

**APPLICATION OF MACCORMACK SCHEME TO
THE STUDY OF AGGRADATION-DEGRADATION
IN ALLUVIAL CHANNELS**

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**APPLICATION OF MACCORMACK SCHEME TO
THE STUDY OF AGGRADATION-DEGRADATION
IN ALLUVIAL CHANNELS**

**A Thesis Submitted by
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**In partial fulfillment of the requirements for the degree
of
Master of Science in Engineering (Water Resources)**

**Department of Water Resources Engineering
Bangladesh University of Engineering and Technology (BUET)
DHAKA**

July 1998

CERTIFICATE

This is to certify that the thesis on **“Application of MacCormack Scheme to the Study of Aggradation-Degradation in Alluvial Channels”** has been performed by me and neither this thesis nor any part thereof has been submitted elsewhere for the award of any other degree or diploma.

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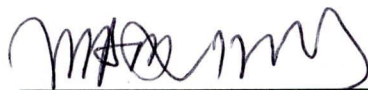
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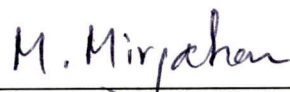
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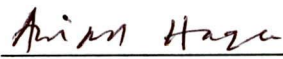
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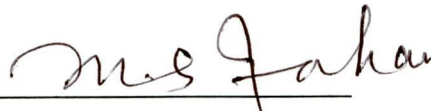
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ABSTRACT

Aggradation and degradation are ubiquitous phenomena that occur in most alluvial channels. The complete Saint Venant equations describe the unsteady open channel flow conditions. The continuity equation for the conservation of sediment mass needs to be solved along with the hydrodynamic equations to determine aggradation and/or degradation of channel bottom. In these viewpoints, a one-dimensional mathematical model has been developed that computes the aggradation-degradation of channel bed. The MacCormack explicit finite difference scheme has been used in the model. This scheme is second-order accurate, handles shocks and discontinuities in the solution without any special treatment, and allows simultaneous solution of the water and sediment equations, thereby obviating the need for iterations.

The developed model has been applied to two different case studies. The first case is a laboratory investigation where experimental results were obtained due to imposing extra sediment load. The sediment overloading produces aggradation in the channel. It has been found that the model simulates most of the test runs with a reasonable accuracy.

In order to verify the ability of the model to real field situation, it has been applied to simulate Jamuna river reach from Bahadurabad to Sirajgang. A test run for three years has been done using field data for that period. The agreement between the computed results and measured data is quite satisfactory.

The successful application of the model strongly demonstrates that the model is valid in solving unsteady open channel flow problems in conjunction with sediment transport. The different problems just require incorporating the appropriate initial and boundary conditions.

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LIST OF NOTATIONS

Notation	Meaning
A	area of flow
Q	water discharge
x	spatial variable
t	time variable
q_l	lateral inflow per unit length of channel
β	momentum flux correction factor
g	acceleration due to gravity
h	depth of flow
S_o	channel bed slope
S_f	friction/energy slope
V_x	x-component (longitudinal direction) of the inflow velocity vector
Q_s	sediment discharge
λ	porosity
P	wetted perimeter
Z_b	bed elevation
C_t	mean sediment concentration
q_{sl}	lateral sediment inflow per unit length of channel
Cn	Courant number
C^+	positive characteristics
C^-	negative characteristics
q	fresh water discharge per unit width
q_s	unit sediment discharge
n	the Manning roughness coefficient
i	the node in spatial grid
k	the node in time grid
Δx	the distance step
Δt	the time step
u	flow velocity
ρ_s	density of sediment

ρ	density of water
d_{50}	median particle size
C	Chezy's roughness coefficient
K	aggradation coefficient
K_0	theoretical value of aggradation coefficient

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CHAPTER ONE

INTRODUCTION



1.1 General

The fresh water and sediment discharges in natural alluvial streams, which have evolved over geologic times, are in equilibrium and produce no objectionable scour or deposition. Because such alluvial rivers or natural streams have reached a state of dynamic equilibrium when the governing factors such as water and sediment discharges, channel geometry, slope, water and sediment properties have remained unchanged for a long enough period of time. However, if this delicate balance is disturbed by changing any of these properties through natural or man made factors, e.g., the construction of weirs, barrages, dams, sudden change in sediment supply rate due to heavy erosion or landslide in the river basin, narrowing of river for navigation, river dredging, bend cutting, lowering of the channel bottom, migration of knick points, etc. a process of achieving another state of equilibrium begins. The process is inevitably accompanied by aggradation and/or degradation along the riverbed.

Engineers, planners and researchers are highly interested in predicting accurately as well as reliably the quantitative estimates of channel bottom aggradation or degradation. These information are heavily correlated with the behavior of the river channel which in turn leads to proper planning and design of river control structures, water management projects and flood protection measures.

Most of the river courses in Bangladesh are of alluvial nature. In fact, Bangladesh is one of the biggest Deltas in the world which is composed of sediments carried by the three major river systems, namely, the Ganges, the Brahmaputra-Jamuna and the Meghna. The huge catchment area of these river systems supplies

enormous amount of sediment to the rivers. Proper quantification of aggradation and/or degradation and changes in channel form of such alluvial rivers still has been a subject of considerable research. A systematic study on the aggradation-degradation phenomenon of these alluvial rivers of Bangladesh is, therefore, of utmost importance from the viewpoint of improved planning and design of various water development projects.

1.2 Necessity of Mathematical Tools

Problems related to alluvial rivers can be studied in the following four ways:

- field investigation
- laboratory study
- physical scale model
- mathematical model

Field methods are directly associated with the analysis and computation based on field measurements. It is not always possible to have field measurements of all parameters for a sustainable period of time. However, data interpretation studies can be made. Laboratory studies are based on experimental runs and data collection and their analysis that demonstrate a particular phenomenon. Laboratory set up and environment can seldom reach field condition because it is difficult to take all the degrees of freedom (DOF's) as variables at a time. Newton (1951), Lane and Borland (1954), Brush and Wolman (1960), Suryanarayana (1969), Soni et al. (1980), Begin et al. (1981) etc. conducted laboratory tests to study bed level changes in alluvial channels.

In the context of many uncertainties involved in the complex process of flow pattern, physical scale models have become very indispensable tools for the solution of alluvial stream problems in the last few decades. When properly built and run, scale models provide the answer to the engineer's eternal question

qualitatively as well as quantitatively 'what will happen if.....'. Unfortunately, they can only model the immediate neighborhood of the structures being studied and when large scale and long term consequences of man's intervention are concerned (e.g., hundreds of kilometers of river course, tens of years of morphological variations), a study made by an expert consultant possibly using a mathematical model is the only solution. These models can play very important role in providing qualitative as well as quantitative results on which design decisions may be done. Now a days water resource projects are becoming larger in volume and economic considerations are more pronounced, and so physical scale models have practical limitation to the scope of their application. Hence, mathematical models can serve very effectively in the planning and design of river-control engineering.

Various researchers have developed analytical solutions after suitable simplifications of the set of basic governing equations. Jain (1981), Jaramillo and Jain (1984) presented analytical solutions for bed aggradation of alluvial channels. Soni et al. (1980), Begin et al. (1981), Gill (1983 a,b) used linear diffusion models while Zhang and Kahawita (1987) and Gill (1987) presented non linear solution for aggradation-degradation phenomenon. However, a closed-form solution of the partial differential equations describing the unsteady flows in open channels coupled with sediment continuity equation is not available, except in few specially simplified cases. Therefore, numerical methods have been used effectively to solve the system of equations. Finite element methods have been found unsuitable to solve the non-linear hyperbolic governing equations due to their complexity in the formulation of the phenomena. Finite difference methods both implicit and explicit schemes are used thereof. Explicit numerical methods have not been used extensively because of its stability restriction on the time step. Even though several second-order accurate explicit methods have been reported which are more accurate than first-order methods and are easier to apply than many implicit schemes (Fennema and Chaudhry, 1986). In this view point MacCormack explicit finite difference scheme has been chosen to study aggradation-degradation phenomenon in alluvial channels.

1.3 Objectives of the Study

The main objectives of the study are:

- i. To develop a one-dimensional, unsteady, coupled hydrodynamic model to compute aggradation and degradation using MacCormack second-order accurate explicit finite difference scheme;
- ii. To verify the developed model to a published laboratory based experimental data;
- iii. To apply the model to an alluvial river (i.e. Brahmaputra-Jamuna) to verify the capability of handling real field situation.

1.4 Organization of Contents

The first chapter of the study gives a brief introduction of the aggradation-degradation phenomenon in an alluvial river reach. It emphasizes the need of mathematical tools for analyzing the phenomenon. It also includes the organization and main objectives of the study.

The second chapter gives a short account of previous studies and literature available for the study. Some of the important numerical schemes (both finite difference and finite element) are illustrated thereof.

The next chapter deals with the MacCormack scheme in details. It shows the gradual development of the mathematical model using the governing hydrodynamic and sediment continuity equation.

Two different case studies are presented in chapter four to verify the model. These are, "laboratory study of aggradation due to sediment overloading" and "model run for Jamuna river simulation".

The fifth chapter incorporates the results of the model application in the two different cases. The study comes to an end in chapter six by drawing conclusions, limitations and recommendations for further study.

LITERATURE REVIEW

2.1 General

Aggradation occurs in an alluvial channel when the sediment supply rate exceeds the sediment transport capacity of the flow. Conversely, the flow tends to degrade the channel when its sediment transport capacity is in excess of the sediment supply rate. A sediment laden flow is generally very complex in nature and, therefore, the problem of aggradation-degradation phenomenon in an alluvial river reach has attracted many researchers for the last several decades. The principal methods of investigation include experimental studies, simplified analytical solutions and numerical techniques.

2.2 Experimental Investigations

A preliminary laboratory study can serve as a useful guide for future field investigations. For instance, if the time variation of bed profile with a specific flow condition is found in the laboratory, one might hope to use this as a basis to compare different field results. Perhaps through more studies in the laboratory and in the field one can establish the usefulness or the limitations of laboratory investigations.

Quite a countable number of experimental studies had been conducted to study the effect of the long term and short term bed level changes in alluvial channels with different flow conditions. Newton (1951) conducted a series of degradation experiments due to sediment diminution using uniform sediment and concluded that the bed elevation and the bed slope decreased asymptotically with time. Lane and Borland (1954) obtained laboratory data to study the riverbed scour during

floods. Brush and Wolman (1960) measured the time variation of bed levels in a laboratory channel due to migration of the knick points or the points of abrupt change in the longitudinal profile. Begin et al. (1981) experimentally studied degradation of alluvial channels in response to lowering of the base level.

Some of the most important degradation studies in rivers were presented by Lane (1955), Stanley (1951), and Livesey (1963). These studies attempted to find the rate of degradation which was obtained by taking periodic water surface readings below the dam. Simons (1965) gave various methods of estimating the magnitude of degradation from stable channel concepts, experimental observations, and the concept of critical traction force. Harrison (1950) concluded from his experiments that the bed degrades as a unit, and the final profile is parallel to the initial profile if nonmoving particles are evenly distributed along the bed.

Bhamidipaty and Shen (1971) studied degradation of the bed profile below a dam due to release of relatively sediment free water from a reservoir and aggradation in a canal (by withdrawing water from the river) due to the difference in sediment transport capabilities between the canal and river. Their study was made in a laboratory flume with rigid banks and mainly relatively uniform size sediment. They developed an equation of the bed profile of a degrading channel which depicts that the bed elevation decreases exponentially with time. They also concluded that during aggradation incoming sediment deposits at the upstream reach with a resulting bed slope increase and depth decrease.

Soni et al. (1980) conducted an experiment that covered a wide range of flow and sediment loading conditions. They have also developed an analytical mathematical model for aggradation in an infinitely long channel where the results from the analytical study have been suitably modified on the basis of carefully controlled laboratory experiments. They observed that after a long time, the aggradation downstream of the section of increased sediment supply closed once the hydraulic conditions became compatible with the increased sediment load.

Yen et al. (1988a) performed a series of overloading experiments with uniform coarse sediment and found that both the aggradation wave speed and the mean sediment transport velocity increase with the initial equilibrium bed slope and with decreasing loading ratio. As to the study of aggradation and degradation of non-uniform sediment, Little and Mayer (1972) performed a series of degradation experiments. They focused on the variation in sediment gradation of the bed surface during the armoring process. Ribberink (1983) experimentally studied the vertical sorting phenomenon of sediment having an idealized gradation under equilibrium conditions and proposed a transport layer concept. Wilcock and Southard (1989) investigated the interaction between the transport, bed surface texture, and bed configuration by making careful measurements and observations in the startup and equilibrium states in a recirculating laboratory flume.

The fundamental behaviors of the whole process of bed aggradation due to overloading and then followed by degradation due to under-loading have been studied by Yen et al. (1992). Series of experiments were conducted by Yen et al. (1989) to study the irreversibility of an alternating aggradation/degradation process. The results show that the recovery rate decreases as standard deviation of sediment gradation increases. They derived an equation for aggradation-degradation wave resulting from the change in sediment loading which characterizes the response of the channel reach at a pace depending on the response time. Experimental results show that the channel bed roughness increases in all cases of sediment under-loading preceded by overloading and most of the medium-size fraction grains are transported downstream in the aggradation period. They concluded that the response time for the aggradation process is greater than that for degradation process and the aggradation-degradation cycle in the alluvial channel composed of non-uniform bed material is irreversible due to the effects of hydraulic sorting and armoring.

2.3 Analytical Solutions

The fundamental notions and hypothesis used in the mathematical modeling of rivers are formulized in the equations of unsteady open channel flow. Three conservation of laws - mass, momentum, and energy - are used to describe the equations. These equations are simple models of extremely complex phenomenon. Two governing equations may be used to analyze a typical flow situation. The continuity and momentum equations, referred to as de Saint Venant equations, are usually used because the momentum equation is applicable to both discontinuous and continuous flow situations. The differential form of the de Saint Venant equations are:

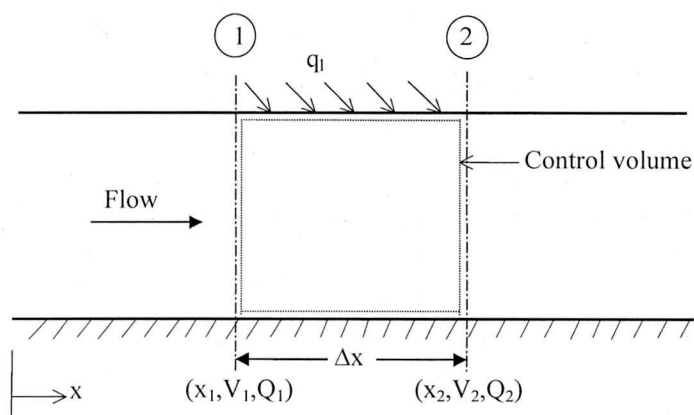


Fig. 2.1 Definition sketch for continuity equation

$$\text{continuity equation} \quad \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l \quad (2.1)$$

Where,

- A = area of flow
- Q = water discharge
- x = spatial variable
- t = time variable
- q_l = lateral inflow per unit length of channel

momentum equation
$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{\beta}{A} Q^2 \right) + gA \left(\frac{\partial h}{\partial x} - S_o \right) + gAS_f = q_l V_x \quad (2.2)$$

Where, β = momentum flux correction factor
 g = acceleration due to gravity
 h = depth of flow
 S_o = channel bed slope
 S_f = friction/energy slope
 V_x = x-component (longitudinal direction) of the inflow velocity vector

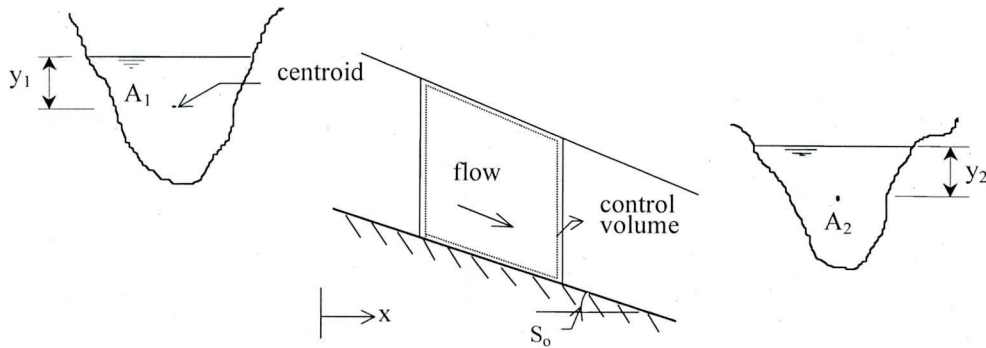


Fig. 2.2 Definition sketch for momentum equation

When aggradation and degradation are concerned a third equation is necessary for the computation of bed elevation. In fact the sediment transport and water flow are interrelated in such a way that they can never be completely disassociated. There are many formulations expressing the interrelation of the two phenomena in unsteady situations; one of the acceptable mathematical descriptions is presented by the following equation.

sediment continuity equation
$$\frac{\partial Q_s}{\partial x} + (1 - \lambda) \frac{\partial}{\partial t} (PZ_b) + \frac{\partial}{\partial t} (AC_t) = q_{sl} \quad (2.3)$$

Where, Q_s = sediment discharge

λ	=	porosity
P	=	wetted perimeter
Z_b	=	bed elevation
C_t	=	mean sediment concentration
q_{sl}	=	lateral sediment inflow per unit length of channel

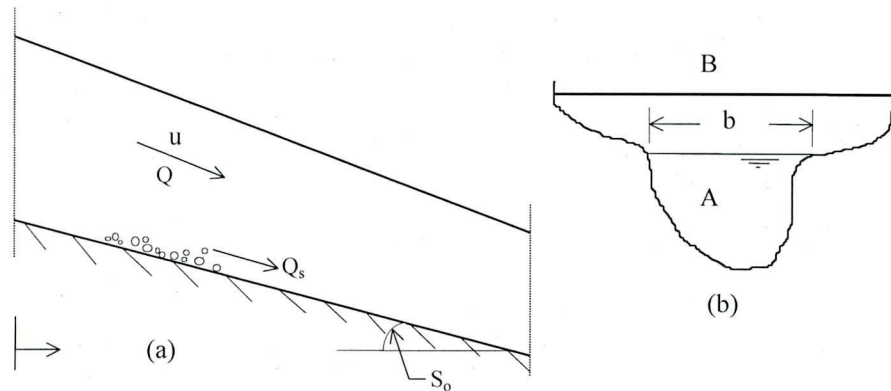


Fig. 2.3 Definition sketch for one-dimensional flow in a movable bed river. (a) longitudinal profile (b) cross section

The fundamental assumptions made in the derivation of these equations are:

1. The flow is assumed to be one-dimensional, i.e., the channel is fairly straight such that the centrifugal effect due to channel curvature is negligible, the velocity is uniform over the cross section, and the free surface is taken to be a horizontal line across the section.
2. The pressure distribution is hydrostatic along a vertical, i.e., the streamlines do not have sharp curvatures, and the vertical accelerations are negligible.
3. The head losses in unsteady flow may be simulated by using the steady state resistance laws, such as the Mannings or Chezy equation, i.e., effects

of boundary friction and turbulence during unsteady flow are the same as that during steady flow.

4. The channel bed slope is small, so that the flow depths measured normal to the channel bottom and measured vertically are approximately the same.
5. The density of the sediment laden water is assumed to be homogeneous.

The governing equations are of first order, nonlinear partial differential equations of the hyperbolic type. It is extremely difficult and almost impossible at present to develop analytical solutions altogether of the system. Useful insight into the complex problem can however be gained by studying an idealized and simple situation that is amenable to analytical solution. Alternatively, for a given stretch of a river and a set of flow conditions, a mathematical model can be constructed. While such a mathematical model is usually valid and useful for the given reach and the specified flow conditions only, it is often possible to deduce far reaching conclusions of a general nature from an analytical solution.

Soni et al. (1980) developed a linear diffusion model to predict the transient bed profile due to sediment overloading. Drastic simplifications made in arriving at the equation of the bed profile could not predict the transient bed profiles satisfactorily. The writers, however, empirically modified the aggradation coefficient suitably in the equation to obtain satisfactory prediction of bed profiles retaining the essential form of the solution. Jain (1981) pointed out an error in the boundary condition of their analytical model. He presented the analytical solution of the aggradation problem for a more appropriate boundary condition. The analytical results were in agreement with the experimental data.

Gill (1983 a,b) prepared linear diffusion model for aggrading and degrading channel. With a suitable value of diffusion coefficient, the diffusion model quite adequately predicts bed level changes due to aggradation and degradation. He

obtained two types of solutions, namely, Fourier series solution and Error function solution and concluded that the Fourier series solution is best suited for computations at large values of time and the Error function solution at small values of time.

A nonlinear parabolic model for nonequilibrium processes in alluvial river was presented by Jaramillo and Jain (1984) in an attempt to circumvent some of the limitations of the classical linear parabolic model. They used the method of weighted residuals for the solution to the nonlinear parabolic sediment transport (diffusion) equation which is in concert with sediment continuity equation and compared the model for the case of aggradation due to sediment augmentation data collected by Soni et al. (1980) and for degradation due to sediment diminution observation by Newton (1951).

Zhang and Kahawita (1987) and Gill (1987) also developed nonlinear solution for aggradation and degradation in alluvial channels and showed better accuracy with the experimental data than the linear solutions. Gill (1987) used the method of perturbation in which the "pseudo-diffusion" coefficient was a function of the local sediment transport but was regarded a constant in the earlier linear solutions.

Ribberink and Sande (1984, 1985) discussed the aggradation problem using a hyperbolic partial differential equation. The proposed hyperbolic model is presumably better than the parabolic (diffusion) model in that some of the acceleration terms that are neglected in the parabolic model are preserved in it. They could not obtain the general solution of the partial differential equation that they used. They presented exact solution for some specified values of distance x and approximate solutions for small and large values of time. A general solution of the proposed hyperbolic equation was developed by Gill (1988) and yielded the solutions previously given by Ribberink and Sande (1984, 1985) as special cases. Qualitatively, the general solutions were capable of predicting the physical conditions more accurately than any other presently available solutions.

2.4 Numerical Simulations

The linear as well as nonlinear parabolic models are based on the assumption of quasi-steady water flow. This assumption does not hold good while estimating the bed level changes during floods or other unsteady water flow conditions. The kinematic wave assumption which assumes that the friction slope can be approximated by the bed slope is not also strictly valid even if the discharge is constant, since the water surface profile computations depend on bed slope. The error encountered in the flow calculation due to the kinematic wave assumption increases with the decrease of channel bed slope (Lu and Shen, 1986). For example, when the back-water effect is significant due to the existence of a barrage, the error is not negligible. Analytical solutions thus developed fail to simulate unsteady flows in open channels with sediments present, except in few simple cases. Therefore, the complete unsteady water flow equations and the sediment continuity equation are generally solved by numerical techniques.

There are several numerical methods available for the solution of unsteady open channel problems with movable bed. The methods can be classified in to the following three categories.

1. Method of characteristics
2. Finite element method
3. Finite difference method

2.4.1 Method of Characteristics

Monge developed a graphical procedure in 1789 for the integration of partial differential equations. He called this procedure the method of characteristics. It was used by Massau (1889) and Craya (1946) for analyzing surges in open channels and it was utilized to investigate the propagation of flood waves

(Isaacson et al., 1954) and other unsteady flow problems (Chaudhry, 1993). The characteristic method has two important fields of application (Cunge et al. 1980):

- as a standard for other methods, since it is possible to prove that its solutions may be brought as close to the solution of the basic equations as one desires;
- as a means of representing the boundary conditions in methods which cannot compute all flow variables at exterior or interior boundary points of the model.

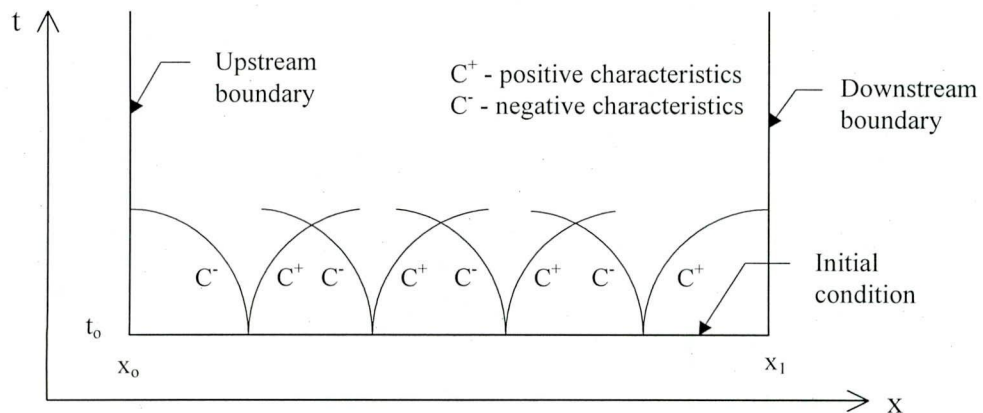


Fig 2.4 Computational domain for a one-dimensional system

In applying the method of characteristics, one must transform the basic partial differential equations into equivalent ordinary differential equations. The transformed equations, known as the compatibility equations, are valid only along the characteristics, namely, the positive characteristic, C^+ and the negative characteristic, C^- . After the proper inclusion of initial and boundary conditions one can reach at the solution by two different formulations. One of the formulations is the method of specified intervals, where interpolations become necessary either in time or in space, since all characteristics do not pass through the grid points. The other one is characteristic grid method where one integrates

along the characteristics and does not use specified intervals for time and distance. This procedure gives satisfactory results for continuous flows.

Although the method of characteristics has become a standard method for the analysis of transients in closed conduits, its application to open channel flow has become almost negligible. However, the concept of characteristics curves is helpful in understanding the propagation of waves and the development of boundary conditions for explicit finite difference methods.

The original method of characteristics on a variable computational grid is not widely used for industrial modeling because of its complexity and the need to interpolate both the original topographic data and the final results. Characteristics on specified intervals are employed somewhat more frequently, but this method is complex and more costly than finite difference methods without offering any improvement in accuracy. The principal advantage of characteristic methods would seem to be their capability of dealing easily with supercritical flow, which is however rarely encountered in river modeling problems. A notable exception is the study of dam break waves for which the variable grid characteristics method is often used (Chervet and Dallèves, 1970; after Cunge et al., 1980).

2.4.2 Finite Difference Method

The finite difference method (FDM) has been used very extensively in the solution of de Saint Venant equation. The method appears, at first sight, as exceedingly simple. Integration of the partial differential equations is performed merely by changing the differentials into their divided differences. Thus, the differential equations, i.e., the laws describing the evolution of the continuum, are replaced by algebraic finite difference relationships. These relationships aid to obtain an arithmetic analog equation that can be solved numerically on the computer.

The different ways in which derivatives and integrals are expressed by discrete functions are called finite difference schemes. These schemes can be classified into explicit schemes or implicit schemes. The implicit schemes permit numerical solutions over large time steps but require the solution of large sets of simultaneous algebraic equations at each time step. On the contrary, explicit schemes are easy to understand and formulate but require small time steps in order to maintain stability.

2.4.2.1 Finite Difference Approximations

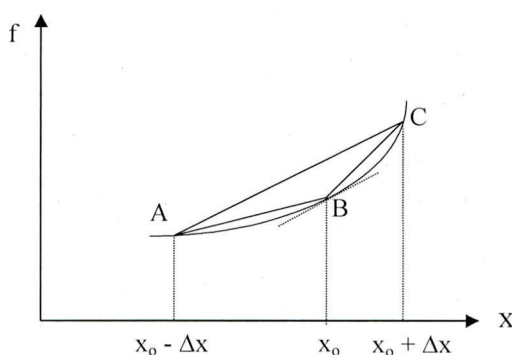


Fig 2.5 Graphical representation of finite difference approximations

forward finite difference
$$\left. \frac{df}{dx} \right|_{x=x_0} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \quad (2.4)$$

backward finite difference
$$\left. \frac{df}{dx} \right|_{x=x_0} = \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x} \quad (2.5)$$

central finite difference
$$\left. \frac{df}{dx} \right|_{x=x_0} = \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x} \quad (2.6)$$

Both backward and forward finite difference approximations are first-order accurate. Central finite difference approximation is referred to as second-order accurate. It is clear from the Fig. 2.5 that the central difference approximation is more accurate than the forward or backward finite difference approximations.

Now, let us consider finite difference approximations for a partial derivative. Let us consider a function $f(x,t)$. We may divide the x - t plane into a grid as shown in Fig. 2.6. To refer to different variables at these grid points, we will use the number of the spatial grid as a subscript and that of the time grid as a superscript. For example, the flow depth, y , at the i th spatial grid point and k th time grid point will be denoted by y_i^k . We will denote the known time level by superscript k and the unknown time level by $k+1$. If the computations progress from one step to the next, then the procedure is referred to as a marching procedure. Most of the phenomena described by hyperbolic partial differential equations are solved by using marching procedures. The conditions specified at time $t = 0$ are referred to as the initial conditions. The conditions specified at the channel ends are called end, or boundary conditions.

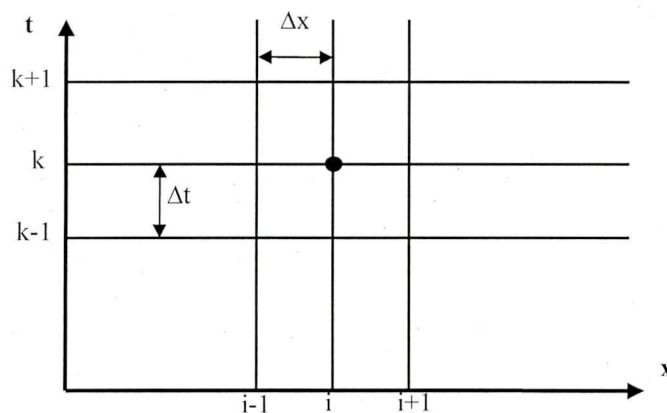


Fig. 2.6 Finite difference computational grid

There are several possibilities for approximating the partial derivatives. The spatial partial derivatives replaced in terms of the variables at the known time level are referred to as the explicit finite differences, whereas those in terms of the variables at the unknown time level are called implicit finite differences. Some typical finite difference approximations for the spatial partial derivative,

$\frac{\partial f}{\partial x}$, at the grid point (i,k) are as follows:

Explicit finite differences

$$\text{Backward: } \frac{\partial f}{\partial x} = \frac{f_i^k - f_{i-1}^k}{\Delta x} \quad (2.7)$$

$$\text{Forward: } \frac{\partial f}{\partial x} = \frac{f_{i+1}^k - f_i^k}{\Delta x} \quad (2.8)$$

$$\text{Central: } \frac{\partial f}{\partial x} = \frac{f_{i+1}^k - f_{i-1}^k}{2\Delta x} \quad (2.9)$$

Implicit finite differences

$$\text{Backward: } \frac{\partial f}{\partial x} = \frac{f_i^{k+1} - f_{i-1}^{k+1}}{\Delta x} \quad (2.10)$$

$$\text{Forward: } \frac{\partial f}{\partial x} = \frac{f_{i+1}^{k+1} - f_i^{k+1}}{\Delta x} \quad (2.11)$$

$$\text{Central: } \frac{\partial f}{\partial x} = \frac{f_{i+1}^{k+1} - f_{i-1}^{k+1}}{2\Delta x} \quad (2.12)$$

Many other finite difference approximations are possible if the values at three or more grid points, instead of just two grid points, are used.

2.4.2.2 Some Explicit Schemes

There are several explicit finite difference schemes proposed for the solution of hyperbolic partial differential equations. These schemes do not lead to a system of algebraic equations, since each point can be computed separately. Some typical explicit schemes employed in hydraulic engineering are discussed below:

i. Leap-frog Scheme

The leap-frog method is the earliest scheme used for numerical solution of one dimensional wave equation. In this method finite difference approximations are:

$$\frac{\partial f}{\partial t} = \frac{f_i^{k+1} - f_i^{k-1}}{2\Delta t} \quad (2.13)$$

$$\frac{\partial f}{\partial x} = \frac{f_{i+1}^k - f_{i-1}^k}{2\Delta x} \quad (2.14)$$

The scheme approximates the derivative with an accuracy of second order in Δx as well as in Δt . Matsutomi (1983) used this scheme to compute dam-break flow profiles over a dry bed. He employed shock fitting to track the bore and used Ritter's solution for the first few time steps.

ii. Lax Scheme

Lax developed this method in 1954. This scheme is one of the simplest to program and yields satisfactory results (Chaudhry, 1987) for typical hydraulic engineering applications. The partial derivatives and other variables are approximated in this scheme as follows:

$$\frac{\partial f}{\partial x} = \frac{f_{i+1}^k - f_{i-1}^k}{2\Delta x} \quad (2.15)$$

$$\frac{\partial f}{\partial t} = \frac{f_i^{k+1} - f^*}{\Delta t} \quad (2.16)$$

$$f^* = \frac{1}{2}(f_{i-1}^k + f_{i+1}^k) \quad (2.17)$$

$$R^* = \frac{1}{2}(R_{i-1}^k + R_{i+1}^k) \quad (2.18)$$

$$S_f^* = \frac{1}{2} (S_{f_{i-1}}^k + S_{f_{i+1}}^k) \quad (2.19)$$

For the stability of the scheme, it must satisfy Courant number, C_n condition.

$$C_n = \frac{\text{actual wave velocity}}{\text{numerical wave velocity}} = \frac{|V| \pm c}{\Delta x / \Delta t} \quad (2.20)$$

For stability, $C_n \leq 1$

iii. MacCormack Scheme

An interesting and simpler variation of the lax-wendroff scheme was introduced by MacCormack and has been widely used in computational fluid dynamics.

The detail discussion of the scheme has been presented in Chapter 3.

iv. Lambda Scheme

The governing equations are transformed into λ -form and then they are discretized according to the sign of the characteristic directions, thereby enforcing the correct signal direction. This allows analysis of flows having sub- and super-critical flows. This scheme was proposed by Moretti in 1979. The spatial finite differences are written in terms of values at three grid points and the scheme comprises the predictor and corrector parts.

Predictor Part

$$f_x^+ = \frac{2f_i^k - 3f_{i-1}^k + f_{i-2}^k}{\Delta x} \quad (2.21)$$

$$f_x^- = \frac{f_{i+1}^k - f_i^k}{\Delta x} \quad (2.22)$$

$$\frac{\partial f}{\partial t} = \frac{f_i^* - f_i^k}{\Delta t} \quad (2.23)$$

Corrector Part

$$f_x^+ = \frac{f_i^* - f_{i-1}^*}{\Delta x} \quad (2.24)$$

$$f_x^- = \frac{-2f_i^* + 3f_{i+1}^* - f_{i+2}^*}{\Delta x} \quad (2.25)$$

$$\frac{\partial f}{\partial t} = \frac{f_i^{**} - f_i^k}{\Delta t} \quad (2.26)$$

Values at k+1 time step are determined from

$$f_i^{k+1} = \frac{1}{2}(f_i^* + f_i^{**}) \quad (2.27)$$

For the scheme to be stable, Courant number, C_n must be less than or equal to 1 at all grid points during each time step.

Fennema and Chaudhry (1986) used this scheme to the analysis of unsteady free surface flows having shocks or bores.

v. Gabutti Scheme

The Lambda scheme has the disadvantage that it does not follow the characteristics exactly for a linear problem (Anderson et al., 1984). Gabutti

extended the scheme in order to rectify this problem (Gabutti, 1983). The Gabutti scheme is identical to the Lambda scheme in its development of the equations in the λ form; however, it differs in its approximation for the partial derivatives by finite differences. In the Gabutti scheme, a two-part predictor step and a single corrector step are used with the derivatives approximated as follows:

Predictor step, part 1

$$\bar{f}_x^+ = \frac{f_i^k - f_{i-1}^k}{\Delta x} \quad (2.28)$$

$$\bar{f}_x^- = \frac{f_{i+1}^k - f_i^k}{\Delta x} \quad (2.29)$$

Predictor step, part 2

$$\bar{f}_x^+ = \frac{2f_i^k - 3f_{i-1}^k + f_{i-2}^k}{\Delta x} \quad (2.30)$$

$$\bar{f}_x^- = \frac{-2f_i^k + 3f_{i+1}^k - f_{i+2}^k}{\Delta x} \quad (2.31)$$

Corrector step

$$\hat{f}_x^+ = \frac{\bar{f}_i - \bar{f}_{i-1}}{\Delta x} \quad (2.32)$$

$$\hat{f}_x^- = \frac{\bar{f}_{i+1} - \bar{f}_i}{\Delta x} \quad (2.33)$$

the value of f at the new time $k+1$ becomes

$$f_i^{k+1} = \frac{1}{2} \left(f_i^k + \bar{f}_i + \hat{f}_i - \bar{f}_i \right) \quad (2.34)$$

For the scheme to be stable, C_n must be less than or equal to 1 at all grid points during each time step. However, if the unsteady flow equations are solved until the solution converges to a steady state, then the stability criterion may be relaxed to $C_n \leq 2$. This scheme was used by Fennema and Chaudhry (1986, 1987).

2.4.2.3 Some Implicit Schemes

In implicit finite difference schemes, the spatial partial derivatives and/or the coefficients are replaced in terms of the values at the unknown time level. The unknown variables, therefore, appear implicitly in the algebraic equations, and the methods are called implicit methods. The algebraic equations for the entire system have to be solved simultaneously in these methods. Some of the implicit schemes are discussed below:

i. Preissmann Scheme

The Preissmann scheme has been extensively used since the early 1960s (Cunge et al., 1980). It has the advantages that a variable spatial grid may be used; steep wave fronts may be properly simulated by varying the weighting coefficient; and the scheme yields an exact solution of the linearized form of the governing equations for a particular value of Δx and Δt . The partial derivatives and other coefficients are approximated as follows:

$$\frac{\partial f}{\partial t} = \frac{(f_i^{k+1} + f_{i+1}^{k+1}) - (f_i^k + f_{i+1}^k)}{2\Delta t} \quad (2.35)$$

$$\frac{\partial f}{\partial x} = \frac{\alpha(f_{i+1}^{k+1} - f_i^{k+1})}{\Delta x} + \frac{(1-\alpha)(f_{i+1}^k - f_i^k)}{\Delta x} \quad (2.36)$$

$$f = \frac{1}{2}\alpha(f_{i+1}^{k+1} + f_i^{k+1}) + \frac{1}{2}(1-\alpha)(f_{i+1}^k + f_i^k) \quad (2.37)$$

in which α is a weighting coefficient; in the partial derivative f refers to both V and y , and f as a coefficient stands for S_f and V . By selecting a suitable value for α , the scheme may be made totally explicit ($\alpha=0$) or implicit ($\alpha=1$). The scheme is unconditionally stable if $0.55 < \alpha \leq 1$. An unconditional stability means that there is no restriction on the size of Δx and Δt for stability. However, for accuracy C_n , should be close to 1. Fennema and Chaudhry (1986), Park and Jain (1986), Lyn (1987) etc. used this scheme to solve unsteady open channel flow equation.

ii. Vasiliev Scheme

The Vasiliev scheme was developed by a team of researchers at the Institute of Hydrodynamics, Novosibirsk, Russia (Vasiliev et al., 1965). The scheme is a fully implicit one in which both dependent variables are computed at all grid points. It uses the following approximation of time and space derivatives:

$$\frac{\partial f}{\partial t} = \frac{f_i^{k+1} - f_i^k}{\Delta t} \quad (2.38)$$

$$\frac{\partial f}{\partial x} = \frac{f_{i+1}^{k+1} - f_{i-1}^{k+1}}{\Delta x} \quad (2.39)$$

iii. Gunaratnam-Perkins Scheme

This scheme was developed by Gunaratnam and Perkins in 1970. The finite difference approximation of the flow equations are written in a linearized

homogeneous characteristics form in this scheme. The derivatives are replaced by:

$$\frac{\partial f}{\partial t} = \frac{1}{6} \frac{f_{i-1}^{k+1} - f_{i-1}^k}{\Delta t} + \frac{2}{3} \frac{f_i^{k+1} - f_i^k}{\Delta t} + \frac{1}{6} \frac{f_{i+1}^{k+1} - f_{i+1}^k}{\Delta t} \quad (2.40)$$

$$\frac{\partial f}{\partial x} = \frac{f_{i+1}^{k+1} - f_{i-1}^{k+1}}{2\Delta x} \quad (2.41)$$

iv. Abbott-Ionescu Scheme

This scheme was first proposed by Abbott and Ionescu (1967) at the International Institute for Hydraulic and Environmental Engineering in Delft, the Netherlands. In this scheme two dependent variables (e.g., discharge Q and depth h) are computed at different grid points as shown in Fig. 2.7.

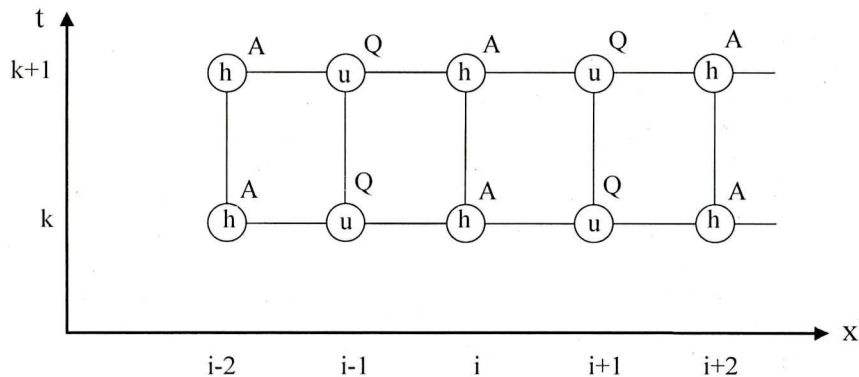


Fig. 2.7 Computational grid for the Abbott-Ionescu scheme

Consider the following system of homogeneous flow equations:

$$g \frac{\partial h}{\partial t} - u \frac{\partial u}{\partial t} - (u^2 - gh) \frac{\partial u}{\partial x} = 0 \quad (2.42)$$

$$h \frac{\partial u}{\partial t} - u \frac{\partial h}{\partial t} - (u^2 - gh) \frac{\partial h}{\partial x} = 0 \quad (2.43)$$

Eq. (2.42) is replaced by following derivative approximations:

$$\frac{\partial h}{\partial t} = \frac{h_i^{k+1} - h_i^k}{\Delta t} \quad (2.44)$$

$$\frac{\partial u}{\partial t} = \frac{1}{2} \left(\frac{u_{i+1}^{k+1} - u_{i+1}^k}{\Delta t} + \frac{u_{i-1}^{k+1} - u_{i-1}^k}{\Delta t} \right) \quad (2.45)$$

$$\frac{\partial u}{\partial x} = \frac{1}{2} \left(\frac{u_{i+1}^{k+1} - u_{i-1}^{k+1}}{2\Delta x} + \frac{u_{i+1}^k - u_{i-1}^k}{2\Delta x} \right) \quad (2.46)$$

and in Eq. (2.43)

$$\frac{\partial u}{\partial t} = \frac{u_i^{k+1} - u_i^k}{\Delta t} \quad (2.47)$$

$$\frac{\partial h}{\partial t} = \frac{1}{2} \left(\frac{h_{i+2}^{k+1} - h_{i+2}^k}{\Delta t} + \frac{h_i^{k+1} - h_i^k}{\Delta t} \right) \quad (2.48)$$

$$\frac{\partial h}{\partial x} = \frac{1}{2} \left(\frac{h_{i+2}^{k+1} - h_i^{k+1}}{2\Delta x} + \frac{h_{i+2}^k - h_i^k}{2\Delta x} \right) \quad (2.49)$$

v. Delft Hydraulics Laboratory Scheme

This scheme was described by Vreugdenhil (1973). It is based on the concept of nodes, or computational cells at the center of which water stages are computed, and which are linked to other cells on the left and on the right through discharge

laws. The space derivatives are weighted between k and $k+1$ time level using a weighting coefficient θ :

$$\frac{\partial f}{\partial x} = \theta \frac{f_{i+1}^{k+1} - f_{i-1}^{k+1}}{2\Delta x} + (1 - \theta) \frac{f_{i+1}^k - f_{i-1}^k}{2\Delta x} \quad (2.50)$$

where, $0.5 \leq \theta \leq 1.0$

The Delft scheme is identical to the Abbott-Ionescu scheme, differences appearing at the level of definition of the equation coefficients, coefficient θ , convective momentum term, and solution algorithm.

2.4.2.4 Related Previous Studies

Tinney (1955) presented a method of predicting the bed profile of a degrading channel by combining a sediment transport continuity equation and sediment transport equation. He also predicted the final depth of flow and final slope at the end of degradation for uniform sediments by solving simultaneously the flow and the critical tractive force equations. It was also assumed that the bed was plane at the end of degradation. His results were in close agreement with Newton's (1951) experimental results.

Lu and Shen (1986) analyzed three different numerical models for degradation studies. The models are: uncoupled solution with kinematic wave assumption, uncoupled solution based on the quasi-steady assumption, and known discharge implicit model. They compared and modified the models with Suryanarayana's (1969) laboratory experiment data.

Lyn (1987) used perturbation techniques to identify multiple time scales in the governing equations, and suggested that complete coupling between the full unsteady water flow equations and the sediment continuity equation is desirable

in cases where the conditions change rapidly at the boundaries. He proposed use of the Preissmann linearized implicit scheme for a simultaneous solution of the governing equations.

Ponce et al. (1979) developed a mathematical model to simulate bed transient formation and propagation in alluvial channels. The model was of the known-discharge type, i.e., within one time step, the water discharge was considered steady. This technique allowed for the Four-Point implicit finite difference scheme to use larger time steps. Using a power relation between bed material discharge and mean flow velocity circumvented instability due to ill posing of the sediment transport function. They analyzed stability and convergence, and established criteria to ensure a successful model operation.

An unsteady, uncoupled model for determining river bed profiles with imposed sediment load was developed by Park and Jain(1986) using Preissmann implicit scheme. The iteration scheme was necessary in the model whenever the spatial gradient of change in bed level became very large. They compared the model with analytical and laboratory data prepared by Soni et al. (1980). The agreement between the numerical results and available experimental data was satisfactory. The results of the numerical analysis were presented in the form of algebraic relations that could be easily applied by practicing engineers to predict the river-bed aggradation due to an increased sediment load larger than the sediment transport capacity of the river.

Dawdy and Vanoni (1986) described the characteristics of several presently available numerical simulation models along with their similarities and differences. They pointed out the general problems in present models which include the choice of sediment transport function, assessment of resistance to flow and how it varies with changes in flow and cross section, modeling of bed armoring and its effect on sediment transport, the allocation of net scour and fill in a stream cross section, and the handling of upstream and downstream boundary conditions.

Momentum and continuity equations for water and sediment flows in a wide prismatic channel were solved numerically using a verified model for the armoring process by Jain and Park (1989). The changes in bed level and mean sediment size, and the length of degradation, determined from the numerical experiments, were correlated by multiple regression analysis to the independent dimensionless variables. The details of the numerical simulation model, which was verified by using the classical experimental data of Little and Mayer (1972), were given by Park and Jain(1987). They concluded that the depth of degradation depends on Froude number, the geometric standard deviation, and the normalized median size of the bed material. The proposed relationship for bed degradation was verified with the field data of Missouri river near Yankton, South Dakota following the closure of the Gavins Point Dam.

Bhallamudi and Chaudhry (1991) developed a mathematical tool by using MacCormack finite difference scheme and applied to predict (1) Bed level changes due to sediment overloading; (2) development of longitudinal profile due to base-level lowering; (3) and bed-level changes associated with the migration of knickpoints. The computed results were compared with the available experimental data obtained on laboratory flumes. The agreement between the computed and experimental result was satisfactory.

2.4.3 Finite Element Method

The finite element method (FEM) is an alternative technique for solving the same partial differential equations by means of piecewise approximations. It is a newer technique, has not been as extensively used as the finite difference method, and does not offer any significant advantage for one dimensional problems.

The main advantage of the finite element method is its ability to handle irregular boundaries and grid refinement. The finite difference solutions have grids with

square corners, straight edges, and mostly uniform grid spacing. The finite element method adapts easily to problems where these boundary characteristics are unsuitable. The computational grids for the finite difference method are usually defined by parallel lines and cannot easily be used to simulate natural boundaries. Smaller grid spacings are frequently required with either method for areas where variables change rapidly (e.g., near edges and corners). These grid refinements are easily handled by the finite element method but require additional effort when using the finite difference method.

The basic steps required to analyze a physical phenomenon by finite element method are as follows:

- i. The domain is subdivided into many small-interconnected sub-domains or finite elements; the process is called discretization.
- ii. A matrix known as element matrix is formed to relate the nodal variables of each element.
- iii. The element matrices are assembled to form a set of algebraic equations that describe the entire domain. The coefficient matrix of the final set of equations is called global matrix. The formulation is done in such a manner that certain compatibility conditions are met at each node.
- iv. Boundary conditions are properly incorporated.
- v. The resulting set of simultaneous algebraic equations is solved then to obtain the solution at every nodal point.

The essential step in solving a differential equation is to convert it into an integral equation. For this purpose, the following three finite element approaches are available: direct, variational, and weighted residual methods. Each results in an identical integral equation for linear partial differential equations.

2.4.3.1 Variational Approach

In this method, the problem of finding a solution of the governing differential equation is replaced by an equivalent variational problem that consists of finding a known function that extremizes a certain integral quantity, subject to the prescribed boundary conditions. Such an integral is called a functional. A general expression for this functional, U over a one-dimensional domain R is given by

$$U = \int_R F \left(x, u, \frac{du}{dx}, \frac{d^2u}{dx^2}, \dots \right) dx \quad (2.51)$$

Within each sub-domain, the unknown function u is approximated by a trial function $\hat{u}$,

$$\hat{u} = \sum_{j=1}^{N^e} \phi_j(x) u_j \quad (2.52)$$

where, $\phi_j(x)$ are linearly independent functions, known as basis functions.

The total functional, U

$$U = \sum_{e=1}^m U^e \quad (2.53)$$

$$\text{where,} \quad U^e = \int_{R^e} F \left(x, \hat{u}, \frac{d\hat{u}}{dx}, \frac{d^2\hat{u}}{dx^2}, \dots \right) dx \quad (2.54)$$

Applying nodal compatibility,

$$U = U(u_1, u_2, u_3, \dots, u_N) \quad (2.55)$$

After assembly of all elements, the global matrix equation takes the form

$$[C]\{u\} = \{F\} \quad (2.56)$$

where,
$$[C] = \sum_{e=1}^m [C]^e \quad (2.57)$$

$$\{F\} = \sum_{e=1}^m \{F\}^e \quad (2.58)$$

2.4.3.2 Method of Weighted Residuals

The weighted residual methods are general methods that can be applied in cases where direct and variational methods does not work. In this method, the desired function, say $u(*)$ is replaced by a finite series approximation, $\hat{u}(*)$

$$u(*) \approx \hat{u}(*) = \sum_{j=1}^{N^e} u_j \phi_j(*) \quad (2.59)$$

where, $\phi_j(*)$ are denoted as basis functions.

The basis functions are chosen to satisfy all boundary conditions. Substitution of $\hat{u}(*)$ in the partial differential equations of the general form

$$Lu(*) - f = 0 \quad (2.60)$$

results in a residual, R

$$R(*) = L \hat{u}(*) - f \quad (2.61)$$

the objective that the residual is minimized by introducing N^e weighting functions, $W(*)$

$$\int_V R(*) W_i(*) dV dt = 0 \quad ; \quad i = 1, 2, \dots, N^e \quad (2.62)$$

The method of weighted residuals may be classified into the following three approaches: Galerkin, Collocation, and Sub-domain method.

i. Galerkin method

When the weighting function is chosen to be the basis function, the method results in Galerkin method.

$$\int_V R(*) \phi_i(*) dV dt = 0 \quad ; \quad i = 1, 2, \dots, N^e \quad (2.63)$$

In this case, $R(*)$ and $\phi_i(*)$ are orthogonal to each other and the residual is forced to zero.

ii. Sub-domain Method

In the sub-domain method the weighting function is chosen to be unity in the sub-domain V_i and zero elsewhere.

$$\int_V R(*) W_i(*) dV = 0 \quad ; \quad i = 1, 2, \dots, N^e \quad (2.64)$$

where, $W_i = 1, (x, y, z)$ within V_i
 $= 0, (x, y, z)$ outside V_i

The integration is less tedious in this method than the Galerkin method because of simplicity of the form of the $W_i(*)$ function.

iii. Collocation Method

Collocation method is the simplest of the method of weighted residuals to implement. The weighting function, W_i is chosen here to be the Dirac delta, that is,

$$W_i = \delta(x - x_i) \quad (2.65)$$

The Dirac delta function has the following important property

$$\int_V a(*) \delta(x - x_i, y - y_i, z - z_i, t - t_i) dV dt = a|_{x_i, y_i, z_i, t_i} \quad (2.66)$$

Thus, the orthogonality constraints for the collocation method are

$$\int_V R(*) \delta_i(*) dV dt = 0 \quad ; \quad i = 1, 2, \dots, N^e \quad (2.67)$$

The residual vanishes at each collocation points (x_i, y_i, z_i, t_i) . Hence, the accuracy of this method depends heavily on the location of the collocation points.

MODEL DEVELOPMENT

3.1 General

The prediction technique of aggradation and degradation in an alluvial river reach is based on the procedure used to develop a model. The successful application of a model largely depends on the search for the correct path to develop the model for a particular problem. The assumptions used in the model development to simplify a phenomenon are sometimes crucial to the extent of their validity. Considering all these constraints, a model is developed from the basic equations describing the problem.

3.2 Governing Equations

The basic one-dimensional partial differential equations describing unsteady flow in a wide rectangular channel with movable bed are given by:

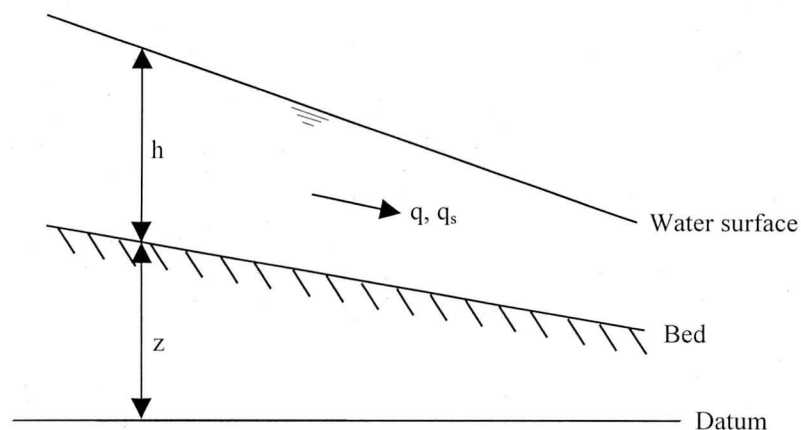


Fig. 3.1 Definition sketch of flow variables

- Continuity equation for water

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (3.1)$$

- Momentum equation for water

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{q^2}{h} + \frac{gh^2}{2} \right) + gh \frac{\partial z}{\partial x} + ghS_f = 0 \quad (3.2)$$

- Continuity equation for sediment

$$\frac{\partial}{\partial t} \left[(1 - \lambda)z + \frac{q_s h}{q} \right] + \frac{\partial q_s}{\partial x} = 0 \quad (3.3)$$

Where,

q	=	fresh water discharge per unit width
h	=	depth of flow
z	=	bed elevation
q_s	=	unit sediment discharge
g	=	acceleration due to gravity
S_f	=	friction slope
x	=	longitudinal distance along the channel
t	=	time
λ	=	porosity of the bed layer

The friction slope, S_f is determined using the Manning equation (SI unit):

$$S_f = \frac{q^2 n^2}{h^{3.333}} \quad (3.4)$$

Where, n = the Manning roughness coefficient

The sediment discharge is estimated by an empirical power function of the flow velocity, u :

$$q_s = au^b$$

or, $q_s = a\left(\frac{q}{h}\right)^b$ (3.5)

Where, a & b are empirical constants for which the values depend on the sediment properties.

The relationship for the friction slope and sediment discharge, defined by Eq. (3.4) and Eq. (3.5) are used in the model for the sake of simplicity. More elaborate or different relations can be included in the model.

3.3 Numerical Scheme

It was previously stated that Eq. (3.1) through Eq. (3.3) represent nonlinear hyperbolic partial differential equations and closed form solutions of these equations are not possible except for a few simplified cases. Therefore, they are solved by numerical schemes. In this model, a finite difference scheme developed by MacCormack (1969) is used. The scheme is chosen for its following reasons:

- i. The MacCormack scheme is simple, easy to implement and does not require solution of large matrices.
- ii. There is no need for iteration.
- iii. The scheme has excellent capability of capturing the shocks without isolating them.

- iv. Empirical relationship for roughness and sediment discharge can be incorporated in the scheme.
- v. The scheme is second order accurate in both space and time. Therefore, longer distance steps can be used to obtain the same accuracy as that obtained using first order schemes.

The MacCormack scheme is a two level predictor-corrector scheme. For one-dimensional flow two alternatives of this scheme are possible. In one alternative, backward finite differences are used to approximate the spatial partial derivatives in the predictor part and forward finite differences are utilized in the corrector part. The values of the variables so determined during the predictor part are used during the corrector part. In the second alternative, forward finite differences are used in the predictor part and backward finite differences are used in the corrector part.

With reference to the finite difference grid shown in Fig. 2.6, the finite difference approximations for the derivatives of a variable 'f' are as follows:

predictor step

$$\frac{\partial f}{\partial t} = \frac{f_i^* - f_i^k}{\Delta t} \quad (3.6)$$

$$\frac{\partial f}{\partial x} = \frac{f_{i+1}^k - f_i^k}{\Delta x} \quad (3.7)$$

Where,

- i = the node in spatial grid;
- k = the node in time grid;
- Δx = the distance step;

Δt = the time step.

An asterisk refers to the predicted values of the variable.

Using these approximations in Eq. (3.1) to Eq. (3.5) one obtains the predicted values of the variables h_i^* , q_i^* and z_i^* .

$$h_i^* = h_i^k - \frac{\Delta t}{\Delta x} (q_{i+1}^k - q_i^k) \quad (3.8)$$

$$q_i^* = q_i^k - \frac{\Delta t}{\Delta x} \left\{ \frac{(q_{i+1}^k)^2}{h_{i+1}^k} - \frac{(q_i^k)^2}{h_i^k} + \frac{g}{2} [(h_{i+1}^k)^2 - (h_i^k)^2] \right\} - gh_i^k \frac{\Delta t}{\Delta x} (z_{i+1}^k - z_i^k) - gh_i^k \Delta t \frac{(q_i^k)^2}{(h_i^k)^{3.33}} \quad (3.9)$$

$$(q_s)_i^* = a \left(\frac{q_i^*}{h_i^*} \right)^b \quad (3.10)$$

$$z_i^* = z_i^k + \frac{1}{1-\lambda} \left[\left(\frac{q_s h}{q} \right)_i^k - \left(\frac{q_s h}{q} \right)_i^* \right] - \frac{\Delta t}{(1-\lambda)\Delta x} [(q_s)_{i+1}^k - (q_s)_i^k] \quad (3.11)$$

Corrector step

$$\frac{\partial f}{\partial t} = \frac{f_i^{**} - f_i^*}{\Delta t} \quad (3.12)$$

$$\frac{\partial f}{\partial x} = \frac{f_i^* - f_{i-1}^*}{\Delta x} \quad (3.13)$$

Where, the two asterisks denote the value of the variables calculated in the corrector step.

After substituting these finite difference approximations in Eq. (3.1) to Eq. (3.5), the corrected values of the variables h_i^{**} , q_i^{**} and z_i^{**} are obtained:

$$h_i^{**} = h_i^* - \frac{\Delta t}{\Delta x} (q_i^* - q_{i-1}^*) \quad (3.14)$$

$$q_i^{**} = q_i^* - \frac{\Delta t}{\Delta x} \left\{ \frac{(q_i^*)^2}{h_i^*} - \frac{(q_{i-1}^*)^2}{h_{i-1}^*} + \frac{g}{2} [(h_i^*)^2 - (h_{i-1}^*)^2] \right\} - gh_i^* \frac{\Delta t}{\Delta x} (z_i^* - z_{i-1}^*) - gh_i^* \Delta t \frac{(q_i^*)^2}{(h_i^*)^{3.33}} \quad (3.15)$$

$$(q_s)_i^{**} = a \left(\frac{q_i^{**}}{h_i^{**}} \right)^b \quad (3.16)$$

$$z_i^{**} = z_i^* + \frac{1}{1-\lambda} \left[\left(\frac{q_s h}{q} \right)_i^* - \left(\frac{q_s h}{q} \right)_i^{**} \right] - \frac{\Delta t}{(1-\lambda)\Delta x} [(q_s)_i^* - (q_s)_{i-1}^*] \quad (3.17)$$

Finally, the values of the unknown variables at time level $k+1$, i.e., at the end of the time interval Δt , are given by

$$h_i^{k+1} = \frac{1}{2} (h_i^k + h_i^{**}) \quad (3.18)$$

$$q_i^{k+1} = \frac{1}{2} (q_i^k + q_i^{**}) \quad (3.19)$$

$$(q_s)_i^{k+1} = \frac{1}{2} [(q_s)_i^k + (q_s)_i^{**}] \quad (3.20)$$

$$z_i^{k+1} = \frac{1}{2} (z_i^k + z_i^{**}) \quad (3.21)$$

3.4 Boundary Conditions

The preceding equations determine the values of h , q , and z at the new time level $k+1$ at every interior grid points ($i = 2, 3 \dots N-1$). The values at the boundary grid points ($i = 1$ and N) can not be determined by using these equations, since there are no grid points outside the flow domain. Hence, they are determined by using

boundary conditions. For one-dimensional flows, the specified interval approach of characteristic method gives acceptable results and is employed herein.

By applying the characteristic theory, it can be shown that for subcritical flow conditions, two boundary conditions must be imposed at the upstream boundary, and one boundary condition must be imposed at downstream boundary. The values of the dependent variables that are not specified through boundary conditions can be determined by using the characteristic equations. Their values may also be determined by interpolation from the known values at the interior nodes (Roache, 1972). The inclusion of these boundary conditions into the finite difference scheme is problem specific and has to deal with individually for specific studies.

3.5 Stability

Stability of a numerical scheme is very essential as the solution progresses. When an error grows continuously the scheme becomes unstable. The MacCormack scheme must satisfy the Courant-Friedrichs-Lewy (CFL) condition for stability. Since the water waves travel at a much higher velocity than the bed transients, this condition is given by:

$$C_n = \left(\frac{q}{h} + \sqrt{gh} \right) \frac{\Delta t}{\Delta x} \quad (3.22)$$

For stability, $C_n \leq 1$

This criterion must be satisfied at every grid point of each time step for the scheme to be stable.

3.6 Artificial Viscosity

The solution obtained by a finite difference scheme has dissipative errors if the leading term of the truncation error in the scheme has even derivatives, and the solution has dispersive errors if the leading term has odd derivatives. The dispersive errors usually produce oscillations in the computed results in the vicinity of steep wave fronts. These oscillations are purely due to numerical errors (mainly due to large grid spacing Δx and Δt for a particular Courant number) and have nothing to do with the physical phenomenon being simulated. This phenomenon is more pronounced in case of higher order schemes like the MacCormack scheme. To smooth these oscillations, artificial viscosity is introduced. There are several ways to add artificial viscosity to a scheme. Jameson et al. (1981) developed a procedure to dampen the oscillations produced by large gradients while leaving the relatively smooth areas unaffected.

The values of the variables at the end of the time step computed by MacCormack scheme are modified using the following parameter based on the normalized form of the computed water surface gradients:

$$v_i = \frac{|h_{i+1} - 2h_i + h_{i-1}|}{|h_{i+1}| + 2|h_i| + |h_{i-1}|} \quad (3.23)$$

$$\varepsilon_{i+1/2} = \kappa \max(v_{i+1}, v_i) \quad (3.24)$$

$$\varepsilon_{i-1/2} = \kappa \max(v_i, v_{i-1}) \quad (3.25)$$

Where, κ = a dissipation constant which is used to regulate the amount of artificial viscosity.

At grid points where h_{i+1} and h_{i-1} do not exist, the following expressions are used instead of Eq. (3.23)

$$v_i = \frac{|h_i - h_{i-1}|}{|h_i| + |h_{i-1}|} \quad (3.26)$$

$$v_i = \frac{|h_{i+1} - h_i|}{|h_{i+1}| + |h_i|} \quad (3.27)$$

The corrected values of f to be used at the new time step are given by:

$$f_i^{k+1} = f_i^{k+1} + \varepsilon_{i+1/2} (f_{i+1}^{k+1} - f_i^{k+1}) - \varepsilon_{i-1/2} (f_i^{k+1} - f_{i-1}^{k+1}) \quad (3.28)$$

The value of κ should be chosen such that it is as small as possible, but at the same time smoothens the high frequency oscillations.

3.7 Methodological Steps

Before the start of the model application, the topographic data (z_i^k at $t = 0$) and hydraulic data (h_i^k and q_i^k at $t = 0$) should be known as initial conditions at all the grid points. The new values of h_i^{k+1} , q_i^{k+1} and z_i^{k+1} after the time interval Δt are calculated as follows:

- i. The predicted values of h_i^* , q_i^* , $(q_s)_i^*$ and z_i^* are computed by applying Eq. (3.8) through Eq. (3.11) at the interior nodes ($i = 2, \dots, N-1$).
- ii. The corrected values of h_i^{**} , q_i^{**} , $(q_s)_i^{**}$ and z_i^{**} at the interior nodes ($i = 2, \dots, N-1$) are determined by using Eq. (3.14) to Eq. (3.17).
- iii. The new values of h , q , q_s and z at the end of the time step Δt , i.e., h_i^{k+1} , q_i^{k+1} , $(q_s)_i^{k+1}$ and z_i^{k+1} are determined from Eq. (3.18) to Eq. (3.21) at the

interior nodes. The values at the boundary ($i = 1$ and $i = N$) are determined by utilizing proper boundary conditions.

- iv. The values determined from previous steps are modified (if required) to dampen higher order oscillations by adding artificial viscosity.
- v. h_i^k , q_i^k , $(q_s)_i^k$ and z_i^k for the next time step are set equal to h_i^{k+1} , q_i^{k+1} , $(q_s)_i^{k+1}$ and z_i^{k+1} , respectively.
- vi. The time interval Δt for the next time step is determined from Courant-Friedrichs-Lewy stability criterion (Eq. 3.22).

The procedure is repeated until the required time is reached.

3.8 Coupling of the Scheme

The MacCormack scheme is an explicit numerical scheme, and the unknowns in each algebraic equation representing the finite difference form of the governing equations are those resulting from the temporal derivative in these equations. The coupling of flow equations and the sediment continuity equation is achieved in this method because it uses a two-level predictor-corrector approach. In fact, there is no coupling during the predictor part. However, the predicted values of bed elevation are used to determine the corrected values of discharge through the term $\partial z / \partial x$ in Eq. (3.15). Similarly, the predicted values of h and q are used to both determine q_s in Eq. (3.10) and evaluate the spatial derivative term in Eq. (3.13). Thus, for one time step, the ultimate computation of each dependent variable at the end of the time step takes into account the changes in all the other variables. In this sense, this procedure is coupled indirectly. In fact the methodological steps of the scheme are such that coupling is achieved automatically. It does not require any special treatment for that. On the other hand coupling is not achieved if Eq. (3.3) is solved after completed solving Eq.

(3.1) and Eq. (3.2), i.e., after corrector step for Eq. (3.14) and Eq. (3.15). The latter approach is similar to the uncoupled implicit method. Park and Jain (1986) pointed out, this approach resulted in numerical instabilities whenever the bed level gradient became too large. They, however, achieved coupling and numerical stability by iterating the solution during each time step.

MODEL APPLICATION

4.1 Introduction

The developed mathematical model has been applied to two different case studies in order to demonstrate its capability. The first one is a laboratory investigation of aggradation in streams due to sediment overloading. This experimental data was obtained by Soni et al. (1980) in a laboratory flume. The problem has been identified here to see the performance of the model and ability to capture shock. The sediment supply rate was increased to four times the equilibrium sediment transport capacity at one run, which creates a huge shock in aggradation front. A sound model stabilizes the transient condition and gives stable results.

The second case is the field application of the model. Aggradation and degradation are regular phenomena in an alluvial river like the Jamuna in Bangladesh. The river Jamuna has been chosen for the fact that it carries enormous volume of silts each year and eventually, due to its braided nature, it is changing continuously its planform and other parameters. Thus, the model is tested for its ability to cope up with real field data.

4.2 Laboratory Application

4.2.1 Aggradation due to Sediment Overloading

In order to obtain information on bed and water surface profiles under transient conditions, extensive experiments were carried out. The experiments were conducted in a 200 mm (8 inch) wide, 500 mm (20 inch) deep and 30 m (98.4

feet) long recirculatory tilting flume located in the Hydraulics Laboratory of the University of Roorkee, Roorkee, India. Water discharge was controlled by a valve and discharge was measured by means of a calibrated orifice-meter provided in the supply line. The sand forming the bed and injected material had a median diameter of 0.32 mm (0.012 in) and geometric standard deviation of 1.30. After the establishment of uniform flow conditions for a given discharge and slope, the sediment supply rate was increased to a predetermined value by continuously feeding excess sediment at the upstream end of the flume at a constant rate. The excess sediment load was progressively deposited in the flume. It was estimated that about 5% of the injected material was deposited upstream of the section of sediment addition. In view of the very small amount of this upstream deposition, the amount of material actually added is taken to be equal to the material deposited downstream. The area under each transient bed profile was determined and Δq_s thus calculated during the time interval. In this calculation λ was taken to be 0.4. The average of the values as determined for all transient bed profiles of a run was taken as Δq_s for that run. The maximum difference between the individual Δq_s values during a run was 15%.

The bed and water surface profiles were recorded by using a point gage with a flat bottom at every 2.5 m (8.2 ft) length at intervals generally varying from 10 min - 20 min. The runs were continued until the aggradation front reached close to the end of the flume. The total sediment transport rate was measured at the downstream end of the flume by means of a specially designed sampler. The sediment transport rate so measured was practically equal to that before addition of sediment thereby indicating that all the added sediment had been deposited in the flume itself. A typical plot of sediment transport rates measured at the end of the flume before and after sediment addition is shown in Fig. 4.1. It is important to note that the average sediment transport rate at the end of the flume is practically unaffected by addition of excess sediment during the period of observation.

A series of 24 uniform flow runs were conducted. The aggradation experiments were conducted using two discharges 4.0 L/s and 7.0 L/s and ranges of slopes from 0.00212-0.00652 under different rates of sediment addition. The rate of sediment addition was varied from $0.30 q_{so}$ to $4.0 q_{so}$. A total of 11 such runs were conducted to obtain 30 transient profiles of bed and water surface. Table 4.1 gives a brief summary of these 11 runs.

SUMMARY OF THE TEST RUNS

Test Run Nos.	Ratio of Sediment increment $\Delta q_s/q_{so}$	Discharge per unit width q (m^2/s)	Depth h (m)	Velocity V (m/s)	Bed Slope S_o
1	4.00	0.020	0.050	0.400	0.00356
2	3.00	0.020	0.052	0.385	0.00330
3	1.80	0.020	0.053	0.378	0.00360
4	3.50	0.035	0.086	0.413	0.00225
5	2.70	0.035	0.075	0.467	0.00338
6	1.35	0.035	0.085	0.417	0.00263
7	0.90	0.035	0.072	0.493	0.00427
8	1.00	0.035	0.075	0.473	0.00363
9	0.90	0.035	0.092	0.386	0.00212
10	0.63	0.035	0.062	0.572	0.00652
11	0.30	0.035	0.058	0.612	0.00482

Table 4.1 Details of the test conditions (after Soni et al., 1980)

4.2.2 Determination of Sediment Transport Constants

The data from the uniform flow experiments were utilized to determine the values of 'a' and 'b' in the sediment transport law, $q_s = a*(q/h)^b$, used in the

mathematical model. The experimental data were plotted on a log-log paper as q_{so} versus (q/h) as shown in Fig. 4.2. The relatively small scatter of data permits fitting a straight line through them justifying the form of Eq. (3.5). The best fit line drawn through these points yielded values of 'a' and 'b' as 1.45×10^{-3} and 5.0, respectively for the sediment size used in the study. The correlation coefficient is found to be 0.958.

4.2.3 Model Calibration

It has been mentioned earlier that two boundary conditions at upstream boundary and one boundary condition at downstream end are required for the proper functioning of the model.

In addition to these, initial conditions are needed at every computational finite difference grid point. Uniform unit discharge (q_0), uniform flow depth (h_0) and bed levels as calculated from initial bed slope (S_0) were provided as initial conditions, i.e.,

$$q(x,0) = q_0 \quad (4.1a)$$

$$h(x,0) = h_0 \quad (4.1b)$$

$$z(x,0) = z_0 \quad \text{for } x = 0 \quad (4.1c)$$

Constant measured discharge per unit width was imposed as an upstream boundary condition. Another boundary condition came from increment of sediment discharge. However, this was not as straight forward as the former one. It was required to be translated into an equation by which the bed elevation at the upstream end could be calculated. This was achieved by assuming a fictitious node upstream from node 1 and specifying the sediment discharge at that node equal to $q_{so} + \Delta q_s$. Then, applying the backward finite difference on the spatial differential term of the sediment continuity equation, one can get the bed level at node 1 for the unknown time level $k+1$.

$$\frac{\partial}{\partial t} \left[(1-\lambda)z + \frac{q_s h}{q} \right] + \frac{\partial q_s}{\partial x} = 0 \quad (4.2)$$

$$\frac{\left[(1-\lambda)z + \frac{q_s h}{q} \right]_1^{k+1} - \left[(1-\lambda)z + \frac{q_s h}{q} \right]_1^k}{\Delta t} + \frac{[q_s]_1^k - [q_s]_f^k}{\Delta x} = 0 \quad (4.3)$$

$$\left[(1-\lambda)z + \frac{q_s h}{q} \right]_1^{k+1} = \left[(1-\lambda)z + \frac{q_s h}{q} \right]_1^k + \frac{\Delta t}{\Delta x} \left[(q_{so} + \Delta q_s) - (q_s)_1^k \right] \quad (4.4)$$

$$z_1^{k+1} = z_1^k + \frac{1}{(1-\lambda)} \left(\frac{q_s h}{q} \right)_1^k - \frac{1}{(1-\lambda)} \left(\frac{q_s h}{q} \right)_1^{k+1} + \frac{1}{(1-\lambda)} \frac{\Delta t}{\Delta x} \left[(q_{so} + \Delta q_s) - (q_s)_1^k \right] \quad (4.5)$$

The flow depth, h at node 1 was determined by extrapolation from the already calculated values at the interior nodes using the MacCormack scheme. The downstream boundary condition was the constant depth, which was specified by

$$h(x_n, t) = h_0 \quad \text{for all } t \geq 0 \quad (4.6)$$

This was based on the assumption that the channel was long and the bed transients would not reach the downstream end within the period for which conditions were computed and that the variation in flow depth would be negligible. The discharge and the bed elevation at the downstream end were determined by extrapolation from the values at the interior nodes.

The mathematical model was used by dividing the channel with a total length of 30 m into 30 reaches ($\Delta x = 1.0$ m). The computational time step Δt was selected according to the Courant-Friedrichs-Lewy (CFL) condition for stability, for this case it was taken to be 0.85 ($C_n = 0.85$).

4.3 Field Application

4.3.1 The River Jamuna

The Brahmaputra-Jamuna river system is the largest and most important in Bangladesh, accounting for more than 50 percent of the total inflow into the country from all cross border rivers. The river is a braided one without fixed banks and with frequently shifting channels. Thus, it is inherently unstable and small changes in the boundary conditions may result in major changes in geometry over a matter of one or two seasons.

4.3.2 Hydrological Characteristics

The Brahmaputra river originates from the Manas Sarowar in Chemayungaung glacier in Tibet on the north slope of Himalayas as Tsangpo, flows eastward for about 1250 km, turns south in to the Indian province of Assam, and then turns sharply west for about 640 km to the border of Bangladesh. At the border, the river curves to the south and continues on this course to its confluence with the Ganges about 240 km downstream. The river takes the name Jamuna at the principal distributary the Old Brahmaputra, the former course of the Brahmaputra that was abandoned over 200 years ago. Three major tributaries, the Teesta, Dudhkumar and Dharla, rise in the Himalayas and flow south-west across the north Bengal plains. The total length of the Tsangpo-Brahmaputra-Jamuna River up to Goalundo is 2575 km. The total catchment area is approximately 580,000 sq. km. Snowmelt accounts for most of the flow of the Brahmaputra, but rainfall in Assam and in the northern part of Bangladesh also contribute significantly. Consequently, the river flow has high seasonal variability.

4.3.3 Sediment Transport Characteristics

The Brahmaputra-Jamuna catchment supplies vast quantities of sediment from the actively uplifting fold mountains in the Himalayas, the erosive foot hills of the Himalayan Foredeep and the great alluvial deposits stored in the Assam valley. As a result, the Jamuna in Bangladesh carries a heavy sediment load with an average of 600 million tones annually. Coleman (1969) gave some figures based on suspended load measurements at Bahadurabad during the period 1958-1962. The average particle size ranges between 0.21 mm at Chilmari to 0.15 mm at the Ganges confluence. The gradation is about 1.3, which indicates fairly uniform sediments. The silt fraction is carried in suspension by the river and the sand fraction is transported through a combination of true bed load and a complex pattern of partial suspension that occurs mainly in the zone close to the bed.

4.3.4 Channel Morphology and Planform Trends

The river course constitutes two different types of chars in terms of their elevations. The lower chars get inundated in the dominant discharge and adjust themselves. The upper chars, or islands, divide the flow even at the dominant discharge and their tops are inactive except during high magnitude, out of bank flow events.

Jamuna is a braided river with a braiding index that varies spatially as well as with time. In general, the braiding index and the overall width are larger at the upstream part than further downstream, probably due to the effects of higher slope and grain sizes. The overall width of the river exhibits an increasing trend, and there is tendency of shifting westwards, especially at the upstream part of the river within Bangladesh (FAP 1, 1993). The widening can be attributed to an advancing alluvial fan or to the not yet completed adaptation process after the shift to its new course (FAP 24, study Report 3, 1994).

Bristow (1987) proposed a classification of river channels into different orders. The entire channel is first-order channel and comprises a number of second order channels, which in turn have smaller channels. The second order channels have slightly different characteristics, such as slightly different slopes, different water and sediment carrying capacities, and as a result, they behave differently. For the study of channel dimensions, the second order channels are of main interest. The shifting rate of the first order channel of the Jamuna river is 75 m to 150 m per year (FAP 24, Special Report 7, 1996). The second-order channels change continuously, large channels being abandoned, and new ones developing in a few years only. The bank erosion rate of the second order channels, vary generally 250 m to 300 m/year and in extreme cases, it can be more than 800 m/year.

4.4 Jamuna River Simulation

The most difficult part of the study is to apply the model in the river Jamuna. Recognizing the complex and profound nature of flow behavior, sediment transport mechanism and unstable formulation, endeavor has been made to prove the ability of mathematical tools as an inseparable portion of analyzing alluvial river problems. It is quite obvious simplification of the phenomenon to a one dimensional problem may lead to easy but undesirable outputs. The model has been applied to simulate aggradation-degradation from Bahadurabad to Sirajgang along the river reach. A map of the study area has been shown in Fig. 4.3.

The model is based on a schematization of the physical system subject to certain restrictive assumption. Therefore, the results must be interpreted with a good understanding of the physical processes, and a proper awareness of the assumptions made during the schematization.

4.4.1 Data and its Source

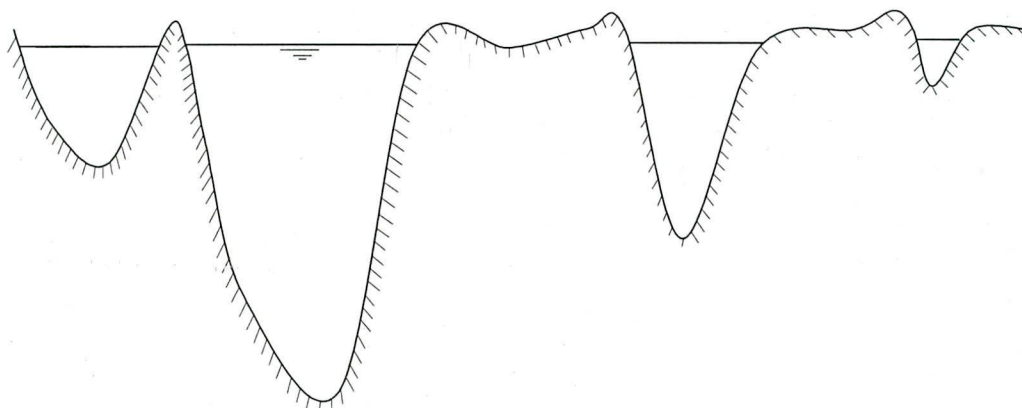
The following data are required for the application of the model.

- cross-sectional data at various stations along the river reach,
- daily discharge data at Bahadurabad for the study period,
- daily water level data at Bahadurabad and Sirajgang for the study period
- sediment transport data, i.e., median particle size (D_{50}) for the river reach.

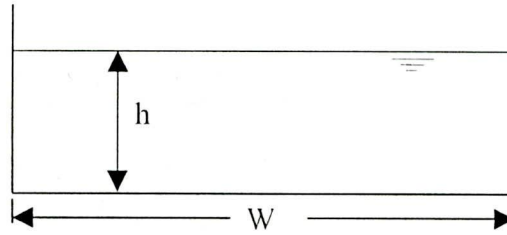
Bangladesh Water Development Board (BWDB) collects these data at various selected stations along the river. All the required data were collected from Surface Water Hydrology Directorate-II, BWDB.

4.4.2 Schematization of the Cross-section

The cross-sectional shape is assumed to be rectangular. The average depth and width of section is computed from at a station width and depth relations (at different discharges) of channels of Jamuna river using FAP 24 survey data (River Survey Project, Special Report No. 7, 1996).



(a) real cross section



(b) idealized cross section

Fig. 4.4 Typical real and idealized cross section

The width and depth relationships are given in Fig. 4.5. These are:

$$\text{Width, } W = 10.46 Q^{0.54} \quad (4.7)$$

$$\text{Flow depth, } h = 0.84 Q^{0.21} \quad (4.8)$$

4.4.3 Initial and Boundary Conditions

The initial bed level (at $t = 0$) has been provided with an average bed slope at a particular time. Steady water depth and water discharge at all grid points are provided as initial condition. Water discharge and bed elevation at $t > 0$ at the upstream node, i.e., Bahadurabad are defined as the boundary condition. The water level at the downstream node, i.e., Sirajgang is defined as the downstream boundary condition.

4.4.4 Sediment Transport Relation

The Engelund-Hansen sediment transport formula has been used in the model for simulation of sediment transport. The formula is expressed as

$$q_s = \frac{0.05 u^5}{\left(\frac{\rho_s}{\rho} - 1\right)^2 \sqrt{g} d_{50} C^3} \quad (4.9)$$

Where,

- q_s = sediment discharge
- u = flow velocity
- ρ_s = density of sediment
- ρ = density of water
- g = acceleration due to gravity
- d_{50} = median particle size
- C = Chezy's roughness coefficient

RESULTS AND DISCUSSIONS

5.1 Laboratory Study Evaluation

Two different types of plot have been produced after the successful application of the model to the prementioned laboratory study. The first type depicts transient bed and water surface profiles downstream of the section of increased sediment supply rate. While the other type is a dimensionless plot that shows variation of relative aggradation with distance downstream.

Fig. 5.1 through Fig.5.11 have been plotted from the model output of 11 different test conditions. Each of the figure shows equilibrium and transient water level and bed profile for 15 min., 30 min., 1 hr. and 2 hr. The following results are evident from these plots.

1. Length of aggradation increases spatially with time.
2. The shape of the transient bed profile is concave upwards and the ordinate of the profile is monotonically decreasing with increase in distance.
3. The depth of aggradation as well as amount of aggradation at a location increases with time.
4. Water surface profiles at small time intervals (i.e., up to 30 min.) after sediment addition show a considerable change in curvature and undulations indicating probably that up to a certain time the deposit of injected sediment acts like a local hump on the bed. Similar comment was also made by Soni et al. (1980) during their observation.

The only exception is found in Fig. 5.11 where degradation occurs after a certain distance. This particular test condition has relatively high stream power (due to high velocity and very steep slope) and might have a tendency of degrading the channel. As the sediment overloading is low (30%), sediment gets deposited within a short reach of the channel. But the stream has still power to degrade. As a consequence degradation starts thereafter.

A typical plot of bed and water surface profiles with distance is shown in Fig. 5.12 for the test run 4. This figure depicts the actual data points, observed mean bed and water surface profiles along with computed profiles from the model. Averaging of the actual data points of these profiles was required because of the presence of ripples and dunes on the bed. It was seen that the scatter of the data around the averaged bed profiles was generally smaller in the aggraded reach than in the region of uniform flow indicating smaller undulations in the aggradation reach (Soni et al., 1980). The comparison with the computed profiles show good result with the average transient profile.

Figs. 5.13 to 5.15 show experimental and computed transient bed and water surface profiles along the channel for the test run 1. Three different profiles of 15 min, 30 min and 40 min. have been represented. Here the experimental points are not the actual data points. These are the points of average transient profiles as reported by Soni et al. (1980). Close agreements are found between the experimental and computed values of the bed and water surface profiles.

The non dimensional bed profile $[z/z_0]$ as a function of non dimensional distance $[x/(2\sqrt{Kt})]$ for the 11 test case are shown in Figs. 5.16 to 5.26. To obtain these profiles the following procedure has been followed as described by Soni et al. (1980).

$$\begin{aligned} z &= \Delta z(x,t) \text{ for all } x \text{ and } t > 0 \\ z_0 &= \Delta z(0,t) \text{ for } x = 0 \text{ and } t > 0 \end{aligned}$$

- $K = (0.885/m)^2 * K_o$ = aggradation coefficient
 $K_o = bq_{so}/(3S_o(1-\lambda))$ = theoretical value of aggradation coefficient
 m = slope of the plot for different test conditions, as shown in Fig. 4.6

It is observed from these plots that the computed results of Figs. 5.16 to 5.18 and Fig. 5.21 show excellent agreement with the experimental observations. The result from the model exhibits an increasing trend in Figs. 5.19 to 5.20, Figs. 5.22 to 5.23 and Figs. 5.25 to 5.26, thereby indicating slightly higher relative aggradation. The result shows good agreement at the initial part of Fig. 5.24 for the test condition no. 9. Later the model value underestimates the relative aggradation to a certain amount. But at the end it conforms with the observed points. It is apparent that the model predicts good for higher ratios of sediment overloading (i.e., test run 1, 2, 3, 5 etc.). This is probably due to the fact that the large changes in bed elevations that occur for higher values of $\Delta q_s/q_{so}$ can be more accurately measured. Similar finding was also made by Jain (1981) who developed another analytical solution for the same problem and used test runs 1 to 6 ($\Delta q_s/q_{so}$ values greater than 1.0) for verification. However, the overall performance of the model in this laboratory observation is satisfactory.

5.2 Real Field Evaluation

The model used actual measured data from April, 1993 to March, 1996 for the simulation. However, the interpretation of the results is qualitative rather than quantitative. Nevertheless, there lies no confusion that the mathematical model can play very vital role in predicting the change associated with aggradation-degradation.

However, problems encountered during the comparison of the model when scarcity of measured data resulted to a very limited verification of results. Due to unavailability of measured sediment measurement data for the sustainable period, it was not possible to verify sediment transport results with measured data. The

verification was done with the generated sediment transport data based on a rating curve using FAP 24 survey data. Discharge is measured at Bahadurabad location only and this was given as an upstream boundary condition. So, discharge at Sirajgang is also verified with generated discharge from the rating curve of Jamuna at Sirajgang based on 1994-95 data. Water level is compared only at Bahadurabad as water level at Sirajgang is provided as a downstream boundary condition. Change in bed levels associated with aggradation degradation during the simulation period of 3 years along the river reach has been given.

Fig. 5.27 compares the simulated water level with the actual measured data at Bahadurabad for the period 1993-94 to 1995-96. The computed output fits very well (i.e., the trend of the curve is excellent) with the measured one. It is seen that the discrepancy between these two curves is large during high flow and computed values are lower than actual data. However, it diminishes towards the low flow regions where computed values are slightly over-estimated to the measured values. The model computes water level as the summation of bed level and depth of flow. The deviation might be due to low value of flow depth during peak, which is based on an average roughness used throughout the period.

The variation of the bed level at Bahadurabad with discharge and water level is shown in Fig. 5.28. A plot of sediment discharge variation at Bahadurabad has been presented in Fig. 5.29. The generated sediment discharge curve has been produced using the sediment rating curve at Bahadurabad using the sediment transport data of 1966-1994. The sediment rating curve has been taken from FAP24, River Survey Project, special report no. 18. The rating curve has been shown in Fig. 5.30. The direction of these two curves is very good. The computed sediment discharge from the model over-estimates the generated sediment discharge curve to a certain extent. This may be due to the fact sediment discharge has been computed from Engelund-Hansen formula, which has not been calibrated. In this formula, the exponent of velocity is 5. So a small error in computation may result large deviation. A properly calibrated sediment transport

equation may result in better conformity. Moreover, generated sediment rating curve gives an average value that may be underestimated during high peaks. However, the trend and agreement between these two curves are fair.

For the comparison of discharge at Sirajgang another discharge rating curve (Shown in Fig. 5.31) based on 1994-95 data at Sirajgang has been used. This rating curve has been taken from the Mathematical Morphological Model of the Jamuna River, Jamuna Bridge Site, First Forecast Report, February 1996, DHI/SWMC. The accompanying result has been shown in Fig. 5.32. The trend and agreement with the computed results are quite satisfactory. The computed values depict slight accretion during the periods of high flows and downward trend during lean flow season.

Fig. 5.33 is a plot of variation of bed level at Sirajgang during the periods of model run. The computed discharge is also shown along with the computed bed level. It is observed that there is a trend of bed level rising at this station. The variation in sediment discharge at the same station is shown in Fig. 5.34. It follows from the graph that as the water discharge increases, sediment discharge also increases. The good correlation between the water and sediment discharge is justified through this plot.

Change of bed level along the river reach has been plotted in Fig. 5.35. The figure shows initial assumed bed level along with the bed levels at one year intervals. It is clearly understood from the plot that there is a tendency of bed level rising through out the whole reach under study except slight irregularities at the upstream boundary. The average bed elevation increment is about 25 cm during these three years. However, this should not be treated as an outcome of the study. The river Jamuna is braided and flow behavior is three-dimensional. Hence, conversion to a one-dimensional situation may not produce realistic results. The river may aggrade at some places of a station and at the same time it may degrade at some other places of the same station. The overall effect may be aggradation which is given by a one-dimensional model.

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

This study includes the development and application of a one-dimensional mathematical model based on MacCormack scheme. The following conclusions can be made after summarizing the present study:

- i. The second-order accurate MacCormack explicit finite difference scheme is found to be efficient in handling large and small-scale morphological phenomena. All three governing equations are solved simultaneously during any computational step so that water-flow equations and sediment continuity equation are coupled. Any form of sediment transport equation can be incorporated easily, as it does not require any iteration.
- ii. The developed model is capable of predicting aggradation-degradation in an alluvial channel.
- iii. The model has simulated the resulting aggradation caused by sediment overloading at the upstream section very nicely. This demonstrates the model's ability to capture shock in the aggradation profile.
- iv. The model is proved to be efficient when it handles real field situation moderately well. Close agreement between the computed results and measured data justifies that the scheme can be applied to any alluvial river problems.

- v. The model, when applied to Jamuna river, predicts that there is a tendency of bed level rising along the whole reach (i.e., from Bahadurabad to Sirajgang in the model).

6.2 Limitations

The following limitations can be drawn from this study:

- i. All field or laboratory observations are three-dimensional in reality. These observations are required to be converted into one-dimensional form to apply in a one-dimensional model. So, three-dimensional variables are reduced to one-dimensional for the sake of model application. The outcomes obtained thus are not very good in visualizing the real field situation.
- ii. The one-dimensional momentum equation of water considers only the advective transport. There is no Reynolds stress term or momentum transport term, which may influence the result considerably.
- iii. The hydrodynamic and morphological time scales are not same. These two differs a lot. Hydrodynamic scale is small while the other is very large. Hence, coupling between the water-flow and sediment continuity equations in the same computational time step may not be treated efficient.
- iv. In both predictor and corrector step, non-linear terms were evaluated at the known time level. So the scheme might have some artificial viscosity. Fourier analysis would reveal this.
- v. As the model is one dimensional, drastic simplification was required while schematizing the cross-section of the river Jamuna into a

rectangular form. This is a considerable deviation from the real field situation.

- vi. The verification of the model at an interim place between Bahadurabad and Sirajgang along the river reach was not possible due to lack of field data.

6.3 Recommendations

Based on the present study the following recommendations can be made for future study that may produce better results.

- i. The selection of a suitable sediment transport equation is very important since it affects the computed results considerably. Hence, the model needs to be calibrated against a proper sediment transport prediction formula.
- ii. The model has been developed using prismatic cross-section of a channel. The analysis for a non-prismatic channel can be made using the same governing equations written for the entire cross-section rather than for unit width.
- iii. The MacCormack scheme can be applied to develop a two-dimensional model to study the two-dimensional problems.
- iv. The Scheme can also be used for the simulation of channel networks.

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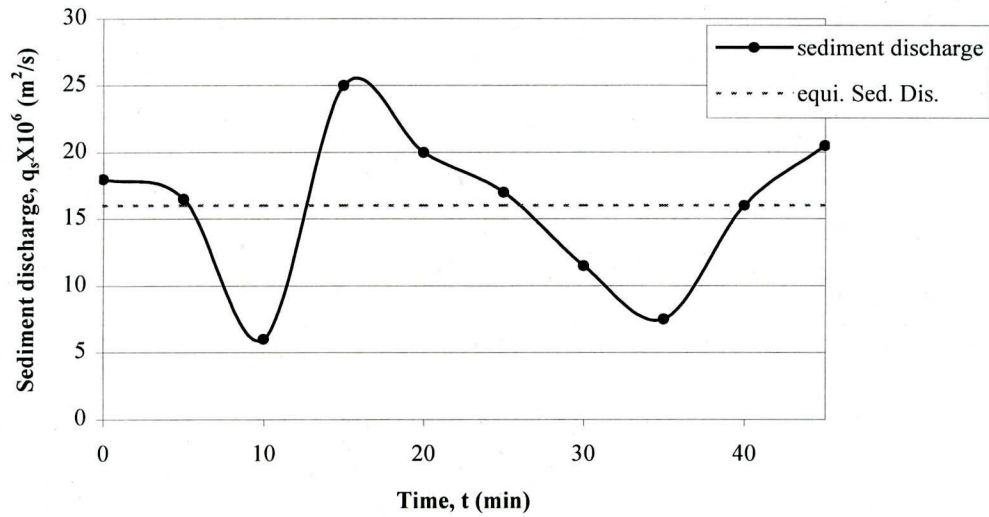
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Variation of sediment transport rate at end of flume
(before injection of sediment)



Variation of sediment transport rate at end of flume
(after injection of sediment)

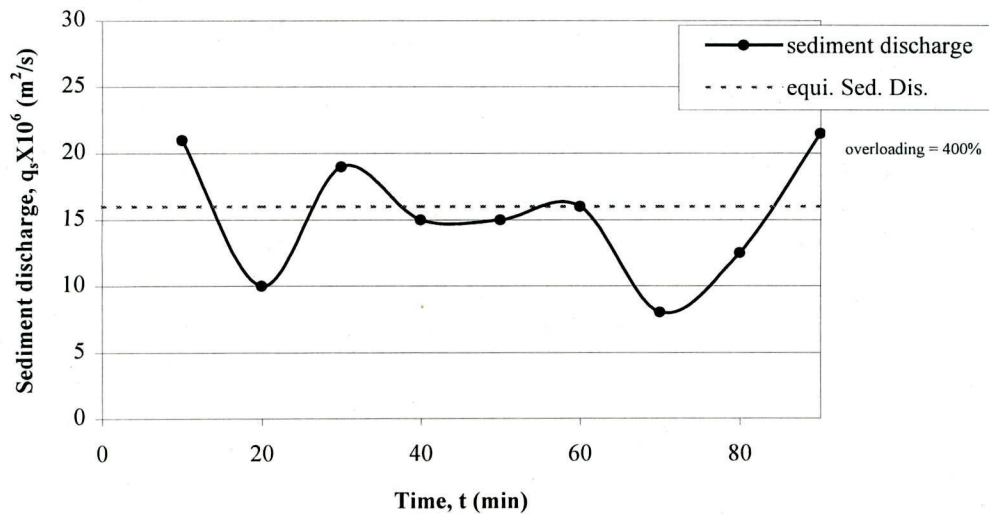


Fig. 4.1 Sediment transport rates before and after sediment injection
(After Soni et al., 1980)

SEDIMENT TRANSPORT LAW FOR UNIFORM FLOW

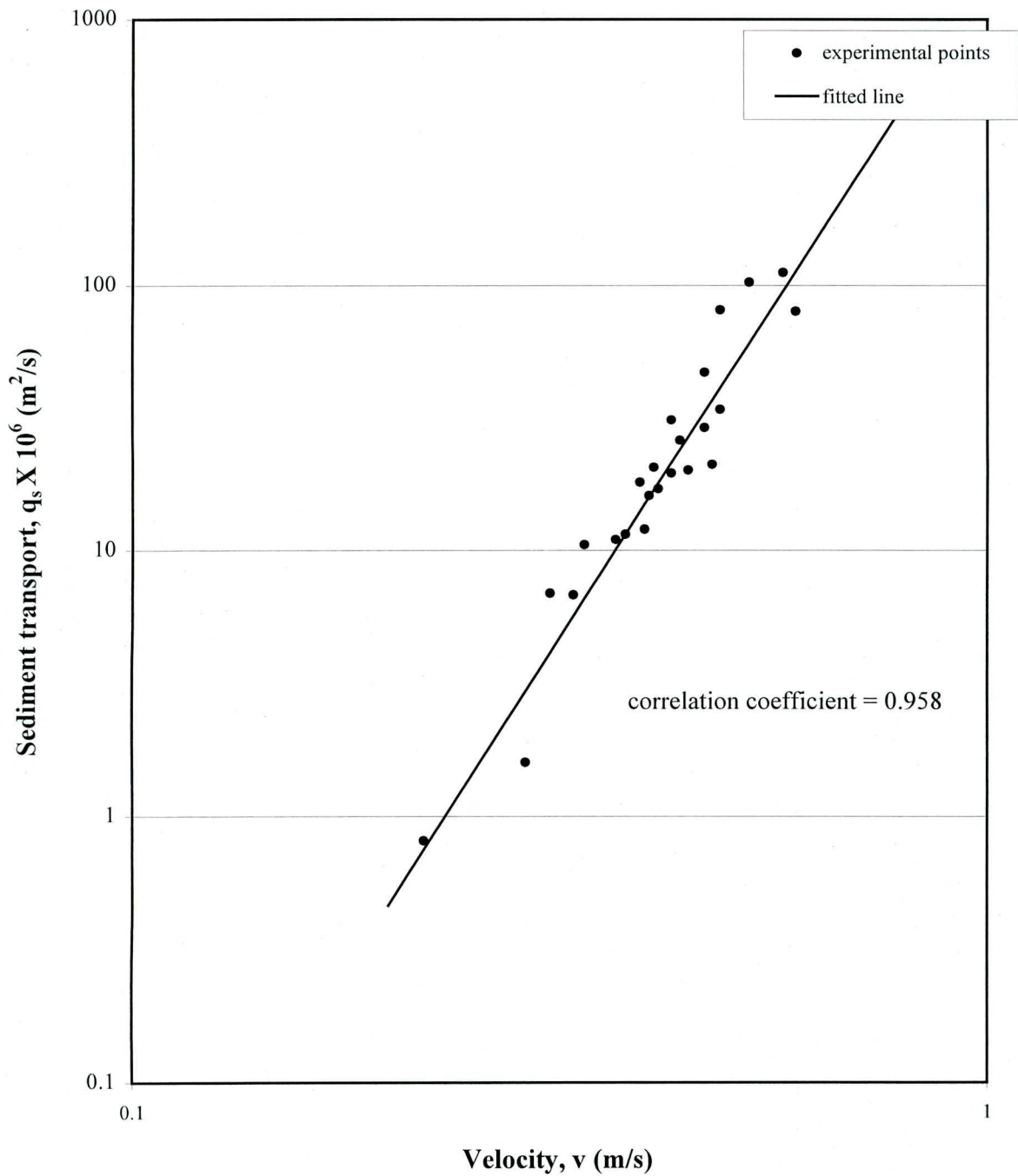
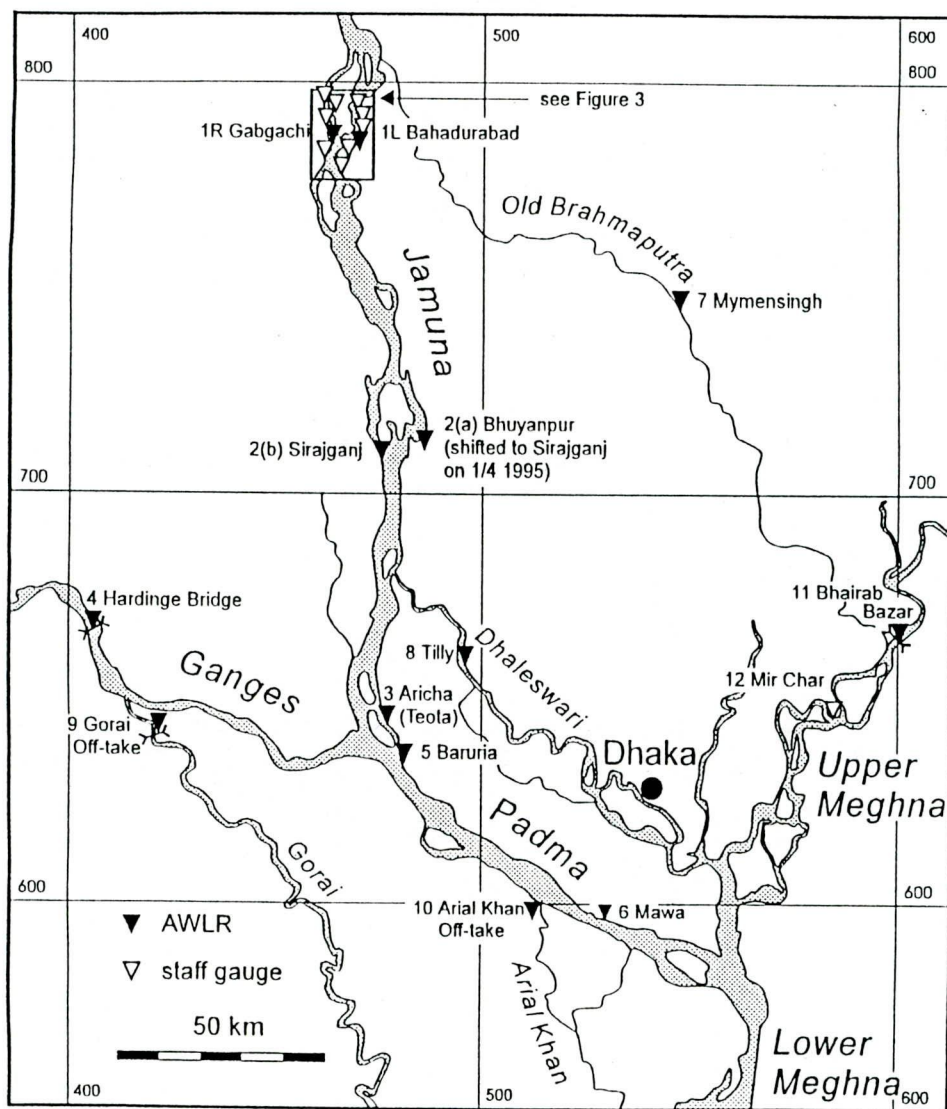


Fig. 4.2 Determination of 'a' and 'b' for sediment transport formula
(after Soni et al., 1980)

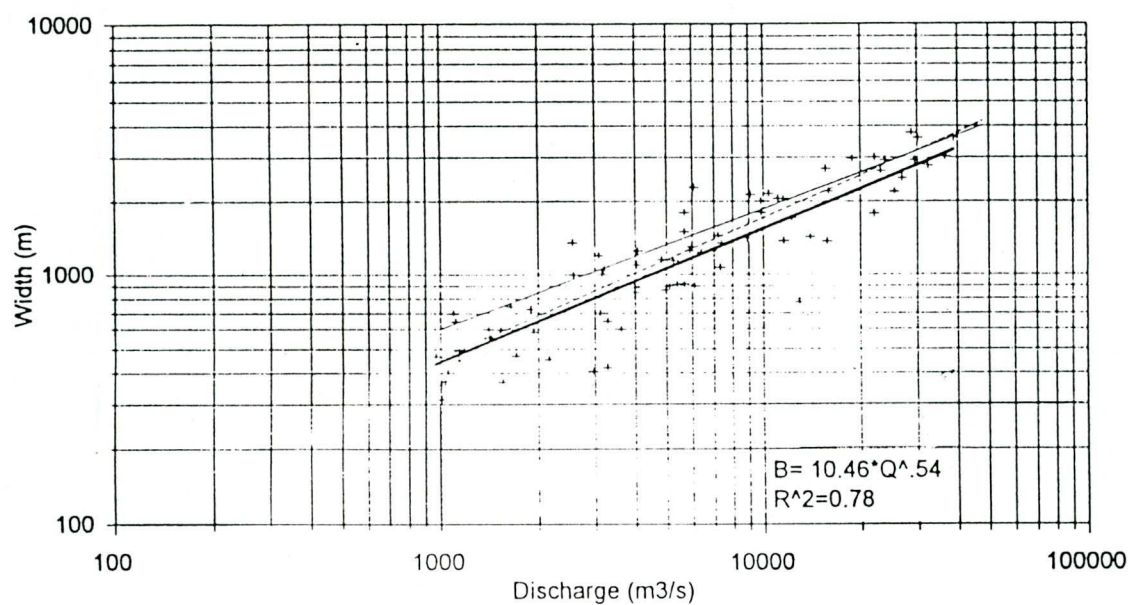


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Fig. 4.3

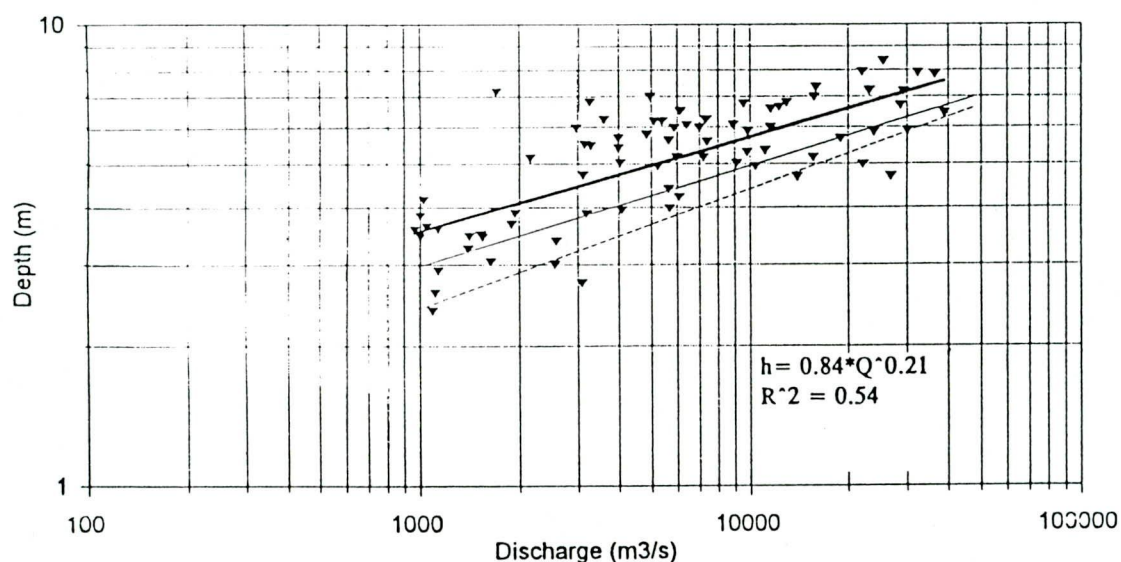
Location map of the study area

(Source: River Survey Project FAP24, Special report No. 22, 1996)



— At-a-station, FAP24 data Regime relation — At-a-station, BWDB data

a. Width relation to discharges



— At-a-station, FAP 24 data ----- Regime relations — At-a-station, BWDB data

b. Depth relation to discharges

Fig. 4.5 At a station width and depth relations (at different discharges) of channels of the Jamuna River (using FAP24 survey data)
(Source: River Survey Project FAP24, Special report No. 7, 1996)

CALCULATION OF 'm'

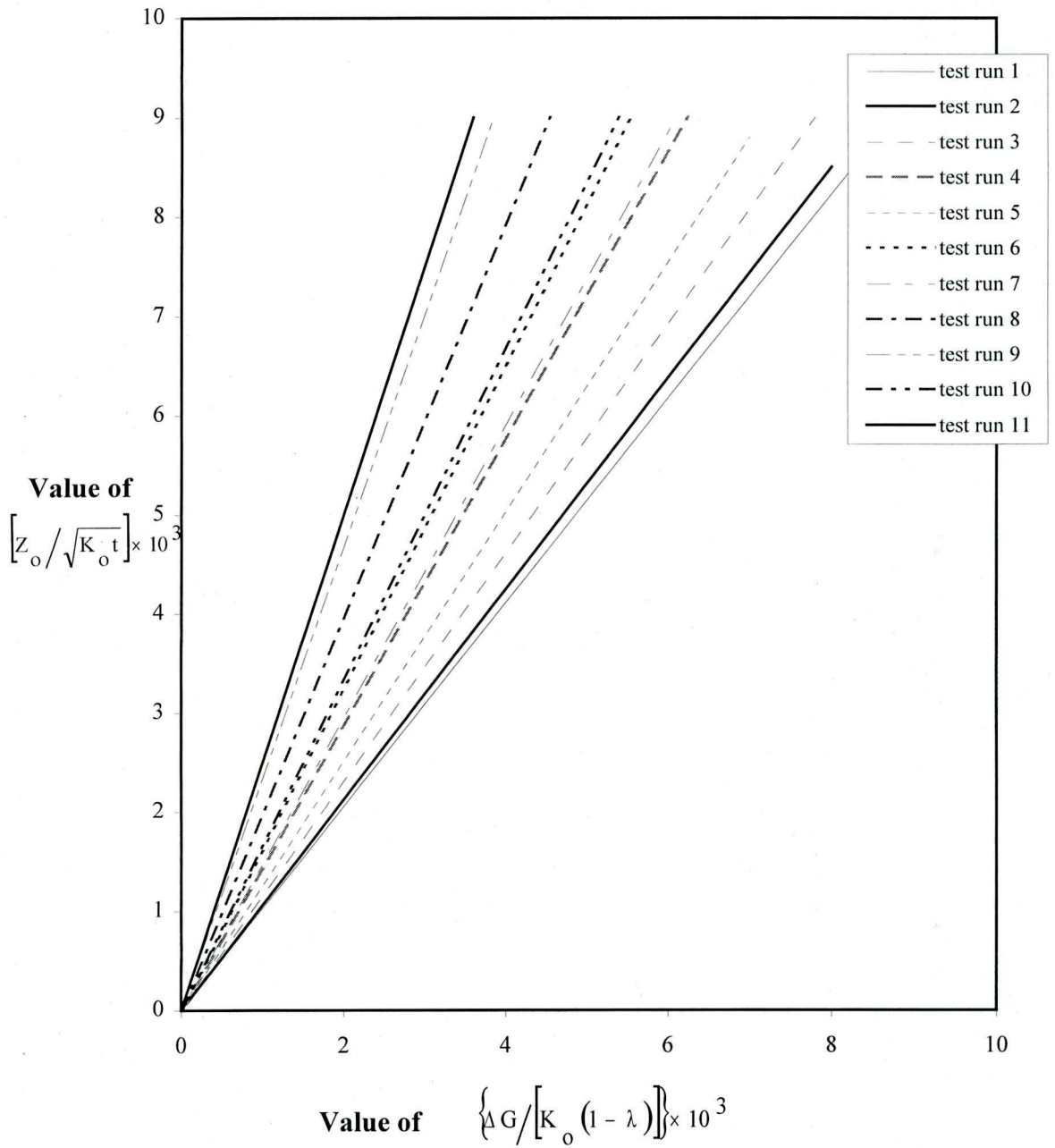


Fig. 4.6 Variation of $\left[\frac{Z_o}{\sqrt{K_o t}} \right] \times 10^3$ with $\left\{ \frac{\Delta G}{[K_o (1 - \lambda)]} \right\} \times 10^3$
(after Soni et al., 1980)

Aggradation due to sediment overloading

test run 1

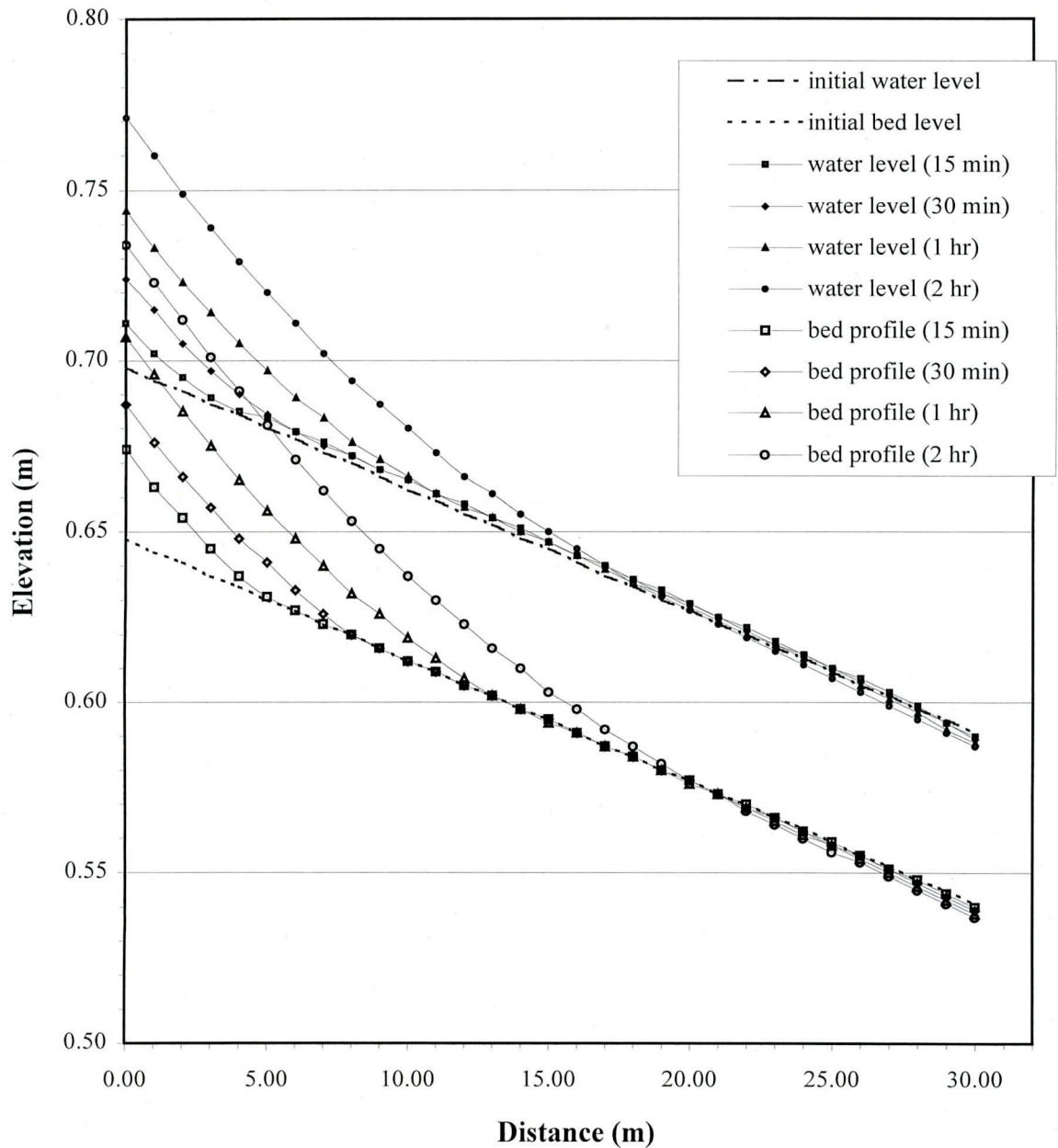


Fig. 5.1 Transient bed and water surface profiles (test run 1)

Aggradation due to sediment overloading

test run 2

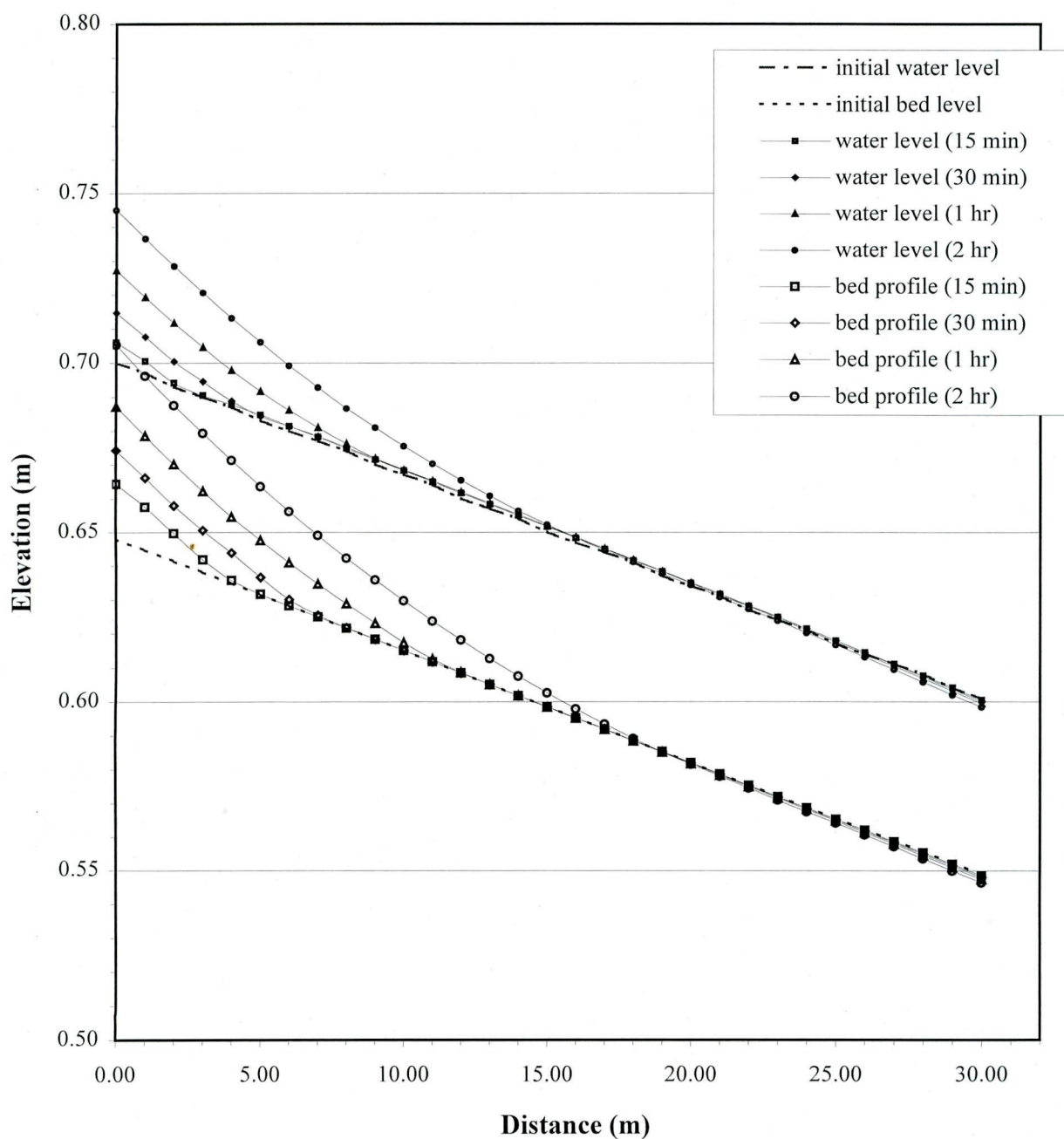


Fig. 5.2 Transient bed and water surface profiles (test run 2)

Aggradation due to sediment overloading

test run 3

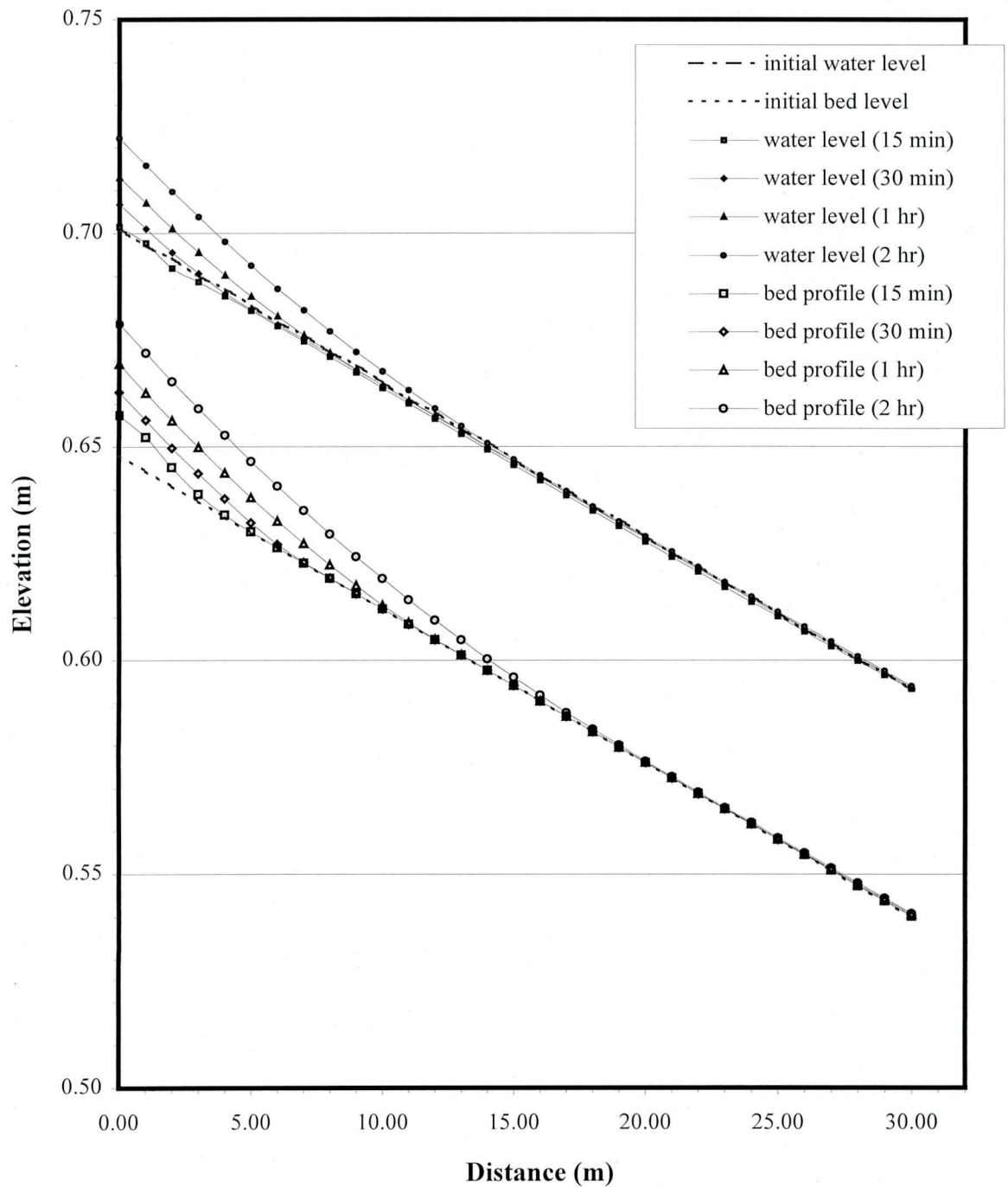


Fig. 5.3

Transient bed and water surface profiles (test run 3)

Aggradation due to sediment overloading

test run 4

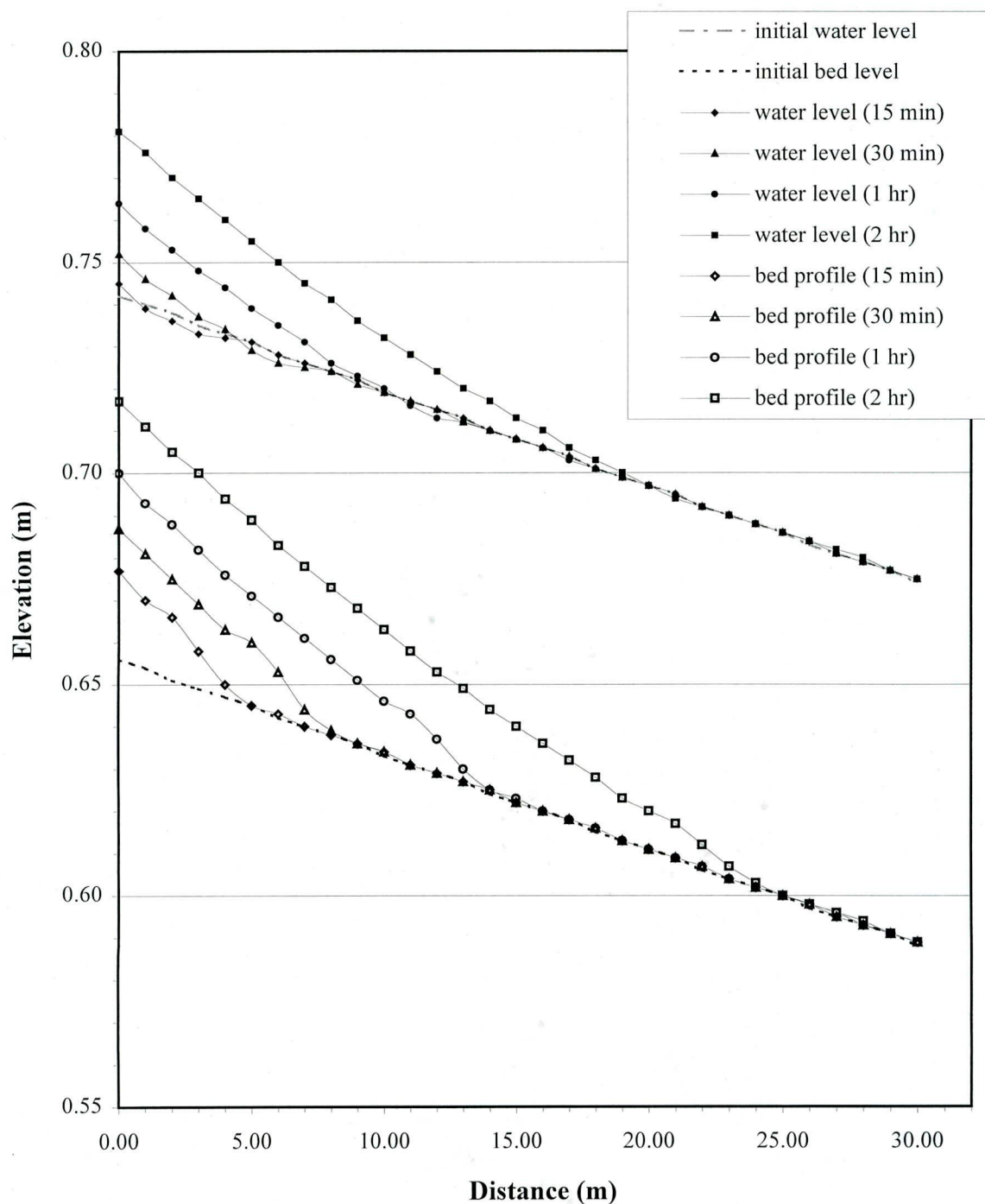


Fig. 5.4

Transient bed and water surface profiles (test run 4)

Aggradation due to sediment overloading test run 5

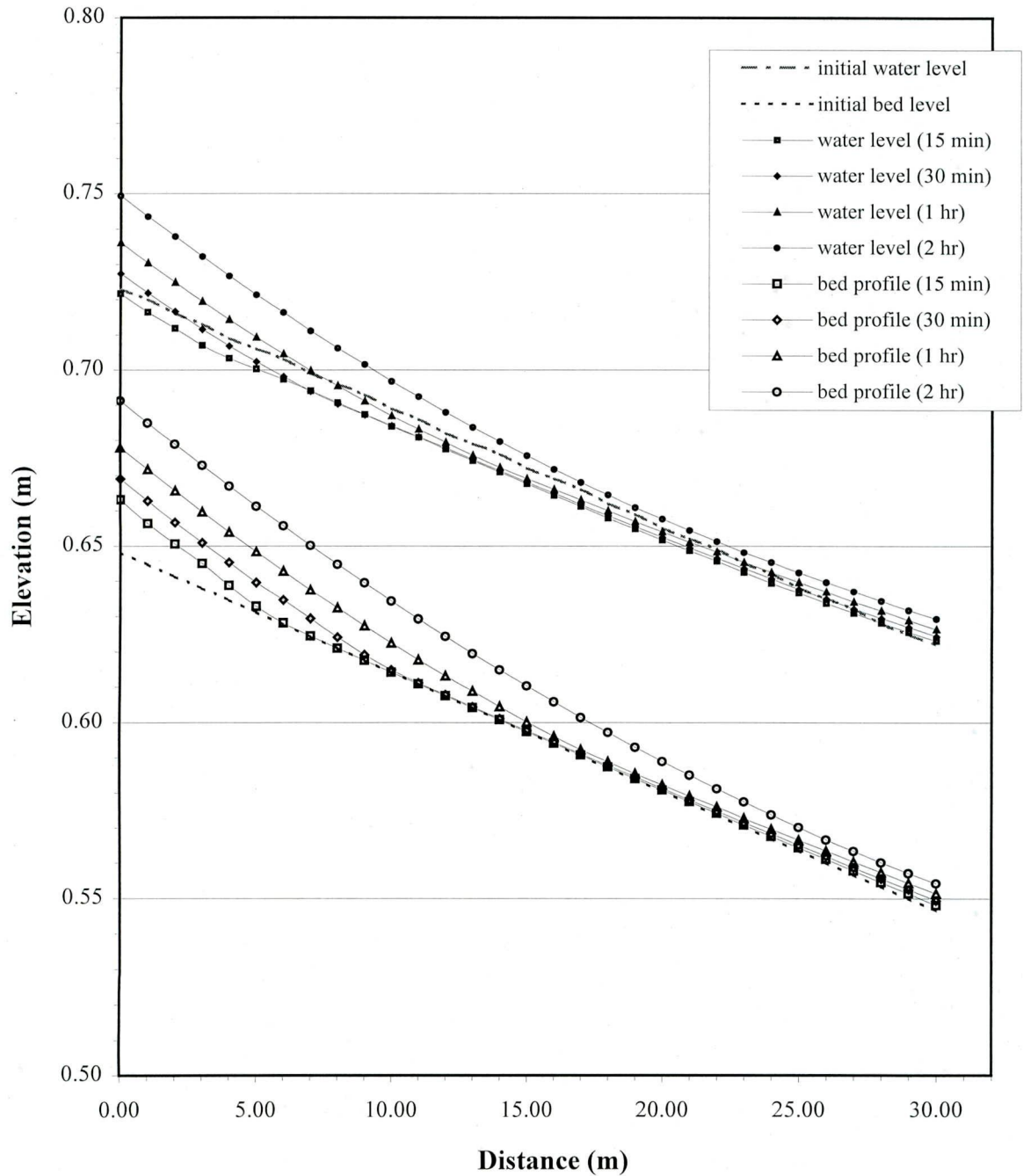


Fig. 5.5 Transient bed and water surface profiles (test run 5)

Aggradation due to sediment overloading

test run 6

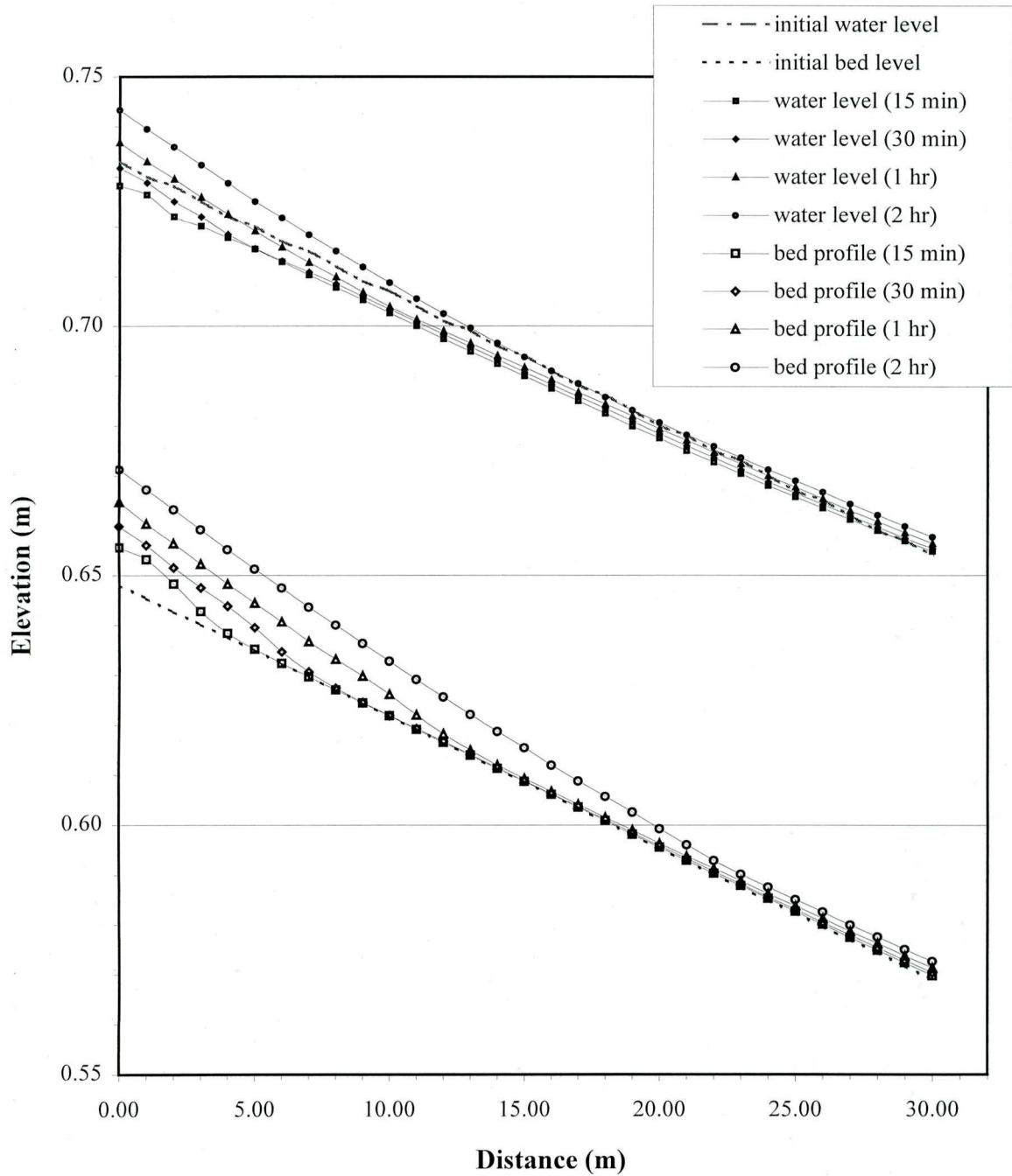


Fig. 5.6 Transient bed and water surface profiles (test run 6)

Aggradation due to sediment overloading

test run 7

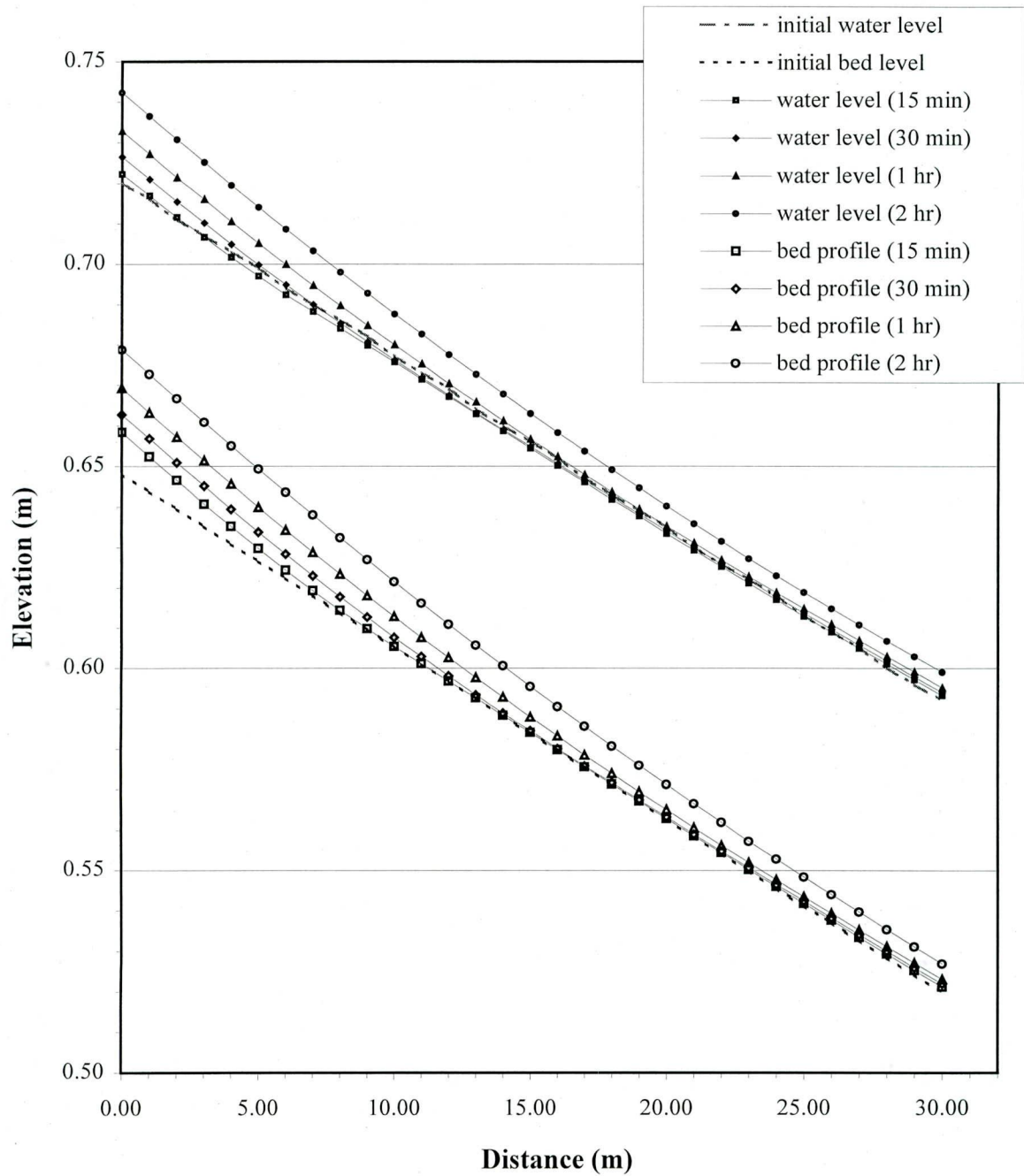


Fig. 5.7 Transient bed and water surface profiles (test run 7)

Aggradation due to sediment overloading

test run 8

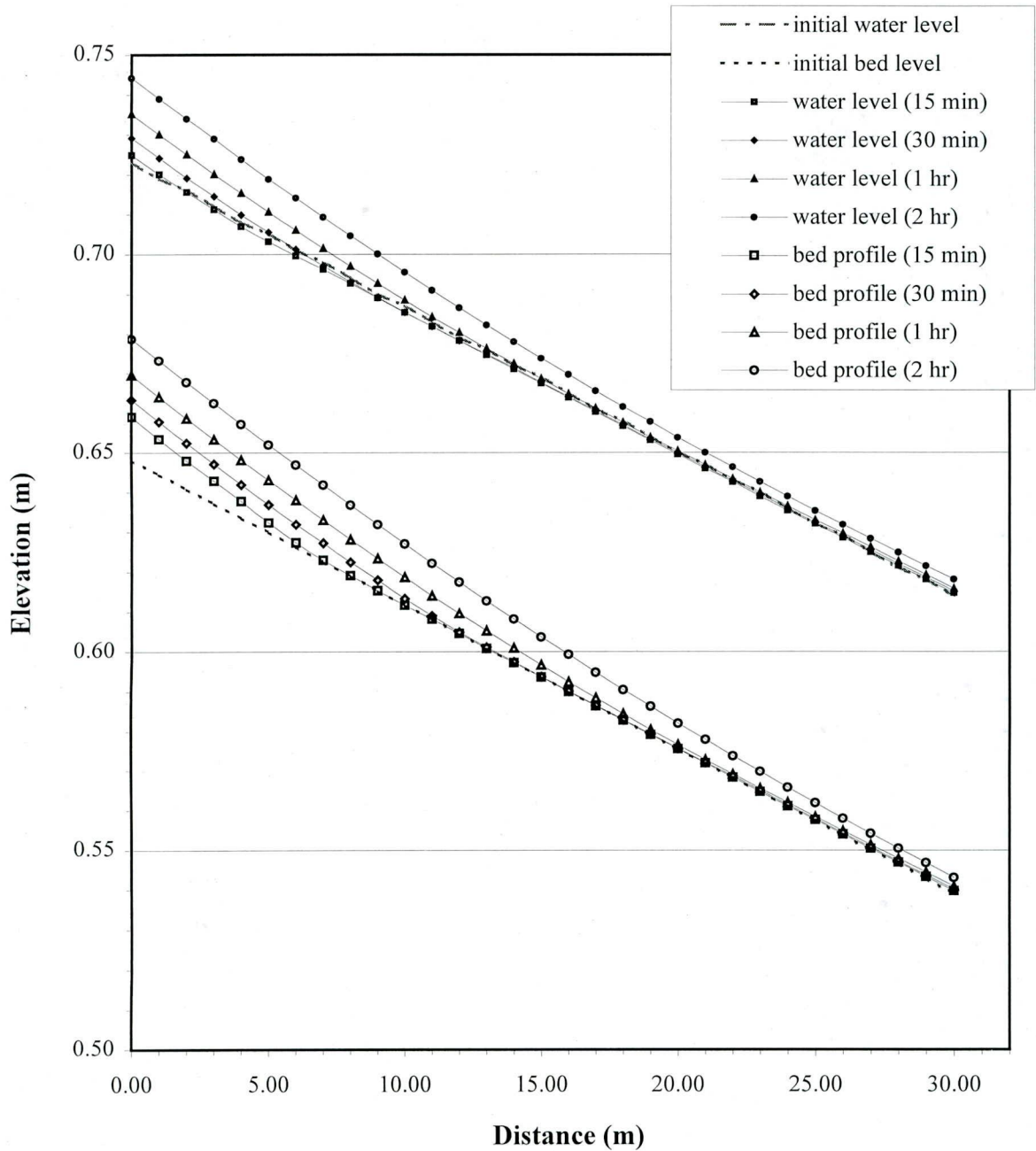


Fig. 5.8 Transient bed and water surface profiles (test run 8)

Aggradation due to sediment overloading

test run 9

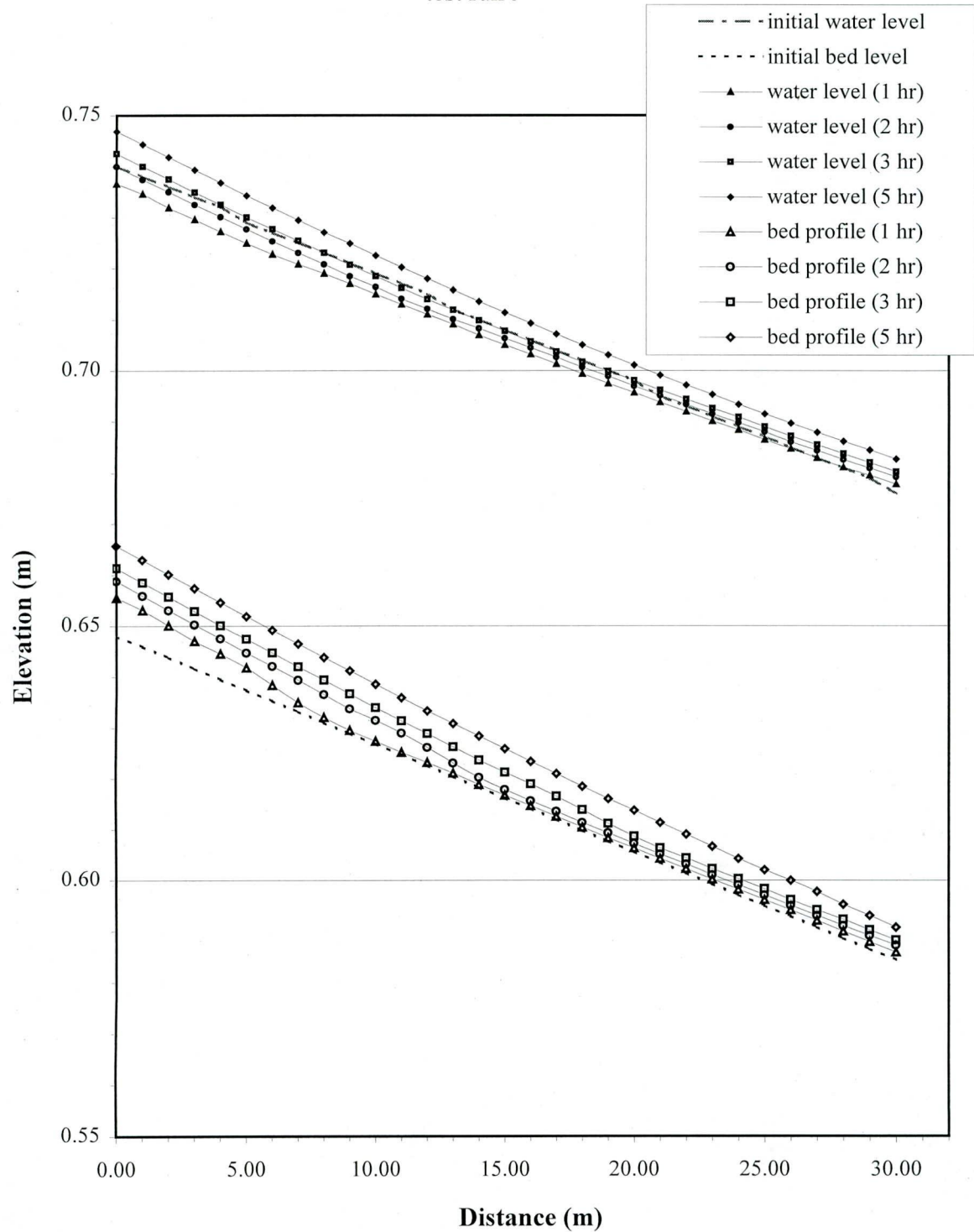


Fig. 5.9 Transient bed and water surface profiles (test run 9)

Aggradation due to sediment overloading

test run 10

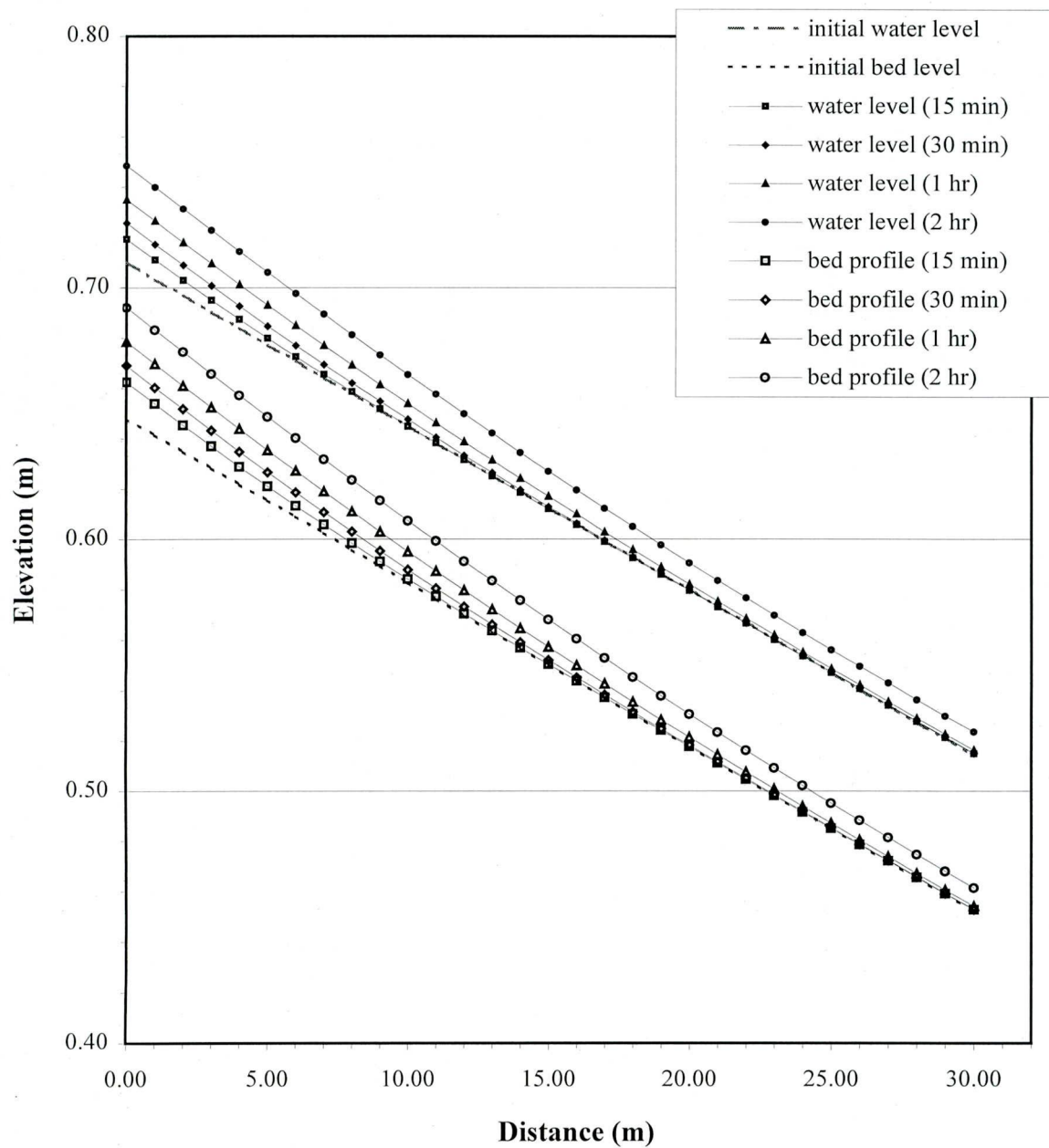


Fig. 5.10 Transient bed and water surface profiles (test run 10)

Aggradation due to sediment overloading

test run 11

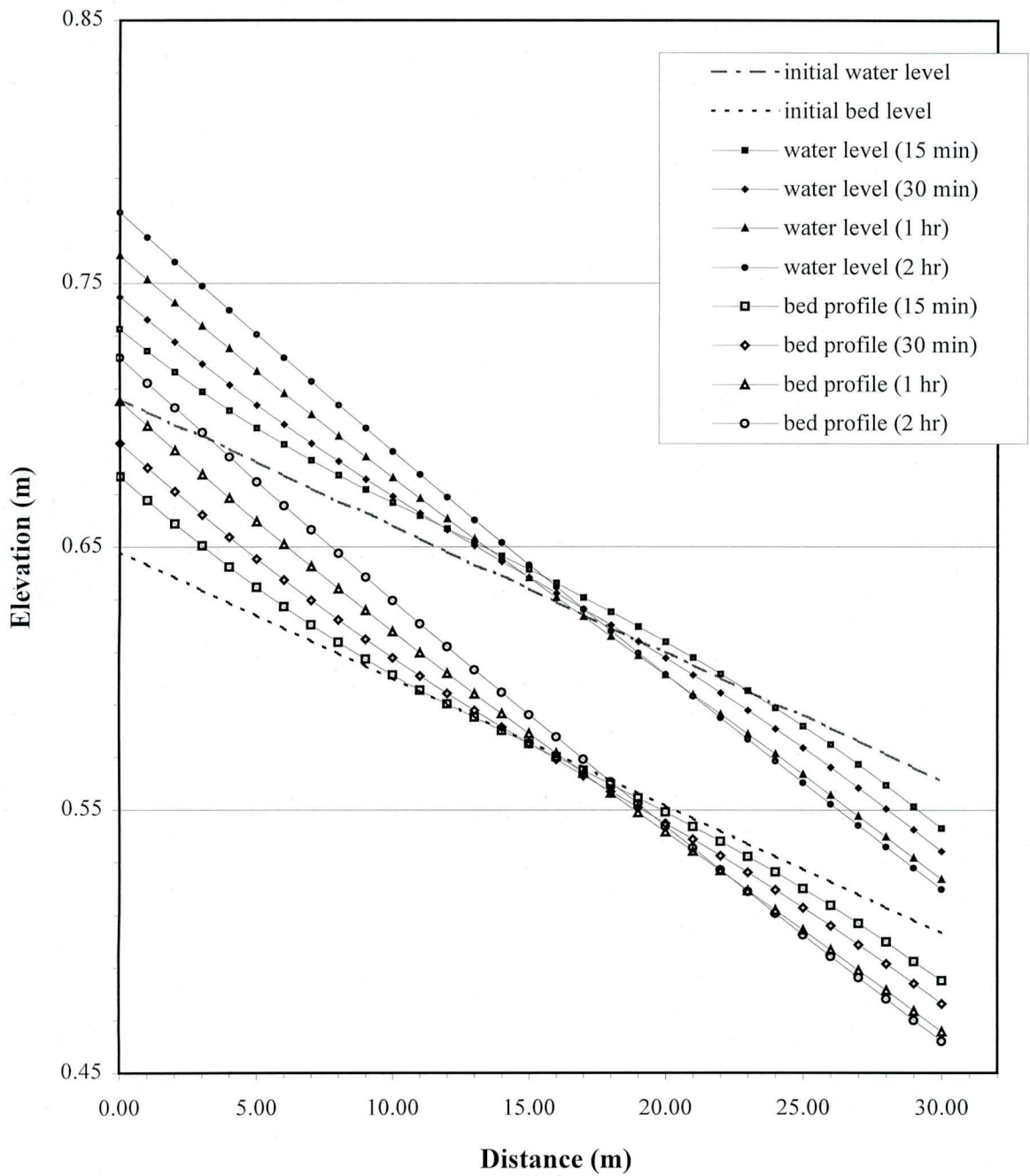


Fig. 5.11 Transient bed and water surface profiles (test run 11)

Aggradation due to Sediment Overloading

test run 4

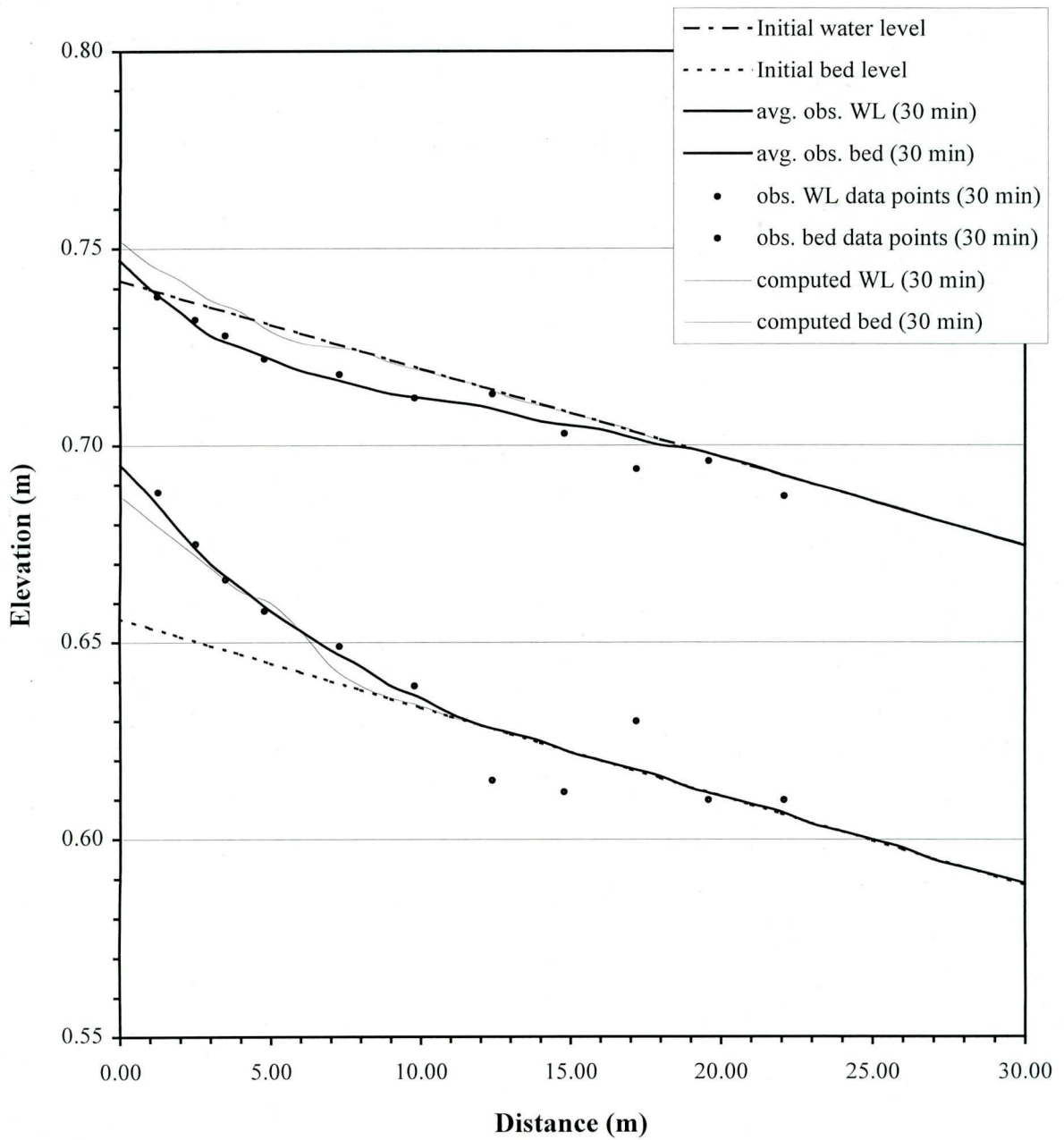


Fig. 5.12 Typical averaging of transient bed and water surface profiles and comparison with computed result (test run 4)

Comparison of water and bed profiles

test run 1
Time = 15 min.

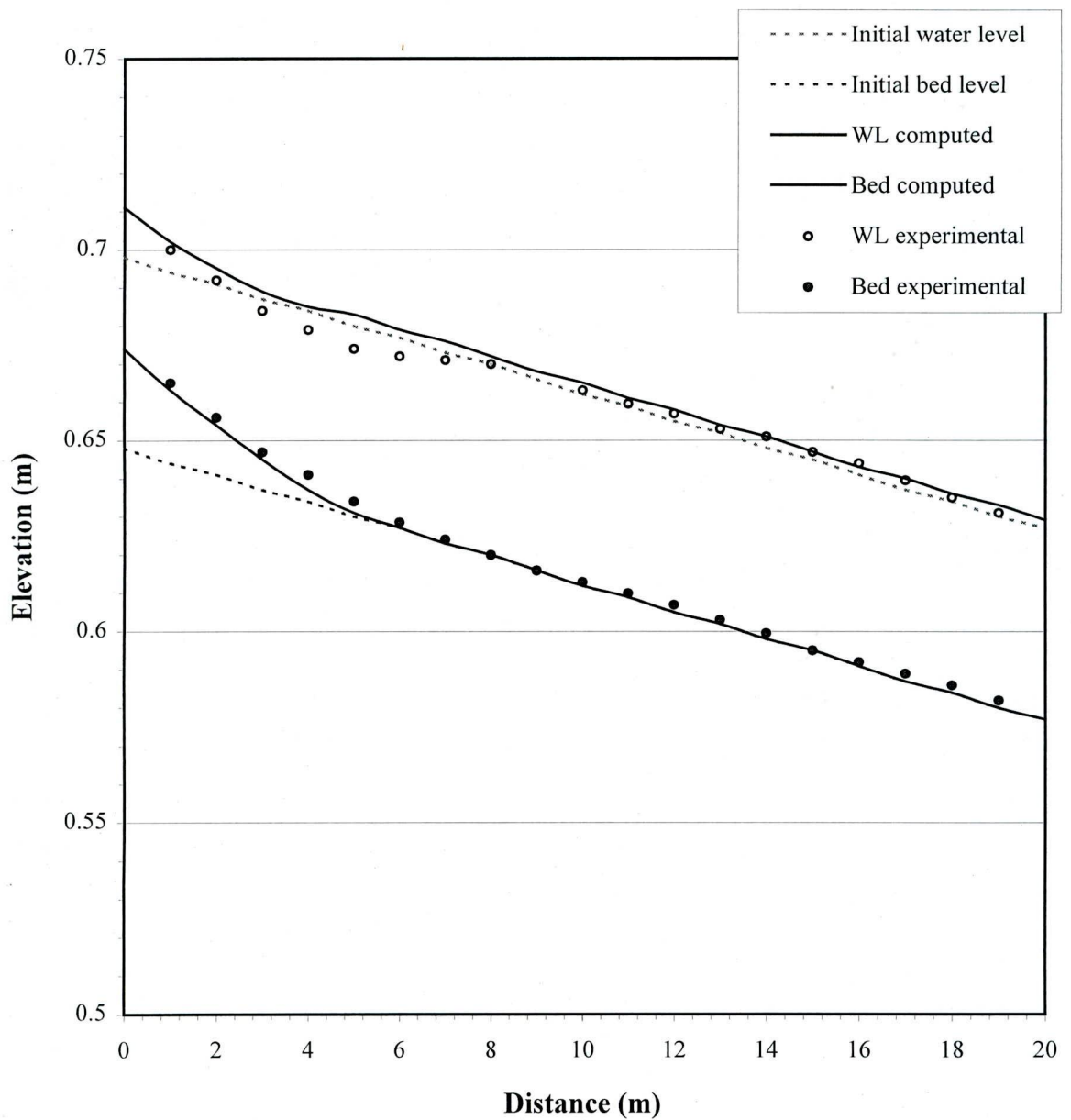


Fig. 5.13 Comparison of bed and water surface profile (test run 1, 15 min.)

Comparison of water and bed profiles

test run 1
Time = 30 min.

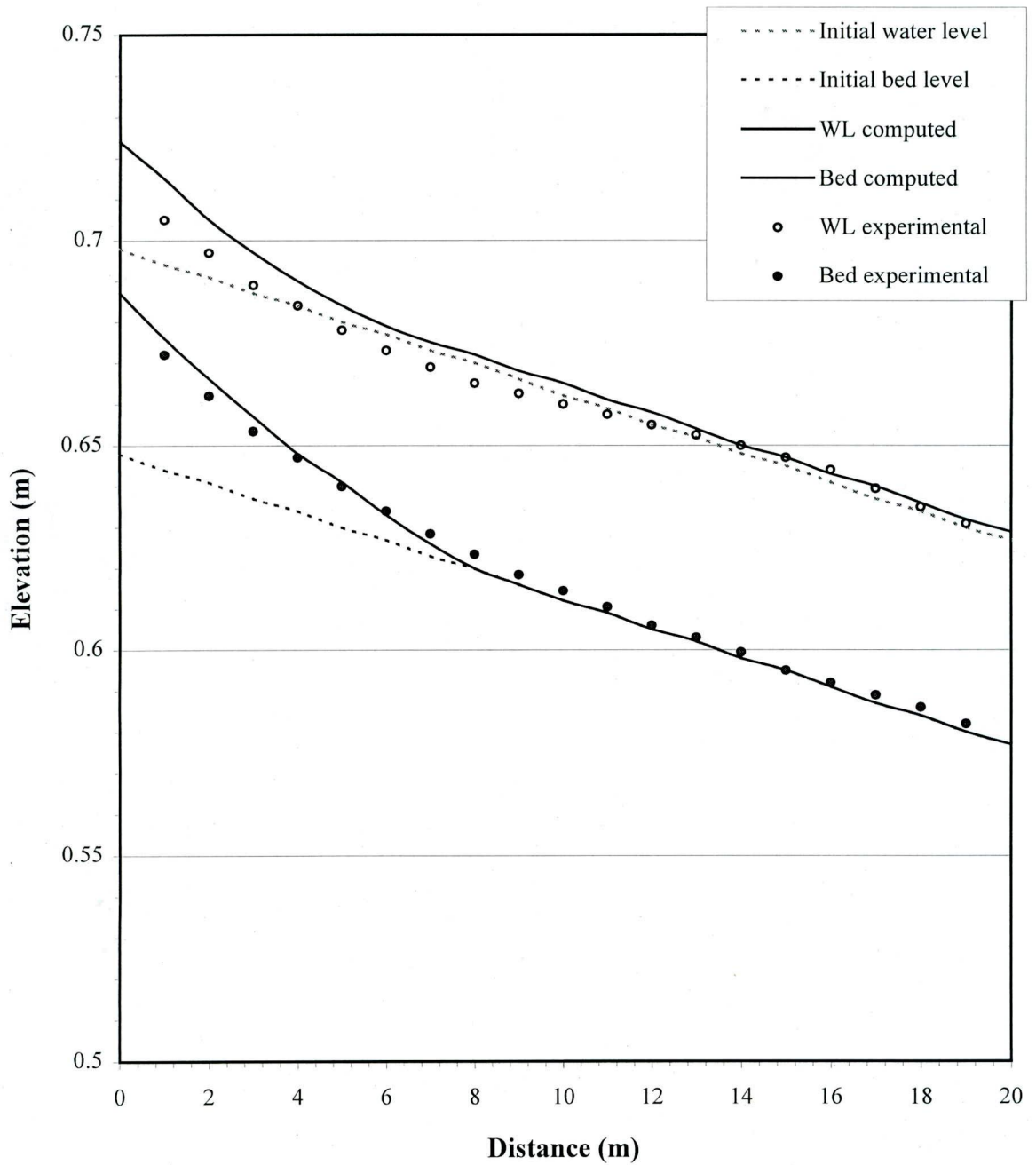


Fig. 5.14 Comparison of bed and water surface profile (test run 1, 30 min.)

Comparison of water and bed profiles

test run 1
Time = 40 min.

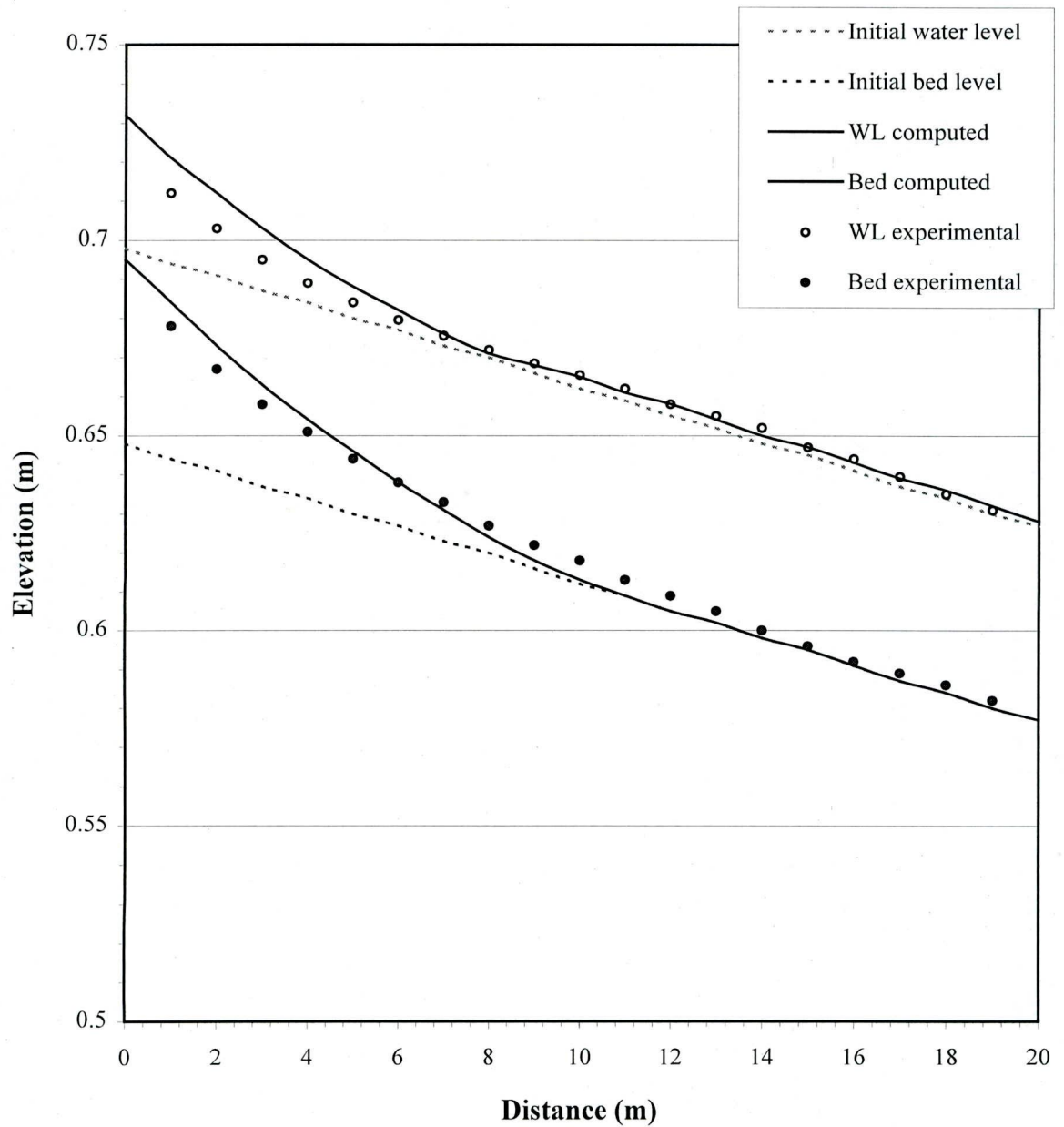


Fig. 5.15 Comparison of bed and water surface profile (test run 1, 40 min.)

RELATIVE AGGRADATION

test run 1

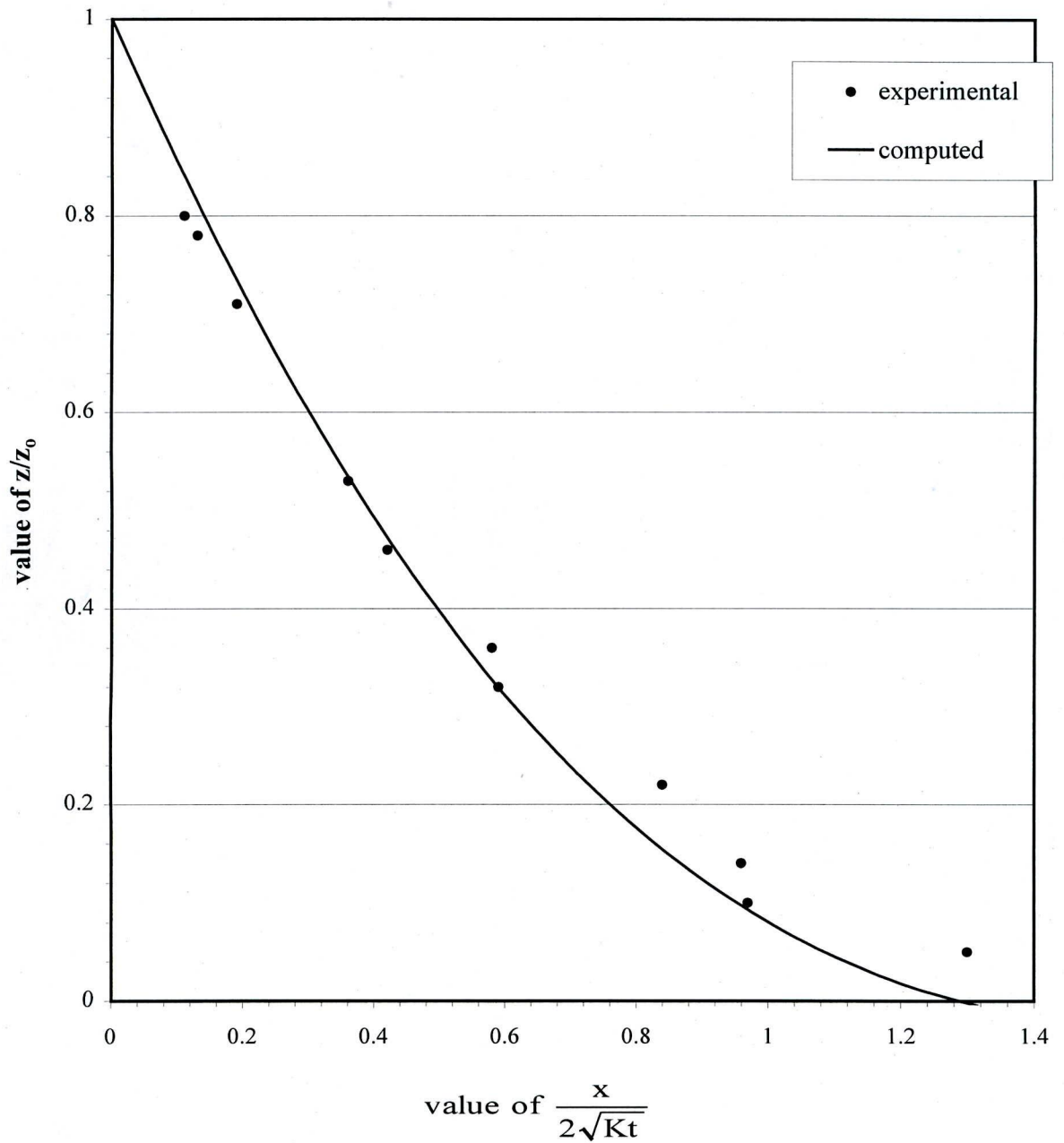


Fig. 5.16 Comparison of relative aggradation (test run 1)

RELATIVE AGGRADATION

test run 2

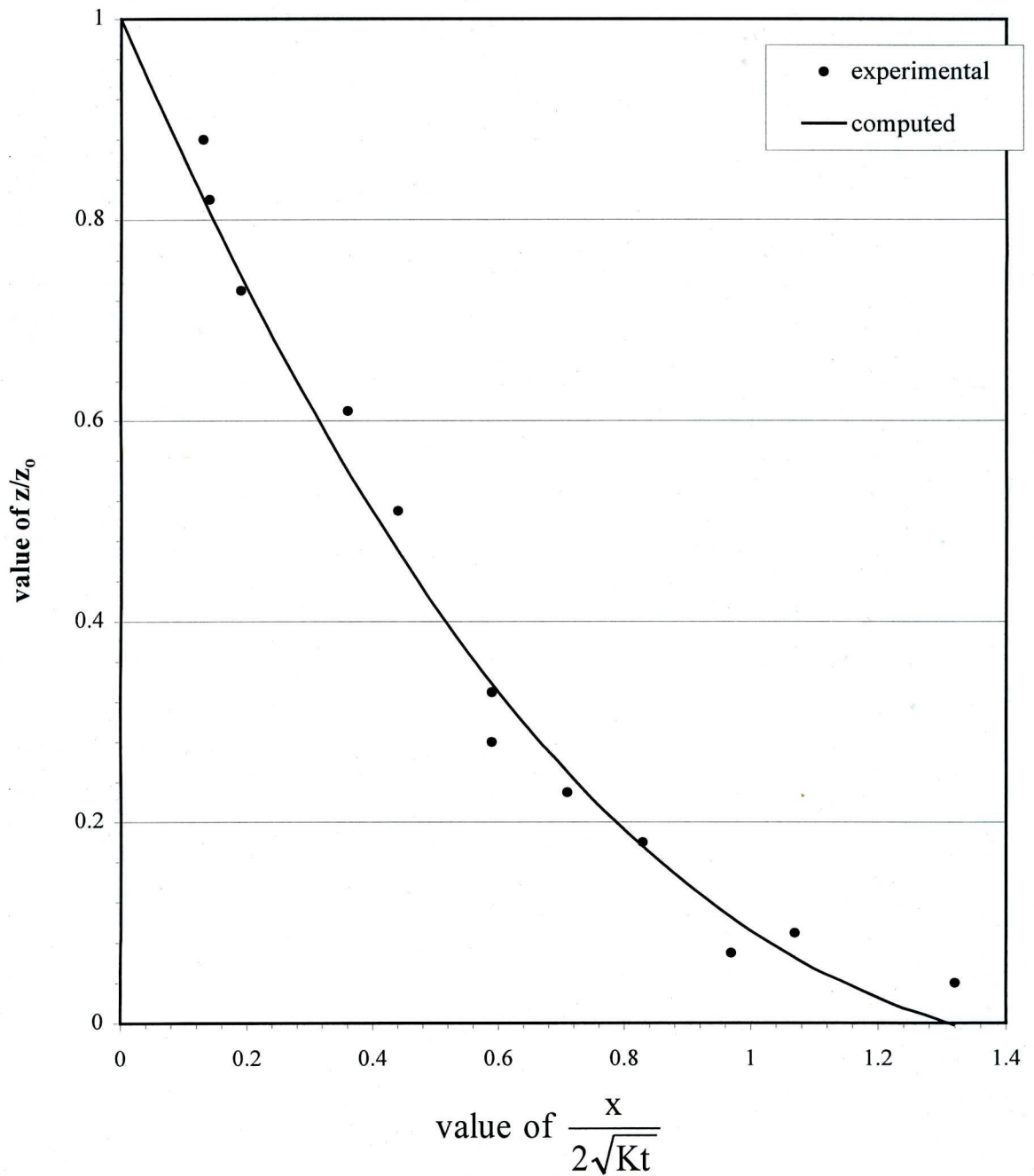


Fig. 5.17 Comparison of relative aggradation (test run 2)

RELATIVE AGGRADATION

test run 3

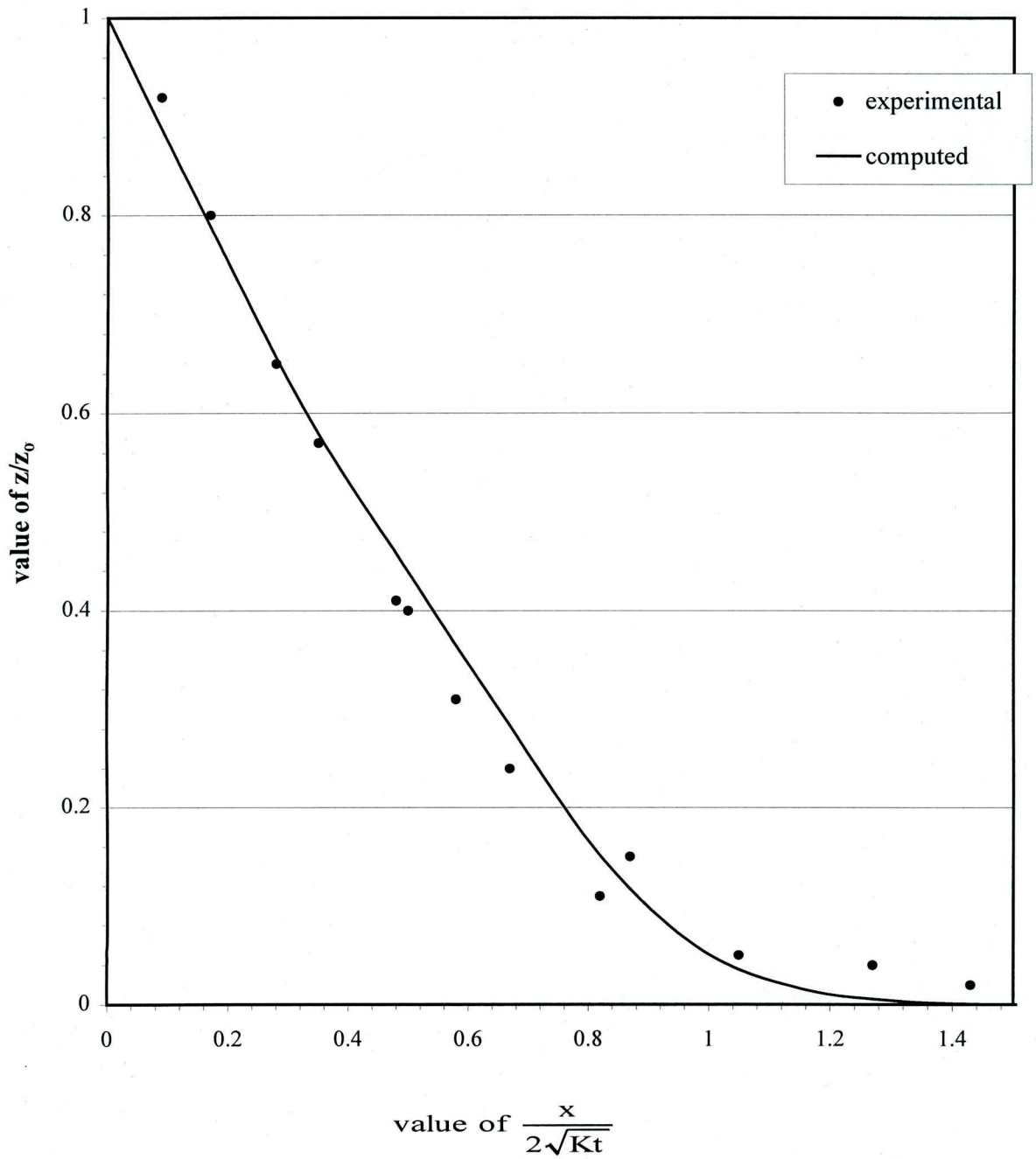


Fig. 5.18

Comparison of relative aggradation (test run 3)

RELATIVE AGGRADATION

test run 4

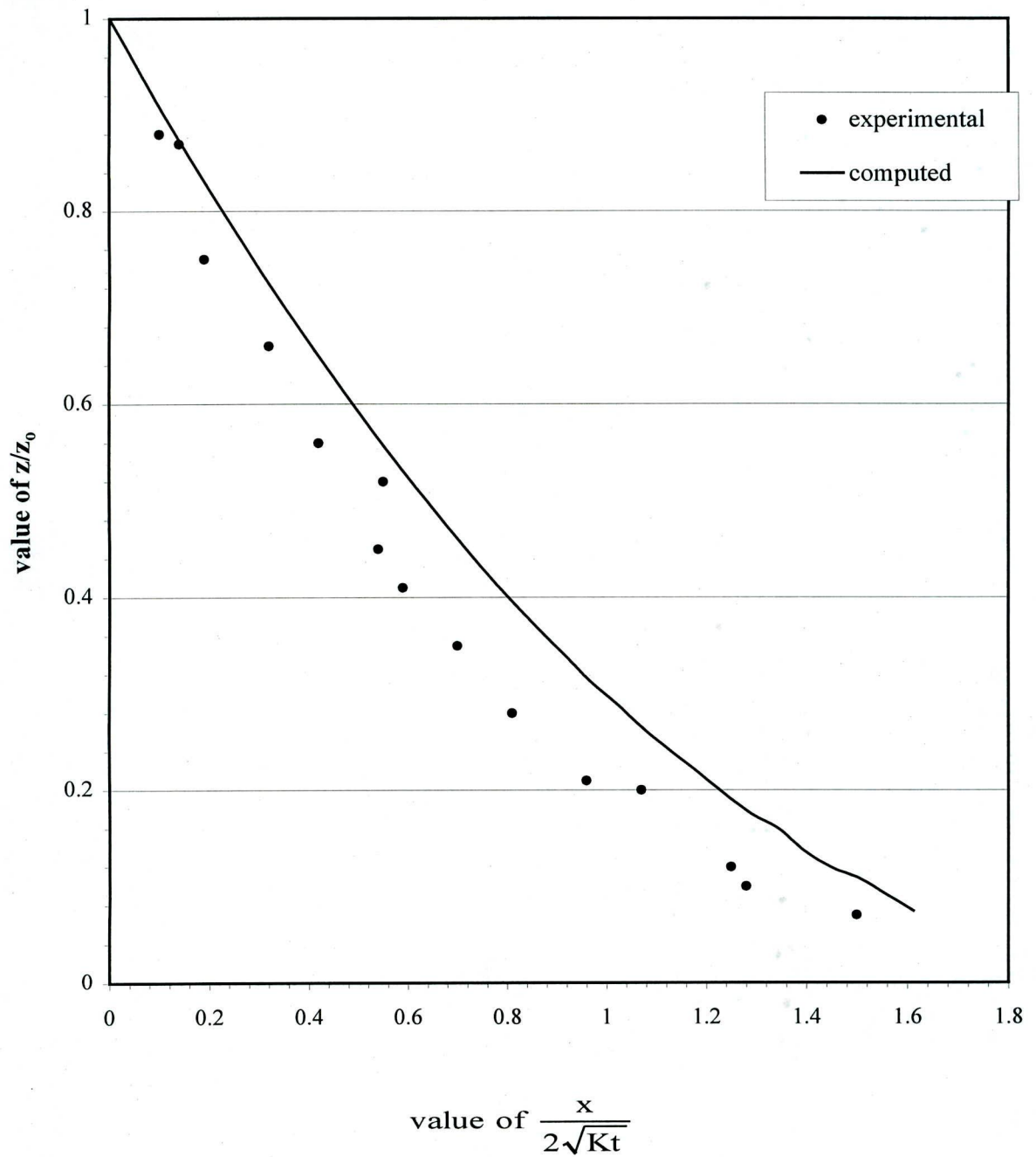


Fig. 5.19 Comparison of relative aggradation (test run 4)

RELATIVE AGGRADATION

test run 5

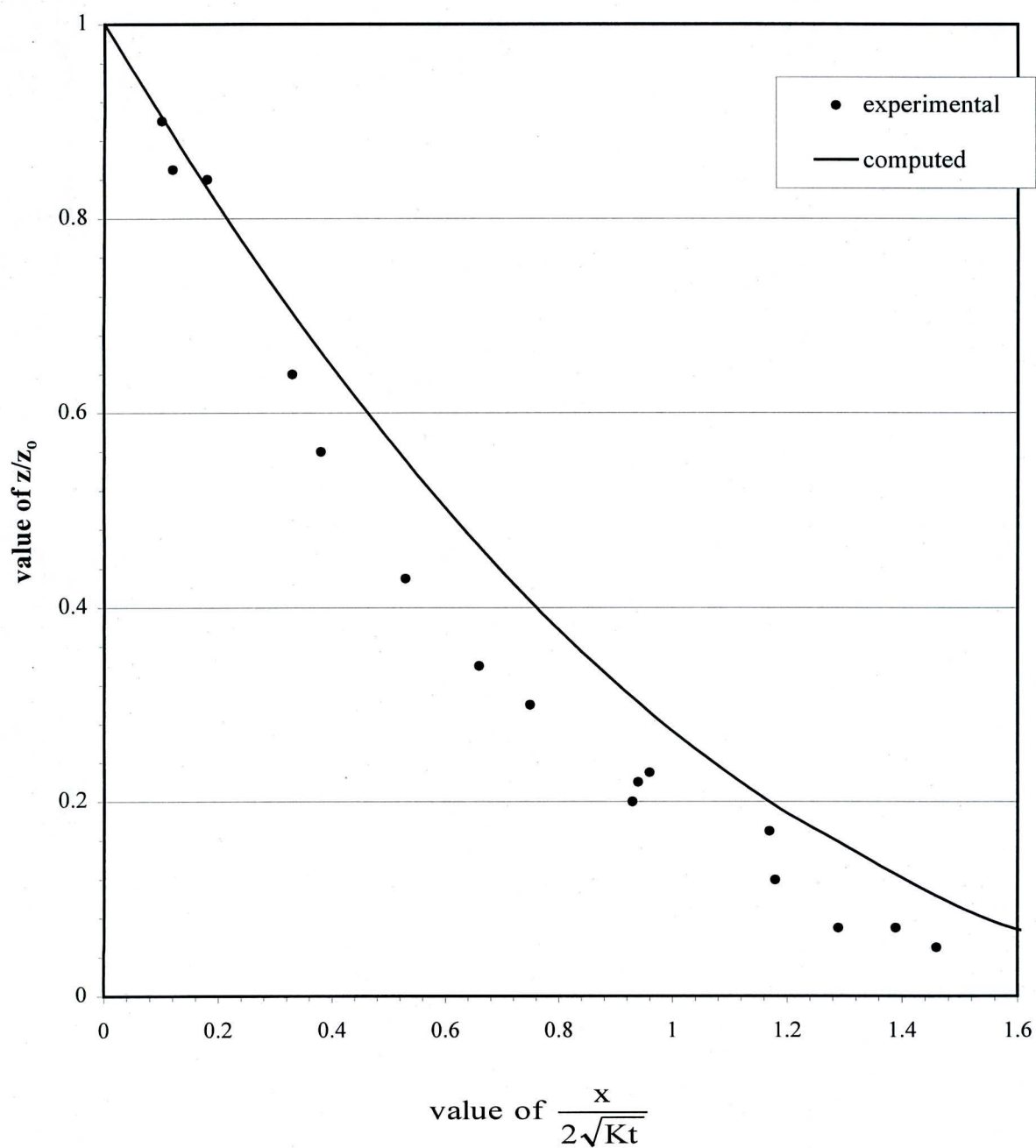


Fig. 5.20 Comparison of relative aggradation (test run 5)

RELATIVE AGGRADATION

test run 6

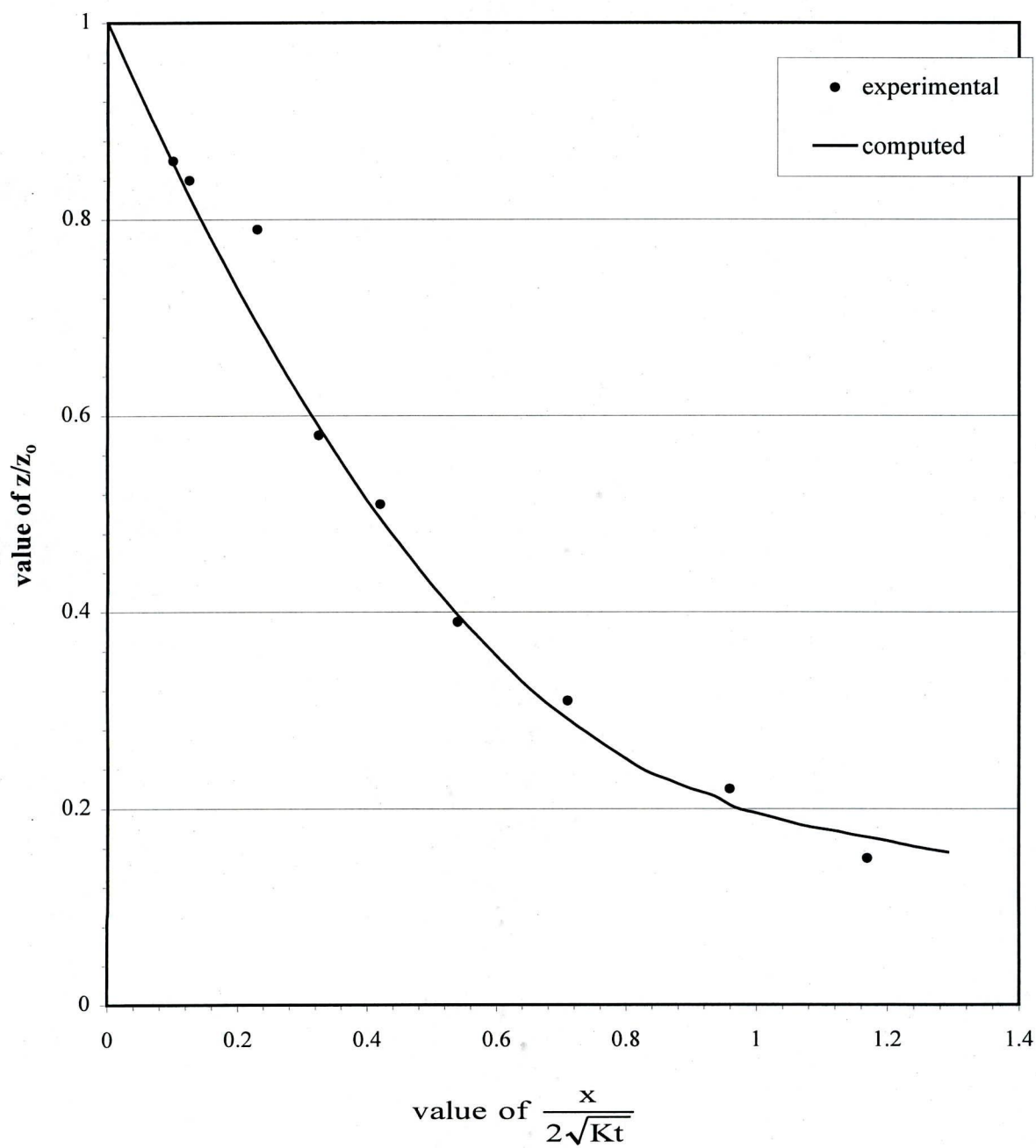


Fig. 5.21 Comparison of relative aggradation (test run 6)

RELATIVE AGGRADATION

test run 7

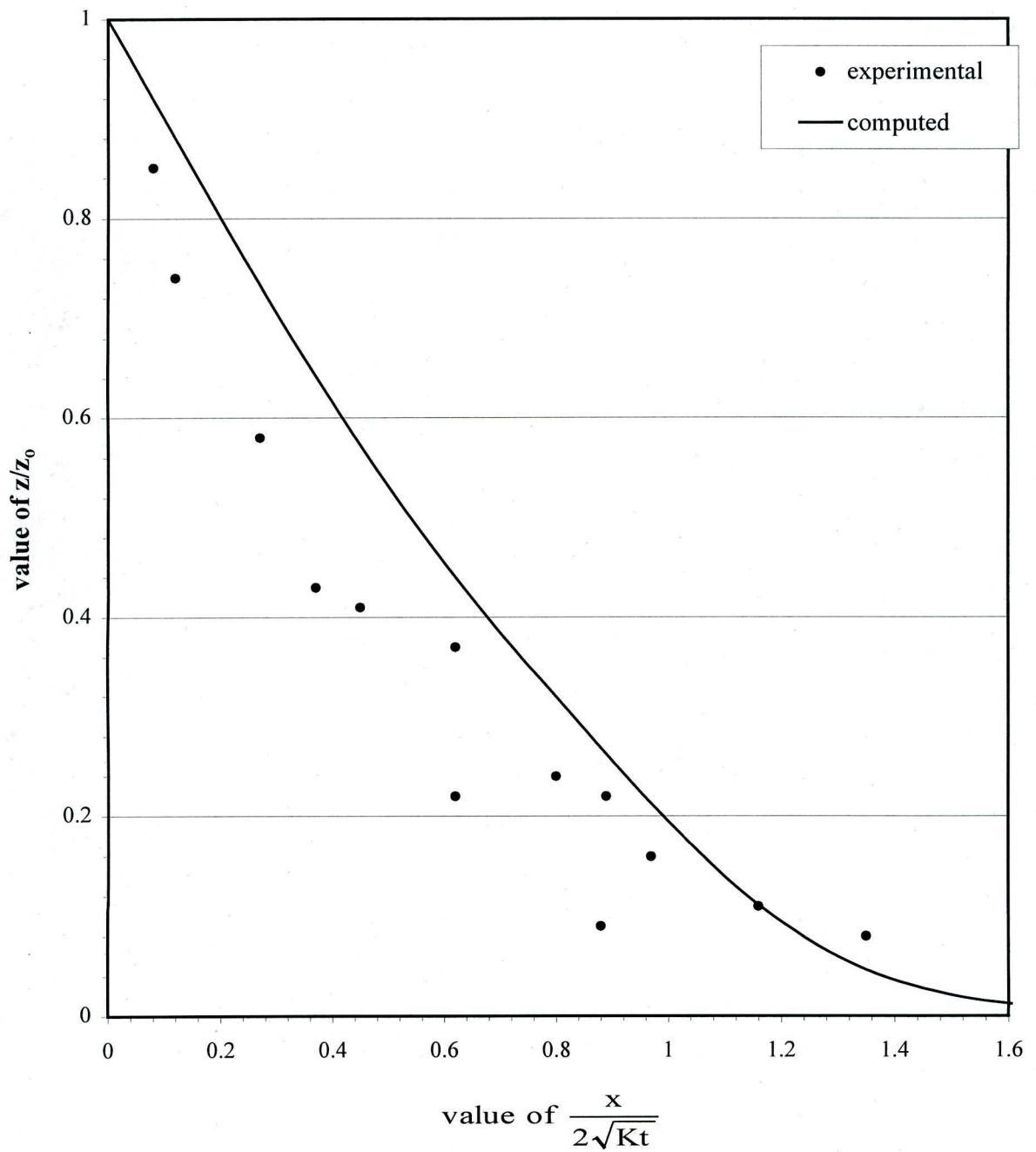


Fig. 5.22 Comparison of relative aggradation (test run 7)

RELATIVE AGGRADATION

test run 8

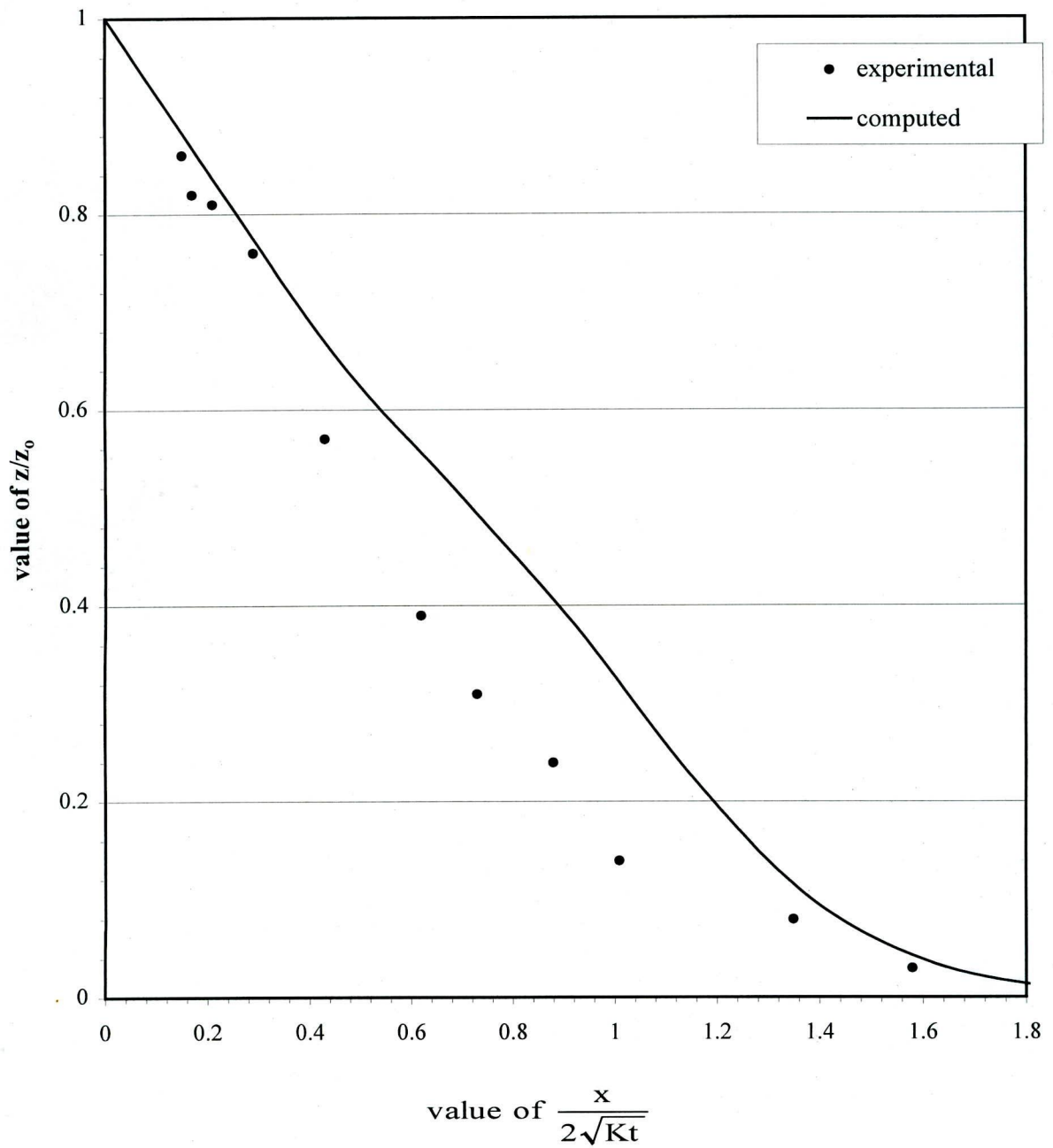


Fig. 5.23

Comparison of relative aggradation (test run 8)

RELATIVE AGGRADATION

test run 9

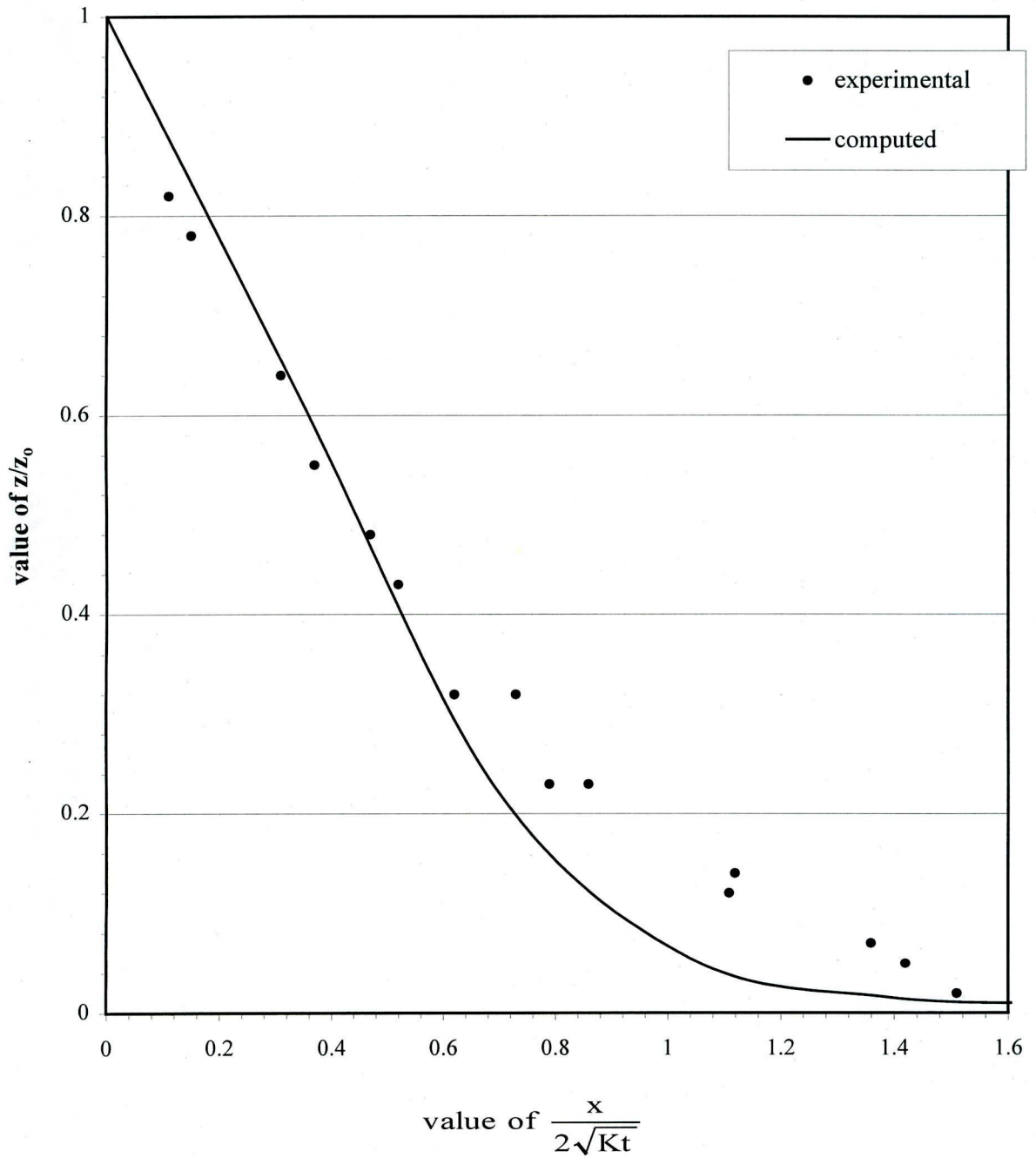


Fig. 5.24

Comparison of relative aggradation (test run 9)

RELATIVE AGGRADATION

test run 10

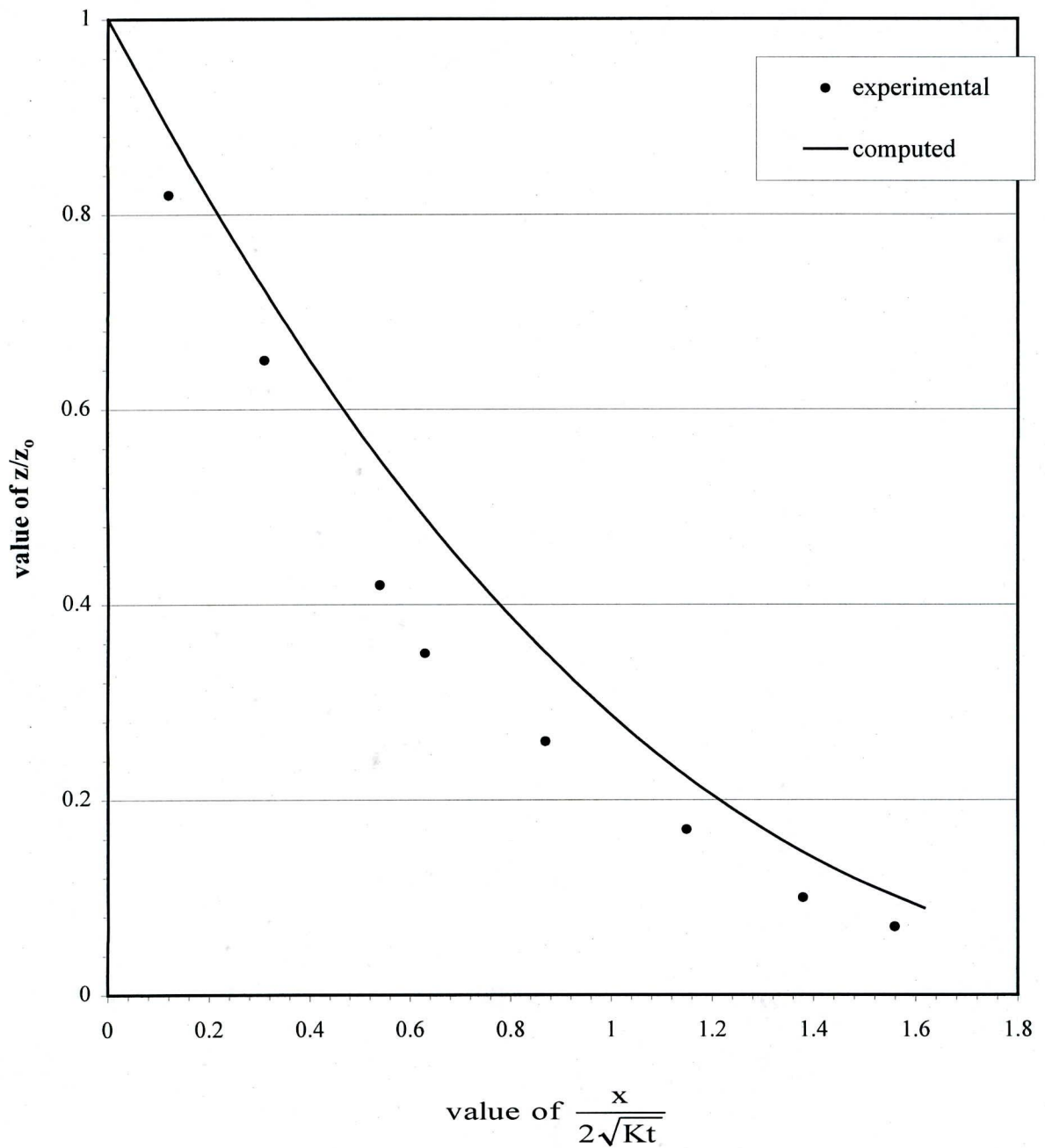


Fig. 5.25

Comparison of relative aggradation (test run 10)

RELATIVE AGGRADATION

test run 11

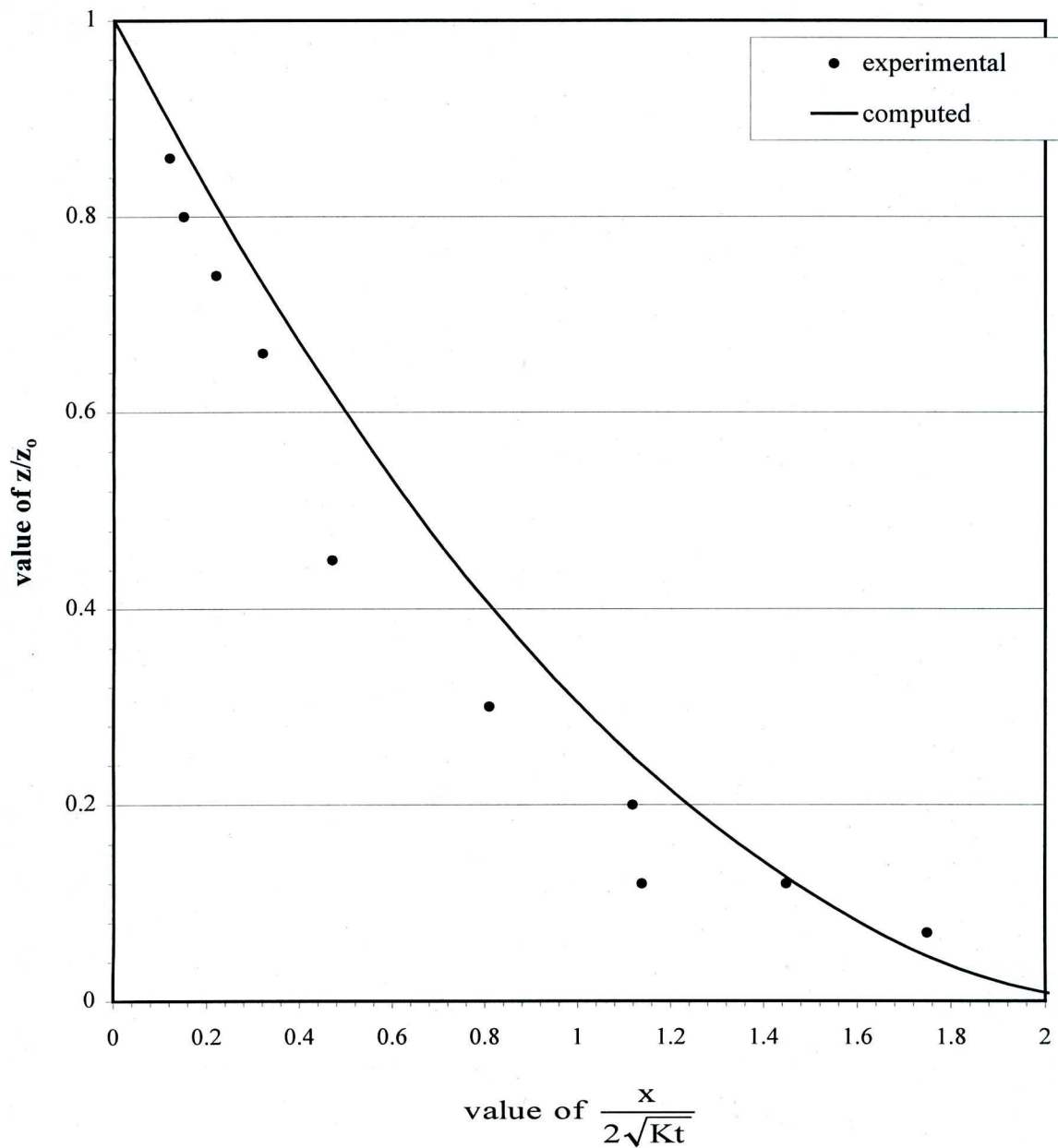


Fig. 5.26

Comparison of relative aggradation (test run 11)

Water level comparison at Bahadurabad from 93-94 to 95-96

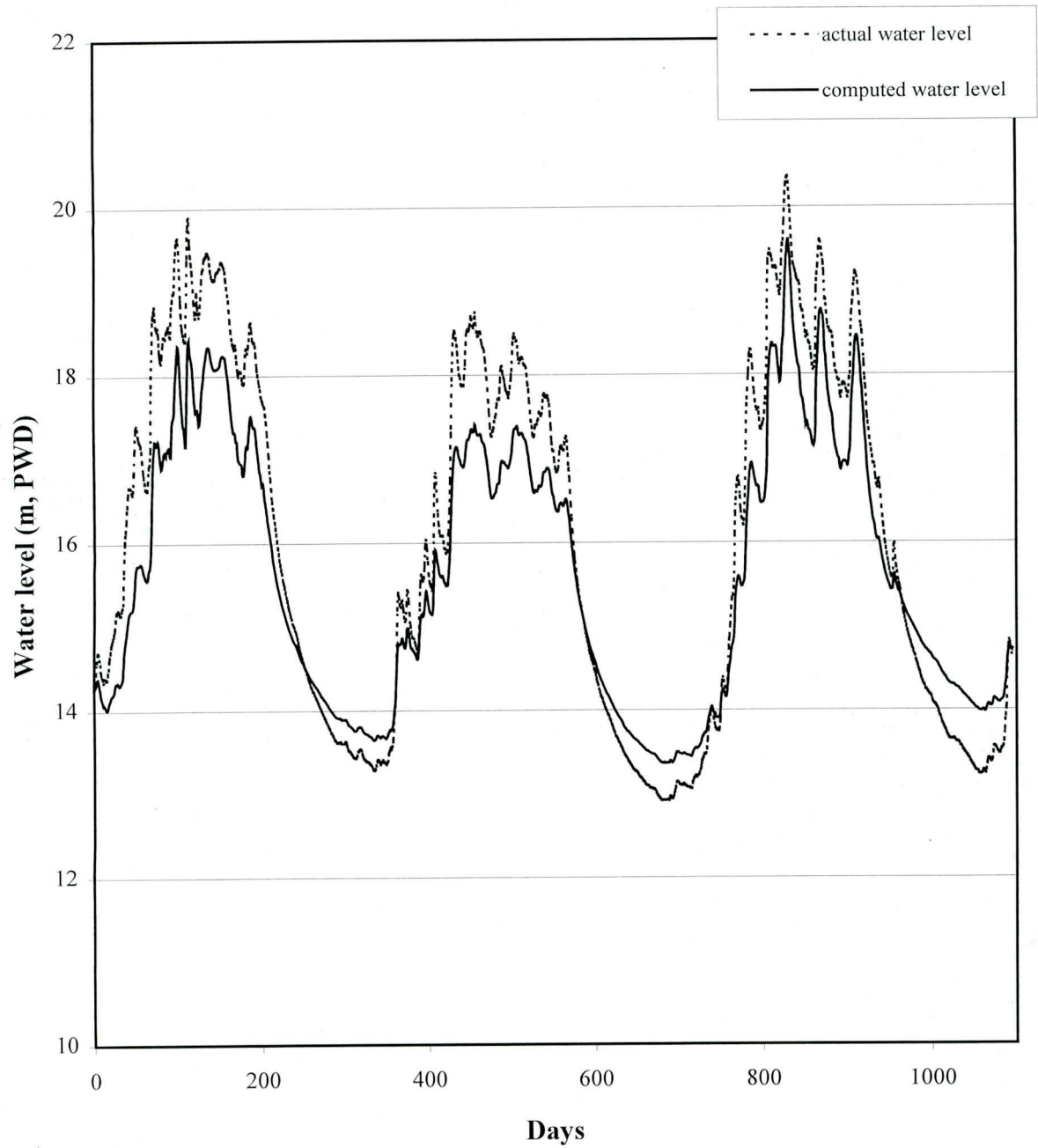


Fig. 5.27 Comparison of water level at Bahadurabad from 93-94 to 95-96

Variation of bed and water level at Bahadurabad from 93-94 to 95-96

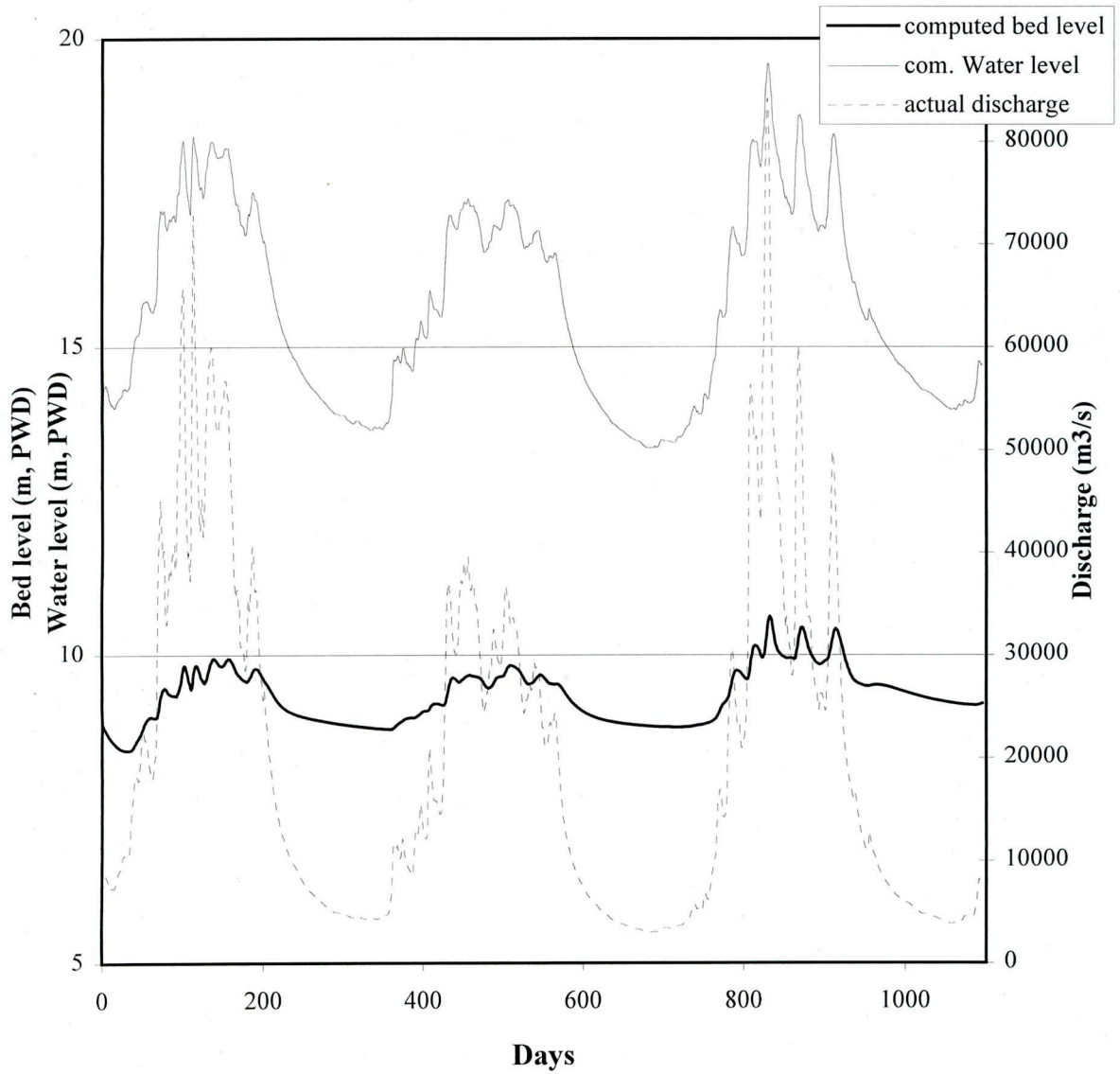


Fig. 5.28 Change of bed and water level with discharge at Bahadurabad from 93-94 to 95-96

Comparison of sediment discharge at Bahadurabad from 93-94 to 95-96

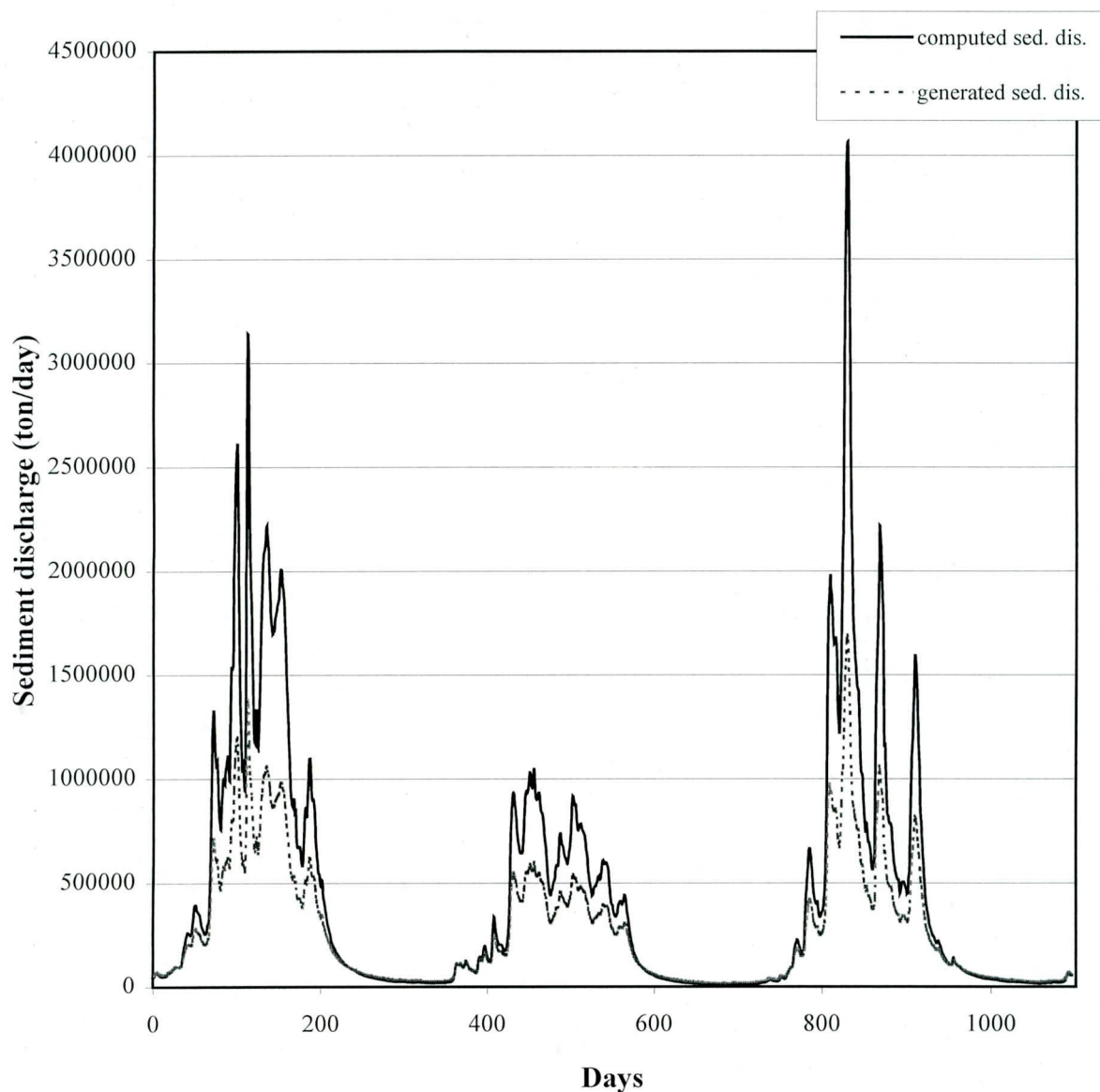


Fig. 5.29 Comparison of sediment discharge at Bahadurabad with generated sediment discharge from 93-94 to 95-96

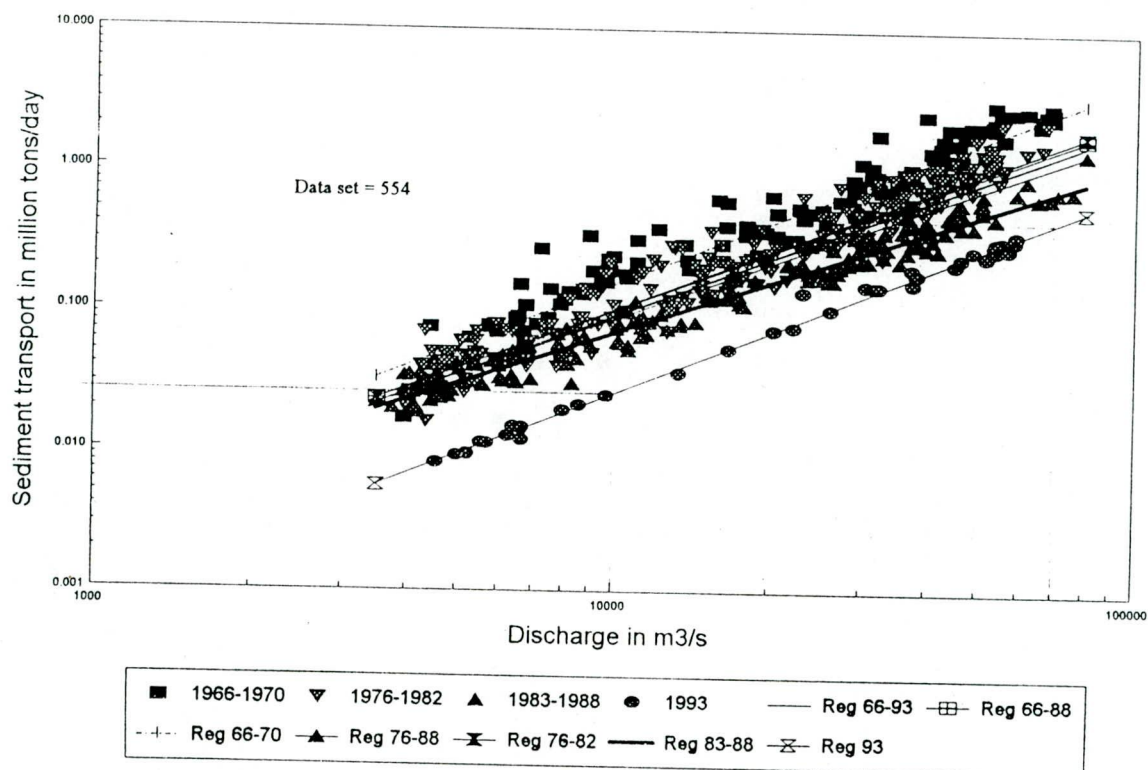


Fig. 5.30 The sediment discharge rating curve at Bahadurabad
(Source: River Survey Project FAP24, Special report No. 18, 1996)

Sirajganj rating curve
1994-95 data

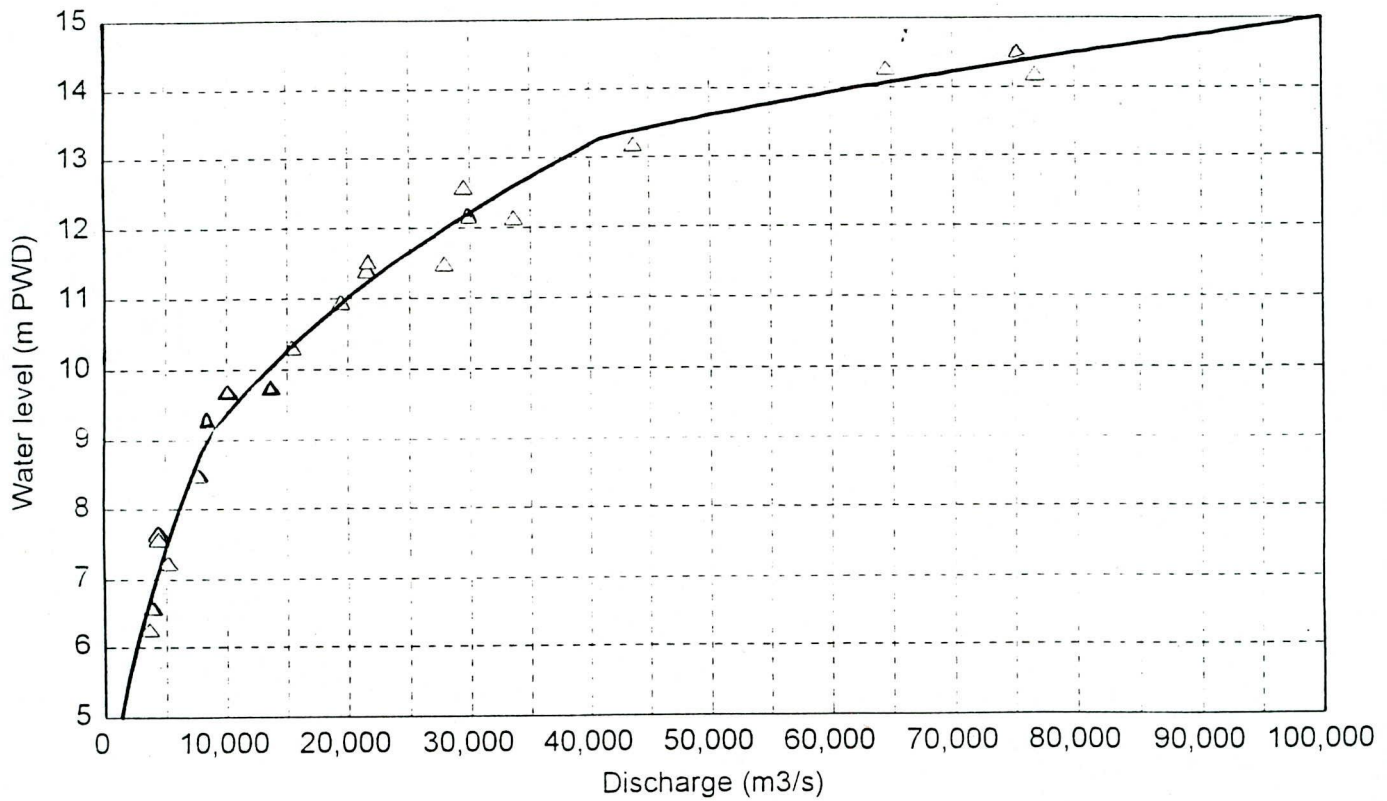


Fig. 5.31 The water discharge rating curve at Sirajganj developed from 1994-95 FAP24 survey data
(Source: Mathematical Morphological Model of the Jamuna River, Jamuna Bridge Site, First Forecast Report, February 1996, DHI/SWMC)

Comparison of discharge at Sirajgang
from 93-94 to 95-96

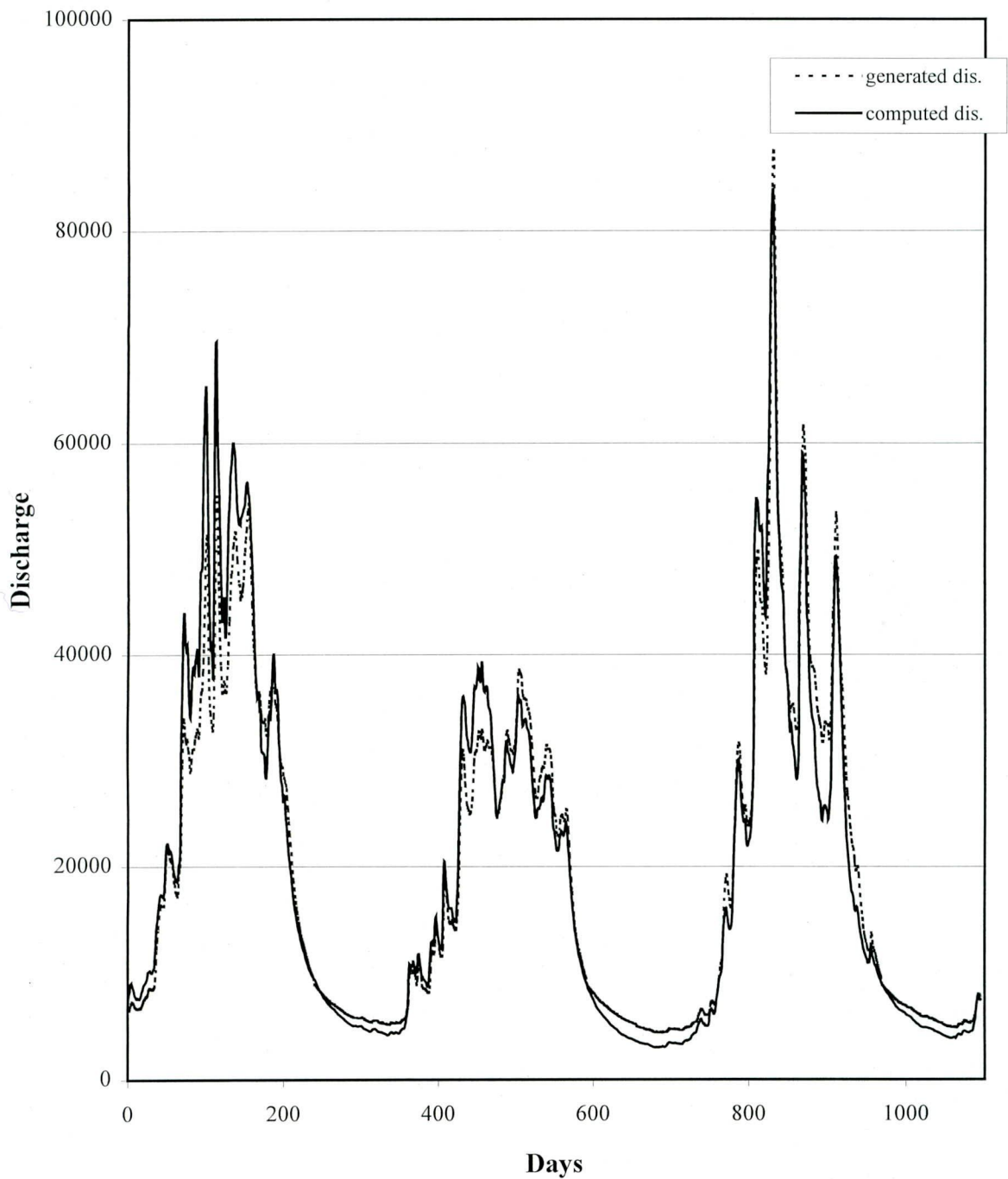


Fig. 5.32 Comparison of water discharge at Sirajgang with generated discharge from 93-94 to 95-96

Variation of bed level at Sirajgang
from 93-94 to 95-96

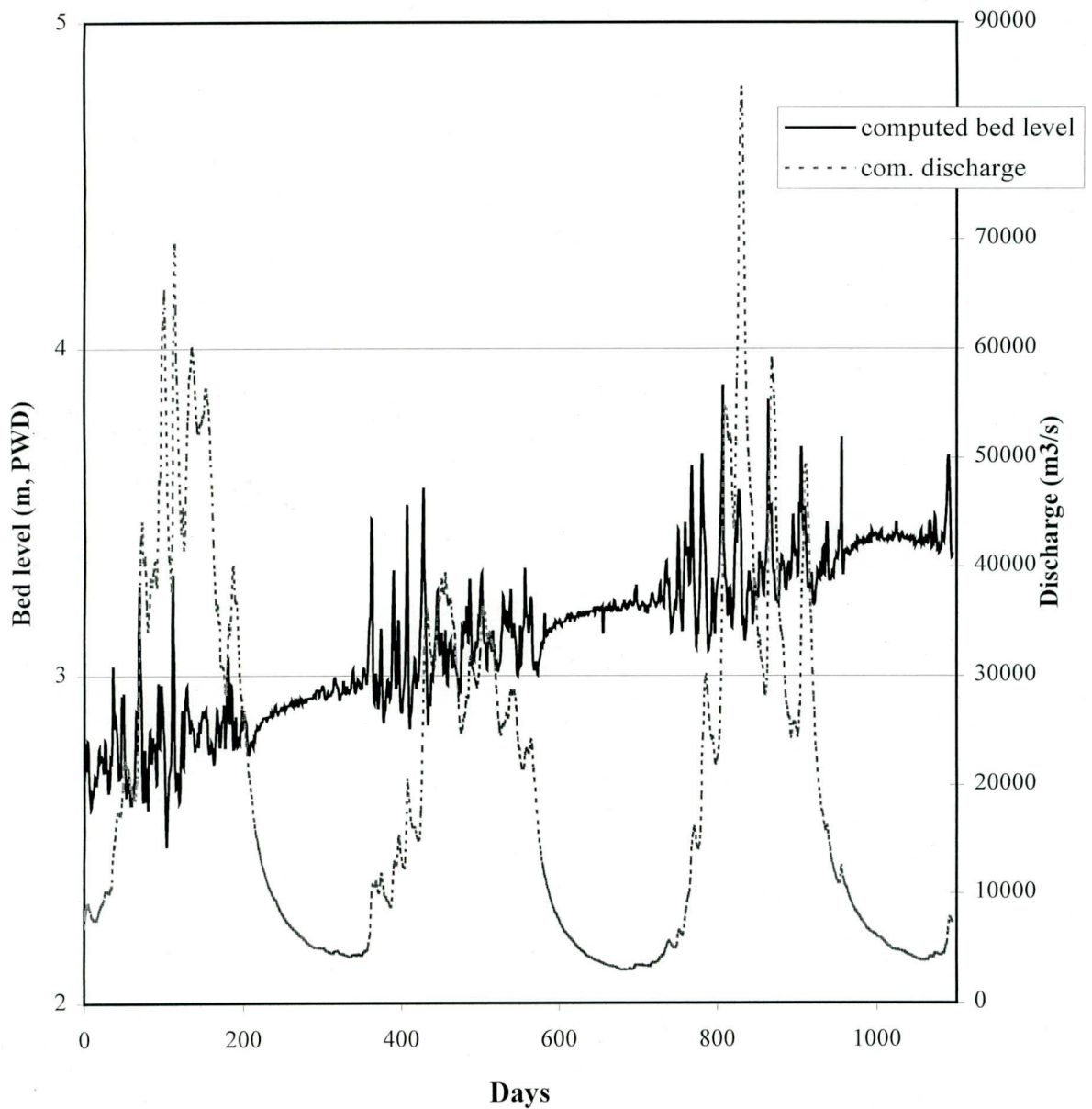


Fig. 5.33 Change of bed level with discharge at Sirajgang from 93-94 to 95-96

Variation of sediment discharge at Sirajgang from 93-94 to 95-96

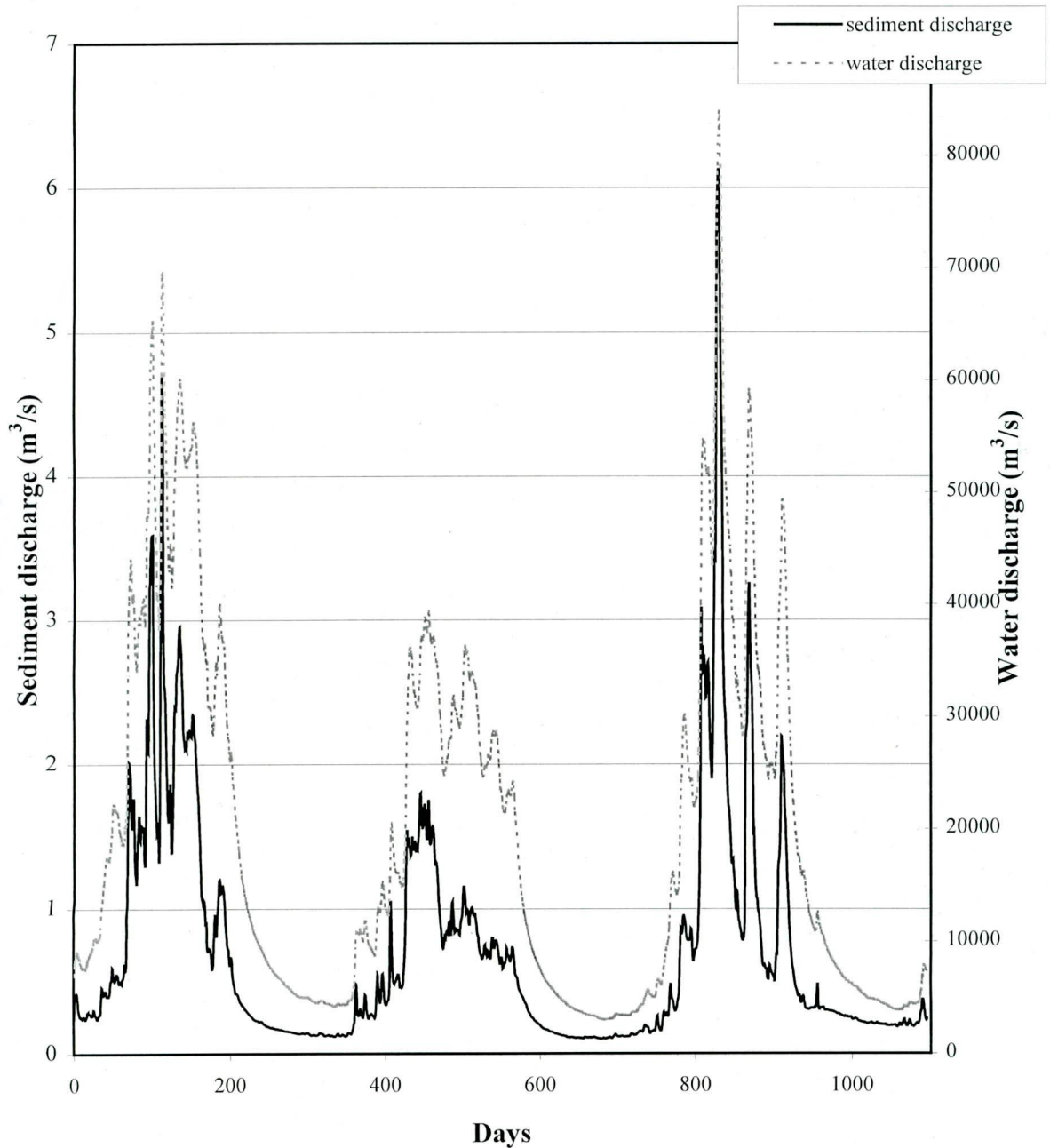


Fig. 5.34 Change of sediment discharge with water discharge at Sirajgang from 93-94 to 95-96

Change of bed level
from 93-94 to 95-96

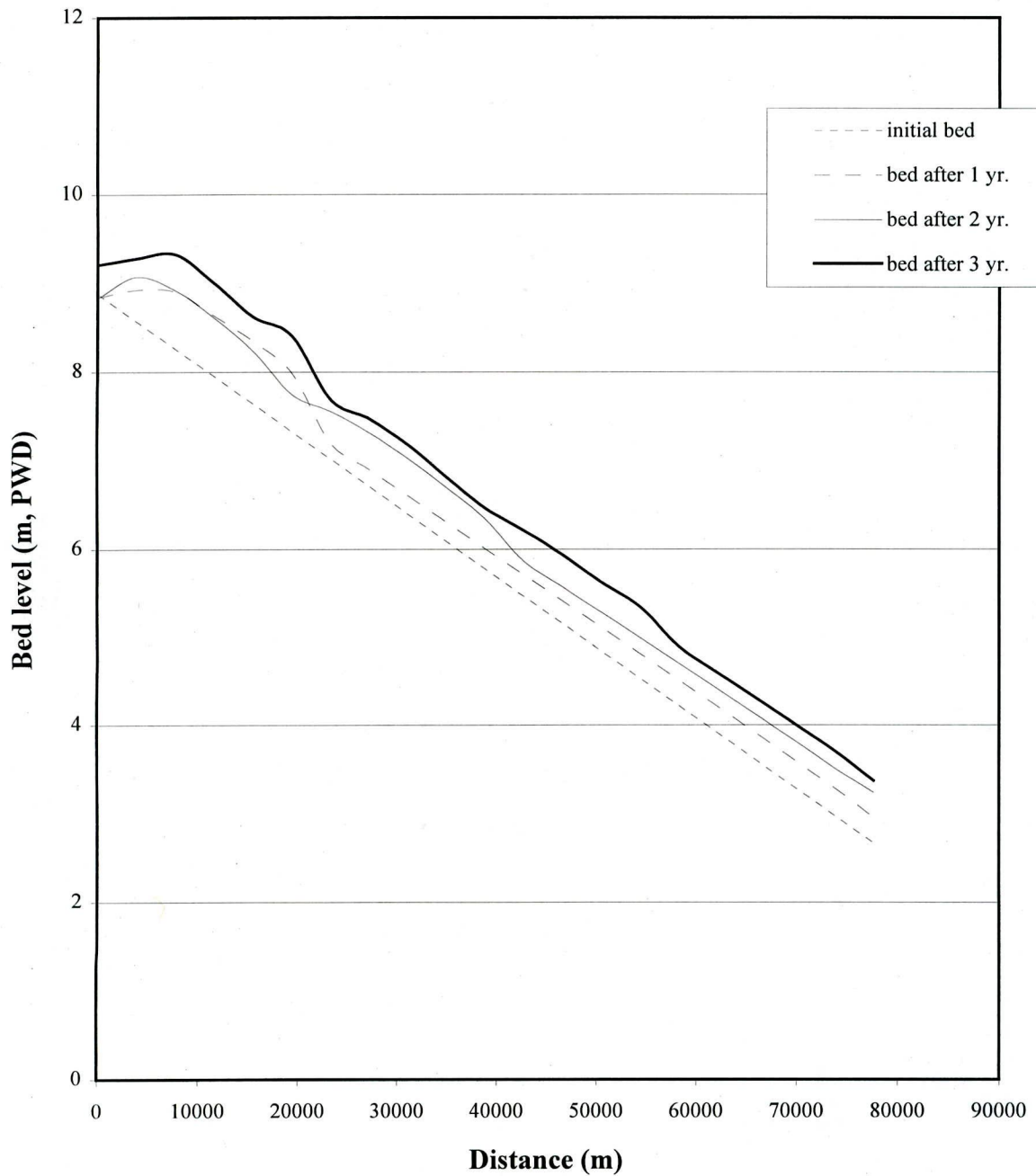


Fig. 5.35 Change in bed level in 3 years along the Jamuna river reach from Bahadurabad to Sirajgang

APPENDIX

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C  COMPUTATION OF AGGRADATION PROFILE DUE TO SEDIMENT OVERLOADING
C  BY MACCORMACK EXPLICIT SCHEME
C  CONSTANT FLOW DEPTH AND WATER DISCHARGE ALONG THE CHANNEL IS USED
C  AS INITIAL CONDITION.
C  CONSTANT WATER DISCHARGE AT UPSTREAM BOUNDARY AND CONSTANT FLOW
C  DEPTH AT DOWNSTREAM BOUNDARY
C  TRANSIENT CONDITIONS ARE PRODUCED BY SUPPLYING EXTRA SEDIMENT
C  LOAD AT UPSTREAM SECTION
C
C  ***** NOTATION *****
C
C  CHL = CHANNEL LENGTH
C  Cmn = MANNING'S COEFFICIENT
C  g = ACCELERATION DUE TO GRAVITY
C  NSEC = NUMBER OF CHANNEL SECTIONS
C  q = FRESH WATER DISCHARGE
C  qs = SEDIMENT DISCHARGE
C  So = CHANNEL BOTTOM SLOPE
C  TLAST = TIME FOR TRANSIENT FLOW COMPUTATIONS
C  y = FLOW DEPTH AT DIFFERENT SECTIONS
C
C  *****
C
C  DIMENSION y(60),q(60),qs(60),z(60),yp(60),qp(60),qsp(60),zp(60),
+ynew(60),qnew(60),znew(60),qsnew(60),zi(60)
C  DOUBLE PRECISION y,yp,yc,ynew,q,qp,qc,qnew,DT,
+      DTNEW,qs,qsp,qsc,qsnew,DTDx,exp
C  OPEN(5,FILE='ADMS11.IN')
C  OPEN(6,FILE='ADMS116.OUT')
C  READ(5,*) g,NSEC,TLAST
C  READ(5,*) CHL,Cmn,So,qo,yo,zo,p,a,b,Cn,sedratio
C      READ(5,*) SI
C  qso = a*(qo/yo)**b
C  delqs = sedratio*qso
C  exp = 10./3
C  T = 0
C
C  STEADY-STATE CONDITIONS
C
C  DX = CHL/NSEC
C  DT = Cn*DX/(qo/yo + SQRT(g*yo))
C  DTDx = DT/DX
C  I = 1
C  zi(I) = zo
C  NP = NSEC + 1
C  DO WHILE(I.LE.NP)
C      q(I) = qo
C      y(I) = yo
C      qs(I) = qso
C      I = I+1
C      zi(I) = zi(I-1) - So*DX
C  END DO
C  WRITE(6,15)CHL,p,sedratio,So,Cmn,qo,yo,NSEC,TLAST,Cn
C
C  COMPUTE TRANSIENT CONDITIONS
C

```



```

I = 1
DO WHILE(I.LE.NP)
    z(I) = zi(I)
    I = I+1
END DO

k = 0
DO WHILE(T.LE.TLAST)
c  print*, T, '***** PROGRAM RUNNING *****'
    If((k.eq.0).or.(k.eq.900).or.(k.eq.1800).or.(k.eq.2400).or.
3   (k.eq.3600).or.(k.eq.5400).or.(k.eq.7200).or.(k.eq.9000).or.
4   (k.eq.10800).or.(k.eq.14400).or.(k.eq.18000).or.
5   (k.eq.21600).or.(k.eq.28800).or.(k.eq.36000))Then
        PRINT*, T
        dc = b*qso/(3*So*(1-p))
        dcl = dc*(0.885/SI)**2
c        IF (k.eq.9000) zog = delqs*T/(1.13*(1-p)*SQRT(dc*T))
        WRITE(6,30) T
        WRITE(6,60)
        i = 1
        x = 0.0
        DO WHILE (I.LE.NP)
            IF((k.eq.900).or.(k.eq.1800).or.(k.eq.2400).or.(k.eq.3600)
+           .or.(k.eq.5400).or.(k.eq.7200).or.(k.eq.9000).or.
+           (k.eq.10800).or.(k.eq.14400).or.(k.eq.18000).or.
+           (k.eq.21600).or.(k.eq.28800).or.(k.eq.36000)) Then
c                print*,zog,dc
                eta = x/(2*SQRT(dc*T))
                etal = x/(2*SQRT(dcl*T))
                delz = z(I) - zi(I)
                zog = z(I) - zi(I)
                coeff1 = delz/zog
                w1 = y(I) + z(I)
                WRITE(6,40) x,y(I),q(I),z(I),w1,eta,coeff1,etal,coeff1
            ELSE
                w1 = y(I) + z(I)
                WRITE(6,50) x,y(I),q(I),z(I),w1
            ENDIF
            i = i+1
            x = x+DX
        END DO
        Else
        Endif
        T = T+DT
C
C  INTERIOR NODES
C
C  PREDICTOR STEP
C
    I = 1
    DO WHILE(I.LE.NSEC)
        yp(I) = y(I) - DTDX*(q(I+1)-q(I))
        qp(I) = q(I) - DTDX*(q(I+1)*ABS(q(I+1))/y(I+1) -
+        q(I)*ABS(q(I))/y(I) + g/2*(y(I+1)**2-y(I)**2)) -
+        g*y(I)*DTDX*(z(I+1)-z(I)) -
+        g*y(I)*DT*(q(I)*ABS(q(I)))*Cmn**2/(y(I)**exp)
        qsp(I) = a*(qp(I)/yp(I))**b
        zp(I) = z(I) + 1./(1-p)*(qs(I)*y(I)/q(I) - qsp(I)*yp(I)/qp(I))
+        - DTDX/(1-p)*(qs(I+1)-qs(I))
    
```

```

      I = I+1
      END DO
C
C   CORRECTOR STEP
C
      I = 2
      DO WHILE(I.LE.NSEC)
        yc = yp(I) - DTDX*(qp(I)-qp(I-1))
        qc = qp(I) - DTDX*(qp(I)*ABS(qp(I))/yp(I) -
+      qp(I-1)*ABS(qp(I-1))/yp(I-1) + g/2*(yp(I)**2-yp(I-1)**2))
+      - g*yp(I)*DTDX*(zp(I)-zp(I-1)) -
+      g*yp(I)*DT*(qp(I)*ABS(qp(I))*Cmn**2/(yp(I)**exp)
        qsc = a*(qc/yc)**b
        zc = zp(I) + 1./(1-p)*(qsp(I)*yp(I)/qp(I) - qsc*yc/qc) -
+      DTDX/(1-p)*(qsp(I)-qsp(I-1))
        ynew(I) = (y(I) + yc)*0.5
        qnew(I) = (q(I) + qc)*0.5
        znew(I) = (z(I) + zc)*0.5
        qsnew(I) = a*(qnew(I)/ynew(I))**b
        I = I+1
      END DO
C
C   UPSTREAM END
C
        qnew(1) = qo
        ynew(1) = 2*ynew(2) - ynew(3)
        qsnew(1) = a*(qnew(1)/ynew(1))**b
        znew(1) = z(1) + 1./(1-p)*((qs(1)*y(1)/q(1) - qsnew(1)*ynew(1)/
+      qnew(1)) + DTDX*(qso + delqs - qs(1)))
C
C   DOWNSTREAM END
C
        ynew(NP) = yo
        qnew(NP) = 2*qnew(NSEC) - qnew(NSEC-1)
        znew(NP) = 2*znew(NSEC) - znew(NSEC-1)
        qsnew(NP) = a*(qnew(NP)/ynew(NP))**b
C
C   CHECK FOR STABILITY
C
        J = 1
        DO WHILE(J.LE.NP)
          DTNEW = Cn*DX/(qnew(J)/ynew(J) + SQRT(g*ynew(J)))
          IF (DTNEW.LE.DT) DT = DTNEW
          J = J+1
        END DO
        DTDX = DT/DX
C
C   REDUCE DT FOR STABILITY AND RECALCULATE
C
        I = 1
        DO WHILE(I.LE.NP)
          q(I) = qnew(I)
          Y(I) = ynew(I)
          z(I) = znew(I)
          qs(I) = qsnew(I)
          I = I+1
        END DO
        kk = T

```

```

If(kk.EQ.k)Then
  k = kk+1
Else
  k = kk
Endif
END DO
15  FORMAT(5X,'MACCORMACK SCHEME TO COMPUTE AGGRADATION-
DEGRADATION',
1  /5X,'CHANNEL LENGTH = ',F7.2,/5X,'CHANNEL BEDSOIL POROSITY = ',
2  F5.2,/5X,'RATIO OF SEDIMENT INCREMENT = ',F5.2,/5X,
3  'CHANNEL BOTTOM SLOPE = ',F10.6,/5X,'MANNING"S n = ',F10.5,/5X,
4  'INITIAL STEADY STATE DISCHARGE = ',F6.3,/5X,
5  'INITIAL UNIFORM FLOW DEPTH = ',F5.3,/5X,
6  'NUMBER OF CHANNEL SECTIONS = ',I3,/5X,
7  'TIME FOR WHICH TRANSIENTS WILL BE COMPUTED = ',F8.2,/5X,
8  'COURANT NUMBER = ',F4.2)
30  FORMAT(/1X,'T= ',F8.2,' Seconds')
40  FORMAT(F12.2,8F8.3)
50  FORMAT(F12.2,4F8.3)
60  FORMAT(9x,'x',7x,'y',7x,'q',7x,'z',6x,'wl',5x,
+      'eta',5x,'z/zo',5x,'etal',5x,'z/zo')
Close(5)
Close(6)
STOP
END

```

