac{4ECL(1)}{H(1)} + \frac{4EBL(1)}{X(1)} * \left[1 + \frac{3S2(1)}{X(1)}\left(1 + \frac{S2(1)}{X(1)}\right)\right]$$

$$S_{43} = -\frac{6EBL(1)}{X(1)^2}\left(1 + \frac{2S2(1)}{X(1)}\right)$$

$$S_{53} = \frac{2EBL(1)}{X(1)}\left[1 + \frac{3}{X(1)}(S2(1) + S3(1) + 2S2(1)\,S3(1)/X(1)\right]$$

$$S_{44} = \frac{EAR(1)}{H(1)} + \frac{12EBL(1)}{X(1)^3}; \quad S_{54} = -\frac{6EBL(1)}{X(1)^2}\left(1 + 2S3(1)/X(1)\right)$$

$$S_{55} = \frac{4ECR(1)}{H(1)} + \frac{4EBL(1)}{X(1)}\left[1 + \frac{3S3(1)}{X(1)}\left(1 + \frac{S3(1)}{X(1)}\right)\right]$$

The above matrix is a symmetric matrix. Hence the other elements of the matrix can be generated from the above elements

$$[C]_1 = \begin{bmatrix} S_{16} & S_{17} & S_{18} & S_{19} & S_{1,10} \\ S_{26} & S_{27} & S_{28} & S_{29} & S_{2,10} \\ S_{36} & S_{37} & S_{38} & S_{39} & S_{3,10} \\ S_{46} & S_{47} & S_{48} & S_{49} & S_{4,10} \\ S_{56} & S_{57} & S_{58} & S_{59} & S_{5,10} \end{bmatrix}$$

The elements of matrix $[c]_1$ are:

$S_{16} = -\frac{12E}{H(1)^3}$ (CL (1) + CR (1); $S_{26} = 0$; $S_{36} = -\frac{6ECL(1)}{H(1)^2}$

$S_{46} = 0$; $S_{56} = -\frac{6ECR(1)}{H(1)^2}$; $S_{17} = 0$; $S_{27} = -\frac{EAL(1)}{H(1)}$

$S_{37} = S_{47} = S_{57} = 0$

$S_{18} = \frac{6ECL(1)}{H(1)^2}$; $S_{28} = 0$; $S_{38} = \frac{2ECL(1)}{H(1)}$; $S_{48} = S_{58} = 0$

$S_{19} = S_{29} = S_{39} = 0$; $S_{49} = -\frac{EAR(1)}{H(1)}$; $S_{59} = 0$

$S_{1,10} = \frac{6ECR(1)}{H(1)^2}$; $S_{2,10} = S_{3,10} = S_{4,10} = 0$; $S_{5,10} = \frac{2ECR(1)}{H(1)}$

FROM MATRIX $[C]_1$ GENERATE MATRIX $[C]_1^T$

IN GENERAL (IF NI = I − 1, WHERE I IS STORY LEVEL FROM TOP) THE MATRIX $[C]_{N1}$ IS GIVEN BY: [TAKING "E" COMMON FOR ALL]

C (1,1) = 12.* (CL (NI) + CR (NI))/H (NI)**3
C (3,1) = −6.* CL(NI)/H (NI) ** 2
C (5,1) = 6.* CR (NI) /H (NI) ** 2
C (2,2) = −AL (NI) /H(NI)
C (1,3) = 6.*CL (NI) /H(NI)**2
C (3,3) = 2.*CL (NI)/H (NI)
C (4,4) = −AR (NI)/H (NI)
C (1,5) = 6.* CR (NI)/ H(NI) ** 2
C (5,5) = 2.* CR (NI) /H(NI)

$[B]_2 =$

S_{66}	S_{67}	S_{68}	S_{69}	$S_{6,10}$
S_{76}	S_{77}	S_{78}	S_{79}	$S_{7,10}$
S_{86}	S_{87}	S_{88}	S_{89}	$S_{8,10}$
S_{96}	S_{97}	S_{98}	S_{99}	$S_{9,10}$
$S_{10,6}$	$S_{10,7}$	$S_{10,8}$	$S_{10,9}$	$S_{10,10}$

$$S_{66} = \frac{12E}{H(1)^3}\left(CL(1) + CR(1)\right) + \frac{12E}{H(2)^3}\left(CL(2) + CR(2)\right)$$

$$S_{76} = 0 \; ; \; S_{86} = \frac{6ECL(2)}{H(2)^2} - \frac{6ECL(1)}{H(1)^2} \; ; \; S_{96} = 0$$

$$S_{10,6} = \frac{6ECR(2)}{H(2)^2} - \frac{6ECR(1)}{H(1)^2}$$

$$S_{77} = \frac{EAL(1)}{H(1)} + \frac{EAL(2)}{H(2)} + \frac{12EBL(2)}{X(2)^3}$$

$$S_{87} = \frac{6EBL(2)}{X(2)^2}\left(1 + \frac{2S2(2)}{X(2)}\right) ; \; S_{97} = -\frac{12EBL(2)}{X(2)^3}$$

$$S_{10,7} = \frac{6EBL(2)}{X(2)^2}\left(1 + \frac{2S3(2)}{X(2)}\right)$$

$$S_{88} = \frac{4ECL(1)}{H(1)} + \frac{4ECL(2)}{H(2)} + \frac{4EBL(2)}{X(2)}\left[1 + \frac{3S2(2)}{X(2)}\left(1 + \frac{S2(2)}{X(2)}\right)\right]$$

$$S_{98} = -\frac{6EBL(2)}{X(2)^2}\left(1 + 2S2(2)/X(2)\right)$$

$$S_{10,8} = \frac{2EBL(2)}{X(2)}\left[1 + \frac{3}{X(2)}\left(S2(2) + S3(2) + 2S2(2)\,S3(2)/X(2)\right)\right]$$

$$S_{99} = \frac{EAR(1)}{H(1)} + \frac{EAR(2)}{H(2)} + \frac{12EBL(2)}{X(2)^3}$$

$$S_{10,9} = -\frac{6EBL(2)}{X(2)^2}\left(1 + \frac{2S3(2)}{X(2)}\right)$$

$$S_{10,10} = \frac{4ECR(1)}{H(1)} + \frac{4ECR(2)}{H(2)} + \frac{4EBL(2)}{X(2)}\left[1 + \frac{3S3(2)}{X(2)}\left(1 + \frac{S3(2)}{X(2)}\right)\right]$$

The above matrix is a symmetric matrix. Therefore all other elements can be generated from the above elements.

In general (IF N1=I-1, where I is story level from top) the matrix $[B]_I$ is given by as follows: where I > 1 (Take "E" common)

Let: Z_1 = 1.+2.* S2 (I)/X(I)

Z_2 = 1.+2.* S3 (I)/X (I)

Z_3 = 1.+3.* S2 (I)/X (I) *(1.+S2 (I) /X(I))

Z_4 = 1.+3 * S3 (I)/X (I) *(1.+ S3 (I) /X (I))

Z_5 = 1.+3./X(I) * (S2 (I)+ S3 (I)+2.* S2 (I) * S3(I)/X (I))

B (1,1)= 12.* ((CL(I)+ CR(I))/H (I)**3 + (CL (N1) + CR (N1)) /H(N1)**3

B (2,1)= 0

B (3,1)= 6.*(CL (I)/H(I)**2 − CL (N1)/H(N1)**2

B (4,1)= 0

B (5,1)= 6.*(CR (I)/H (I)**2 − CR(N1) /H(N1)**2

B (2,2)= AL(N1)/H (N1) + AL(I) /H (I) + 12.* BL (I) X (I)**3

B (3,2)= 6.* BL (I)/X(I)**2 * Z1

B (4,2)=−12 * BL (I)/X(I) **3

B (5,2)= 6.* BL (I)/X (I)**2 * Z2

B (3,3)= 4.*(CL (N1)/H(N1) + CL(I)/H(I) + BL (I)/X(I) * Z 3)

B(4,3)=−6.* BL(I) /X(I)**2 * Z1

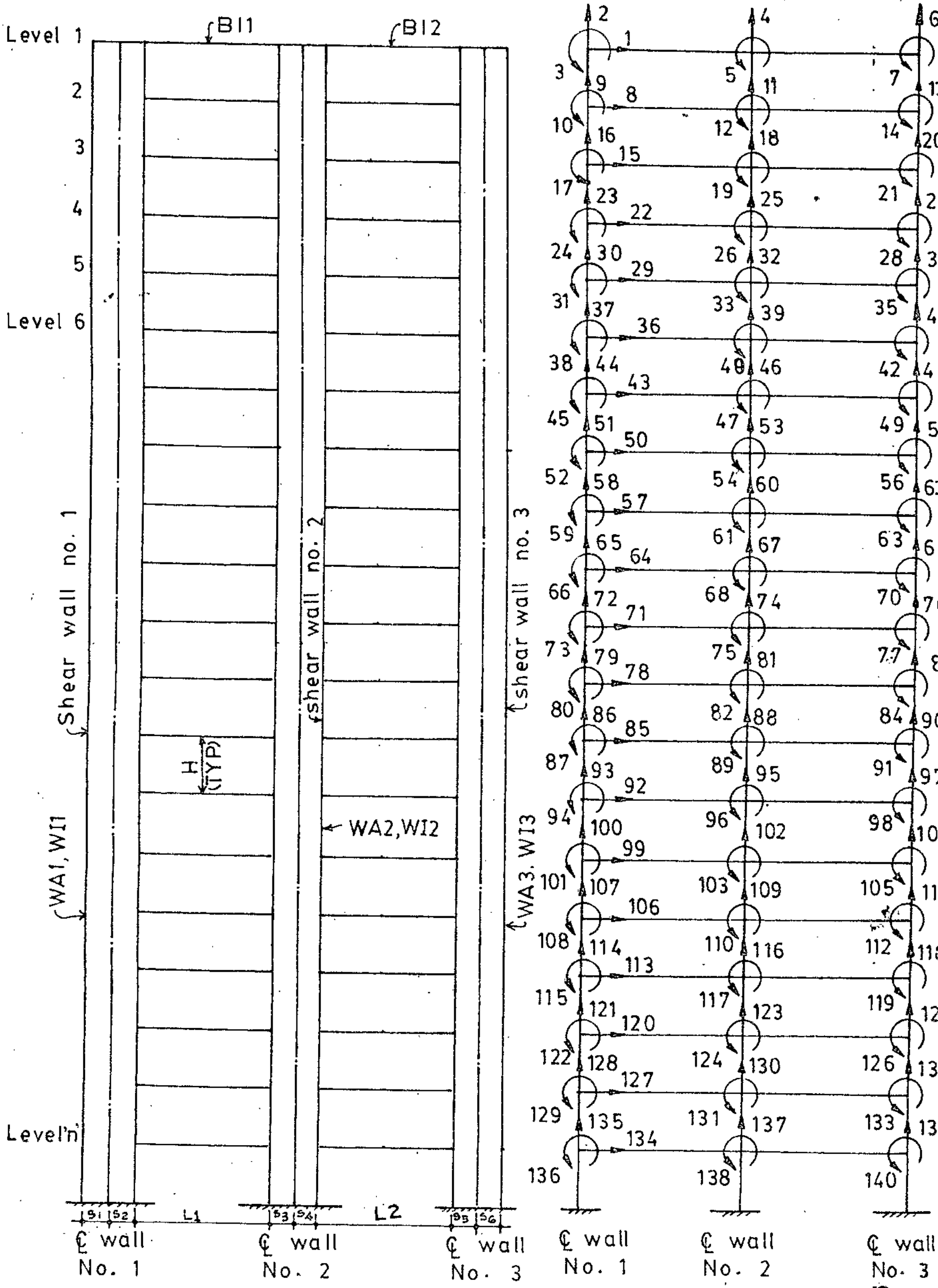
B(5,3)= 2.* BL(I)/X(I) * Z5

B(4,4)= AR (N1)/H(N1) + AR(I) /H(I) + 12.* BL (I)/X(I)**3

B(5,4)=−6.* BL(I)/X(I)**2 * Z2

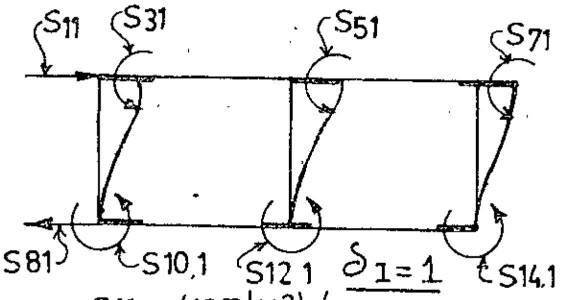
S(5,5)= 4.*(CR (N1)/H(N1) + CR(I)/H(I) + BL(I)/X(I) * Z4)

4.6. TWO BAY FRAME ANALYSIS (WITH SHEAR WALL)



SHEAR WALL FRAME

COORDINATES



$\delta_1 = 1$

$S11 = (12E/H^3)(WI1 + WI2 + WI3)$

$S31 = 6E\,WI1/H^2$

$S51 = 6E\,WI2/H^2$

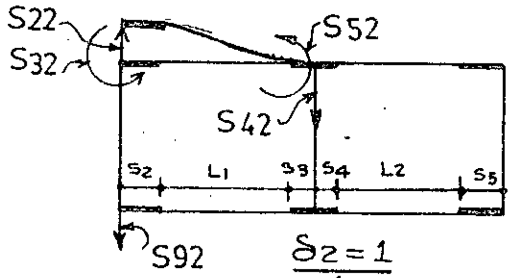
$S71 = 6E\,WI3/H^2$

$S81 = -(12E/H^3)(WI1 + WI2 + WI3)$

$S10{,}1 = 6E\,WI1/H^2$

$S12{,}1 = 6E\,WI2/H^2$

$S14{,}1 = 6E\,WI3/H^2$



$\delta_2 = 1$

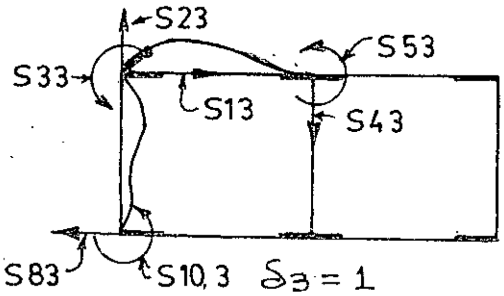
$S22 = E\,WA1/H + 12E\,BI1/L1^3$

$S32 = (6E\,BI1/L1^2)(1 + 2S2/L1)$

$S42 = -(12E\,BI1/L1^3)$

$S52 = (6E\,BI1/L1^2)(1 + 2S3/L1)$

$S92 = -(E\,WA1/H)$



$\delta_3 = 1$

$S13 = 6E\,WI1/H^2$

$S23 = (6E\,BI1/L1^2)(1 + 2S2/L1)$

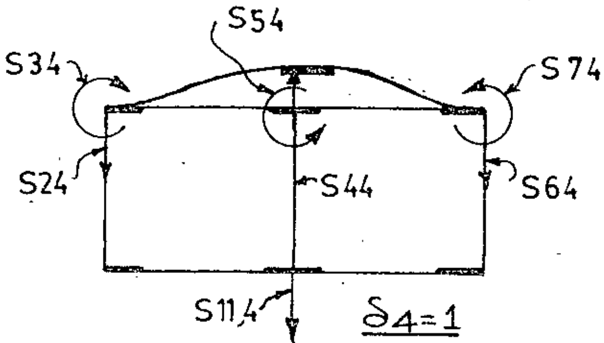
$S33 = 4E\,WI1/H + (4E\,BI1/L1)[1 + 3S2/L1$
$* (1 + S2/L1)]$

$S43 = -(6E\,BI1/L1^2)(1 + 2S2/L1)$

$S53 = (2E\,BI1/L1\,[1 + 3/L1$
$* (S2 + S3 + 2S2S3/L1)]$

$S83 = -(6E\,WI1/H^2)$

$S10{,}3 = (2E\,WI1/H)$



$\delta_4 = 1$

$S24 = -(12E\,BI1/L1^3)$

$S34 = -(6E\,BI1/L1^2)(1 + 2S2/L1)$

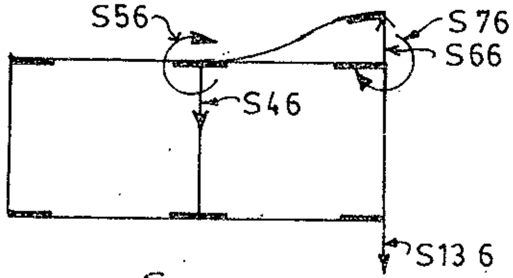
$S44 = E\,WA2/H + 12E\,BI1/L1^3 + 12E\,BI2/L2^3$

$S54 = (6E\,BI2/L2^2)(1 + 2S4/L2)$
$-(6E\,BI1/L1^2)(1 + 2S3/L1)$

$S64 = -12E\,BI2/L2^3$

$S74 = (6E\,BI2/L2^2)(1 + 2S5/L2)$

$S11{,}4 = -(E\,WA2/H)$



$\delta_6 = 1$

$S46 = -(12E\,BI2/L2^3)$

$S56 = -(6E\,BI2/L2^2)(1 + 2S4/L2)$

$S66 = E\,WA3/H + 12E\,BI2/L2^3$

$S76 = -(6E\,BI2/L2^2)(1 + 2S5/L2)$

$S13{,}6 = -(E\,WA3/H)$

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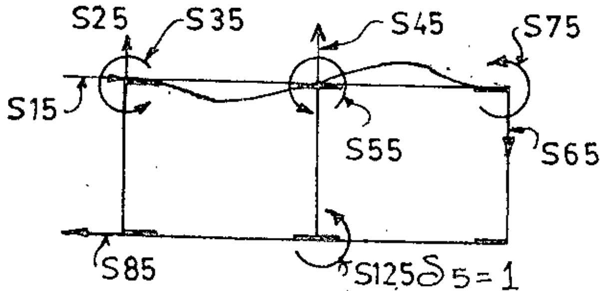


$\underline{\delta 5=1}$

$S15 = 6EWI2/H^2$

$S25 = (6EBI1/L1^2)(1+2S3/L1)$

$S35 = (2EBI1/L1)[1+3/L1(S2+S3 + 2S2S3/L1)]$

$S45 = (6EBI2/L2^2)(1+2S4/L2) - (6EBI1/L1^2)(1+2S3/L1)$

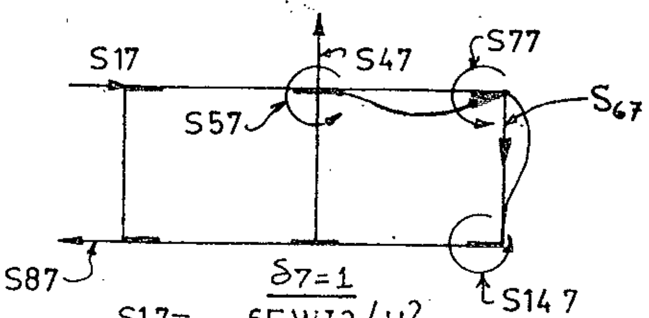
$S55 = (4EWI2/H) + (4EBI1/L1)[1+3S3/L1(1+S3/L1)] + (4EBI2/L2)[1+3S4/L2$ $*(1+S4/L2)]$

$S65 = -(6EBI2/L2^2)(1+2S4/L2)$

$S75 = (2EBI2/L2[1+3/L2(S4+S5 + 2S4.S5/L2)]$

$S85 = -(6EWI2/H^2)$

$S12,5 = (2EWI2/H)$



$\underline{\delta 7=1}$

$S17 = 6EWI3/H^2$

$S47 = (6EBI2/L2^2)(1+2S5/L2)$

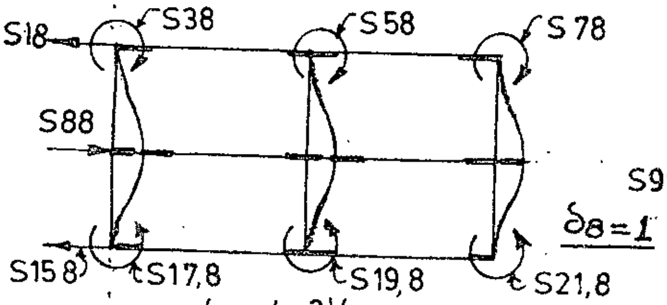
$S57 = (2EBI2/L2)[1+3/L2(S4+S5 + 2S4S5/L2)]$

$S67 = -(6EBI2/L2^2)(1+2S5/L2)$

$S77 = (4EWI3/H) + (4EBI2/L2)[1+3S5/L2(1+S5/L2)]$

$S87 = -(6EWI^3/H^2)$

$S14,7 = (2EWI3/H)$



$\underline{\delta 8=1}$

$S18 = -(12E/H^3)(WI1+WI2+WI3)$

$S38 = -(6EWI1/H^2)$

$S58 = -(6EWI2/H^2)$

$S78 = -(6EWI3/H^2)$

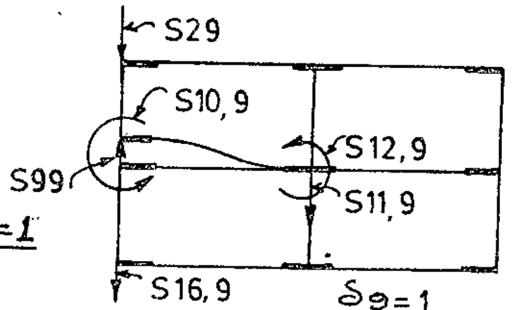
$S88 = (24E/H^3)(WI1+WI2+WI3)$

$S15,8 = (12E/H^3)(WI1+WI2+WI3)$

$S17,8 = (6EWI1/H^2)$

$S19,8 = (6EWI2/H^2)$

$S21,8 = (6EWI3/H^2)$



$\underline{\delta 9=1}$

$S29 = -(EWA1/H)$

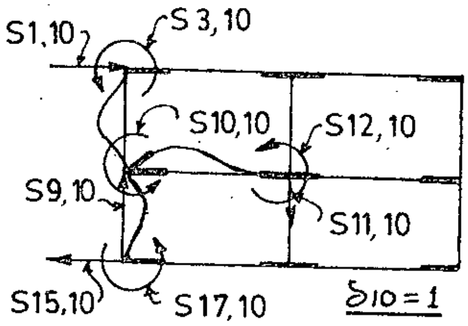
$S99 = (2EWA1/H) + (12EBI1/L1^3)$

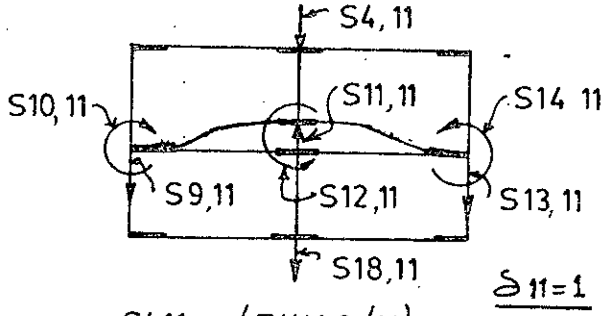
$S10,9 = (6EBI1/L1^2)(1+2S2/L1)$

$S11,9 = -(12EBI1/L1^3)$

$S12,9 = (6EBI1/L1^2)(1+2S3/L1)$

$S16,9 = -(EWA1/H)$





$S1,10 = (6EWI1/H^2)$

$S3,10 = (2EWI1/H)$

$S9,10 = (6EBI1/L1^2)(1+2S2/L1)$

$S10,10 = (8EWI1/H)+(4EBI1/L1)$
$[1+3S2/L1\ (1+S2/L1)]$

$S11,10 = -(6EBI1/L1^2)(1+2S2/L1)$

$S12,10 = (2EBI1/L1)[1+3/L1\ (S2+S3$
$+2S2S3/L1)]$

$S15,10 = -(6EWI1/H^2)$

$S17,10 = (2EWI1/H)$

$S4,11 = -(EWA2/H)$

$S9,11 = -(12EBI1/L1^3)$

$S10,11 = -(6EBI1/L1^2)(1+2S2/L1)$

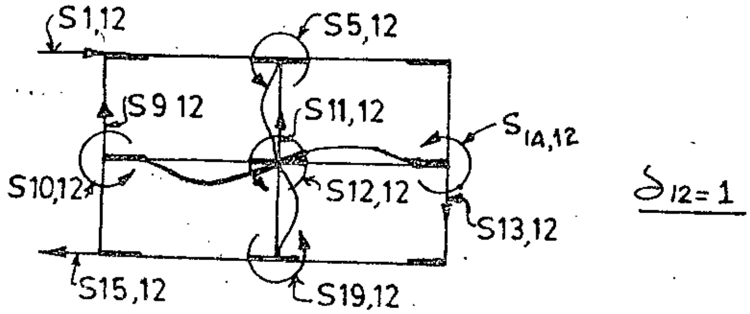
$S11,11 = 2EWA2/H + 12EBI1/L1^3$
$+(12EBI2/L2^3)$

$S12,11 = (6EBI2/L2^2)(1+2S4/L2)$
$-(6EBI1/L1^2)(1+2S3/L1)$

$S13,11 = -(12EBI2/L2^3)$

$S14,11 = +(6EBI2/L2^2)(1+2S5/L2)$

$S18,11 = -(EWA2/H)$



$S1,12 = (6EWI2/H^2)$

$S5,12 = (2EWI2/H)$

$S9,12 = (6EBI1/L1^2)(1+2S3/L1)$

$S10,12 = (2EBI1/L1)[1+3/L1\ (S2+S3+2S2S3/L1)]$

$S11,12 = (6EBI2/L2^2)(1+2S4/L2)-(6EBI1/L1^2)(1+2S3/L1)$

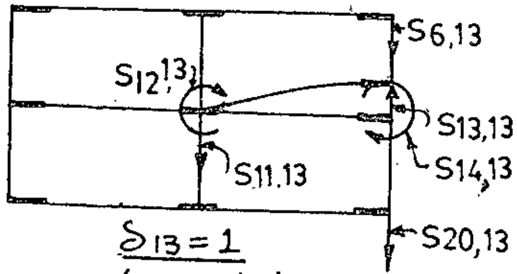
$S12,12 = (8EWI2/H)+(4EBI1/L1)\ [1+3S3/L1\ (1+S3/L1)]+(4EBI2/L2)[1+3S4/L2(1+S4/L2$

$S13,12 = -(6EBI2/L2^2)(1+2S4/L2)$

$S14,12 = (2EBI2/L2)\ [1+3/L2\ (S4+S5+2S4S5/L2)]$

$S15,12 = -6EWI2/H^2$

$S19,12 = 2EWI2/H$



$S6,13 = -(EWA3/H)$

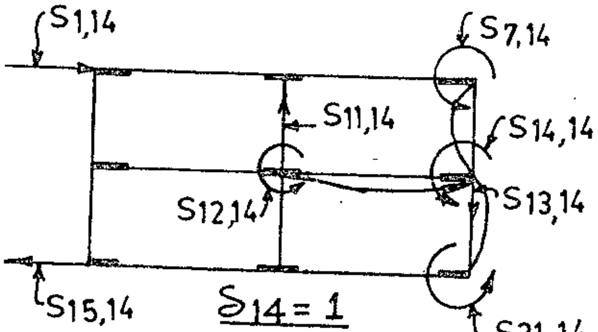
$S11,13 = -(12EBI2/L2^3)$

$S12,13 = -(6EBI2/L2^2)(1+2S4/L2)$

$S13,13 = (2EWA3/H)+(12EBI2/L2^3)$

$S14,13 = -(6EBI2/L2^2)(1+2S5/L2)$

$S20,13 = -(EWA3/H)$



$S1,14 = (6EWI3/H^2)$

$S7,14 = (2EWI3/H)$

$S11,14 = (6EBI2/L2^2)(1+2S5/L2)$

$S12,14 = (2EBI2/L2)[1+3/L2(S4+S5+2S4S5/L2)]$

$S13,14 = -(6EBI2/L2^2)(1+2S5/L2)$

$S14,14 = (8EWI3/H)+(4EBI2/L2)[1+3S5/L2(1+S5/L2)]$

$S15,14 = -(6EWI3/H^2)$

$S21,14 = (2EWI3/H)$

B0 =

S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}
S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}
S_{31}	S_{32}	S_{33}	S_{34}	S_{35}	S_{36}	S_{37}
S_{41}	S_{42}	S_{43}	S_{44}	S_{45}	S_{46}	S_{47}
S_{51}	S_{52}	S_{53}	S_{54}	S_{55}	S_{56}	S_{57}
S_{61}	S_{62}	S_{63}	S_{64}	S_{65}	S_{66}	S_{67}
S_{71}	S_{72}	S_{73}	S_{74}	S_{75}	S_{76}	S_{77}

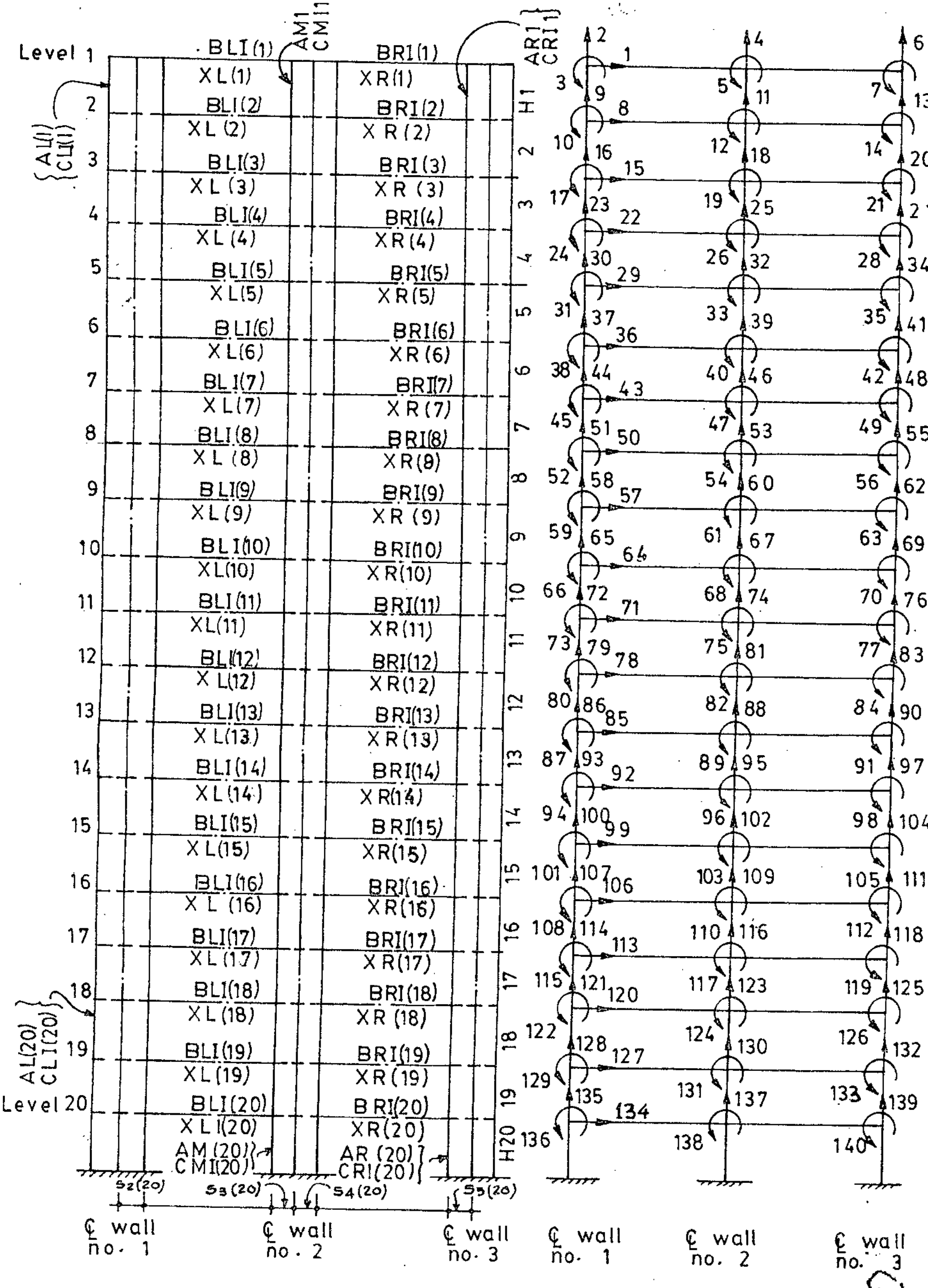
B =

$S_{8,8}$	$S_{8,9}$	$S_{8,10}$	$S_{8,11}$	$S_{8,12}$	$S_{8,13}$	$S_{8,14}$
$S_{9,8}$	$S_{9,9}$	$S_{9,10}$	$S_{9,11}$	$S_{9,12}$	$S_{9,13}$	$S_{9,14}$
$S_{10,8}$	$S_{10,9}$	$S_{10,10}$	$S_{10,11}$	$S_{10,12}$	$S_{10,13}$	$S_{10,14}$
$S_{11,8}$	$S_{11,9}$	$S_{11,10}$	$S_{11,11}$	$S_{11,12}$	$S_{11,13}$	$S_{11,14}$
$S_{12,8}$	$S_{12,9}$	$S_{12,10}$	$S_{12,11}$	$S_{12,12}$	$S_{12,13}$	$S_{12,14}$
$S_{13,8}$	$S_{13,9}$	$S_{13,10}$	$S_{13,11}$	$S_{13,12}$	$S_{13,13}$	$S_{13,14}$
$S_{14,8}$	$S_{14,9}$	$S_{14,10}$	$S_{14,11}$	$S_{14,12}$	$S_{14,13}$	$S_{14,14}$

C =

$S_{1,8}$	$S_{1,9}$	$S_{1,10}$	$S_{1,11}$	$S_{1,12}$	$S_{1,13}$	$S_{1,14}$
$S_{2,8}$	$S_{2,9}$	$S_{2,10}$	$S_{2,11}$	$S_{2,12}$	$S_{2,13}$	$S_{2,14}$
$S_{3,8}$	$S_{3,9}$	$S_{3,10}$	$S_{3,11}$	$S_{3,12}$	$S_{3,13}$	$S_{3,14}$
$S_{4,8}$	$S_{4,9}$	$S_{4,10}$	$S_{4,11}$	$S_{4,12}$	$S_{4,13}$	$S_{4,14}$
$S_{5,8}$	$S_{5,9}$	$S_{5,10}$	$S_{5,11}$	$S_{5,12}$	$S_{5,13}$	$S_{5,14}$
$S_{6,8}$	$S_{6,9}$	$S_{6,10}$	$S_{6,11}$	$S_{6,12}$	$S_{6,13}$	$S_{6,14}$
$S_{7,8}$	$S_{7,9}$	$S_{7,10}$	$S_{7,11}$	$S_{7,12}$	$S_{7,13}$	$S_{7,14}$

4.7 GENERAL TWO BAY FRAME ANALYSIS



4.7.1 STIFFNESS MATRIX OF GENERAL FRAME

$$[K] = \left[\begin{array}{cccccccccccccccccccc}
[S]_1 & [C]_1 \\
[C]_1^T & [B]_2 & [C]_2 \\
 & [C]_2^T & [B]_3 & [C]_3 \\
 & & [C]_3^T & [B]_4 & [C]_4 \\
 & & & [C]_4^T & [B]_5 & [C]_5 \\
 & & & & [C]_5^T & [B]_6 & [C]_6 \\
 & & & & & [C]_6^T & [B]_7 & [C]_7 \\
 & & & & & & [C]_7^T & [B]_8 & [C]_8 \\
 & & & & & & & [C]_8^T & [B]_9 & [C]_9 \\
 & & & & & & & & [C]_9 & [B] & [C]_{10} \\
 & & & & & & & & & [C]_{10} & [B] & [C]_{11} \\
 & & & & & & & & & & [C]_{11} & [B] & [C]_{12} \\
 & & & & & & & & & & & [C]_{12} & [B] & [C]_{13} \\
 & & & & & & & & & & & & [C]_{13} & [B] & [C]_{14} \\
 & & & & & & & & & & & & & [C]_{14} & [B] & [C]_{15} \\
 & & & & & & & & & & & & & & [C]_{15} & [B] & [C]_{16} \\
 & & & & & & & & & & & & & & & [C]_{16} & [B] & [C]_{17} \\
 & & & & & & & & & & & & & & & & [C]_{17} & [B] & [C]_{18} \\
 & & & & & & & & & & & & & & & & & [C]_{18} & [B] & [C]_{19} \\
 & & & & & & & & & & & & & & & & & & [C]_{19}^T & [B]_{20}
\end{array}\right]$$

$[S]_1 =$

S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}
S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}
S_{31}	S_{32}	S_{33}	S_{34}	S_{35}	S_{36}	S_{37}
S_{41}	S_{42}	S_{43}	S_{44}	S_{45}	S_{46}	S_{47}
S_{51}	S_{52}	S_{53}	S_{54}	S_{55}	S_{56}	S_{57}
S_{61}	S_{62}	S_{63}	S_{64}	S_{65}	S_{66}	S_{67}
S_{71}	S_{72}	S_{73}	S_{74}	S_{75}	S_{76}	S_{77}

$[C]_1$

S_{18}	S_{19}	$S_{1,10}$	$S_{1,11}$	$S_{1,12}$	$S_{1,13}$	$S_{1,14}$
S_{28}	S_{29}	$S_{2,10}$	$S_{2,11}$	$S_{2,12}$	$S_{2,13}$	$S_{2,14}$
S_{38}	S_{39}	$S_{3,10}$	$S_{3,11}$	$S_{3,12}$	$S_{3,13}$	$S_{3,14}$
S_{48}	S_{49}	$S_{4,10}$	$S_{4,11}$	$S_{4,12}$	$S_{4,13}$	$S_{4,14}$
S_{58}	S_{59}	$S_{5,10}$	$S_{5,11}$	$S_{5,12}$	$S_{5,13}$	$S_{5,14}$
S_{68}	S_{69}	$S_{6,10}$	$S_{6,11}$	$S_{6,12}$	$S_{6,13}$	$S_{6,14}$
S_{78}	S_{79}	$S_{7,10}$	$S_{7,11}$	$S_{7,12}$	$S_{7,13}$	$S_{7,14}$

$[C]_1^T =$

S_{81}	S_{82}	S_{83}	S_{84}	S_{85}	S_{86}	S_{87}
S_{91}	S_{92}	S_{93}	S_{94}	S_{95}	S_{96}	S_{97}
$S_{10,1}$	$S_{10,2}$	$S_{10,3}$	$S_{10,4}$	$S_{10,5}$	$S_{10,6}$	$S_{10,7}$
$S_{11,1}$	$S_{11,2}$	$S_{11,3}$	$S_{11,4}$	$S_{11,5}$	$S_{11,6}$	$S_{11,7}$
$S_{12,1}$	$S_{12,2}$	$S_{12,3}$	$S_{12,4}$	$S_{12,5}$	$S_{12,6}$	$S_{12,7}$
$S_{13,1}$	$S_{13,2}$	$S_{13,3}$	$S_{13,4}$	$S_{13,5}$	$S_{13,6}$	$S_{13,7}$
$S_{14,1}$	$S_{14,2}$	$S_{14,3}$	$S_{14,4}$	$S_{14,5}$	$S_{14,6}$	$S_{14,7}$

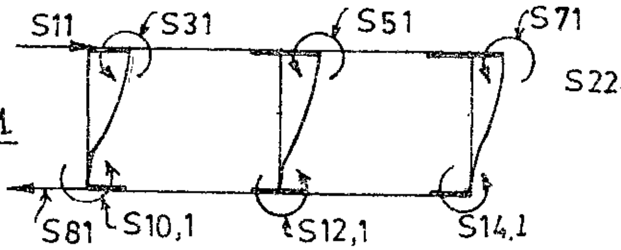
$[B]_2 =$

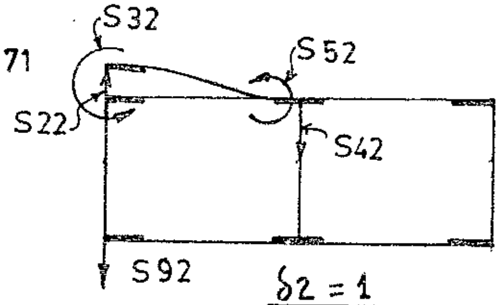
S_{88}	S_{89}	$S_{8,10}$	$S_{8,11}$	$S_{8,12}$	$S_{8,13}$	$S_{8,14}$
S_{98}	S_{99}	$S_{9,10}$	$S_{9,11}$	$S_{9,12}$	$S_{9,13}$	$S_{9,14}$
$S_{10,8}$	$S_{10,9}$	$S_{10,10}$	$S_{10,11}$	$S_{10,12}$	$S_{10,13}$	$S_{10,14}$
$S_{11,8}$	$S_{11,9}$	$S_{11,10}$	$S_{11,11}$	$S_{11,12}$	$S_{11,13}$	$S_{11,14}$
$S_{12,8}$	$S_{12,9}$	$S_{12,10}$	$S_{12,11}$	$S_{12,12}$	$S_{12,13}$	$S_{12,14}$
$S_{13,8}$	$S_{13,9}$	$S_{13,10}$	$S_{13,11}$	$S_{13,12}$	$S_{13,13}$	$S_{13,14}$
$S_{14,8}$	$S_{14,9}$	$S_{14,10}$	$S_{14,11}$	$S_{14,12}$	$S_{14,13}$	$S_{14,14}$

$[C]_2^T$

S15,8	S15,9	S15,10	S15,11	S15,12	S15,13	S15,14
S16,8	S16,9	S16,10	S16,11	S16,12	S16,13	S16,14
S17,8	S17,9	S17,10	S17,11	S17,12	S17,13	S17,14
S18,8	S18,9	S18,10	S18,11	S18,12	S18,13	S18,14
S19,8	S19,9	S19,10	S19,11	S19,12	S19,13	S19,14
S20,8	S20,9	S20,10	S20,11	S20,12	S20,13	S20,14
S21,8	S21,9	S21,10	S21,11	S21,12	S21,13	S21,14

$\delta_1 = 1$





$\delta_2 = 1$

$S11 = (12E/H1^3)(CLI(1)+(CMI(1)+CRI(1))$

$S31 = 6ECLI(1)/H1^2$

$S51 = 6ECMI(1)/H1^2$

$S71 = 6ECRI(1)/H1^2$

$S81 = -(12E/H1^3)(CLI(1)+CMI(1)+CRI(1))$

$S10,1 = 6ECLI(1)/H1^2$

$S12,1 = 6ECMI(1)/H1^2$

$S14,1 = 6ECRI(1)/H1^2$

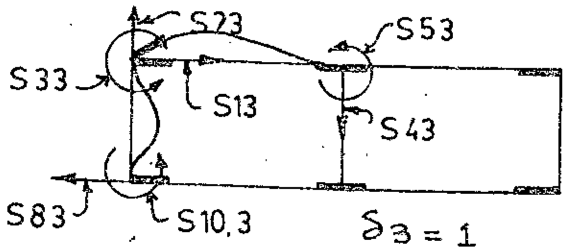
$S22 = EAL(1)/H1 + 12EBLI(1)/XL(1)^3$

$S32 = (6EBLI(1)/(XL(1)^2)(1+2S2(1)/XL(1)$

$S42 = -12EBLI(1)/(XL(1)^3)$

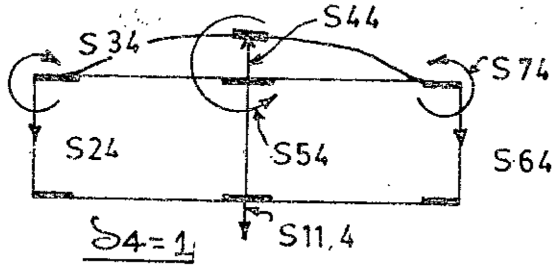
$S52 = (6EBLI(1)/(XL(1)^2)(1+2S3(1)/XL(1))$

$S92 = -EAL(1)/H1$



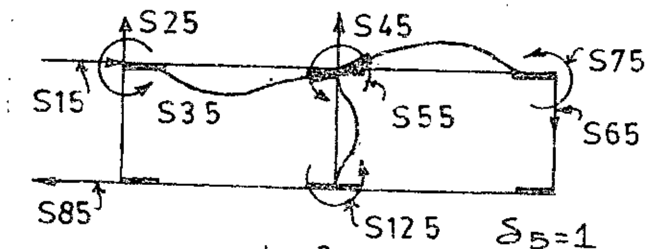
$\delta_3 = 1$

$$S13 = 6ECLI(1)/H1^2$$
$$S23 = 6EBLI(1)/X\,L(1)^2)(1+2S2(1)/X\,L(1))$$
$$S33 = 4ECLI(1)/H1 + (4EBLI(1)/X\,L(1)\left[1+\frac{3S2(1)}{XL(1)}\left(1+\frac{S2(1)}{XL(1)}\right)\right.$$
$$S43 = -(6EBLI(1)/X\,L(1)^2)(1+2S2(1)X\,L(1)$$
$$S53 = (2EBLI(1)/X\,L(1))\left[1+\frac{3}{X\,L(1)}(S2(1)+S3(1)+2S2(1)S3(1)/X\,L(1))\right]$$
$$S83 = -6ECLI(1)/H1^2$$
$$S10{,}3 = 2ECLI(1)/H1$$



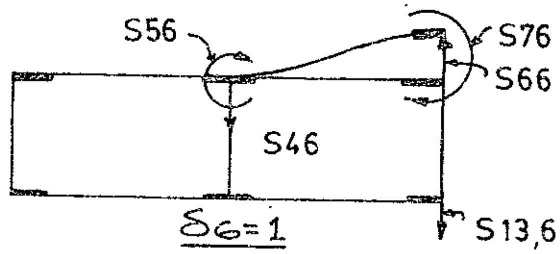
$\delta_4 = 1$

$$S24 = -12EBLI(1)/XL(1)^3$$
$$S34 = -(6EBLI(1)/XL(1)^2(1+2S2(1)/X\,L(1))$$
$$S44 = EAM(1)/H1 + 12EBLI(1)/X\,L(1)^3 + 12EBRI(1)/X\,R(1)^3$$
$$S54 = (6EBRI(1)/XR(1)^2)(1+2S4(1)/XR(1)) - (6EBLI(1)/X\,L(1)^2)(1+2S3(1)/X\,L(1))$$
$$S64 = -12EBRI(1)/X\,R(1)^3$$
$$S74 = (6EBRI(1)/X\,R(1)^2)(1+2S5(1)/X\,R(1)$$
$$S11{,}4 = -(EAM(1)/H1)$$



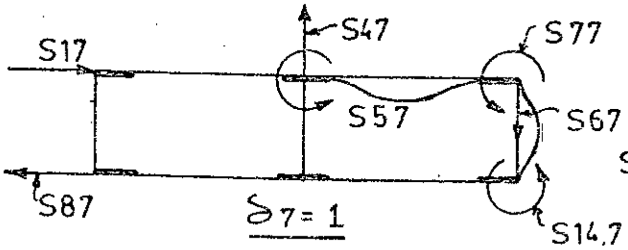
$\delta_5 = 1$

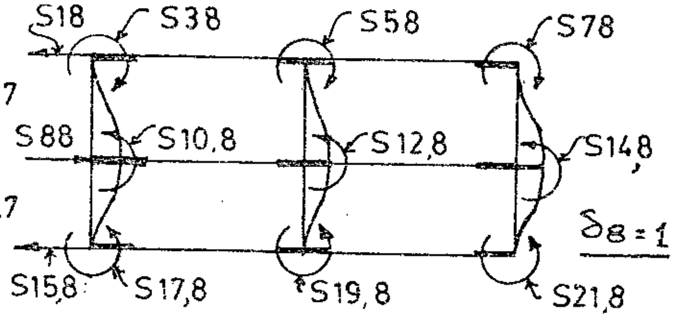
$$S15 = 6ECMI(1)/H1^2$$
$$S25 = (6EBLI(1)/X\,L(1)^2)(1+2S3(1)/X\,L(1))$$
$$S35 = (2EBLI(1)/X\,L(1)\left[1+\frac{3}{XL(1)}(S2(1)+S3(1)+2S2(1)S3(1)/X\,L(1))\right]$$
$$S45 = (6EBRI(1)/X\,R(1)^2)(1+2S4(1)/X\,R(1) - 6EBLI(1)/X\,L(1)^2)(1+2S3(1)/X\,L(1)$$
$$S55 = (4ECMI(1)/H1) + (4EBLI(1)/X\,L(1))\left[1+\frac{3S3(1)}{XL(1)}(1+S3(1)/X\,L(1))\right] + (4EBRI(1)/X\,L(1)\left[1+3S4(1)/X\,R(1)(1+S4(1)/X\,R(1))\right]$$
$$S65 = -(6EBRI(1)/X\,R(1)^2)(1+2S4(1)/X\,R(1)$$
$$S75 = (2EBRI(1)/X\,R(1))\left[1+\frac{3}{XR(1)}(S4(1)+S5(1)+2S4(1)S5(1)/X\,R(1))\right]$$
$$S85 = -(6ECMI(1)/H1^2)$$
$$S12{,}5 = (2ECMI(1)/H1)$$



$\delta_6 = 1$

$$S46 = -(12EBRI(1)/X\,R(1)^3)$$
$$S56 = -(6EBRI(1)/X\,R(1)^2)(1+2S4(1)/X\,R(1))$$
$$S66 = EAR(1)/H1 + 12EBRI(1)/X\,R(1)3)$$
$$S76 = -6EBRI(1)/X\,R(1)^2)(1+2S5(1)/X\,R(1))$$
$$S13{,}6 = -(EAR(1)/H1)$$





$S17 = (6ECRI(1)/H1^2)$

$S47 = (6EBRI(1)/XR(1)^2) + (1 + 2S5(1)/XR(1))$

$S57 = (2EBRI(1)/XR(1))[1 + \frac{3}{XR(1)}(S4(1) + S5(1) + 2S4(1)S5(1)/XR(1)]$

$S67 = -(6EBRI(1)/XR(1)^2)(1 + 2S5(1)/XR(1))$

$S77 = (4ECRI(1)/H1) + (4EBRI(1)/XR(1))[1 + 3S5(1)/XR(1)(1 + S5(1)/XR(1))]$

$S87 = -(6ECRI(1)/H1^2)$

$S14,7 = (2ECRI(1)/H1)$

$S18 = -(12E/H1^3)(CLI(1) + CMI(1) + CRI(1))$

$S38 = -(6ECLI(1)/H1^2)$

$S58 = -(6ECMI(1)/H1^2)$

$S78 = -(6ECRI(1)/H1^2)$

$S88 = -(12E/H1^3)(CLI(1) + CMI(1) + CRI(1) + 12E/H2^3)(CLI(2) + CMI(2) + CRI(2))$

$S10,8 = (6ECLI(2)/H2^2) - (6ECLI(1)/H1^2)$

$S12,8 = (6ECMI(2)/H2^2) - (6ECMI(1)/H1^2)$

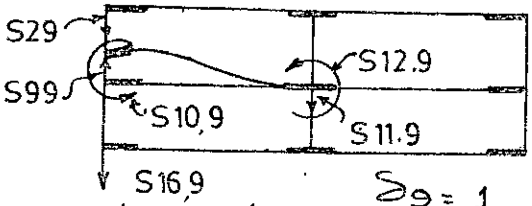
$S14,8 = (6ECRI(2)/H2^2) - (6ECRI(1)/H1^2)$

$S15,8 = (12E/H2^3)(CLI(2) + CMI(2) + CRI(2))$

$S17,8 = (6ECLI(2)/H2^2)$

$S19,8 = (6ECMI(2)/H2^2)$

$S21,8 = (6ECRI(2)/H2^2)$



$S29 = -(EAL(1)/H1)$

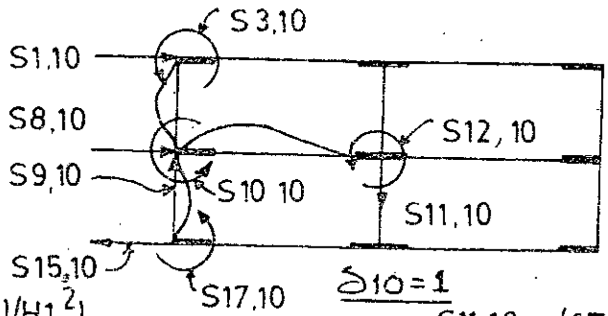
$S99 = EAL(1)/H1 + EAL(2)/H2 + 12EBLI(2)/XL(2)^3)$

$S10,9 = (6EBLI(2)/XL(2)^2)(1 + 2S2(2)/XL(2))$

$S11,9 = -(12EBLI(2)/XL(2)^3)$

$S12,9 = (6EBLI(2)/XL(2)^2)(1 + 2S3(2)/XL(2))$

$S16,9 = -(EAL(2)/H2)$



$S1,10 = (6ECLI(1)/H1^2)$

$S3,10 = (2ECLI(1)/H1)$

$S8,10 = (6ECLI(2)/H2^2) - (6ECLI(1)/H1^2)$

$S9,10 = (6EBLI(2)/XL(2)^2)(1 + 2S2(2)/XL(2))$

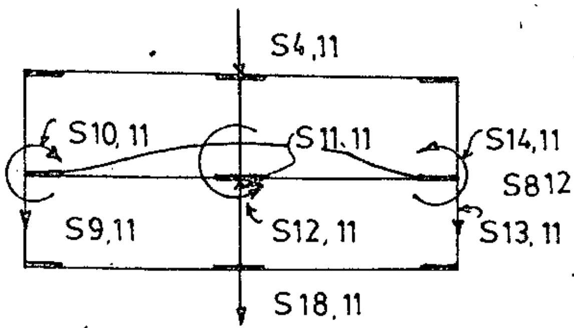
$S10,10 = 4ECLI(1)/H1 + 4ECLI(2)/H2 + 4EBLI(2)/XL(2) * [1 + 3S2(2)/XL(2)(1 + S2(2)/XL(2))]$

$S11,10 = -(6EBLI(2)/XL(2)^2)(1 + 2S2(2)/XL(2))$

$S12,10 = (2EBLI(2)/XL(2)[1 + \frac{3}{XL(2)}(S2(2) + S3(2) + \frac{2S2(2)S3(2)}{XL(2)})]$

$S15,10 = -(6EBLI(2)/H2^2)$

$S17,10 = (2ECLI(2)/H2)$



$\delta_{11} = 1$

$S4,11 = -EAM(1)/H1$

$S9,11 = -(12EBLI(2)/XL(2)^3)$

$S10,11 = -(6EBLI(2)/XL(2)^2)(1+2S2(2)/XL(2))$

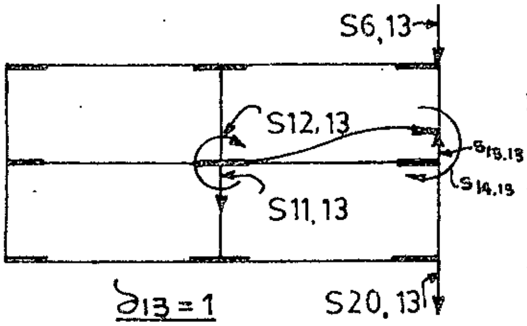
$S11,11 = EAM(1)/H1 + EAM(2)/H2 + (12EBLI(2)/XL(2)^3) + (12EBRI(2)/XR(2)^3)$

$S12,11 = (6EBRI(2)/XR(2)^2)(1+2S4(2)/XR(2)) - (6EBLI(2)/XL(2)^2)(1+2S3(2)/XL(2))$

$S13,11 = -(12EBRI(2)/XR(2)^3)$

$S14,11 = (6EBRI(2)/XR(2)^2)(1+2S5(2)/XR(2))$

$S18,11 = -(EAM(2)/H2)$



$\delta_{13} = 1$

$S6,13 = -(EAR(1)/H1)$

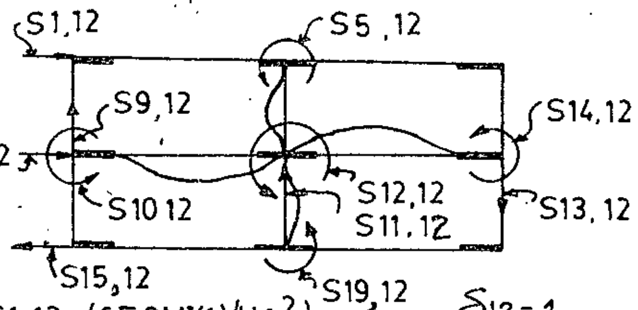
$S11,13 = -(12EBRI(2)XR(2)^3)$

$S12,13 = -\left(\frac{6EBRI(2)}{XR(2)^2}\right)(1+2S4(2)/XR(2))$

$S13,13 = EAR(1)/H1 + EAR(2)/H2 + (12EBRI(2)/XR(2)^3$

$S14,13 = -\left(\frac{6EBRI(2)}{XR(2)^2}\right)(1+2S5(2)/XR(2))$

$S20,13 = -(EAR(2)/H2)$



$\delta_{12} = 1$

$S1,12 = (6ECMI(1)/H1^2)$

$S5,12 = (2ECMI(1)/H1)$

$S8,12 = (6ECMI(2)/H2^2) - (6ECMI(1)/H1^2)$

$S9,12 = (6EBLI(2)/XL(2)^2)(1+2S3(2)XL(2))$

$S10,12 = (2EBLI(2)XL(2))\left[1+\frac{3}{XL(2)}\left(S2(2)+S3(2) + \frac{2S2(2)S3(2)}{XL(2)}\right)\right]$

$S11,12 = (6EBRI(2)/XR(2)^2)(1+2S4(2)/XR(2)) - (6EBLI(2)/XL(2)^2)(1+2S3(2)/XL(2))$

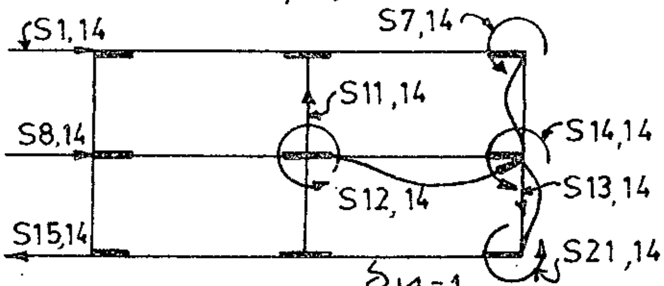
$S12,12 = (4ECMI(1)/H1) + (4ECMI(2)/H2 + (4EBLI(2)/XL(2)$
$*\left[1+\frac{3S3(2)}{XL(2)}\left(1+\frac{S3(2)}{XL(2)}\right)\right] + \left(\frac{4EBRI(2)}{XR(2)}\right)\left[1+\frac{3S4(2)}{XR(2)}\left(1+\frac{S4(2)}{XR(2)}\right.\right.$

$S13,12 = -(6EBRI(2)/XR(2)^2)(1+2S4(2)/XR(2))$

$S14,12 = \left(\frac{2EBRI(2)}{XR(2)}\right)\left[1+\frac{3}{XR(2)}\left(S4(2)+S5(2)+\frac{2S4(2)S5(2)}{XR(2)}\right.\right.$

$S15,12 = -(6ECMI(2)/H2^2$

$S19,12 = (2ECMI(2)/H2)$



$\delta_{14} = 1$

$S1,14 = (6ECRI(1)/H1^2)$

$S7,14 = (2ECRI(1)/H1)$

$S8,14 = (6ECRI(2)/H2^2) - (6ECRI(1)/H1^2)$

$S11,14 = (6EBRI(2)XR(2)^2)(1+2S5(2)/XR(2))$

$S12,14 = (2EBRI(2)/XR(2)\left[1+\frac{3}{XR(2)}\left(S4(2) + S5(2) + \frac{2S4(2)S5(2)}{XR(2)}\right)\right]$

$S13,14 = -(6EBRI(2)/XR(2)^2(1+2S5(2)/XR(2))$

$S14,14 = (4ECRI(1)/H1) + (4ECRI(2)/H2) + (4EBRI(2)/R(2))\left[1+\frac{3S5(2)}{XR(2)}\left(1+S5(2)/XR(2)\right)\right]$

$S15,14 = -(6ECRI(2)/H2^2)$

$S21,14 = 2ECRI(2)/H2$

CHAPTER 5

COMPUTER PROGRAMS FOR STRUCTURES

5.1 Introduction

In chapter 4 the stiffness method and recursion procedure of analysis was presented in a form which is suitable for computer programming. In this chapter flow charts of computer programs are presented. For each program, a table containing the identifiers of the program is given to facilitate the reader to identify the names of variables, constants, or other entities which are used in the program. The programs are written in Fortran. The listings of the programms are given in Appendix 'A'.

In the programs, all of the structures to be analyzed are assumed to consist of straight prismatic members. The material properties for a given structure are taken to be constant throughout the structure. Only the effects of loads are considered, and no other influences, such as temperature change, are taken into account.

All the programs are written for twenty storied frame. However the number of storey may be increased to any number depending on the storage capacity of computer.

5.2 Outline of Programs

Since the detailed steps in a computer program are related to the manner in which data are prepared each program is preceded by a summary which explains the preparation of the necessary data. In addition, some of the variables in the program will be of integer type while others are of floating-point type. Therefore, a listing of the variables of integer type is given in each identifier table. The variables which are subscripted in the program (and which require blocks of storage in the computer) are given in the dimension statement of each program.

5.2.1 General outline of Programs

The general outline of all the programs is shown in the following five steps.

1. Input and print structure data.
   a) Number of storey, number of storey loaded
   b) Structure parameters and elastic moduli
   c) Member designations, properties and orientations
2. Structures stiffness matrix
   a) Generation of stiffness matrix
   b) Inversion of stiffness matrix
3. Input and print load data
   a) Actions applied at joints
   b) Actions at ends of restrained members due to loads
4. Construction of vectors associated with loads
   a) Equivalent joint loads
   b) Combined joint loads

5) Calculation and out put of results

a) Joint displacement and support reactions

b) Member and actions.

5.2.2 Units

The units of input data used for the programs.are loads in kips, lengths in inches, areas in square inches, modulus of elasticity in kips per square inch, and so on. The units of final results are in kips, inches and radians.

5.3 Computer Storage and Time

IBM 1620 computer of Atomic Energy Center was used in the development and test run of all the programs. After the programs were developed the production run were made in the IBM 360-30 computer available in the Bureau of Statistics. The storage capacity of IBM 1620 is 60K and the existing storage capacity of IBM 360-30 is 65K. Time required by IBM 1620 computer for the compilation and execution of a program is much more than the time required by IBM 360-30 computer. To cite an example : for a two bay frame program, the compilation time required by IBM 1620 is 18 min., while the compilation time required by IBM 360-30 is $3\frac{1}{2}$ min., and the execution time for the same program for twenty storey is 15 min. for IBM 1620 and approximately 48 Sec. for IBM 360-30.

Originally the programs were tested and run with single precision. The results were found to be satisfactory; and the equilibrium was satisfied at all joints and the whole frame. However in the final production run, when the value of λ was varied from 0.5 to 100, it was found that if the value of λ goes beyond certain range, then neither the equilibrium of joints nor of the whole frame does satisfy. To overcome this problem, all the programs were subsequently written for double precision. It is found that double precision gives very accurate result for any frame irrespective of the value of λ.

A chart showing the computer core used and the time required by IBM 360-30 computer for the compilation and execution of each program in double precision is given below:

Chart showing computer core used and time required in each program

Name of program	Core used (Hexadecimal Unit) (Bytes)	Compilation time (min.)	Execution time for one 20 storey frame (min.)
One bay frame analysis	00A567	03.10	00.24
General one bay frame analysis	00A2FF	03.20	01.22
Two bay frame analysis	00C587	03.38	00.48
General two bay frame analysis	00BB27	04.40	02.05
GA for two bay frame	00B7B7	03.08	00.28
GA for one bay frame	009CFF	02.53	00.18

Note: Time :- 03.38 min. means 3 min. 38 sec., 00.24 min. means 0 min. 24 sec. etc.

5.4 FLOW DIAGRAM

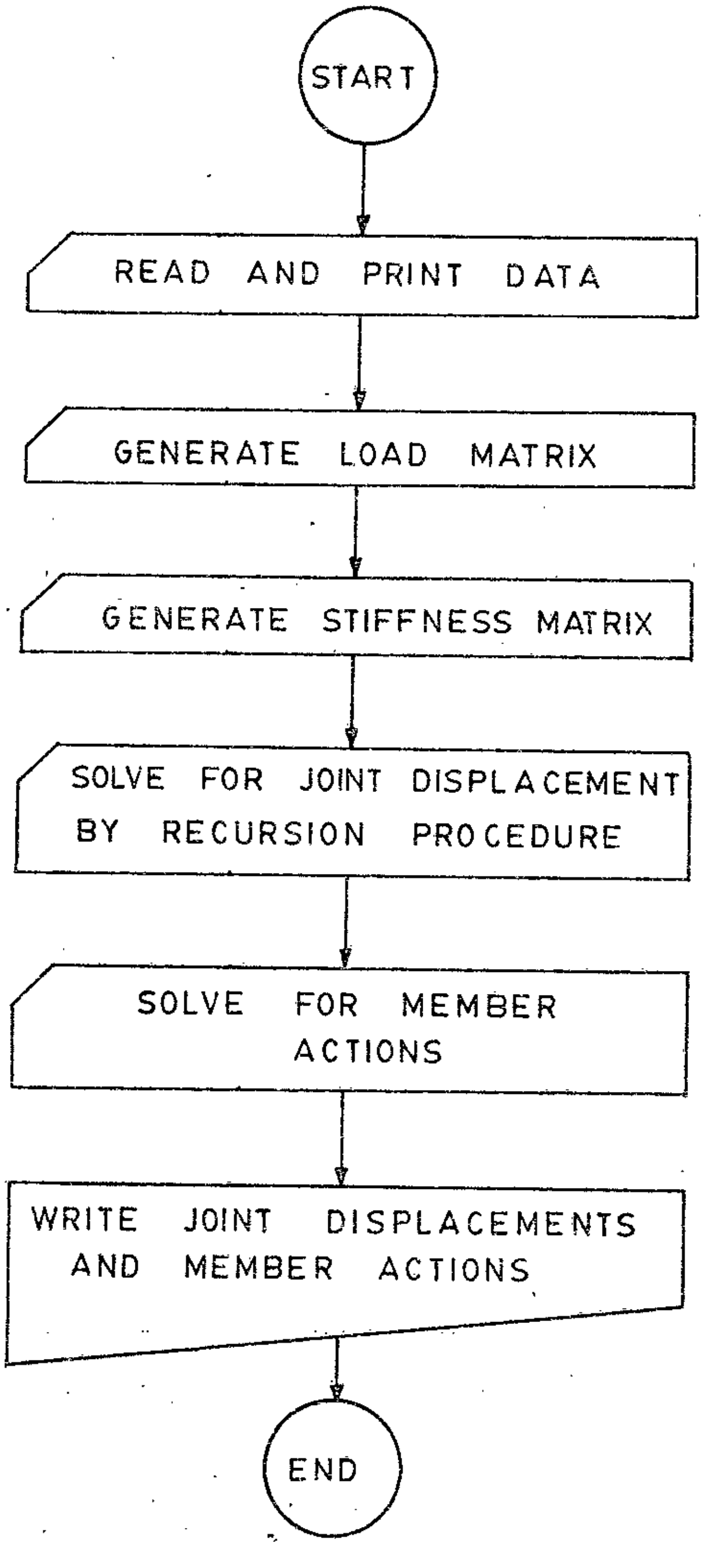


FIG. 5.1 FLOW DIAGRAM OF THE COMPUTER PROGRAM FOR THE ANALYSIS OF PLANE FRAME

5.5 One Bay Frame Program

One bay frame program was written to analyze frame with or without shear wall. The identifiers used in this program are listed in Table 5A. The input data required in the program are summarized in Table 5B.

TABLE 5A

Idenfier used in one Bay frame computer programs

Integer type variables are: N, NSLOAD, LS

Identifier	Definition
N	Number of story
NSLOAD	Number of story loaded
XI1	Moment of Inertia of left column (equal for all floors)
XI2	Moment of inertia of right column (equal for all floors
XI3	Moment of inertia of beam (equal for all floors)
A1	Area of cross-section - Left column (equal for all floors)
A2	Area of cross-section-right column (equal for all floors)
EM	Young's modulus of Elasticity
XL	Beam span from center to center of columns or beam length from face to face of shear walls
H	Storey height (equal for all story)
XL2	Distance from c.g. of left wall to end of beam. It is zero in case of column.
XL3	Distance from c.g. of right wall to end of beam. It is zero in case of column.

TABLE 5A (Contd.)

Identifier	Definition
LS	Loaded storey level from top
XP( )	Loads applied at joints of a storey.
FER( )	Fixed end reactions on storey beams. The order of FER( ) is left F.E. shear, Left F.E.M, Right F.E. shear, Right F.E.M.

TABLE 5B

Preparation of Data for one Bay frame

Data	Number of cards	Items on data cards
a) Number of storey of storey loaded, member properties, structure parameters and elastic modulus.	1	N, NSLOAD, XI1, XI2, XI3, A1, A2, EM, XL, H, XL2, XL3
a) Storey level, actions applied at joints, actions at end of restrained members due to loads.	N	LS, XP( ), FER ( )

5.6 General One Bay Frame Program

While one bay frame program can be used for constant storey height, constant moment of inertia of Beams etc., the General one bay Frame program can take care of variable storey height, moment of inertia, area etc. Identifiers used for general one bay frame is given in Table 5D and preparation of data is given in Table 5C.

TABLE 5C

Data	Number of cards	Items on data card
a) Number of story, Number of story loaded, Modulus of Elasticity	1	N, NSL, E
a) Member properties, span, storey height etc.	N	CL(I),CR(I),BL(I), AL(I),AR(I),X(I),H(I), S2(I),S3(I).
a) Loaded story level, Actions applied at joint, Actions at ends of restrained members due to loads.	NSL	LS, XP( ), FER( )

TABLE 5D

Identifiers used in general one Bay Computer Programs

Integer type variables:- N, NSL, LS

Identifier	Definition
N	Number of story
NSL	Number of story loaded
E	Modulus of Elasticity
CL( )	Moment of Inertia of left column or wall
CR( )	Moment of Inertia of Right column or wall
BL( )	Moment of Inertia of Beam
AL( )	Area of left column or wall
AR( )	Area of right column or wall
X( )	Beam span C/C of column or face to face of wall

TABLE 5D (Contd...)

Identifier	Definition
H( )	Storey height
S2( )	Distance from e.g. of left wall to Beam end.
S3( )	Distance from c.g. of right wall to Beam end.
LS	Loaded storey level counted from top
XP( )	Actions (Loads) applied at joints on a storey.
FER( )	Actions at ends of restrained members due to loads. Order of FER( ) is left F.E. shear, Left F.E.M., Right F.E. shear, Right F.E.M.

5.7 Two Bay Frame Program

Two bay frame program was written to analyze frames of columns or walls or combination of both. Storey height, Moment of inertia of left beams, right beams, left vertical members, center vertical members right vertical members etc. have been kept constant throughout the full height. The identifiers used in the program are listed in Table 5E and the input data required are summerized in Table 5F.

TABLE 5E

Identifiers used in two Bay Frame Computer Program

Integer type variables are:- N, NSLOAD, LS

Identifier	Definition
N	Number of storey
NSLOAD	Number of storey loaded
WI1	Moment of inertia of left column or wall
WI2	Moment of inertia of center column or wall
WI3	Moment of inertia of right column or wall
BI_1	Moment of inertia of left beam
BI_2	Moment of inertia of right beam
WA_1	Area of left column or wall
WA2	Area of center column or wall
WA3	Area of right column or wall
EM	Modulus of elasticity
XL_1	Span c/c of columns or face to face of walls of left beam
XL_2	Span c/c of columns or face to face of walls of right beam
H	Storey height
S2	Distance from c.g. of left wall to beam end
S3	Distance from c.g. of center wall to end of left beam
S4	Distance from c.g. of center wall to end of right beam
LS	Loaded storey level from top
SP( )	Actions (loads) applied at joints on a storey
FER( )	Actions at ends of restr ined members due to loads. order of FER( ) is same as for one bay frame.

TABLE 5F

Preparation of Data for two Bay Program

Data	Number of Cards	Items on data cards
a) Number of storey, number of storey loaded, section properties of members etc.	1	N, NSLOAD, WI1, WI2, WI3, BI1, BI2, WA1, WA2, WA3.
b) Elastic modulus, span, storey height, wall widths from c.g. etc.	1	EM, XL1, XL2, H, S2, S3, S4, S5.
a) Storey level, actions applied at joints	NSLOAD	LS, XP( )
b) Actions at ends of restrained members due to loads	NSLOAD	FER( )

5.8 General two Bay Frame Program

General two Bay frame program can take care of variable storey height, variable spans and section properties of all members. The identifiers are listed in Table 5G and the input data required are summerized in Table 5H.

TABLE 5G

Identifiers used in General Two Bay Computer Program

Integer type variables are:- N, NSL, LS

Identifier	Definition
N	Number of storey
NSL	Number of storey loaded
E	Modulus of Elasticity
LS	Storey level from top
SP( )	Actions (loads) applied at joints on a storey
FER( )	Actions at ends of restrained members due to loads. Order of FER( ) is same as two Bay frame
H( )	Storey height
AL( )	Area of left column or wall
AM( )	Area of center column or wall
AR( )	Area of right column or wall
CL( )	Moment of inertia of left column or wall
CM( )	Moment of inertia of right column or wall
BL	Moment of inertia of left beam
BR	Moment of inertia of right beam
X	Span left beam
Y	Span right beam

S2, S3, S4 and S5 are same as two bay frame

TABLE 5H

Preparation of Data for General two Bay frame Program

Data	Number of cards	Items on data cards
a) Number of storey, number of storey loaded, Modulus of Elasticity	1	N, NSL, E
a) Loaded storey level, actions applied at joint	NSL	LS, XP( )
b) Loaded storey level, actions at ends of restrained members due to loads	NSL	LS, FER( )
a) Storey height, properties of vertical members	N	H(I), AL(I), AM(I) AR(I), CL(I), CM(I) CR(I)
b) Beam properties, spans, wall depths from c.g.	2N	BL, BR, X, Y, S2, S3, S4, S5

5.9 Program for GA for One Bay Frame

Ga parameter of a frame is defined as the concentrated load required atkthe top of the frame top cause unit lateral displacement. GA for one bay frame was written to compare the actual GA value of a frame evaluated by stiffness method and recursion procedure and the approximate GA value evaluated by Heidebrecht and Smith method (HSM). Identifiers used in this program is same as the identifiers of first data card of one bay frame program. Only one card is required in this program to evaluate the GA value.

5.10 Program for GA for Two Bay Frame

This program was written to find out the GA value by exact as well as HSM for two Bay frame. Only two data cards are required for this program. Identifiers used in this program are same as the identifiers of the first two cards of two bay frame except that the identifiers are arranged in different manners in this program.

5.11 Program for Approximate Analysis by Heidebrecht and Smith Method

This program was written to find out the deflection, B.M, shear and the interacting forces in flexural and shear beam by HSM. Identifiers used in this program is same as the identifiers of the first two cards of two bay frame program.

CHAPTER 6

COMPARISON OF EXACT METHOD WITH APPROXIMATE METHOD

6.1 Introduction

Tall building frame is one of the highly redundant structure encountered in Civil Engineering practice. Numerical solution of the exact analysis, without the help of high speed digital computers, is too time consuming and sometimes impossible. Designers with no access to high speed digital computers go for approximate analysis. Here in this chapter an attempt has been made to compare the different parameters or assumptions of approximate method with that of "exact" method.

For academic interest the value of λ $(= I_c.\ell / I_b.h$ or $\sum I_c.\ell /2 \sum I_b.h)$ has been used between 0.5 and 100. However, the realistic range of is between 1 and 10.

6.2 Comparison of GA Parameter of One and Two Bay Frames

GA parameter of a frame may be defined as the lateral load required at the top of the frame to cause unit lateral translation of the frame at top. In Chapter 3, GA parameter by Heidebrecht and Smith was given in equation (3.2.11).

To compare the value of GA by Heidebrecht and Smith method (HSM) and the exact method (EM) programs were developed for one bay and two bay frames.

For different values of λ ($= I_c/h \big/ I_b/\ell$)
GA value by the above two methods were evaluated. The results are given in tabular form.

From the results it is seen that for a two Bay frame, the GA value by Heidebrecht and Smith method may be used in the following range of storeys for different values of λ.

λ =	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6
No. of storey	7-10	10-12	12-13	13-14	14-16

Similarly for one Bay frame the GA value may be used in the following the range of storey for different values of λ.

λ =	1 - 2	2 - 3	3 - 4	4 - 5	5 - 7	7 - 9	9 - 10
No. of storey	4 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10-11	11 - 12

GA FOR ONE BAY FRAME

$$\lambda = \frac{I_c \cdot \ell}{I_b \cdot h}$$, where

h = Storey height

ℓ = Span

I_c = Moment of inertia of column

I_b = Moment of inertia of Beam

HSM = Heidebrecht and Smith method

EM = Exact Method

λ =	1.0	1.92	2.0	3.0	4.0	4.16	4.17
GA BY HSM	185185	21421	111111	79365	61728	15064	20089
NO. OF STOREY	GA BY EM	GA BY EM	GA BY EM	GA BY EM	GA BY EM	GA BY EM	GA BY EM
20	42412	10446	37149	33089	29857	7509	13343
19	45923	11034	39833	35218	31595	7935	13883
18	49841	11658	42770	37516	33450	8390	14443
17	54221	12321	45986	39993	34528	8873	15021
16	59126	13022	49505	42662	37536	93876	15620
15	64628	13763	53355	45534	39779	9932	16238
14	70803	14544	57562	48620	42162	10510	16876
13	77739	15366	62155	51931	44961	11122	17536
12	85526	16227	67157	55476	47374	11770	18220
11	94256	17128	72595	59269	50220	N/A	18932
10	104021	18070	78491	63323	53246	13185	19681
9	114904	19057	84874	67669	56485	13965	20481
8	126981	20098	91784	72353	59993	14811	21356
7	140324	21213	99304	77481	63884	15751	22353
6	155039	22447	107619	83270	68385	16844	23553
5	171401	23895	117173	90201	73980	18212	25131
4	190263	25793	129110	99441	81803	20139	27475
3	214501	28783	146744	114237	94974	23411	31641
2	255590	35185	182352	146496	124970	30911	41488

GA FOR ONE BAY FRAME (CONTD.)

λ =	5.0	6.0	7.0	8.0	9.0	10.0
GA BY HSM	50505	42735	37037	32680	29240	26455
NO. OF STOREY	GA BY EM	GA BY EM	GA BY EM	GA BY EM	GA BY EM	GA BY EM
20	27220	25027	23173	21585	20208	19003
19	28671	26260	24237	22515	21030	19737
18	30207	27558	25352	23486	21886	20499
17	31832	28922	26518	24498	22776	21290
16	33548	30355	27739	25554	23703	22112
15	35361	31860	29015	26656	24668	22968
14	37273	33439	30350	27806	25675	23861
13	39289	35098	31749	29011	26730	24788
12	41415	36843	33221	30280	27842	25788
11	43663	38687	34778	31625	29026	26845
10	46052	40657	36444	33071	30305	27995
9	48616	42772	38254	34655	31718	29273
8	51419	45114	40275	36442	33328	30746
7	54577	47793	42623	38548	35249	32523
6	58316	51036	45518	41187	37693	34812
5	63126	55303	49409	44797	41084	38028
4	70060	61652	55321	50375	46399	43129
3	82157	72982	66072	60673	56332	52763
2	110530	100136	92283	86132	81181	77107

GA VALUE FOR TWO BAY FRAME

$$\lambda = \frac{\sum I_c \cdot \ell}{2\sum I_b \cdot h'}$$

where: h = Storey height

ℓ = Span

$\sum I_c$ = Sum of moment of inertia of columns

$\sum I_b$ = Sum of moment of inertia of Beams

HSM = Heidebrecht and Smith Method

EM = Exact method

λ =	1.0	1.44	1.60	1.842	1.98	2.965	3.95	4.95	5.93
GA BY HSM	63746	39175	33547	35288	38308	27382	21305	17437	14756
NO. OF STOREY	GA BY EM	GA BY EM	GA BY EM	GA BY EM	GA BY EM	GA BY EM	GA BY EM	GA BY EM	GA BY EM
20	40305	26664	26577	25664	28918	22604	18585	15800	13753
19	41835	27623	27248	26491	29777	23146	18965	16086	13979
18	43617	28606	27925	27333	30650	23694	19248	16376	14209
17	45346	29610	28606	28190	31534	24247	19738	16669	14443
16	47120	30634	29291	29059	32430	24806	20131	16968	14682
15	48934	31676	29980	29942	33337	25373	20532	17274	14929
14	50784	32735	30673	30836	34254	25948	20941	17590	15186
13	52664	33809	31372	31744	35184	26535	21363	17918	15455
12	54572	34899	32080	32665	36130	27138	21802	18265	15744
11	56505	36007	32803	33606	37097	27765	22267	18638	16058
10	58464	37138	33547	34576	38095	28428	22769	19047	16408
m9	60460	38304	34330	35583	39144	29146	23326	19512	16813
8	62512	39529	35173	36661	40273	29948	23968	20060	17289
7	64666	40850	36127	37853	41537	30889	24745	20738	17910
6	67016	42351	37250	39247	43036	32065	25749	21634	18731
5	69752	44189	38713	41017	44971	33664	27159	22918	19926
4	73316	46727	40848	43555	47790	36115	29378	24976	21861
3	78886	50933	44554	47910	52692	40556	33491	28843	25538
2	90596	60241	53020	57802	63936	51088	43436	38331	34668

6.3 Comparison of "Exact" Method with Portal and Cantilever Methods

Probably the most popular method of analyzing a rigid rectangular frame under lateral load is either the portal method or the cantilever method. These are, in effect, the methods generally used in the preliminary analysis of tall buildings.

The two common assumptions made in portal and cantilever methods are:

1. There is a point of inflection at the center of each girder.

2. There is a point of inflection at the center of each column.

The third assumption for portal method is "The total horizontal shear on each storey is divided between the columns of that storey in a manner such that each interior column carries twice as much shear as each exterior column".

While the third assumption for cantilever method is "The intensity of axial stress in each column of a storey is proportion l to the horizontal distance of the column from the center of gravity of all the columns of the storey under consideration".

Several programs for lateral loads with different structure data were run to check the validity of the above simplified assumptions.

6.3.1 One Bay Frame

By varying the value of λ $(=\frac{I_c \cdot \ell}{I_b \cdot h})$ from 0.5 to 100, programs for lateral load for one bay frame were run with a view to assessing the range of λ for which the above assumptions of portal and cantilever methods may safely be used in the design office. By analyzing the results it is found that for symmetrical frames the assumptions 1 and 3 of the above methods are valid for all values of λ, while assumption 2 does not hold good for any value of λ. Point of inflection of each column is dependent on λ and varies from storey to storey for same value of λ. Point of inflection of column for different storey with different valuesof λ are given in Tabular and graphical form.

6.3.1.1 Discussion on point of inflection in columns

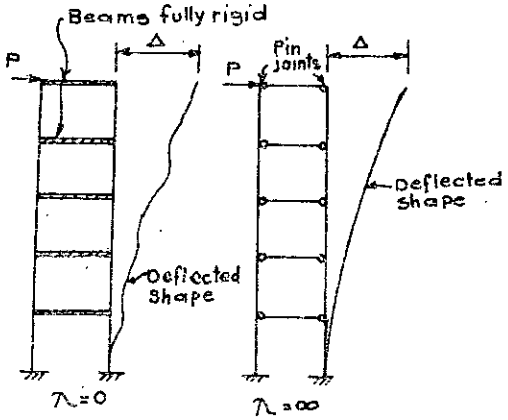
The amount by which the points inflection in the columns move off-center can be estim·ted using the tables or graphs as given after this section.

If λ is low, then the beams are stiff and the frame deflects purely in a shear mode as shown in Fig. 7(a). Points of inflection are at midheight throughout.

As λ increases the beams become less effective until, when they have no bending stiffness, the load is resisted by the columns alone, as in Fig. 7(b). The frame then assumes a bent shape similar to that of a single cantilever under lateral load, that is, it deflects in a bending mode. There are no points of inflection in the columns.

Neither of the ideal conditions illustrated in Fig. 7 are normally achieved in reality. Beams can never be fully rigid and even a very low beam stiffness can have a significant effect in restraining the frame deflection. What does happen is that within a realistic range of λ (say 1 to 10) the points of inflection in the columns (except those in the lower and upperr stories) are close to midheight, and shear mode deformation predominates.

If significant axial strains do occur in the columns, bending mode deformations becomes more prominent. From the point of view of stiffness calculations, it is important to note that as λ decreases, deflection due to axial deformation of the columns will become more prominent.



(a) shear mode deformation (b) bending mode deformation

FIG.7 EFFECT OF λ ON FRAME DEFORMATION

POINT OF INFLECTION AT COLUMN
(ONE BAY FRAME)

$\lambda = \dfrac{I_c \cdot \ell}{I_b \cdot h}$, h = storey height, ℓ = span, I_c = Moment of Inertia of column

I_b = Moment of Inertia of Beam

X = Distance of point of inflection from bottom of column.

NPC = No point of inflection

NUMBER OF STOREY = 10

STOREY LEVEL FROM TOP	λ=0.5	λ=1.0	λ=2.0	λ=4.0	λ=5.0	λ=8.0	λ=10.0	λ=20.0	λ=40.0	λ=80.0	λ=100.0
	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h
1	0.3866	0.2778	0.0856	NPC	NPC	NPC	NPC	NPC	NPC	NPC	NPC
2	0.4514	0.3997	0.3025	0.1259	0.0448	NPC	NPC	NPC	NPC	NPC	NPC
3	0.4755	0.4426	0.3787	0.2565	0.1991	0.0398	NPC	NPC	NPC	NPC	NPC
4	0.4878	0.4643	0.4177	0.3271	0.2837	0.1640	0.0936	NPC	NPC	NPC	NPC
5	0.4961	0.4779	0.4416	0.3719	0.3392	0.2517	0.2022	0.0376	NPC	0.0432	0.1348
6	0.5027	0.4878	0.4588	0.4061	0.3830	0.3268	0.2986	0.2298	0.2572	0.4739	0.5936
7	0.5082	0.4961	0.4738	0.4409	0.4297	0.4110	0.4074	0.4430	0.5969	0.9233	NPC
8	0.5136	0.5052	0.4952	0.5954	0.5017	0.5338	0.5611	0.7123	0.9895	NPC	NPC
9	0.5240	0.5293	0.5523	0.6163	0.6506	0.7520	0.8159	NPC	NPC	NPC	NPC
10	0.6022	0.6645	0.7702	0.9373	NPC	NPC	NPC	NPC	NPC	NPC	NPC

POINT OF INFLECTION AT COLUMN (CONTD..)

NUMBER OF STOREY = 15

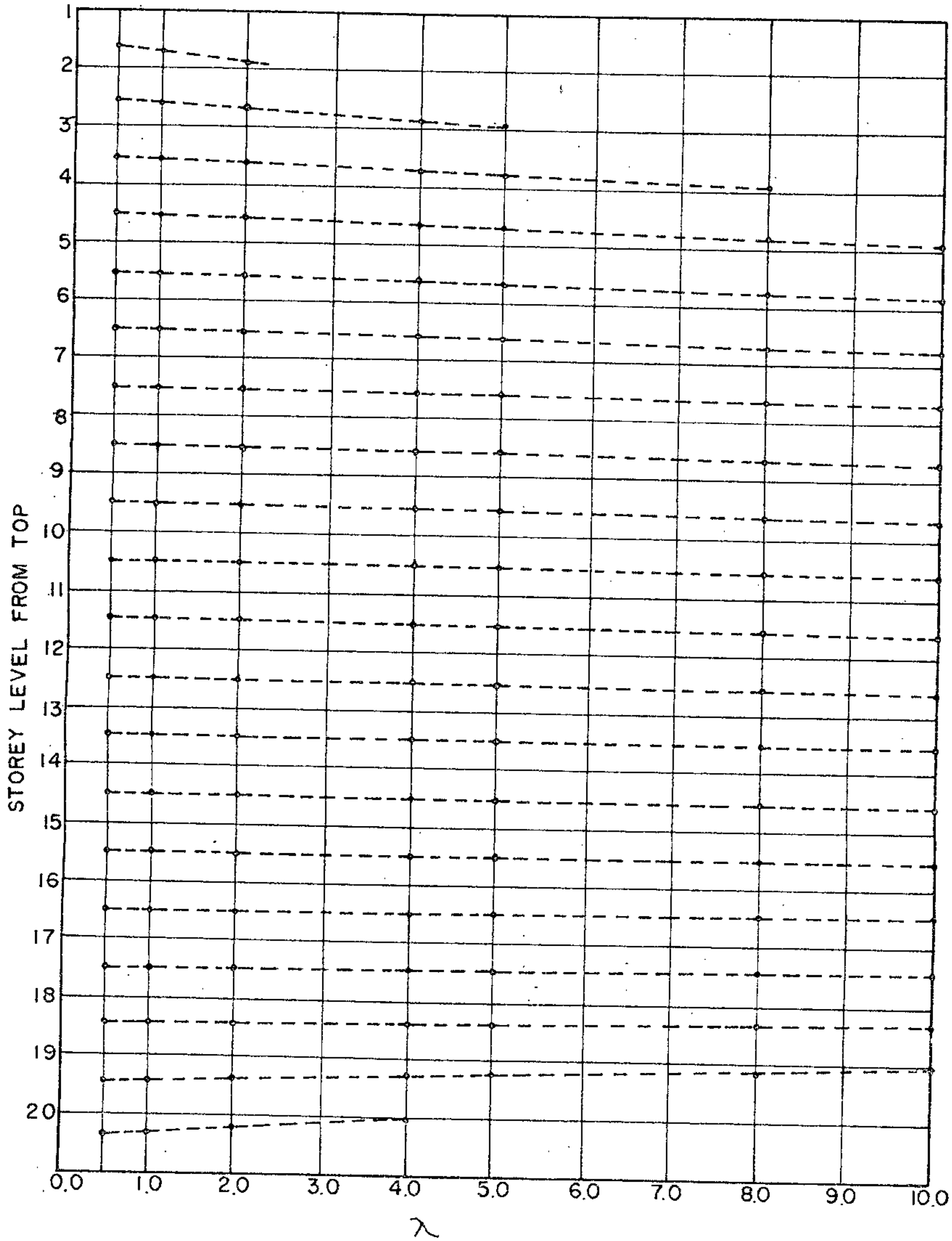
Storey level from top	λ=0.5	λ=1.0	λ=2.0	λ=4.0	λ=5.0	λ=8.0	λ=10.0	λ=20.0	λ=40.0	λ=80.0	λ=100.0
	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h
1	0.3866	0.2778	0.0856	NPC	NPC	NPC	NPC	NPC	NPC	NPC	NPC
2	0.4514	0.3997	0.3025	0.1258	0.0440	NPC	NPC	NPC	NPC	NPC	NPC
3	0.4755	0.4426	0.3787	0.2560	0.1977	0.0319	NPC	NPC	NPC	NPC	NPC
4	0.4878	0.4643	0.4177	0.3257	0.2811	0.1508	0.0681	NPC	NPC	NPC	NPC
5	0.4961	0.4779	0.4414	0.3688	0.3331	0.2276	0.1597	NPC	NPC	NPC	NPC
6	0.5027	0.4878	0.4579	0.3981	0.3687	0.2811	0.2247	NPC	NPC	NPC	NPC
7	0.5082	0.4955	0.4703	0.4197	0.3948	0.3210	0.2739	0.0682	NPC	NPC	NPC
8	0.5131	0.5022	0.4802	0.4366	0.4151	0.3529	0.3139	0.1531	NPC	NPC	NPC
9	0.5176	0.5079	0.4886	0.4506	0.4321	0.3809	0.3502	0.2369	0.1443	0.1645	0.2486
10	0.5218	0.5132	0.4960	0.4633	0.4482	0.4094	0.3884	0.3299	0.3356	0.5011	0.6050
11	0.5258	0.5180	0.5030	0.4771	0.4667	0.4450	0.4371	0.4457	0.5590	0.8505	0.9928
12	0.5297	0.5228	0.5113	0.4976	0.4952	0.5002	0.5109	0.6040	0.8340	NPC	NPC
13	0.5339	0.5296	0.5272	0.5405	0.5528	0.6013	0.6386	0.8356	NPC	NPC	NPC
14	0.5431	0.5512	0.5790	0.6517	0.6902	0.8042	0.8765	NPC	NPC	NPC	NPC
15	0.6197	0.6837	0.7933	0.9689	NPC	NPC	NPC	NPC	NPC	NPC	NPC

POINT OF INFLECTION AT COLUMN (CONTD..)

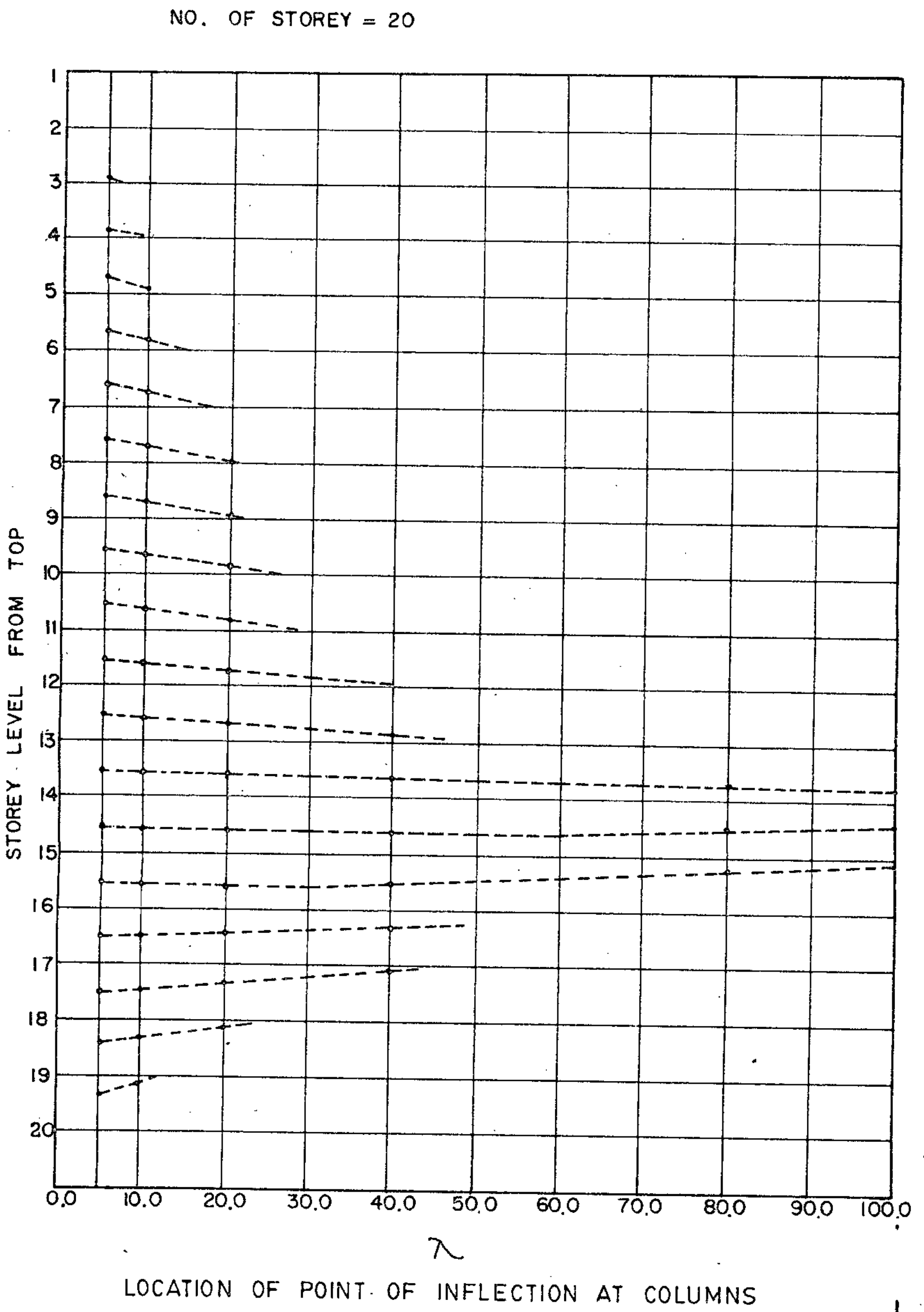
NUMBER OF STOREY = 20

Storey level from top	λ=0.5	λ=1.0	λ=2.0	λ=4.0	λ=6.0	λ=8.0	λ=10.0	λ=20.0	λ=40.0	λ=80.0	λ=100.0
	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h
1	0.3866	0.3778	0.0856	NPC	NPC	NPC	NPC	NPC	NPC	NPC	NPC
2	0.4514	0.4000	0.3025	0.1258	0.0440	NPC	NPC	NPC	NPC	NPC	NPC
3	0.4755	0.4429	0.3787	0.2560	0.1977	0.0315	NPC	NPC	NPC	NPC	NPC
4	0.4878	0.4644	0.4177	0.3258	0.2811	0.1505	0.0675	NPC	NPC	NPC	NPC
5	0.4963	0.4781	0.4416	0.3688	0.3331	0.2271	0.1584	NPC	NPC	NPC	NPC
6	0.5028	0.4878	0.4578	0.3981	0.3685	0.2803	0.2222	NPC	NPC	NPC	NPC
7	0.5082	0.4956	0.4703	0.4197	0.3944	0.3191	0.2694	0.0304	NPC	NPC	NPC
8	0.5131	0.5022	0.4801	0.4363	0.4144	0.3489	0.3056	0.0969	NPC	NPC	NPC
9	0.5176	0.5079	0.4886	0.4498	0.4304	0.3727	0.3344	0.1506	NPC	NPC	NPC
10	0.5218	0.5131	0.4957	0.4611	0.4438	0.3923	0.3583	0.1985	NPC	NPC	NPC
11	0.5258	0.5179	0.5022	0.4710	0.4554	0.4090	0.3787	0.2409	0.0403	NPC	NPC
12	0.5297	0.5225	0.5082	0.4797	0.4655	0.4239	0.3972	0.2823	0.1372	0.0485	0.0568
13	0.5334	0.5269	0.5137	0.4876	0.4747	0.4380	0.4153	0.3263	0.2431	0.2592	0.3075
14	0.5371	0.5310	0.5188	0.4950	0.4837	0.4528	0.4349	0.3782	0.3658	0.4931	0.5822
15	0.5407	0.5350	0.5238	0.5029	0.4935	0.4709	0.4600	0.4455	0.5157	0.7601	0.8902
16	0.5443	0.5390	0.5289	0.5127	0.5069	0.4978	0.4977	0.5402	0.7064	NPC	NPC
17	0.5478	0.5431	0.5356	0.5297	0.5307	0.5454	0.5619	0.6813	0.9569	NPC	NPC
18	0.5517	0.5492	0.5500	0.5689	0.5838	0.6394	0.6812	0.8999	NPC	NPC	NPC
19	0.5606	0.5699	0.5999	0.6765	0.7170	0.8368	0.9131	NPC	NPC	NPC	NPC
20	0.6365	0.7013	0.8129	0.9925	NPC	NPC	NPC	NPC	NPC	NPC	NPC

NO. OF STOREY = 20



LOCATION OF POINT OF INFLECTION AT COLUMNS


NO. OF STOREY = 20
STOREY LEVEL FROM TOP
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
0.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100.0
λ
LOCATION OF POINT OF INFLECTION AT COLUMNS

6.3.1 Two Bay Frame

Comparison of exact method with portal and cantilever methods where made for two Bay frame. Two types of data were used for this comparison. In one set the moment of inertia of all the columns were kept equal, while in the other set the moment of inertia of interior Column was kept double the moment of inertia of exterior column. The frames are symmetrical and the values of λ ($\Sigma I_c . \ell$ / $2 \Sigma I_b . h$) varies from 0.5 to 100.0.

From the computer result it is found that for the frames where the moment of inertia of interior column is double the moment of inertia of exterior column, the assumptions 1 and 3 of portal and cantilever methods hold good. The point of inflection at columns varies from storey to storey. For the upper stories it is below the center of columns and for lower stories it is above the center of columns. For both exterior and interior columns the point of inflections are at the same level in a storey.

For the frames where the moment of inertia of all the columns are same, only the assumption 1 of portal method and assumptions 1 and 3 of cantilever method hold good. The point of inflection on column varies from storey to storey and depends on λ . The point of inflection at exterior and interior columns on a storey do not fall on the same level. Except at the top and bottom storey, the ratio of shear at interior column to shear at exterior columns varies from 1.665 to 2.40 depending on λ . The ratio increases with the increase of λ .

Point of inflection at column and ratio of column shear are given in the tabular form.

POINT OF INFLECTION AT COLUMN

Area and moment of inertia of interior column is double that of exterior column

$$\lambda = \frac{\sum I_c \cdot \ell}{2\sum I_b \cdot h},$$ where:- $\sum I_c$ = Sum of moment of inertia of columns

$\sum I_b$ = Sum of moment of inertia of beams

ℓ = Span of beams , h = Storey height

X = Distance of point of inflection from bottom of column, NPC = No point of inflection

Number of Storey = 15

Storey level from top	λ=0.5	λ=1.0	λ= 2.0	λ=2.5	λ=4.0	λ=5.0	λ=10.0	λ=20.0	λ=40.0	λ=50.0
	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h
1	0.3843	0.2754	0.0791	NPC	NPC	NPC	NPC	NPC	NPC	NPC
2	0.4491	0.3972	0.2983	0.2523	0.1206	0.0387	NPC	NPC	NPC	NPC
3	0.4713	0.4384	0.3738	0.3423	0.2505	0.1918	NPC	NPC	NPC	NPC
4	0.4821	0.4583	0.4113	0.3881	0.3188	0.2737	0.0598	NPC	NPC	NPC
5	0.4889	0.4706	0.4338	0.4152	0.3606	0.3247	0.1504	NPC	NPC	NPC
6	0.4939	0.4787	0.4486	0.4336	0.3887	0.3589	0.2139	NPC	NPC	NPC
7	0.4977	0.4850	0.4593	0.4469	0.4087	0.3835	0.2620	0.0543	NPC	NPC
8	0.5011	0.4901	0.4679	0.4569	0.4241	0.4042	0.3005	0.1382	NPC	NPC
9	0.5039	0.4941	0.4748	0.4650	0.4364	0.4180	0.3354	0.2214	0.1365	0.1290
10	0.5065	0.4978	0.4805	0.4721	0.4476	0.4326	0.3724	0.3139	0.3177	0.3560
11	0.5089	0.5012	0.4860	0.4789	0.4599	0.4495	0.4197	0.4299	0.5469	0.6179
12	0.5114	0.5045	0.4927	0.4880	0.4790	0.4766	0.4924	0.5877	0.8189	0.9300
13	0.5139	0.5095	0.5071	0.5086	0.5207	0.5330	0.6195	0.8181	NPC	NPC
14	0.5217	0.5298	0.5577	0.5748	0.6309	0.6698	0.8570	NPC	NPC	NPC
15	0.5972	0.6615	0.7717	0.8202	0.9481	NPC	NPC	NPC	NPC	NPC

POINT OF INFLECTION AT COLUMN (CONTD.,)

Area and moment of inertia of interior column is double that of exterior column

Number of storey = 20

Storey level from top	λ=0.5	λ=1.0	λ=2.0	λ=2.5	λ=4.0	λ=5.0	λ=10.0	λ=20.0	λ=40.0	λ=50.0
	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h	X/h
1	0.3843	0.2755	0.0790	NPC	NPC	NPC	NPC	NPC	NPC	NPC
2	0.4491	0.3966	0.2988	0.2515	0.1204	0.0386	NPC	NPC	NPC	NPC
3	0.4713	0.4384	0.3788	0.3426	0.2500	0.1917	NPC	NPC	NPC	NPC
4	0.4821	0.4583	0.4114	0.3882	0.3188	0.2738	0.0589	NPC	NPC	NPC
5	0.4889	0.4706	0.4337	0.4152	0.3606	0.3246	0.1492	NPC	NPC	NPC
6	0.4937	0.4787	0.4487	0.4335	0.3885	0.3590	0.2111	NPC	NPC	NPC
7	0.4979	0.4850	0.4595	0.4468	0.4085	0.3832	0.2571	0.0171	NPC	NPC
8	0.5009	0.4901	0.4678	0.4568	0.4237	0.4018	0.2923	0.0821	NPC	NPC
9	0.5038	0.4943	0.4746	0.4649	0.4356	0.4162	0.3198	0.1356	NPC	NPC
10	0.5065	0.4978	0.4803	0.4717	0.4455	0.4281	0.3421	0.1813	NPC	NPC
11	0.5089	0.5011	0.4853	0.4775	0.4535	0.4379	0.3611	0.2224	0.0209	NPC
12	0.5133	0.5041	0.4897	0.4824	0.4609	0.4466	0.3781	0.2625	0.1169	0.0757
13	0.5134	0.5068	0.4936	0.4869	0.4672	0.4543	0.3945	0.3052	0.2220	0.2104
14	0.5155	0.5094	0.4972	0.4911	0.4732	0.4618	0.4127	0.3560	0.3440	0.3645
15	0.5176	0.5118	0.5006	0.4951	0.4794	0.4071	0.4365	0.4220	0.4931	0.5490
16	0.5195	0.5142	0.5041	0.4995	0.4877	0.4819	0.4726	0.5158	0.6834	0.7769
17	0.5215	0.5167	0.5092	0.5065	0.5032	0.5042	0.5359	0.6563	0.9336	NPC
18	0.5238	0.5212	0.5221	0.5250	0.5412	0.5561	0.6544	0.8744	NPC	NPC
19	0.5311	0.5405	0.5707	0.5888	0.6479	0.6887	0.8860	NPC	NPC	NPC
20	0.6095	0.6711	0.7832	0.8327	0.9638	NPC	NPC	NPC	NPC	NPC

POINT OF INFLECTION AND RATIO OF SHEAR AT EXTERIOR AND INTERIOR COLUMNS

All columns are of same section

SIC/SEC = Shear interior column/shear exterior column

Number of Storey = 15

Storey level from top	$\lambda = 0.5$			$\lambda = 1.0$			$\lambda = 2.0$			$\lambda = 4.0$		
	Ext. Col.	Int. col.	SIC/SEC	Ext. Col.	Int. col.	SIC/SEC	Ext. col.	Int. col.	SIC/SEC	Ext. col.	Int. col.	SIC/SEC
	X/h	X/h		X/h	X/h		X/h	X/h		X/h	X/h	
1	0.3088	0.4406	1.986	0.1371	0.3753	2.368	NPC	0.2706	3.039	NPC	0.1252	4.559
2	0.4180	0.4732	1.659	0.3455	0.4399	1.791	0.2129	0.3760	1.887	NPC	0.2605	1.970
3	0.4524	0.4847	1.666	0.4072	0.4629	1.800	0.3204	0.4200	1.895	0.1544	0.3385	1.964
4	0.4685	0.4905	1.665	0.4359	0.4748	1.797	0.3723	0.4438	1.887	0.2487	0.3821	1.941
5	0.4781	0.4942	1.665	0.4529	0.4821	1.798	0.4035	0.4575	1.887	0.3056	0.4087	1.940
6	0.4848	0.4968	1.665	0.4642	0.4869	1.798	0.4237	0.4667	1.887	0.3433	0.4266	1.939
7	0.4899	0.4989	1.665	0.4726	0.4905	1.798	0.4384	0.4734	1.886	0.3704	0.4393	1.938
8	0.4939	0.5008	1.665	0.4791	0.4933	1.798	0.4495	0.4785	1.886	0.3909	0.4491	1.938.
9	0.4974	0.5023	1.665	0.4844	0.4957	1.798	0.4584	0.4825	1.886	0.4075	0.4570	1.938
10	0.5004	0.5038	1.665	0.4889	0.4977	1.798	0.4660	0.4861	1.886	0.4222	0.4640	1.936
11	0.5032	0.5050	1.665	0.4929	0.4995	1.797	0.4730	0.4893	1.884	0.4388	0.4717	1.929
12	0.5057	0.5064	1.666	0.4968	0.5016	1.798	0.4816	0.4936	1.882	0.4644	0.4842	1.913
13	0.5092	0.5075	1.658	0.5043	0.5041	1.783	0.5021	0.5023	1.853	0.5210	0.5111	1.855
14	0.5163	0.5149	1.684	0.5284	0.5020	1.791	0.5672	0.5387	1.808	0.6607	0.5898	1.742
15	0.6256	0.5569	1.308	0.7010	0.6069	1.344	0.8221	0.6990	1.337	NPC	0.8550	1.294

POINT OF INFLECTION AND RATIO OF SHEAR AT INTERIOR AND EXTERIOR COLUMNS

All columns are of same section

Number of Storey = 15

Storey level from top	$\lambda = 5.0$			$\lambda = 8.0$			$\lambda = 10.0$			$\lambda = 20.0$		
	Ext. col. X/h	Int. col. X/h	SIC/SEC	Ext. col. X/h	Int. col. X/h	SIC/SEC	Ext. col. X/h	Int. col. X/h	SIC/SEC	Ext. col. X/h	Int. col. X/h	SIC/SEC
1	NPC	0.0710	5.568	NPC	NPC	10.221	NPC	NPC	17.608	NPC	NPC	-15.11
2	NPC	0.2090	1.997	NPC	0.0704	2.077	NPC	NPC	2.122	NPC	NPC	2.402
3	0.0739	0.2997	1.986	NPC	0.1923	2.039	NPC	0.1259	2.067	NPC	NPC	2.224
4	0.1884	0.3522	1.954	0.0138	0.2656	1.979	NPC	0.2110	1.994	NPC	NPC	2.049
5	0.2573	0.3847	1.952	0.1162	0.3143	1.972	0.0246	0.2689	1.981	NPC	0.0664	2.008
6	0.3035	0.4067	1.950	0.1863	0.3481	1.968	0.1107	0.3101	1.974	NPC	0.1402	1.979
7	0.3367	0.4225	1.949	0.2380	0.3730	1.966	0.1751	0.3411	1.969	NPC	0.2014	1.963
8	0.3620	0.4346	1.949	0.2787	0.3927	1.963	0.2270	0.3663	1.964	0.0159	0.2559	1.945
9	0.3829	0.4446	1.947	0.3144	0.4098	1.958	0.2741	0.3890	1.950	0.1278	0.3098	1.920
10	0.4023	0.4539	1.944	0.3509	0.4273	1.948	0.3239	0.4129	1.939	0.2510	0.3705	1.884
11	0.4252	0.4646	1.934	0.3973	0.4496	1.926	0.3878	0.4439	1.913	0.4024	0.4480	1.830
12	0.4618	0.4824	1.911	0.4697	0.4855	1.884	0.4843	0.4926	1.860	0.6031	0.5584	1.747
13	0.5373	0.5196	1.841	0.5998	0.5539	1.788	0.6463	0.5810	1.753	0.8813	0.7314	1.619
14	0.7081	0.6182	1.708	0.8432	0.7057	1.622	0.9262	0.7633	1.578	NPC	NPC	1.440
15	NPC	0.9229	1.276	NPC	NPC	1.235	NPC	NPC	1.215	NPC	NPC	1.158

POINT OF INFLECTION AND RATIO OF SHEAR AT EXTERIOR AND INTERIOR COLUMNS

All columns are of same section

Number of storey = 15

Storey level from top	λ = 40.0			λ = 80.0			λ = 100.0		
	Ext. col. X/h	Int. col. X/h	SIC/SEC	Ext. col. X/h	Int. col. X/h	SIC/SEC	Ext. col. X/h	Int. col. X/h	SIC/SEC
1	NPC	NPC	-5.893	NPC	NPC	-4.065	NPC	NPC	-4.041
2	NPC	NPC	2.892	NPC	NPC	3.466	NPC	NPC	3.603
3	NPC	NPC	2.481	NPC	NPC	2.771	NPC	NPC	2.831
4	NPC	NPC	2.149	NPC	NPC	2.217	NPC	NPC	2.209
5	NPC	NPC	2.038	NPC	NPC	2.022	NPC	NPC	2.004
6	NPC	NPC	1.962	NPC	NPC	1.901	NPC	NPC	1.870
7	NPC	0.0133	1.916	NPC	NPC	1.816	NPC	NPC	1.773
8	NPC	0.1235	1.875	NPC	0.0487	1.743	NPC	0.064	1.694
9	0.0297	0.2391	1.824	0.091	0.257	1.674	0.167	0.297	1.624
10	0.2670	0.3694	1.764	0.4794	0.4874	1.604	0.5993	0.5639	1.550
11	0.5471	0.5278	1.689	0.8910	0.7553	1.526	NPC	0.8703	1.474
12	0.8766	0.7344	1.594	NPC	NPC	1.436	NPC	NPC	1.389
13	NPC	NPC	1.469	NPC	NPC	1.333	NPC	NPC	1.294
14	NPC	NPC	1.315	NPC	NPC	1.215	NPC	NPC	1.188
15	NPC	NPC	1.110	NPC	NPC	1.074	NPC	NPC	1.064

POINT OF INFLECTION AND RATIO OF SHEAR AT INTERIOR AND EXTERIOR COLUMN

All columns are of same section. Number of story = 20

Story level from top	λ = 0.5			λ = 1.0			λ = 2.0			λ = 4.0		
	Ext. col.	Int. col.	SIC/SEC	Ext. col.	Int. col.	SIC/SEC	Ext. col.	Int. col.	SIC/SEC	Ext. col.	Int. col.	SIC/SEC
	X/h	X/h		X/h	X/h		X/h	X/h		X/h	X/h	
1	0.3088	0.4395	1.974	0.1371	0.3753	2.371	NPC	0.2706	3.027	NPC	0.1250	4.557
2	0.4175	0.4732	1.659	0.3455	0.4399	1.790	0.2129	0.3760	1.884	NPC	0.2607	1.967
3	0.4525	0.4845	1.666	0.4076	0.4631	1.800	0.3204	0.4203	1.895	0.1542	0.3383	1.964
4	0.4685	0.4905	1.665	0.4359	0.4750	1.797	0.3728	0.4436	1.886	0.2489	0.3822	1.940
5	0.4780	0.4942	1.665	0.4529	0.4821	1.797	0.4033	0.4575	1.887	0.3057	0.4087	1.939
6	0.4848	0.4969	1.665	0.4642	0.4868	1.798	0.4237	0.4667	1.886	0.3433	0.4267	1.939
7	0.4897	0.4989	1.665	0.4724	0.4905	1.797	0.4384	0.4734	1.886	0.3703	0.4392	1.939
8	0.4939	0.5008	1.665	0.4792	0.4932	1.797	0.4495	0.4785	1.886	0.3904	0.4489	1.938
9	0.4974	0.5023	1.665	0.4844	0.4957	1.798	0.4585	0.4825	1.886	0.4063	0.4564	1.939
10	0.5004	0.5038	1.665	0.4889	0.4977	1.798	0.4658	0.4859	1.886	0.4192	0.4626	1.937
11	0.5031	0.5051	1.665	0.4929	0.4996	1.798	0.4719	0.4889	1.886	0.4298	0.4677	1.938
12	0.5057	0.5063	1.665	0.4965	0.5012	1.798	0.4774	0.4914	1.886	0.4391	0.4721	1.938
13	0.5081	0.5075	1.665	0.4996	0.5027	1.798	0.4822	0.4937	1.886	0.4472	0.4759	1.938
14	0.5104	0.5087	1.665	0.5026	0.5042	1.798	0.4866	0.4958	1.886	0.4548	0.4796	1.937
15	0.5125	0.5098	1.665	0.5054	0.5056	1.798	0.4906	0.4977	1.886	0.4628	0.4835	1.936
16	0.5145	0.5108	1.665	0.5080	0.5068	1.797	0.4950	0.4998	1.885	0.4738	0.4886	1.930
17	0.5165	0.5120	1.666	0.5109	0.5084	1.798	0.5014	0.5031	1.882	0.4944	0.4988	1.915
18	0.5195	0.5130	1.658	0.5172	0.5106	1.784	0.5195	0.5108	1.854	0.5455	0.5236	1.859
19	0.5262	0.5200	1.683	0.5401	0.5260	1.791	0.5819	0.5460	1.811	0.6802	0.6000	1.747
20	0.6335	0.5624	1.310	0.7101	0.6129	1.348	0.8334	0.7064	1.341	NPC	0.8657	1.298

POINT OF INFLICTION AND RATIO OF SHEAR AT INTERIOR AND EXTERIOR COLUMNS

All columns are of same section. Number of storey = 20

Storey level from top	λ = 5.0			λ = 8.0			λ = 10.0			λ = 20.0		
	Ext. col. X/h	Int. col. X/h	SIC/SEC	Ext. col. X/h	Int. col. X/h	SIC/SEC	Ext. col. X/h	Int. col. X/h	SIC/SEC	Ext. col. X/h	Int. col. X/h	SIC/SEC
1	NPC	0.0709	5.542	NPC	NPC	10.500	NPC	NPC	18.53	NPC	NPC	-14.53
2	NPC	0.2085	1.996	NPC	0.0700	2.078	NPC	NPC	2.132	NPC	NPC	2.400
3	0.0739	0.2997	1.985	NPC	0.1917	2.041	NPC	0.1251	2.074	NPC	NPC	2.230
4	0.1884	0.3521	1.954	0.0122	0.2653	1.981	NPC	0.2102	1.995	NPC	NPC	2.062
5	0.2576	0.3847	1.952	0.1150	0.3136	1.972	0.0221	0.2680	1.983	NPC	0.0558	2.023
6	0.3034	0.4066	1.950	0.1850	0.3473	1.969	0.1073	0.3085	1.976	NPC	0.1246	2.000
7	0.3364	0.4223	1.949	0.2353	0.3716	1.967	0.1684	0.3384	1.974	NPC	0.1780	1.991
8	0.3612	0.4341	1.949	0.2730	0.3901	1.966	0.2150	0.3610	1.973	NPC	0.2207	1.985
9	0.3804	0.4434	1.950	0.3024	0.4046	1.967	0.2515	0.3788	1.974	0.0065	0.2556	1.981
10	0.3961	0.4510	1.950	0.3264	0.4162	1.966	0.2812	0.3933	1.973	0.0678	0.2855	1.976
11	0.4090	0.4572	1.949	0.3466	0.4260	1.966	0.3064	0.4056	1.974	0.1230	0.3124	1.971
12	0.4200	0.4625	1.949	0.3644	0.4345	1.965	0.3289	0.4165	1.970	0.1768	0.3385	1.962
13	0.4300	0.4673	1.949	0.3810	0.4426	1.963	0.3508	0.4271	1.965	0.2344	0.3664	1.950
14	0.4396	0.4719	1.948	0.3985	0.4510	1.959	0.3750	0.4388	1.959	0.3023	0.3997	1.929
15	0.4504	0.4771	1.944	0.4206	0.4617	1.950	0.4066	0.4542	1.946	0.3905	0.4436	1.896
16	0.4661	0.4846	1.935	0.4549	0.4783	1.930	0.4552	0.4780	1.919	0.5128	0.5072	1.843
17	0.4961	0.4994	1.941	0.5164	0.5093	1.890	0.5385	0.5205	1.868	0.6899	0.6063	1.762
18	0.5651	0.5339	1.846	0.6364	0.5733	1.796	0.6884	0.6060	1.763	0.9500	0.7708	1.634
19	0.7300	0.6298	1.714	0.8722	0.7217	1.630	0.9600	0.7824	1.587	NPC	NPC	1.452
20	NPC	0.9353	1.280	NPC	NPC	1.240	NPC	NPC	1.220	NPC	NPC	1.163

POINT OF INFLECTION AND RATIO OF SHEAR AT INTERIOR AND EXTERIOR COLUMN

All columns are of same section

Number storey = 20

Storey level from top	$\lambda = 40.0$			$\lambda = 80.0$			$\lambda = 100.0$		
	Ext. col. X/h	Int. col. X/h	SIC/SEC	Ext. col. X/h	Int. col. X/h	SIC/SEC	Ext. col. X/h	Int. col. X/h	SIC/SEC
1	NPC	NPC	-5.738	NPC	NPC	-3.959	NPC	NPC	-3.688
2	NPC	NPC	2.921	NPC	NPC	3.900	NPC	NPC	4.330
3	NPC	NPC	2.535	NPC	NPC	3.156	NPC	NPC	3.243
4	NPC	NPC	2.215	NPC	NPC	2.391	NPC	NPC	2.463
5	NPC	NPC	2.101	NPC	NPC	2.200	NPC	NPC	2.223
6	NPC	NPC	2.046	NPC	NPC	2.076	NPC	NPC	2.079
7	NPC	NPC	2.014	NPC	NPC	2.008	NPC	NPC	1.992
8	NPC	NPC	1.996	NPC	NPC	1.955	NPC	NPC	1.934
9	NPC	0.0498	1.977	NPC	NPC	1.912	NPC	NPC	1.889
10	NPC	0.1125	1.957	NPC	NPC	1.880	NPC	NPC	1.837
11	NPC	0.1735	1.938	NPC	0.0334	1.843	NPC	NPC	1.793
12	NPC	0.2337	1.913	NPC	0.165	1.800	NPC	0.1666	1.747
13	0.1321	0.3033	1.881	0.1668	0.3100	1.752	0.2322	0.3428	1.698
14	0.2926	0.3899	1.840	0.4599	0.4750	1.698	0.5670	0.5425	1.642
15	0.4844	0.4916	1.786	0.7843	0.6702	1.633	0.9365	0.7759	1.580
16	0.7211	0.6268	1.713	NPC	0.9150	1.554	NPC	NPC	1.503
17	NPC	0.8178	1.617	NPC	NPC	1.463	NPC	NPC	1.415
18	NPC	NPC	1.490	NPC	NPC	1.345	NPC	NPC	1.315
19	NPC	NPC	1.330	NPC	NPC	1.230	NPC	NPC	1.203
20	NPC	NPC	1.116	NPC	NPC	1.079	NPC	NPC	1.069

CHAPTER 7

TWO BAY FRAME WITH SHEAR WALL

7.1 Introduction

In Chapter 3 an approximate method of analysing a shear wall-frame by Heidebrecht and Smith method was given. In this method the frame is reduced to a shear beam and tied to the shear wall, which is termed as flexural beam. The stiffness of the "link beam" between the shear beam and flexural beam is assumed to be zero. In this chapter several structures were analysed with different stiffness of the "link beam" to make a comparison between the exact method and the Heidebrecht and Smith method.

7.2 Comparison Between Exact Method and Heidebrecht and Smith Method

Two Bay frame program was modified to calculate the translation, bending moments, shear etc. by exact as well as Heidebrecht and Smith method. The right vertical member was taken to be shear wall and the other two vertical members as columns. Several programs were run by keeping the sectional properties of all members, except the link beams, same for all programs. Moment of inertia of link beam ($I_{\ell b}$) was varied in such a way that $\eta (= I_{\ell b} \, h/I_c . \ell)$ varied from 0.0 to 8.0. Computer results obtained are given in Tabular form.

Except for η = 0.0, the translation obtained by Heidebrecht and Smith method (HSM) is found to be approximately 50% more than the exact method while for η = 0.0, the translation by exact method is approximately 50% more than HSM. Bending moments at the wall in the middle storeis differs considerably from one system to other, while in the top and bottom stories they differ from 10% to 50% depending on η and α H. Shear on wall differs considerably from one system to other in the middle stories, while in top and bottom stories they are within the range of 20%.

Note: $I_{\ell b}$ = moment of inertia of link beam

h = storey height

I_c = moment of inertia of column

ℓ = length of link beam

α^2 = GA/EI as given in equation (3.2.10)

H = Structure height

COMPARISON BETWEEN EXACT METHOD AND HEIDEBRECHT AND SMITH METHOD

EM = Exact method   HSM = Heidebrecht and Smith method   Number of storey = 20

Storey level from top	η = 0.0 αH= 3.086						η = 0.5 αH = 4.095					
	Translation(in)		B.M. on wall (K-In)		Shear on wall (K)		Translation (in.)		B.M. on wall (K-IN)		Shear on wall (K)	
	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM
1	14.222	9.384	- 7742	- 7166	-53.76	-55.56	3.951	6.129	- 4705	- 6468	-22.51	-50.58
2	13.455	8.984	- 10666	-12774	-20.31	-44.18	3.820	5.921	- 7094	-11675	-3.14	-39.57
3	12.680	8.576	-12967	-16954	-15.98	-33.85	3.683	5.707	-8764	-15229	3.94	-30.21
4	11.892	8.153	-13823	-19807	-5.94	-24.32	3.536	5.478	-9675	-17891	12.59	-22.12
5	11.090	7.710	-13496	-21402	+2.27	-15.38	3.378	5.232	-10032	-19571	20.32	-14.97
6	10.271	7.243	-11976	-21775	10.56	-6.81	3.206	4.965	-9927	-20340	27.86	-8.44
7	9.437	6.752	- 9281	-20960	18.72	+1.61	3.020	4.674	-9418	-20230	35.22	- 2.27
8	8.588	6.234	- 5393	-18866	27.00	10.06	2.820	4.360	-8527	-19238	42.56	+ 3.80
9	7.729	5.692	- 274	-15514	35.55	18.75	2.606	4.021	-7243	-17320	50.00	10.04
10	6.863	5.128	6144	-10800	44.57	27.89	2.379	3.660	-5522	-14397	57.65	16.69
11	5.998	4.546	13957	- 4614	54.25	37.70	2.139	3.279	-3279	-10345	65.67	24.05
12	5.140	3.950	23289	3194	64.81	48.40	1.889	2.880	-381	-4994	74.24	32.42
13	4.299	3.350	34301	12810	76.47	60.26	1.631	2.469	3364	1880	83.58	42.15
14	3.486	2.753	47193	24463	89.53	73.56	1.367	2.053	8232	10568	94.00	53.66
15	2.714	2.172	62209	38431	100.28	88.61	1.103	1.639	14607	21434	105.90	67.42
16	1.998	1.620	79467	55048	120.10	105.78	0.843	1.238	23023	34936	119.82	84.02
17	1.357	1.114	99630	74708	140.39	125.46	0.596	0.862	34220	51641	136.52	104.16
18	0.810	0.673	123291	97883	162.70	148.15	0.371	0.528	49223	72253	157.06	128.67
19	0.382	0.322	150427	125124	188.45	174.36	0.184	0.256	69419	97637	182.63	158.60
20	0.102	0.087	181195	157083	218.93	204.74	0.051	0.070	96886	128862	216.23	195.2

COMPARISON BETWEEN EXACT METHOD AND HEIDEBRECT AND SMITH METHOD (CONTD.)

Number of storey = 20

Storey level from top	η = 1.0				αH = 4.666		η = 1.5				αH = 5.044	
	Translation (in.)		B.M. on wall (K-in)		Shear on wall (K)		Translation (in.)		B.M. on wall (K-in)		Shear on wall (K)	
	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM
1	3.214	4.979	-3585	-5949	-19.30	-46.91	2.920	4.387	-3071	-5605	-17.77	-44.48
2	3.097	4.830	-5177	-10486	- 1.5	-36.08	2.806	4.266	-4280	-9832	-0. 63	-33.78
3	2.976	4.675	-6123	-13862	5.89	-27.23	2.689	4.139	-4893	-12950	6.90	-25.23
4	2.848	4.507	-6455	-16260	14.50	-19.87	2.566	4.000	-4964	-15160	15.55	-18.30
5	2.711	4.324	-6376	-17811	22.33	-13.58	2.435	3.846	-4687	-16600	23.50	-12.54
6	2.565	4.120	-5974	-18599	30.01	- 8.05	2.298	3.675	-4144	-17364	31.32	- 7.58
7	2.410	3.896	-5309	-18669	37.53	- 2.95	2.152	3.483	-3388	-17500	39.03	- 3.10
8	2.244	3.650	-4407	-18024	45.05	1.98	1.999	3.271	-2440	-17020	46.73	1.18
9	2.070	3.382	-3268	-16629	52.63	7.02	1.839	3.039	-1309	-15890	54.49	5.53
10	1.886	3.093	-1868	-14406	60.36	12.45	1.672	2.786	26	-14040	62.39	10.19
11	1.695	2.784	- 152	-11236	68.36	18.56	1.499	2.515	1599	-11350	70.49	15.60
12	1.496	2.458	1970	- 6944	76.76	25.68	1.322	2.227	3478	- 7648	78.92	21.96
13	1.293	2.118	4644	- 1296	85.74	34.40	1.141	1.926	5773	- 2698	87.82	29.73
14	1.086	1.771	8092	+ 6018	95.58	44.59	0.958	1.615	8665	3816	97.43	39.39
15	0.879	1.422	12654	15396	106.64	57.43	0.776	1.302	12444	12312	108.09	51.57
16	0.676	1.081	18841	27351	119.52	73. 41	0.597	0.993	17579	23332	120.36	67.05
17	0.481	0.758	27431	42538	135.06	93.40	0.426	0.700	24823	37580	135.11	86.62
18	0.303	0.468	39602	61787	154.57	118.49	0.269	0.434	35403	55969	153.79	112.13
19	0.151	0.228	57106	86150	179.77	150.06	0.136	0.213	51266	79674	178.46	144.62
20	0.043	0.063	82806	116959	214.83	189.84	0.039	0.059	75798	110210	214.11	186.35

COMPARISON BETWEEN EXACT METHOD AND HEIDEBRECHT AND SMITH METHOD (CONTD.)

Number of storey = 20

Storey level from top	η = 2.0		αH = 5.314				η = 2.5		αH = 5.518			
	Translation(in)		B.M. on wall (K-in)		Shear on wall (K)		Translation (in.)		B.M. on wall (K-in)		Shear on wall (K)	
	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM
1	2.757	4.025	- 2771	- 5366	-16.78	-43.80	2.675	3.781	- 2571	- 5192	-16.04	-41.56
2	2.644	3.920	- 3742	- 9376	- 0.01	-32.18	2.538	3.686	- 3378	- 9042	0.50	-31.00
3	2.528	3.809	- 4166	-12313	7.61	-23.84	2.423	3.586	- 3674	-11847	8.19	-22.82
4	2.407	3.688	- 4087	-14386	16.34	-17.19	2.303	3.475	- 3496	-13820	17.00	-16.38
5	2.280	3.551	- 3699	-15743	24.39	-11.77	2.177	3.350	- 3038	-15113	25.16	-11.20
6	2.146	3.398	- 3076	-16480	32.35	- 7.18	2.046	3.209	- 2361	-15825	33.25	- 6.87
7	2.006	3.226	- 2263	-16650	40.23	- 3.11	1.909	3.050	- 1510	-16011	41.26	- 3.08
8	1.859	3.035	- 1281	-16262	48.10	0.75	1.767	2.872	- 502	-15684	49.29	0.48
9	1.707	2.824	- 135	-15293	56.03	4.66	1.619	2.676	660	-14819	57.38	4.08
10	1.549	2.593	1190	-13671	64.09	8.90	1.467	2.461	1985	-13352	65.58	8.00
11	1.386	2.345	2712	-11281	72.32	13.77	1.311	2.228	3493	-11168	73.94	12.52
12	1.220	2.081	4491	- 7955	80.83	19.61	1.152	1.979	5226	- 8101	82.55	18 .00
13	1.051	1.803	6613	- 3456	89.75	26.86	0.992	1.718	7257	- 3916	91.52	24.86
14	0.882	1.516	9226	2537	99.27	36.00	0.831	1.447	9710	1708	101.02	33.63
15	0.714	1.225	12581	10447	109.71	47.71	0.672	1.171	12804	9202	111.34	44.98
16	0.550	0.937	17103	20838	121.59	62.80	0.517	0.898	16928	19140	122.97	59.77
17	0.393	0.662	23513	34447	135.79	82.36	0.370	0.635	22763	32284	136.75	79.14
18	0.249	0.412	33040	52241	153.79	107.76	0.235	0.396	31527	49640	154.20	104.57
19	0.126	0.203	47746	75483	177.91	140.82	0.119	0.196	45355	72538	177.78	138.01
20	0.036	0.056	71404	105824	213.73	183.88	0.034	0.055	68307	102733	213.56	182.03

COMPARISON BETWEEN EXACT METHOD AND HEIDEBRECHT AND SMITH METHOD (CONTD.)

Number of storey = 20

Storey level from top	$\eta = 3.0$				$\alpha H = 5.678$		$\eta = 3.5$				$\alpha H = 5.807$	
	Translation (in.)		B.M. on wall (K-in)		Shear on wall (K)		Translation (in.)		B.M. on wall (K-in)		Shear on wall (K)	
	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM
1	2.574	3.605	- 2426	- 5060	-15.45	-40.62	2.515	3.472	- 2314	- 4959	-14.95	-39.85
2	2.462	3.518	- 3109	- 8790	0.10	-30.11	2.404	3.390	- 2899	- 8593	+ 1.32	-29.39
3	2.347	3.425	- 3311	-11492	8.70	-22.04	2.289	3.302	-3028	-11219	9.14	-21.42
4	2.228	3.321	-3064	-13388	17.58	-15.76	2.171	3.205	- 2728	-13051	18.10	-15.26
5	2.104	3.205	- 2555	-14632	25.84	-10.76	2.048	3.094	- 2181	-14254	26 .45	-10.40
6	1.975	3.072	- 1841	-15323	34.05	- 6.63	1.920	2.968	- 1440	-14930	34.78	- 6.42
7	1.840	2.922	- 963	-15518	42.19	- 3.04	1.787	2.825	- 539	-15129	43.05	- 2.99
8	1.700	2.754	67	-15232	50.36	0.31	1.649	2.664	507	-14873	51.33	0.19
9	1.556	2.568	1244	-14442	58.58	3.68	1.508	2.485	1698	-14139	59.68	3.39
10	1.408	2.363	2576	-13086	66.92	7.35	1.362	2.289	3039	-12864	68.14	6.87
11	1.256	2.142	4081	-11051	75.41	11.62	1.214	2.076	4545	-10942	76.75	10.94
12	1.103	1.905	5792	- 8174	84.12	16.83	1.065	1.848	6246	- 8209	85.56	15.94
13	0.948	1.655	7770	- 4220	93.16	23.40	0.914	1.607	8192	- 4433	94.68	22.29
14	0.793	1.396	10122	1131	102.67	31.88	0.764	1.357	10478	707	104.23	30.53
15	0.641	1.131	13044	8313	112.93	42.92	0.617	1.101	13278	7647	114.45	41.36
16	0.493	0.869	16888	17909	124.38	57.48	0.474	0.846	16915	16977	125.78	55.71
17	0.353	0.616	22301	30699	137.84	76.70	0.339	0.601	22004	29488	138.99	74.78
18	0.224	0.385	30479	47720	154.83	102.13	0.215	0.376	29714	46243	155.58	100.20
19	0.114	0.190	43604	70352	177.92	135.85	0.110	0.186	42254	68663	178.21	134.13
20	0.033	0.053	65960	100433	213.51	180.60	0.023	0.052	64098	98653	213.55	179.45

COMPARISON BETWEEN EXACT METHOD AND HEIDEBRECHT AND SMITH METHOD (CONTD.)

Number of Storey = 20

Storey level from top	η = 4.0				αH = 5.913		η = 4.5				αH = 6.002	
	Translation (in.)		B.M. on wall (K-in)		Shear on wall (K)		Translation (in.)		B.M. on wall (K-in)		Shear on wall (K)	
	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM
1	2.469	3.368	-2225	- 4870	- 14.53	-39.23	2.431	3.284	- 2151	- 4816	-14.15	-38.75
2	2.357	3.290	- 2728	- 8430	+ 1.67	-28.85	2.319	3.210	- 2585	- 8306	1.99	-28.37
3	2.243	3.207	- 2799	-10990	9.55	-20.92	2.206	3.129	- 2608	-10809	9.92	-20.52
4	2.125	3.113	- 2456	-12780	18.58	-14.86	2.088	3.040	- 2230	-12556	19.02	-14.53
5	2.003	3.007	- 1880	-13947	27.02	-10.10	1.966	2.937	- 1629	-13699	27.55	- 9.88
6	1.876	2.886	-1116	-14605	35.45	- 6.25	1.840	2.820	- 848	-14342	36.07	- 6.11
7	1.744	2.748	- 199	-14810	43.83	- 2.95	1.710	2.686	85	-14547	44.55	- 2.91
8	1.609	2.593	863	-14576	52.23	0.10	1.575	2.536	1158	-14332	53.06	0.04
9	1.469	2.420	2065	-13886	60.70	3.16	1.437	2.368	2371	-13675	61.64	2.98
10	1.326	2.231	3415	-12677	69.27	6.50	1.296	2.183	3730	-12518	70.31	6.21
11	1.181	2.024	4926	-10844	77.98	10.41	1.153	1.982	5247	-10756	79.13	9.99
12	1.034	1.803	6622	- 8225	86.90	15.24	1.009	1.767	6942	- 8229	88.14	14.67
13	0.887	1.567	8549	- 4589	96.09	21.41	0.865	1.538	8857	- 4707	97.41	20.69
14	0.741	1.326	10789	384	105.68	29.46	0.722	1.300	11065	129	107.05	28.59
15	0.598	1.077	13499	7131	115.90	40.11	0.582	1.057	13706	6718	117.28	39.08
16	0.459	0.828	16978	16246	127.14	54.28	0.446	0.814	17059	15658	128.46	53.12
17	0.328	0.589	21808	28532	140.15	73.24	0.319	0.579	21679	27758	141.30	71.97
18	0.209	0.369	29132	45071	156.40	98.64	0.203	0.363	28678	44118	157.36	97.35
19	0.106	0.183	41174	67318	178.61	132.73	0.103	0.180	40287	66222	179.07	131.57
20	0.031	0.051	62562	97234	213.64	178.51	0.030	0.506	61263	96074	213.76	177.72

COMPARISON BETWEEN EXACT METHOD AND HEIDEBRECHT AND SMITH METHOD (CONTD.)

Number of storey = 20

Storey level from top	$\eta = 5.0$		$\alpha H = 6.077$				$\eta = 6.0$		$\alpha H = 6.198$			
	Translation (in.)		B.M. on wall (K-in)		Shear on wall (K)		Translation (in.)		B.M. on wall (K-in)		Shear on wall (K)	
	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM
1	2.398	3.216	- 2089	- 4740	-13.82	-38.38	2.746	3.110	- 1988	- 4652	-13.25	-37.74
2	2.287	3.144	- 2463	- 8186	2.27	-28.00	2.235	3.042	- 2260	- 8016	+ 2.79	-27.38
3	2.174	3.066	- 2444	-10652	10.26	-20.19	2.122	2.968	- 2175	-10410	10.87	-19.66
4	2.056	2.979	- 2037	-12365	19.43	-14.27	2.005	2.885	- 1720	-12066	20.18	-13.85
5	1.935	2.879	- 1415	-13800	28.04	- 9.69	1.885	2.790	- 1068	-13156	28.94	- 9.38
6	1.810	2.765	- 608	-14120	36.66	- 6.01	1.761	2.681	- 246	-13769	37.72	- 5.82
7	1.680	2.635	326	-14328	45. 23	- 2.88	1.633	2.556	719	-13978	46.47	- 2.82
8	1.547	2.488	1410	-14125	53.84	- 0.02	1.501	2.414	1821	-13797	55.26	- 0.09
9	1.410	2.324	2633	-13497	62.51	+ 2.84	1.367	2.257	3061	-13211	64.11	+ 2.62
10	1.271	2.144	4000	-12382	71.29	5.96	1.230	2.083	4443	-12162	73.05	5.59
11	1.130	1.948	5522	-10678	80.20	9.64	1.092	1.893	5977	-10547	82.14	9.10
12	0.988	1.737	7219	- 8226	89.29	14.21	0.953	1.689	7680	- 8213	91.40	13.49
13	0.846	1.513	9126	- 4798	98.64	20.10	0.815	1.473	9580	- 4931	100.89	19.19
14	0.705	1.280	11310	- 76	108.34	27.87	0.679	1.247	11732	- 385	110.70	26.75
15	0.568	1.041	13898	6382	118.58	38.23	0.546	1.015	14245	5865	121.00	36.89
16	0.436	0.802	17152	15174	129.72	52.14	0.418	0.783	17345	14425	132.08	50.61
17	0.311	0.571	21594	27119	142.43	70.91	0.298	0.558	21511	26124	144.59	69.23
18	0.198	0.358	28314	43328	158.13	96.21	0.190	0.351	27774	42094	159.86	94.55
19	0.101	0.178	39540	65310	179.57	130.59	0.097	0.175	38352	63881	180.64	129.03
20	0.030	0.050	60142	95110	213.91	177.06	0.028	0.049	58283	93596	214.26	176.00

COMPARISON BETWEEN EXACT METHOD AND HEIDEBRECHT AND SMITH METHOD (CONTD.)

Number of storey = 20

Storey level from top	$\eta = 7.0$		$\alpha H = 6.292$				$\eta = 8.0$		$\alpha H = 6.366$			
	Translation (in.)		B.M. on wall (K-in)		Shear on wall (K)		Translation (in.)		B.M. on wall (K-in)		Shear on wall (K)	
	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM	EM	HSM
1	2.305	3.032	- 1908	- 4583	-12.77	-37.26	2.272	2.972	- 1843	- 4530	-12.37	-36.87
2	2.195	2.966	- 2099	- 7880	+ 3.24	-26.93	2.162	2.908	- 1965	- 7778	+ 3.64	-26.55
3	2.082	2.900	- 1960	-10220	11.41	-19.26	2.049	2.840	- 1783	-10078	11.89	-18.93
4	1.966	2.816	- 1470	-11842	20.84	-13.54	1.933	2.763	- 1263	-11663	21.43	-13.27
5	1.846	2.724	- 791	-12899	29.74	- 9.15	1.814	2.673	- 564	-12704	30.45	- 8.96
6	1.722	2.618	49	-13499	38.67	- 5.69	1.691	2.587	292	-13296	39.51	- 5.56
7	1.596	2.497	1030	-13710	47.57	- 2.78	1.566	2.452	1287	-13507	48.55	- 2.73
8	1.466	2.360	2147	-13545	56.51	- 0.15	1.437	2.318	2416	-13353	57.63	- 0.19
9	1.333	2.207	3401	-12991	65.52	+ 2.46	1.306	2.168	3682	-12818	66.78	+ 2.34
10	1.198	2.038	4796	-11990	74.62	5.32	1.173	2.003	5089	-11853	76.02	5.12
11	1.063	1.853	6342	-10442	83.86	8.71	1.039	1.822	6645	-10357	85.40	8.41
12	0.926	1.654	8052	- 8193	93.26	12.96	0.904	1.627	8363	- 8175	94.93	12.56
13	0.791	1.443	9952	- 5021	102.89	18.51	0.771	1.420	10264	- 5086	104.68	17.99
14	0.658	1.223	12086	- 607	112.81	25.91	0.641	1.204	12388	- 775	114.71	25.27
15	0.528	0.996	14549	5488	123.18	35.90	0.514	0.981	14816	+ 5201	125.14	35.12
16	0.404	0.769	17540	13874	134.25	49.46	0.393	0.758	17726	13450	136.22	48.57
17	0.288	0.549	21496	25386	146.61	67.96	0.280	0.541	21519	24817	148.48	66.97
18	0.163	0.345	27397	41174	161.53	93.25	0.178	0.341	27125	40461	163.12	92.22
19	0.093	0.173	37437	62818	181.74	127.83	0.091	0.170	36707	61982	182.82	126.89
20	0.028	0.050	56783	92461	214.64	175.18	0.027	0.048	55530	91579	215.02	174.53

CHAPTER 8

BUILDING SWAY

8.1 Introduction

It is exceedingly difficult to formulate the simple answer to the question "How much can we allow a building to sway in the wind ?".

The engineer in his design, establishes the nature of the wind that is the environment of the particular building in question. For a very tall building the engineer would like the structure of the wind to be described in relatively great detail. The engineer must also determine the response of the structure to the turbulent wind environment.

Great steps forward are being made both in describing the nature of the wind and the effects of that wind on a structural system. Deflection and particularly total deflection or sway has little value in an engineering sense. It does not really matter if a building sways 1 inch, 10 inches or 10 feet so long its performance is satisfactory. The three things that do matter are the integrity of the structural system, the integrity of the architectural finishes and the comfort of the building occupants.

8.2 Sway Control by Using Heavier Crown Coupling Beams

For the satisfactory performance of the tall buildings, it is customery for the engineers to ~~to~~ reduce the sway of the buildings as much as possible without any substantial increase in cost of the structure. One economic way of reducing the sway is by using heavier crown coupling beam. In this chapter the role of heavier crown coupling beam in reducing the sway of building is investigated.

8.2.1 Description of Structures

General one Bay frame program was run for two different twenty storied frames to evaluate the percentage of sway reduction by using heavier crown coupling Beam. Vertical members of one of the frames are shear walls while the vertical number of the other frame are columns. For each frame two sets of data are used. Sectional properties of frame elements for both sets of data were same except the Beam of the top storey. Lateral load is assumed to 1^k/ft height. The sections and length of the frame elements as used in the program are as follows:

8.2.2 Frame with Shear Wall

Set:-1 For all stories

Left vertical member = 12"x30'-0"

Right vertical member = 12"x10'-0"

Beam section = 12"x2'-0"

Beam length = 10'-0"   Storey height = 10'-0"

Set:2

For all stories:- Same as above except the x-section of the top storey beam, which is = 1'x4"-0".

The maximum sway reduction occured for this structure is 2.35%. The results of sway (translation) is given in Tabular form.

8.2.3 Frame without Shear Wall

Set:1 For all Stories

Left and right vertical members = 30"x40"

Moment of inertia of beam = 32000 in^4

Beam length = 24'-0" Storey height = 12'-0"

Set:2

For all stories: Same as above except the moment of inertia of top storey beam, which is = 64000 in^4.

The maximum sway reduction occured for this structure is 0.50%. The results of sway (translation) is given in Tabular form.

PERCENTAGE OF SWAY REDUCTION WITH HEAVIER CROWN COUPLING BEAM

Number of storey = 20

Storey level from top	Frame with shear wall			Frame with column		
	Translation (in.)		Percent of sway Reduction	Translation (in.)		Percent of of sway Reduction
	Data Set-1	Data Set-2		Data Set-1	Data Set-2	
1	0.8772	0.8565	2.353	16.065	15.9865	0.487
2	0.8279	0.8112	2.020	15.6684	15.6318	0.234
3	0.7782	0.7646	1.737	15.2321	15.2149	0.113
4	0.7278	0.7169	1.498	14.7431	14.7351	0.054
5	0.6767	0.6680	1.290	14.1961	14.1924	0.026
6	0.6249	0.6179	1.125	13.5991	13.5873	0.013
7	0.5724	0.5668	0.982	12.9217	12.9209	0.006
8	0.5194	0.5150	0.851	12.1947	12.1943	0.003
9	0.4661	0.4626	0.755	11.4095	11.4093	0.002
10	0.4127	0.4100	0.666	10.5676	10.5675	0.001
11	0.3597	0.3575	0.589	9.6714	9.6711	0.001
12	0.3074	0.3058	0.524	8.7228	8.7228	0.000
13	0.2565	0.2553	0.468	7.7255	7.7255	0.000
14	0.2075	0.2066	0.424	6.6832	6.6832	0.000
15	0.1612	0.1606	0.385	5.6017	5.6017	0.000
16	0.1185	0.1181	0.355	4.4899	4.4899	0.000
17	0.0803	0.0801	0.324	3.3636	3.3636	0.000
18	0.0479	0.0477	0.292	2.2525	2.2525	0.000
19	0.0226	0.0225	0.266	1.2165	1.2165	0.000
20	0.0060	0.0060	0.000	0.3792	0.3792	0.000

CHAPTER 9

LOAD TRANSFER FROM A VERTICALLY LOADED COLUMN

9.1 One Bay Frame

Several programs were run for a one-bay frame to find out the percentage of load transfer from a vertically loaded column to another column. The ratio of V_b/V_c varies from 0.0 to 1.0. V_b is the shear stiffness of the beam and is equal to $12EI/L^3$ (assuming equivalent shear stiffness of beam $C_v = 1.0$) and V_c is the axial stiffness of column and is equal to AE/H.

For V_b/V_c from 0.1 to 1.0, the maximum percentage of load transfer is 1.0204%, which takes place within the top three stories. The results are given in Tabular form.

PERCENTAGE OF LOAD TRANSFER FROM ONE VERTICALLY LOADED COLUMN TO OTHER COLUMN

$V_b = 12EI_b/L^3$ = Shear stiffness of the beam

$V_c = AE/H$ = Axial stiffness of column

PCL = Percentage of load transfer.

Storey level from top	$V_b/V_c = .1$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	PCL	PCL	PCL	PCL	PCL	PCL	PCL	PCL	PCL	PCL
1	0.9333	0.9731	0.9879	0.9956	1.0004	1.0036	1.0060	1.0077	1.0091	1.0102
2	1.0130	1.0182	1.0194	1.0198	1.0200	1.0201	1.0202	1.0203	1.0203	1.0203
3	1.0198	1.0203	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
4	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
5	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
6	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
7	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
8	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
9	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
10	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
11	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
12	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
13	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
14	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
15	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
16	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
17	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
18	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
19	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204
20	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204	1.0204

CHAPTER 10

CONCLUSIONS AND SUGGESTIONS

10.1 Conclusion

From the comparison of exact analysis with that of approximate analysis of a frame structure the following conclusions may be drawn.

1. Calculation of GA parameter of one bay frame by Heidebrecht and Smith method (HSM) shall be satisfactory provided for different value of λ , the storey number is limited to the following:-

For $\lambda = 2$ storey number = 6

For $\lambda = 3$ Storey number = 7

For $\lambda = 4$ Storey number = 8

For $\lambda = 5$ Storey number = 9

For $\lambda = 7$ Storey number = 10

For $\lambda = 9$ Storey number = 11

For $\lambda = 10$ Storey number = 12

2. Calculation of GA parameter of two bay frame by HSM shall be satisfactory provided for different value of λ the storey number is limited to the following:-

For $\lambda = 2$ Storey number = 10

For $\lambda = 3$ Storey number = 12

For $\lambda = 4$ Storey number = 13

For $\lambda = 5$ Storey number = 14

For $\lambda = 6$ Storey number = 16

3. For a one bay symmetrical frame there is a point of inflection at the center of each beam and the total horizontal shear on each storey is equally divided between the columns of that storey. Point of inflection at columns do not fall at the center of each column in each storey. In the upper stories the point of inflection is nearer to the bottom of column and in the lower stories it is nearer to the top of column. As the value of λ increases the point of inflection shifts towards the bottom of column in the upper stories and towards the top of column in the lower stories. For $\lambda > 4.0$ certain upper storey/stories or certain upper and lower stories may not have any point of inflection at all.

4. For a two bay symmetrical frame there is a point of inflection at the center of each beam. If the moment of in-rtia of interior column is double the moment of inertia of exterior column, then the total horizontal shear on each storey is divided between the columns of that storey in such a manner that the shear at interior column is double the shear at exterior column. The behaviour of point of inflections at columns are similar to that of one bay frame. Point of inflection for both interior and exterior columns at a storey fall on same level.

5. For a two bay symmetrical frame if the moment of inertia of all the columns are same, the point of inflection at exterior and interior columns on a storey do not fall on same level. Except at the top and bottom storey, the ratio of shear at interior column to shear at exterior column varies from 1.6 to 2.4 depending on λ. The ratio increases with the increase of λ. For $\lambda \geq 20$, the shear

at exterior columns of top storey acts in the opposite direction to the shear at interior column.

For the most two bay frame structures with interconnected shear wall, the analysis by Heidebrecht and Smith method is quite conservative in relation to Sway of the structure. Except for the stiffness of "link beam" is zero, the deflection by HSM is nearly 50% more than the exact value. The B.M. and shear force in the middle stories differs considerably from the exact analysis. Shears at top and bottom stories are within the range of 20% depending on the value of η and αH.

10.2 Suggestions for Future Study

The following approximate analysis of tall shear wall-frame structures may be compared with "exact" analysis to draw a conclusion as to which method of approximate analysis is best suited for a particular structure.

1. Khan and Sbarounia[1] method
2. McLeod[2] method

Programs may be developed for multibay frames and validity of the assumptions of portal and cantilever methods may be verified for the following:

1. Unsymmetrical one bay frames
2. Unsymmetrical two bay frames
3. Multibay frames.

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APPENDIX 'A'

LISTING OF COMPUTER PROGRAMS

1. One bay frame analysis
2. General one bay frame analysis
3. Two bay frame analysis
4. Gneral two bay frame analysis
5. GA for one bay frame
6. GA for two bay frame
7. Approximate analysis of two bay shear-wall frame by Heidebrecht and Smith method.

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// JOB UEC91664   ONE BAY FRAME ANALYSIS, M.A.MANNAN  DEC.1977
// OPTION LINK,LIST,LOG
// EXEC FFORTRAN
C     THIS IS A PROGRAM FOR ONE BAY FRAME ANALYSIS
C     BY STIFFNESS METHOD AND RECURSION PROCEDURE
C     PROGRAM MAY BE USED FOR FRAME OF WALLS OR COLUMNS OR OF BOTH
      DOUBLE PRECISION BO,B,C,A,FM,FRED,DISP,D,E,FI,BI,U,FS,P,Q,XP,FER,
     1XI1,XI2,XI3,A1,A2,XL,H,XL2,XL3,X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,
     2X12,X13,X14,X15,X16,TL,TR,DEL,XML,XMR,VM,VL,VR,TT,TB,X20,XMT,XMB,
     3VT,VB,AXIAL,EM
      DIMENSION BO(5,5),B(5,5),C(5,5),A(5,5),FM(5,1),FRED(5,1),DISP(5,1)
     1,D(5,5),E(5,5),FI(5,1),BI(95,5),U(105),P(100),Q(80),FS(95)
     2,XP(5),FER(4)
      WRITE (3,111)
  111 FORMAT (6X66HONE BAY FRAME ANALYSIS BY STIFFNESS MATRIX AND RECURS,
     1ION PROCEDURE)
  501 READ(1,1,ERR=99,END=99)N,NSLOAD,XI1,XI2,XI3,A1,A2,EM,XL,H,XL2,XL3
      WRITE (3,101)
  101 FORMAT (//6X17HBEGINNING OF DATA)
C     IF XL2=0. LEFT WALL IS A COLUMN. IF XL3=0. RIGHT WALL IS A COLUMN.
      WRITE (3,1) N,NSLOAD,XI1,XI2,XI3,A1,A2,EM,XL,H,XL2,XL3
    1 FORMAT (2I4,3F10.0,7F6.0)
      N1=5*(N+1)
      N2=5*N
      N3=4*N
      DO 4 I=1,N1
    4 U(I)=0.
      DO 5 I=1,N2
    5 P(I)=0.
      DO 6 I=1,N3
    6 Q(I)=0.
C     NSLOAD= NUMBER OF STORY LOADED. LS=LOADED STORY LEVEL FROM TOP
C     XP(K)=LOAD AT EACH STORY   FER(J)=FIXED END REACTION ON STORY BEAM
C     ORDER OF FER(J)=LEFT SHEAR,LEFT F.E.M.,RIGHT SHEAR,RIGHT F.E.M.
      DO 7 I=1,NSLOAD
      READ (1,3) LS,(XP(K),K=1,5),(FER(J),J=1,4)
      WRITE (3,3) LS,(XP(K),K=1,5),(FER(J),J=1,4)
      DO 8 K1=1,5
      I2=5*(LS-1)+K1
    8 P(I2)=XP(K1)
      DO 13 J1=1,4
      I2=4*(LS-1)+J1
   13 Q(I2)=FER(J1)
    7 CONTINUE
    3 FORMAT(I3,5F8.2,4F8.2)
      DO 50 J=1,5
      DO 50 I=1,5
      BO(I,J)=0.
      B(I,J)=0.
   50 C(I,J)=0.
      X1=12.*XI1/H**3
      X2=12.*XI2/H**3
      X3=6.*XI1/H**2
      X4=6.*XI2/H**2
      X5=4.*XI1/H
      X6=4.*XI2/H
```

```
      X7=A1/H
      X8=A2/H
      X9=12.*XI3/XL**3
      X10=6.*XI3/XL**2
      X11=4.*XI3/XL
      X12=1.+2.*XL2/XL
      X13=1.+2.*XL3/XL
      X14=1.+3.*XL2/XL*(1.+XL2/XL)
      X15=1.+3.*XL3/XL*(1.+XL3/XL)
      X16=1.+3./XL*(XL2+XL3+2.*XL2*XL3/XL)
      BO(1,1)=X1+X2
      BO(3,1)=X3
      BO(5,1)=X4
      BO(2,2)=X7+X9
      BO(3,2)=X10*X12
      BO(4,2)=-X9
      BO(5,2)=X10*X13
      BO(3,3)=X5+X11*X14
      BO(4,3)=-X10*X12
      BO(5,3)=X11/2.*X16
      BO(4,4)=X8+X9
      BO(5,4)=-X10*X13
      BO(5,5)=X6+X11*X15
      B(1,1)=(X1+X2)*2.
      B(2,2)=2.*X7+X9
      B(3,2)=X10*X12
      B(4,2)=-X9
      B(5,2)=X10*X13
      B(3,3)=2.*X5+X11*X14
      B(4,3)=-X10*X12
      B(5,3)=X11/2.*X16
      B(4,4)=2.*X8+X9
      B(5,4)=-X10*X13
      B(5,5)=2.*X6+X11*X15
      C(1,1)=-(X1+X2)
      C(3,1)=-X3
      C(5,1)=-X4
      C(2,2)=-X7
      C(1,3)=X3
      C(3,3)=X5/2.
      C(4,4)=-X8
      C(1,5)=X4
      C(5,5)=X6/2.
      DO 51 J=1,4
      J1=J+1
      DO 52 I=J1,5
      BO(J,I)=BO(I,J)
   52 B(J,I)=B(I,J)
   51 CONTINUE
      CALL MATTRA (C,A,5,5)
      CALL MATINV (BO,5,D)
      DO 666 J=1,5
      DO 666 I=1,5
  666 BI(I,J)=D(I,J)
      CALL MATMLY (A,D,E,5,5,5)
      DO 9 I=1,5
```

```
      FS(I)=P(I)
    9 FRED(I,1)=P(I)
      DO 53 I=2,N
      I4=5*(I-1)
      DO 54 K=1,5
      IK=I4+K
   54 FI(K,1)=P(IK)
      CALL MATMLY (E,FRED,FM,5,5,1)
      CALL MATSUB (FI,FM,FRED,5,1)
      CALL MATMLY (E,C,D,5,5,5)
      CALL MATSUB (B,D,E,5,5)
      CALL MAT.INV(E,5,D)
      IF (I-N) 10,11,1C
   10 CALL MATMLY (A,D,E,5,5,5)
      DO 541 J=1,5
      IJ=I4+J
  541 FS(IJ)=FRED(J,1)
      DO 888 J=1,5
      DO 888 L=1,5
      IL=I4+L
  888 BI(IL,J)=D(L,J)
   53 CONTINUE
   11 CALL MATMLY (D,FRED,DISP,5,5,1)
      K=5*(N-1)
      DO 14 J=1,5
      KJ=K+J
   14 J(KJ)=DISP(J,1)
      DO 645 I=2,N
      DO 998 J=1,5
      DO 998 K=1,5
      KJ=5*(N-I)+K
  998 D(K,J)=BI(KJ,J)
      DO 646 J=1,5
      JJ=5*(N-I)+J
  646 FI(J,1)=FS(JJ)
      CALL MATMLY (C,DISP,FM,5,5,1)
      CALL MATSUB (FI,FM,FRED,5,1)
      CALL MATMLY (D,FRED,DISP,5,5,1)
      DO 647 K=1,5
      KK=5*(N-I)+K
  647 J(KK)=DISP(K,1)
  645 CONTINUE
      WRITE (3,777)
  777 FORMAT(//1X5HSTORY,9X11HTRANSLATION,8X12HLT VER.DISP.,9X11HLT ROTA
     1TION,8X12HRT VER.DISP.,9X11HRT ROTATION/)
      DO 20 I=1,N
      DO 200 J=1,5
      K=5*(I-1)
      JL=K+J
  200 FRED(J,1)=U(JL)/EM
   20 WRITE (3,12) I,(FRED(J,1),J=1,5)
   12 FORMAT (2X,I4,5D20.10)
C     CALCULATE INTERNAL FORCES ON BEAMS.
      WRITE (3,222)
  222 FORMAT (/6X38HBENDING MOMENT AND SHEAR FORCE ON BEAM)
      WRITE (3,88)
```

```
   88 FORMAT(1X5HSTORY,12X8HB.M.LEFT,11X9HB.M.RIGHT,10X10HSHEAR LEFT,9X1
     11HSHEAR RIGHT)
      DO 21 I=1,N
      K=5*(I-1)
      K1=4*(I-1)
      TL=-U(K+3)
      TR=-U(K+5)
      DEL=U(K+2)-U(K+4)
      XML=Q(K1+2)-X11/2.*(2.*X14*TL+X16*TR-3.*X12*DEL/XL)
      XMR=Q(K1+4)-X11/2.*(X16*TL+2.*X15*TR-3.*X13*DEL/XL)
      VM=(XML+XMR)/(XL+XL2+XL3)
      VL=Q(K1+1)+VM
      VR=Q(K1+3)-VM
   21 WRITE (3,22) I,XML,XMR,VL,VR
   22 FORMAT (2X,I4,4D20.10)
C     CALCULATE INTERNAL FORCES ON LEFT COLUMNS
      WRITE (3,333)
  333 FORMAT (/6X51HBENDING MOMENT SHEAR AND AXIAL FORCE ON LEFT COLUMN)
      WRITE (3,77)
      DO 23 I=1,N
      K=5*(I-1)
      TT=-U(K+3)
      TB=-U(K+8)
      DEL=U(K+1)-U(K+6)
      X20=TT+TB-3.*DEL/H
      XMT=-X5/2.*(X20+TT)
      XMB=-X5/2.*(X20+TB)
      VT=(XMT+XMB)/H
      VB=-VT
      AXIAL=X7*(U(K+7)-U(K+2))
   23 WRITE (3,24) I,XMT,XMB,VT,VB,AXIAL
C     CALCULATE INTERNAL FORCES ON RIGHT COLUMNS
      WRITE (3,444)
  444 FORMAT(/6X52HBENDING MOMENT SHEAR AND AXIAL FORCE ON RIGHT COLUMN)
      WRITE (3,77)
   77 FORMAT(1X5HSTORY,13X7HB.M.TOP,10X10HB.M.BOTTOM,11X9HSHEAR TOP,8X12
     1HSHEAR BOTTOM,9X11HAXIAL FORCE)
      DO 25 I=1,N
      K=5*(I-1)
      TT=-U(K+5)
      TB=-U(K+10)
      DEL=U(K+1)-U(K+6)
      X20=TT+TB-3.*DEL/H
      XMT=-X6/2.*(X20+TT)
      XMB=-X6/2.*(X20+TB)
      VT=(XMT+XMB)/H
      VB=-VT
      AXIAL=X8*(U(K+9)-U(K+4))
   25 WRITE (3,24) I,XMT,XMB,VT,VB,AXIAL
   24 FORMAT (2X,I4,5D20.10)
      GO TO 501
   99 STOP
      END
      SUBROUTINE MATINV (D,N,A)
      DOUBLE PRECISION D,A,PIVOT,T
      DIMENSION A(N,N),IPVOT(7),INDEX(7,2),PIVOT(7),D(N,N)
```

```
      DO 10 I=1,N
      DO 10 J=1,N
   10 A(I,J)=D(I,J)
      DO 17 J=1,N
   17 IPVOT(J)=0
      DO 135 I=1,N
      T=0.
      DO 9 J=1,N
      IF(IPVOT(J)-1) 13,9,13
   13 DO 23 K=1,N
      IF(IPVOT(K)-1) 43,23,81
   43 IF (DABS(T)-DABS(A(J,K))) 83,23,23
   83 IROW=J
      ICOL=K
      T=A(J,K)
   23 CONTINUE
    9 CONTINUE
      IPVOT(ICOL)=IPVOT(ICOL)+1
      IF(IROW-ICOL) 73,109,73
   73 DO 12 L=1,N
      T=A(IROW,L)
      A(IROW,L)=A(ICOL,L)
   12 A(ICOL,L)=T
  109 INDEX(I,1)=IROW
      INDEX(I,2)=ICOL
      PIVOT(I)=A(ICOL,ICOL)
      A(ICOL,ICOL)=1.
      DO 205 L=1,N
  205 A(ICOL,L)=A(ICOL,L)/PIVOT(I)
  347 DO 135 LI=1,N
      IF(LI-ICOL) 21,135,21
   21 T=A(LI,ICOL)
      A(LI,ICOL)=0.
      DO 89 L=1,N
   89 A(LI,L)=A(LI,L)-A(ICOL,L)*T
  135 CONTINUE
  222 DO 3 I=1,N
      L=N-I+1
      IF(INDEX(L,1)-INDEX(L,2)) 19,3,19
   19 JROW=INDEX(L,1)
      JCOL=INDEX(L,2)
      DO 549 K=1,N
      T=A(K,JROW)
      A(K,JROW)=A(K,JCOL)
      A(K,JCOL)=T
  549 CONTINUE
    3 CONTINUE
   81 RETURN
      END
      SUBROUTINE MATTRA(A,B,M,N)
C     MATRIX TRANSPOSE A TO B
      DOUBLE PRECISION A,B
      DIMENSION A(5,5),B(5,5)
      DO 3 J=1,N
      DO 3 I=1,M
    3 B(J,I)=A(I,J)
```

```
      RETURN
      END
      SUBROUTINE MATSUB(A,B,C,M,N)
C     MATRIX SUBTRACTION A-B=C
      DOUBLE PRECISION A,B,C
      DIMENSION A(5,5),B(5,5),C(5,5)
      DO 3 J=1,N
      DO 3 I=1,M
    3 C(I,J)=A(I,J)-B(I,J)
      RETURN
      END
      SUBROUTINE MATMLY(A,B,C,M,L,N)
C     MATRIX MULTIPLICATION A*B=C   A=M BY L   B=L BY N
      DOUBLE PRECISION A,B,C
      DIMENSION A(5,5),B(5,5),C(5,5)
      DO 3 J=1,N
      DO 3 I=1,M
      C(I,J)=0.0
      DO 3 K=1,L
    3 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
/*
// EXEC LNKEDT
// EXEC
/*
/&
```

```
// JOB UEC91664   M.A.MANNAN        GENERAL ONE BAY FRAME  JAN 1978
// OPTION LINK,LIST,LOG
// ASSGN SYS001,X'192'
// EXEC FFORTRAN
C     THIS IS A PROGRAM FOR GENERAL ONE BAY FRAME ANALYSIS
C     BY STIFFNESS METHOD AND RECURSION PROCEDURE
C     PROGRAM MAY BE USED FOR FRAME OF WALLS OR OF COLUMNS OR OF BOTH
C     PROGRAM CAN TAKE CARE OF VARIABLE HEIGHT,SPAN,MOMENT OF INERTIA.
      DOUBLE PRECISION S,B,C,D,FM,FR,FI,U,FS,Z1,Z2,Z3,Z4,Z5,TL,TR,DEL,XM
     1L,XMR,VM,VL,VR,XMLR,XP,FER,P,Q,H,AL,AR,CL,CR,X,BL,S2,S3,E
      DIMENSION S(5,5),B(5,5),C(5,5),D(5,5),FM(5,1),FR(5,1),FI(5,1),XP(5
     1),FER(4),U(105),P(100),Q(80),FS(95),H(20),AL(20),AR(20),CL(20),CR(
     220),X(20),BL(20),S2(20),S3(20)
      DEFINE FILE4 (140,40,L,ID)
      WRITE (3,1)
    1 FORMAT (1X30HGENERAL ONE BAY FRAME ANALYSIS)
C     N=NUMBER OF STORY,NSL=NUMBER OF LOADED STORY,E=MODU. OF ELASTICITY
    2 READ (1,4,ERR=51,END=51) N,NSL,E
      ID=1
      FIND (4'ID)
      WRITE (3,3) N,NSL,E
    3 FORMAT(/1X10HNO.OF STY=,I3,9X17HNO.OF STY LOADED=,I3,5X2HE=,F6.0/)
    4 FORMAT (2I3,F6.0)
      NN=5*(N+1)
      DO 5 I=1,NN
    5 U(I)=0.
      NN=5*N
      DO 6 I=1,NN
    6 P(I)=0.
      NN=4*N
      DO 7 I=1,NN
    7 Q(I)=0.
      DO 12 J=1,5
      DO 12 I=1,5
      S(I,J)=0.
   12 C(I,J)=0.
      WRITE (3,102)
  102 FORMAT(/6X10HMOI LT COL,5X10HMOI RT COL,7X8HMOI BEAM,4X11HAREA LT
     1COL,4X11HAREA RT COL,4X11HBEAM LENGTH,9X6HHEIGHT,7X10HWALL DEPTH)
C     H=STORY HEIGHT, AL=AREA OF LEFT COLUMN, AR=AREA OF RIGHT COLUMN
C     CL=MOMENT OF INERTIA OF LT COL., CR=MOMENT OF INERTIA OF RT COLUMN
C     X=BEAM LENGTH, BL=MOMENT OF INERTIA OF BEAM, S2 AND S3 WALL DEPTH
      DO 15 I=1,N
      READ (1,13) CL(I),CR(I),BL(I),AL(I),AR(I),X(I),H(I),S2(I),S3(I)
   15 WRITE (3,14) CL(I),CR(I),BL(I),AL(I),AR(I),X(I),H(I),S2(I),S3(I)
   13 FORMAT (3F10.0,6F7.0)
   14 FORMAT (1X,3(5X,F10.0),4(8X,F7.0),2(5X,F7.0))
      WRITE (3,100)
  100 FORMAT(//1X5HSTORY,7X7HLOAD P1,7X7HLOAD P2,7X7HLOAD P3,7X7HLOAD P4
     1,7X7HLOAD P5,8X6HFES LT,8X6HFEM LT,8X6HFES RT,8X6HFEM RT)
C      LS=LOADED STORY LEVEL FROM TOP
C     XP(K)=STORY LOAD,FER(J)=FIXED END REACTION ON STORY BEAM
C     ORDER OF FER(J)=LT FESHEAR,LT FEM,RT FES,RT FEM.
      DO 11 I=1,NSL
      READ (1,8) LS,(XP(K),K=1,5),(FER(J),J=1,4)
      WRITE (3,17) LS,(XP(K),K=1,5),(FER(J),J=1,4)
```

```
      DO 9 K=1,5
      NN=5*(LS-1)+K
    9 P(NN)=XP(K)
      DO 10 J=1,4
      NN=4*(LS-1)+J
   10 Q(NN)=FER(J)
   11 CONTINUE
    8 FORMAT (I3,9F8.1)
   17 FORMAT (I6,9(4X,F10.3))
      Z1=1.+2.*S2(1)/X(1)
      Z2=1.+2.*S3(1)/X(1)
      Z3=1.+3.*S2(1)/X(1)*(1.+S2(1)/X(1))
      Z4=1.+3.*S3(1)/X(1)*(1.+S3(1)/X(1))
      Z5=1.+3./X(1)*(S2(1)+S3(1)+2.*S2(1)*S3(1)/X(1))
      S(1,1)=12.*(CL(1)+CR(1))/H(1)**3
      S(3,1)=6.*CL(1)/H(1)**2
      S(5,1)=6.*CR(1)/H(1)**2
      S(2,2)=AL(1)/H(1)+12.*BL(1)/X(1)**3
      S(3,2)=6.*BL(1)/X(1)**2*Z1
      S(4,2)=-12.*BL(1)/X(1)**3
      S(5,2)=6.*BL(1)/X(1)**2*Z2
      S(3,3)=4.*(CL(1)/H(1)+Z3*BL(1)/X(1))
      S(4,3)=-6.*BL(1)/X(1)**2*Z1
      S(5,3)=2.*BL(1)/X(1)*Z5
      S(4,4)=AR(1)/H(1)+12.*BL(1)/X(1)**3
      S(5,4)=-6.*BL(1)/X(1)**2*Z2
      S(5,5)=4.*(CR(1)/H(1)+BL(1)/X(1)*Z4)
      DO 16 J=1,4
      NN=J+1
      DO 16 I=NN,5
   16 S(J,I)=S(I,J)
      CALL MATINV (S,5,D)
      DO 18 I=1,5
      WRITE (4'ID) (D(I,J),J=1,5)
      FS(I)=P(I)
   18 FR(I,1)=P(I)
      DO 27 I=2,N
      NN=5*(I-1)
      N1=(I-1)
      DO 20 K=1,5
      KK=NN+K
   20 FI(K,1)=P(KK)
      Z1=1.+2.*S2(I)/X(I)
      Z2=1.+2.*S3(I)/X(I)
      Z3=1.+3.*S2(I)/X(I)*(1.+S2(I)/X(I))
      Z4=1.+3.*S3(I)/X(I)*(1.+S3(I)/X(I))
      Z5=1.+3./X(I)*(S2(I)+S3(I)+2.*S2(I)*S3(I)/X(I))
      C(1,1)=-12.*(CL(N1)+CR(N1))/H(N1)**3
      C(3,1)=-6.*CL(N1)/H(N1)**2
      C(5,1)=-6.*CR(N1)/H(N1)**2
      C(2,2)=-AL(N1)/H(N1)
      C(1,3)=6.*CL(N1)/H(N1)**2
      C(3,3)=2.*CL(N1)/H(N1)
      C(4,4)=-AR(N1)/H(N1)
      C(1,5)=6.*CR(N1)/H(N1)**2
      C(5,5)=2.*CR(N1)/H(N1)
```

```
      CALL MATTRA (C,S,5,5)
      CALL MATMLY (S,D,B,5,5,5)
      CALL MATMLY (B,FR,FM,5,5,1)
      CALL MATSUB (FI,FM,FR,5,1)
      CALL MATMLY (B,C,D,5,5,5)
      DO 60 J=1,5
      DO 60 K=1,5
   60 B(K,J)=0.
      B(1,1)=12.*((CL(I)+CR(I))/H(I)**3+(CL(N1)+CR(N1))/H(N1)**3)
      B(3,1)=6.*(CL(I)/H(I)**2-CL(N1)/H(N1)**2)
      B(5,1)=6.*(CR(I)/H(I)**2-CR(N1)/H(N1)**2)
      B(2,2)=AL(N1)/H(N1)+AL(I)/H(I)+12.*BL(I)/X(I)**3
      B(3,2)=6.*BL(I)/X(I)**2*Z1
      B(4,2)=-12.*BL(I)/X(I)**3
      B(5,2)=6.*BL(I)/X(I)**2*Z2
      B(3,3)=4.*(CL(N1)/H(N1)+CL(I)/H(I)+BL(I)/X(I)*Z3)
      B(4,3)=-6.*BL(I)/X(I)**2*Z1
      B(5,3)=2.*BL(I)/X(I)*Z5
      B(4,4)=AR(N1)/H(N1)+AR(I)/H(I)+12.*BL(I)/X(I)**3
      B(5,4)=-6.*BL(I)/X(I)**2*Z2
      B(5,5)=4.*(CR(N1)/H(N1)+CR(I)/H(I)+BL(I)/X(I)*Z4)
      DO 22 J=1,4
      KK=J+1
      DO 22 K=KK,5
   22 B(J,K)=B(K,J)
      CALL MATSUB (B,D,S,5,5)
      CALL MATINV (S,5,D)
      IF (I-N) 24,28,24
   24 DO 25 K=1,5
      WRITE (4'ID) (D(K,J),J=1,5)
      KK=NN+K
   25 FS(KK)=FR(K,1)
   27 CONTINUE
   28 CALL MATMLY (D,FR,FM,5,5,1)
      K=5*(N-1)
      DO 29 J=1,5
      KK=K+J
   29 U(KK)=FM(J,1)
      DO 32 I=2,N
      N1=N-I+1
      C(1,1)=-12.*(CL(N1)+CR(N1))/H(N1)**3
      C(3,1)=-6.*CL(N1)/H(N1)**2
      C(5,1)=-6.*CR(N1)/H(N1)**2
      C(2,2)=-AL(N1)/H(N1)
      C(1,3)=6.*CL(N1)/H(N1)**2
      C(3,3)=2.*CL(N1)/H(N1)
      C(4,4)=-AR(N1)/H(N1)
      C(1,5)=6.*CR(N1)/H(N1)**2
      C(5,5)=2.*CR(N1)/H(N1)
      ID=5*(N-I)+1
      FIND (4'ID)
      DO 31 J=1,5
      READ (4'ID) (D(J,K),K=1,5)
      KK=5*(N-I)+J
   31 FI(J,1)=FS(KK)
      CALL MATMLY (C,FM,FR,5,5,1)
```

```
      CALL MATSUB (FI,FR,FM,5,1)
      DO 33 J=1,5
   33 FR(J,1)=FM(J,1)
      CALL MATMLY (D,FR,FM,5,5,1)
      DO 32 K=1,5
      KK=5*(N-I)+K
   32 U(KK)=FM(K,1)
      WRITE (3,34)
   34 FORMAT(//1X5HSTORY,9X11HTRANSLATION,8X12HLT VER DISP.,9X11HLT ROTA
     1TION,8X12HRT VER DISP.,9X11HRT ROTATION)
      DO 36 I=1,N
      DO 35 J=1,5
      KK=5*(I-1)+J
   35 FR(J,1)=U(KK)/E
   36 WRITE (3,37) I,(FR(J,1),J=1,5)
   37 FORMAT (1X,I5,5D20.10)
C     CALCULATE INTERNAL FORCES ON BEAMS
      WRITE (3,38)
   38 FORMAT (/21X39HBENDING MOMENT AND SHEAR FORCE ON BEAMS)
      WRITE (3,40)
   40 FORMAT(1X5HSTORY,9X9HB.M. LEFT,8X10HB.M. RIGHT,8X10HSHEAR LEFT,7X1
     11HSHEAR RIGHT)
      DO 41 I=1,N
      K=5*(I-1)
      KK=4*(I-1)
      Z1=1.+2.*S2(I)/X(I)
      Z2=1.+2.*S3(I)/X(I)
      Z3=1.+3.*S2(I)/X(I)*(1.+S2(I)/X(I))
      Z4=1.+3.*S3(I)/X(I)*(1.+S3(I)/X(I))
      Z5=1.+3./X(I)*(S2(I)+S3(I)+2.*S2(I)*S3(I)/X(I))
      TL=-U(K+3)
      TR=-U(K+5)
      DEL=U(K+2)-U(K+4)
      XML=Q(KK+2)-2.*BL(I)/X(I)*(2.*Z3*TL+Z5*TR-3.*Z1*DEL/X(I))
      XMR=Q(KK+4)-2.*BL(I)/X(I)*(Z5*TL+2.*Z4*TR-3.*Z2*DEL/X(I))
      VM=(XML+XMR)/(X(I)+S2(I)+S3(I))
      VL=Q(KK+1)+VM
      VR=Q(KK+3)-VM
   41 WRITE (3,42) I,XML,XMR,VL,VR
   42 FORMAT (1X,I5,4D18.10)
C     CALCULATE INTERNAL FORCES ON LEFT COLUMNS
      WRITE (3,43)
   43 FORMAT (/6X51HBENDING MOMENT SHEAR AND AXIAL FORCE ON LEFT COLUMN)
      WRITE (3,48)
      DO 44 I=1,N
      K=5*(I-1)
      Z1=-U(K+3)
      Z2=-U(K+8)
      Z3=U(K+1)-U(K+6)
      Z4=Z1+Z2-3.*Z3/H(I)
      XML=-2.*CL(I)/H(I)*(Z4+Z1)
      XMR=-2.*CL(I)/H(I)*(Z4+Z2)
      VL=(XML+XMR)/H(I)
      VR=-VL
      XMLR=AL(I)/H(I)*(U(K+7)-U(K+2))
   44 WRITE (3,50) I,XML,XMR,VL,VR,XMLR
```

```
C       CALCULATE INTERNAL FORCES ON RIGHT COLUMNS
        WRITE (3,47)
   47   FORMAT(/6X52HBENDING MOMENT SHEAR AND AXIAL FORCE CN RIGHT COLUMN)
        WRITE (3,48)
   48   FORMAT(1X5HSTORY,9X7HB.M.TOP,9X10HB.M.BOTTOM,9X9HSHEAR TCP,6X12HSH
       1EAR BOTTOM,7X11HAXIAL FORCE)
        DO 49 I=1,N
        K=5*(I-1)
        Z1=-U(K+5)
        Z2=-U(K+10)
        Z3=U(K+1)-U(K+6)
        Z4=Z1+Z2-3.*Z3/H(I)
        XML=-2.*CR(I)/H(I)*(Z4+Z1)
        XMR=-2.*CR(I)/H(I)*(Z4+Z2)
        VL=(XML+XMR)/H(I)
        VR=-VL
        XMLR=AR(I)/H(I)*(U(K+9)-U(K+4))
   49   WRITE (3,50) I,XML,XMR,VL,VR,XMLR
   50   FORMAT (1X,I5,5D18.1C)
        GO TO 2
   51   STOP
        END
        SUBROUTINE MATINV (D,N,A)
        DOUBLE PRECISION D,A,PIVOT,T
        DIMENSION A(N,N),IPVOT(7),INDEX(7,2),PIVOT(7),D(N,N)
        DO 10 I=1,N
        DO 10 J=1,N
   10   A(I,J)=D(I,J)
        DO 17 J=1,N
   17   IPVOT(J)=0
        DO 135 I=1,N
        T=0.
        CO 9 J=1,N
        IF(IPVOT(J)-1) 13,9,13
   13   DO 23 K=1,N
        IF(IPVOT(K)-1) 43,23,81
   43   IF (DABS(T)-DABS(A(J,K))) 83,23,23
   83   IROW=J
        ICOL=K
        T=A(J,K)
   23   CONTINUE
    9   CONTINUE
        IPVOT(ICOL)=IPVOT(ICOL)+1
        IF(IROW-ICOL) 73,1C9,73
   73   DO 12 L=1,N
        T=A(IROW,L)
        A(IROW,L)=A(ICOL,L)
   12   A(ICOL,L)=T
  109   INDEX(I,1)=IROW
        INDEX(I,2)=ICOL
        PIVOT(I)=A(ICOL,ICOL)
        A(ICOL,ICOL)=1.
        DO 205 L=1,N
  205   A(ICOL,L)=A(ICOL,L)/PIVOT(I)
  347   DO 135 LI=1,N
        IF(LI-ICOL) 21,135,21
```

```
   21 T=A(LI,ICOL)
      A(LI,ICOL)=0.
      DO 89 L=1,N
   89 A(LI,L)=A(LI,L)-A(ICOL,L)*T
  135 CONTINUE
  222 DO 3 I=1,N
      L=N-I+1
      IF(INDEX(L,1)-INDEX(L,2)) 19,3,19
   19 JROW=INDEX(L,1)
      JCOL=INDEX(L,2)
      DO 549 K=1,N
      T=A(K,JROW)
      A(K,JROW)=A(K,JCOL)
      A(K,JCOL)=T
  549 CONTINUE
    3 CONTINUE
   81 RETURN
      END
      SUBROUTINE MATMLY(A,B,C,M,L,N)
      DOUBLE PRECISION A,B,C
C     MATRIX MULTIPLICATION A*B=C   A=M BY L   B=L BY N
      DIMENSION A(5,5),B(5,5),C(5,5)
      DO 3 J=1,N
      DO 3 I=1,M
      C(I,J)=0.0
      DO 3 K=1,L
    3 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
      SUBROUTINE MATSUB(A,B,C,M,N)
      DOUBLE PRECISION A,B,C
C     MATRIX SUBTRACTION A-B=C
      DIMENSION A(5,5),B(5,5),C(5,5)
      DO 3 J=1,N
      DO 3 I=1,M
    3 C(I,J)=A(I,J)-B(I,J)
      RETURN
      END
      SUBROUTINE MATTRA(A,B,M,N)
      DOUBLE PRECISION A,B
C     MATRIX TRANSPOSE A TO B
      DIMENSION A(5,5),B(5,5)
      DO 3 J=1,N
      DO 3 I=1,M
    3 B(J,I)=A(I,J)
      RETURN
      END
/*
// EXEC LNKEDT
// DLBL UOUT,'IJSYS01'
// EXTENT SYS001,,1,0,40,20
// EXEC CLRDSK
// UCL B=(K=0,D=40),X'CC',DY
// END
// DLBL IJSYS01
// EXTENT SYS001,,1,0,40,20
```

```
//  EXEC
/*
/&
```

```
// JOB UEC91664     TWO BAY FRAME ANALYSIS    M.A.MANNAN  DEC.1977
// OPTION LINK,LIST,LOG
// EXEC FFORTRAN
C        THIS IS A PROGRAM FOR TWO BAY FRAME ANALYSIS
C        BY STIFFNESS METHOD AND RECURSION PROCEDURE
C        PROGRAM MAY BE USED FOR FRAME OF WALLS OR OF COLUMNS OR OF BOTH
         DOUBLE PRECISION BO,B,C,A,FM,FRED,DISP,D,E,FI,BI,U,FS,P,Q,XP,FER,
        1WI1,WI2,WI3,BI1,BI2,WA1,WA2,WA3,XL1,XL2,H,S2,S3,S4,S5,X1,X2,X3,X4,
        2X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18,X19,X20,X21,X22
        3,X23,X24,X25,X26,X27,X28,TL,TR,TRR,DEL,DELR,X30,XML,XMR,VM,VL,VR,
        4XMLR,XMRR,VMR,VLR,VRR,TT,TB,XMT,XMB,VT,VB,AXIAL,EM
         DIMENSION BO(7,7),B(7,7),C(7,7),A(7,7),FM(7,1),FRED(7,1),DISP(7,1)
        1,D(7,7),E(7,7),FI(7,1),BI(133,7),U(147),P(140),Q(160),FS(133)
        2,XP(7),FER(8)
         WRITE (3,111)
  111    FORMAT (6X66HTWO BAY FRAME ANALYSIS BY STIFFNESS MATRIX AND RECURS
        1ION PROCEDURE)
C        N=NO.OF STORY, WI1=MOI LT COL., WI2=MOI CTR COL., WI3=MOI WALL
C        BI1=MOI LT BEAM, BI2=MOI RT BEAM, WA1=AREA LT COL.,
C        WA2=AREA CTR COL., WA3=AREA WALL, FM=MODULUS OF ELASTICITY
C        XL1=SPAN LT BEAM, XL2=SPAN RT BEAM, H=STORY HEIGHT
  501    READ(1,1,ERR=99,END=99)N,NSLOAD,WI1,WI2,WI3,BI1,BI2,WA1,WA2,WA3
         WRITE (3,101)
  101    FORMAT (//6X17HBEGINNING OF DATA)
         WRITE (3,1) N,NSLOAD,WI1,WI2,WI3,BI1,BI2,WA1,WA2,WA3
    1    FORMAT (2I4,3F11.0,2F9.0,3F7.0)
         READ (1,30) EM,XL1,XL2,H,S2,S3,S4,S5
C        IF S2=0.1ST WALL IS COL.,IF S3=S4=0.CENTER WALL IS COL., AND SO ON
         WRITE (3,30) EM,XL1,XL2,H,S2,S3,S4,S5
   30    FORMAT (8F9.2)
         N1=7*(N+1)
         N2=7*N
         N3=8*N
         DO 4 I=1,N1
    4    U(I)=0.
         DO 5 I=1,N2
    5    P(I)=0.
         DO 6 I=1,N3
    6    Q(I)=0.
C        NSLOAD=NUMBER OF STORY LOADED    LS=LOADED STORY LEVEL FROM TOP
C        XP(K)=LOAD AT EACH STORY, FER(J)=FIXED END REACTION ON STORY BEAM
C        ORDER OF FER(J)=LT SHEAR,LT FEM,RT SHEAR,RT FEM.(FOR BOTH SIDES)
         DO 7 I=1,NSLOAD
         READ (1,3) LS,(XP(K),K=1,7)
         WRITE (3,3) LS,(XP(K),K=1,7)
         READ (1,30) (FER(J),J=1,8)
         WRITE (3,30) (FER(J),J=1,8)
         DO 8 K1=1,7
         I2=7*(LS-1)+K1
    8    P(I2)=XP(K1)
         DO 13 J1=1,8
         I2=8*(LS-1)+J1
   13    Q(I2)=FER(J1)
    7    CONTINUE
    3    FORMAT (I4,7F10.2)
         DO 50 J=1,7
```

```
      DO 50 I=1,7
      BO(I,J)=0.
      B(I,J)=0.
   50 C(I,J)=0.
      X1=12.*WI1/H**3
      X2=6.*WI1/H**2
      X3=4.*WI1/H
      X4=WA1/H
      X5=12.*WI2/H**3
      X6=6.*WI2/H**2
      X7=4.*WI2/H
      X8=WA2/H
      X9=12.*WI3/H**3
      X10=6.*WI3/H**2
      X11=4.*WI3/H
      X12=WA3/H
      X13=12.*BI1/XL1**3
      X14=6.*BI1/XL1**2
      X15=4.*BI1/XL1
      X16=12.*BI2/XL2**3
      X17=6.*BI2/XL2**2
      X18=4.*BI2/XL2
      X19=1.+2.*S2/XL1
      X20=1.+2.*S3/XL1
      X21=1.+3.*S2/XL1*(1.+S2/XL1)
      X22=1.+3./XL1*(S2+S3+2.*S2*S3/XL1)
      X23=1.+2.*S4/XL2
      X24=1.+2.*S5/XL2
      X25=1.+3.*S3/XL1*(1.+S3/XL1)
      X26=1.+3.*S4/XL2*(1.+S4/XL2)
      X27=1.+3./XL2*(S4+S5+2.*S4*S5/XL2)
      X28=1.+3.*S5/XL2*(1.+S5/XL2)
      BO(1,1)=X1+X5+X9
      BO(3,1)=X2
      BO(5,1)=X6
      BO(7,1)=X10
      BO(2,2)=X4+X13
      BO(3,2)=X14*X19
      BO(4,2)=-X13
      BO(5,2)=X14*X20
      BO(3,3)=X3+X15*X21
      BO(4,3)=-X14*X19
      BO(5,3)=X15/2.*X22
      BO(4,4)=X8+X13+X16
      BO(5,4)=X17*X23-X14*X20
      BO(6,4)=-X16
      BO(7,4)=X17*X24
      BO(5,5)=X7+X15*X25+X18*X26
      BO(6,5)=-X17*X23
      BO(7,5)=X18/2.*X27
      BO(6,6)=X12+X16
      BO(7,6)=-X17*X24
      BO(7,7)=X11+X18*X28
      C(1,1)=-(X1+X5+X9)
      C(3,1)=-X2
      C(5,1)=-X6
```

```
      C(7, 1)=-X10
      C(2, 2)=-X4
      C(1, 3)=X2
      C(3, 3)=X3/2.
      C(4, 4)=-X8
      C(1, 5)=X6
      C(5, 5)=X7/2.
      C(6, 6)=-X12
      C(1, 7)=X10
      C(7, 7)=X11/2.
      B(1, 1)= 2.*(X1+X5+X9)
      B(2, 2)= 2.*X4+X13
      B(3, 2)=X14*X19
      B(4, 2)=-X13
      B(5, 2)=X14*X20
      B(3, 3)= 2.*X3+X15*X21
      B(4, 3)=-X14*X19
      B(5, 3)=X15/2.*X22
      B(4, 4)= 2.*X8+X13+X16
      B(5, 4)=X17*X23-X14*X20
      B(6, 4)=-X16
      B(7, 4)=X17*X24
      B(5, 5)= 2.*X7+X15*X25+X18*X26
      B(6, 5)=-X17*X23
      B(7, 5)=X18/2.*X27
      B(6, 6)= 2.*X12+X16
      B(7, 6)=-X17*X24
      B(7, 7)= 2.*X11+X18*X28
      DO 51 J= 1, 6
      J1=J+1
      DO 52 I=J1, 7
      BO(J, I)= BO(I, J)
   52 B(J, I)=B(I, J)
   51 CONTINUE
      CALL MATTRA (C, A, 7, 7)
      CALL MATINV (BO, 7, D)
      DO 666 J= 1, 7
      DO 666 I= 1, 7
  666 BI(I, J)=D(I, J)
      CALL MATMLY (A, D, E, 7, 7, 7)
      DO 9 I= 1, 7
      FS(I)=P(I)
    9 FRED(I, 1)=P(I)
      DO 53 I= 2, N
      I4= 7*(I-1)
      DO 54 K= 1, 7
      IK= I4+K
   54 FI(K, 1)=P(IK)
      CALL MATMLY (E, FRED, FM, 7, 7, 1)
      CALL MATSUB (FI, FM, FRED, 7, 1)
      CALL MATMLY (E, C, D, 7, 7, 7)
      CALL MATSUB (B, D, E, 7. 7)
      CALL MATINV (E, 7, D)
      IF (I-N) 10, 11, 10
   10 CALL MATMLY (A, D, E, 7, 7, 7)
      DO 541 J= 1, 7
```

```
      IJ= I4+J
  541 FS(IJ)=FRED(J,1)
      DO 888 J=1,7
      DO 888 L=1,7
      IL=I4+L
  888 BI(IL,J)=D(L,J)
   53 CONTINUE
   11 CALL MATMLY (D,FRED,DISP,7,7,1)
      K=7*(N-1)
      DO 14 J=1,7
      KJ=K+J
   14 U(KJ)=DISP(J,1)
      DO 645 I=2,N
      DO 998 J=1,7
      DO 998 K=1,7
      KJ=7*(N-I)+K
  998 D(K,J)=BI(KJ,J)
      DO 646 J=1,7
      JJ=7*(N-I)+J
  646 FI(J,1)=FS(JJ)
      CALL MATMLY (C,DISP,FM,7,7,1)
      CALL MATSUB (FI,FM,FRED,7,1)
      CALL MATMLY (D,FRED,DISP,7,7,1)
      DO 647 K=1,7
      KK=7*(N-I)+K
  647 J(KK)=DISP(K,1)
  645 CONTINUE
      WRITE (3,777)
  777 FORMAT (//1X5HSTORY,6X11HTRANSLATION,6X11HLT VER DISP,6X11HLT ROTA
     1TION,5X12HCTR VER DISP,5X12HCTR ROTATION,6X11HRT VER DISP,6X11HRT
     2ROTATION/)
      DO 20 I=1,N
      DO 200 J=1,7
      K=7*(I-1)
      JL=K+J
  200 FRED(J,1)=U(JL)/EM
   20 WRITE (3,12) I,(FRED(J,1),J=1,7)
   12 FORMAT (2X,I4,7D17.10)
C     CALCULATE INTERNAL FORCES ON BEAMS
      WRITE (3,222)
  222 FORMAT (/43X39HBENDING MOMENT AND SHEAR FORCE ON BEAMS)
      WRITE (3,555)
  555 FORMAT (26X15HLEFT SIDE BEAMS,44X16HRIGHT SIDE BEAMS)
      WRITE (3,88)
   88 FORMAT(1H ,5HSTORY,6X9HB.M. LEFT,5X10HB.M. RIGHT,5X10HSHEAR LEFT,4
     1X11HSHEAR RIGHT,6X9HB.M. LEFT,5X10HB.M. RIGHT,5X10HSHEAR LEFT,4X11
     2HSHEAR RIGHT/)
      DO 21 I=1,N
      K=7*(I-1)
      K1=8*(I-1)
      TL=-U(K+3)
      TR=-U(K+5)
      TRR=-U(K+7)
      DEL=U(K+2)-U(K+4)
      DELR=U(K+4)-U(K+6)
      XML=Q(K1+2)-X15/2.*(2.*X21*TL+X22*TR-3.*X19*DEL/XL1)
```

```
      XMR=Q(K1+4)-X15/2.*(X22*TL+2.*X25*TR-3.*X20*DEL/XL1)
      VM=(XML+XMR)/(XL1+S2+S3)
      VL=Q(K1+1)+VM
      VR=Q(K1+3)-VM
      XMLR=Q(K1+6)-X18/2.*(2.*X26*TR+X27*TRR-3.*X23*DELR/XL2)
      XMRR=Q(K1+8)-X18/2.*(X27*TR+2.*X28*TRR-3.*X24*DELR/XL2)
      VMR=(XMLR+XMRR)/(XL2+S4+S5)
      VLR=Q(K1+5)+VMR
      VRR=Q(K1+7)-VMR
   21 WRITE (3,22) I,XML,XMR,VL,VR,XMLR,XMRR,VLR,VRR
   22 FORMAT (2X,I4,8D15.8)
C     CALCULATE INTERNAL FORCES ON LEFT COLUMNS
      WRITE (3,333)
  333 FORMAT (/6X51HBENDING MOMENT SHEAR AND AXIAL FORCE ON LEFT COLUMN)
      WRITE (3,77)
      DO 23 I=1,N
      K=7*(I-1)
      TT=-U(K+3)
      TB=-U(K+10)
      DEL=U(K+1)-U(K+8)
      X30=TT+TB-3.*DEL/H
      XMT=-X3/2.*(X30+TT)
      XMB=-X3/2.*(X30+TB)
      VT=(XMT+XMB)/H
      VB=-VT
      AXIAL=X4*(U(K+9)-U(K+2))
   23 WRITE (3,24) I,XMT,XMB,VT,VB,AXIAL
C     CALCULATE INTERNAL FORCES ON CENTER COLUMNS
      WRITE (3,890)
  890 FORMAT(/6X52HBENDING MOMENT SHEAR AND AXIAL FORCE ON CENTR COLUMN)
      WRITE (3,77)
      DO 26 I=1,N
      K=7*(I-1)
      TT=-U(K+5)
      TB=-U(K+12)
      DEL=U(K+1)-U(K+8)
      X30=TT+TB-3.*DEL/H
      XMT=-X7/2.*(X30+TT)
      XMB=-X7/2.*(X30+TB)
      VT=(XMT+XMB)/H
      VB=-VT
      AXIAL=X8*(U(K+11)-U(K+4))
   26 WRITE (3,24) I,XMT,XMB,VT,VB,AXIAL
C     CALCULATE INTERNAL FORCES ON RIGHT COLUMNS
      WRITE (3,444)
  444 FORMAT(/6X52HBENDING MOMENT SHEAR AND AXIAL FORCE ON RIGHT COLUMN)
      WRITE (3,77)
   77 FORMAT(1H ,5HSTORY,6X8HB.M. TOP,8X11HB.M. BOTTOM,8X9HSHEAR TOP,8X1
     12HSHEAR BOTTOM,6X11HAXIAL FORCE/)
      DO 25 I=1,N
      K=7*(I-1)
      TT=-U(K+7)
      TB=-U(K+14)
      DEL=U(K+1)-U(K+8)
      X30=TT+TB-3.*DEL/H
      XMT=-X11/2.*(X30+TT)
```

```
      XMB=-X11/2.*(X30+TB)
      VT=(XMT+XMB)/H
      VB=-VT
      AXIAL=X12*(U(K+13)-U(K+6))
   25 WRITE (3,24) I,XMT,XMB,VT,VB,AXIAL
   24 FORMAT (2X,I4,5D18.10)
      GO TO 501
   99 STOP
      END
      SUBROUTINE MATINV (D,N,A)
      DOUBLE PRECISION D,A,PIVOT,T
      DIMENSION A(N,N),IPVOT(7),INDEX(7,2),PIVOT(7),D(N,N)
      DO 10 I=1,N
      DO 10 J=1,N
   10 A(I,J)=D(I,J)
      DO 17 J=1,N
   17 IPVOT(J)=0
      DO 135 I=1,N
      T=0.
      DO 9 J=1,N
      IF(IPVOT(J)-1) 13,9,13
   13 DO 23 K=1,N
      IF(IPVOT(K)-1) 43,23,81
   43 IF (DABS(T)-DABS(A(J,K))) 83,23,23
   83 IROW=J
      ICOL=K
      T=A(J,K)
   23 CONTINUE
    9 CONTINUE
      IPVOT(ICOL)=IPVOT(ICOL)+1
      IF(IROW-ICOL) 73,109,73
   73 DO 12 L=1,N
      T=A(IROW,L)
      A(IROW,L)=A(ICOL,L)
   12 A(ICOL,L)=T
  109 INDEX(I,1)=IROW
      INDEX(I,2)=ICOL
      PIVOT(I)=A(ICOL,ICOL)
      A(ICOL,ICOL)=1.
      DO 205 L=1,N
  205 A(ICOL,L)=A(ICOL,L)/PIVOT(I)
  347 DO 135 LI=1,N
      IF(LI-ICOL) 21,135,21
   21 T=A(LI,ICOL)
      A(LI,ICOL)=0.
      DO 89 L=1,N
   89 A(LI,L)=A(LI,L)-A(ICOL,L)*T
  135 CONTINUE
  222 DO 3 I=1,N
      L=N-I+1
      IF(INDEX(L,1)-INDEX(L,2)) 19,3,19
   19 JROW=INDEX(L,1)
      JCOL=INDEX(L,2)
      DO 549 K=1,N
      T=A(K,JROW)
      A(K,JROW)=A(K,JCOL)
```

```
      A(K,JCOL)=T
  549 CONTINUE
    3 CONTINUE
   81 RETURN
      END
      SUBROUTINE MATTRA(A,B,M,N)
C     MATRIX TRANSPOSE A TO B
      DOUBLE PRECISION A,B
      DIMENSION A(7,7),B(7,7)
      DO 3 J=1,N
      DO 3 I=1,M
    3 B(J,I)=A(I,J)
      RETURN
      END
      SUBROUTINE MATMLY(A,B,C,M,L,N)
C     MATRIX MULTIPLICATION A*B=C  A=M BY L  B=L BY N
      DOUBLE PRECISION A,B,C
      DIMENSION A(7,7),B(7,7),C(7,7)
      DO 3 J=1,N
      DO 3 I=1,M
      C(I,J)=0.0
      DO 3 K=1,L
    3 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
      SUBROUTINE MATSUB(A,B,C,M,N)
C     MATRIX SUBTRACTION A-B=C
      DOUBLE PRECISION A,B,C
      DIMENSION A(7,7),B(7,7),C(7,7)
      DO 3 J=1,N
      DO 3 I=1,M
    3 C(I,J)=A(I,J)-B(I,J)
      RETURN
      END
/*
// EXEC LNKEDT
// EXEC
/*
/&
```

```
// JOB UEC91664   M.A.MANNAN              GENERAL TWO BAY FRAME  JAN 1978
// OPTION LINK,LIST,LOG
// ASSGN SYS001,X'192'
// EXEC FFORTRAN
C       THIS IS A PROGRAM FOR GENERAL TWO BAY FRAME ANALYSIS
C       BY STIFFNESS METHOD AND RECURSION PROCEDURE
C       PROGRAM MAY BE USED FOR FRAME OF WALLS OR OF COLUMNS OR OF BOTH
C       PROGRAM CAN TAKE CARE OF VARIABLE HEIGHT,SPAN,MOMENT OF INERTIA.
        DOUBLE PRECISION S,B,C,D,FM,FR,FI,XP,FER,U,P,Q,FS,H,AL,AM,AR,CL,CM
       1,CR,E,BL,BR,X,Y,S2,S3,S4,S5,Z1,Z2,Z3,Z4,Z5,Z6,Z7,Z8,Z9,Z10,X11,TL,
       2TR,TRR,DEL,DELR,XML,XMR,VM,VL,VR,XMLR,XMRR,VMR,VLR,VRR
        DIMENSION S(7,7),B(7,7),C(7,7),D(7,7),FM(7,1),FR(7,1),
       1FI(7,1),XP(7),FER(8),U(147),P(140),Q(160),FS(133),
       2H(20),AL(20),AM(20),AR(20),CL(20),CM(20),CR(20)
        DEFINE FILE4 (140,56,L,ID)
        WRITE (3,1)
      1 FORMAT (1X30HGENERAL TWO BAY FRAME ANALYSIS)
C       N=NUMBER OF STORY,NSL=NUMBER OF LOADED STORY,E=MODU. OF ELASTICITY
      2 READ (1,4,ERR=51,END=51) N,NSL,E
        ID=1
        FIND (4'ID)
        WRITE (3,3) N,NSL,E
      3 FORMAT(/1X10HNO.OF STY=,I3,9X17HNO.OF STY LOADED=,I3,5X2HE=,F6.0/)
      4 FORMAT (2I3,F6.0)
        NN=7*(N+1)
        DO 5 I=1,NN
      5 U(I)=0.
        NN=7*N
        DO 6 I=1,NN
      6 P(I)=0.
        NN=8*N
        DO 7 I=1,NN
      7 Q(I)=0.
C       LS=LOADED STORY LEVEL FROM TOP
C       XP(K)=STORY LOAD,FER(J)=FIXED END REACTION ON STORY BEAM
C       ORDER OF FER(J)=LT FESHEAR,LT FEM,RT FES,RT FEM.(FOR BOTH BEAMS)
        WRITE (3,200)
    200 FORMAT (1H ,5HSTORY,4X7HLOAD P1,4X7HLOAD P2,4X7HLOAD P3,4X7HLOAD P
       14,4X7HLOAD P5,4X7HLOAD P6,4X7HLOAD P7)
        DO 9 I=1,NSL
        READ (1,8) LS,(XP(K),K=1,7)
        WRITE (3,201) LS,(XP(K),K=1,7)
        DO 9 K=1,7
        NN=7*(LS-1)+K
      9 P(NN)=XP(K)
      8 FORMAT (I3,7F11.2)
    201 FORMAT (1H ,I5,7F11.2)
        WRITE (3,202)
    202 FORMAT(/18X14HLEFT SIDE BEAM,30X15HRIGHT SIDE BEAM)
        WRITE (3,203)
    203 FORMAT (1X5HSTORY,3X6HFES LT,3X6HFEM LT,3X6HFES RT,3X6HFEM RT,13X6
       1HFES LT,3X6HFEM LT,3X6HFES RT,3X6HFEM RT)
        DO 10 I=1,NSL
        READ (1,11) LS,(FER(J),J=1,8)
        WRITE (3,204) LS,(FER(J),J=1,8)
        DO 10 J=1,8
```

```
      NN=8*(LS-1)+J
   10 Q(NN)=FER(J)
   11 FORMAT(I3,8F9.1)
  204 FORMAT (1X,I5,4F9.1,10X,4F9.1)
      DO 12 J=1,7
      DO 12 I=1,7
      S(I,J)=0.
   12 C(I,J)=0.
C     H=STORY HEIGHT,AL=AREA OF LEFT COL.,AM=AREA OF CENTER CCLUMN
C     AR=AREA OF RIGHT COLUMN,CL=MOM.OF INERTIA OF LT CCL.
C     CM=M.O.I.OF CENTER COLUMN, CR=M.O.I. OF RIGHT COLUMN
      WRITE (3,205)
  205 FORMAT(/1X6HHEIGHT,3X11HAREA LT COL,3X12HAREA CTR COL,3X11HAREA RT
     1 COL,4X10HMOI LT COL,3X11HMOI CTR COL,4X10HMOI RT COL)
      DO 15 I=1,N
      READ (1,13) H(I),AL(I),AM(I),AR(I),CL(I),CM(I),CR(I)
   15 WRITE (3,206) H(I),AL(I),AM(I),AR(I),CL(I),CM(I),CR(I)
   13 FORMAT (F6.1,3F8.1,3F11.1)
  206 FORMAT (1X,F6.1,6(3X,F11.1))
C     BL AND BR=M.O.I.OF LT AND RT BEAM. X AND Y=SPAN LT AND RT BEAM.
C     S2,S3,S4,S5=DEPTH OF WALL FROM C.G. (FOR COLUMNS THEY ARE ZERO)
      WRITE (3,207)
  207 FORMAT (/1X42HSECTION PROPERTIES OF BEAMS AND WALL DEPTH)
      WRITE (3,208)
  208 FORMAT (4X11HMOI LT BEAM,3X11HMOI RT BEAM,3X12HSPAN LT BEAM,3X12HS
     1PAN RT BEAM,10X42HWALL DEPTH FROM C.G. OF WALL LEFT TO RIGHT)
      READ (1,14) BL,BR,X,Y,S2,S3,S4,S5
      WRITE (3,209) BL,BR,X,Y,S2,S3,S4,S5
   14 FORMAT (2F10.1,6F6.1)
  209 FORMAT (1X,4(3X,F11.1),10X,4F11.1)
      Z1=1.+2.*S2/X
      Z2=1.+2.*S3/X
      Z3=1.+2.*S4/Y
      Z4=1.+2.*S5/Y
      Z5=1.+3.*S2/X*(1.+S2/X)
      Z6=1.+3.*S3/X*(1.+S3/X)
      Z7=1.+3.*S4/Y*(1.+S4/Y)
      Z8=1.+3.*S5/Y*(1.+S5/Y)
      Z9=1.+3./X*(S2+S3+2.*S2*S3/X)
      Z10=1.+3./Y*(S4+S5+2.*S4*S5/Y)
      S(1,1)=12.*(CL(1)+CM(1)+CR(1))/H(1)**3
      S(3,1)=6.*CL(1)/H(1)**2
      S(5,1)=6.*CM(1)/H(1)**2
      S(7,1)=6.*CR(1)/H(1)**2
      S(2,2)=AL(1)/H(1)+12.*BL/X**3
      S(3,2)=Z1*6.*BL/X**2
      S(4,2)=-12.*BL/X**3
      S(5,2)=Z2*6.*BL/X**2
      S(3,3)=4.*CL(1)/H(1)+Z5*4.*BL/X
      S(4,3)=-Z1*6.*BL/X**2
      S(5,3)=Z9*2.*BL/X
      S(4,4)=AM(1)/H(1)+12.*(BL/X**3+BR/Y**3)
      S(5,4)=6.*(Z3*BR/Y**2-Z2*BL/X**2)
      S(6,4)=-12.*BR/Y**3
      S(7,4)=Z4*6.*BR/Y**2
      S(5,5)=4.*CM(1)/H(1)+Z6*4.*BL/X+Z7*4.*BR/Y
```

```
      S(6,5)=-Z3*6.*BR/Y**2
      S(7,5)=Z10*2.*BR/Y
      S(6,6)=AR(1)/H(1)+12.*BR/Y**3
      S(7,6)=-Z4*6.*BR/Y**2
      S(7,7)=4.*CR(1)/H(1)+Z8*4.*BR/Y
      DO 16 J=1,6
      NN=J+1
      DO 16 I=NN,7
   16 S(J,I)=S(I,J)
      CALL MATINV (S,7,D)
      DO 18 I=1,7
      WRITE (4'ID)(D(I,J),J=1,7)
      FS(I)=P(I)
   18 FR(I,1)=P(I)
      DO 27 I=2,N
      READ (1,14) BL,BR,X,Y,S2,S3,S4,S5
      WRITE (3,209) BL,BR,X,Y,S2,S3,S4,S5
      NN=7*(I-1)
      N1=(I-1)
      DO 20 K=1,7
      KK=NN+K
   20 FI(K,1)=P(KK)
      Z1=1.+2.*S2/X
      Z2=1.+2.*S3/X
      Z3=1.+2.*S4/Y
      Z4=1.+2.*S5/Y
      Z5=1.+3.*S2/X*(1.+S2/X)
      Z6=1.+3.*S3/X*(1.+S3/X)
      Z7=1.+3.*S4/Y*(1.+S4/Y)
      Z8=1.+3.*S5/Y*(1.+S5/Y)
      Z9=1.+3./X*(S2+S3+2.*S2*S3/X)
      Z10=1.+3./Y*(S4+S5+2.*S4*S5/Y)
      Z11=(CL(I)+CM(I)+CR(I))/H(I)**3+(CL(N1)+CM(N1)+CR(N1))/H(N1)**3
      C(1,1)=-12.*(CL(N1)+CM(N1)+CR(N1))/H(N1)**3
      C(3,1)=-6.*CL(N1)/H(N1)**2
      C(5,1)=-6.*CM(N1)/H(N1)**2
      C(7,1)=-6.*CR(N1)/H(N1)**2
      C(2,2)=-AL(N1)/H(N1)
      C(1,3)=6.*CL(N1)/H(N1)**2
      C(3,3)=2.*CL(N1)/H(N1)
      C(4,4)=-AM(N1)/H(N1)
      C(1,5)=6.*CM(N1)/H(N1)**2
      C(5,5)=2.*CM(N1)/H(N1)
      C(6,6)=-AR(N1)/H(N1)
      C(1,7)=6.*CR(N1)/H(N1)**2
      C(7,7)=2.*CR(N1)/H(N1)
      CALL MATTRA (C,S,7,7)
      CALL MATMLY (S,D,B,7,7,7)
      CALL MATMLY (B,FR,FM,7,7,1)
      CALL MATSUB (FI,FM,FR,7,1)
      CALL MATMLY (B,C,D,7,7,7)
      DO 100 J=1,7
      DO 100 K=1,7
  100 B(K,J)=0.
      B(1,1)=12.*Z11
      B(3,1)=6.*(CL(I)/H(I)**2-CL(N1)/H(N1)**2)
```

```
      B(5, 1)=6.*(CM(I)/H(I)**2-CM(N1)/H(N1)**2)
      B(7, 1)=6.*(CR(I)/H(I)**2-CR(N1)/H(N1)**2)
      B(2, 2)=AL(N1)/H(N1)+AL(I)/H(I)+12.*BL/X**3
      B(3, 2)=Z1*6.*BL/X**2
      B(4, 2)=-12.*BL/X**3
      B(5, 2)=Z2*6.*BL/X**2
      B(3, 3)=4.*(CL(N1)/H(N1)+CL(I)/H(I)+Z5*BL/X)
      B(4, 3)=-Z1*6.*BL/X**2
      B(5, 3)=Z9*2.*BL/X
      B(4, 4)=AM(N1)/H(N1)+AM(I)/H(I)+12.*(BL/X**3+BR/Y**3)
      B(5, 4)=6.*(Z3*BR/Y**2-Z2*BL/X**2)
      B(6, 4)=-12.*BR/Y**3
      B(7, 4)=Z4*6.*BR/Y**2
      B(5, 5)=4.*(CM(N1)/H(N1)+CM(I)/H(I)+Z6*BL/X+Z7*BR/Y)
      B(6, 5)=-Z3*6.*BR/Y**2
      B(7, 5)=Z10*2.*BR/Y
      B(6, 6)=AR(N1)/H(N1)+AR(I)/H(I)+12.*BR/Y**3
      B(7, 6)=-Z4*6.*BR/Y**2
      B(7, 7)=4.*(CR(N1)/H(N1)+CR(I)/H(I)+Z8*BR/Y)
      DO 22 J=1,6
      KK=J+1
      DO 22 K=KK,7
   22 B(J,K)=B(K,J)
      CALL MATSUB (B,D,S,7,7)
      CALL MATINV (S,7,D)
      IF (I-N) 24,28,24
   24 DO 25 K=1,7
      WRITE (4'ID)(D(K,J),J=1,7)
      KK=NN+K
   25 FS(KK)=FR(K,1)
   27 CONTINUE
   28 CALL MATMLY (D,FR,FM,7,7,1)
      K=7*(N-1)
      DO 29 J=1,7
      KK=K+J
   29 J(KK)=FM(J,1)
      DO 32 I=2,N
      N1=N-I+1
      C(1, 1)=-12.*(CL(N1)+CM(N1)+CR(N1))/H(N1)**3
      C(3, 1)=-6.*CL(N1)/H(N1)**2
      C(5, 1)=-6.*CM(N1)/H(N1)**2
      C(7, 1)=-6.*CR(N1)/H(N1)**2
      C(2, 2)=-AL(N1)/H(N1)
      C(1, 3)=6.*CL(N1)/H(N1)**2
      C(3, 3)=2.*CL(N1)/H(N1)
      C(4, 4)=-AM(N1)/H(N1)
      C(1, 5)=6.*CM(N1)/H(N1)**2
      C(5, 5)=2.*CM(N1)/H(N1)
      C(6, 6)=-AR(N1)/H(N1)
      C(1, 7)=6.*CR(N1)/H(N1)**2
      C(7, 7)=2.*CR(N1)/H(N1)
      ID=7*(N-I)+1
      FIND (4'ID)
      DO 31 J=1,7
      READ (4'ID) (D(J,K),K=1,7)
      KK=7*(N-I)+J
```

```
   31 FI(J,1)=FS(KK)
      CALL MATMLY (C,FM,FR,7,7,1)
      CALL MATSUB (FI,FR,FM,7,1)
      DO 33 J=1,7
   33 FR(J,1)=FM(J,1)
      CALL MATMLY (D,FR,FM,7,7,1)
      DO 32 K=1,7
      KK=7*(N-I)+K
   32 U(KK)=FM(K,1)
      WRITE (3,34)
   34 FORMAT (//1X5HSTORY,6X11HTRANSLATION,5X12HLT VER DISP.,6X11HLT ROT
     1ATION,4X13HCTR VER DISP.,5X12HCTR ROTATION,5X12HRT VER DISP.,6X11H
     2RT ROTATION)
      DO 36 I=1,N
      DO 35 J=1,7
      K=7*(I-1)
      KK=K+J
   35 FR(J,1)=U(KK)/E
   36 WRITE (3,37) I,(FR(J,1),J=1,7)
   37 FORMAT (3X,I3,7D17.10)
C     CALCULATE INTERNAL FORCES ON BEAMS
      WRITE (3,38)
   38 FORMAT (/21X39HBENDING MOMENT AND SHEAR FORCE ON BEAMS)
      WRITE (3,210)
  210 FORMAT(20X25HL E F T  S I D E  B E A M,35X27HR I G H T  S I D E  B
     1 E A M)
      WRITE (3,40)
   40 FORMAT (1X5HSTORY,8X7HB.M. LT,8X7HB.M. RT,7X8HSHEAR LT,7X8HSHEAR R
     1T,10X7HB.M. LT,8X7HB.M. RT,7X8HSHEAR LT,7X8HSHEAR RT)
      DO 41 I=1,N
      READ (1,14) BL,BR,X,Y,S2,S3,S4,S5
      K=7*(I-1)
      KK=8*(I-1)
      Z1=1.+2.*S2/X
      Z2=1.+2.*S3/X
      Z3=1.+2.*S4/Y
      Z4=1.+2.*S5/Y
      Z5=1.+3.*S2/X*(1.+S2/X)
      Z6=1.+3.*S3/X*(1.+S3/X)
      Z7=1.+3.*S4/Y*(1.+S4/Y)
      Z8=1.+3.*S5/Y*(1.+S5/Y)
      Z9=1.+3./X*(S2+S3+2.*S2*S3/X)
      Z10=1.+3./Y*(S4+S5+2.*S4*S5/Y)
      TL=-U(K+3)
      TR=-U(K+5)
      TRR=-U(K+7)
      DEL=U(K+2)-U(K+4)
      DELR=U(K+4)-U(K+6)
      XML=Q(KK+2)-2.*BL/X*(2.*Z5*TL+Z9*TR-3.*Z1*DEL/X)
      XMR=Q(KK+4)-2.*BL/X*(Z9*TL+2.*Z6*TR-3.*Z2*DEL/X)
      VM=(XML+XMR)/(X+S2+S3)
      VL=Q(KK+1)+VM
      VR=Q(KK+3)-VM
      XMLR=Q(KK+6)-2.*BR/Y*(2.*Z7*TR+Z10*TRR-3.*Z3*DELR/Y)
      XMRR=Q(KK+8)-2.*BR/Y*(Z10*TR+2.*Z8*TRR-3.*Z4*DELR/Y)
      VMR=(XMLR+XMRR)/(Y+S4+S5)
```

```
      VLR=Q(KK+5)+VMR
      VRR=Q(KK+7)-VMR
   41 WRITE(3,42) I,XML,XMR,VL,VR,XMLR,XMRR,VLR,VRR
   42 FORMAT (3X,I3,4D15.8,2X,4D15.8)
C       CALCULATE INTERNAL FORCES ON LEFT COLUMNS
      WRITE (3,43)
   43 FORMAT (/6X51HBENDING MOMENT SHEAR AND AXIAL FORCE ON LEFT COLUMN)
      WRITE (3,48)
      DO 44 I=1,N
      K=7*(I-1)
      Z1=-U(K+3)
      Z2=-U(K+10)
      Z3=U(K+1)-U(K+8)
      Z4=Z1+Z2-3.*Z3/H(I)
      XML=-2.*CL(I)/H(I)*(Z4+Z1)
      XMR=-2.*CL(I)/H(I)*(Z4+Z2)
      VL=(XML+XMR)/H(I)
      VR=-VL
      XMLR=AL(I)/H(I)*(U(K+9)-U(K+2))
   44 WRITE (3,50) I,XML,XMR,VL,VR,XMLR
C     CALCULATE INTERNAL FORCES ON CENTER COLUMN
      WRITE (3,45)
   45 FORMAT(/6X52HBENDING MOMENT SHEAR AND AXIAL FORCE ON CENTR COLUMN)
      WRITE (3,48)
      DO 46 I=1,N
      K=7*(I-1)
      Z1=-U(K+5)
      Z2=-U(K+12)
      Z3=U(K+1)-U(K+8)
      Z4=Z1+Z2-3.*Z3/H(I)
      XML=-2.*CM(I)/H(I)*(Z4+Z1)
      XMR=-2.*CM(I)/H(I)*(Z4+Z2)
      VL=(XML+XMR)/H(I)
      VR=-VL
      XMLR=AM(I)/H(I)*(U(K+11)-U(K+4))
   46 WRITE (3,50) I,XML,XMR,VL,VR,XMLR
C     CALCULATE INTERNAL FORCES ON RIGHT COLUMNS
      WRITE (3,47)
   47 FORMAT(/6X52HBENDING MOMENT SHEAR AND AXIAL FORCE ON RIGHT COLUMN)
      WRITE (3,48)
   48 FORMAT (1X5HSTORY,9X7HB.M TOP,9X10HB.M BOTTOM,9X9HSHEAR TOP,7X12HS
     1HEAR BOTTOM,7X11HAXIAL FORCE)
      DO 49 I=1,N
      K=7*(I-1)
      Z1=-U(K+7)
      Z2=-U(K+14)
      Z3=U(K+1)-U(K+8)
      Z4=Z1+Z2-3.*Z3/H(I)
      XML=-2.*CR(I)/H(I)*(Z4+Z1)
      XMR=-2.*CR(I)/H(I)*(Z4+Z2)
      VL=(XML+XMR)/H(I)
      VR=-VL
      XMLR=AR(I)/H(I)*(U(K+13)-U(K+6))
   49 WRITE (3,50) I,XML,XMR,VL,VR,XMLR
   50 FORMAT (3X,I3,5D18.10)
      GO TO 2
```

```
   51 STOP
      END
      SUBROUTINE MATINV (D,N,A)
      DOUBLE PRECISION D,A,PIVOT,T
      DIMENSION A(N,N),IPVOT(7),INDEX(7,2),PIVOT(7),D(N,N)
      DO 10 I=1,N
      DO 10 J=1,N
   10 A(I,J)=D(I,J)
      DO 17 J=1,N
   17 IPVOT(J)=0
      DO 135 I=1,N
      T=0.
      DO 9 J=1,N
      IF(IPVOT(J)-1) 13,9,13
   13 DO 23 K=1,N
      IF(IPVOT(K)-1) 43,23,81
   43 IF (DABS(T)-DABS(A(J,K))) 83,23,23
   83 IROW=J
      ICOL=K
      T=A(J,K)
   23 CONTINUE
    9 CONTINUE
      IPVOT(ICOL)=IPVOT(ICOL)+1
      IF(IROW-ICOL) 73,109,73
   73 DO 12 L=1,N
      T=A(IROW,L)
      A(IROW,L)=A(ICOL,L)
   12 A(ICOL,L)=T
  109 INDEX(I,1)=IROW
      INDEX(I,2)=ICOL
      PIVOT(I)=A(ICOL,ICOL)
      A(ICOL,ICOL)=1.
      DO 205 L=1,N
  205 A(ICOL,L)=A(ICOL,L)/PIVOT(I)
  347 DO 135 LI=1,N
      IF(LI-ICOL) 21,135,21
   21 T=A(LI,ICOL)
      A(LI,ICOL)=0.
      DO 89 L=1,N
   89 A(LI,L)=A(LI,L)-A(ICOL,L)*T
  135 CONTINUE
  222 DO 3 I=1,N
      L=N-I+1
      IF(INDEX(L,1)-INDEX(L,2)) 19,3,19
   19 JROW=INDEX(L,1)
      JCOL=INDEX(L,2)
      DO 549 K=1,N
      T=A(K,JROW)
      A(K,JROW)=A(K,JCOL)
      A(K,JCOL)=T
  549 CONTINUE
    3 CONTINUE
   81 RETURN
      END
      SUBROUTINE MATTRA(A,B,M,N)
C     MATRIX TRANSPOSE A TO B
```

```
      DOUBLE PRECISION A,B
       DIMENSION A(M,N),B(M,N).
      DO 3 J=1,N
      DO 3 I=1,M
    3 B(J,I)=A(I,J)
      RETURN
      END
      SUBROUTINE MATMLY(A,B,C,M,L,N)
C     MATRIX MULTIPLICATION A*B=C  A=M BY L  B=L BY N
      DOUBLE PRECISION A,B,C
      DIMENSION A(M,L),B(L,N),C(M,N)
      DO 3 J=1,N
      DO 3 I=1,M
      C(I,J)=0.0
      DO 3 K=1,L
    3 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
      SUBROUTINE MATSUB(A,B,C,M,N)
C     MATRIX SUBTRACTION A-B=C
      DOUBLE PRECISION A,B,C
      DIMENSION A(M,N),B(M,N),C(M,N)
      DO 3 J=1,N
      DO 3 I=1,M
    3 C(I,J)=A(I,J)-B(I,J)
      RETURN
      END
/*
// EXEC LNKEDT
// DLBL UOUT,'IJSYS01'
// EXTENT SYS001,,1,0,40,20
// EXEC CLRDSK
// UCL B=(K=0,D=56),X'CC',OY
// END
// DLBL IJSYS01
// EXTENT SYS001,,1,0,40,20
// EXEC
/*
/&
```

```
// JOB UEC91664      GA FOR ONE BAY FRAME       M.A.MANNAN  JAN.1978
// OPTION LINK,LIST,LOG
// EXEC FFORTRAN
      DOUBLE PRECISION BO,B,C,A,FM,FRED,DISP,D,E,FI,BI,U,FS,P,Q,GA,EM,
     1XI1,XI2,XI3,XL,H,A1,A2,XL2,XL3,X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,
     2X12,X13,X14,X15,X16
      DIMENSION BO(5,5),B(5,5),C(5,5),A(5,5),FM(5,1),FRED(5,1),DISP(5,1)
     1,D(5,5),E(5,5),FI(5,1),BI(95,5),U(105),P(100),FS(95),Q(20)
      WRITE (3,1)
    1 FORMAT(/1X35HCALCULATION OF GA FOR ONE BAY FRAME)
  501 READ(1,2,ERR=99,END=99)N,XI1,XI2,XI3,XL,H,A1,A2,EM,XL2,XL3
      WRITE (3,8) N,XI1,XI2,XI3,XL,H,A1,A2,EM,XL2,XL3
    2 FORMAT (I5,3F8.0,7F7.1)
    8 FORMAT (//1X,I5,3F8.0,7F7.1)
C     IF XL2=0. LEFT WALL IS A COLUMN. IF XL3=0. RIGHT WALL IS A COLUMN.
      X1=H*XI3/XL
      X2=1.+2.*XI1/X1
      X3=1.+2.*XI2/X1
      GA=12.*EM/H**2*(XI1/X2+XI2/X3)
      WRITE (3,3) GA
    3 FORMAT (/1X41HGA VALUE BY HEIDEBRECHT AND SMITH METHOD=,D17.10/)
      N1=5*(N+1)
      N2=5*N
      DO 4 I=1,N1
    4 U(I)=0.
      DO 5 I=1,N2
    5 P(I)=0.
      P(1)=1.
      DO 50 J=1,5
      DO 50 I=1,5
      BO(I,J)=0.
      B(I,J)=0.
   50 C(I,J)=0.
      X1=12.*XI1/H**3
      X2=12.*XI2/H**3
      X3=6.*XI1/H**2
      X4=6.*XI2/H**2
      X5=4.*XI1/H
      X6=4.*XI2/H
      X7=A1/H
      X8=A2/H
      X9=12.*XI3/XL**3
      X10=6.*XI3/XL**2
      X11=4.*XI3/XL
      X12=1.+2.*XL2/XL
      X13=1.+2.*XL3/XL
      X14=1.+3.*XL2/XL*(1.+XL2/XL)
      X15=1.+3.*XL3/XL*(1.+XL3/XL)
      X16=1.+3./XL*(XL2+XL3+2.*XL2*XL3/XL)
      BO(1,1)=X1+X2
      BO(3,1)=X3
      BO(5,1)=X4
      BO(2,2)=X7+X9
      BO(3,2)=X10*X12
      BO(4,2)=-X9
      BO(5,2)=X10*X13
```

```
      BO(3,3)=X5+X11*X14
      BO(4,3)=-X10*X12
      BO(5,3)=X11/2.*X16
      BO(4,4)=X8+X9
      BO(5,4)=-X10*X13
      BO(5,5)=X6+X11*X15
      B(1,1)=(X1+X2)*2.
      B(2,2)=2.*X7+X9
      B(3,2)=X10*X12
      B(4,2)=-X9
      B(5,2)=X10*X13
      B(3,3)=2.*X5+X11*X14
      B(4,3)=-X10*X12
      B(5,3)=X11/2.*X16
      B(4,4)=2.*X8+X9
      B(5,4)=-X10*X13
      B(5,5)=2.*X6+X11*X15
      C(1,1)=-(X1+X2)
      C(3,1)=-X3
      C(5,1)=-X4
      C(2,2)=-X7
      C(1,3)=X3
      C(3,3)=X5/2.
      C(4,4)=-X8
      C(1,5)=X4
      C(5,5)=X6/2.
      DO 52 J=1,4
      J1=J+1
      DO 52 I=J1,5
      BO(J,I)=BO(I,J)
   52 B(J,I)=B(I,J)
      CALL MATTRA (C,A,5,5)
      CALL MATINV (BO,5,D)
      DO 666 J=1,5
      DO 666 I=1,5
  666 BI(I,J)=D(I,J)
      CALL MATMLY (A,D,E,5,5,5)
      DO 9 I=1,5
      FS(I)=P(I)
    9 FRED(I,1)=P(I)
      DO 53 I=2,N
      I4=5*(I-1)
      DO 54 K=1,5
      IK=I4+K
   54 FI(K,1)=P(IK)
      CALL MATMLY (E,FRED,FM,5,5,1)
      CALL MATSUB (FI,FM,FRED,5,1)
      CALL MATMLY (E,C,D,5,5,5)
      CALL MATSUB (B,D,F,5,5)
      CALL MATINV(E,5,D)
      IF (I-N) 10,11,10
   10 CALL MATMLY (A,D,E,5,5,5)
      DO 541 J=1,5
      IJ=I4+J
  541 FS(IJ)=FRED(J,1)
      DO 888 J=1,5
```

```
      DO 888 L=1,5
      IL=I4+L
  888 BI(IL,J)=D(L,J)
   53 CONTINUE
   11 CALL MATMLY (D,FRED,DISP,5,5,1)
      K=5*(N-1)
      DO 14 J=1,5
      KJ=K+J
   14 U(KJ)=DISP(J,1)
      DO 647 I=2,N
      DO 998 J=1,5
      DO 998 K=1,5
      KJ=5*(N-I)+K
  998 D(K,J)=BI(KJ,J)
      DO 646 J=1,5
      JJ=5*(N-I)+J
  646 FI(J,1)=FS(JJ)
      CALL MATMLY (C,DISP,FM,5,5,1)
      CALL MATSUB (FI,FM,FRED,5,1)
      CALL MATMLY (D,FRED,DISP,5,5,1)
      DO 647 K=1,5
      KK=5*(N-I)+K
  647 U(KK)=DISP(K,1)
      AN=N
      GA=AN*H*EM/U(1)
      WRITE (3,7) GA
    7 FORMAT (17X25HGA VALUE BY EXACT METHOD =,D17.10)
      WRITE (3,777)
  777 FORMAT(/1X39HTRANSLATION OF DIFFERENT STORY FROM TOP)
      DO 20 I=1,N
      K=5*(I-1)+1
   20 Q(I)=U(K)/EM
      WRITE (3,24) (Q(I),I=1,N)
   24 FORMAT (1X,7D18.10)
      GO TO 501
   99 STOP
      END
      SUBROUTINE MATINV (D,N,A)
      DOUBLE PRECISION D,A,PIVOT,T
      DIMENSION A(N,N),IPVOT(7),INDEX(7,2),PIVOT(7),D(N,N)
      DO 10 I=1,N
      DO 10 J=1,N
   10 A(I,J)=D(I,J)
      DO 17 J=1,N
   17 IPVOT(J)=0
      DO 135 I=1,N
      T=0.
      DO 9 J=1,N
      IF(IPVOT(J)-1) 13,9,13
   13 DO 23 K=1,N
      IF(IPVOT(K)-1) 43,23,81
   43 IF (DABS(T)-DABS(A(J,K))) 83,23,23
   83 IROW=J
      ICOL=K
      T=A(J,K)
   23 CONTINUE
```

```
    9 CONTINUE
      IPVOT(ICOL)=IPVOT(ICOL)+1
      IF(IROW-ICOL) 73,109,73
   73 DO 12 L=1,N
      T=A(IROW,L)
      A(IROW,L)=A(ICOL,L)
   12 A(ICOL,L)=T
  109 INDEX(I,1)=IROW
      INDEX(I,2)=ICOL
      PIVOT(I)=A(ICOL,ICOL)
      A(ICOL,ICOL)=1.
      DO 205 L=1,N
  205 A(ICOL,L)=A(ICOL,L)/PIVOT(I)
  347 DO 135 LI=1,N
      IF(LI-ICOL) 21,135,21
   21 T=A(LI,ICOL)
      A(LI,ICOL)=0.
      DO 89 L=1,N
   89 A(LI,L)=A(LI,L)-A(ICOL,L)*T
  135 CONTINUE
  222 DO 3 I=1,N
      L=N-I+1
      IF(INDEX(L,1)-INDEX(L,2)) 19,3,19
   19 JROW=INDEX(L,1)
      JCOL=INDEX(L,2)
      DO 549 K=1,N
      T=A(K,JROW)
      A(K,JROW)=A(K,JCOL)
      A(K,JCOL)=T
  549 CONTINUE
    3 CONTINUE
   81 RETURN
      END
      SUBROUTINE MATTRA(A,B,M,N)
C     MATRIX TRANSPOSE A TO B
      DOUBLE PRECISION A,B
      DIMENSION A(5,5),B(5,5)
      DO 3 J=1,N
      DO 3 I=1,M
    3 B(J,I)=A(I,J)
      RETURN
      END
      SUBROUTINE MATMLY(A,B,C,M,L,N)
C     MATRIX MULTIPLICATION A*B=C   A=M BY L   B=L BY N
      DOUBLE PRECISION A,B,C
      DIMENSION A(5,5),B(5,5),C(5,5)
      DO 3 J=1,N
      DO 3 I=1,M
      C(I,J)=0.0
      DO 3 K=1,L
    3 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
      SUBROUTINE MATSUB(A,B,C,M,N)
C     MATRIX SUBTRACTION A-B=C
      DOUBLE PRECISION A,B,C
```

```
      DIMENSION A(5,5),B(5,5),C(5,5)
      DO 3 J=1,N
      DO 3 I=1,M
    3 C(I,J)=A(I,J)-B(I,J)
      RETURN
      END
/*
// EXEC LNKEDT
// EXEC
/*
/&
```

```
// JOB UEC91664     GA FOR TWO BAY FRAME      M.A.MANNAN  JAN.1978
// OPTION LINK,LIST,LOG
// EXEC FFORTRAN
      DOUBLE PRECISION BO,B,C,A,FM,FRED,DISP,D,E,FI,BI,U,FS,P,Q,GA,EM,
     1WI1,WI2,WI3,BI1,BI2,WA1,WA2,WA3,XL1,XL2,H,S2,S3,S4,S5,X1,X2,X3,X4,
     2X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18,X19,X20,X21,X22
     3,X23,X24,X25,X26,X27,X28
      DIMENSION BO(7,7),B(7,7),C(7,7),A(7,7),FM(7,1),FRED(7,1),DISP(7,1)
     1,D(7,7),E(7,7),FI(7,1),BI(133,7),U(147),P(140),FS(133),Q(20)
      WRITE (3,1)
    1 FORMAT(/1X35HCALCULATION OF GA FOR TWO BAY FRAME)
  501 READ(1,2,ERR=99,END=99)N,WI1,WI2,WI3,BI1,BI2,XL1,XL2,H,EM
    2 FORMAT (I5,9F8.0)
      WRITE (3,8) N,WI1,WI2,WI3,BI1,BI2,XL1,XL2,H,EM
    8 FORMAT (//1X,I5,9F8.0)
C     IF S2=0.1ST WALL IS COL.,IF S3=S4=0.CENTER WALL IS COL., AND SO ON
      READ (1,3) WA1,WA2,WA3,S2,S3,S4,S5
      WRITE (3,3) WA1,WA2,WA3,S2,S3,S4,S5
    3 FORMAT (5X,7F8.0)
      X1=H*BI1/XL1
      X2=1.+2.*WI1/X1
      X3=H*BI2/XL2
      X4=1.+2.*WI3/X3
      X5=1.+2.*WI2/(X1+X3)
      GA=12.*EM/H**2*(WI1/X2+WI2/X5+WI3/X4)
      WRITE (3,6) GA
    6 FORMAT (/1X41HGA VALUE BY HEIDEBRECHT AND SMITH METHOD=,D17.10/)
      N1=7*(N+1)
      N2=7*N
      DO 4 I=1,N1
    4 U(I)=0.
      DO 5 I=1,N2
    5 P(I)=0.
      P(1)=1.
      DO 50 J=1,7
      DO 50 I=1,7
      BO(I,J)=0.
      B(I,J)=0.
   50 C(I,J)=0.
      X1=12.*WI1/H**3
      X2=6.*WI1/H**2
      X3=4.*WI1/H
      X4=WA1/H
      X5=12.*WI2/H**3
      X6=6.*WI2/H**2
      X7=4.*WI2/H
      X8=WA2/H
      X9=12.*WI3/H**3
      X10=6.*WI3/H**2
      X11=4.*WI3/H
      X12=WA3/H
      X13=12.*BI1/XL1**3
      X14=6.*BI1/XL1**2
      X15=4.*BI1/XL1
      X16=12.*BI2/XL2**3
      X17=6.*BI2/XL2**2
```

```
X18=4.*BI2/XL2
X19=1.+2.*S2/XL1
X20=1.+2.*S3/XL1
X21=1.+3.*S2/XL1*(1.+S2/XL1)
X22=1.+3./XL1*(S2+S3+2.*S2*S3/XL1)
X23=1.+2.*S4/XL2
X24=1.+2.*S5/XL2
X25=1.+3.*S3/XL1*(1.+S3/XL1)
X26=1.+3.*S4/XL2*(1.+S4/XL2)
X27=1.+3./XL2*(S4+S5+2.*S4*S5/XL2)
X28=1.+3.*S5/XL2*(1.+S5/XL2)
BO(1,1)=X1+X5+X9
BO(3,1)=X2
BO(5,1)=X6
BO(7,1)=X10
BO(2,2)=X4+X13
BO(3,2)=X14*X19
BO(4,2)=-X13
BO(5,2)=X14*X20
BO(3,3)=X3+X15*X21
BO(4,3)=-X14*X19
BO(5,3)=X15/2.*X22
BO(4,4)=X8+X13+X16
BO(5,4)=X17*X23-X14*X20
BO(6,4)=-X16
BO(7,4)=X17*X24
BO(5,5)=X7+X15*X25+X18*X26
BO(6,5)=-X17*X23
BO(7,5)=X18/2.*X27
BO(6,6)=X12+X16
BO(7,6)=-X17*X24
BO(7,7)=X11+X18*X28
C(1,1)=-(X1+X5+X9)
C(3,1)=-X2
C(5,1)=-X6
C(7,1)=-X10
C(2,2)=-X4
C(1,3)=X2
C(3,3)=X3/2.
C(4,4)=-X8
C(1,5)=X6
C(5,5)=X7/2.
C(6,6)=-X12
C(1,7)=X10
C(7,7)=X11/2.
B(1,1)=2.*(X1+X5+X9)
B(2,2)=2.*X4+X13
B(3,2)=X14*X19
B(4,2)=-X13
B(5,2)=X14*X20
B(3,3)=2.*X3+X15*X21
B(4,3)=-X14*X19
B(5,3)=X15/2.*X22
B(4,4)=2.*X8+X13+X16
B(5,4)=X17*X23-X14*X20
B(6,4)=-X16
```

```
      B(7,4)=X17*X24
      B(5,5)=2.*X7+X15*X25+X18*X26
      B(6,5)=-X17*X23
      B(7,5)=X18/2.*X27
      B(6,6)=2.*X12+X16
      B(7,6)=-X17*X24
      B(7,7)=2.*X11+X18*X28
      DO 52 J=1,6
      J1=J+1
      DO 52 I=J1,7
      BO(J,I)=BO(I,J)
   52 B(J,I)=B(I,J)
      CALL MATTRA (C,A,7,7)
      CALL MATINV (BO,7,D)
      DO 666 J=1,7
      DO 666 I=1,7
  666 BI(I,J)=D(I,J)
      CALL MATMLY (A,D,E,7,7,7)
      DO 9 I=1,7
      FS(I)=P(I)
    9 FRED(I,1)=P(I)
      DO 53 I=2,N
      I4=7*(I-1)
      DO 54 K=1,7
      IK=I4+K
   54 FI(K,1)=P(IK)
      CALL MATMLY (E,FRED,FM,7,7,1)
      CALL MATSUB (FI,FM,FRED,7,1)
      CALL MATMLY (E,C,D,7,7,7)
      CALL MATSUB (B,D,E,7,7)
      CALL MATINV (E,7,D)
      IF (I-N) 10,11,10
   10 CALL MATMLY (A,D,E,7,7,7)
      DO 541 J=1,7
      IJ=I4+J
  541 FS(IJ)=FRED(J,1)
      DO 888 J=1,7
      DO 888 L=1,7
      IL=I4+L
  888 BI(IL,J)=D(L,J)
   53 CONTINUE
   11 CALL MATMLY (D,FRED,DISP,7,7,1)
      K=7*(N-1)
      DO 14 J=1,7
      KJ=K+J
   14 J(KJ)=DISP(J,1)
      DO 647 I=2,N
      DO 998 J=1,7
      DO 998 K=1,7
      KJ=7*(N-I)+K
  998 D(K,J)=BI(KJ,J)
      DO 646 J=1,7
      JJ=7*(N-I)+J
  646 FI(J,1)=FS(JJ)
      CALL MATMLY (C,DISP,FM,7,7,1)
      CALL MATSUB (FI,FM,FRED,7,1)
```

```
      CALL MATMLY (C,FRED,DISP,7,7,1)
      DO 647 K=1,7
      KK=7*(N-I)+K
  647 U(KK)=DISP(K,1)
      AN=N
      GA=AN*H*EM/U(1)
      WRITE (3,7) GA
    7 FORMAT (17X25HGA VALUE BY EXACT METHOD =,D17.10)
      WRITE (3,777)
  777 FORMAT(/1X39HTRANSLATION OF DIFFERENT STORY FROM TOP)
      DO 20 I=1,N
      K=7*(I-1)+1
   20 Q(I)=U(K)/EM
      WRITE (3,24) (Q(I),I=1,N)
   24 FORMAT (1X,7D18.10)
      GO TO 501
   99 STOP
      END
      SUBROUTINE MATINV (D,N,A)
      DOUBLE PRECISION D,A,PIVOT,T
      DIMENSION A(N,N),IPVOT(7),INDEX(7,2),PIVOT(7),D(N,N)
      DO 10 I=1,N
      DO 10 J=1,N
   10 A(I,J)=D(I,J)
      DO 17 J=1,N
   17 IPVOT(J)=0
      DO 135 I=1,N
      T=0.
      DO 9 J=1,N
      IF(IPVOT(J)-1) 13,9,13
   13 DO 23 K=1,N
      IF(IPVOT(K)-1) 43,23,81
   43 IF (DABS(T)-DABS(A(J,K))) 83,23,23
   83 IROW=J
      ICOL=K
      T=A(J,K)
   23 CONTINUE
    9 CONTINUE
      IPVOT(ICOL)=IPVOT(ICOL)+1
      IF(IROW-ICOL) 73,109,73
   73 DO 12 L=1,N
      T=A(IROW,L)
      A(IROW,L)=A(ICOL,L)
   12 A(ICOL,L)=T
  109 INDEX(I,1)=IROW
      INDEX(I,2)=ICOL
      PIVOT(I)=A(ICOL,ICOL)
      A(ICOL,ICOL)=1.
      DO 205 L=1,N
  205 A(ICOL,L)=A(ICOL,L)/PIVOT(I)
  347 DO 135 LI=1,N
      IF(LI-ICOL) 21,135,21
   21 T=A(LI,ICOL)
      A(LI,ICOL)=0.
      DO 89 L=1,N
   89 A(LI,L)=A(LI,L)-A(ICOL,L)*T
```

```
  135 CONTINUE
  222 DO 3 I=1,N
      L=N-I+1
      IF(INDEX(L,1)-INDEX(L,2)) 19,3,19
   19 JROW=INDEX(L,1)
      JCOL=INDEX(L,2)
      DO 549 K=1,N
      T=A(K,JROW)
      A(K,JROW)=A(K,JCOL)
      A(K,JCOL)=T
  549 CONTINUE
    3 CONTINUE
   81 RETURN
      END
      SUBROUTINE MATTRA(A,B,M,N)
C     MATRIX TRANSPOSE A TO B
      DOUBLE PRECISION A,B
      DIMENSION A(7,7),B(7,7)
      DO 3 J=1,N
      DO 3 I=1,M
    3 B(J,I)=A(I,J)
      RETURN
      END
      SUBROUTINE MATMLY(A,B,C,M,L,N)
C     MATRIX MULTIPLICATION A*B=C  A=M BY L  B=L BY N
      DOUBLE PRECISION A,B,C
      DIMENSION A(7,7),B(7,7),C(7,7)
      DO 3 J=1,N
      DO 3 I=1,M
      C(I,J)=0.0
      DO 3 K=1,L
    3 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
      SUBROUTINE MATSUB(A,B,C,M,N)
C     MATRIX SUBTRACTION A-B=C
      DOUBLE PRECISION A,B,C
      DIMENSION A(7,7),B(7,7),C(7,7)
      DO 3 J=1,N
      DO 3 I=1,M
    3 C(I,J)=A(I,J)-B(I,J)
      RETURN
      END
/*
// EXEC LNKEDT
// EXEC
/*
/&
```

```
// JOB UEC91664      APPROX.ANALYSIS OF FRAME M.A.MANNAN   JAN.1978
// OPTION LINK,LIST,LOG
// EXEC FFORTRAN
C     APPROXIMATE WALL-FRAME ANALYSIS BY HEIDEBRECHT AND SMITH METHOD
C     N=NO.OF STORY,NSL=NO.OF STORY LOADED,WI1=MOI LT COL.,
C     WI2=MOI CTR COL.,  WI3=MOI RT WALL
C     BI1=MOI LT BEAM, BI2=MOI RT BEAM, WA1=AREA LT COL.,
C     WA2=AREA CTR COL., WA3=AREA WALL, EM=MODULUS OF ELASTICITY
C     XL1=SPAN LT BEAM, XL2=SPAN RT BEAM, H=STORY HEIGHT
C     S2=S3=S4=0.  S5=WALL DEPTH FROM C.G. TO BEAM END
      DOUBLE PRECISION WI1,WI2,WI3,BI1,BI2,WA1,WA2,WA3,EM,XL1,XL2,H,S2,
     1S3,S4,S5,Z1,Z2,Z3,Z4,Z5,GA,ASQ,A,TH,AH,SINHAH,COSHAH,CONS,XMO,X,Y,
     2AX,SINHAX,COSHAX,XMB,XMS,SB,SS,Q
      WRITE (3,1)
    1 FORMAT(6X46HFRAME ANALYSIS BY HEIDEBRECHT AND SMITH METHOD/)
    2 READ (1,4,ERR=15,END=15) N,NSL,WI1,WI2,WI3,BI1,BI2,WA1,WA2,WA3
    4 FORMAT (2I4,3F11.0,2F9.0,3F7.0)
      WRITE (3,5) N,NSL,WI1,WI2,WI3,BI1,BI2,WA1,WA2,WA3
    5 FORMAT (1X,2I5,8F15.4)
      READ (1,7) EM,XL1,XL2,H,S2,S3,S4,S5
    7 FORMAT (8F9.2)
      WRITE (3,8) EM,XL1,XL2,H,S2,S3,S4,S5
    8 FORMAT (11X,8F15.4)
      Z1=H*BI1/XL1
      Z2=1.+2.*WI1/Z1
      Z3=H*BI2/XL2
      Z4=1.+2.*WI2/(Z1+Z3)
      GA=12.*EM/H**2*(WI1/Z2+WI2/Z4)
      ASQ=GA/(EM*WI3)
      A=DSQRT(ASQ)
      AN=N
      TH=AN*H
      AH=A*TH
      WRITE (3,9) GA,A,AH
    9 FORMAT (/1X3HGA=,D17.10,9X6HALPHA=,D17.10,8X7HALPHAH=,D17.10)
      Z1=DEXP(AH)
      Z2=DEXP(-AH)
      SINHAH=(Z1-Z2)/2.
      COSHAH=(Z1+Z2)/2.
      CONS=(1.+AH*SINHAH)/COSHAH
C     XMO=MOMENT (K-IN.) AT BASE DUE TO UNIT LATERAL U.D.L.
      XMO=TH**2/24.
      X=TH
      N1=N+1
      WRITE (3,10)
   10 FORMAT (/1X11HSTORY LEVEL,9X11HTRANSLATION,7X13HB.M.FLEX.BEAM,6X14
     1HB.M.SHEAR BEAM,5X15HSHEAR FLEX.BEAM,4X16HSHEAR SHEAR BEAM,3X17HIN
     2TERACTION FORCE)
      DO 11 I=1,N1
      AX=A*X
      Z1=DEXP(AX)
      Z2=DEXP(-AX)
      SINHAX=(Z1-Z2)/2.
      COSHAX=(Z1+Z2)/2.
      Z3=EM*WI3*A**4
      Z4=X/TH*(1.-X/(2.*TH))
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      Y=1./(12.*Z3)*(CONS*(COSHAX-1.)-AH*SINHAX+Z4*AH**2)
      XMB=2.*XMO/AH**2*(CONS*COSHAX-AH*SINHAX-1.)
      Z5=1.-X/TH
      XMS=XMO*Z5**2-XMB
      SB=1./(12.*A)*(AH*COSHAX-CONS*SINHAX)
      SS=TH*Z5/12.-SB
      Q=XMB*AH**2/(2.*XMO)+1.
      X=X-H
   11 WRITE (3,12)  I,Y,XMB,XMS,SB,SS,Q
   12 FORMAT (8X,I4,6D20.10)
      WRITE (3,13)
   13 FORMAT (1X18HEND OF CALCULATION//)
      GO TO 2
   15 STOP
      END
/*
// EXEC LNKEDT
// EXEC
/*
/&
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