DETERMINATION OF BATCH SIZE FOR A PRODUCTION SYSTEM OPERATING UNDER A PERIODIC DELIVERY POLICY

A Thesis
By

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Submitted to the department of Industrial and Production Engineering, Bangladesh University of Engineering and Technology, Dhaka in partial fulfillment of the requirement for the degree M Sc. Engg. (IP).

December , 1995

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CERTIFICATE

This is to certify that this work has been done by me and it was not submitted elsewhere for the award of any degree or diploma or for any publication.

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Author

ABSTRACT

A manufacturing system which procures raw materials from suppliers in a lot and process them to convert to finished products is considered. An ordering policy has been proposed for raw materials in orderto meet the requirements of a production facility. This must deliver the finished products demanded by the outside consumers at a fixed interval. Firstly, a general cost model is developed considering both raw materials and finished products. This model has been used to develop a simple procedure in order to determine an optimal ordering policy for procurement of raw materials, and the manufacturing batch size to minimize the total cost for meeting the customer demand in time. The results of the proposed method were compared with the solution of a search algorithm. It indicates that the proposed method produces very good solution with a little computational effort. Numerical examples and sensitivity analysis are also presented. The proposed method can be applied in a real world situation where a batch production environment exists with a periodic delivery policy.

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NOMENCULTURE

D_p = demand rate of a product p, units per year

P_p = production rate, units per year (here, $P_p > D_p$)

Q_p = production lot size

H_p = annual inventory holding cost, $/unit/year

A_p = setup cost for a product p ($/setup)

TC_p = total cost of production per year

r_i = amount/quantity of raw material i required in producing one unit of a product

f = $1/r_i$

D_i = demand of raw material i for the product p in a year, $D_i = r_i D_p$

Q_i = ordering quantity of raw material i

A_i = ordering cost of a raw material i

H_i = annual inventory holding cost for raw material i

PR_i = price of raw material i

TC_i = total cost in purchasing raw material i in a year

Q'_p = optimum production lot size

Q'_i = optimum ordering quantity of raw material i

x = Shipment quantity to customer at a regular interval (units/shipment)

L = time between successive shipments.

	=	D_p/X
T	=	Cycle time measured in year = Q_p/D_p
m	=	number of full shipments during the cycle time = $[T/L]$
TC	=	total cost of the system

CHAPTER – 1

INTRODUCTION

1.1 GENERAL INTRODUCTION

1.2 PROBLEM STATEMENT

1.3 OBJECTIVE OF THE THESIS

1.4 APPLICATION

1.5 ORGANIZATION OF THE THESIS

Chapter-1

INTRODUCTION

1.1 GENERAL INTRODUCTION

Inventory is one of the known traditional problems in industrial, retailer and wholesalers system. Inventory being simply stock keeping isolates one part of the system from the next to permits each to work independently, absorbs the shock of forecast errors and permit the effective utilization of resources when demand undulations are experienced. The objective of inventory management is to have the appropriate amount of raw materials, packaging materials, supplies, and finished goods at the right time at the right place. The inventory quantity (ordering / production) is normally determined based on minimum system cost incurred. The general inventory models are purchase model and production model for single or multiple items. In purchase model, economic order quantity for raw materials, packaging materials, office supplies etc. are determined. By production model the economic production quantity for a manufacturing system is determined. Both purchase and production models are analyzed and discussed in the literature considering shortage & back order, price discount, price break inflation etc. [Silver and Peterson(1985), Chikan(1990), Ahuja(1993)]. It is a common practice to determine the economic lot sizes for manufacturing a product and purchasing raw materials separately. In reality, inventory of raw materials, inprocess goods, packaging materials, finished products, supplies etc. are integral part of the total production / retailing/ distribution system. These inventories are dependent on each other. Therefore, they should be treated integratedly. This aspect would be discussed later.

1.2 PROBLEM STATEMENT

Traditionally economic lot sizes for manufacturing a product and purchasing raw material are determined separately. However, when the raw materials are used in production, their ordering quantities are dependent on the batch quantity of the product. Therefore, it is not desirable to isolate the problem of economic purchase of raw materials from economic batch quantity. As a result, it is required to determine the optimum production lot size of a product and the ordering quantities of associated raw materials together. These could be done treating the production and purchasing as components of a single system, minimizing the total cost of the system.

Consider the product-raw material inventory system of an organization. Normally since the amount of raw material used for a product is known, the amount of raw material to be used in a lot is also known. The raw materials are consumed at a given rate only during the production uptime of batch, and not throughout the whole cycle time, as is commonly assumed. The ordering policy of a raw material can either be based on its economic ordering quantity (EOQ) or on the requirement of the raw material in a lot of economic production quantity (EPQ). The latter reflects the dependence relationship between raw material requirement and production quantity. The relationship between products and there raw material has been the basic consideration in the model as developed in the present work. Moreover the problem is considered from the point of view of the benefit to the manufacturing firm. The proposed systemic view supports the fact that the optimum purchasing-production policy should be determined by considering the system as a whole and not by treating its components individually [Sarker et at(1993)]. The finished product supply policy could be continuous or periodic as practiced. The fixed quantity periodic supply policy has been considered in this research. However, the supply of raw material will be lot by lot.

The problem considered in this thesis is summarized as follows:

- a finite production-rate environment uses raw materials taking from outside suppliers

- only the product lot sizing and their associated raw material supply quantities are under consideration

- supply policy of product is to deliver equal quantity at fixed intervals

- product cannot be delivered until the whole lot is finished and quality certification is ready.

The above situation exists in many industrial units under certain assumptions / conditions. The objective is to determine the optimum batch size of the product and the ordering quantities of raw materials minimizing the overall system cost.

1.3 OBJECTIVE OF THE THESIS

Considering the problem discussed in section 1.2, the objectives of the thesis are set as follows:

i) Formulate a problem for a batch size of a production system operating under a fixed-quantity, periodic delivery policy, under different conditions, as an unconstrained nonlinear optimization problem

ii) Determine the optimal batch size by traditional optimization approach

iii) Judge the quality of solution comparing the result of the proposed method with the solution of a search algorithm

iv) Analyze the results and demonstrate the applicability of the developed models.

1.4 ORGANIZATION OF THE THESIS

The general aspects of the problem and the objectives of the thesis are described in Chapter-1 A literature survey illustrating the various raw material delivery policies (Lot for Lot, Multiple Lot for a Lot) and finished product delivery polices (continuous,periodic) has been discussed in chapter-2. Chapter-3 provides the development of different mathematical models and solution approach of the models. The data, experimentation. results, discussion and sensitivity analysis are presented in chapter-4. Chapter-5 contains conclusion and recommendation for future work. The necessary appendices and references have been given at the end of the thesis.

CHAPTER – 2

LITERATURE REVIEW

2.1 INTRODUCTION

2.2 INDEPENDENT LOT SIZING

2.3 DEPENDENT LOT SIZING

1.4 THE PRESENT PROBLEM

Chapter-2

LITERATURE REVIEW

2.1 INTRODUCTION:

The demand for finished products from a manufacturing system could be continuous at constant rate or fixed quantity at fixed interval points in time. The research as observed in the literature on both continuous and periodic delivery policy are discussed in this chapter.

The materials presented in this chapter can be divided into Independent Lot Sizing and Dependent Lot Sizing. These are briefly discussed below:

2.2 INDEPENDENT LOT SIZING

The well known cost equations and lot sizing formulae [Tersine(1985)] for lot sizing policy for both the raw material and the product are given below:

2.2.1 Raw Material

Total cost of purchasing raw material i in terms of ordering quantity is

$$TC_i = A_i \frac{D_i}{Q_i} + H_i \frac{Q_i}{2} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

TC_i = yearly ordering cost + inventory holding cost

Differentiating TC_i with respect to Q_i and then equating to zero, it becomes,

Economic ordering quantity, $Q_i^* = \sqrt{\frac{2A_iD_i}{H_i}}$ ....(2)

As mentioned earlier, the raw material usage is not constant through out the cycle time. Therefore, the above quantity may not be the actual economic ordering quantity for the raw material in the case considered in this research.

2.2.2 Product

Total cost of the system in terms of production quantity is

$$TC_p = A_p \frac{D_p}{Q_p} + H_p \frac{Q_p}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

= yearly set up cost + inventory holding cost.

Differentiating TC_p with respect to Q_p and then equating to zero, it becomes,

Economic production quantity, $Q_p^* = \sqrt{\frac{2A_pD_p}{H_p}}$.

Though the ordering quantity of raw material and production batch size are determined independently, the usage of raw material is dependent on production rate in reality. Therefore, the total cost of the system cannot be just the simple sum of equation (1) and (3).

2.3 DEPENDENT LOT SIZING

Different models for dependent lot sizing [Sarker et al(1993 & 1995), Sarker and Parija(1994), Zahir and Sarker(1991)] can be classified as follows:

i) Product-Raw material system

According to the supply of raw material it can be classified as follows:

a) Lot for a lot

b) Multiple lot for a lot

According to the delivery policy of the product it can be classified as follows:

a) Continuous supply

b) Periodic supply

- whole lot in single installment
- in multiple installment

ii) Producer-wholesalers system

These models are described briefly in the following subsections.

2.3.1 Continuous Supply system

Sarker et al (1995 & 1993) developed a model operating under continuous supply of raw material at a constant rate for both lot for lot and multiple lots for a lot. These are briefly discussed since the solution approach is similar to the case considered in the present work.

2.3.1a Lot for lot supply of raw material:

Here the ordering quantity of raw material is assumed to be equal to the raw material required for one batch of the production system. This is case-I as shown in figure-1. The raw material which is replenished at the beginning of a production cycle will be fully consumed at the end of this production run.

As shown in figure-1:

T	=	length of production cycle (time)
t	=	production time in a production cycle

In Fig-1, raw material inventory in a cycle time T

$= \text{Area (abc)} = \frac{1}{2}Q_1 t$

therefore average raw material inventory in a cycle = $\frac{Q_1 D_p}{2P_r}$

Finished product inventory in a cycle = Area (efh) = Area (efg) +

$(gfh) = \frac{Q_p}{2}(1-\frac{D_p}{P_p})\,t + \frac{Q_p}{2}(1-\frac{D_p}{P_p})\,(T-t)$

The finished product inventory in a period (0-t) is the modified form of raw material of the same period. The total inventory of raw material in a cycle is calculated from the total stock (say area abc in period 0-t) multiplying by its unit price. Part of these physical units of raw material stock appears in finished product stock (area efg) during the production uptime, since the system under consideration involves the production of finished products using the raw materials to be purchased. In a production system,

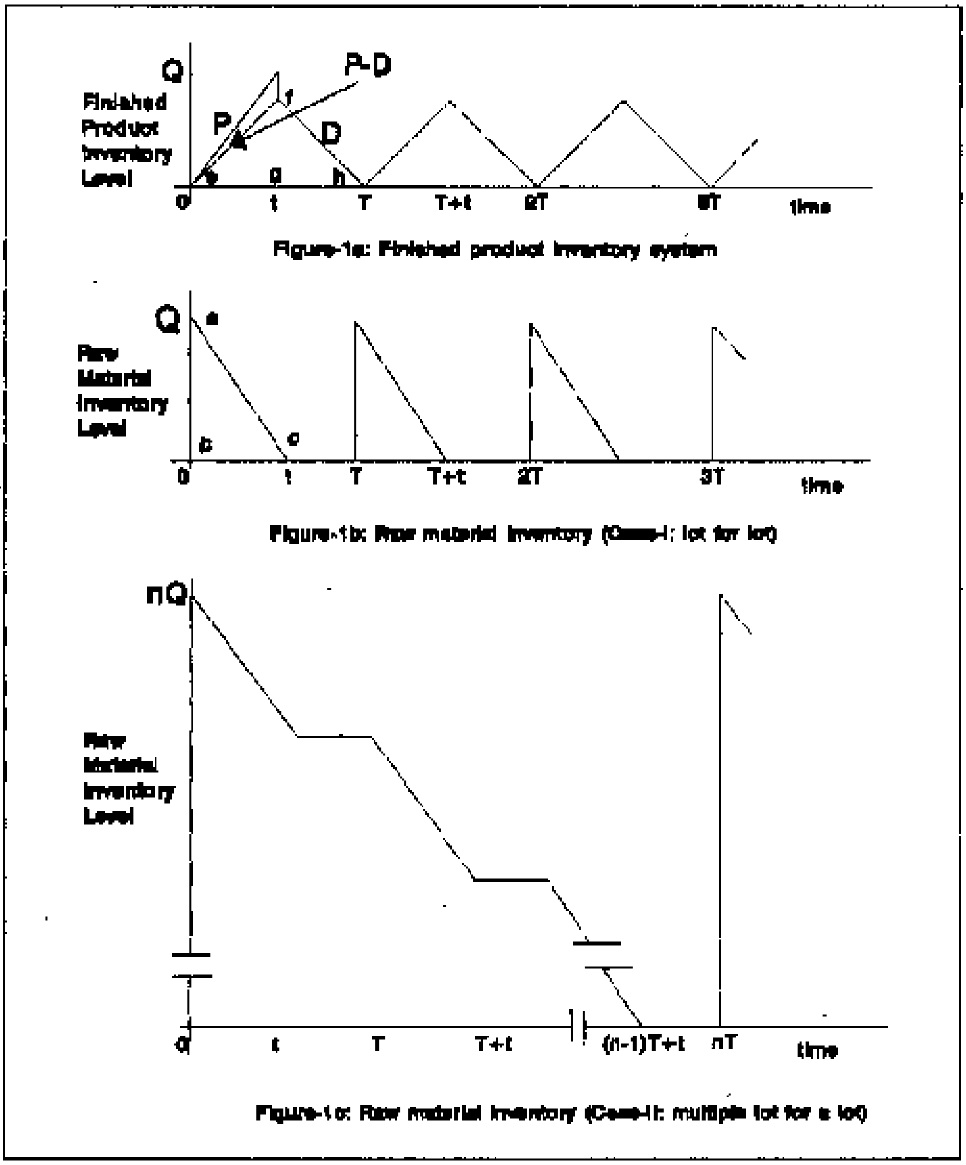

Figure-1a: Finished product inventory system

Figure-1b: Raw material inventory (Case-I: lot for lot)

Figure-1c: Raw material inventory (Case-II: multiple lot for a lot)

(In figure-1a: $D=D_p$, $Q=Q_p$, $P=P_p$ and in figure-1b & c: $Q=Q_i$ are assumed) one single working capital runs for both raw material and finished good. In this case, to avoid double counting of some costs for the same unit at both stages, the finished product inventory in period 0-t (or area efg) will be calculated only on the value added part. Therefore the total inventory holding cost in a year for finished product is

$$= H_p \frac{Q_p}{2}\left(1-\frac{D_p}{P_p}\right)\left(1-\frac{f_i r_i D_p}{P_p}\right) \qquad \text{where, } H_i = f_i H_p$$

The term $(1- f_i r_i D_p/P_p)$ can be avoided. However, the user must revise the holding cost in the light of integrated system approach.

Consider $i = 1$ initially, that is for one raw material only.
Total cost of the system = TC = set up cost + inventory holding cost of finished product + ordering cost for raw material + inventory holding cost for raw material

$$TC = A_p \frac{D_p}{Q_p} + H_p \frac{Q_p}{2}\left(1 - \frac{D_p}{P_p}\right)\left(1 - \frac{f_1 r_1 D_p}{P_p}\right) + A_1 \frac{D_1}{Q_1} + H_1 \frac{Q_1 D_1}{2 P_p} \qquad \ldots \qquad (5)$$

Considering the dependent relationship (ordering quantity of raw material for one lot of a product) and the various assumptions, it can be written that $Q_1 = r_1 Q_p$ and $D_1 = r_1 D_p$. Then differentiating TC with respect to Q_p and equating to zero, it is found that

$$\text{Joint economic lot size, } Q_p^* = \sqrt{\frac{2D_p(A_p + A_1)}{H_p[1 - \frac{D_p}{P_p} + \frac{f_1 r_1 D_p^2}{P_p^2}]}} \qquad (6)$$

For multiple raw materials (i > 1), the joint economic lot size becomes:

$$Q_p^* = \sqrt{\frac{2D_p(A_p + \sum_i A_i)}{H_p[1 - \frac{D_p}{P_p} + \frac{D_p^2}{P_p^2}\sum_i f_i r_i]}} \qquad \ldots \quad \ldots \quad \ldots \qquad (7)$$

When i = 1, the equations (7) and (6) become the same.

2.3.1b Multiple lot for a lot supply of raw material:

Here, we assume the ordering quantity of raw material i to be n_i times the quantity required for one lot of a product, where n_i is an integer. If n_I is continuous, it becomes complicated in formulating the average inventory in a cycle. The raw material inventory is shown in figure-1. In this case, the raw material inventory in a cycle of $n_i T$ periods is

$$= \frac{1}{2}Q_i t + (n_i - 1)Q_i T + \frac{1}{2}Q_i t + (n_i - 2)Q_i T + \frac{1}{2}Q_i t + (n_i - 3)Q_i T +$$

$$= \frac{n_i r_i Q_p^2}{2}\left(\frac{1}{P_p} + \frac{n_i - 1}{D_p}\right)$$

Average raw material inventory per cycle

$$= \frac{n_i r_i Q_p^2}{2}\left(\frac{1}{P_p} + \frac{n_i - 1}{D_p}\right)/n_i T$$

$$= \frac{r_i Q_p}{2}\left(\frac{D_p}{P_p} + n_i - 1\right)$$

Consider now i = 1.

Total cost of the system = TC = set up cost + inventory holding cost of finished product + ordering cost for raw material + inventory holding cost for raw material

$$TC = A_p\frac{D_p}{Q_p} + H_p\frac{Q_p}{2}\left(1 - \frac{D_p}{P_p}\right)\left(1 - \frac{f_1 r_1 D_p}{P_p}\right) + A_1\frac{D_1}{Q_1} + H_1\frac{r_1 Q_p}{2}\left(\frac{D_p}{P_p} + n_1 - 1\right) \quad (8)$$

$$= \frac{D_p}{Q_p}(A_p + A_1/n_1) + H_p\frac{Q_p}{2}(KK)$$

where, $KK = CC + f_1 r_1 n_1$ and

$$CC = (1 - \frac{D_p}{P_p})(1 - \frac{f_1 r_1 D_p}{P_p}) + f_1 r_1(\frac{D_p}{P_p} - 1)$$

The function TC is an unconstrained nonlinear optimization problem with a continuous variable Q_p and an integer variable n_i. In order to find optimal production quantity Q_p^*, it is necessary to differentiate relation (8) with respect to variable Q_p. However, the function TC is nondifferentiable since it is a function of integer variable n_i. As such a closed-form solution for Q_p^* cannot be obtained. However, it can be shown that TC is a piecewise convex function of Q_p. A simple procedure is developed here to obtain an optimal or near optimal solution. A search algorithm like Hooke and Jeeves' Algorithm (Hooke and Jeeves, 1961) has also been applied to solve the problem and compare the results.

When the optimum integer value of n_i is known the function TC becomes a convex function for a given n_i. Now differentiating TC (with known n_i) with respect to Q_p and then equating to zero, it is found that

Joint economic lot size, $Q_p^* = \sqrt{\dfrac{2D_p(A_p + A_1/n_1)}{H_p(KK)}}$ ... (9)

If $n_1 = 1$, Q_p^* of case-II is same as Q_p^* of case-I. Substituting Q_p^* of equation (9) as Q_p in equation (8), the total cost becomes as:

$$TC = D_p(A_p + A_1/n_1)\sqrt{\frac{H_p(KK)}{2D_p(A_p + A_1/n_1)}} + \frac{H_p}{2}(KK)\sqrt{\frac{2D_p(A_p + A_1/n_1)}{H_p(KK)}}$$

$$= \sqrt{2D_p\ (A_p + A_1/n_1)\ H_p\ (KK)}$$

TC can be minimized by minimizing TC^2

$$TC^2 = 2D_p(A_p + A_1/n_1)H_p(KK) = 2D_pH_p(A_p + A_1/n_1)[(CC) + f_1r_1n_1]$$

For the time being, n_1 is considered as a continuous variable. Then differentiating TC^2 (which is independent of Q_p) with respect to n_1 and equating to zero we get the optimum value of continuous n_1 is found as follows:

$$n_1^* = \sqrt{\frac{A_1(CC)}{A_pf_1r_1}} \qquad \ldots \quad \ldots \quad \ldots \quad (10)$$

The value of n_1^* should be integer. If the calculated n_1^* is not integer (as equation-10), the neighboring integer values must be taken corresponding to which the total cost is minimum. If n_1 is not an integer, the inventory and cost calculation for raw material are not correct. However, the above approach is a heuristic to find an approximate value of n_1. This heuristic approach produces good solutions as compared to search method.

For multiple raw materials, the equations for batch size and number of batches are given in Sarker et al(1995).

2.3.2 Periodic supply system

Sarker and Parija(1994) developed a model operating under periodic delivery system of product for lot for lot supply system of raw materials. A graphical representation of their inventory system is shown in fig-2.2. A demand for x units of finished goods at the end of every L time units forces inventory build-up in a saw-tooth fashion during the production up time T_1. The on hand inventory at time t, $Q_f(t)$, depletes sharply at a regular interval after the production period T_1 until the end of cycle time. The later one form a stair case pattern. The algorithm for solving this problem is given below:

Algorithm: finding batch size

Step 0. Initialize and store P_p, D_p, A_1, A_p, H_p, H_r, f and x.

Step 1. Compute A and B by applying the following equations:

$$A=\frac{D_p(A_1+A_p)}{(1-D_p/2P_p)H_px^2+(D_p/2P_pf)H_1x^2}$$

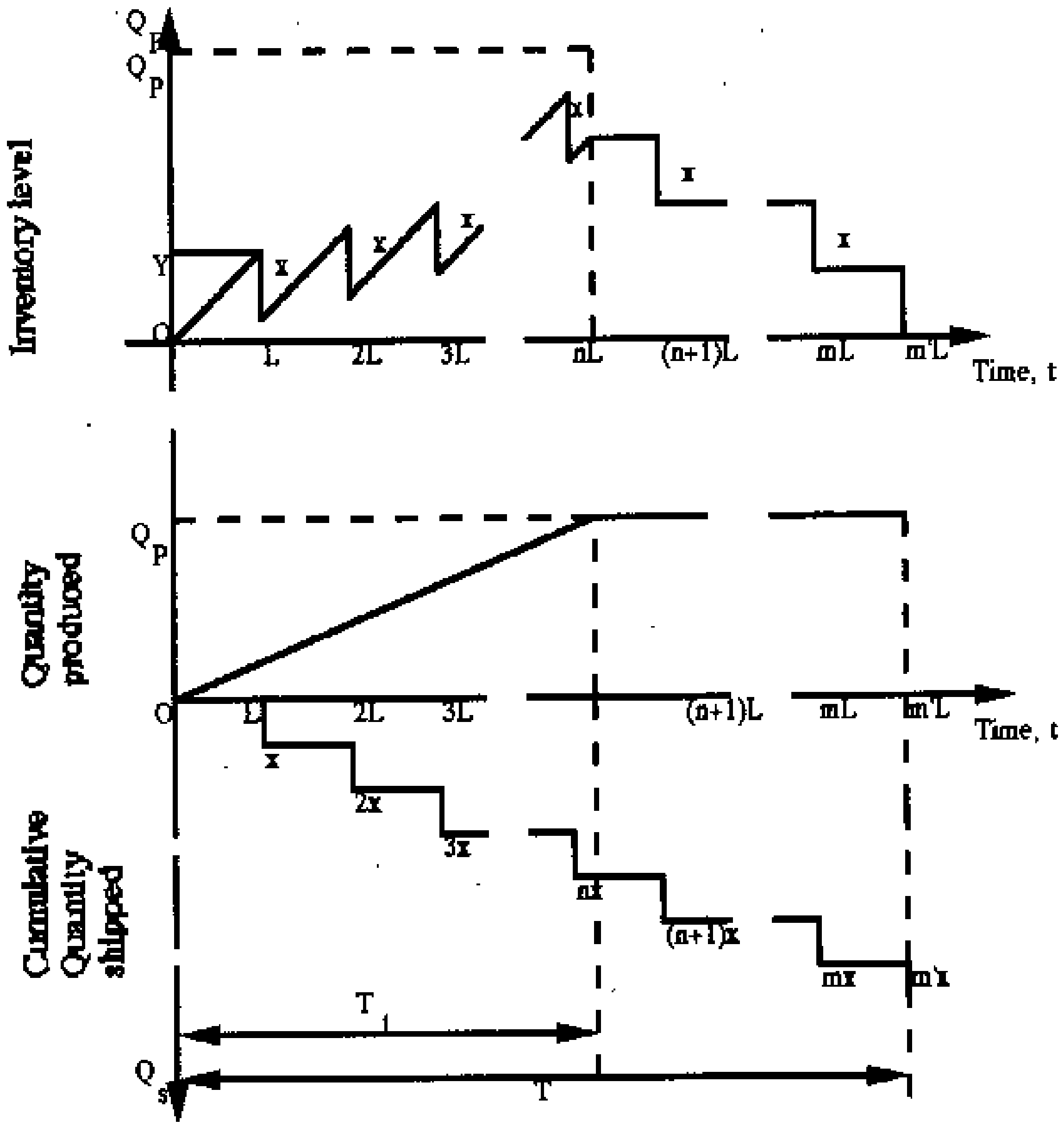

Fig. 2.2 Inventory system for periodic delivery system.

$$B=\frac{H_p}{2(1-D_p/2P_p)H_p+(D_p/P_pf)H_1}=\frac{1}{2+(H_1/fH_p-1)D_p/P_p}$$

Step 2. Compute k^* and Q_p^* using the following equation

$k^* = [1/2\ (1 + (1-(4A - 1)/(B - 1))^{1/2})]$ or

$[1/2\ (1 - (1-(4A - 1)/(B - 1))^{1/2})]$

$Q_p^* = (k - 1)x = x[A + Bk^*(k^* - 1)]^{1/2}$

Step 3. Find $Q_{avg} = Q_p(1-D_p/2P_p) - (k^* - 1)x + k^*(k^* - 1)x^2/2Q_p^*$ and

$$TC = \left(\frac{D_p}{Q_p^*}\right)(A_p+A_1)+\frac{D_p}{P_p}\left(\frac{Q_p^*}{2f}\right)H_1+Q_{avg}^*H_p$$

Step 4. Stop.

Once Q_p is determined, the optimal order quantity for raw material, Q_1^*, may be obtained immediately using the conversion factor, $f = Q_p^*/Q_1^*$

2.3.3 Producer Wholesalers system

Zahir and Sarker (1991) developed a model for determining the optimum production lot size of a single manufacturer supplying a network of regional wholesalers. They treat producer and the wholesalers/retailers as one system and therefore their goal is to maximize the profit of the wholesalers and at the same time minimize the cost of production. They consider demands for each region as function of the retail sale prices which vary from region to region. Therefore, the total demand for the manufacturer is the sum of the all regional demands. This system is so much similar to the raw material-product system.

2.4 THE PRESENT PROBLEM:

In this thesis, the problem considered is similar to Sarker and Parija(1994), but the nature of periodic supply is different. In this case product cannot be supplied until the whole lot is produced because of quality check/control. So there is no saw tooth pattern during the production up-time. Instead there is a stair pattern during production down time of the cycle. The solution approach is some what similar to Sarker et al (1995).

CHAPTER – 3

MATHEMATICAL FORMULATION

Chapter-3

MATHEMATICAL FORMULATION

3.1 INTRODUCTION:

A general cost model is developed considering both the supplier (of raw material) and the buyer (of finished products) together. This model is used as described in chapter-1 to determine an optimal ordering policy for procurement of raw materials, and the manufacturing batch size to minimize the total cost for meeting equal shipments of the finished products, at fixed intervals, to the buyers. The cost models are developed for the production system under consideration and their corresponding optimal batch sizes are determined.

To simplify the model, as mentioned earlier, the following three options(relationship) have been considered:

i) ordering quantity of raw material is equal to the raw material for one lot of a product(i.e. $Q_i = r_i Q_p$)

where,

Q_p = production lot size

Q_i = ordering quantity of raw material i

r_i = amount of raw material i required in producing one unit of product

ii)ordering quantity of raw material is equal to the raw material required for multiple lots of the product(i.e. $Q_i = n_i r_i Q_p$

where,

n_i = number of production lots (integer) forming one lot of raw material i

iii) multiple order of a raw material for one lot of a product to be manufactured.

Two different dependent relationships in developing models for the problem considered in the present work are provided. These are (i) lot for lot and (ii) multiple lot for a lot.

To determine the model, the following assumptions have been made:

i. the production rate is finite and constant

ii. the production capacity is greater than demand

iii. no shortage are permitted

iv. the time horizon is infinite

v. fixed quantity of product are delivered after fixed intervals

3.2 PROBLEM FORMULATION:

The formulation of the problems are presented in the following sub-sections.

3.2.1 Lot for Lot supply of product

We consider the ordering quantity of raw material to be equal to the raw material required for a batch of the production system is considered. The raw material which is replenished at the beginning of a production cycle will be fully consumed at the end of this production run. As mentioned earlier.Two different delivery policies (a) delivery of whole production lot at a time and (b) delivery of a production lot in multiple installments are considered.

3.2.1a. DELIVERY OF WHOLE PRODUCTION LOT AT A TIME, LOT FOR LOT SUPPLY SYSTEM OF PRODUCT

In this type of delivery system, the whole production lot is supplied in one shipment or distributed to one customer or among different wholesalers / customers at a time. This is shown in figure-3.1

As in figure-3.1

T = length of production cycle (time)

Raw material inventory in a cycle time T = Area (abl)

$= 1/2\ Q_i\ T$

Therefore average raw material inventory in a cycle

$= (1/2\ Q_i T)/T$

$= 1/2\ Q_i$

Finish product inventory in a cycle = area (efh)

$= 1/2\ Q_p T$

Therefore average finish product inventory in a cycle

$= (1/2\ Q_p T)/T$

$= 1/2\ Q_p$

Let it is considered, $i = 1$ initially, that is for one raw material only. In this case the total cost of the system is:

Total cost = setup cost of the finished product + inventory holding cost of finished product + ordering cost for raw material + inventory holding cost for raw material

$$= A_p \frac{Q_p}{Q_p} + H_p \frac{Q_p}{2} + A_1 \frac{D_1}{Q_1} + \frac{Q_1}{2} H_1 \quad \ldots\ldots\ldots(5)$$

Considering the assumption (dependent relationship-i) made earlier it can be written as,

$Q_1 = r_1 Q_p$ and $D_1 = r_1 D_p$

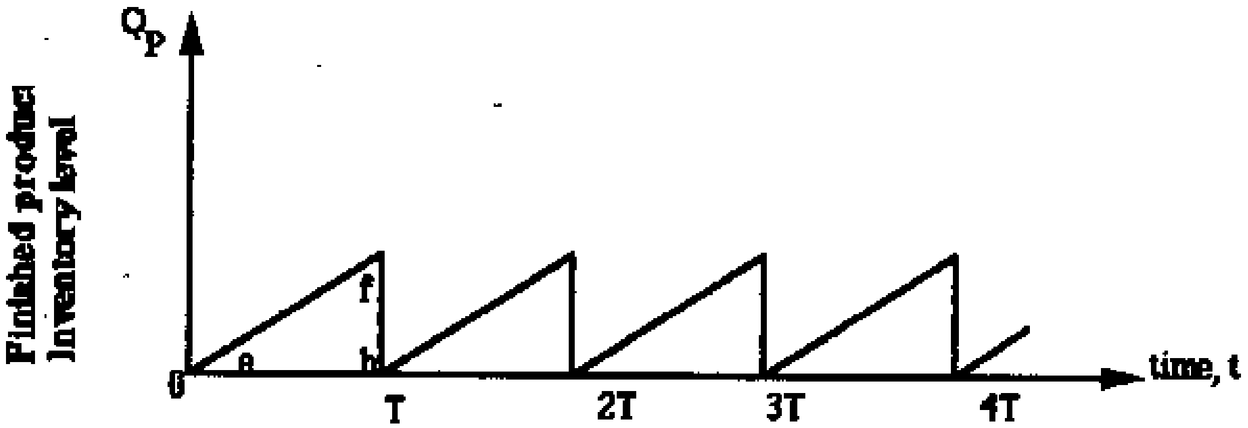

Fig. 3.1b Finished product inventory system

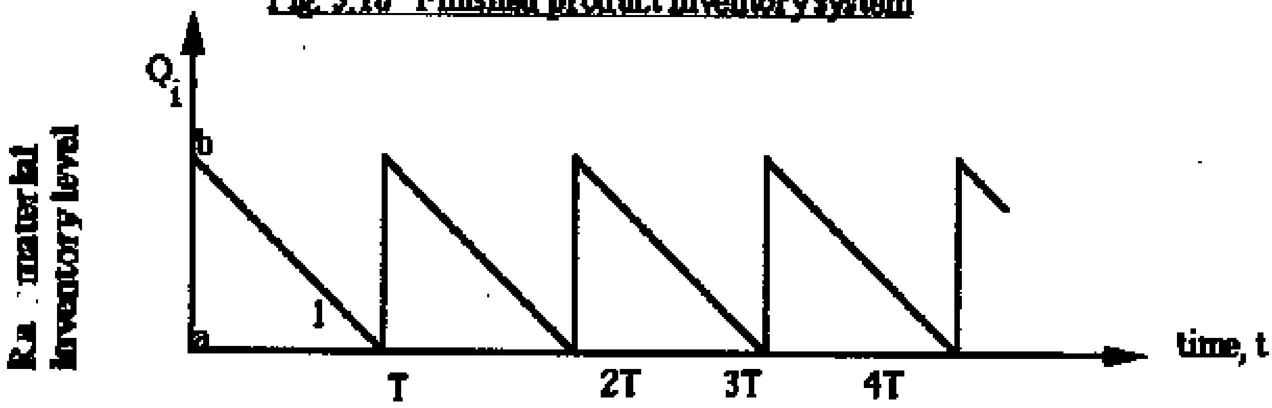

Fig. 3.1a Raw material inventory system

So, TC = $$A_p\frac{D_p}{Q_p}+H_p\frac{Q_p}{2}+A_1\frac{r_1D_p}{r_1Q_p}+\frac{r_1Q_p}{2}H_1$$

$$= A_p\frac{D_p}{Q_p}+H_p\frac{Q_p}{2}+A_1\frac{D_p}{Q_p}+\frac{r_1Q_p}{2}H_1$$

$$= \frac{D_p}{Q_p}(A_p+A_1)+\frac{Q_p}{2}(H_p+r_1H_1) \quad \ldots\ldots\ldots\ldots\ldots(6)$$

Differentiating TC with respect to Q_p and then equating to zero, it gives

$$\frac{d(TC)}{dQ_p}=-\frac{D_p}{Q_p^2}(A_p+A_1)+\frac{1}{2}(H_p+r_1H_1)=0$$

$$Q_p^2=\frac{2D_p(A_p+A_1)}{H_p+r_1H_1}$$

Therefore the economic lot size, $Q_p^*=\sqrt{\dfrac{2D_p(A_p+A_1)}{H_p+r_1H_1}}$ $\ldots\ldots\ldots\ldots\ldots(7)$

Now for i>1, that is for multiple raw materials, the ordering cost of raw materials = $\sum_i \frac{D_i}{Q_i}A_i=\frac{D_p}{Q_p}\sum_i A_i$

Inventory holding cost of raw material = $\sum_i \frac{Q_i}{2}H_i$ = $\frac{Q_p}{2}\sum r_iH_i$

Thus the total cost of the system

$$TC=\frac{D_p}{Q_p}(A_p+\sum_i A_i)+\frac{Q_p}{2}(H_p+\sum_i r_iH_i) \quad \ldots\ldots\ldots\ldots\ldots(8)$$

Differentiating TC with respect to Q_p and then equating to zero, it gives

$$Q_p^* = \sqrt{\frac{2D_p(A_p+\sum_i A_i)}{H_p+\sum_i r_i H_i}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Where i = 1, the equation (9) and (7) become the same.

3.2.1b <u>Delivery of a production lot in multiple installments, Lot for supply system of product</u>

In this type of delivery system, products are delivered to a buyer whose demand is constant. The products are delivered at fixed interval points of time. This is shown in figure-3.2.

From figure-3.2,

t_s = production time in a cycle

Raw material inventory in a cycle time T = Area (abc) = $Q_i t_s/2$

From figure-3.2, $t_s = Q_p/P_p$ and $Q_p/D_p = mL = T$

Therefore, average raw material inventory in a cycle = $Q_i t_s/(2T)$

$$= \frac{1}{2}(r_i Q_p)\left(\frac{Q_p}{P_p}\right)\left(\frac{D_p}{Q_p}\right)$$

$$= \frac{r_i Q_p D_p}{2P_p}$$

Finished product inventory in a cycle = Area (efg + ijk)

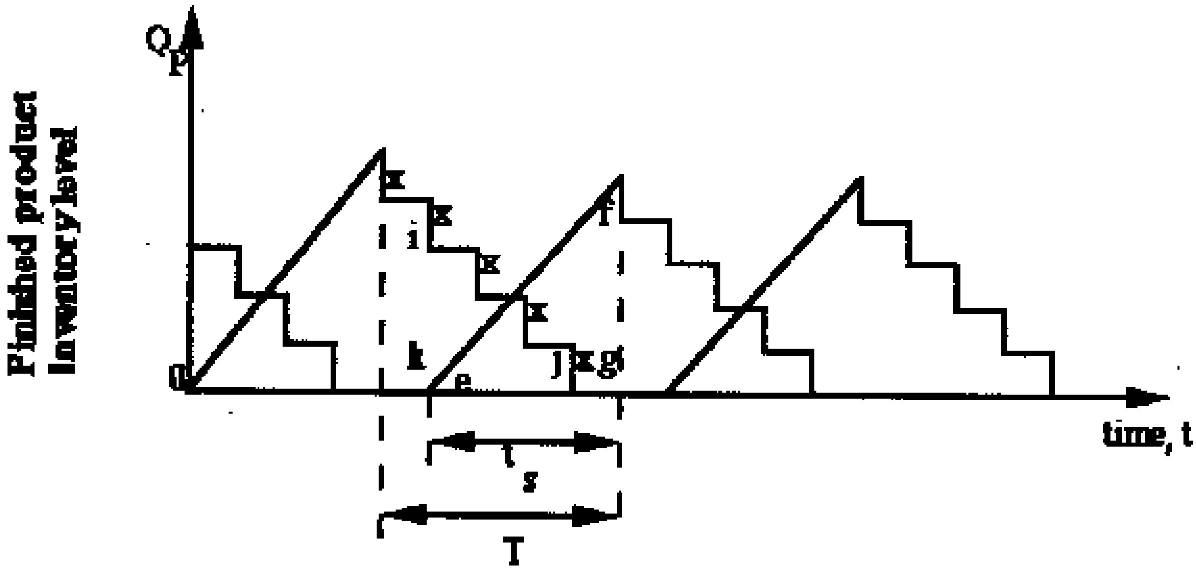

Fig. 3.2b. Finished prodduct inventory system

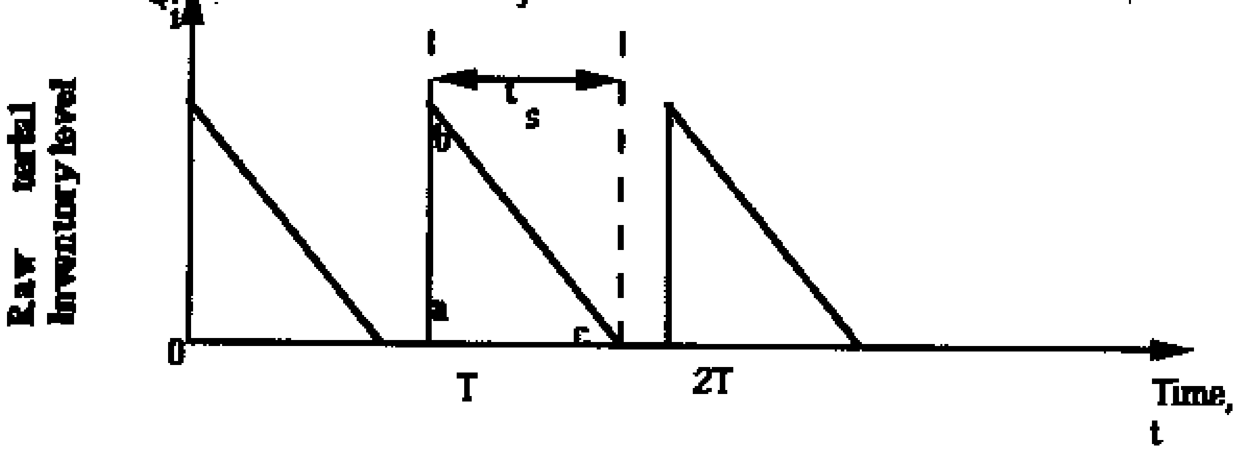

Fig. 3.2a. Raw material inventory system

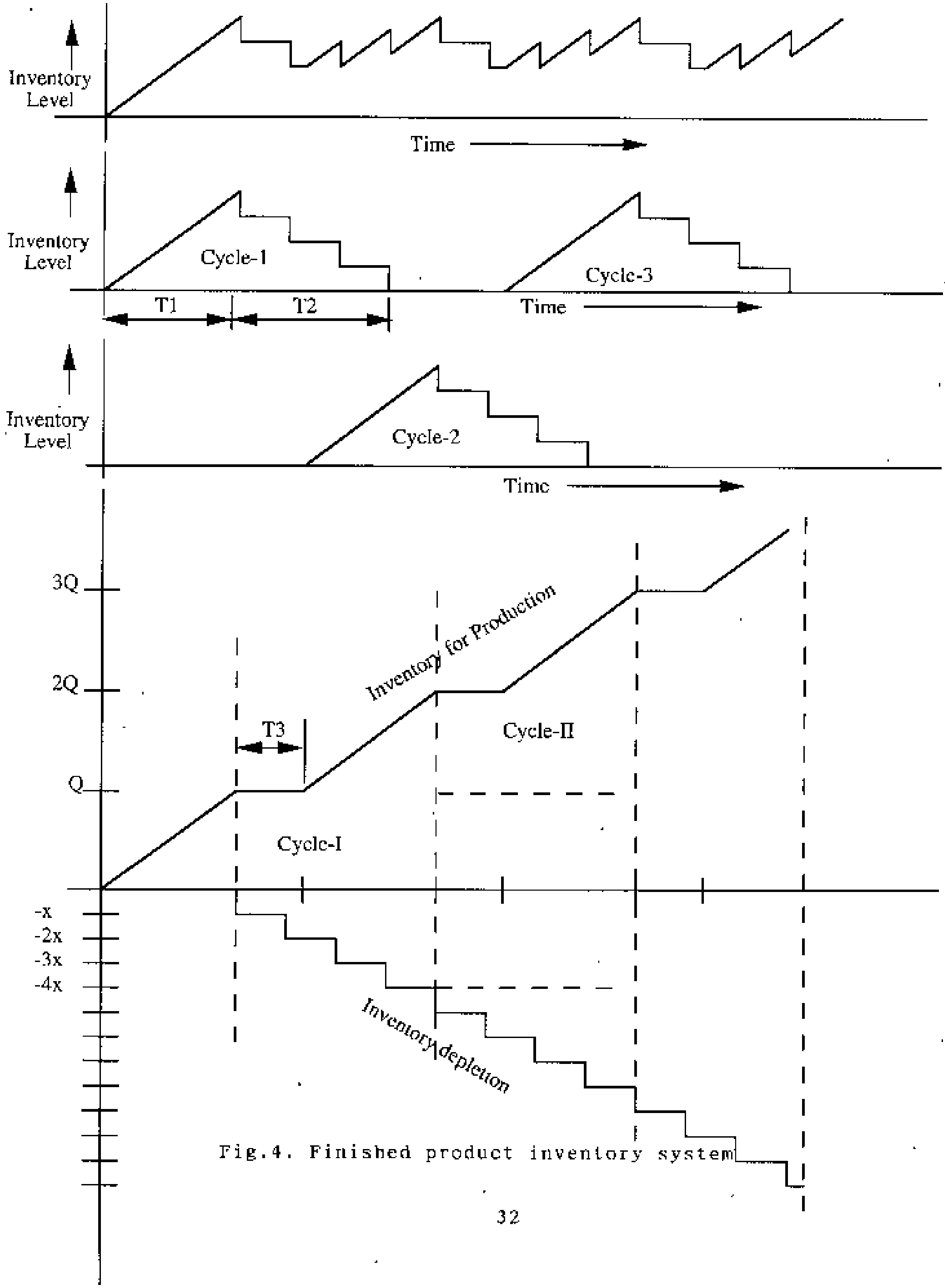

Fig.4. Finished product inventory system

$$\text{Area }(ijk) = x(m-1)L + x(m-2)L + \dots\dots\dots + x.2L + x.L$$

$$= x\frac{(m-1)m}{2}L$$

Thus the finished product inventory in a cycle = $\frac{1}{2}Q_p t_s + x\frac{(m-1)}{2}mL$

and the average finished product inventory in a cycle

$$= \frac{\frac{1}{2}Q_p . t_s + \frac{x(m-1)}{2}mL}{T}$$

$$= \frac{1}{2}Q_p\frac{t_s}{T} + x\frac{m-1}{2}\frac{mL}{T}$$

$$= \frac{1}{2}Q_p . \frac{D_p}{P_p} + \frac{mx}{2} - \frac{x}{2}$$

$$= \frac{1}{2}Q_p(\frac{D_p}{P_p}+1) - \frac{x}{2}$$

Therefore total cost of the system,

$$TC = \frac{D_p}{Q_p}.A_p + [\frac{1}{2}Q_p(\frac{D_p}{P_p}+1) - \frac{x}{2}]H_p + \frac{D_1}{Q_1}A_1 + r_1\frac{Q_p}{2}\frac{D_p}{P_p}H$$

$$= \frac{D_p}{Q_p}(A_p+A_1) + \frac{Q_p}{2}[(\frac{D_p}{P_p}+1)H_p + \frac{D_p}{P_p}r_1H_1] - \frac{x}{2}H_1 \quad \dots\dots\dots(10)$$

The equation-10 is not differentiable since m is integer. For a given m, TC is a convex function of Q_p (piecewise convex).

Differentiating TC with respect to Q_p and then equating to zero, it can be find out the value of Q_p^*.

But $Q_p = mx$

$\Rightarrow \quad m = Q_p/x$

Since x is a preassigned value, the value of m calculated from $m=Q_p/x$ may not be a integer value. In such case, the neighboring integer value of m which incurs minimum cost will be taken as the best m. Alternatively, m can be found out directly. The equation (10) can be written in terms of m as follows-

$$TC = \frac{D_p}{mx}(A_p+A_1)+\frac{mx}{2}[(\frac{D_p}{P_p}+1)H_p+\frac{D_p}{P_p}r_1H_1]-\frac{x}{2}H_l \quad \ldots\ldots\ldots\ldots\ldots (11)$$

Relaxing m integer to continuous and then differentiating TC with respect to m and then equating zero, it gives

$$\frac{d(TC)}{dm}=-\frac{D_p}{m^2x}(A_p+A_1)\ \frac{x}{2}[(\frac{D_p}{P_p}+1)H_p+\frac{D_p}{P_p}r_1H_1]=0$$

$$=> \qquad m^2=\frac{2D_p(A_p+A_1)}{x^2[(\frac{D_p}{P_p}+1)H_p+r_1H_1\frac{D_p}{P_p}]}$$

Therefore, $$m^*=\sqrt{\frac{2D_p(A_p+A_1)}{x^2[(\frac{D_p}{P_p}+1)H_p+r_1H_1\frac{D_p}{P_p}]}} \quad \ldots\ldots\ldots\ldots\ldots (12)$$

Here m also be a real number. But here a integer value of m can be searched, neighboring its real value for which total cost is minimum.

So, $Q_p^* = [m^*]x$

Now for multiple raw materials (i>1)

$$m' = \sqrt{\frac{2D_p(A_p+\sum_i A_i)}{x^2[(\frac{D_p}{P_p}+1)H_p+\frac{D_p}{P_p}\sum_i r_i H_i]}} \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

and $Q_p' = [m']x$

3.2.2 **Multiple Lots for a Lot supply system of raw material**

It is assumed that the ordering quantity of raw material i to be n_i times the quantity required for one lot of a product where n_i is an integer. If n_i is continuous, it would be difficult to formulate the average inventory in a cycle. This case is discussed, for both the

delivery policies presented in the previous section, below

3.2.2a <u>Delivery of whole production lot at a time</u>

The delivery of the production lot is considered at a time and the ordering quantity of raw material i to be n_i times the quantity required for one lot of a product. This is shown in figure-3.3. The raw material inventory, as shown in figure-3.3, in a cycle of

$n_i T$ periods is = $\frac{1}{2} n_i Q_i . n_i T$

Average raw material inventory per cycle

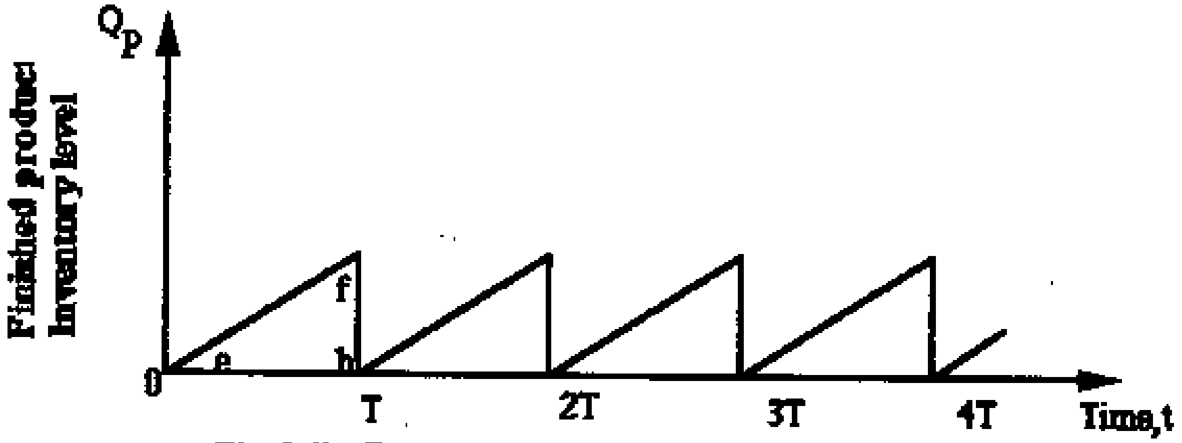

Fig. 3.4b Finished product inventory system

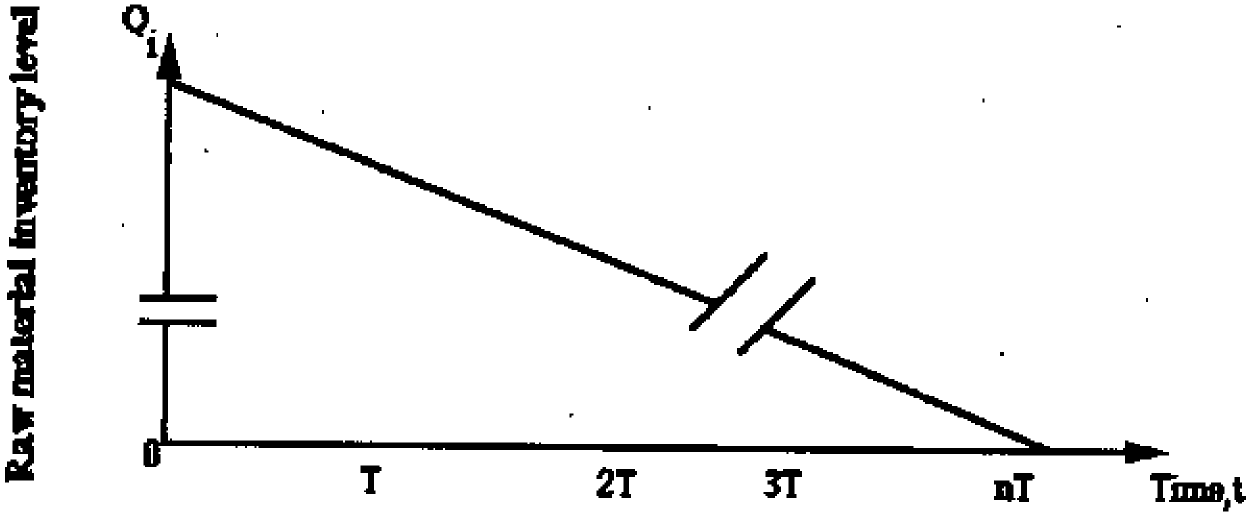

Fig. 3.4a. Raw material inventory system

$$= \frac{\frac{1}{2} n_1 Q_1 . n_1 T}{n_1 T}$$

$$= \frac{1}{2} n_1 r_1 Q_p$$

Let, i = 1

Total cost of the system,

TC = set up cost + inventory holding cost of finished product
+ ordering cost of raw material + inventory holding cost
for raw material

$$= A_p \frac{D_p}{Q_p} + H_p \frac{Q_p}{2} + A_1 \frac{D_1}{nQ_1} + \frac{1}{2} n_1 r_1 Q_p H_1$$

$$= \frac{D_p}{Q_p} (A_p + \frac{A_1}{n}) + \frac{Q_p}{2} (H_p + n_1 r_1 H_1) \quad \ldots\ldots\ldots\ldots\ldots (14)$$

The function TC is an unconstrained nonlinear optimization problem with a continuous variable Q_p and integer variable n_1. In order to find optimal production quantity Q_p^*, it is necessary to differentiate relation (14) with respect to Q_p. However, the function TC is nondifferentiable since it is a function of integer variable n_1. As such a closed form solution for Q_p cannot be obtained. However, it can be shown that TC is a piecewise convex function of Q_p. This is shown in figure-3.4. Algorithms may be applied to solve this problem by using a discrete optimization technique. An algorithm has recently been proposed by Moinzadeh and Aggarwal (1990) to obtain a global minimum for such a function. Although it may not be efficient, a simple procedure is developed here to obtain a optimal or near optimal solution. A

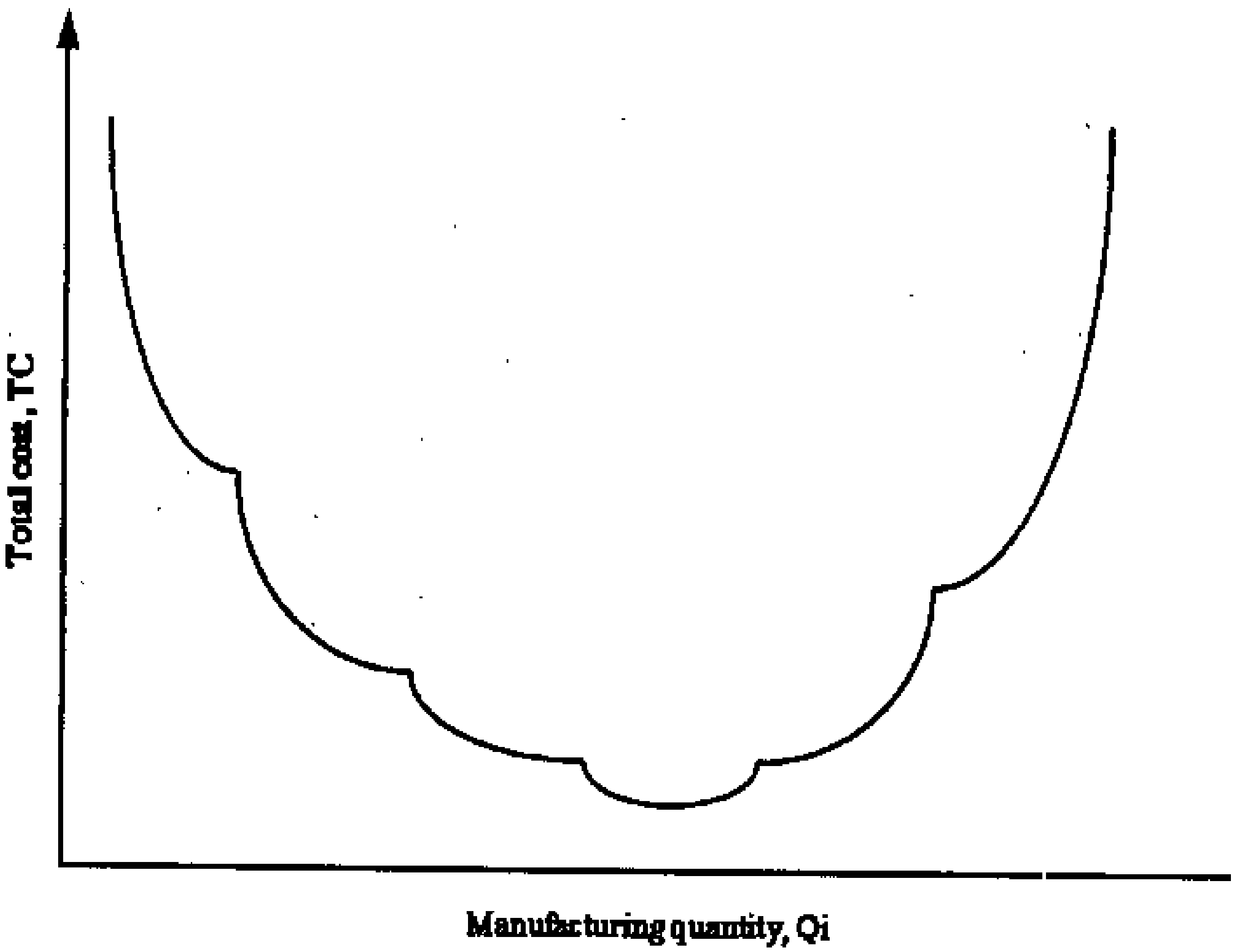

Fig. 3.5 : Total cost function for fixed interval delivery system

search algorithm of Hooke and Jeeves' (Hook and Jeeves, 1961) has also been applied to solve the problem and compared the result.

The function TC is a Convex function for a given n_1. Now differentiating TC (with known n_1) with respect to Q_p and the equating to Zero it becomes

$$\frac{dTC}{dQ_p}=-\frac{D_p}{Q_p^2}\left(A_p+\frac{A_1}{n_1}\right)+\frac{1}{2}\left(H_p+n_1r_1H_1\right)=0$$

$$\Rightarrow \quad Q_p^2=\frac{2D_p\left(A_p+\frac{A_1}{n_1}\right)}{H_p+n_1r_1H_1}$$

Joint economic lot size ,

$$Q_p^*=\sqrt{\frac{2D_p\left(A_p+\frac{A_1}{n_1}\right)}{H_p+n_1r_1H_1}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(15)$$

If $n_1 = 1$, Q_p^* of equation (15) becomes same as equation (7).

Substituting Q_p^* of equation (15) as Q_p in equation (14), it gives the total cost as,

$$TC \quad = \quad D_p\left(A_p+\frac{A_1}{n_1}\right)\sqrt{\frac{H_p+n_1r_1H_1}{2D_p\left(A_p+\frac{A_1}{n_1}\right)}}+\frac{1}{2}\sqrt{\frac{2D_p\left(A_p+\frac{A_1}{n_1}\right)}{H_p+n_1r_1H_1}}\cdot\left(H_p+n_1r_1H_1\right)$$

$$= \quad \sqrt{\frac{1}{2}D_p\left(A_p+A_1n_1\right)\left(H_p+n_1r_1H_1\right)}+\sqrt{\frac{1}{2}D_p\left(A_p+\frac{A_1}{n_1}\right)\left(H_1+n_1r_1H_1\right)}$$

=

$$\sqrt{2D_p\left(A_p+\frac{A_1}{n_1}\right)\left(H_p+n_1 r_1 H_1\right)}$$

TC can be minimized by minimizing TC^2

Now $TC^2 = 2D_p\left(A_p+\frac{A_1}{n_1}\right)\left(H_p+n_1 r_1 H_1\right)$

$$= 2D_p\left(A_p H_p+n_1 A_p r_1 H_1+\frac{A_1 H_p}{n_1}+A_1 r_1 H_1\right)$$

For the time being, let consider that n_1 is a continuous variable. Then differentiating TC^2(which is independent of Q_p) with respect to n_1 and equating zero, it gives

$$\frac{d(TC^2)}{dn_1} = 2D_p\left(A_p r_1 H_1-\frac{A_1 H_p}{n_1^2}\right) = 0$$

$$\Longrightarrow \quad n_1^2 = \frac{A_1 H_p}{A_p r_1 H_1}$$

$$n_1^* = \sqrt{\frac{A_1 H_p}{A_p r_1 H_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The value of n_1^* should be integer. If the calculated n_1^* is not integer (as equation-16) we take the neighboring integer values corresponding to which total cost is minimum.

For multiple raw material the equation for batch size and number of batches are as follows:

Joint economic lot size, $Q_p^* = \sqrt{\frac{2D_p\left(A_p+\sum_i \frac{A_i}{n_i}\right)}{H_p+\sum_i n_i r_i H_i}} \quad \ldots\ldots(17)$

and number of lot size

$$n_i^* = \sqrt{\frac{H_p \sum_i A_i}{A_p \sum_i r_i H_i}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

If $i = 1$, the equations (17) and (18) become similar to equation (15) and (16) respectively.

3.2.2b <u>Delivery of a Production Lot in Multiple Installments</u>

The delivery of the production lot is considered in multiple installments to a buyer whose demand is constant and the ordering quantity of raw material i to be n_i times the quantity required for one lot of a product. This is shown in figure-3.5.

In this case the raw material inventory in a cycle of $n_i T$ periods is

$$= \frac{1}{2}Q_i t_s + Q_i T(n_i - 1) + \frac{1}{2}Q_i t_s + Q_i T(n_i - 2) + \ldots\ldots\ldots + \frac{1}{2}Q_i t_s + Q_i T + \frac{1}{2}Q_i t_s$$

$$= \frac{1}{2}n_i Q_i t_s + \frac{n_i(n_i - 1)}{2} Q_i T$$

Average raw material inventory per cycle

$$= \frac{[\frac{1}{2}n_i Q_i t_s + \frac{n_i(n_i-1)}{2} Q_i T]}{n_i T}$$

$$= \frac{1}{2}Q_i \frac{t_s}{T} + \frac{n_i - 1}{2} Q_i$$

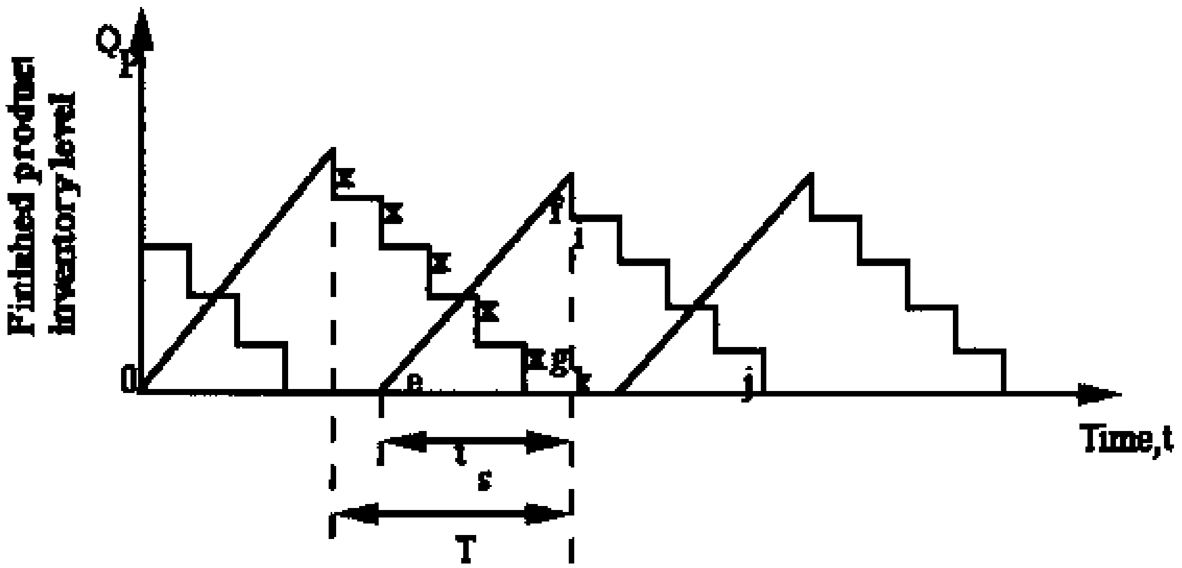

Fig. 3.6b. Finished product inventory system

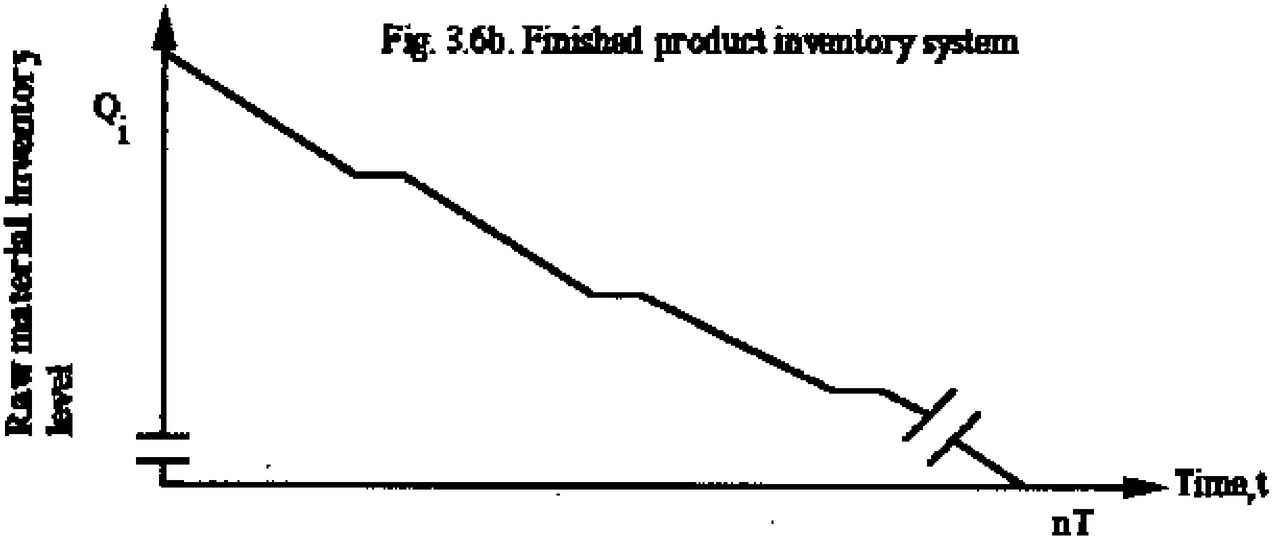

fig.3.6a. Raw material inventory system

$$= \quad \frac{1}{2} r_i Q_p \frac{D_p}{P_p} + \frac{n_i - 1}{2} \cdot r_i Q_i$$

$$= \quad \frac{1}{2} r_i Q_p \left(\frac{D_p}{P_p} + n_i - 1 \right)$$

Let us consider now i = 1

Total cost of the system,

$$TC = \frac{D_p}{Q_p} A_p + \left[\frac{1}{2} Q_p \left(\frac{D_p}{P_p} + 1 \right) - \frac{x}{2} \right] H_p + \frac{D_1}{n_1 Q_1} A_1 + \frac{1}{2} r_1 Q_p \left(\frac{D_p}{P_p} + n_1 - 1 \right) H$$

$$= \quad \frac{D_p}{Q_p} \left(A_p + \frac{A_1}{n_1} \right) + \frac{Q_p}{2} (kk) - \frac{x}{2} H_j$$

Where, kk = $\left(\frac{D_p}{P_p} + 1 \right) H_p + r_1 H_1 \left(\frac{D_p}{P_p} + n_1 - 1 \right) = CC + n_1 r_1 H_1$

now, Q_p = mx

$$TC = \quad \frac{D_p}{mx} \left(A_p + \frac{A_1}{n_1} \right) + \frac{mx}{2} (kk) - \frac{x}{2} H_i \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

Differentiating TC with respect to m and equating to zero. We have,

$$\frac{d(TC)}{dm} = - \frac{D_p}{m^2 x} \left(A_p + \frac{A_1}{n_1} \right) + \frac{x}{2} (kk) = 0$$

$$\Rightarrow \quad m^2 = \frac{2 D_p \left(A_p + \frac{A_1}{n_1} \right)}{x^2 (kk)}$$

$$\text{Therefore, } m^* = \sqrt{\frac{2 D_p \left(A_p + \frac{A_1}{n_1} \right)}{x^2 (kk)}} \quad \ldots\ldots\ldots\ldots (20)$$

Substituting m^* in equation (19) the total cost can be written as:

$$TC \quad \frac{D_p}{x}\left(A_p+\frac{A_1}{n_1}\right)\sqrt{\frac{x^2(kk)}{2D_p\left(A_p+\frac{A_1}{n_1}\right)}}+\frac{x}{2}(kk)\sqrt{\frac{2D_p\left(A_p+\frac{A_1}{n_1}\right)}{x^2(kk)}}-\frac{x}{2}H_I$$

$$= 2\sqrt{\frac{1}{2}D_p\left(A_p+\frac{A_1}{n_1}\right)(kk)}-\frac{x}{2}H_p$$

$$= \sqrt{2D_p\left(A_p+\frac{A_1}{n_1}\right)\left[(CC)+n_1 r_1 H_1\right]}-\frac{x}{2}H_p$$

Differentiating TC with respect to n_1 and equating to zero, it gives,

$$\frac{d(TC_s)}{dn_1}=-\frac{2D_p\left[A_p r_1 H_1-\frac{A_1}{n_1^2}(CC)\right]}{2D_p\left(A_p+\frac{A_1}{n_1}\right)\left[(CC)+n_1 r_1 H_1\right]}=0$$

$$\Rightarrow \quad n_1^2=\frac{A_1(CC)}{A_p r_1 H_1}$$

Therefore, $$n_1^*=\sqrt{\frac{A_1(CC)}{A_p r_1 H_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

Both m and n_1 should be integers.

3.3 NUMERICAL EXAMPLES:

Numerical examples for all the four cases are provided in this section to show the applicability of the models.

<u>Example-1</u>: Consider an example of lot for lot of raw material for

a lot of product. The required data are as follows:

$D_p = 4X10^6$ units, $P_p = 5X10^6$ units, $A_p = \$\ 50.00$ per setup, $A_i = \$3000$ per order, $H_p = \$\ 1.20$ per unit per year, x = 1000 units $H_i = \$\ 1.08$ per unit per year.

Solution:

Let $r_i = 1.00$

From equation- 7,

the economical lot size $Q_i^* = 103449$ units

From equation- 6,

Total cost TC = $235864.35.

<u>Example-2</u>: Consider an example of lot for lot of raw material for multiple lot of product. The required data are as follows:

$D_p = 4X10^6$ units, $P_p = 5X10^6$ units, $A_p = \$\ 50.00$ per setup, $A_i = \$3000$ per order, $H_p = \$\ 1.20$ per unit per year, x = 1000 units $H_i = \$\ 1.08$ per unit per year.

Solution:

Let $r_i = 1.00$

From equation- 12 ,

number of full shipment during the cycle time = 10.628

the economical lot size $Q_i^* = 106280$ units

From equation- 11 ,

Total cost TC = $269988.20

To find the integer value of m, we try for m = 10 & 11

Value of m	Q^*	TC	Remarks
10	100000	267200	Optimum
11	110000	271229	

So number of full shipment during the cycle time m =10, and lot size = 100000 units.

Example-3: Consider an example of multiple lot of raw material for a lot of product. The required data are as follows:
D_p = $4X10^6$ units, P_p= $5X10^6$ units, A_p= \$ 50.00 per setup, A_r= \$3000 per order, H_p= \$ 1.20 per unit per year, H_j= \$ 1.08 per unit per year, x = 1000 units.

Solution:

Let r_j = 1.00

From equation-16 ,

number of lot size n_j = 8.16

From equation-15,

the economical lot size Q_j^* = 18257.41 units

From equation-14,

Total cost TC = $182905.79

To find the integer value of n_l, we try for n_l = 8 & 9

Value of n_l	Q^*	TC	Remarks
8	18588	182909.81	Optimum
9	16758	182997.27	

So number of lots n_i = 8 and lot size =18588 units.

<u>Example-4</u>: Consider an example of multiple lot of raw material for a multiple lot of product. The required data are as follows:
$D_p = 4X10^6$ units, $P_p= 5X10^6$ units, A_p= \$ 50.00 per setup, A_i= \$3000 per order, H_p= \$ 1.20 per unit per year, H_i= \$ 1.08 per unit per year, x = 1000 units.

Solution:

Let r_i = 1.00

From equation-20,

number of full shipment during the cycle time m = 1.37

From equation-21,

number of lot size n_i = 10.958

From equation-20,

the economical lot size m = 1.3714

From equation-19,

Total cost TC = \$182914.3

To find the integer value of m & n_i, we try for m = 1 & 2 and n_i = 10 & 11

Value of m	Value of n_i	TC	Remarks
1	10	197720	
2	10	191440	Optimum
1	11	192211	
2	11	196785.5	

So number of full shipment during the cycle time m = 2, number of lots n_i = 10 and lot size = 20000 units.

3.4 DISCUSSION:

To avoid the complexity in formulation, the variables m and n are assumed to be integers for all cases in section 3.2. However, this is relaxed in the solution approach which is done with the help of traditional optimization method. As a result, it is not expected that the solution will be optimal. This method can be considered as a simple heuristic to solve the stated problem with little computational effort and an acceptable level of solution. To check the quality of solution of the proposed method, a search algorithm would be used as given in the next Chapter.

CHAPTER - 4

COMPUTATIONAL EXPERIENCES

4.1 INTRODUCTION

4.2 SEARCH ALGORITHM

4.3 INPUT DATA

4.4 RESULT

4.5 QUALITY OF SOLUTION OF THE PROPOSED METHOD

4.6 SENSITIVITY ANALYSIS

Chapter-4

COMPUTATIONAL EXPERIENCES

In the previous chapter, several mathematical models for batch sizes of finished product and ordering quantities of raw materials under different conditions have been formulated. The input data of these models are annual demand of finished product(D_p).set-up cost of finished product(A_p), inventory holding cost of finished product(H_p), ordering cost of raw material(A_j), inventory holding cost of finished product(H_j). In this chapter, the computational analysis has been carried out with various input data.

Also presented in this chapter presents input data, results and discussion, a brief discussion on quality of solution of the proposed method, and analysis of the results.

4.1 SEARCH ALGORITHM

To compare the result of the proposed method, the total cost function (which is a nonlinear function with one or multiple variables) is solved using a Search Algorithm. The well known Hooke and Jeeves' method (Hooke and Jeeves, 1961) is applied in this case. This is a very efficient and ingenious procedure. The search consists of a sequence of exploration steps about a base point which if successful are followed by pattern moves. The flow chart for Hooke and Jeeves' method is given in the appendix-A. This method is coded in FORTRAN-77 on a IBM compatible PC 80386 with a PROFOR compiler.

4.2 INPUT DATA:

A number of data group have been designed to study the effect of various input parameters on inventory system response. The total cost is considered as the system response. Though the data were somewhat similar to Sarker et al(1995) and Sarker and Parija(1994), the arbitrary demand, setup cost, ordering cost and holding cost data are randomly generated for different test problems. The sample data set are given below.

D_p units/year	A_p \$/setup	H_p \$/unit/year	A_r \$/order	H_r \$/unit/year
$2X10^6$, $3X10^6$, $4x10^6$, $5X10^6$	50	1.2	3000	1.08

Table 4.1. Data set for annual demand

In table-4.1, setup cost, ordering cost and holding cost arre kept fixed while the demand data are varied. In table-4.2, demand, ordering cost and holding cost are kept fixed while the setup cost data are varied. In table-4.3, fixing demand, setup cost, ordering cost, and holding cost of raw material, the holding cost of product data are varied.

D_p units/year	A_p \$/setup	H_p \$/unit/year	A_i \$/order	H_i \$/unit/year
$4X10^6$	50, 500, 1000, 3000	1.2	3000	1.08

Table 4.2. Data set for setup cost of finished product

D_p units/year	A_p \$/setup	H_p \$/unit/year	A_i \$/order	H_i \$/unit/year
$4X10^6$	50	1.08, 1.2, 2, 3	3000	1.08

Table 4.3. Data set for inventory holding cost of finished product

D_p units/year	A_p \$/setup	H_p \$/unit/year	A_i \$/order	H_i \$/unit/year
$4X10^6$	50	1.2	1000, 2000, 3000, 3500	1.08

Table 4.4. Data set for ordering cost of raw material

D_p units/year	A_p \$/setup	H_p \$/unit/year	A_i \$/order	H_i \$/unit/year
$4X10^6$	50	1.2	3000	1.08, 1.2, 2, 3.

Table 4.5. Data set for inventory holding cost of raw material

In table-4.4, fixing demand, setup cost and holding cost the ordering cost of raw material data are varied. In table-4.5, fixing demand, setup cost, ordering cost, and holding cost of product, the holding cost of raw material data are varied.

4.3 RESULTS:

The different models developed in the previous Chapter are used to solve the different test problems presented in section-4.2. The results are shown in the following sub-sections. For convenience of analysis, the different models are defined as follows:

Case-1: Lot for Lot-Delivery of Product in Single Installments

Case-2: Lot for Lot-Delivery of Product in Multiple Installments

Case-3: Multiple Lot for a Lot-Delivery of product in Single Installment

Case-4: Multiple Lot for a Lot- Delivery of Product in Multiple Installments

4.4.1 CASE-1 (Lot for Lot):

The total costs of the inventory system and ordering quantities using both proposed method (equation-6 & 7 of Chapter-3) and search algorithm with a data set given in table-4.1 are shown in table-

4.6.

D_p	Q (Proposed method)	TC(proposed method)	Q(Search alg.)	TC(Search alg.)
$2X10^6$	73150	166781.29	73132	166781.29
$3X10^6$	89589	204264.53	89772	204264.95
$4X10^6$	103449	235864.35	103385	235866.40
$5X10^6$	115659	263704.40	115539	263704.50

Table 4.6: Variation of total cost for variation of annual demand of the product

The total costs of the inventory system and ordering quantities using both proposed method (equation-6 & 7 of Chapter-3) and search algorithm with a data set given in table-4.2 are shown in table-4.7.

A_p	Q (Proposed method)	TC(proposed method)	Q(Search alg.)	TC(Search alg.)
50	103449	235864.35	103385	235866.40
100	104293	237789.82	104293	237789.82
200	105962	241596.50	106129	241595.00
300	107605	245340.57	107515	247340.17

Table 4.7: Variation of total cost for variation of ordering cost of product.

The total costs of the inventory system and ordering quantities using both proposed method (equation-6 & 7 of Chapter-3) and search algorithm with a data set given in table-4.4 are shown in table-4.8

A_i	Q (Proposed method)	TC(proposed method)	Q(Search alg.)	TC(Search alg.)
1000	60698	138390.74	60699	138390.74
2000	84811	193370.11	84805	193370.11
3000	103449	235864.35	103444	235864.37
3500	111607	254464.14	111569	254464.14

Table 4.8: Variation of total cost for variation of ordering cost of raw material.

The total costs of the inventory system and ordering quantities using both proposed method (equation-6 & 7 of Chapter-3) and search algorithm with a data set given in table-4.3 are shown in table-4.9.

H_p	Q (Proposed method)	TC(proposed method)	Q(Search alg.)	TC(Search alg.)
0.5	124270	196346.64	124269	196346.62
1.0	108309	225282.04	108318	22528.04
1.2	103449	235864.35	103444	23586.37
2.0	89006	274138.70	88986	274138.70

Table 4.9: Variation of total cost for variation of inventory carrying cost of the finished product.

The total costs of the inventory system and ordering quantities using both proposed method (equation-6 & 7 of Chapter-3) and search algorithm with a data set given in table-4.5 are shown in table-4.10.

H_i	Q (Proposed method)	TC(proposed method)	Q(Search alg.)	TC(Search alg.)
0.5	119804	203666.40	119799	203666.38
1.08	103449	235864.35	103444	235864.37
1.2	100830	241991.74	100820	241991.72
1.5	95063	256671.01	95061	256671.01

Table 4.10: Variation of total cost for variation of inventory carrying cost of raw materials.

4.4.2 CASE-2 (Lot for lot):

The total costs of the inventory system, ordering quantities and the lot number 'm' using the proposed method (equation-11, 12 & 13 of Chapter-3) with a data set given in table-4.1 are shown in table-4.11. The search algorithm produces similar results.

D_p	m	Q	TC
$2X10^6$	8	80000	154730
$3X10^6$	9	90000	211226
$4X10^6$	10	100000	267200
$5X10^6$	10	100000	320500

Table 4.11: Variation of total cost for variation of annual demand of the product.

The total costs of the inventory system, ordering quantities and lot number 'm' using the proposed method (equation-11, 12 & 13 of Chapter-3) with a data set given in table-4.2 are shown in table-4.12.

A_p	m	Q	TC
50	10	100000	267200
500	10	100000	289112.20
1000	10	100000	309988.20
3000	13	130000	375175.4

Table 4.12: Variation of total cost for variation of ordering cost of product.

The total costs of the inventory system, ordering quantities and lot number 'm' using the proposed method (equation-11, 12 & 13 of Chapter-3) with a data set given in table-4.4 are shown in table-4.13.

A_i	m	Q	TC
1000	6	60000	154720
2000	7	70000	216982.90
3000	10	100000	267200
3500	10	100000	287200

Table 4.13: Variation of total cost for variation of ordering cost of raw material.

The total costs of the inventory system, ordering quantities and lot number 'm' using the proposed method (equation-11, 12 & 13 of Chapter-3) with a data set given in table-4.3 are shown in table-4.14.

H_n	m	Q	TC
1.08	10	100000	257000
1.2	10	100000	267200
2	7	70000	320525
3	6	60000	76253

Table 4.14: Variation of total cost for variation of inventory carrying cost of the finished product..

The total costs of the inventory system, ordering quantities and lot number 'm' using the proposed method (equation-11, 12 & 13 of Chapter-3) with a data set given in table-4.5 are shown in table-4.15.

H_l	m	Q	TC
1.08	10	100000	267200
1.2	10	100000	272000
2	8	80000	296900
3	7	70000	327885

Table 4.15: Variation of total cost for variation of inventory carrying cost of raw materials.

4.4.3 CASE-3 (Multiple lot for a lot):

The total costs of the inventory system and ordering quantities using both proposed method (equation-14 & 15 of Chapter-3) and search algorithm with a data set given in table-4.2 are shown in table-4.16.

A_p	n (Prop. method)	Q	TC (proposed method)	n (Search alg.)	Q	TC (Search alg.)
50	8	18588	182909.81	8	18626	182910.18
100	6	25000	192000.00	6	25074	192000.85
200	4	37105	204821.86	4	37226	204822.95
600	2	70711	237587.88	2	70600	237588.16
1000	2	77152	270111.10	2	77127	270111.10
3000	1	145095	330817.20	1	145088	330817.20

Table 4.16: Variation of total cost for variation of ordering cost of product.

The total costs of the inventory system and ordering quantities using both proposed method (equation-14 & 15 of Chapter-3) and search algorithm with a data set given in table-4.4 are shown in table-4.17.

A_1	n (Prop. method)	Q	TC (proposed method)	n (Search alg.)	Q	TC (Search alg.)
1000	5	17407	114891.26	5	17489	114892.50
2000	7	17510	153384.67	7	17490	153841.76
3000	8	18588	182909.81	8	18626	182910.18
3500	9	17931	195809.00	9	17866	195810

Table 4.17: Variation of total cost for variation of ordering cost of raw material.

The total costs of the inventory system and ordering quantities using both proposed method (equation-14 & 15 of Chapter-3) and search algorithm with a data set given in table-4.1 are shown in table-4.18.

D_p	n (Proposed method)	Q	TC (proposed method)	n (Search alg.)	Q	TC (Search alg.)
$2x10^6$	8	13144	129336.78	8	13141	129336.78
$3x10^6$	8	16095	158404.54	8	16039	158405.91
$4x10^6$	8	18588	182909.81	8	18626	182910.18
$5x10^6$	8	20782	204499.39	8	20804	204499.49

Table 4.18: Variation of total cost for variation of annual demand of the product.

The total costs of the inventory system and ordering quantities using both proposed method (equation-14 &15 of Chapter-3) and search algorithm with a data set given in table-4.3 are shown in table-4.19.

H_p	n (Prop. method)	Q	TC (proposed method	n (Search alg.)	Q	TC (Search alg.)
1.2	8	18588	182909.81	8	18626	182910.18
1.0	7	21149	181031.97	7	21213	181032.79
0.5	5	29688	175157.08	5	29743	175157.39
2.0	11	13639	189303.02	11	13630	189303.05

Table 4.19 : Variation of total cost for variation of inventory carrying cost of the finished product.

The total costs of the inventory system and ordering quantities using both proposed method (equation-14 & 15 of Chapter-3) and search algorithm with a data set given in table-4.5 are shown in table-4.20.

H_1	n (Proposed method)	Q	TC (proposed method)	n (Search alg.)	Q	TC (Search alg.)
1.08	8	18588	182909.81	8	18626	182910.18
1.2	8	17743	191624.64	8	17678	191625.93
1.5	7	18089	211646.61	7	18045	211646.98
0.5	12	18257	131453.3	12	18243	131453.45

Table 4.20: Variation of total cost for variation of inventory carrying cost of raw materials.

4.4.4 CASE-4 (MULTIPLE LOT FOR A LOT):

The total costs of the inventory system and ordering quantities using both proposed method (equation-19 & 20 of Chapter-3) and search algorithm with a data set given in table-4.1 are shown in table-4.21.

D_p	m (proposed method)	n (Props. method)	TC (proposed method)	m (Search alg.)	n (Sea. alg.)	TC (Seach alg.)
$2x10^6$	1	9	124426.7	1	9	124426.7
$3x10^6$	1	11	157658.2	1	11	157658.2
$4x10^6$	2	10	191440	2	10	191440
$5x10^6$	2	11	217481.8	2	11	217481.8

Table 4.21: Variation of total cost for variation of annual demand of finished product.

The total costs of the inventory system and ordering quantities using both proposed method (equation-19 & 20 of Chapter-3) and search algorithm with a data set given in table-4.2 are shown in table-4.22.

A_p	m (proposed method)	n (Props. method)	TC (proposed method)	m (Search alg.)	n (Search alg.)	TC (Search alg.)
50	2	10	191440	2	10	191440
500	5	3	243600	5	3	243600
1000	7	2	280497	7	2	280497
3000	11	2	308337	11	2	308335

Table 4.22: Variation of total cost for variation of ordering cost for finished product.

The total costs of the inventory system and ordering quantities using both proposed method (equation-19 & 20 of Chapter-3) and search algorithm with a data set given in table-4.4 are shown in table-4.23.

A_l	m (proposed method)	n (Props. method)	TC (proposed method)	m (Search alg.)	n (Search alg.)	TC (Search algh)
1000	1	7	118663	1	7	118663
2000	2	8	159840	2	8	159840
3000	2	10	191440	2	10	191440
3500	1	12	205187	1	12	205187

Table 4.23: Variation of total cost for variation of ordering cost of cost of raw materials.

The total costs of the inventory system and ordering quantities using both proposed method (equation-19 & 20 of Chapter-3) and search algorithm with a data set given in table-4.3 are shown in table-4.24.

H_p	m (prop. method)	n (Props. method)	TC (proposed method)	m (Search alg.)	n (Search alg.)	TC (Search alg.)
1.08	2	9	185746.67	2	9	185746.67
1.2	2	10	191440	2	10	191440
2	1	15	187920	1	15	187920
3	1	16	192320	1	16	192320

Table 4.24: Variation of total cost for variation of inventory carrying cost of raw materials

The total costs of the inventory system and ordering quantities using both proposed method (equation-19 & 20 of Chapter-3) and search algorithm with a data set given in table-4.5 are shown in table-4.25.

H_i	m (prop. method)	n (Props. method)	TC (proposed method)	m (Search alg.)	n (Search alg.)	TC (Search alg.)
1.08	2	10	191440.00	2	10	191440.00
1.2	2	9	197866.67	2	9	197866.67
2	2	7	247314.30	2	7	247314.30
3	2	5	315314.30	2	5	315314.30

Table 4.25: Variation of total cost for variation of inventory carrying cost of raw materials

4.5 QUALITY OF SOLUTION OF THE PROPOSED METHOD

To check the quality of the proposed method, Hooke and Jeeves'

Search Algorithm was applied. The results produced by Search algorithm are presented in the previous section. The variations in the total costs in percentage for different models developed in Chapter-3, with different data set presented in Section 4.2, are given below.

Variable	Model	% of maxm. variation of TC from Search Algorithm
D_p	Lot for Lot: Delivery of product in single installment.	$0.896X10^{-3}$
A_p		$0.896X10^{-3}$
A_i		$0.896X10^{-3}$
H_p		$0.896X10^{-3}$
H_i		$0.896X10^{-3}$
D_p	Lot for Lot: Delivery of product in multiple installments.	0
A_p		0
A_i		0
H_p		0
H_i		0
D_p	Multiple Lot for a Lot: Delivery of product in single installment.	$0.864X10^{-3}$
A_p		$.053 X 10^{-3}$
A_i		0.297
H_p		0
H_i		$0.673 X 10^{-3}$
D_p	Multiple Lot for a Lot: Delivery of product in multiple installments.	0
A_p		$0.648 X 10^{-3}$
A_i		0
H_p		0
H_i		0

Table 4.26. % of maximum variation of total cost from search algorithm for different models and different variables.

The above table indicates that the proposed method produces very much similar results to the search algorithm. This method is very easy to understand and it takes little computational time as compared to the search algorithm. Therefore the proposed method could be useful for practical use.

4.6 SENSITIVITY ANALYSIS

The behavior of total costs with different parameters are analysed in this section. Figure 4.1 shows the behavior of total costs with annual demand for all four cases. The total cost increases with demand for all cases if other parameters are fixed. However the rate of change of total cost is the highest for case-2. It is clear that Case-1 is better than Case-2 if the demand is greater than $2.5x10^6$ and Case-3 is better than Case-4 if the demand is greater than $3.0x10^6$.

The total cost increases with the increase of setup cost, as shown in fig.-4.2, if other parameters are fixed. Case-1 is always better than Case-2 within the given range. The rate of change of total cost decreases for Case-3 & 4 after setup 1000 and Case-4 is better than Case-3 for setup cost grater than 2000.

The total cost increases with the product holding cost if other parameters are fixed. This is shown in Figure-4.3. Here, Case-1

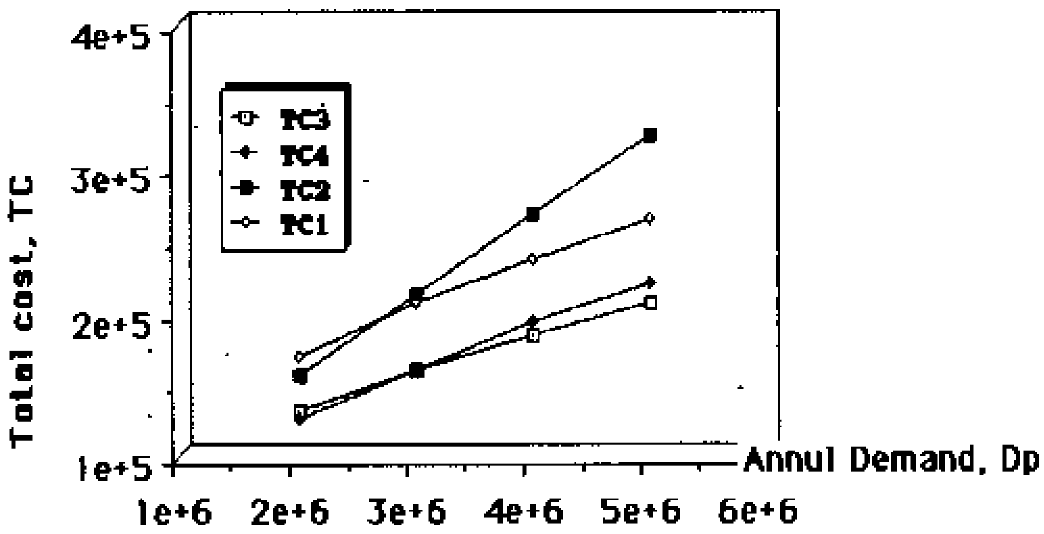

Fig. 4.1 : Annul Demand VS Total Cost

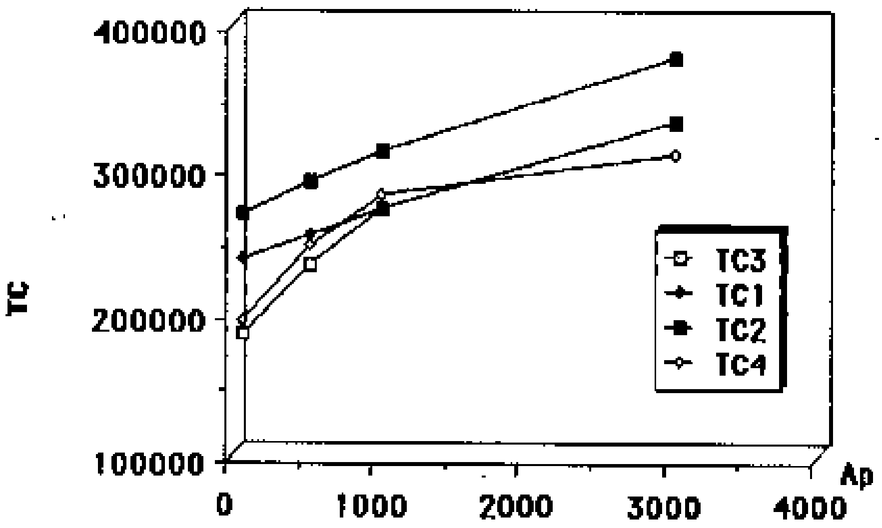

Fig.4.2: Setup cost of product VS TC

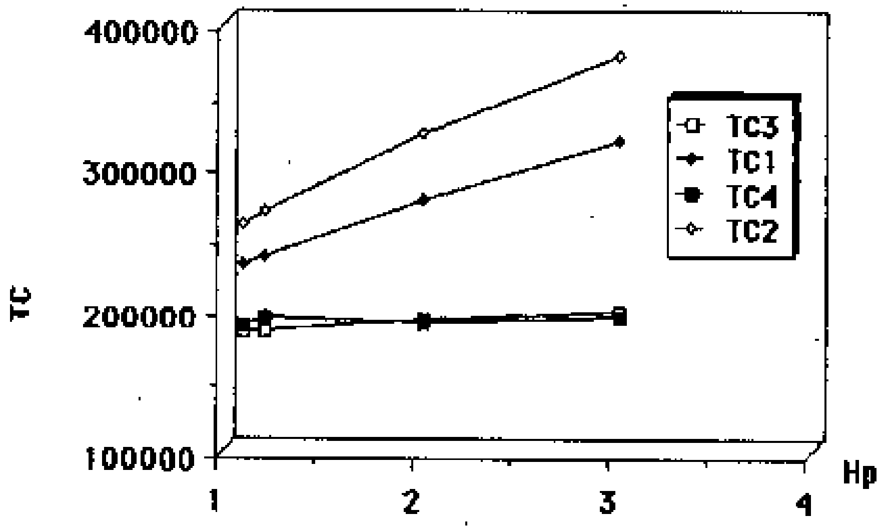

Fig.4.3:Holding cost of product VS TC

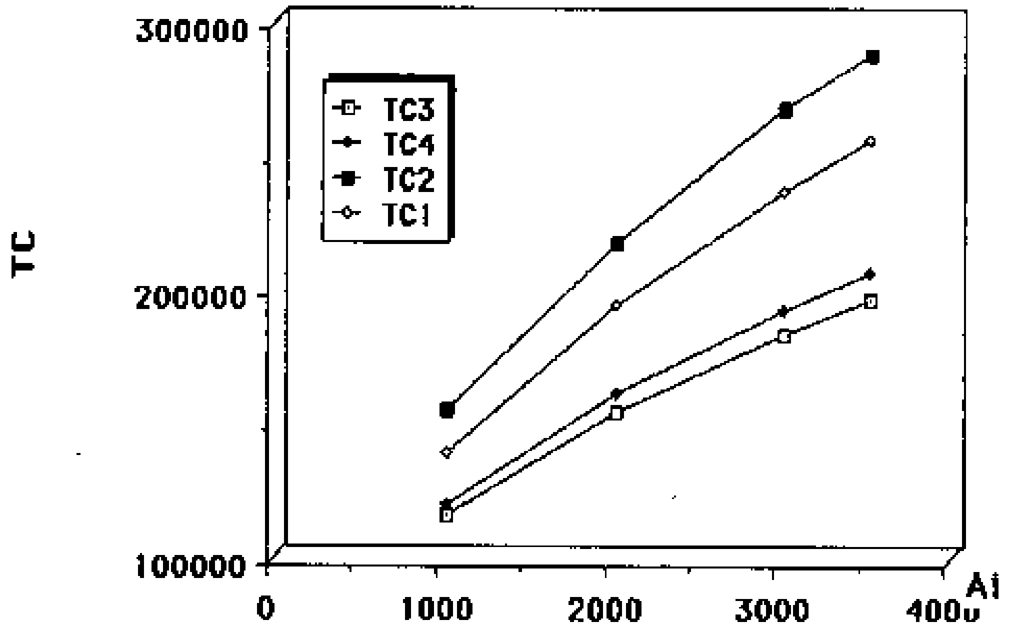

Fig. 4.4: Setup cost VS Total cost

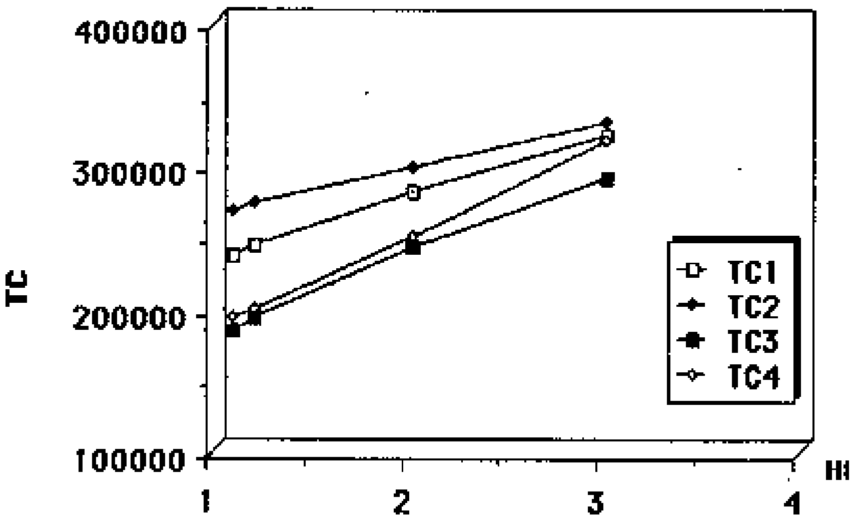

Fig.4.5: Inventory holding cost of raw material VS TC

is always better than Case-2 within the given range. However Case-4 is better than Case-3 for holding cost higher than 2.0.

With the increase of ordering cost of raw material, the total cost increases which is shown in fig.4.4. Here Case-1 is better than Case-2 and Case-3 is better than Case-4 within the range shown in figure. And the difference between Case-1 & 2 and Case-3 & 4 increase with the increase of ordering cost.

Figure-4.5 shows total cost increases with the increase of holding cost of raw material if other parameters are fixed. The difference of total cost between Case-1 & 2 decreases with the increase of holding cost of raw material. Case-1 is better than Case-2 and similarly Case-3 is better than Case-4 within the range considered in this thesis.

CHAPTER - 5

CONCLUSION AND FUTURE WORK

5.1 CONCLUSION

5.2 RECOMMENDATION FUTURE WORK

Chapter - 5

CONCLUSION AND FUTURE WORK

5.1 CONCLUSION

In this research, a manufacturing system which procures raw materials from outside suppliers and processes them to convert to finished products is considered. Traditionally economic lot sizes for manufacturing a product and purchasing raw material are determined separately. However, when the raw materials are used in production, their ordering quantities are dependent on the batch quantity of the product. Therefore, it is not desirable to isolate the problem of economic purchase of raw materials from economic batch quantity. As a result, it is required to determine the optimum production lot size of a product and the ordering quantities of associated raw materials together. This is done treating the production and purchasing as components of a single system, minimizing the total cost of the system.

The ordering policy of a raw material can either be based on its economic ordering quantity (EOQ) or on the requirement of the raw material in a lot of economic production quantity (EPQ). The latter reflects the dependent relationship between raw material requirement and production quantity. The relationship between products and its raw material has been the basic consideration for the model developed. The problem is considered from the point of view of the benefit to the manufacturing firm. The proposed systemic view supports the fact that the optimum purchasing-production policy should be determined by considering the system as a whole and not by treating its components individually.

In this thesis, the total cost equations are formulated for batch size of a production system operating under a fixed-quantity, periodic delivery policy as an unconstrained nonlinear optimization problem for four different cases. Here, it is assumed that the finished product can not be delivered until the whole lot is finished and the quality certification is ready. The optimal batch sizes are determined for all the cases with the help of traditional optimization approach. The proposed is a hueristic or approximate method to produce a near optimal solution. This method does not guarantee an optimal solution because of

- dependent relationships considered in the formulation
- relaxation of integer varaible to continuous variable in determining the optimum batch sizes

However, the solutions produced, for a substantial number of test problems, are compared with the results of a search algorithm (Hooke and Jeeves' algorithm) to judge the performance of the proposed method. The comparison of results are presented in section-4.4. It indicates that the proposed method produces very much similar results to the search algorithm. This method is very easy to understand and it takes little computational time as compared to the search algorithm. The proposed method could be helpful for easy implementation. The sensitivity analysis is also performed.

The model developed in this thesis could be applied in real life cases, where, a finite production-rate environment uses raw materials taking from outside suppliers with a supply policy of delivering equal quantity at fixed intervals.
Case-2 is more sensative with the variation of annual demand (fig.4.1), setup cost (fig.4.2), holding cost of product (fig.4.3) and ordering cost of raw material (4.4) than other cases. Fig.4.5 shows case-4 is more sensative with holding cost of raw material than other cases. The variation of holding cost of product (fig.4.3) has a little effect on case-3 and case-4.

5.2 RECOMMENDATION FOR FUTURE WORK:

Though the present area of research is not new, only a few research papers have been observed in the literature. As a result, there are many open topics and/or unansweredable questions for further research. Some of them are described below:

- Real world data can be used to study and compare the performance of the proposed methods.

- Experimentation can be done with continuous delivery system of product at a constant rate, after the completion of the batch production.

- Using large number of data with closer step size and wide range. the performance of the proposed methods can be compared.

- Experimentation can be done with Multiple raw materials for single product

- Experimentation can be done with Multiple Products with single raw material (common) or multiple raw material (common) or multiple raw material of different type from the same or different suppliers

- Experimentation can be done with Multiple customers
- Experimentation can be done with the production system with aging period before delivery of the finished product.

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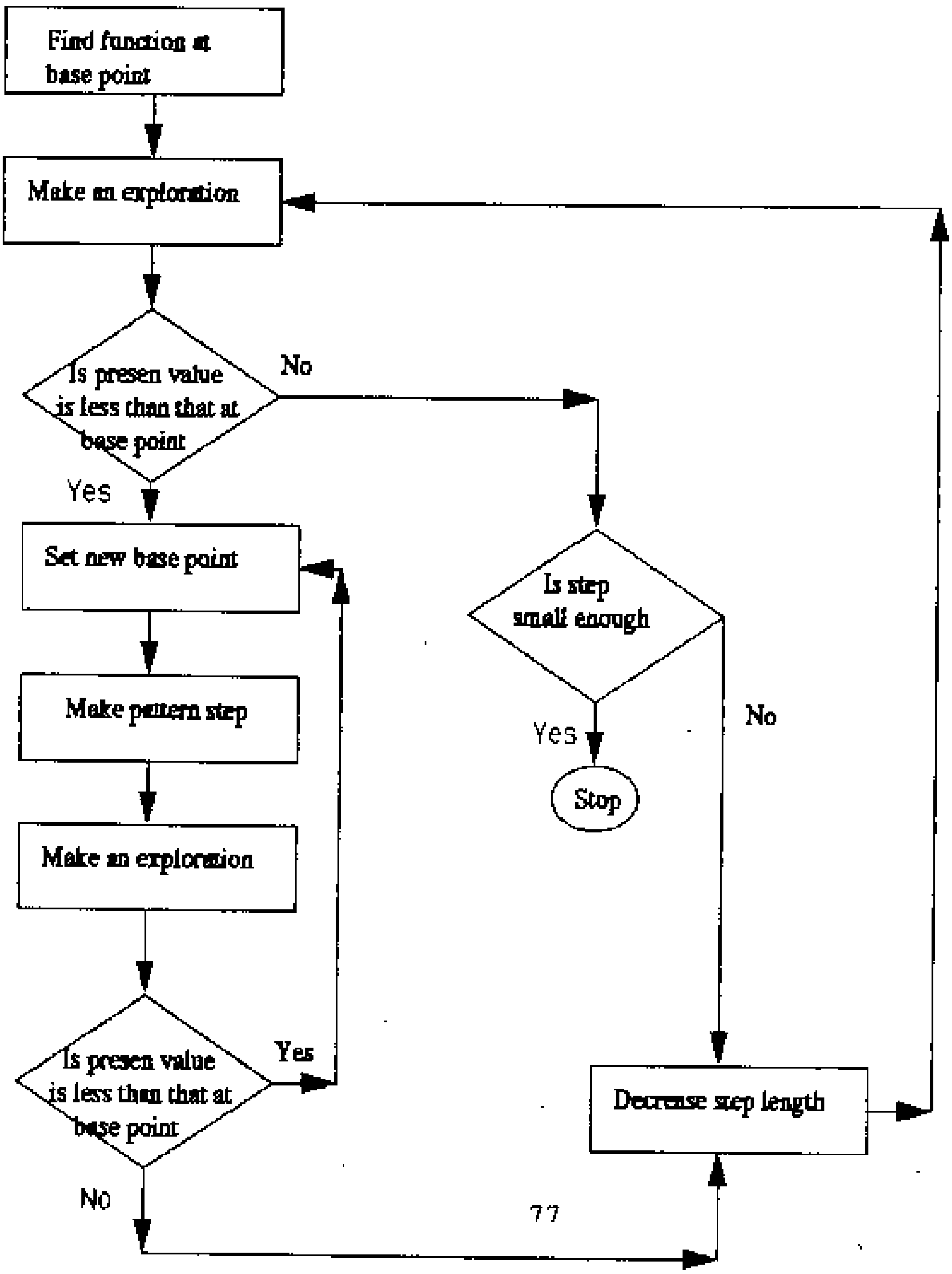
APPENDIX

Flow Chart for Hooke Jeeves



Flow Chart for an Exploration