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THE DIAGONAL COMPRESSION FIELD THEORY FOR
REINFORCED CONCRETE BEAMS SUBJECTED TO COMBINED
TORSION, FLEXURE AND AXIAL LOAD

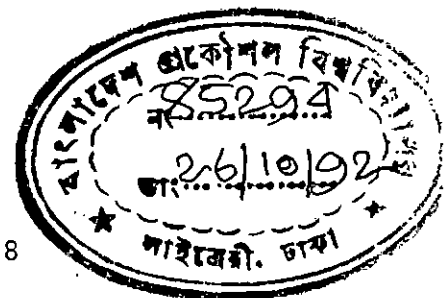
by

WINSTON M. ONSONGO

Department of Civil Engineering

A Thesis submitted in conformity with the requirements
for the Degree of Doctor of Philosophy in the
University of Toronto

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SCHOOL OF GRADUATE STUDIES

PROGRAM OF THE FINAL ORAL EXAMINATION
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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WINSTON MARASI ONSONGO

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THE DIAGONAL COMPRESSION FIELD THEORY FOR REINFORCED CONCRETE BEAMS

SUBJECTED TO COMBINED TORSION, FLEXURE AND AXIAL LOAD

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The Diagonal Compression Field Theory is formulated by considering equilibrium conditions, compatibility of geometric deformations and the stress-strain characteristics of the steel and the concrete. It is assumed that plane sections remain plane and that after cracking the concrete can not resist tension. The concrete is assumed to resist the torsional shear flow by means of diagonal compressive stresses. The direction of these principal stresses at any particular point on the section is assumed to coincide with the direction of principal compressive strains at this point.

Also presented in this thesis are the test results of fourteen large, heavily reinforced rectangular beams tested in combined torsion and bending. The detailed measurements of deformation made during these tests (over 250 strain readings were taken on each beam at each load stage) verified the detailed predictions of the compression field theory.

Numerical techniques which enable a solution to be obtained for rectangular beams subjected to torsion and flexure

have been incorporated into a computer program which is listed and explained in the thesis.

The thesis restricts its attention to beams in which torsion is resisted by shear flow (St. Venant torsion) and in which the reinforcement is properly detailed and consists of closed hoops perpendicular to the axis of the member and longitudinal reinforcing steel (which may be prestressed) parallel to the axis of the member.

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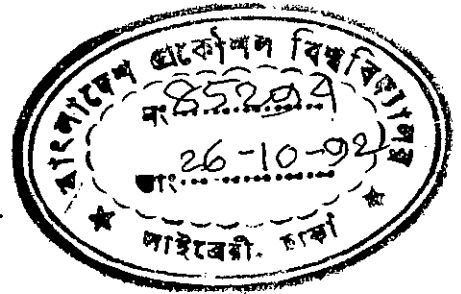
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CHAPTER 1 - INTRODUCTION

Many modern structures such as elevated guideways are subjected to combined flexure, torsion, shear and axial load. Designing such structures requires an understanding of the behaviour of structural concrete under such combined actions. A long-term research program is in progress in the Department of Civil Engineering of the University of Toronto whose overall objective is to develop simple, general and rational models capable of predicting the response of structural concrete members under combined loading. Eventually it is hoped to replace the complex, restricted, empirical, design rules now used for shear and torsion by procedures comparable in rationality with those available for flexure and axial loads.

As part of this program, a rational model called the Diagonal Compression Field Theory capable of predicting the behaviour of symmetrically reinforced structural concrete members subjected to pure torsion, was developed.¹ This theory like the well known theory for flexure was developed by considering equilibrium conditions, compatibility of geometric deformations and the stress-strain characteristics of the steel and the concrete. Further, after cracking, it was assumed that the concrete could not resist tensile stresses and that the concrete cover would spall off down to the hoop centreline.

This thesis extends the scope of the Diagonal Compression Field Theory so that it is capable of predicting the post-cracking behaviour of reinforced concrete members subjected to combined torsion,

flexure and axial load.

The important difference between the Compression Field Theory as presented in this thesis and variable angle space truss models for beams in torsion and flexure such as that presented by Rabbat⁹ is that in these space truss models empirical assumptions are made as to the effective geometry of the section. In the Compression Field Theory the thickness of the concrete diagonals, the position of the resultant steel and concrete forces and the path of the shear flow are all determined from equilibrium and compatibility requirements.

In addition to presenting the theoretical model this thesis will describe an experimental program in which fourteen heavily reinforced, intensively instrumented rectangular beams were loaded in combined torsion and flexure. Heavily reinforced beams were chosen as they would provide the most sensitive test of the theory. Detailed deformation measurements were made during the tests so that the detailed predictions of the theory could be checked. While the detailed solutions described in this thesis will be for rectangular sections subjected to combined torsion and flexure the general theory presented can be applied to other cross-sectional shapes. The theory is however applicable only to St. Venant torsion (i.e., torsion resisted by shear flow) and hence cannot predict the response of sections where warping torsion dominates.

The thesis restricts its attention to beams in which the

reinforcement is properly detailed and consists of closed hoops perpendicular to the axis of the member and longitudinal steel (which may be prestressed) parallel to the axis of the member.

In the available English literature the only other attempt to develop a theory capable of predicting the complete post-cracking behaviour of under-reinforced and over-reinforced concrete beams subjected to combined torsion and flexure is that by Below, Rangan and Hall.¹⁴ However, their theory, which is based on the skew-bending model, contains a number of arbitrary assumptions and does not predict well the behaviour of over-reinforced beams nor the behaviour of beams loaded predominantly in torsion. Further, their theory is not capable of treating torsion and biaxial flexure nor torsion, bending and axial load.

It is emphasized that the objective of this thesis is to develop a model for structural concrete beams in combined torsion, flexure and axial load which is comparable in rationality and generality to the well known theory for flexure and axial load.

CHAPTER 2

THE DIAGONAL COMPRESSION FIELD THEORY FOR
COMBINED FLEXURE, TORSION AND AXIAL LOAD2.1 ASSUMPTIONS

The following basic assumptions are made in the development of the theoretical model.

1. The longitudinal strains along any line in a section vary linearly (Bernoulli-Navier hypothesis).
2. All strains at a point in a plane are compatible.
3. The principal direction of diagonal compression strain evaluated from a set of compatible strains at a point in a plane of loading is coincident with the direction of principal compression stress at that point in the plane.
4. In the post-cracking loading range the concrete can carry no tension and the applied actions will be resisted by a field of diagonal compression in the concrete and the reinforcing steel will resist the tension.
5. The stress-strain relationships for the concrete and the reinforcing steel in a structural concrete beam under loading are assumed the same as the stress-strain relationships obtained from a standard cylinder test for concrete and the stress-strain relationship obtained from a standard tension test for the steel.

2.2 EQUILIBRIUM CONDITIONS

Figure 2.1 shows the assumed cracked model for a rectangular beam section subjected to uniform flexural moments M_y and M_z , a torque, T , and an axial force N_x . The shear flow set up by the torque is resisted in a field of diagonal compression in the concrete after cracking. The inclination of the concrete diagonals and their effective depths in the beam section are determined by the equilibrium requirements and requirements of compatibility of deformations.

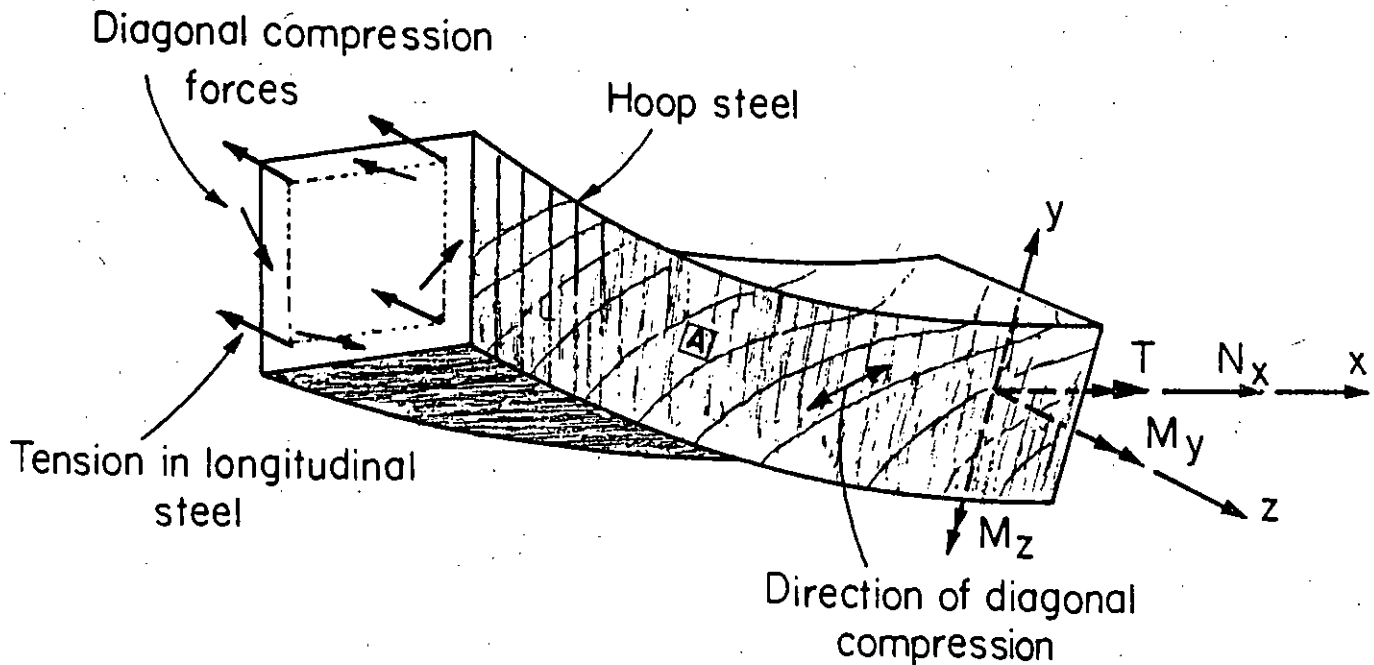


FIG. 2.1 CRACKED REINFORCED CONCRETE BEAM MODEL.

2.2.1 Concrete Stresses

Consider the state of stress in a concrete element A in the cracked model (Fig. 2.1). Such an element which is assumed to be in the plane of shear flow will be subjected to a general set of stresses in the plane shown in Fig. 2.2 (a). If it is assumed that concrete has no tensile strength the stresses on the element can be considered equivalent to a principal compression stress f_d as shown in Fig. 2.2(b). This state of stress can be represented by Mohr's circle shown in Figure 2.2 (c) and the following relationships are deduced:

$$\tau = f_d \tan \theta / (1 + \tan^2 \theta) \dots \dots \dots (2.1)$$

$$\sigma_{ch} = \tau \tan \theta \dots \dots \dots (2.2)$$

$$\sigma_{cl} = \tau / \tan \theta \dots \dots \dots (2.3)$$

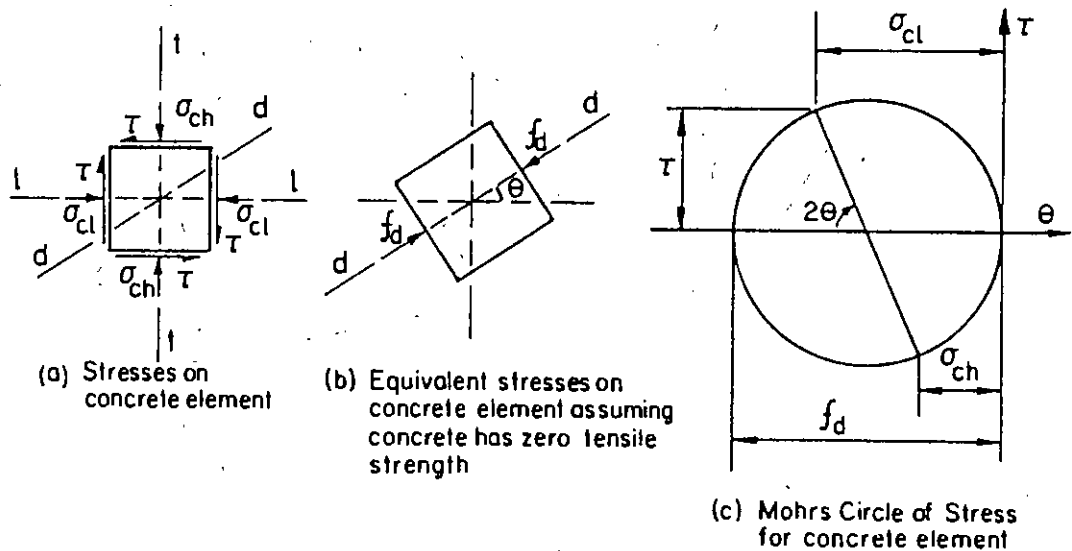


FIG. 2.2 STRESSES ON CONCRETE ELEMENT

The shear stress τ when integrated over the wall thickness, t , at a location on the section, will result in the shear flow q which is constant around the section and balances the applied torque, T :

$$q = \int_t \tau dt \dots \dots \dots (2.4)$$

$$\text{and } T = 2qA_o \dots \dots \dots (2.5)$$

where A_o is the area enclosed by the effective path of shear flow around the section.

2.2.2. Steel Stresses

The diagonal concrete stresses, f_d , give rise to forces which are equilibrated by the tension developed in the reinforcing steel. The concrete and the steel stresses can be related by considering the equilibrium of appropriate free body diagrams. For simplicity a rectangular section shall be used to derive these relationships.

2.2.2.1 Longitudinal Steel Stresses

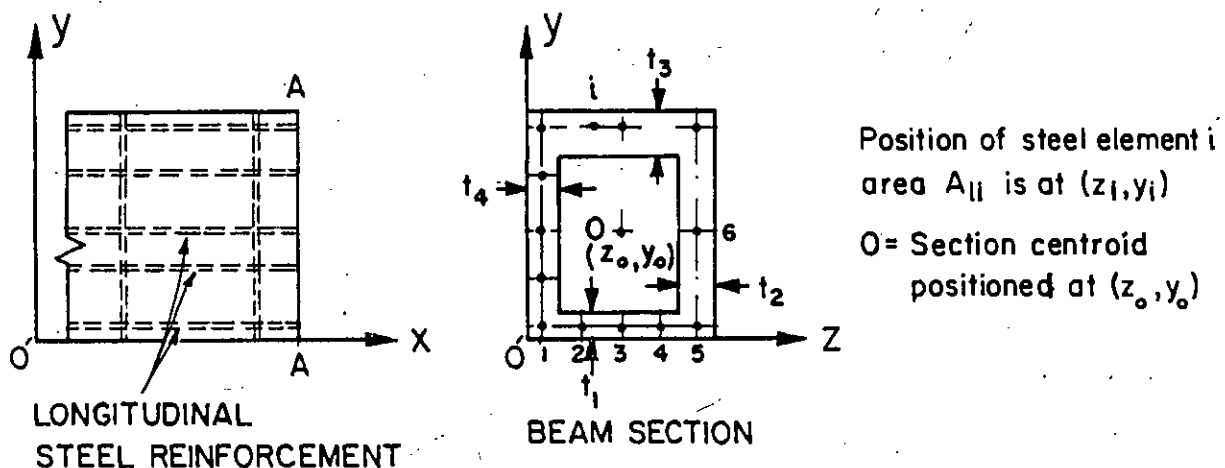


FIG. 2.4 LONGITUDINAL STEEL IN SECTION

Consider section A-A of an unsymmetrically reinforced rectangular section shown in Fig. 2.4. The longitudinal stresses can be evaluated by considering the equilibrium of longitudinal forces at the section.

If given the longitudinal strain ϵ_{l0} at the origin (0,0), then using assumption (1) the strains of the steel element at position (z_i, y_i) can be evaluated as:

$$\epsilon_{li} = \epsilon_{l0} + \phi_{xy} y_i + \phi_{xz} z_i \dots \dots \dots (2.6)$$

Where: ϕ_{xy} = curvature in Y - direction
 ϕ_{xz} = curvature in Z - direction

With the longitudinal strains the corresponding steel stresses are evaluated using the stress-strain relationships for the steel obtained from standard tension tests of representative specimens of the steel. Thus the resultant tension force in the reinforcing steel in the longitudinal direction can be evaluated as:

$$F_{sl} = \sum f_{li} A_{li} \dots \dots \dots (2.7)$$

where f_{li} is the stress of the steel element of area A_{li} at (z_i, y_i) and is a function of ϵ_{li} .

The position at which the resultant steel tension force, F_{sl} , acts with respect to the origin chosen is $(\bar{z}_s, \bar{y}_s)$ where

$$\left. \begin{aligned} \bar{y}_s &= \sum f_{li} A_{li} y_i / F_{sl} \\ \bar{z}_s &= \sum f_{li} A_{li} z_i / F_{sl} \end{aligned} \right\} \dots \dots \dots (2.8)$$

The resultant compression force, F_{cl} , in the concrete in the longitudinal direction at section A-A is evaluated by

integrating the compression concrete stresses σ_{cl} over the cross section:

$$F_{cl} = \oint_p \left(\int_t \sigma_{cl} dt \right) dp \dots \dots \dots (2.9)$$

where the stresses are summed over the wall thickness, t , and around the entire section perimeter, p .

The position at which the force F_{cl} acts with respect to the origin chosen is $(\bar{z}_c, \bar{y}_c)$, where

$$\begin{aligned} \bar{y}_c &= \oint_p \left(\int_t \sigma_{cl} dt \right) y_p dp / F_{cl} \\ \bar{z}_c &= \oint_p \left(\int_t \sigma_{cl} dt \right) z_p dp / F_{cl} \end{aligned} \dots \dots \dots (2.10)$$

The internal forces F_{sl} and F_{cl} and the moments they induce will be in equilibrium with the applied actions. If moments are evaluated about the section centroid (z_o, y_o) and if it is assumed that the applied axial load N_x is tensile and applied along the longitudinal centroidal axis then equilibrium requires:

$$\left. \begin{aligned} N_x &= F_{sl} - F_{cl} \\ M_y &= F_{sl} (\bar{y}_c - \bar{y}_s) + N_x (y_o - \bar{y}_c) \\ M_z &= F_{sl} (\bar{z}_c - \bar{z}_s) + N_x (z_o - \bar{z}_c) \end{aligned} \right\} \dots \dots \dots (2.11)$$

If $F_{sl} = F_{cl}$, $\bar{y}_c = \bar{y}_s$ and $\bar{z}_c = \bar{z}_s$, a state of pure torsion exists.

2.2.2.2. Hoop Steel Stresses

The hoop steel stresses can be related to the concrete diagonal stresses, f_d , by considering the equilibrium of forces in the transverse direction for free bodies obtained by taking sections parallel to the longitudinal axis as in Figure 2.5.

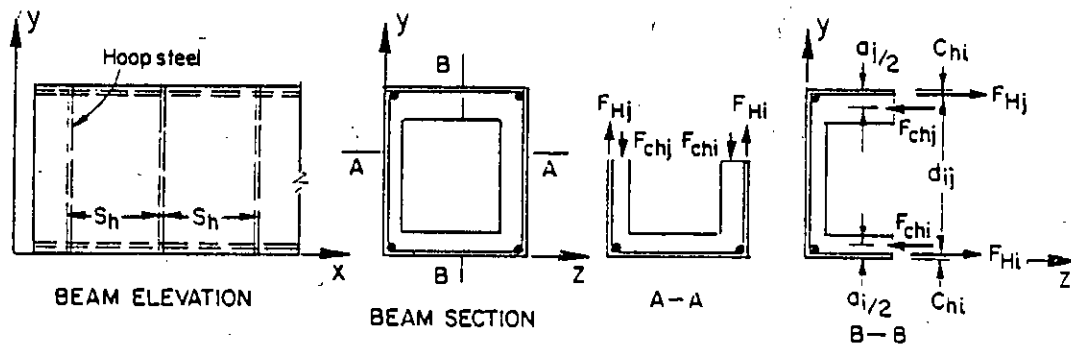


FIG. 2.5 EVALUATION OF STRESSES IN HOOP STEEL

For simplicity assume the hoop steel spacing, s_h , is constant over the entire beam span for all the beam faces. Then in equilibrium considerations in the direction of hoop steel, only the stresses over a typical length, s_h , of the span are considered. With reference to Fig. 2.5 section B-B:

$F_{Hi} = A_{hf} f_{hi}$ = resultant tension force in hoop steel at position 'i'

A_{hf} = area of hoop steel section for face 'f' where 'i' is positioned

f_{hi} = hoop steel stress at position 'i'

$$F_{chi} = s_h \int \sigma_{chi} dt \dots \dots \dots (2.12)$$

= resultant compression force in concrete in the direction of hoop steel at position 'i' which effectively acts at a distance $a_i/2$ from the external surface.

σ_{chi} is given by equation (2.2) and the integral

in equation 2.12 is evaluated over the wall thickness of the section at position 'i'.

Applying the conditions for equilibrium ($\Sigma F=0$ and $\Sigma M=0$) of the forces shown in Section B-B, deduce:

$$\left. \begin{aligned} F_{Hi} &= F_{chi}[1-(a_i/2 - c_{hi})/d_{ij}] + F_{chj}(a_j/2 - c_{hj})/d_{ij} \\ F_{Hj} &= F_{chi}(a_i/2 - c_{hi})/d_{ij} + F_{chj}[1-(a_j/2 - c_{hj})/d_{ij}] \end{aligned} \right\} \dots(2.13)$$

The evaluation of a_i and a_j is described in section 2.3.4.

2.3 GEOMETRIC CONDITIONS

2.3.1 Compatibility requirements for strains

While the local strains of the diagonally cracked beam may exhibit local discontinuities the average strains in the various directions are related to one another by the requirements of compatibility. Thus, if at a point in a plane the following average strains are measured (Fig. 2.6 (a)):

ϵ_ℓ in the longitudinal direction $\ell-\ell$;

ϵ_t in the transverse direction $t-t$;

ϵ_d in the diagonal direction $d-d$ at angle θ to $\ell-\ell$;

and $\gamma_{\ell t}$ is the shear strain between $\ell-\ell$ and $t-t$;

then these set of strains will be compatible. Mohr's circle of strain, Fig. 2.6 (b), can be used to obtain a relationship between these strains.

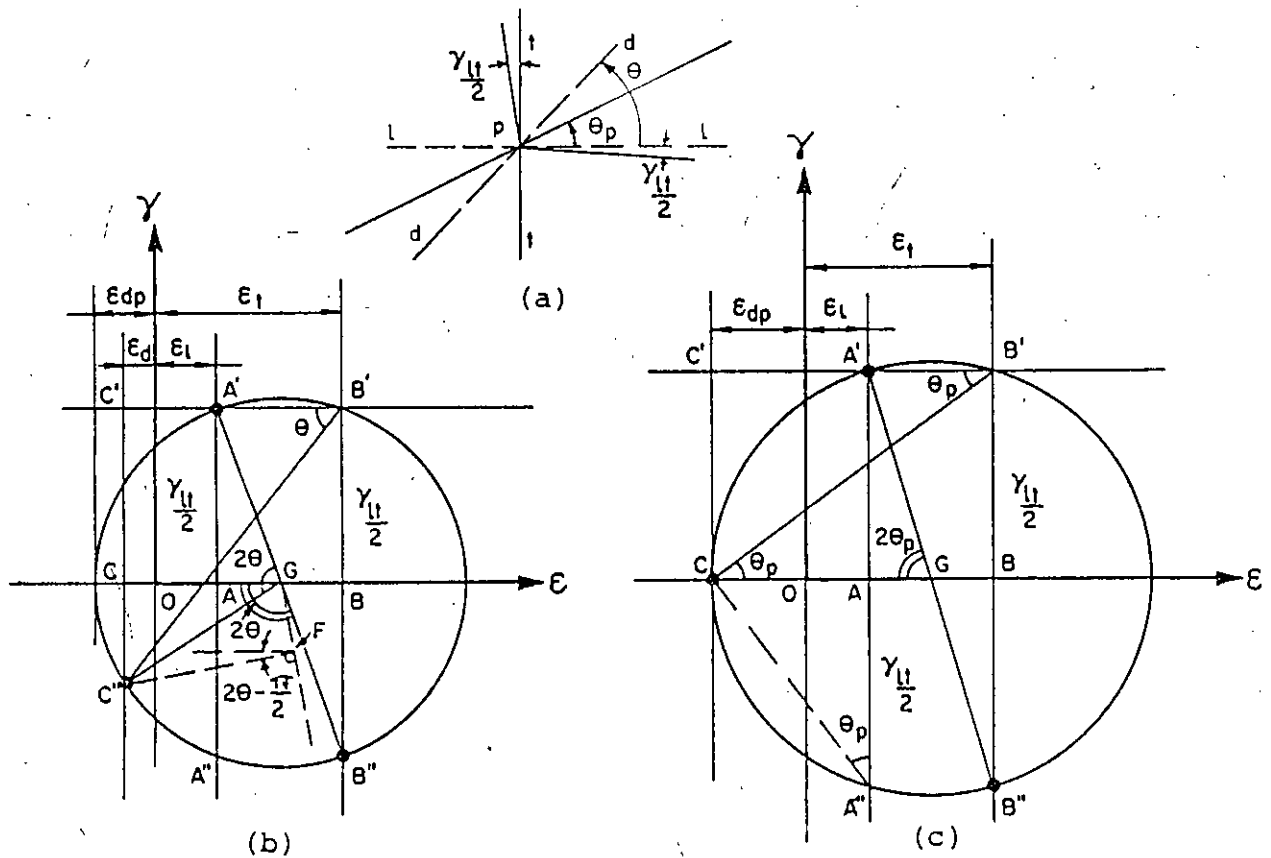


FIG. 2.6 GEOMETRY OF STRAINS

With reference to Figure 2.6(b):

$$OG = 1/2 (\epsilon_t + \epsilon_l)$$

$$GA = 1/2 (\epsilon_t - \epsilon_l)$$

By constructing $\triangle GFC''$ congruent to $\triangle GAA'$ such that $GF = GA$ and $FC'' = AA' = 1/2 \gamma_{lt}$, we deduce

$$OC = OG + GF \cos (2\theta - 2\theta_p) + FC'' \cos (2\theta - \pi/2)$$

$$\text{hence } \epsilon_d = (\epsilon_t + \epsilon_l)/2 - (\epsilon_t - \epsilon_l) \cos 2\theta/2 + \gamma_{lt} \sin 2\theta/2$$

$$\text{or } \epsilon_d = \epsilon_t \sin^2 \theta + \epsilon_l \cos^2 \theta - \gamma_{lt} \sin 2\theta/2 \dots \dots \dots (2.14)$$

Equation (2.14) gives the relationship for compatible strains at a point in a plane.

SIGN CONVENTION FOR STRAINS

The longitudinal strain ϵ_l and the transverse strain

ϵ_t shall be considered positive when in tension. The concrete strains of interest are the principal diagonal compression strains denoted by ϵ_{dp} . Because these strains will always be compressive it is convenient to consider them positive when compressive.

From Mohr's circle of strain (Fig. 2.6 (b)) it is deduced that the principal diagonal compressive strain, ϵ_{dp} , will occur at an angle θ_p with respect to the direction $\ell-\ell$. If this average principal diagonal strain (ϵ_{dp}) is used to construct Mohr's circle instead of ϵ_d , the point C" in Fig. 2.6 (b) will coincide with C and with reference to Figure 2.6(c) the following simple relationships are deduced:

$$\text{From } \triangle BBC: \tan \theta_p = \gamma_{\ell t} / 2(\epsilon_t + \epsilon_{dp}) \quad (a)$$

$$\text{From } \triangle AA'C: \tan \theta_p = 2(\epsilon_\ell + \epsilon_{dp}) / \gamma_{\ell t} \quad (b)$$

$$(a) \times (b) \text{ gives: } \tan^2 \theta_p = \frac{\epsilon_\ell + \epsilon_{dp}}{\epsilon_t + \epsilon_{dp}} \dots\dots\dots (2.15)$$

$$(a) \div (b) \text{ gives: } \gamma_{\ell t} = 2\sqrt{(\epsilon_\ell + \epsilon_{dp})(\epsilon_t + \epsilon_{dp})} \dots\dots (2.16)$$

Thus for given values of the three strains ϵ_ℓ , ϵ_t and ϵ_{dp} the direction of average principal compression strain can be evaluated using equation (2.15). It will be assumed that for the diagonally cracked beam the direction of average principal compressive stress will coincide with the direction of the average principal diagonal strain defined by equation (2.15). It is of interest to note that equation (2.15) is essentially identical to the expression for the direction of the principal tensile stresses in thin walled elastic metal beams derived from energy

considerations by Wagner in 1929.

2.3.2. EVALUATION OF TWIST

Figure 2.7(a) shows a portion of a prismatic beam of length δx subjected to a torque T which causes a change in the torsional rotation of section B relative to section A of $\delta\phi$ radians. From the beam section following any contour 's' around the beam longitudinal axis, isolate a thin walled tube element (Fig. 2.7(b)) with a perimeter equal to the length of the contour 's' and length δx . For compatibility of displacement the thin walled tube element will also have a total change of rotation of $\delta\phi$ radians over the length δx .

The compatibility condition for the thin walled tube element is that the warping around its perimeter is zero. Suppose the tube element is cut along $A_1 B_1$. The resulting open section will warp under the change in rotation of section B relative to A. Consider the displacement of the 'slice' element $A_1 B_1 A_2 B_2$ which is cut out of the tube element. Under the change in rotation of $\delta\phi$ the slice element if free to displace will do so as indicated in Fig. 2.7(c), and an out of plane displacement dw_1 will result. If this element is also subjected to a shearing strain, γ , as indicated in Fig. 2.7(d), the shearing strain will also cause an out of plane displacement, dw_2 . The compatibility condition requires that for the closed tube element the sum of the out of plane displacements for all slice elements around the section will be zero. Hence,

$$\oint_S dw_1 + \oint_S dw_2 = 0$$

Therefore $-\oint r \frac{\delta\phi}{\delta x} ds + \oint \gamma ds = 0$

Recognizing that $\frac{\delta\phi}{\delta x}$ is the twist ψ_x and is constant for all the slice elements, deduce

$$\psi_x = \frac{1}{2A_s} \oint \gamma ds \dots\dots\dots(2.17)$$

where $2A_s = \oint r ds$

A_s = area enclosed by the contour 's'

Thus, if the shear strain, at every point along any contour 's' in any section where St Venant torsion dominates, is known, equation (2.17) can be used to evaluate the twist.

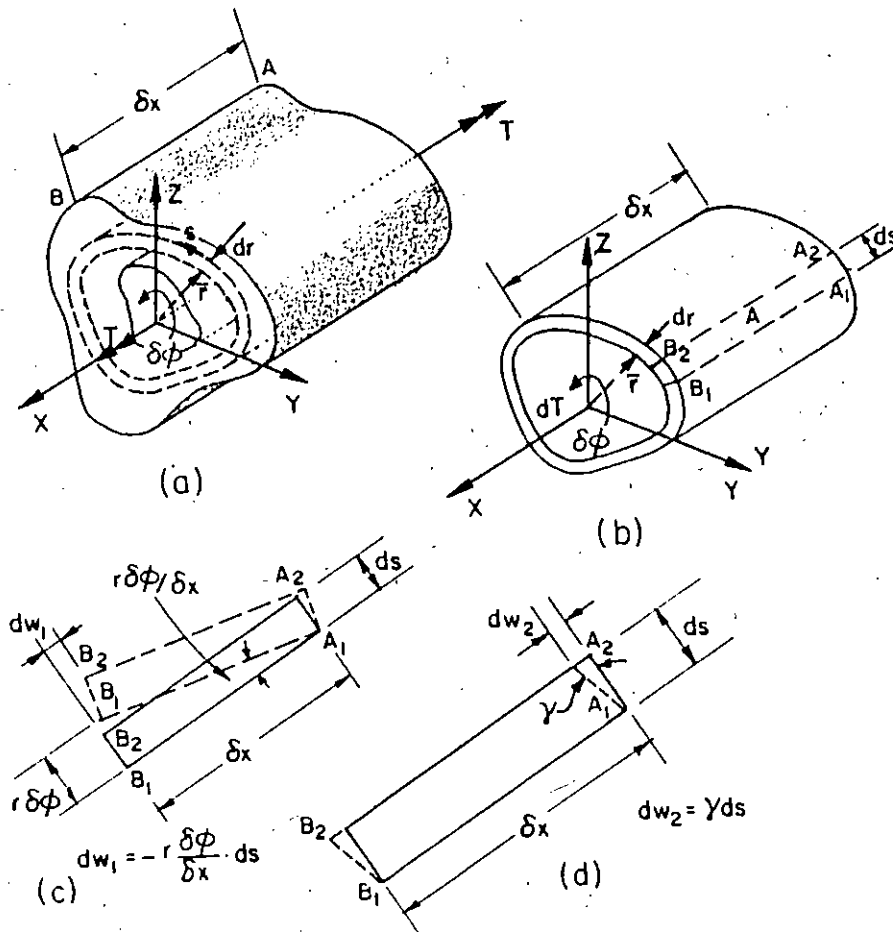


FIG. 2.7 EVALUATION OF TWIST

2.3.3 Geometry of Curvature and Twist

When a concrete beam is twisted and bent it is observed that the beam surfaces do not remain plane. The twist, Ψ_x , and the curvature, ϕ_ℓ , of the beam's longitudinal axis and the curvature, ϕ_t , of the beam's transverse axis induce curvature, ϕ_d , in a diagonal direction in a given beam face. For small deformations Ψ_x , ϕ_ℓ , ϕ_t and ϕ_d will be compatible and they can be represented by Mohr's circle (Fig. 2.8). Curvature shall be considered positive when it tends to induce compression in the surface fibre. It is convenient to measure the curvatures in the same directions as those used for measurement of strains in section 2.3.1.

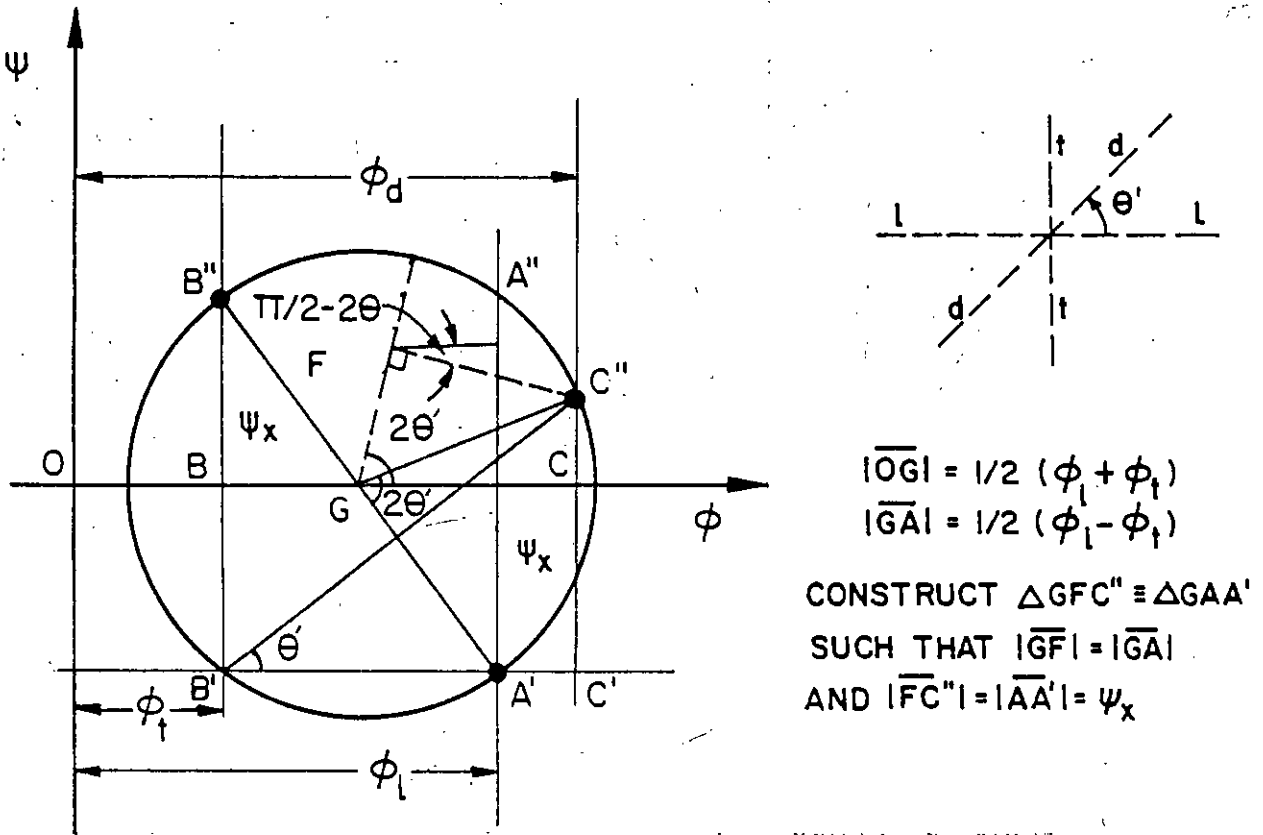


FIG. 2.8 GEOMETRY OF CURVATURE AND TWIST

From Mohr's circle in Figure 2.8 deduce:

$$\bar{OC} = \bar{OG} + \bar{GF} \cos 2\theta' + \bar{FC}'' \cos (\pi/2 - 2\theta') /$$

$$\text{Hence } \phi_d = \frac{1}{2}(\phi_\ell + \phi_t) + \frac{1}{2}(\phi_\ell - \phi_t) \cos 2\theta' + \psi_x \sin 2\theta'$$

$$\text{or } \phi_d = \phi_t \sin^2 \theta' + \phi_\ell \cos^2 \theta' + \psi_x \sin 2\theta'$$

Of particular interest is the diagonal curvature in the direction of principal diagonal strain which is at an angle θ_p to the longitudinal direction ℓ - ℓ . This curvature can be evaluated by setting $\theta' = \theta_p$ and hence.

$$\phi_{dp} = \phi_t \sin^2 \theta_p + \phi_\ell \cos^2 \theta_p + \psi_x \sin 2\theta_p \dots \dots \dots (2.18)$$

Due to the curvature ϕ_{dp} , the concrete compression strains will vary through the thickness, t_d , of the concrete diagonal. This results in the maximum average compressive strain, ϵ_{ds} , occurring on the surface. Experimental evidence⁽¹⁾ suggests it is reasonable to assume a linear compressive strain variation down the concrete diagonal depth with ϵ_{ds} on the surface of the diagonal and zero at the depth t_d which is the effective diagonal depth. Hence, with reference to Figure 2.9, the effective depth can be evaluated from the curvature ϕ_{dp} and the principal compression strain on the surface ϵ_{ds} as:

$$t_d = \epsilon_{ds} / \phi_{dp} \dots \dots \dots (2.19)$$

2.3.4. Path of shear flow

If the concrete strain distribution is known over the effective depth of the compression diagonal, the magnitude and line of action of the resultant compression force can be evaluated using the stress strain characteristics of the concrete.

A convenient approach is to replace the concrete stress distribution with an equivalent rectangular stress block having an equivalent uniform stress over a depth 'a' where depth 'a/2' from the surface defines the position of the line of action of the resultant compression force as illustrated in Figure 2.9.

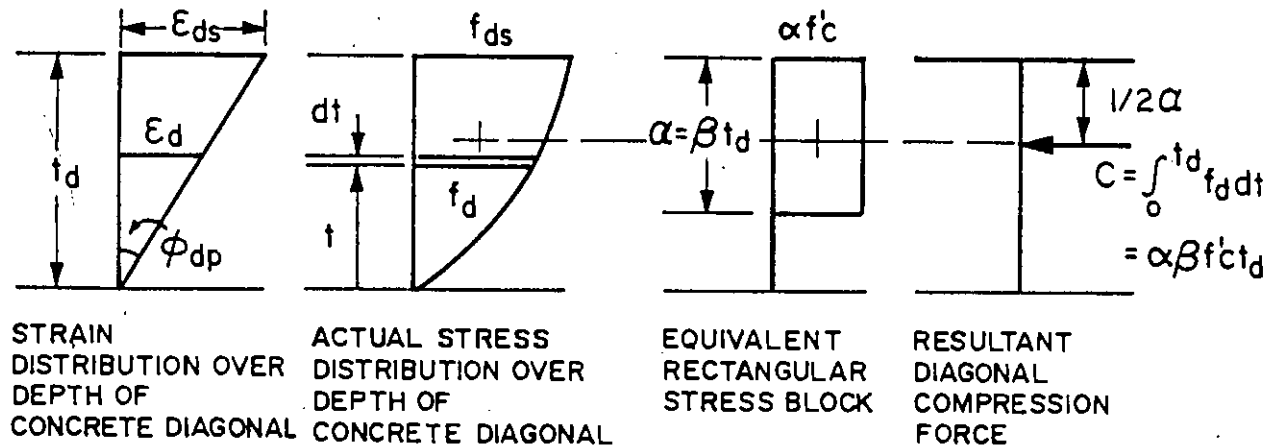


FIG. 2.9 EVALUATION OF COMPRESSION FORCE IN CONCRETE

For a simple and accurate stress-strain relationship for concrete, a parabola which has been found to provide a good fit for the stress-strain relationship obtained from a standard concrete cylinder compression test, will be used. This assumes that the concrete diagonals in a cracked concrete beam are virtually loaded in compression is the principal direction and this state of loading is closely approximated by the loading on concrete in the standard compression test. Thus the compression stress f_d , can be evaluated from a given strain, ϵ_d , as:

$$f_d = f'_c (2 \Omega_d - \Omega_d^2) \dots\dots\dots (2.20)$$

$$\Omega_d = \epsilon_d / \epsilon_o$$

ϵ_o = compression strain in concrete corresponding to the maximum stress achieved f'_c for a standard cylinder test.

Using equation (2.20) the factors α and β which define the equivalent stress block in Figure 2.9 can be evaluated as follows:

$$\alpha = \frac{6\Omega_{ds} (1 - \Omega_{ds}/3)^2}{4 - \Omega_{ds}} \dots\dots\dots(2.21)$$

$$\beta = \frac{4 - \Omega_{ds}}{6 - 2\Omega_{ds}} \dots\dots\dots(2.22)$$

$$\alpha\beta = \Omega_{ds} - \Omega_{ds}^2/3 \dots\dots\dots(2.23)$$

where $\Omega_{ds} = \epsilon_{ds}/\epsilon_o$

With the factor, β , the depth of the equivalent stress block, $a = \beta t_d$, can be evaluated. The effective path of shear flow can be taken to be at $a/2$ from the surface along the plane of action of the resultant compression force (Fig. 2.9).

In the evaluation of the resultant diagonal compression force as presented above, it is assumed that the effective depth of compression at a given point, t_d , is less than the section wall thickness, t . If the depth t_d , is greater than the wall thickness, t , correction terms (see Appendix A and B) can be introduced in the derived equations to obtain α and β factors. It has also been assumed that the compression strains in the concrete diagonals are those introduced by the applied loads.

CHAPTER 3
APPLICATION TO RECTANGULAR CROSS-SECTION
WITH ONE AXIS OF SYMMETRY

The theory developed in chapter two will now be applied to analyse a rectangular structural concrete section with one axis of symmetry shown in Figure 3.1. The section will be analysed for the particular loading of a torque T in the $x - z$ plane and a moment M causing curvature of the $x - z$ plane. Hoop steel content is the same in all faces with the spacing of s_h along the entire span of the beam.

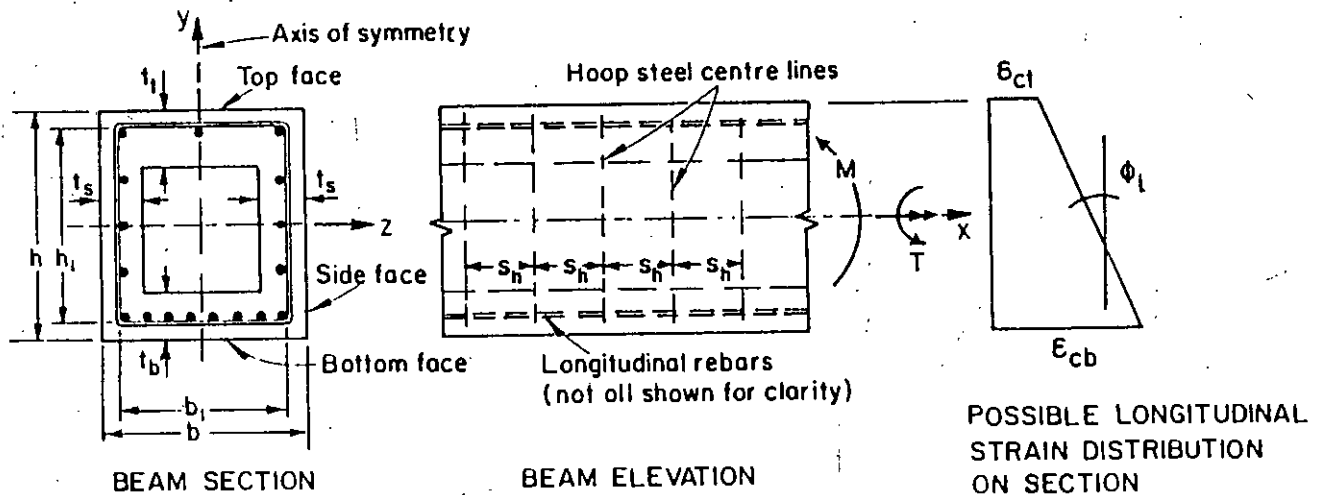


FIG. 3.1 RECTANGULAR SECTION FOR ANALYSIS

3.1 EQUILIBRIUM REQUIREMENTS

3.1.1 Equilibrium in the Longitudinal Direction

With a longitudinal strain profile at a cross section and given the stress-strain relationships for the longitudinal steel the resultant tensile force in the steel, F_{cl} , and the position at which it acts defined by $\bar{Y}_s$ can be evaluated easily and exactly using equations (2.7) and (2.8). On the other hand the resultant compression force, F_{cl} , given by equation (2.9) is not so easy to evaluate as it requires the evaluation of the principal compression angles, θ_p , which change from point to point on the perimeter along the path of shear flow.

Using equation (2.3) rewrite equation (2.9) as

$$F_{cl} = \oint_p (f_t \cot \theta_p dt) dp \dots \dots \dots (3.1)$$

If it is assumed that θ_p remains constant down the depth of a diagonal at a particular point on the perimeter, equation (3.1) can be rewritten as:

$$F_{cl} = \oint_p \cot \theta_p (f_t dt) dp$$

and using equation (2.4), this becomes

$$F_{cl} = \oint_p q \cot \theta_p dp \dots \dots \dots (3.2)$$

Because of symmetry of the section shown in Figure 3.1 and for the loading under consideration the angle of principal compression, θ_p , will be constant for the top face and for the bottom face but will vary over the side faces with the variation of longitudinal strains. For the side faces the θ_p -angles can be evaluated at a finite number of points and hence obtain

$$F_{cl} = \sum_i q \cot \theta_{pi} \Delta p_i \dots \dots \dots (3.3)$$

Hence
$$F_{cl} = q \cot \theta_t b_{ot} + q \cot \theta_b b_{ob} + 2q \sum_{\text{side}} \cot \theta_{pi} \Delta p_i \dots (3.3a)$$

where θ_t = angle of principal compression strain in top face

b_{ot} = length of shear flow perimeter in top face

θ_b = angle of principal compression strain in bottom face

b_{ob} = length of shear flow perimeter in bottom face

To evaluate the resultant compression force in the longitudinal direction for a side face sufficient accuracy is obtained if the angles θ_{pi} are evaluated over five points ($i = 1+5$) equally spaced along the side, then, using Simpson's integration formula for the sides:

$$2q \sum \cot \theta_{pi} \Delta p_i = \frac{qh_o}{6} [\cot \theta_{s1} + 4\cot \theta_{s2} + 2\cot \theta_{s3} + 4\cot \theta_{s4} + \cot \theta_{s5}] \quad (3.3b)$$

where h_o = height of shear flow perimeter in the side faces

θ_{si} = angle of principal compression strain at point 'i' in side face.

Equations (3.3a) and (3.3b) can thus be used to evaluate the resultant compression force, F_{cl} , in the longitudinal direction. The position at which this force acts denoted by $\bar{y}_c$ can be evaluated for the section in Figure 3.1 as

$$\bar{y}_c = \frac{\sum \cot \theta_{pi} \Delta p_i y_i}{F_{cl}} \dots (3.4)$$

For flexure and torsion only the equilibrium requirements for forces in the longitudinal direction is that the resultant axial force is zero. This requires $F_{sl} = F_{cl}$.

3.1.2. Equilibrium in the Transverse Direction

With the assumption that θ_p will remain constant over the effective depth of a compression diagonal at a particular point of the section perimeter, equations (2.2) and (2.4) can be used to rewrite equation (2.12) as

$$F_{chi} = s_h q \tan \theta_{pi} \dots \dots \dots (3.5)$$

Due to section symmetry (Figure 3.1) the hoop stresses at corresponding points in the side face will be the same; hence for the sides, the equilibrium equations (2.13) give

$$F_{Hi} = A_h f_{hi} = F_{chi} = s_h q \tan \theta_{pi}$$

$$\text{or } t_h f_{hi} = q \tan \theta_{pi} \dots \dots \dots (3.6)$$

$$\text{where } t_h = A_h / s_h$$

For the top and bottom faces the equilibrium equation (2.13) give

$$t_h f_{ht} = q \tan \theta_t [1 - (\frac{1}{2} a_t - c_{ht}) / h_1] + q \tan \theta_b (\frac{1}{2} a_b - c_{hb}) / h_1 \quad (3.7)$$

$$t_h f_{hb} = q \tan \theta_t [1 - (\frac{1}{2} a_b - c_{hb}) / h_1] + q \tan \theta_b (\frac{1}{2} a_t - c_{ht}) / h_1 \quad (3.8)$$

where a_t = depth of the equivalent rectangular stress block for the top face compression diagonal (Fig. 2.8)

a_b = depth of the equivalent rectangular stress block for the bottom face compression diagonal

f_{ht}, f_{hb} = hoop stress in the top and bottom faces, respectively

c_{ht}, c_{hb} = cover to centreline of hoop steel in the top and bottom faces, respectively

3.2 COMPATIBILITY REQUIREMENTS

The strain compatibility equations (2.15) and (2.16) can be used to evaluate the principal angles, θ_p , and the shearing strains. It is convenient to evaluate the strains in the plane defined by the centrelines of hoop steel so that the transverse strain, ϵ_y , will be equal to the hoop strain and the shearing strain evaluated will be in the plane defined by the hoop steel centrelines.

3.2.1 Evaluation of Twist

As indicated in section 2.3.2 the twist can be evaluated using equation (2.17) evaluated over any contour 's' within the wall thickness of the section. For the section shown in Figure 3.1 it is convenient to choose the contour 's' to coincide with the path of the hoop centreline around the section. Due to section symmetry and for the loading under consideration, the shear strains γ_t and γ_b , for the top and bottom faces respectively, will be constant over the width b_1 in the faces. Suppose $\gamma_1, \gamma_2, \gamma_3, \gamma_4$, and γ_5 , are the shear strains evaluated at the five points selected over the height h_1 of a side face along the hoop centreline. Then by rewriting equation 2.17 as:

$$\psi_x = \frac{1}{2A_s} \sum_i \gamma_i \Delta s_i \dots\dots\dots(3.9)$$

the twist can be evaluated to sufficient accuracy using Simpson's integration rule:

$$\psi_x = \frac{1}{2A_1} [b_1 (\gamma_b + \gamma_t) + \frac{h_1}{6} (\gamma_1 + 4\gamma_2 + 2\gamma_3 + 4\gamma_4 + \gamma_5)] \dots\dots(3.10)$$

where $A_s = A_1 = b_1 h_1$

b_1 = width of hoop centreline (for top and bottom faces)

h_1 = height of hoop centreline (for side faces)

3.2.2 Evaluation of Diagonal Curvature and Effective Diagonal Depth

Equation (2.18) is used to evaluate the diagonal curvature ϕ_{dp} and equation (2.19) to evaluate the effective depth t_d of the concrete compression diagonal. The longitudinal and transverse curvatures together with the twist are required for the evaluation of the diagonal curvature at a given position using equation (2.18). Due to section symmetry there will be no longitudinal curvature nor transverse curvature for the side surfaces. Hence for the side faces only, at any given position, the diagonal curvature can be evaluated as:

$$\phi_{dp} = \psi_x \sin 2\theta_p$$

on the other hand, the top and bottom surfaces will be subjected to longitudinal curvature, ϕ_ℓ , and transverse curvature, ϕ_t , together with the twist, ψ_x , where

$$\phi_\ell = \frac{\epsilon_{cb} - \epsilon_{ct}}{h} \dots \dots \dots (3.11)$$

where ϵ_{ct} , ϵ_{cb} are longitudinal strains in the top and bottom surfaces, respectively, for the section shown in

Figure 3.1,

$$\text{and } \phi_t = \frac{\epsilon_{hb} - \epsilon_{ht}}{h_1} \dots \dots \dots (3.12)$$

where ϵ_{ht} and ϵ_{hb} are the hoop steel strains in the top and bottom faces, respectively.

3.3 SOLUTION TECHNIQUE

The equilibrium and geometric equations which have been derived are adequate for a solution to predict the complete post-cracking response of a structural concrete beam subjected to flexure and torsion, given the geometry of the beam cross-section and the stress-strain characteristics of the concrete and of the reinforcing steel.

3.3.1 Pure Flexure Case

A convenient solution technique for the complete response of a given structural concrete beam in pure flexure using the well known theory for flexure can be summarized as follows:

- 1) Select a trial strain profile over the cross section
- 2) With the strains use the given stress-strain relationships for the concrete and steel to evaluate the stress in every fibre and hence obtain resultant forces.
- 3) Check that the forces evaluated in step (2) satisfy equilibrium. If equilibrium is not satisfied, revise the strain profile.
- 4) Repeat steps (2) and (3) until equilibrium is satisfied.

The basic principle in the outlined technique is that one startsoff with strains then get stresses from the strains and finally forces from the stresses. The forces are then checked for equilibrium. This basic approach will be used for the case of pure torsion and for the combined loading cases. It is also of interest to show that the theory for pure flexure is a special case of the more general theory presented.

In the equations developed, the equilibrium equations (2.7) and (2.8) are directly applicable for the pure flexure case for evaluation of the resultant tension force in the steel and its line of action. With reference to the section in Figure 3.1 the concrete compression stresses will be distributed over the top face down to a depth t_M if the moment M alone were acting. Then using the rectangular stress block theory and assuming that t_M is less than t_t the resultant compression force for M alone acting is given by

$$C = \alpha \beta f'_c t_M b \dots \dots \dots (3.13a)$$

where α and β are functions of the top longitudinal strain ϵ_{ct} which will be compressive

If besides the moment a torque is also applied then the concrete compression stresses are distributed around the section perimeter as has been discussed. For this case the contribution of the top face to the total resultant compression force in the section is (see equation 3.3a)

$$C_T = q \cot \theta_t b_{ot} \dots \dots \dots (3.13b)$$

Imagine reducing the applied torque, T , until only the moment M is left acting on the section. In the limit state of pure flexure the theoretical model suggests that the total resultant compression force will be given by equation (3.13b). Using equation (2.4) and (2.1), q can be expressed as:

$$q = \int_0^{td} f_d \frac{\tan \theta}{1 + \tan^2 \theta} dt$$

With reference to the top face and assuming

$$\theta = \theta_t = \text{constant over the effective depth } t_{dt}$$

and

$$q = \frac{\tan \theta_t}{1 + \tan^2 \theta_t} \int_0^{t_{dt}} f_d dt = \frac{\tan \theta_t}{1 + \tan^2 \theta_t} \alpha \beta f'_c t_{dt} \dots (3.14)$$

where $\alpha \beta f'_c t_{dt}$ is equivalent to $\int_0^{t_d} f_d dt$ (Fig. 2.9)

Hence equation (3.13b) can be rewritten as:

$$C_T = \frac{\alpha \beta f'_c t_{dt} b_{ot}}{1 + \tan^2 \theta_t}$$

In the limit $T = 0$, the angle of principal compression is the top face $\theta_t = 0$, hence for pure flexure the theoretical model predicts:

$$C_T = \alpha \beta f'_c t_{dt} b_{ot}$$

This equation is identical with equation (3.13a) for $t_{dt} = t_M$ and $b_{ot} = b$. Thus equation (3.13a) can be considered a special case of equation (3.3) used in the evaluation of the resultant compression force in the section for the combined loading case.

It is also of interest to note that equation (2.15) applies for the case of pure flexure for the special case when

$$(a) \text{ for } \epsilon_\ell \leq 0 : \quad \epsilon_t = 0 \text{ and } \epsilon_\ell + \epsilon_{dp} = 0,$$

$$(b) \text{ for } \epsilon_\ell > 0 : \quad \epsilon_t = 0 \text{ and } \epsilon_{dp} = 0.$$

Then equation (2.15) gives:

$$\text{for case (a) } \tan^2 \theta_p = 0, \text{ and hence } \theta_p = 0;$$

$$\text{and for case (b) } \tan^2 \theta_p = \infty, \text{ and hence } \theta_p = 90^\circ$$

3.3.2 Pure Torsion Case

As in the case of pure flexure, a convenient solution technique for pure torsion on the section shown in Figure 3.1 is to fix the longitudinal strain in the top face, ϵ_{ct} , then for any chosen curvature ϕ_ℓ the steps for solution can be as summarized below. It is assumed that the stress-strain relationships for the concrete and steel are given. The solution is basically an iterative procedure and to initiate the iteration process:

- (a) set all principal diagonal angles at all points to be 45°
- (b) set all values of the effective diagonal thicknesses for all points to be equal to $0.375 bh / (b+h)$ where b and h are the effective outside dimensions for the section.
- (c) set $b_{ot} = b_{ob} = 0.9b$
and $h_o = 0.9h$

STEP 1

With the chosen curvature, ϕ_ℓ , evaluate strains at each fibre and using the given stress-strain relationships for the steel evaluate the resultant tension force in the longitudinal steel ($F_{s\ell}$) using equation (2.7) and the position at which this force acts ($\bar{y}_s$) using equation (2.8)

STEP 2

For unit shear flow ($q=1.0$), using equations (3.3a)

and (3.3b) evaluate the resultant compression force (F_{cl}) in the longitudinal direction. The shear flow required for equilibrium of forces in the longitudinal direction can be evaluated as $q = F_{sl}/F_{cl}$.

STEP 3

With the shear flow, q , from Step 2 and the longitudinal strain, ϵ_ℓ , at a given point evaluate the compatible strains ϵ_t , ϵ_{dp} and $\gamma_{\ell t}$ and determine the angle of principal compression, θ_p . The procedure outlined in Appendix B can be used in this step.

STEP 4

With the shear strains from Step 3, evaluate the twist ψ_x using equation (3.10).

STEP 5

Evaluate the diagonal curvatures using equation (2.18) and with equation (2.19) obtain new values for the effective depths (t_d) of the concrete compression diagonals.

STEP 6

Iterate with steps 2 to 5 until convergence is obtained in the t_d -values.

STEP 7

Using equation (2.22) evaluate β factors and hence evaluate the values of $a = \beta t_d$ which determine the shear flow path and hence obtain the area A_o enclosed by the path of shear flow.

STEP 8

Using equation 3.4 evaluate $\bar{y}_C$. If $\bar{y}_C = \bar{y}_S$ the solution for pure torsion has been obtained, otherwise a new value of ϕ_ℓ (with ϵ_{ct} still fixed) is chosen and steps 1 to 8 repeated. Note that for $\bar{y}_C \neq \bar{y}_S$ a moment $M = F_{sl} (\bar{y}_C - \bar{y}_S)$ is also acting.

STEP 9

Evaluate the torque $T = 2qA_O$

By repeating steps 1 to 9 for a number of values of ϵ_{ct} a complete response of a beam section in pure torsion is obtained.

3.3.3 Combined Torsion and Flexure

In the solution for pure flexure and pure torsion it was only necessary to fix the longitudinal strain profile for a section in order to make the problem statically determinate. In the combined loading case the problem becomes statically determinate if besides specifying the longitudinal strain profile at a section the shear flow, q , is also fixed. The shear flow can be fixed by either specifying the applied torque, T , or by fixing the principal compression diagonal strain in concrete at some point in the section. A procedure which can be used to evaluate the shear flow for the latter case is outline in Appendix A.

Assuming that q is determined and the longitudinal strain profile is chosen by fixing the longitudinal strain in

the top face, ϵ_{ct} , with a chosen curvature ϕ_ℓ , then with the initial values of the variables as given for the pure torsion case, the steps to obtain a solution are as follows:

STEP 1

Same as for the Pure Torsion Case

STEP 2

With the specified shear flow q this step is same as Step 3 of the Pure Torsion Case

STEP 3

Same as Step 4 of the Pure Torsion Case

STEP 4

Same as Step 5 of the Pure Torsion Case

STEP 5

Same as Step 7 of the Pure Torsion Case. With the value of A_0 found in this step a re-evaluation of q may be necessary if the torque T is specified so that $q = T/2A_0$

STEP 6

Iterate with Steps 2 to 5 until convergence is obtained in the t_d -values.

STEP 7

Using equations (3.3a) and (3.3b) evaluate the resultant compression force, $F_{c\ell}$, in the longitudinal direction

STEP 8

Check equilibrium of forces in the longitudinal direction.

If $F_{s\ell} = F_{c\ell}$ then the equilibrium of forces is satisfied.

If $F_{s\ell} \neq F_{c\ell}$, then there is an axial force acting,

$N_x = F_{s\ell} - F_{c\ell}$. For the case of flexure and torsion

loading only, new curvatures, ϕ_l , have to be selected and iterate with steps 1 to 8 until $F_{sl} = F_{cl}$

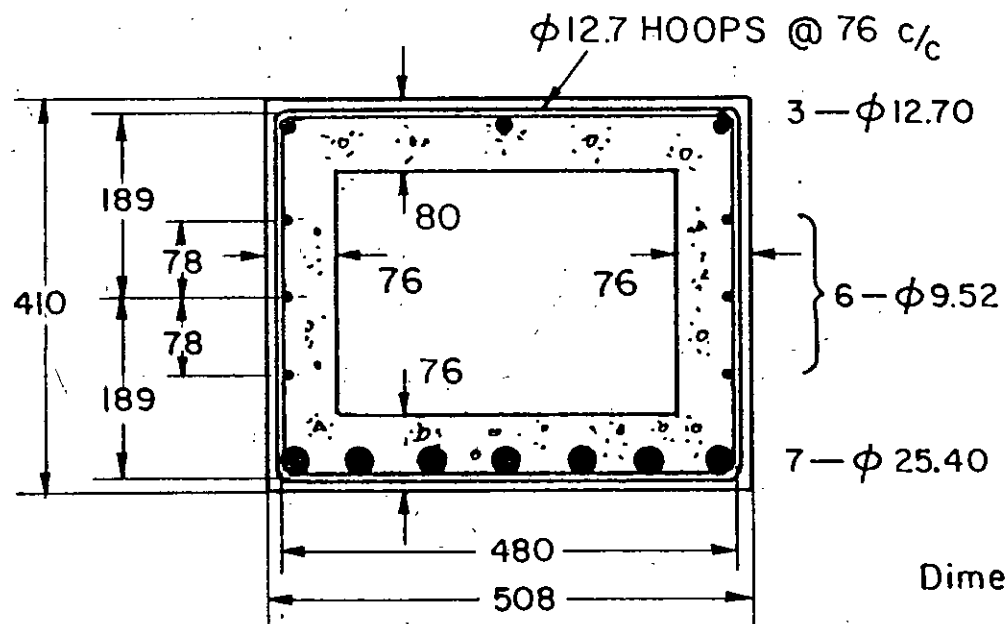
STEP 9

Using equation (3.4) evaluate $\bar{y}_c$ and hence evaluate the applied moment M using equation (2.11). If the torque was not specified, then it can be evaluated from the shear flow using equation (2.5).

By selecting new values of q and ϵ_{ct} and repeating Steps 1 to 9 provides the complete response prediction of the structural concrete section subjected to combined torsion and bending.

3.4 RESPONSE PREDICTION FOR A RECTANGULAR SECTION HAVING ONE AXIS OF SYMMETRY

The solution technique outlined in section 3.3.3 has been programmed to enable a computer aided solution and the program is given in Appendix D. Moreover in Appendix C an example is worked out to show how the solution technique is applied for a specified case. In this section the capability of the theoretical model presented is briefly shown by looking at the response prediction for the section shown in Figure 3.2 subjected to flexure and torsion. This beam section was one of the fourteen beam sections tested to verify the validity of the predictions of the theoretical model.



MATERIAL PROPERTIES

CONCRETE

$$f'_c = 34.8 \text{ N/mm}^2$$

$$\epsilon_o = 0.0031$$

STEEL

The properties for steel are given in table 4.2

Dimensions are in mm.

FIG. 3.2 CROSS SECTION OF BEAM TBU3

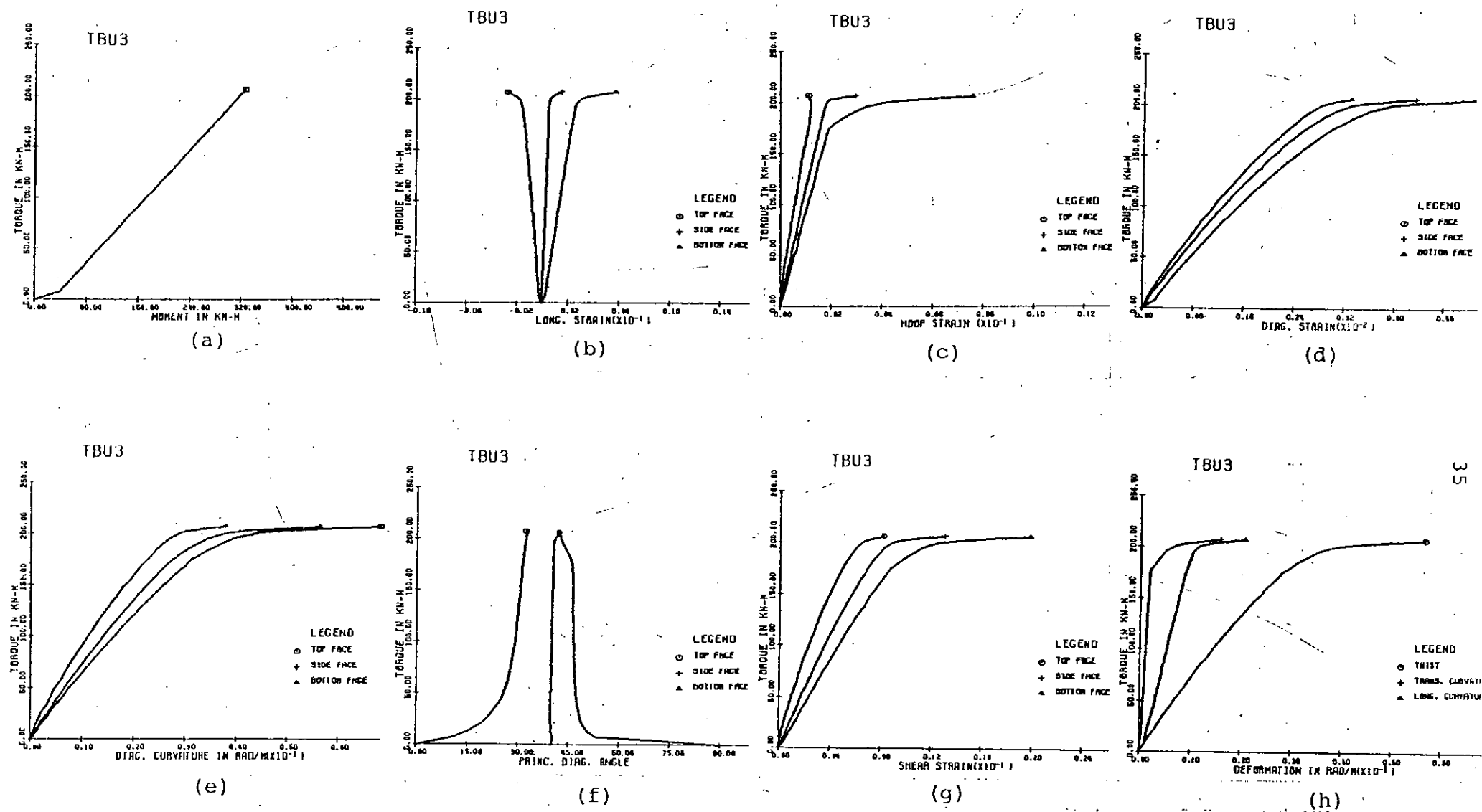


FIG. 3.3 RESPONSE PREDICTIONS FOR TBU3

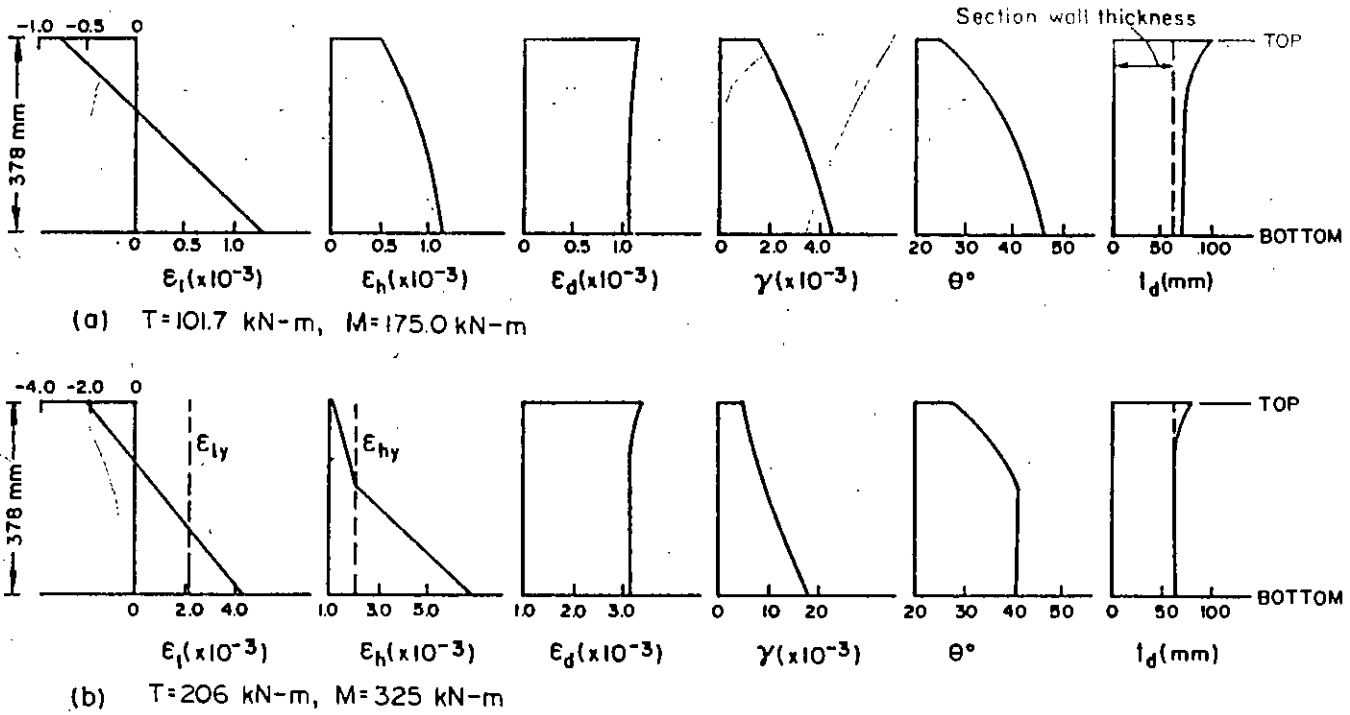


FIG. 3.4 PREDICTED STRAIN VARIATIONS ON SIDE FACES FOR TBU3

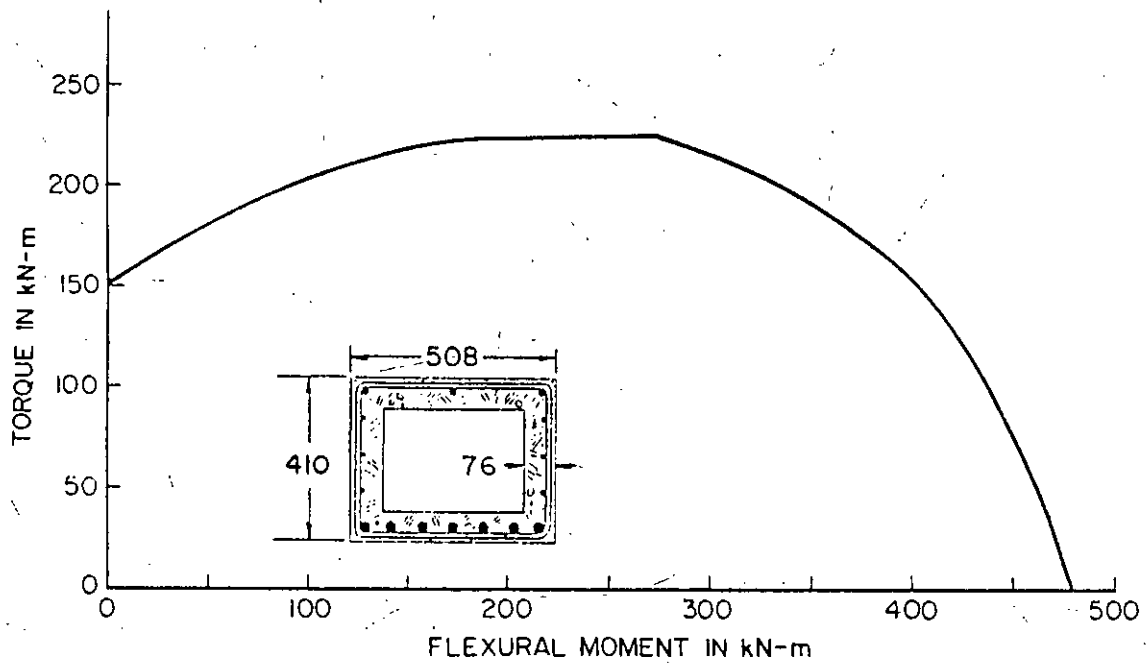


FIG. 3.5 PREDICTED FAILURE INTERACTION CURVE FOR SPALLED SECTION DIMENSIONS

Figure 3.3 shows a computer plot of the results of the model prediction for TBU3. The initial flexural moment and torque of the actual test specimen as tested is taken into account. In addition to these initial values the loading for analysis was increased in small steps at a fixed ratio of torque to moment of 0.696 as seen in Figure 3.3(a). For each load increment a complete analysis using the solution technique outlined in section 3.3.3 was done and the result of the analysis is plotted in Fig. 3.3(b) - (h). The analysis was terminated when the maximum diagonal strain in the section was $\geq 1.5\epsilon_0$.

The prediction results shown are based on spalled* section dimensions whereby it is assumed that the effective external section dimensions are those described by the centreline of hoop steel. It should also be noted that in Fig. 3.3 (b) - (g), the values plotted are for points at the middle of the faces.

A closer look at Fig. 3.3 (b)-(h) reveals the following:

1) Longitudinal strains (Fig. 3.3(b))

In (b) it is seen that for all load levels the bottom face longitudinal strains at the level of the centreline of hoop steel are tensile whereas the corresponding strains in the top face are compressive. It is also seen that the bottom face strains exceeded the yield strain of bottom face longitudinal steel bars of 0.00225 before the attainment of capacity loads. After the yield of bottom face

* The phenomenon of spalling is discussed in Chapter 5

longitudinal steel there was little load carrying capacity left in the beam.

2) Hoop strains (Fig. 3.3(c))

In (c) the transverse strains which are also equal to hoop steel strains are plotted. The model predicts that at any load level the bottom face will have the largest hoop steel strains and the top face the minimum. Comparing (a) and (b) it is seen that the model predicts that the hoop steel in the bottom face will yield before the bottom face longitudinal steel bars yield. It is also seen from (b) that at capacity load the hoop strains at the midheight of the side faces will also be in the post yield range but the model predicts that the yielding of hoop steel will not extend to the top face.

3) Principal Compression diagonal strains (Fig. 3.3(d)) and Diagonal curvatures (Fig. 3.3(e)).

The predicted principal diagonal strains and the curvatures in the corresponding directions are plotted in (d) and (e) respectively. As expected from the longitudinal strain profile, at any load level, the highest principal diagonal strains and the corresponding curvatures occur in the top face and the lowest in the bottom face. It is observed that after the yield of the reinforcing steel the principal diagonal strains and the curvatures increase rapidly with little gain in the load carrying capacity of the beam.

4) Principal Diagonal Angles (Fig. 3.3(f))

In (f) is shown the pattern of variation of the angles

defining the direction of principal diagonal strain with respect to the beam's longitudinal axis. It is seen that the angles are much more acute in the top face than the angles in the bottom face for the same load level. Notice that because of the effects of the initial dead weights which were predominantly flexural, the angles at the low load levels approached 90° in the bottom face and 0° at the top face. With more load added at a fixed ratio of torque to moment, it is seen that the angles quickly stabilised and changed little until the onset of steel yielding.

5) Shear Strains (Fig. 3.3(g))

In (h) the shear strains predicted are plotted. At any load level the bottom face had the greatest shear strains and the top face the least. A rapid increase in the shear strains is noticed when the reinforcing steel gets into the post-yield range.

6) Angular Deformations (Fig. 3.3(h))

In (h) the twist, the longitudinal curvature and the transverse curvature are plotted against the torque. The model predicts greater twist than there is longitudinal curvature. It also predicts that the transverse curvatures will be small until the bottom face hoop steel yields after which all the angular deformation increase rapidly with little gain in load carrying capacity of the section.

It is of interest to look at the variation of the pattern of

deformations on the side faces where a linear variation of longitudinal strains is assumed. Figure 3.4 shows a plot of the strains ϵ_ℓ , ϵ_h , ϵ_d , and γ_{xt} and the principal angles, θ_p , and the effective depths of the compression diagonals, t_d , for the side faces for the torque and bending moment values shown. The plots in (a) are at a load level approximately half the capacity load with all the reinforcing steel in the elastic range. The plots in (b) are for the capacity loading level as predicted by the model, and, at this load level, some of the reinforcing steel has yielded and the yield strains are indicated.

From these plots the following is observed:

- a) If the ϵ_ℓ -strains vary over a given face of a section there will be a corresponding variation in the ϵ_h , ϵ_d , and γ strains, and in the θ angles and the t_d values. The variation in the ϵ_h , ϵ_d , γ , θ , and t_d values is generally non-linear. However in those regions where the ϵ_ℓ -strains are tensile the ϵ_d strains and the t_d values vary little. The ϵ_d strains and t_d strains show significant variation when the varying ϵ_ℓ strains are compressive.
- b) For those regions in the section where the hoop steel strain is within the plateau (if present) of the stress-strain curve of the hoop steel, the θ -angles assume a constant value despite the variation in ϵ_ℓ strains for the given load level.

- c) For the bottom face and in those regions where the longitudinal strains are tensile the concrete is active in compression as indicated by the principal compressive strains, ϵ_d , in these regions.
- d) The plot of t_d values in Figure 3.4 indicates that the spalled section wall thickness was smaller than the effective wall thickness as predicted by the theoretical model. This is taken into account in the analysis using the theoretical model.

Figure 3.5 shows the failure interaction curve for the section shown in Fig. 3.2 as predicted by the model. The curve is obtained by fixing the maximum principal diagonal compression strain in the section at approximately $1.5\epsilon_o$ and systematically varying the curvature θ_o and evaluating the applied torque and flexural moment at each step. It is important to note that the interaction curve shown contains no empirical fitting factors.

The trend of behaviour illustrated in Figures 3.3 and 3.4 with the observations noted and the failure interaction prediction shown in Figure 3.5 need experimental verification. This is the purpose of the experimental programme described in chapter 4.

3.5 PREDICTION OF THE BALANCED FAILURE MODE

FOR COMBINED TORSION AND BENDING /

Unlike pure flexure which induces compression in one region of the section and tension in the rest of the section, combined flexure and torsion will induce diagonal compression in concrete in all faces with tensile strains at right angles to the direction of the compression diagonals. Moreover, flexure alone will only stress the longitudinal reinforcing steel but combined flexure and torsion will stress both longitudinal steel and the hoop steel. In pure flexure, the balanced failure is normally defined when the longitudinal steel in the farthest face from the compression face attains yield strains simultaneously with the attainment of the capacity load for the beam section. This will occur, say, when the extreme compression fibre strain is ϵ_{cu} . In the case of combined flexure and torsion, a balanced failure can be considered defined when both the longitudinal and hoop steel reach their respective yield strains simultaneously with the attainment of capacity loads. It is assumed that this will occur when the diagonal compression strain in the surface fibres of all the sides is ϵ_{cu} . Assuming that longitudinal steel yield strain is ϵ_{ly} at a yield stress of f_{ly} and that the hoop steel content is the same in all faces with a yield strain of ϵ_{ly} at a yield stress of f_{hy} , then balanced failure will require uniform beam elongation ($\phi_l = 0$). Then, using the equations already derived and assuming adequate concrete wall thickness is provided in the section the following is deduced:

Equation (2.16) $\gamma = 2\sqrt{(\epsilon_{ly} + \epsilon_{cu})(\epsilon_{hy} + \epsilon_{cu})}$ For all points

Equation (3.9) $\psi_x = \frac{p}{2A}\gamma$

where $p = 2(b + h) =$ surface perimeter of section

$A = bh =$ Area enclosed by surface perimeter.

Equation (2.15) $\tan^2 \theta_p = \frac{\epsilon_{ly} + \epsilon_{cu}}{\epsilon_{hy} + \epsilon_{cu}}$ for all points

Equation (2.18) $\phi_{dp} = \psi_x \sin 2\theta_p$

Equation (2.19) $t_d = \frac{\epsilon_{cu}}{\phi_{dp}} = \frac{A}{2p} \frac{\epsilon_{cu}(\epsilon_{ly} + \epsilon_{hy} + 2\epsilon_{cu})}{(\epsilon_{ly} + \epsilon_{cu})(\epsilon_{hy} + \epsilon_{cu})} \dots (3.16)$

Equation (3.14) $q = \alpha \beta f'_c t_d \sin \theta \cos \theta$

where $\alpha \beta = \Omega_{cu} - \frac{1}{3}\Omega_{cu}^2$ and $\Omega_{cu} = \epsilon_{cu}/\epsilon_o$

Equation (3.6) $\frac{A_h f_{hy}}{s_h} = q \tan \theta_p = \frac{A}{2p} \alpha \beta f'_c \frac{\epsilon_{cu}}{\epsilon_{hy} + \epsilon_{cu}} \dots (3.17)$

Equation (3.3) $\frac{A_T f_{ly}}{p_o} = q \cot \theta_p = \frac{A}{2p_o} \alpha \beta f'_c \frac{\epsilon_{cu}}{\epsilon_{ly} + \epsilon_{cu}} \dots (3.18)$

where $p_o = p - \frac{10}{3}t_d$ for a rectangular section, and $\Omega_{cu} = 1.5$

By defining

$t_l = A_T/p_o$, $t_h = A_h/s_h$, and $t_c = A/p$, then:

From (3.17) $\frac{t_h}{t_c} = \rho_{hb} = \frac{f'_c}{f_{hy}} \cdot \frac{\alpha \beta}{2} \cdot \frac{\epsilon_{cu}}{\epsilon_{hy} + \epsilon_{cu}} \dots (3.19)$

= balanced hoop steel ratio.

$$\text{From (3.18)} \quad \frac{t_l}{t_c} = \rho_{lb} = \frac{f'_c}{f_{ly}} \cdot \frac{\alpha\beta}{2} \cdot \frac{\epsilon_{cu}}{\epsilon_{ly} + \epsilon_{cu}} \dots\dots\dots (3.20)$$

= balanced longitudinal steel ratio.

The shear flow can be evaluated as $q = \sqrt{t_h f_{hy} t_l f_{ly}}$ and for balanced failure the shear flow can also be evaluated as

$$q_b = t_c f'_c \frac{\alpha\beta}{2} \epsilon_{cu} / \sqrt{(\epsilon_{ly} + \epsilon_{cu})(\epsilon_{hy} + \epsilon_{cu})} \dots\dots\dots (3.21)$$

The torque, T_b , and the bending moments M_{bz} and M_{by} for the balanced failure condition can be evaluated for a section for which the resultant compression force in concrete can be considered to act at $(\bar{z}_c, \bar{y}_c)$ as follows:

$$T_b = 2q_b A_o$$

$$M_{bz} = \rho_{lb} t_c f_{ly} p_o \bar{z}_b \dots\dots\dots (3.22)$$

$$M_{by} = \rho_{lb} t_c f_{ly} p_o \bar{y}_b$$

where $\bar{z}_b = \bar{z}_c - \bar{z}_s$

$$\bar{y}_b = \bar{y}_c - \bar{y}_s$$

and $\bar{z}_s$ and $\bar{y}_s$ are evaluated using equations (2.8).

It should be noted that for a symmetrically reinforced case $\bar{z}_b = \bar{y}_b = 0$; hence a pure torsion case will result for the balanced failure condition. The moments M_{bz} and M_{by} are greatest when the longitudinal steel is positioned so that the position $(\bar{z}_s, \bar{y}_s)$ of the resultant steel force is furthest

from the position at which the resultant concrete compression force acts.

The evaluation of the balanced failure mode as developed above is not restricted to a rectangular section but is applicable to any section where St. Venant torsion dominates. The loading ratio for balanced failure is evaluated as:

$$T_b/M_b = \frac{2A_o}{p_o j d} \sqrt{\frac{\epsilon_{ly} + \epsilon_{cu}}{\epsilon_{hy} + \epsilon_{cu}}} \dots \dots \dots (3.23)$$

where $j d$ = moment lever arm.

For the special case when $\epsilon_{ly} = \epsilon_{hy} = \epsilon_y$ and

$f_{ly} = f_{hy} = f_y$ equations (3.19) and (3.20) give

$$\rho_{hb} = \rho_{lb} = \frac{f'_c}{f_y} \cdot \frac{\alpha \beta}{2} \cdot \frac{\epsilon_{cu}}{\epsilon_y + \epsilon_{cu}} \dots \dots \dots (3.24)$$

CHAPTER 4

EXPERIMENTAL STUDY

4.1 OBJECTIVES AND CHOICE OF TEST SPECIMENS

The theoretical model presented is closely related to and similar in rationality and generality to the well known model for flexural analysis. Indeed it has been shown that the pure flexure case can be treated as a special case of the model presented. It is therefore relevant to note the comment made by Rüsch in his discussion of the various flexural theories:⁵ "Only tests of over-reinforced members can furnish a true measure of the validity of a flexural theory." In accordance with Rüsch's comment a test programme consisting of 14 heavily reinforced beams was designed to test the validity of the theoretical model.

All the specimens had identical steel cages and the same outside section dimensions. Four of the specimens were solid in contrast to the rest which were hollow. A typical section of the hollow beams as designed is shown in Figure 4.1. It should be noted that English units were used in designing the specimen sections but in the testing of the specimens the loads were measured in metric units. The actual measured dimensions of the specimens are given in metric units in Table E-1 of Appendix E,

and Figure 3.2 shows the metric dimensions for one of the specimens.

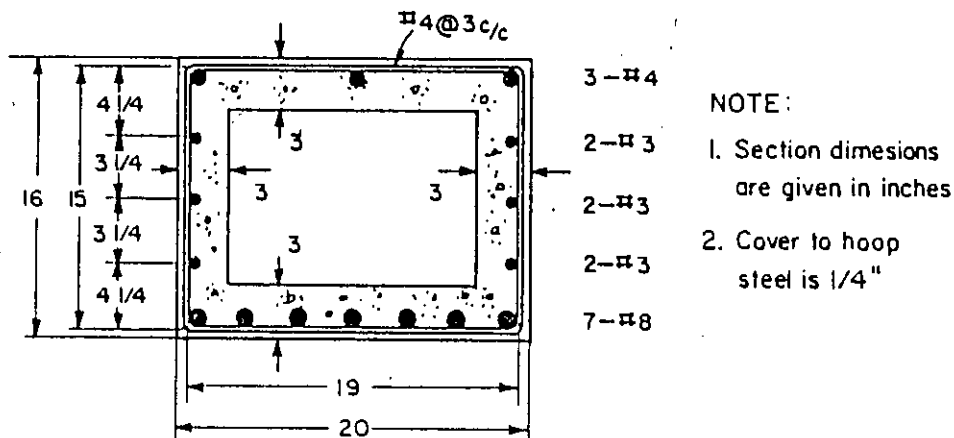


FIG. 4.1 CROSS-SECTION OF HOLLOW SPECIMENS AS DESIGNED

Heavy instrumentation of the specimens was necessary for the purpose of obtaining enough strain measurements to give us reasonable representative strain patterns to enable a comparison with the corresponding patterns predicted by the model, described in section 3.4. Furthermore, basic to the development of the theoretical model is the Bernoulli-Navier hypothesis concerning the variation of longitudinal strains at a section. This assumption, which is also basic to the development of the pure flexure theory, was to be verified for the case of combined flexure and torsion.

Besides the need to have experimental verification for the complete response prediction of a given beam section loaded in a specific manner, it was also the object of the test programme to verify how well the model prediction of the failure interaction curve for a given concrete section (such as that given in Fig. 3.5) was in agreement with test results. This necessitated testing the specimens of the same concrete strength under widely varying torsion to bending moment ratios. The effect of the variation of concrete strength was also to be experimentally investigated.

In section 3.5 of the theoretical development it was shown that the theoretical model can predict the steel ratios for a given section which will lead to balanced failure for the special conditions outlined in the section. This prediction needs experimental verification and the test programme was also aimed at doing this.

4.2 TEST SPECIMENS

4.2.1 Manufacture of Test Specimens

The hollow beams were cast in two groups of five so that for each group all the specimens were not only cast at the same time but also were made of the same concrete mix and they were cured together under the same conditions. One of these groups of beams made up the TBO series and the other group made up the TBU series. Figure 4.2 shows a complete steel cage for TBO4. Figure 4.3 shows the forms with five steel cages in position during the casting of the TBO series.

The hollow section was created by using styrofoam six feet (1.829m) long with a cross section of 10" X 14" (254mm x 356mm). The bottom wall thickness of the section shown in Figure 4.1 was concreted before the styrofoam was placed in position. The styrofoam was positioned by sliding it into the steel cage through one end of the cage with appropriate steel end plates designed for this purpose. The styrofoam was guided into and held in position by five equally spaced welded steel collars made out of $\frac{1}{4}$ " ϕ steel bars. These collars were designed to be anchored to the bottom cage steel and were to restrain the styrofoam from the tendency to float upwards under the pressure of the almost fluid concrete during casting. Loss of this anchorage for some of the collars for TBU4 and TBU2 led to upward displacement of the styrofoam with a reduced wall thickness for the top face of these specimens.

The four solid specimens constituted the TBS-series. Three of these specimens were also cast at the same time with the same concrete mix. The fourth was cast alone at a later time with a different concrete mix.

The capacity of the MTS actuators, used in loading during the testing, dictated the loading spans and consequently the total length of a specimen particularly for the pure flexure loading case. It was found that on the basis of the load carrying capacity of the chosen test section, as predicted by the model the total length of the specimen required which would be failed by the available MTS actuators was not convenient in terms of handling and the available casting space in the laboratory. To solve this problem it was decided to cast shorter concrete beams basically to provide the necessary test span length and to provide appropriate extensions to the concrete beams using structural steel sections. An available hollow steel sections 12" x 12" x $\frac{1}{4}$ " (305mm x 305mm x 13mm) was used for this purpose. The moment and shear connection between the steel beams and the concrete beam which involved the welding of reinforcing bars to steel end plates turned out to be the trouble spot in the testing programme.

Figure 4.4 shows the end plate with nuts and anchor bars welded in position. This plate enabled a successful moment connection between the steel beams and the concrete beam for each specimen of the TBO and TBS series. Figure 4.5 shows the moment connection used in the testing of the TBU series specimens which were cast and tested before the

TBO and TBS series specimens were fabricated. The connection in Fig. 4.5 had to be replaced by that in Fig. 4.6 for the testing of the TBO and TBS series specimens. This was because the connection shown in Fig. 4.5 failed before the failure load was reached for the pure flexure specimen TB01.

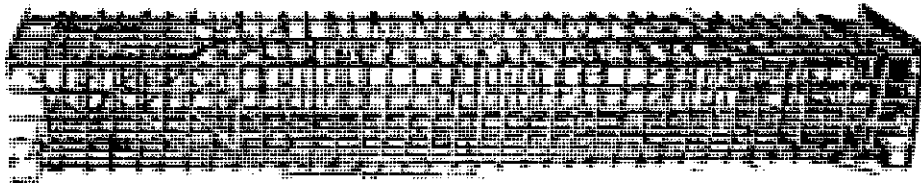


FIG. 4.2 REINFORCING STEEL CAGE FOR TB04



FIG. 4.3 CASTING OF TBO SERIES

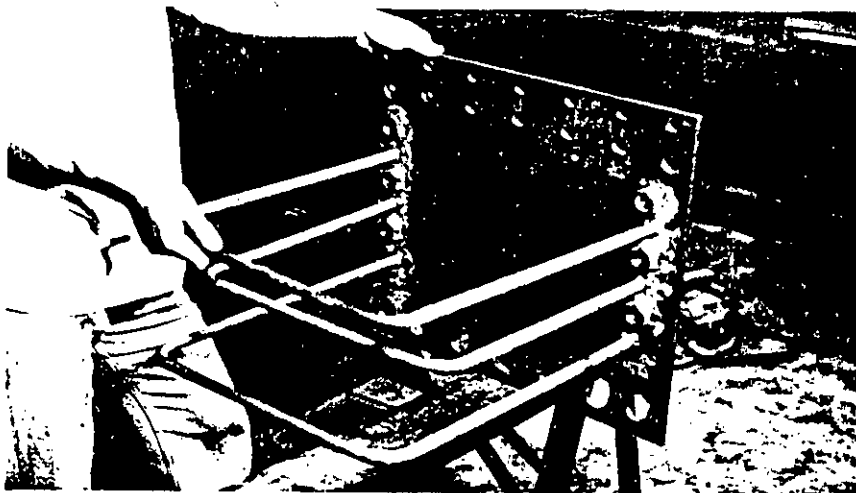


FIG. 4.4
END PLATE FOR CONCRETE
BEAM



FIG. 4.5
CONNECTION FOR TBU
SERIES



FIG. 4.6
CONNECTION FOR TBO
AND TBS SERIES

4.2.2 MATERIALS

4.2.2.1 Concrete

In Table 4.1 are summarized, the results of standard compression cylinder tests for the concrete used in casting the test specimens. The loading rate used was a stroke movement of 20mm in 3600 seconds corresponding to a strain of .064mm/mm per second. A continuous load stroke plot was automatically plotted during the test which lasted about 5 minutes. More accurate load deformation characteristics were obtained from some of the cylinder tests by plotting the applied compression load against the movement of the two LVDT's shown in Figure 4.7. The LVDT's measured the deformation of the middle part of the cylinder between the two collars shown.

During the testing of the beam specimens it was necessary to stop the loading by holding the stroke of the MTS actuators at several stages to enable the readings of deformations before proceeding with the loading. The stops generally lasted 20 to 30 minutes. During these stops there was a drop in the loads due to the creep in the concrete. These stops were simulated in some cylinder tests by holding the stroke of the cylinder testing machine for either 20 minutes for some tests or 30 minutes for others. This was done at several load levels and the results of these tests are shown in Figure 4.7 and 4.8. Also shown in these Figures are the curves for the standard cylinder tests. It was found that the maximum compression cylinder stress as obtained in the standard cylinder tests was always attained in the short term 'creep' tests as

well. However, with the more accurate load deformation characteristics obtained using the LVDT's as shown in Fig. 4.7, it was found that the strains at peak stress for the 'creep' tests was always greater than the corresponding strain for the standard test.

The parabolic stress-strain relationships used in the theoretical model to approximate the actual stress-strain relationship of the concrete are also plotted in Figure 4.7 and 4.8. It is noted that the representation is excellent up to the peak stress, at a strain of ϵ_0 ; however for strains much greater than ϵ_0 the parabola tends to under-estimate the stress especially for low concrete strengths.

TABLE 4.1 CONCRETE PROPERTIES

BEAM	AGE DAYS	E'_c N/mm ²	ϵ_0 $\times 10^{-3}$	TOTAL NO. OF CYLINDERS TESTED
TBO1	25	19.5	2.4	28
TBO2	28	19.7	2.4	
TBO3	20	19.1	2.4	
TBO4	33	20.4	2.4	
TBO5	36	20.6	2.4	
TBU1	102	34.3	3.1	34
TBU2	113	34.3	3.1	
TBU3	109	34.3	3.1	
TBU4	92	34.3	3.1	
TBU5	88	34.3	3.1	
TBS1	3	29.0	2.5	4
TBS2	7	32.9	2.5	4
TBS3	45	45.3	2.5	10
TBS4	31	15.5	2.0	19

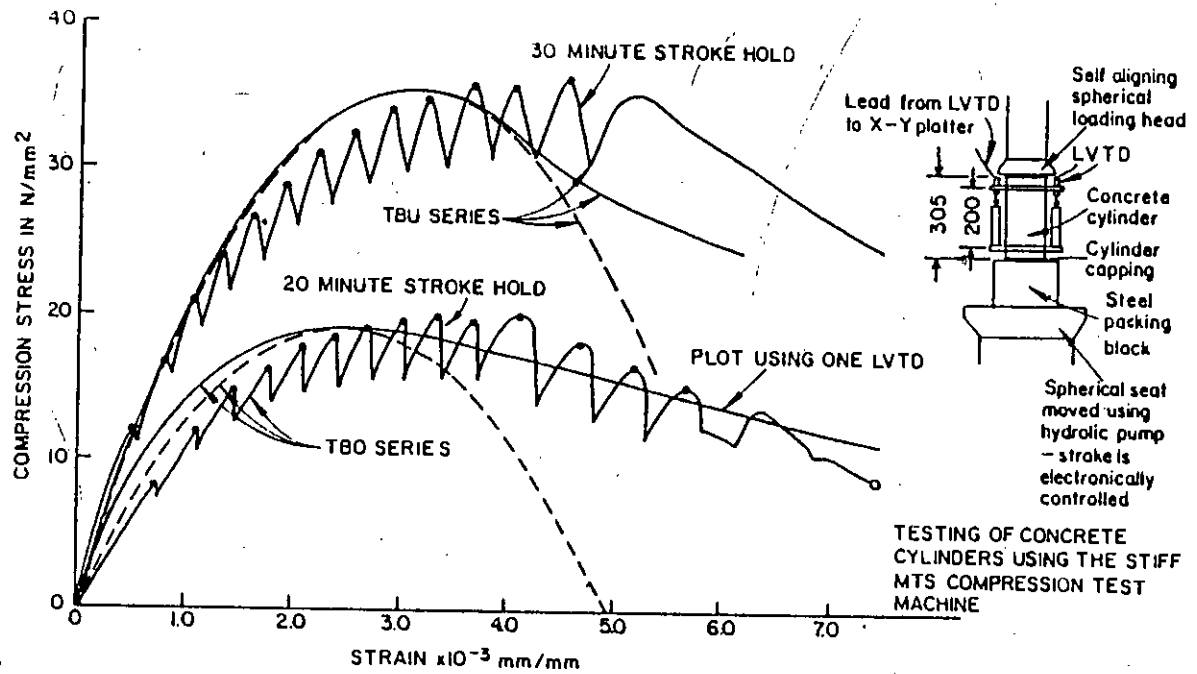


FIG. 4.7 STRESS-STRAIN CURVES OF CONCRETE FOR TBO AND TBU SERIES

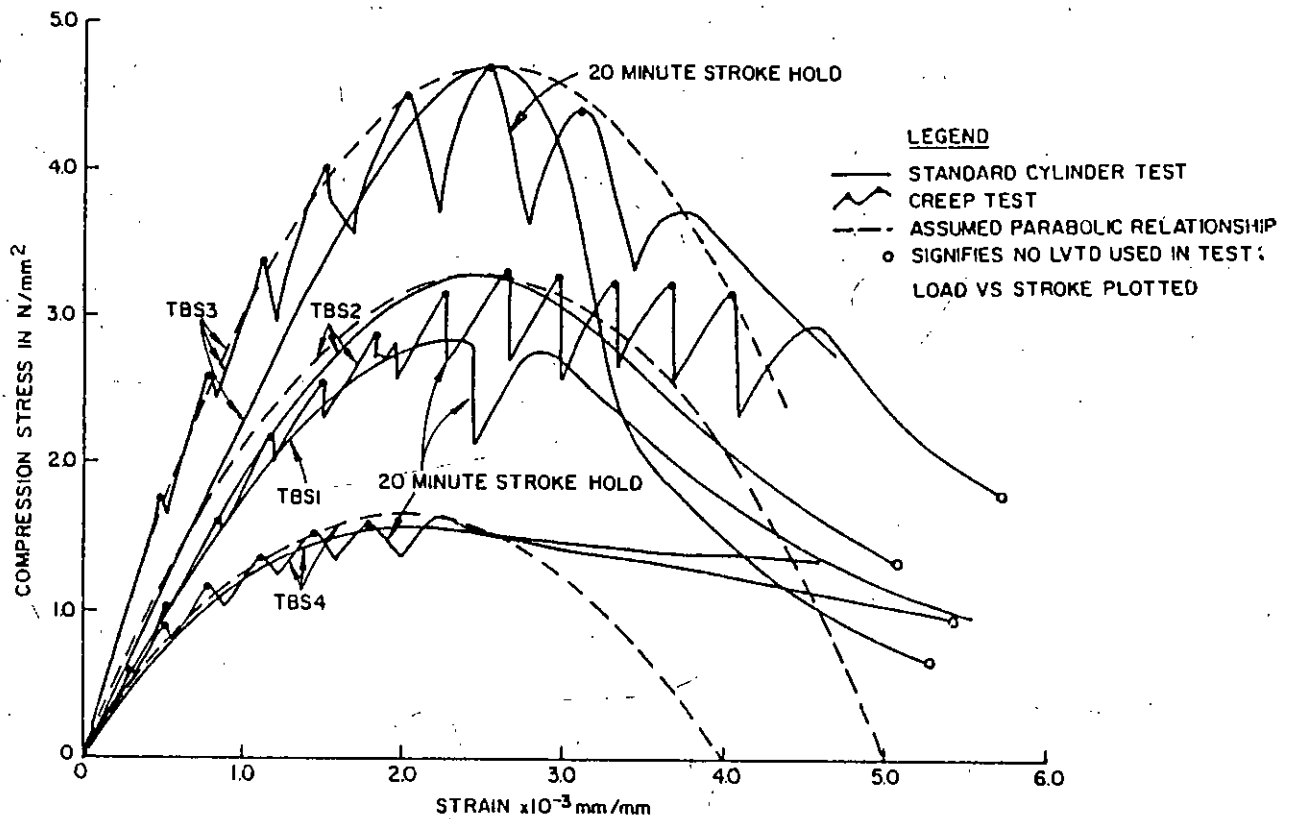


FIG. 4.8 STRESS-STRAIN CURVES OF CONCRETE FOR TBS SERIES

4.2.2.2 Steel

In Table 4.2 are summarized the steel properties for all the reinforcing steel used in all the specimens. The strain rate was 0.125 mm/mm per 9000 seconds from zero load up to the onset of strain hardening. This strain rate was doubled after the onset of strain hardening in order to cut down the total test time. The load was automatically plotted against the strain and the average moduli of elasticity in the elastic range and after the onset of strain hardening were evaluated from the plots.

Figure 4.9 shows a plot of the representative stress-strain curves for the reinforcing steel up to a strain of 0.015. The strain signal was obtained by a sensitive extensometer attached to the test specimen and calibrated so as to give a plot of direct strain on an X-Y plotter. The test specimens were representative pieces cut from the actual reinforcing bars used to make the steel cages for the specimens.

TABLE 4.2 PROPERTIES OF REINFORCING STEEL

BAR* TYPE	BAR SIZE	NO.	AREA mm ²	f_y N/mm ²	f_u N/mm ²	$E_s \times 10^3$ N/mm ²	$E_{sh} \times 10^3$ N/mm ²	$\epsilon_{sh} \times 10^{-3}$ mm/mm	WHERE USED
LONGITUDINAL	#8	16	510	436	758	194.2	9.0	6.13	ALL BEAMS.
	#4	8	129	393	620	201.9	4.9	15.18	TBO&TBU SERIES
	#4	5	129	433	723	185.7	11.2	2.00	TBS SERIES
	#3	7	71	552	763	205.1	9.0	2.00	TBU SERIES
	#3	5	71	401	646	201.7	7.5	2.00	TBO SERIES
	#4	3	71	406	606	200.0	3.9	15.67	TBS SERIES
Hoop	#4	9	129	379	605	198.5	5.2	15.4	TBO&TBU SERIES
	#4	6	129	443	717	190.4	9.6	7.96	TBS SERIES

* Bar sizes are in eighths of an inch (3.2 mm)

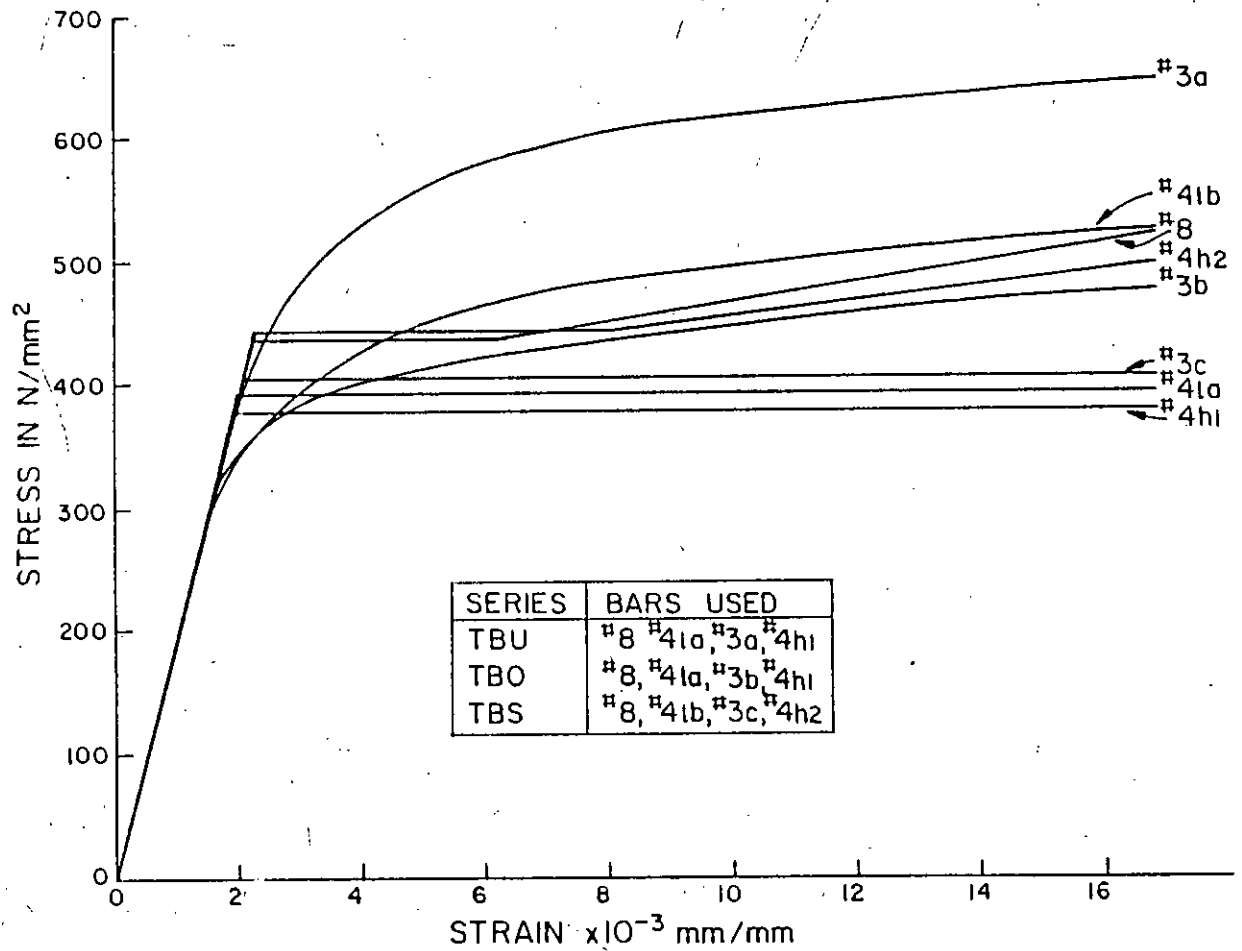


FIG. 4.9 STRESS-STRAIN CURVES FOR REINFORCING STEEL

NOTE: The yield indicated in Table 4.2 for those bars which did not have a distinct yield point was taken as the proof stress using the 0.2% set method.

4.3 PARAMETERS OF THE TEST PROGRAM

The concrete compression strength, f'_c , and the ratio of torque to flexural moment, R , were the basic parameters varied in the tests. The concrete strength for the TBO series was selected so as to obtain failure of the section without yielding any of the reinforcing steel for both the special loading case that produces uniform beam elongation and for pure flexure. Thus according to the criteria for balanced failure given in section 3.5, the TBO series specimens were designed to be over-reinforced. On the other hand the concrete strength for the TBU series was chosen so that at least some of the reinforcing steel would be in the post-yield range at failure. The TBS series was designed to investigate the effect of the variation of f'_c for a fixed loading ratio, R , and to test the validity of the theoretical model for solid sections.

The loading ratios, R , for the TBO and TBU series were chosen so as to span out the torsion-flexural interaction domain in such a way as to reveal important representative response. The schematic loading arrangement is shown in Figure 4.10. Statical analysis of the beam AB under the loads shown in Figure 4.10 reveals the applied actions shown in Figure 4.11. The lever arm lengths x_T and x_M shown in Figure 4.10 were chosen so as to achieve the desired R ratios which are summarized in Table 4.3.

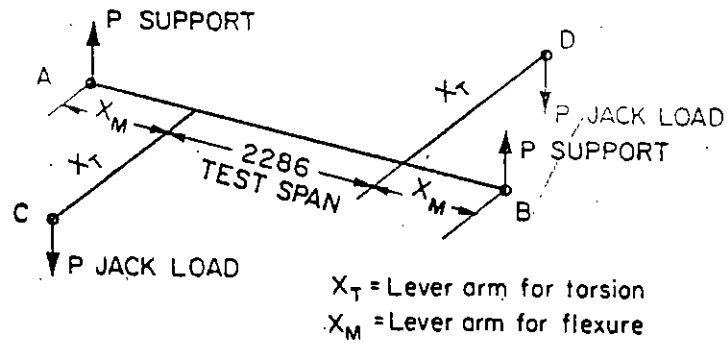


FIG. 4.10 SCHEMATIC LOADING ARRANGEMENT

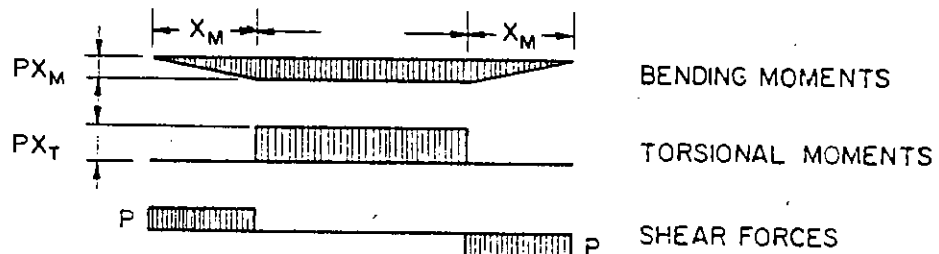


FIG. 4.11 APPLIED ACTIONS ON TEST BEAM

TABLE 4.3 LOADING RATIOS

	TBO SERIES					TBU SERIES					TBS SERIES
	TBO1	TBO2	TBO3	TBO4	TBO5	TBU1	TBU2	TBU3	TBU4	TBU5	
$X_T(m)$	0.000	0.610	1.200	1.911	1.911	0.000	0.616	1.200	1.911	1.911	1.911
$X_M(m)$	2.321	2.321	1.711	1.254	0.378	2.359	2.359	1.724	1.267	0.378	1.254
R	0.000	0.261	0.701	1.524	5.059	0.000	0.261	0.696	1.509	5.059	1.254

4.4 THE TEST RIG

Figure 4.12 shows a view of the test rig and Fig. 4.13 shows a corresponding schematic representation. Except for specimens TBO5 and TBU5 all the specimens tested in the test rig shown had the steel extension beams connected to the ends of the concrete specimens by a carefully designed moment

and shear connection. At the ends of the steel extension beams frictionless universal bearings were attached. These bearings, shown in Fig. 4.15, were designed and constructed specifically for the tests to enable free rotation at the supports due to the applied torque as well as free rotation at the supports due to the curvature of the specimens under flexure. For specimens TBO5 and TBU5 these bearings were attached directly to the ends of the specimens. Which had specially designed end plates for the purpose.

The torsion arms were provided by the heavy welded steel section which were attached 90" (2.286m) apart on the central part of the concrete beam. They were clamped to the specimens as seen in Fig. 4.14 using $1\frac{1}{3}" \phi$ (34mm) dwidag bars secured with the appropriate nuts and plates.

The test beam with the attached extension beams and the attached torsion arms was supported overhead at its ends. A load cell was positioned at each support end, as indicated in Figure 4.13, to monitor the support reaction. These load cells were specially made for this purpose. The height of the specimen above the floor was chosen to enable convenient and easy attachment of the MTS actuators which were used to apply the loads and also to enable strain readings in all the faces particularly the bottom face.

The torque and flexural moment were applied to the test specimen by the two MTS actuators synchronized and programmed to apply equal displacements downwards at the ends of the torsion arms. The actuators were anchored in position

to the floor by a system of floor beams shown in Figure 4.16. Enough hinges enabling free rotation in all directions were provided so that the actuators were self-aligning in application of the load to the torsion arms. So well were the friction effects eliminated in the entire test set up that at any load level an additional load applied by the actuators and monitored by the special MTS load cells attached to the actuators were equal and also reflected by the overhead support load cells with a discrepancy of less than $\pm 4\%$ between the support loads and actuator loads.

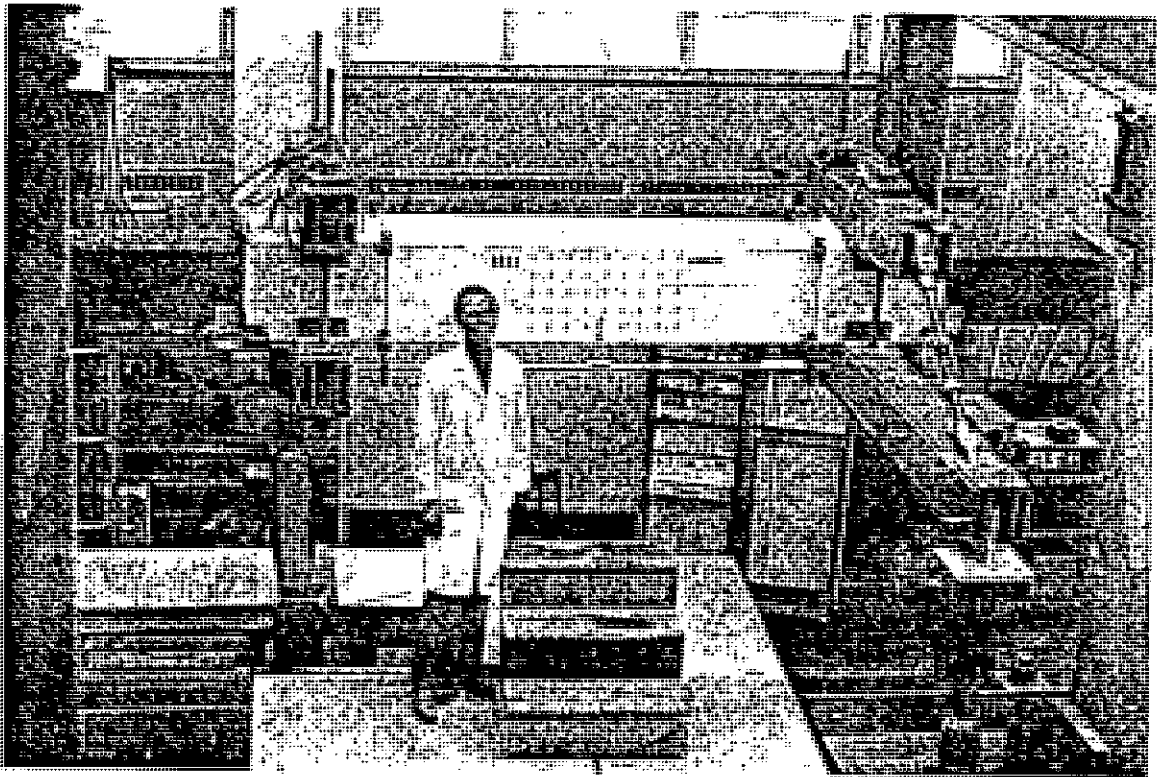


FIG. 4.12 TEST RIG

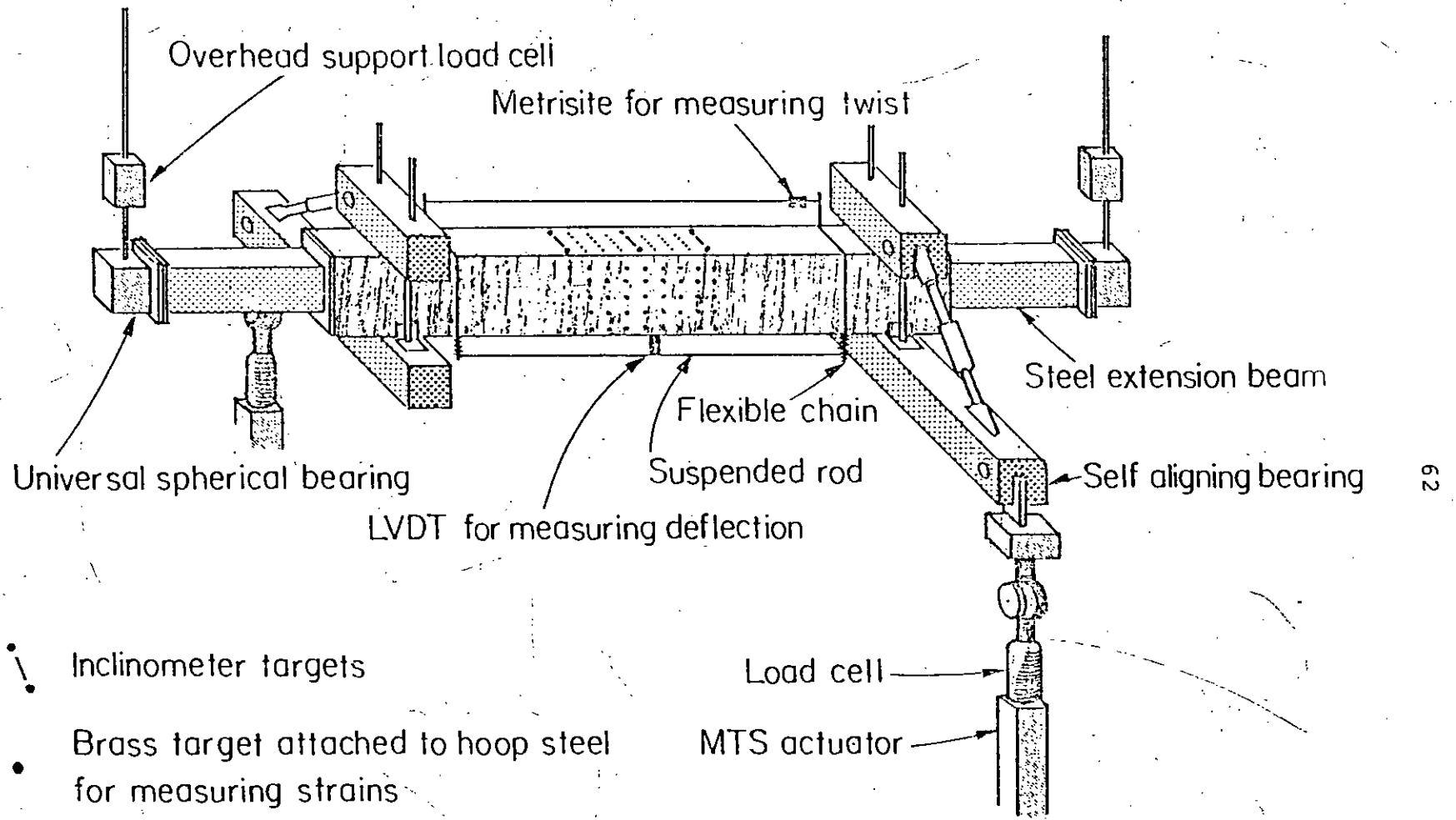


FIG. 4.13 SCHEMATIC REPRESENTATION OF TESTING RIG SHOWING INSTRUMENTATION

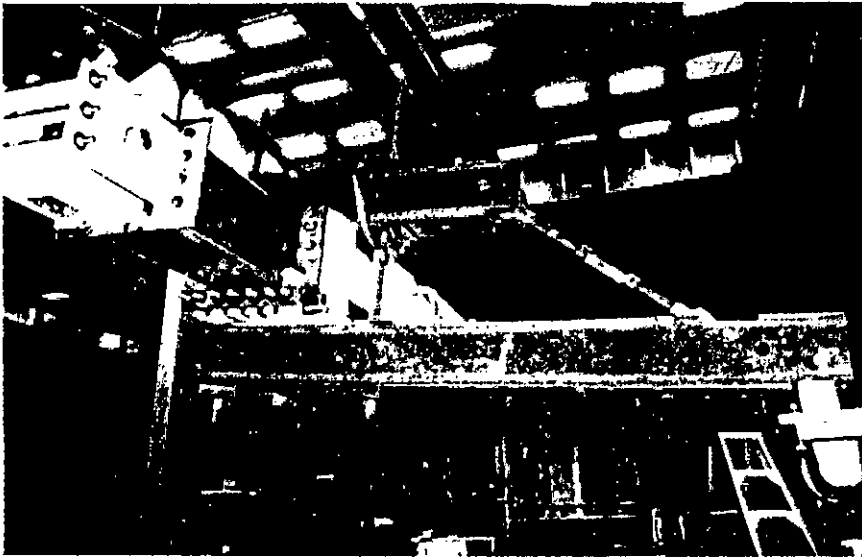


FIG. 4.14 TORSION ARMS



FIG. 4.15 UNIVERSAL BEARING SUPPORT ATTACHED TO END OF EXTENSION BEAM

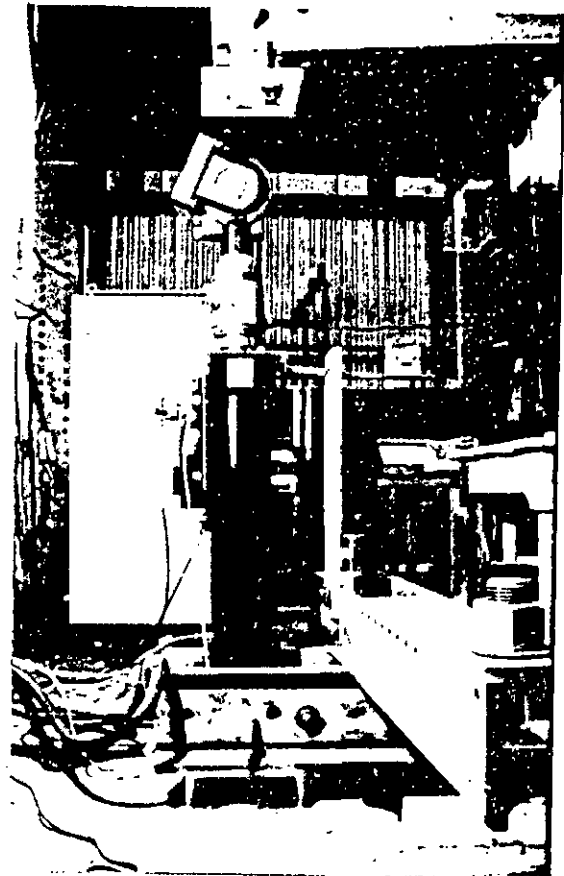


FIG. 4.16 MTS ACTUATOR ANCHORING ARRANGEMENT

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4.5 INSTRUMENTATION

The loads applied by the MTS actuators and the support reactions were monitored by the load cells indicated in Fig. 4.13. A continuous plot of load versus rotation due to the applied torque was obtained on an X-Y plotter driven by a load signal from one of the actuators and a rotation signal from a rotational transducer attached to the specimen to measure relative rotation over the central 1.75 metres of the specimen. As a check on the torsional rotation obtained using the metrisite three inclinometer readings were taken over three positions 12 inches (305mm) apart in the central part of the top face. A measure of the amount of torsional rotation was also given by the continuous plot of load versus the stroke of the synchronized actuators.

A continuous plot of load versus the midspan deflection of the specimen was also recorded on another X-Y plotter driven by a load signal from one of the actuators and the deflection signal was obtained from an LVDT positioned at the beam midspan in an arrangement enabling the relative deflection of the midspan section with respect to sections 0.875 metres away on either side of the midspan. This arrangement is shown in Figure 4.13. From the deflection measured the overall curvature over the central 1.75 metres of the specimen was evaluated and checked against the curvature obtained from the longitudinal strain profile.

Most of the time during the testing was taken up in reading the longitudinal, transverse and diagonal strains in

the top, bottom and side faces of a specimen at each load stage. The strains were measured from targets attached on to the hoop steel in regular patterns using rapid hardening epoxy glue. The pattern of targets used for the top and bottom faces is shown in Figure 4.17 and that for the north and south faces is shown in Figure 4.18. Mechanical strain measuring gauges (1 div = 0.01mm) were used in the measurement of strains. The longitudinal strains were measured over targets 6 inches (152 mm) apart the hoop strains over 3 inches (76 mm), and the diagonal strains over $6\sqrt{2}$ inches (216 mm) apart.

With the target patterns shown in Figures 4.17 and 4.18, 258 strain readings were taken at every load stage. This large number of strain readings was necessary to establish experimentally the strain variation pattern in the faces so as to enable a reasonable comparison with the corresponding strain patterns predicted by the theoretical model. It should be noted that due to the actual orientation of the specimens in the testing position, the side faces were appropriately designated North face and South face.

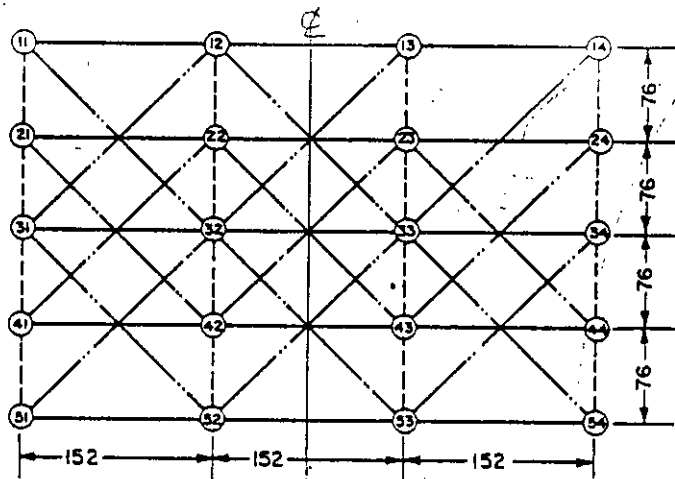
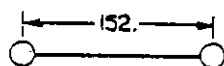
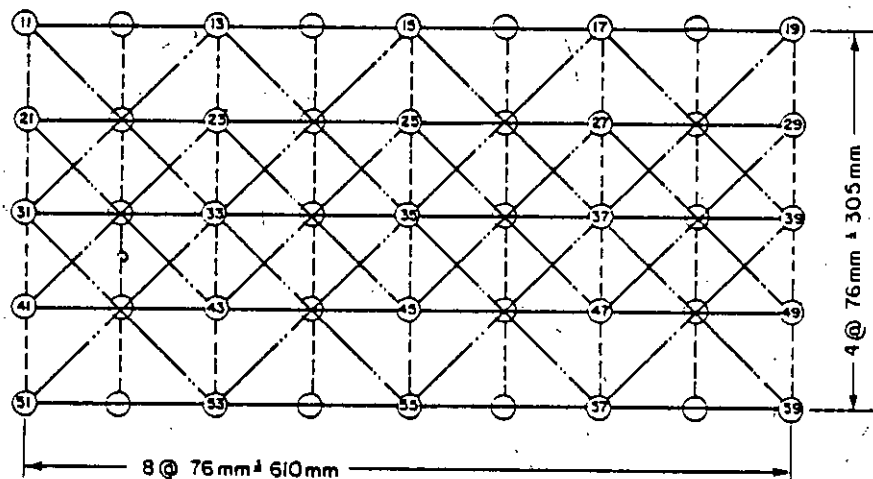
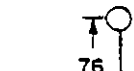


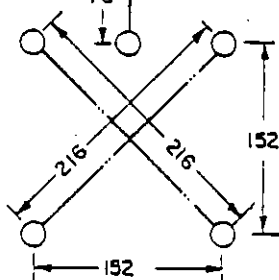
FIG. 4.17 TARGET PATTERN FOR TOP & BOTTOM FACES



LONG STRAIN MEASUREMENT



HOOP STRAIN MEASUREMENT



DIAGONAL STRAIN MEASUREMENTS

NOTE-DIAGONAL AND LONGITUDINAL STRAINS WERE TAKEN OVER THE NUMBERED TARGETS INDICATED

FIG. 4.18 TARGET PATTERN FOR NORTH AND SOUTH FACES

4.6 TESTING PROCEDURE

Before the start of each test, the theory developed in Chapter 3 was used to predict the expected response using the known or estimated material properties for the specimen, with the theoretical model prediction it was possible to decide at approximately what load levels or deformation levels at which it would be appropriate to have load stages for the purpose of strain readings.

All the strain readings were taken before the steel extension beams and the torsion arms were attached. The strains were also read again when the specimen was in position in the test rig before the first load increment. This ensured that the initial readings taken were correct.

The specimen was then loaded in steps using the two MTS actuators whose stroke movements were synchronized and controlled by the same MTS control unit. The actuators were programmed to pull down the torsion arms (Fig. 4.13) by equal amounts. With the friction-free universal joints provided in the test set up, the loads applied by the two actuators were always exactly equal. It is of interest to note that, in loading the specimens using stroke control, a consistent loading procedure was achieved in the testing of the materials (concrete and reinforcing steel) used in making the test specimen, and in the testing the specimen itself.

In a typical loading stage the load was steadily increased by actuator stroke movement at rate of 127 mm³ per

3600 seconds to a desired load value or deformation value and the stroke was then held until all the strain readings and other readings were done. The crack patterns were marked and photographed during this time interval. The actuator loads and the support reactions as given by the load cells were recorded at the beginning and at the end of each load stage. During a load stage the loads were observed to drop. The rotation due to torsion and the deflection due to flexure remained reasonably constant during each load stage but with a tendency to increase as the load dropped. This tendency would be eliminated with a stiffer testing rig. Because the torsional rotation and the deflection due to flexure remained close to constant, it is reasonable to assume that the strains in the beam remained close to constant for the load stage.

In the testing of the pure flexure specimens it was not necessary to take the transverse and diagonal strains. However, to ensure that there were no torsional effects these strains were taken at almost half capacity load and at a load stage close to capacity loading and no significant transverse and diagonal strains were found. Thus the load stages for these specimens lasted a comparatively short time.

In the case of pure flexure the test set up shown in Figure 4.13 with zero level arm for torsion did not produce uniform flexure in the test region as planned. This was due to the fact that unequal settlements and unequal deformations at the ends of the specimens resulted in unequal loads in the

two actuators which were programmed to apply equal stroke movements. TBO1 which was tested to failure using this arrangement failed with one actuator exerting a load 28% higher than the other actuator. Specimens TBUI was not failed with the test arrangement because the connection between the concrete specimens and the extension beam failed prematurely. The beam was tested to failure under regular knife-edge end-supports with two equal point loads applied at $1/3$ span point and $2/3$ span point from one end. The total span between the supports was 12 feet (3.048mm).

CHAPTER 5EXPERIMENTAL RESULTS AND THEORETICAL PREDICTIONS5.1 INTRODUCTION

In Chapter 4 a comprehensive test programme has been described which was designed to provide test data for comparison with the predictions of the theoretical model presented in Chapter 2 and Chapter 3. The comparison is made in this Chapter. The Chapter first reviews the assumed validity of the Bernoulli-Navier hypothesis with the test data. Also discussed is the assumption that concrete has no tensile strength and the phenomenon of the spalling of concrete cover. It is then shown that the model predicts the deformations well. The failure loads for the specimens tested as well as the failure loads for other tests reported in the literature are shown to be also well predicted. The failure modes of the specimens tested are then discussed and the criteria for balanced failure based on the model is shown to be in good agreement with the test results. The chapter closes with a discussion of torque-twist relationships and moment curvature relationships with reference to the parameters varied in the tests and good agreement of test data with the model prediction is noted.

5.2 REVIEW OF THE BERNOULLI-NAVIER HYPOTHESIS WITH EXPERIMENTAL RESULTS

The assumption that longitudinal strains vary linearly across the section is basic in the development of the theoretical model . Fig. 5.1 shows the measured average longitudinal strains for each face of the section for the TBO series specimens at approximately half capacity loading level (Fig. 5.1(a)) and at approximately capacity loading level (Fig. 5.1(b)). Similar plots are shown in Appendix E.4 for the specimens in the TBU and TBS series.

These figures indicate a linear variation of the measured longitudinal strains in the North and South faces for all the specimens. It is further observed that the longitudinal strains remained virtually constant over the top and bottom faces and the strain measurements in the North face are in good agreement with the corresponding measurements in the South face. However, comparatively noticeable oscillations of the strains measured in any given face are noted in those specimens loaded at the ratio R (Table 4.3) of about 1.5.

The strains plotted in Fig. 5.1 are averaged for the section and plotted in Fig. 5.2 for the indicated load levels. The corresponding average strains for the TBU series are also plotted in Fig. 5.2 for comparison. Also shown in Fig. 5.2 are the correlation coefficients, r , obtained from a linear regression analysis, using all the measured strains on each section at the indicated load levels.

Fig. 5.1(a)

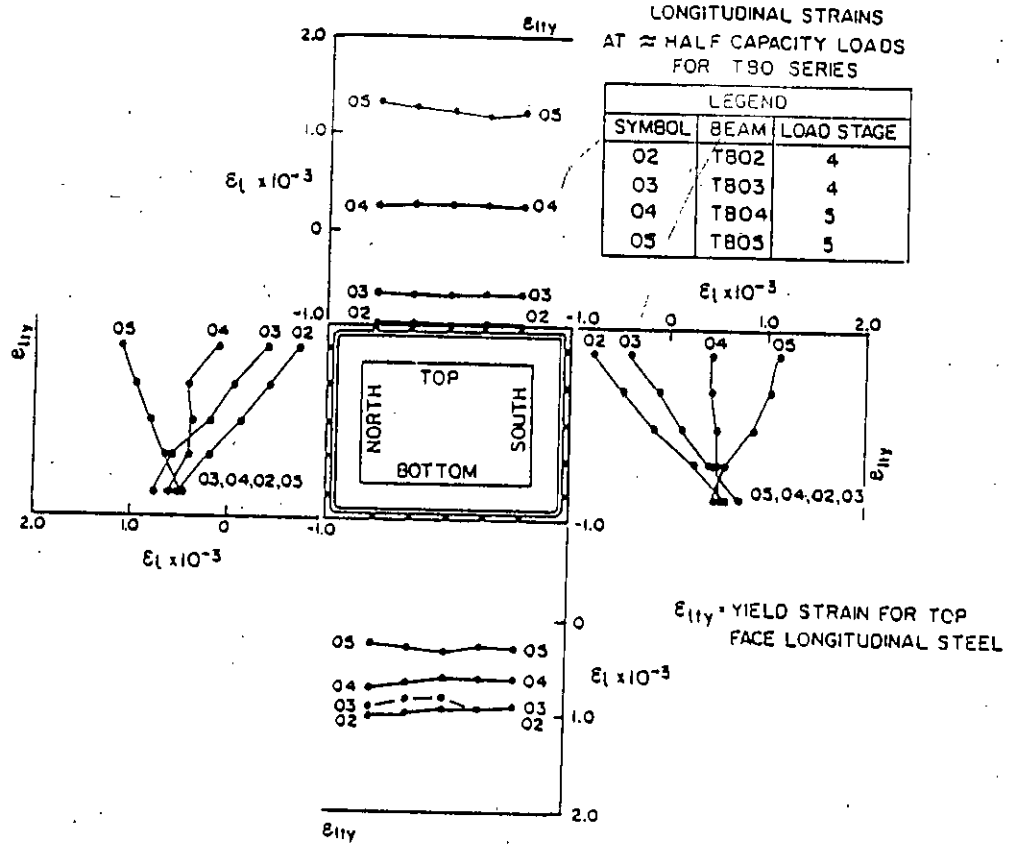
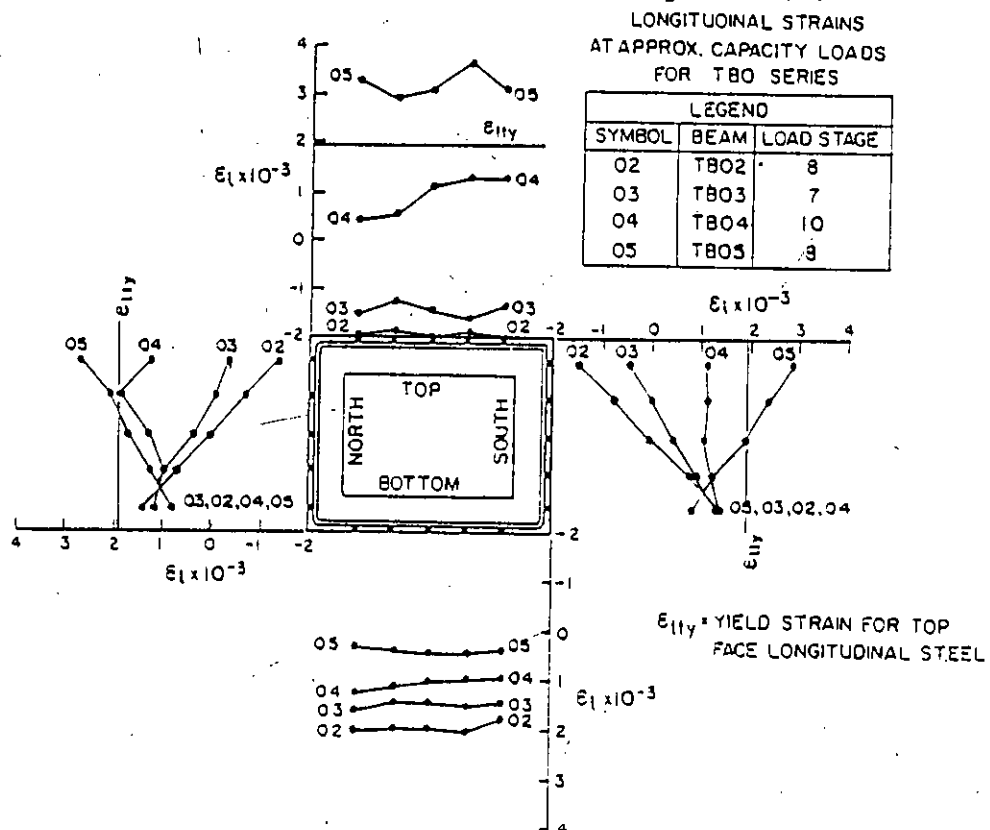
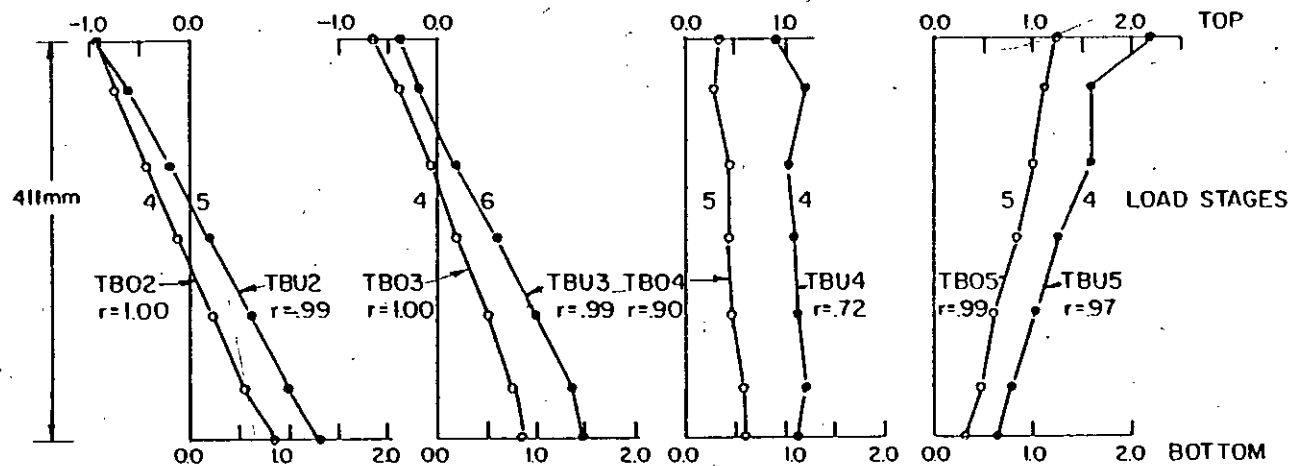
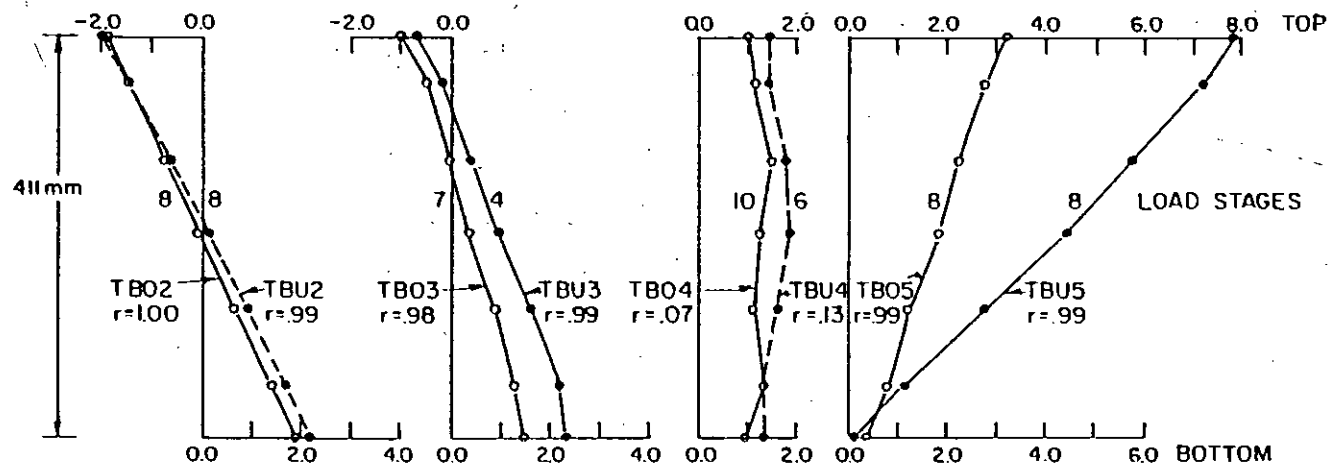


Fig. 5.1(b)





(a) LONGITUDINAL STRAINS ($\epsilon_l \times 10^{-3}$) AT $\approx$ HALF CAPACITY LOADING LEVEL



(b) LONGITUDINAL STRAINS ($\epsilon_l \times 10^{-3}$) AT $\approx$ CAPACITY LOADING LEVEL

FIG. 5.2 LONGITUDINAL STRAIN PROFILES FOR TBO & TBU SERIES
(r=Linear Correlation Coefficient)

Except for the specimens TBO4 and TBU4 the linearity of the longitudinal strain profiles is unquestionable. For specimens TBO4 and TBU4 the strains, particularly at capacity loads, were practically the same in all faces and statistical analysis using the students t-test confirmed the noted uniform beam elongation at a confidence level of 99%. This was also the case with the variation of the longitudinal strains for the TBS series particularly at approximately capacity loading levels.

Thus on the basis of experimental evidence it is valid to assume a linear longitudinal strain distribution across the section for the combined loading case.

5.3 DISCUSSION OF THE ASSUMPTION OF ZERO TENSILE STRENGTH OF CONCRETE AND SPALLING OF CONCRETE COVER

A simplifying assumption used in the development of the theoretical model is that concrete has no tensile strength after cracking. Thus it is assumed that diagonal cracks will form in all the faces of a beam subjected to combined torsion and bending as soon as the load level causing the first cracking as evaluated by elastic analysis is reached. From the test results it was observed that the specimens subjected to a predominantly torsion loading did indeed have diagonal cracks in all faces quite early in their loading history, whereas those specimens with predominantly flexural loading had diagonal cracking, if any, forming at much higher

load levels. For example beams TBO5 and TBU5, which were loaded with a high torsion to moment ratio, had noticeable diagonal cracks forming in all faces by the second load stage, whereas beams TBO2 and TBU2, which had a predominantly flexural loading, did not have any diagonal cracks forming in their top faces and noticeable diagonal cracks did not form in the bottom and side faces till the third load stage. It should be noted that in the testing procedure much greater loading increments were used in the early load stages than for those loads stages close to capacity loading.

A further consequence of ignoring the tensile strength of concrete is the spalling of the concrete cover to hoop steel. Reference (1) discusses in detail the phenomenon of spalling of concrete cover to hoop steel and its effects on the response of a structural concrete beam subjected to pure torsion. It is suggested that the spalling can be assumed to occur to the level of hoop steel centrelines. On the other hand, for the case of beams subjected to pure flexure, spalling as discussed in reference (1) does not occur and is not considered in the well established theory of flexure. It is therefore logical to expect that in the case of combined torsion and flexure the effects of spalling will become increasingly significant as the torsion to moment ratio is increased in the loading of a beam.

This was indeed found to be the case in the tests. Signs of spalling were very evident close to capacity load

for these specimens loaded with a high torsion to moment ratio. For example the spalled concrete cover could be easily picked off any of the faces of beams TBO4 (Fig. 5.3), TBO5, TBU4 and TBU5 at failure. On the other hand TBO2 (Fig. 5.4) and TBU2 showed no such clearly marked signs of spalling although some signs of spalling were evident at the corners in the bottom face. No signs of spalling were seen in the pure flexure specimens TBO1 and TBU1.

Evidently torsion induces spalling. Whereas pure flexure will induce a principal compression field in a localized region of a beam section that is aligned to the beam longitudinal axis, torsion when applied alone or in combination with flexure will align the direction of principal compression to create a field of diagonal compression in all the beam faces. The orientation with respect to the beam longitudinal axis of the principal compression is governed by the requirements of equilibrium and compatibility of deformations. The resulting compression diagonals meeting at the section corner will initiate the spalling of the concrete cover to hoop steel at the corners in the manner described in reference (1).

Complete spalling of the concrete cover to hoop steel is prevented by the tensile stresses in concrete. However, if the tensile strength of concrete is assumed to be zero then it is consistent to assume that the spalling initiated at the corners will spread around the section and the concrete outside the hoop centreline perimeter can be considered ineffective in

carrying load.

Finally, ignoring the tensile strength of concrete also means ignoring the stiffening of the reinforcing steel bars by the uncracked concrete between the crack spacing. This tension stiffening effect will be particularly significant in the low levels of loadings after the formation of the first cracks.

It is therefore evident that by assuming that concrete has zero tensile strength and using the spalled section dimensions in the model prediction, the predicted deformations should be larger than those measured for a given loading level. The test results to be discussed do indeed confirm this. It is possible to get predictions of deformations a little closer to those measured by averaging the spalled predictions and the unspalled predictions. Such a procedure is used in reference (1). However, this still fails to take into account the tension stiffening effects of the concrete on the reinforcement.

It is considered reasonable to use the spalled prediction as the basis of evaluation because it is conservative and it best predicts the behaviour at failure for a section loaded with a significant amount of torsion. For loading cases which induce diagonal cracks in all the beam faces and at high load levels, the conditions assumed in the spalled prediction will be closely approximated and the measured load deformation response will then approach that predicted by the model.

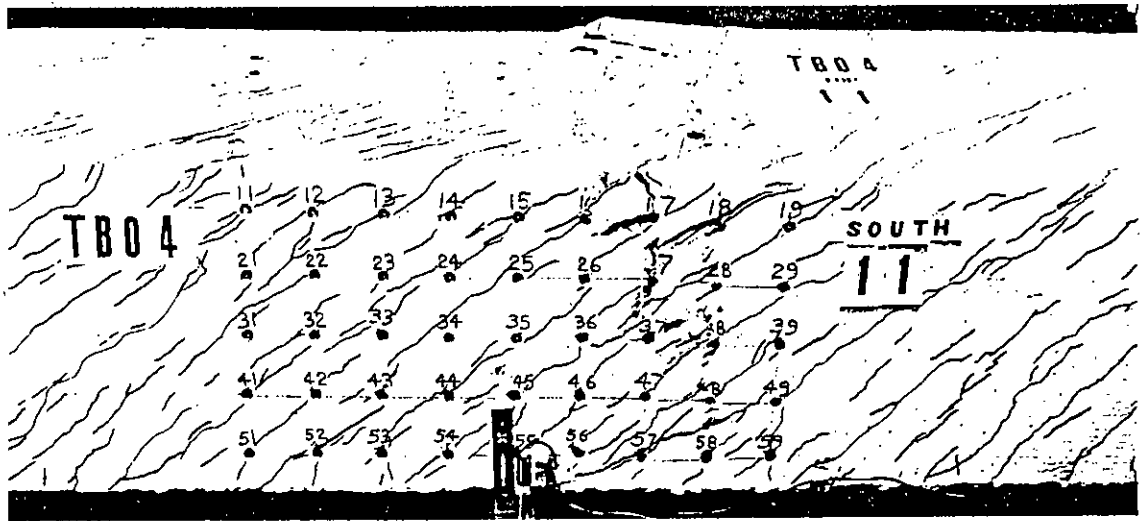


FIG. 5.3 TB04 AT FAILURE LOAD

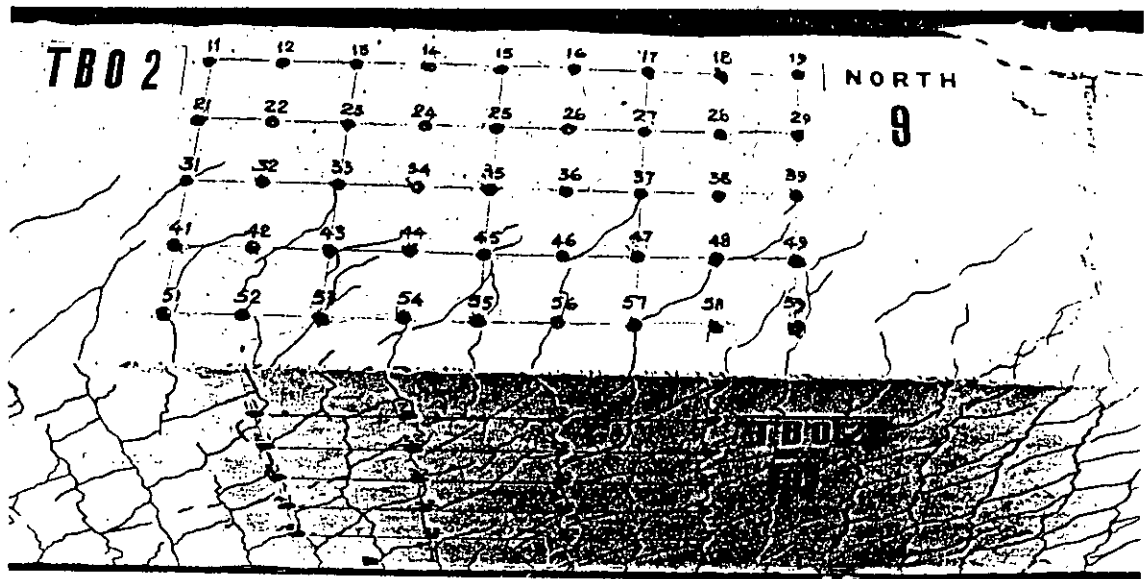
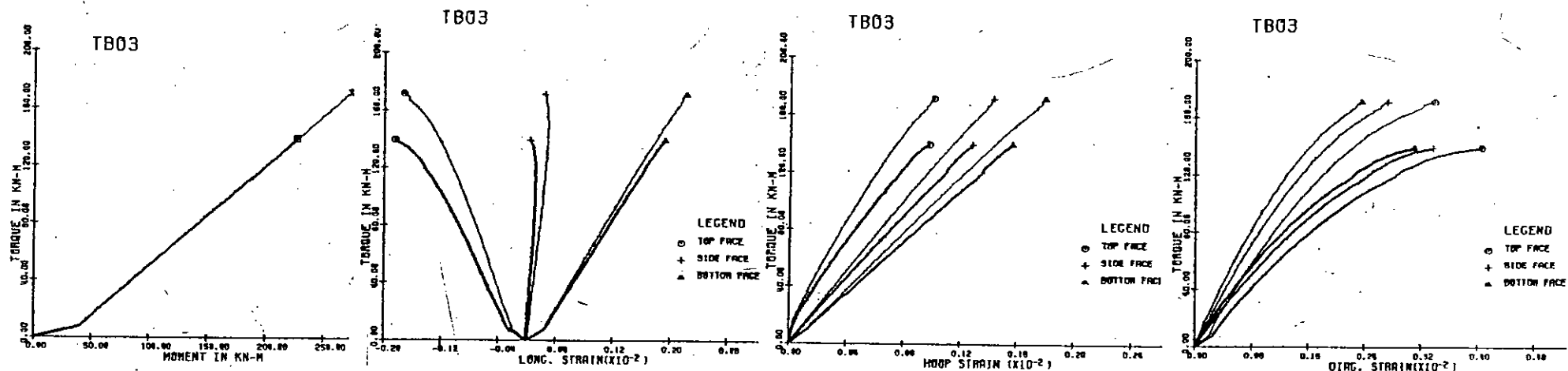


FIG. 5.4 TB02 AT FAILURE LOAD

5.4 STRAIN PATTERNS

Fig. 5.5 shows the computer plot of the results of the model prediction for beam TBO3 and the corresponding plot of the measured values obtained from the test of the specimen. The model predictions for the spalled and unspalled section dimensions are plotted and these plots are similar to the plots in Fig. 3.3 described in details in section 3.4. A complete set of the plots similar to that shown in Fig. 5.5 for all the beams tested is given in Appendix E.5.

These plots show that for all the specimens tested covering widely varying loading ratios ($R = 0.0$ to 5.06) and for the ranging concrete strengths used ($f'_c = 15.5 - 45.8 \text{ N/mm}^2$) the model predicts the load-deformation responses for the rectangular section tested well. With reference to Fig. 5.5 it is seen that the model predicted deformations are higher than those measured but at close to capacity loads the measured strains show a rapid increase approaching the values of the model prediction for the spalled section condition. It is also of interest to note that for TBO3 the spalled model prediction shows that at capacity loading the longitudinal and hoop steel will not yield and this is confirmed by the test results. For those specimens for which either hoop steel or some longitudinal steel was yielded, the load level at which the yielding was initiated is well predicted by the model for spalled section conditions as indicated by the plots in Appendix E.5.



08

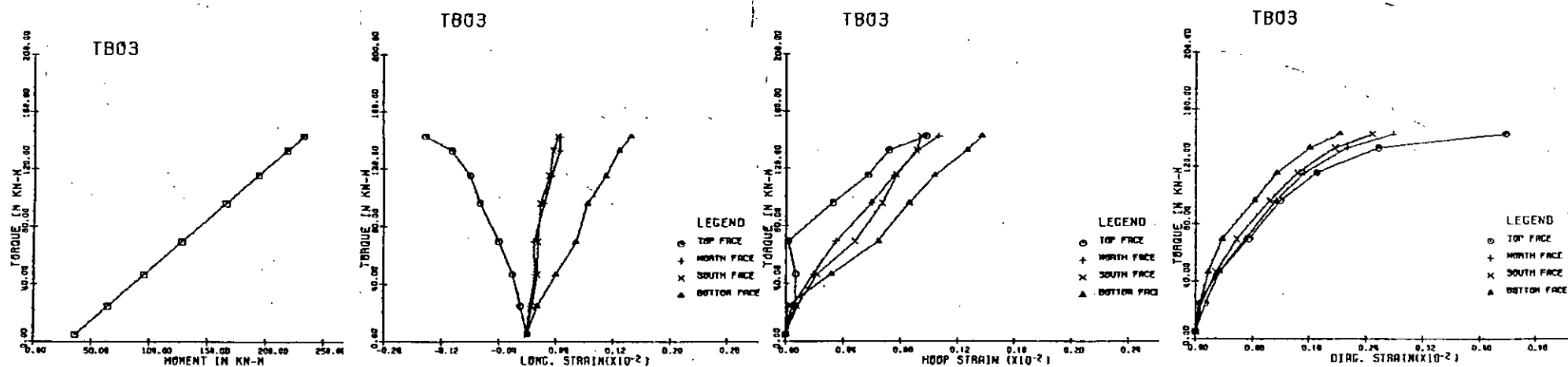
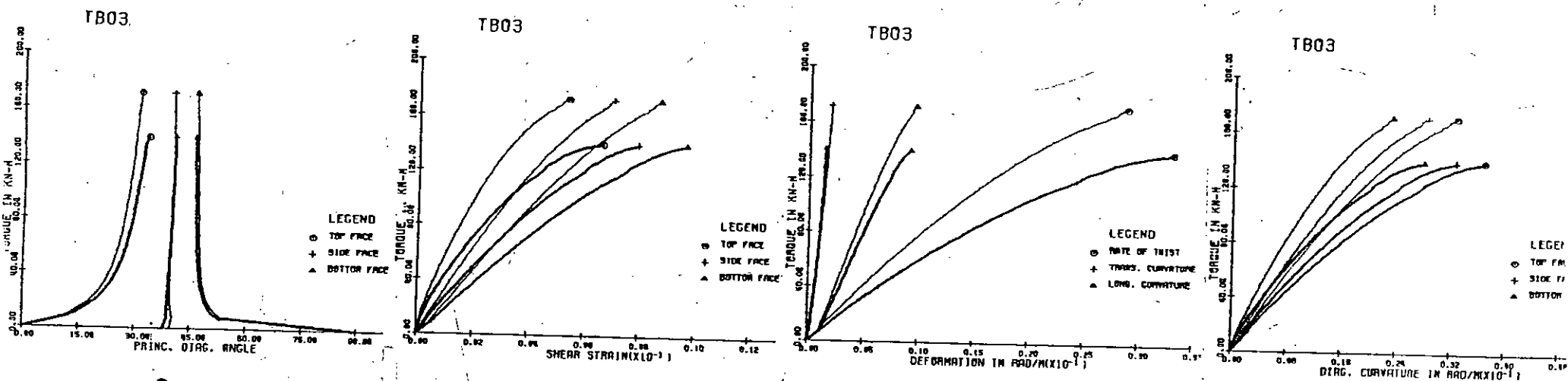


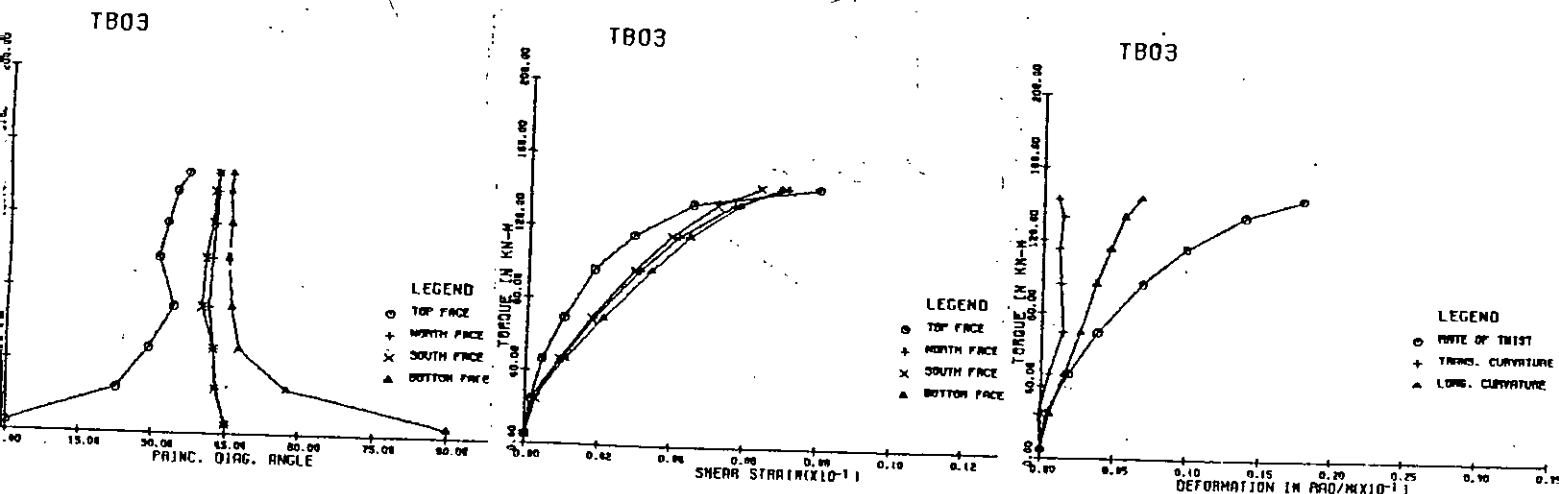
FIG. 5.5 COMPARISON OF THEORETICAL PREDICTION WITH TEST RESULTS



THEORETICAL PREDICTION

Thick lines = Spalled Prediction
Thin lines = Unspalled Prediction

81



TEST RESULTS

FIG. 5.5 COMPARISON OF THEORETICAL PREDICTION WITH TEST TESTS (cont'd)

Besides the overall load-deformation responses such as those shown in Fig. 5.5 it is of interest to investigate how well the strain patterns across a face of a section as predicted by the model agree with those obtained from the test data.

5.4.1 Variation of Longitudinal Strains

Fig. 5.6 shows a plot of the longitudinal strain profiles at failure as predicted by the model for the TBO series beams assuming spalled section dimensions. The strains measured at the last load stage before failure are also plotted for comparison. It is seen that beams TBO4 and TBO5, whose condition at failure closely approximated the assumed model conditions, have their measured longitudinal strain profiles in very good agreement with the predicted profiles. On the other hand TBO2 which did not have any diagonal cracks in the top face at failure did not show such a good agreement. Notice however, that the model predicts greater curvature for TBO2 than that measured and this is as expected.

The measured longitudinal strains were used to evaluate flexural curvatures at the various load stages. The curvatures thus evaluated were compared with those evaluated from the continuously monitored midspan beam deflection and very good agreement was revealed.

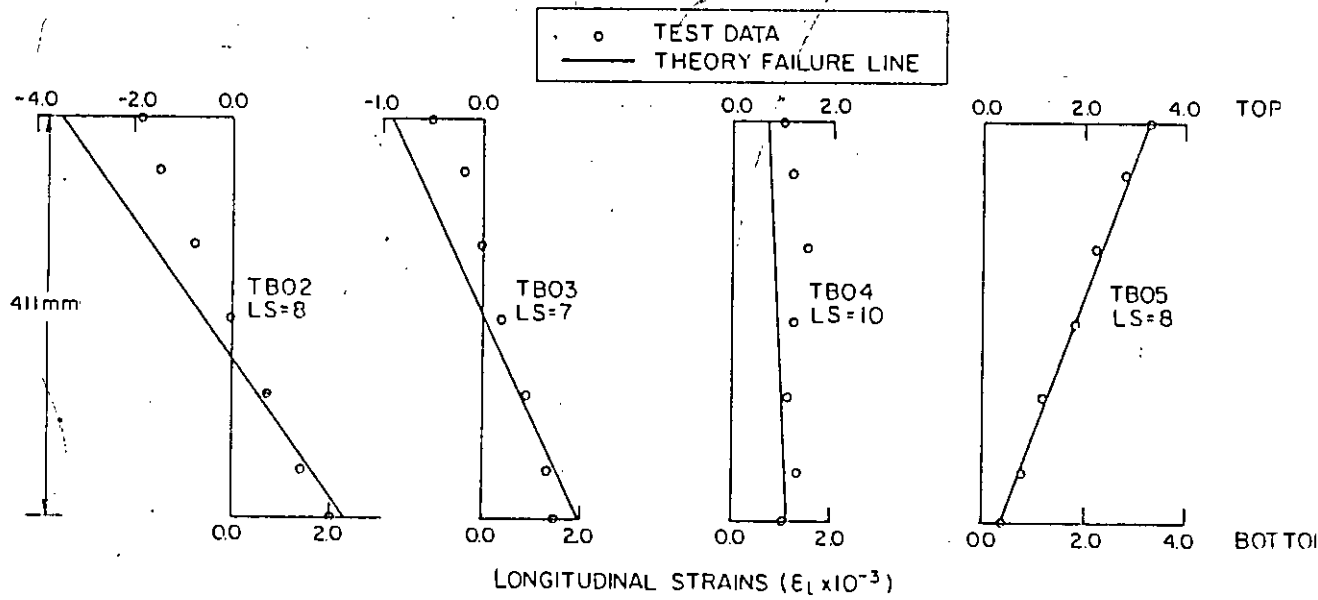


FIG. 5.6 PREDICTED & MEASURED LONGITUDINAL STRAIN PROFILES FOR TBO SERIES

5.4.2 Variation of Transverse Strains

Fig. 5.7 shows the measured average transverse strains for each face of the section for the TBO series specimens at approximately half capacity loading level (Fig. 5.7(a)) and at approximately capacity loading level (Fig. 5.7(b)). Similar plots are shown in Appendix E.4 for the TBU and TBS series beams.

These figures indicate that there is some significant variation in the transverse strains in the side faces if there is longitudinal strain variation in the faces. Fig. 5.8 shows a plot of the model predictions of the transverse strains at load levels corresponding to those of the average measured transverse strains which are also plotted for comparison. It is seen that the model predicts greater transverse strains than those measured in Fig. 5.8(a). However at approximately capacity loading level (Fig. 5.8(b)) when the condition of the test specimens closely approximates the conditions assumed for the model prediction the agreement between the test values and the predicted values is very good.

5.4.3 Variation of Diagonal Strains

Fig. 5.9 shows the average principal compression strains evaluated from the test data and plotted for each face of the section for the TBO series at approximately half capacity loading level (Fig. 5.9(a)) and at approximately capacity load-

Fig. 5.7(a)

TRANSVERSE STRAINS
AT APPROX. HALF CAPACITY LOADS
FOR TBO SERIES

LEGEND		
SYMBOL	BEAM	LOAD STAGE
02	TB02	4
03	TB03	4
04	TB04	5
05	TB05	5

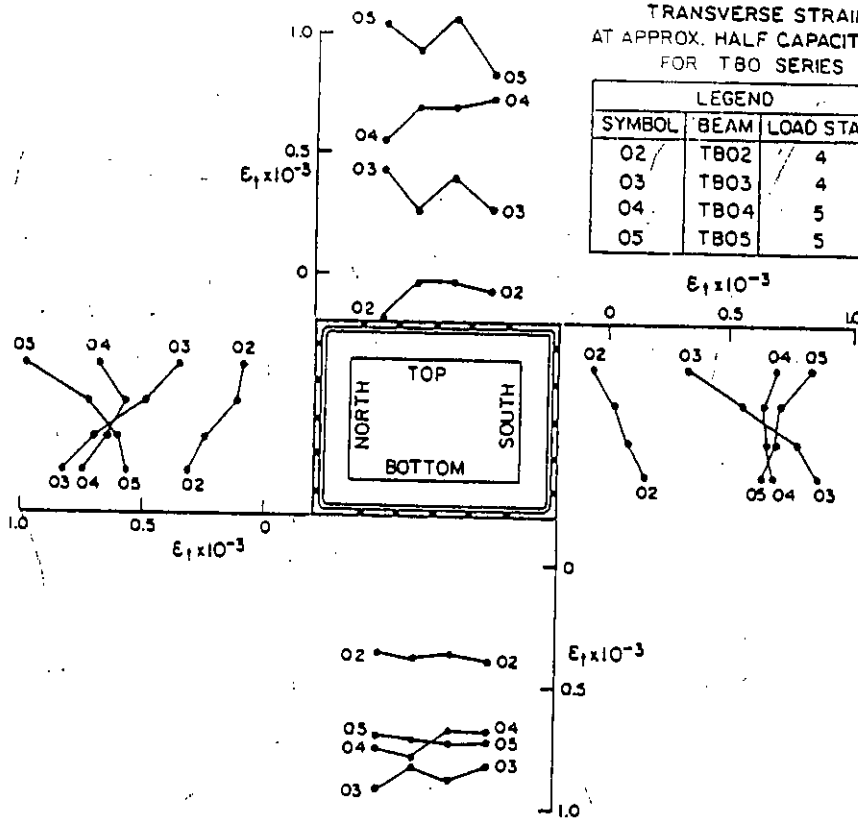
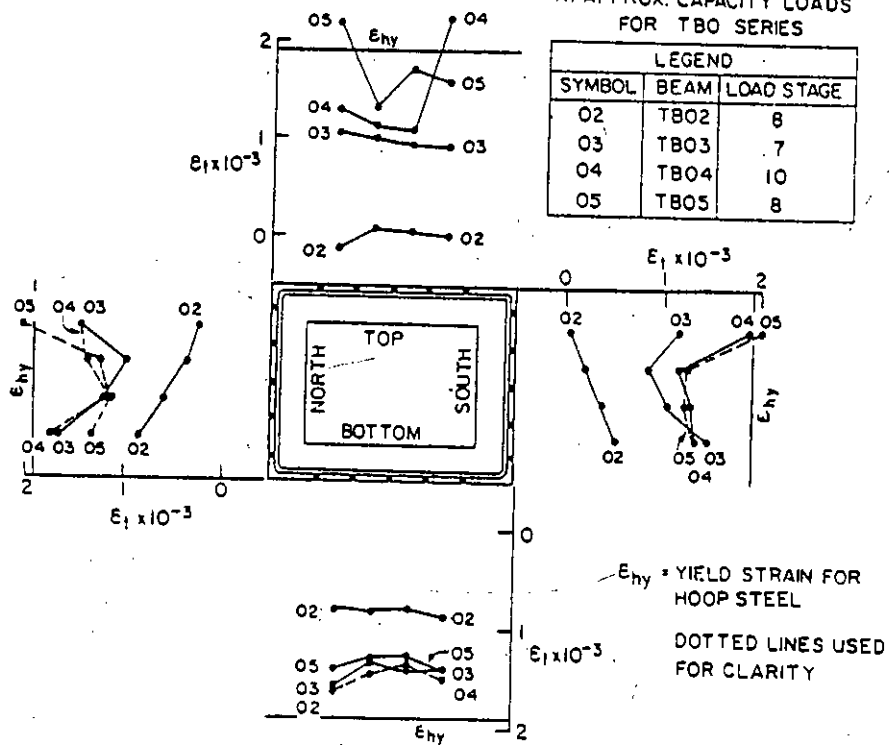
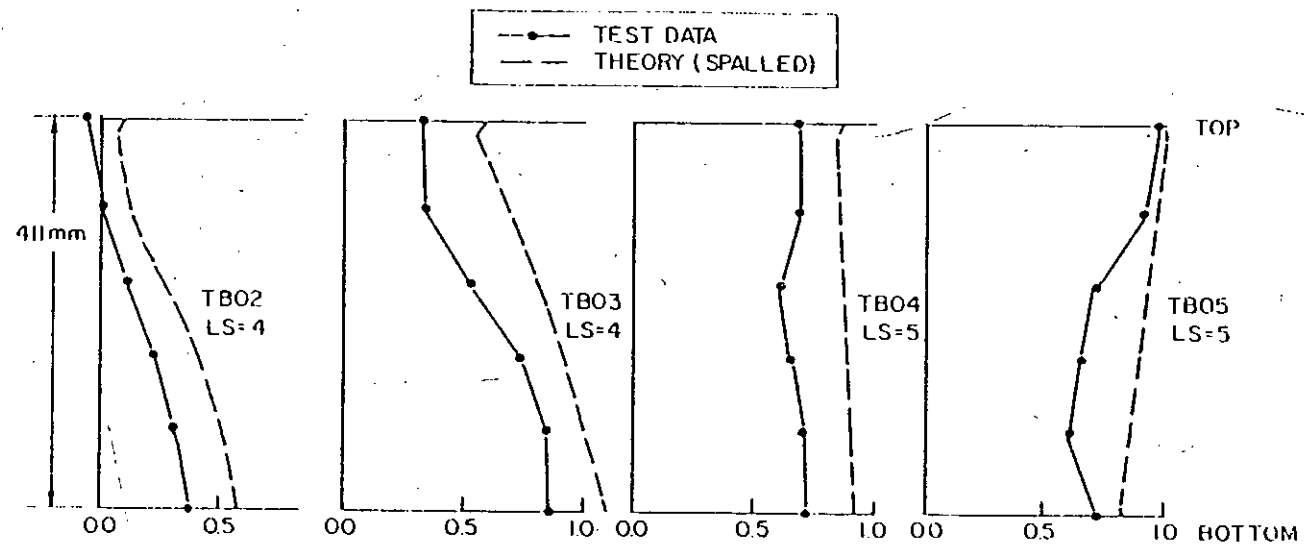


Fig. 5.7(b)

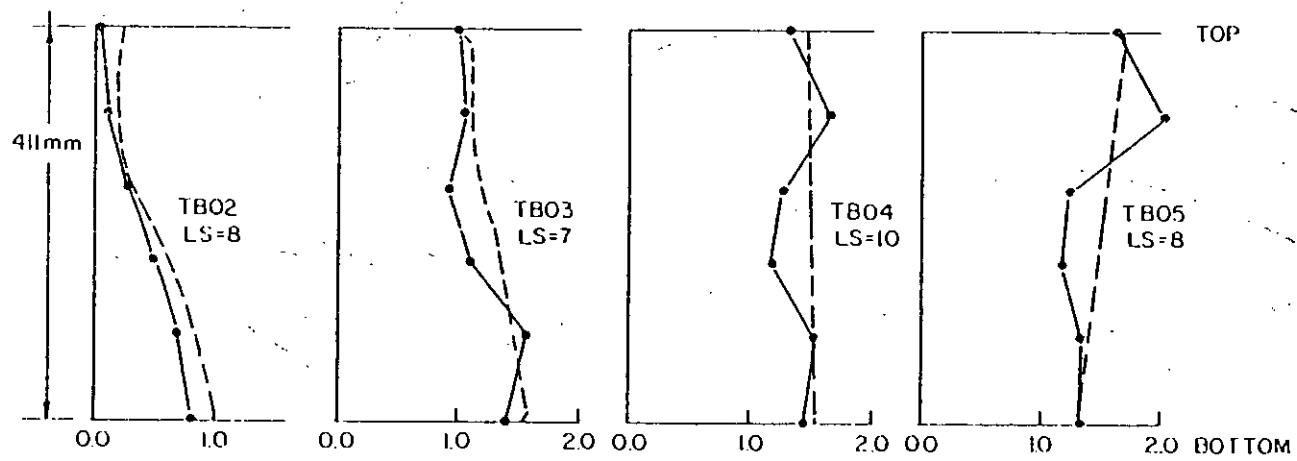
TRANSVERSE STRAINS
AT APPROX. CAPACITY LOADS
FOR TBO SERIES

LEGEND		
SYMBOL	BEAM	LOAD STAGE
02	TB02	8
03	TB03	7
04	TB04	10
05	TB05	8





(a) TRANSVERSE STRAINS ($\epsilon_t \times 10^{-3}$) AT $\approx$ HALF CAPACITY LOAD LEVEL



(b) TRANSVERSE STRAINS ($\epsilon_t \times 10^{-3}$) AT CAPACITY LOAD LEVELS

FIG. 5.8 TRANSVERSE STRAIN PROFILES FOR TBO SERIES

Fig. 5.9(a)
PRINCIPLE DIAGONAL STRAINS
AT APPROX. HALF CAPACITY LOADS
FOR TBO SERIES

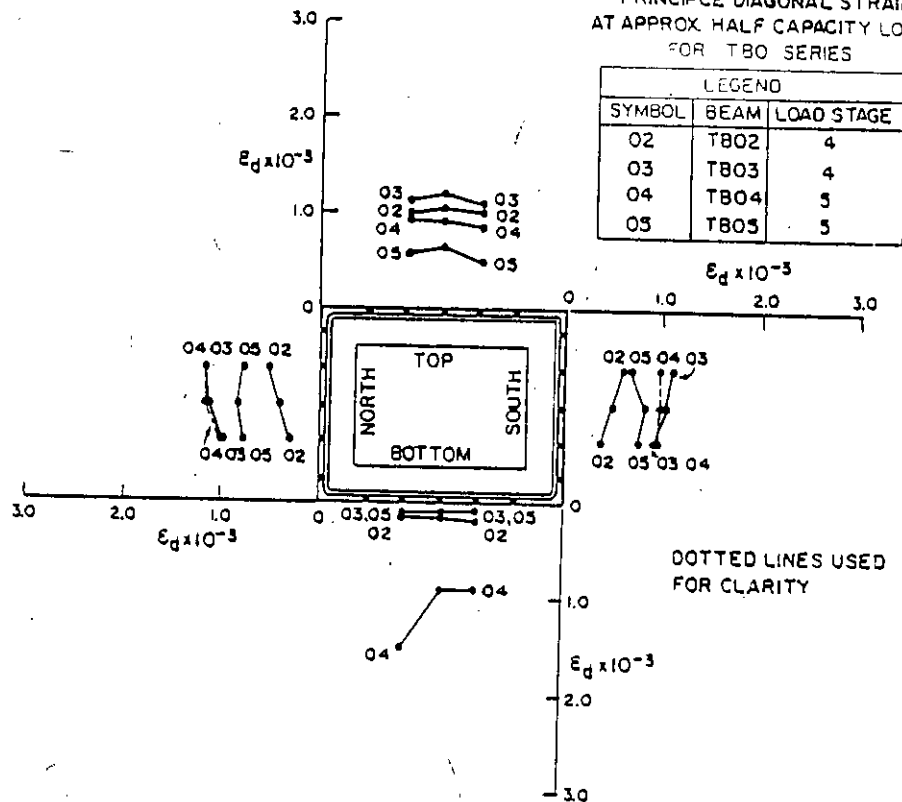
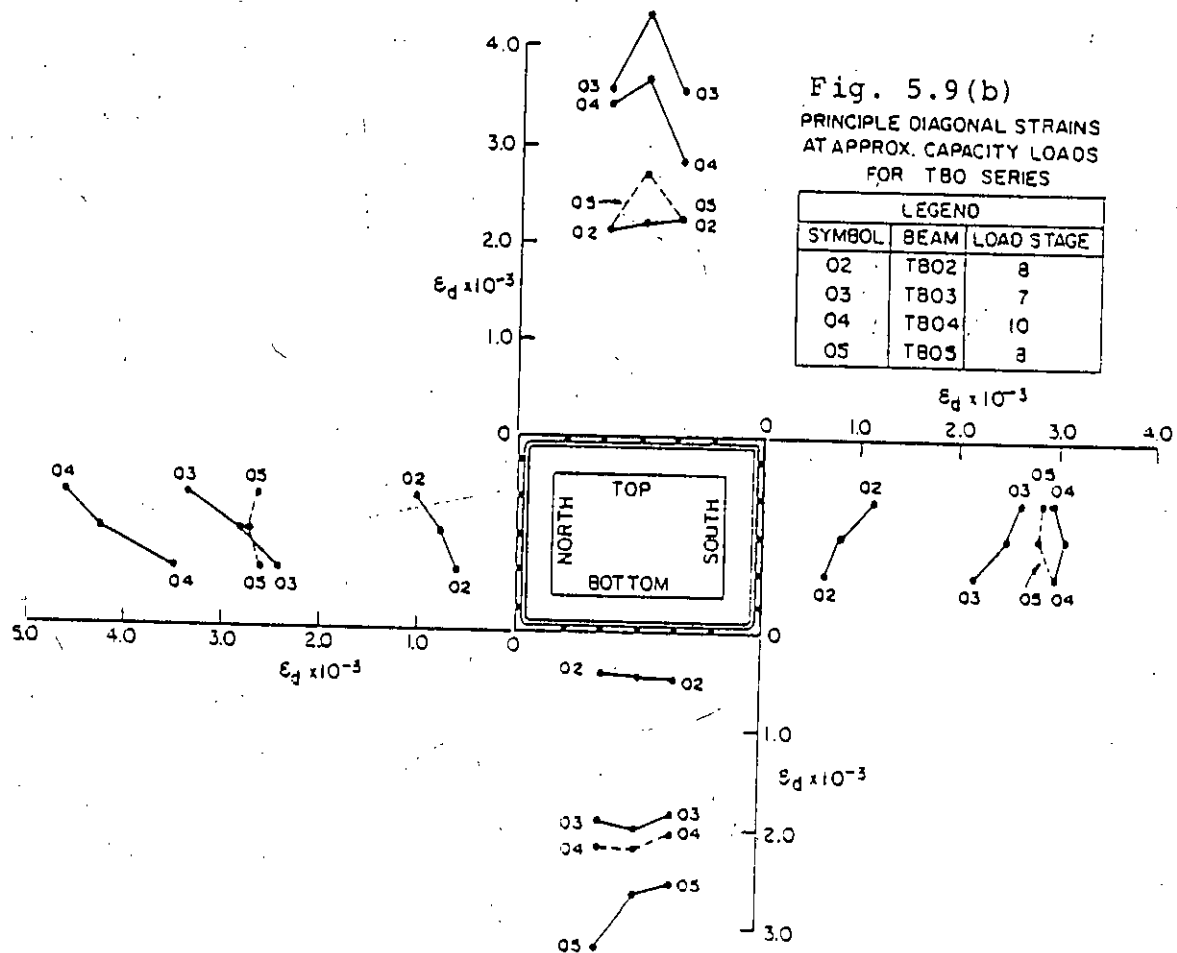


Fig. 5.9(b)
PRINCIPLE DIAGONAL STRAINS
AT APPROX. CAPACITY LOADS
FOR TBO SERIES



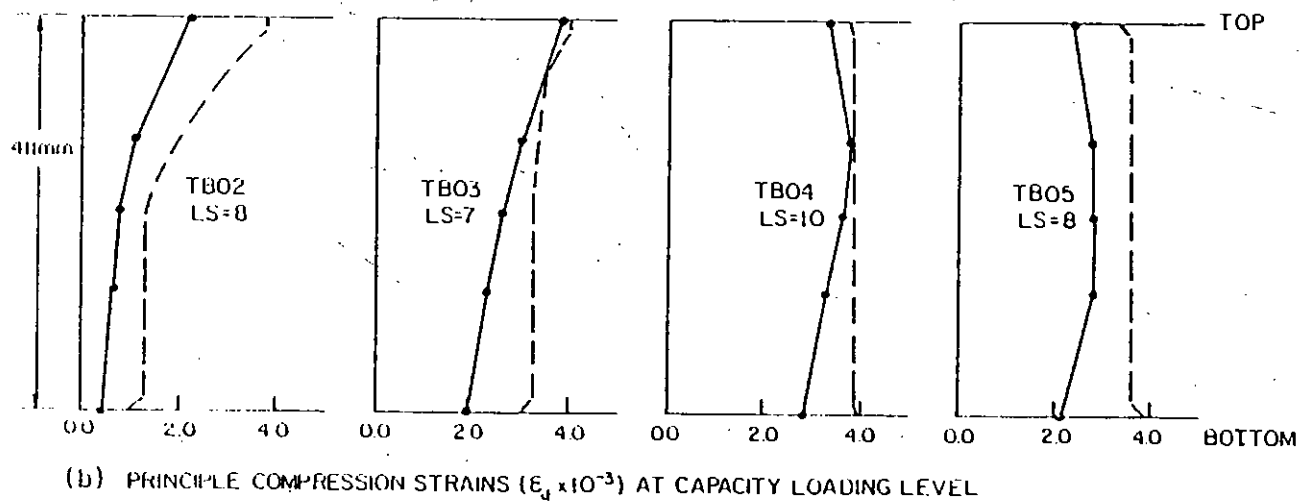
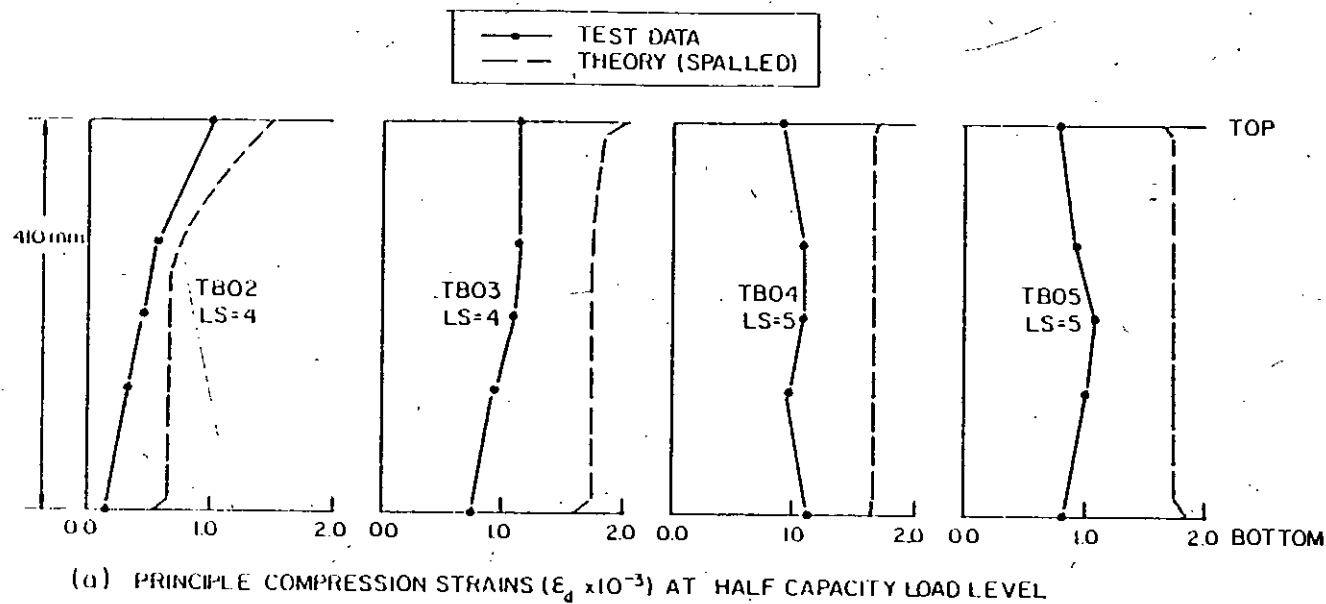


FIG. 5.10 PRINCIPAL COMPRESSION STRAIN PROFILES FOR TBO SERIES

level (Fig. 5.9(b)). Similar plots are shown in Appendix E.4 for the TBU and TBS series specimens.

These strains were evaluated from strains measured over a grid of targets making 6" (152 mm) squares. Since to evaluate the principal compression strain from a given square needed not only the measurement of the longitudinal and transverse strains but also the diagonal strains (see Appendix E.2) it was only feasible for the available test time to use the 6" squares with the mechanical measuring gauges. Smaller squares would enable a more realistic profile of the principal compression strain to be obtained for any given face, however, considerably more time would be required per load stage.

Fig. 5.10 shows a plot of the principal compression strains predicted by the model at load levels corresponding to those of the average measured principal compression strains which are also plotted for comparison. Again we notice in Fig. 5.10 (a) that the model predicts greater compression strains than those measured. However at approximately capacity loading level (Fig. 5.10(b)) the agreement between the predicted and measured is much better. With reference to Fig. 5.10(b), it should be noted that the measured principal compression strains were for a load stage close to capacity loading whereas the model prediction is for capacity loading condition. Recognizing that close to capacity loading the principal compression strains increase rapidly with little gain in load carrying capacity (see in Fig. 5.5), it is

concluded that agreement between the test data and the theory as seen in Fig. 5.10(b) is very good.

5.4.4 Variation of Angles of Principal Diagonal Compression and Crack Patterns

In Fig. 5.11 are plotted the average angles of principal compression evaluated from the strains of the TBO series beams for the load stages indicated. Appendix E.2 shows how these angles were evaluated from the measured strains. Also shown in Fig. 5.11 is the model predictions for the angles for the specimens at the corresponding load levels.

These plots indicate that the model predicts the angles of principal compression well. Moreover, both test data and model predictions indicate that principal angles vary little for a wide range of loading levels particularly for those regions of the beam where the longitudinal strains are tensile. The principal compression angles vary relatively rapidly down the depth of the section when the varying longitudinal strains are compressive and particularly for large curvatures. However when the longitudinal strains in a section are all tensile, the variation is relatively slow even for large curvatures.

Fig. 5.12 shows crack patterns for the beams in the TBO series for the same load stages used in the plotting of the principal compression angles in Fig. 5.11. Although the

inclination of cracks at any load stage do not necessarily indicate the exact direction of principal compression the crack directions and the pattern of variation of their angles of inclination give an indication of the change in the variation of the principal compression directions. For example the pattern of the cracks in the North face of TBO2 is seen to be quite different from that for TBO5 for the same face as shown in Fig. 5.12. The trend of the variation of the crack patterns as the torsion to moment ratio increases from 0.0 for TBO1 to over 4.0 for TBO5 is in good agreement with the trend of variation of the predicted angles of principal compression shown in Fig. 5.11.

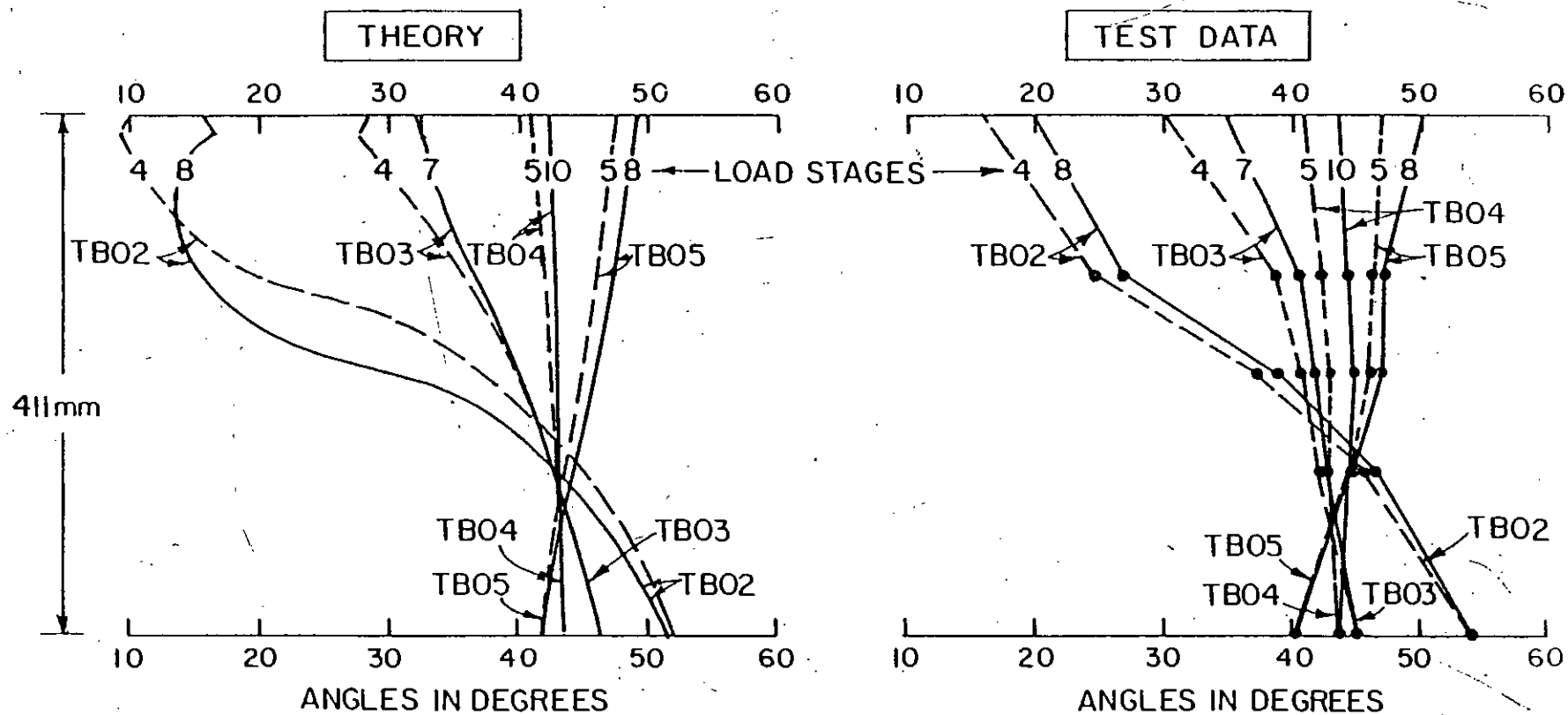


FIG. 5.11 VARIATION OF PRINCIPAL COMPRESSION ANGLES FOR TBO SERIES

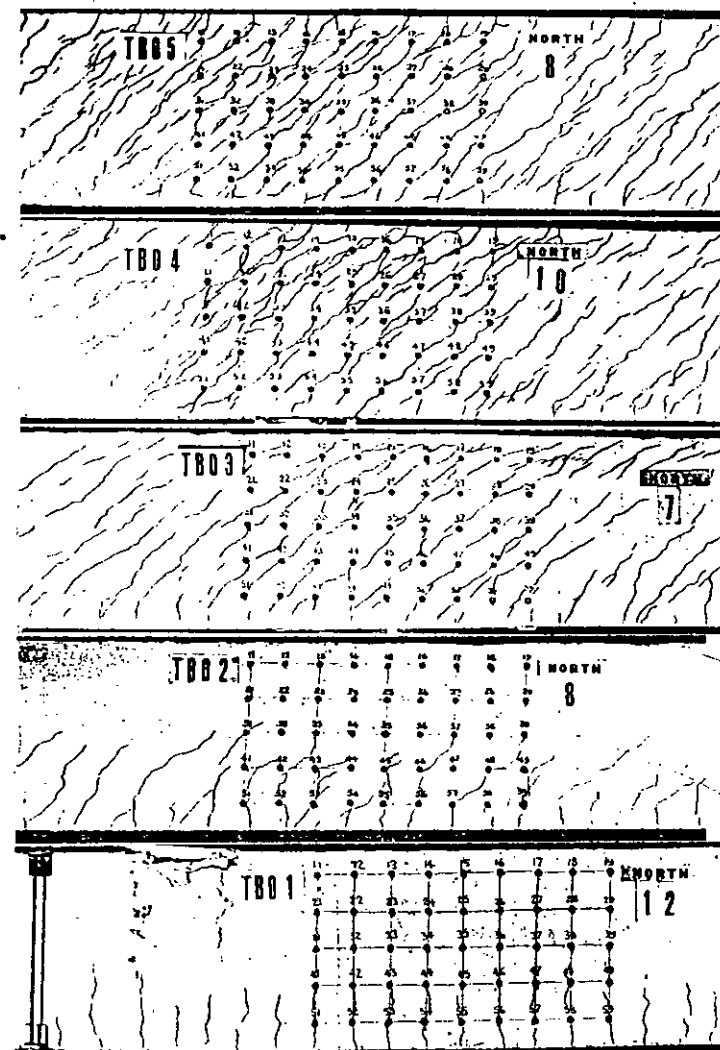
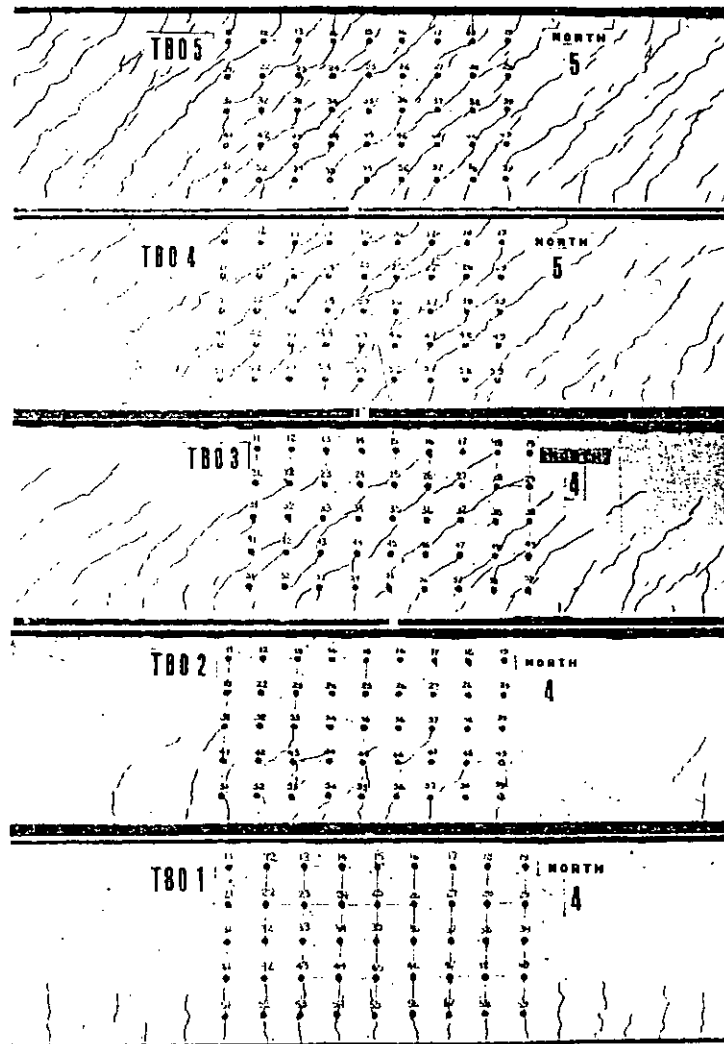


FIG. 5.12 CRACK PATTERNS FOR TBO SERIES

5.5 PREDICTION OF CAPACITY LOADS

5.5.1 Capacity Loads for Heavily Reinforced Beams

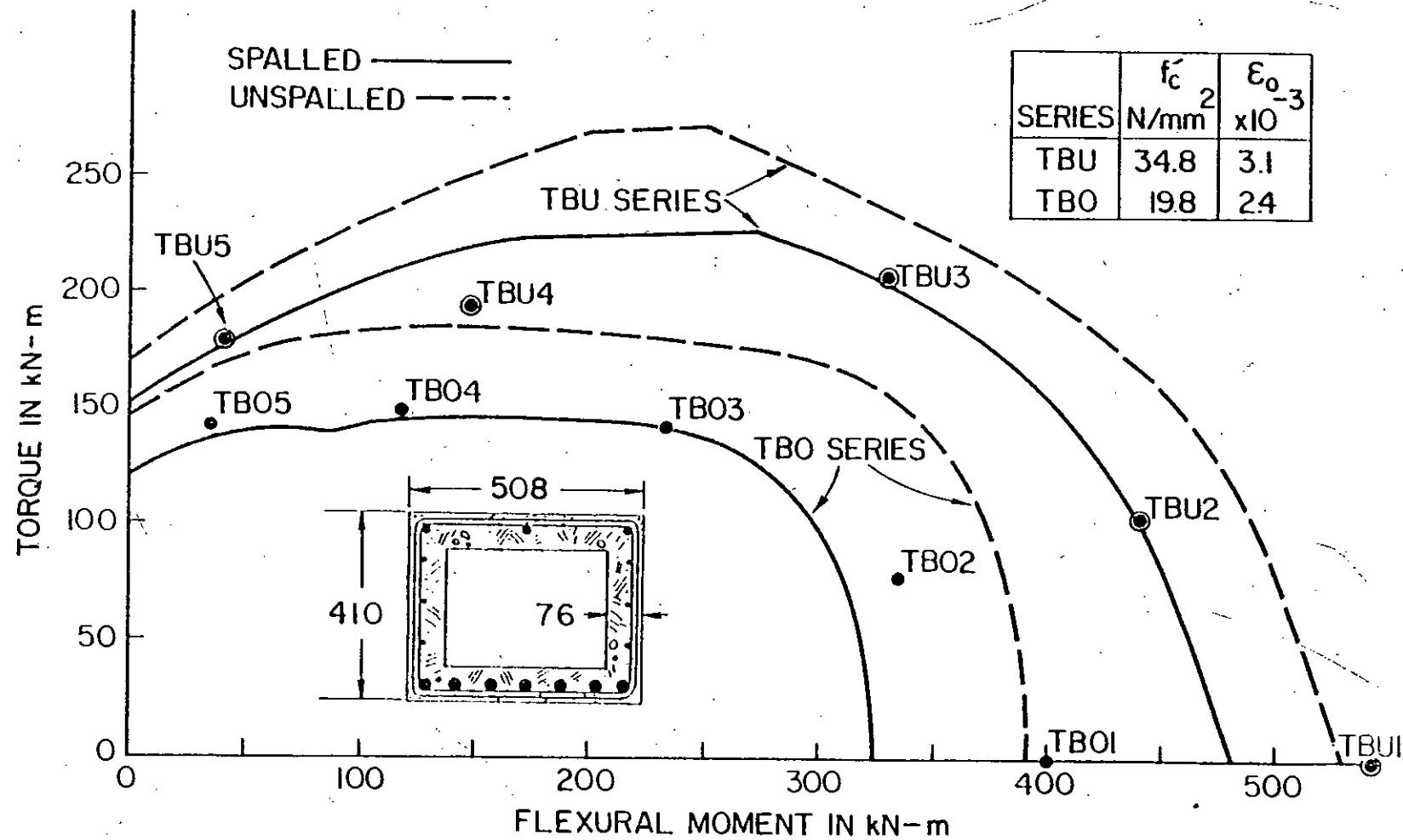
The TBO series specimens were designed such that under pure flexure and for the loading ratio causing uniform beam elongation, the capacity loads would be achieved before yielding of the reinforcing steel. Thus the TBO series section was overreinforced. The TBU series specimens were designed such that, at capacity loads, some of the reinforcing steel would be yielded. Thus they were partially overreinforced. In any case, both these series of specimens were relatively heavily reinforced in comparison to the content of reinforcement used in sections normally used in practice.

Fig. 5.13 shows the theoretical predictions of the failure interaction curves for the TBO and TBU series for both the spalled and the unspalled section dimensions. The capacity loads of the specimens obtained from the tests are also plotted in Fig. 5.13.

For the pure flexure case it is evident that the model predictions for the unspalled section conditions agree well with the test results whereas the spalled predictions underestimate the beam flexural capacities. However, the unspalled prediction quickly becomes unconservative with the application of torsion and the capacity loads are best predicted by the spalled prediction for the test specimen failing

at a ratio of T_u/M_u greater than 0.6.

Specimen TBU4 failed prematurely because of the problem in casting mentioned in Section 4.2.1. The interaction prediction is based on a top face spalled thickness of 64 mm whereas TBU4 had a local minimum top wall thickness of 39 mm. It was in this local region where failure suddenly occurred. Specimen TBU2 suffered from the same problem but not as severely. TBU2 had a minimum wall thickness of 44 mm compared to the 64 mm used in the spalled prediction. Fig. 5.13 shows that the model predicts well the capacity loads for the heavily reinforced specimens for the widely varying loading ratios and for the concrete strengths shown.



FLEXURE - TORSION INTERACTION FOR TBU AND TBO SERIES

FIG. 5.13 FAILURE CURVES FOR HEAVILY REINFORCED BEAMS UNDER FLEXURE AND TORSION

5.5.2 Capacity Loads for Under-reinforced Beams

It has been demonstrated that the theoretical model predicts well the capacity loads for heavily reinforced specimens. The model is now used to predict the failure interaction lines for under-reinforced beams tested in combined torsion and bending in Zurich.² Fig. 5.14 shows the section used in the series and the average values of the properties of the materials used are indicated.

The predicted failure interaction curves are shown for the spalled and unspalled section dimensions and the failure loads for the specimens tested are plotted for comparison. As before, it is again noted that as the ratio, T_u/M_u , (at failure) increases, the capacity of the beams tends towards the spalled prediction value. The peak point in the interaction curve with the maximum attainable torsion capacity is characteristic of under-reinforced specimens which have an unsymmetrical longitudinal steel reinforcement pattern such as that shown for the section. Such a peak is not seen for similar over-reinforced specimens.

It should be noted that specimen TB4, unlike the other specimens indicated, was not hollow. It was a solid section with the same outside dimensions and reinforcement layout as for the section shown. The theoretical model indicates that this specimen will have the same load carrying capacity as a hollow one with a wall thickness of 80 mm

because the effective depths of the compression diagonals at failure are less than the provided wall thickness. As indicated in the failure interaction diagram the capacity of TB4 and the capacity of the other specimens are well predicted.

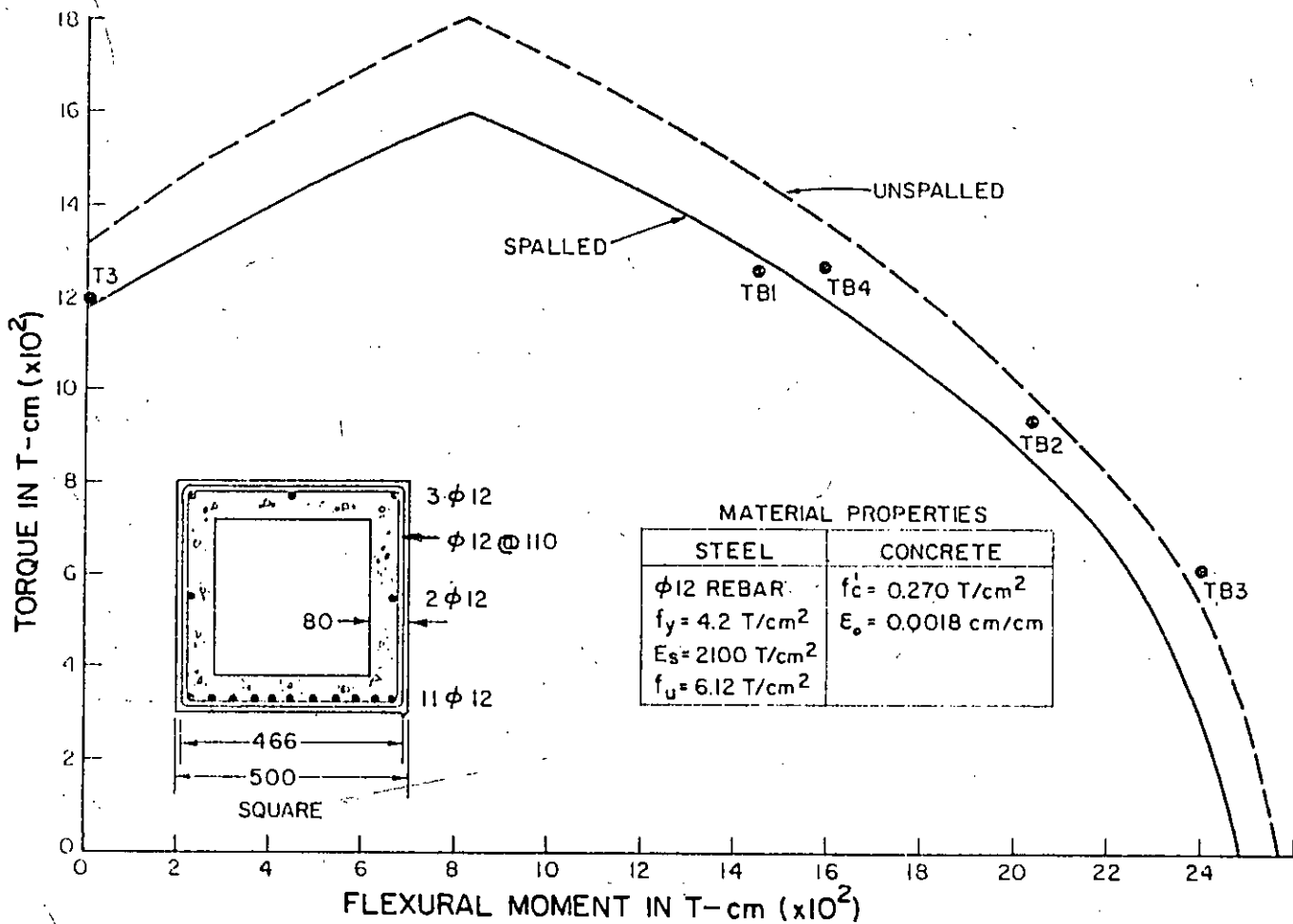


FIG. 5.14 FAILURE INTERACTION CURVES FOR UNDER-REINFORCED BEAM TESTED IN ZURICH UNDER COMBINED FLEXURE AND TORSION

5.5.3 Capacity Loads for Prestressed Beams

The theoretical model can also be applied to predict the complete response of prestressed beams loaded in flexure and torsion. In the case of the presence of prestressed steel in a section the resultant force in the reinforcing steel in the longitudinal direction (F_{sl}) is evaluated for a given longitudinal strain profile as

$$F_{sl} = \sum_i f_{li} A_{li} + \sum_j f_{pj} A_{pj} \dots \dots \dots (5.1)$$

where f_{pj} = stress in prestressing steel area, A_{pj} , at position 'j' with a strain of ϵ_{pj}

$$\epsilon_{pj} = \epsilon_{lj} + \Delta\epsilon_{pj} \dots \dots \dots (5.2)$$

ϵ_{lj} = longitudinal strain in the concrete at the position 'i' obtained from given strain profile

$\Delta\epsilon_{pj}$ = initial prestrain in the prestressed steel area A_{pj} .

If given the initial prestress f_{pI} , then the prestress $\Delta\epsilon_p$ for a symmetrically prestressed section can be evaluated as:

$$\Delta\epsilon_p = f_{pI} \left[\frac{1}{E_s} + \frac{A_p}{A_{Tl} E_s + A_c E_c} \right] \dots \dots \dots (5.3)$$

With these expressions the theoretical model was used to predict the failure interaction curve for a series of prestressed beams tested in torsion and bending in Toronto.³

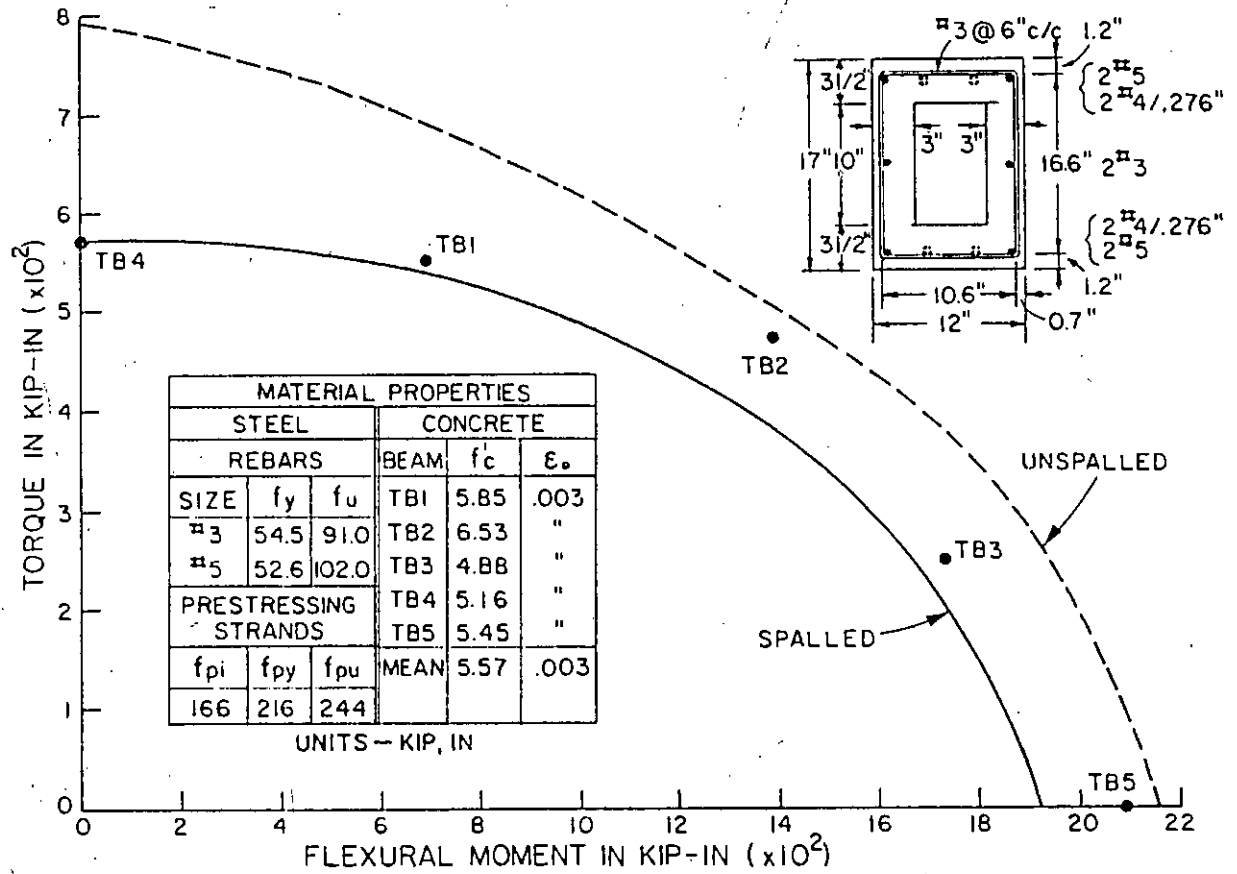


FIG. 5.15 FAILURE INTERACTION CURVES FOR PRESTRESSED BEAMS UNDER COMBINED FLEXURE AND TORSION TESTED IN TORONTO

The cross section details and the material properties of the specimens tested are shown in Fig. 5.15. The failure interaction lines predicted are again shown for the spalled and unspalled section dimensions. The failure loads of the beams are plotted and it is seen that the model predicts the capacities well.

It is of interest to note that for the pure torsion case the unspalled prediction overestimates the torsional capacity by about 39%. This is because of the large cover to hoop steel in the top and bottom faces. The spalled prediction for the pure torsion case is in excellent agreement with the test failure loads, thus confirming that the cover to hoop steel can be considered ineffective in carrying loads at failure for this case.

5.6 PREDICTION OF FAILURE MODE

As discussed in Section 3.5 the loading ratio which will lead to uniform stress in concrete in all faces at capacity loads will also induce uniform longitudinal strains in the section. Using the equations developed in Section 3.5 and for spalled section dimensions the amounts of steel reinforcement in the transverse and longitudinal directions for balanced failure have been evaluated and the results are summarized in Table 5.1.

TABLE 5.1

	f_c^* N/mm ²	ϵ_o $\times 10^{-3}$	f_{hy} N/mm ²	ϵ_{hy} $\times 10^{-3}$	f_{ly}^* N/mm ²	ϵ_{ly}^* $\times 10^{-3}$	t_{db} (mm) (Eq. 3.16)	t_{min} mm	ρ_{hb} (Eq. 3.19)	ρ_{lb} (Eq. 3.20)	ρ_h	ρ_l	Test T_u/M_u	Theory (T/M)**
TBO4	20.4	2.4	379	1.91	429	2.19	67.6	60.0	.0132	.0111	.0160	.0292	1.281	1.394
TBU4	34.8	3.1	379	1.91	443	2.26	73.1	39.0	.0244	.0198	.0160	.0296	1.328	1.248
TBS1	28.0	2.5	443	2.33	433	2.23	66.2	206.5	.0146	.0152	.0159	.0289	1.271	1.420
TBS2	32.9	2.5	443	2.33	433	2.23	66.2	206.5	.0172	.0179	.0159	.0289	1.277	1.391
TBS3	45.8	2.5	443	2.33	433	2.23	66.2	206.5	.0239	.0249	.0159	.0289	1.312	1.266
TBS4	15.5	2.0	443	2.33	433	2.23	66.2	206.5	.0074	.0077	.0159	.0268	1.157	1.411

t_{min} = Minimum concrete wall thickness in section

$\epsilon_{cu} = 1.5\epsilon_o$ used in evaluation of ρ_{hb} , ρ_{lb} and (T/M)**

$\rho_h = t_h/t_c$ = supplied hoop steel ratio

$\rho_l = t_l/t_c$ = supplied longitudinal steel ratio

f_{ly}^* = Equivalent yield stress for all longitudinal steel

ϵ_{ly}^* = Equivalent yield strain for all longitudinal steel

T_u/M_u = Failure Torsion to Moment ratio in test

(T/M)** = Ratio of loading for zero flexural curvature

As is evident from Table 5.1 all the specimens tested were over-reinforced with respect to the longitudinal steel content. With respect to hoop steel content, beams TBO4, TBS1 and TBS4 were over-reinforced whereas TBU4, TBS2 and TBS3 were under-reinforced. It is also noted that beams TBS1 and TBS2 had hoop steel content close to that of balanced failure.

The actual ratio of torque to flexural moment (T_u/M_u) at failure of the beams is also indicated in Table 5.1. Due to the presence of dead loads the overall T/M ratios in a test varied during the loading even though the increments in the loads were at the constant ratios, R, shown in Table 4.3. Shown beside the T_u/M_u ratios in Table 5.1 are the loading ratios determined using expressions given in Section 3.5 for uniform beam elongation and it is seen that these ratios agree closely with the test loading ratios. It was indeed observed from the continuously plotted midspan deflection which gave a measure of the average flexural curvature over the central 1.75 metres of the specimen that there was negligible curvature up to capacity loads.

5.6.1 Steel and Concrete Strains

Fig. 5.16 shows a plot of the longitudinal strains for the beams shown in Table 5.1 at the last load stage for which strains were measured before attainment of capacity

loads. The yield strain of the main longitudinal reinforcing bars is also plotted in Figure 5.16. The figure shows that for all the specimens, the average longitudinal strains at capacity loads were less than the steel yield strain. The strain profiles also indicate that the beam sections had almost uniform strains at the capacity loads. These observations are in good agreement with the model predictions.

Fig. 5.17 shows a plot of hoop strains for the specimens at the load stage closest to capacity loading. The uniformity of the hoop strains profiles is comparable to that of the corresponding longitudinal strain profiles in Fig. 5.16.

The theory predicts that for TBO4 and TBS4 the average transverse strains at capacity load will be less than the yield strain of the hoop steel. A close look at the transverse strains given in Appendix E.3 reveals that strains in excess of yield strain were measured locally. However the overall average transverse strains measured at close to capacity loads for beams TBO4 and TBS4 are indeed less than the yield strain of the hoop steel used. This is in agreement with the model prediction.

Fig. 5.18 shows the average principal compression strains evaluated from the test data for the specimens listed in Table 5.1. The premature failure of TBU4 is indicated by the rather low principal compression strains at capacity loading. The average principal compression strains for the top faces of TBS1, TBS2, TBS3 and TBO4 are close to

the $1.5\epsilon_0$ values used in the theoretical prediction for failure. However, TBS4 shows principal compression strain in excess of $3\epsilon_0$ in the top face at capacity loading. Large principal compression strains were also indicated for TBS2 and TBS3 for decreasing loads after attainment of capacity loading.

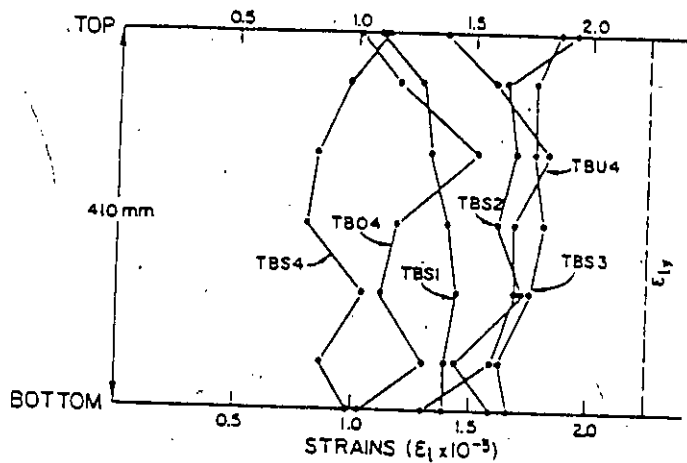


FIG. 5.16

LONGITUDINAL STRAINS
AT $\approx$ CAPACITY LOADS

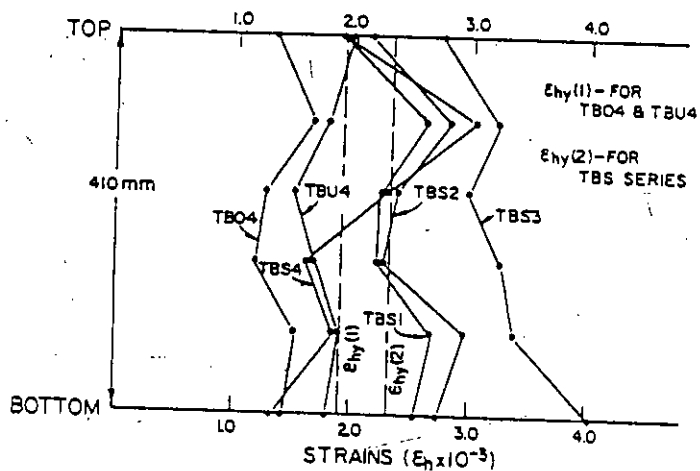


FIG. 5.17

TRANSVERSE STRAINS
AT $\approx$ CAPACITY LOADS

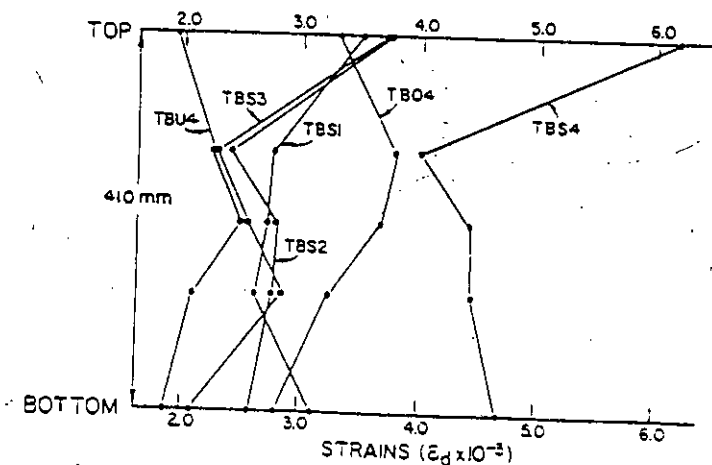


FIG. 5.18

PRINCIPAL COMPRESSION
STRAINS AT $\approx$
CAPACITY LOADS

5.6.2 Effect of Variation of Concrete Strength on Failure Mode

The TBS series was specifically designed to investigate the effect of varying concrete strength on the failure loads for a fixed loading ratio, R , of 1.524 (see Table 4.3). This loading ratio was chosen because at failure the specimen had nearly zero flexural curvature. For uniform beam elongation expressions for equilibrium and compatability requirements at failure can be derived as follows:

(a) Equilibrium requirements

$$q_u \tan \theta = t_h f_h \dots \dots \dots (5.4)$$

$$q_u \cot \theta = t_\ell f_\ell \dots \dots \dots (5.5)$$

$$q_u = \alpha \beta f'_c t_d \sin \theta \cos \theta \dots \dots \dots (5.6)$$

and (5.4) and (5.5) also give

$$q_u = \sqrt{t_h f_h t_\ell f_\ell} \dots \dots \dots (5.7)$$

(b) Geometric Requirements

$$\tan^2 \theta = \frac{\epsilon_\ell + \epsilon_{cu}}{\epsilon_h + \epsilon_{cu}} \dots \dots \dots (5.8)$$

$$\gamma = 2\sqrt{(\epsilon_\ell + \epsilon_{cu})(\epsilon_h + \epsilon_{cu})} \dots \dots \dots (5.9)$$

$$\psi_x = \frac{p}{2A} \gamma = \gamma / 2t_c \dots \dots \dots (5.10)$$

$$\phi_d = \psi_x \sin 2\theta = \epsilon_{cu} / t_d \dots \dots \dots (5.11)$$

From the above equations of equilibrium and compatability the following expressions are derived:

$$t_d = \frac{t_c \epsilon_{cu} (\epsilon_l + \epsilon_h + 2\epsilon_{cu})}{2(\epsilon_l + \epsilon_{cu})(\epsilon_h + \epsilon_{cu})} \dots\dots\dots (5.12)$$

$$q_u = \frac{\alpha\beta}{2} t_c f'_c \frac{\epsilon_{cu}}{\sqrt{(\epsilon_l + \epsilon_{cu})(\epsilon_h + \epsilon_{cu})}} \dots\dots\dots (5.13)$$

$$\epsilon_h = \frac{\alpha\beta}{2} \left(\frac{t_c f'_c}{t_h f'_h} - 1 \right) \epsilon_{cu} \dots\dots\dots (5.14)$$

$$\epsilon_l = \frac{\alpha\beta}{2} \left(\frac{t_c f'_c}{t_l f'_l} - 1 \right) \epsilon_{cu} \dots\dots\dots (5.15)$$

If the parabolic stress-strain relationships is used for concrete then:

$$\alpha\beta = \Omega_u - \Omega_u^2/3$$

where $\Omega_u = \epsilon_{cu}/\epsilon_o$

Expressions developed in section 3.5 enable the determination in advance of whether hoop steel or longitudinal steel will yield at capacity loading for a given section and for the given material properties. Whichever is the case, equations (5.14) and (5.15) together with the given stress-strain relationships for the reinforcing steel can be used to evaluate the strains ϵ_h and ϵ_l . It should be noted that the longitudinal strain ϵ_l will be evaluated using an iterative procedure since $t_l = A_{Tl}/p_o$ requires knowledge of p_o which in turn requires knowledge of t_d . To initiate the

iteration procedure p_o can be guessed and the iterations are carried out to obtain convergence in p_o .

With the strains ϵ_ℓ , ϵ_h and ϵ_{cu} , the equations given above can be used to obtain θ , γ , ψ_x , f_h , f_ℓ and q_h . The area A_o , enclosed by the path of shear flow can be evaluated for the spalled section dimensions, and hence the torque $T_{up} = 2q_u A_o$ can be evaluated. The moment acting together with this torque, M_{up} , is evaluated as:

$$M_{up} = A_{T\ell} f_\ell j_d \dots \dots \dots (5.16)$$

where j_d is the moment lever arm.

The procedure outlined above was used to obtain failure torques and moments for uniform beam elongation of the section used in the TBS series for $\epsilon_{cu} = 0.00375$ and for a series of concrete strengths. The torques and moments obtained are plotted versus the concrete strengths in Figure 5.19. Also plotted in Fig. 5.19 are the failure torques and moments for the TBS series specimens which had different concrete strengths as shown. The lines joining the failure points predicted by the model for each individual specimen are also shown.

The theory indicates that if sufficiently low concrete strength is used the longitudinal steel and hoop steel will not yield at the attainment of capacity loads. this is considered an over-reinforced case, and beams TBS4 and TBS1 were therefore over-reinforced.

There is a range of concrete strengths over which the model predicts only hoop steel will yield for the section chosen. For the TBS series beams the hoop steel content was less than the longitudinal steel content ($\rho_h < \rho_\ell$). For such a case hoop steel will yield before the longitudinal steel does for a loading that produces zero flexural curvature. Note that if $\rho_\ell < \rho_h$ then the longitudinal steel will yield before the hoop steel yields in this range of concrete strengths. This can be considered to be a partially over-reinforced range and beams TBS2 and TBS3 were in this range.

If sufficiently high concrete strength is used both longitudinal steel and hoop steel will be yielded at the attainment of capacity loads. This constitutes the under-reinforced case.

It is emphasized here that the failure mode as discussed here pertains to a unique loading case producing zero flexural curvature and zero transverse curvature. Unsymmetry in the reinforcing pattern of longitudinal and/or hoop steel and differences in properties of steel in the various faces may lead to yield of some of the steel in some local area of the section for some loading ratio with some curvature. Note that the unique loading case of interest results in the maximum possible torque to be resisted by the section at failure.

Figure 5.19 indicates that the model predicts well the effect of the variation of concrete on the capacity loads for the specimens tested.

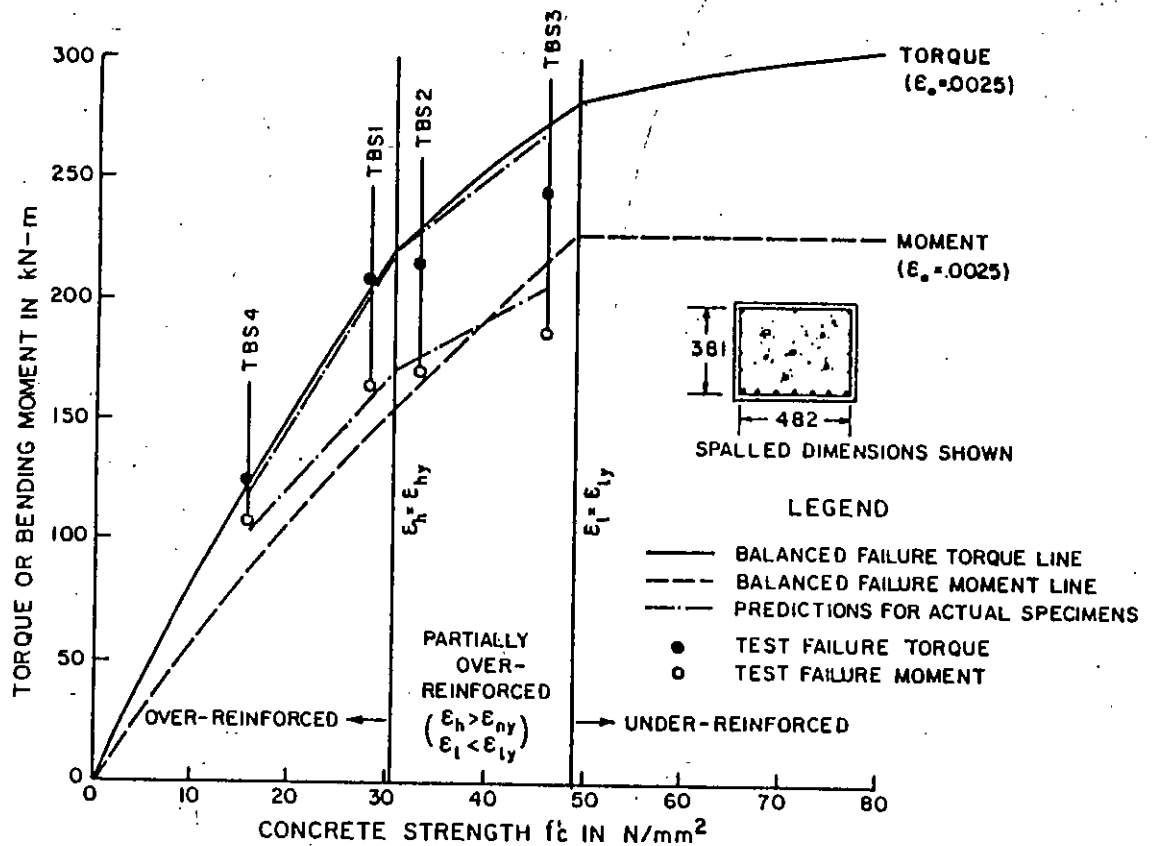


FIG. 5.19 EFFECT OF CONCRETE STRENGTH ON FAILURE MODE

5.6.3 Observed Behaviour of Beams Tested at Failure

Table 5.2 summarizes the experimental failure loads and the predicted failure loads for all the specimens tested. Except for TBU4 the agreement between theory and experiment is good.

Beam TBU4 failed prematurely due to a casting defect explained in Chapter 4. The beam failed suddenly

during a loading stage and the failure was located at the region with the least wall thickness in the top face.

Failure occurred after the yielding of hoop steel.

All the specimens with the hollow section failed in a brittle manner with a sudden drop in load. TBU2 and TBU3 failed suddenly as the main longitudinal steel in the bottom face reached yield strain. On the other hand the solid section of the TBS series showed remarkable ductility even for TBS4 which had the lowest concrete strength in the series. However, the ductility displayed was not at sustained loads but at decreasing lower load levels than the capacity loading. The confining effect of the closely spaced hoops on the solid concrete core of these specimen is largely responsible for the ductility displayed and the very large concrete strains measured. The hollow sections did not have such a concrete core for effective confinement. Moreover, the thickness of the spalled sections was smaller than the effective depths predicted by the model for the compression diagonals at failure. Thus the thin concrete walls were heavily stressed over their entire depth and at failure they crushed leading to the sudden drop in load observed for the hollow sections.

TBS2 and TBS3 failed at slightly lower loads than those predicted. Both of these specimens failed after the yield of hoop steel in all faces. As the capacity loads were approached there was a rapid increase in the measured diagonal

strains and the transverse strains, and this was accompanied by a relatively little increase in load carrying capacity. The predicted principal compression strains at the onset of the yield of hoop steel were greater than ϵ_0 for TBS2 and greater than $0.85\epsilon_0$ for TBS3. The large shearing strains at the high levels of compression in the concrete diagonals is believed to have led to the deterioration in the effective concrete strength leading to an early failure.

It should also be noted that for all the specimens tested the sections had a large amount of longitudinal steel in the bottom face. The tendency of this longitudinal steel was to weaken the concrete in the compression diagonals in the bottom face particularly for high levels of tensile strain in the bars such as was the case for TBS2 and TBS3 at close to their attainment of capacity loads. All TBS series specimens showed a marked upward deflection after the attainment of capacity loads. This upward deflection increased rapidly with further loading.

TABLE 5.2

BEAM	f'_c N/mm ²	EXPERIMENTAL		THEORETICAL*		FAILURE MODE**
		T_u kN-m	M_u kN-m	T_u kN-m	M_u kN-m	
TBO1	19.5	-	401	-	379	over-reinforced; brittle failure
TBO2	19.7	78	334	72	309	" "
TBO3	19.1	143	232	139	227	" "
TBO4	20.4	149	117	147	114	" "
TBO5	20.6	143	35	140	34	" "
TBU1	34.8	0	551	-	519	under-reinforced: rig not able to fail it
TBU2	34.8	104	439	104	440	Partially over-reinforced: brittle failure
TBU3	34.8	207	327	207	326	" " "
TBU4	34.8	195	147	221	164	" " "
TBU5	34.8	175	41	181	42	" " "
TBS1	28.0	209	164	202	159	over-reinforced: ductile failure
TBS2	32.9	216	169	229	176	Partially over-reinforced: ductile failure
TBS3	45.8	245	186	269	206	" " "
TBS4	15.5	125	108	119	104	over-reinforced-ductile failure

* Except for pure flexure specimen the theoretical prediction given are those based on spalled section dimensions

**The decision as to whether a beam was over-reinforced or under reinforced is based on the theoretical consideration given in Section 3.5.

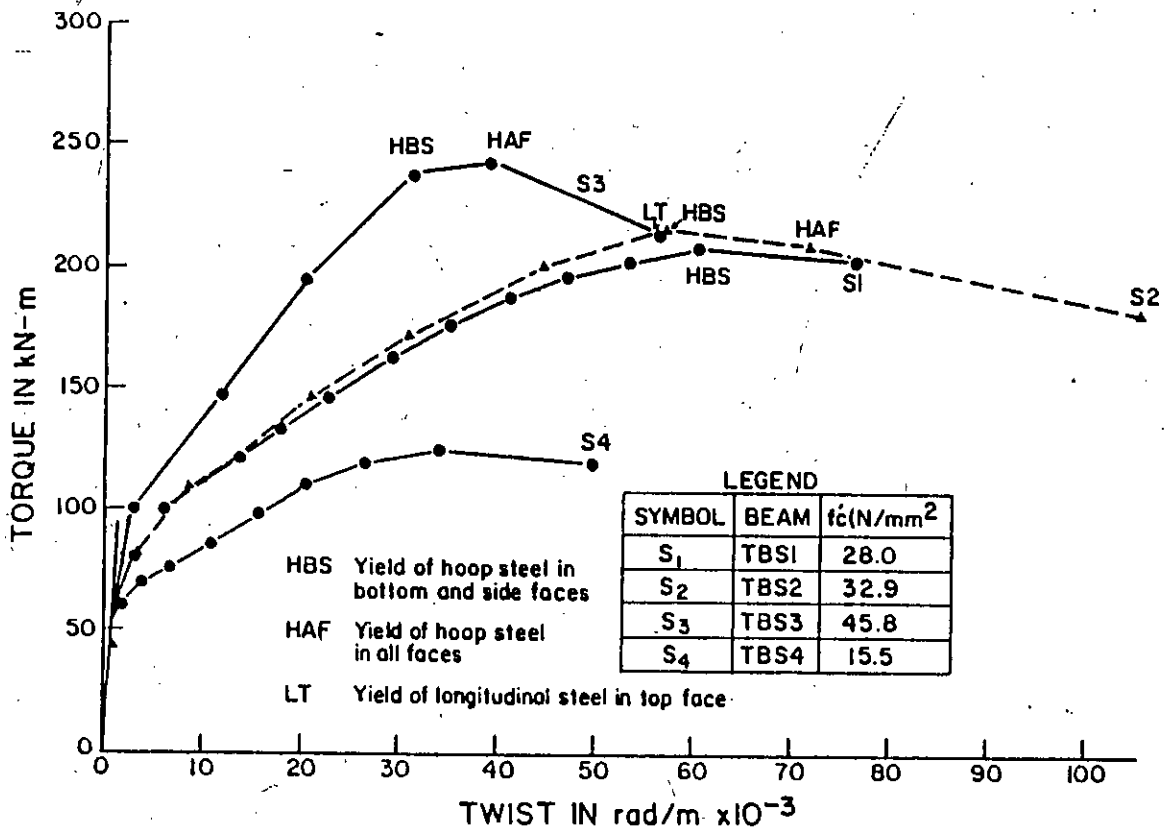
5.7 TORQUE-TWIST RELATIONSHIPS

5.7.1 Effect of Variation of Concrete Strength on Torque-Twist Curves

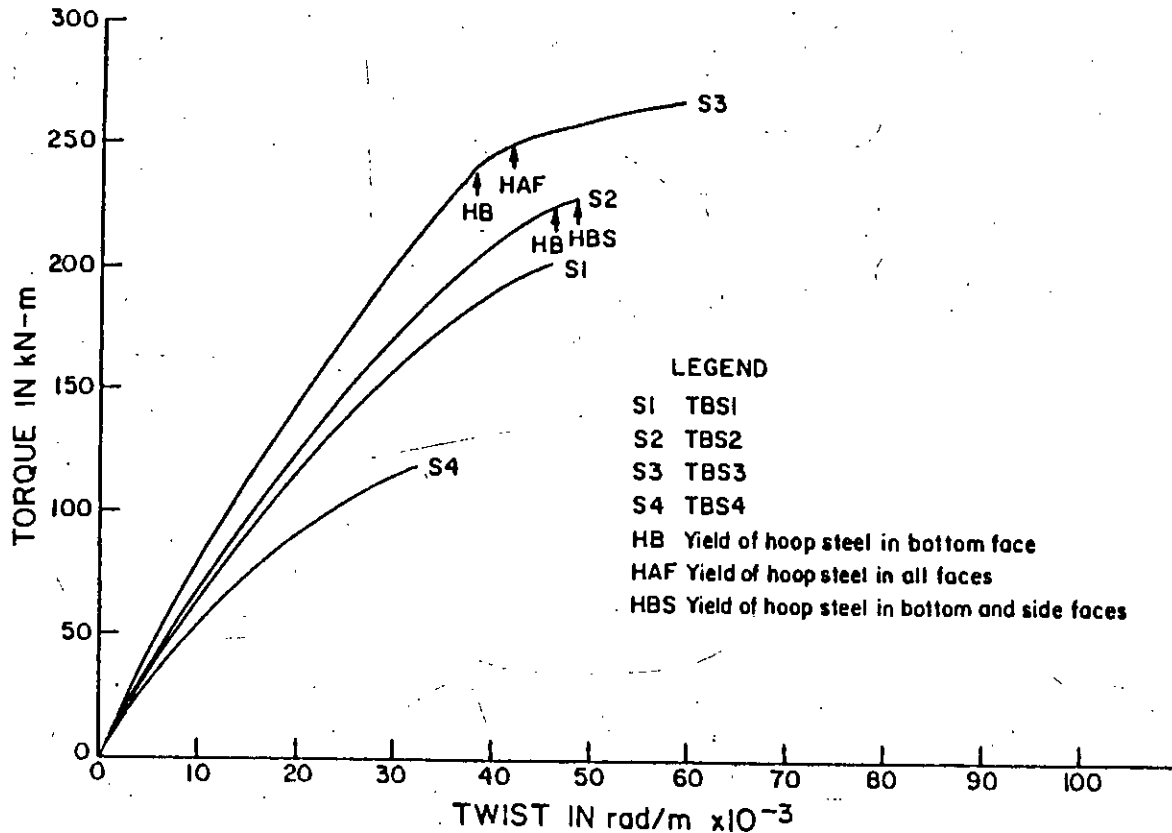
Fig. 5.20(a) shows the test torque-twist curves for the TBS series beams for which the varied parameter in the tests was the concrete strength. Fig. 5.20(b) shows the predicted torque-twist curves for the same specimens for spalled section dimensions. The predicted curves were obtained using the computer program in Appendix D in which the termination of analysis is done when the maximum principal compression strain was approximately $1.5\epsilon_0$ according to the criteria described in Appendix B-5.

The predicted plots in Fig. 5.20(b) indicate that the effect of increasing the concrete strength is to increase the capacity torque as well as the twist at capacity torque for a given section. The load levels at which the hoop steel yields are indicated.

The test curves in Fig. 5.20(a) confirm the increase in torsional capacity with the increase in concrete strength as predicted. In regard to the yielding of hoop steel, Table 5.3 shows good agreement between the model prediction and test results at failure.



(a) TEST TORQUE-TWIST CURVES FOR TBS SERIES



(b) PREDICTED TORQUE-TWIST CURVES FOR TBS SERIES

FIG. 5.20 TORQUE-TWIST CURVES FOR TBS SERIES

TABLE 5.3

YIELD OF HOOP STEEL AT FAILURE*

BEAM	THEORY		TEST
	SPALLED	UNSPALLED	
TBS1	-	HBS	HBS
TBS2	HBS	HBS	HBS
TBS3	HAF	HAF	HAF
TBS4	-	-	-

* see legend in Fig. 5.20

The test torque-twist curves for TBS1 and TBS2 indicate much greater twists at close to capacity loading than those predicted. Also TBS1 and TBS2 showed greater twist at failure than TBS3, contrary to theoretical prediction. However, it is of interest to note with reference to Table 4.1 that TBS1 and TBS2 were tested at 3 and 7 days respectively after casting whereas TBS3 and TBS4 were tested 45 and 31 days respectively after casting. Thus TBS1 and TBS2 were tested when the concrete was still curing and the strength gain per day was substantial. On the other hand TBS3 and TBS4 were tested when the curing process was practically complete and the strength gain per day was negligible. The creep effects on the young concrete of TBS1 and TBS2 due to the sustained high load levels as capacity loading was approached in the loading programme which lasted over 3.5 hours is believed to be largely responsible for the greater torsional

ductility for TBS1 and TBS2. Furthermore the effective stress-strain curve for the concrete for TBS1 and TBS2 with the effects of creep was no longer well represented by the assumed parabola used in the model analysis.

A comparatively rapid load drop was observed soon after the yield of hoop steel in all the faces, for TBS3. The greater capacity load predicted for TBS3 was not achieved because of the large shearing strains generated after the yielding of hoop steel at the high levels of principal compression in the concrete. This tended to reduce the effective concrete strength to much lower values than those obtained from the standard compression cylinder test.

5.7.2 Effect of the Variation of Loading Ratio on Torque-Twist Curves

Fig. 5.21 and Fig. 5.22 show plots of the test and predicted torque-twist curves for the TBO series beams and TBU series beams, respectively. For low torque levels these plots indicate, as expected, that the measured twist values are less than the corresponding predicted twist values. However, as capacity torque is approached there is an increase in the measured twists tending to the values predicted for the spalled dimensions of the sections especially for those specimens when diagonal cracks formed in all faces. The predicted torque-twist curves for the pure torsion cases are also shown in Fig. 5.21 and Fig. 5.22 for comparison.

These plots show that as the torque to moment ratio, R , in the loading of a beam is increased from zero to the ratio causing uniform beam elongation, the torque capacity is steadily increased to its maximum at the balanced failure point. A further increase in the torque to moment ratio results in a decrease in the torque capacity. With regard to the twist at failure, the effect of increasing R is always to increase the twist.

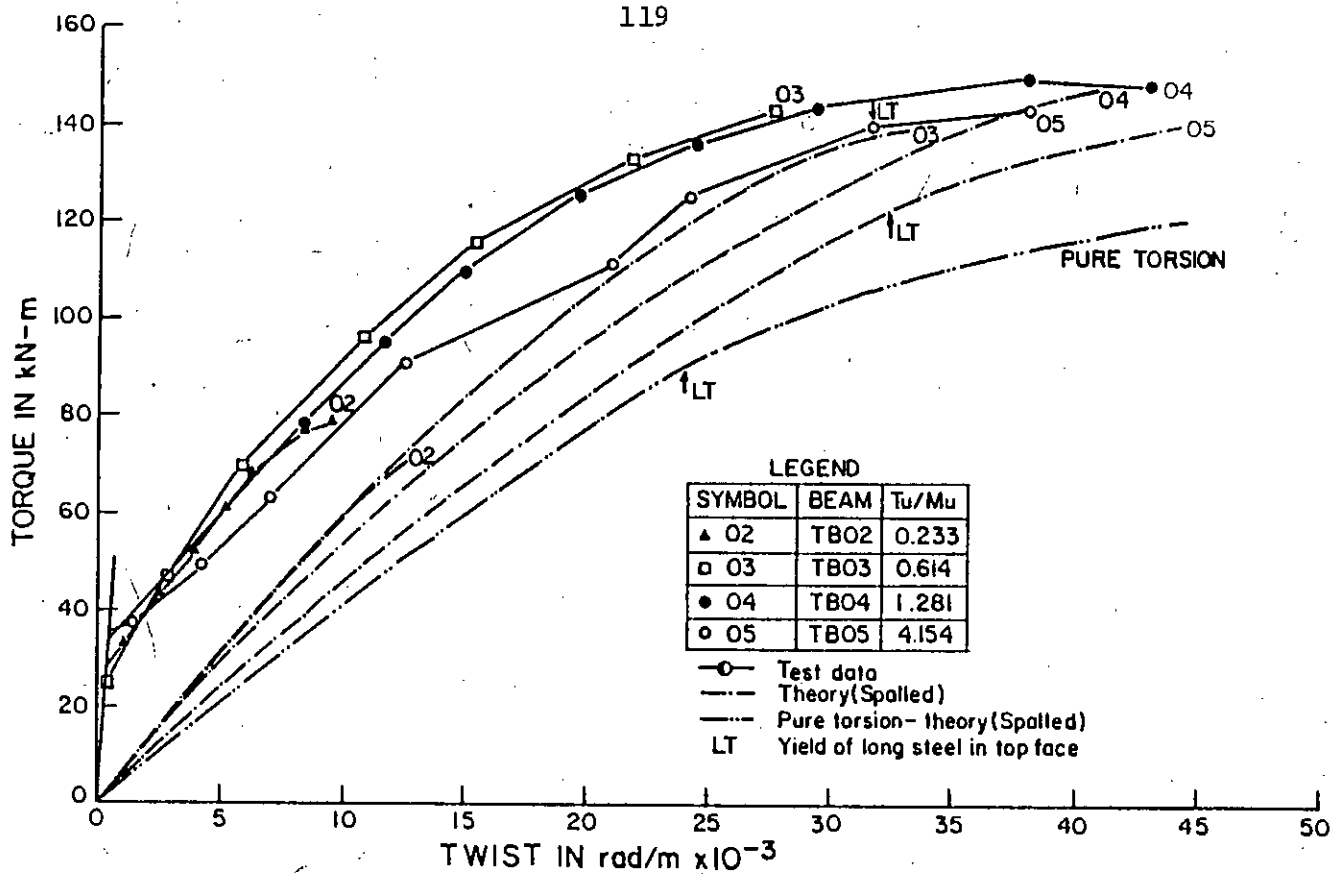


FIG. 5.21 TORQUE-TWIST CURVES FOR TBO-SERIES

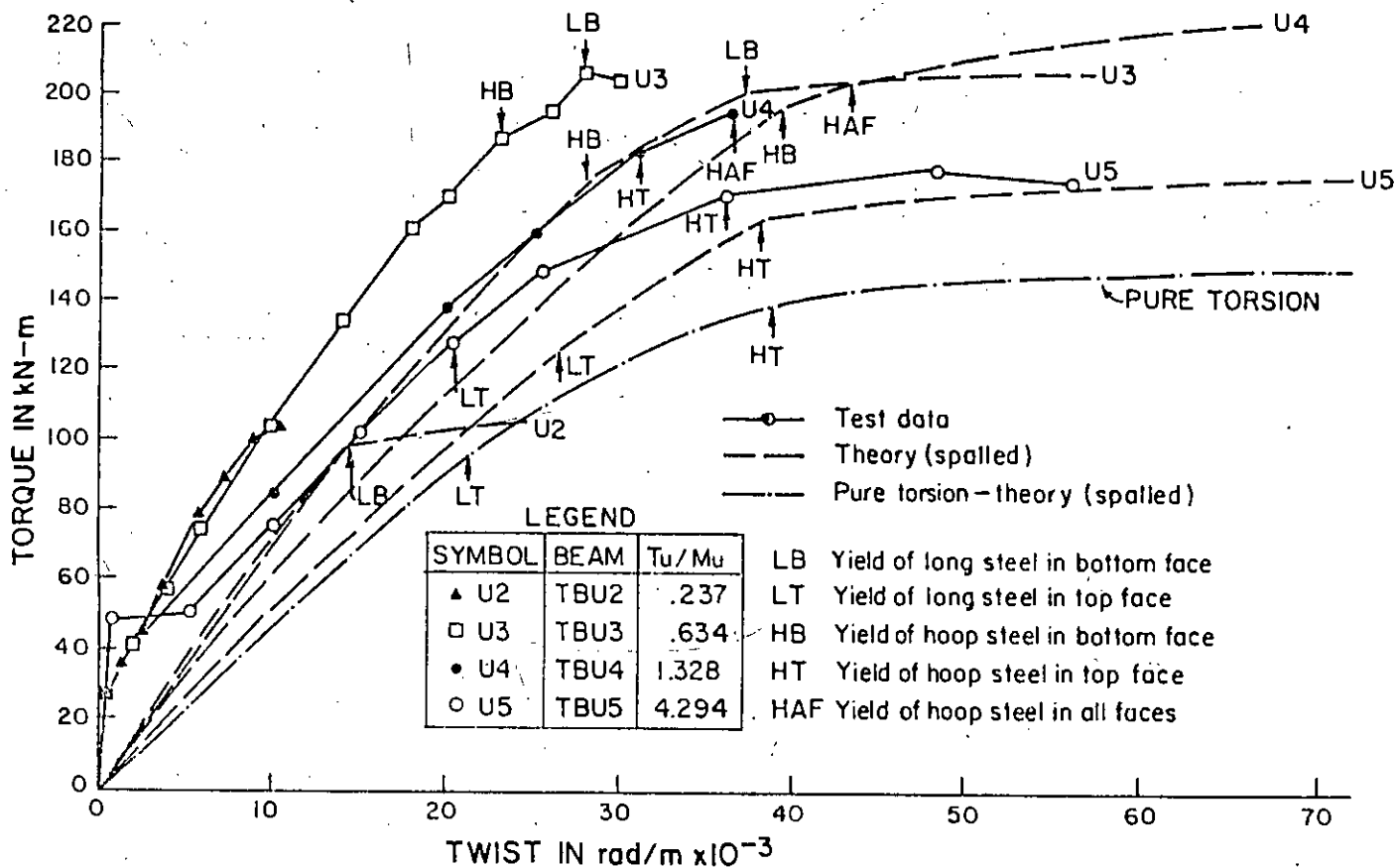


FIG. 5.22 TORQUE-TWIST CURVES FOR TBU-SERIES

5.8 MOMENT-CURVATURE RELATIONS

5.8.1 Effect of the Variation of Loading Ratio on Moment-Curvature Curves

Fig. 5.23 and Fig. 5.24 show plots of the test and predicted moment-curvature curves for the TBO series beams and the TBU series beams respectively. The well known theory of pure flexure is used for the prediction curves of the pure flexure specimens TB01 and TBU1. The accuracy of the flexural theory in predicting the moment-curvature response is shown by the plots for TB01 and TBU1. The theoretical prediction for the rest of the specimens is made using the model for spalled section dimensions. It is seen that the model does predict the moment-curvature responses for the specimens tested well.

These plots show that the effect of increasing the loading torque to moment ratio, R , is always to reduce the flexural capacity. With regard to curvature, if curvature under pure flexure is taken to be positive, then as R increases from zero to the value at balanced loading the curvature is steadily reduced to zero. A further increase in R results in negative curvature which increases as R is increased. It is important to note that the discussion is restricted to applied flexural moments that tend to cause sagging in a beam.

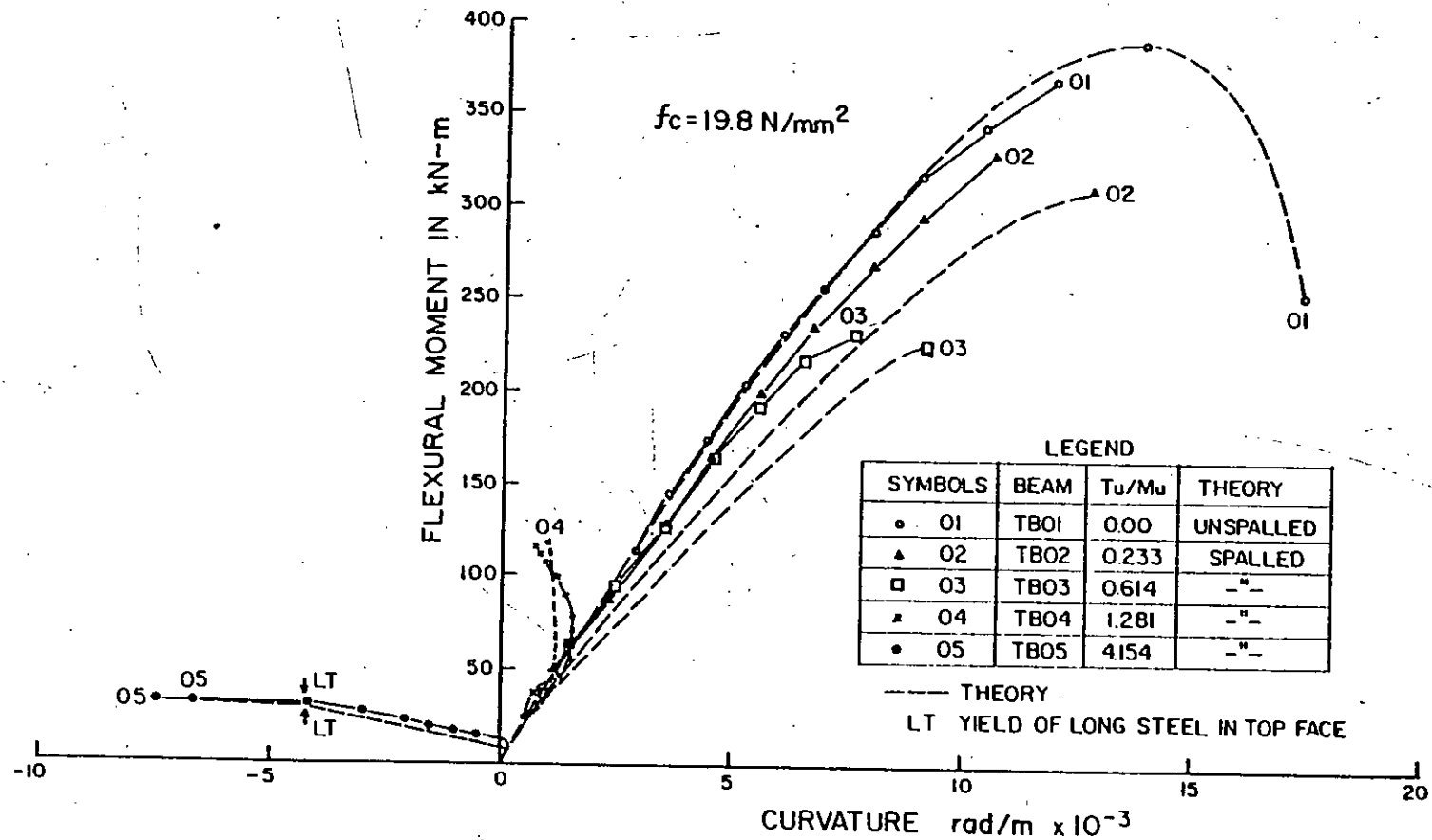


FIG. 5.23 MOMENT-CURVATURE RELATIONSHIPS FOR TBO SERIES

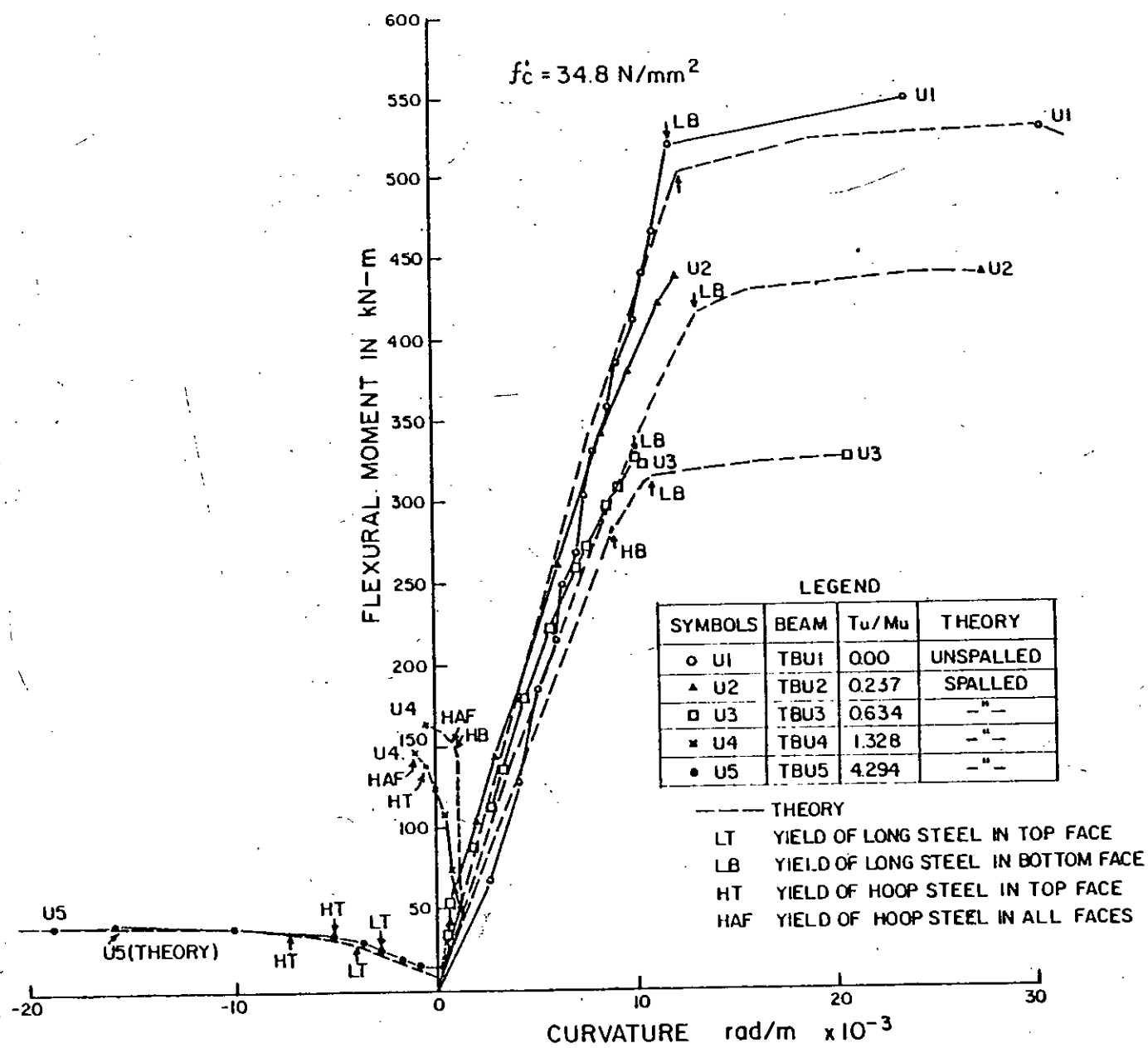


FIG. 5.24 MOMENT-CURVATURE RELATIONSHIPS FOR TBU SERIES

5.8.2 Effect of the Variation of Concrete Strength on Moment-Curvature Curve

A comparison of the moment-curvature curves in Fig. 5.23 and Fig. 5.24 reveals that the change in the concrete strength from 19.8 N/mm^2 for TBO series to 34.8 N/mm^2 for TBU series resulted in an increase in the flexural capacities and curvatures of the corresponding beams with a common loading ratio. The trend of variation as predicted by the model is in good agreement with the test results.

5.9 REVIEW OF BASIC ASSUMPTIONS WITH EXPERIMENTAL RESULTS

To develop the theoretical model five basic assumptions were made. These assumptions were necessary to enable the formulation of the equilibrium and geometric conditions needed in the solution for the problem of a structural concrete beam loaded in combined torsion, bending and axial load. The first basic assumption regarding the distribution of longitudinal strains at a section has been shown to be well verified by test results. The rest of the basic assumptions can be considered valid because the model predictions have been shown to be in good agreement with experimental results in the preceeding sections of this chapter for the widely varying changes in the parameters examined. In particular the simple parabolic stress-strain curve used to approximate the behaviour of concrete in a loaded beam has been shown to give good predictions for the heavily reinforced sections tested.

CHAPTER 6CONCLUSIONS

In this thesis a theoretical model has been presented which has the capability of predicting the response of a structural concrete section subjected to combined torsion, flexure and axial load. The model is comparable in rationality to the well known model for flexural analysis and it has been shown that pure flexure and pure torsion can be treated as special cases of the more general formulation presented.

The capability of the model to predict the complete response of reinforced concrete sections under combined torsion and flexure for the post-cracking loading range of the sections has been demonstrated for rectangular beam sections. However the basic equilibrium and geometric conditions can be applied to any cross-section shape for which the torsion resisted is predominantly St. Venant torsion. Moreover, since the actual stress-strain characteristics of the steel and the concrete are accounted for in the theory, the theory can be extended to cover such factors as repeated loading, reversed loading, and sustained loading which affect the stress strain characteristics of the materials.

The theory has been verified by comparing its detailed predictions with the results of a test programme involving fourteen heavily reinforced, heavily instrumented beams reported in the thesis. The model has also been shown to predict well the behaviour of prestressed concrete beams tested under combined

torsion and flexure at the University of Toronto.³ Accurate predictions were also obtained for a series of beams tested under combined torsion and flexure at the Swiss Federal Institute of Technology in Zurich.²

In the theoretical model presented in this thesis loading which produces a change of moment in the span of a beam has not been considered. The next development of the model should be to include the effects of such shearing loads. However, results of beams tested in shear suggest that large shearing strains affect the stress-strain characteristics of diagonally cracked concrete.⁷ Thus, a realistic stress-strain relationship for concrete which takes into account the effect of the shearing strains generated by torsion and shear will be required before accurate predictions of behaviour can be made.

ACKNOWLEDGEMENTS

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Finally, I am thankful to my mother and my father who sacrificed much to send me to school. To them I humbly dedicate this thesis.

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NOTATION

A	Area enclosed by external surface of section
A_c	Cross sectional area of concrete
A_h	Cross sectional area of one hoop leg
$A_{\ell i}$	Cross section area of longitudinal reinforcing bar 'i' located at (z_i, y_i)
A_{pj}	Cross section area of prestressing steel 'j' located at (z_i, y_i)
$A_{T\ell}$	Total area of longitudinal rebars in the section
A_o	Area enclosed by shear flow
A_1	Area enclosed by the perimeter described by the hoop centrelines
a_i	Depth of equivalent rectangular stress block at 'i'
a_b	Depth of equivalent rectangular stress block for bottom face
a_t	Depth of equivalent rectangular stress block for top face
b, h	External dimensions of rectangular section
b_1, h_1	Centreline dimensions of rectangular hoop
b_o, h_o	Dimensions defining the area enclosed by shear flow
C, C_T	Resultant compression force in concrete in top face
C_{hi}	Concrete cover to hoop steel centreline at 'i'
d_{ij}	Distance between two points 'i' and 'j' on the perimeter defined by the hoop centrelines
E_c	Modulus of elasticity of concrete

E_s	Modulus of elasticity of steel
F_{chi}	Resultant compression force in concrete in the direction of hoop steel at 'i'
F_{cl}	Resultant Compression Force in the concrete in the longitudinal direction at a section
F_{Hi}	Resultant tension force in hoop steel at 'i'
F_{sl}	Resultant tension force in reinforcing steel in the longitudinal direction
f'_c	Compressive strength of concrete (cylinder strength)
f_d	Average compressive stress in concrete diagonals
f_{hi}	Stress in hoops at 'i' (f_{hy} = yield stress)
f_{li}	Stress in longitudinal reinforcing steel at 'i' (f_{ly} = yield stress)
f_{pj}	Stress in prestressing steel at 'j' (f_{py} = yield stress f_{pI} = initial stress)
h, h_1, h_0	Dimensional parameters defined with b, b_1, b_0
jd	Lever arm for flexural moment
M_y	Moment causing curvature to a plane perpendicular to the y-axis (M_{by} = moment at balanced failure)
M_z	Moment causing curvature to a plane perpendicular to the z-axis (M_{bz} = moment at balanced failure)
N_x	Axial force in the x-axis
p	Perimeter of external surface of section
p_o	Perimeter of shear flow path
q	Shear flow (q_u = shear flow at failure)
q_b	Shear flow for balanced loading
R	Ratio of applied torque to applied moment

- r Linear correlation coefficient; or magnitude of a vector defining position of an element.
- s_h Hoop spacing
- T Torque (T_b = Torque at balanced failure)
- t Section wall thickness (t_t, t_s, t_b for top, side and bottom faces, respectively)
- t_c Ratio of section area to surface perimeter ($t_c = A/p$)
- t_d Depth of compression strains in a concrete diagonal ($t_d = t_M$ for pure flexure)
- t_h Hoop steel area in beam face per unit length of beam ($t_h = A_h/s_h$)
- t_ℓ Longitudinal steel area per unit length of perimeter ($t_\ell = A_{T\ell}/p_o$)
- w Measure of warping displacement for open section
- x Longitudinal axis of the beam
- (z_o, y_o) Point of action of N_x in section
- $(\bar{z}_c, \bar{y}_c)$ Point of action of F_{c1} in section
- $(\bar{z}_s, \bar{y}_s)$ Point of action of F_{s1} in section
- α An equivalent stress block factor (Equ. 2.21)
- β An equivalent stress block factor (Equ. 2.22)
- γ_i Shearing strain at 'i'
- ϵ_o Strain in concrete corresponding to maximum stress
- $\epsilon_{cb}, \epsilon_{ct}$ Longitudinal strains in bottom surface and top surface
- ϵ_{cu} Maximum compression strain in concrete at capacity loads
- ϵ_{dp} Principal compression diagonal strain in concrete (compressive positive) (ϵ_{ds} = Principal compression strain at surface)
- ϵ_{hi} Strain in hoop steel at 'i' (tensile positive)

$\epsilon_{hb}, \epsilon_{ht}$	Strain in hoop steel in bottom face and top face for rectangular section
ϵ_{li}	Strain in longitudinal direction at 'i' (tensile positive)
ϵ_{pj}	Strain in prestressing steel at 'j' (tensile positive)
ϵ_t	Strain in transverse direction (tensile positive)
$\Delta\epsilon_p$	Difference in strain between the bonded tendon and the surrounding concrete
θ	Angle of diagonal compression with respect to longitudinal axis (θ_p for principal compression direction)
ρ_h	Hoop reinforcement ratio ($\rho_h = t_h/t_c$)
ρ_ℓ	Longitudinal reinforcement ratio ($\rho_\ell = t_\ell/t_c$)
ρ_{hb}	Balanced value of ρ_h
$\rho_{\ell b}$	Balanced value of ρ_ℓ
σ_{ch}	Compressive stress on concrete element in direction of hoop steel
$\sigma_{c\ell}$	Compressive stress on concrete element in direction of longitudinal steel
τ	Shearing stress
ϕ	Angular deformation over a given length
ϕ_{dp}	Curvature of concrete diagonal in direction of principal compression
ϕ_ℓ	Longitudinal axis curvature (Eqn. 3.11)
ϕ_t	Transverse axis curvature (Eqn. 3.12)
ϕ_{xy}	Longitudinal axis curvature in y direction
ϕ_{xz}	Longitudinal axis curvature in z direction
ψ_x	Twist in longitudinal axis (torsional rotation/length)
Ω_d	Ratio of principal compression strain to the ϵ_o value of cylinder test ($\Omega_d = \epsilon_d/\epsilon_o$)

APPENDIX AEvaluation of Shear Flow Given ϵ_{ds}

The equilibrium equations (3.7) and (3.8) can be written as

$$(t_h f_h)_b = q \tan \theta_b r_{bb} + q r_{bt}$$

$$(t_h f_h)_t = q \tan \theta_t r_{tt} + q r_{tb}$$

where $r_{bb} = 1 - (\frac{1}{2}a_b - c_{hb})/h_1$

$$r_{bt} = \tan \theta_t (\frac{1}{2}a_t - c_{ht})/h_1$$

$$r_{tt} = 1 - (\frac{1}{2}a_t - c_{ht})/h_1$$

$$r_{tb} = \tan \theta_b (\frac{1}{2}a_b - c_{hb})/h_1$$

In general

$$(t_h f_h)_i = q (\tan \theta_i r_{ii} + r_{ij})$$

where i and j are the two walls cut by an appropriate section, e.g. Fig. 2.5. Rewrite simply as

$$t_h f_h = q (\tan \theta_i r_i + r_j) \dots \dots \dots (A1)$$

$$\text{Therefore } q \tan \theta_i = \frac{t_h f_h - q r_j}{r_i} \dots \dots \dots (A2)$$

The shear flow q is related to the principal concrete compression strain ϵ_{ds} by equation (3.14) which can be rewritten as

$$k_1 = \alpha \beta = \sigma / f'_c t_d \sin \theta \cos \theta \quad (\theta > 0) \quad (A3)$$

where $\alpha\beta = (1 - \mu^2)\Omega_{ds} - \frac{1}{3}(1 - \mu^3)\Omega_{ds}^2$

$$\Omega_{ds} = \epsilon_{ds}/\epsilon_0$$

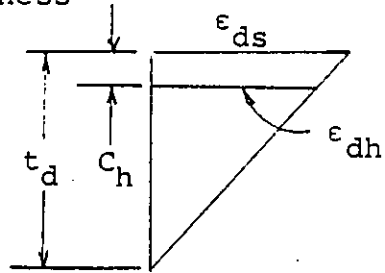
$$\mu = \frac{t_d - t}{t_d} \geq 0 \text{ and } \mu < 1.0, \mu = 0 \text{ for } t_d \leq t$$

t_d = effective diagonal thickness

t = actual section wall thickness

Compatibility of strains requires that

$$\tan^2 \theta_i = \frac{\epsilon_{lh} + \epsilon_{dh}}{\epsilon_h + \epsilon_{dh}} \quad (A4)$$



where

$$\epsilon_{dh} = f\epsilon_{ds}$$

f = concrete principal diagonal strain at level of hoop steel

ϵ_{lh} = longitudinal steel strain at level of hoop steel

$$f = (t_d - c_h)/t_d \leq 1.0$$

c_h = cover to centreline of hoop steel

Recognizing that $\sin\theta\cos\theta = \tan\theta/(1 + \tan^2\theta)$, equation A3

becomes

$$k_1 = \frac{q \tan \theta_i (1 + \tan^2 \theta_i)}{f_c t_d \tan^2 \theta_i}$$

Then using equations (A1) and (A4) gives

$$k_1 = \frac{(t_h f_h - q r_i) (\epsilon_h + \epsilon_{lh} + 2\epsilon_{dh})}{f_c t_d r_i (\epsilon_{lh} + \epsilon_{dh})} \quad (A5)$$

In equation (A5) the unknowns are ϵ_h , f_h and q , but the stress strain relationship between ϵ_h and f_h is known. Assuming that $\epsilon_h = \epsilon_{hy}$, then $f_h = f_{hy}$. The

terms r_i and r_j are also unknown initially but in general r_i is close to 1.0 and r_j is close to zero. Thus by initially ignoring the term qr_j and setting $r_i = 1$, the RHS* of equation A5 can be evaluated. It is assumed that the quantities ϵ_{lh} , ϵ_{ds} , t_d and t are given, hence k_1 of equation A5 can be determined.

If the $\text{RHS} < k_1$ then hoop steel is in the poor yield range

If the $\text{RHS} > k_1$ then hoop steel is in the elastic range.

Case A: Hoop Steel in Elastic Range

If hoop steel is in elastic range then $f_h = E_s \epsilon_h$, hence equation A5 becomes

$$k_1 = \frac{(t_h E_s \epsilon_h - qr_j)(\epsilon_h + \epsilon_{lh} + 2\epsilon_{dh})}{f_c' t_d r_i (\epsilon_{lh} + \epsilon_{dh})}$$

This equation can be rewritten as a quadratic in ϵ_h as

$$\epsilon_h^2 + \frac{[t_h E_s (\epsilon_{lh} + 2\epsilon_{dh}) - qr_j]}{t_h E_s} \epsilon_h - \frac{[k_1 f_c' t_d r_i (\epsilon_{lh} + \epsilon_{dh}) + qr_j (\epsilon_{lh} + 2\epsilon_{dh})]}{t_h E_s} = 0 \dots (A6)$$

This equation can be written in the form

$$\epsilon_h^2 + b\epsilon_h - c = 0$$

and hence obtain a solution for the hoop strain as

$$\epsilon_h = \frac{1}{2}(-b \pm \sqrt{b^2 + 4c})$$

Clearly the positive root is the correct one, hence

$$\epsilon_h = \frac{b}{2} \left[\sqrt{1 + \frac{4c}{b^2}} - 1 \right] \dots \dots \dots (A7)$$

With ϵ_h determined, $\tan \theta_i$ can be evaluated using (A4) and then using A2 the shear flow q can be evaluated. Clearly the value of q has to be guessed initially if $r_j \neq 0$, so as to solve equation (A6). The final solution is obtained iteratively.

Case B: Hoop Steel in the Post Yield Range

For this case rewrite equation A5 as

$$\epsilon_h = \frac{k_1 f'_c t_d r_i (\epsilon_{lh} + \epsilon_{dh})}{(t_h f_h - q r_j)} - (\epsilon_{lh} + 2\epsilon_{dh}) \quad (A8)$$

Knowing that $\epsilon_h \geq \epsilon_{hy}$, equation (A8) is solved iteratively using the given stress strain relationship for hoop steel.

APPENDIX BEvaluation of Compatible Strains Given q

The equilibrium and compatibility equations required for a solution are:

I. Equilibrium Equations

$$\text{From Appendix A: } q \tan \theta_i = \frac{t_h^f r_h - q r_j}{r_i} \dots \dots \dots (A2)$$

$$(1 - \mu^2) \Omega_{ds} - \frac{1}{3} (1 - \mu^3) \Omega_{ds}^2 = q / (f_c' t_d \sin \theta \cos \theta) \quad (\theta \geq 0) \dots (A3)$$

II. Compatibility Requirement

$$\tan^2 \theta_i = \frac{\epsilon_{lh} + \epsilon_{dh}}{\epsilon_h + \epsilon_{dh}} \quad (A4)$$

There are thus 3 equations A2, A3 and A4 to solve for three unknowns θ , ϵ_h and ϵ_d . It is assumed that values are given for all the other variables.

B-1 Determination of Limits for Strains ϵ_h and ϵ_d

It is convenient to evaluate ϵ_h in terms of the hoop yield strain ϵ_{hy} so that

$$\epsilon_h = n \epsilon_{hy} \dots \dots \dots (B1)$$

If the hoop steel is in the elastic range then rewrite equation (A2) as

$$\tan \theta_i = \frac{n t_h^f r_{hy} - q r_j}{q r_i} \quad \text{for } 0 \leq n \leq 1.0; \text{ and}$$

$$\text{define } C = \frac{t_h^f r_{hy}}{q} \dots \dots \dots (B2)$$

Then $\tan \theta_i = \frac{\eta C - r_j}{r_i} \dots \dots \dots (B3)$

Substituting (B3) into (A4), obtain

$$\tan^2 \theta_i = \left(\frac{\eta C - r_j}{r_i} \right)^2 = \frac{\Omega_{\ell h} + \Omega_{dh}}{\eta \Omega_{hy} + \Omega_{dh}}$$

where $\Omega_{\ell h} = \epsilon_{\ell h} / \epsilon_o$, $\Omega_{hy} = \epsilon_{hy} / \epsilon_o$, and $\Omega_{dh} = \epsilon_{dh} / \epsilon_o$

Thus $\Omega_{dh} = \left[\left(\frac{\eta C - r_j}{r_i} \right)^2 \eta \Omega_{hy} - \Omega_{\ell h} \right] / \left[1 - \left(\frac{\eta C - r_j}{r_i} \right)^2 \right] \dots (B4)$

For a practical solution $\Omega_{dh} \geq 0$. This simple requirement can be used to define limits for η for use in an iteration scheme. For example for the case $r_j = 0$ and $r_i = 1$ which is the case for the side faces of the section in Fig. 3.1, equation (B4) becomes

$$\Omega_{dh} = [\eta^3 C^2 \Omega_{hy} - \Omega_{\ell h}] / [1 - \eta^2 C^2] \dots \dots \dots (B5)$$

For equation (B5) to give $\Omega_{dh} \geq 0$

$$\left[\frac{\Omega_{\ell h}}{\Omega_{hy}} \frac{1}{C^2} \right]^{1/3} \leq \eta < \frac{1}{C} \quad \text{or} \quad \frac{1}{C} < \eta \leq \left[\frac{\Omega_{\ell h}}{\Omega_{hy}} \frac{1}{C^2} \right]^{1/3} \dots (B6)$$

Thus for hoop steel in the elastic range η must always lie between $1/C$ and $\left[\frac{\Omega_{\ell h}}{\Omega_{hy}} \frac{1}{C^2} \right]^{1/3}$.

These limits shall be referred to as ℓ_1 and ℓ_2 respectively.

B-2 Determination of Compatible Strains for Hoop Steel in Post Yield Range.

If both ℓ_1 and ℓ_2 are greater than 1.0 then it is certain that $\epsilon_h > \epsilon_{hy}$. In this case for a

bilinear stress strain relationship for the hoop steel
a solution for Ω_{ds} is obtained as follows:

Equation (A2) gives $\tan\theta_i = (C - r_j)/r_i$
and rewrite equation (A3) as

$$\Omega_{ds}^2 - \frac{3(1 - \mu^2)}{1 - \mu^3} \Omega_{ds} + \frac{3}{1 - \mu^3} \frac{q}{f_c t_d \sin\theta_i \cos\theta_i} = 0 \dots (B7)$$

and let $z = \sin\theta_i \cos\theta_i = \frac{\tan\theta_i}{1 + \tan^2\theta_i} = \frac{(C - r_j)/r_i}{1 + [(C - r_j)/r_i]^2}$

Equation (B7) can be written in the form

$$\Omega_{ds}^2 - b\Omega_{ds} + g = 0$$

For which the solution for Ω_{ds} is obtained as

$$\Omega_{ds} = \frac{1}{2}[b \pm \sqrt{b^2 - 4g}]$$

For minimum strain energy the correct root is the smaller
of the two roots i.e.

$$\Omega_{ds} = \frac{1}{2}[b - \sqrt{b^2 - 4g}] \dots \dots \dots (B8)$$

and $\epsilon_{dh} = f\Omega_{ds}\epsilon_o$

The value of hoop strain ϵ_h can now be evaluated
using equation (A4):

$$\epsilon_h = [\epsilon_{\lambda h} + \epsilon_{dh} (1 - \tan^2\theta_i)] \tan^2\theta_i \dots \dots \dots (B9)$$

B-3/ Determination of Compatible strains for Hoop Steel in Elastic Range

If both ℓ_1 and ℓ_2 are less than 1.0 then it is certain that $\epsilon_h < \epsilon_{hy}$. To solve for this case rewrite equation (A3) as

$$f(\eta) = (1 - \mu^2)\Omega_{ds}z - \frac{1}{3}(1 - \mu^3)\Omega_{ds}^2z - q/(f'_c t'_d) = 0$$

$$(\text{for } \theta > 0) \dots \dots \dots (B10)$$

$$\text{where } z = \sin\theta_i \cos\theta_i = \frac{\tan\theta_i}{1 + \tan^2\theta_i} = \frac{(\eta C - r_j)/r_i}{1 + [(\eta C - r_j)/r_i]^2}$$

$$\Omega_{ds} = \Omega_{dh}/f \text{ and } \Omega_{dh} \text{ is given by equation (B4)}$$

The problem here is to find η to satisfy equations (B4) and (B10). The solution is to be sought using an iterative procedure.

$$\begin{aligned} \text{Now } f'(\eta) &= (1 - \mu^2) \left[\frac{d\Omega_{ds}}{d\eta} z + \Omega_{ds} \frac{dz}{d\eta} \right] \\ &- \frac{1}{3}(1 - \mu^3) \left[2\Omega_{ds} \frac{d\Omega_{ds}}{d\eta} z + \Omega_{ds}^2 \frac{dz}{d\eta} \right] \dots \dots \dots (B11) \end{aligned}$$

$$\frac{d\Omega_{ds}}{d\eta} = \frac{1}{f} \frac{d\Omega_{ds}}{d\eta} = \frac{2N \frac{C}{r_i} \eta \Omega_{hy} + N^2 \Omega_{hy}}{f(1 - N^2)} + \frac{2N \frac{C}{r_i} \Omega_{ds}}{(1 - N^2)}$$

$$\text{where } N = \frac{\eta C - r_j}{r_i}$$

$$\frac{dz}{d\eta} = \frac{C}{r_i} (1 - N^2) / (1 + N^2)^2$$

The iteration process to solve equation (B10) can be initiated by assuming $\eta_i = \frac{1}{2}(\ell_1 + \ell_2)$ for $i = 1$.

For subsequent iteration steps, use the recursion formula:

$$\eta_{i+1} = \eta_i - f(\eta_i)/f'(\eta_i) \dots \dots \dots (B12)$$

This gives quadratic convergence close to the root.

B-4 Limitation on t_d Value

Equations (B7) and (B10) are identical if $\sin\theta_i \cos\theta_i$ is replaced by z so that:

$$\Omega_{ds}^2 - \frac{3(1 - \mu^2)}{1 - \mu^3} \Omega_{ds} + \frac{3}{1 - \mu^3} \frac{q}{f'_c t_d z} = 0$$

This equation is solved directly to give

$$\Omega_{ds} = \frac{3}{2} \left(\frac{1 - \mu^2}{1 - \mu^3} \right) \left[1 - \sqrt{1 - \frac{4}{3} \frac{q}{f'_c t_d z} \frac{1 - \mu^3}{(1 - \mu^2)^2}} \right] \dots (B13)$$

For a real solution

$$\frac{q}{f'_c t_d} \leq 0.75z \frac{(1 - \mu^2)^2}{1 - \mu^3} \dots \dots \dots (B14)$$

Taking $\mu = 0$ and recognizing that the maximum possible value of z is 0.5, obtain the limitation for t_d for a given q as

$$t_d \geq \frac{q}{.375 f'_c} \dots \dots \dots (B15)$$

B-5 Limitation on the Value of Ω_{ds}

Rewrite equation (B10) as

$$f(\eta) = f^*(\Omega_d) - k$$

where $f^*(\Omega_d) = (1 - \mu^2)\Omega_{ds}z - \frac{1}{3}(1 - \mu^3)\Omega_{ds}^2z$

and $k = q/f'_c t_d$

For a given k , $f(\eta)$ will be a maximum when $f^{**}(\Omega_d)$ is maximum. Now for a maximum value for $f^*(\Omega_d)$, $\frac{\partial}{\partial \Omega_d} f^*(\Omega_d)$ will be zero. Now

$$\begin{aligned} \frac{\partial}{\partial \Omega_d} (f^*(\Omega_d)) &= (1 - \mu^2)z - \frac{2}{3}(1 - \mu^3)\Omega_{ds}z \\ &+ \frac{dz}{d\Omega_d} [(1 - \mu^2)\Omega_{ds} - \frac{1}{3}(1 - \mu^3)\Omega_{ds}^2] = 0 \end{aligned}$$

$$\Omega_{ds} = \frac{3}{2} \frac{1 - \mu^2}{1 - \mu^3} - \frac{dz}{d\Omega_d} [(1 - \mu^2)\Omega_{ds} - \frac{1}{3}(1 - \mu^3)\Omega_{ds}^2] \frac{3}{2(1 - \mu^3)}$$

When close to the maximum of $f(\eta)$ it is found that $dz/\partial \Omega_d$ is very small and hence can be neglected. In fact in the limit when $\eta = \epsilon_1$, $dz/\partial \Omega_d = 0$ and $\theta_1 = 45^\circ$. Thus for a maximum value of $f(\Omega)$ the value of Ω_{ds} is given by

$$\Omega_{ds} = \frac{3}{2} \frac{1 - \mu^2}{1 - \mu^3} \dots \dots \dots (B16)$$

Figure B-1 shows the possible cases for which a solution is to be sought for equation (B10). It is indicated that there are two solutions of Ω_{ds} for each case. The smaller solution value is chosen. The other solution value is generally found to be quite large often giving ϵ_{ds} values in excess of $2\epsilon_0$. Such large strain values will only occur after a failure mechanism has developed. By limiting the value of Ω_{ds} as given by equation B16, the search for a solution

is restricted to be for compression strain up to capacity loading. It should however be noted that for some reinforced concrete sections it is possible at capacity loading to have Ω_{ds} values in excess of the value given by equation (B16) for the highly compressed zone, however the rest of the section will have Ω_{ds} values less than those given by (B16). Using the limitation on Ω_{ds} as given by B16 is convenient for the iteration scheme used and the capacity loading is always practically predicted with this limitation.

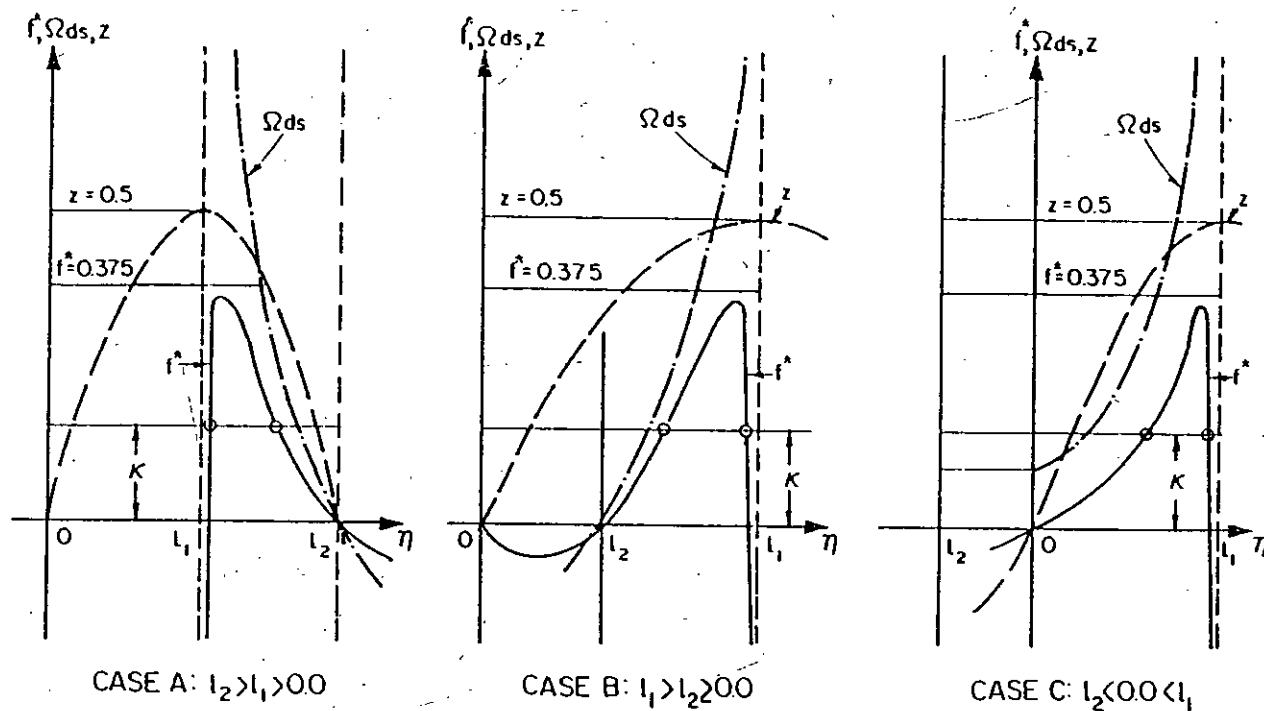


FIG. B.1

$$\Omega_{ds} = \frac{1}{f} \Omega_{dh} = \frac{1}{f} (\eta^3 c^2 \Omega_{hy} - \Omega_{lh}) / (1 - \eta^2 c^2), \quad \kappa = q / f'_c t_d$$

$$z = \tan \theta_i / (1 + \tan^2 \theta_i); \quad \tan \theta_i = (\eta c - r_i) / r_i; \quad \text{maximum value of } z \text{ is } 0.5$$

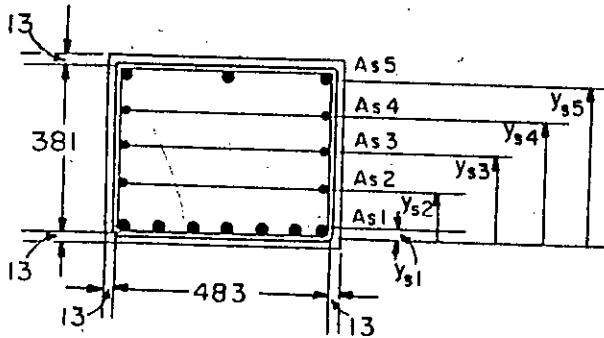
$$f^* = (1 - \mu^2) \Omega_{ds} z - \frac{1}{3} (1 - \mu^3) \Omega_{ds}^2 z; \quad \text{maximum value of } f^* \text{ with } z = 0.5 \text{ is } 0.375$$

The circles show the two possible solutions for each case. The solution giving the smaller value of Ω_{ds} is chosen.

APPENDIX C

EXAMPLE: Combined Flexure, Torsion and Axial Load

Given



Steel - (a) Longitudinal

I	A_s mm ²	y_s mm	f_{ly} N/mm ²	$E_s \cdot 10^3$ N/mm ²	ϵ_{ly} $\cdot 10^{-3}$
5	387	354.0	393	201.9	1.95
4	142	270.5	552	205.1	2.69
3	142	190.5	552	205.1	2.69
2	142	110.5	552	205.1	2.69
1	3570	2.30	436	194.2	2.25

Concrete

$$f'_c = 34.8 \text{ N/mm}^2$$

$$\epsilon_o = 3.1 \cdot 10^{-3}$$

(b) Hoop Steel

$$A_h = 129 \text{ mm}^2$$

$$S_h = 76.2 \text{ mm} \quad t_h = 1.693$$

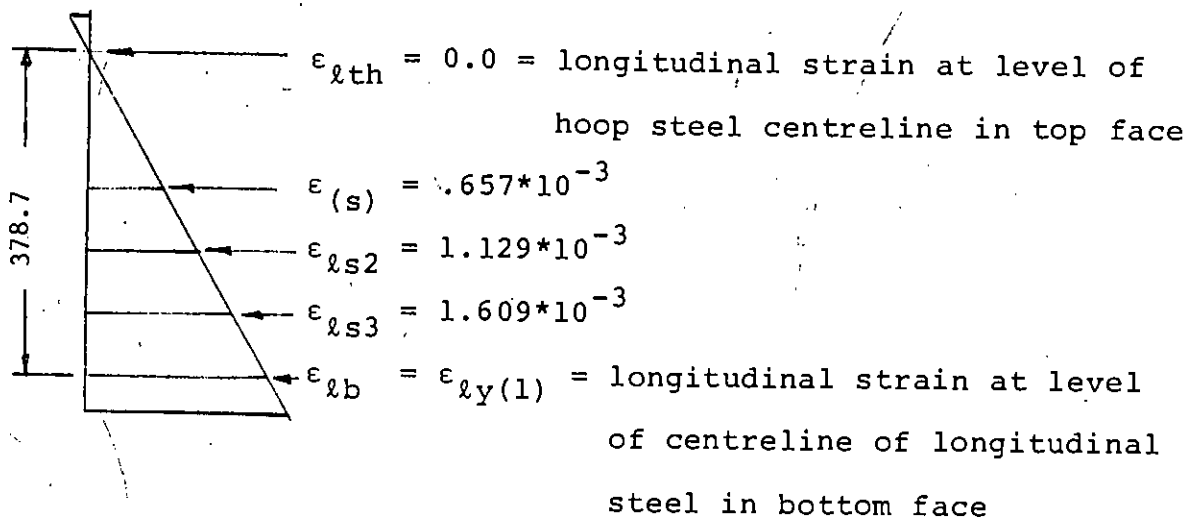
$$f_{hy} = 379 \text{ N/mm}^2 \quad E_s = 198.5 \cdot 10^3 \text{ N/mm}^2$$

$$\epsilon_{hy} = 1.91 \cdot 10^{-3}$$

Assume all steel has a bilinear stress-strain relationship.

Problem

Assuming spalled section dimensions effective, evaluate the moment and axial load acting together with a torque $T = 200 \text{ kN-m}$ so as to induce the following longitudinal strain profile with a curvature of $5.94 \cdot 10^{-6} \text{ rad/mm}$



Solution

To illustrate how the theory presented is used in the solution of this problem the procedure outlined in section 5.3.3 shall be used. To simplify the problem we shall take three points on the side faces for the evaluation of concrete forces and strains. Two of these points have the same longitudinal strains as the centrelines of hoop steel at the top and bottom faces respectively. The third point is taken at the midheight of the hoop steel.

Using spalled section dimensions equations (3.7) and (3.8):

$$t_{hfb} = q \tan \theta_b r_{bb} + q r_{bt} \dots \dots \dots (3.7)$$

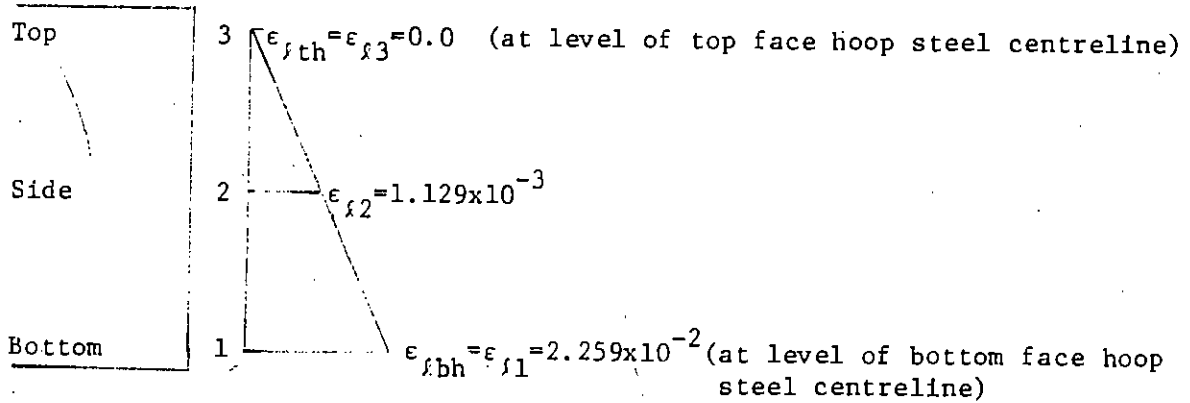
$$t_{hft} = q \tan \theta_t r_{tt} + q r_{tb} \dots \dots \dots (3.8)$$

are used in the first iteration with $r_{bb} = r_{tt} = 1.0$ and $r_{ht} = r_{tb} = 0.0$. This is a good approximation since h_1 is always very much larger than either a_b or a_t . For subsequent iterations the values of r_{bb} , r_{bt} , r_{tt} , and r_{tb} are evaluated

using the appropriate values from the previous iteration.

Initial Values

With the given strain profile, evaluate the strains at the various points selected



To start off the iteration process, assume the effective depths of the concrete compression diagonals to be:

$$t_{db} = t_{dt} = t_{d1} = t_{d2} = t_{d3} = \frac{.375b_1h_1}{b_1+h_1} = 79.84 \text{ mm}$$

To evaluate the shear flow due to the torque of 200 kN-m, the area enclosed by the path of shear flow is required.

Assume

$$A_o = b_o h_o$$

where $b_o = .9b_1$ and $h_o = .9h_1$

therefore $A_o = .81b_1h_1 = 148935 \text{ mm}^2$

and $q = T/2A_o = 671.433 \text{ N/mm}$

Step 1 Evaluation of Resultant Tension Force in Steel in Long Direction (F_{sl}) and its Point of Action

A_s mm ²	y_s mm	ϵ_l mm/mm	f_s N/mm ²	$T_s = A_s T_s$ N	$T_s y_s$ N-mm
3570	2.30	2.245	436.00	1556520	3579996
142	110.5	1.604	328.91	46705	5160901
142	190.5	1.129	231.63	32892	6265936
142	270.5	.655	134.36	19079	5160901
387	354.0	.160	32.39	12507	4427477
				1667703	24595210

$$F_{sl} = 1,667,703 \text{ N}$$

$$\bar{y}_s = 14.748 \text{ mm}$$

Step 2 Evaluation of Compatible Strains at a Point in a Plane Given q and ϵ_l Using Appendix B.

(i) Determine Limits ℓ_1 and ℓ_2

$$C = \frac{t_h f_{hy}}{q} = \frac{1.693 \cdot 379}{671.433} = 0.956$$

$$\ell_1 = \frac{1}{C} = 1.046$$

$$\ell_2 = \left[\frac{\epsilon_l}{\epsilon_{hy}} \frac{1}{C^2} \right]^{1/3}$$

Point	ϵ_l $\times 10^{-3}$	ℓ_2	Comments
Bottom	2.259	1.090	As both ℓ_1 and $\ell_2 > 1.0$, $\epsilon_{hb} > \epsilon_{hy}$
1	2.259	1.090	
2	1.129	0.865	
3	0.0	0.0	Hoop Steel $< \epsilon_{hy}$
Top	0.0	0.0	

(ii) Bottom Face (Use equation in Appendix B-2)

$$\tan \theta_b = C = 0.956$$

Hence $\theta_b = 43.7^\circ$

and $z = \sin \theta \cos \theta = 0.499$

Now $k = q/f_c' t_d = \frac{671.433}{348 \times 79.84} = 0.242$

$$\Omega_{ds}^2 - 3\Omega_{ds} + 3 \cdot 0.242 / 0.499 = 0$$

Solving, obtain $\Omega_{ds} = 0.606 = \Omega_{dh}$

$$\epsilon_{dnh} = \Omega_{dh} \epsilon_o = 1.88 \times 10^{-3}$$

$$\epsilon_{hb} = [2.259 + 1.88(1 - .956^2)] \times 10^{-3} / 0.956^2 = 2.652 \times 10^{-3}$$

$$\gamma_b = \frac{2\sqrt{(2.259 + 1.88)(2.652 + 1.88)} \times 10^{-3}}{1} = 8.662 \times 10^{-3}$$

(iii) Top Face - Hoop Steel in Elastic Range

Use iteration procedure outlined in Appendix B to obtain ϵ_{dst} and ϵ_{ht} with the following equation

$$N = \eta C$$

$$\Omega_{ds} = \frac{N^2 \eta \Omega_{hy} - \Omega_{lh}}{1 - N^2}$$

$$z = N / (1 + N^2)$$

$$\frac{dz}{d\eta} = C(1 - N^2) / (1 + N^2)^2$$

$$\frac{d\Omega_{ds}}{d\eta} = [2NC(\eta \Omega_{hy} + \Omega_{ds}) + N^2 \Omega_{hy}] / (1 - N^2)$$

$$f(\eta) = \Omega_{ds} z - \frac{1}{3} \Omega_{ds}^2 z - \sigma / f_c' t_d$$

$$f'(\eta) = z \frac{d\Omega_{ds}}{d\eta} + \Omega_{ds} \frac{dz}{d\eta} - \frac{1}{3} [2\Omega_{ds} z \frac{d\Omega_{ds}}{d\eta} + \Omega_{ds}^2 \frac{dz}{d\eta}]$$

Iterate with

$$\eta_{i+1} = \eta_i - f(\eta_i) / f'(\eta_i)$$

where

$$\Omega_{ly} = \frac{\epsilon_{hy}}{\epsilon_o} = \frac{1.909}{3.1} = .616$$

$$\Omega_h = \frac{\epsilon_l}{\epsilon_o} = 0.0$$

$$C = 0.956$$

Iter.	η	Ω_{ds}	$f(\eta)$
1	.523	0.107	-0.20
2	.829	0.863	0.058
3	.790	0.644	0.001
4	.789	0.639	2.77×10^{-6}
5	.789	0.639	-2.0×10^{-10}

$$\epsilon_{ht} = \eta \epsilon_{hy} = 0.789 \times 1.909 \times 10^{-3} = 1.506 \times 10^{-3}$$

$$\epsilon_{dst} = \Omega_{ds} \epsilon_o = 0.639 \times 3.1 \times 10^{-3} = 1.980 \times 10^{-3}$$

$$\tan^2 \theta_t = \frac{0 + 1.980}{1.506 + 1.980} = 0.568; \text{ hence } \theta_t = 37.005^\circ$$

$$\gamma_t = 2 \sqrt{[(0 + 1.98)(1.506 + 1.980)]} \times 10^{-3} = 5.255 \times 10^{-4}$$

(iv) Side Face

- Level 1 - same as for Bottom Face
- Level 3 - same as for Top Face
- Level 2

Hoop steel will be in elastic range and start iteration with $\eta = \frac{1}{2}(.865 + 1.046)$ i.e. $\eta = 0.955$.

Convergence in η is obtained in two iterations with the following results.

η	$\epsilon_{h2} * 10^{-3}$	$\epsilon_{l2} * 10^{-3}$	$\epsilon_{d2} * 10^{-3}$	$\gamma_2 * 10^{-3}$	θ_2
.946	1.806	1.129	1.890	6.681	42.11°

Step 3 - Evaluate Twist

$$\begin{aligned}
 \psi_x &= \frac{1}{2A_1} [b_1(\gamma_b + \gamma_t) + \frac{h_1}{3} (\gamma_1 + 4\gamma_2 + \gamma_3)] \\
 &= \frac{1}{2 \times 381 \times 482.6} [482.6(8.662 + 5.255) \\
 &\quad + \frac{381}{3}(8.662 + 4 \times 6.681 + 5.255)] * 10^{-3} \\
 &= 0.0323 * 10^{-3} \text{ rad/mm}
 \end{aligned}$$

Step 4 Evaluation of Diagonal Curvatures and New t_d Values

$$\phi_d = \phi_t \sin^2 \theta + \phi_l \cos^2 \theta + \psi_x \sin 2\theta$$

$$t_d = \epsilon_{ds} / \phi_d$$

$$(t_d)_{\text{new}} = \frac{1}{2} (t_d + (t_d)_{\text{old}})$$

Position	θ°	$\phi_t * 10^{-3}$	$\phi_l * 10^{-3}$	$\psi_x * 10^{-3}$	$\phi_d * 10^{-3}$	t_d	$t_{d_{\text{old}}}$	$t_{d_{\text{new}}}^*$
Bottom	43.7	-.00301	-.00594	.0323	.0277	67.81	79.84	73.82
Top	37.0	.00301	.00594	"-	.0359	55.11	"-	67.48
1	43.7	0	0	"-	.03227	58.26	"-	69.05
2	42.11	0	0	"-	.03214	58.82	"-	69.33
3	37.0	0	0	"-	.03105	60.55	"-	70.19

* t_d values to be used in the next iteration

Step/ 5 Evaluation of Area A_o Enclosed by Path of Shear Flow
and Re-evaluation of q

Position	Ω_{ds}	β	t_d	$a=\beta t_d$	$h_o = h_1 - \frac{1}{2}(a_b + a_t) = 324.29 \text{ mm}$
Bottom	.606	.709	79.84	56.60	$b_{01} = b_1 - a_1 = 426.00 \text{ mm}$
Top	.639	.712	"-	56.83	$b_{02} = b_1 - a_2 = 425.98 \text{ mm}$
1	.606	.709	"-	56.60	$b_{03} = b_1 - a_3 = 425.77 \text{ mm}$
2	.610	.709	"-	56.62	$A_o = \frac{h_o}{4} [b_{01} + 2b_{02} + b_{03}] = 138126 \text{ mm}^2$
3	.639	.712	"-	56.83	

$r_{bb} = .926, r_{bt} = .075, r_{tt} = .925,$
 $r_{tb} = .074$

For evaluation of q take $A_o = \frac{1}{2}(138126 + 148935) = 143531 \text{ mm}^2$

For next iteration $q = T/2A_o = 696.716 \text{ N/mm}$

Step 6 Iterations with Steps 1-5 to get Convergence in t_d Values

Iteration		1	2	3	4	5	
q N/mm		671.433	696.433	699.330	697.821	696.370	
BOTTOM	t _{db} mm	79.84	73.82	70.18	69.53	69.37	
	θ _b deg	43.70	43.19	43.09	43.08		
	k _b	.242	0.271	.286	.2884	.2884	
	ε _{db} *10 ⁻³	1.880	2.208	2.394	2.425		
	ε _{hb} *10 ⁻³	2.652	2.893	2.966	2.972		
	γ _b *10 ⁻³	8.662	9.547	9.988	10.055		
	SIDE P A C E	LEVEL 1	t _{d1} mm	79.84	69.05	66.90	66.28
θ ₁ deg			43.70	42.64	42.54	42.60	
k ₁			0.242	0.290	.300	.3025	.3024
ε _{d1} *10 ⁻³			1.880	2.448	2.586	2.621	
ε _{h1} *10 ⁻³			2.652	3.102	3.700	3.151	
γ ₁ *10 ⁻³			8.662	10.223	10.562	10.614	
LEVEL 2		t _{d2} mm	79.84	69.33	66.92	66.36	66.22
		θ ₂ deg	42.11	42.26	42.32	42.35	
		k ₂	0.242	0.289	.300	.3022	.3023
		ε _{d2} *10 ⁻³	1.890	2.437	2.589	2.620	
		ε _{h2} *10 ⁻³	1.805	1.884	1.895	1.893	
		γ ₂ *10 ⁻³	6.681	7.851	8.166	8.225	
LEVEL 3		t _{d3} mm	79.84	70.19	68.95	67.94	67.90
		θ ₃ deg	37.00	37.92	38.04	38.14	
		k ₃	0.242	0.285	.291	.2952	.2947
		ε _{d3} *10 ⁻³	1.980	2.491	2.568	2.625	
		ε _{h3} *10 ⁻³	1.506	1.615	1.628	1.631	
		γ ₃ *10 ⁻³	5.255	6.397	6.565	6.684	
TOP		t _{dt} mm	79.84	67.48	65.45	64.97	64.77
		θ _t deg	37.00	38.12	38.35	38.40	
		k _t	0.242	0.297	.307	.3086	.3091
		ε _{dt} *10 ⁻³	1.980	2.647	2.798	2.821	
		ε _{ht} *10 ⁻³	1.506	1.652	1.673	1.671	
		γ _t *10 ⁻³	5.255	6.748	7.073	7.119	
	φ _t *10 ⁻³ rad/mm		.00301	.00326	.00339	.00342	
φ _x *10 ⁻³ rad/mm		.03230	.03797	.03959	.03987		
A ₀ mm ²		148935	143531	142994	143303	143602	

It is seen that by the end of 4 iterations an accuracy better than .5% in t_d values have been achieved. With good initial guesses of the t_d values for the various points in the section selected an accuracy of .1% is achieved in 3 to 4 iterations. This will be realized, for example, if a new longitudinal strain profile is taken which only differs slightly from the strain profile for which an iteration process has been carried out and convergence obtained for the t_d values which are then used as initial values for the new strain profile.

Steps 7 and 8 Evaluation of Resultant Compression Force and its Point of Action

Use $q = 696.37 \text{ N/mm}$

Position	θ_i	y_{ci}	Δp_i	$\omega_i = q \cot \theta_i \Delta p_i$	$\omega_i y_{ci}$
Bottom	43.08	25'	434	323200	8080000
1	42.60	25	} $p_s = 332$	$\Sigma^* = 520130$	$\Sigma^{**} = 101556000$
2	42.35	190.5			
3	38.14	357			
Top	38.40	357 ²	433	380400	135815000
				1223800	245450000

$$1 \quad y_{cb} = a_{b/2} = y_{c1}$$

$$2 \quad y_{ct} = h_1 - a_{t/2} = y_{c3}$$

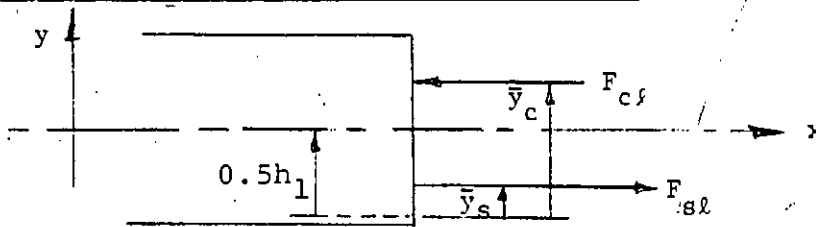
$$\Sigma^* = \frac{p_s}{3} (q \cot \theta_1 + 4q \cot \theta_2 + q \cot \theta_3)$$

$$\Sigma^{**} = \frac{p_s}{3} (q \cot \theta_1 y_{c1} + 4q \cot \theta_2 y_{c2} + q \cot \theta_3 y_{c3})$$

$$F_c = 1223800 \text{ N}$$

$$\bar{y}_c = 245450000 / 1223800 = 200.57 \text{ mm}$$

Step 9 Evaluation of Applied Actions



$$N_x = F_{sl} - F_{cl} = 1667700 - 1223800 \text{ N} = 443.9 \text{ KN}$$

N_x is assumed to act as the section centroid

$$M_y = F_{sl} (\bar{y}_c - \bar{y}_s) + N_x (0.5h_1 - \bar{y}_c)$$

$$\begin{aligned} &= 1667.70 (200.57 - 14.75) + 443.9 (381/2 - 200.57) \text{ KN} - \text{mm} \\ &= 305.42 \text{ kN} - \text{m} \end{aligned}$$

$$\psi_x = .040 \text{ rad/m}, \phi_\ell = .006 \text{ rad/m}, \phi_t = .003 \text{ rad/m}$$

Thus for the given strain profile and for the section, internal resisting actions have been evaluated. If a strain profile, which will result in $N_x = 0$ for $\epsilon_{lth} = 0.0$, is required then ϵ_{lbh} is varied until this is achieved. Furthermore if the strains for a given T/M ratio with $T = 200 \text{ kN} - \text{m}$ and $N_x = 0$ are required, then with the given moment the top longitudinal strain is varied to get the desired internal moment and the bottom longitudinal strain is varied to ensure $N_x = 0$ for any chosen top face longitudinal strain. This procedure is used in the computer program given in Appendix D to analyze the test specimens.

APPENDIX D

Computer program for analysis of beams under combined flexure and torsion by the diagonal compression field theory.

D.1 Program Documentation

Program Name: Diagonal Compression Field Analysis

Language: Fortran IV

Purpose and Limitations

To analyse the complete response of a rectangular structural concrete beam under combined flexure and torsion. The theory is applicable only to St. Venant torsion loading and the program is specifically for a section with one axis of symmetry.

Program Capabilities

- (1) For a given flexural moment and torque and for given section dimensions (spalled or unspalled) the program predicts the twist, the longitudinal and transverse strain profiles in the section, the concrete principal compression strains at chosen points, the angles of the principal compression strains, the thicknesses of the concrete compression diagonals at the chosen points, the shear flow and the area enclosed by the shear flow.
- (2) The effective thicknesses of the concrete compression diagonals, t_d , can be greater than that provided by the section wall thickness.
- (3) The reinforcing steel in the section analysed can be initially unstressed or prestressed or a combination of both.

- (4) The stress-strain relationships for the concrete (parabolic relationship assumed) and the steel (option of bilinear or trilinear or continuous stress-strain curves for steel) are accounted for.
- (5) The response of under-reinforced, partially over-reinforced and over-reinforced beams can be predicted.

Solution Technique

The solution technique is outlined in section 3.3.3. The sequence of operations in the program are as follows

- (1) Input beam data
- (2) Print input data
- (3) Input the analysis control parameters which includes the flexural moment and torque for analysis and the initial guess of the longitudinal strain profile
Also, initialize analysis dimensions.
- (4) Calculate the resultant tension force in the reinforcing steel
- (5) Evaluate the shear flow and hence obtain the compatible strains at chosen points
- (6) Evaluate the twist and the diagonal curvatures at chosen points
- (7) Evaluate t_d - values
- (8) Determine path of shear flow and evaluate the area A_o
- (9) Evaluate the resultant compression force in the longitudinal direction in concrete

- (10) Evaluate strain in reinforcing steel in bottom face
- (11) Evaluate the internal flexural moment and the torque
- (12) Evaluate strain in top face surface
- (13) Output results. A summarized output of results or a detailed output can be specified.

Commentary is given in the appropriate sections of the program giving in greater details the operations in the program.

D.2 NOTATION USED TO INPUT DATA

D.2.1 Section Geometry

B,H	Outside section dimensions
B ₁ ,H ₁	Dimensions of hoop centrelines
CHB,CHS,CHT	Cover to hoop steel centrelines in bottom, side and top faces respectively
CLB	Cover to centroid of longitudinal steel in bottom face
TB,TS,TT	Wall thickness in section for bottom, side and top faces respectively
TGH	Level at which test strains are measured with respect to hoop centrelines.

D.2.2 Material Properties

Concrete

FC	Concrete cylinder compression strength (f'_c)
EPSO	Strain, ϵ_o , corresponding to the peak stress, f'_c , in a standard cylinder compression test.

Reinforcing Steel(a) Longitudinal Steel

AS(I), YS(I)	Area of steel bar(s) positioned at distance YS(I) with respect to the bars in the bottom face
EPLY(I), EPSH(I)	Yield strain and strain at onset of strain hardening, respectively
ES(I), ESH(I)	Moduli of elasticity before yield and after onset of strain hardening, respectively
FLY(I), FU(I)	Yield and ultimate stresses, respectively

(b) Prestressing Steel

AP(I), YP(I)	Area of tendon(s) positioned at distance YP(I) with respect to the reinforcing steel bars in bottom face
EP	Modulus of elasticity
EPPO, EPPY	Prestrain and strain at .2% proof stress
FPI, FPY, FPU	Prestress, .2% proof stress and ultimate stress respectively for the tendons.

(c) Hoop Steel

AH	Cross section area of one hoop leg
SH	Spacing of hoop steel
EPHY, EPSHH	Yield strain and strain at onset of strain hardening respectively
ESS, ESHH	Moduli of elasticity before yield and after onset of the strain hardening respectively
FHY, FUH	Yield and ultimate stresses respectively

D.2.3 Control Parameters

BMNO	Beam Number
DEADMO	Initial moment acting due to dead weights
DEADTO	Initial torque acting due to dead weights
DMO	Amount by which the flexural moment is to be increased in the analysis
EPCCB, EPCCT	Longitudinal strains in bottom and top face defining the longitudinal strain profile
IOUT	Parameter controlling printout If IOUT = 0 complete printout of the strains, angles, t_d -values and curvatures for all the points for each load increment as well as a table at the end summarizing the analysis results for mid-points of each side at the level of strain measurements in a test. If IOUT > 0 only the final summary of results is given
INTV	Number of load increments required. If the loading is increased beyond the capacity of the section the analysis is terminated when the maximum principal compression strain is $\approx 1.5\epsilon_0$
IPRE	Indicator for the presence of prestressed steel. If there is no prestressed steel IPRE is set to zero
LSIMP	Number of points on the side face at which evaluations of strains, t_d -values and angles are done. LSIMP takes the values $2n+1$ ($n=1,2,3,\dots$)

MES Indicator of uniformity of the reinforcing steel properties in the section. If all longitudinal steel bars have the same stress-strain characteristics then set MES=0 otherwise set MES=1

N Number of layers of reinforcing bars along the side faces

NPRE Number of layers of prestressed steel along the side faces

SOURCE Identification of origin of beam BMNO being analysed

TAGMO Specified initial target moment for analysis

TMRO Fixed ratio at which the torque and flexural moment applied to the beam are increased

TTMM Value of moment above which further increments in moments are reduced to DMO/4 in an attempt to approach the failure loads more closely

D.3 DATA INPUT

DATA GROUP	INPUT FORMAT	PARAMETERS
1	(I3,2I2,I3,1X,16A4)	N,MES,IPRE,NPRE,BMNO,SOURCE
2	((8F10.0))	B, H, TB, TS, TT, B1, H1, CLB, CHS, CHT, TGH
3	((8F10.0))	(AS(I),I=1,N), (YS(I),I=1,N)
4**	((8F10.0))	(AP(I),I=1,NPRE), (YP(I),I=1,NPRE), FPY, FPI, EP, FPU
5*	((8F10.0))	(FLY(I),I=1,N), (FU(I),I=1,N), (ES(1),I=1,N), (ESH(I),I=1,N), (EPSH(I),I=1,N)
6	((8F10.0))	AH, SH, FHY, FUH, ESS, ESHH, EPSHH
7	((8F10.0))	FC,EPSO, DEADMO, DEADTO
8	I5, 6F10.0, 215	INTV, TMRO, TAGMO, DMO, EPCCT, EPCCB, TTMM, IOUT, LSIMP

* If MES = 0 for group 5 we read FY, FULTM, EMOD, EMODH, EPSSH where these variables denote the common values of the parameters in group 5 for any 'I' value.

**If IPRE = 0 group 4 cards are omitted.

D.4 NOTATION USED IN PRINTING RESULTS

The notation used in the input of data is also used in the printout of the input data. Besides, the following further symbols are used in the printout of analysis results. Note that the values of the indices IJK, IIM, ITR and KK which are printed in the detailed printout of results refer to the value at the end of the last iteration step.

AO	Area enclosed by path of shear flow
CN	Depth from top face to plane with zero longitudinal strains
EPLB, EPLT	Strains in the longitudinal steel bars in the bottom face and the top face respectively
FCC	Resultant compression force in concrete in the direction of the beam's longitudinal axis
FSS	Resultant tension force in the reinforcing steel in the direction of the beam's longitudinal axis
HO	Height of the path of shear flow
III	General counter index
IJK	Counter of number of load increments
IIM	Counter of number of iterations done on EPCCT to get the internal flexural moment to converge to the applied moment (maximum value is 7)
ITR	Counter of number of iterations done on EPCCB for a fixed EPCCT to get equilibrium of forces in the longitudinal axis direction (maximum value is 8)
KK	Counter of number of iterations to obtain convergence in t_d -values for a fixed longitudinal strain profile

(maximum allowed is 7)

M	Evaluated internal flexural moment about the geometric centroid of the section
PHIX	Curvature due to variation in longitudinal strains from top face to bottom face (rad/unit length)
PHIZ	Curvature due to variation of hoop strains from top face to bottom face (rad/unit length)
PSIX	Twist (rad/unit length)
Q	Shear flow
T	Evaluated internal torque
TAGMOM	Given applied moment
TAGTQQ	Given applied torque
TDMIN	Minimum t_d -value for the given shear flow according to the theoretical model
YC	Defines the position at which FCC acts with respect to the rebars in the bottom face
YSB	Defines the position at which FSS acts with respect to the longitudinal steel bars in the bottom face

The indices indicated in the printout of the detailed output of results are as follows

INDEX HOOP = 0 for hoop steel strain $\epsilon_h \leq \epsilon_{hy}$
 = 1 for hoop steel strain $\epsilon_h \leq \epsilon_{hy}$
 = 2 for hoop steel strain in strain hardening range.

INDEX CONC = 0 indicates normal convergence on principal diagonal strain, ϵ_{ds} .

$\neq 0$ indicates problem in convergence on principal diagonal strain, ϵ_{ds} .

D-5 CONVERGENCE ACCURACY

In the program an iterative procedures are used to obtain convergence of t_d -values, at various points in the section, the shear flow q , the resultant axial force which is required to be zero for torsion and flexure loading and convergence of the applied internal actions to the corresponding external ones. The iteration process is terminated when the following criteria are met:

(a) For t_d -values:

top face and bottom face - error $\leq .1\%$

side faces at selected points - error $\leq .2\%$

(b) For shear flow - error $\leq .1\%$

(c) Axial force $(F_{ss} - F_{cc})$ - error $\leq .2\%$ for IIM ≤ 1

$\leq .3\%$ for ITR ≥ 5

- maximum number of iterations is ITR=8

(d) Flectural moment - error $\leq .2\%$

$\leq .5\%$ for IIM ≥ 5

- maximum number of iterations is IIM=7

D.6 PROGRAM LISTING

```

C
C PROGRAM ANALYSES RECT. R.C. SECTION IN TORSION AND BENDING
C
0001 REAL AS(10),YS(10),ES(10),FLY(10),FU(10),EO(10)
0002 REAL AP(10),YP(10),EPP(10),EPPN(10),FPI(10),DEP(10),FPX(10)
0003 REAL TO(10),TDS(10),AB(10),FS(10),FSX(10),ESH(10)
0004 REAL THETA(10),ATH(10),DMOS(10),GAM(10),PL(10)
0005 REAL EPSH(10),EPLY(10),EPLS(10),EPLL(10),EPH(10)
0006 REAL EPDS(10),PHIO(10),SOURCE(15),ECT(10),DMOM(10)
0007 REAL SST(10),EPCH(10),FOIFF(10),JO,WW(10)
0008 DIMENSION INDEX(10),IC(10)
0009 REAL TDO(90),MON(90)
0010 REAL TEPL(90),SEPL(90),BEPL(90),TEPH(90),SEPH(90),BEPH(90),
      *TEPD(90),SEPD(90),BEPD(90),TPHID(90),SPHID(90),BPHID(90),
      *TTHETA(90),STHETA(90),BTHETA(90),PSIXB(90),PHIXB(90),PHIZB(90),
      *TGAMA(90),SGAMA(90),BGAMA(90),XAR(20)
      COMMON /BLOCK1/FC,EPGQ,TH,ESS,ESHH,FHY,FUH,EPSSH
0011
C
C SECTION GEOMETRY
C
C N=NO. OF LAYERS OF REBARS ON SIDE FACE
C MES=INDICATOR FOR SAME STEEL PROPERTIES FOR ALL LAYERS OF REBARS
C IF ALL LONG. REBARS HAVE SAME STRESS-STRAIN CHARACTERISTICS THEN
C SET MES=0 OTHERWISE SET MES=1
C IPRE=INDICATOR FOR PRESENCE OF PRESTRESSED STEEL
C IF NO PRESTRESSING SET IPRE=0
C IF THERE IS PRESTRESSED STEEL SET IPRE=1
C NPRES=NO. OF LAYERS OF PRESTRESSED STEEL
C IT IS ASSUMED THAT PROPERTIES OF ALL PRESTRESSED STEEL ARE SAME
C CHB,CHS,CHT =COVER TO HOOP CENTRE-LINES IN BOTTOM, SIDES AND TOP FACES
C CLB=COVER TO CENTRE-LINE OF BOTTOM REBARS
C FOR SPALLED CASE CHB,CHT,CHS ARE ALL ZERO
C TGH=TARGET HEIGHT FROM HOOP CENTRELINE
C
0012 1002 ISPALL=0
0013 1000 READ(5,5000)N,MES,IPRE,NPRE,BMNO,SOURCE
0014 IF(N.EQ.0)GO TO 999
0015 1 READ(5,5100)B,H,TB,TS,TT,B1,H1,CLB,CHB,CHS,CHT,TGH
0016 IF(B.EQ.0.0) GO TO 999
0017 IF(ISPALL.NE.0) GO TO 1007
C
C MATERIAL PROPERTIES
C AMOUNT AND LOCATION OF STEEL IN SECTION
C POSITION OF STEEL IS TAKEN W.R.T. BOTTOM-CORNER LONG. REBARS
C
0018 READ(5,5100)(AS(I),I=1,N),(YS(I),I=1,N)
0019 IF(IPRE.EQ.0)GO TO 2
0020 READ(5,5100)(AP(I),I=1,NPRE),(YP(I),I=1,NPRE),FPI,EP,EPN
C
C FPY=0.2XPROOF STRESS
C FPU=ULTIMATE STRENGTH OF TENDON
C FPI=PRESTRESS
C EPPD=PRESTRAIN
      EPPY=FPY/EP
0021
0022 2 IF(MES.EQ.0)GO TO 3
0023 READ(5,5100)(FLY(I),I=1,N),(FU(I),I=1,N),(ES(I),I=1,N),(ESH(I),I=1,N),
      *(EPSH(I),I=1,N)
      GO TO 4
0024 3 READ(5,5100)F,FULTM,EMOD,EMODH,EPSSH
0025 4 READ(5,5100) AM,SH,FHY,FUH,ESS,ESHH,EPSSH
0026
C
C DEADMO=INITIAL MOMENT ACTING DUE TO DEAD WEIGHT
C DEADTO=INITIAL TORQUE ACTING DUE TO DEAD WEIGHT
C
0027 READ(5,5100) FC,EPGQ,DEADMO,DEADTO
0028 DO A K=1,N
0029 IF(MES.EQ.1)GO TO 5
0030 FLY(K)=FPY
0031 FU(K)=FULTM
0032 ES(K)=EMOD
0033 ESH(K)=EMODH
0034 EPSH(K)=EPSSH
0035 EPLY(K)=FLY(K)/ES(K)
0036 5 IF(FU(K).EQ.FLY(K))EPSH(K)=EPLY(K)*10.
0037 EPHY=FHY/ESS
0038 IF(EPSSH.EQ.0.)EPSSH=10.*EPHY
0039 TH=AH/SH
0040 A1=H1*H1
0041 HL=YS(N)
0042 1007 D=H-CLB
0043 YO=0.5*H-CLB
0044 IF(IPRE.EQ.0)GO TO 9
0045 DIN=B-2.*TS
0046 HIN=H-TT-TB
0047 AC=B *H -BIN*HIN
0048 EC=2.*FC/EPGQ
0049 ASTOT=0.0
0050 APTOT=0.0
0051 ALES=0.0
0052 DO 7 K=1,N
0053 ALES=ALES+AS(K)*ES(K)
0054 7 ASTOT=ASTOT+AS(K)
0055 DO 8 K=1,NPRE
0056 APTOT=APTOT+AP(K)
0057 AC=AC-APTOT-ASTOT
0058 EPPQ=FPI*(1./EP+APTOT/(ALES+AC*EC))
0059 TIN=0.375*B*H/(D*H)
C
C FACTOR FOR CONVERTING DEGREES TO RADIANS
      DEGREE=45.0/ATAN(1.0)
      A45=ATAN(1.00)
C
C PRINT INPUT DATA
C
0062 WRITE(6,5000)
0063 WRITE(6,6050)BMNO,SOURCE
0064 WRITE(6,6100)
0065 WRITE(6,5140)B,H,TB,TS,TT,B1,H1,CLB,D,TIN
0066 WRITE(6,6510)
0067 WRITE(6,6110)

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0068      DO 10 K=1,N
0069      10 WRITE(6,5110)K,AS(K),YS(K),FLY(K),FU(K),ES(K),ESH(K),EPLY(K),EPSH
      +K)
0070      IF(IPRE.EQ.0)GO TO 13
0071      WRITE(6,6115)
0072      DO 12 K=1,NPRE
0073      12 WRITE(6,5110)K,AP(K),YP(K),FPY,FPU,FPI,EP,EPPY,EPPQ
0074      13 WRITE(6,6120)
0075      WRITE(6,5115)AH,SH,FHY,FUH,ESS,ESHH,EPSHH,EPHY,CHT,CMB,CHS
0076      WRITE(6,6125)
0077      WRITE(6,5116)FC,EPSQ,DEADMO,DEADTO
0078      WRITE(6,6490)
0079      KK=0
0080      IDMO=0
0081      15 ILL=0
0082      ISKIP=0

C
C INPUT ASSUMED LONG. STRAIN PROFILE
C TAGMO=INITIAL SPECIFIED MOMENT VALUE FOR ANALYSIS
C THRO=SPECIFIED T/M RATIO FOR ANALYSIS
C DMO=SPECIFIED MOMENT INCREMENT
C INTV=MAXIMUM NO. OF INCREMENTS OF MOMENT REQUIRED
C FOR TERMINATION PROVIDE 2 BLANK CARDS
C EPCCT = LONG. STRAIN IN TOP FACE
C EPCCB = LONG. STRAIN IN BOTTOM FACE
C TTMM = VALUE OF MOMENT ABOVE WHICH DMO IS QUARTERED
C IOUT =OUTPUT INDEX : IF IOUT=0 ->COMPLETE OUTPUT IS PRINTED
C IF IOUT>0 SUMMARIZED OUTPUT ONLY PRINTED
C LSIMP = NO. OF POINTS FOR EVALUATION OF STRAINS ON SIDE FACES WHICH TAKES
C VALUES 2*N+1 FOR N=1,2,.... IF NOT GIVEN DEFAULT VALUE IS 5

0083      18 READ(5,5050)INTV,THRO,TAGMO,DMO,EPCCT,EPCCB,TTMM,IOUT,LSIMP
0084      IF(EPCCT.EQ.0.0.AND.EPCCB.EQ.0.0) GO TO 1000
0085      20 IJK=1
0086      IJK=IJK+1
0087      ILM=0
0088      INY=0
0089      IF(IJK.EQ.1) GO TO 16
0090      IF(TAGMOH,LF,TTMM.AND.IDMO.EQ.0) GO TO 16
0091      IF(IDMO.GE.1) GO TO 16
0092      IDMO=IDMO+1
0093      DMO=.25*DMO
0094      16 IF(IJK.LE.(INTV.AND.ISKIP.EQ.0))GO TO 25
0095      17 NJK=IJK
0096      IF(ISKIP.NE.0)NJK=NJK-1

C
C ** PRINT FINAL RESULTS SUMMARY **
C
0097      WRITE(6,6200)
0098      6200 FORMAT(1H1,21X,13HLONG. STRAINS,6X,12HHOOP STRAINS,5X,13HDIAG. STR
      +AINS,4X,13HSHEAR STRAINS,23X,13HPRINC. ANGLES,74X,517H*10**3,11X
      +J,7HDEGREES,78X,3HTOO,4X,3HMMOM,4(3X,3HTOP,3X,3HMID,3X,3HBOT),
      +2X,4HPSIX,2X,4HPHIX,2X,4HPHIZ,3X,3HTOP,3X,3HMID,3X,3HBOT)
0099      DO 1701 K=1,NJK
0100      XAR(1)=TEPL(K)*1000.0
0101      XAR(2)=SEPL(K)*1000.0
0102      XAR(3)=REPL(K)*1000.0
0103      XAR(4)=TEPH(K)*1000.0
0104      XAR(5)=SEPH(K)*1000.0
0105      XAR(6)=BEPH(K)*1000.0
0106      XAR(7)=TEPD(K)*1000.0
0107      XAR(8)=SEPD(K)*1000.0
0108      XAR(9)=REPD(K)*1000.0
0109      XAR(10)=TGAMA(K)*1000.0
0110      XAR(11)=SGAMA(K)*1000.0
0111      XAR(12)=BGAMA(K)*1000.0
0112      XAR(13)=PS(XH(K)*1000.0
0113      XAR(14)=PHIXB(K)*1000.0
0114      XAR(15)=PHIZB(K)*1000.0
0115      1701 WRITE(6,5150)K,TOO(K),MMOM(K),(XAR(J),J=1,15),TTHETA(K),STHETA(K),
      +BTHETA(K)
0116      5150 FORMAT(14,1X,2F7.2,1BF6.2)
0117      IF(ISPALL.EQ.1) GO TO 1002
0118      ISPALL=1
0119      GO TO 1
0120      25 IF(IJK.EQ.1)GO TO 26
0121      TAGMM=TAGMM+DMO
0122      TAGTOO=(TAGMM-DEADMO)*THRO+DEADTO
0123      TMMR=TAGTOO/TAGMM
0124      GO TO 28
0125      26 TAGMM=TAGMO
0126      TAGTOO=(TAGMM-DEADMO)+THRO+DEADTO
0127      TMMR=TAGTOO/TAGMM
0128      28 ITR=0
0129      MM=0
0130      MN=0
0131      IF(KK.GE.1)GO TO 77
C CHOOSE STEP SIZE FOR SIDE FACES
C L=DIVISIONS OF SIDE FACE (ODD NUMBER)
0132      L=5
0133      IF(LSIMP.NE.0)L=LSIMP
0134      LM=L-1
0135      LMID=0.5*LM+1
0136      DIVH=1./LM

C
C INITIALIZE ANALYSIS DIMENSIONS
C
0137      BOB=B1
0138      BOT=B1
0139      HJ=H1
0140      AO=0.81*B*BH
0141      TOB=TIN
0142      TOT=TIN
0143      AUB=2.*CHB
0144      ABT=2.*CHT
0145      THETAT=A45
0146      DO 35 K=1,L
0147      35 TD(K)=TIN

C
C EVALUATE CURVATURE AND STRAINS IN ALL LONG. STEEL
C
0148      40 PHIX=(EPCCB-EPCCT)/H
0149      50 EPLT=EPCCB-PHIX*(LM+CLB)
0150      EPLB=EPCCB-PHIX*CLB
0151      EPLBH=EPCCB-PHIX*CHB
0152      EPLTH=EPLBH-PHIX*H

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0153      DO 60 K=1,L
0154      EK=K-1
0155      60 EPL(K)=EPLB-DIVH*EK*H1*PHIX
0156      IF(IPRE.EQ.0)GO TO 63
0157      DO 62 K=1,NPRE
0158      62 EPP(K)=EPP0+EPLB-PHIX*YF(K)
0159      63 DO 64 K=1,N
0160      64 EPLS(K)=EPLB-PHIX*YS(K)

C
C EVALUATE RESULTANT FORCE DUE TO STRESSES IN LONG. STEEL (FSS)
C ESTABLISH THE POSITION AT WHICH THIS FORCE ACTS W.R.T. BOTTOM-CORNER
C LONG. REBARS (YSB)
C
0161      FPP1=0.0
0162      FPPH=0.0
0163      IF(IPRE.EQ.0) GO TO 73
0164      71 DO 72 K=1,NPRE
0165      71 F(K)=STF(AP(K),EP,EPP(K),FPU,FPY)
0166      FPP1=FPP1+F(K)
0167      FPPH=FPPH+FPX(K)
0168      72 FPPH=FPPH+FPX(K)
0169      73 FSI=0.0
0170      FSIH=0.0
0171      DO 75 K=1,N
0172      74 F(K)=STF(AS(K),ES(K),EPLS(K),EPLY(K),FLY(K),FU(K),EPSH(K),ESH(K))
0173      FSI=FSI+F(K)
0174      FSIH=FSIH+F(K)*YS(K)
0175      75 FSIH=FSIH+F(K)*YS(K)
0176      FSS=FPPH+FSI
0177      FSH=FSIH+FSI
0178      YSB=FSH/FSS
0179      77 KK=0
0180      CN=0.0
0181      IF(EPCCT.LT.0.0)CN=-EPCCT*0/(EPLB-EPCCT)

C
C EVALUATE SHEAR FLOW
C
0182      100 Q=FACT00/(2.*AO)
0183      100=0

C
C EVALUATE HOOP STRAINS AND CONCRETE STRAINS
C EVALUATE ANGLES FROM STRAINS
C
0184      150 CAHB=(0.5*ABH-CHB)/H1
0185      CAHT=(0.5*ABT-CHT)/H1
0186      MOD=1.-CAHB
0187      HOT=1.-CAHT
0188      152 CALL STRAIN(0,THETAB,TB,TDB,EPLB,CHB,EPHB,OMOSB,EPDSB,EPCHB,
0189      +INDEXB,ICD,HOB,CAHT,THETAT,GAMB,KK)
0190      153 CALL STRAIN(0,THETAT,TT,TDT,EPLT,CHT,EPHT,OMOST,EPOST,EPCHT,
0191      +INDEXT,ICT,HOT,CAHB,THETAB,GAMT,KK)
0192      IF(ICB.EQ.0.AND. ICT.EQ.0) GO TO 155
0193      IF(ITR.LE.2.AND.KK.LE.2) GO TO 155
0194      154 SKIP=1
0195      GO TO 20
0196      155 PHIZ=(EPHT-EPHT)/H1
0197      DO 160 K=1,L
0198      160 CALL STRAIN(0,THETA(K),TS ,TD(K),EPL(K),CHS,EPH(K),OMOS(K),
0199      +EPDS(K),EPCH(K),INDEX(K),IC(K),0.0,0.0,0.1,0,GAM(K),KK)

C
C EVALUATE TWIST
C
0197      GAMS=SIMP(GAM,L,DIVH,H1)
0198      360 PSIX=(GAMB*H1+GAMT*H1+2.0*GAMS)/(2.0*H1)

C
C EVALUATION OF DIAGONAL CURVATURES
C
0199      365 PHIDB=PSIX*SIN(2.*THETAB)-PHIX*(COS(THETAB))*2-PHIZ*(SIN(THETAB))
0200      PHIDT=PSIX*SIN(2.*THETAT)+PHIX*(COS(THETAT))*2+PHIZ*(SIN(THETAT))
0201      DO 375 K=1,L
0202      375 PHID(K)=PSIX*SIN(2.*THETA(K))

C
C CALCULATE TO-VALUES AND USE THESE TO GET NEW BETTER ESTIMATES
C
0203      400 KK=KK+1
0204      TDMIN=0/(0.375*FC)
0205      IF(PHIDB.LE.0.0)GO TO 403
0206      TDB1=EPOSB/PHIDB
0207      GO TO 404
0208      403 TDB1=TIN
0209      404 DED0=ABS(TDB1-TDB)/TDB
0210      TDB=0.5*(TDB+TDB1)
0211      IF(TDB.LT.TDMIN)TDB=1.05*TDMIN
0212      IF(PHIDT.LE.0.0) GO TO 405
0213      TDT1=EPOST/PHIDT
0214      GO TO 407
0215      405 TDT1=TIN
0216      407 DEDT=ABS(TDT1-TDT)/TDT
0217      TDT=0.5*(TDT+TDT1)
0218      IF(TDT.LT.TDMIN)TDT=1.05*TDMIN
0219      DO 415 K=1,L
0220      410 IF(PHID(K).EQ.0.0)GO TO 411
0221      TDS(K)=EPOS(K)/PHID(K)
0222      GO TO 412
0223      411 TDS(K)=2.0*TIN*KK
0224      412 DED(K)=ABS(TDS(K)-TD(K))/TD(K)
0225      TD(K)=0.5*(TD(K)+TDS(K))
0226      415 IF(TD(K).LT.TDMIN)TD(K)=1.05*TDMIN
0227      IF(DEDB.LE.0.0010.AND.OEDT.LE.0.0010.AND.DED(1).LE.0.0020.AND.DED(
0228      +LM10).LE.0.0020.AND.DED(LM).LE.0.0020)GO TO 430
0229      422 IF(KK.GT.7) GO TO 430
0230      GO TO 150

C
C DETERMINE PATH OF SHEAR FLOW AND EVALUATE AREA ENCLOSED AO
C
0231      430 ABH=ARB(OMOSB,TDB,TB,FC)
0232      ADT=ARB(OMOST,TDT,TT,FC)
0233      DO 432 K=1,L
0234      AB(K)=ARB(OMOS(K),TD(K),TS,FC)
0235      IF(THETA(K).EQ.0.0)AB(K)=TS
0236      432 BD(K)=H-AB(K)
0237      HD=H-0.5*(AB0+ABT)
0238      HOB=BD(1)
0239      HOT=BD(L)

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0239      DELH=DIVH*H0
0240      ADX=0.5*(B0(1)+B0(L))
0241      DO 435 K=2,L,M
0242      435 ADX=ADX+B0(K)
0243      AO=DELH*ADX
0244      QNEW=TAGTOQ/(2.*AO)
0245      IF (ABS(QNEW-0)/O.LE.0.001) GO TO 442
0246      O=0.5*(O+QNEW)
0247      IQQ=IQQ+1
0248      IF (IQQ.GE.4) GO TO 442
0249      KK=0
0250      GO TO 150

C
C
C      EVALUATE RESULTANT FORCE IN LONG. DIRECTION (FCC) DUE TO CONCRETE STRESSES
C
0251      442 PL0=Q/TAN(THETAB)
0252      IF (THETAT.EQ.0.0) GO TO 443
0253      PLT=Q/TAN(THETAT)
0254      GO TO 444
0255      443 PLT=(OMOST-OMDST**2/3.0)*FC*ABT
0256      444 DO 448 K=1,L
0257      IF (THETA(K).EQ.0.0) GO TO 447
0258      PL(K)=O/TAN(THETA(K))
0259      GO TO 448
0260      447 PL(K)=(OMDS(K)-OMDS(K)**2/3.0)*FC*TS
0261      448 CONTINUE
0262      450 FCC1=PLONG(PLB,PLT,PL,L,DIVH,B0B,BOT,H0)

C
C
C      EVALUATE NEW VALUE OF EPCCB FOR NEXT ITERATION
C
0263      ITR=ITR+1
0264      IF (ITR.EQ.8) GO TO 480
0265      MM=MM+1
0266      FDIFF(MM)=FSS-FCC1
0267      451 SST(MM)=EPCCB
0268      453 IF (ABS(FDIFF(MM)).LE.0.001+FSS) GO TO 480
0269      IF (ABS(FDIFF(MM)).LE.0.002+FSS.AND.(1M.LE.1)) GO TO 480
0270      IF (ABS(FDIFF(MM)).LE.0.003+FSS.AND.(ITR.GE.5)) GO TO 480
0271      454 IF (MM.LE.1) GO TO 461
0272      NN=MM-1
0273      IF (MM.GT.2) GO TO 455
0274      GO TO 465
0275      455 LL=MM-2
0276      457 IF (FDIFF(MM).EQ.FDIFF(NN).AND.FDIFF(MM).EQ.FDIFF(LL)) GO TO 461
0277      IF (ABS(1.0-FDIFF(MM)/FDIFF(NN)).LT.0.125.AND.EPCCB.GT.EPLY(1))
0278      *GO TO 461
0279      EPCCB=PARAB(FDIFF(MM),FDIFF(NN),FDIFF(LL),SST(MM),SST(NN),SST(LL),
0280      *FSS)
0281      GO TO 40
0282      461 EPCCB=GUESS(FSS,FDIFF(MM),SST(MM),EPLY(1))
0283      GO TO 40
0284      465 EPCCB=SST(MM)-FDIFF(MM)*((SST(MM)-SST(NN))/(FDIFF(MM)-FDIFF(NN)))
0285      IF (EPCCB.LT.SST(MM).AND.FDIFF(MM).LT.0.0) GO TO 461
0286      IF (EPCCB.GT.SST(MM).AND.FDIFF(MM).GT.0.0) GO TO 461
0287      GO TO 40

C
C
C      ESTABLISH THE POSITION AT WHICH FCC ACTS W.R.T. BOTTOM FACE LONG. REBARS
C
0288      480 DU=0.5*ABB-CLB
0289      WB=DU*PLB
0290      WT=(DU+H0)*PLT
0291      DO 500 K=1,L
0292      EK=K-1
0293      500 WW(K)=(DU+DIVH*EK*H0)*PL(K)
0294      WS=SIMP(WW,L,DIVH,H0)
0295      520 WSM=BOT*WT+BOT*WB+2.0*WS
0296      YC=WSM/FCC1

C
C
C      APPLIED ACTIONS
C
C      EVALUATION OF TORQUE AND FLEXURAL MOMENT
C
0297      600 T=2.0*Q*AO
0298      JD=YC-Y50
0299      FM=FSS*JD*(FSS-FCC1)*(Y0-YC)
0300      IF (FM.EQ.0.0) GO TO 603
0301      TM=T/FM
0302      GO TO 660
0303      603 TM=99999.0

C
C
C      EVALUATE NEW VALUE OF EPCCT FOR NEXT ITERATION
C
0304      660 II=II+1
0305      IIM=IIM+1
0306      IF (IIM.GT.7.OR.(SKIP.NE.0)) GO TO 700
0307      ECT(IIM)=EPCCT
0308      DMOM(IIM)=TAGMOM-FM
0309      IF (ABS(DMOM(IIM)).LE.0.002*TAGMOM) GO TO 700
0310      IF (ABS(DMOM(IIM)).LE.0.005*TAGMOM.AND.(1M.GE.5)) GO TO 700
0311      ITR=0
0312      MM=0
0313      IF (1M.GE.3) GO TO 680
0314      IF (1M.EQ.2) GO TO 670
0315      IF (ABS(EPCCT).GT.0.00005) GO TO 664
0316      EPCCT=-.0001*DMOM(IIM)/ABS(DMOM(IIM))
0317      FRC=DMOM(IIM)/TAGMOM
0318      IF (FRC.LT.-1.0) EPCCT=0.0002
0319      IF (FRC.GT.1.0) EPCCT=-0.0002
0320      GO TO 40
0321      664 EPCCT=GUESS(TAGMOM,DMOM(IIM),ECT(IIM),EPLY(1))
0322      IF (EPCCT.LT.-1.35*EPS0) EPCCT=-1.35*EPS0
0323      GO TO 40
0324      670 II=IIM-1
0325      FRC=DMOM(IIM)/(DMOM(IIM)-DMOM(II))
0326      IF (ABS(FRC).GT.2.0) FRC=2.0*FRC/ABS(FRC)
0327      EPCCT=EPCCT-FRC*(EPCCT-ECT(II))
0328      IF (EPCCT.LT.ECT(II).AND.DMOM(IIM).LT.0.0) GO TO 664
0329      IF (EPCCT.GT.ECT(II).AND.DMOM(IIM).GT.0.0) GO TO 664
0330      IF (EPCCT.LT.-1.35*EPS0) EPCCT=-1.35*EPS0
0331      GO TO 40
0332      680 II=IIM-1
0333      I=IIM-2
0334      IF (1M.EQ.3) GO TO 681
0335      IF (ABS(DMOM(IIM)).LT.DMST0) GO TO 681
0336      IMY=IMY+1
0337      IF (IMY.EQ.1) EPCCT=0.5*(ECT(IIM)+ECT(II))
0338      IF (IMY.EQ.2) EPCCT=0.25*ECT(IIM)+0.75*ECT(II)

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0337       IF(IHY.EQ.3) GO TO 700
0338       GO TO 40
0339       681 OMSTO=ABS(DMOM(IIM))
0340       IHY=0
0341       IF(ABS(1.0-DMOM(IIM)/DMOM(1)).LT..125.AND.EPLT.GT.EPLY(N))
0342       +GO TO 664
0343       EPCCT=PARAB(DMOM(IIM),DMOM(1),DMOM(1),EPCCT,ECT(1),ECT(1),
0344       +TAGMOM)
0345       683 IF(EPCCT.GT.-1.4*EPS0) GO TO 40
0346       IF(IIM.GE.5) ISKIP=1
0347       IF(ECT(IIM).EQ.-1.4*EPS0) GO TO 20
0348       EPCCT=-1.4*EPS0
0349       GO TO 40
0350
0351 C
0352 C CONVERT ANGLES FROM RADIANS TO DEGREES
0353 C
0354 700 ATHB=THETAB*DEGREE
0355       ATHT=THETAT*DEGREE
0356       ISKIP=ICR+ISKIP
0357       DO 710 K=1,L
0358       ISKIP=ISKIP+IC(K)
0359       710 ATH(K)=THETA(K)*DEGREE
0360       ISKIP=ISKIP+ICT
0361       IF(1OUT.NE.0) GO TO 745
0362
0363 C
0364 C ** DETAILED PRINT OUT OF RESULTS **
0365 C
0366 C
0367 723 WRITE(6,6550)
0368       WRITE(6,5140)Q,TAGTOQ,TAGMOM,HQ,AD,FSS,FCC1,Y5B,YC,TDMIN,CN
0369       WRITE(6,6660)
0370       WRITE(6,6670)KK,ITR,IIM,III,IJK,TMRO,DEADTO,DEADMO,EPCCB
0371       736 WRITE(6,6400)
0372       738 WRITE(6,5130)T,FM,TH,PSIX,PHIX,EPCCT,EPLT,EPLB,PHIZ
0373       WRITE(6,6500)
0374       WRITE(6,6600)
0375       WRITE(6,6620)ATHB,INDEXB,EPHB,EPLBH,ICB,EPOSB,EPCHB,GAMB,PHIDB,TDB
0376       +,ABB,ROB,TB
0377       DO 740 K=1,L
0378       740 WRITE(6,6630)K,ATH(K),INDEX(K),EPH(K),EPLL(K),IC(K),EPUS(K),EPCH(K)
0379       +,GAM(K),PHID(K),TD(K),AB(K),RO(K),TS
0380       WRITE(6,6610)ATHT,INDEXT,EPHT,EPLTH,ICT,EPDST,EPCHT,GAMT,PHIDT,TDT
0381       +,ART,BOT,TT
0382       WRITE(6,6480)
0383
0384 C
0385 C STORE RESULTS FOR PLOTTING AND PRINTING OF FINAL RESULTS IN SUMMARY
0386 C
0387 C
0388 745 IF(IJK.GT.1)GO TO 750
0389       TOQ(1)=0.0
0390       MOM(1)=0.0
0391       TEPL(IJK)=0.0
0392       SEPL(IJK)=0.0
0393       DEPL(IJK)=0.0
0394       TEPH(IJK)=0.0
0395       SEPH(IJK)=0.0
0396       REPH(IJK)=0.0
0397
0398       TEPO(IJK)=0.0
0399       SEPO(IJK)=0.0
0400       BEPO(IJK)=0.0
0401       TPHID(IJK)=0.0
0402       SPHID(IJK)=0.0
0403       BPHID(IJK)=0.0
0404       TGAMA(IJK)=0.0
0405       SGAMA(IJK)=0.0
0406       RGAMA(IJK)=0.0
0407       PSIXB(IJK)=0.0
0408       PHIXB(IJK)=0.0
0409       PHIZB(IJK)=0.0
0410       TTHETA(1)=0.0
0411       STHETA(1)=ATH(LM10)
0412       BTHETA(1)=90.0
0413
0414 750 IJS=IJK+1
0415       TOQ(IJS)=T
0416       MOM(IJS)=FM
0417       TEPL(IJS)=EPLTH-PHIX*TGH
0418       SEPL(IJS)=EPLL(LM10)
0419       BEPL(IJS)=EPLBH+PHIX*TGH
0420       TEPH(IJS)=EPHT-PHIZ*TGH
0421       SEPH(IJS)=EPH(LM10)
0422       BEPH(IJS)=EPHB+PHIZ*TGH
0423       TEPO(IJS)=EPOST-PHIOT*(CHT-TGH)
0424       SEPO(IJS)=EPOS(LM10)-PHID(LM10)*(CHS-TGH)
0425       BEPO(IJS)=EPOSB-PHI0B*(CHB-TGH)
0426       TPHID(IJS)=PHIDT
0427       SPHID(IJS)=PHID(LM10)
0428       BPHID(IJS)=PHIDB
0429       CALL GAMO(TEPL(IJS),TEPH(IJS),TEPO(IJS),TGAMA(IJS),TTHETA(IJS))
0430       CALL GAMO(SEPL(IJS),SEPH(IJS),SEPO(IJS),SGAMA(IJS),STHETA(IJS))
0431       CALL GAMO(BEPL(IJS),BEPH(IJS),BEPO(IJS),BGAMA(IJS),BTHETA(IJS))
0432       TTHETA(IJS)=TTHETA(IJS)*DEGREE
0433       STHETA(IJS)=STHETA(IJS)*DEGREE
0434       BTHETA(IJS)=BTHETA(IJS)*DEGREE
0435       PSIXB(IJS)=PSIX
0436       PHIXB(IJS)=PHIX
0437       PHIZB(IJS)=PHIZ
0438       IF(IJK.EQ.1) GO TO 20
0439       IF(TEPO(IJS).LT.1.35*EPS0.AND.BEPO(IJS).LT.1.35*EPS0) GO TO 30
0440       IJS=IJS-1
0441       IF(TEPO(IJS).LT.1.35*EPS0.OR.BEPO(IJS).LT.1.35*EPS0) GO TO 30
0442       +,LT,BEPO(IJS)) ISKIP=ISKIP+1
0443       GO TO 20
0444
0445 C
0446 C FORMAT STATEMENTS
0447 C
0448 5000 FORMAT(13,12,12,13,1X,A4,15A4)
0449 5050 FORMAT(15,6F10.0,2I5)
0450 5100 FORMAT((AF10.0))
0451 5110 FORMAT(2X,15,6F10.3,1P2E12.3)
0452 5115 FORMAT(6F10.3,2F10.7,3F10.3)
0453 5116 FORMAT(F10.3,F10.5,2F10.3)
0454 5130 FORMAT(3F10.3,1P6E15.4)
0455 5140 FORMAT(13F10.3)
0456 6000 FORMAT(1H1/21X,30(1H*)/
0457       +21X,30H* STRUCTURAL CONCRETE

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121X,30H* BEAM IN TORSION & BENDING */
121X,30H* DIAGONAL COMPRESSION FIELD */
121X,30H* ANALYSIS JULY 1978 */ 21X,30(1H*)
0431 6050 FORMAT(1H0,18X,11HBEAM NO. - ,A4,5X,10H SOURCE - ,15A4,/,
118X,7H=====,15X,6H=====)
0432 6100 FORMAT(1H0,4X,1HB,10X,1HH,9X,2HTB,8X,2HTY,8X,2HTS,
+8X,2HB1,8X,2HM1,8X,3HCLB,7X,3H/ 0,7X,3HTIN)
0433 6110 FORMAT(1H0,5X,1H1,5X,5HAS(1),5X,5HYS(1),4X,6HFLY(1),4X,
+5HES(1),5X,6HESH(1),4X,7HEPLY(1),5X,7HEPSH(1))
0434 6115 FORMAT(1H0,5X,1H1,5X,5HAP(1),5X,5HYP(1),7X,3HFPY,7X,3HFPU,7X,3HFP1
+7X,2HEP,8X,4HEPPY,8X,4HEPP0)
0435 6120 FORMAT(1H0,5X,2HAH,8X,2HSH,8X,3HFHY,5X,3HFUH,7X,3HESS,7X,4HESHH,6X
+5HEPSHH,5X,4HEPHY,8X,3HCHT,7X,3HCHB,7X,3HCHS)
0436 6125 FORMAT(1H0,5X,2HFC,8X,4HEPS0,4X,6HDEADMO,4X,6HDEADTO)
0437 6400 FORMAT(1H0,6X,1HT,9X,1HM,9X,3HT/M,7X,4HPS1X,11X,4HPH1X,11X,5HEPCCT
+10X,4HEPLT,11X,4HEPLB,11X,4HPLI2)
0438 6480 FORMAT(1H0,2X,128(1H,1))
0439 6490 FORMAT(5X,105(1H-1))
0440 6500 FORMAT( 2X,10(1H=),93(1H*),15(1H=))
0441 6510 FORMAT( 2X,100(1H=))
0442 6530 FORMAT(1H0,6X,1H0,4X,6HTAGTOQ,4X,6HTAGMOM,7X,2HHO,8X,2HAD,8X,3HFSS
+7X,3HFCC,7X,3HYS8,7X,2HVC,8X,5HTDHN,6X,2HCN1)
0443 6600 FORMAT(1H0,5HLEVEL,2X,5HPRINC,2X,5HINOEX,3X,4HHOOP,8X,5HLONG,4X,
+4HINO,4X,16HCONCRETE STRAINS,7X,5HSHCAR,10X,8HDIAGONAL,8X,7HSTR.B
+LL,5X,7H ACTIVE/7X,5HANGLE,3X,4HHOOP,3X,6HSTRAIN,6X,6HSTRAIN,3X,4H
+CONC,4X,7HSURFACE,3X,8HAH-LEVEL,6X,7HSTRAIN,6X,5HCURV,2X,8HTHI
+CK,TD,2X,8HDEPTH,1X,3X,5HW10TH,3X,6HTHICK,1)
0444 6610 FORMAT(2X,1TOP,1F9.4,1A,1P2E12.3,1A,1PAE12.3,0P2F9.4,2F9.3)
0445 6620 FORMAT(1X,1ROT,1F9.4,1A,1P2E12.3,1A,1PAE12.3,0P2F9.4,2F9.3)
0446 6630 FORMAT(15,F9.4,1A,1P2E12.3,1A,1PAE12.3,0P2F9.4,2F9.3)
0447 6660 FORMAT(1H0,2X,2HKK,2X,3HITR,2X,3H11M,2X,3H111,2X,3H1JK,6X,4HTMRO,
+4X,6HDEADTO,4X,6HDEADMO,8X,5HEPCCB)
0448 6670 FORMAT(515,3F10.3,1P2E15.4)
0449 999 STOP
0450 END

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C
C FUNCTIONS AND SUBROUTINES
0001 C FUNCTION STF(AS,ES,EPS,EPY,FY,FULT,EP SH,ESH)
C
C FUNCTION TO COMPUTE FORCE IN STEEL BAR GIVEN STRAIN
C
0002 IF(ESH.GT.0.0.OR.FULT.EQ.FY)GO TO 10
0003 IF(EPS.LT.0.0)GO TO 5
0004 R=ALOG10(2.0)/ALOG10(FULT/FY)
0005 STF=AS*ES*EPS*(1.+(ES*EPS/FULT)**R)**(-1./R)
0006 RETURN
0007 5 STF=AS*ES*EPS
0008 IF(EPS.LT.-EPY)STF=-AS*FY
0009 RETURN
0010 10 STF=AS*ES*EPS
0011 IF(EPS.GE.EPY.AND.EPS.LE.EPSH)STF=AS*FY
0012 IF(EPS.GT.EPSH)STF=AS*(FY+(EPS-EP SH)*ESH)
0013 IF((FY*(EPS-EP SH)*ESH).GT.FULT)STF=AS*FULT
0014 IF(EPS.LT.0.0.AND.ABS(EPS).GE.EPY)STF=-AS*FY
0015 RETURN
0016 END

0001 C FUNCTION STP(AP,EP,EPS,FPU,FPY)
C
C FUNCTION TO COMPUTE THE FORCE IN TENDON
C NOTE FUNCTION NOT VALID FOR -VE STRAINS
C
0002 IF(FPU.EQ.FPY)GO TO 10
0003 IF(EPS.GT.0.0.AND.FPU.GT.FPY)GO TO 20
0004 C CASE OF -VE STRAIN IN TENDON
0005 WRITE(6,30)EPS
0006 30 FORMAT(5X,'**TENDON IS IN COMPRESSION**EPS=',F12.6)
0007 STP=0.0
0008 RETURN
0009 20 R=ALOG10(2.0)/ALOG10(FPU/FPY)
0010 STP=AP*EP*EPS*(1.+(EP*EPS/FPU)**R)**(-1./R)
0011 RETURN
0012 C BILINEAR CASE
0013 10 EPSY=FPY/EP
0014 STP=AP*EP*EPS
0015 IF(EPS.GT.EPSY)STP=AP*FPY
0016 RETURN
0017 END

0001 C FUNCTION FLONG(PB,PT,PL,L,DIVH,XB,XT,XS)
C
C FUNCTION TO COMPUTE RESULTANT CONCRETE FORCE IN LONG. DIRECTION
C
0002 REAL PL(10)
0003 PBX=PB*XB
0004 PTX=PT*XT
0005 IF(L.EQ.1)GO TO 10
0006 PSX=SIMP(PL,L,DIVH,XS)
0007 GO TO 20
0008 10 PSX=PL(1)*XS
0009 20 FLONG=PBX+PTX+2.*PSX
0010 RETURN
0011 END

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0001      SUBROUTINE STRAIN(O,THETA,THK,TD,EPL,CH,EPH,OMDS,EPDS,EPCH,
COMMON /BLOCK1/FC,EP50,TH,ESS,ESHH,FHY,FUH,EP5HM
0002      DIMENSION X(20),FX(20),FPRIME(20),FXX(20),XX(20)
0003
C
C
C
C
0004      I=0
0005      K=0
0006      KK=0
0007      TNN=TAN(ANG)
0008      RO=TNN*Y2
0009      EPHY=PHY/ESS
0010      FHYTH=FHY*TH
0011      IF( (TR.EQ.0) ) GO TO 5
0012      ETA=EPH/EPHY
0013      OMDSI=OMDS
0014      GAMAI=GAMA
0015      THETA1=THETA
0016      5 F=(TD-CH)/TD
0017      IF(F.LE.0.0)F=0.0
0018      7 F2=F**2
0019      TR=0.0
0020      IF(TD.GT.THK)TR=(TD-THK)/TD
0021      TR2=1.-TR**2
0022      TR3=1.-TR**3
0023      9 RR=0.0
0024      RY1=1.
0025      CO=FHYTH/O
0026      C1=CO
0027      IF(Y1.EQ.0.0)GO TO 10
0028      RR=RO/Y1
0029      HY1=Y1
0030      C1=C1/Y1
0031      10 C1=C1-RR
0032      C12=C1**2
0033      C11=1.-C12
0034      QTD=O/(FC*TD)
0035      OML=EPL/EP50
0036      OMH=EPHY/EP50
0037      35 BH=3.*TR2/TR3
0038      BOUND1=(1.+RR)/C1
0039      IF(OML.LT.0.0)GO TO 37
0040      IF(RR.EQ.0.0) GO TO 36
0041      BOUND2=BOUNDP(C1,RR,OML,OMH)
0042      GO TO 38
0043      36 BOUND2=(OML/(OMH*C12))**(.1/.3.)
0044      GO TO 38
0045      37 BOUND2=0.0
0046      38 BD1F=BOUND2-BOUND1
0047      IF(BOUND2.LT.1.0.AND.BOUND1.LT.1.0)GO TO 60
0048      CX=C1
0049      CX2=C12
0050      CXM=C11
0051      CXM=C1*FUH/FHY-RR
0052      IF(ESHH.EQ.0.0)R=ALOG10(2.0)/ALOG10(FUH/FHY)
0053
0054      41 K=K+1
0055      IF(K.GE.10)GO TO 55
0056      42 CC=3.*OTD*CX/(TR3*CX)
0057      BBC=BB**2-4.*CC
0058      IF(BBC.GE.0.0) GO TO 45
0059      IF(CX.EQ.CXM) GO TO 43
0060      CX2=CX**2
0061      CXM=1.-CX2
0062      GO TO 42
0063      43 OMD5=0.5*BB
0064      TD=12.*O*CXX/(BB**2*FC*TR3*CX)
0065      F=(TD-CH)/TD
0066      IF(F.LE.0.0)F=0.0
0067      INDEX=4
0068      IC=1
0069      GO TO 111
0070      45 SOQ=SQRT(BBC)
0071      OMD5=0.5*(BB-SOQ)
0072      EPDS=OMDS*EP50
0073      EPCH=F*EPDS
0074      EPH=(EPL+EPCH*(1.-CX2))/CX2
0075      IF(EPH.GE.EPHY.OR.I.EQ.1)GO TO 50
0076      GO TO 60
0077      50 IF(FHY.EQ.FUH)GO TO 55
0078      I=1
0079      IF(ESHH.EQ.0.0)GO TO 54
0080      IF(EPH.LE.EPSHH) GO TO 55
0081      CY=C1*((EPH-EP5HM)*ESHH/FHY+1.)-RR
0082      IF(EPH.LT.EPSHH)CY=C1
0083      IF(CY.GT.CXM)CY=CXM
0084      53 IF(ABS((CY-CX)/CY).LE.1.0E-3) GO TO 55
0085      CX=0.5*(CX+CY)
0086      IF(CY.EQ.C1.AND.CX.GT.C1)CX=0.5*(CX+CY)
0087      CX2=CX**2
0088      CXM=1.-CX2
0089      GO TO 41
0090      54 PHN=ES*EPH*(1.+(ES=EPH/FUH)*R)**(-1./R)
0091      CY=C1*PHN/FHY-RR
0092      IF(EPH.LT.EPHY)CY=C1
0093      GO TO 53
0094      55 IF(K.GE.10)WRITE(6,655)O,TD,BOUND1,BOUND2,EPH,EPL,EPDS,CX,CY
0095      655 FORMAT(1H0,'** NO CONVERGENCE AFTER TEN ITERATIONS **',/1P9E12.3,
+/'** HOOPS IN POST-YIELD RANGE IN SUBROUTINE STRAIN **')
0096      CALL GAMQ(EPL,EPH,EPCH,GAMA,THETA)
0097      INDEX=0
0098      IC=0
0099      IF(EPH.GT.EPHY.AND.EPH.LE.EPSHH) INDEX=1
0100      IF(EPH.GT.EPSHH)INDEX=2
0101      IF(OMDS.GT.1.5)IC=2
0102      IF( (TR.GE.1) )GO TO 140
0103      RETURN
0104      60 I=1
0105      L=1
0106      INDEX=0
0107      IB=0
0108      IC=0
0109      JJ=0

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0110      IF(F.LE.0.1)GO TO 160
0111      OMDX=0.5*BD
0112      IF(OTD.LT.0.375) GO TO 62
0113      61 INDEX=8
0114      GO TO 86
0115      62 IF(EPL.LE.0.0) GO TO 65
0116      OTDTR=4.0*OTD*TR3/(3.0*TR2**2)
0117      SQQ=1.0-2.0*OTDTR
0118      IF(SQQ.LT.0.0) GO TO 61
0119      EPH=BOUND1*EPHY
0120      DEPHL=ABS(EPH-EPL)
0121      IF(DEPHL/EPL.GE.0.1.OR.DEPHL/EPH.GE.0.1) GO TO 65
0122      OMD5=OMDX*(1.0-SQRT(SQQ))
0123      EPSO=EPSO+OMD5
0124      EPCH=F*EPSO
0125      TANT=SQRT((EPL+EPCH)/(EPH+EPCH))
0126      IF(ABS(TANT-1.0).GT.0.02) GO TO 65
0127      THETA=ATAN(TANT)
0128      GAMA=2.0*SQRT((EPL+EPCH)*(EPH+EPCH))
0129      RETURN
0130      65 IF(BOUND2.LT.BOUND1)GO TO 70
0131      I1=1
0132      GO TO 71
0133      70 I1=2
0134      71 X(I1)=BOUND1+0.5*BDIF
0135      IF(BOUND2.EQ.0.0)X(I1)=0.001
0136      X10=X(I1)
0137      IF(1TR.EQ.0) GO TO 81
0138      IF(ETA.GE.1.0.OR.ABS(ETA-X10).GE.0.5*ABS(BDIF))GO TO 81
0139      IF(1TR.GE.1)X(I1)=0.5*(X(I1)+ETA)
0140      IF(1TR.GE.2)X(I1)=0.5*(X(I1)+ETA)
0141      GO TO 81
0142      74 IB=IB-1
0143      75 X(I1)=0.5*(X(I1)+BOUND2)
0144      IB=IB+1
0145      GO TO 81
0146      76 X(I1)=0.5*(X(I1)+BOUND1)
0147      L=4
0148      GO TO 81
0149      77 JJ=JJ+1
0150      J1=JJ-2
0151      J2=JJ-2
0152      IF(JJ.GE.20)L=2
0153      XX(JJ)=X(I1)
0154      IF(JJ.GE.3) GO TO 78
0155      X(I1)=X(I1)-0.025*BDIF
0156      IF(JJ.EQ.1) GO TO 80
0157      IF(JJ.EQ.2.AND.ABS(1.-FX(I1)/FXX(J1)).LT.0.25)X(I1)=X(I1)-0.075*BDIF
0158      GO TO 79
0159      78 IF(ABS(1.-FX(I1)/FXX(J1)).LT.0.25) GO TO 781
0160      X(I1)=PARAB(FX(I1),FXX(J1),FXX(J2),XX(JJ),XX(J1),0.0)
0161      IF(ABS(X(I1)-XX(JJ)).GT.0.10*ABS(BDIF))X(I1)=XX(JJ)-0.10*BDIF
0162      GO TO 79
0163      781 X(I1)=XX(JJ)-0.10*BDIF
0164      79 IF(FX(I1).GT.FXX(J1)) GO TO 80
0165      L=2
0166      IF(JJ.GE.3) GO TO 80
0167      X(I1)=X(I1)+0.0375*BDIF

0168      80 FXX(JJ)=FX(I1)
0169      81 IF(X(I1).LT.1.0E-6) GO TO 150
0170      C1=X(I1)*C1-RR
0171      PROO1=C1**2
0172      DD2=1.+PROO1
0173      DD1=1.-PROO1
0174      PROO2=PROO1*X(I1)*OMH
0175      DD3=PROO2-OML
0176      IF(ABS(F*DD1)*2.0.GT.ABS(DD3))GO TO 83
0177      IF(JJ.EQ.0.AND.L.EQ.1.AND.(B.EQ.0) GO TO 74
0178      WRITE(6,881)I,X(I1),0.0,BOUND1,BOUND2,EPL,DD1,DD3
0179      681 FORMAT(2X,'** OMD5=GT.2.0 **',15,1PBE12.3,'/'** HOOPS IN ELASTIC RA
0180      OMD5=2.0
0181      INDEX=9
0182      GO TO 100
0183      83 OMD5=DD3/(F*DD1)
0184      IF(JJ.NE.0.AND.L.NE.1)GO TO 85
0185      IF(OMD5.GT.0.0.AND.OMD5.LE.1.5) GO TO 85
0186      IF(L.NE.4.AND.(B.EQ.0) GO TO 74
0187      85 OMD52=OMD5**2
0188      GMM=C1/DD2
0189      FX(I1)=TR2*OMD5*GMM-TR3*OMD52*GMM/3.-OTD
0190      IF(ABS(FX(I1)).LE.1.0E-6*OTD) GO TO 100
0191      IF(L.EQ.1.AND.FX(I1).LT.-.01*OTD.AND.(B.NE.1) GO TO 77
0192      IF(EPL.LT.0.0.AND.OMD5.LT.-OML)GO TO 76
0193      DIVGM=C1*DD1/(DD2**2)
0194      DIVOM=(2.*C1*C1*X(I1)+PROO1)*OMH/(F*DD1)+2.*OMD5*C1*C1/DD1
0195      FPRIME(I1)=TR2*(DIVOM*GMM+OMD5*DIVGM)-TR3*(2.*OMD5*DIVOM*GMM+
0196      +OMD52*DIVGM)/3.
0197      IF(ABS(FPRIME(I1)).LE.0.0001*OTD) GO TO 88
0198      IF(I1.EQ.1.AND.FPRIME(I1).LT.0.0) GO TO 87
0199      IF(I1.EQ.2.AND.FPRIME(I1).GT.0.0) GO TO 87
0200      IF(L.EQ.4) GO TO 87
0201      IF(L.EQ.1.AND.(B.LE.1) GO TO 75
0202      INDEX=7
0203      GO TO 87
0204      86 OMD5=OMDX
0205      EPSO=OMD5*EPSO
0206      EPCH=EPSO*F
0207      TO=0*TR3/(0.375*FC*TR2**2)
0208      GO TO 120
0209      87 DELTX=FX(I1)/FPRIME(I1)
0210      GO TO 89
0211      88 IF(FX(I1).LT.0.0.AND.OMD5.GE.1.4) GO TO 86
0212      DELTX=-X(I1)*FX(I1)/OTD
0213      IF(I1.EQ.1)DELTX=-DELTX
0214      89 I=I+1
0215      L=3
0216      J=-1
0217      IF(ABS(DELTX).GT..03*ABS(BDIF).AND.JJ.NE.0)
0218      +DELTX=DELTX*0.03*ABS(BDIF/DELTX)
0219      IF(ABS(DELTX).GT..10*ABS(BDIF).AND.JJ.EQ.0)
0220      +DELTX=DELTX*0.10*ABS(BDIF/DELTX)
0221      X(I1)=X(I1)-DELTX
0222      ERROR=ABS(DELTX)/X(J)
0223      IF(I.GE.8) GO TO 99
0224      GO TO (90,92),11

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0222 90 IF(X(1).LT.BOUND2.AND.X(1).GT.BOUND1)GO TO 95
0223 INDEX=5
0224 GO TO 100
0225 92 IF(X(1).LT.BOUND1.AND.X(1).GT.BOUND2)GO TO 95
0226 INDEX=6
0227 GO TO 100
0228 95 IF(1.LE.2) GO TO 96
0229 IF(ERROR.LE.1.0E-4.AND.ABS(OMDS-OMDS5)/OMDS.LE.1.0E-4) GO TO 100
0230 OMD5=OMDS
0231 96 IF(ABS(FX(J)).GT.1.0E-5*OTD.OR.ERROR.GT.1.0E-5) GO TO 81
0232 GO TO 100
0233 99 INDEX=3
0234 100 EPDS=OMDS*EPSO
0235 EPC=EPDS
0236 EPH=X(1)*EPHY
0237 IF(EPH.LT.0.0)EPH=0.0
0238 IF(EPH.GE.EPHY)INDEX=1
0239 CALL GAMO(EPL,EPH,EPC,GAMA,THETA)
0240 IF(THETA.EQ.0.0)GO TO 150
0241 105 IF(OMDS.GT.1.5)IC=2
0242 IF(ITR.GE.1)GO TO 140
0243 RETURN
0244 111 EPDS=OMDS*EPSO
0245 EPC=EPDS
0246 EPH=(EPL+EPC*(1.-CX2))/CX2
0247 IF(EPH.LT.EPHY)EPH=EPHY
0248 GO TO 130
0249 120 IC=4
0250 W1=EPL+2.*EPC
0251 W2=0.75*FC*TD*(EPL+EPC)/(TH*ESS)
0252 W3=W1**2+4.*W2
0253 IF(W3.GT.0.0) GO TO 125
0254 GO TO 150
0255 125 EPH=0.5*(SORT(W3)-W1)
0256 IF(EPH.LT.0.0)EPH=0.0
0257 130 CALL GAMO(EPL,EPH,EPC,GAMA,THETA)
0258 IF(THETA.EQ.0.0)GO TO 150
0259 IF(ITR.EQ.0)RETURN
0260 GO TO 145
0261 140 OMD5=0.5*(OMDS+OMDS1)
0262 EPDS=OMDS*EPSO
0263 145 GAMA=0.5*(GAMA+GAMA1)
0264 THETA=0.5*(THETA+THETA1)
0265 RETURN
0266 150 EPH=0.0
0267 OMD5=-OML
0268 EPDS=-EPL
0269 EPC=EPDS
0270 GAMA=0*EP50/(FC*THK*0.5)
0271 THETA=0.0
0272 IC=0
0273 RETURN
0274 160 EPH=BOUND2*EPHY
0275 IF(EPH.EQ.0.0)GO TO 150
0276 THETA=ATAN(SORT(EPL/EPH))
0277 GAMA=2.0*SORT(EPL*EPH)
0278 CC=8.0*OTD*(EPL+EPH)/(3.0*GAMA)
0279 IF(CC.GT.1.0) GO TO 162

0280 OMD5=1.5*(1.-SORT(1.-CC))
0281 GO TO 164
0282 162 OMD5=1.5
0283 164 EPDS=OMDS*EPSO
0284 EPC=0.0
0285 IC=6
0286 RETURN
0287 END

```

```

0001 FUNCTION BOUNDP(C1,RR,OML,OMH)
0002 DIMENSION X(20),FX(20),FPRIME(20)
0003 C2=C1**2
0004 RR2=RR**2
0005 OMR=OML/OMH
0006 I=1
0007 X(I)=(OMR/C2)**(1./3.)
0008 1 X1=X(I)
0009 X2=X1**2
0010 X3=X1**3
0011 FX(I)=X3*C2-2.*X2*C1*RR+X1*(RR2-OMR)
0012 FPRIME(I)=3.*X2*C2-4.*X1*C1*RR+RR2
0013 I=I+1
0014 J=I-1
0015 IF(ABS(FPRIME(J)).LE.ABS(FX(J)))GO TO 5
0016 DELTX=FX(J)/FPRIME(J)
0017 GO TO 10
0018 5 DELTX=X(I)*FX(I)/OMR
0019 IF(ABS(DELTx).GT.0.200*X(I))DELTx=DELTx*0.200*ABS(X(I)/DELTx)
0020 X(I)=X(J)-DELTx
0021 IF(1.GT.10)GO TO 12
0022 IF(X(J).LE.1.0E-8) GO TO 1
0023 ERROR=ABS(DELTx)/X(J)
0024 IF(ERROR.GT..0001)GO TO 1
0025 GO TO 15
0026 12 WRITE(6,13)
0027 13 FORMAT(1H0,10X,'** NO CONVERGENCE AFTER 10 ITERATIONS IN FUNCTION
0028 +BOUNDP **')
0029 15 BOUNDP=X(I)
0030 IF(BOUNDP.LT.0.0) GO TO 20
0031 RETURN
0032 20 BOUNDP=0.0
0033 RETURN
0034 END

```

```

SUBROUTINE GAMQ(EPL,EPH,EPD,GAMA,THETA)
FUNCTION TO COMPUTE SHEAR STRAIN AND ANGLE OF PRINCIPAL COMPRESSION GIVEN
LONG., TRANSVERSE, AND PRINCIPAL COMPRESSION STRAINS AT A POINT IN A PLANE
0001
0002 ELD=EPL+EPD
0003 IF (ELD.LE.0.0) GO TO 160
0004 EHD=EPH+EPD
0005 GAMA=2.*SORT(ELD*EHD)
0006 THETA=ATAN(SORT(ELD/EHD))
0007 RETURN
0008 160 THETA=0.0
0009 GAMA=0.0
0010 RETURN
0011 END

0001 FUNCTION ARB(OMDS,TD,THK,FC)
FUNCTION TO COMPUTE DEPTH OF THE EQUIVALENT RECTANGULAR STRESS BLOCK FOR
TD LESS THAN OR = THE WALL THICKNESS
0002 IF (OMDS.GT.2.0) GO TO 10
0003 OMD2=OMDS**2
0004 BETA=(4.0-OMDS)/(6.0-2.0*OMDS)
0005 ARB=BETA*TD
0006 IF (ARB.GT.THK) ARB=ARB*(FC,THK,TD,OMDS,OMD2)
0007 RETURN
0008 10 ARB=TD*(2.-2./OMDS)
0009 IF (ARB.GT.THK) ARB=THK
0010 RETURN
0011 END

0001 FUNCTION ARBN(FE,THK,TD,OMDS,OMD2)
FUNCTION TO COMPUTE DEPTH OF THE EQUIVALENT RECTANGULAR STRESS BLOCK FOR
TD GREATER THAN THE WALL THICKNESS
0002 TDK=TD-THK
0003 TR=TDK/TD
0004 TR2=TR**2
0005 ALBT1=OMDS-OMD2/3.0
0006 ALBT2=TR*(OMDS-TR*OMD2/3.0)
0007 BETA1=(4.0-OMDS)/(6.0-2.0*OMDS)
0008 BETA2=(4.0-TR*OMDS)/(6.0-2.0*TR*OMDS)
0009 RF1=ALBT1*FE*TD
0010 RF2=ALBT2*FE*TDK
0011 ARBN=2.0*(0.5*RF1*BETA1*TD-RF2*(THK+0.5*BETA2*TDK))/(RF1-RF2)
0012 IF (ARBN.GT.THK) ARBN=THK
0013 RETURN
0014 END

0001 FUNCTION SIMP(X,L,DVH,H)
INTEGRATION USING SIMPSONS RULE
0002 REAL X(10)
0003 SUMX=0.0
0004 DO 350 K=2,L.2
0005 K1=K-1
0006 K2=X+1
0007 350 SUMX=SUMX+X(K1)+4.0*X(K)+X(K2)
0008 SIMP=D(VH)*H*SUMX/3.
0009 RETURN
0010 END

0001 FUNCTION GUESS(F,DF,STR,EPY)
FRC=DF/F
SIGN=FRC/ABS(FRC)
IF (ABS(FRC).GT.1.0) FRC=1.0*SIGN
IF (ABS(FRC).LT.0.1) FRC=0.1*SIGN
IF (STR.GT.EPY.AND.ABS(FRC).LT.0.2) FRC=0.2*SIGN
IF (STR.LT.0.0) GO TO 10
GUESS=STR*(1.0-FRC)
RETURN
10 GUESS=STR*(1.0+FRC)
RETURN
END

0001 FUNCTION PARAB(X3,X2,X1,Y3,Y2,Y1,F)
CONVERGENCE TECHNIQUES USED IN ITERATION ARE :
... FITTING A PARABOLA THROUGH 3 POINTS
... SECANT METHOD
0002 IF (ABS(1.-X3/X2).LT..125.AND.F.NE.0.0) GO TO 460
0003 DX3=X3-X1
0004 SX2=X3-X2
0005 SX1=X2-X1
0006 GRD23=(Y3-Y2)/(X3-X2)
0007 GRD21=(Y2-Y1)/(X2-X1)
0008 A=(GRD23-GRD21)/DX3
0009 B=(GRD21+5*X2-GRD23*SX21)/DX3
0010 PARAB=Y3-A*X3**2-B*X3
0011 IF (F.EQ.0.0) RETURN
0012 DY=X3-GRD23
0013 IF (ABS(PARAB-Y3).GT.ABS(DY)) GO TO 430
0014 GO TO 450
0015 430 PARAB=Y3-DY
0016 450 IF (X3.GT.0.0.AND.PARAB.GT.Y3) GO TO 460
0017 IF (X3.LT.0.0.AND.PARAB.LT.Y3) GO TO 460
0018 RETURN
0019 460 FRC=X3/F
0020 SIGN=FRC/ABS(FRC)
0021 IF (ABS(FRC).GT.0.5) FRC=0.5*SIGN
0022 IF (ABS(FRC).LT.0.05) FRC=0.05*SIGN
0023 IF (Y3.LT.0.0) GO TO 463
0024 PARAB=Y3*(1.-FRC)
0025 RETURN
0026 463 PARAB=Y3*(1.+0.5*FRC)
0027 RETURN
0028 END

```

INPUT DATA FOR TBU3

```

A 1 0 0 TRUS TORONTO-ONSONGO-UNITS:KN-CM
4A.0 37.6 6.02 6.22 6.38 4A.0 37.6 2.30
25.30 10.20 1.42 1.42 1.42 3.87 0.0 0.55
9.35 17.35 24.95 33.45 35.2 34.3 74.4 74.4
43.6 43.6 55.2 55.2 10420.0 10420.0 20510.0 20510.0
76.3 76.3 76.3 76.3 76.3 76.3 76.4 76.4
20510.0 20510.0 900.0 900.0 900.0 900.0 20510.0 20510.0
0.00013 0.00013 0.002 0.002 0.002 0.002 0.0151 0.0151
1.2 7.62 37.9 60.5 10450.0 10450.0 0.0154 0.0154
3.44 0.0031 3074.0 109.0 109.0 109.0 0.0154 0.0154
30.8 70.636 4000.0 750.0 -0.00024 0.0002 31000.0 31000.0
1.60 1.40 7.62 7.62 7.62 7.62 17.4 17.4
60 0.694 4000.0 750.0 -0.00025 0.0002 34000.0 34000.0

```

PRINTOUT OF INPUT DATA AND START OF OUTPUT OF THE
DETAILED PRINTOUT

```

*****
* STRUCTURAL CONCRETE *
* BEAM IN TORSION & BENDING *
* DIAGONAL COMPRESSION FIELD *
* ANALYSIS *
*****

```

BEAM NO. - TBU3 SOURCE - TORONTO-ONSONGO-UNITS:KN-CM

```

*****
A 48.000 M 37.600 TS 6.020 TT 6.380 TS 6.220 81 48.000 M1 37.600 CL8 2.300 D 35.500 TIN 7.930
1 45111 Y5111 PLY111 PU111 ES111 ESH111 EPLY111 EPSH111
1 25.500 0.0 43.600 75.400 19420.000 900.000 2.245E-03 6.130E-03
2 10.200 0.350 43.600 75.400 19420.000 900.000 2.245E-03 6.130E-03
3 1.420 0.350 55.200 76.300 20510.000 900.000 2.691E-03 2.000E-03
4 1.420 17.350 55.200 76.300 20510.000 900.000 2.691E-03 2.000E-03
5 1.420 24.950 55.200 76.300 20510.000 900.000 2.691E-03 2.000E-03
6 3.870 33.450 55.200 76.300 20510.000 900.000 2.691E-03 1.514E-02
AM 3M PHV FUM ESS ESHM EPSHM EPHY CNF CHD CHS
1 1.290 7.620 37.900 60.500 19450.000 520.000 0.0154000 0.0019093 0.0 0.0 0.0
PC EPSD EPADM DEATO DEATO
1 480 0.00310 1059.000 109.000
0 TAGTQO TAGNOM HQ AO P88 FCC Y88 YC TOWIN CN
0 0.275 763.936 4000.000 32.314 1390.047 130.578 130.491 -3.081 27.569 0.210 16.575
AX ITR IIM IJK IJL TMRD DEATO DEATO DEATO
1 3 3 3 1 0.696 109.000 1059.000 2.3037E-04
0 0.275 TAGTQO TAGNOM HQ AO P88 FCC Y88 YC TOWIN CN
763.495 4001.235 0.191 1.172E-05 PHIX 1.0863E-05 EPCC1 -1.5779E-04 EPL8 2.0530E-04 PHIZ 2.2288E-04
LEVEL PRINC INDOX MOOP LONG. CONC. CONCRETE STRAINS SHEAR STRAIN DIAGONAL STRAIN STR. d.L. ACTIVE
BOT. 33.2276 0 1.020E-04 2.306E-04 0 5.920E-05 5.920E-05 4.704E-04 3.448E-04 5.448E-04 3.248E-04 43.408 6.020
1 52.4081 0 1.062E-04 2.306E-04 0 7.491E-05 7.491E-05 4.704E-04 3.448E-04 5.448E-04 3.248E-04 43.408 6.020
2 47.9181 0 6.833E-05 2.306E-04 0 7.491E-05 7.491E-05 3.660E-04 1.121E-05 6.6643 4.4519 43.548 6.220
4 17.8998 0 2.602E-05 -7.740E-05 0 4.907E-05 4.907E-05 2.393E-04 1.109E-05 6.7415 4.5034 43.497 6.220
5 17.8998 0 2.602E-05 -7.740E-05 0 4.907E-05 4.907E-05 2.393E-04 1.109E-05 6.7415 4.5034 43.497 6.220
6 6.701 0 9.802E-06 -1.801E-04 0 1.827E-04 1.827E-04 4.519E-05 2.613E-05 70.0061 6.1206 41.673 6.220
TOP 6.701 0 9.802E-06 -1.801E-04 0 1.827E-04 1.827E-04 4.519E-05 2.613E-05 70.0061 6.1206 41.673 6.220

```

SUMMARY PRINTOUT

```

LONG. STRAINS      HOOP STRAINS      DIAG. STRAINS      SHEAR STRAINS      PRINC. ANGLES
*10**-1            *10**-1            *10**-1            *10**-1            *10**-1
FIB  MID  ROT  FIB  MID  ROT  FIB  MID  ROT  FIB  MID  ROT  PSIX  PSIX  PSIX  PSIX  PSIX  PSIX
1  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
2  7.63  40.01  -0.27  0.03  0.25  0.01  0.07  0.11  0.21  0.07  0.09  0.12  0.13  1.07  0.22  11.31  49.75
3  12.87  47.50  -0.24  0.03  0.11  0.01  0.11  0.16  0.26  0.15  0.12  0.16  0.44  0.69  1.74  1.34  30.32  19.70
4  14.07  44.75  -0.29  0.05  0.37  0.04  0.16  0.22  0.32  0.21  0.17  0.25  0.61  1.01  2.49  1.60  4.40  19.70
5  23.32  62.49  -0.32  0.06  0.84  0.05  0.20  0.28  0.48  0.27  0.22  0.34  0.78  1.14  3.09  1.85  20.17  32.75
6  24.50  70.01  -0.37  0.07  0.39  0.11  0.25  0.34  0.42  0.33  0.27  0.44  0.96  1.34  3.41  2.13  21.62  39.71
7  33.71  77.39  -0.40  0.08  0.67  0.14  0.29  0.39  0.52  0.39  0.33  0.56  1.14  1.61  4.51  2.37  0.63  32.74
8  34.93  85.01  -0.44  0.09  0.67  0.14  0.34  0.45  0.59  0.45  0.39  0.63  1.11  1.45  4.79  2.40  0.66  34.20
9  44.14  92.50  -0.47  0.11  0.67  0.21  0.39  0.50  0.67  0.52  0.45  0.62  1.32  2.04  6.03  2.34  0.74  35.74
10  43.42  90.44  -0.51  0.12  0.75  0.24  0.41  0.56  0.74  0.59  0.41  0.75  1.32  2.04  6.03  2.34  0.74  35.74
11  54.67  107.44  -0.55  0.13  0.81  0.27  0.45  0.61  0.81  0.65  0.47  1.07  1.49  2.55  7.53  1.12  0.41  36.43
12  59.89  114.76  -0.54  0.13  0.81  0.31  0.52  0.67  0.89  0.72  0.62  1.21  2.08  2.77  8.30  1.36  0.44  37.44
13  65.12  122.44  -0.62  0.16  0.94  0.34  0.57  0.73  0.96  0.79  0.69  1.34  2.27  3.33  9.07  1.40  0.74  38.44
14  70.34  129.77  -0.65  0.15  1.00  0.37  0.62  0.78  1.04  0.85  0.75  1.48  2.44  3.27  9.45  1.40  0.74  39.44
15  74.56  137.44  -0.69  0.17  1.06  0.41  0.66  0.84  1.12  0.92  0.81  1.62  2.66  3.91  10.64  1.27  1.04  40.44
16  79.78  144.75  -0.72  0.20  1.13  0.44  0.71  0.89  1.20  0.99  0.87  1.75  2.86  3.76  11.45  1.51  1.10  41.44
17  85.01  152.44  -0.75  0.22  1.19  0.48  0.76  0.95  1.29  1.07  0.94  1.91  3.05  4.01  12.25  1.59  1.19  42.44
18  91.23  159.76  -0.77  0.23  1.25  0.51  0.80  1.00  1.36  1.15  1.00  2.04  3.25  4.25  13.09  1.67  1.20  43.44
19  94.45  167.44  -0.83  0.24  1.32  0.55  0.85  1.05  1.44  1.21  1.07  2.21  3.44  4.51  13.71  1.24  1.25  44.44
20  101.68  174.76  -0.87  0.26  1.38  0.54  0.90  1.12  1.53  1.27  1.14  2.36  3.67  4.75  14.75  1.48  1.30  45.44
21  106.90  182.44  -0.90  0.27  1.44  0.62  0.94  1.17  1.61  1.36  1.21  2.52  3.88  5.02  15.60  1.72  1.35  46.44
22  112.12  190.77  -0.94  0.29  1.51  0.66  0.99  1.23  1.70  1.44  1.24  2.67  4.09  5.24  16.37  1.97  1.40  47.44
23  117.35  197.44  -0.94  0.30  1.57  0.67  1.04  1.29  1.77  1.53  1.35  2.84  4.30  5.54  17.35  1.61  1.45  48.44
24  122.57  205.31  -1.04  0.31  1.63  0.73  1.09  1.34  1.84  1.60  1.42  3.01  4.52  5.81  18.23  1.64  1.40  49.44
25  127.79  212.52  -1.05  0.32  1.70  0.77  1.13  1.40  1.91  1.67  1.49  3.13  4.74  6.09  19.13  1.53  1.30  50.44
26  133.02  220.92  -1.07  0.33  1.76  0.80  1.18  1.45  2.07  1.74  1.57  3.25  4.97  6.35  20.05  1.60  1.40  51.44
27  138.24  228.33  -1.13  0.35  1.82  0.84  1.23  1.51  2.17  1.85  1.65  3.51  5.19  6.63  20.99  1.72  1.54  52.44
28  143.46  235.04  -1.17  0.36  1.88  0.88  1.28  1.56  2.27  1.94  1.73  3.71  5.42  6.91  21.93  1.77  1.69  53.44
29  148.69  242.55  -1.21  0.37  1.94  0.92  1.33  1.63  2.37  2.02  1.81  3.90  5.64  7.19  22.87  1.71  1.71  54.44
30  153.91  250.96  -1.25  0.38  2.00  0.96  1.38  1.69  2.47  2.11  1.89  4.04  5.90  7.44  23.80  1.73  1.73  55.44
31  159.13  257.44  -1.30  0.39  2.04  1.00  1.43  1.75  2.59  2.21  1.98  4.29  6.15  7.74  24.74  1.73  1.73  56.44
32  164.35  264.76  -1.34  0.40  2.09  1.04  1.48  1.81  2.69  2.30  2.07  4.44  6.39  7.99  25.69  1.73  1.73  57.44
33  169.57  272.36  -1.37  0.41  2.11  1.09  1.53  1.86  2.73  2.31  2.15  4.60  6.59  8.24  26.64  1.73  1.73  58.44
34  174.79  279.99  -1.43  0.42  2.14  1.12  1.55  1.92  2.73  2.31  2.25  4.72  6.72  8.49  27.59  1.73  1.73  59.44
35  179.91  287.44  -1.47  0.43  2.18  1.17  1.62  2.05  2.85  2.35  2.34  4.85  6.90  8.74  28.54  1.73  1.73  60.44
36  185.03  294.76  -1.55  0.44  2.23  1.19  1.67  2.10  2.90  2.37  2.41  4.97  7.04  8.99  29.49  1.73  1.73  61.44
37  190.15  302.57  -1.62  0.45  2.32  1.21  1.72  2.15  2.95  2.37  2.47  5.07  7.17  9.19  30.44  1.73  1.73  62.44
38  195.27  309.99  -1.69  0.46  2.41  1.22  1.77  2.20  3.00  2.37  2.54  5.19  7.30  9.39  31.39  1.73  1.73  63.44
39  200.39  317.51  -1.77  0.47  2.51  1.25  1.85  2.25  3.05  2.37  2.61  5.31  7.41  9.59  32.34  1.73  1.73  64.44
40  205.51  324.93  -1.84  0.48  2.60  1.27  1.90  2.30  3.10  2.37  2.68  5.43  7.52  9.79  33.29  1.73  1.73  65.44
41  210.63  332.35  -1.92  0.49  2.69  1.29  1.95  2.35  3.15  2.37  2.75  5.55  7.63  9.99  34.24  1.73  1.73  66.44
42  215.75  339.77  -1.99  0.50  2.78  1.31  2.00  2.40  3.20  2.37  2.82  5.67  7.74  10.19  35.19  1.73  1.73  67.44
43  220.87  347.19  -2.07  0.51  2.87  1.33  2.05  2.45  3.25  2.37  2.89  5.79  7.85  10.39  36.14  1.73  1.73  68.44
44  225.99  354.61  -2.14  0.52  2.96  1.35  2.10  2.50  3.30  2.37  2.96  5.91  7.96  10.59  37.09  1.73  1.73  69.44

```

APPENDIX EEXPERIMENTAL DATA AND PLOTS

- E.1 TEST SPECIMENS
- E.2 EVALUATION OF STRAINS FROM EXPERIMENTAL DATA
- E.3 SUMMARY OF STRAINS AND LOADS OBTAINED FROM TEST DATA
- E.4 VARIATION OF STRAINS IN BEAM FACES
- E.5 PLOTS OF MEASURED STRAINS AND PREDICTED STRAINS AT MID-POINTS OF BEAM FACES AGAINST APPLIED TORQUE

E.1 Test Specimens

The stirrups used in making the steel cages were bent careful and their dimensions were checked after bending to ensure uniformity in the dimensions to within ± 2 mm.

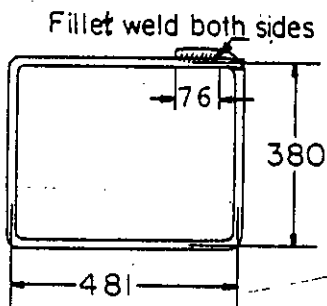


FIG. E.0

Three inches of overlap in the bent hoop were fillet welded as shown. In the fabrication of the cages the welded parts were in the top face and they were staggered on plan (see Fig. E.3).

To make the cages the hoops were tied to longitudinal steel, with tying wire so that the hoop legs were spaced at 76 mm centres. The central nine hoops were positioned with great care and using an accurately marked pattern plate positions were marked for placement of targets to form the grids

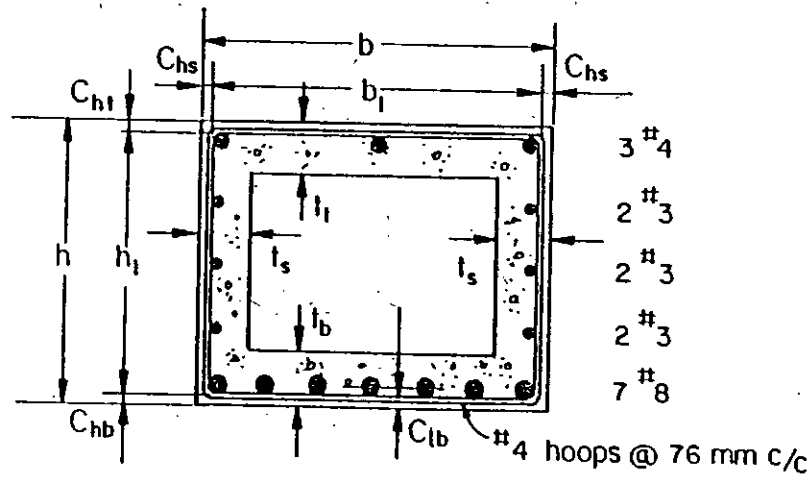
TABLE E.1 MEASURED DIMENSIONS

SERIES	b mm	h mm	b _l mm	h _l mm	t _b mm	t _s mm	t _t mm	c _{lb} mm	c _{hb} mm	c _{hs} mm	c _{ht} mm
TBO	508	411	480	380	76	76	81	39	16	14	15
TBU	508	410	480	378	76	76	80*	39	16	14	16
TBS**	508	413	482	381	206.5	254	206.5	39	16	13	16

* for TBU2 minimum $t_t = 60$ mm

* for TBU4 minimum $t_t = 55$ mm

** TBS series specimens were solid.

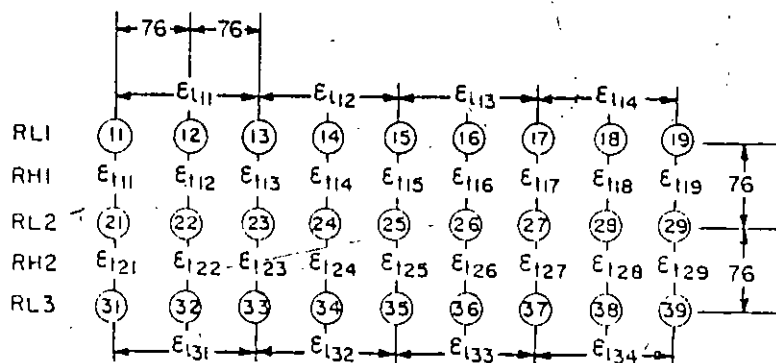


shown in Figure 4.17 and 4.18. The hoop bar ribs on these positions were filed off and cleaned in readiness for the placing of the targets used in the measurement of strains.

After the cages were complete the final overall measurements over the central 1 metre of the span were taken. Also after the casting of the specimens the outside dimensions of the specimens were measured. The average values of these measurements for each series of specimens are recorded in Table E.1.

E.2 Evaluation of Strains from Experimental Data

Figure E.1 is part of the grid of targets for measuring strains shown in Figure 4.15. Indicated are the strains measured.



DIAGONAL STRAINS WERE MEASURED OVER THE FOLLOWING TARGETS:

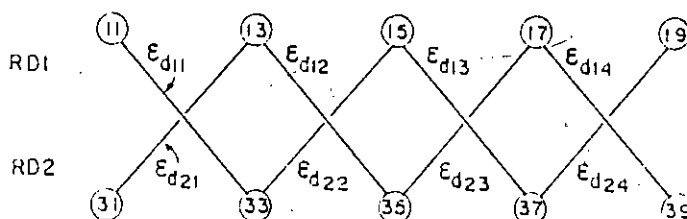


FIG. E.1 EVALUATION OF AVERAGE MEASURED STRAINS

A summary of all the strains measured for all the specimens is tabulated in Appendix E.3. With reference to

Fig. E.1 the average strains were evaluated as follows:

The average longitudinal strains for row RL1 is $\epsilon_{\ell 1} = \frac{1}{4} \sum_{i=1}^4 \epsilon_{\ell i}$

The average longitudinal strains for rows RL2 and RL3

were evaluated in a similar way.

The average transverse strain for row RH1 is $\epsilon_{t1} = \frac{1}{9} \sum_{i=1}^9 \epsilon_{tli}$

The average transverse strain for row RH2 was

similarly evaluated.

The average diagonal strain for row RD1 is $\epsilon_{d1} = \frac{1}{4} \sum_{i=1}^4 \epsilon_{dli}$

The average diagonal strain for row RD2 was similarly obtained.

The average longitudinal strain $\epsilon_{\ell} = \frac{1}{3}(\epsilon_{\ell 1} + \epsilon_{\ell 2} + \epsilon_{\ell 3})$;

The average transverse strain $\epsilon_t = \frac{1}{2}(\epsilon_{t1} + \epsilon_{t2})$; and the

average diagonal strains ϵ_{d1} and ϵ_{d2} constitute a set of

compatible strains which can be used to draw Mohr's circle of

strain. Note that average diagonal strains ϵ_{d1} and ϵ_{d2} are

measured in directions which are mutually perpendicular.

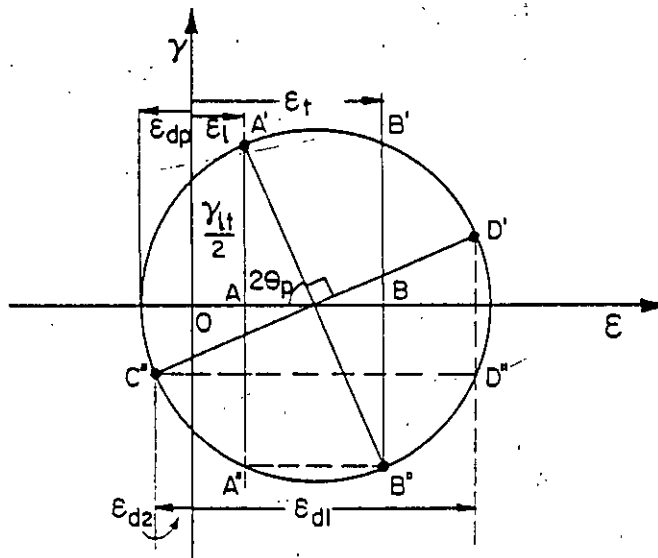


FIG. E.2 MOHR'S CIRCLE FOR MEASURED STRAINS

From the circle it can be seen that $\Delta A'A''B''$ is similar to $\Delta C''D''D'$ and hence $C''D''=A'A''=\gamma_{lt}$. If tension strains are taken to be positive then ϵ_{dp} will be negative; and assuming ϵ_{d2} is compressive, deduce: $\epsilon_{d1} + \epsilon_{d2} = \epsilon_l + \epsilon_t$ (E.1)
 and the shear strain γ_{lt} is evaluated as $\gamma_{lt} = \epsilon_{d1} - \epsilon_{d2}$ (E.2)
 Using equation (2.16) it follows that

$$\left(\frac{\gamma_{lt}}{2}\right)^2 = (\epsilon_l - \epsilon_{dp})(\epsilon_t - \epsilon_{dp})$$

Hence
$$\epsilon_{dp}^2 - (\epsilon_t + \epsilon_l)\epsilon_{dp} + \epsilon_t\epsilon_l - \left(\frac{\gamma_{lt}}{2}\right)^2 = 0$$

$$\begin{aligned}\epsilon_{dp} &= \frac{1}{2}[(\epsilon_t + \epsilon_l) - \sqrt{(\epsilon_t + \epsilon_l)^2 - 4[\epsilon_t\epsilon_l - \frac{\gamma_{lt}^2}{4}]}] \\ &= \frac{1}{2}(\epsilon_t + \epsilon_l) - \sqrt{(\epsilon_t - \epsilon_l)^2 + \gamma_{lt}^2} \dots\dots\dots (E.3)\end{aligned}$$

Thus from the measured diagonal strains the shear strain with respect to the transverse and longitudinal axes is directly obtained, using equation (E.2). With the shear strain determined and the measured average strains in the longitudinal and transverse directions, the principal compression strain is evaluated by equation (E.3). Finally the angle of principal compression is evaluated as

$$\tan 2\theta_p = \frac{\gamma_{lt}}{\epsilon_t - \epsilon_l} = \frac{\epsilon_{d1} - \epsilon_{d2}}{\epsilon_t - \epsilon_l} \dots\dots\dots (E.4)$$

E.3 SUMMARY OF STRAINS AND LOADS OBTAINED FROM TEST DATA

E.3.1 MEASURED TORQUE, FLEXURAL MOMENT, TWIST AND CURVATURE

In the following tables are listed the torque, moment, twist and curvature values measured for all the specimens for all the load stages. The units used are kilonewtons and meters. The angular deformations are given in radians per meter $\times 10^{-3}$. The twist was measured by the metrisite over 1.75 meters in the central part of the specimen; and by inclinometer readings on the top face over the central 0.61 meters of the beam. The curvature was measured over the central 1.75 meters and also the curvature was evaluated from the average longitudinal strains measured over the central 0.61 meters.

DATA FOR BEAM TB01

LS=	0	1	2	3	4	5	6	7	8	9	10	11
MOMENT	START	41.18	32.05	115.68	146.14	175.96	205.29	232.97	256.90	287.59	319.56	344.75
	END	45.18	73.28	111.56	140.45	170.01	198.88	225.69	249.42	278.17	307.24	331.93
CURV.	START	1.12	2.55	3.86	5.17	6.48	7.78	9.09	10.39	12.35	14.31	16.27
	END	1.12	2.56	3.93	5.24	6.54	7.85	9.22	10.72	12.48	14.44	16.27
STR.	START	1.12	2.00	2.91	3.64	4.45	5.22	6.06	6.95	8.05	9.06	10.47
	END	1.12	2.00	2.91	3.64	4.45	5.22	6.06	6.95	8.05	9.06	10.47

DATA FOR BEAM TB02

LS=	0	1	2	3	4	5	6	7	8
TORQUE	START	2.46	12.84	23.69	33.18	42.48	51.95	60.49	67.99
	END	2.46	11.87	22.16	31.03	40.06	49.14	57.34	64.56
MOMENT	START	49.43	88.55	129.42	165.20	200.21	235.90	268.07	296.33
	END	49.43	84.87	123.66	157.07	191.10	225.34	256.23	283.42
TWIST	START	0.01	0.11	0.51	1.16	1.95	2.75	3.35	3.90
	END	0.01	0.13	0.51	1.23	2.00	2.78	3.35	3.85
INCLN.	START	0.01	0.15	0.83	2.06	3.43	4.93	6.16	7.25
CURV.	START	1.22	2.06	3.10	4.15	5.19	6.24	7.28	8.33
	END	1.22	2.06	3.10	4.15	5.19	6.24	7.28	8.48
STR.	START	1.22	2.34	3.58	4.50	5.58	6.73	8.00	9.07
	END	1.22	2.34	3.58	4.50	5.58	6.73	8.00	9.07

DATA FOR BEAM TB03

LS=	0	1	2	3	4	5	6	7
TORQUE	START	4.95	24.59	46.52	69.32	96.08	115.36	132.84
	END	4.95	22.04	42.93	64.61	90.17	106.93	121.21
MOMENT	START	36.10	64.10	95.37	127.88	166.04	193.53	218.45
	END	36.10	60.47	90.25	121.16	157.61	181.51	201.87
TWIST	START	0.02	0.37	1.86	3.86	6.90	9.89	13.93
	END	0.02	0.37	1.91	4.01	7.20	10.39	14.83
INCLN.	START	0.02	1.25	3.71	6.85	11.64	16.29	22.71
CURV.	START	0.89	1.52	2.40	3.42	4.55	5.61	6.55
	END	0.89	1.48	2.36	3.34	4.55	5.50	6.42
STR.	START	0.89	0.95	2.48	3.57	4.61	5.56	6.52
	END	0.89	0.95	2.48	3.57	4.61	5.56	6.52

DATA FOR BEAM TB04

LS=	0	1	2	3	4	5	6	7	8	9	10
TORQUE	START	6.83	25.63	46.57	60.76	78.80	95.16	109.71	125.50	136.14	143.81
	END	6.83	23.12	41.31	54.59	70.72	84.29	98.70	112.51	124.36	132.00
MOMENT	START	22.12	34.58	48.46	57.86	69.82	80.66	90.31	100.77	107.82	112.90
	END	22.12	32.91	44.97	53.78	64.46	73.46	83.01	92.16	100.01	105.08
TWIST	START	0.02	0.23	2.02	4.11	7.06	10.05	12.99	17.03	21.02	25.01
	END	0.02	0.23	2.12	4.21	7.11	10.10	13.19	17.38	21.47	25.56
INCLN.	START	0.02	0.16	2.76	5.22	8.36	11.65	14.93	19.71	24.49	29.42
CURV.	START	0.55	0.70	1.38	1.59	1.54	1.54	1.38	1.17	0.96	0.86
	END	0.55	0.65	1.38	1.46	1.49	1.36	1.23	0.96	0.86	0.70
STR.	START	0.55	0.69	1.43	1.49	1.46	1.39	1.30	0.96	0.73	0.55
	END	0.55	0.69	1.43	1.49	1.46	1.39	1.30	0.96	0.73	0.55

DATA FOR BEAM TB05

LS=	0	1	2	3	4	5	6	7	8
TORQUE	START	11.66	37.58	49.58	62.88	76.86	90.60	111.15	129.81
	END	11.66	29.39	41.05	54.40	68.12	80.68	99.03	116.28
MOMENT	START	8.47	13.59	15.96	18.59	21.35	24.08	28.14	31.93
	END	8.47	11.97	14.28	16.92	19.63	22.11	25.74	29.16
TWIST	START	0.04	0.79	3.63	5.63	7.72	9.62	12.61	15.60
	END	0.04	0.84	3.66	5.66	7.72	9.72	12.68	15.75
INCLN.	START	0.04	1.41	4.83	7.70	10.43	13.17	21.64	24.79
CURV.	START	0.21	0.31	-0.31	-0.63	-1.12	-1.63	-2.43	-3.27
	END	0.21	0.31	-0.34	-0.64	-1.14	-1.63	-2.24	-3.25
STR.	START	0.21	0.15	-0.57	-1.02	-1.55	-2.10	-3.01	-4.18
	END	0.21	0.15	-0.57	-1.02	-1.55	-2.10	-3.01	-4.18

DATA FOR BEAM TB01

LS=	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
MOM.	0.04	63.66	120.06	185.61	257.61	330.09	404.07	479.32	555.44	633.55	713.67	795.79	879.90	967.02	1057.25	1150.37
CURV.	0.05	2.71	4.16	5.39	6.41	7.34	8.19	8.92	9.58	10.14	10.62	11.04	11.41	11.76	12.09	12.39
	0.08	3.68	5.11	6.47	7.73	8.91	9.99	10.94	11.78	12.52	13.17	13.75	14.28	14.76	15.20	15.60

DATA FOR BEAM TB02

LS=	0	1	2	3	4	5	6	7	8
TORQUE	START	0.50	5.73	16.54	27.00	36.47	57.70	78.65	88.90
	END	0.50	4.78	15.59	25.52	34.78	55.08	75.81	85.52
MOMENT	START	42.01	62.07	103.47	143.51	179.81	261.11	341.34	380.59
	END	42.01	59.42	99.84	137.88	173.32	251.08	330.46	367.65
TWIST	START	0.01	0.04	0.16	0.51	1.36	3.70	5.80	7.19
	END	0.01	0.04	0.16	0.61	1.51	3.95	6.34	8.04
INCLN.	START	0.01	0.15	0.56	1.51	2.47	5.75	8.62	10.54
	END	0.01	0.15	0.56	1.51	2.47	5.75	8.62	10.54
CURV.	START	0.59	0.98	2.03	3.07	4.12	6.21	8.51	9.86
	END	0.59	0.98	2.03	3.07	4.12	6.21	8.51	9.86
STR.	START	0.59	0.97	2.02	3.04	4.12	6.21	8.51	10.02
	END	0.59	0.97	2.02	3.04	4.12	6.21	8.51	10.02

DATA FOR BEAM TB03

LS=	0	1	2	3	4	5	6	7	8	9	10	11
TORQUE	START	4.08	17.83	41.34	57.90	74.12	104.83	134.34	160.56	170.04	187.51	194.88
	END	4.08	15.72	38.02	53.95	69.49	98.92	127.51	152.26	162.49	179.44	186.76
MOMENT	START	36.18	54.64	88.41	112.20	135.51	179.63	222.04	259.69	273.31	298.41	309.00
	END	36.18	51.61	83.65	106.53	128.86	171.14	212.21	247.77	262.47	286.82	297.34
TWIST	START	0.11	0.30	1.95	3.94	5.94	9.88	13.92	17.86	19.90	22.89	24.89
	END	0.11	0.30	2.10	4.09	6.19	10.13	14.27	18.41	20.35	23.49	25.44
INCLN.	START	0.11	0.51	2.70	5.16	7.49	11.86	17.06	21.43	23.75	26.90	29.36
	END	0.11	0.51	2.70	5.16	7.49	11.86	17.06	21.43	23.75	26.90	29.36
CURV.	START	0.51	0.77	1.86	2.73	3.39	4.57	5.92	7.19	7.82	8.76	9.35
	END	0.51	0.77	1.86	2.73	3.39	4.57	5.92	7.19	7.82	8.76	9.35
STR.	START	0.51	0.84	1.85	2.49	3.12	4.19	5.13	6.07	6.51	7.14	7.27
	END	0.51	0.84	1.85	2.49	3.12	4.19	5.13	6.07	6.51	7.14	7.27

DATA FOR BEAM TB04

LS=	0	1	2	3	4	5	6	7
TORQUE	START	6.28	25.16	44.40	84.51	137.90	160.24	183.64
	END	6.28	22.82	40.52	77.86	129.18	148.73	170.66
MOMENT	START	21.76	34.27	47.02	73.61	109.00	123.81	139.32
	END	21.76	32.72	44.45	69.21	102.56	116.18	130.72
TWIST	START	0.13	0.43	2.53	10.10	20.08	25.07	31.05
	END	0.13	0.43	2.63	10.31	20.48	25.46	31.65
INCLN.	START	0.13	0.43	2.35	10.00	20.39	25.17	31.87
	END	0.13	0.43	2.35	10.00	20.39	25.17	31.87
CURV.	START	0.31	0.39	0.94	0.85	0.49	0.08	-0.43
	END	0.31	0.39	0.94	0.76	0.21	-0.15	-0.70
STR.	START	0.31	0.65	1.25	1.14	0.81	0.71	0.14
	END	0.31	0.65	1.25	1.14	0.81	0.71	0.14

DATA FOR BEAM TB05

LS=	0	1	2	3	4	5	6	7	8	9
TORQUE	START	5.52	49.00	51.70	76.34	101.68	128.04	148.58	171.12	178.48
	END	5.52	37.26	46.64	69.24	94.25	119.74	139.60	159.50	166.07
MOMENT	START	7.25	15.85	16.38	21.25	26.27	31.48	35.54	40.00	41.46
	END	7.25	13.55	15.38	19.86	24.80	29.84	33.77	37.70	39.00
TWIST	START	0.12	0.78	5.10	10.09	15.08	0.00	0.00	0.00	0.00
	END	0.12	1.03	5.27	10.42	15.24	20.23	25.38	36.02	48.22
INCLN.	START	0.12	0.78	5.15	10.49	15.55	20.47	26.21	36.73	48.90
	END	0.12	0.78	5.15	10.49	15.55	20.47	26.21	36.73	48.90
CURV.	START	0.10	0.19	-0.86	-1.73	-2.82	-3.65	-5.02	-10.09	-15.99
	END	0.10	0.19	-0.95	-1.82	-2.73	-3.78	-5.11	-10.50	-16.49
STR.	START	0.10	0.84	-1.57	-1.21	-3.60	-5.20	-5.74	-11.57	-18.66
	END	0.10	0.84	-1.57	-1.21	-3.60	-5.20	-5.74	-11.57	-18.66

DATA FOR BEAM TBS1

	LS=	0	1	2	3	4	5	6	7	8	9	10	11
TORQUE	START	12.71	80.64	100.56	111.77	122.61	133.99	147.03	163.47	176.71	188.05	196.77	202.91
	END	12.71	71.60	91.63	102.02	114.01	123.95	136.40	151.21	162.97	175.13	182.03	188.13
MOMENT	START	34.37	79.39	92.59	100.02	107.21	114.75	123.39	134.29	143.06	150.58	156.36	160.43
	END	34.37	73.40	86.67	93.56	101.51	108.10	116.35	126.16	133.95	142.01	146.59	150.63
TWIST	START	0.01	3.00	6.09	10.08	14.07	18.06	23.04	29.43	35.41	41.40	47.38	53.86
	END	0.01	3.05	6.39	10.48	14.47	18.46	23.39	29.98	35.96	41.89	48.08	54.81
INCLN.	START	0.01	2.20	5.21	8.08	10.27	13.27	15.87	19.70	24.07	27.50	32.41	37.20
	END	0.52	1.75	2.46	2.69	2.85	2.98	3.06	3.19	3.20	3.10	2.89	2.51
CURV.	START	0.52	1.73	2.38	2.61	2.80	2.82	2.95	3.02	2.97	2.84	2.54	2.12
	END	0.52	1.77	2.35	2.49	2.36	2.35	2.43	2.21	1.93	1.89	1.58	1.15
	STR.												

DATA FOR BEAM TBS2

	LS=	0	1	2	3	4	5	6	7	8	9	10
TORQUE	START	11.28	44.39	82.24	109.95	123.16	146.67	171.74	201.00	215.74	208.96	180.77
	END	11.28	40.14	75.09	100.44	115.45	136.23	159.89	186.09	197.68	190.51	165.07
MOMENT	START	33.42	55.36	80.45	98.82	107.57	123.16	139.77	159.16	168.93	164.44	145.75
	END	33.42	52.55	75.71	92.51	102.46	116.23	131.91	149.28	156.96	152.21	136.01
TWIST	START	0.01	0.63	3.00	8.73	13.97	21.32	31.17	44.88	57.43	72.31	105.72
	END	0.01	0.63	3.12	9.23	14.34	21.70	31.79	45.63	58.35	73.56	106.96
INCLN.	START	0.01	0.28	2.33	7.80	9.72	14.50	21.06	29.95	40.47	62.76	0.00
	END	0.43	0.67	1.69	2.37	2.63	2.55	2.23	1.59	0.54	-3.67	-22.80
CURV.	START	0.43	0.59	1.66	2.31	2.37	2.27	1.97	1.16	0.07	-5.24	-24.90
	END	0.43	0.77	1.73	2.19	2.17	2.09	1.47	0.72	-0.19	-2.11	-3.10
	STR.											

DATA FOR BEAM TBS3

	LS=	0	1	2	3	4	5	6	7
TORQUE	START	5.52	52.54	100.85	148.06	195.53	238.97	244.61	214.69
	END	5.52	48.81	93.32	137.95	184.40	223.08	229.41	199.14
MOMENT	START	29.60	60.45	92.15	123.13	154.28	182.78	186.48	166.85
	END	29.60	58.01	87.21	116.50	146.97	172.35	176.51	156.64
TWIST	START	0.00	0.50	2.99	11.97	20.70	31.67	39.27	56.60
	END	0.00	0.50	2.99	12.10	21.20	32.04	40.40	57.47
INCLN.	START	0.00	0.50	2.41	11.71	20.87	33.45	39.73	49.17
	END	0.23	0.49	1.64	2.16	1.64	1.01	0.00	0.00
CURV.	START	0.23	0.49	1.64	2.00	1.48	0.85	0.00	0.00
	END	0.23	0.67	2.03	2.14	1.55	0.72	-0.11	-2.21
	STR.								

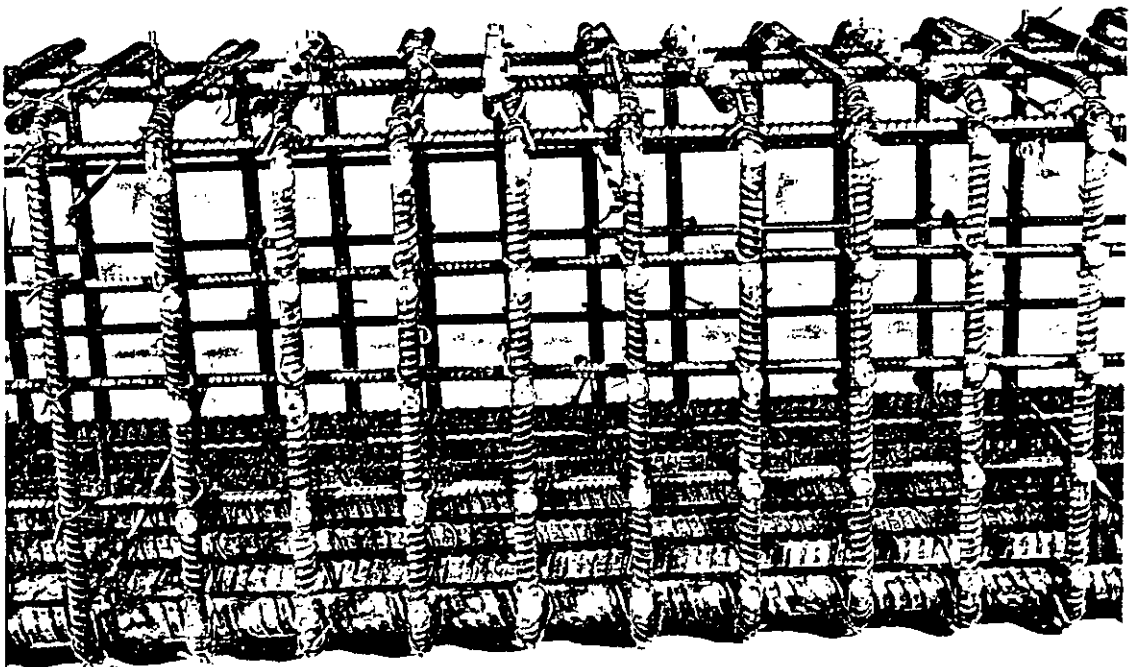
DATA FOR BEAM TBS4

	LS=	0	1	2	3	4	5	6	7	8	9	10
TORQUE	START	11.66	30.06	61.38	71.10	76.96	86.23	99.10	110.72	119.79	124.88	119.87
	END	17.16	27.76	54.59	63.53	69.00	77.27	88.81	97.91	106.20	110.00	104.93
MOMENT	START	33.62	45.70	66.25	72.63	76.47	82.56	91.01	98.62	104.58	107.92	104.63
	END	37.23	44.19	61.80	67.66	71.25	76.68	84.25	90.22	95.66	98.15	94.83
TWIST	START	0.02	0.22	1.92	4.01	6.90	10.90	15.88	20.87	26.85	34.43	49.79
	END	0.02	0.27	2.02	4.31	7.35	11.29	16.13	21.42	27.35	35.13	50.49
INCLN.	START	0.02	0.57	2.90	5.77	9.87	14.10	20.26	26.27	35.29	45.82	75.21
	END	0.81	0.96	1.54	1.95	2.11	2.11	2.01	1.90	1.70	1.54	-2.22
CURV.	START	0.81	0.96	1.59	1.96	1.96	1.96	1.96	1.85	1.85	1.54	-2.22
	END	0.81	0.98	1.92	2.25	1.55	1.06	1.28	0.63	0.70	0.45	0.46
	STR.											

E.3.2 MEASURED STRAINS

In the following tables are listed the strains measured from the target patterns shown in Figures 4.17 and 4.18 for all the section faces of all the beams at all the load stages. For any particular type of strain such as longitudinal strain in the top face, the strains at a given load stage for a particular row of targets, the number of strain readings in that row are listed below the title of the row. If it was not possible to obtain readings of strain over any two targets in a row, the listing will have a zero entry for that position. All the strains are listed in units of 10^{-3} .

Figure E.3 shows the positions of the targets on the steel cage for a side face and the top face. Steel studs, inside plastic tubes with springs to enable easy extraction, were glued on to the hoops and waxed to keep moisture out. These studs were replaced after casting by brass targets from which strains were measured.



T-104M TWIN - TOP FACE - LONGITUDINAL STRAINS										T-104M TWIN - BOTTOM FACE - LONGITUDINAL STRAINS									
1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
1.00	0.11	0.47	0.64	0.74	1.01	1.15	1.44	1.63	2.35	1.00	0.01	0.20	0.57	0.74	0.74	0.24	0.24	1.02	1.1
0.00	0.20	0.47	0.64	0.74	1.01	1.15	1.44	1.63	2.35	0.00	0.01	0.20	0.57	0.74	0.74	0.24	0.24	1.02	1.07
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.00	0.								

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HEAM TONIS - SPININ FACE - DIAGONAL STRAINS									
	1	2	3	4	5	6	7		
LSA	0	0.16	0.04	1.00	2.16	0.01	0	1	20
KWA	0	0.16	0.04	1.00	2.16	0.01	0	1	20
LSA	0	0.09	0.04	1.00	2.16	0.01	0	1	20
KWA	0	0.09	0.04	1.00	2.16	0.01	0	1	20
LSA	0	0.09	0.15	1.00	2.06	0.01	0	1	20
KWA	0	0.09	0.15	1.00	2.06	0.01	0	1	20
LSA	0	0.14	0.01	0.00	1.00	0.01	0	1	20
KWA	0	0.14	0.01	0.00	1.00	0.01	0	1	20
LSA	0	0.14	0.01	0.00	1.00	0.01	0	1	20
KWA	0	0.14	0.01	0.00	1.00	0.01	0	1	20
LSA	0	0.10	0.01	0.00	1.00	0.01	0	1	20
KWA	0	0.10	0.01	0.00	1.00	0.01	0	1	20
LSA	0	0.14	0.01	0.00	1.00	0.01	0	1	20
KWA	0	0.14	0.01	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
KWA	0	0.05	0.04	0.00	1.00	0.01	0	1	20
LSA	0	0.05	0.04	0.00					

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DEAM TROJ - NORTH FACE - DIAGONAL STRAINS							
103	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0
0	0	0.33	0.11	1.00	1.33	3.00	2.00
0	0.33	0	0.05	1.00	2.00	3.00	1.67
0	0.05	0.05	0	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00	1.00	1.33	3.00	2.00
0	0.00	0.00	0.00				

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[illegible][illegible]

HEAM FROM - NORTH FACE - DIAGONAL STRAINS										HEAM FROM - SOUTH FACE - DIAGONAL STRAINS										
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ROW	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
US	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
COL	1	2	3	4	5	6	7	8	9											

[illegible]

HEAM TENDS - SOUTH FACE - 100MP STRAINS	HEAM TENDS - SOUTH FACE - 100MP STRAINS	HEAM TENDS - SOUTH FACE - 100MP STRAINS	HEAM TENDS - SOUTH FACE - 100MP STRAINS
LS	LS	LS	LS
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10
11	11	11	11
12	12	12	12
13	13	13	13
14	14	14	14
15	15	15	15
16	16	16	16
17	17	17	17
18	18	18	18
19	19	19	19
20	20	20	20
21	21	21	21
22	22	22	22
23	23	23	23
24	24	24	24
25	25	25	25
26	26	26	26
27	27	27	27
28	28	28	28
29	29	29	29
30	30	30	30
31	31	31	31
32	32	32	32
33	33	33	33
34	34	34	34
35	35	35	35
36	36	36	36
37	37	37	37
38	38	38	38
39	39	39	39
40	40	40	40
41	41	41	41
42	42	42	42
43	43	43	43
44	44	44	44
45	45	45	45
46	46	46	46
47	47	47	47
48	48	48	48
49	49	49	49
50	50	50	50
51	51	51	51
52	52	52	52
53	53	53	53
54	54	54	54
55	55	55	55
56	56	56	56
57	57	57	57
58	58	58	58
59	59	59	59
60	60	60	60
61	61	61	61
62	62	62	62
63	63	63	63
64	64	64	64
65	65	65	65
66	66	66	66
67	67	67	67
68	68	68	68
69	69	69	69
70	70	70	70
71	71	71	71
72	72	72	72
73	73	73	73
74	74	74	74
75	75	75	75
76	76	76	76
77	77	77	77
78	78	78	78
79	79	79	79
80	80	80	80
81	81	81	81
82	82	82	82
83	83	83	83
84	84	84	84
85	85	85	85
86	86	86	86
87	87	87	87
88	88	88	88
89	89	89	89
90	90	90	90
91	91	91	91
92	92	92	92
93	93	93	93
94	94	94	94
95	95	95	95
96	96	96	96
97	97	97	97
98	98	98	98
99	99	99	99
100	100	100	100

[illegible]

MEAN THIN - SIX FACE - LONGITUDINAL STRAINS

THIN	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.00	-0.20	-0.40	-0.60	-0.80	-1.00	-1.20	-1.40	-1.60	-1.80	-2.00	-2.20	-2.40	-2.60
2	0.00	-0.47	-0.94	-1.41	-1.88	-2.35	-2.82	-3.29	-3.76	-4.23	-4.70	-5.17	-5.64	-6.11
3	0.00	-0.94	-1.88	-2.82	-3.76	-4.70	-5.64	-6.58	-7.52	-8.46	-9.40	-10.34	-11.28	-12.22
4	0.00	-1.41	-2.82	-4.23	-5.64	-7.05	-8.46	-9.87	-11.28	-12.69	-14.10	-15.51	-16.92	-18.33
5	0.00	-1.88	-3.76	-5.64	-7.52	-9.40	-11.28	-13.16	-15.04	-16.92	-18.80	-20.68	-22.56	-24.44
6	0.00	-2.35	-4.70	-7.05	-9.40	-11.75	-14.10	-16.45	-18.80	-21.15	-23.50	-25.85	-28.20	-30.55
7	0.00	-2.82	-5.64	-8.46	-11.28	-14.10	-16.92	-19.74	-22.56	-25.38	-28.20	-31.02	-33.84	-36.66
8	0.00	-3.29	-6.58	-9.87	-13.16	-16.45	-19.74	-23.03	-26.32	-29.61	-32.90	-36.19	-39.48	-42.77
9	0.00	-3.76	-7.52	-11.28	-15.04	-18.80	-22.56	-26.32	-30.08	-33.84	-37.60	-41.36	-45.12	-48.88
10	0.00	-4.23	-8.46	-12.69	-16.92	-21.15	-25.38	-29.61	-33.84	-38.07	-42.30	-46.53	-50.76	-54.99
11	0.00	-4.70	-9.40	-14.10	-18.80	-23.50	-28.20	-32.90	-37.60	-42.30	-46.99	-51.69	-56.39	-61.09
12	0.00	-5.17	-9.87	-15.51	-20.68	-25.85	-31.02	-36.19	-41.36	-46.53	-51.69	-56.86	-62.03	-67.19
13	0.00	-5.64	-10.34	-16.92	-22.56	-28.20	-33.84	-39.48	-45.12	-50.76	-56.39	-62.03	-67.67	-73.31
14	0.00	-6.11	-10.81	-17.39	-24.44	-30.55	-36.66	-42.77	-48.88	-54.99	-61.09	-67.19	-73.31	-79.42

MEAN THIN - SIX FACE - LONGITUDINAL STRAINS

THIN	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.00	-0.20	-0.40	-0.60	-0.80	-1.00	-1.20	-1.40	-1.60	-1.80	-2.00	-2.20	-2.40	-2.60
2	0.00	-0.47	-0.94	-1.41	-1.88	-2.35	-2.82	-3.29	-3.76	-4.23	-4.70	-5.17	-5.64	-6.11
3	0.00	-0.94	-1.88	-2.82	-3.76	-4.70	-5.64	-6.58	-7.52	-8.46	-9.40	-10.34	-11.28	-12.22
4	0.00	-1.41	-2.82	-4.23	-5.64	-7.05	-8.46	-9.87	-11.28	-12.69	-14.10	-15.51	-16.92	-18.33
5	0.00	-1.88	-3.76	-5.64	-7.52	-9.40	-11.28	-13.16	-15.04	-16.92	-18.80	-20.68	-22.56	-24.44
6	0.00	-2.35	-4.70	-7.05	-9.40	-11.75	-14.10	-16.45	-18.80	-21.15	-23.50	-25.85	-28.20	-30.55
7	0.00	-2.82	-5.64	-8.46	-11.28	-14.10	-16.92	-19.74	-22.56	-25.38	-28.20	-31.02	-33.84	-36.66
8	0.00	-3.29	-6.58	-9.87	-13.16	-16.45	-19.74	-23.03	-26.32	-29.61	-32.90	-36.19	-39.48	-42.77
9	0.00	-3.76	-7.52	-11.28	-15.04	-18.80	-22.56	-26.32	-30.08	-33.84	-37.60	-41.36	-45.12	-48.88
10	0.00	-4.23	-8.46	-12.69	-16.92	-21.15	-25.38	-29.61	-33.84	-38.07	-42.30	-46.53	-50.76	-54.99
11	0.00	-4.70	-9.40	-14.10	-18.80	-23.50	-28.20	-32.90	-37.60	-42.30	-46.99	-51.69	-56.39	-61.09
12	0.00	-5.17	-9.87	-15.51	-20.68	-25.85	-31.02	-36.19	-41.36	-46.53	-51.69	-56.86	-62.03	-67.19
13	0.00	-5.64	-10.34	-16.92	-22.56	-28.20	-33.84	-39.48	-45.12	-50.76	-56.39	-62.03	-67.67	-73.31
14	0.00	-6.11	-10.81	-17.39	-24.44	-30.55	-36.66	-42.77	-48.88	-54.99	-61.09	-67.19	-73.31	-79.42

MEAN THIN - SIX FACE - LONGITUDINAL STRAINS

THIN	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.00	-0.20	-0.40	-0.60	-0.80	-1.00	-1.20	-1.40	-1.60	-1.80	-2.00	-2.20	-2.40	-2.60
2	0.00	-0.47	-0.94	-1.41	-1.88	-2.35	-2.82	-3.29	-3.76	-4.23	-4.70	-5.17	-5.64	-6.11
3	0.00	-0.94	-1.88	-2.82	-3.76	-4.70	-5.64	-6.58	-7.52	-8.46	-9.40	-10.34	-11.28	-12.22
4	0.00	-1.41	-2.82	-4.23	-5.64	-7.05	-8.46	-9.87	-11.28	-12.69	-14.10	-15.51	-16.92	-18.33
5	0.00	-1.88	-3.76	-5.64	-7.52	-9.40	-11.28	-13.16	-15.04	-16.92	-18.80	-20.68	-22.56	-24.44
6	0.00	-2.35	-4.70	-7.05	-9.40	-11.75	-14.10	-16.45	-18.80	-21.15	-23.50	-25.85	-28.20	-30.55
7	0.00	-2.82	-5.64	-8.46	-11.28	-14.10	-16.92	-19.74	-22.56	-25.38	-28.20	-31.02	-33.84	-36.66
8	0.00	-3.29	-6.58	-9.87	-13.16	-16.45	-19.74	-23.03	-26.32	-29.61	-32.90	-36.19	-39.48	-42.77
9	0.00	-3.76	-7.52	-11.28	-15.04	-18.80	-22.56	-26.32	-30.08	-33.84	-37.60	-41.36	-45.12	-48.88
10	0.00	-4.23	-8.46	-12.69	-16.92	-21.15	-25.38	-29.61	-33.84	-38.07	-42.30	-46.53	-50.76	-54.99
11	0.00	-4.70	-9.40	-14.10	-18.80	-23.50	-28.20	-32.90	-37.60	-42.30	-46.99	-51.69	-56.39	-61.09
12	0.00	-5.17	-9.87	-15.51	-20.68	-25.85	-31.02	-36.19	-41.36	-46.53	-51.69	-56.86	-62.03	-67.19
13	0.00	-5.64	-10.34	-16.92	-22.56	-28.20	-33.84	-39.48	-45.12	-50.76	-56.39	-62.03	-67.67	-73.31
14	0.00	-6.11	-10.81	-17.39	-24.44	-30.55	-36.66	-42.77	-48.88	-54.99	-61.09	-67.19	-73.31	-79.42

MEAN THIN - SIX FACE - LONGITUDINAL STRAINS

THIN	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.00	-0.20	-0.40	-0.60	-0.80	-1.00	-1.20	-1.40	-1.60	-1.80	-2.00	-2.20	-2.40	-2.60
2	0.00	-0.47	-0.94	-1.41	-1.88	-2.35	-2.82	-3.29	-3.76	-4.23	-4.70	-5.17	-5.64	-6.11
3	0.00	-0.94	-1.88	-2.82	-3.76	-4.70	-5.64	-6.58	-7.52	-8.46	-9.40	-10.34	-11.28	-12.22
4	0.00	-1.41	-2.82	-4.23	-5.64	-7.05	-8.46	-9.87	-11.28	-12.69	-14.10	-15.51	-16.92	-18.33
5	0.00	-1.88	-3.76	-5.64	-7.52	-9.40	-11.28	-13.16	-15.04	-16.92	-18.80	-20.68	-22.56	-24.44
6	0.00	-2.35	-4.70	-7.05	-9.40	-11.75	-14.10	-16.45	-18.80	-21.15	-23.50	-25.85	-28.20	-30.55
7	0.00	-2.82	-5.64	-8.46	-11.28	-14.10	-16.92	-19.74	-22.56	-25.38	-28.20	-31.02	-33.84	-36.66
8	0.00	-3.29	-6.58	-9.87	-13.16	-16.45	-19.74	-23.03	-26.32	-29.61	-32.90	-36.19	-39.48	-42.77
9	0.00	-3.76	-7.52	-11.28	-15.04	-18.80	-22.56	-26.32	-30.08	-33.84	-37.60	-41.36	-45.12	-48.88
10	0.00	-4.23	-8.46	-12.69	-16.92	-21.15	-25.38	-29.61	-33.84	-38.07	-42.30	-46.53	-50.76	-54.99
11	0.00	-4.70	-9.40	-14.10	-18.80	-23.50	-28.20	-32.90	-37.60	-42.30	-46.99	-51.69	-56.39	-61.09
12	0.00	-5.17	-9.87	-15.51	-20.68	-25.85	-31.02	-36.19	-41.36	-46.53	-51.69	-56.86	-62.03	-67.19
13	0.00	-5.64	-10.34	-16.92	-22.56	-28.20	-33.84	-39.48	-45.12	-50.76	-56.39	-62.03	-67.67	-73.31
14	0.00	-6.11	-10.81	-17.39	-24.44	-30.55	-36.66	-42.77	-48.88	-54.99	-61.09	-67.19	-73.31	-79.42

	0	1	2	3	4	5	6	7	8	9
L	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
1	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
2	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
3	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
4	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
5	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
6	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
7	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
8	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
9	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

RE:AM THU2 - THE FACE - 10101 SIGAINS

[illegible]

THE AM TIMEZ - YDP FACE - PLACENTAL STRAINS

[illegible]

[illegible][illegible][illegible][illegible][illegible][illegible]

DEAFAM 10118 - SOUTH PACIFIC - CRIMINAL STRAINS

[illegible]

REF AND TITLE - SOURCE FAC - INVS - SPREADS

[illegible]

THE AM TREATY - STORY OF FACT - (U.S. GOVT. STAINS

[illegible]

104 PM YORKT - 100TH EAST - 1.00:11 PM 57RA0105

[illegible]

STAIN 100X - WIDTH FACE - 100X STAINS

[illegible]

REF ID: A67146 - NATIONAL ARCHIVES

[illegible]

STREAM T0114 - TOP FACE - LONGITUDINAL STRAINS

[illegible]

MEAN 1994 - FDP FACE - 10UP STRAINS

[illegible]

SEAM TENSILE - TOP FACE - DIAGONAL STRAINS.

[illegible]

MEAN T3114 - INITIAL STRAINS - INITIAL FACE - 1 (PAGE 10)

[illegible]

NO. 100-7689 - DIVISION PAGE - 144349 STOKES

Run	1	2	3	4	5	6	7
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0

TESTIM PAGE - DIAGONAL STRIPS

[illegible]

[illegible][illegible]

BEAM TUNIA - SOUTH FACE - DIAGONAL STRAINS									
	1	2	3	4	5	6	7		
MS-1	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-2	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-3	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-4	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-5	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-6	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-7	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-8	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-9	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-10	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-11	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-12	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-13	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-14	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-15	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-16	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-17	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-18	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-19	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-20	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-21	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-22	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-23	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-24	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-25	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-26	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-27	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-28	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-29	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-30	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-31	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-32	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-33	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-34	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-35	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-36	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-37	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-38	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-39	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-40	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-41	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-42	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-43	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-44	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-45	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-46	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-47	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-48	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-49	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-50	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-51	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-52	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-53	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-54	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-55	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-56	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-57	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-58	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-59	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-60	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-61	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-62	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-63	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-64	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-65	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-66	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-67	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-68	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-69	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-70	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-71	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-72	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-73	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
MS-74	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00

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		HELM FIBRE - SOUTH FACE - LONGITUDINAL STRAINS										HELM FIBRE - NORTH FACE - LONGITUDINAL STRAINS																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																													
LS	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999	1000

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DEAM IN3J - TOP FACE - LONGITUDINAL STRAINS									
1	2	3	4	5	6	7	DEAM IN3J - BOTTOM FACE - LONGITUDINAL STRAINS		
1	0.07	-0.13	0.00	0.00	2.71	0.0	1	0	0
2	-0.06	-0.10	0.52	0.00	1.97	0.0	2	0	0
3	-0.13	-0.20	0.00	0.00	1.93	0.0	3	0	0
4	-0.13	-0.07	0.13	1.02	2.03	0.0	4	0	0
5	-0.26	-0.40	0.63	1.30	2.53	0.0	5	0	0
6	-0.26	-0.33	-0.13	0.20	1.57	2.14	6	0	0
7	0.0	-0.13	0.74	1.28	1.02	-0.54	7	0	0
8	-0.13	-0.10	0.13	0.85	1.41	3.44	8	0	0
9	-0.07	-0.20	0.33	0.47	1.15	5.33	9	0	0
10	-0.07	-0.13	0.00	0.00	1.31	2.29	10	0	0
11	-0.06	-0.13	0.00	0.00	1.43	1.30	11	0	0
12	-0.06	-0.13	0.00	0.00	1.43	2.00	12	0	0
13	-0.06	-0.13	0.00	0.00	1.43	2.00	13	0	0
14	-0.06	-0.13	0.00	0.00	1.43	2.00	14	0	0
15	-0.06	-0.13	0.00	0.00	1.43	2.00	15	0	0
16	-0.06	-0.13	0.00	0.00	1.43	2.00	16	0	0
17	-0.06	-0.13	0.00	0.00	1.43	2.00	17	0	0
18	-0.06	-0.13	0.00	0.00	1.43	2.00	18	0	0
19	-0.06	-0.13	0.00	0.00	1.43	2.00	19	0	0
20	-0.06	-0.13	0.00	0.00	1.43	2.00	20	0	0

DEAM IN3J - TOP FACE - DIAGONAL STRAINS									
1	2	3	4	5	6	7	DEAM IN3J - BOTTOM FACE - DIAGONAL STRAINS		
1	0.0	0.13	0.70	1.71	2.10	2.00	1	0	0
2	-0.13	0.00	0.00	1.00	2.00	2.00	2	0	0
3	-0.13	0.00	0.00	1.00	2.00	2.00	3	0	0
4	-0.13	0.00	0.00	1.00	2.00	2.00	4	0	0
5	-0.13	0.00	0.00	1.00	2.00	2.00	5	0	0
6	-0.13	0.00	0.00	1.00	2.00	2.00	6	0	0
7	-0.13	0.00	0.00	1.00	2.00	2.00	7	0	0
8	-0.13	0.00	0.00	1.00	2.00	2.00	8	0	0
9	-0.13	0.00	0.00	1.00	2.00	2.00	9	0	0
10	-0.13	0.00	0.00	1.00	2.00	2.00	10	0	0
11	-0.13	0.00	0.00	1.00	2.00	2.00	11	0	0
12	-0.13	0.00	0.00	1.00	2.00	2.00	12	0	0
13	-0.13	0.00	0.00	1.00	2.00	2.00	13	0	0
14	-0.13	0.00	0.00	1.00	2.00	2.00	14	0	0
15	-0.13	0.00	0.00	1.00	2.00	2.00	15	0	0
16	-0.13	0.00	0.00	1.00	2.00	2.00	16	0	0
17	-0.13	0.00	0.00	1.00	2.00	2.00	17	0	0
18	-0.13	0.00	0.00	1.00	2.00	2.00	18	0	0
19	-0.13	0.00	0.00	1.00	2.00	2.00	19	0	0
20	-0.13	0.00	0.00	1.00	2.00	2.00	20	0	0

DEAM IN3J - TOP FACE - DIAGONAL STRAINS									
1	2	3	4	5	6	7	DEAM IN3J - BOTTOM FACE - DIAGONAL STRAINS		
1	0.0	0.13	0.70	1.71	2.10	2.00	1	0	0
2	-0.13	0.00	0.00	1.00	2.00	2.00	2	0	0
3	-0.13	0.00	0.00	1.00	2.00	2.00	3	0	0
4	-0.13	0.00	0.00	1.00	2.00	2.00	4	0	0
5	-0.13	0.00	0.00	1.00	2.00	2.00	5	0	0
6	-0.13	0.00	0.00	1.00	2.00	2.00	6	0	0
7	-0.13	0.00	0.00	1.00	2.00	2.00	7	0	0
8	-0.13	0.00	0.00	1.00	2.00	2.00	8	0	0
9	-0.13	0.00	0.00	1.00	2.00	2.00	9	0	0
10	-0.13	0.00	0.00	1.00	2.00	2.00	10	0	0
11	-0.13	0.00	0.00	1.00	2.00	2.00	11	0	0
12	-0.13	0.00	0.00	1.00	2.00	2.00	12	0	0
13	-0.13	0.00	0.00	1.00	2.00	2.00	13	0	0
14	-0.13	0.00	0.00	1.00	2.00	2.00	14	0	0
15	-0.13	0.00	0.00	1.00	2.00	2.00	15	0	0
16	-0.13	0.00	0.00	1.00	2.00	2.00	16	0	0
17	-0.13	0.00	0.00	1.00	2.00	2.00	17	0	0
18	-0.13	0.00	0.00	1.00	2.00	2.00	18	0	0
19	-0.13	0.00	0.00	1.00	2.00	2.00	19	0	0
20	-0.13	0.00	0.00	1.00	2.00	2.00	20	0	0

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NEAR TUSA - TOP FACE - LONGITUDINAL STRAINS										NEAR TUSA - BOTTOM FACE - LONGITUDINAL STRAINS									
13	0	1	2	3	4	5	6	7	8	13	0	1	2	3	4	5	6	7	8
ROW 1	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	ROW 1	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
2	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	2	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
3	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	3	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
4	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	4	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
5	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	5	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
6	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	6	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
7	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	7	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
8	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	8	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
9	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	9	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
10	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	10	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
11	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	11	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
12	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	12	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10
13	0.00	-0.07	-0.13	-0.23	-0.33	-0.40	-0.20	-0.04	0.10	13	0.00	0.00	0.07	0.14	0.23	0.29	0.28	0.23	0.10

[illegible][illegible]

BEAM T054 - NORTH FACE - LONGITUDINAL STRAINS[illegible]

10	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000
9	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000
8	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000
7	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000
6	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000
5	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000
4	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000
3	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000
2	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000
1	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000
0	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000	0000 0000 0000 0000

NEAR THS - SOUTH FACE - HIGH STRAINS

TEAM TUSA - (W)RIN FACE - (W)RIN STAINING

[illegible][illegible]

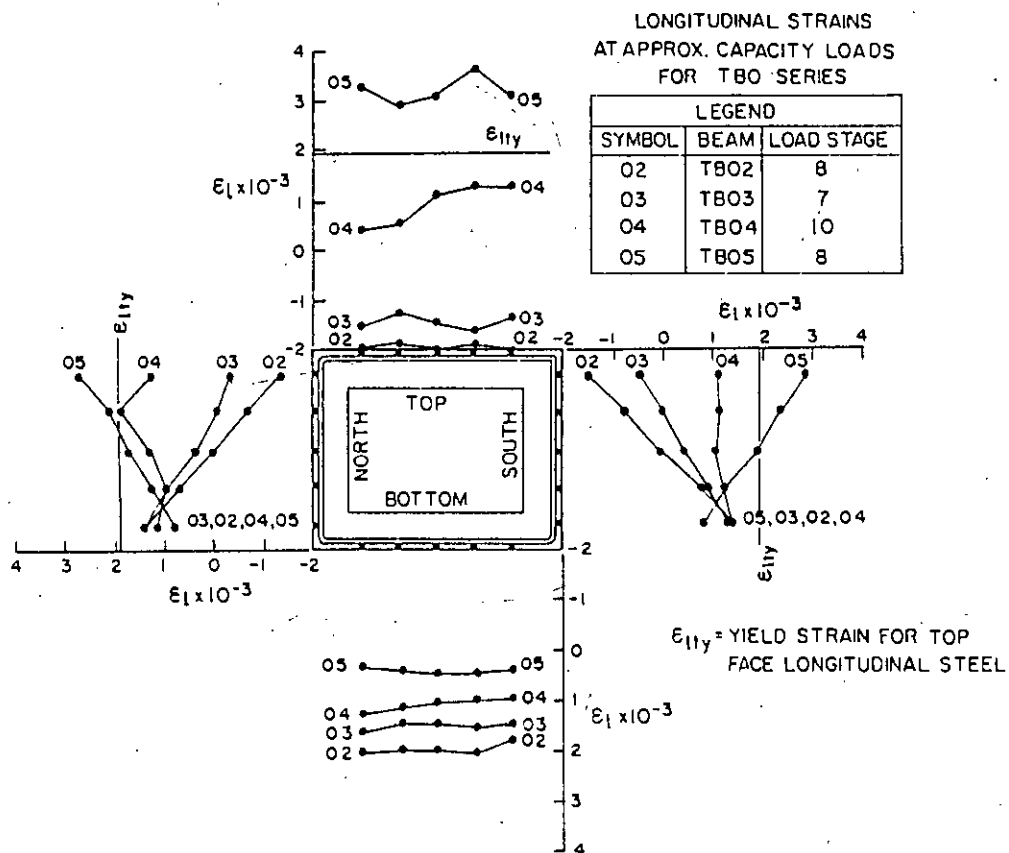
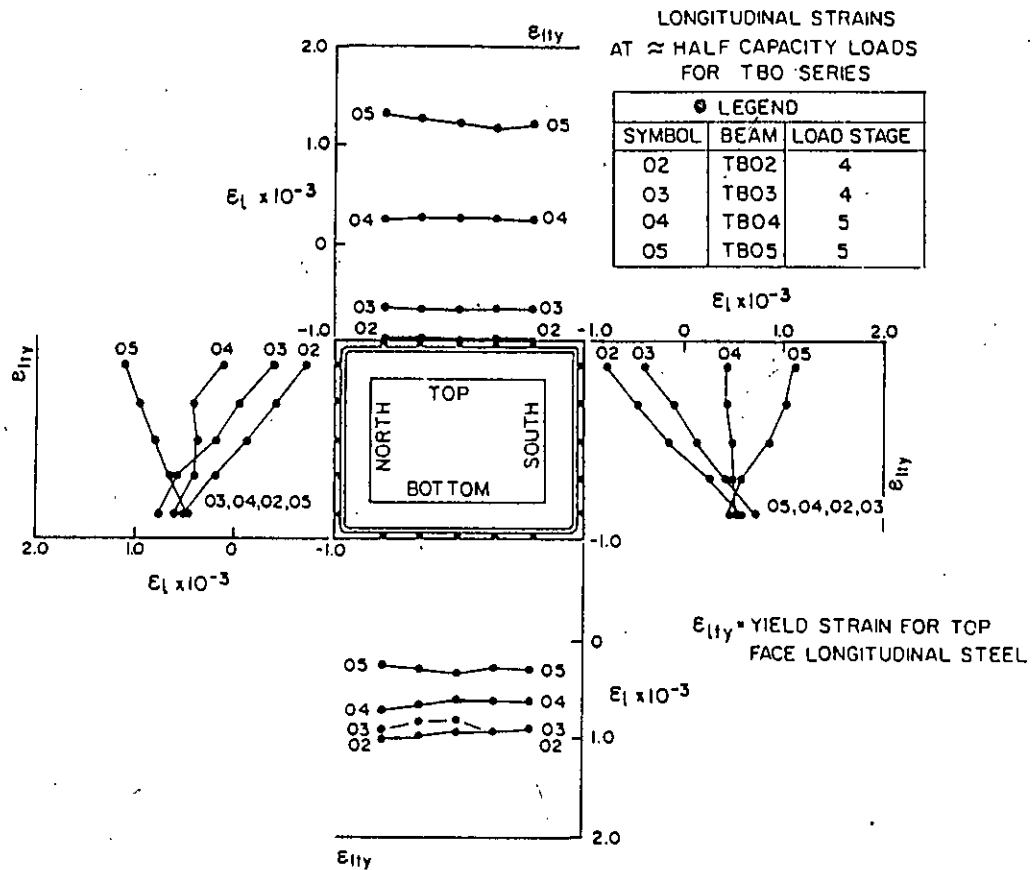
HEAM 1054 - SOUTH FACE - DIAGONAL STRAINS

MEAN TIME - EIGHTH FIVE - DIAGONAL STRAINS

[illegible][illegible]

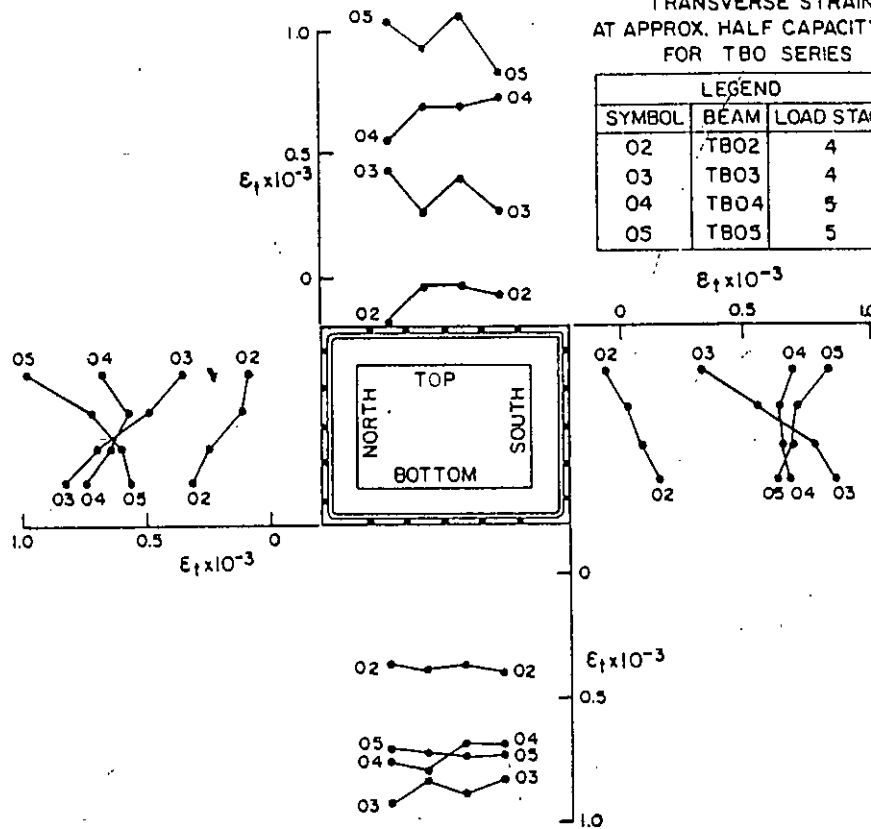
E.4 VARIATION OF STRAINS IN BEAM FACES

In this section the average strains for any beam face are plotted so as to reveal the variation trends of the measured strains on the sides. Plots are made for all the TBO and TBU series beams at approximately half capacity loading and at approximately capacity loading. For the TBS series plots are given for the near capacity loading case.



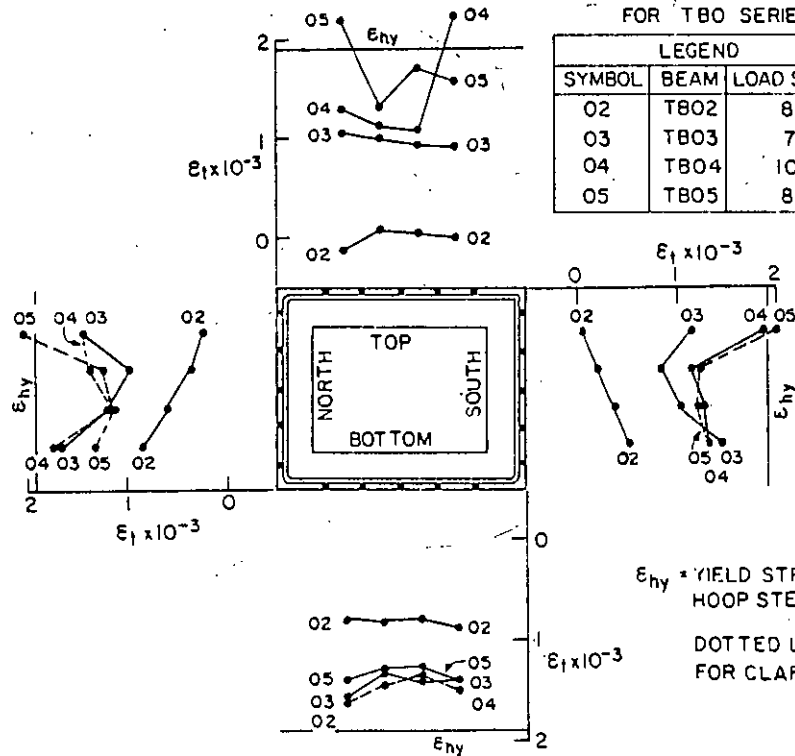
TRANSVERSE STRAINS
AT APPROX. HALF CAPACITY LOADS
FOR TBO SERIES

LEGEND		
SYMBOL	BEAM	LOAD STAGE
02	TB02	4
03	TB03	4
04	TB04	5
05	TB05	5

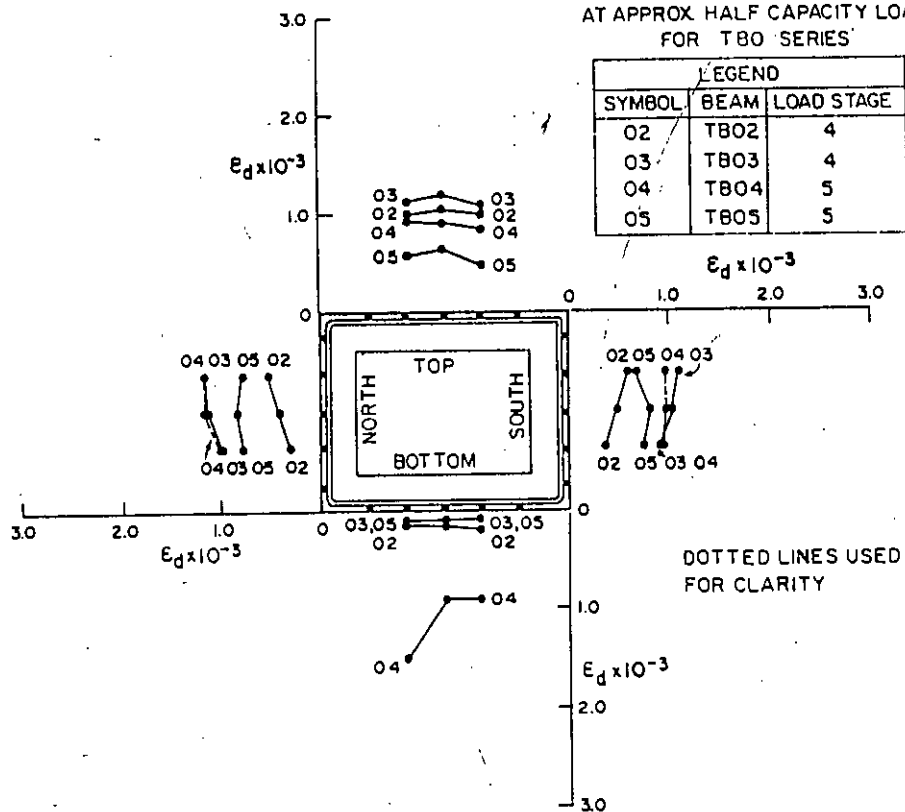


TRANSVERSE STRAINS
AT APPROX. CAPACITY LOADS
FOR TBO SERIES

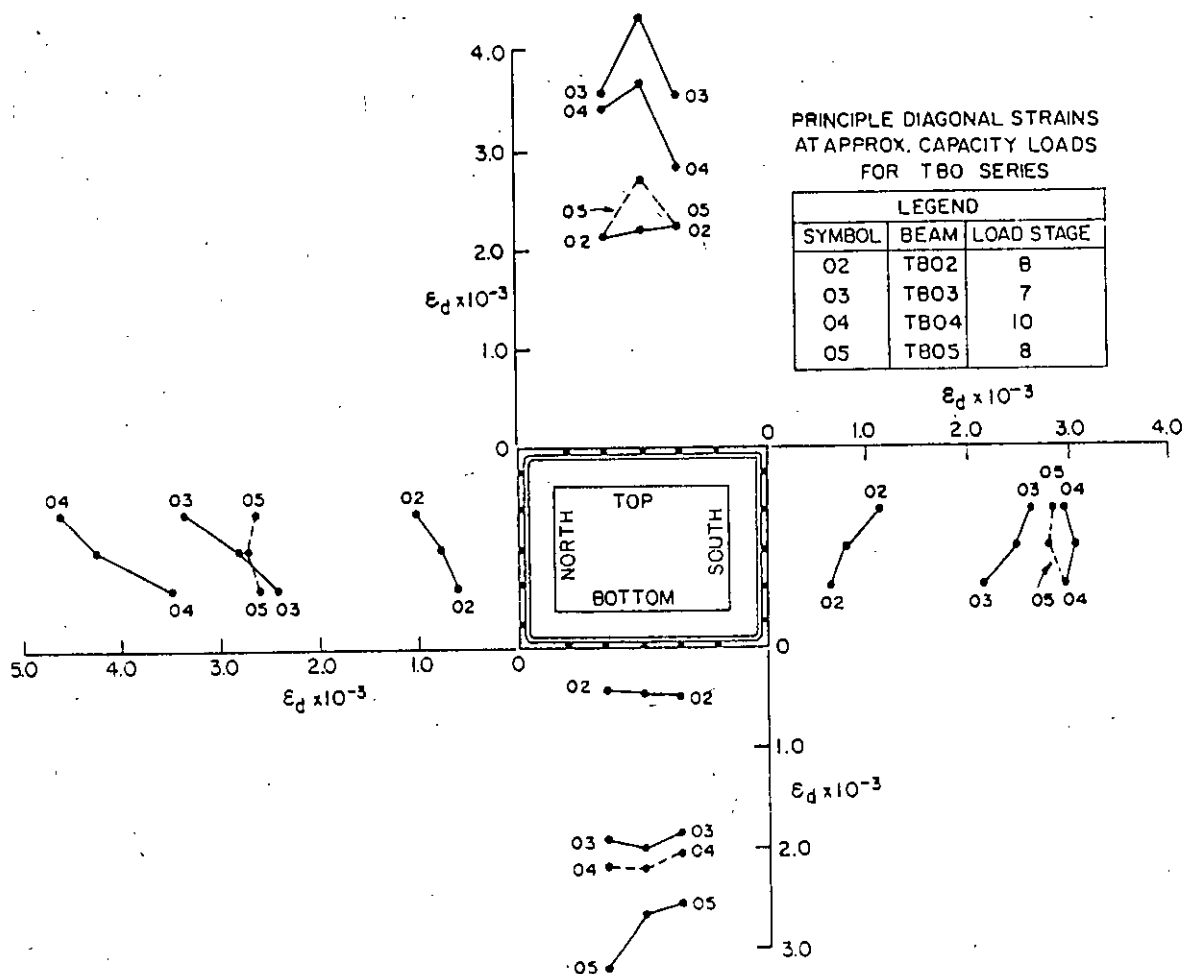
LEGEND		
SYMBOL	BEAM	LOAD STAGE
02	TB02	8
03	TB03	7
04	TB04	10
05	TB05	8

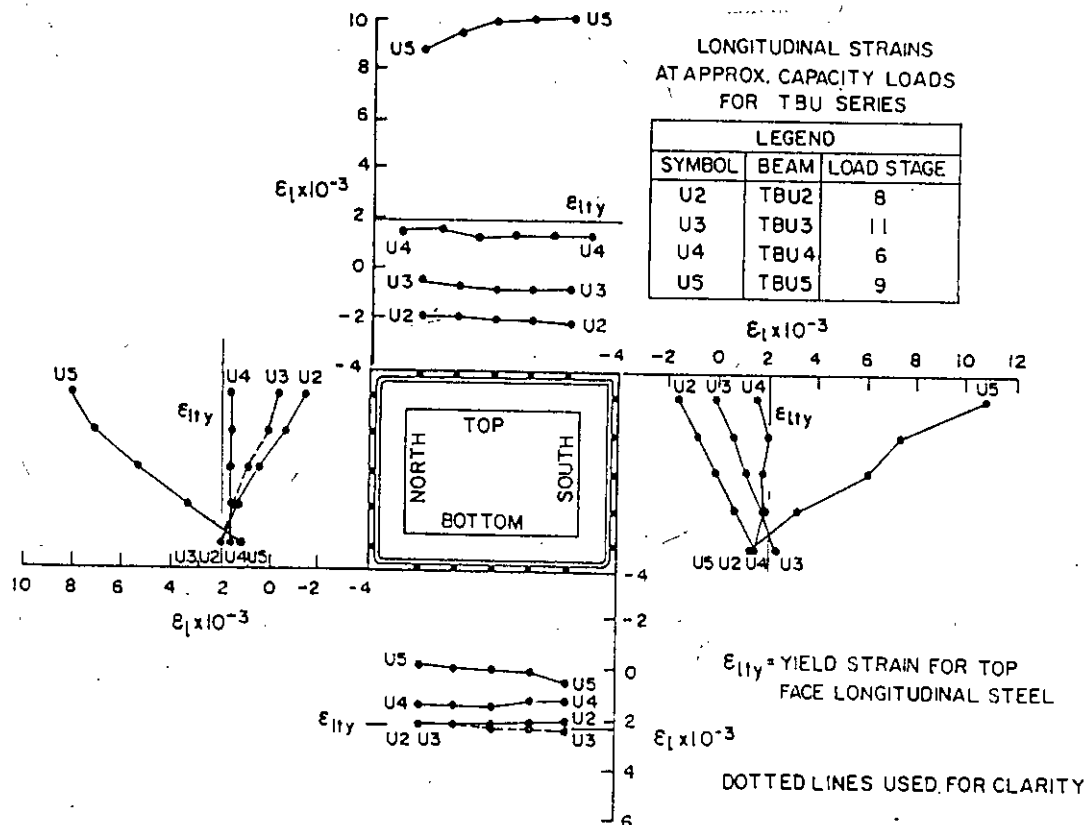
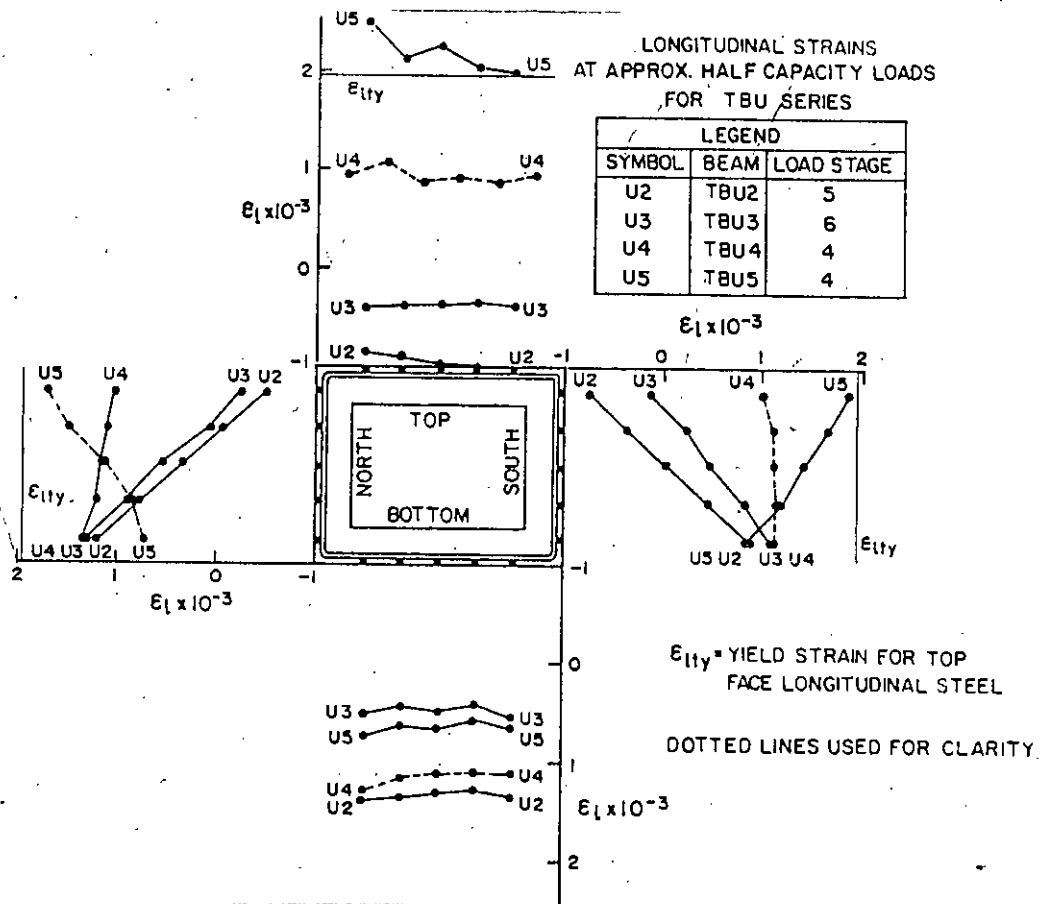


PRINCIPLE DIAGONAL STRAINS
AT APPROX. HALF CAPACITY LOADS
FOR T80 SERIES

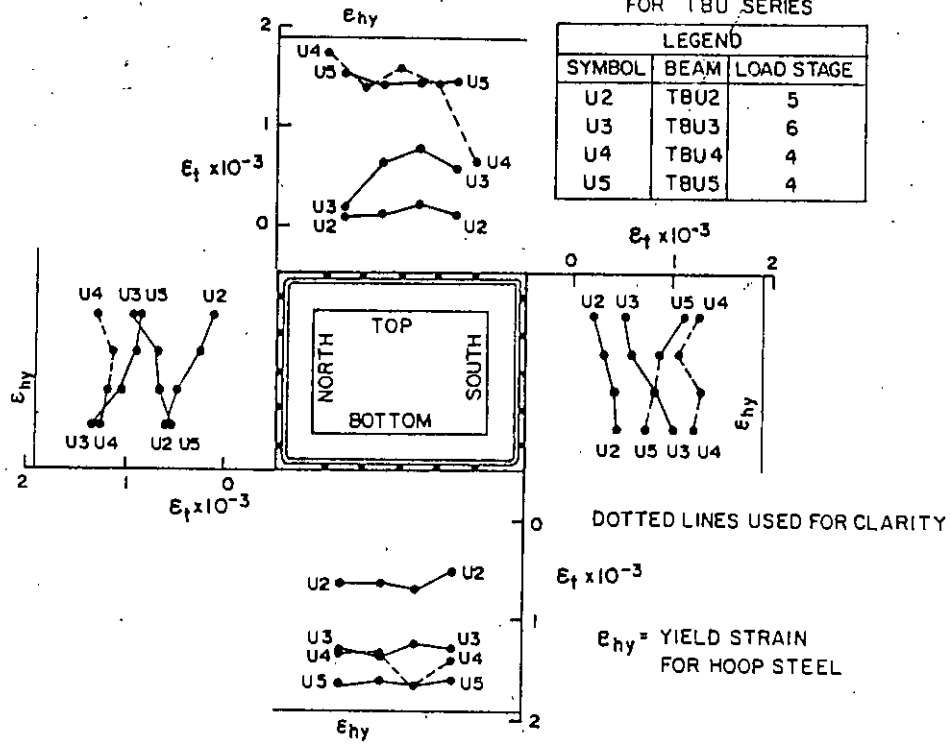


PRINCIPLE DIAGONAL STRAINS
AT APPROX. CAPACITY LOADS
FOR T80 SERIES

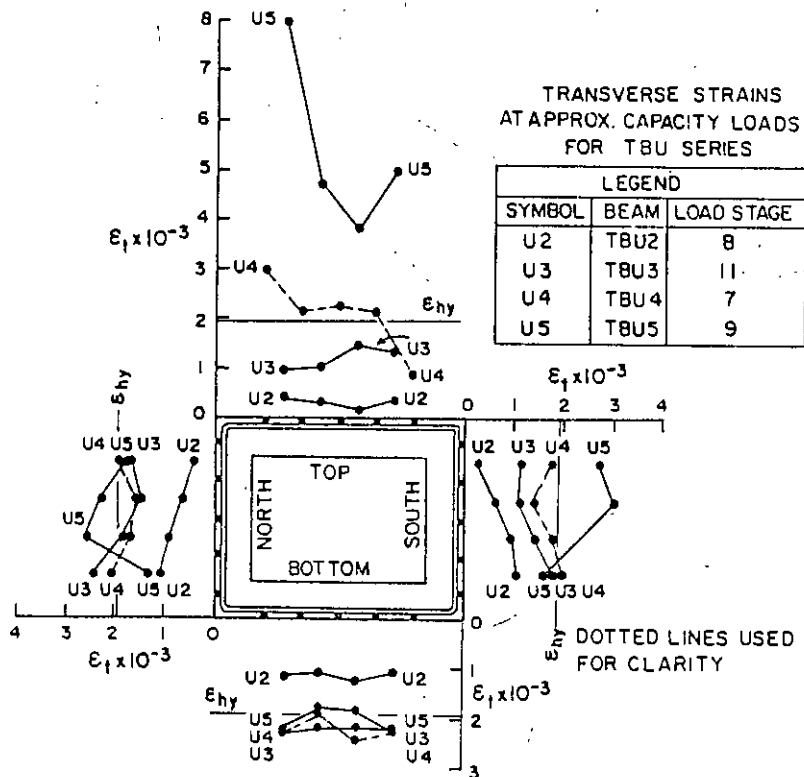




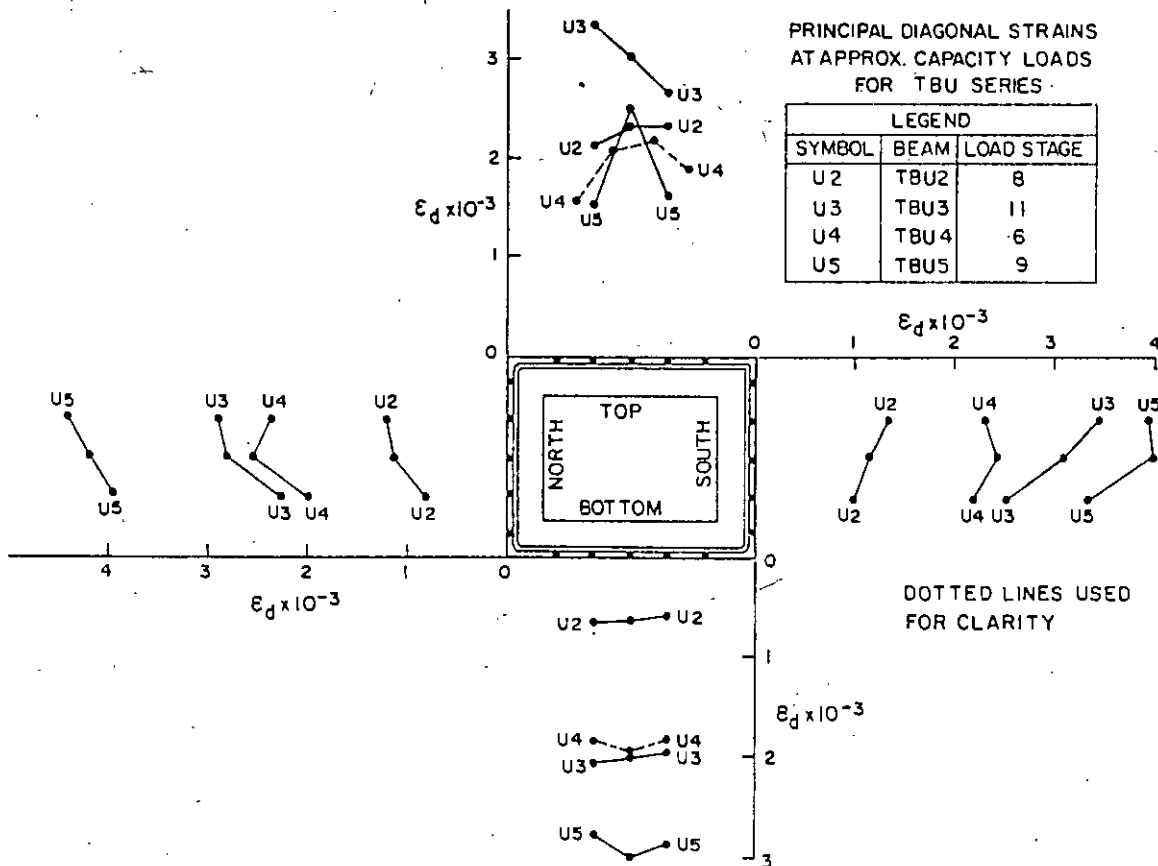
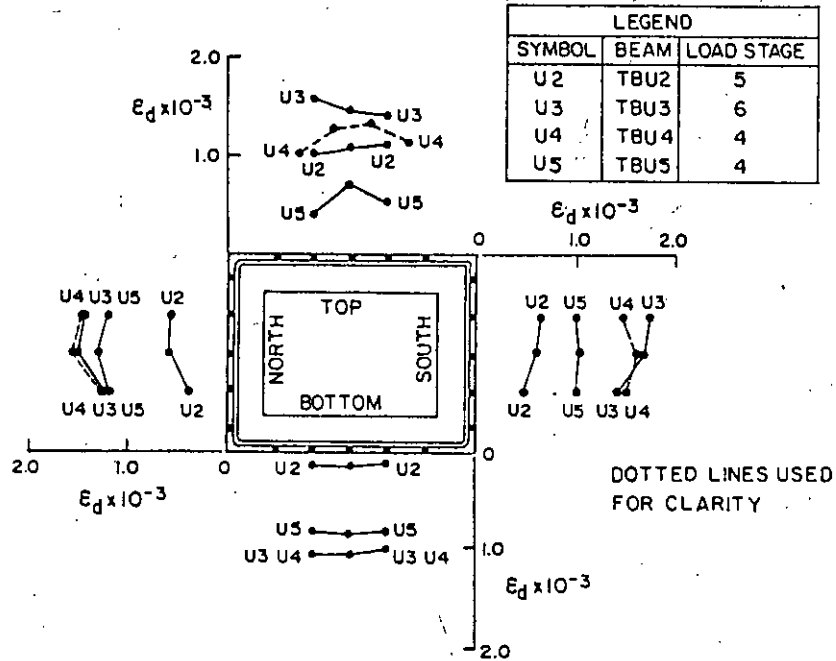
TRANSVERSE STRAINS
AT APPROX. HALF CAPACITY LOADS
FOR TBU SERIES



TRANSVERSE STRAINS
AT APPROX. CAPACITY LOADS
FOR TBU SERIES

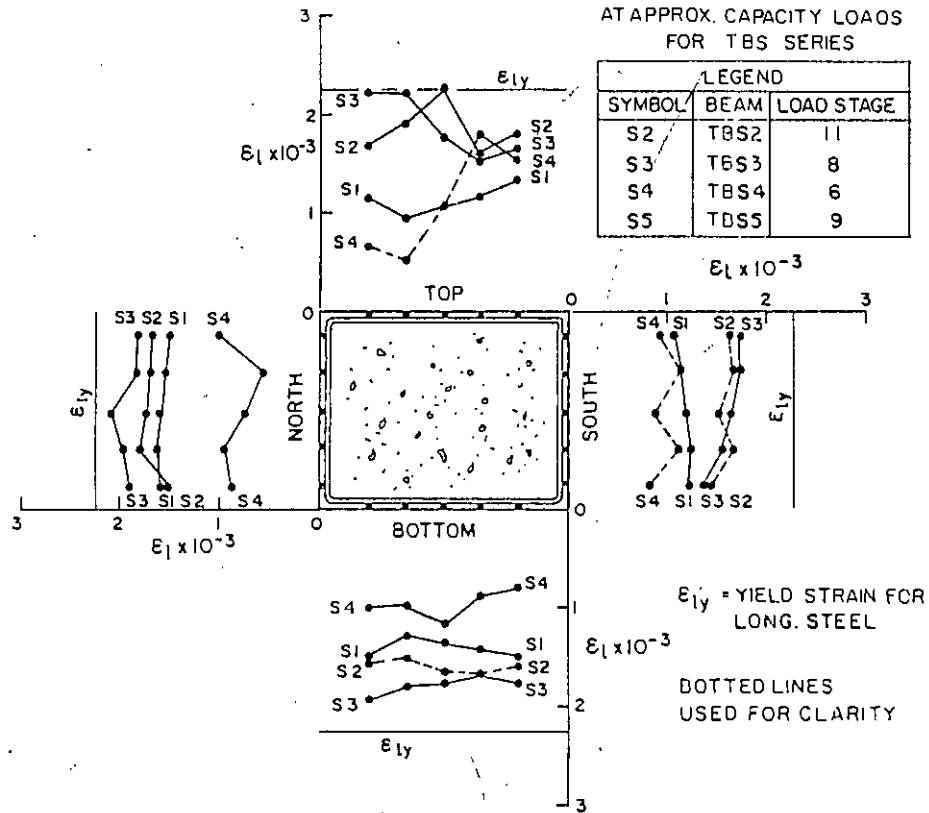


PRINCIPAL DIAGONAL STRAINS
AT APPROX. HALF-CAPACITY LOADS
FOR TBU SERIES



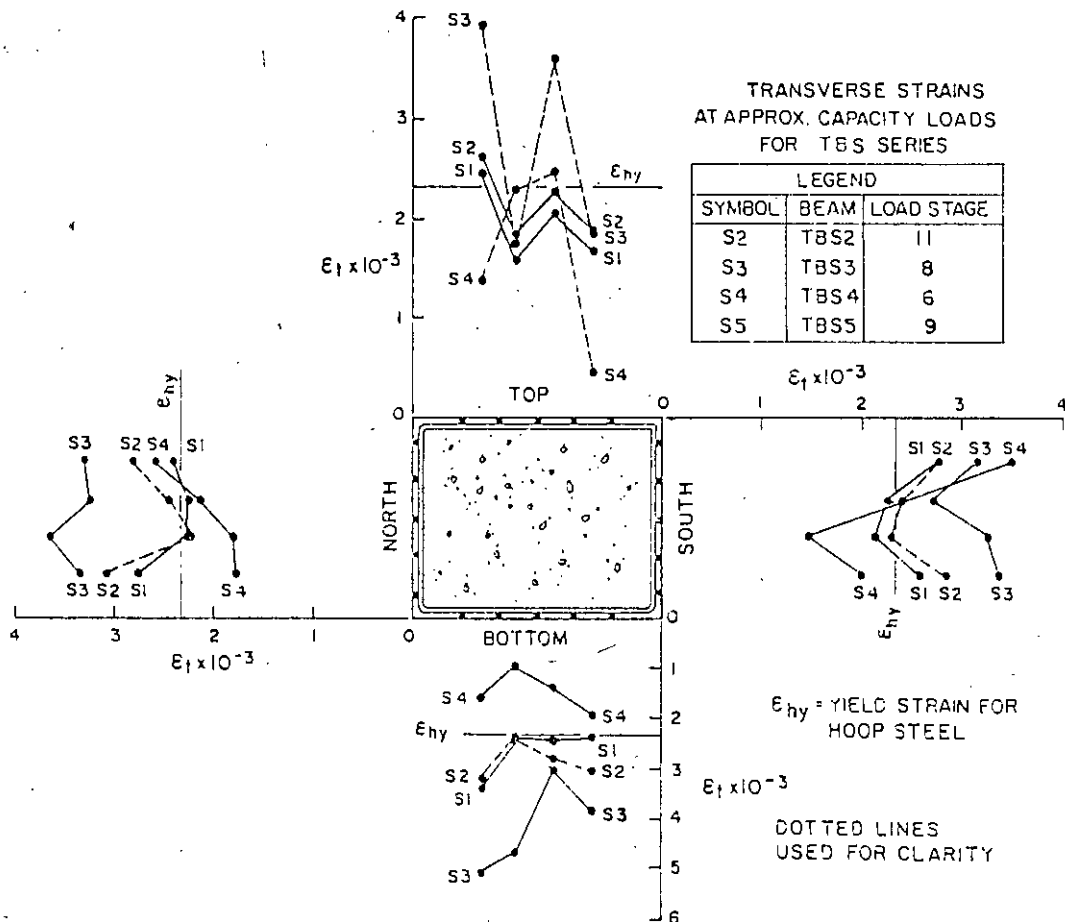
LONGITUDINAL STRAINS
AT APPROX. CAPACITY LOADS
FOR TBS SERIES

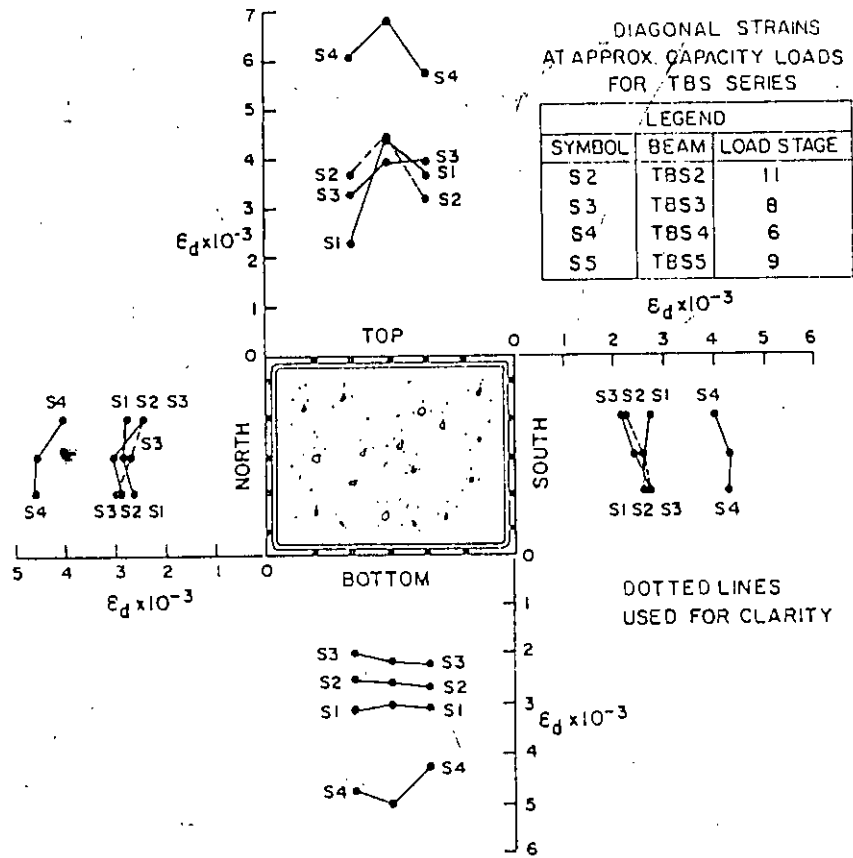
LEGEND		
SYMBOL	BEAM	LOAD STAGE
S2	TBS2	11
S3	TBS3	8
S4	TBS4	6
S5	TBS5	9



TRANSVERSE STRAINS
AT APPROX. CAPACITY LOADS
FOR TBS SERIES

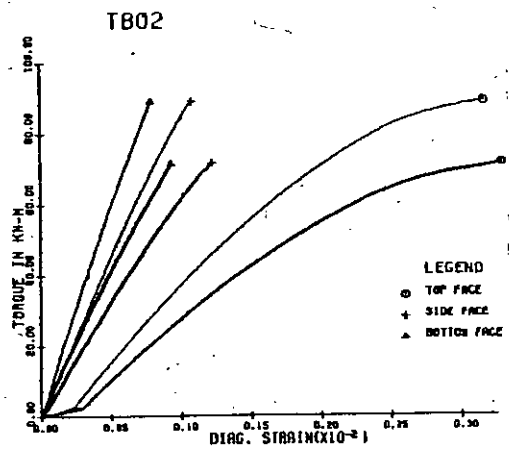
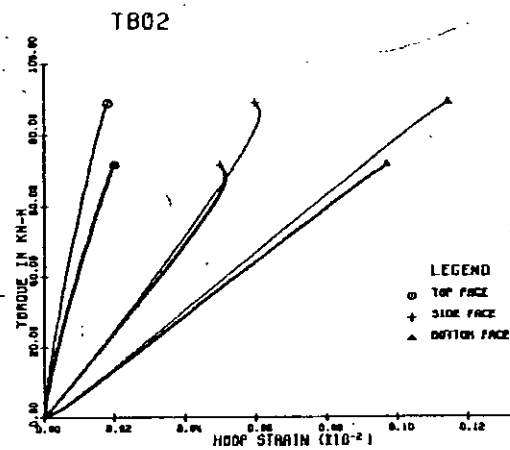
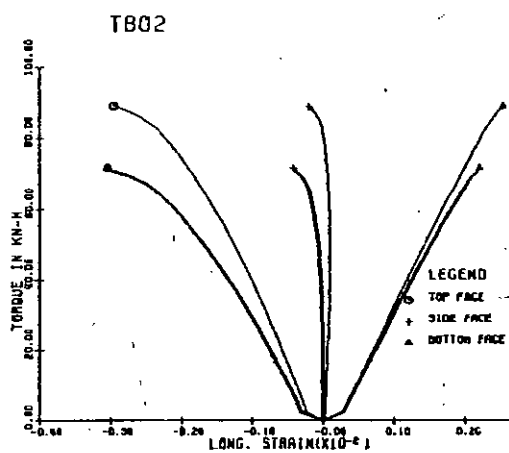
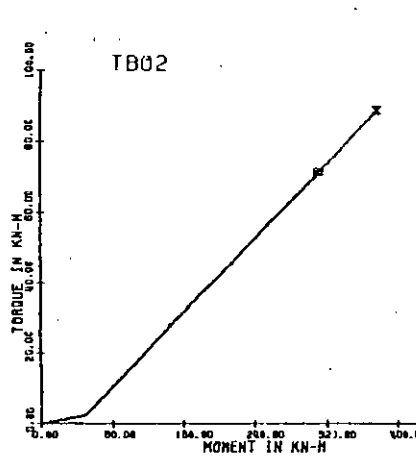
LEGEND		
SYMBOL	BEAM	LOAD STAGE
S2	TBS2	11
S3	TBS3	8
S4	TBS4	6
S5	TBS5	9



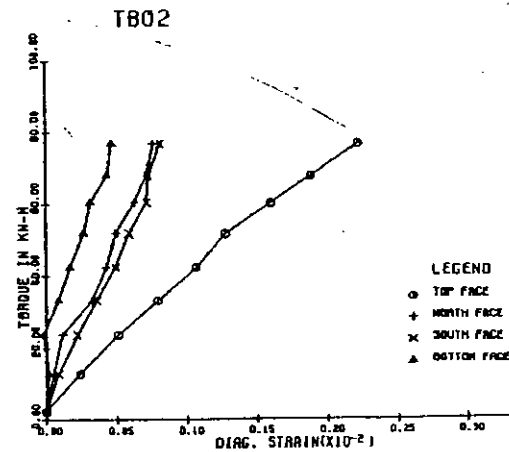
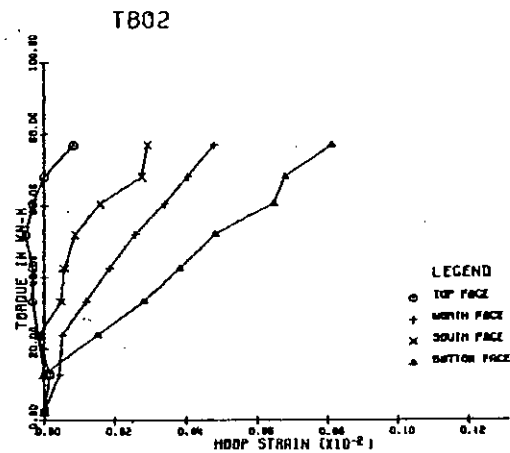
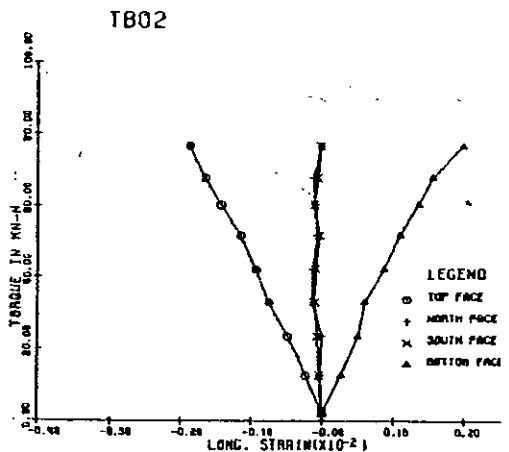
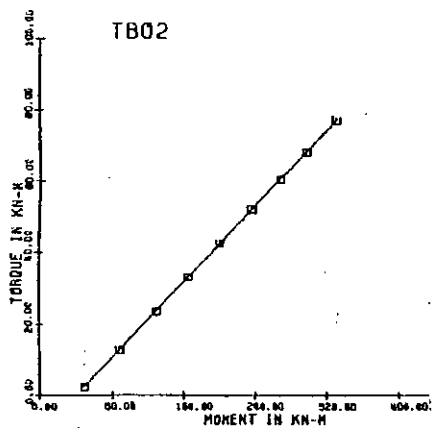


E.5 PLOTS OF MEASURED STRAINS AND PREDICTED STRAINS AT MID-POINTS OF SECTION FACES AGAINST APPLIED TORQUE

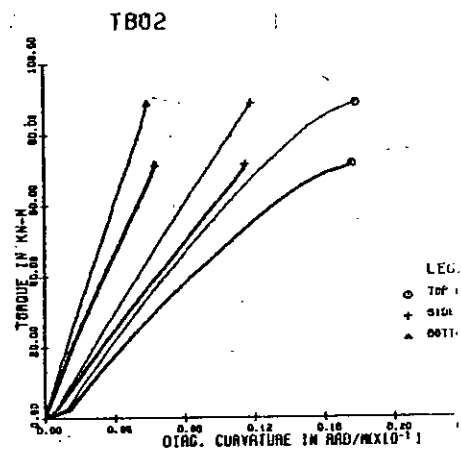
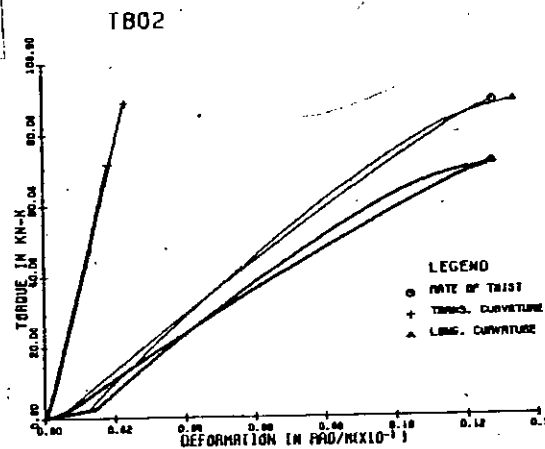
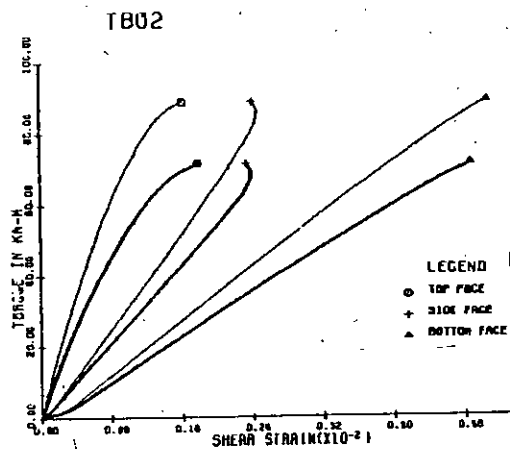
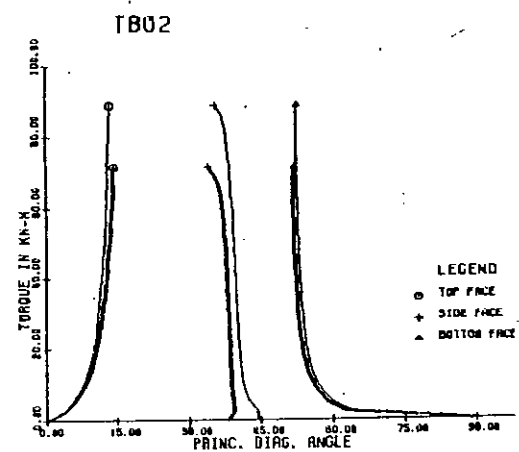
The following plots give the complete range of the response predictions for all the beams tested and the corresponding measured values for comparison. In the plots of test data separate lines are plotted for the North and South faces. These lines are evidently in close agreement. For the model predictions for the section with one axis of symmetry, the values for the mid-points of the North face and South face are identical and have been referred to as side face values. It should also be noted that response predictions were made for the spalled and unspalled section dimensions for each beam. The spalled predictions are plotted in thick lines and the unspalled in thin lines.



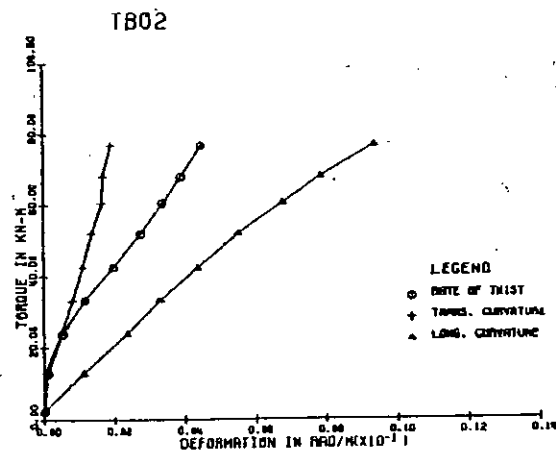
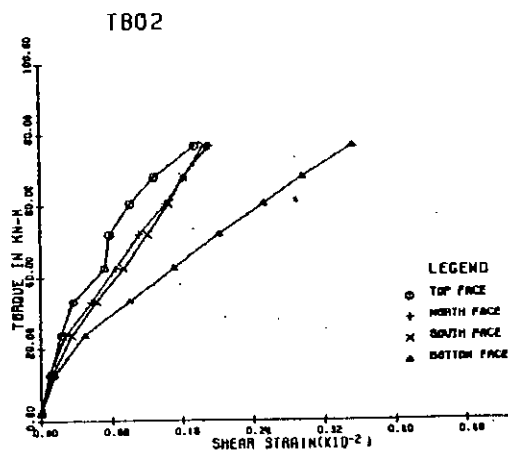
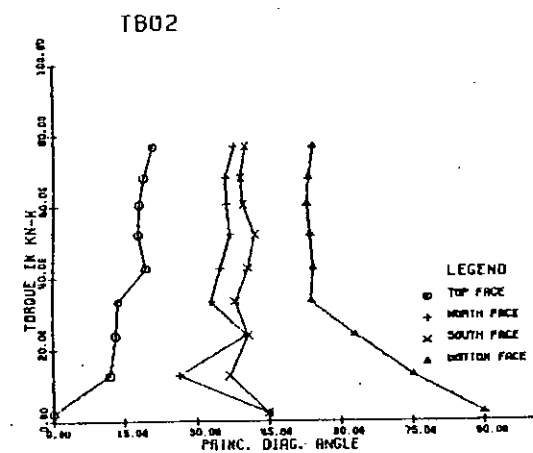
THEORETICAL PREDICTION



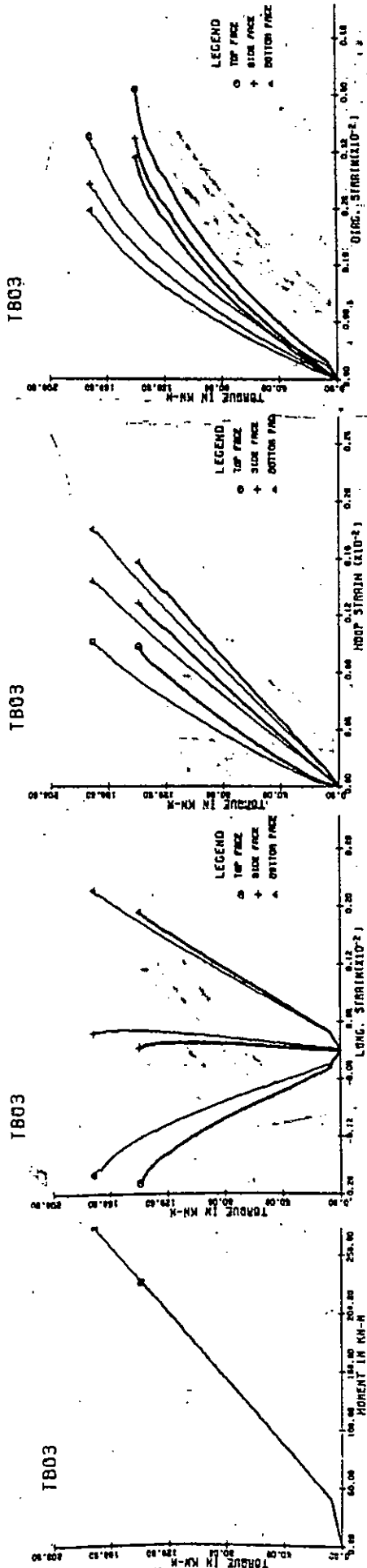
TEST RESULTS



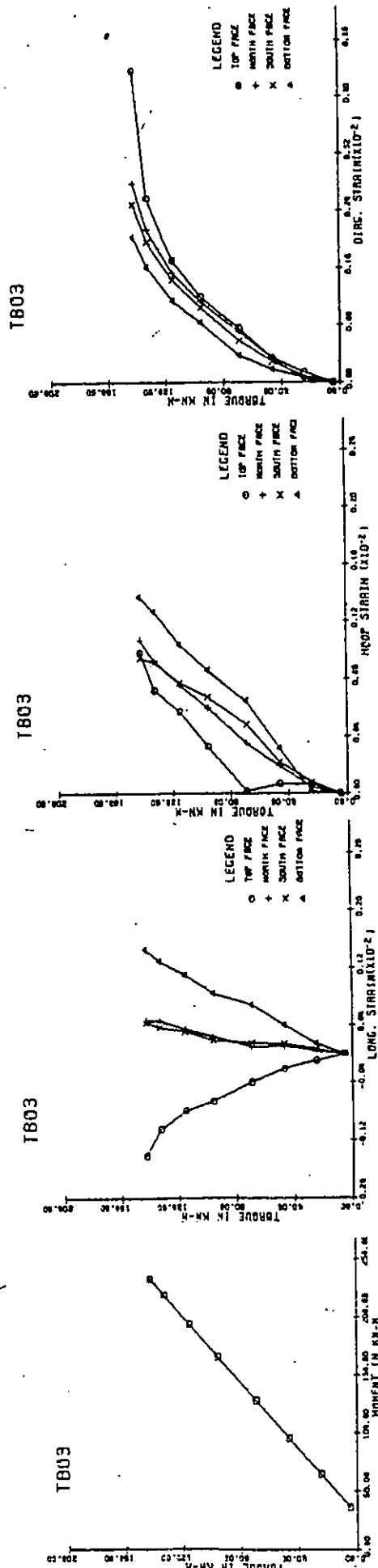
THEORETICAL PREDICTION



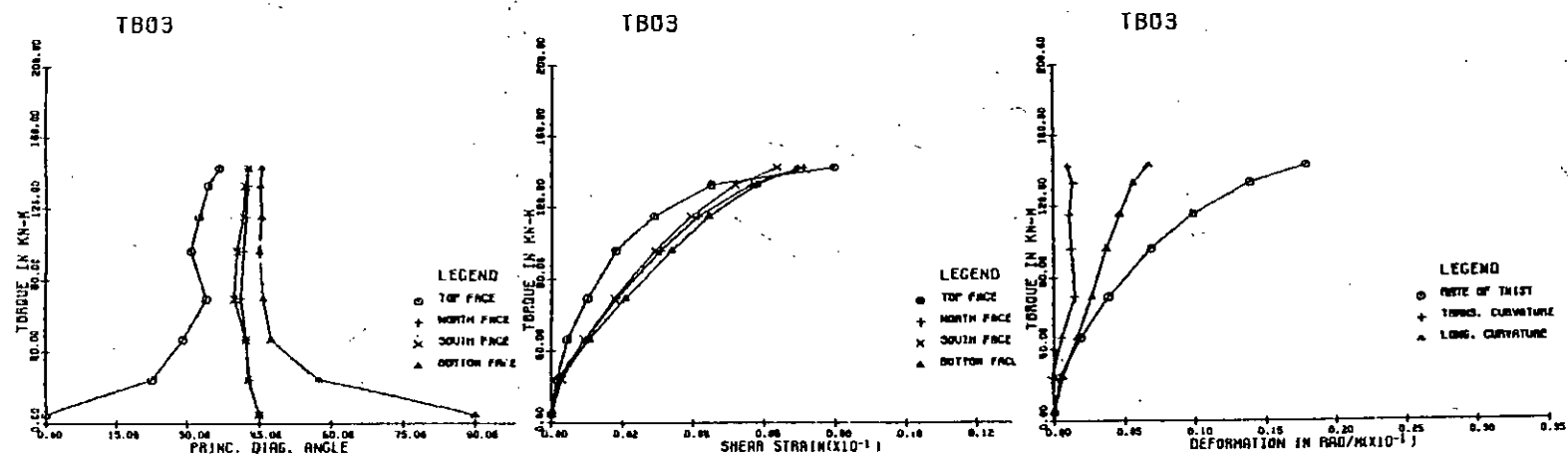
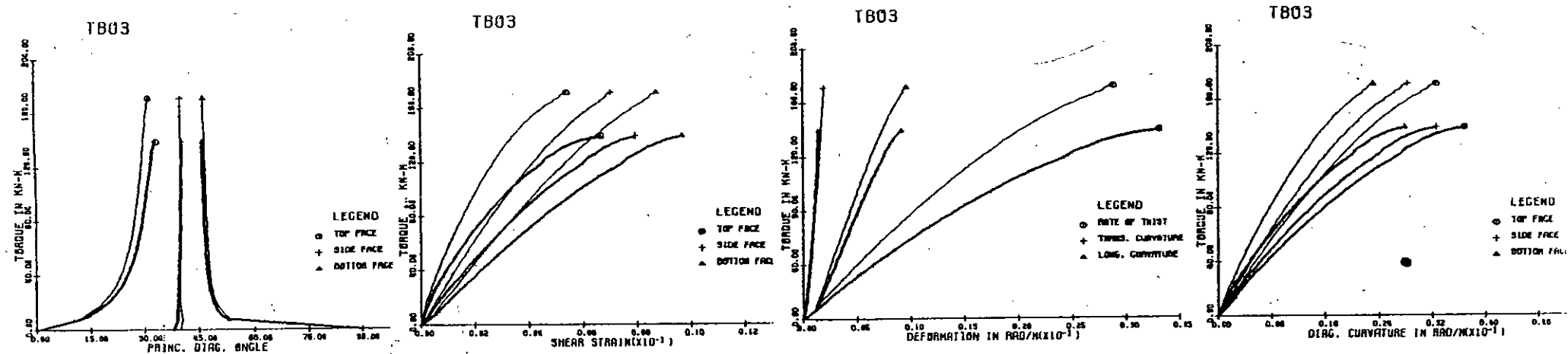
TEST RESULTS

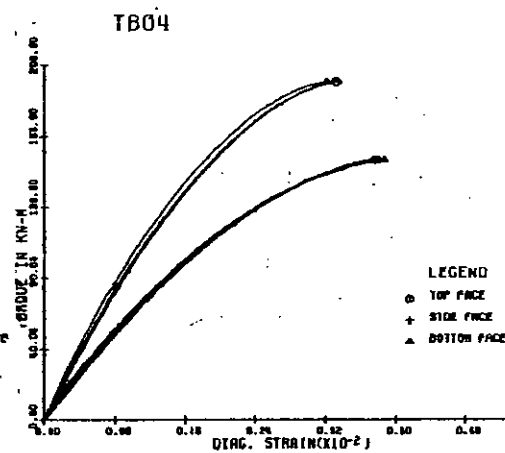
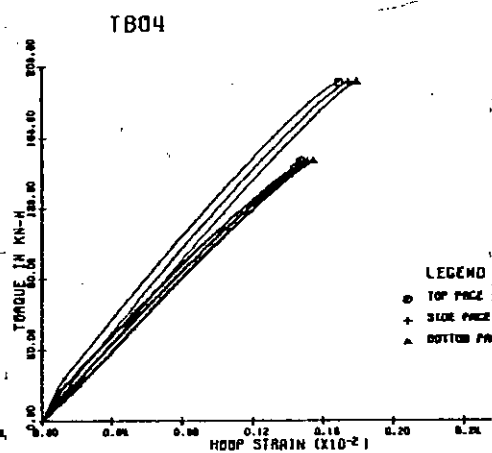
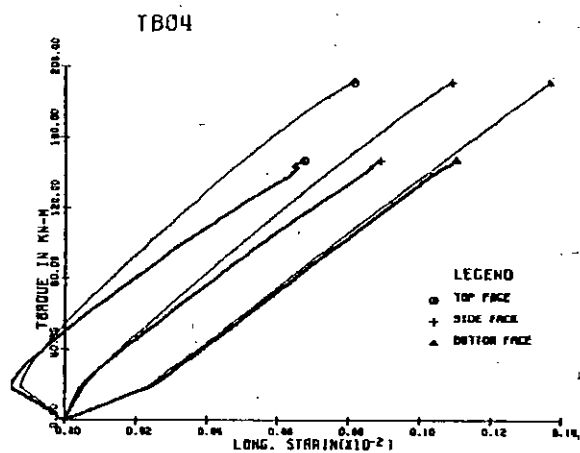
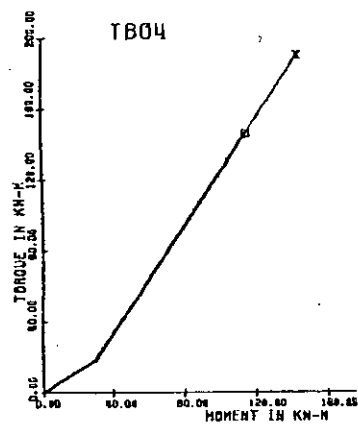


THEORETICAL PREDICTION

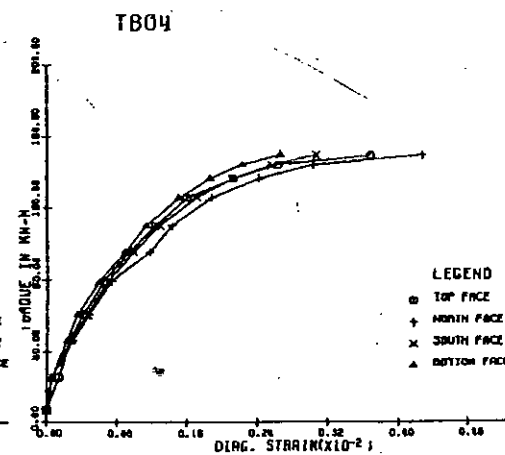
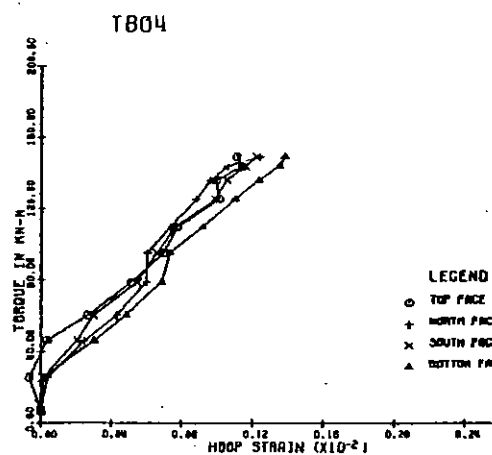
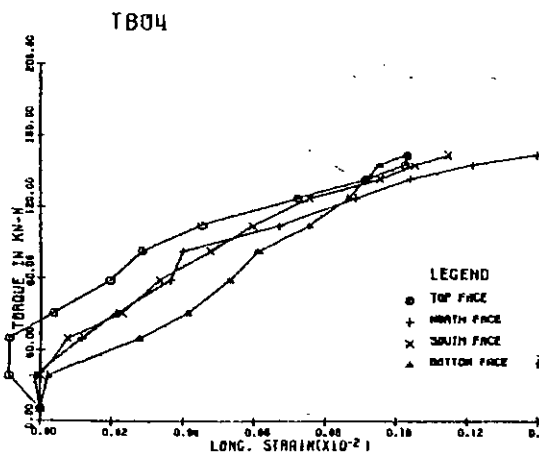
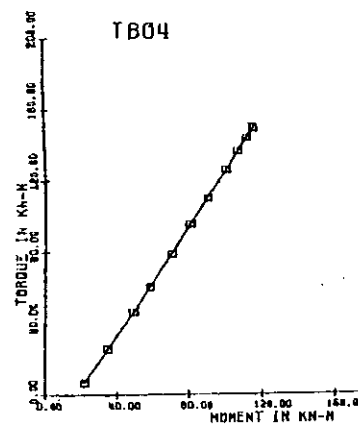


TEST RESULTS



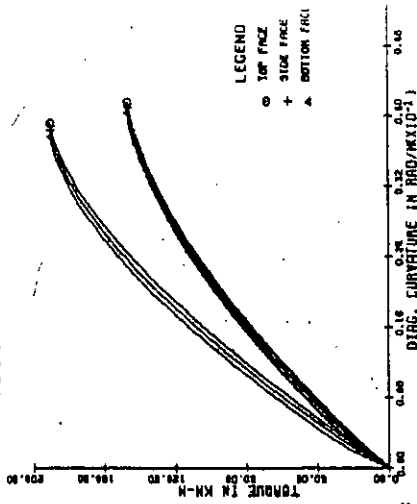


THEORETICAL PREDICTION

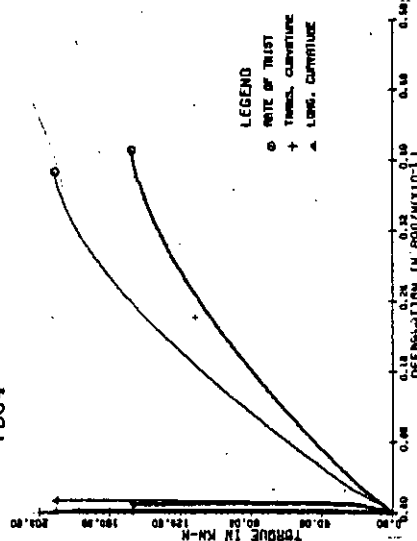


TEST RESULTS

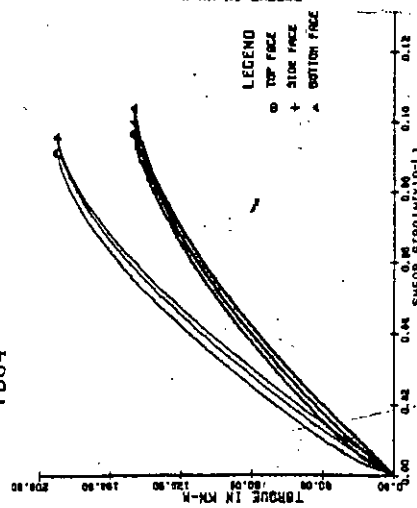
TB04



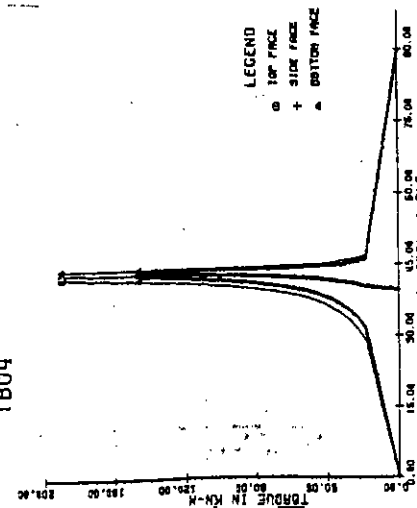
TB04



TB04

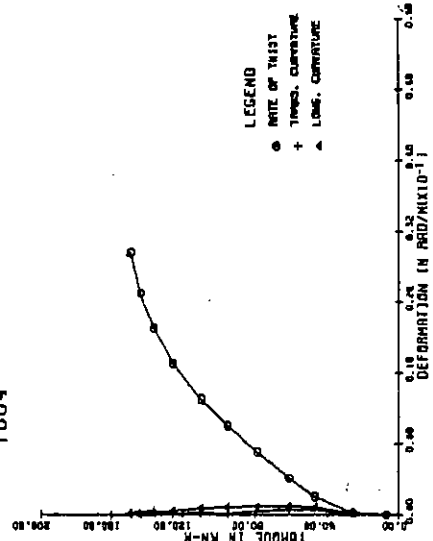


TB04

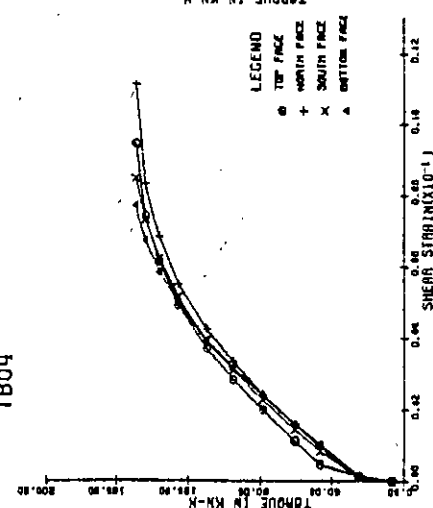


THEORETICAL PREDICTION

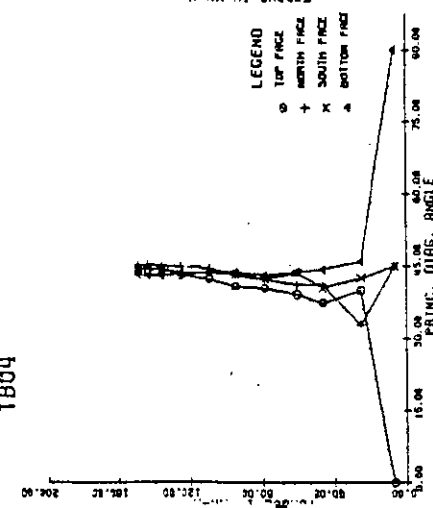
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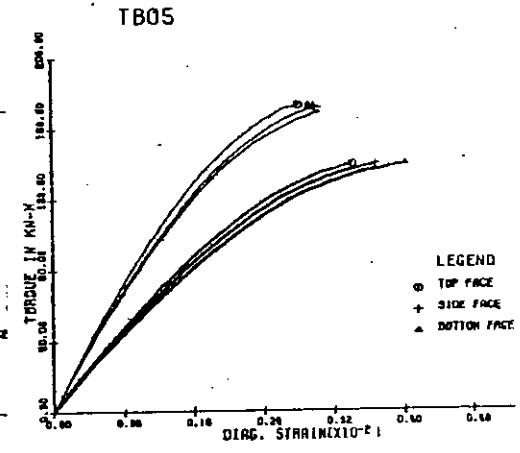
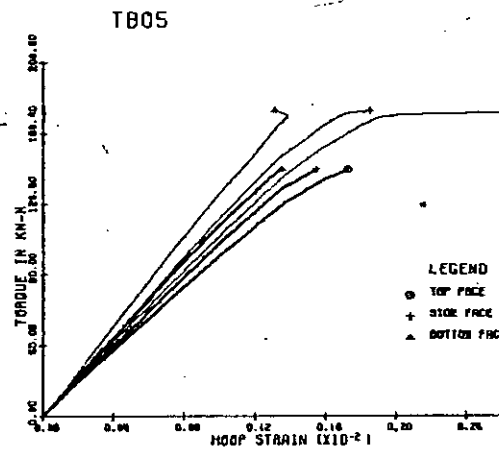
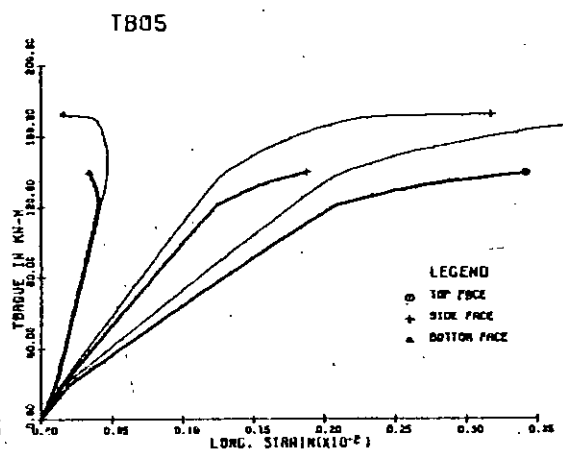
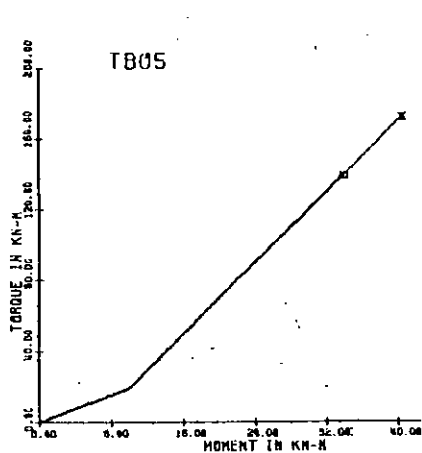
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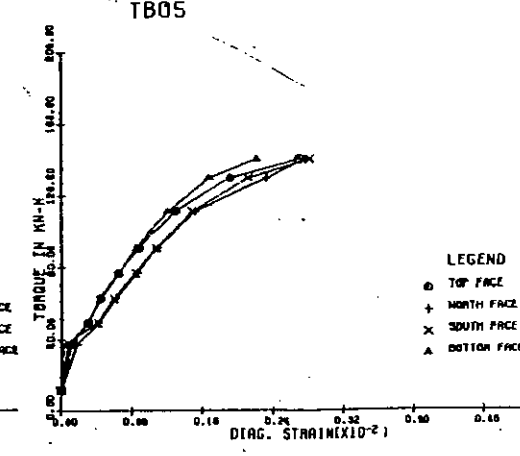
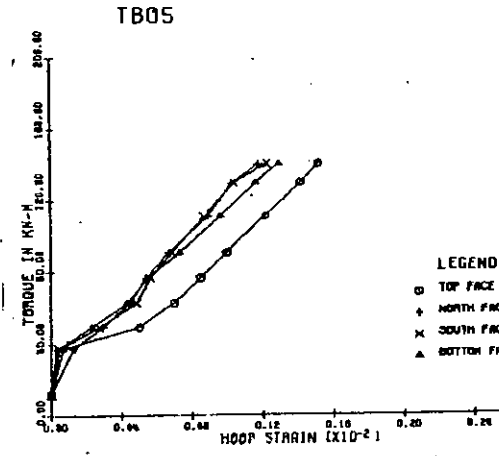
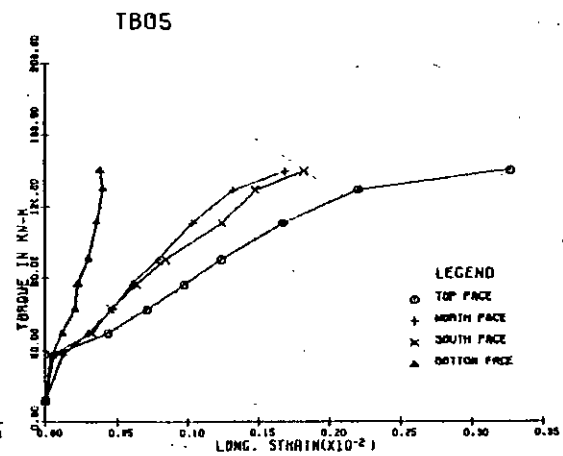
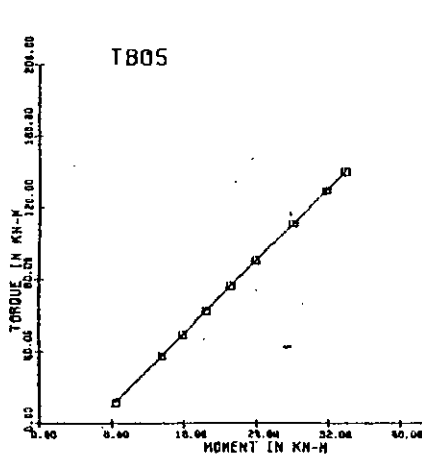
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TEST RESULTS

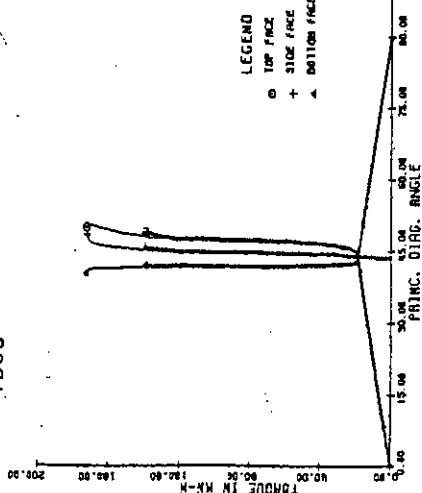


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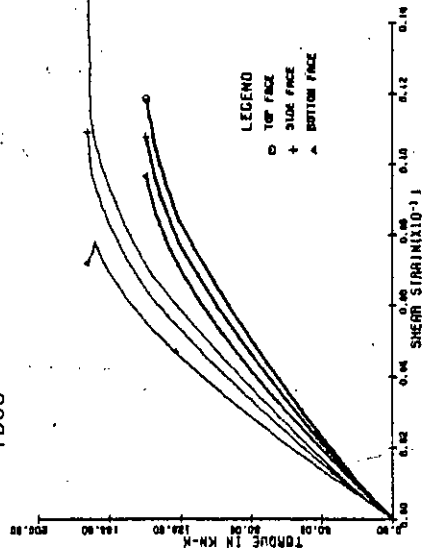


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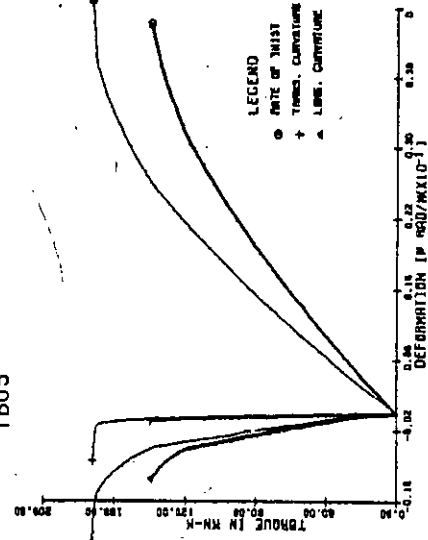
TB05



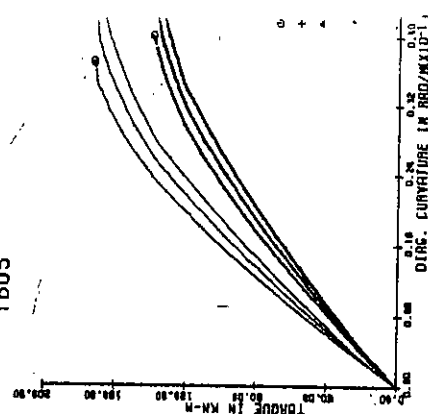
TB05



TB05

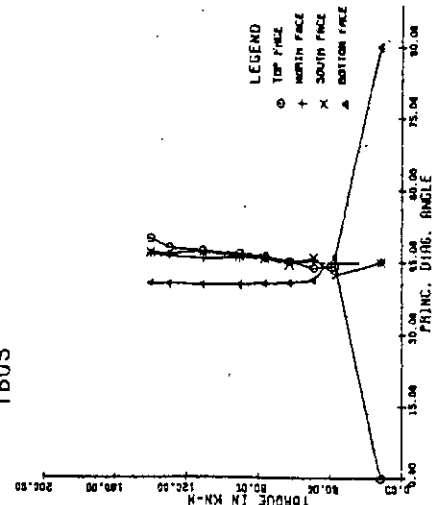


TB05

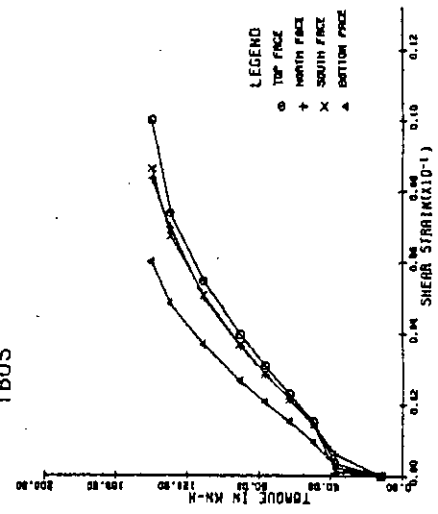


THEORETICAL PREDICTION

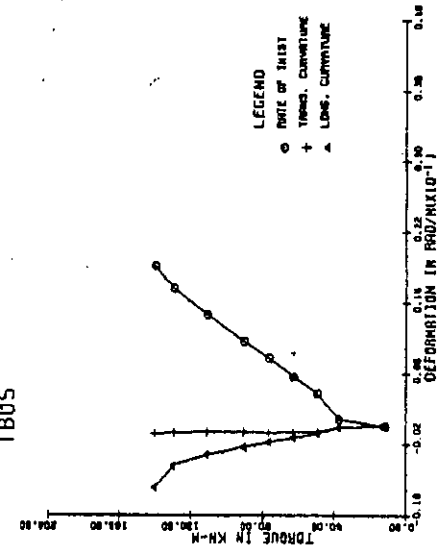
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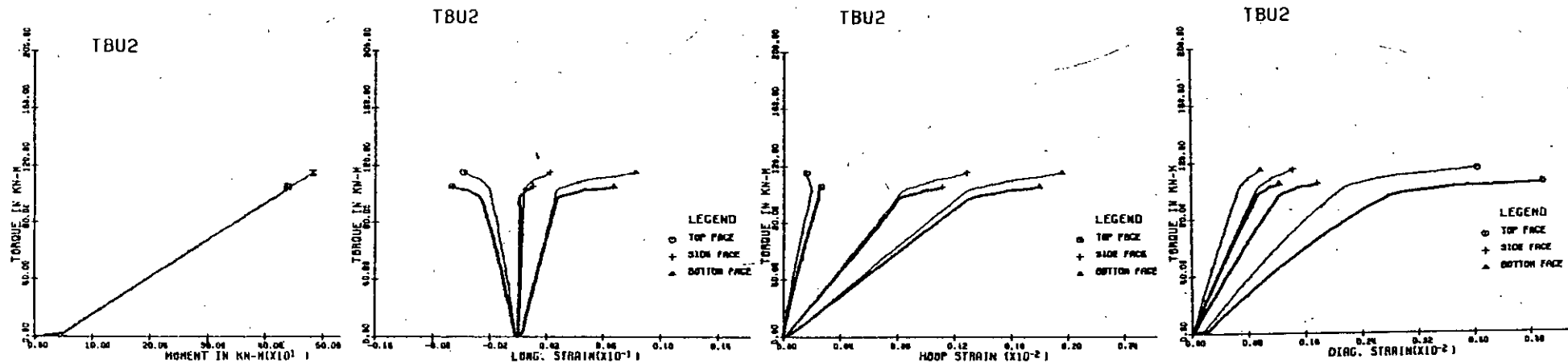
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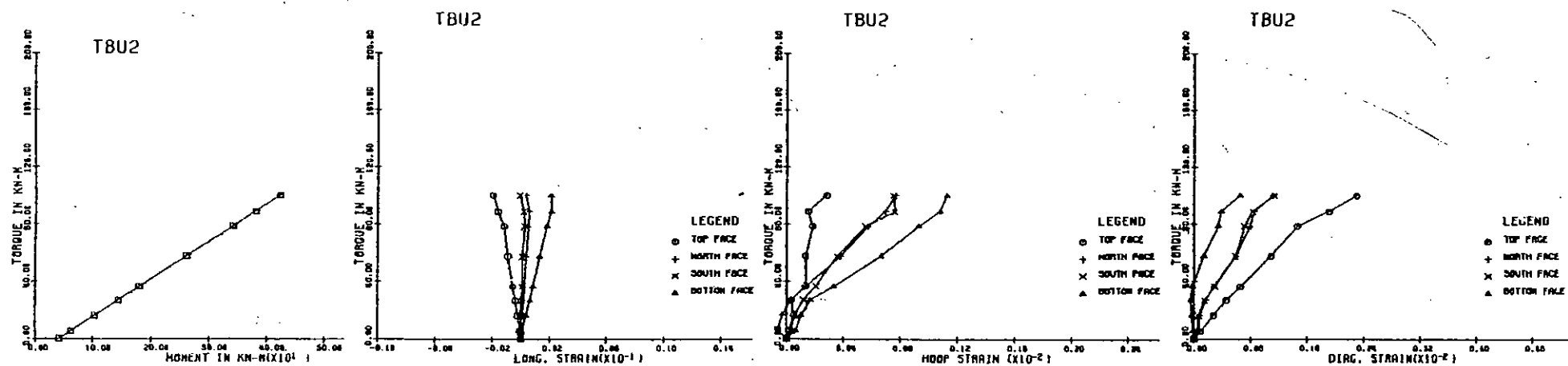
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TEST RESULTS

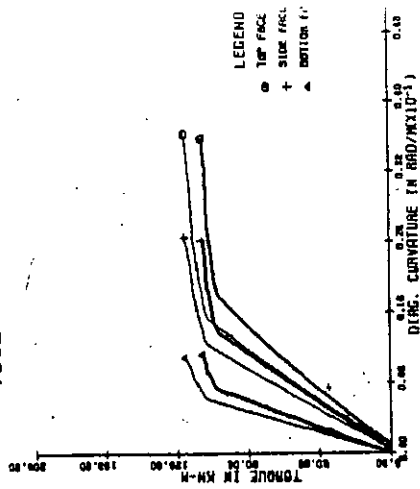


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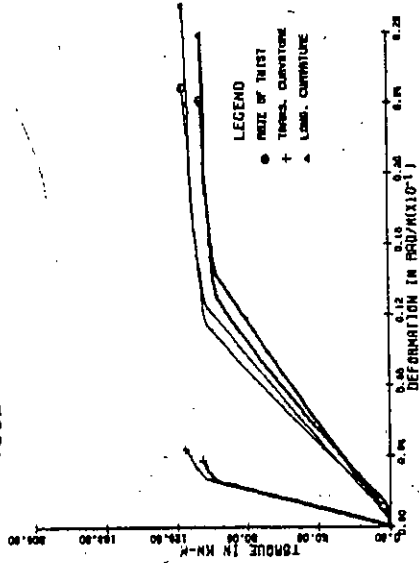


TEST RESULTS

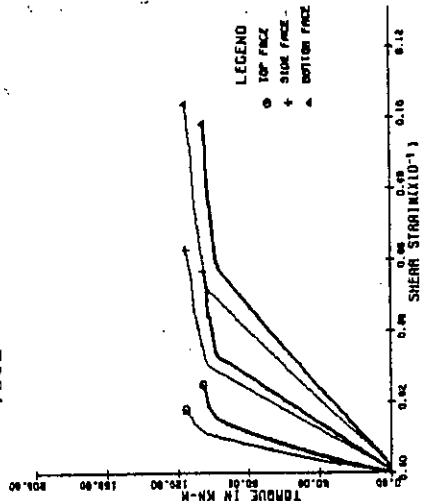
TBU2



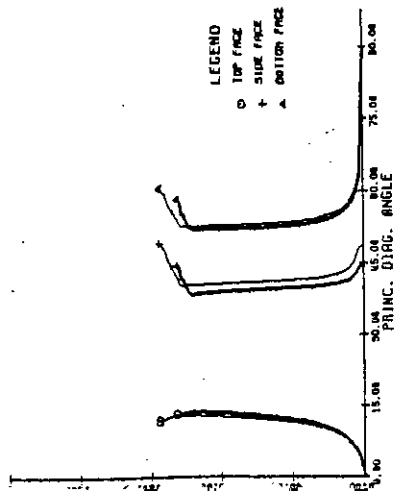
TBU2



TBU2

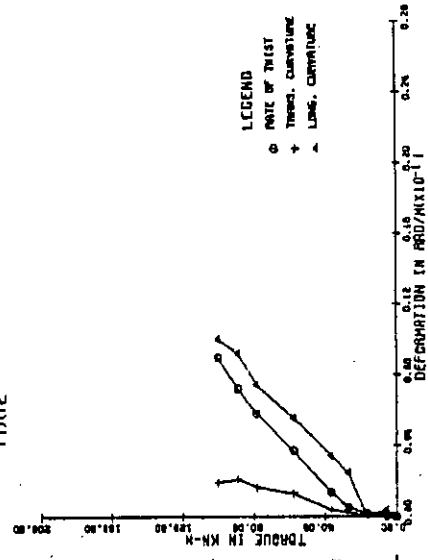


TBU2

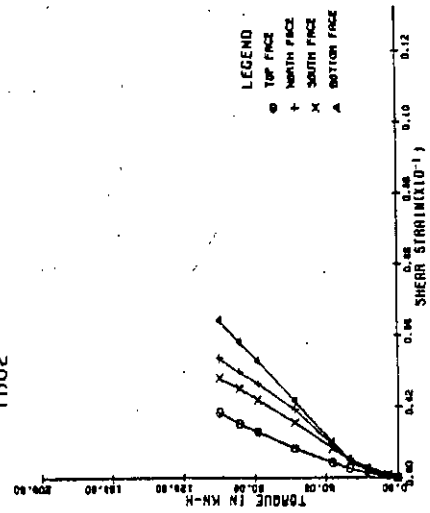


THEORETICAL PREDICTION

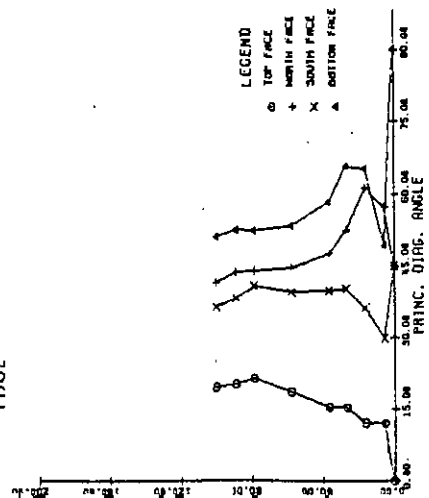
TBU2



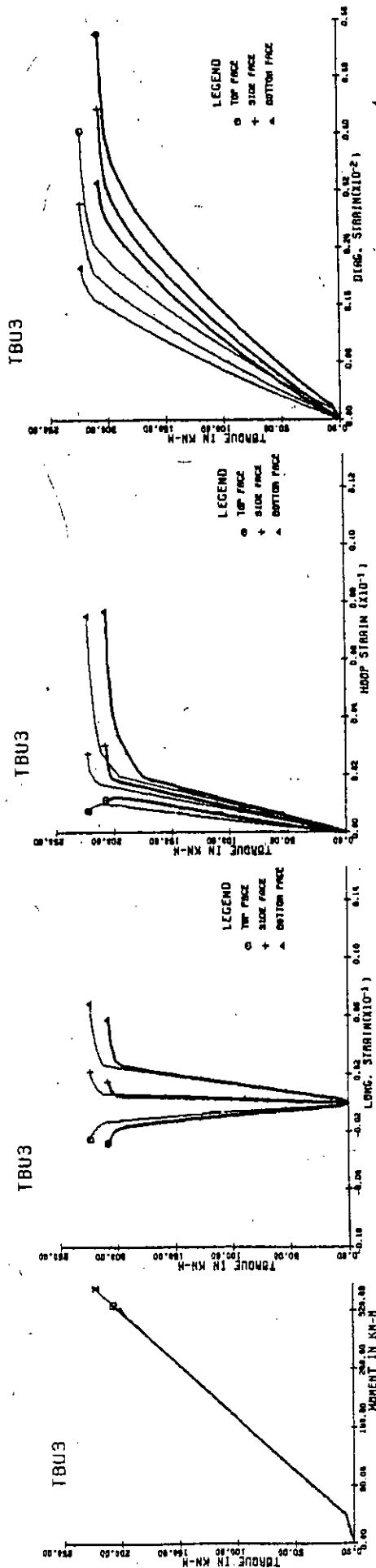
TBU2



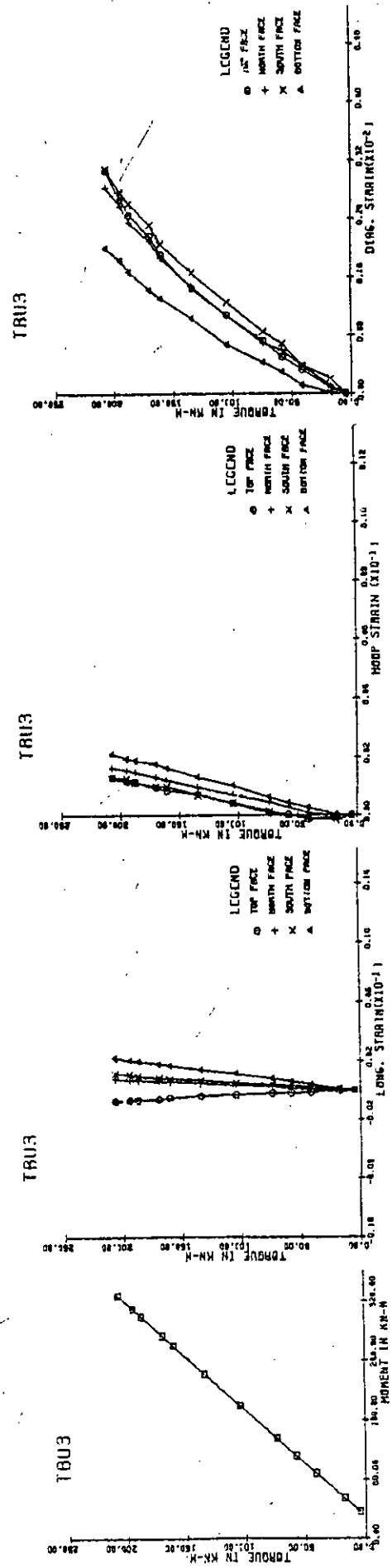
TBU2



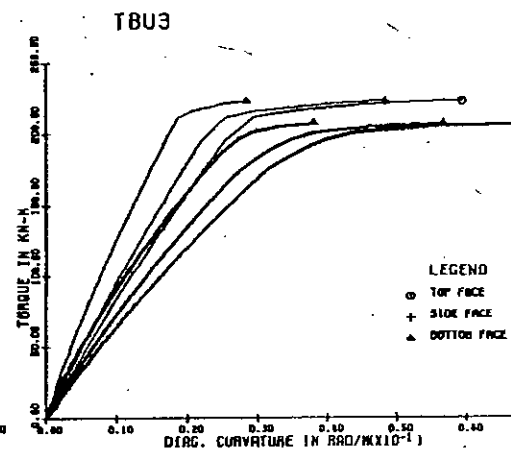
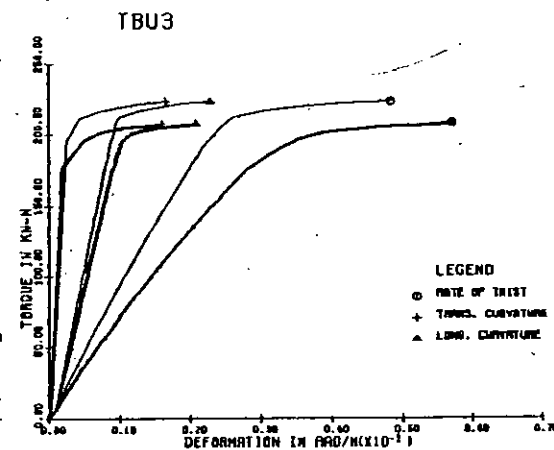
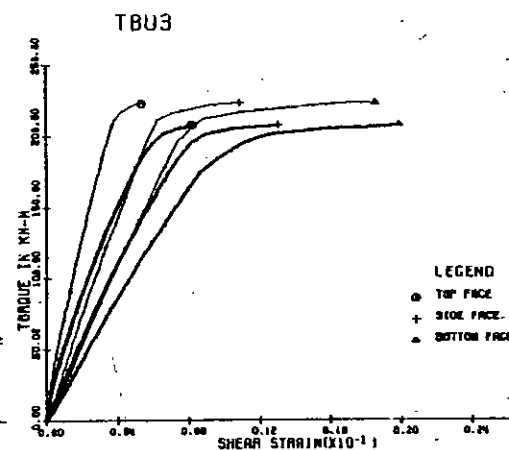
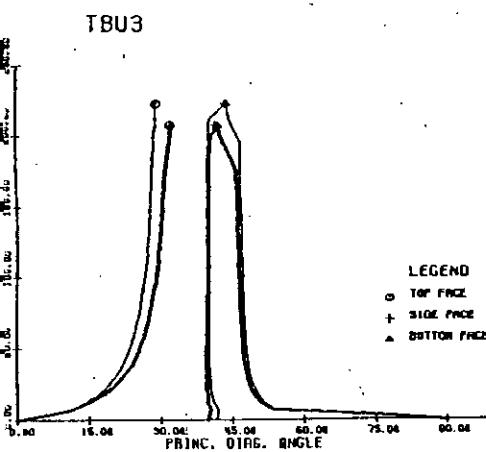
TEST RESULTS



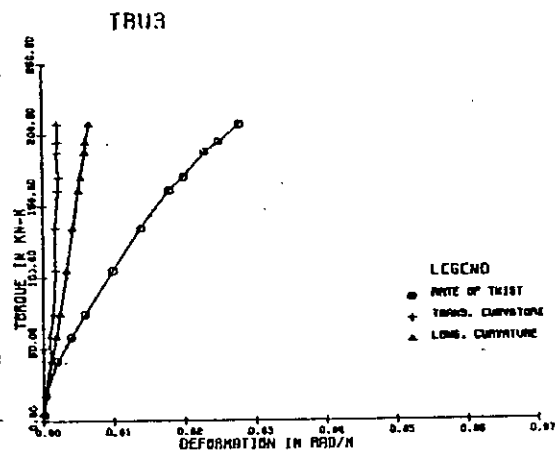
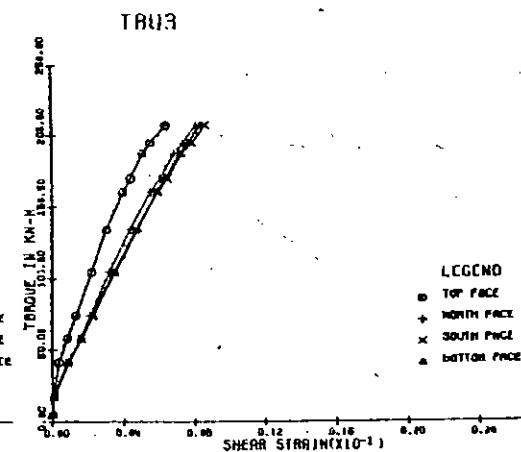
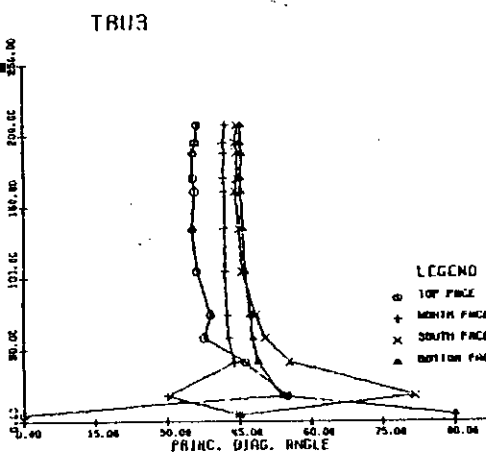
THEORETICAL PREDICTION



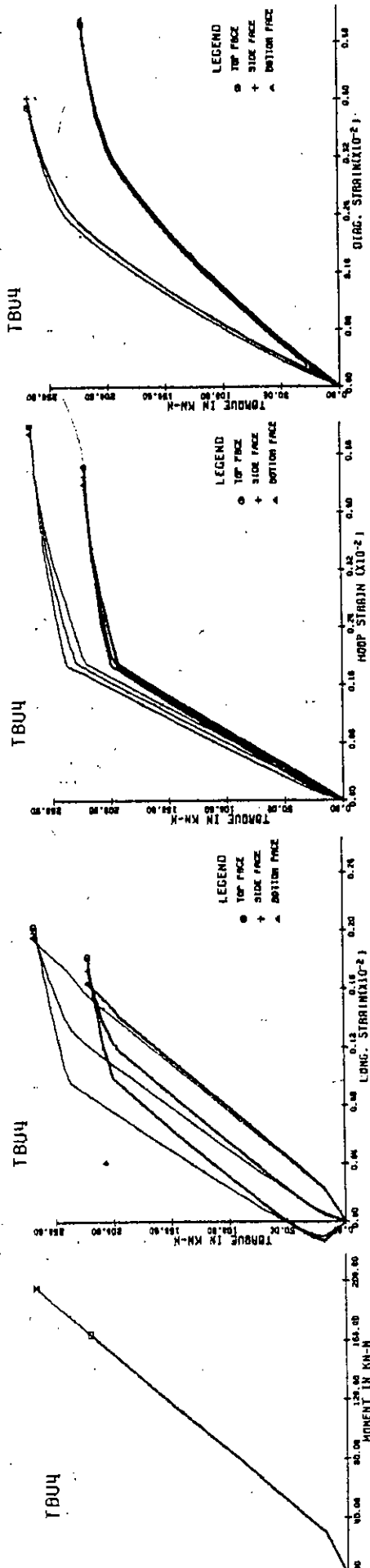
TEST RESULTS



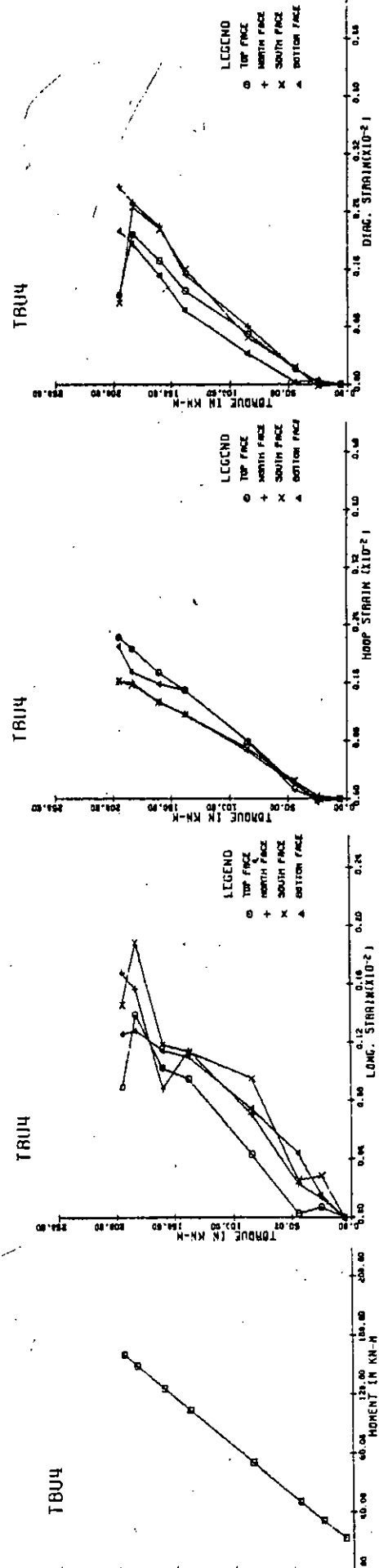
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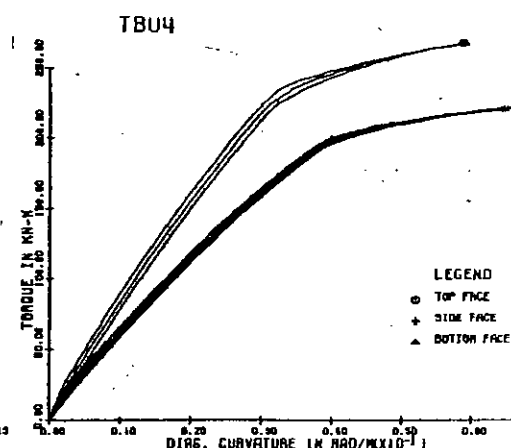
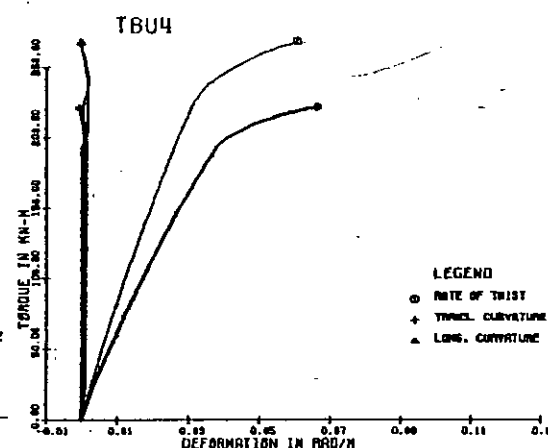
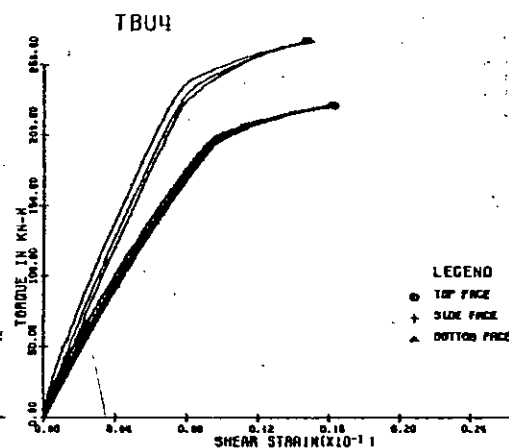
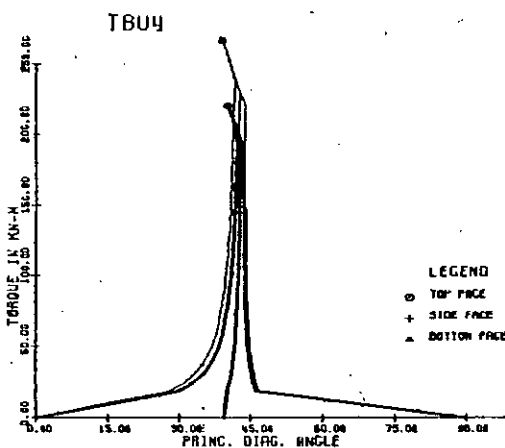
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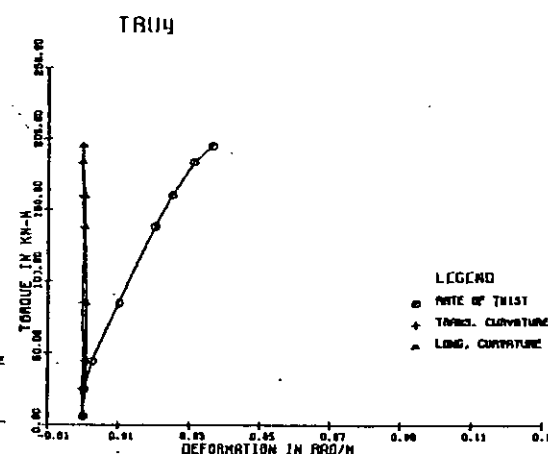
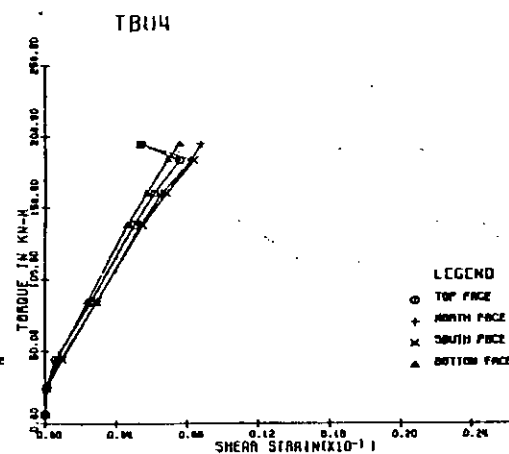
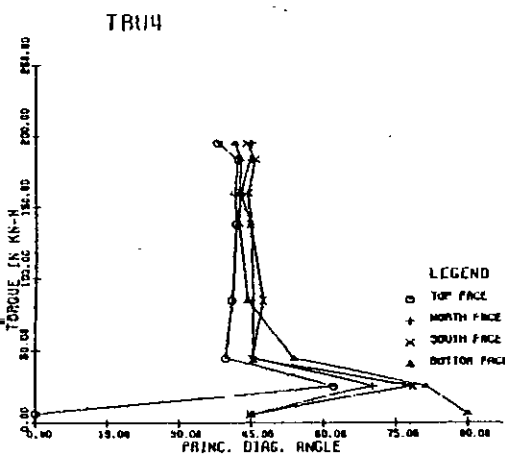
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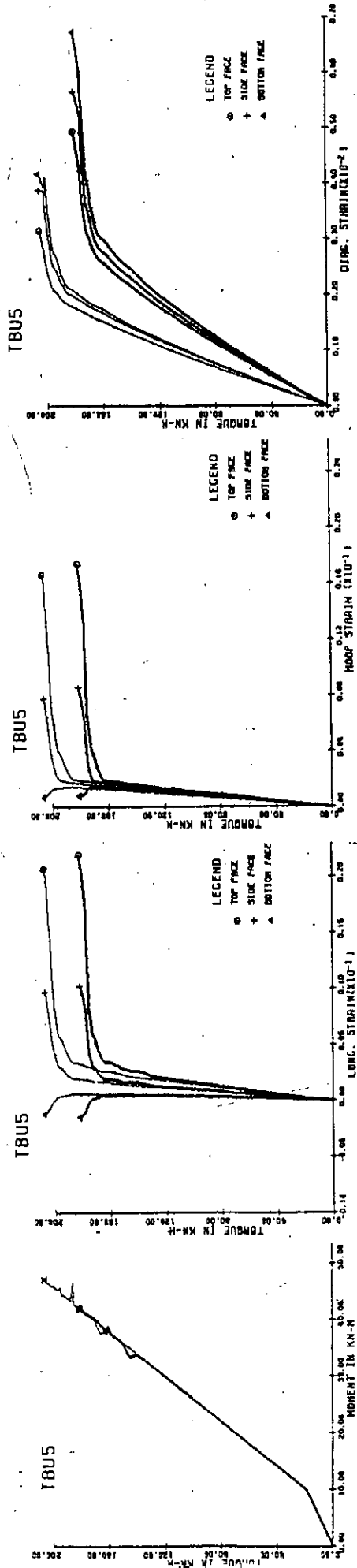
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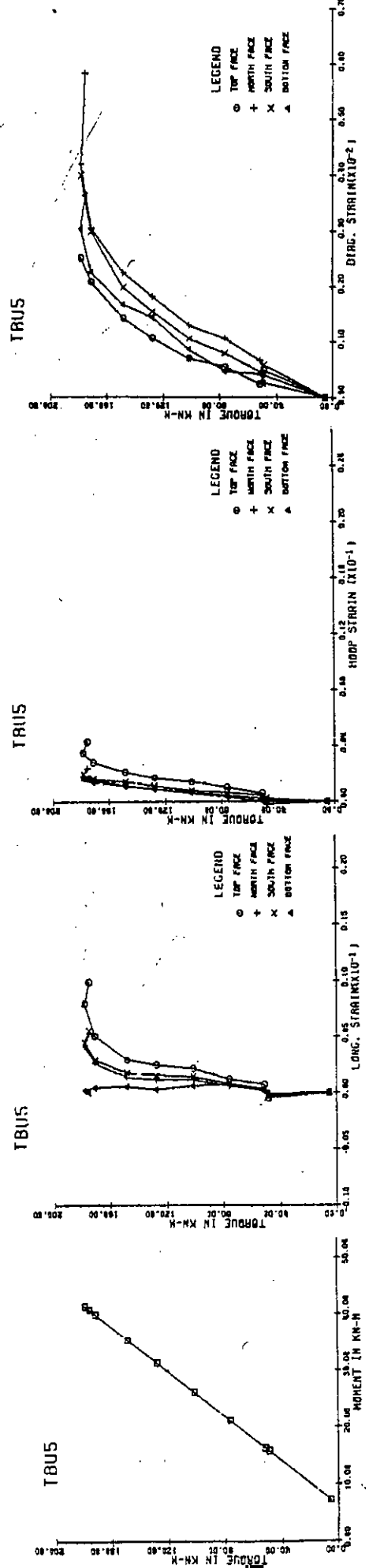
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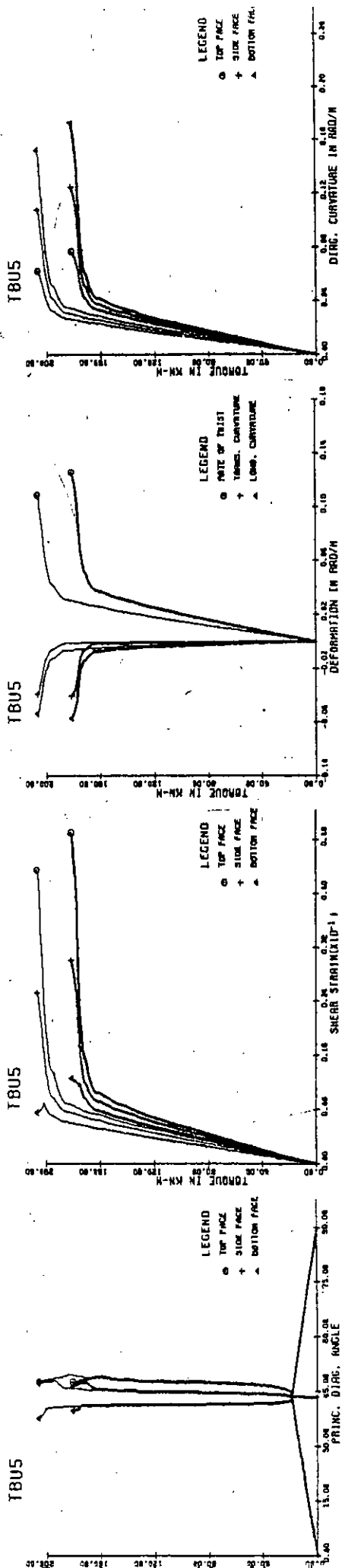
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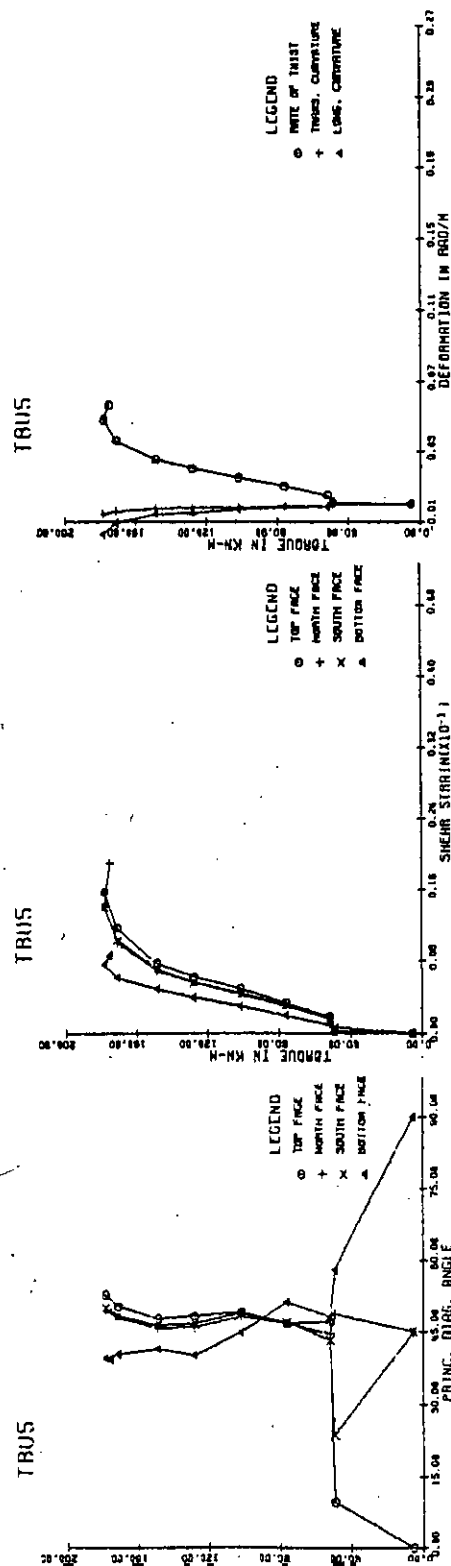
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TEST RESULTS

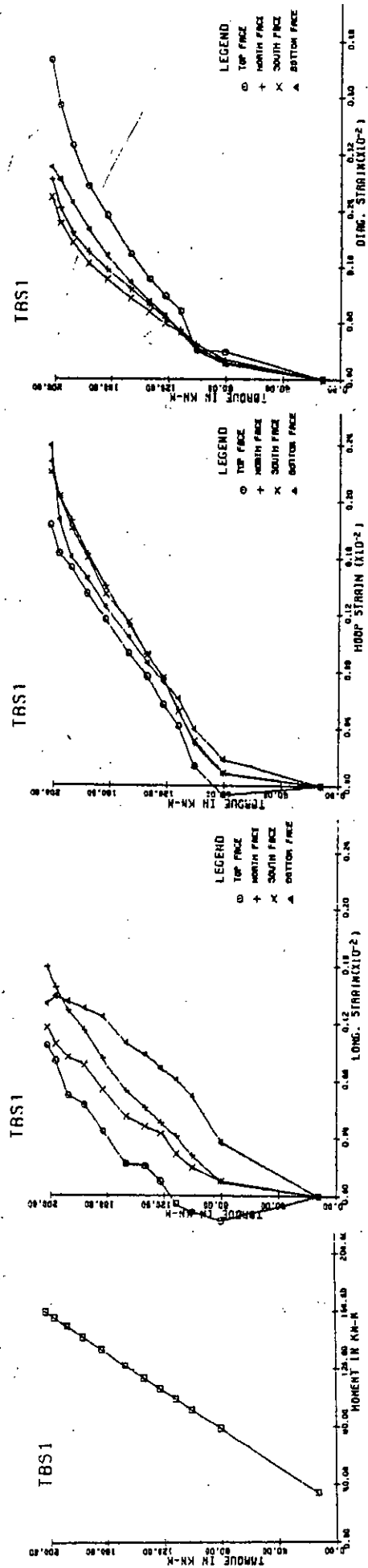
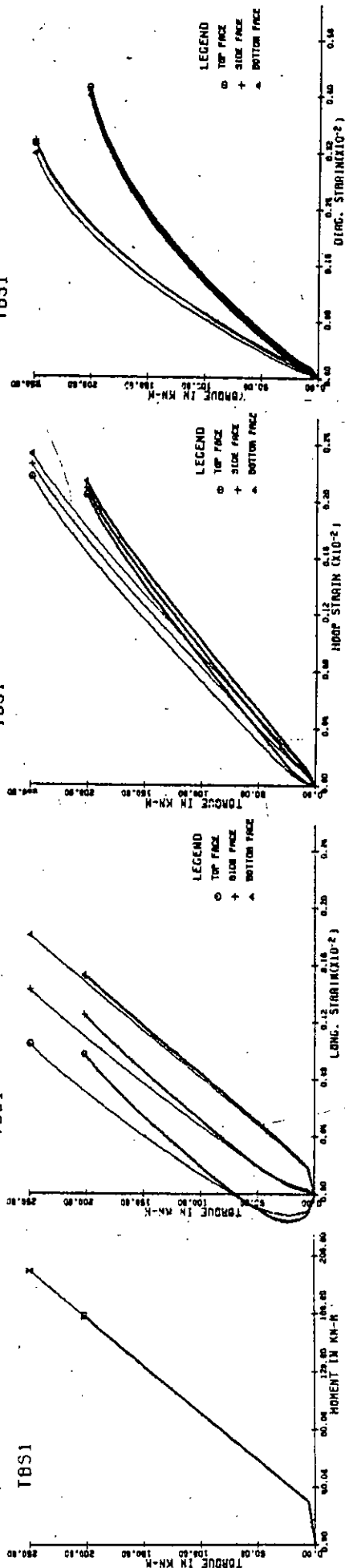


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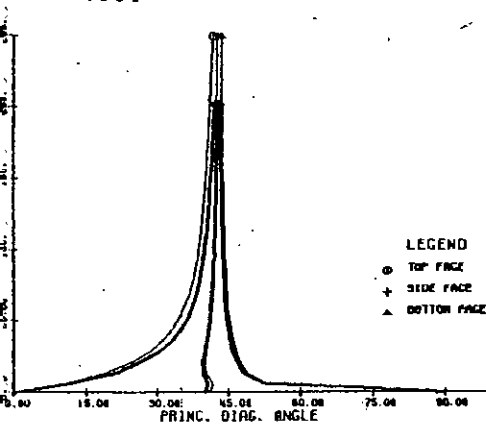


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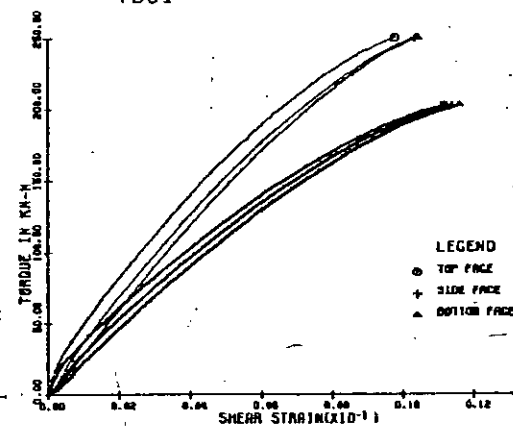
TEST RESULTS



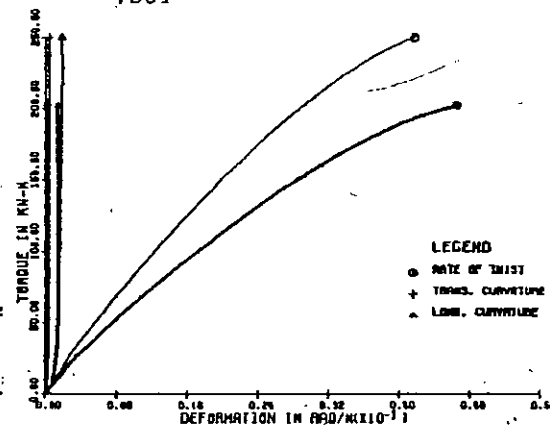
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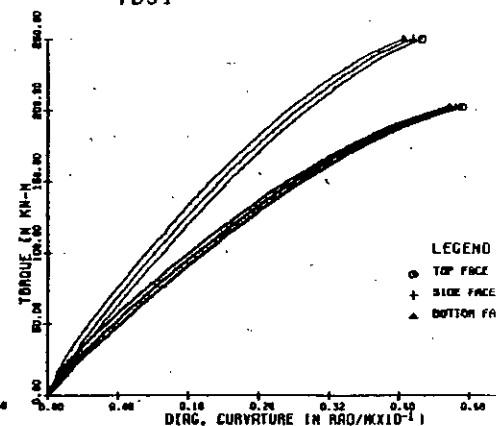
TBS1



TBS1

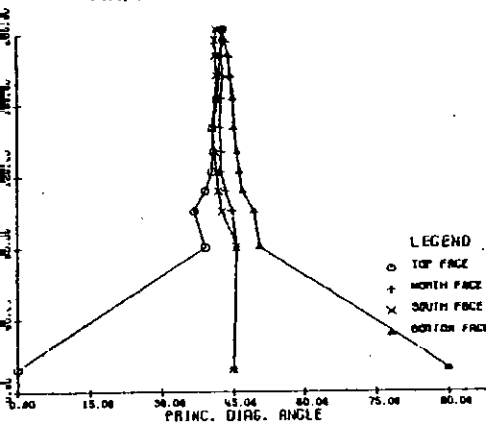


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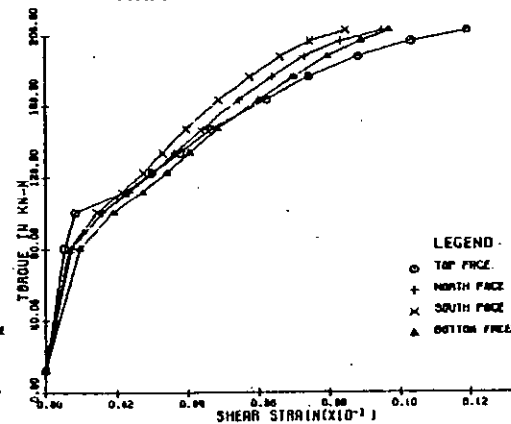


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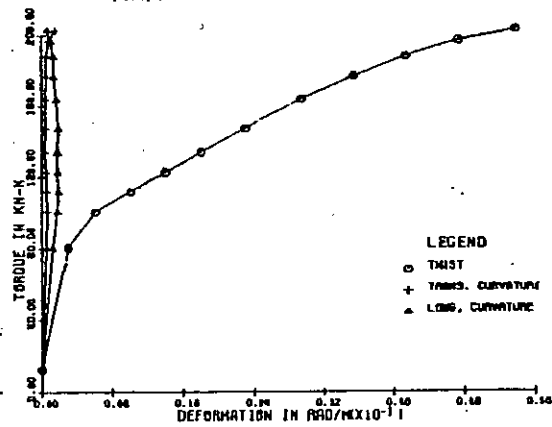
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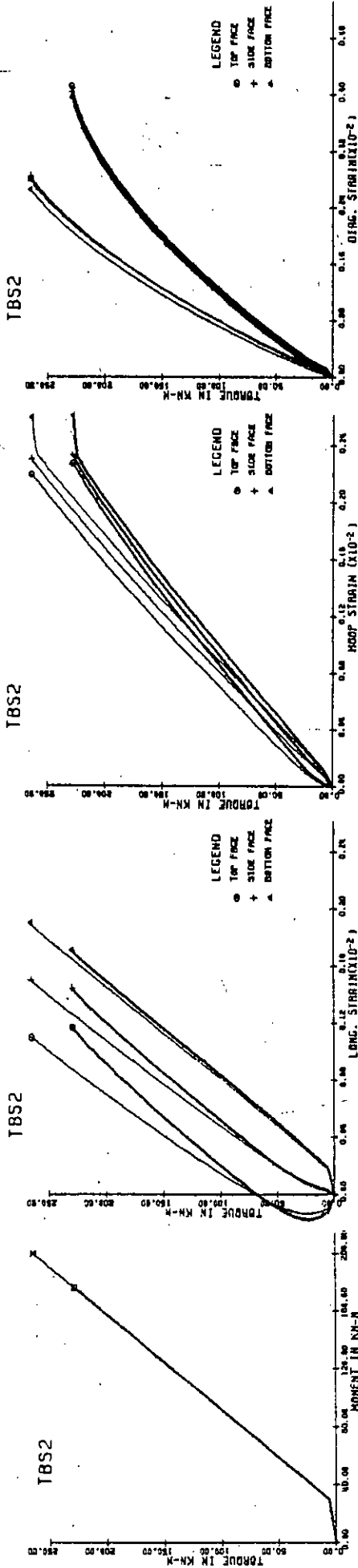
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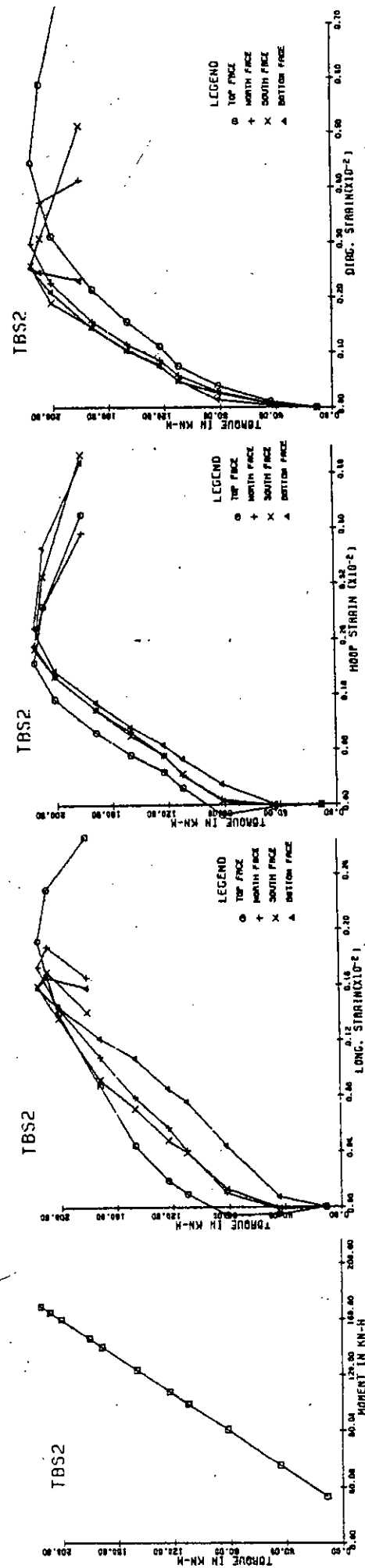
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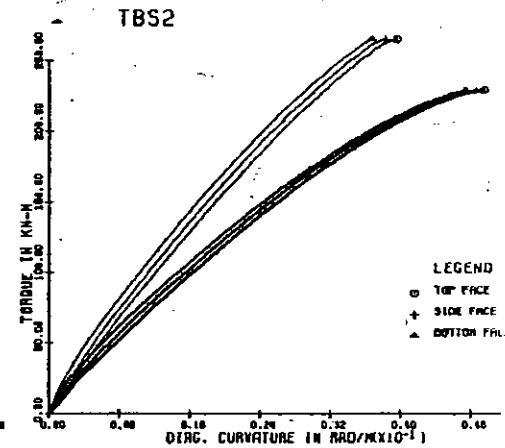
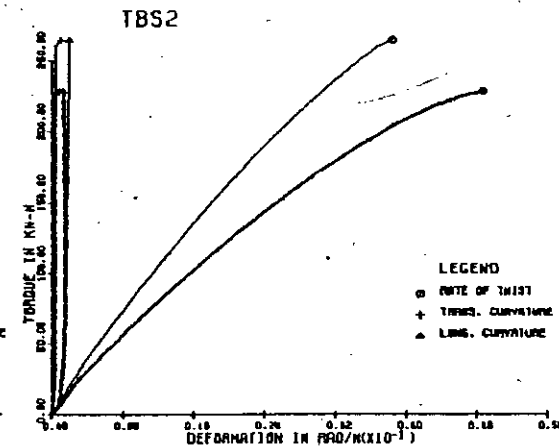
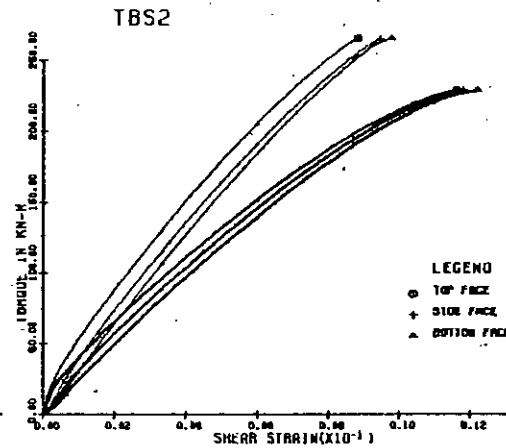
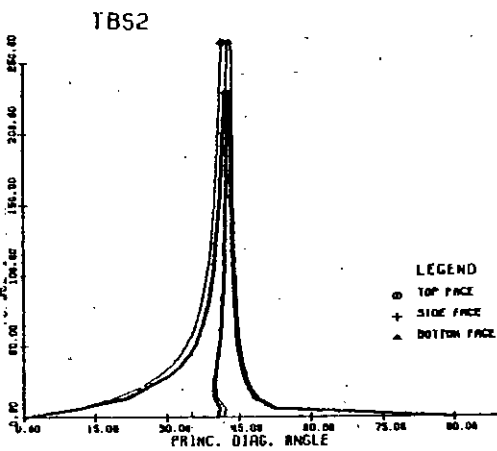
TEST RESULTS



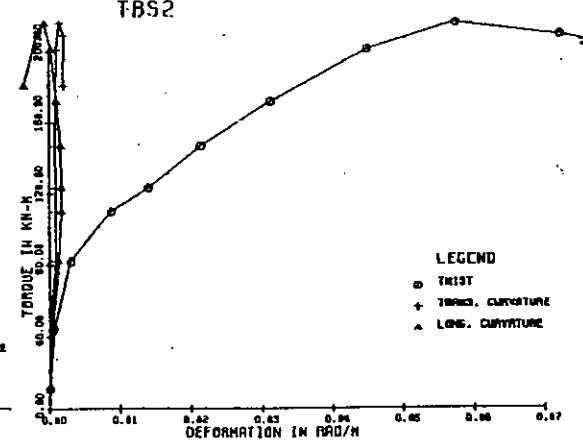
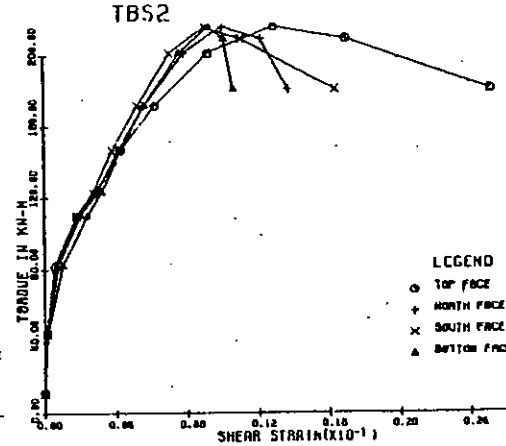
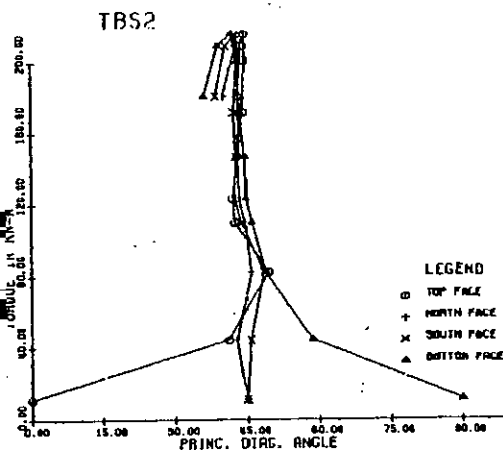
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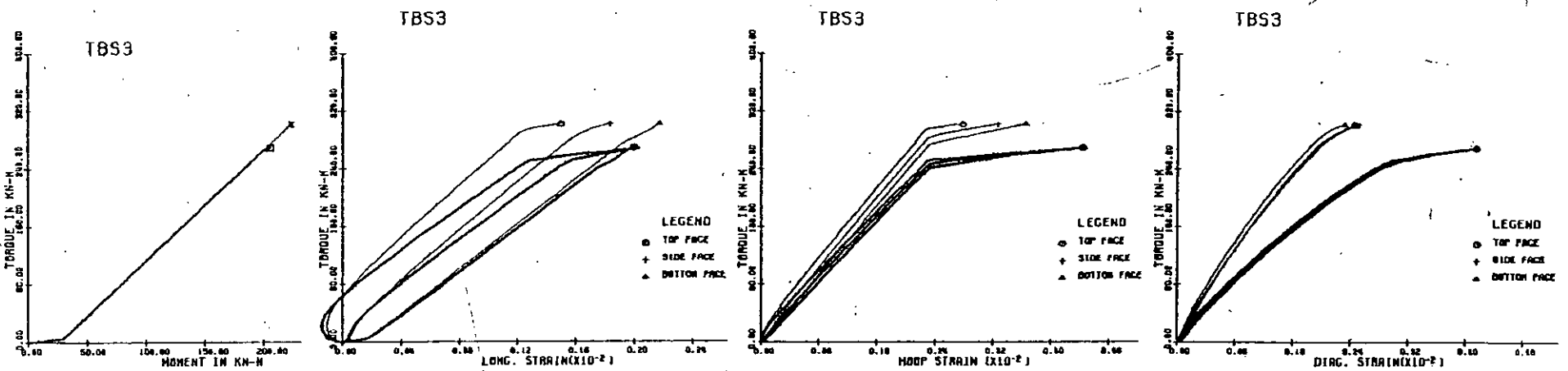
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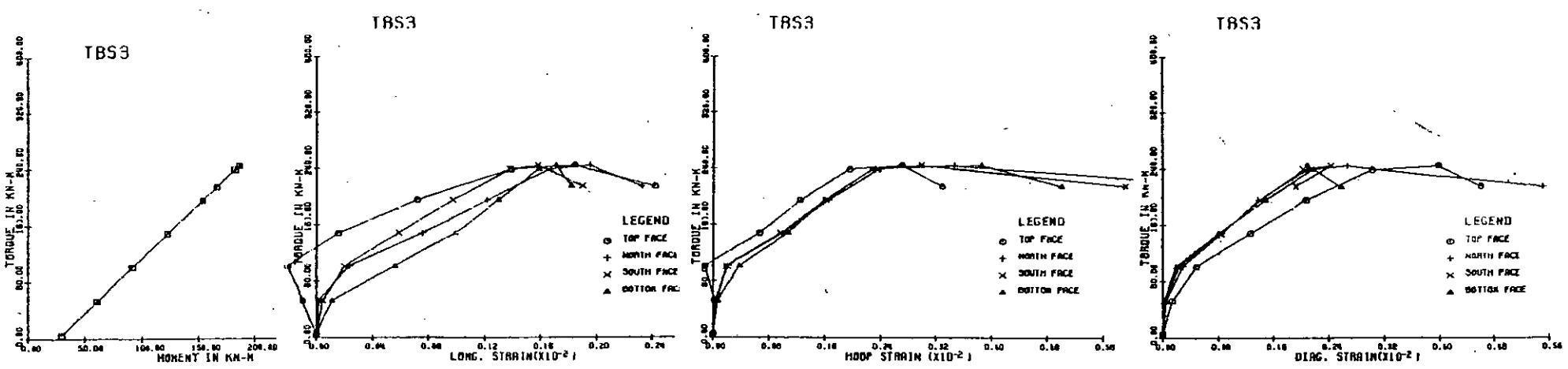
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TEST RESULTS

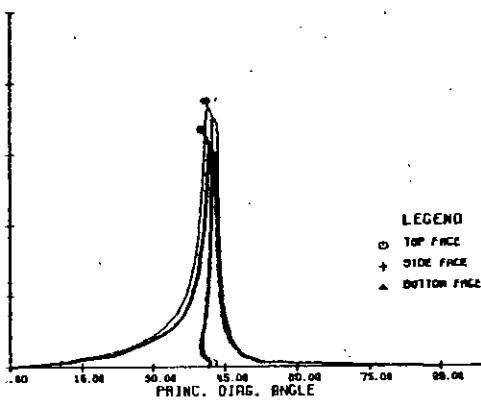


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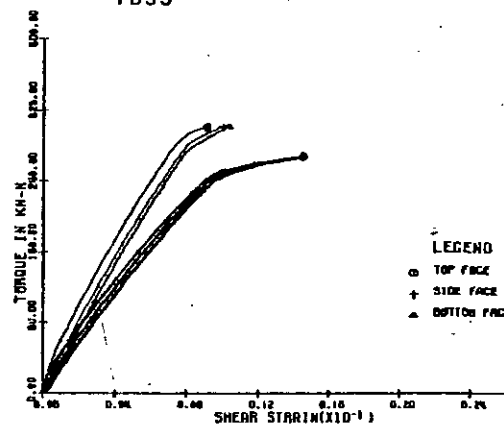


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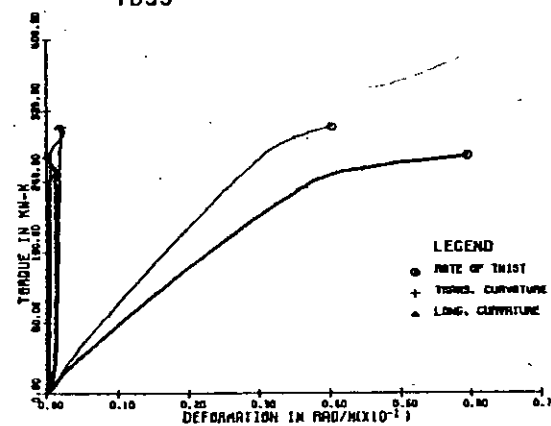
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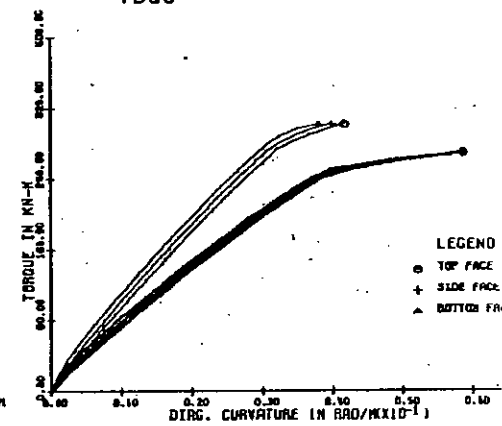
TBS3



TBS3

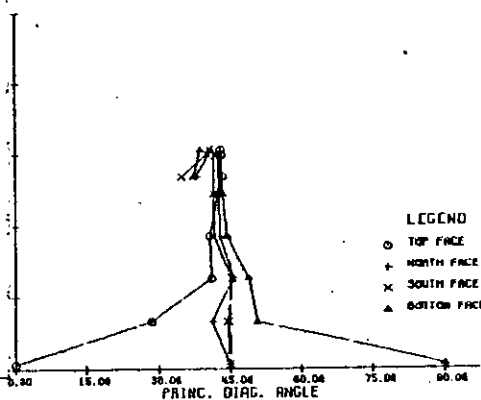


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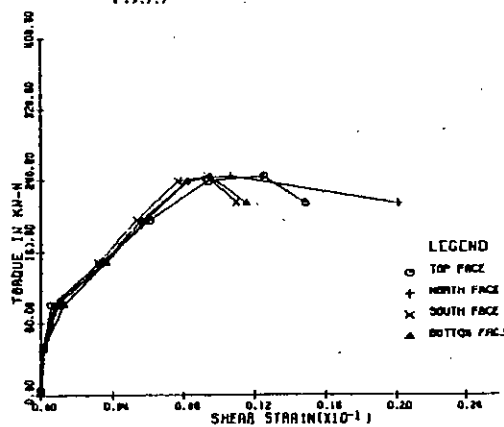


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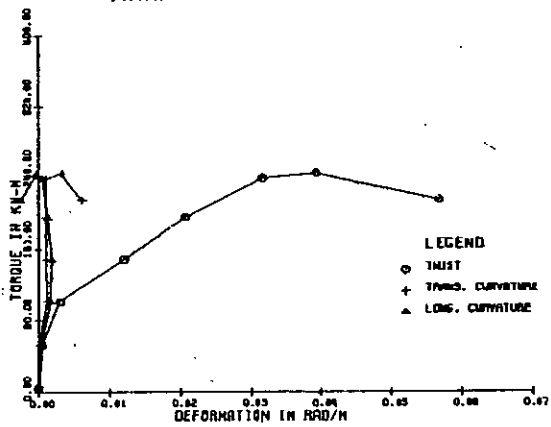
TBS3



TBS3



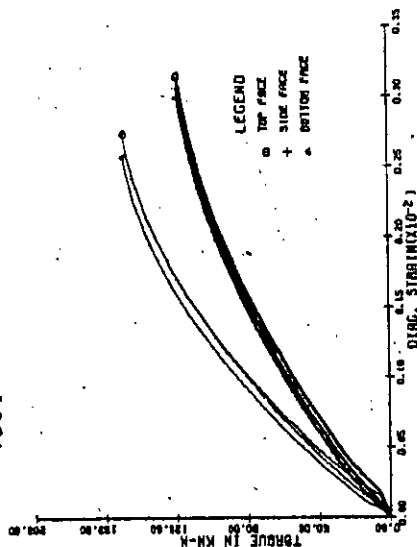
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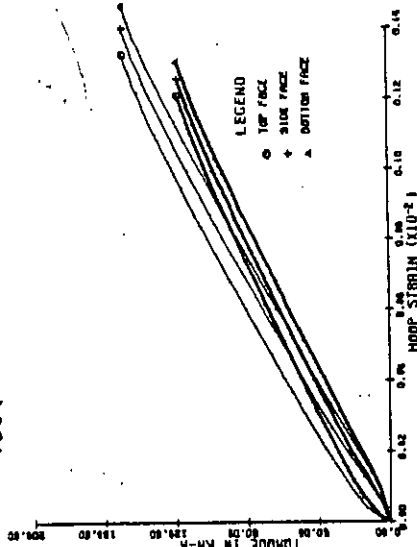
TEST RESULTS

TEST RESULTS

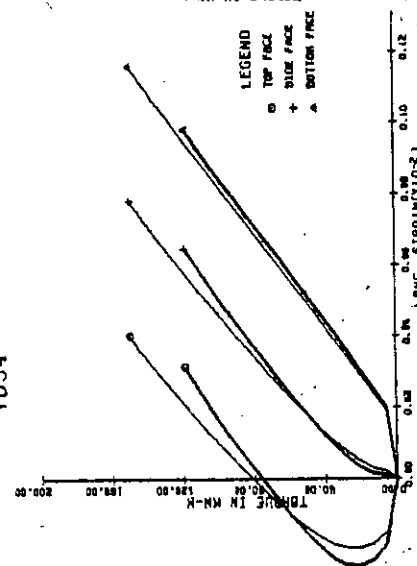
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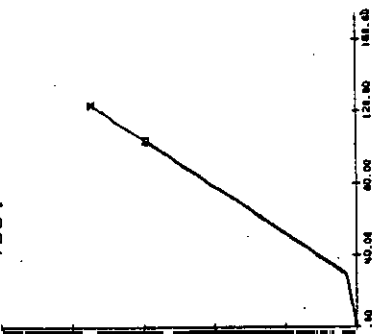
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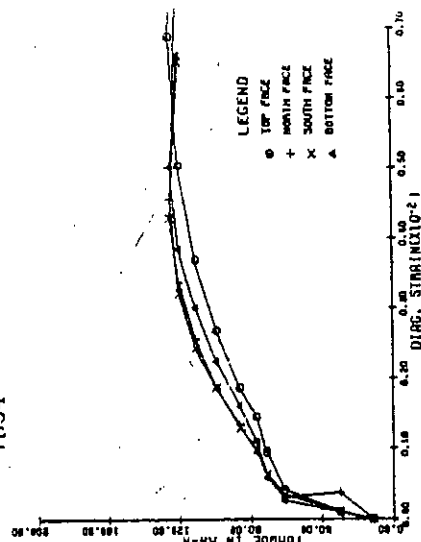
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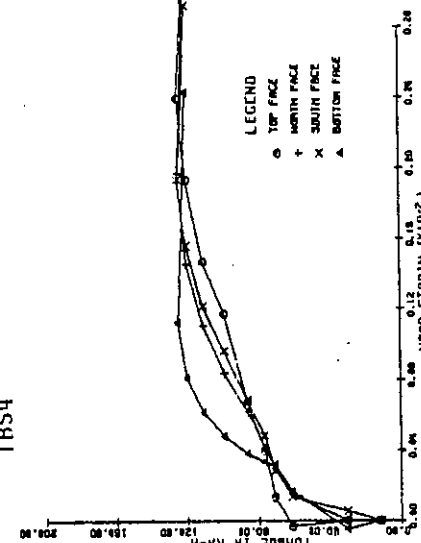
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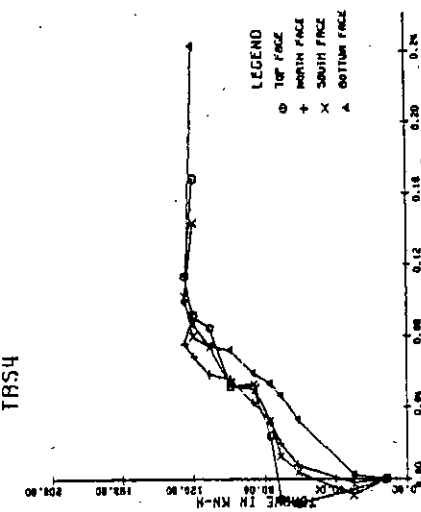
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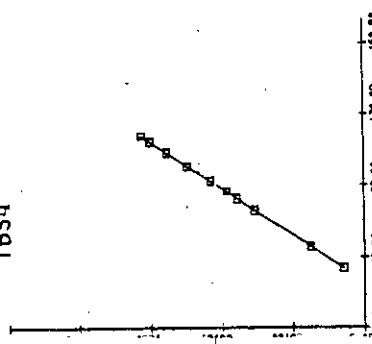
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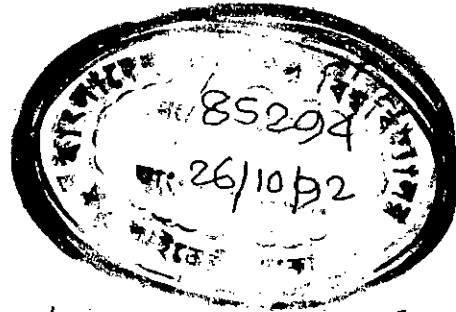
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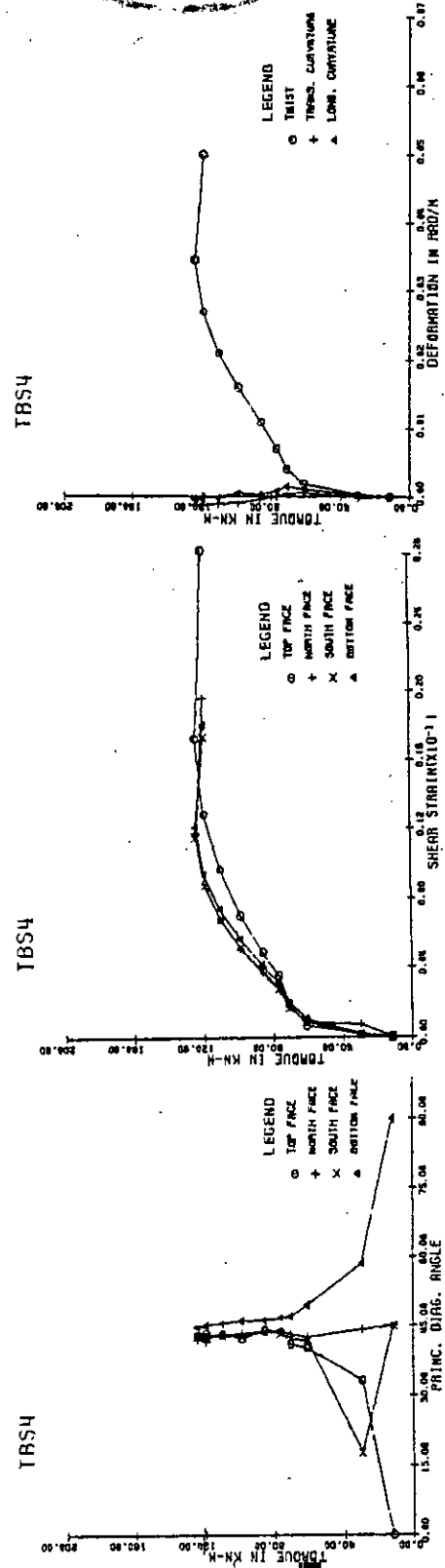
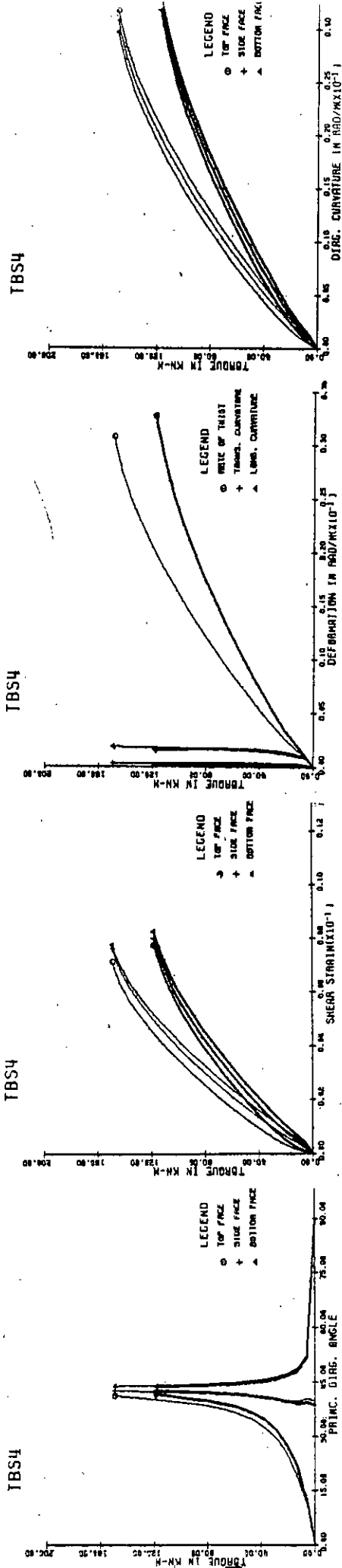
TBS4



THEORETICAL PREDICTION



THEORETICAL PREDICTION



TEST RESULTS