

A SIMPLE MATHEMATICAL MODEL OF ALTERNATORS FOR
USE IN POWER SYSTEM STABILITY STUDIES

BY

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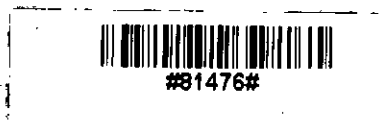
A THESIS

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DEDICATED TO MY TEACHERS

DECLARATION

No portion of this work has been submitted to any other University or similar institution for the award of any degree.

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ABSTRACT

The work describes the development of a simple but accurate model for alternators suitable for the use in power system stability studies. The model is developed from a detail model whose validity was checked by comparing its simulation with some test results. Since the rotor angle movement following a fault is the determining factor in deciding the stability of a machine, accurate prediction of rotor angle was judged as goodness of the model. The speed of the simple model was gained by neglecting the rate of change of stator fluxes. But due to neglecting the rates, the oscillatory component of torque disappeared from the model. The effect of the missing component of torque was compensated by introducing an analytical expression for rotor angle. The compensation has to be applied at each time step of the solution. This is necessary because of representing the component of infinite bus voltage accurately. The accuracy of the developed model was checked by simulating rotor movement during short circuit conditions both from no-load and preloaded machine. The validity of the model was further established by simulating fault throwing tests.

List of Symbols

X_a	=	Armature reactance.
R_a	=	Armature resistance.
X_{md}	=	Direct axis magnetising reactance.
X_{kd}	=	Direct axis damper reactance.
R_{kd}	=	Direct axis damper resistance.
X_f	=	Field reactance.
R_f	=	Field resistance.
X_{mq}	=	Quadrature axis magnetising reactance.
X_{kq}	=	Quadrature axis damper resistance.
V_d	=	Direct axis component of voltage.
V_q	=	Quadrature axis component of voltage.
V_{bd}	=	Direct axis component of bus-bar voltage.
V_q	=	Quadrature axis component of bus-bar voltage.
V_f	=	Field voltage.
X_d'	=	D-axis transient reactance.
X_d''	=	D-axis subtransient reactance.
X_q''	=	Q-axis subtransient reactances.
T_d'	=	D-axis transient time constant.
T_d''	=	D-axis subtransient time constant.
T_q''	=	Q-axis subtransient time constant.
$\Phi's$	=	Flux terms.
$Y's$	=	Admittance terms.
I_a	=	Terminal current.
I_d	=	Direct axis component of current.

I_q = Quadrature axis component of current.

P_T = Terminal power of the machine.

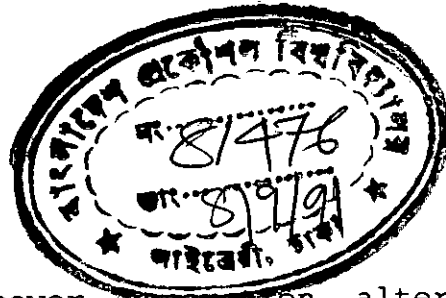
p = Derivative with respect to time (d/dt).

ω_o = Synchnous speed.

δ = Rotor angle.

Chapter 1
Introduction

1.1 General



In order to achieve economic power generation alternators are now-a-days built for large output power. Compared to small machines, large generators are relatively small in size and have low inertia. These machines are very sensitive dynamically. This sensitivity increases with the development of transmission system. Since power is now being transmitted over longer distances, the generators are called upon to provide reactive compensation involving operations with larger rotor angle. Such operations are always prone to instability in transient conditions. The increased sensitivity of present power system requires the development of improved control systems. The development is only possible if accurate prediction of disturbances can be made. On the other hand, online control becomes only successful when alternators behaviour can be modelled accurately.

The first attempt to model power systems involved the construction of network analyser [1]. The simplest analyser being dc driven, ac sources of variable magnitude and phase were used in its improved versions. The network analysers were found to be satisfactory for steady state analysis but not for transients. Their failure in respect to transient studies was due to inadequate representation of synchronous machines. Two approaches emerged to overcome these

inadequacy. The first involved [2] using analogue computers to model the machines. This proved quite successful, particularly with the introduction of operational amplifier technology which enabled very complex studies to be undertaken. In the second approach, the machines were modelled using scaled down replicas having ratings of order 1 kVA. Hence entitlement 'micromachines' [3]. These machines when interconnected with model transformers and transmission lines, offered instantaneous results in real time.

The use of network analysers and dynamic models declined with the advent of the digital computer which offered flexibility in operation. Parameters can be changed and system configuration modified with the addition of protection equipment and other auxiliaries, almost instantaneously. The digital computer also provides power system engineers with a very compact powerful tool having a modelling capacity in terms of number of generators and transmission links well beyond the most complex network analyser and dynamic models. Although the digital computer provided adequate means of simulating steady state operation, it has been found to give disappointing results in transient studies. In this respect, there are two main difficulties. The first difficulty arises from the parameters of the machine which is very difficult to extract

and the second arises from the inclusion of variation of machine parameters due to saturation.

A number of equivalent circuits and procedure of extracting machine parameters have been suggested [4]. Until recently , one equivalent circuit whilst modelling a particular test accurately was found to give inaccurate results for another different test. A rigorous test of different equivalent circuit has been performed [5] and Canay's [6] equivalent circuit is termed as universal representation of alternator which is capable of modelling a wide range of tests.

Inclusion of saturation has been the subject of much work [7] during the last decade. Saturation during steady state can easily be included in the alternator model. Inclusion of saturation during transient is divided into two part. One is for mutual path saturation and other is for leakage path saturation. Much attention is given for saturation in the mutual path. Since leakage path remains unaffected in most of the transient time of consideration, saturation in this path remained almost unattended. But recent study [5] shows that dynamics of an alternator may severely be affected by the leakage path saturation due to large current flowing during initial portion of transients. A means of including saturation has also been formulated.

1.2 Evolution of Digital Model

The simplest digital model of an alternator is based on the equivalent circuit [8] often referred to as the 'constant emf behind constant reactance' representation as illustrated in Figure 1.1 where V_t , E_g and I_a are the terminal voltage, internal emf and armature current respectively whilst X_s is the synchronous reactance of the machine. The model relates to the steady state analysis of synchronous machines. In order to account for transients, X_s is modified to equal the subtransient and transient reactances, X_s'' and X_s' , according to the period of interest. Although the arrangement is still being used to represent machines in some of the complex multimachine studies, it is no longer employed in the detailed study of a transient phenomena, specially involving accurate dynamics analysis of the machines.

The failure of the simple model under transient conditions arises from it not accounting for variations in terminal quantities due to changes in rotor velocity. This is overcome by expressing the inductances of the coils of the machine as a function of rotor position. The concept is illustrated by considering a hypothetical two winding machine as shown in Figure 1.2. At a particular instant of its rotation, the rotor coil occupies a position making an

angle θ with respect to the stationary stator coil. Since flux linkage associated with the coils per unit current varies with position of the rotor, the transfer impedances between field and stator coils can be expressed as a function of angle ' θ ' [9,10]. This expression may include the higher harmonics to describe nonsinusoidal flux distribution. It follows that a machine's terminal voltage and current can be derived under steady state and transient conditions using impedance values calculated according to the rotor position. To calculate a change in terminal voltage and current requires the inversion of the machines impedances matrix. Since the machines impedance changes with the rotor position, the inversion has to be made very many times in a typical transient calculation and so demanding large computer time.

Park [9,10] eliminated these impedance changes by reducing the stator of synchronous machine into a two coil equivalent turning in synchronism with its rotor. One of the coils is aligned with the field or 'direct axis' whilst the other is displaced by 90° to lie along the quadrature axis. The coils are assigned fixed parameters to maintain equality of the airgap mmf with 3-phase machine. This reduction into two axis was based on the assumption that the flux distribution within the machine is sinusoidal, which is reasonable since to avoid the generation of harmonics, the windings are

usually distributed sinusoidally. The concept was refined by Kron [11] to yield a basic comprehensive and practical model of a synchronous machine.

The early work of Park and Kron was of an absolute mathematical kind. But since they did describe a practical system attempts were made to resolve their formulation into equivalent circuits to provide a more tangible picture of the physical interactions that take place in synchronous machines. The simplest diagrammatic representation of the mathematical form is shown in Figure 1.3. Fig 1.3a is the equivalent circuit of direct axis and Fig. 1.3b of the quadrature axis. Referring to Fig 1.3, V_f corresponds to excitation voltage of the machine. Reactances associated with the various windings are represented by X_a , X_f , X_{kd} , X_{kq} corresponding to armature, field and damper leakages respectively. X_{md} and X_{mq} represent mutual coupling between stator and rotor circuit whilst the letter 'R' represents the various resistances.

The equivalent circuits of Fig.1.3 provides a very ready understanding of a machine and are convenient to manipulate mathematically. But problems arise in establishing values for the various parameters. Three phase short circuit test results are usually employed with stator current and on some occasion field current oscillograms. Conventionally

[12] the quantities X_d' , X_d'' , T_d' and T_d'' are obtained from the stator current envelope of the test record. It is from these values the equivalent circuit parameters are extracted. In this analysis, the usual practice is to assume the damper windings to be acting only during the sub-transient period. The parameters thus obtained when used to simulate stator quantities produce reasonably accurate results, but gives erroneous simulation of the field phenomena. Error in field simulation emerged relatively recently with the need to predict field changes in order to design fast acting excitation control system.

The problem of field current simulation has been considered by many people. Adkins [13] recognised it to be caused by falsely assuming equal mutual coupling between stator and rotor circuits and that between field and damper circuits. But his analysis was still based on the conventional assumption that damper circuits are only effective during the sub-transient period. Shackshaft [14] disagreed with this assumption. On the other hand, he agreed with the conventional view that mutual coupling between the various circuits can be made equal by choosing appropriate rotor base currents. Canay [6], like Adkins, represented unequal mutual coupling in the equivalent circuit whilst agreeing with Shackshaft on the action of damper circuit during transient conditions. His solution gave the accurate

prediction of the field current.

The parameters of an equivalent circuit be it of the Adkins, Shackshaft or Canay type are conventionally obtained from the three phase short circuit tests. Since operation under this condition is confined to the machines direct axis, only direct axis parameters are obtainable. Quadrature axis parameters are extrapolated from these with modification according to the geometry of the generator concerned. This is not an entirely satisfactory state of affairs. To determine q-axis parameters directly requires alternative tests. One of these is the stator decrement test [12].

On the basis of the accurate equivalent circuits, the differential equations for the flux linkages on different coils and equation of motion are written and arranged in state space form [15]. Using digital computer, the equations are solved in steps. The changes in parameter due to any type of non-linearity can be incorporated in the solution.

The step by step procedure of solving machine's differential equations consumes large time and becomes impracticable to be used for multimachine study. Approximations are employed to reduce the computing time. In most cases the approximate model is used to obtain an idea about the study, an exact

assessment is not possible. But some of the power system studies e.g., stability analysis requires precise prediction under different fault condition. For example, in order to design the fault clearing time for a circuit breaker of a line, the maximum rotor angle movement and time to reach that maximum value are two important information that needs to know accurately beforehand. Therefore, a simple but accurate alternator model is always sought.

1.3 Proposed Work

Previous works show that the Canay's representation of equivalent circuit gives accurate results in predicting 3-phase stator and field current. The accuracy of the representation can only be established which when applied to simulate dynamic behaviour of a machine. In order to model a machine's dynamic behaviour the interchange of electrical and mechanical energy is required to be represented accurately. Again, since the simulation process in the detail model has to be gone through step by step to solve the machine's differential equations, it involves large computing time. This puts a serious restriction in its use in real time control or even for off line prediction for a disturbance on a system involving multi-machine. A Simple model has to be devised which would consume less computing time for the use in multi-machine study and real time

control.

Considering the first, the author's task was to investigate effectiveness of the Canay's equivalent circuit representation for use in predicting dynamic behaviour of the machine. This involved development of a detail machine model using a digital computer. The model was based on machine's differential equations and used with the parameters obtained from Canay's representation. The author's second task was to develop a technique of simplifying the model in order to reduce computing time so that it becomes suitable for use in multimachine study. The simplification was tested against the results obtained from the detailed model developed earlier.

Chapter - 2

Mathematical Formulation

2.1 Introduction

State space modelling [15] is a method of solving the complicated system equations in a step by step manner. First, the system is represented by a set of first order differential equations. The rate of change of the variables are expressed as a function of the variables themselves. In a particular step the rate of changes are determined from the knowledge of the values of the variables obtained in the previous step. The new value of the variables are determined from the previous values and rate of their changes. Time is advanced by one step and the procedure is repeated to obtain a complete time domain solution of the variables.

2.2 Formulation of A State Space Model

2.2.1 Machine at Isolation

For simplicity, it was decided first to develop a state space model for a machine in isolation. The model will describe the behavior of a machine alone under any terminal condition. The differential equations written for the machine are arranged so that the machine fluxes constitute state variable of the model.

The direct and quadrature axis voltages of a synchronous

machine V_d and V_q are given by : [15]

$$V_d = p\Phi_d + w\Phi_q + R_a i_d \quad (2.1)$$

$$V_q = p\Phi_q - w\Phi_d + R_a i_q \quad (2.2)$$

The ' $w\Phi$ ' terms represent voltage induced in one axis by virtue of flux in the other axis due to rotational effect whilst the ' $p\Phi$ ' terms represent voltage induced in the one axis due to rate of change of flux in the same axis often referred to 'transformer voltage'.

The voltages on the rotor circuits are expressed as:

$$\text{field : } V_f = p\Phi_f + R_f i_f \quad (2.3)$$

$$\text{d-damper : } 0 = p\Phi_{kd} + R_{kd} i_{kd} \quad (2.4)$$

$$\text{q-damper : } 0 = p\Phi_{kq} + R_{kq} i_{kq} \quad (2.5)$$

Equations (2.1) to (2.5) can be rearranged to find the expression for ' $p\Phi$ ' terms. The rearranged form is :

$$p\Phi_d = V_d - w\Phi_q - R_a i_d + \Phi_q p\delta \quad (2.6)$$

$$p\Phi_q = V_q + w\Phi_d - R_a i_q - \Phi_d p\delta \quad (2.7)$$

$$p\Phi_f = V_f - R_f i_f \quad (2.8)$$

$$p\Phi_{kd} = -R_{kd} i_{kd} \quad (2.9)$$

$$p\Phi_{kq} = -R_{kq} i_{kq} \quad (2.10)$$

where rotor speed, w is related to synchronous speed, w_0 by the equation, $w = w_0 - p\delta$.

The currents are related to the fluxes by two sets of equations in the form of $[w_0\Phi] = [Z] \cdot [I]$, where $[w_0\Phi]$, $[Z]$ and $[I]$ are the flux, impedance and current matrices respectively. The first set of the equations is :

$$\begin{bmatrix} w_0\Phi_d \\ w_0\Phi_f \\ w_0\Phi_{kd} \end{bmatrix} = \begin{bmatrix} X_{md}+X_a & X_{md} & X_{md} \\ X_{md} & X_{md}+X_f+X_{kf} & X_{md}+X_{kf} \\ X_{md} & X_{md}+X_{kf} & X_{md}+X_{kd}+X_{kf} \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{kd} \end{bmatrix} \quad (2.11)$$

The second set is :

$$\begin{bmatrix} w_0\Phi_q \\ w_0\Phi_{kq} \end{bmatrix} = \begin{bmatrix} X_{mq}+X_a & X_{mq} \\ X_{mq} & X_{mq}+X_{kq} \end{bmatrix} \begin{bmatrix} i_q \\ i_{kq} \end{bmatrix} \quad (2.12)$$

Inversion of the impedance matrices of the above relationship gives the currents in terms of fluxes. So,

$$\begin{bmatrix} i_d \\ i_f \\ i_{kq} \end{bmatrix} = \begin{bmatrix} Y_d(1,1) & Y_d(1,2) & Y_d(1,3) \\ Y_d(2,1) & Y_d(2,2) & Y_d(2,3) \\ Y_d(3,1) & Y_d(3,2) & Y_d(3,3) \end{bmatrix} \begin{bmatrix} w_0\Phi_d \\ w_0\Phi_f \\ w_0\Phi_{kq} \end{bmatrix} \quad (2.13)$$

And, the currents in the quadrature axis are obtained from equation (2.12). The q-axis currents are :

$$\begin{bmatrix} i_q \\ i_{kq} \end{bmatrix} = \begin{bmatrix} Y_q(1,1) & Y_q(1,2) \\ Y_q(2,1) & Y_q(2,2) \end{bmatrix} \begin{bmatrix} w_0 \Phi_q \\ w_0 \Phi_{kq} \end{bmatrix} \quad (2.14)$$

The mechanical input torque, T_m is related to electrical developed torque, T_e by the equation :

$$p^2 \delta = - [T_m - T_e] / J \quad (2.15)$$

where, 'J' is the moment of inertia of the machine.

The electrical torque is produced by the cross axis flux and currents and given by :

$$T_e = w_0 (\Phi_d i_q - \Phi_q i_d) / 2. \quad (2.16)$$

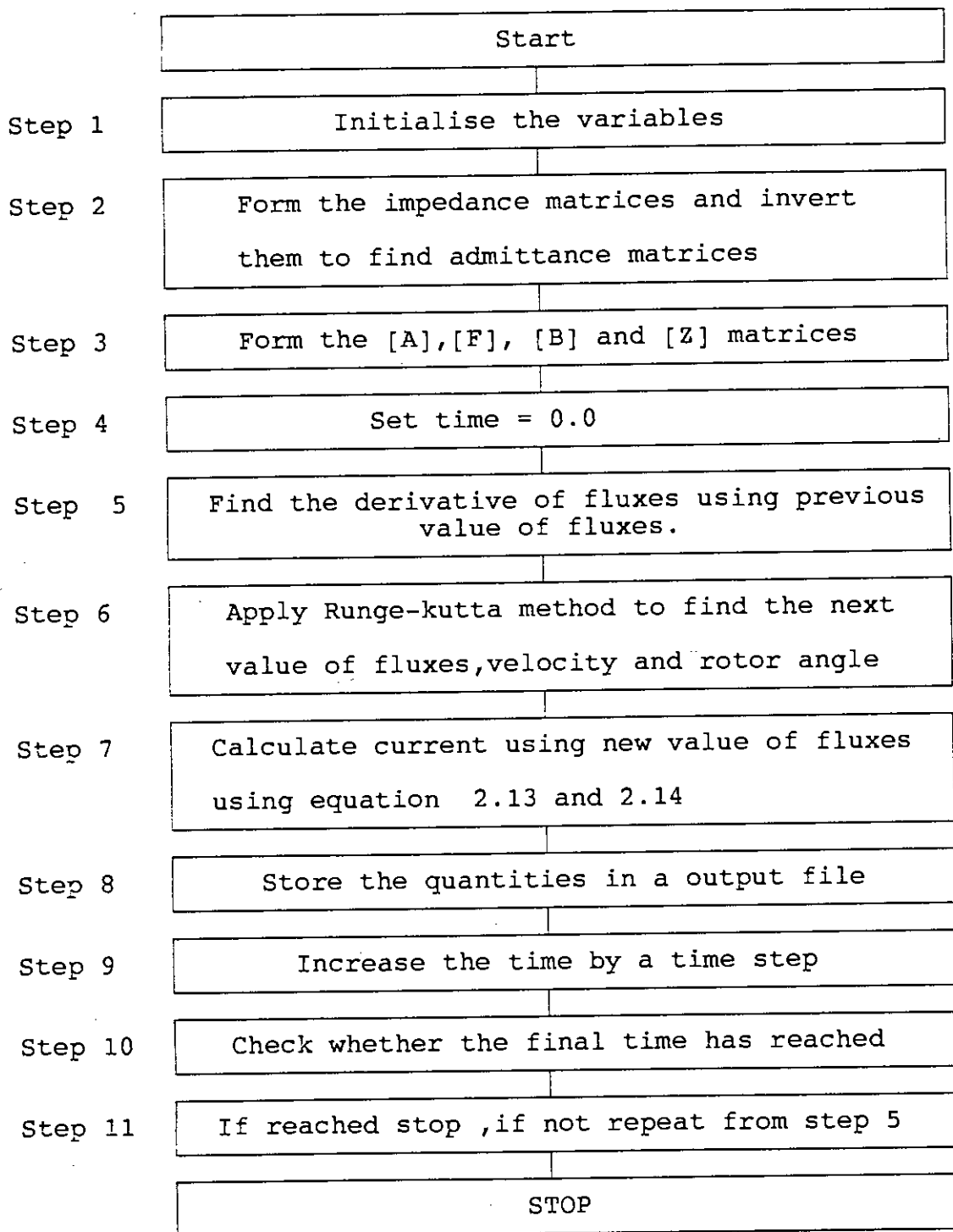
Replacing all the current terms of the equations (2.6) to (2.9) by the equation (2.13) and (2.14) and rearranging in matrix form -

$$\begin{bmatrix} \dot{X} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} X \end{bmatrix} + \begin{bmatrix} F(X) \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} Z \end{bmatrix} \quad (2.17)$$

The elements of the matrices in the above equation are given in Appendix - I.

A flow diagram showing the steps to be followed in solving

the state space form of the machine equation is described below.



2.2.2 Machine connected to Infinite bus-bar through a link

In practice a machine, at few instant, is operated in isolation, rather it is always connected to a system. The connection of a machine to a system is described by system impedance looking at the bus where the former is connected. There may be a transmission link between the machine and the bus. For the sake of generalisation a system as described in Figure 2.1 is considered here for the mathematical formulation of a generator connected to an infinite bus-bar through a transmission line.

The differential equations that govern the terminal conditions of the machine are same as those expressed by equations (2.6) to (2.10) :

$$p\Phi_d = v_d - \omega_0\Phi_q - R_a i_d + \Phi_q p\delta \quad (2.18)$$

$$p\Phi_q = v_q + \omega_0\Phi_d - R_a i_q - \Phi_d p\delta \quad (2.19)$$

$$p\Phi_f = v_f - R_f i_f \quad (2.20)$$

$$p\Phi_{kd} = - R_{kd} i_{kd} \quad (2.21)$$

$$p\Phi_{kq} = - R_{kq} i_{kq} \quad (2.22)$$

Again the terminal voltages of the machine can be related to the voltage of the infinite bus-bar as follows :

$$v_d = v_{bd} - (X/w_o) p i_d - R i_d - X i_q + (X/w_o) p \delta i_q \quad (2.23)$$

$$v_q = v_{bq} - (X/w_o) p i_q - R i_q + X i_d - (X/w_o) p \delta i_d \quad (2.24)$$

where, R and X are the resistive and reactive parts of the line impedance whilst v_{bd} and v_{bq} are the direct and quadrature axis components of the infinite bus-bar voltage.

Replacing v_d and v_q of equations (2.18) and (2.19) by equations (2.23) and (2.24) respectively and finally replacing currents in terms of fluxes from equation (2.13) and (2.14), we have,

$$\begin{aligned} p\Phi_d = & v_{bd} - X [Y_d(1,1) p\Phi_d + Y_d(1,2) p\Phi_f + Y_d(1,3) + p\Phi_{kd}] \\ & - w_o (R + R_a) [Y_d(1,1) \Phi_d + Y_d(1,2) + \Phi_f + Y_d(1,3)\Phi_{kd}] \\ & - w_o X [Y_q(1,1) \Phi_q + Y_q(1,2) \Phi_{kq}] + \Phi_q p\delta + X p\delta \cdot \\ & [Y_q(1,1) \Phi_q + Y_q(1,2) \Phi_{kq}] - w_o \Phi_q \end{aligned} \quad (2.25)$$

$$\begin{aligned} p\Phi_q = & v_{bd} + X [Y_q(1,1) p\Phi_q + Y_q(1,2) p\Phi_{kq}] - w_o (R + R_a) \\ & [Y_q(1,1)\Phi_q + Y_q(1,2) p\Phi_{kq}] + w_o X [Y_d(1,1) \Phi_d + \\ & Y_d(1,2)\Phi_f + Y_d(1,3)\Phi_{kd}] - \Phi_d p\delta - X p\delta [Y_d(1,1)\Phi_d \\ & + Y_d(1,2) \Phi_f + Y_d(1,3) \Phi_{kd}] + w_o \Phi_d \end{aligned} \quad (2.26)$$

$$p\Phi_f = v_f - w_o R_{f} [Y_d(2,1)\Phi_d + Y_d(2,2) \Phi_f + Y_d(2,3) \Phi_{kd}] \quad (2.27)$$

$$p\Phi_{kd} = - w_o R_{kd} [Y_d(3,1) \Phi_d + Y_d^{(3,2)} \Phi_f + Y_d(3,3) \Phi_{kd}] \quad (2.28)$$

$$p\Phi_{kq} = - w_o R_{kq} [Y_q(2,1) \Phi_q + Y_q(2,2) \Phi_{kq}] \quad (2.29)$$

Rearranging the above equations (2.25) to (2.29), the equations for the rate of change of fluxes can be obtained. Those equations along with the equation of motion given by equation (2.15) can be arranged in matrix form,

$$\begin{bmatrix} \dot{x} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x \end{bmatrix} + \begin{bmatrix} F(x) \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} z \end{bmatrix} \text{ similar}$$

to that described in section 2.2.1. The elements of the matrices are described in Appendix -II.

2.3 Computer Program

Based on the analysis given in the previous section a computer program was developed for the study. The program was written in FORTRAN-77. Two files for input data and one file for output were used in the program. One of the input files contained the machine parameters whilst the terminal conditions are stored in a separate input file. The results obtained from the program are kept in the output file which

was retrieved from a graphics package in order to obtain graphical output of the results under investigation. The program used a number of subroutine as necessary for the development of an efficient model. A time step of 0.0001 second was used to avoid any source error from the Runge-Kutta integration routine. Depending on the type of study, duration of 0.5 to 2.0 seconds were considered for the total time of solution.

Chapter - 3

Detail Model of an Alternator

3.1 Introduction

A detail model for an alternator has been developed on the basis of the analysis described in the previous chapter. The model was applied to simulate some practical tests performed in a micromachine. The test results and the machine parameters were collected from the work of Kabir, S. M. L. [5]. The parameters of the machine and that of external circuit were used in the detail model in predicting the tests. First three phase short circuit responses from no-load was simulated. The simulation of short circuit for the machine with pre-load was also compared. The currents and rotor angle movement following the short circuits were investigated in order to verify the accuracy of the model. The interaction of the machine with an infinite bus bar through a transmission link was tested by simulating fault throwing test at the machine terminal.

3.2 Parameters of the Machine and System Configuration

The direct axis parameters were obtained from a 3-phase short circuit test. In the test, according to standardized IEEE test code [12] for synchronous machine, it requires the generator to be driven at rated speed with the field winding excited to give the desired per unit open circuit stator voltage. Then the terminals are short circuited and

the phase currents and field currents are monitored. The applied field voltage are kept constant. The waves are analyzed and unidirectional and alternating currents are separated. By applying standard procedure X_d , X_d' , X_d'' , T_d' and T_d'' are obtained which can be related to the equivalent circuit parameter X_a , X_{md} , X_f , X_{kd} , R_f , R_{kd} . The quadrature axis parameters can not be obtained from the 3-phase short circuit test. A stator decrement test is applied to obtain the q-axis parameters. In this case, the machine is operated at no load with 1.0 pu terminal voltage and rotor is aligned to the q-axis by applying mechanical power. The stator supply is then suddenly disconnected and the fall in terminal voltage is recorded. From the step change and the rate of decay of terminal voltage the the q-axis parameters are obtained [12]. The value of the d-axis and q-axis parameters of the micromachine are given in Figure 3.1.

The system shown in Figure 2.1 was used to represent the link between the machine and infinite bus via a transmission line. The line parameters are as indicated in the Figure 2.1. The point, F on the line represents the fault point. In the case of fault throwing tests, three phase short circuit was applied at that point for a short period of time and then removed.

3.3 Responses of Machine at Isolation

3.3.1 From Noload Condition

The machine was excited to 1 pu voltage at no load and a three phase short circuit was applied at the machine terminal and oscillogram of the stator and field currents and the rotor angle movement were recorded. To enable clear interpretation of the decay of current during subtransient and transient periods, only the envelope of the alternating stator current has been plotted. On the other hand the instantaneous values of field current are plotted. The test results are plotted in Figure 3.2. Figure 3.2a shows the envelope of stator current and Figure 3.2b represents the instantaneous values of field current. The machine parameters of Figure 3.1 were used for the simulation of the short circuit. The results obtained from detail model are shown along side the corresponding test results. The close agreement of the predicted results with the test proves the effectiveness of the model developed.

Apart from the stator and field current transients, the machine suffers rotor angle movement during any disturbance. Transient conditions involve changes in electrical output and losses so that the imbalance between electrical and mechanical power is resolved in terms of movement of the

alternator rotor. The rate of change of speed is directly proportional to any discrepancy in the power balance. The rotor movement resulting from the interchange of electrical and mechanical energy is continuous. Hence the prediction of rotor angle at any instant is determined by the previous movement, so any error at any point in the transient period reflects in the overall result. Therefore the simulation of dynamic response of a machine following a disturbance is the most revealing test for a model.

The rotor movement following the 3-phase short circuit at the machine terminal was simulated. The machine retards under this condition due to the influence of the transient losses. The test and predicted results showing the rotor angle versus time is shown in Figure 3.3. The detail model predicted accurately.

3.3.2 Machine With Preload

Having demonstrated the effectiveness of the detail model in the simulation of the simplest of dynamic disturbances, complexity was introduced with the inclusion of a steady preload prior to application of a three phase short circuit. For the detail model to simulate dynamic response in the presence of an initial load, the initial conditions impressed by this load on the flux distribution between the

machine's 'd'- and 'q' axes are obtained from the steady state equations in conjunction with two-axis vector diagram of the machine.

In the test, a steady power of 0.615 pu was generated with a rotor angle of 61 degrees between the machine and the infinite bus. The input conditions are given in Table 3.1. The rotor angle movement following the three phase short circuit at the machine terminal is shown in Figure 3.4. The machine exhibited back swing. This phenomena is also observed in the large machines [17] and happens due to high initial losses exceeding the mechanical input power. The predicted result is shown in Figure 3.4 alongside the test result. A reasonable accuracy is obtained in the prediction.

The effect of preload is to accelerate the machine after a short duration of back swing. The accelerating torque increases as the preload is increased. This was tested by simulating four cases of short circuit with and without preload. The relative rotor movement is demonstrated in Figure 3.5. Curve (i) of the figure is for no preload whilst other curves for various degree of preload. The acceleration of the rotor was increasing as the preload was increased. It can be seen that a delay before any net forward movement of the rotor is obtained, even for high

preload. It can also be stated, without qualification that, when the alternator is subjected to simultaneous three phase fault, the rotor will always swing backwards during the first 20 to 40 milliseconds.

3.4 Machine Connected to Infinite Bus-bar

The dynamic behaviour of a machine connected to an infinite bus bar is of practical importance since it is a determining factor in system stability. To be useful therefore a digital model must be capable of accurately predicting this behaviour. Of particular importance in the ability to simulate fault throwing tests used to replicate practical fault conditions. In these tests, transmission of power to the infinite bus bar is disturbed for a short period by the application of a transitory fault. The machine's dynamic movement during the fault period is determined by the fault currents in the machine. Movement following clearance of fault is determined by the manner in which the imbalance of electrical and mechanical energy build up during the fault period and their redistribution between the machine and infinite bus bar in the presence of synchronising torque when the fault is removed.

To calculate the rotor movement during a fault throwing test required the equation used previously to be expanded into a

generalised form to accommodate the presence of the transmission link and bus bar during the post fault period. The development of the generalised form is described in the previous chapter. The modified equations allow for the simulation of fault throwing tests with the fault applied at any point on the transmission link.

The ability to simulate fault throwing test was assessed in terms of its prediction of a test having a fault duration of 0.21 seconds. In this case, the rotor angle was set at 22.5 degree with a power output from the machine of 0.144 pu. A three phase short circuit of 210 milliseconds duration was applied at machine's terminal. The test result is shown by solid line of the Figure 3.6. The detail model's prediction of the test is shown by dotted line in the same figure. The calculated rotor angle curve compares favourably with the test results. This proves the accuracy of the model developed for the machine connected with an infinite bus-bar through a transmission link.

Chapter - 4

Simplification of the Model

4.1 Introduction

Having an accurate model of an alternator been developed, it becomes important to investigate whether the model is applicable to a large scale study involving multi-machine. A time step of 0.0001 second was used in the detail model developed in the previous chapter. The computing time, even for a simple case of short circuit study is few seconds when a personal computer is used. The time of computation becomes increasingly high if the complexity of the system is increased. It becomes apparent that the application of the detail model in a multimachine study is not feasible. Therefore, it seems very important to devise a method of simplifying the model so that it becomes suitable for its use in large scale study.

4.2 The Classical Model

The simplest multimachine model for stability study is the so-called 'classical model'. The model is based on the principle of constant flux linkage of the field circuit during the transient. In the classical model, the synchronous machine is represented by a voltage behind transient reactance. The voltage is calculated from the initial condition at the machine terminal prior to a fault. During the faulted condition the steady state power equation

applicable for a particular duration is used to represent instantaneous power of the synchronous machine. The instantaneous imbalance between electrical and mechanical power is obtained by subtracting mechanical power from the electrical power. This discrepancy is integrated twice to calculate the instantaneous rotor angle following a disturbance. The results obtained from classical model does not give accurate prediction. This is due to the fact that the following inherent assumptions are made in the development of classical model :

(i) The model assumes constant flux linkage all over the time of computation which is not strictly valid because the field flux changes in accordance with its open circuit time constant. The assumption of constant flux linkage is only valid during initial period of time.

(ii) The steady state power equation is used to represent instantaneous power of the machine. During transient, the power equation should include the component due to the rate of change of energy arises from the instantaneous imbalance.

(iii) The saliency is not properly accounted for. The reactances in the axes do not remain constant all over the time. The variation of the reactances should also be included in the model.

There is no doubt that the classical model gives an estimate about the stability margin of a system under transient fault condition, but never capable of giving an accurate prediction about the stability. Since an accurate model which involves minimum computer time is to be developed, it was decided to investigate the means of simplifying the detail model already developed.

4.3 Effect of Increasing Integration Time Step in the Detail Model.

The speed of computation can be increased by increasing the time step in solving the differential equations of the detail model. To see the effect of increasing step size the model was reapplied to simulate the no load short circuit test with different time steps. The stator and field current results are shown in Figure 4.1. Four curves are shown for time steps of 0.0001, 0.001, 0.005 and 0.01 second. It is observed that the accuracy of prediction decreases as the time step is increased beyond 0.0001 seconds. The maximum time step to give an accurate result is determined by the slope of the curve over which the integration of the rate of change of the state space variable has to be carried out. The short circuit fluxes contain both the unidirectional and fundamental components. When there is a fault at the stator side the stator flux has

to change very sharply. But since the flux can not change instantaneously, the impact is translated into an alternating component which eventually decay out exponentially with armature time constant. It is the alternating component which produces stiff slopes during the transients. The slopes represented by the rate of change of fluxes when calculated by Runge-Kutta method produces wrong estimate when time step is increased beyond 0.0001 seconds.

4.4 Effect of Neglecting Rate of Change of Stator Flux Terms

When there is a fault at the machine's stator side the armature fluxes readjust itself to accommodate the terminal condition. Over the transient period the rate of change of d.c. fluxes are very small. So, the rate of change of stator fluxes in the detail model can be neglected [15]. The effect of neglecting the rate of changes is that the dc component in the three phase stator current or ac component in the resolved axes current is eliminated. The fact was investigated by simulating the 3-phase short circuit at the machine terminal from the no-load. The model was simplified by neglecting rate of change of stator fluxes. But the instantaneous d- and q-axis fluxes were obtained at each step from the values of other fluxes. The procedure of obtaining the relationship is described in the Appendix-III.

The simple model's prediction along with that obtained from the detail model is exhibited in Figure 4.2. The currents obtained from the simplified model gives accurate dc components of the total response. Since the component is of actual interest for the design, the outcome of the simple model may be treated as sufficient.

It is interesting to note that while the purpose of predicting fault current is served even with neglecting rate of change of stator fluxes, the time step can greatly be increased to solve the state space model by the Runge-kutta method. In this instant, a time step of 0.005 sec was used for the simulation which means a 20 times increase in time step is possible by neglecting the rate of change of fluxes.

4.5 Rotor Angle Simulation by the Simple Model

As stated in the previous chapter, since the simulation of the rotor movement is the confirmatory test of a model, the simple model was used to simulate rotor angle of the short circuit test. The simulated results of the model along with that obtained from the detail model is shown in Figure 4.3. As seen from the figure, unlike current simulation, the prediction of rotor angle is totally unsatisfactory.

Since in the no load case the net mechanical torque was zero only the electrical developed torque is responsible for the rotor movement. The electrical torque in this case is that due to the resistive losses within the machine. For the purpose of investigation, the electrical torque developed by the machine is obtained both from the simple model and that from the detail model. They are plotted in Figure 4.4. The difference between the curves is an oscillatory component which is eliminated in the case of simple model. It may appear that the oscillatory component of electrical torque does not produce any net movement of rotor because the movement in half of the cycle due to positive torque may cancel the opposite movement in the other half cycle due to negative torque. This is not true as seen from Figure 4.3. The discrepancy in the rotor angle obtained from the two model must have arisen due to the oscillatory component. The difference in the rotor angle prediction should provide an estimate of the component of rotor angle movement due to the oscillatory component of the torque. The difference between the two rotor angle curves of Figure 4.3 is plotted in the Figure 4.5. The difference represents a straight line.

An oscillatory component of torque which when integrated once gives an step change in rotor speed which when integrated further produces a straight line change in rotor

angle. This fact can be established from the analysis as described in Appendix-IV. The evaluation of straight line change in rotor angle is illustrated in Figure 4.6. The straight line change in rotor angle due to the oscillatory component of the torque is given by -

$$\Delta \delta' = \frac{V^2 * t}{J * X_d''} \quad (4.1)$$

where V , J and X_d'' are the machines internal voltage, moment of inertia and subtransient reactance. The machines internal emf is assumed to be 1 pu which is approximately correct in most of the cases. In fact the straight line deceleration is superimposed by a very small oscillatory component of the rotor angle change. For simplicity the oscillatory component was not simulated. The straight line approximation as obtained from the analysis exactly matches the discrepancies between the results of the detail model and that of the simple model.

4.6 Modified Model's Prediction of Rotor Angle Movement for Short Circuit on a Preloaded Machine.

A modified model is developed by incorporating the discrepancy in the rotor angle term into the simplified model. The new model should be able to predict the rotor

movement during short circuit accurately. Three short circuit cases were simulated by the modified model. Different values of preload were used for the three cases. The effect of preload as demonstrated in the previous chapter is to accelerate the rotor due to the presence of constant mechanical input torque. The initial backswing obtained from the cases is due to the high initial losses within the machine. Figure 4.7, 4.8 and 4.9 represent the angular movement from the initial rotor angle of 22.5° , 61° and 81.5° respectively. The movement of rotor obtained from the detail, simple and modified model are superimposed for the purpose of comparison. The error between the detail and simple model are compensated by the extra term in the modified model. The accuracy obtained by the modified model is excellent.

4.7 Application of Modified Model for the Fault Throwing Simulation.

Since the simple model when modified gives responses as good as those obtained from detail model, the possibility of applying the modified model for the fault throwing cases were investigated. When the rate of change of stator fluxes are neglected, the electrical developed torque is represented by the average component only both during and after removal of fault. The traces of electrical torque in a fault

throwing simulation are shown in Figure 4.10. The simulation was made for a machine initially at 76.5° of rotor angle and a fault duration of 100 milliseconds. This simulation is more accurate than a previous study [15]. It is seen from the Figure 4.10 that the fault when removed produces another oscillatory component of torque with shorter decaying time constant. The rate of decay is the governed by the machine and line parameters in together. The initial step of the oscillatory torque just after the removal is determined by the total of machine and line reactances instead of machine's reactance during the fault. Therefore it was apparent to simulate the corrected rotor angle by straight line with slope applicable for the particular duration of time. Accordingly the fault throwing case was simulated by the modified model. But a disappointing result was obtained as seen from the Figure 4.11. When the response obtained from the simple model is compared with that obtained from the detail model it is found that the rotor angle movement following the fault removal, unlike during fault, does not produce straight line discrepancies. The differences in the rotor angle prediction for this case are plotted in Figure 4.12. There must be another source of error introduced in the simple model apart from those due to the oscillatory component of torque.

The rotor angle after the fault removal is determined by the interaction between the machine and the infinite bus. The component of the infinite bus voltage in the two axes is dependent on the rotor angle. Since the simple model does not produce correct rotor angle, the resolved parts of the infinite bus voltage at each instant become erroneous giving further error in the rotor angle. The discrepancies in the component voltages are illustrated by plotting the instantaneous d-axis bus voltage as obtained from the detail and the simple model. The curve is shown in Figure 4.13. The error can only be eliminated if the correction of rotor angle for the oscillatory component is made at each time step and the corrected rotor angle is used for the resolution of the bus voltage. During the faulted period it is immaterial whether the correction is applied at each step or globally because the infinite bus remains isolated when the fault is on.

The correction for rotor angle when applied at each time step in the modified model gave an accurate simulation as can be seen from the Figure 4.14 where the modified model's prediction favourably agrees ^{with} that obtained from the detail model. For the sake of further validation another simulation of fault throwing case was made. In this instant, the rotor angle was aligned initially at 90.5° and a fault of 100 millisecond was applied on the line. The simulation is

exhibited in Figure 4.15. The errors in the simple model is eliminated when the modified model was applied for the simulation.

Chapter - 5

Conclusion and Discussion

5.1 Conclusion and Discussion

A detail model of an alternator was developed on the basis of a state space model. The five winding fluxes, rotor angle and speed were chosen as state variable. The model when applied to simulate some practical tests was proved to be satisfactory. Its effectiveness was established by it being capable of simulating alternator currents and, more importantly, rotor angle movement both for sustained three phase fault and fault throwing tests. Since the model provides an accurate representation of an alternator, the output from the the detail model substitute the test results.

The model being established as an accurate model of an alternator was used as the basis of the development of a simple mathematical model. The objective of the simplification was to reduce the computing time. The prime factor in order to achieve the fastness was identified as to neglect the stator transients. This was done by eliminating the rate of change of stator flux terms from the detail model. The stator fluxes were obtained from the knowledge of other fluxes at a particular instant. The simple model when applied to simulate the electrical quantities gave accurate results but fails to simulate mechanical quantity i,e rotor

angle. The failure of the simple model is due to the omission of oscillatory component of torque following a disturbance. This omission was due to neglecting rate of change of stator fluxes. The oscillatory component of electrical torque produces a straight line deceleration of rotor in addition to that determined by the resultant of the losses of the machine and mechanical torque. The simple model is thus modified by adding the discrepancy term with the rotor angle that obtained from the simple model.

This concept of modification when applied to fault throwing cases gave erroneous results. The error in rotor angle prediction is not only due to neglecting oscillatory component of torque but also due to wrong resolution of infinite bus bar voltage using the erroneous value of rotor angle. The two errors are added cumulatively during each time step. Therefore in the modified model an instantaneous correction has to be applied and the modified rotor angle has to be used in resolving the infinite bus bar voltage. The model thus developed produces an accurate estimate of rotor angle movement following a disturbance in the system. The accuracy is complimented by its fast speed of computation and thus making it suitable for large scale study.

5.2 Further Work

The development of the simple model has opened up few area of research in the field of Power system stability. Some of them are discussed below.

- (i) The modified model can be made faster if time step used can be increased further. The only restriction is imposed by point of discontinuity. The sharp changes in fluxes just at the moment of application of fault and at the moment of removal of fault can not be accurately obtained when time step is increased. A variable time step routine could be applied in this case. A small time step of 0.005 second may be used in the region of sharp changes and a longer steps in the order of 0.01 second for other region. This would make the model as fast as 'Classical model' whilst providing accuracy in the results.
- (ii) The model now works on a matrix form of equations. This involves a large computer storage. This storage will be increased when multimachines are involved in the study. Therefore, like classical model, an analytical equation should be developed that will describe the instantaneous electrical torque during disturbances. This will enable one to use the model efficiently in

the case of multimachine study.

- (iii) Since the oscillatory component of electrical developed torque produces a negative rotor angle movement, the component is a blessing for the stability of the machine. Would it be absent, the machine will tend to be more unstable. This fact may be utilised in the development of a new kind of stabilizer. The new stabilizer would create an impulse of torque when machine would tend to accelerate. The impulse of torque would retard the machine and thus reducing the chance of instability.

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Appendices

Appendix - I

$$\begin{aligned}
 & \begin{bmatrix} \dot{\delta} \\ \ddot{\delta} \\ \dot{\delta} \\ w_o \dot{\phi}_d \\ w_o \dot{\phi}_f \\ w_o \dot{\phi}_{kd} \\ w_o \dot{\phi}_q \\ w_o \dot{\phi}_{kq} \end{bmatrix} = w_o \begin{bmatrix} 0 & 1/w_o & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -Y_d(1,1)R_a & -Y_a(1,2)R_a & -Y_d(1,3)R_a & -1 & 0 & 0 \\ 0 & 0 & -Y_d(2,1)R_a & -Y_d(2,2)R_a & -Y_d(2,3)R_a & 0 & 0 & 0 \\ 0 & 0 & -Y_d(3,1)R_a & -Y_d(3,2)R_a & -Y_d(3,3)R_a & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -Y_q(1,1)R_a & -Y_q(1,2)R_a & 0 \\ 0 & 0 & 0 & 0 & 0 & -Y_q(2,1)R_a & -Y_q(2,2)R_a & 0 \end{bmatrix} \begin{bmatrix} \delta \\ \dot{\delta} \\ \delta \\ w_o \phi_d \\ w_o \phi_f \\ w_o \phi_{kd} \\ w_o \phi_q \\ w_o \phi_{kq} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{T_e/J}{w_o^2 \phi_q \dot{\delta} + w_o v_d} \\ 0 \\ 0 \\ 0 \\ -w_o^2 \phi_d \dot{\delta} + w_o v_q \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1/J & 0 \\ 0 & 0 \\ 0 & w_o \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_m \\ v_f \end{bmatrix} \quad (2.16) \\
 & \begin{bmatrix} \dot{x} \end{bmatrix} \quad \begin{bmatrix} A \end{bmatrix} \quad \begin{bmatrix} x \end{bmatrix} \quad \begin{bmatrix} F(x) \end{bmatrix} \quad \begin{bmatrix} B \end{bmatrix} \quad \begin{bmatrix} z \end{bmatrix}
 \end{aligned}$$

Appendix - II

Elements of A-matrix of equation 2.17 for the machine connected to infinite bus-bar.

$$A(1,2) = 1 / w_o$$

$$A(3,3) = [C_1 R_f Y_d(2,1) + C_2 R_{kd} Y_d(3,1) + C_3 Y_d(1,1)] / C_o$$

$$A(3,4) = [C_1 R_f Y_d(2,2) + C_2 R_{kd} Y_d(3,2) + C_3 Y_d(1,2)] / C_o$$

$$A(3,5) = [C_1 R_f Y_d(2,3) + C_2 R_{kd} Y_d(3,3) + C_3 Y_d(1,3)] / C_o$$

$$A(3,7) = [-1 - X Y_q(1,1)] / C_o$$

$$A(4,3) = - R_f Y_d(2,1)$$

$$A(4,4) = - R_f Y_d(2,2)$$

$$A(5,3) = - R_f Y_d(2,3)$$

$$A(5,4) = - R_{kd} Y_d(3,2)$$

$$A(5,5) = - R_{kd} Y_d(3,3)$$

$$A(6,3) = [1 + X Y_d(1,1)] / C_o$$

$$A(6,4) = X Y_d(1,3) / C_o$$

$$A(6,5) = X Y_d(1,3) / C_o$$

$$A(6,6) = [d_1 R_{kq} Y_q(2,1) + d_2 Y_q(1,1)] / d_o$$

$$A(6,7) = [d_1 R_{kq} Y_q(2,2) + d_2 Y_q(1,2)] / d_o$$

$$A(7,6) = - R_{kq} Y_q(2,1)$$

$$A(7,7) = - R_{kq} Y_q(2,2)$$

$$F(2) = 0.5 [w_o \Phi_d \{ w_o \Phi_q Y_q(1,1) + w_o \Phi_{kq} Y_q(1,2) \} - w_o \Phi_q \{ w_o \Phi_d Y_d(1,1) + w_o \Phi_f Y_d(1,2) + w_o \Phi_{kd} Y_d(1,3) \}]$$

$$F(3) = [\{ (X Y_q(1,1) + 1) w_o \Phi_q + X Y_q(1,2) w_o \Phi_{kq} \} \delta + w_o v_{bd}] / C_o$$

$$F(6) = [\{ (X Y_d(1,1) + 1) \omega_0 \Phi_q + X Y_d(1,2) \omega_0 \Phi_f + \\ X Y_d(1,3) \omega_0 \Phi_{kd} \} \delta - \omega_0 v_{bq}] / C_0$$

$$B(2,1) = -1 / J$$

$$B(3,2) = - \omega_0 C_1 / C_0$$

$$B(4,2) = - \omega_0$$

where-

$$C_0 = 1 + X Y_d(1,1)$$

$$C_1 = X Y_d(1,2)$$

$$C_2 = X Y_d(1,3)$$

$$C_3 = - (R_a + R)$$

$$d_0 = 1 + X Y_q(1,1)$$

$$d_1 = X Y_q(1,1)$$

$$d_2 = C_3$$

As described in the section 2.2.1 a flow diagram was followed for the study of synchronous machine when connected to an infinite bus-bar.

Appendix-III

The Expression for Stator fluxes when 'pΦ' terms are neglected

Combining equation (2.18) with (2.23) and (2.19) with (2.24) and neglecting the rate of change of flux terms i.e., putting $p\Phi_d = 0$ and $p\Phi_q = 0$ we can write-

$$0 = u_{bd} - w_o \Phi_q - X_{LL} i_q - (R_a + R_{LL}) i_d + \Phi_q \dot{\delta} \quad (1)$$

and

$$0 = u_{bq} + w_o \Phi_q + X_{LL} i_d - (R_a + R_{LL}) i_q - \Phi_q \dot{\delta} \quad (2)$$

Now putting

$$- (R_a + R_{LL}) = R_3$$

$$\dot{\delta} = X(2)$$

And replacing stator currents in terms of fluxes, we have :

$$i_q = Y_q(1,1) X(6) + Y_q(1,2) X(7)$$

$$i_d = Y_d(1,1) X(3) + Y_d(1,2) X(4) + Y_d(1,3) X(5)$$

From equation (1) we get

$$\begin{aligned} 0 = & u_{bd} - X(6) - X_{LL} \{ Y_q(1,1) X(6) + Y_q(1,2) X(7) \} \\ & + R_3 \{ Y_d(1,2) X(3) + Y_d(1,2) X(4) + Y_d(1,3) X(5) \} \\ & + X(6) \cdot X(2)/w_o \end{aligned}$$

which can be expressed as-

$$A_1 X(3) + B_1 X(6) = C_1 \quad (3)$$

where,

$$A_1 = - R_3 Y_d(1,1)$$

$$B_1 = 1 + X_{LL} - X(2)/w_o$$

$$C_1 = u_{bd} - X_{LL} Y_q(1,2) X(7) + R_3 Y_3 Y_d(1,2) X(4) \\ + R_3 Y_d(1,3) X(5)$$

Again from equation (2) we get

$$0 = U_{bq} + X_{LL} Y_d(1,2) X(4) + X_{LL} Y_d(1,3) X(5) + \\ R_3 Y_q(1,2) X(7) + X(3) + X_{LL} Y_d(1,1) X(3) - X(3)X(2)/w_o \\ + R_3 Y_q(1,1) X(6)$$

which in short form :

$$A_2 X(3) + B_2 X(6) = C_2 \quad (4)$$

where,

$$A_2 = -1 - X_{LL} Y_d(1,1) + X(2)/w_o$$

$$B_2 = - R_3 Y_q(1,1)$$

$$C_2 = u_{bq} + X_{LL} Y_d(1,2) X(4) + Y_d(1,3) X(5) + R_3 Y_q(1,2) X(7)$$

Now solving equation (3) and (4) we can write-

$$X(3) = \frac{A_1 B_2 - A_2 B_1}{C_1 B_2 - C_2 B_1}$$

And

$$X(6) = \frac{B_1 A_2 - B_2 A_1}{C_1 A_2 - C_2 A_1}$$

These two equations are used in simple model to calculate the unidirectional component of d- and q-axis fluxes.

Appendix- IV

Derivation of rotor angle due to oscillatory torque

When a rotor is subjected to an oscillatory torque, such as shown in Fig 4.6, it is deaccelerated during the first half cycle of torque and is then accelerated during the second half cycle and so on during subsequent cycles. The net effect of this is a reduction of the mean speed of the rotor.

Adkins [15] derives an expression for the electrical torque developed in an ideal generator when it is subjected to a 3-phase short circuit at its terminals from an open circuit condition. The fundamental component of the torque decay exponentially and is described by-

$$T_e = (V/X_d'') \cdot e^{-(t/T_a)} \sin \omega_0 t$$

Now when the generator rotor is subjected to this torque (it is assumed that the primover torque is zero), it moves in a manner dictated by-

$$- T_e = J \frac{d^2 \delta}{dt^2}$$

Substituting the electrical torque,

$$\frac{d^2 \delta}{dt^2} = - \frac{1}{J} \frac{V}{X_d''} e^{-(t/T_a)} \sin \omega_0 t$$

$\delta' = \text{error} = \Delta \theta$

Integrating once and simplifying, the speed is given by

$$w' = \frac{d\delta'}{dt} = - \frac{V^2}{J X_{d''}} \frac{\omega_0}{\{(1/Ta)^2 + \omega_0^2\}} [1 - e^{-t/Ta} \cos(\omega_0 t - \gamma)]$$

where γ is approximately equal to zero.

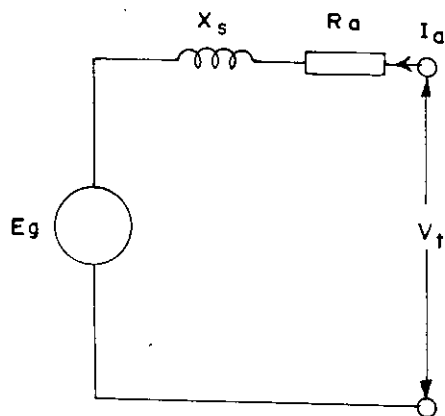
And, integrating further and simplifying the expression for the δ' is obtained as-

$$\delta' = - \frac{V^2}{J X_{d''}} \frac{\omega_0^2}{\{(1/Ta)^2 + \omega_0^2\}} \left[t - \frac{e^{-t/Ta} \sin(\omega_0 t - \beta)}{\omega_0} \right]$$

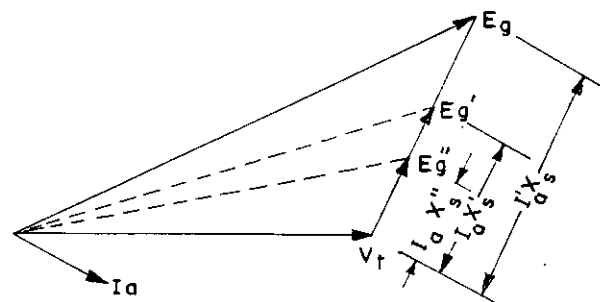
where the angle β is again approximately zero.

The plot of δ' , w' and T_e are shown in figure 4.6. Since the fundamental component of the rotor angle is small, it was neglected in the simple model.

Figures



a) Equivalent circuit



b) Phasor diagram

Fig 1.1 'Constant emf behind constant reactance' model.

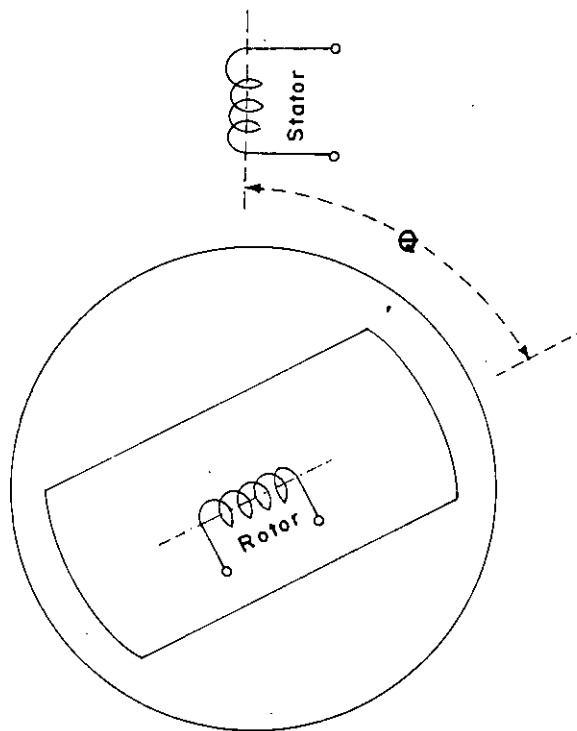
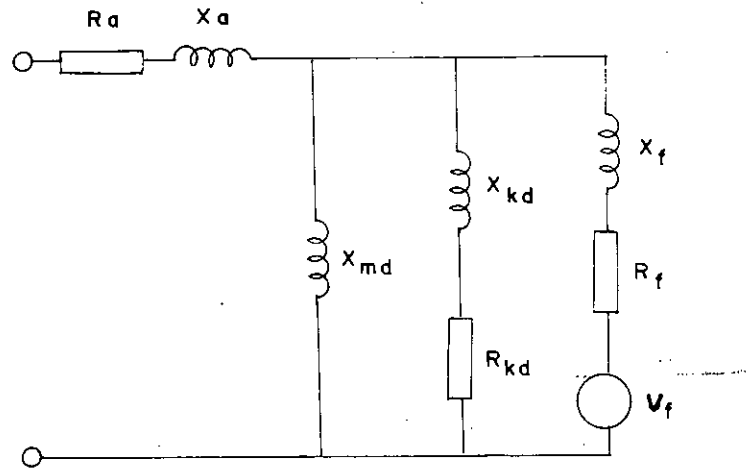
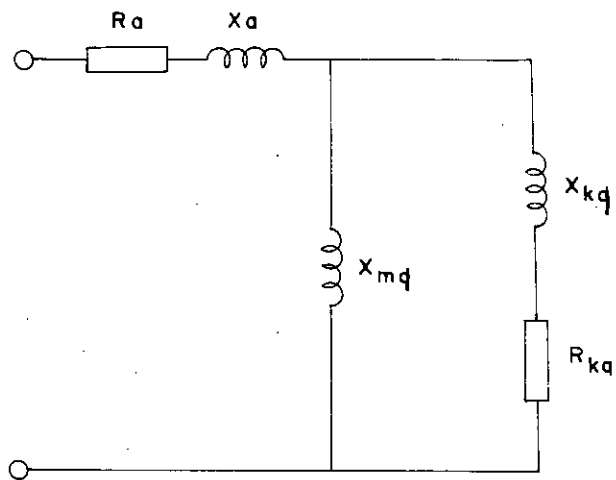


Fig 1.2 A hypothetical two winding machine.



(a) Direct axis



(b) Quadrature axis

Fig 1.3 Conventional equivalent circuit of an alternator

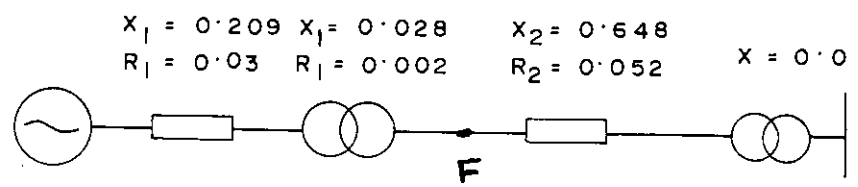


Fig 2.1 A machine connected to an infinite bus.

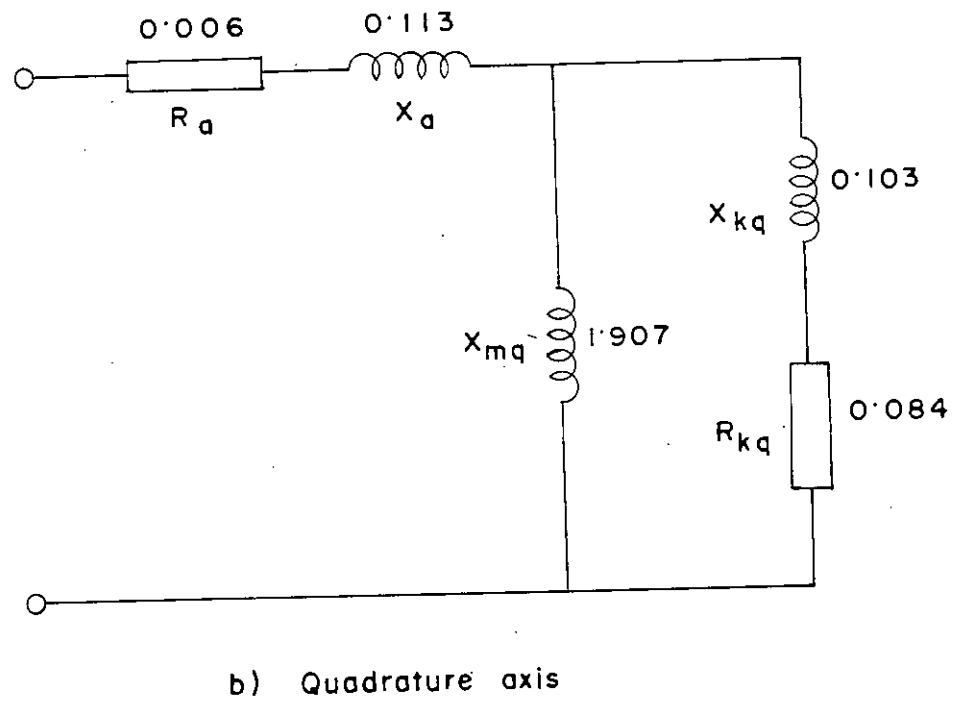
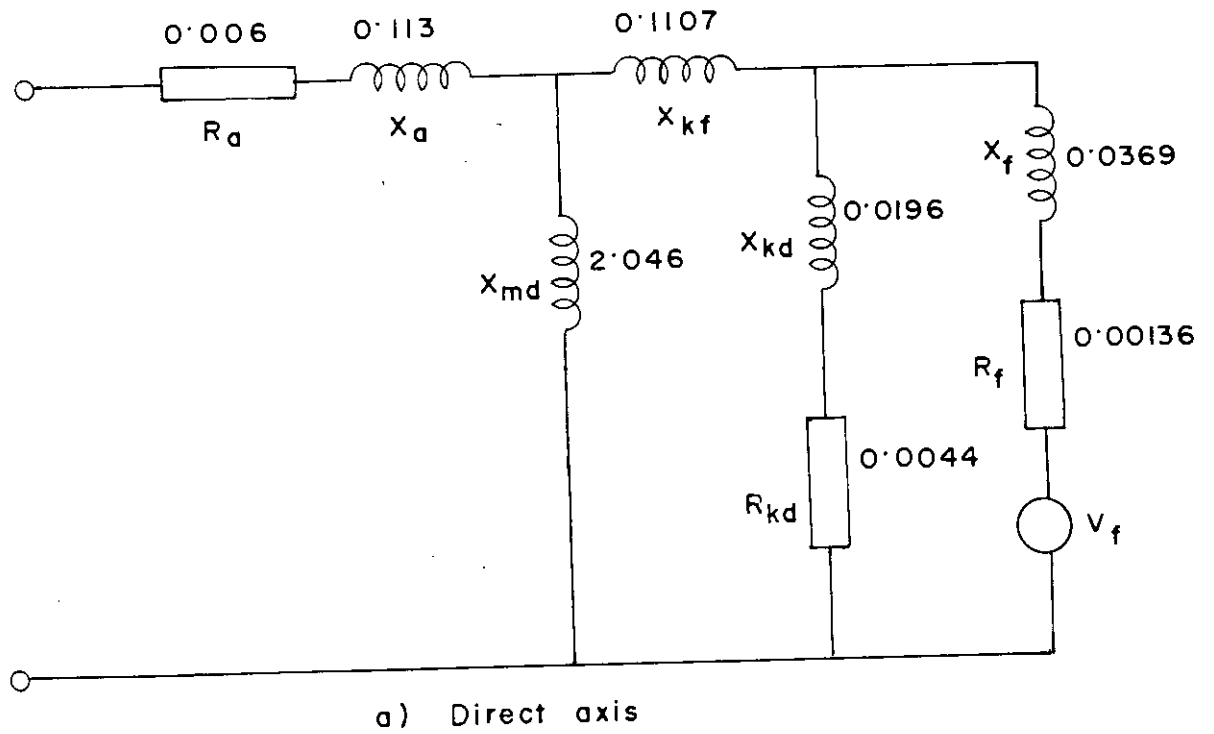


Fig 3.1 d- and q-axis equivalent circuit of a micromachine

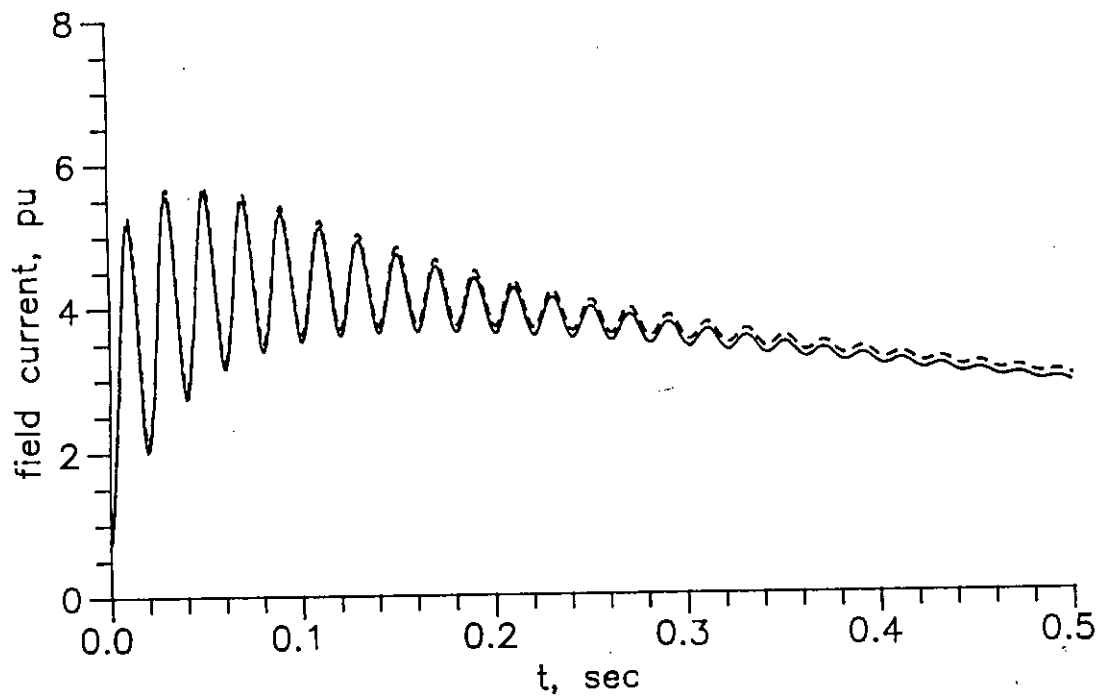
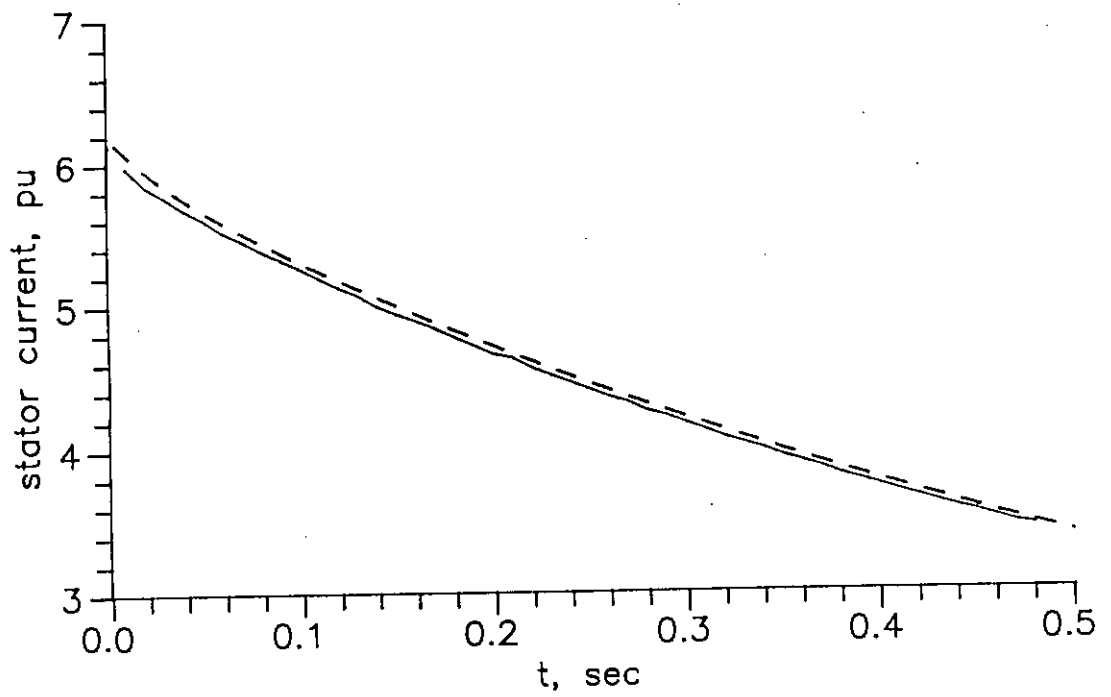


Fig 3.2 Currents during a 3-phase short circuit at machine terminal.

— test. - - - detail model's prediction.

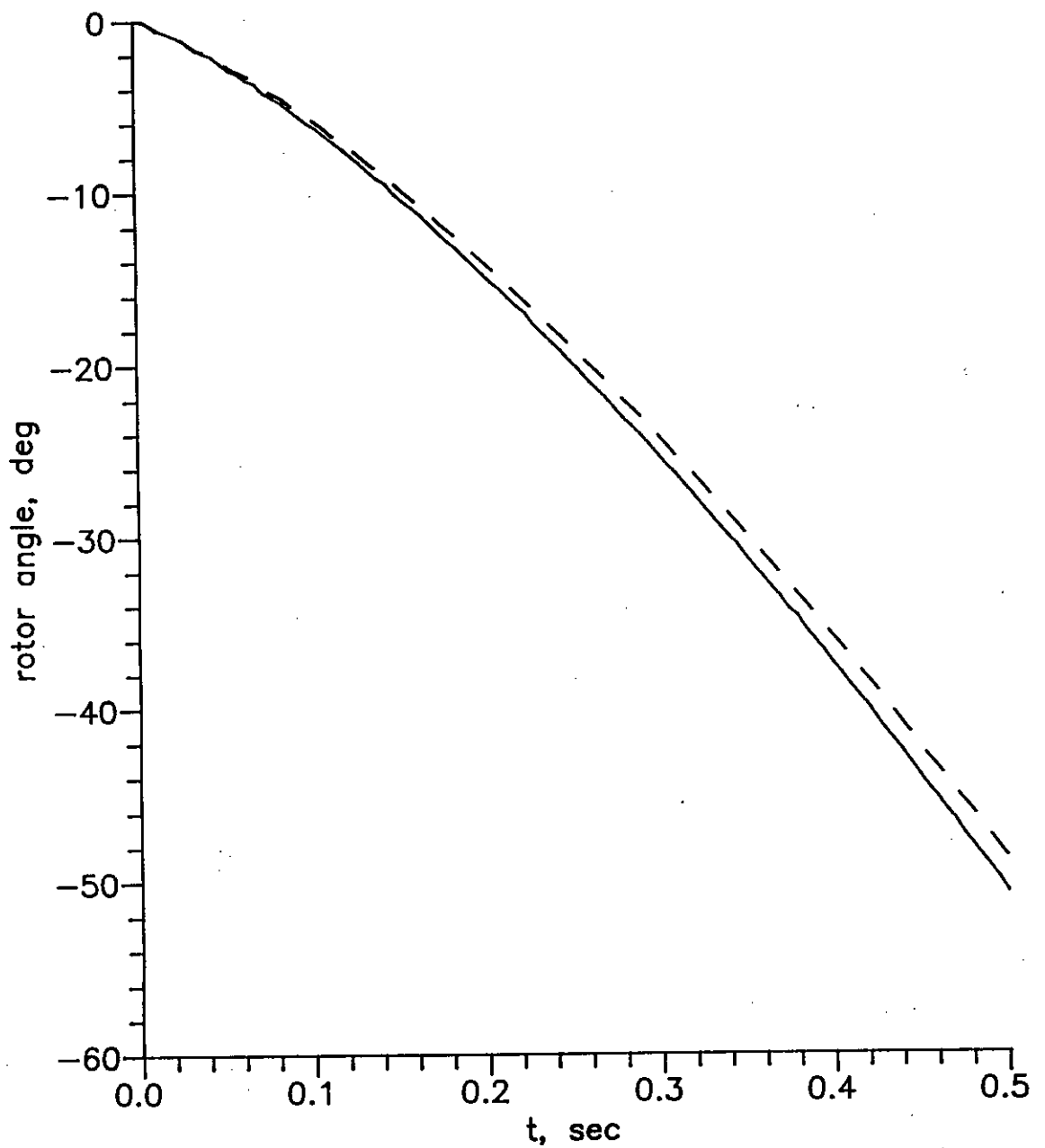


Fig 3.3 Rotor movement of a no-load machine following a 3-phase short circuit at its terminal.

— test. - - - detail model's prediction.

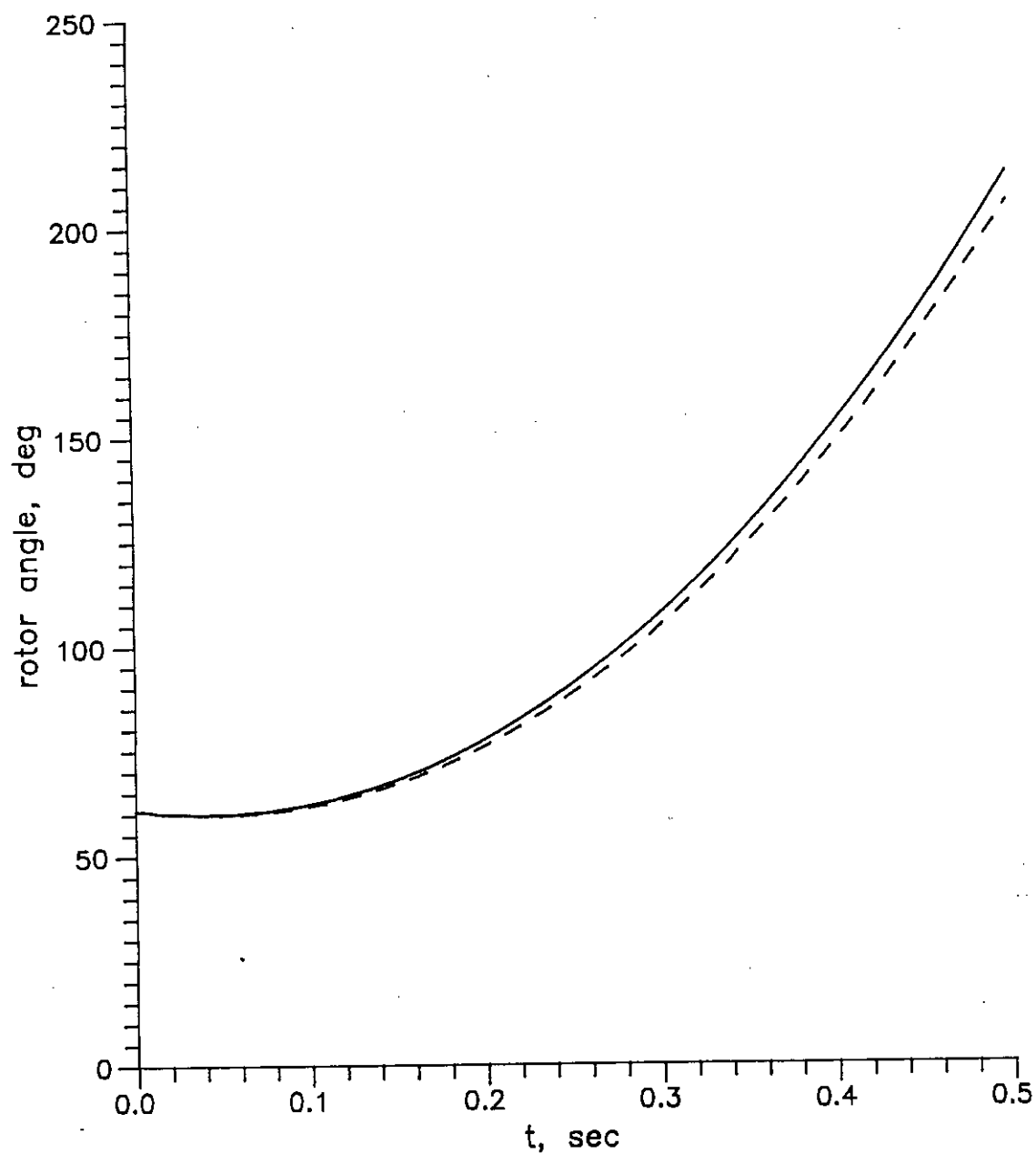


Fig 3.4 Rotor movement of a loaded machine following a 3-phase short circuit at machine terminal. $P_T = 0.615$ pu
— test. - - - detail model's prediction.

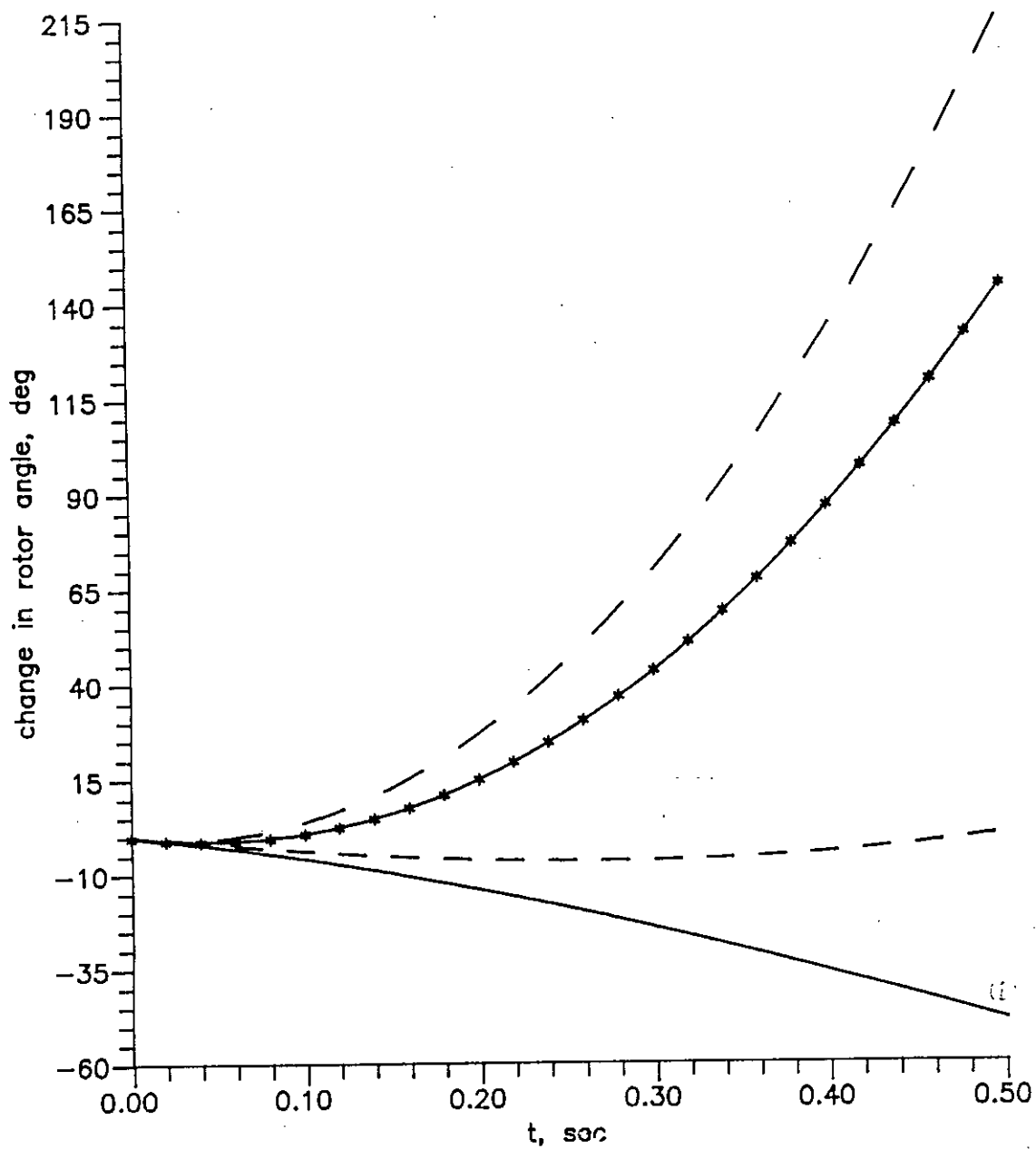


Fig 3.5 Effect of preloaded power on the rotor movement following a 3-phase short circuit.

—	PT= 0.0 pu	—*—*	PT= 0.615 pu
- - -	PT= 0.146 pu	— — —	PT= 0.813 pu

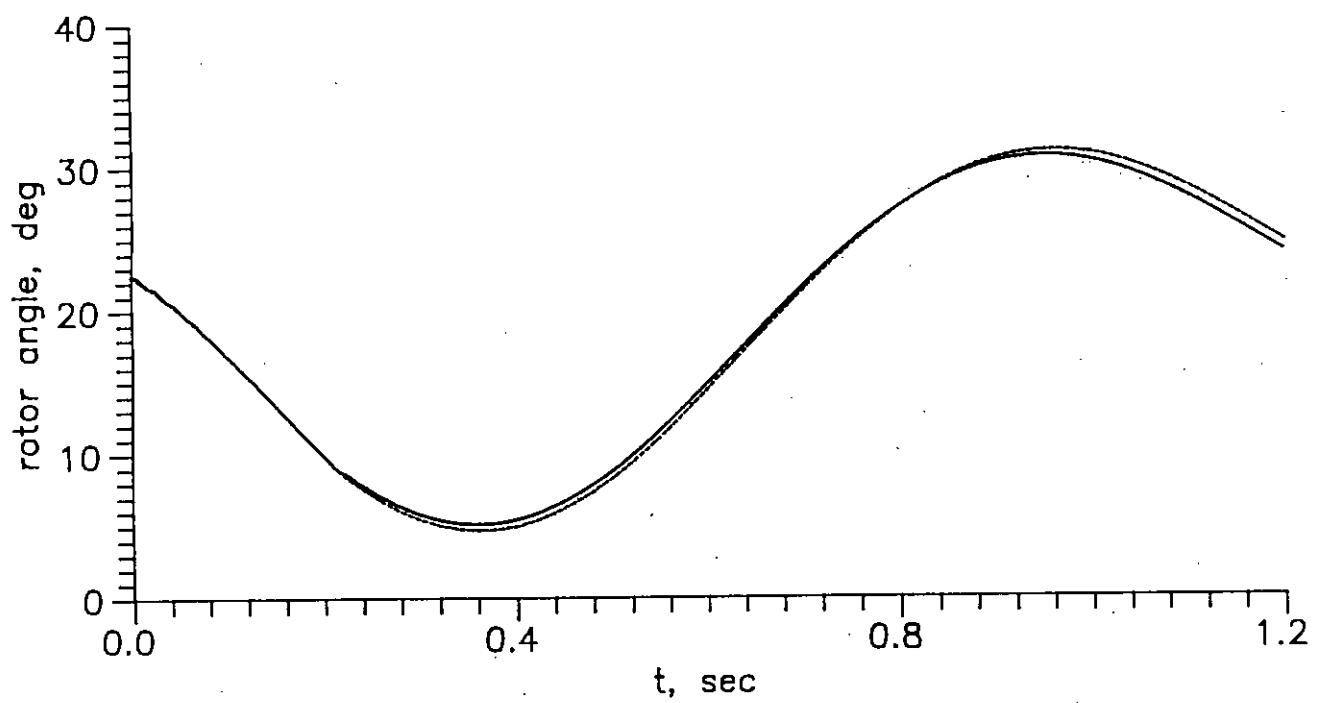


Fig 3.6 Result of a fault throwing test.

— test result.
- - - detail model's prediction.

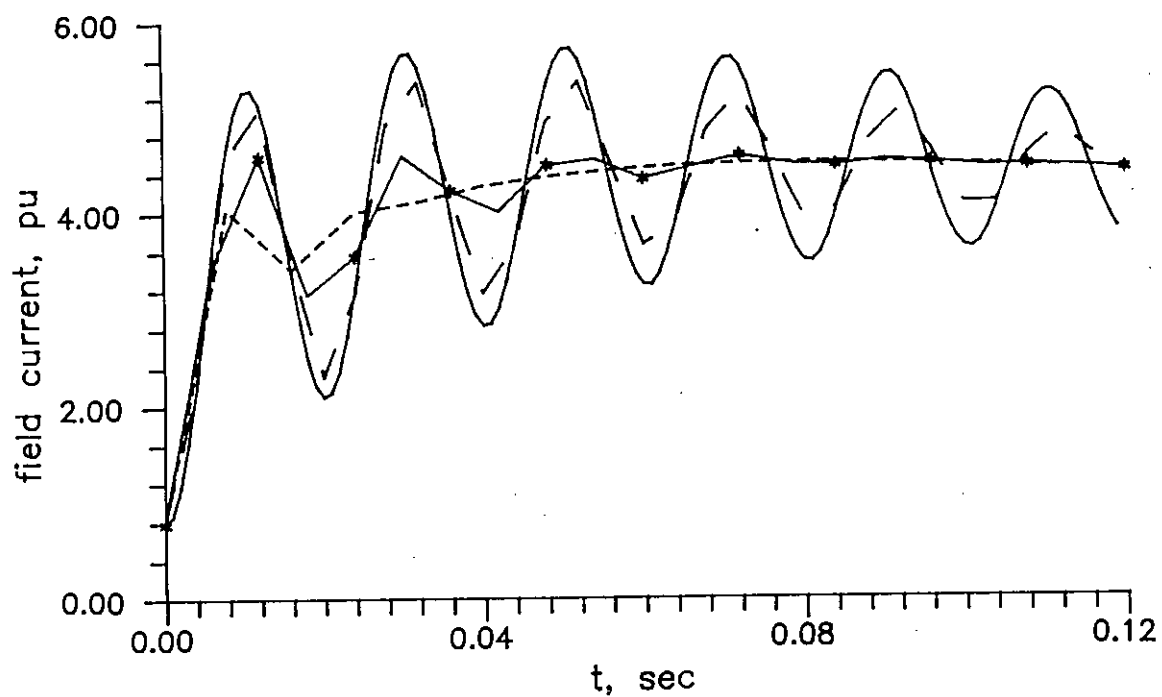
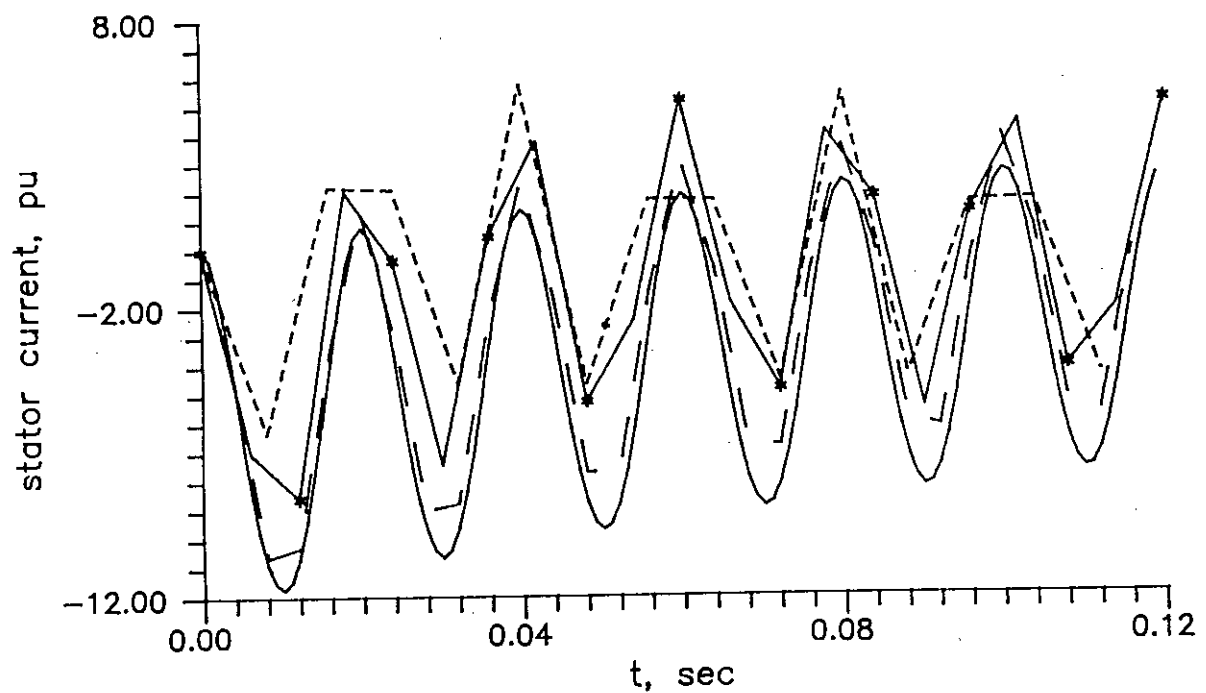


Fig 4.1 Effect of time step in the prediction of currents.

—	0.0001 sec.	— —	0.001 sec.
—*—*	0.0005 sec.	- - -	0.01 sec.

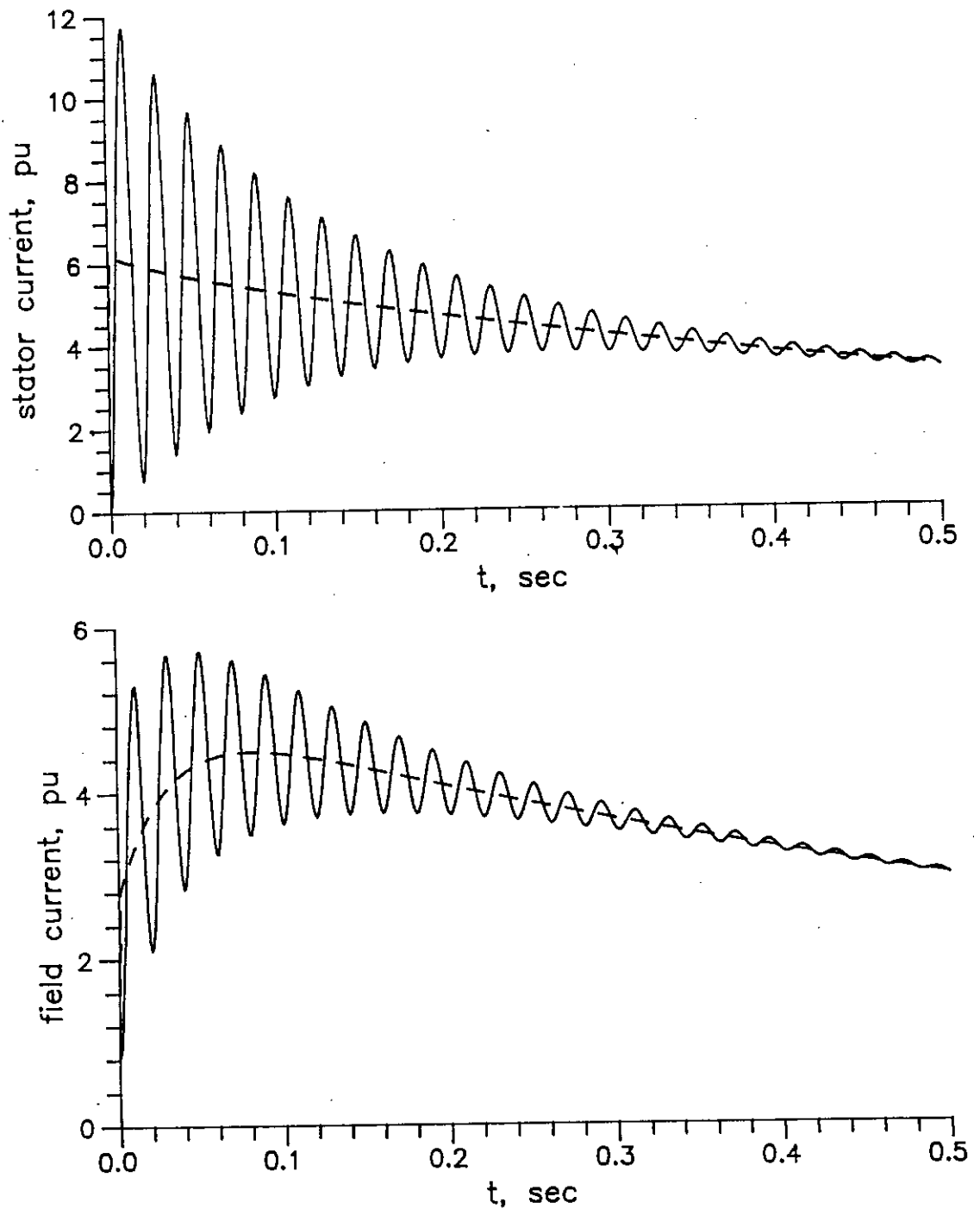


Fig 4.2 Current wave shapes following a 3-phase short circuit at machine terminal.

—— detail model's simulation.
--- simple model's simulation.

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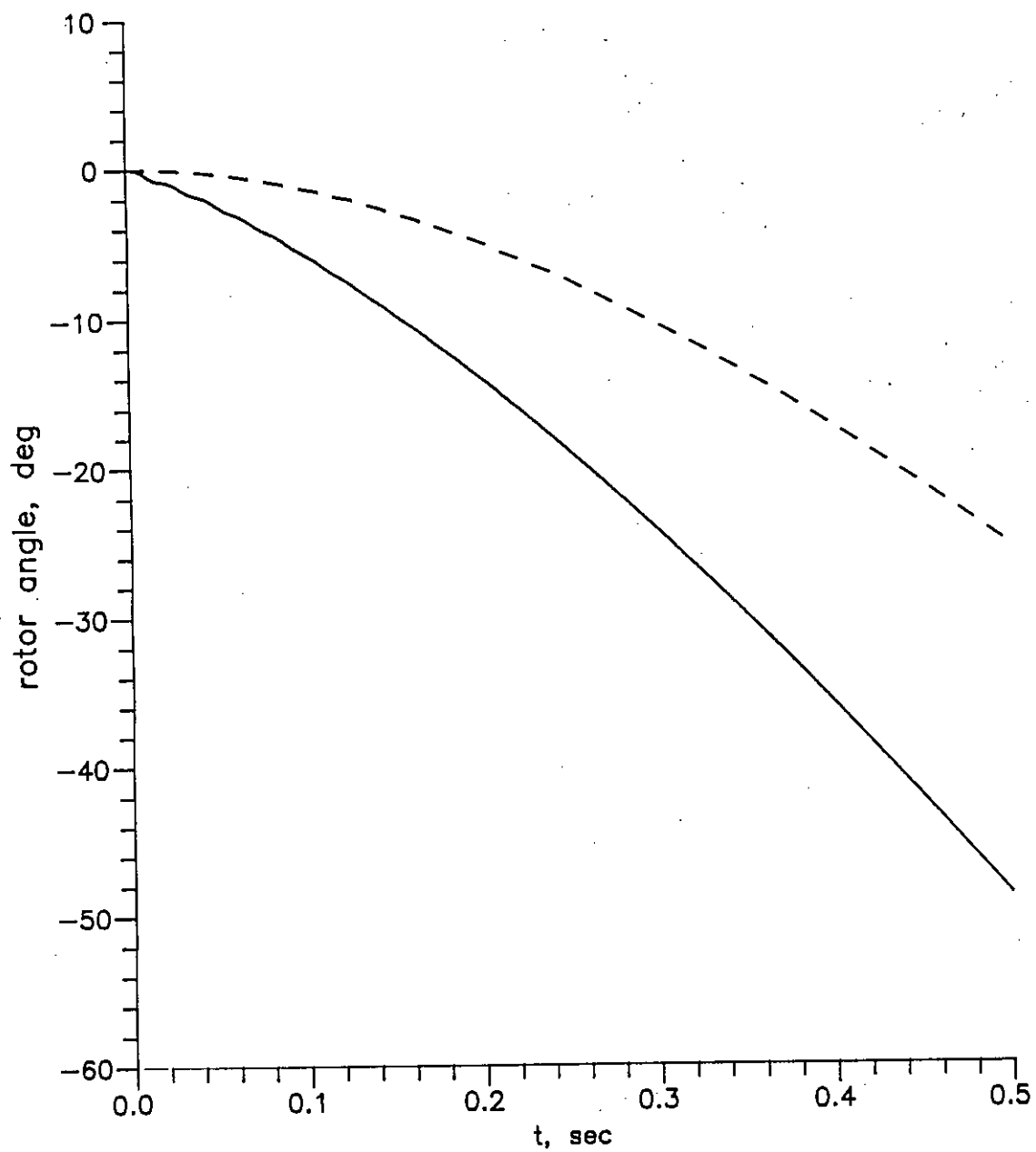


Fig 4.3 Prediction of rotor angle movement following a three phase short circuit at machine terminal.

— detail model's prediction.
--- simple model's prediction.

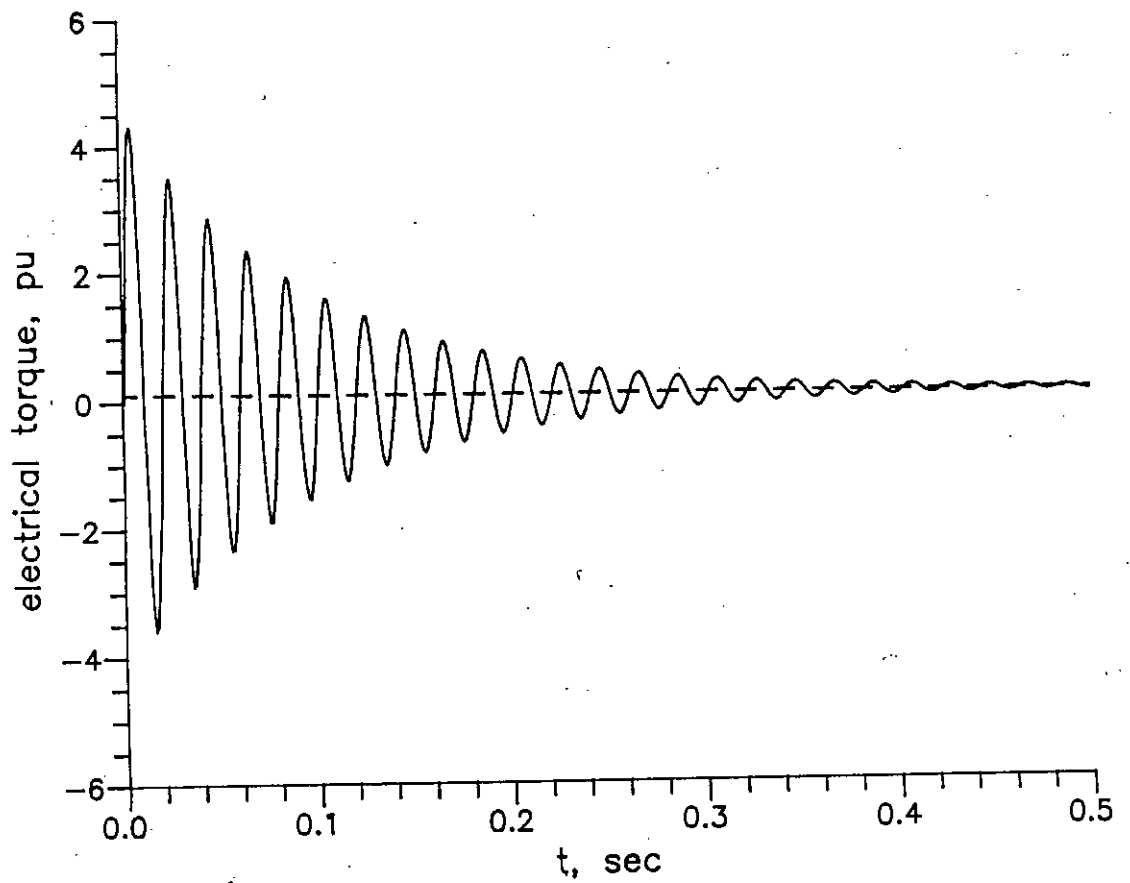
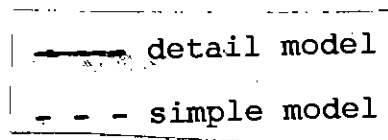


Fig 4.4 Electrical torque following a 3-phase short circuit.



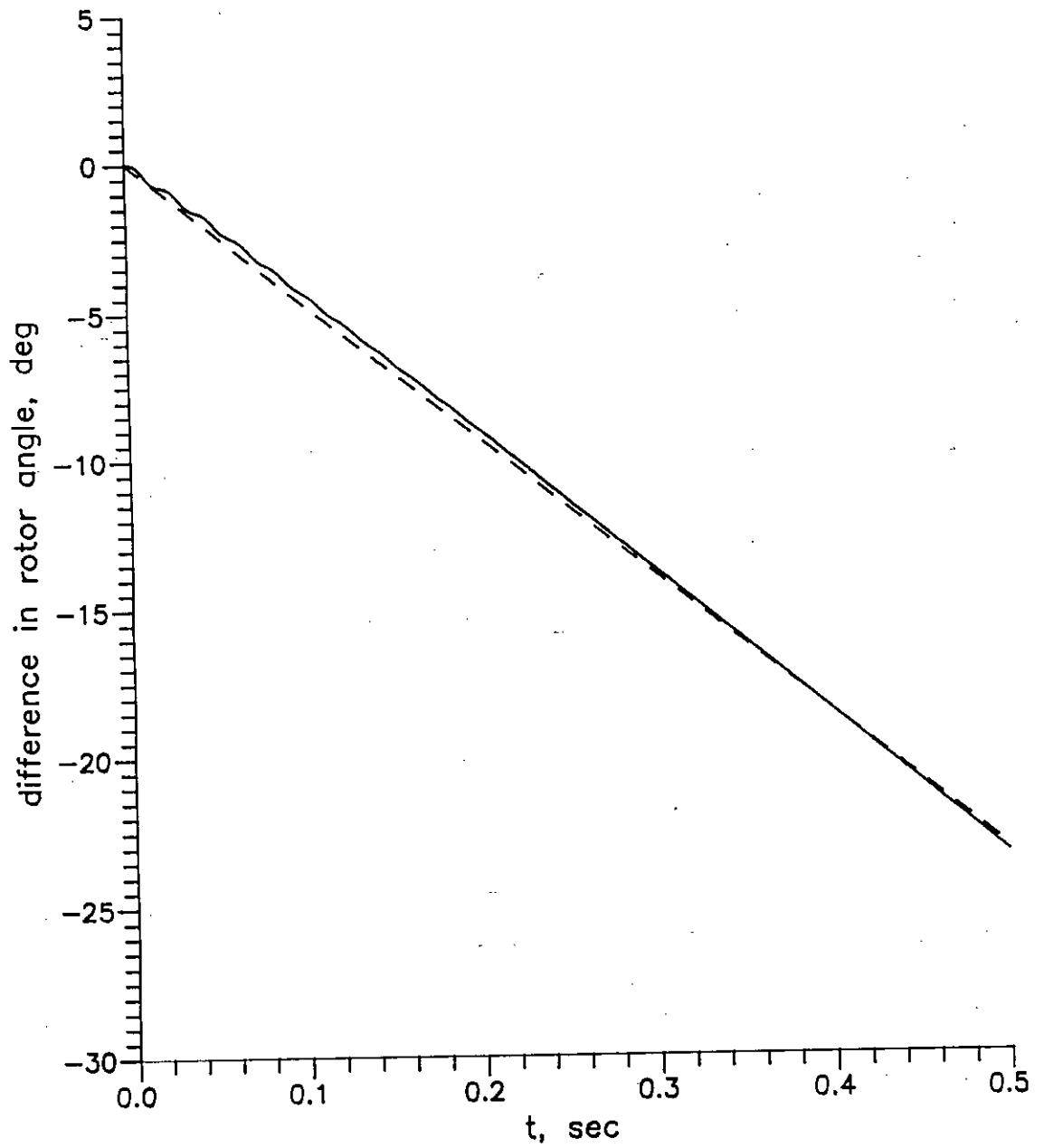


Fig 4.5 Error in rotor angle movement following a 3-phase short circuit.

— Obtained from the difference between the models.
 - - - Derived from oscillatory component of electrical torque.

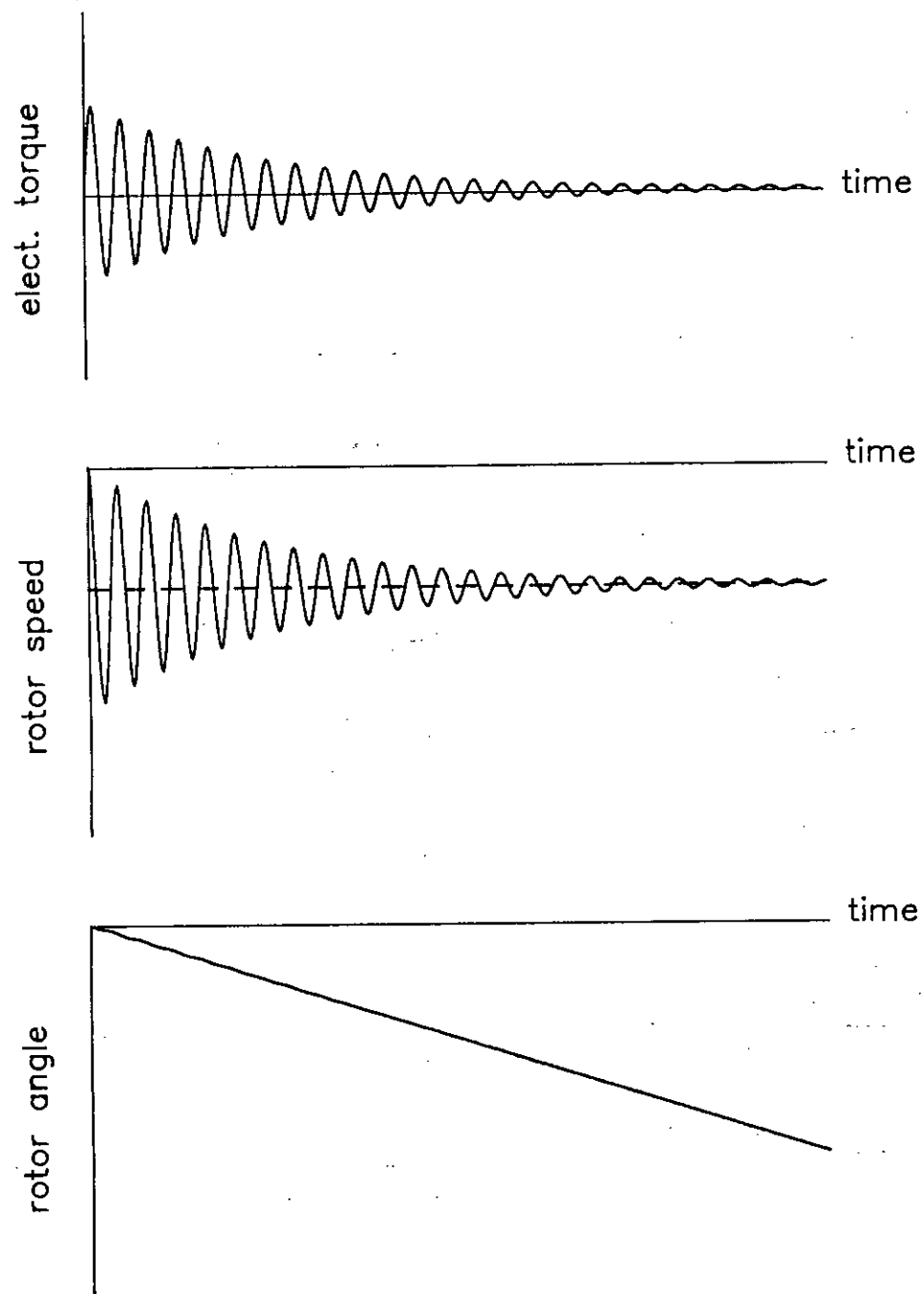


Fig 4.6 Derivation the rotor angle change from the oscillatory component of torque.

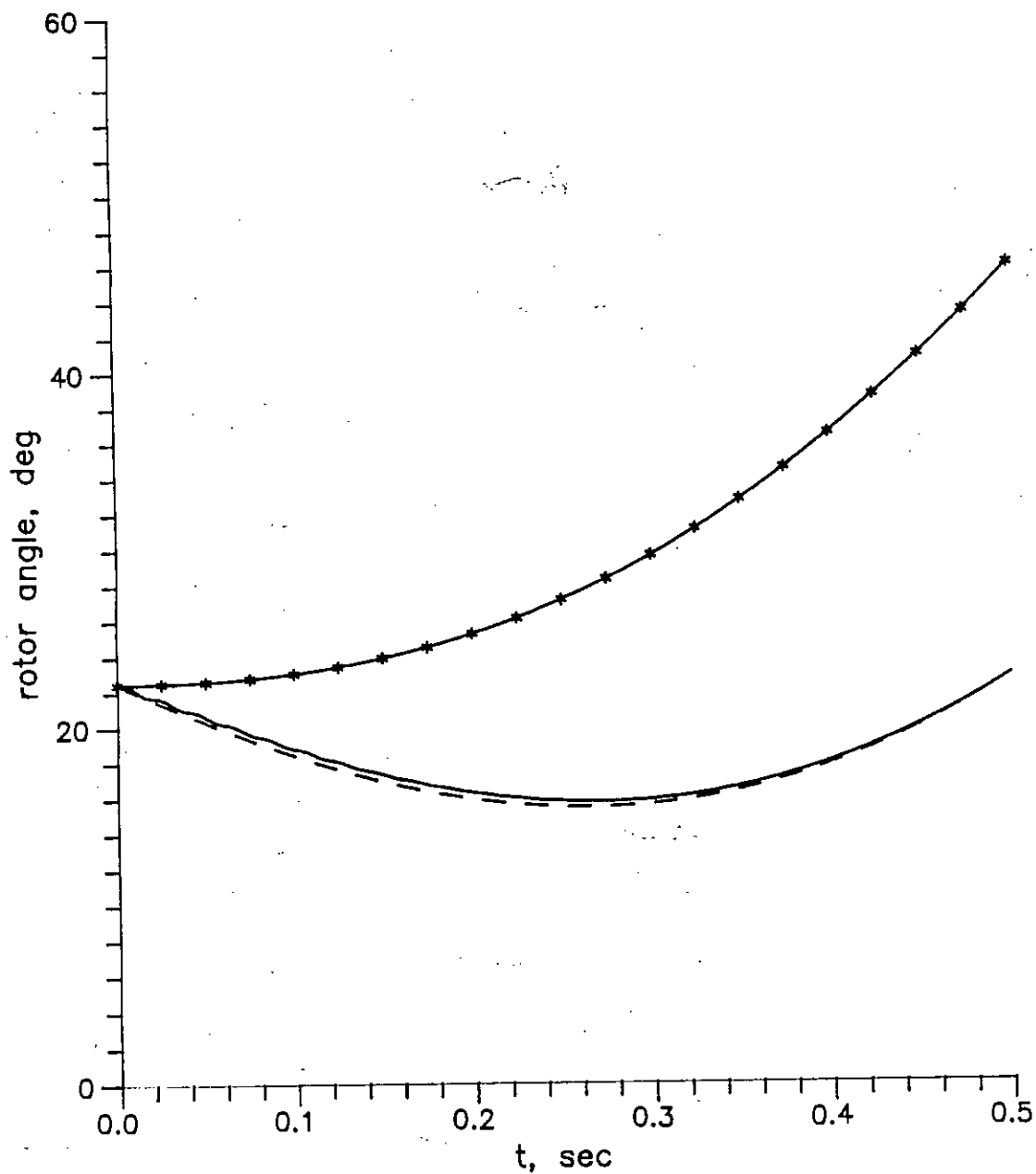


Fig 4.7 Prediction of rotor movement of a machine with a preload of 0.146 pu.

—— detail model. -*- simple model.
 --- modified model.

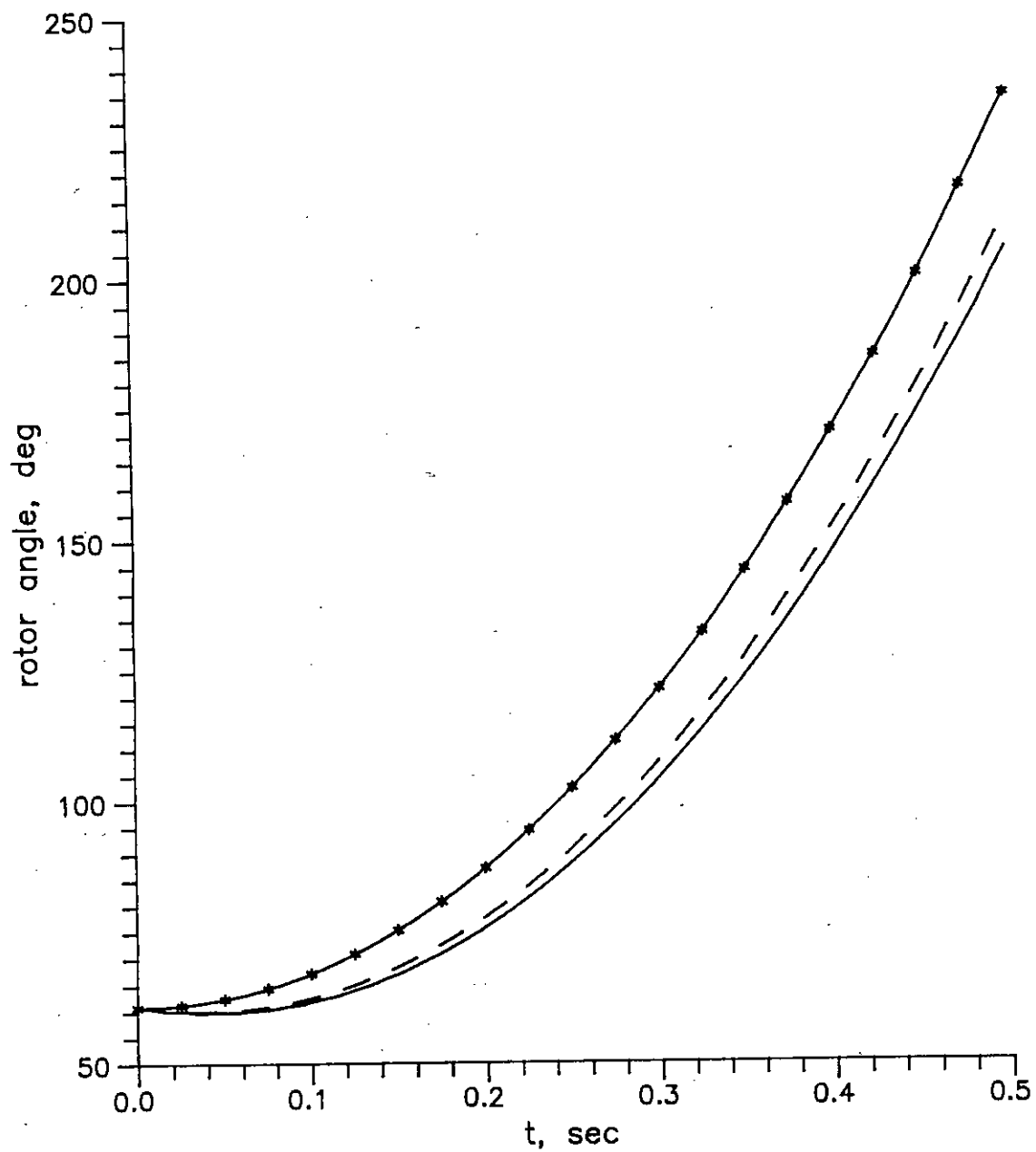


Fig 4.8 Prediction of rotor movement of a machine with a preload of 0.615 pu.

— detail model -*- simple model
 --- modified model

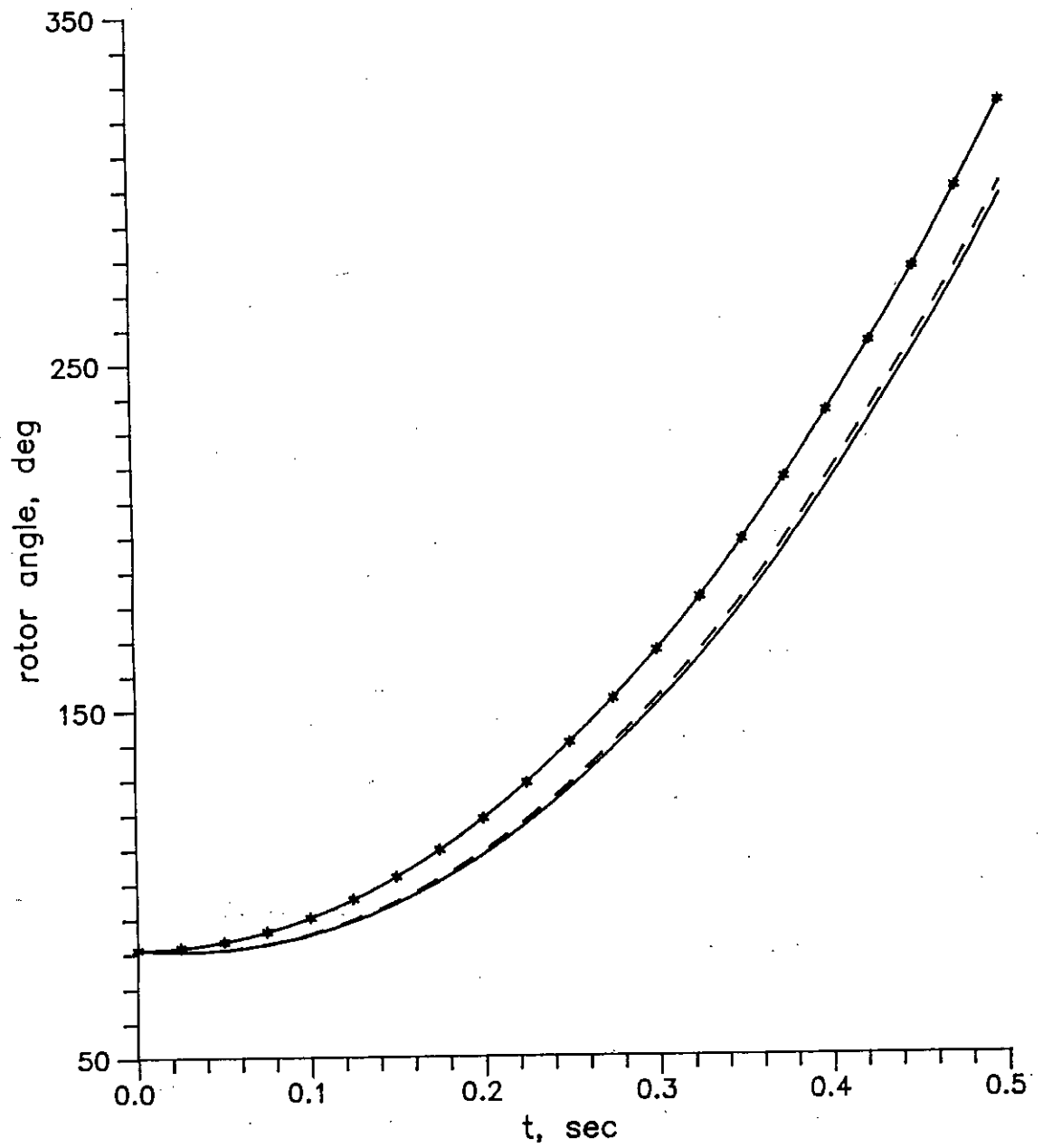


Fig 4.9 Prediction of rotor movement of a machine with a preload of 0.813 pu

— detail model -*- simple model
 --- modified model

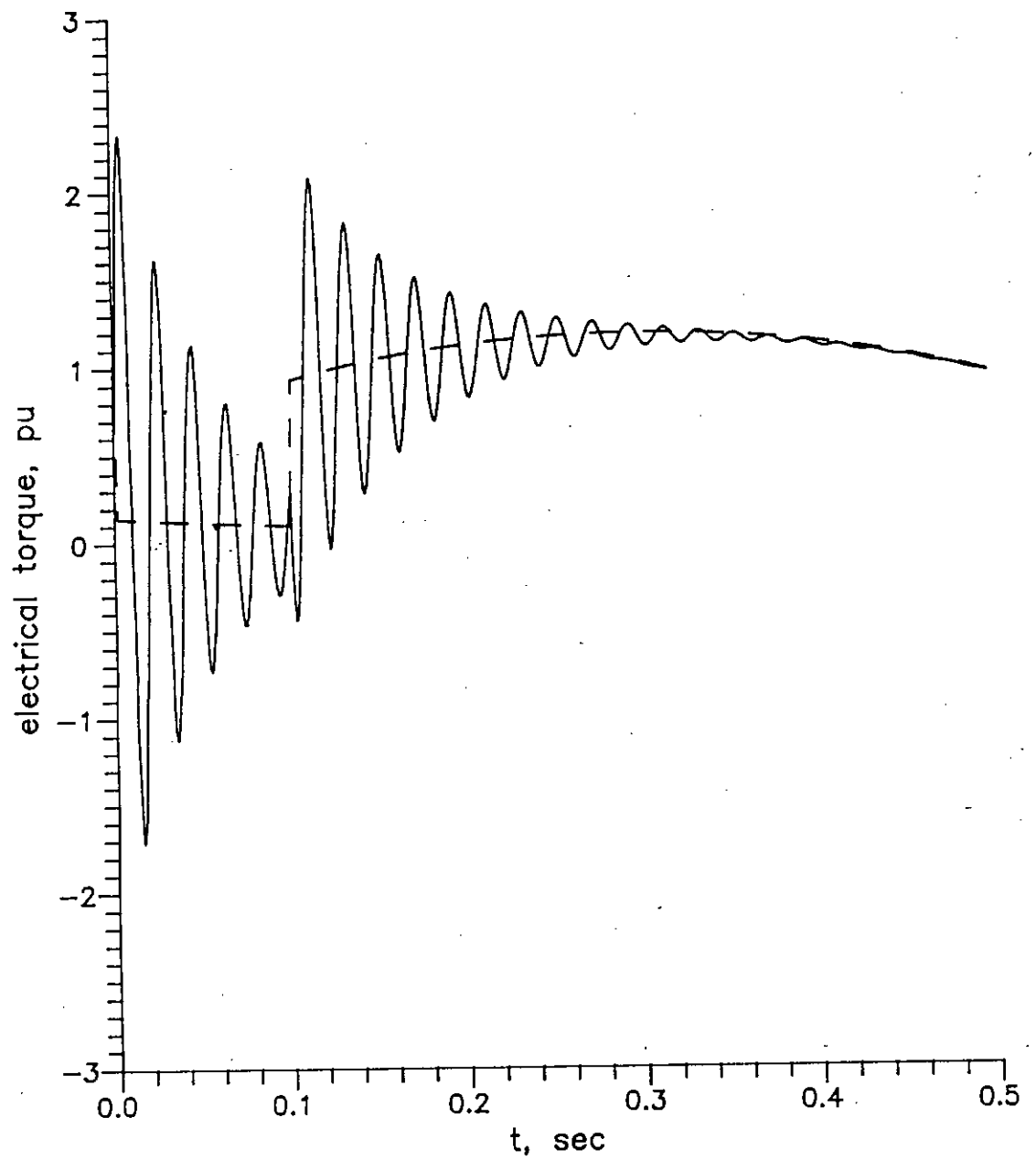


Fig 4.10 Electrical torque during a fault throwing test from 76.5°

—— detail model's simulation
 --- simple model's simulation

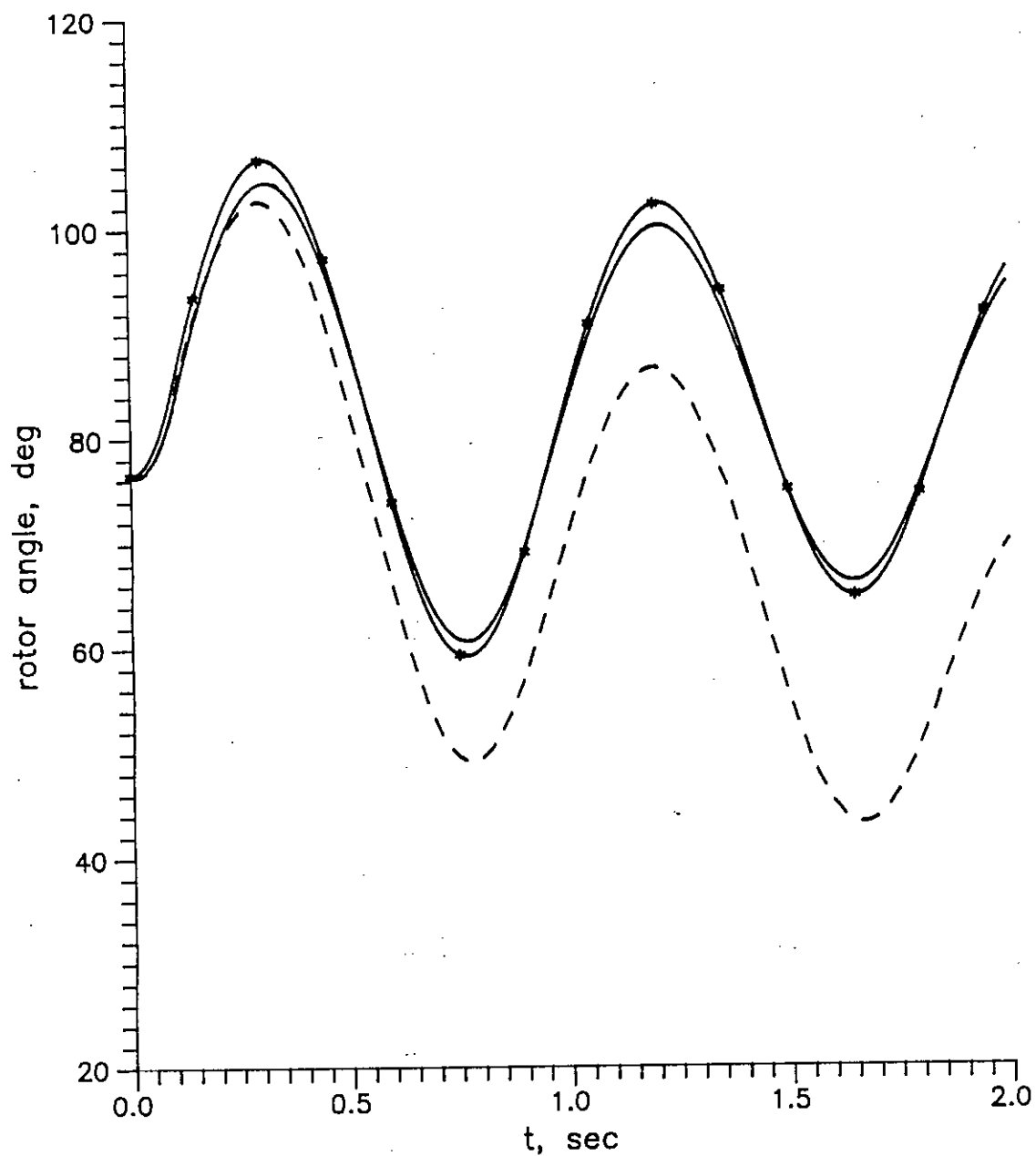


Fig 4.11 Simulation of rotor angle following short circuit fault of 100 milliseconds.

- detail model's simulation
- *- simple model's prediction
- - - considering the effect of oscillatory torque globally.

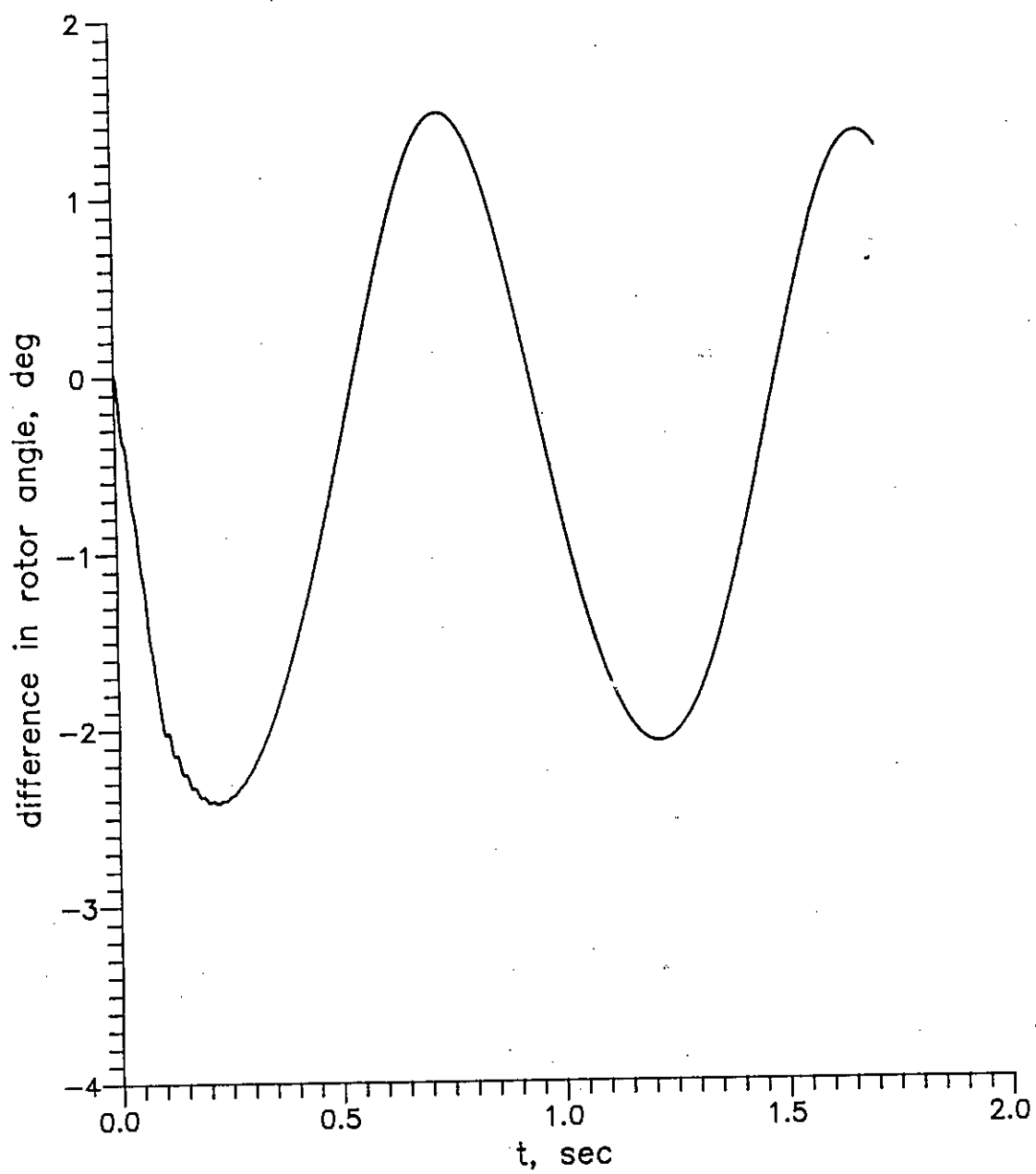


Fig 4.12 Error in the rotor angle prediction by the simple model for a fault throwing test.

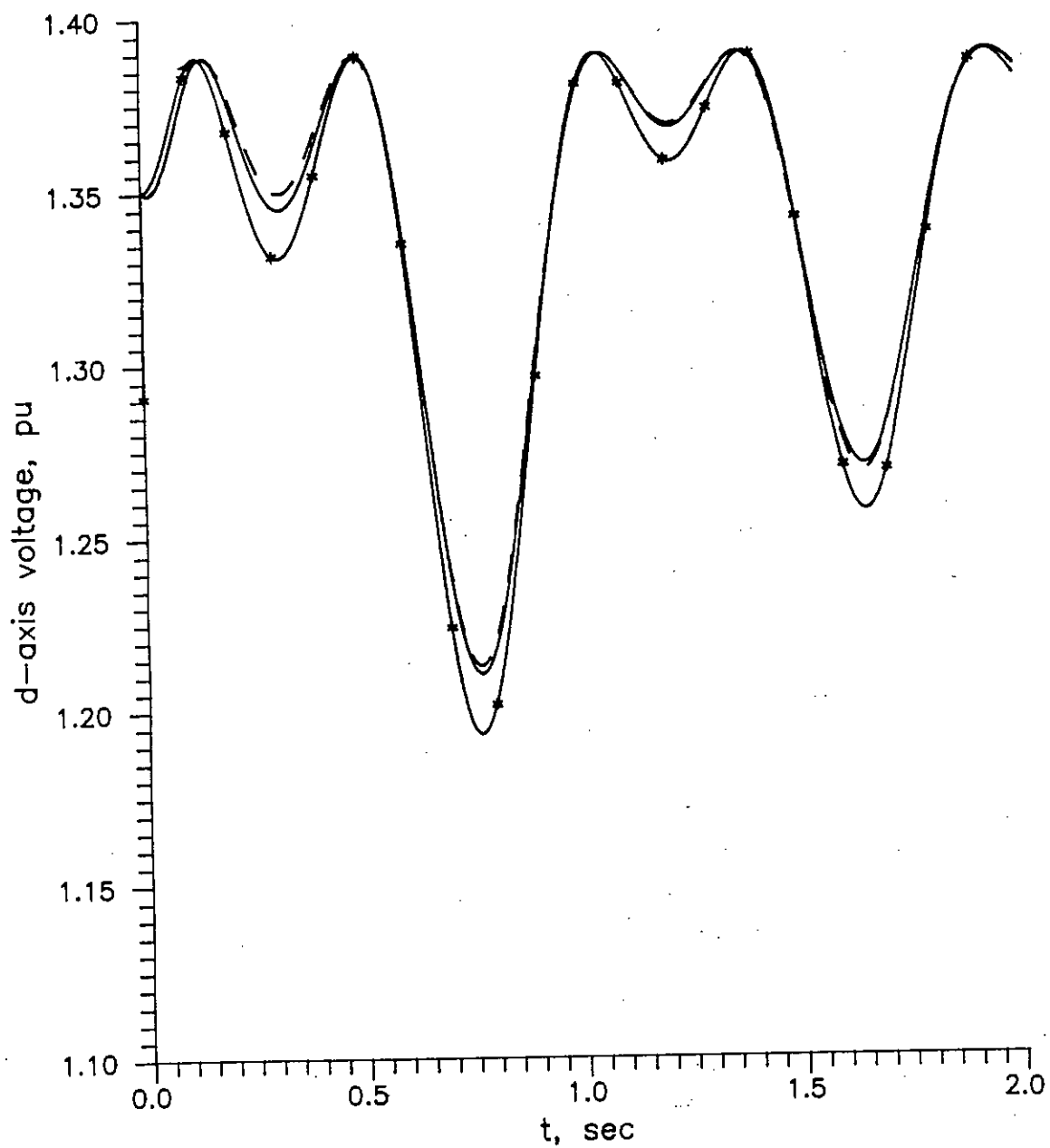


Fig 4.13 D-axis component of infinite bus voltage.

——— predicted by detail model
 -*-*- predicted by simple model.
 - - - predicted by modified model.

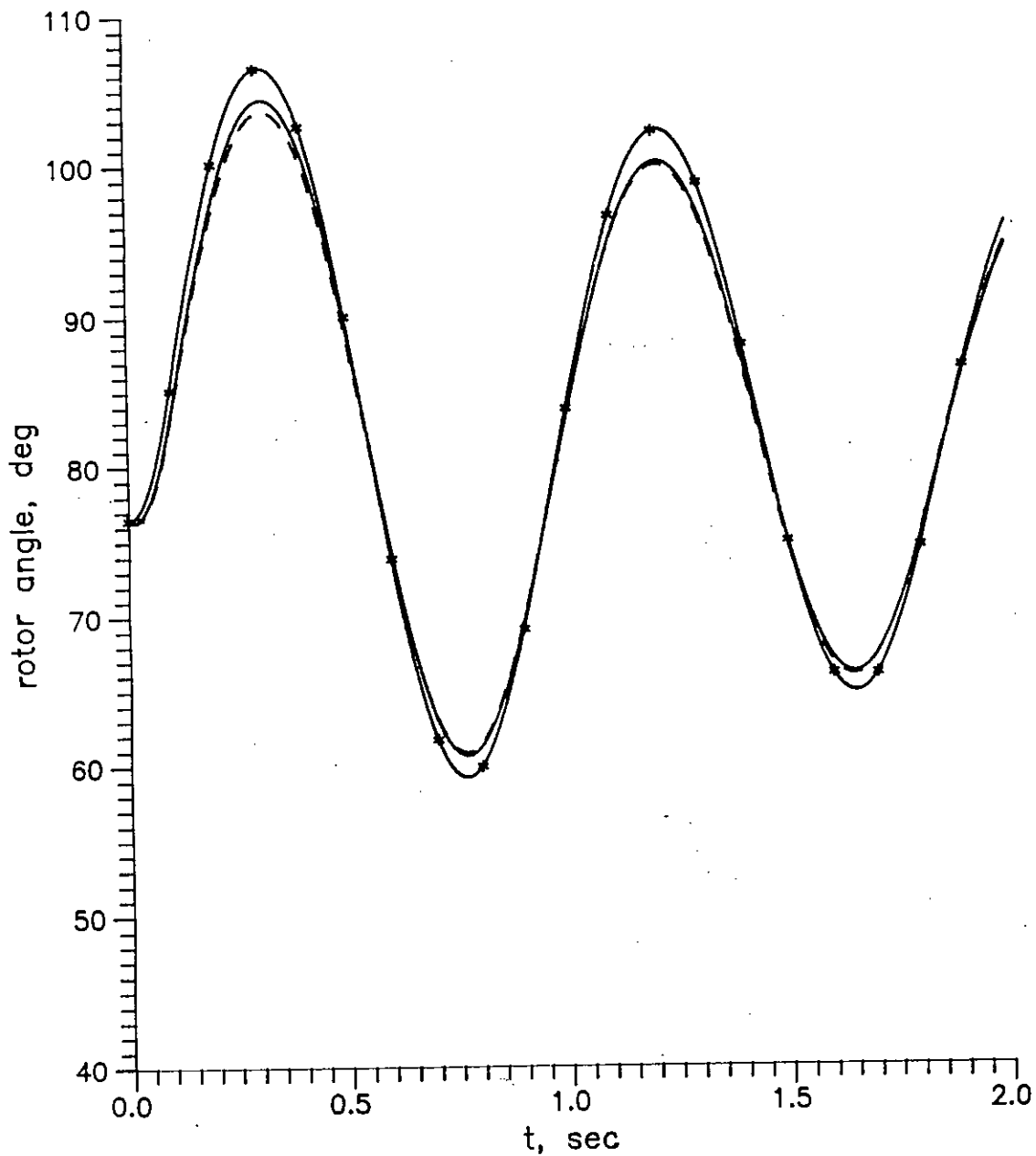


Fig 4.14 Comparison of rotor angle simulation obtain from detail, simple and modified model.

— detail model. —*— simple model.
 - - modified model.

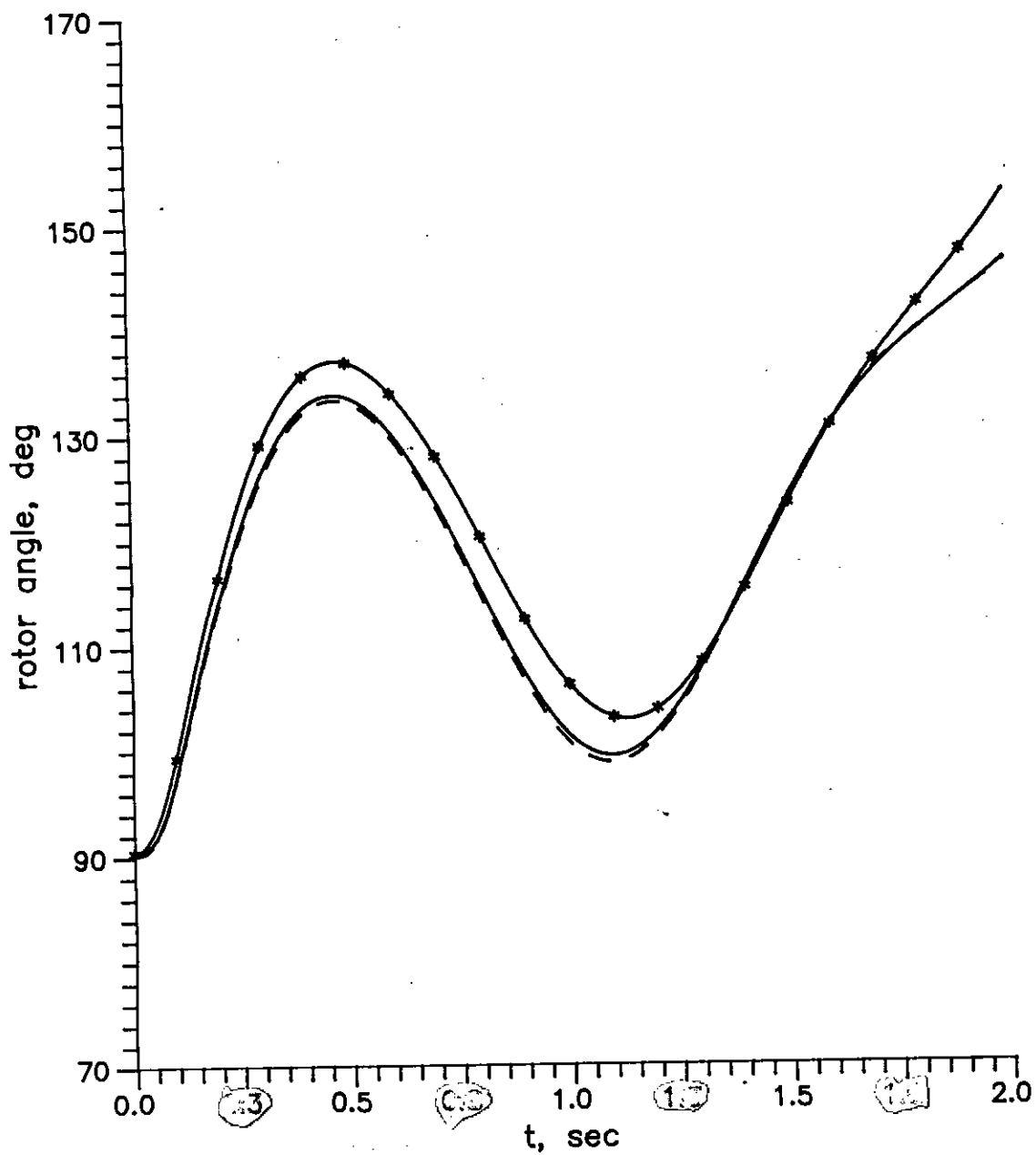


Fig 4.15 Simulation of a fault throwing test from an initial rotor angle of 90.5°

— detail model. -*- simple model.
 - - modified model.

TABLE 3.1
INITIAL CONDITIONS OF DYNAMIC TESTS

Quantities	Tests					
	1	2	3	4	5	6
Terminal power, pu	0	0.144	0.615	0.813	0.787	0.8
Rotor angle, deg	0	22.5	61.0	81.5	76.5	90.5
Terminal voltage, pu	1.0	0.995	1.095	1.04	1.0	1.004
Terminal current, pu	0.0	0.146	0.598	0.823	0.797	0.87
Field current, pu	0.79	0.8115	1.544	1.769	1.386	1.25

