A NEW GRAPHIC TECHNIQUE FOR AGE BASED PREVENTIVE REPLACEMENT POLICY DETERMINATION

A PROJECT THESIS

BY

EMAM UDDIN AHMED

বাংলাদেশ প্রকৌশল বিশ্ববিদ্যালয়
ঢাকা

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CERTIFICATE

This is to certify that this work has been done by me and it has not been submitted elsewhere for the award of any degree or diploma or for publication.

Counter signed
by

Supervisor

Signature of the candidate

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SUMMARY

In the present work, cost-optimal age-based replacement model has been studied because of its importance in the application of preventive maintenance. The model has been solved by a computer package written in FORTRAN language. Optimal replacement ages have been determined for a wide range of cost ratios (i.e. ratio of failure replacement cost to preventive replacement cost) from 1.1 to 100 and for a wide range of shape parameters (β values) from 1.1 to 5.0. Total cost of optimal replacement policy in multiples of failure replacement cost has also been determined for each value of cost ratio and β . Results have been presented in both tabular and graphical forms. Two sets of graphs have been produced, in the semi-log plotting; one for optimal age determination and the other for determination of associated replacement cost. In the earlier set of curves, the so-called Z-curves are plotted; where Z indicates the closeness of optimal age to the characteristic life. A comparison of the present solution with that of Glasser has also been made. Following Glasser, Weibull failure distribution has been considered to express failure statistic because of the generality of this distribution.

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NOTATIONS

C_f	- the cost of failure replacement i.e. cost of replacement after failure
C_p	- the cost of preventive replacement i.e. cost of replacement before failure
$C(t_p)$	- the total expected cost per unit time, for preventive replacement at time t_p
$f(t)$	- the probability density function (p.d.f) of the failure times of the equipment
k	- the cost ratio (C_f/C_p) i.e ratio of failure replacement cost to preventive replacement cost
$M(t_p)$	- the mean time to failure when preventive replacement occurs at time t_p (or length of the failure cycle at time t_p)
$r(t_p)$	- the hazard rate (or, instantaneous failure rate or, force of mortality) at time t_p
$R(t_p)$	- the reliability of the equipment in preventive replacement time t_p
t	- failure times
t_p	- the preventive replacement time or, planned replacement age or, specified age
t_p^*	- optimal replacement age
V	- ratio of mean life to standard deviation units, which is the average service life of the component in standard deviation units

β - shape parameter

η - scale parameter

γ - location parameter

ρ - standardized cost ratio

μ - mean failure time

σ - standard deviation unit

Γ - evaluation of Gamma function

CHAPTER 1

INTRODUCTION

CHAPTER-1

INTRODUCTION

1.1 Introduction to Replacement Problems:

Preventive maintenance is an important part of maintenance management. It deals with efficient utilization of equipment with some risk of failure. Preventive maintenance refers to the planned replacement of pieces of equipment or parts that may fail in operation unless replacement is made in time. Any type of equipment may break down during the operation due to mechanical failure or any other cause. Apart from break downs, equipments or machines tend to wear or deteriorate with the increase in time and use. This condition may decrease the efficiency and lower the quality of the product. On the other hand it has two important implications.

1) The increasing cost of the maintenance necessary to obtain continuing reliability of the equipment.

2) The obsolescence of the equipment.

Replacement problems are a particular class of maintenance problems, which in general, include inspection, overhaul and repair, organizational structure, reliability, scheduling and sequencing besides the replacement ones. Usually replacement problems can be classified as either deterministic or probabilistic (stochastic). In the case of deterministic problems, the timing and outcome of replacement policies are assumed to be known with certainty.

One typical and perhaps most common problem under this category is finding the economic life of capital equipment. Here, the equipment is not subject to failure but the operating cost increases with its use. To reduce this operating cost, a replacement is to be performed. The 'best' time at which the total cost incurred is minimized, is called the economic life. It is assumed that after the replacement, the trend in operating costs is known in the sense that the replacement action returns the equipment to the as new condition and continues to provide the same services as the replaced equipment. If this assumption holds then various costs used in the analysis are always invariant between replacements. Of course, this assumption does not hold where technological improvement of the equipment is taken into account in the model.

In the case of probabilistic problems, the timing and outcome of the replacement action can depend on chance. So, maintenance of stochastically failing equipment involves problems of decision making under uncertainty. Two sources of uncertainty are distinguished. First, it is impossible to predict with certainty when a failure will occur or more generally, when the transition from one state of the equipment to another will occur. Second, it may be impossible to determine the actual state of the equipment with any accuracy unless some form of inspection is taken. During operation the equipment under consideration is assumed to occupy one of several states, the two extreme states being 'new (or good)' and 'failed'. These two boundary states enclose a set of intermediate states that denote different degrees of deterioration. The movement from state to state is governed by a probability mechanism whose law may be completely known, partially known or unknown, by the decision-maker. In the simplest situation, of course, the equipment may be described as being good or failed, and the corresponding probability law may be described by the distribution of time between completion of the replacement action and failure. The time to failure is a random variable whose distribution may be termed the equipment's failure distribution.

Corresponding to these sources of uncertainty mentioned above, are two distinct models for stochastically failing equipment: 'preventive' maintenance models in which the state of the equipment is always known with certainty; and 'preparedness' maintenance models in which the state of the equipment (at least for some of its components) is not known with certainty unless some definite maintenance action is taken. In this work, only the first type of model has been considered and so the subsequent discussion will be related to this.

With the preventive maintenance model, the assumption is that the equipment operates continuously until it has failed. Continuous operation provides assurance that the actual state of the equipment is known with certainty.

Alternatively, it may be assumed that the equipment is kept in storage for its later use (particularly when emergency arises) but is under continuous surveillance. It makes it possible to determine the actual state of the equipment without any further maintenance action.

The maintenance action contemplated for the preventive maintenance model is replacing a given piece of equipment or component with another of the same type which provides the same services. So, like the deterministic model, the relevant pieces of equipment are assumed identical at the time of replacement.

In developing a replacement policy, the interest lies mainly in determining a sequence of times at which the replacement actions should take place, and the optimal replacement policies are the ones which maximize or minimize some criterion, such as profit, total cost, down time, etc.

For the justification of preventive replacement actions, two important conditions must be satisfied:

a) The total cost of replacement must be greater after a failure than before, where 'cost' is the appropriate criterion. If some other criterion, such as down time is taken as the appropriate one, then that criterion is substituted in place of cost. However, this sort of situation may be caused by a greater loss of production since replacement after failure is unplanned, or failure of one piece of plant may cause damage to other equipment.

b) Preventive replacement may be adantageous only if the failure rate is increasing. As an illustration of this point, a case can be considered where the failure rate of the equipment is constant. By the definition of failure rate, replacement before failure does not affect the probability that the equipment will fail in the next instant, given that it is good now. Since replacement costs are positive, it will never be advisable to do preventive replacement, i.e. replacing the equipment before failure. Obviously, when equipment fails according to the hyperexponential distribution its failure rate is decreasing and preventive replacement should not be applied in this case as well.

Basically, there are several types of preventive maintenance policies of interest; One is the periodic policy (age-replacement) and another is the sequential policy. A periodic policy is one in which the equipment is replaced as soon as it fails, or as soon as it reaches a specified age, whichever occurs first. This is the simplest and best known of the periodic maintenance policies and is appropriate for an investment process with an 'unlimited' life time. A sequential policy, on the other hand, is always more advantageous when the life time of the investment process is 'finite'. It differs from the periodic policy in that the mandatory replacement age is re-calculated after each replacement. Applications of sequential policies include situations in which the equipment is subject to rapid technological change and therefore possesses a relatively short life time.

In case of maintenance policies for equipment with several parts, the analysis is the same as for the single part equipment only if parts are stochastically and economically independent. In this case maintenance decisions are also independent and the correct procedure is to assign a separate periodic or sequential policy to each part. But, if the failure distributions of the several parts are not stochastically independent or if the cost of replacing several parts jointly is less than the cost of their respective separate replacements, then decisions of replacement will not be independent. The condition of economic dependence reflects the presence of economies of scale in maintenance. Under either condition, it may be advantageous for the maintenance decisions for one part to depend on the states of the remaining parts of the system. This last described replacement policy, in which all items or a group of items are replaced together at periodic intervals, whether or not they have failed since the last replacement, is called a 'Block replacement'. The classic example of this sort of situation is that of maintenance of street lamps. A block replacement may not be optimal in most situations. The policy is very easy to administer however, since no information is required concerning the age of individual items, as in the periodic preventive maintenance policy for a single item. A natural extension of this type of block replacement policy is in the development of a replacement policy for a single item where preventive replacements occur at fixed intervals of time, and failure replacements occur when necessary.

The nature of the replacement policies has so far been discussed in general. It should be mentioned here that certain procedures are already available for the solution of the replacement models mentioned earlier. In case of preventive replacement problems, first the failure statistics have to be analysed by using fairly standard statistical techniques and the life time distribution is then estimated in terms of population parameters, say, the mean life and standard deviation. Then, with these failure characteristics and the ratio of the cost of a failure replacement to that of a preventive replacement, the optimal preventive replacement time to minimize total cost per unit time can be calculated. Now, the usual procedures for doing this are:

a) Using mathematical models that are well developed

b) Using a graphical solution procedure or,

c) Using a computer package.

1.2 Objective of the Study:

The objective of this research are the followings:

1) To study age-based periodic replacement model for individual item.

2) To derive a mathematical model to determine optimal preventive replacement age so as to minimize replacement cost per unit time.

3) To determine optimal replacement ages for a range of failure to preventive replacement cost ratios and for a range of failure distribution parameter values.

4) To present the resulting policies in a graphical form so that replacement interval can be readily determined.

5) To compare the graphical solution with other similar existing solutions.

CHAPTER 2

LITERATURE REVIEW

CHAPTER 2

LITERATURE REVIEW

2.1 Literature Survey:

Literature involving the theory of equipment maintenance is now abundant. In many works, optimal maintenance policies have been found and characterized for a variety of situations. Deterministic problems were the earliest ones to draw the attention of many authors on the theory of maintenance but, within the past few years a substantial body of literature has developed for probabilistic maintenance problems as well. Impetus for the development of a theory of maintenance for the latter situations has come largely from the practical problems of maintaining complex electronic equipment, i.e. modern aircraft, missiles, spacecraft, communication equipments,computers, etc. The main reason is that operational requirements can be achieved only by the observance of sophisticated maintenance policies. For the most part, these stringent operational requirements are also economically justified. However, application of these maintenance theories have now been extended to manufacturing industries.

Stochastic problems in maintenance are closely connected with the reliability area. So, many early literatures which considered the mathematical theory of reliability are also relevant to replacement problems. Khintchine, Palm, Erlang and many other authors published considerable works in this field. Weiss (1), in a series of reports, considered the effects on system reliability and on maintenance costs of both age replacement and random age replacement policies. In one of the reports, he was particularly interested in the time to first system failure. He proposed that an age replacement policy would be beneficial if the expected time to an inservice failure were a decreasing function of the age of replacement. Herd (2) also considered the question of the beneficiality of a replacement policy. He based his considerations solely on the failure rate. In his opinion, replacement should be considered if the failure rate is increasing. A detailed study by Flehinger (3) considered the expected number of failures and the expected number of planned replacements in a specified interval.

Takacs (4) studied a model considering the time for replacement as non-negligible and the stochastic process as simple 'on' and 'off' state. An expository paper directed towards reliability applications appeared in Operations Research by Barlow and Hunter (5).

It seems that age replacement for an infinite time span has received most attention in the literature. Morse (6) shows how to determine the replacement interval minimizing expected cost per unit time, when such an optimal interval exists. The derivation of an optimum age replacement interval corresponding to a finite time span is basically a much more difficult problem. Barlow and Proschan (7) prove the existence of such an optimal policy. A treatment of the comparison of replacement policies have been studied by Barlow and Proschan (8,9).

Renewal theory is a very important subject in the study of preventive maintenance. Most of the general theory may be found in the expository paper by Smith (10). The more recent results based on an underlying increasing failure rate distribution may be found in Barlow, Marshal and Proschan (11) and Barlow and Proschan (8).

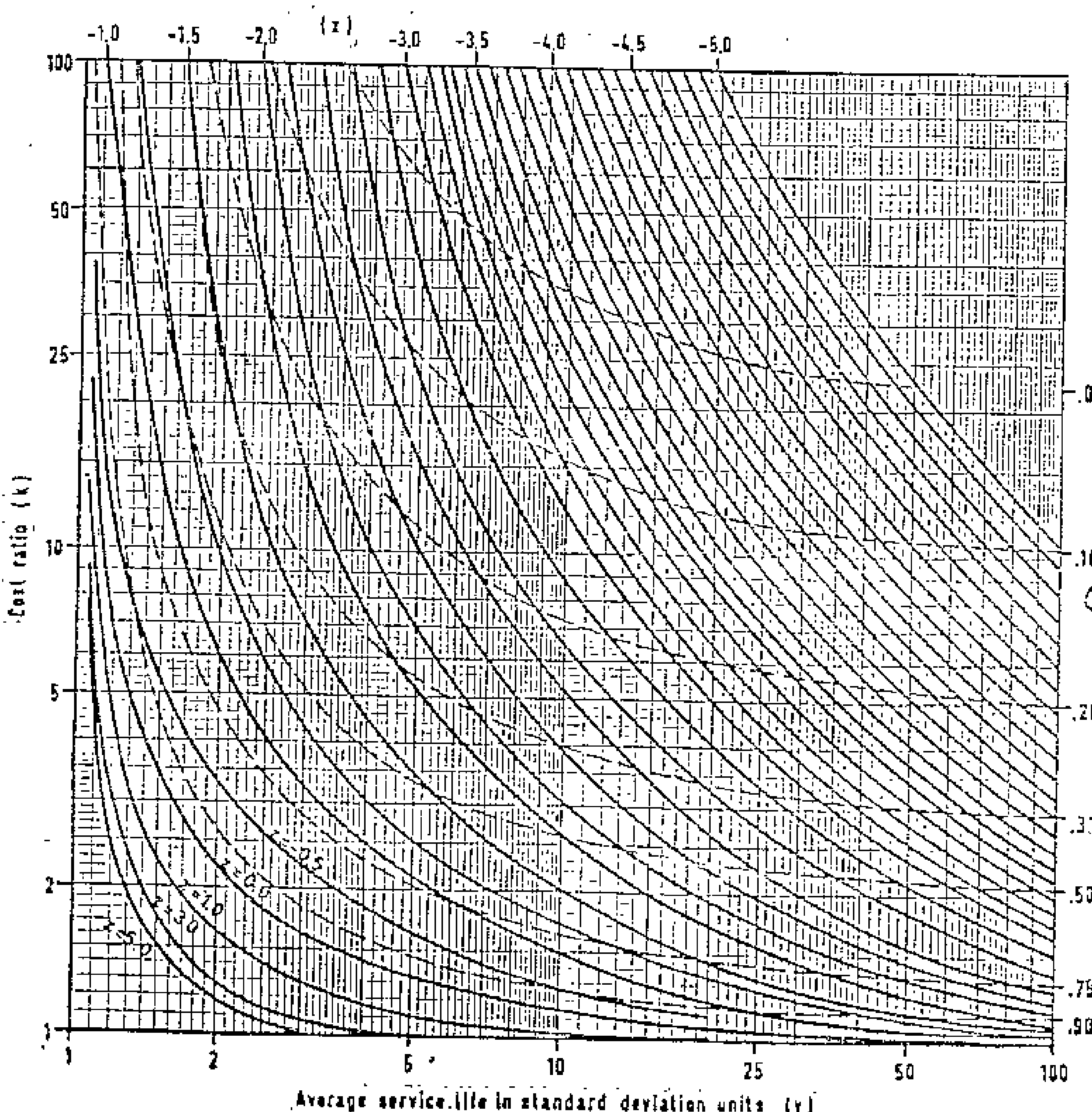
The preventive maintenance models have also been studied, in general, by Jorgenson, McCall, and Radner (12) and Jardine (13). Barlow, Proschan, and Hunter (9) also discussed various theoretical aspects in their book.

2.2 Glasser's Graph:

It has been mentioned in the objective of this research that a set of graphs will be constructed, so that maintenance intervals can be readily determined. From the literature, one such graphical presentation could be obtained and this was developed by Glasser (14).

Glasser's solution is based on reading off the graphs (Fig. 2.1) the value of Z (the deviation of optimal replacement age from the mean life expressed in standard deviation unit) corresponding to the intersection of a horizontal line from K (cost ratio i.e ratio of failure replacement cost to preventive replacement cost) and a vertical line V

Fig. 2.1 Optimal policies under age replacement: Weibull Distribution



(ratio of mean life to standard deviation unit, which is the average service life of the component in standard deviation unit). The optimal preventive replacement time (t_p) is obtained from the equation

$$t_p = \text{Mean } (\mu) + Z \times \text{standard deviation } (\sigma)$$

i.e., $t_p = \mu + Z.\ \sigma$ (2.2.1)

From Glasser's graph, the standardized cost ratio(ρ) can also be determined. This gives the cost of the optimal policy as a decimal fraction of the cost of a 'replace on failure only' policy.

It should be pointed out that Glasser set one of the requirements prior to using the graphs that the failure distribution of an item could be desclibed adequately by Weibull distribution. The Weibull distribution is used to accommodate a large number of failure characteristics of equipments. The distribution is characterized by two or three parameters - shape parameter (β); scale parameter (η) and location parameter (γ). β and η are some times called operating characteristics. For failures, the probability density function f(t) for Weibull distribution is given by

$$f(t) = \frac{\beta}{\eta} \cdot \left(\frac{t}{\eta}\right)^{\beta-1} . \exp\left\{ - \left(\frac{t}{\eta} \right)^{\beta} \right\} \text{ for } t \geq 0 \qquad (2.2.2)$$

When $\beta = 1$, the Weibull distribution is equivalent to negative exponential distribution and then the failure rate is constant with time. When $\beta > 1$ the failure is increasing with time and the Weibull distribution represents a case of both random and wearout failures. When $\beta = 3.4$, it approximates normal distribution. When $\beta < 1$ the failure rate is decreasing with time. When $\beta = 0.5$ it approximates Hyper exponential failure distribution. Weibull (15) used this to describe fatigue failure, Kao (16) for vacuum tube failure and Lieblein and Zelen (17) for ball bearing failure.

Before using Glasser's graph for determining optimal replacement age, one must first determine mean (μ) and standard deviation (σ), having obtained failure distribution parameters (β and η) from the item failure history. In terms of β and η, mean and standard deviation can be expressed as

$$\left.\begin{aligned} \mu &= \eta \, . \, \Gamma\left(1 + \frac{1}{\beta}\right) \\ \sigma &= \left[\eta^2 . \Gamma\left(1 + \frac{2}{\beta}\right) - \mu^2 \right]^{\frac{1}{2}} \end{aligned}\right\} \quad \ldots\ldots\ldots (2.2.3)$$

where Γ indicates Gamma function. This entails the use of a Gamma function table. Alternatively, one may evaluate Gamma function by the following approximation (18), which gives near correct values within 0.2 - 0.3 per cent accuracy.

$$\Gamma(x) = x^x . e^{-x} . \left\{\frac{2\pi}{x}\left(1 + \frac{1}{12x} + \frac{1}{288x^2} - \frac{139}{51840x^3} - \frac{571}{2488320x^4}\right)\right\}^{\frac{1}{2}} \quad \ldots(2.2.4)$$

So, it can be realized that Glasser's graph can be a time consuming process.

Another interesting point to note that Glasser plotted the graphs upto a minimum value of 100 for V. Relating V to β , it can be seen that V is a function of β only.

$$V = \frac{\mu}{\sigma} = \frac{\Gamma(1 + \frac{1}{\beta})}{\left[\Gamma(1+\frac{2}{\beta}) - \{\Gamma(1+\frac{1}{\beta})\}^2\right]^{\frac{1}{2}}} \qquad (2.2.5)$$

For all practical purposes, β value can at the most be around 5.0. Special Weibull probability papers also cater for a range of β values, the maximum being 5.0. From equation (2.2.5), it can be seen that for a β of 5.0, V will around 4.35, which is far lower than 100.0. So a wide zone in Glasser's graph (Fig. 2.1) will rarely be used.

CHAPTER 3

FORMULATION OF MATHEMATICAL MODEL

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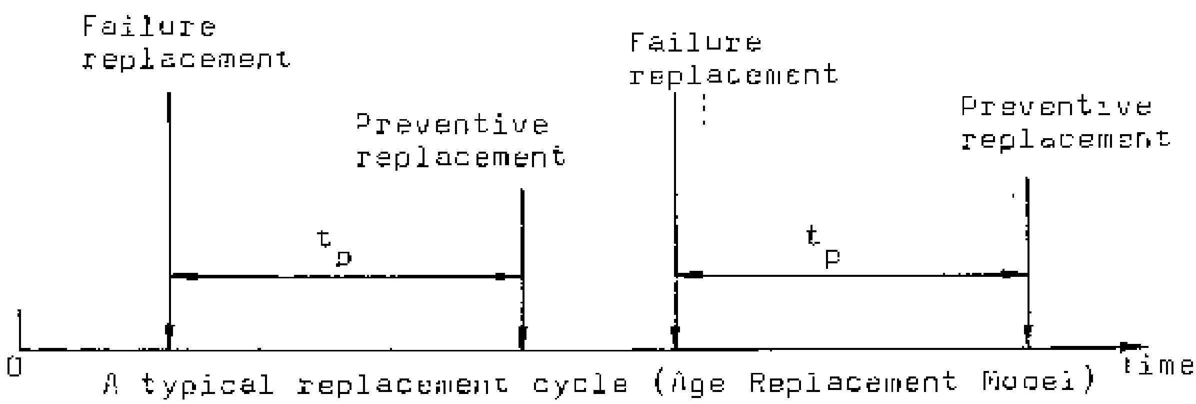
FORMULATION OF MATHEMATICAL MODEL

3.1 Model for Age-Based Preventive Replacement:

An age-based preventive replacement policy specifies that if a unit survives to a specified age then it is replaced, with additional provision that if a unit fails prior to attaining that specified age then it is immediately replaced. Age-replacement is logical if the cost of replacement before failure effects some saving. This would indicate that there is an additional cost to a failure during operation over and above the cost of a new unit.

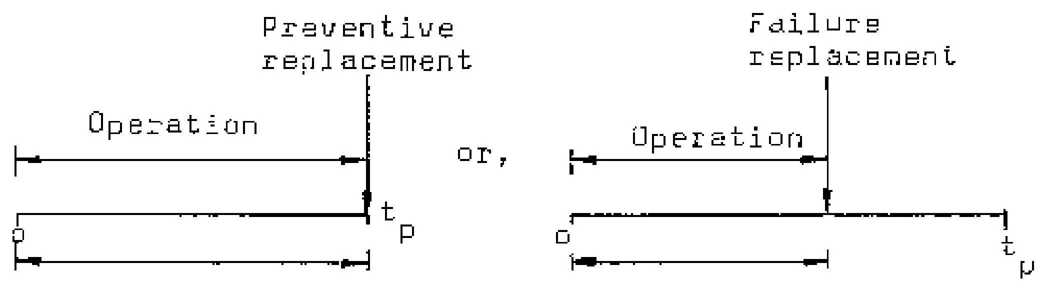
To balance the cost of the preventive replacements against their benefits necessitates determination of the optimal preventive replacement age for the equipment to minimize the total expected cost of replacements per unit time. Age-based preventive replacement model is well developed and a detailed coverage has been given by Jardine (13). This policy is expressed in Fig. 3.1.



A typical replacement cycle (Age Replacement Model)

Fig. 3.1.1

From the figure it can be seen that the Aage-based preventive replacement policy incorporates two possible cycles of operation: one cycle being determined when the equipment reaches its planned replacement age (t_p); and the other is due to a failure occuring before the planned replacement time. These two possible cycles are illustrated in Fig. 3.2.



(a) Preventive replacement cycle

(b) Failure replacement cycle

Fig. 3.2

The total expected replacement cost per unit time $C(t_p)$ is

$$C(t_p) = \frac{\text{Total expected replacement cost per cycle}}{\text{Expected cycle length}}$$

Total expected
replacement cost = {(cost of a preventive replacement cycle)
per cycle x (probability of a preventive cycle)}
+ {(cost of failure cycle)x(probability of a failure cycle)} = $C_p.R(t_p)+C_f[1-R(t_p)]$.

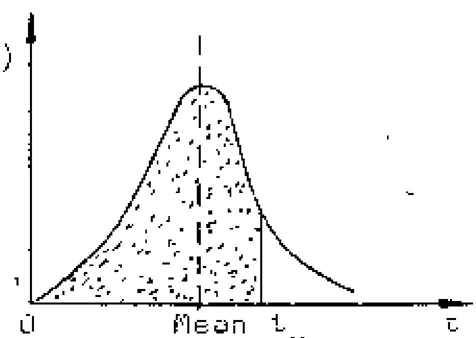
Expected cycle length = (length of a preventive cycle) x(Probability of a preventive cycle) + (Expected length of a failure cycle)x(Probability of a failure cycle)

$= t_p.R(t_p)+(\text{Expected length of a failure cycle})\times\{1-R(t_p)\}.$

Expected length of a failure cycle when a preventive replacement is carried out at time t_p, will be the mean of the shaded zone as demonstrated in Fig. 3.3, since the unshaded area is an impossible region for failures. Mathematically, it is expressed as the mean time to failure within the interval $(0, t_p)$.

That is:

$$M(t_p) = \frac{\int_0^{t_p} t.f(t)dt}{\{1-R(t_p)\}}$$



Operating zone of an item

Fig. 3.3

Therefore,

Expected cycle length $= t_p.R(t_p)+(\text{Expected length of a failure cycle})\times\{1-R(t_p)\}.$

$= t_p.R(t_p)+M(t_p)\cdot\{1-R(t_p)\}.$

Now, the total expected replacement cost per unit time $C(t_p)$ is

$$C(t_p) = \frac{\text{Total expected replacement cost per cycle}}{\text{Expected cycle length}}$$

$$C(t_p) = \frac{C_p \cdot R(t_p) + C_f \{1-R(t_p)\}}{t_p \cdot R(t_p) + M(t_p) \cdot \{1-R(t_p)\}} \qquad (3.1.1)$$

This is the model for an Age replacement policy relating the replacement age (t_p) to the total expected replacement cost per unit time.

The model can be re-arranged and expressed in terms of a relative failure cost (i.e cost ratio) denoted by $k = \frac{C_f}{C_p}$. Considering first, the denominator of equation (3.1.1), we have

$$\text{Expected cycle length} = t_p \cdot R(t_p) + M(t_p) \cdot \{1-R(t_p)\}.$$

$$= t_p \cdot R(t_p) + \frac{\int_0^{t_p} t \cdot f(t)\,dt}{\{1-R(t_p)\}} \cdot \{1-R(t_p)\}$$

$$= t_p \cdot R(t_p) + \int_0^{t_p} t \cdot f(t)\,dt$$

$$= t_p \cdot R(t_p) - \Big[t \cdot R(t)\Big]_0^{t_p} + \int_0^{t_p} R(t)\,dt$$

$$= t_p \cdot R(t_p) - t_p \cdot R(t_p) + \int_0^{t_p} R(t)\,dt$$

$$= \int_0^{t_p} R(t)\,dt.$$

Now substituting the above value in equation (3.1.1) we get

$$C(t_p) = \frac{C_p \cdot R(t_p) + C_f \{1-R(t_p)\}}{t_p \cdot R(t_p) + M(t_p) \cdot \{1-R(t_p)\}} \qquad (3.1.1)$$

$$C(t_p) = \frac{C_p \cdot R(t_p) + C_f \cdot \{1 - R(t_p)\}}{\int_0^{t_p} R(t)dt} \tag{3.1.2}$$

For an optimal replacement age t_p^*, equation (3.1.2) is differentiated with respect to t_p and then equated to zero. This gives:

$$C(t_p) = \frac{C_p \cdot R(t_p) + C_f \cdot \{1 - R(t_p)\}}{\int_0^{t_p} R(t)dt} \tag{3.1.2}$$

$$\frac{dC(t_p)}{dt_p} = \frac{d}{dt_p}\left\{\frac{C_p \cdot R(t_p) + C_f\{1 - R(t_p)\}}{\int_0^{t_p} R(t)dt}\right\} = 0$$

or, $$\frac{d}{dt_p}\left\{\frac{C_p \cdot R(t_p) + C_f\{1 - R(t_p)\}}{\int_0^{t_p} R(t)dt}\right\} = 0$$

or, $$\frac{\int_0^{t_p} R(t)dt \cdot \frac{d}{dt_p}\{C_p R(t_p) + C_f\{1 - R(t_p)\}\} - \{C_p \cdot R(t_p) + C_f \cdot \{1 - R(t_p)\}\} \cdot \frac{d}{dt_p}\{\int_0^{t_p} R(t)dt\}}{\{\int_0^{t_p} R(t)dt\}^2} = 0$$

$$\text{or,}\ \frac{\int_0^{t_p} R(t)dt.\ \{-C_p.f(t_p) + C_f(0) + C_f.f(t_p)\} - \{C_p.R(t_p)+C_f\{1-R(t_p)\}\} .R(t_p)}{\{\int_0^{t_p} R(t)dt\}^2} = 0$$

$$\text{or,}\ \int_0^{t_p} R(t)dt.\{-C_p.f(t_p)+C_f.f(t_p)\} - \{C_p.R(t_p)+C_f[1-R(t_p)]\}.R(t_p)=0$$

$$\text{or,}\ \int_0^{t_p} R(t)dt.f(t_p)(C_f-C_p) - \{C_p.R(t_p)+C_f[1-R(t_p)]\}.R(t_p) = 0$$

$$\text{or,}\ \int_0^{t_p} R(t)dt.f(t_p)(C_f-C_p) = \{C_p.R(t_p)+C_f[1-R(t_p)]\}.R(t_p)$$

$$\text{or,}\ \int_0^{t_p} R(t)dt.\{\frac{f(t_p)}{R(t_p)}\}.(C_f-C_p) = \{C_p.R(t_p)+C_f[1-R(t_p)]\}$$

$$\text{or}\ \int_0^{t_p} R(t)dt.\{\frac{f(t_p)}{R(t_p)}\}.(C_f-C_p) = \{C_f-R(t_p)(C_f-C_p)\}$$

[dividing both side by (C_f-C_p) and we get]

$$\text{or}\ \int_0^{t_p} R(t)dt.\{\frac{f(t_p)}{R(t_p)}\} = \frac{C_f}{C_f-C_p} - R(t_p)$$

Since $r(t_p) = \frac{f(t_p)}{R(t_p)} = \frac{f(t_p)}{1-F(t_p)}$; where $r(t_p)$ is the hazard rate or instantaneous failure rate (or force or mortality) at time t_p and putting this

$$\text{or} \quad \int_0^{t_p} R(t)dt \cdot r(t_p) = \frac{C_f}{C_f - C_p} - R(t_p)$$

$$\text{or,} \quad r(t_p) \cdot \int_0^{t_p} R(t)dt + R(t_p) = \frac{C_f}{C_f - C_p}$$

$$\text{or} \quad r(t_p) \cdot \int_0^{t_p} R(t)\,dt + R(t_p) = \frac{\frac{C_f}{C_p}}{\frac{C_f}{C_p} - \frac{C_p}{C_p}}$$

(Since failure cost ratio $k = C_f/C_p$)

$$\text{or,} \quad r(t_p) \cdot \int_0^{t_p} R(t)dt + R(t_p) = \frac{k}{k - 1} \qquad (3.1.3)$$

which is expressed in terms of cost ratio (i.e relative failure cost ratio).

The model described in equation (3.1.2)can also be expressed in multiples of planned replacement cost, i.e.

$$C(t_p) = \frac{C_p \cdot R(t_p) + C_f\{1 - R(t_p)\}}{\int_0^{t_p} R(t)dt} \qquad (3.1.2)$$

$$\frac{C(t_p)}{C_p} = \frac{R(t_p) + C_f/C_p \cdot \{1 - R(t_p)\}}{\int_0^{t_p} R(t)dt}$$

$$\text{or,} \quad \frac{C(t_p)}{C_p} = \frac{R(t_p) + k\{1 - R(t_p)\}}{\int_0^{t_p} R(t)dt} \qquad (3.1.4)$$

The saving for the optimal age replacement policy over a policy of 'replace on failure only' can be calculated and which is given by 'saving' as percentage of cost of a policy of 'replace on failure only' as follows:

Saving as a percentage of cost of a policy of 'replace on failure only'

$$= \left[\frac{(\text{Cost of a policy of replace on failure only}) - (\text{cost of optimal policy})}{(\text{cost of a policy of replace at failure only})} \right] \times 100\%$$

$$\text{Saving} = \left[\frac{\left(\dfrac{C_f}{\text{Mean failure time}}\right) - C(t_p)}{\left(\dfrac{C_f}{\text{Mean failure time}}\right)} \right] \times 100\%$$

$$= \left[1 - \frac{C(t_p)}{\dfrac{C_f}{\text{Mean failure time}}} \right] \times 100\%$$

$$= \left[1 - \frac{C(t_p)}{C_f} \times \text{Mean failure time } (\mu) \right] \times 100\%$$

$$= \left[1 - \frac{\dfrac{C(t_p)}{C_p} \times \mu}{\dfrac{C_f}{C_p}} \right] \times 100\%$$

$$= \left[1 - \frac{\dfrac{C(t_p)}{C_p} \cdot \mu}{K} \right] \times 100\% \qquad (3.1.5)$$

which is the percentage saving for Age replacement policy.

Standard cost ratio (ρ) is the ratio of cost of optimal policy to the cost of a policy to replace at failure only. This can be formulated as follows:

$$\text{Standard cost ratio } (\rho) = \frac{\text{Cost of optimal policy}}{\text{Cost of a policy to replace at failure only}}$$

$$\rho = \frac{C(t_p)}{\dfrac{C_f}{\text{Mean failure time}}} = \frac{C(t_p)}{\dfrac{C_f}{\mu}} = \frac{\dfrac{C(t_p)}{C_p}}{\dfrac{C_f}{C_p\cdot\mu}} = \frac{\dfrac{C(t_p)}{C_p}}{\dfrac{k}{\mu}}$$

$$\rho = \frac{\dfrac{C(t_p)}{C_p}\cdot\mu}{k} \,. \qquad (3.1.6)$$

which is the standard cost ratio of Age replacement policy.

CHAPTER-4

SOLUTION PROCEDURE FOR THE MODEL

CHAPTER-4

SOLUTION PROCEDURE FOR THE MODEL

4.1 General Comments on the Solution Procedure:

The formulation for the determination of cost optimal preventive replacement age t_p has been given in equation (3.1.4); which is

$$\frac{C(t_p)}{C_p} = \frac{R(t_p) + k[1-R(t_p)]}{\int_0^{t_p} R(t)dt} \qquad (3.1.4)$$

The equation is not explicit in terms of t_p and so can not be solved without the use of an iterative procedure. But before discussing any particular iterative procedure it is essential to consider the nature of the cost curve $C(t_p)$. In figures 4.1.1 and 4.1.2 two possible cost curves are plotted.

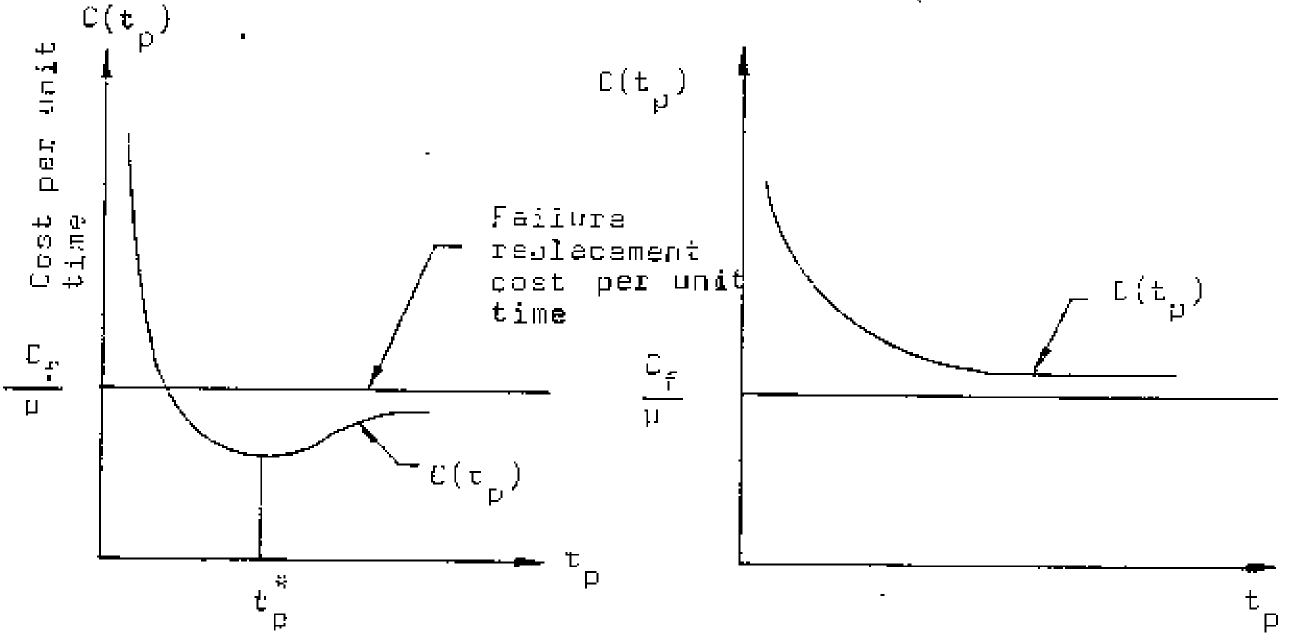


Fig. 4.1.1

Fig. 4.1.2

Typical curves for cost function $C(t_p)$

In Fig. 4.1.1 the curve $C(t_p)$ has got a minimum. Barlow etal (9) has proved that for an increasing failure rate $C(t_p)$ has at most one minimum, and for $C_f \geqslant C_p$, $C(t_p)$ must cross C_f/μ to the right of where C_p/t_p crosses C_f/μ, if it crosses at all. In the second figure, it is seen that $C(t_p)$ does not cross C_f/μ which implies that preventive replacement is not beneficial in this case.

From the above discussion, it is apparent that any iterative procedure should incorporate two important aspects in it. These are:

(i) Identification of the minima in the cost function, and, and

(ii) A stopping rule when a minima does not exist.

Glasser (14) solved the equation by Newton-Raphson technique. Alternatively, optimal t_p which lies at the minima on the cost curve, can be determined by initializing a small value of t_p in the programme and then increasing the value of t_p by discrete number, preferably smallest integer unit, until the minimum cost point has been reached. In this latter case, failure time distribution should be expressed in the smallest time unit.

However, in either procedure, t_p is obtained for a fixed cost ratio K (which is the ratio of the cost of a failure replacement to that of a preventive replacement) and fixed values of failure distribution parameters β and η.

The values of β and η are required for the solution of the reliability function $R(t_p)$. For Weibull distribution, $R(t)$ is expressed by

$$R(t) = \exp \{- (\frac{t}{\eta})^{\beta}\} \qquad (4.1.1)$$

From the cost equation it can be seen that the term $\int_0^{t_p} R(t)dt$ will have to be solved at each iteration. When a large number of iterations is involved, it may take an appreciable amount of computer time. Another problem is that if t_p is not an integer, the solution of the integral by a numerical procedure (either Trapezoidal or Simpson's rule) will involve a difficult computer programming. These two difficulties will definitely arise in case of Newton-Raphson solution procedure. So, the model has been solved by the alternative procedure mentioned above.

However, for a general representation of replacement policies, t_p will have to be obtained for a large range of cost ratio k and for large ranges of β and η. Following Glasser, the range of cost ratio k has been taken from 1.1 to 100.0. The range of β values has been taken from 1.1 to 5.0. This range of β value is quite sufficient for all practical purposes. The situations where $k \leq 1$ and/or $\beta \leq 1$ are impracticable in so far as preventive replacements are concerned, because breakdown maintenance will be more economic.

In the solution procedure, a single value for characteristic life or scale parameter η is considered rather than a range. From Glasser's solution where optimal replacement age t_p is determined for k and $V(=\mu/\sigma)$, it is apparent that t_μ is independent of η, because (μ/σ) is a function of β only (equation 2.2.5). But the value of η should be large enough so as to minimize computational error of the integral $\int_0^{t_p} R(t)dt$. To illustrate this fact, a series of reliability curves have been drawn for $\eta = 10$ and $\eta = 50$ for some values of β (Appendix-A). It can be seen from the graphs that for small η value (i.e. $\eta = 10$), the reliability curve sharply falls to near-zero values. The higher the value of β, the sharper is the fall of R(t). While integrating R(t) over this sharply fallen zone, computational error will be substantially high. So, to minimize the integration error, the value of η has been taken as 100.0.

As far as the shape of the distribution is concerned the choice of η does not change the shape characteristic. It simply flattens or elongates the density curve for large values of η. The effects of various η values on the probability density function f(t) have been shown in Appendix-B. For Weibull distribution, f(t) is given by

$$f(t) = \frac{\beta}{\eta} \cdot \left(\frac{t}{\eta}\right)^{\beta-1} \cdot \exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \qquad (4.1.2)$$

The functional value has been computed by a computer programme for various values of β and η and over a large range of t. In Appendix-B, f(t) curves have been plotted for η values of 10 and 50 for each β value ranging from 1.5 to 5.0 with an increment of 0.5 (i.e β = 1.5, 5.0, 0.5).

4.2 Computer Package for the Solution of the Model:

For solving the mathematical model a computer programme has been written in FORTRAN language using IBM 370 computer. The whole programme is divided into three sections consisting

i) a master programme to determine cost optimal replacement time, replacement cost in multiples of failure replacement cost and savings if any,

ii) a subroutine to carry out Weibull analysis and solve the integral $\int_0^{t_p} R(t)dt$, and

iii) a functional subprogramme to solve the gamma function (Γ) for determining mean time between failures and standard deviation.

The flow chart and the programme have been given in Appendices C and D.

The iterative procedure which has been incorporated in the programme is a very simple and crude one but effective. The value of optimal replacement age has been initialized as t_p = 1, and $C(t_p)$ is evaluated for this t_p and then for comparison $C(t_p)$ is evaluated for next higher t_p, i.e t_p = 2. By simple logic, the minimum $C(t_p)$ is kept in store. If $C(t_p)$ for t_p = 2 is smaller, then $C(t_p)$ for t_p = 3 is calculated and compared with the previously obtained minimum cost. If

$C(t_p)$ for $t_p = 3$ is minimum, this value is kept in temporary storage again and the iteration is proceeded as before until the trend is reversed. This means, for a certain new value of t_p, if it is found that $C(t_p - 1)$ is smaller, then $(t_p - 1)$ is the optimal value since the cost is minimum at this point and the iteration is stopped. The programme then calculates the saving that can be obtained by adopting this policy. If the saving is negative, then break down replacement policy is superior to preventive replacement policy. If the saving is positive, then preventive replacement is beneficial. The policies are printed accordingly.

If the cost curve is one of Fig. 4.1.2, the search for t_p will have to be stopped anyway. In this programme the search is continued upto a value of t_p where $R(t_p)$ is equal to or less than 0.005. Since there is no minima within this range, the optimal replacement policy is taken as failure replacement one.

The optimal replacement policies are determined for a range of values of cost ratio, as mentioned in the previous section. The range and incremental values are as the following:

(1.1, 2.0,0.1);(2.0,10.0,1.0),(10.0,40.0.5.0),(40.0,100.0,10.

In the programme, this has been executed by simple logic statements.

CHAPTER-5

RESULTS AND DISCUSSION

CHAPTER-5

RESULTS AND DISCUSSION

5.1 Results from the Model Solution:

The cost-optimal age replacement model has been solved for a range of cost ratios and a range of β values by a computer package discussed in section 4.2. The results have been presented in both tabular and graphical form. Tabular presentation has been made as a direct print-out from the computer package. It includes informations like β , η , mean time time between failures (μ), standard deviation (σ) and average service life in standard deviation units (μ / σ) in the beginning of each table. These are followed by a table containing relative cost of failure of replacement, resulting optimal policy and associated standardized cost of optimal policy and associated gives 'replace on failure only' in case of break-down replacement being optimal one. In case of preventive replacement being optimal, the print-out gives a 'Z' value which indicates deviation of optimal age from the characteristic life η. The value of 'Z' is calculated by

$$Z = \frac{t_p - \eta}{\eta} \tag{5.1.1}$$

So, the optimal age is

$$t_p = \eta(1 + Z) \tag{5.1.2}$$

The results are given in Table 5.1.1.

Computer print outs

Table 5.1.1

// EXEC

B=1.1 ETA=100.0 FMT= 96.49 SD= 87.83 ASSD= 1.09858

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	REPLACE ON FAILURE ONLY	
1.400	REPLACE ON FAILURE ONLY	
1.500	REPLACE ON FAILURE ONLY	
1.600	REPLACE ON FAILURE ONLY	
1.700	REPLACE ON FAILURE ONLY	
1.800	REPLACE ON FAILURE ONLY	
1.900	REPLACE ON FAILURE ONLY	
2.000	REPLACE ON FAILURE ONLY	
3.000	REPLACE ON FAILURE ONLY	
4.000	REPLACE ON FAILURE ONLY	
5.000	REPLACE ON FAILURE ONLY	
6.000	2.3700	0.9997
7.000	1.5300	0.9987
8.000	1.0200	0.9970
9.000	0.7000	0.9945
10.000	0.4600	0.9923
15.000	-0.1300	0.9774
20.000	-0.3700	0.9629
25.000	-0.5000	0.9500
30.000	-0.5900	0.9387
35.000	-0.6500	0.9286
40.000	-0.6900	0.9197
50.000	-0.7500	0.9043
60.000	-0.7900	0.8915
70.000	-0.8200	0.8806
80.000	-0.8400	0.8710
90.000	-0.8600	0.8626
100.000	-0.8700	0.8550

B=1.2 ETA=100.0 FMT= 94.06 SD= 78.73 ASSD= 1.19479

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	REPLACE ON FAILURE ONLY	
1.400	REPLACE ON FAILURE ONLY	
1.500	REPLACE ON FAILURE ONLY	
1.600	REPLACE ON FAILURE ONLY	
1.700	REPLACE ON FAILURE ONLY	
1.800	REPLACE ON FAILURE ONLY	
1.900	REPLACE ON FAILURE ONLY	
2.000	REPLACE ON FAILURE ONLY	
3.000	3.0000	0.9999
4.000	1.2700	0.9976
5.000	0.5900	0.9912
6.000	0.2400	0.9822
7.000	0.0200	0.9722
8.000	-0.1200	0.9620
9.000	-0.2300	0.9519
10.000	-0.3100	0.9422
15.000	-0.5400	0.9001
20.000	-0.6500	0.8673
25.000	-0.7200	0.8411
30.000	-0.7600	0.8195
35.000	-0.7900	0.8012
40.000	-0.8100	0.7854
50.000	-0.8500	0.7592
60.000	-0.8700	0.7380
70.000	-0.8900	0.7205
80.000	-0.9000	0.7054
90.000	-0.9100	0.6923
100.000	-0.9200	0.6809

B=1.3 ETA=100.0 FMT= 92.35 SD= 71.65 ASSD= 1.28901

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	REPLACE ON FAILURE ONLY	
1.400	REPLACE ON FAILURE ONLY	
1.500	REPLACE ON FAILURE ONLY	
1.600	REPLACE ON FAILURE ONLY	
1.700	REPLACE ON FAILURE ONLY	
1.800	REPLACE ON FAILURE ONLY	
1.900	REPLACE ON FAILURE ONLY	
2.000	REPLACE ON FAILURE ONLY	
3.000	1.0800	0.9967
4.000	0.3400	0.9831
5.000	0.0100	0.9643
6.000	-0.1700	0.9447
7.000	-0.3000	0.9257
8.000	-0.3900	0.9079
9.000	-0.4500	0.8913
10.000	-0.5000	0.8760
15.000	-0.6500	0.8146
20.000	-0.7300	0.7702
25.000	-0.7800	0.7361
30.000	-0.8100	0.7086
35.000	-0.8300	0.6859
40.000	-0.8500	0.6665
50.000	-0.8700	0.6350
60.000	-0.8900	0.6101
70.000	-0.9000	0.5896
80.000	-0.9100	0.5723
90.000	-0.9200	0.5574
100.000	-0.9300	0.5447

B=1.4 ETA=100.0 FMT= 91.14 SD= 65.97 ASSD= 1.38152

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	REPLACE ON FAILURE ONLY	
1.400	REPLACE ON FAILURE ONLY	
1.500	REPLACE ON FAILURE ONLY	
1.600	REPLACE ON FAILURE ONLY	
1.700	REPLACE ON FAILURE ONLY	
1.800	REPLACE ON FAILURE ONLY	
1.900	REPLACE ON FAILURE ONLY	
2.000	2.0600	0.9998
3.000	0.4500	0.9866
4.000	0.0100	0.9588
5.000	-0.2100	0.9286
6.000	-0.3400	0.9000
7.000	-0.4300	0.8740
8.000	-0.4900	0.8505
9.000	-0.5400	0.8293
10.000	-0.5800	0.8101
15.000	-0.7000	0.7359
20.000	-0.7600	0.6845
25.000	-0.8000	0.6459
30.000	-0.8200	0.6156
35.000	-0.8400	0.5907
40.000	-0.8600	0.5697
50.000	-0.8800	0.5360
60.000	-0.8900	0.5100
70.000	-0.9100	0.4897
80.000	-0.9100	0.4709
90.000	-0.9200	0.4555
100.000	-0.9300	0.4423

B=1.5 ETA=100.0 FMT= 90.27 SD= 61.30 ASSD= 1.47260

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	REPLACE ON FAILURE ONLY	
1.400	REPLACE ON FAILURE ONLY	
1.500	REPLACE ON FAILURE ONLY	
1.600	REPLACE ON FAILURE ONLY	
1.700	REPLACE ON FAILURE ONLY	
1.800	1.7600	0.9997
1.900	1.4400	0.9992
2.000	1.1700	0.9984
3.000	0.1600	0.9710
4.000	-0.1600	0.9301
5.000	-0.3200	0.8903
6.000	-0.4300	0.8547
7.000	-0.5000	0.8234
8.000	-0.5500	0.7958
9.000	-0.5900	0.7713
10.000	-0.6200	0.7494
15.000	-0.7200	0.6671
20.000	-0.7700	0.6117
25.000	-0.8100	0.5710
30.000	-0.8300	0.5393
35.000	-0.8500	0.5136
40.000	-0.8600	0.4922
50.000	-0.8800	0.4582
60.000	-0.8900	0.4321
70.000	-0.9100	0.4111
80.000	-0.9100	0.3935
90.000	-0.9200	0.3784
100.000	-0.9300	0.3659

B=1.6 ETA=100.0 FMT= 89.65 SD= 57.39 ASSD= 1.56246

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	REPLACE ON FAILURE ONLY	
1.400	REPLACE ON FAILURE ONLY	
1.500	REPLACE ON FAILURE ONLY	
1.600	1.7600	0.9998
1.700	1.4000	0.9995
1.800	1.1100	0.9986
1.900	0.8900	0.9971
2.000	0.7300	0.9951
3.000	-0.0100	0.9523
4.000	-0.2600	0.9000
5.000	-0.3900	0.8525
6.000	-0.4800	0.8116
7.000	-0.5400	0.7763
8.000	-0.5800	0.7457
9.000	-0.6200	0.7190
10.000	-0.6400	0.6953
15.000	-0.7300	0.6080
20.000	-0.7800	0.5506
25.000	-0.8100	0.5091
30.000	-0.8300	0.4771
35.000	-0.8500	0.4514
40.000	-0.8600	0.4301
50.000	-0.8800	0.3966
60.000	-0.8900	0.3711
70.000	-0.9000	0.3507
80.000	-0.9100	0.3338
90.000	-0.9200	0.3198
100.000	-0.9200	0.3074

B=1.7 ETA=100.0 FMT= 89.22 SD= 54.03 ASSD= 1.65124

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	REPLACE ON FAILURE ONLY	
1.400	REPLACE ON FAILURE ONLY	
1.500	1.6000	0.9998
1.600	1.2300	0.9994
1.700	0.9600	0.9982
1.800	0.7500	0.9962
1.900	0.5900	0.9934
2.000	0.4600	0.9898
3.000	-0.1100	0.9322
4.000	-0.3200	0.8702
5.000	-0.4300	0.8166
6.000	-0.5100	0.7716
7.000	-0.5600	0.7334
8.000	-0.6000	0.7008
9.000	-0.6300	0.6725
10.000	-0.6600	0.6477
15.000	-0.7400	0.5575
20.000	-0.7800	0.4993
25.000	-0.8100	0.4578
30.000	-0.8300	0.4260
35.000	-0.8400	0.4009
40.000	-0.8600	0.3801
50.000	-0.8700	0.3476
60.000	-0.8900	0.3229
70.000	-0.9000	0.3034
80.000	-0.9100	0.2876
90.000	-0.9100	0.2740
100.000	-0.9200	0.2626

B=1.8 ETA=100.0 FMT= 88.92 SD= 51.13 ASSD= 1.73909

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	REPLACE ON FAILURE ONLY	
1.400	REPLACE ON FAILURE ONLY	
1.500	1.1900	0.9995
1.600	0.3900	0.9983
1.700	0.6800	0.9960
1.800	0.5200	0.9926
1.900	0.3900	0.9883
2.000	0.2900	0.9830
3.000	-0.1300	0.9117
4.000	-0.3300	0.8416
5.000	-0.4600	0.7932
6.000	-0.5300	0.7350
7.000	-0.5700	0.6948
8.000	-0.6100	0.6607
9.000	-0.6400	0.6314
10.000	-0.6600	0.6058
15.000	-0.7400	0.5142
20.000	-0.7800	0.4561
25.000	-0.8100	0.4151
30.000	-0.8200	0.3840
35.000	-0.8400	0.3592
40.000	-0.8500	0.3391
50.000	-0.8700	0.3078
60.000	-0.8800	0.2843
70.000	-0.8900	0.2659
80.000	-0.9000	0.2506
90.000	-0.9100	0.2382
100.000	-0.9100	0.2273

B=1.9 ETA=100.0 FMT= 88.73 SD= 48.59 ASSD= 1.82611

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	REPLACE ON FAILURE ONLY	
1.400	1.2200	0.9997
1.500	0.9000	0.9988
1.600	0.6600	0.9965
1.700	0.4900	0.9929
1.800	0.3600	0.9880
1.900	0.2600	0.9820
2.000	0.1800	0.9752
3.000	-0.2300	0.8915
4.000	-0.3700	0.8147
5.000	-0.4300	0.7523
6.000	-0.5400	0.7018
7.000	-0.5800	0.6601
8.000	-0.6100	0.6250
9.000	-0.6400	0.5950
10.000	-0.6600	0.5691
15.000	-0.7300	0.4770
20.000	-0.7700	0.4194
25.000	-0.8000	0.3790
30.000	-0.8200	0.3487
35.000	-0.8300	0.3249
40.000	-0.8500	0.3055
50.000	-0.8600	0.2754
60.000	-0.8800	0.2531
70.000	-0.8900	0.2355
80.000	-0.8900	0.2213
90.000	-0.9000	0.2093
100.000	-0.9100	0.1994

B=2.0 ETA=100.0 FMT= 88.62 SD= 46.34 ASSD= 1.91243

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	REPLACE ON FAILURE ONLY	
1.400	0.9700	0.9994
1.500	0.6900	0.9976
1.600	0.5000	0.9941
1.700	0.3500	0.9890
1.800	0.2500	0.9825
1.900	0.1600	0.9750
2.000	0.0900	0.9666
3.000	-0.2600	0.8719
4.000	-0.4100	0.7894
5.000	-0.4900	0.7240
6.000	-0.5500	0.6718
7.000	-0.5900	0.6290
8.000	-0.6200	0.5932
9.000	-0.6400	0.5629
10.000	-0.6600	0.5367
15.000	-0.7300	0.4443
20.000	-0.7700	0.3880
25.000	-0.8000	0.3486
30.000	-0.8100	0.3191
35.000	-0.8300	0.2960
40.000	-0.8400	0.2773
50.000	-0.8600	0.2486
60.000	-0.8700	0.2272
70.000	-0.8800	0.2106
80.000	-0.8900	0.1972
90.000	-0.8900	0.1861
100.000	-0.9000	0.1765

B=2.1 ETA=100.0 RELATIVE COST OF FAILURE REPLACEMENT	FMT= 88.56 OPTIMAL AGE	SD= 44.32 STD. COST RATIO	ASSD= 1.99808
1.100	REPLACE ON FAILURE ONLY		
1.200	REPLACE ON FAILURE ONLY		
1.300	1.1500	0.9998	
1.400	0.7700	0.9988	
1.500	0.5400	0.9959	
1.600	0.3400	0.9911	
1.700	0.2600	0.9845	
1.800	0.1600	0.9765	
1.900	0.0900	0.9674	
2.000	0.0300	0.9577	
3.000	-0.2900	0.8531	
4.000	-0.4200	0.7659	
5.000	-0.5000	0.6981	
6.000	-0.5500	0.6445	
7.000	-0.5900	0.6010	
8.000	-0.6200	0.5648	
9.000	-0.6400	0.5343	
10.000	-0.6600	0.5081	
15.000	-0.7300	0.4168	
20.000	-0.7600	0.3610	
25.000	-0.7900	0.3225	
30.000	-0.8100	0.2939	
35.000	-0.8200	0.2716	
40.000	-0.8300	0.2537	
50.000	-0.8500	0.2261	
60.000	-0.8600	0.2059	
70.000	-0.8700	0.1901	
80.000	-0.8800	0.1773	
90.000	-0.8900	0.1669	
100.000	-0.8900	0.1580	

B=2.2 ETA=100.0 RELATIVE COST OF FAILURE REPLACEMENT	FMT= 88.55 OPTIMAL AGE	SD= 42.51 STD. COST RATIO	ASSD= 2.08318
1.100	REPLACE ON FAILURE ONLY		
1.200	REPLACE ON FAILURE ONLY		
1.300	0.9400	0.9996	
1.400	0.6300	0.9979	
1.500	0.4400	0.9939	
1.600	0.2900	0.9877	
1.700	0.1300	0.9795	
1.800	0.1000	0.9701	
1.900	0.0300	0.9595	
2.000	-0.0200	0.9485	
3.000	-0.3100	0.8352	
4.000	-0.4500	0.7441	
5.000	-0.5000	0.6744	
6.000	-0.5500	0.6198	
7.000	-0.5900	0.5757	
8.000	-0.6200	0.5394	
9.000	-0.6400	0.5068	
10.000	-0.6600	0.4827	
15.000	-0.7200	0.3922	
20.000	-0.7600	0.3375	
25.000	-0.7800	0.3001	
30.000	-0.8000	0.2723	
35.000	-0.8100	0.2509	
40.000	-0.8300	0.2336	
50.000	-0.8400	0.2072	
60.000	-0.8600	0.1879	
70.000	-0.8700	0.1730	
80.000	-0.8700	0.1609	
90.000	-0.8800	0.1509	
100.000	-0.8900	0.1426	

B=2.3 ETA=100.0 FMT= 83.58 SD= 40.86 ASSD= 2.16776

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	0.7300	0.9993
1.400	0.5100	0.9968
1.500	0.3400	0.9915
1.600	0.2100	0.9838
1.700	0.1200	0.9743
1.800	0.0500	0.9634
1.900	-0.0100	0.9516
2.000	-0.0600	0.9393
3.000	-0.3200	0.8183
4.000	-0.4400	0.7239
5.000	-0.5100	0.6527
6.000	-0.5500	0.5973
7.000	-0.5900	0.5530
8.000	-0.6100	0.5166
9.000	-0.6400	0.4861
10.000	-0.6500	0.4601
15.000	-0.7200	0.3707
20.000	-0.7500	0.3171
25.000	-0.7800	0.2806
30.000	-0.7900	0.2538
35.000	-0.8100	0.2330
40.000	-0.8200	0.2163
50.000	-0.8400	0.1911
60.000	-0.8500	0.1726
70.000	-0.8600	0.1583
80.000	-0.8700	0.1470
90.000	-0.8700	0.1376
100.000	-0.8500	0.1296

B=2.4 ETA=100.0 FMT= 88.64 SD= 39.36 ASSD= 2.25186

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	0.6600	0.9989
1.400	0.4200	0.9954
1.500	0.2700	0.9888
1.600	0.1600	0.9797
1.700	0.0700	0.9688
1.800	0.0100	0.9566
1.900	-0.0500	0.9436
2.000	-0.0900	0.9301
3.000	-0.3300	0.8023
4.000	-0.4400	0.7051
5.000	-0.5100	0.6327
6.000	-0.5500	0.5768
7.000	-0.5800	0.5324
8.000	-0.6100	0.4960
9.000	-0.6300	0.4656
10.000	-0.6500	0.4397
15.000	-0.7100	0.3515
20.000	-0.7400	0.2991
25.000	-0.7700	0.2636
30.000	-0.7900	0.2376
35.000	-0.8000	0.2175
40.000	-0.8100	0.2015
50.000	-0.8300	0.1772
60.000	-0.8400	0.1595
70.000	-0.8500	0.1459
80.000	-0.8600	0.1351
90.000	-0.8700	0.1262
100.000	-0.8700	0.1187

B=2.5 ETA=100.0 FMT= 88.72 SD= 37.99 ASSD= 2.33554

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	REPLACE ON FAILURE ONLY	
1.300	0.5600	0.9984
1.400	0.3500	0.9938
1.500	0.2100	0.9859
1.600	0.1100	0.9754
1.700	0.0400	0.9632
1.800	-0.0200	0.9497
1.900	-0.0700	0.9356
2.000	-0.1200	0.9211
3.000	-0.3400	0.7872
4.000	-0.4500	0.6877
5.000	-0.5100	0.6143
6.000	-0.5500	0.5581
7.000	-0.5800	0.5136
8.000	-0.6100	0.4773
9.000	-0.6300	0.4471
10.000	-0.6500	0.4215
15.000	-0.7000	0.3346
20.000	-0.7400	0.2832
25.000	-0.7600	0.2486
30.000	-0.7800	0.2234
35.000	-0.7900	0.2040
40.000	-0.8000	0.1885
50.000	-0.8200	0.1652
60.000	-0.8300	0.1483
70.000	-0.8400	0.1353
80.000	-0.8500	0.1249
90.000	-0.8500	0.1164
100.000	-0.8600	0.1094

B=2.6 ETA=100.0 FMT= 88.81 SD= 36.72 ASSD= 2.41881

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	0.8200	0.9997
1.300	0.4300	0.9977
1.400	0.2900	0.9920
1.500	0.1700	0.9828
1.600	0.0700	0.9710
1.700	0.0	0.9575
1.800	-0.0500	0.9429
1.900	-0.1000	0.9277
2.000	-0.1400	0.9123
3.000	-0.3500	0.7729
4.000	-0.4500	0.6715
5.000	-0.5100	0.5974
6.000	-0.5500	0.5410
7.000	-0.5800	0.4965
8.000	-0.6000	0.4604
9.000	-0.6200	0.4304
10.000	-0.6400	0.4050
15.000	-0.7000	0.3193
20.000	-0.7300	0.2691
25.000	-0.7500	0.2354
30.000	-0.7700	0.2109
35.000	-0.7800	0.1922
40.000	-0.8000	0.1772
50.000	-0.8100	0.1547
60.000	-0.8300	0.1395
70.000	-0.8400	0.1230
80.000	-0.8400	0.1162
90.000	-0.8500	0.1081
100.000	-0.8600	0.1013

B=2.7 ETA=100.0 FMT= 88.92 SD= 35.54 ASSD= 2.50175

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	0.7100	0.9996
1.300	0.4100	0.9969
1.400	0.2400	0.9901
1.500	0.1300	0.9796
1.600	0.0400	0.9665
1.700	-0.0200	0.9518
1.800	-0.0700	0.9361
1.900	-0.1200	0.9200
2.000	-0.1500	0.9037
3.000	-0.3500	0.7595
4.000	-0.4500	0.6564
5.000	-0.5000	0.5818
6.000	-0.5400	0.5253
7.000	-0.5700	0.4809
8.000	-0.6000	0.4449
9.000	-0.6200	0.4152
10.000	-0.6300	0.3901
15.000	-0.6900	0.3056
20.000	-0.7200	0.2565
25.000	-0.7500	0.2236
30.000	-0.7600	0.1995
35.000	-0.7800	0.1816
40.000	-0.7900	0.1671
50.000	-0.8100	0.1455
60.000	-0.8200	0.1298
70.000	-0.8300	0.1179
80.000	-0.8400	0.1085
90.000	-0.8400	0.1008
100.000	-0.8500	0.0943

B=2.8 ETA=100.0 FMT= 89.03 SD= 34.45 ASSD= 2.58440

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	0.6300	0.9995
1.300	0.3600	0.9960
1.400	0.2000	0.9880
1.500	0.0900	0.9763
1.600	0.0200	0.9620
1.700	-0.0400	0.9461
1.800	-0.0900	0.9294
1.900	-0.1300	0.9124
2.000	-0.1700	0.8954
3.000	-0.3600	0.7468
4.000	-0.4500	0.6423
5.000	-0.5000	0.5673
6.000	-0.5400	0.5107
7.000	-0.5700	0.4665
8.000	-0.5900	0.4308
9.000	-0.6100	0.4013
10.000	-0.6300	0.3764
15.000	-0.6800	0.2933
20.000	-0.7200	0.2451
25.000	-0.7400	0.2130
30.000	-0.7600	0.1899
35.000	-0.7700	0.1722
40.000	-0.7800	0.1582
50.000	-0.8000	0.1373
60.000	-0.8100	0.1223
70.000	-0.8200	0.1108
80.000	-0.8300	0.1017
90.000	-0.8400	0.0944
100.000	-0.8600	0.0832

B=2.9 ETA=100.0 FMT= 89.16 SD= 33.43 ASSD= 2.66670

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	0.5600	0.9993
1.300	0.3100	0.9950
1.400	0.1600	0.9858
1.500	0.0700	0.9729
1.600	-0.0100	0.9574
1.700	-0.0600	0.9405
1.800	-0.1100	0.9229
1.900	-0.1500	0.9051
2.000	-0.1800	0.8873
3.000	-0.3600	0.7349
4.000	-0.4500	0.6292
5.000	-0.5000	0.5538
6.000	-0.5400	0.4973
7.000	-0.5700	0.4533
8.000	-0.5900	0.4178
9.000	-0.6100	0.3886
10.000	-0.6200	0.3640
15.000	-0.6800	0.2821
20.000	-0.7100	0.2349
25.000	-0.7300	0.2036
30.000	-0.7500	0.1810
35.000	-0.7600	0.1639
40.000	-0.7700	0.1503
50.000	-0.7900	0.1301
60.000	-0.8000	0.1156
70.000	-0.8100	0.1046
80.000	-0.8200	0.0958
90.000	-0.8300	0.0887
100.000	-0.8400	0.0829

B=3.0 ETA=100.0 FMT= 89.29 SD= 32.48 ASSD= 2.74873

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	0.5000	0.9990
1.300	0.2700	0.9939
1.400	0.1300	0.9836
1.500	0.0400	0.9694
1.600	-0.0300	0.9529
1.700	-0.0800	0.9350
1.800	-0.1200	0.9165
1.900	-0.1600	0.8979
2.000	-0.1900	0.8795
3.000	-0.3600	0.7236
4.000	-0.4500	0.6170
5.000	-0.5000	0.5413
6.000	-0.5300	0.4850
7.000	-0.5600	0.4411
8.000	-0.5800	0.4059
9.000	-0.6000	0.3769
10.000	-0.6200	0.3526
15.000	-0.6700	0.2719
20.000	-0.7000	0.2256
25.000	-0.7200	0.1951
30.000	-0.7400	0.1731
35.000	-0.7500	0.1564
40.000	-0.7700	0.1433
50.000	-0.7800	0.1236
60.000	-0.8000	0.1096
70.000	-0.8100	0.0990
80.000	-0.8100	0.0906
90.000	-0.8200	0.0838
100.000	-0.8300	0.0781

B=3.1 ETA=100.0 FMT= 89.42 SD= 31.59 ASSD= 2.83056

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	0.4400	0.9988
1.300	0.2300	0.9927
1.400	0.1100	0.9813
1.500	0.0200	0.9660
1.600	-0.0400	0.9484
1.700	-0.0900	0.9296
1.800	-0.1300	0.9103
1.900	-0.1700	0.8910
2.000	-0.2000	0.8719
3.000	-0.3600	0.7129
4.000	-0.4400	0.6054
5.000	-0.4900	0.5297
6.000	-0.5300	0.4734
7.000	-0.5600	0.4298
8.000	-0.5800	0.3949
9.000	-0.6000	0.3662
10.000	-0.6100	0.3422
15.000	-0.6600	0.2627
20.000	-0.7000	0.2173
25.000	-0.7200	0.1873
30.000	-0.7300	0.1659
35.000	-0.7500	0.1496
40.000	-0.7600	0.1368
50.000	-0.7800	0.1178
60.000	-0.7900	0.1042
70.000	-0.8000	0.0939
80.000	-0.8100	0.0859
90.000	-0.8100	0.0794
100.000	-0.8200	0.0739

B=3.2 ETA=100.0 FMT= 89.55 SD= 30.75 ASSD= 2.91206

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	0.4000	0.9984
1.300	0.2000	0.9915
1.400	0.0800	0.9739
1.500	0.0	0.9625
1.600	-0.0600	0.9439
1.700	-0.1100	0.9243
1.800	-0.1400	0.9042
1.900	-0.1300	0.8843
2.000	-0.2100	0.8647
3.000	-0.3700	0.7028
4.000	-0.4400	0.5946
5.000	-0.4900	0.5188
6.000	-0.5300	0.4628
7.000	-0.5500	0.4194
8.000	-0.5700	0.3847
9.000	-0.5900	0.3563
10.000	-0.6100	0.3326
15.000	-0.6600	0.2541
20.000	-0.6900	0.2095
25.000	-0.7100	0.1802
30.000	-0.7300	0.1593
35.000	-0.7400	0.1435
40.000	-0.7500	0.1310
50.000	-0.7700	0.1126
60.000	-0.7800	0.0994
70.000	-0.7900	0.0895
80.000	-0.8000	0.0816
90.000	-0.8100	0.0753
100.000	-0.8100	0.0701

B=3.3 ETA=100.0 FMT= 89.69 SD= 29.95 ASSD= 2.99340

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	0.3600	0.9981
1.300	0.1800	0.9902
1.400	0.0600	0.9765
1.500	-0.0100	0.9590
1.600	-0.0700	0.9395
1.700	-0.1200	0.9191
1.800	-0.1500	0.8983
1.900	-0.1800	0.8778
2.000	-0.2100	0.8576
3.000	-0.3700	0.6932
4.000	-0.4400	0.5845
5.000	-0.4900	0.5087
6.000	-0.5200	0.4528
7.000	-0.5500	0.4096
8.000	-0.5700	0.3753
9.000	-0.5900	0.3472
10.000	-0.6000	0.3236
15.000	-0.6500	0.2463
20.000	-0.6800	0.2025
25.000	-0.7000	0.1738
30.000	-0.7200	0.1534
35.000	-0.7300	0.1379
40.000	-0.7400	0.1258
50.000	-0.7600	0.1078
60.000	-0.7700	0.0951
70.000	-0.7800	0.0855
80.000	-0.7900	0.0779
90.000	-0.8000	0.0717
100.000	-0.8100	0.0667

B=3.4 ETA=100.0 FMT= 49.32 SD= 29.22 ASSD= 3.07450

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	0.3200	0.9977
1.300	0.1500	0.9688
1.400	0.0500	0.9741
1.500	-0.0300	0.9556
1.600	-0.0800	0.9352
1.700	-0.1200	0.9140
1.800	-0.1600	0.8926
1.900	-0.1900	0.8715
2.000	-0.2200	0.8509
3.000	-0.3700	0.6842
4.000	-0.4400	0.5749
5.000	-0.4800	0.4992
6.000	-0.5200	0.4434
7.000	-0.5400	0.4006
8.000	-0.5600	0.3665
9.000	-0.5600	0.3386
10.000	-0.5900	0.3154
15.000	-0.6400	0.2391
20.000	-0.6700	0.1961
25.000	-0.7000	0.1679
30.000	-0.7100	0.1479
35.000	-0.7300	0.1329
40.000	-0.7400	0.1210
50.000	-0.7500	0.1035
60.000	-0.7700	0.0911
70.000	-0.7800	0.0813
80.000	-0.7900	0.0745
90.000	-0.7900	0.0685
100.000	-0.8000	0.0636

B=3.5 ETA=100.0 FMT= 89.96 SD= 28.51 ASSD= 3.15545

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	0.2900	0.9972
1.300	0.1300	0.9874
1.400	0.0300	0.9716
1.500	-0.0400	0.9522
1.600	-0.0900	0.9310
1.700	-0.1300	0.9090
1.800	-0.1700	0.8870
1.900	-0.2000	0.8654
2.000	-0.2200	0.8443
3.000	-0.3600	0.6756
4.000	-0.4400	0.5659
5.000	-0.4800	0.4902
6.000	-0.5100	0.4347
7.000	-0.5400	0.3921
8.000	-0.5600	0.3583
9.000	-0.5700	0.3307
10.000	-0.5900	0.3077
15.000	-0.6400	0.2324
20.000	-0.6700	0.1901
25.000	-0.6900	0.1625
30.000	-0.7100	0.1429
35.000	-0.7200	0.1282
40.000	-0.7300	0.1166
50.000	-0.7500	0.0996
60.000	-0.7600	0.0875
70.000	-0.7700	0.0784
80.000	-0.7800	0.0713
90.000	-0.7900	0.0656
100.000	-0.7900	0.0609

B=3.6 ETA=100.0 FMT= 90.10 SD= 27.84 ASSD= 3.23614

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	REPLACE ON FAILURE ONLY	
1.200	0.2700	0.9967
1.300	0.1100	0.9860
1.400	0.0200	0.9692
1.500	-0.0500	0.9489
1.600	-0.1000	0.9268
1.700	-0.1400	0.9042
1.800	-0.1700	0.8816
1.900	-0.2000	0.8595
2.000	-0.2700	0.8380
3.000	-0.3600	0.6673
4.000	-0.4300	0.5574
5.000	-0.4300	0.4818
6.000	-0.5100	0.4265
7.000	-0.5300	0.3842
8.000	-0.5500	0.3506
9.000	-0.5700	0.3233
10.000	-0.5800	0.3005
15.000	-0.6300	0.2262
20.000	-0.6600	0.1846
25.000	-0.6800	0.1575
30.000	-0.7000	0.1383
35.000	-0.7100	0.1239
40.000	-0.7200	0.1126
50.000	-0.7400	0.0960
60.000	-0.7500	0.0842
70.000	-0.7600	0.0754
80.000	-0.7700	0.0685
90.000	-0.7800	0.0629
100.000	-0.7900	0.0583

B=3.7 ETA=100.0 FMT= 90.23 SD= 27.20 ASSD= 3.31667

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.5400	0.9998
1.200	0.2400	0.9962
1.300	0.1000	0.9845
1.400	0.0100	0.9668
1.500	-0.0600	0.9456
1.600	-0.1100	0.9223
1.700	-0.1500	0.8995
1.800	-0.1800	0.8764
1.900	-0.2000	0.8538
2.000	-0.2300	0.8320
3.000	-0.3600	0.6595
4.000	-0.4300	0.5493
5.000	-0.4700	0.4739
6.000	-0.5000	0.4189
7.000	-0.5300	0.3768
8.000	-0.5500	0.3435
9.000	-0.5600	0.3164
10.000	-0.5800	0.2938
15.000	-0.6300	0.2205
20.000	-0.6500	0.1795
25.000	-0.6800	0.1529
30.000	-0.6900	0.1341
35.000	-0.7100	0.1200
40.000	-0.7200	0.1089
50.000	-0.7300	0.0927
60.000	-0.7500	0.0812
70.000	-0.7600	0.0726
80.000	-0.7700	0.0659
90.000	-0.7700	0.0605
100.000	-0.7800	0.0560

B=3.8 ETA=100.0 FMT= 90.36 SD= 26.60 ASSD= 3.39706

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.5200	0.9997
1.200	0.2200	0.9956
1.300	0.0800	0.9830
1.400	-0.0100	0.9643
1.500	-0.0700	0.9423
1.600	-0.1100	0.9188
1.700	-0.1500	0.8949
1.800	-0.1800	0.8713
1.900	-0.2100	0.8483
2.000	-0.2300	0.8261
3.000	-0.3600	0.6521
4.000	-0.4300	0.5417
5.000	-0.4700	0.4664
6.000	-0.5000	0.4116
7.000	-0.5200	0.3698
8.000	-0.5400	0.3368
9.000	-0.5600	0.3099
10.000	-0.5700	0.2876
15.000	-0.6200	0.2151
20.000	-0.6500	0.1747
25.000	-0.6700	0.1486
30.000	-0.6900	0.1302
35.000	-0.7000	0.1163
40.000	-0.7100	0.1055
50.000	-0.7300	0.0895
60.000	-0.7400	0.0784
70.000	-0.7500	0.0700
80.000	-0.7600	0.0635
90.000	-0.7700	0.0583
100.000	-0.7700	0.0539

B=3.9 ETA=100.0 FMT= 90.49 SD= 26.02 ASSD= 3.47729

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.4800	0.9997
1.200	0.2000	0.9951
1.300	0.0700	0.9815
1.400	-0.0200	0.9619
1.500	-0.0700	0.9391
1.600	-0.1200	0.9149
1.700	-0.1600	0.8905
1.800	-0.1900	0.8664
1.900	-0.2100	0.8430
2.000	-0.2300	0.8205
3.000	-0.3600	0.6450
4.000	-0.4200	0.5345
5.000	-0.4700	0.4593
6.000	-0.5000	0.4048
7.000	-0.5200	0.3632
8.000	-0.5400	0.3304
9.000	-0.5500	0.3038
10.000	-0.5700	0.2817
15.000	-0.6100	0.2101
20.000	-0.6400	0.1703
25.000	-0.6600	0.1447
30.000	-0.6800	0.1265
35.000	-0.6900	0.1129
40.000	-0.7000	0.1024
50.000	-0.7200	0.0868
60.000	-0.7300	0.0759
70.000	-0.7400	0.0677
80.000	-0.7500	0.0613
90.000	-0.7600	0.0562
100.000	-0.7700	0.0520

B=4.0 ETA=100.0 FMT= 90.62 SD= 25.47 ASSD= 3.55740

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.4500	0.9997
1.200	0.1800	0.9945
1.300	0.0500	0.9800
1.400	-0.0300	0.9595
1.500	-0.0800	0.9359
1.600	-0.1200	0.9112
1.700	-0.1600	0.8862
1.800	-0.1900	0.8616
1.900	-0.2100	0.8379
2.000	-0.2300	0.8150
3.000	-0.3600	0.6383
4.000	-0.4200	0.5276
5.000	-0.4600	0.4526
6.000	-0.4900	0.3983
7.000	-0.5100	0.3571
8.000	-0.5300	0.3245
9.000	-0.5500	0.2981
10.000	-0.5600	0.2762
15.000	-0.6100	0.2055
20.000	-0.6400	0.1663
25.000	-0.6600	0.1410
30.000	-0.6700	0.1231
35.000	-0.6900	0.1098
40.000	-0.7000	0.0994
50.000	-0.7100	0.0842
60.000	-0.7300	0.0735
70.000	-0.7400	0.0655
80.000	-0.7500	0.0593
90.000	-0.7500	0.0543
100.000	-0.7600	0.0502

B=4.1 ETA=100.0 FMT= 90.75 SD= 24.95 ASSD= 3.63735

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.4200	0.9996
1.200	0.1600	0.9938
1.300	0.0400	0.9785
1.400	-0.0300	0.9572
1.500	-0.0900	0.9329
1.600	-0.1300	0.9074
1.700	-0.1600	0.8520
1.800	-0.1900	0.8570
1.900	-0.2100	0.8329
2.000	-0.2400	0.8098
3.000	-0.3600	0.6318
4.000	-0.4200	0.5211
5.000	-0.4600	0.4463
6.000	-0.4900	0.3922
7.000	-0.5100	0.3512
8.000	-0.5300	0.3189
9.000	-0.5400	0.2927
10.000	-0.5600	0.2711
15.000	-0.6000	0.2011
20.000	-0.6300	0.1624
25.000	-0.6500	0.1375
30.000	-0.6700	0.1200
35.000	-0.6800	0.1069
40.000	-0.6900	0.0967
50.000	-0.7100	0.0818
60.000	-0.7200	0.0713
70.000	-0.7300	0.0635
80.000	-0.7400	0.0574
90.000	-0.7500	0.0526
100.000	-0.7500	0.0485

B=4.2 ETA=100.0 FMT= 90.88 SD= 24.45 ASSD= 3.71714

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.3900	0.9996
1.200	0.1500	0.9932
1.300	0.0300	0.9770
1.400	-0.0400	0.9548
1.500	-0.0900	0.9298
1.600	-0.1300	0.9038
1.700	-0.1700	0.8779
1.800	-0.1900	0.8525
1.900	-0.2200	0.8281
2.000	-0.2400	0.8047
3.000	-0.3500	0.6257
4.000	-0.4100	0.5149
5.000	-0.4500	0.4403
6.000	-0.4800	0.3865
7.000	-0.5000	0.3457
8.000	-0.5200	0.3136
9.000	-0.5400	0.2877
10.000	-0.5500	0.2662
15.000	-0.6000	0.1970
20.000	-0.6200	0.1588
25.000	-0.6400	0.1343
30.000	-0.6600	0.1170
35.000	-0.6700	0.1042
40.000	-0.6800	0.0942
50.000	-0.7000	0.0795
60.000	-0.7100	0.0693
70.000	-0.7200	0.0616
80.000	-0.7300	0.0557
90.000	-0.7400	0.0509
100.000	-0.7500	0.0470

B=4.3 ETA=100.0 FMT= 91.00 SD= 23.97 ASSD= 3.79677

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.3700	0.9995
1.200	0.1400	0.9925
1.300	0.0200	0.9755
1.400	-0.0500	0.9525
1.500	-0.1000	0.9268
1.600	-0.1400	0.9003
1.700	-0.1700	0.8739
1.800	-0.2000	0.8482
1.900	-0.2200	0.8234
2.000	-0.2400	0.7998
3.000	-0.3500	0.6198
4.000	-0.4100	0.5089
5.000	-0.4500	0.4345
6.000	-0.4800	0.3810
7.000	-0.5000	0.3404
8.000	-0.5200	0.3086
9.000	-0.5300	0.2829
10.000	-0.5500	0.2616
15.000	-0.5900	0.1931
20.000	-0.6200	0.1554
25.000	-0.6400	0.1312
30.000	-0.6500	0.1143
35.000	-0.6700	0.1016
40.000	-0.6800	0.0918
50.000	-0.6900	0.0774
60.000	-0.7100	0.0674
70.000	-0.7200	0.0599
80.000	-0.7300	0.0541
90.000	-0.7300	0.0494
100.000	-0.7400	0.0456

B=4.4 ETA=100.0 FMT= 91.12 SD= 23.51 ASSD= 3.87633

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.3500	0.9995
1.200	0.1200	0.9918
1.300	0.0200	0.9740
1.400	-0.0500	0.9502
1.500	-0.1000	0.9239
1.600	-0.1400	0.8968
1.700	-0.1700	0.8700
1.800	-0.2000	0.8440
1.900	-0.2200	0.8189
2.000	-0.2400	0.7950
3.000	-0.3500	0.6141
4.000	-0.4100	0.5033
5.000	-0.4500	0.4291
6.000	-0.4700	0.3758
7.000	-0.5000	0.3355
8.000	-0.5100	0.3039
9.000	-0.5300	0.2783
10.000	-0.5400	0.2572
15.000	-0.5800	0.1895
20.000	-0.6100	0.1522
25.000	-0.6300	0.1284
30.000	-0.6500	0.1117
35.000	-0.6600	0.0992
40.000	-0.6700	0.0896
50.000	-0.6900	0.0755
60.000	-0.7000	0.0656
70.000	-0.7100	0.0583
80.000	-0.7200	0.0526
90.000	-0.7300	0.0480
100.000	-0.7300	0.0443

B=4.5 ETA=100.0 FMT= 91.24 SD= 23.06 ASSD= 3.95578

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.3300	0.9994
1.200	0.1100	0.9911
1.300	0.0100	0.9725
1.400	-0.0600	0.9480
1.500	-0.1100	0.9211
1.600	-0.1400	0.8935
1.700	-0.1700	0.8663
1.800	-0.2000	0.8399
1.900	-0.2200	0.8145
2.000	-0.2400	0.7904
3.000	-0.3500	0.6085
4.000	-0.4100	0.4979
5.000	-0.4400	0.4239
6.000	-0.4700	0.3708
7.000	-0.4900	0.3308
8.000	-0.5100	0.2994
9.000	-0.5200	0.2740
10.000	-0.5400	0.2531
15.000	-0.5800	0.1860
20.000	-0.6100	0.1492
25.000	-0.6300	0.1257
30.000	-0.6400	0.1092
35.000	-0.6500	0.0970
40.000	-0.6600	0.0875
50.000	-0.6800	0.0736
60.000	-0.6900	0.0640
70.000	-0.7000	0.0568
80.000	-0.7100	0.0512
90.000	-0.7200	0.0467
100.000	-0.7300	0.0430

B=4.6 ETA=100.0 FMT= 91.35 SD= 22.64 ASSD= 4.03496

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.3100	0.9993
1.200	0.1000	0.9904
1.300	0.0	0.9709
1.400	-0.0600	0.9458
1.500	-0.1100	0.9183
1.600	-0.1500	0.8902
1.700	-0.1800	0.8626
1.300	-0.2000	0.8359
1.900	-0.2200	0.8103
2.000	-0.2400	0.7860
3.000	-0.3500	0.6035
4.000	-0.4000	0.4927
5.000	-0.4400	0.4189
6.000	-0.4700	0.3661
7.000	-0.4900	0.3263
8.000	-0.5000	0.2951
9.000	-0.5200	0.2699
10.000	-0.5300	0.2492
15.000	-0.5700	0.1827
20.000	-0.6000	0.1464
25.000	-0.6200	0.1232
30.000	-0.6400	0.1070
35.000	-0.6500	0.0949
40.000	-0.6600	0.0855
50.000	-0.6800	0.0719
60.000	-0.6900	0.0624
70.000	-0.7000	0.0553
80.000	-0.7100	0.0499
90.000	-0.7100	0.0455
100.000	-0.7200	0.0419

B=4.7 ETA=100.0 FMT= 91.47 SD= 22.23 ASSD= 4.11423

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.2900	0.9992
1.200	0.0900	0.9897
1.300	-0.0100	0.9695
1.400	-0.0700	0.9436
1.500	-0.1100	0.9155
1.600	-0.1500	0.8870
1.700	-0.1800	0.8590
1.800	-0.2000	0.8320
1.900	-0.2200	0.8062
2.000	-0.2400	0.7817
3.000	-0.3500	0.5985
4.000	-0.4000	0.4878
5.000	-0.4400	0.4142
6.000	-0.4600	0.3616
7.000	-0.4800	0.3220
8.000	-0.5000	0.2910
9.000	-0.5100	0.2661
10.000	-0.5300	0.2455
15.000	-0.5700	0.1796
20.000	-0.6000	0.1438
25.000	-0.6100	0.1209
30.000	-0.6300	0.1048
35.000	-0.6400	0.0929
40.000	-0.6500	0.0837
50.000	-0.6700	0.0703
60.000	-0.6800	0.0609
70.000	-0.6900	0.0540
80.000	-0.7000	0.0486
90.000	-0.7100	0.0443
100.000	-0.7200	0.0408

B=4.8 ETA=100.0 FMT= 91.58 SD= 21.84 ASSD= 4.19332

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.2700	0.9992
1.200	0.0800	0.9890
1.300	-0.0100	0.9680
1.400	-0.0700	0.9415
1.500	-0.1200	0.9128
1.600	-0.1500	0.8839
1.700	-0.1800	0.8556
1.800	-0.2000	0.8283
1.900	-0.2200	0.8022
2.000	-0.2400	0.7775
3.000	-0.3400	0.5937
4.000	-0.4000	0.4831
5.000	-0.4300	0.4097
6.000	-0.4600	0.3573
7.000	-0.4800	0.3180
8.000	-0.4900	0.2872
9.000	-0.5100	0.2624
10.000	-0.5200	0.2419
15.000	-0.5600	0.1767
20.000	-0.5900	0.1412
25.000	-0.6100	0.1186
30.000	-0.6200	0.1028
35.000	-0.6400	0.0911
40.000	-0.6500	0.0820
50.000	-0.6600	0.0688
60.000	-0.6800	0.0596
70.000	-0.6900	0.0528
80.000	-0.7000	0.0475
90.000	-0.7000	0.0433
100.000	-0.7100	0.0398

B=4.9 ETA=100.0 FMT= 91.69 SD= 21.46 ASSD= 4.27226

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.2600	0.9991
1.200	0.0700	0.9883
1.300	-0.0200	0.9665
1.400	-0.0800	0.9393
1.500	-0.1200	0.9102
1.600	-0.1500	0.8809
1.700	-0.1800	0.8522
1.800	-0.2000	0.8246
1.900	-0.2200	0.7984
2.000	-0.2400	0.7735
3.000	-0.3400	0.5890
4.000	-0.3900	0.4785
5.000	-0.4300	0.4054
6.000	-0.4500	0.3533
7.000	-0.4700	0.3141
8.000	-0.4900	0.2835
9.000	-0.5000	0.2589
10.000	-0.5200	0.2386
15.000	-0.5600	0.1740
20.000	-0.5800	0.1388
25.000	-0.6000	0.1165
30.000	-0.6200	0.1008
35.000	-0.6300	0.0893
40.000	-0.6400	0.0803
50.000	-0.6600	0.0673
60.000	-0.6700	0.0583
70.000	-0.6800	0.0516
80.000	-0.6900	0.0464
90.000	-0.7000	0.0423
100.000	-0.7000	0.0389

B=5.0 ETA=100.0 FMT= 91.80 SD= 21.10 ASSD= 4.35110

RELATIVE COST OF FAILURE REPLACEMENT	OPTIMAL AGE	STD. COST RATIO
1.100	0.2400	0.9990
1.200	0.0700	0.9875
1.300	-0.0200	0.9650
1.400	-0.0800	0.9373
1.500	-0.1200	0.9076
1.600	-0.1500	0.8779
1.700	-0.1800	0.8489
1.800	-0.2000	0.8211
1.900	-0.2200	0.7946
2.000	-0.2400	0.7696
3.000	-0.3400	0.5846
4.000	-0.3900	0.4742
5.000	-0.4300	0.4013
6.000	-0.4500	0.3493
7.000	-0.4700	0.3104
8.000	-0.4900	0.2800
9.000	-0.5000	0.2555
10.000	-0.5100	0.2354
15.000	-0.5500	0.1713
20.000	-0.5800	0.1365
25.000	-0.6000	0.1144
30.000	-0.6100	0.0990
35.000	-0.6300	0.0876
40.000	-0.6400	0.0788
50.000	-0.6500	0.0660
60.000	-0.6600	0.0571
70.000	-0.6800	0.0505
80.000	-0.6800	0.0454
90.000	-0.6900	0.0413
100.000	-0.7000	0.0380

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5.2 Graphical Presentation of Age-Replacement Policies:

Optimal age-replacement policies given in Table 5.1 have been produced in graphical form (Figs. 5.1 and 5.2). For the purpose semi-log papers have been used. The values of relative cost of failure replacement k have been taken as the ordinate in the log scale and β values have been taken as the abscissa in the ordinary scale. The plottings give Z-curves (Fig. 5.1) and standardized cost (ρ) curves (Fig. 5.2). Each curve represents a particular value of Z (Fig. 5.1) or a particular value of standardized cost (ρ) (Fig. 5.2).

Fig. 5.1 Optimal Policies (Z Curves) Under Age Replacement (Weibull distribution)

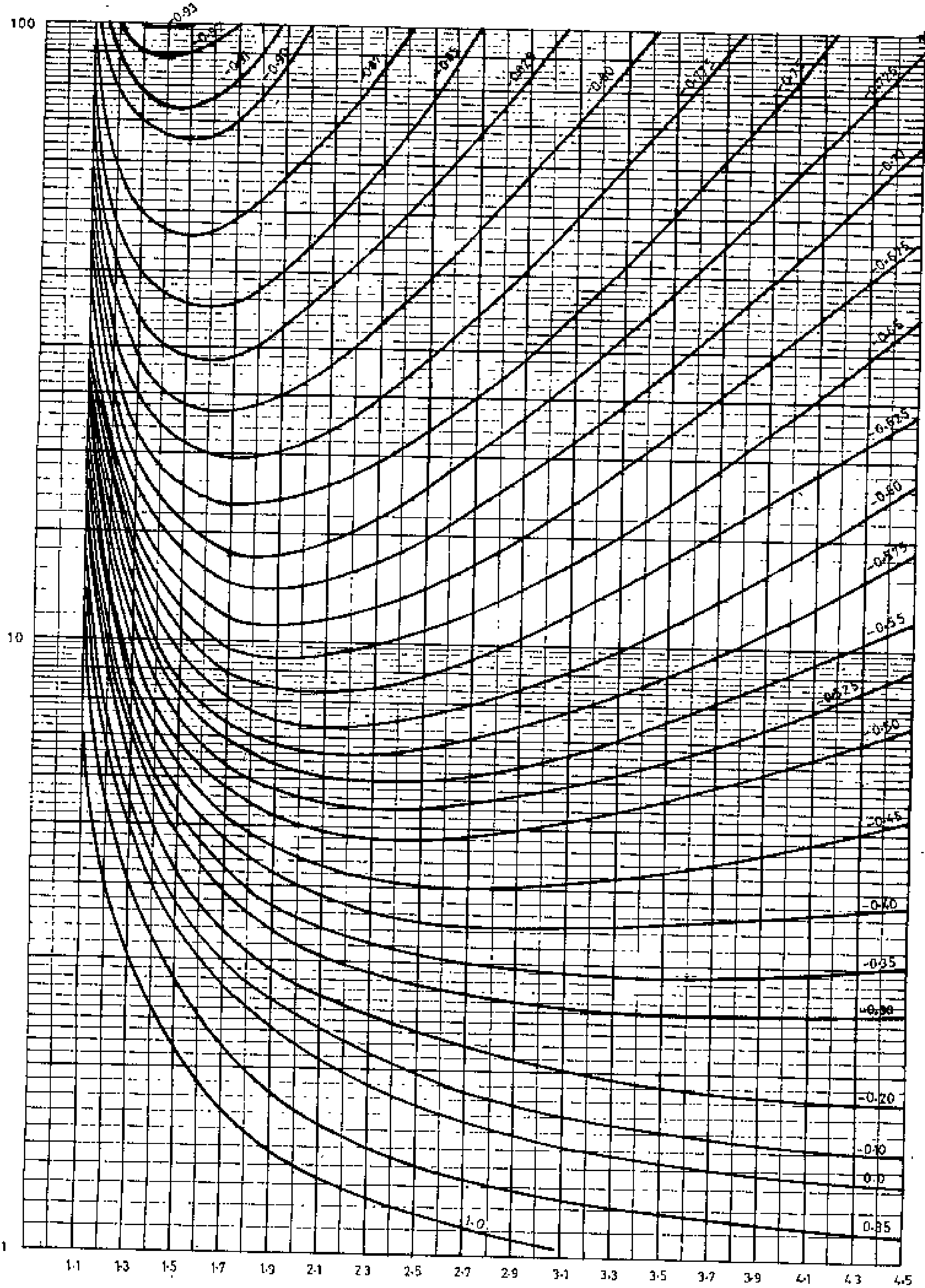
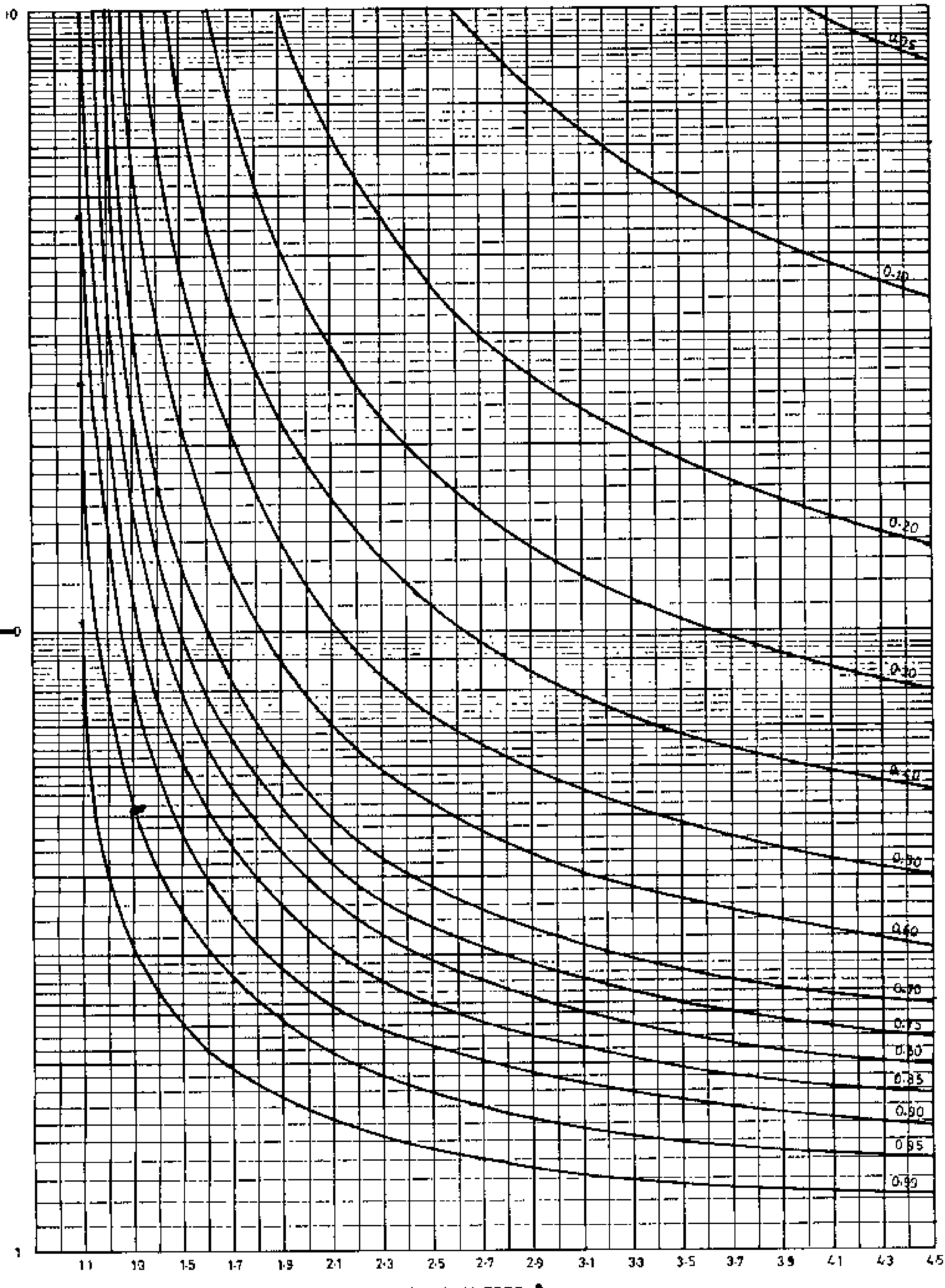


Fig. 5.2 Standardized Cost Curves (β) For Optimal Policies Under Age Replacement (Weibull distribution)



5.3 Discussion:

The new graphical presentation for optimal age replacement policies as given in Figs. 5.1 and 5.2, produces sets of very uniform curves. Although there was no need of two plottings, it has been done so for clear presentation of the curves. Having obtained the value of Z for a certain value of shape parameter β and relative cost of failure replacement κ, (or, R as in the computer programme) optimal replacement age (t_p) can be readily determined by using equation (5.1.2). Ofcourse, failure distribution parameters (β,η) for the respective item for which the replacement policy is being determined, are to be evaluated first from the failure history of the item.

The discretization of t_p values may result in some errors in determining optimal value of t_p, but the choice of smallest time unit like day or hour as appropriate, will minimize this error. So, the methodology applied is fairly adequate.

Comparison of the results of the present solution with those of Glasser has been made. From Glasser's graph, optimal replacement age t_p has been determined for a number of values of cost ratio and average service life in standard deviation units (i.e. μ/σ). μ and σ have, in turn, been evaluated from β and η by the relationships given in equations (2.2.3) and presented in Appendix-E. Glasser's t_p has been converted to Z value as given in the present solution procedure.

Table 5.3.1 shows the comparative Z values and standardized cost values. It is observed that there are some discrepancies in Z values for very small and very high cost ratios for lower values of β. This can only be explained by the fact that in case of Glasser's solution, Z-values have been calculated from Glasser's graph and graph reading could not be done as accurately as would have been desired for a good comparison. Nevertheless most of the Z-values are fairly close. The values of standardized costs are also fairly close. (Table 5.3.1 shows present model solution vs. Glasser's solution).

Table 5.3.1

Present Model Soln. Vs. Glasser's Soln.

$\beta = 1.1$ $\eta = 100$ $\mu = 96.49$ $\sigma = 87.83$ $v = \mu/\sigma = 1.09858$

Cost ratio K	From Glasser's Graph Optimal age Z	Standard Cost ratio ρ	Present Model Solution Optimal age Z	Standard cost ratio ρ
1.1	-	-	BM	-
1.5	-	-	BM	-
2.0	-	-	BM	-
10.0	0.448	0.990	0.46	0.992
15.0	-0.140	0.975	-0.13	0.977
20.0	-0.386	0.960	-0.37	0.962
25.0	-0.473	0.950	-0.50	0.950
35.0	-0.605	0.920	-0.65	0.928
50.0	-0.737	0.895	-0.75	0.904
70.0	-0.820	0.878	-0.82	0.880
100.0	-0.877	0.850	-0.87	0.855

$\beta = 1.2$ $\eta = 100$ $\mu = 94.6$ $\sigma = 78.73$ $v = 1.19479$

Cost ratio K	From Glasser's Graph Optimal age Z	Standard cost ratio ρ	Present Model Solution Optimal age Z	Standard cost ratio ρ
1.1	-	-	BM	-
1.5	-	-	BM	-
2.0	-	-	BM	-
10	-0.334	0.938	-0.31	0.942
15	-0.531	0.900	-0.54	0.900
20	-0.649	0.860	-0.65	0.867
25	-0.720	0.840	-0.72	0.841
35	-0.780	0.806	-0.78	0.801
50	-0.840	0.760	-0.85	0.759
70	-0.885	0.750	-0.89	0.720
100	-0.914	0.708	-0.92	0.680

BM - Break down maintenance.

$\beta = 1.3 \quad \eta = 100 \quad \mu = 92.35 \quad \sigma = 71.65 \quad \nu = 1.28901$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	-	-	BM	-
1.5	-	-	BM	-
2.0	3.504	-	BM	-
10	-0.498	0.88	-0.50	0.8760
15	-0.647	0.82	-0.65	0.814
20	-0.722	0.77	-0.73	0.770
25	-0.763	0.75	-0.78	0.736
35	-0.806	0.69	-0.83	0.585
50	-0.863	0.625	-0.87	0.635
70	-0.928	0.576	-0.90	0.589
100	-0.929	0.53	-0.93	0.544

$\beta = 1.4 \quad \eta = 100 \quad \mu = 91.14 \quad \sigma = 65.97 \quad \nu = 1.38152$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	-	-	BM	-
1.5	-	-	BM	-
2.0	1.989	0.99	2.06	0.999
10	-0.579	0.82	-0.58	0.810
15	-0.711	0.72	-0.70	0.735
20	-0.740	0.68	-0.76	0.684
25	-0.803	0.64	-0.80	0.645
35	-0.833	0.58	-0.84	0.590
50	-0.885	0.52	-0.88	0.536
70	-0.925	0.49	-0.91	0.488
100	-0.921	0.44	-0.93	0.442

$\beta = 1.5$ $\eta = 100$ $\mu = 90.27$ $\sigma = 61.30$ $v = 1.4726$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard-cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	-	-	BM	-
1.5	-	-	BM	-
2.0	1.164	0.99	1.17	0.908
10	-0.516	0.75	-0.62	0.749
15	-0.709	0.67	-0.72	0.667
20	-0.733	0.60	-0.77	0.611
25	-0.782	0.58	-0.81	0.571
35	-0.846	0.50	-0.85	0.513
50	-0.883	0.46	-0.88	0.458
70	-0.908	0.40	-0.91	0.411
100	-0.954	0.36	-0.94	0.365

$\beta = 1.6$ $\eta = 100$ $\mu = 89.65$ $\sigma = 57.38$ $v = 1.56246$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	-	-	BM	-
1.5	1.616	-	BM	
2.0	0.756	0.393	0.73	0.995
10	-0.618	0.70	-0.64	0.695
15	-0.721	0.61	-0.73	0.608
20	-0.790	0.55	-0.78	0.550
25	-0.813	0.51	-0.81	0.501
35	-0.842	0.45	-0.85	0.451
50	-0.882	0.40	-0.88	0.396
70	-0.925	0.35	-0.90	0.350
100	-0.925	0.31	-0.92	0.307

$\beta = 1.7$ $\eta = 100$ $\mu = 89.22$ $\sigma = 54.03$ $\nu = 1.65124$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	-	-	BM	-
1.5	1.511	0.994	1.6	0.999
2.0	0.432	0.982	0.46	0.969
10	-0.646	0.65	-0.66	0.647
15	-0.754	0.55	-0.74	0.557
20	-0.781	0.50	-0.78	0.499
25	-0.808	0.46	-0.81	0.457
35	-0.845	0.40	-0.84	0.400
50	-0.889	0.35	-0.87	0.347
70	-0.911	0.30	-0.90	0.303
100	-0.927	0.25	-0.92	0.262

$\beta = 1.8$ $\eta = 100$ $\mu = 88.92$ $\sigma = 51.13$ $\nu = 1.73969$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	-	-	BM	-
1.5	1.219	0.99	1.19	0.999
2.0	0.298	0.98	0.29	0.983
10	-0.670	0.60	-0.65	0.605
15	-0.748	0.51	-0.74	0.514
20	-0.773	0.46	-0.78	0.456
25	-0.809	0.40	-0.81	0.415
35	-0.835	0.36	-0.84	0.359
50	-0.875	0.30	-0.87	0.307
70	-0.918	0.26	-0.89	0.265
100	-0.926	0.23	-0.91	0.227

$\beta = 1.9$ $\eta = 100$ $\mu = 88.73$ $\sigma = 48.59$ $v = 182611$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	-	-	BM	-
1.5	1.051	0.99	0.90	0.998
2.0	0.178	0.96	0.18	0.975
10	-0.684	0.57	-0.66	0.569
15	-0.727	0.47	-0.73	0.477
20	-0.776	0.42	-0.77	0.419
25	-0.791	0.38	-0.80	0.379
35	-0.839	0.32	-0.83	0.324
50	-0.854	0.27	-0.86	0.275
70	-0.888	0.23	-0.89	0.235
100	-0.907	0.20	-0.91	0.199

$\beta = 2.0$ $\eta = 100$ $\mu = 88.62$ $\sigma = 46.34$ $v = 1.91243$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	-	-	BM	-
1.5	0.681	0.99	0.69	0.997
2.0	0.081	0.96	0.090	0.956
10	-0.676	0.54	-0.66	0.536
15	-0.745	0.44	-0.73	0.444
20	-0.765	0.39	-0.77	0.388
25	-0.797	0.35	-0.80	0.348
35	-0.822	0.30	-0.83	0.296
50	-0.852	0.24	-0.86	0.248
70	-0.869	0.21	-0.88	0.210
100	-0.899	0.18	-0.90	0.176

β = 2.1 $\quad n$ = 100 $\quad \mu$ = 88.56 $\quad \sigma$ = 44.32 $\quad v$ = 199808

Cost ratio K	From Glasser's Graph: Optimal age Z	From Glasser's Graph: Standard cost ratio ρ	Present Model Solution: Optimal age	Present Model Solution: Standard cost ratio ρ
1.1	-	-	BM	-
1.5	0.557	0.99	0.54	0.995
2.0	0.033	0.95	0.03	0.957
10	-0.666	0.50	-0.66	0.508
15	-0.723	0.42	-0.73	0.416
20	-0.754	0.36	-0.76	0.361
25	-0.778	0.30	-0.79	0.322
35	-0.812	0.27	-0.82	0.271
50	-0.854	0.22	-0.85	0.226
70	-0.860	0.20	-0.87	0.190
100	-0.880	0.16	-0.89	0.158

β = 2.2 $\quad n$ = 100 $\quad \mu$ = 88,55 $\quad \sigma$ = 42.51 $\quad v$ = 2.08318

Cost ratio K	From Glasser's Graph: Optimal age Z	From Glasser's Graph: Standard cost ratio ρ	Present Model Solution: Optimal age Z	Present Model Solution: Standard cost ratio ρ
1.1	-	-	BM	-
1.5	0.4221	0.99	0.43	0.993
2.0	-0.03	0.94	-0.02	0.948
10	-0.652	0.47	-0.66	0.482
15	-0.707	0.39	-0.72	0.392
20	-0.749	0.33	-0.76	0.337
25	-0.766	0.30	-0.78	0.300
35	-0.802	0.25	-0.81	0.250
50	-0.825	0.20	-0.84	0.207
70	-0.858	0.17	-0.87	0.173
100	-0.879	0.14	-0.89	0.142

$\beta = 2.3$ $\eta = 100$ $\mu = 88.58$ $\sigma = 40.96$ $v = 2.16776$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	-	-	8M	-
1.5	0.322	0.99	0.34	0.991
2.0	-0.053	0.94	0.06	0.939
10	-0.638	0.46	-0.65	0.450
15	-0.707	0.37	-0.72	0.370
20	-0.739	0.32	-0.75	0.317
25	-0.770	0.28	-0.78	0.290
35	-0.789	0.23	-0.81	0.233
50	-0.805	0.20	-0.84	0.191
70	-0.831	0.161	-0.86	0.159
100	-0.880	0.13	-0.88	0.129

$\beta = 2.4$ $\eta = 100$ $\mu = 88.64$ $\sigma = 39.36$ $v = 2.25186$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	1.848	-	BM	-
1.5	0.279	0.98	0.27	0.988
2.0	-0.116	0.93	-0.09	0.930
10	-0.650	0.429	-0.65	0.439
15	-0.701	0.35	-0.71	0.351
20	-0.734	0.30	-0.74	0.299
25	-0.750	0.26	-0.77	0.263
35	-0.785	0.22	-0.80	0.217
50	-0.829	0.18	-0.83	0.1777
70	-0.846	0.14	-0.85	0.145
100	-0.858	0.12	-0.87	0.118

$\beta = 2.7$ $\eta = 100$ $\mu = 88.99$ $\sigma = 35.54$ $v = 2.50175$

Cost ratio K	From Glasser's Graph: Optimal age Z	From Glasser's Graph: Standard cost ratio ρ	Present Model Solution: Optimal age Z	Present Model Solution: Standard cost ratio ρ
1.1	1.305	-	BM	-
1.5	0.1377	0.98	0.13	0.979
2.0	-0.162	0.90	-0.15	0.903
10	-0.628	0.40	-0.63	0.390
15	-0.682	0.30	-0.69	0.305
20	-0.711	0.25	-0.72	0.250
25	-0.747	0.22	-0.75	0.223
35	-0.786	0.19	-0.79	0.181
50	-0.800	0.14	-0.81	0.145
70	-0.838	0.12	-0.83	0.117
100	-0.842	0.09	-0.85	0.094

$\beta = 2.8$ $\eta = 100$ $\mu = 89.03$ $\sigma = 34.45$ $v = 2.58440$

Cost ratio K	From Glasser's Graph: Optimal age Z	From Glasser's Graph: Standard cost ratio ρ	Present Model Solution: Optimal age Z	Present Model Solution: Standard cost ratio ρ
1.1	1.120	-	BM	-
1.5	0.121	0.97	0.09	0.976
2.0	-0.164	0.89	-0.17	0.895
10	-0.653	0.37	-0.63	0.375
15	-0.679	0.28	-0.68	0.293
20	-0.708	0.24	-0.72	0.245
25	-0.734	0.21	-0.74	0.213
35	-0.757	0.17	-0.77	0.172
50	-0.808	0.14	-0.80	0.137
70	-0.837	0.11	-0.82	0.110
100	-0.852	0.08	-0.84	0.088

$\beta = 2.5$ $\eta = 100$ $\mu = 88.72$ $\sigma = 37.99$ $\nu = 2.33554$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	1.742	-	BM	-
1.5	0.221	0.98	0.21	0.985
2.0	-0.134	0.92	-0.12	0.921
10	-0.653	0.42	-0.65	0.421
15	-0.698	0.33	-0.70	0.334
20	-0.726	0.28	-0.74	0.283
25	-0.747	0.25	-0.76	0.248
35	-0.782	0.20	-0.79	0.204
50	-0.817	0.16	-0.82	0.165
70	-0.826	0.13	-0.84	0.135
100	-0.8690	0.11	-0.86	0.109

$\beta = 2.6$ $\mu = 88.81$ $\sigma = 36.72$ $\eta = 100$ $\nu = 2.41861$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	1.534	-	BM	-
1.5	0.181	0.98	0.17	0.982
2.0	-0.147	0.90	-0.14	0.912
10	-0.647	0.40	-0.64	0.405
15	-0.684	0.32	-0.70	0.319
20	-0.717	0.27	-0.73	0.269
25	-0.742	0.23	-0.75	0.235
35	-0.773	0.19	-0.78	0.192
50	-0.796	0.15	-0.81	0.154
70	-0.834	0.13	-0.84	0.126
100	-0.858	0.10	-0.86	0.101

$\beta = 2.9$ $\eta = 100$ $\mu = 89.16$ $\sigma = 33.43$ $v = 2.66670$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standare cost ratio ρ
1.1	0.827	-	BM	-
1.5	0.082	0.97	0.07	0.972
2.0	-0.171	0.88	-0.18	0.887
10	-0.623	0.36	-0.62	0.364
15	-0.679	0.28	-0.68	0.282
20	-0.702	0.23	-0.71	0.234
25	-0.710	0.20	-0.73	0.202
35	-0.736	0.16	-0.76	0.163
50	-0.790	0.13	-0.79	0.130
70	-0.817	0.10	-0.81	0.104
100	-0.843	0.08	-0.84	0.082

$\beta = 3.0$ $\eta = 100$ $\mu = 89.29$ $\sigma = 32.48$ v 2.74673

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.867	-	BM	-
1.5	0.055	0.96	0.04	0.969
2.0	-0.178	0.88	-0.19	0.879
10	-0.613	0.35	-0.62	0.352
15	-0.662	0.27	-0.67	0.271
20	-0.697	0.22	-0.70	0.225
25	-0.709	0.19	-0.72	0.195
35	-0.753	0.15	-0.75	0.156
50	-0.782	0.12	-0.78	0.123
70	-0.815	0.09	-0.81	0.099
100	-0.837	0.075	-0.83	0.781

$\beta = 3.1$ $\eta = 100$ $\mu = 89.42$ $\sigma = 31.59$ $v = 2.83050$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.841	-	BM	-
1.5	0.020	0.96	0.02	0.966
2.0	-0.194	0.87	-0.20	0.871
10	-0.608	0.34	-0.61	0.342
15	-0.654	0.26	-0.66	0.262
20	-0.679	0.22	-0.70	0.217
25	-0.718	0.18	-0.72	0.187
35	-0.743	0.15	-0.75	0.149
50	=0.775	0.12	-0.78	0.117
70	-0.801	0.09	-0.80	0.093
100	-0.826	0.072	-0.82	0.073

$\beta = 3.2$ $\eta = 100$ $\mu = 89.55$ $\sigma = 30.75$ $v = 2.91206$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.817	-	BM	=
1.5	-0.003	0.96	0.00	0.962
2.0	-0.196	0.85	0.21	0.864
10	-0.602	0.33	-0.61	0.332
15	-0.642	0.25	-0.66	0.254
20	-0.680	0.21	-0.69	0.209
25	-0.716	0.18	-0.71	0.180
35	-0.744	0.14	-0.74	0.143
50	-0.768	0.108	-0.77	0.112
70	-0.784	0.08	-0.79	0.89
100	-0.814	0.070	-0.81	0.070

$\beta = 3.3$ $\eta = 100$ $\mu = 89.69$ $\sigma = 29.96$ $v = 2.99340$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.791	-	BM	-
1.5	-0.010	0.95	-0.01	0.959
2.0	-0.216	0.86	-0.21	0.857
10	-0.589	0.32	-0.60	0.323
15	-0.627	0.25	-0.65	0.246
20	-0.666	0.20	-0.68	0.202
25	-0.702	0.17	-0.70	0.173
35	-0.732	0.14	-0.73	0.137
50	-0.762	0.11	-0.76	0.107
70	-0.789	0.08	-0.78	0.085
100	-0.814	0.65	-0.81	0.066

$\beta = 3.4$ $\eta = 100$ $\mu = 89.82$ $\sigma = 29.22$ $v = 3.07450$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.765	-	BM	-
1.5	-0.029	0.95	-0.03	0.955
2.0	-0.218	0.85	-.22	0.850
10	-0.578	0.32	-0.59	0.315
15	-0.627	0.24	-0.64	0.239
20	-0.664	0.20	-0.67	0.196
25	-0.696	0.17	-0.70	0.167
35	-0.721	0.13	-0.73	0.132
50	-0.745	0.10	-0.75	0.103
70	-0.779	0.08	-0.78	0.081
100	-0.802	0.06	-0.80	0.063

$\beta = 3.5$ $\eta = 100$ $\mu = 89.96$ $\sigma = 28.51$ $v = 3.15545$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.709	-	BM	-
1.5	-0.046	0.94	-0.04	0.952
2.0	-0.219	0.84	-0.22	0.844
10	-0.582	0.30	-0.59	0.307
15	-0.629	0.23	-0.64	0.232
20	-0.670	0.18	-0.67	0.190
25	-0.694	0.16	-0.69	0.162
35	-0.721	0.13	-0.72	0.128
50	-0.729	0.09	-0.75	0.099
70	-0.77	0.08	-0.77	0.078
100	-0.785	0.06	-0.79	0.060

$\beta = 3.6$ $\eta = 100$ $\mu = 90.10$ $\sigma = 27.84$ $v = 3.23814$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.661	-	BM	-
1.5	-0.065	0.94	-0.05	0.948
2.0	-0.217	0.84	-0.22	0.838
10	-0.579	0.30	-0.58	0.300
15	-0.627	0.22	-0.63	0.226
20	-0.661	0.18	-0.66	0.184
25	-0.680	0.16	-0.68	0.157
35	-0.699	0.12	-0.71	0.123
50	-0.739	0.09	-0.74	0.096
70	-0.764	0.07	-0.76	0.075
100	-0.783	-.05	-0.79	0.058

$\beta = 3.7$ $\eta = 100$ $\mu = 90.23$ $\sigma = 27.20$ $v = 3.31667$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standare cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.531	0.99	0.54	0.999
1.5	-0.075	0.94	-0.60	0.945
2.0	-0.220	0.828	-0.23	0.832
10	-0.568	0.28	-0.58	0.293
15	-0.622	0.22	-0.63	0.220
20	-0.660	0.17	-0.65	0.179
25	-0.679	0.15	-0.68	0.152
35	-0.697	0.12	-0.71	0.120
50	-0.735	0.09	-0.73	0.092
70	-0.750	0.07	-0.76	0.073
100	-0.772	0.05	-0.78	0.056

$\beta = 3.8$ $\eta = 100$ $\mu = 90.36$ $\sigma = 26.60$- $v = 3.39706$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.542	0.99	0.52	0.999
1.5	-0.073	0.94	-0.07	0.942
2.0	-0.226	0.83	-0.23	0.826
10	-0.561	0.29	-0.57	0.287
15	-0.623	0.205	-0.62	0.215
20	-0.652	0.17	-0.655	0.174
25	-0.668	0.15	-0.67	0.148
35	-0.694	0.12	-0.70	0.116
50	-0.732	0.08	-0.73	0.089
70	-0.735	0.07	-0.75	0.070
100	-0.7650	0.52	-0.77	0.053

$\beta = 3.9$ $\eta = 100$ $\mu = 90.49$ $\sigma = 25.02$ $v = 3.47720$

Cost ratio K	From Glasser's Graph: Optimal age Z	From Glasser's Graph: Standard cost ratio ρ	Present Model Solution: Optimal age Z	Present Model Solution: Standard cost ratio ρ
1.1	0.509	0.99	0.46	0.999
1.5	-0.074	0.94	-0.07	0.939
2.0	-0.224	0.82	-0.23	0.820
10	-0.551	0.28	-0.57	0.281
15	-0.615	0.21	-0.61	0.210
20	-0.641	0.17	-0.64	0.170
25	-0.664	0.14	-0.66	0.144
35	-0.690	0.11	-0.69	0.112
50	-0.719	0.085	-0.72	0.086
70	-0.742	0.06	-0.74	0.067
100	-0.761	0.050	-0.77	0.052

$\beta = 4.0$ $\eta = 100$ $\mu = 90.62$ $\sigma = 25.47$ $v = 3.55740$

Cost ratio K	From Glasser's Graph: Optimal age Z	From Glasser's Graph: Standard cost ratio ρ	Present Model Solution: Optimal age Z	Present Model Solution: Standard cost ratio ρ
1.1	0.462	0.99	0.45	0.999
1.5	-0.093	0.93	-0.08	0.935
2.0	-0.221	0.80	-0.23	0.815
10	-0.554	0.27	-0.56	0.276
15	-0.613	0.20	-0.61	0.205
20	-0.638	0.17	-0.64	0.166
25	-0.654	0.14	-0.66	0.141
35	-0.687	0.11	-0.69	0.109
50	-0.705	0.08	-0.71	0.084
70	-0.734	0.06	-0.74	0.065
100	-0.753	0.05	-0.76	0.050

β = 4.1 η = 100 μ = 90.75 σ = 24.95 ν = 3.63735

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age	Standard cost ratio ρ
1.1	0.408	0.99	0.42	0.999
1.5	-0.092	0.93	-0.09	0.932
2.0	-0.227	0.81	-0.24	0.809
10	-0.554	0.27	-0.55	0.271
15	-0.686	0.20	-0.60	0.201
20	-0.631	0.16	-0.63	0.162
25	-0.646	0.14	-0.65	0.137
35	-0.688	0.11	-0.68	0.106
50	-0.711	0.08	-0.71	0.081
70	-0.727	0.06	-0.73	0.063
100	-0.740	0.05	-0.75	0.048

β = 4.2 η = 100 μ = 90.88 σ = 24.45 ν = 3.71711

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.397	0.99	0.39	0.999
1.5	-0.088	0.93	-0.09	0.929
2.0	-0.225	0.80	-0.24	0.804
10	-0.549	0.27	-0.55	0.266
15	-0.599	0.20	-0.60	0.197
20	-0.626	0.16	-0.62	0.158
25	-0.648	0.13	-0.64	0.134
35	-0.665	0.10	-0.67	0.104
50	-0.702	0.80	-0.70	0.079
70	-0.7250	0.60	-0.72	0.061
100	-0.746	0.04	-0.75	0.047

$\beta = 4.3$ $\quad n = 100$ $\quad \mu = 91.00$ $\quad \sigma = 23.97$ $\quad v = 3.79677$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.387	0.99	0.37	0.999
1.5	-0.096	0.93	-0.01	0.926
2.0	-0.233	0.81	-0.24	0.799
10	-0.540	0.26	-0.55	0.262
15	-0.593	0.20	-0.590	0.193
20	-0.617	0.15	-0.62	0.755
25	-0.638	0.13	-0.64	0.131
35	-0.667	0.10	-0.67	0.101
50	-0.694	0.07	-0.69	0.077
70	-0.713	0.06	-0.72	0.059
100	-0.737	0.05	-0.74	0.045

$\beta = 4.4$ $\quad n = 100$ $\quad \mu = 91.12$ $\quad \sigma = 23.51$ $\quad v = 3.87633$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.369	0.99	0.135	0.999
1.5	-0.077	0.92	-0.100	0.923
2.0	-0.232	0.80	-0.24	0.795
10	-0.542	0.26	-0.54	0.237
15	-0.588	0.19	-0.58	0.189
20	-0.608	0.15	-0.64	0.152
25	-0.634	0.13	-0.63	0.128
35	-0.664	0.10	-0.66	0.099
50	-0.690	0.07	-0.69	0.075
70	-0.704	0.05	-0.71	0.058
100	-0.735	0.04	-0.73	0.044

$\beta = 4.5$ $\eta = 100$ $\mu = 91.24$ $\sigma = 23.06$ $v = 3.95578$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.328	0.99	0.33	0.999
1.5	-0.097	0.92	-0.11	0.921
2.0	-0.230	0.79	-0.24	0.790
10	-0.546	0.25	-0.54	0.253
15	-0.583	0.19	-0.58	0.186
20	-0.604	0.15	-0.61	0.149
25	-0.627	0.12	-0.63	0.125
35	-0.656	0.09	-0.65	0.097
50	-0.686	0.07	-0.68	0.073
70	-0.703	0.55	-0.70	0.056
100	-0.728	0.04	-0.73	0.043

$\beta = 4.5$ $\eta = 100$ $\mu = 91.35$ $\sigma = 22.64$ $v = 4.03496$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.350	0.99	0.31	0.999
1.5	-0.094	0.92	-0.11	0.918
2.0	-0.233	0.785	-0.24	0.786
10	-0.539	0.25	-0.53	0.249
15	-0.580	0.18	-0.57	0.182
20	-0.600	0.15	-0.60	0.146
25	-0.625	0.12	-0.62	0.123
35	-0.651	0.09	-0.65	0.094
50	-0.675	0.70	-0.68	0.071
70	-0.697	0.05	-0.70	0.055
100	-0.720	0.04	-0.72	0.041

$\beta = 4.7$ $\eta = 100$ $\mu = 91.47$ $\sigma = 22.23$ $v = 4.$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.292	0.99	0.29	0.999
1.5	-0.093	0.92	-0.11	0.915
2.0	-0.236	0.785	-0.24	0.781
10	-0.538	0.24	-0.53	0.245
15	-0.574	0.18	-0.57	0.179
20	-0.605	0.14	-0.60	0.143
25	-0.625	0.12	-0.61	0.120
35	-0.651	0.09	-0.64	0.092
50	-0.678	0.07	-0.67	0.070
70	-0.705	0.54	-0.69	0.054
100	-0.723	0.04	-0.72	0.040

$\beta = 4.8$ $\eta = 100$ $\mu = 91.58$ $\sigma = 21.84$ $v = 4.19332$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.243	0.99	0.27	0.999
1.5	-0.091	0.91	-0.12	0.912
2.0	-0.237	0.78	-0.24	0.777
10	-0.538	0.24	-0.52	0.241
15	-0.573	0.18	-0.56	0.176
20	-0.606	0.14	-0.59	0.141
25	-0.625	0.12	-0.61	0.118
35	-0.652	0.64	-0.64	0.091
50	-0.673	0.065	-0.66	0.068
70	-0.695	0.05	-0.69	0.052
100	-0.726	0.03	-0.71	0.039

$\beta = 4.9$ $\quad \eta = 100$ $\quad \mu = 91.69$ $\quad \sigma = 21.46$ $\quad v = 27226$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.217	0.99	0.26	0.999
1.5	-0.090	0.90	-0.12	0.910
2.0	-0.254	0.78	-0.24	0.773
10	-0.533	0.24	-0.52	0.238
15	-0.572	0.17	-0.56	0.174
20	-0.600	0.14	-0.58	0.138
25	-0.623	0.12	-0.50	0.116
35	-0.660	0.09	-0.63	0.089
50	-0.677	0.06	-0.66	0.057
70	-0.701	0.05	-0.68	0.051
100	-0.726	0.04	-0.70	0.038

$\beta = 5.0$ $\quad \eta = 100$ $\quad \mu = 91.80$ $\quad \sigma = 21.10$ $\quad v = 4.35110$

	From Glasser's Graph		Present Model Solution	
Cost ratio K	Optimal age Z	Standard cost ratio ρ	Optimal age Z	Standard cost ratio ρ
1.1	0.230	0.99	0.24	0.999
1.5	-0.097	0.90	-0.12	0.907
2.0	-0.253	0.77	-0.24	0.769
10	-0.535	0.23	-0.51	0.235
15	-0.571	0.17	-0.55	0.171
20	-0.605	0.135	-0.58	0.136
25	-0.628	0.11	-0.60	0.114
35	-0.651	0.08	-0.63	0.087
50	-0.575	0.06	-0.65	0.066
70	-0.696	0.05	-0.68	0.050
100	-0.695	0.04	-0.70	0.038

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CHAPTER-6

CONCLUSION AND SCOPE OF FUTURE WORK

CHAPTER-6

CONCLUSION AND SCOPE OF FUTURE WORK

6.1 Conclusion:

Cost optimal age replacement model was studied and a graphical presentation of optimal policies has been made. Determination of optimal replacement age for an item is dependent only on cost ratio (i.e. relative cost of a failure replacement) and shape parameter of the failure distribution. This new technique does not require evaluation of meantime between failures and standard deviation from shape and scale parameters by solving gamma functions, which is a particular advantage of the new method.

The use of Weibull distribution as a failure distribution does not impose any limitation on the wide applicability of these new sets of graphs. On the other hand, the simplicity of the distribution is an added advantage.

The new graphical presentation of optimal age replacement policies eliminates the need of cumbersome calculations.

6.2 Scope of Future Works:

In the present study, only age replacement model has been studied. Similar graphical presentation for optimal block replacement policy can also be undertaken.

The objective function of the model in the present study was considered as minimization of cost. The models with other objective functions like minimization of downtime or maximization of availability should also be studied because for many critical components these objectives may be of more relevance. In the later case, time to effect a failure replacement and time to effect a preventive replacement will have to considered.

In many industrial situations, there may be a large number of same component or equipment in operation. Presentation of optimal replacement policy in the form of nomographs is a fertile area for further research works.

In industrial situations, it is not always possible to stick to an optimal maintenance time table. Effects of deviation from optimal policy on cost and availability should also be studied so that management becomes aware of this particular problem.

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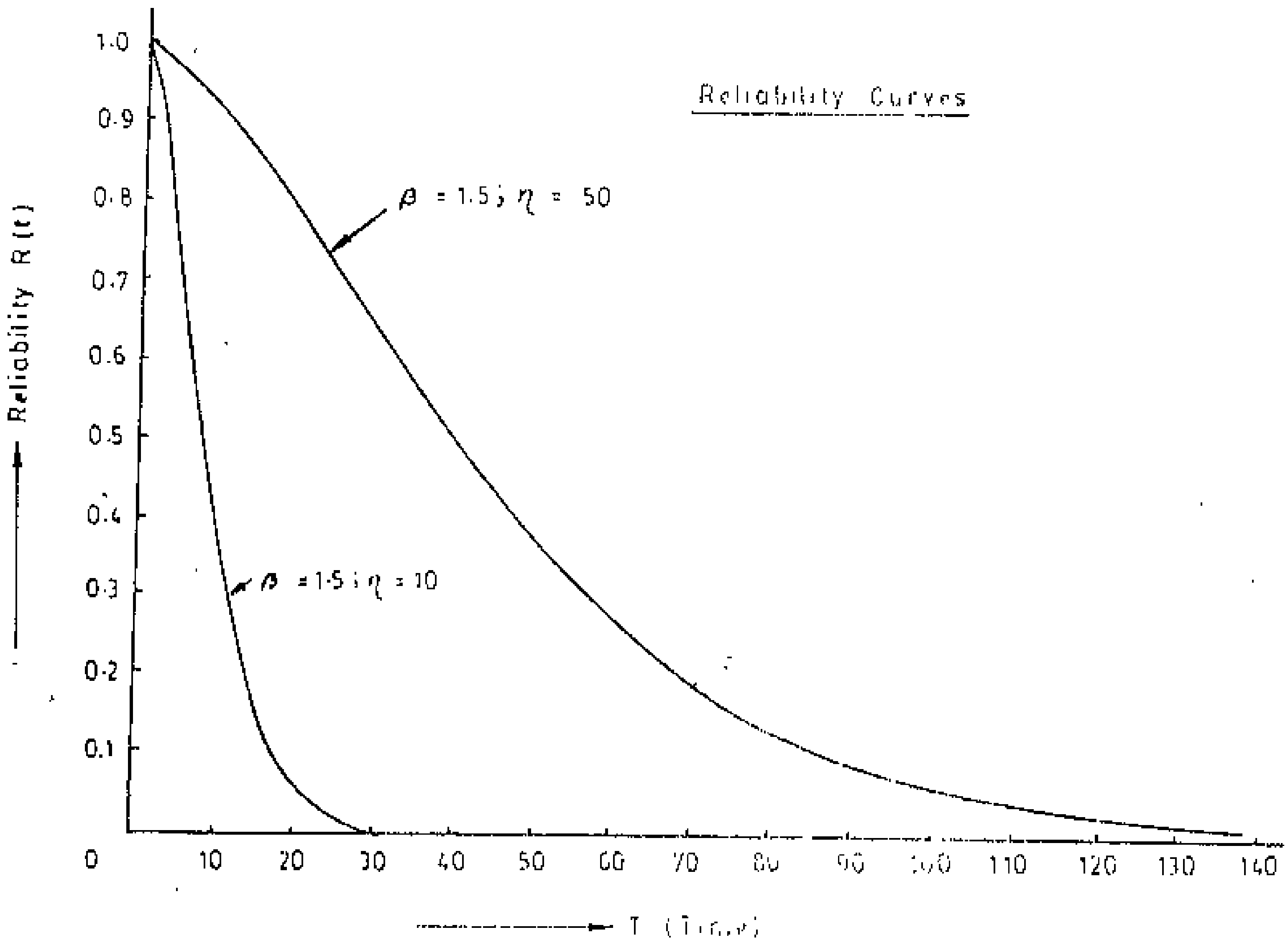
APPENDIX-A

RELIABILITY CURVES

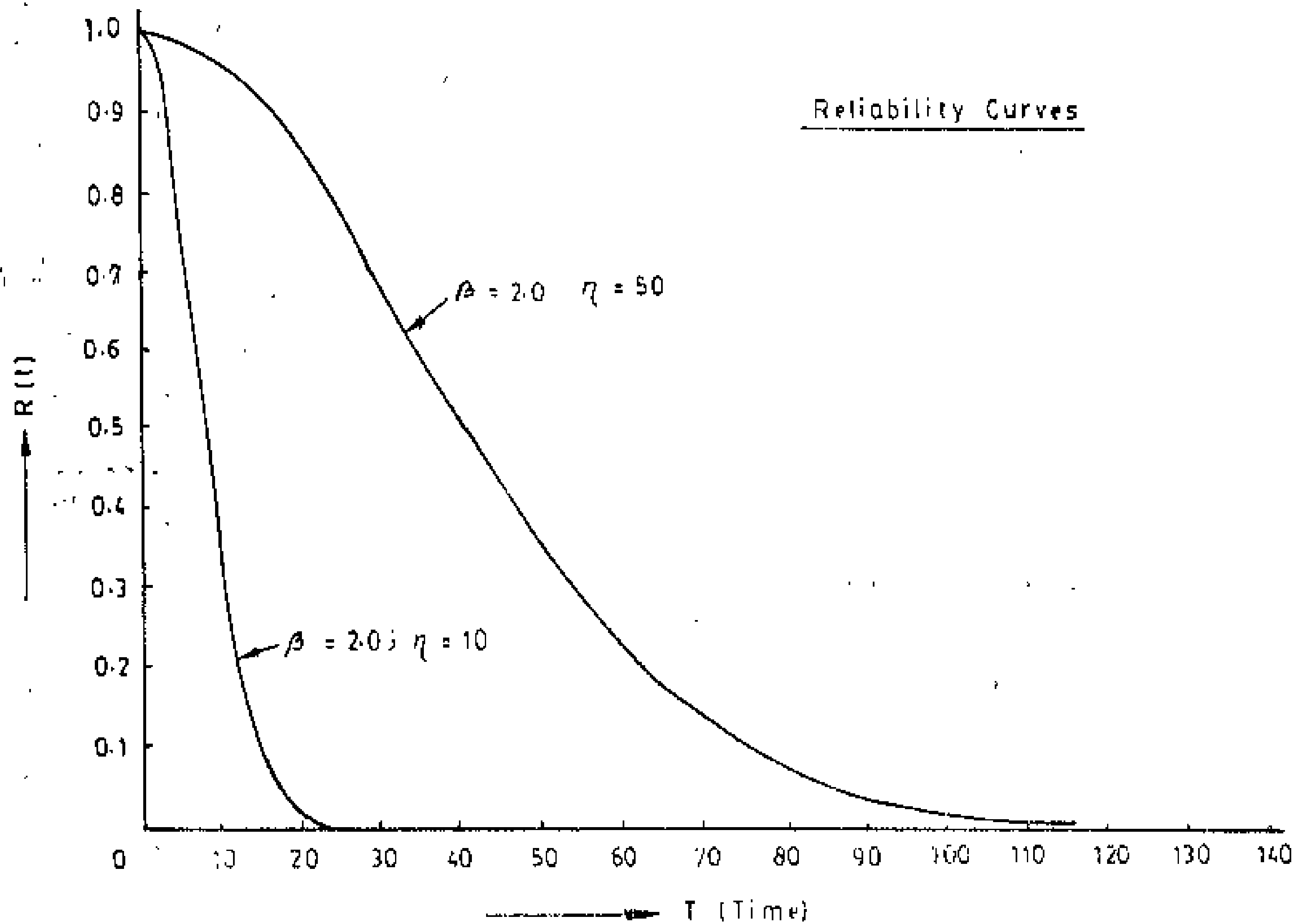
APPENDIX - A

Reliability Curves



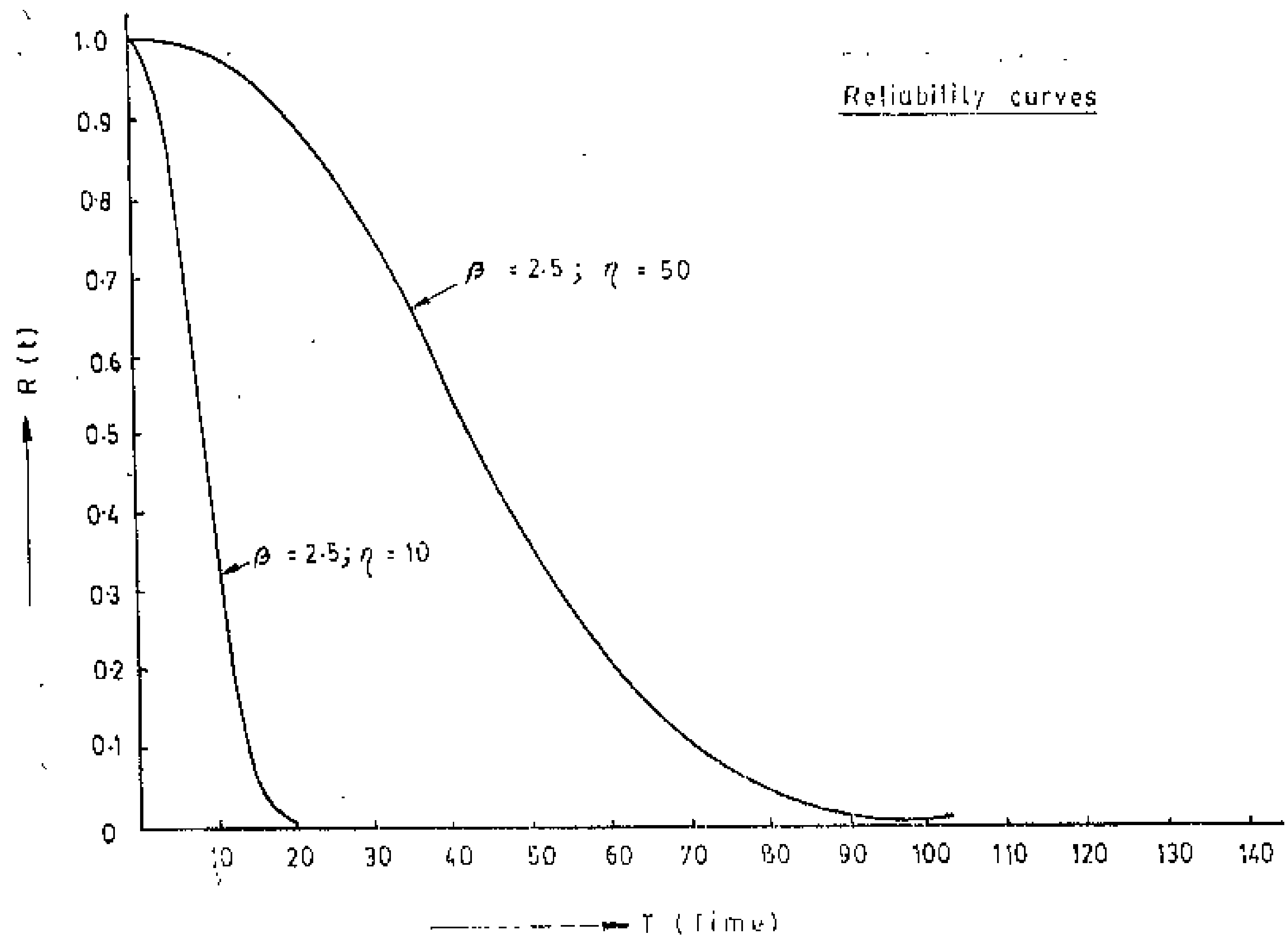
APPENDIX- A

Reliability Curves



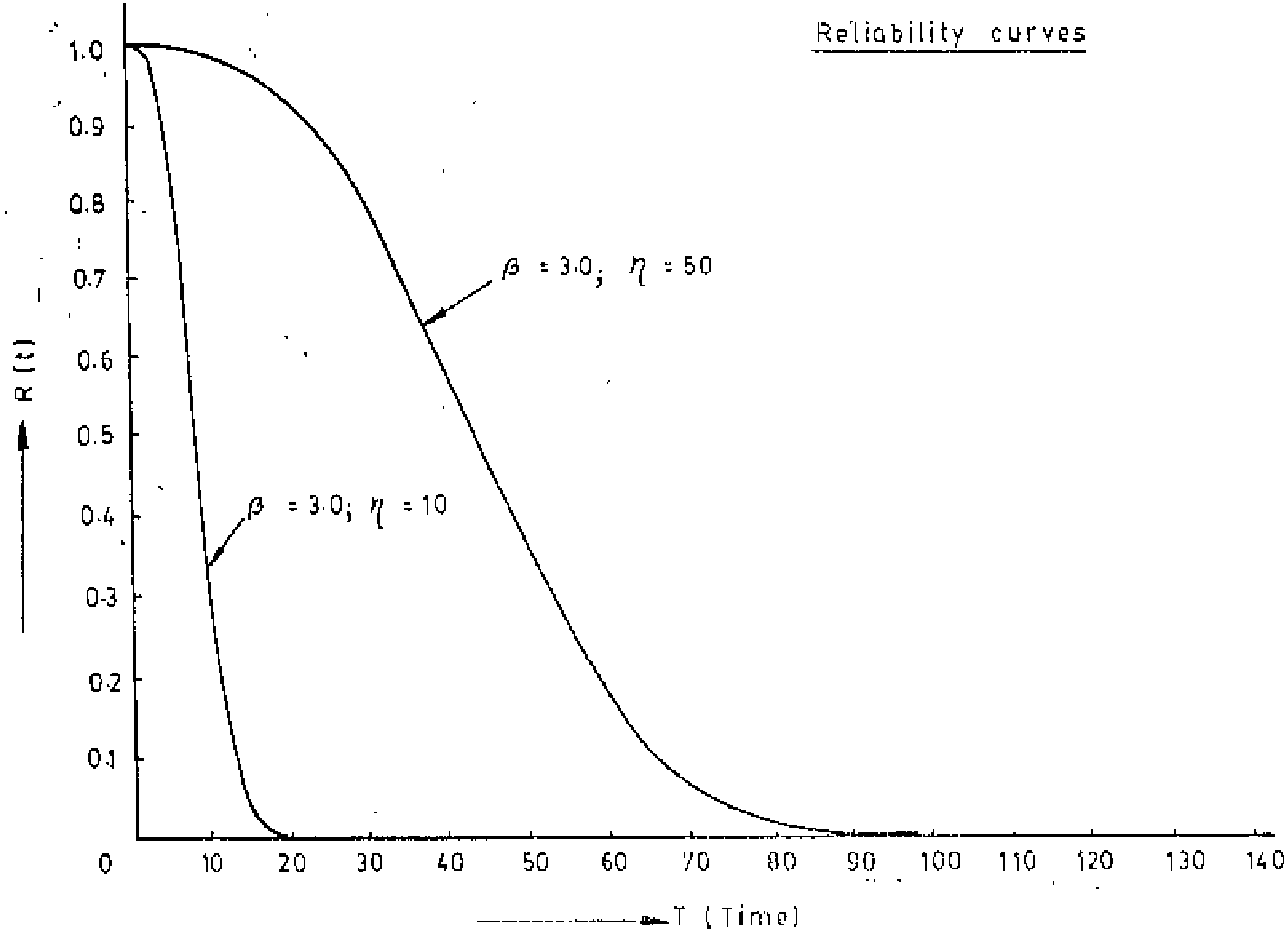
APPENDIX-A

Reliability curves



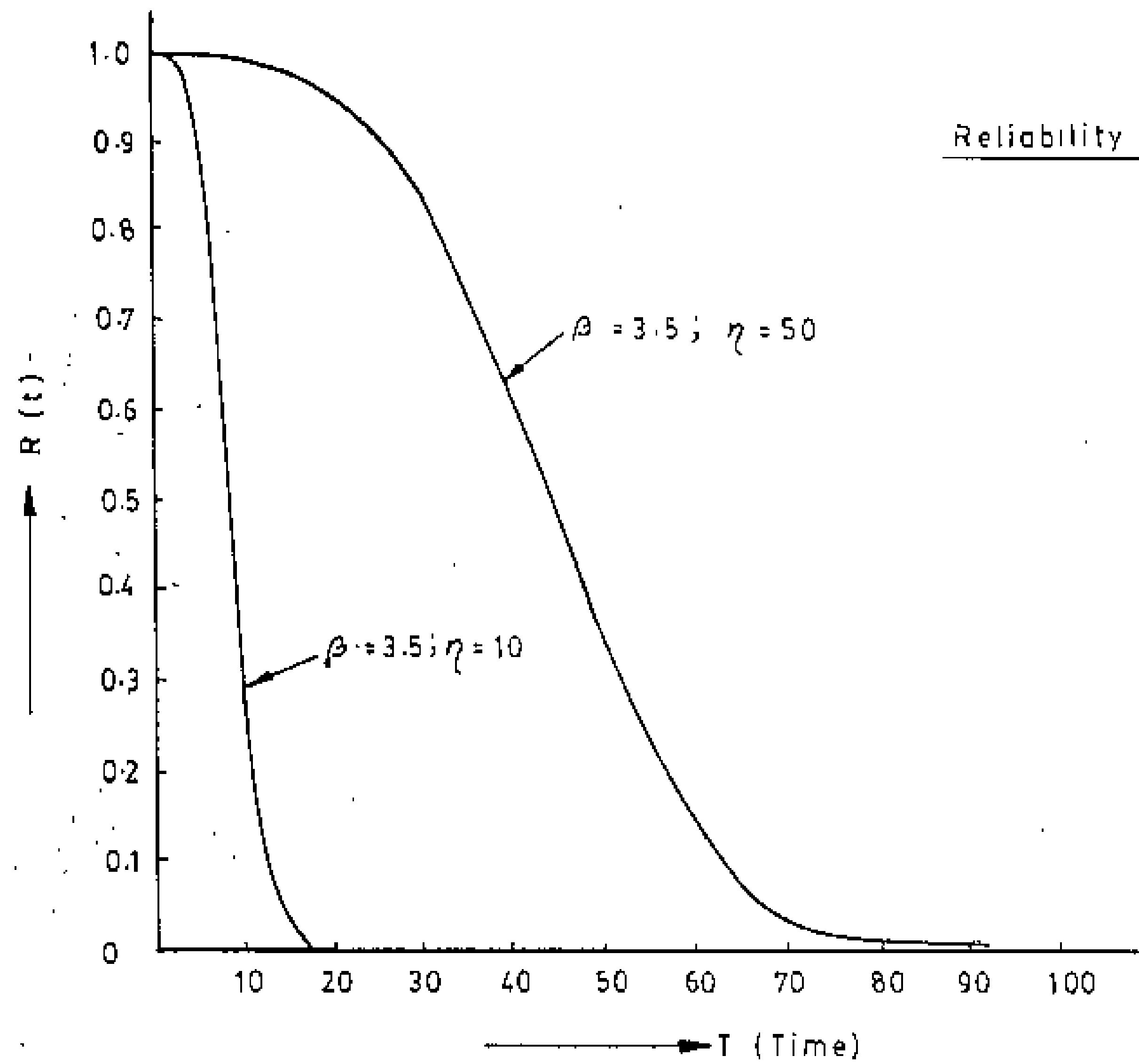
APPENDIX - A

Reliability curves

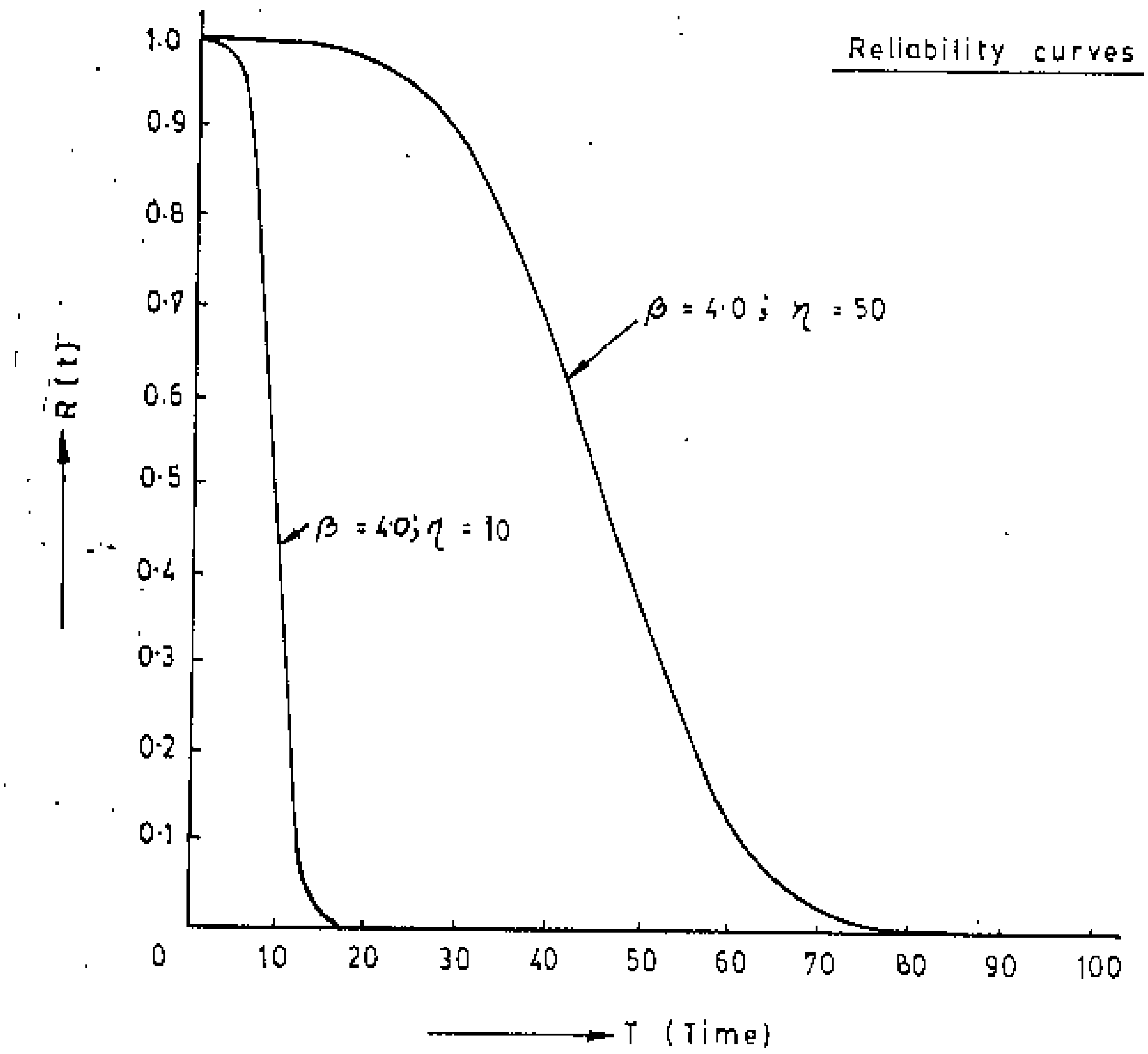


APPENDIX - A

Reliability curves



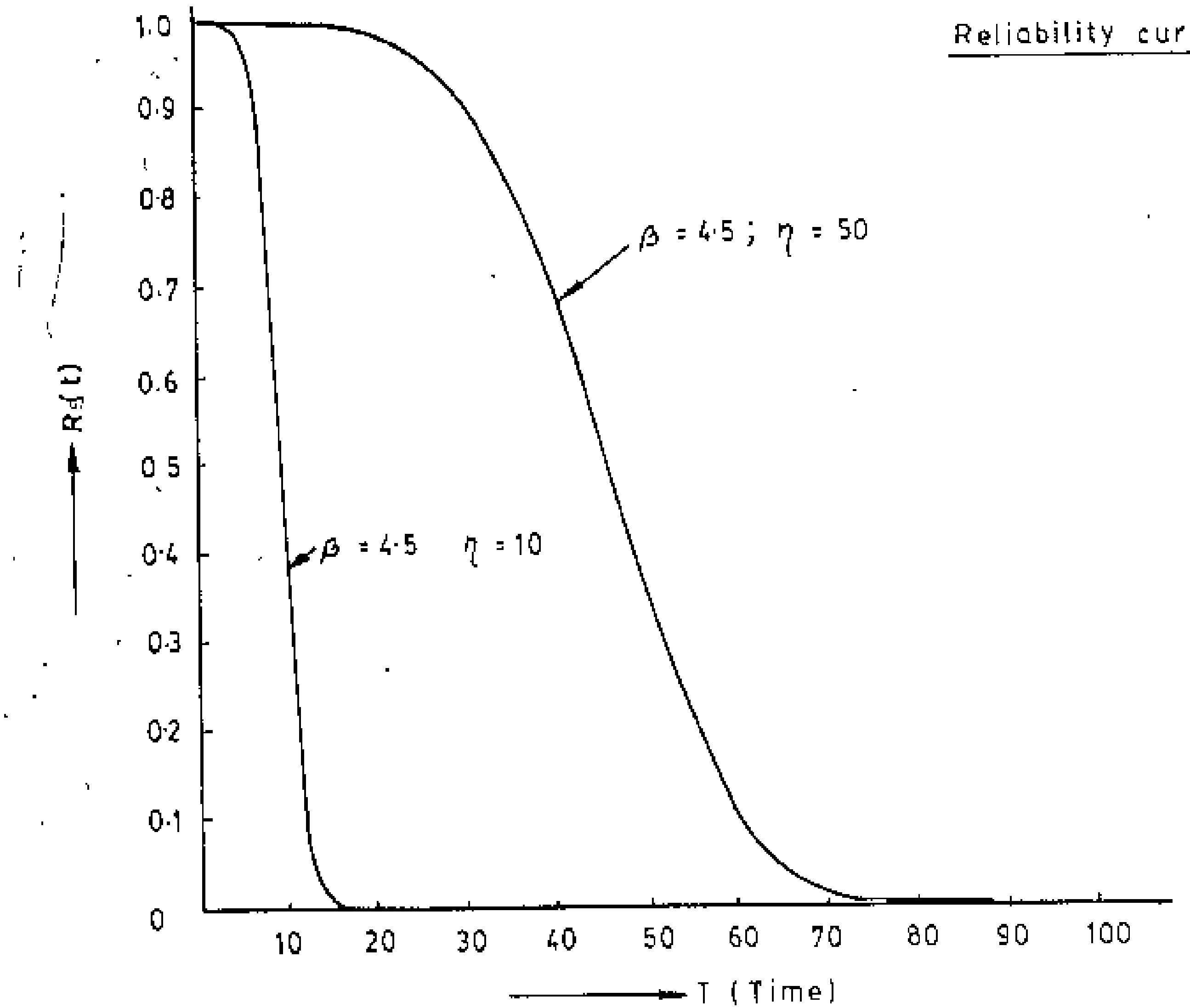
APPENDIX-A



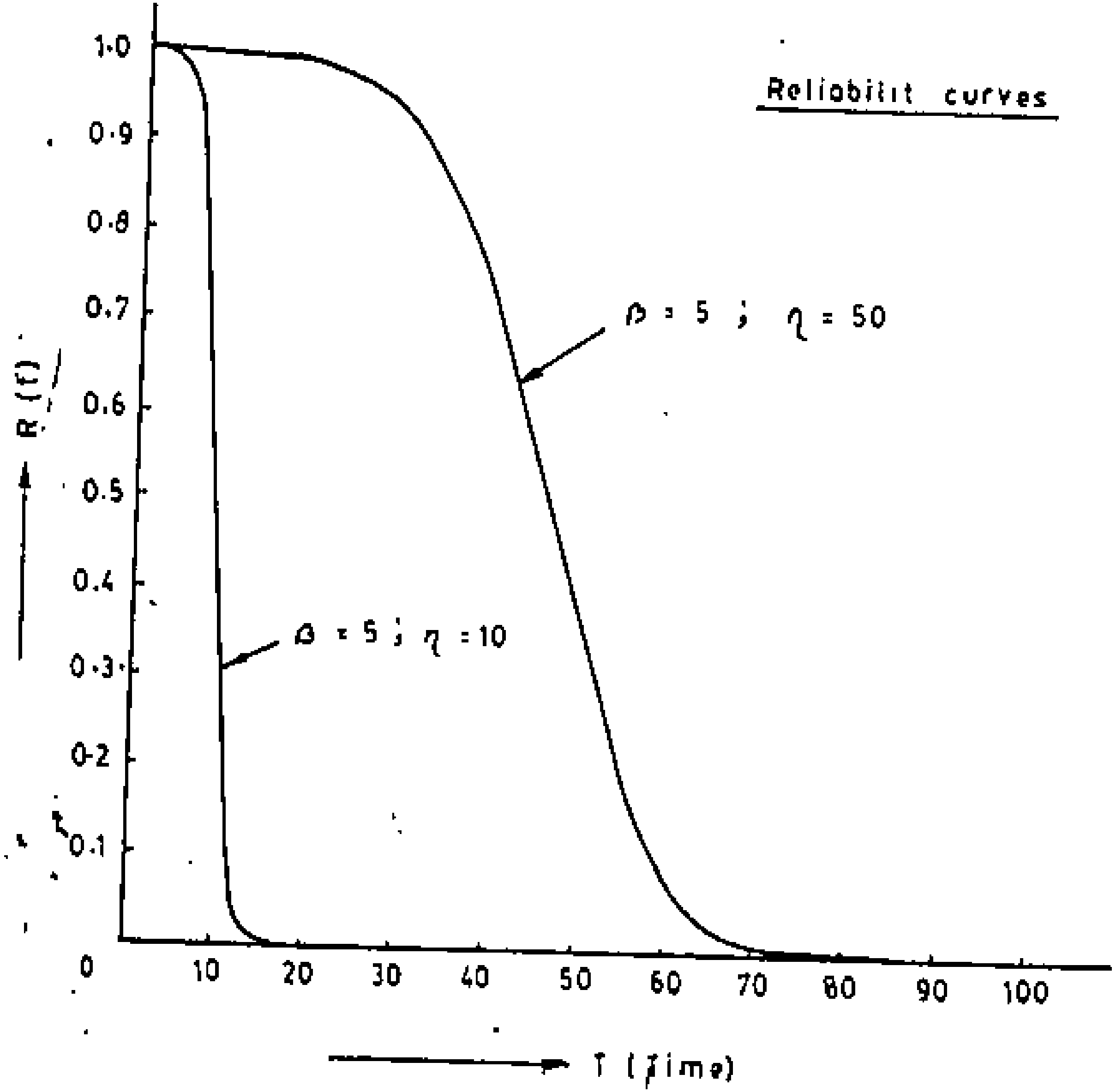
Reliability curves
$\beta = 4.0;\ \eta = 50$
$\beta = 4.0;\ \eta = 10$
R(t)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
60
70
80
90
100
T (Time)

APPENDIX - A

Reliability curves



APPENDIX - A


Reliabilit curves
$\beta = 5$; $\eta = 50$
$\beta = 5$; $\eta = 10$
R (t)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
60
70
80
90
100
T (Time)

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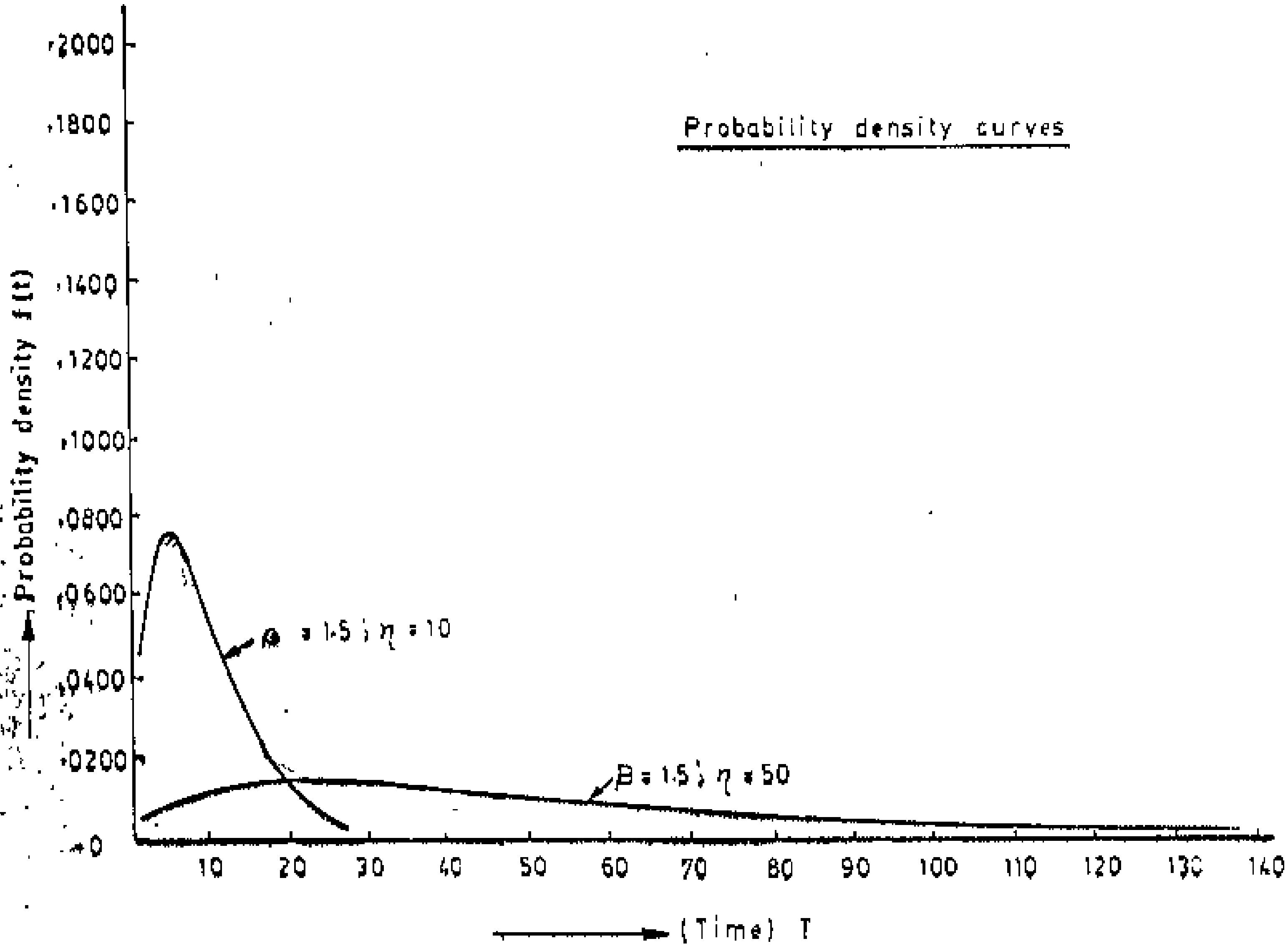
APPENDIX-B

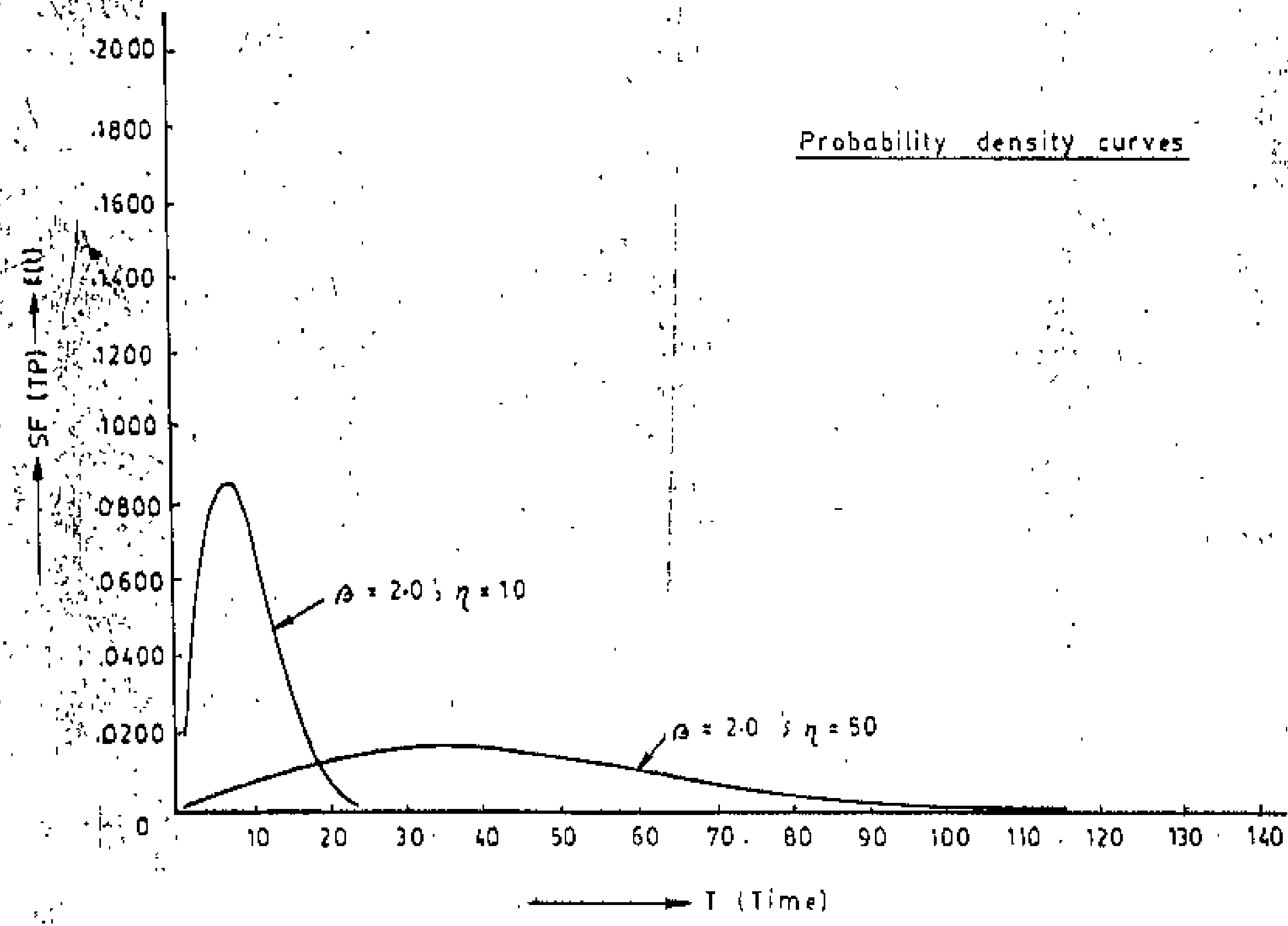
PROBABILITY DENSITY CURVES

APPENDIX – B



Probability density curves
·2000
·1800
·1600
·1400
·1200
·1000
·0800
·0600
·0400
·0200
·0
Probability density f(t)
β = 1.5 ; η = 10
β = 1.5 ; η = 50
10
20
30
40
50
60
70
80
90
100
110
120
130
140
(Time) T

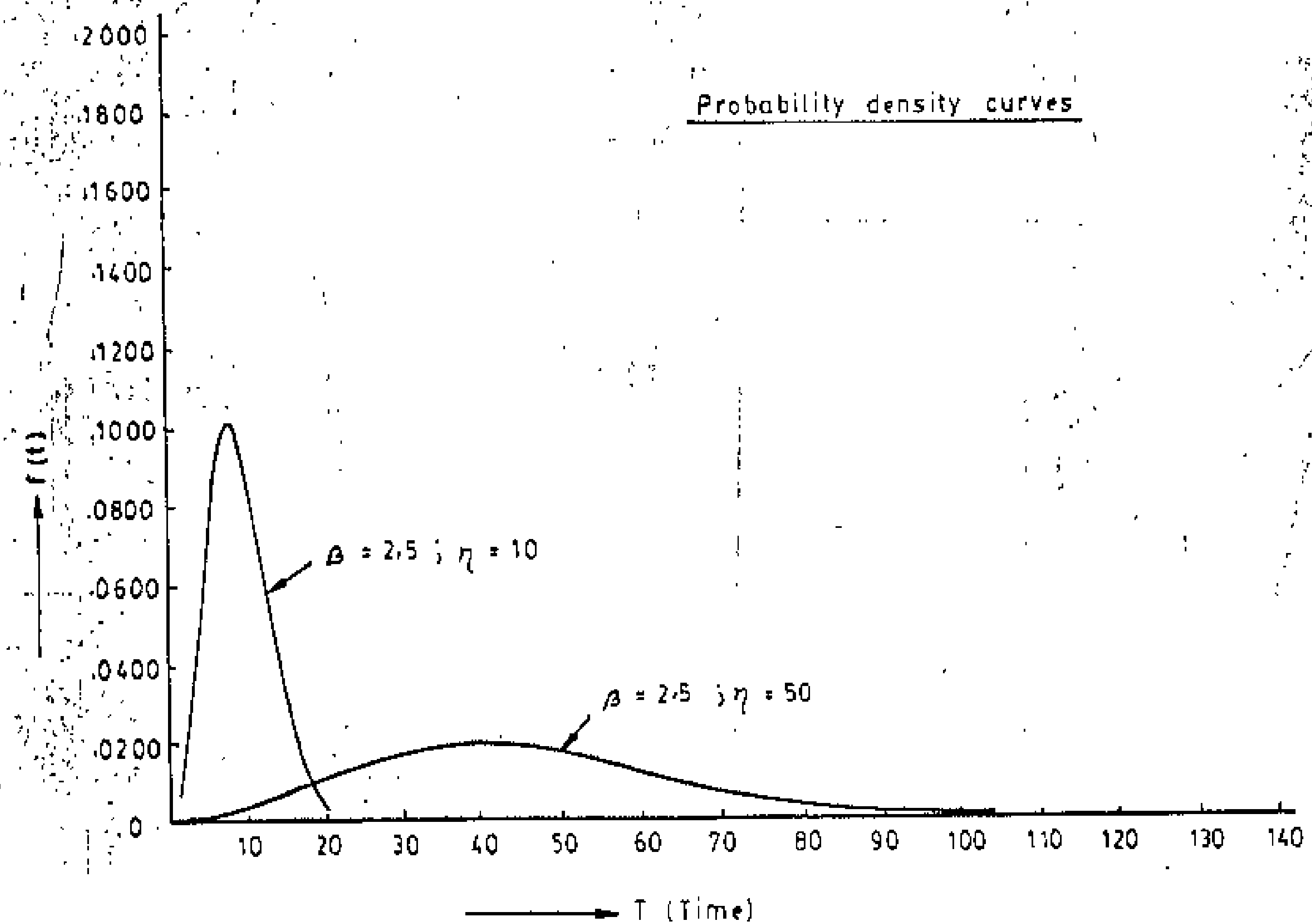
APPENDIX - B



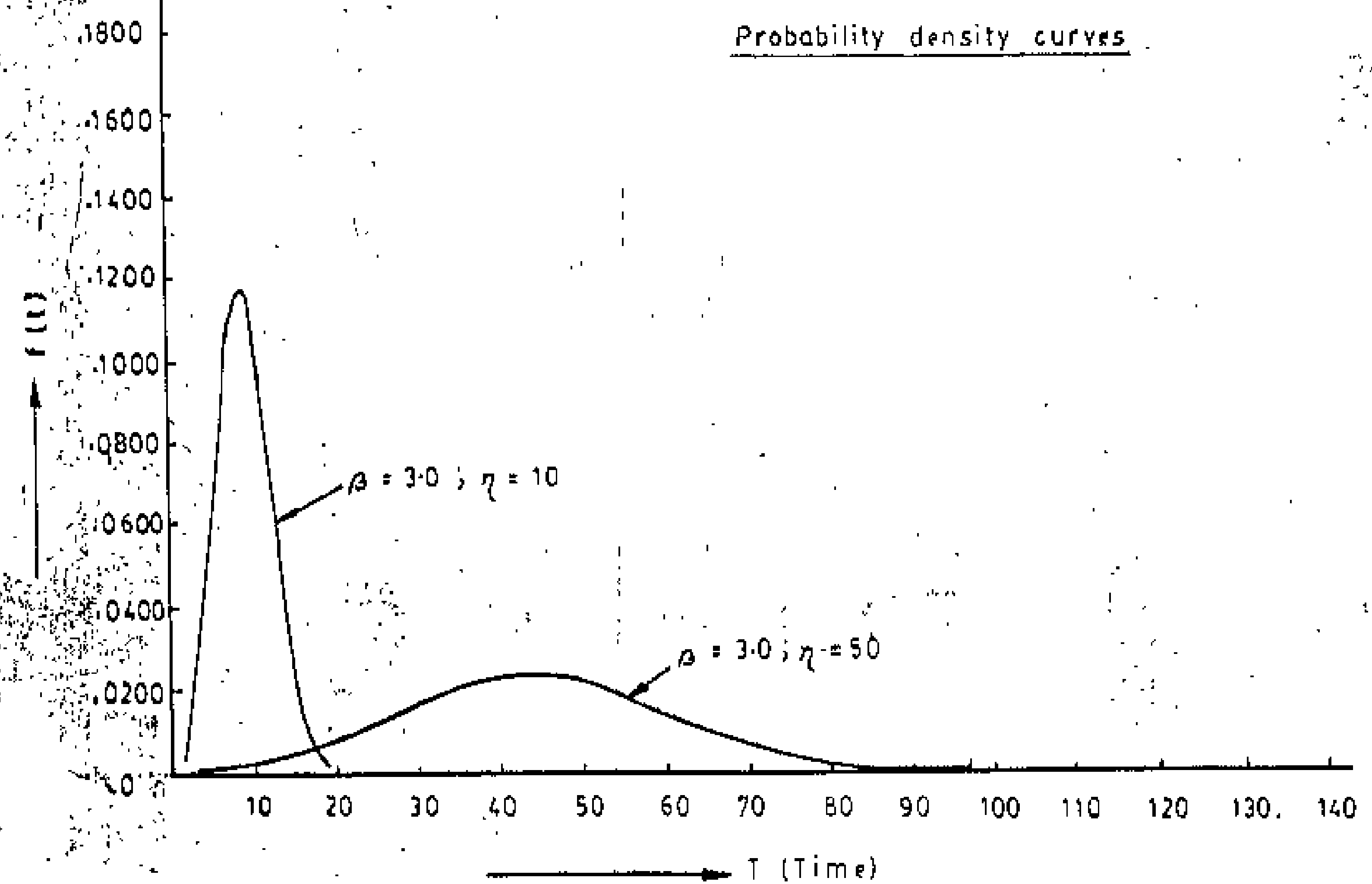
Probability density curves
SF (TP) → f(t)
.2000
.1800
.1600
.1400
.1200
.1000
.0800
.0600
.0400
.0200
0
β = 2·0 ; η = 10
β = 2·0 ; η = 50
10
20
30
40
50
60
70
80
90
100
110
120
130
140
T (Time)

APPENDIX – B

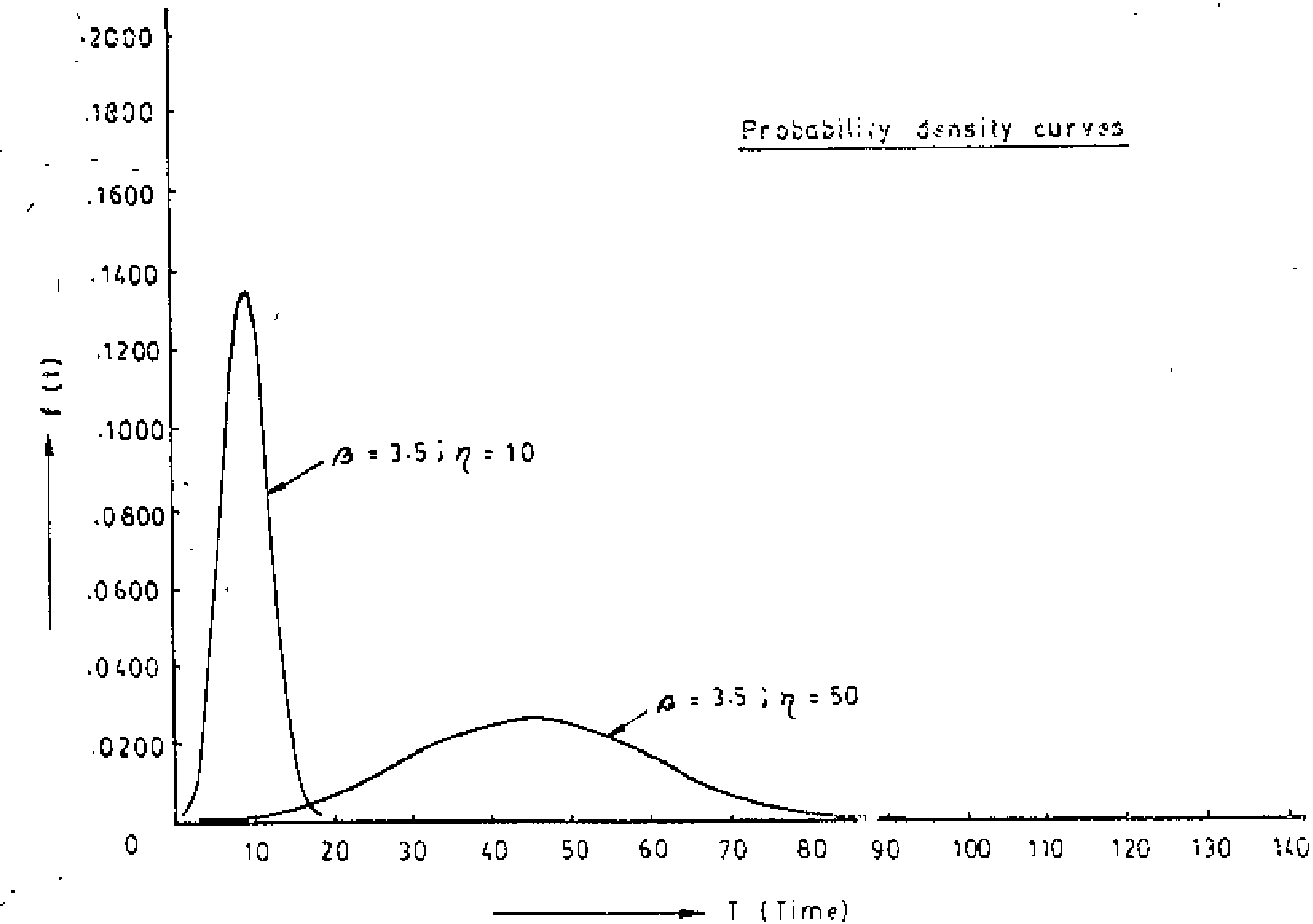
<u>Probability density curves</u>



APPENDIX – B

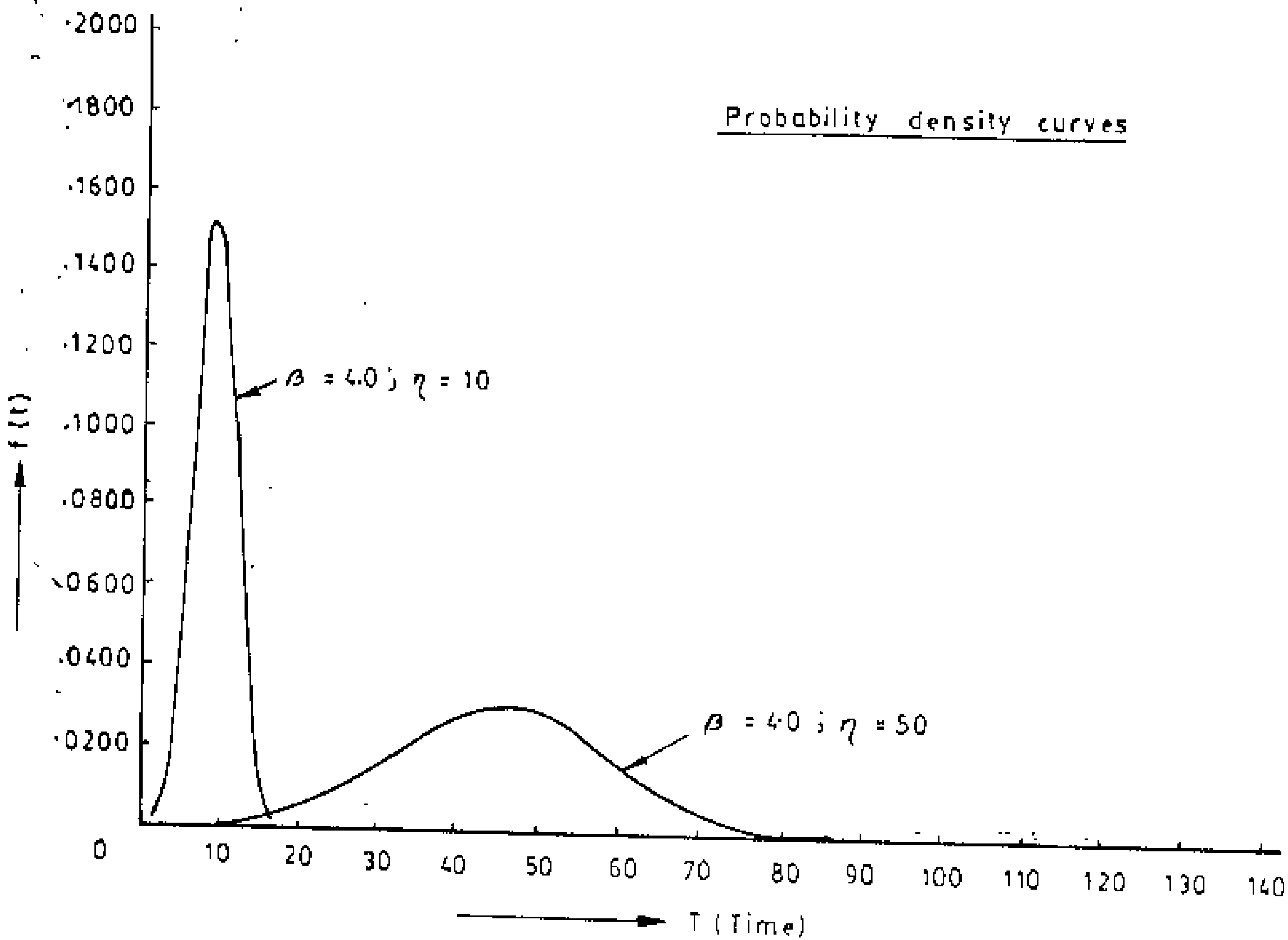

Probability density curves
.2000
.1800
.1600
.1400
.1200
.1000
.0800
.0600
.0400
.0200
0
f(t)
$\beta = 3{\cdot}0$; $\eta = 10$
$\beta = 3{\cdot}0$; $\eta = 50$
10
20
30
40
50
60
70
80
90
100
110
120
130.
140
T (Time)

APPENDIX – B



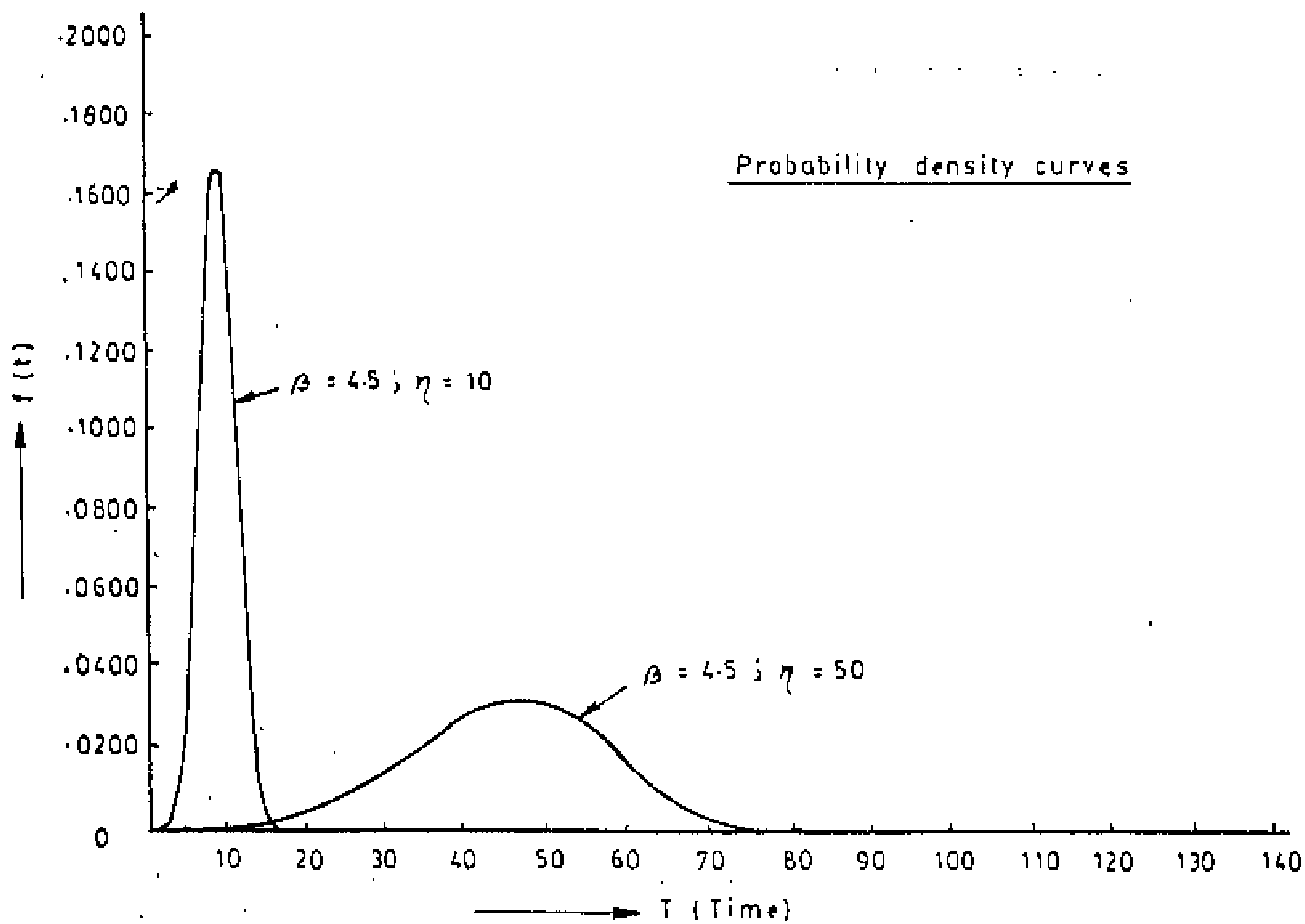
Probability density curves
.2000
.1800
.1600
.1400
.1200
.1000
.0800
.0600
.0400
.0200
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
f (t)
T (Time)
β = 3.5 ; η = 10
β = 3.5 ; η = 50

APPENDIX - B



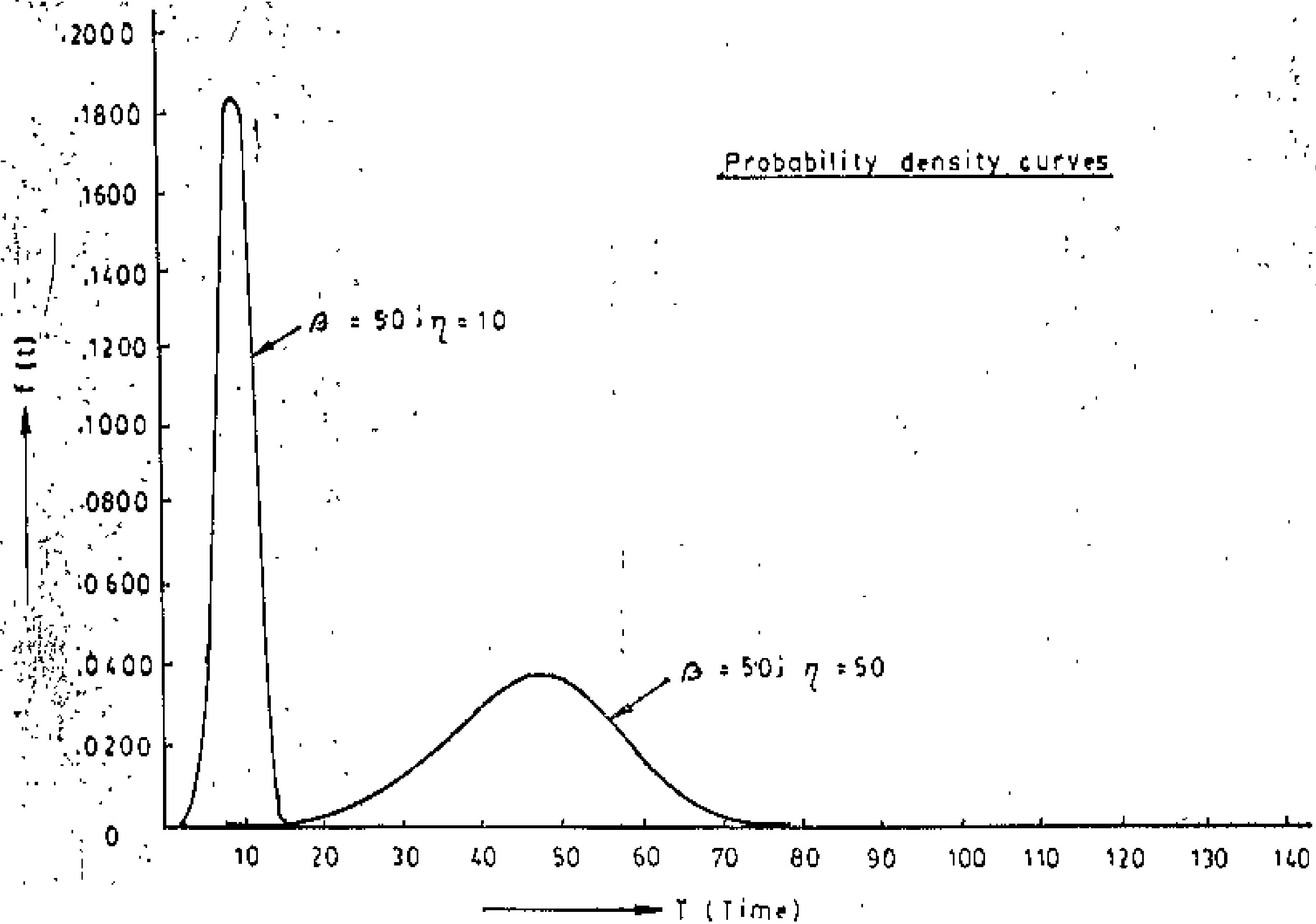
Probability density curves
·2000
·1800
·1600
·1400
·1200
·1000
·0800
·0600
·0400
·0200
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
f (t)
T (Time)
β = 4.0 ; η = 10
β = 4.0 ; η = 50

APPENDIX - B



Probability density curves
.2000
.1800
.1600
.1400
.1200
.1000
.0800
.0600
.0400
.0200
0
f (t)
β = 4.5 ; η = 10
β = 4.5 ; η = 50
10
20
30
40
50
60
70
80
90
100
110
120
130
140
T (Time)

APPENDIX - B


Probability density curves
.2000
.1800
.1600
.1400
.1200
.1000
.0800
.0600
.0400
.0200
0
f (t)
β = 50; η = 10
β = 50; η = 50
10
20
30
40
50
60
70
80
90
100
110
120
130
140
T (Time)

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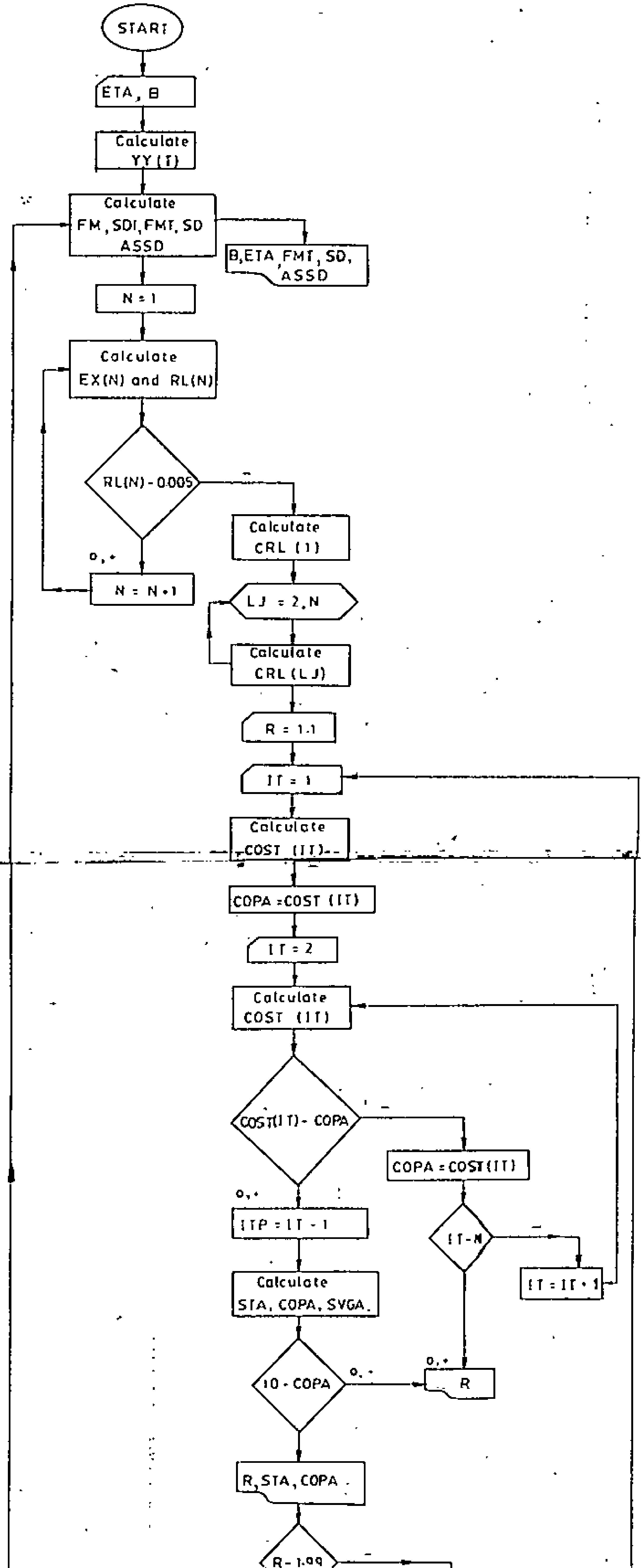
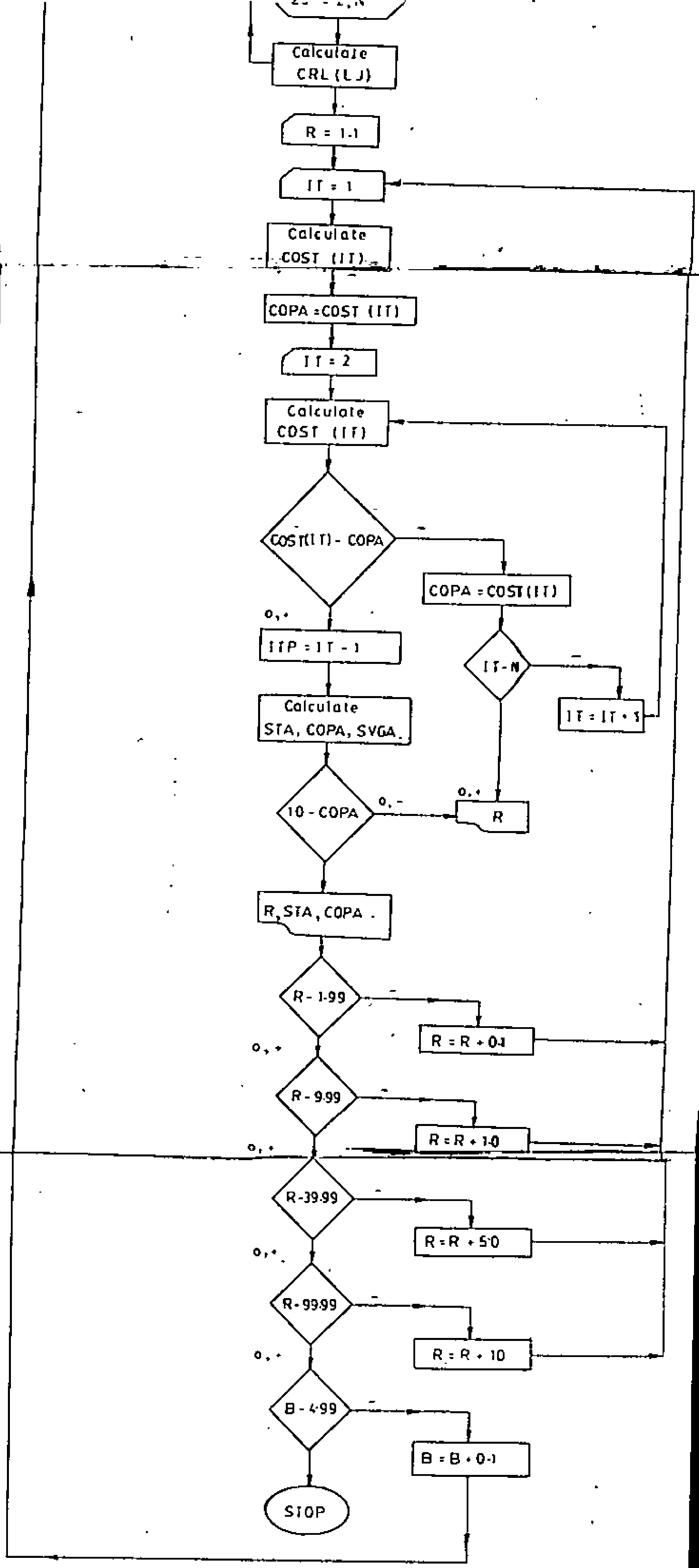
APPENDIX-C

FLOW CHART OF THE COMPUTER PROGRAMME FOR

DETERMINATION OF PREVENTIVE REPLACEMENT AGE

FLOW CHART APPENDIX-





Calculate CRL (L J)
R = 1.1
IT = 1
Calculate COST (IT)
COPA = COST (IT)
IT = 2
Calculate COST (IT)
COST(IT) - COPA
COPA = COST(IT)
IT-N
IT = IT + 1
ITP = IT - 1
Calculate STA, COPA, SVGA
10 - COPA
R
R, STA, COPA
R - 1.99
R = R + 0.1
R - 9.99
R = R + 1.0
R - 39.99
R = R + 5.0
R - 99.99
R = R + 10
B - 4.99
B = B + 0.1
STOP

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APPENDIX-D

COMPUTER PROGRAMME FOR DETERMINATION OF PREVENTIVE REPLACEMENT AGE

APPENDIX-D

```
ANTIV 360N-FO-479 3-8          MAINPGM          DATE    18/12/??        TIME
C       PROBLEM NO 1
C       PROGRAM FOR THE SOLUTION OF THE MODEL
C       FINDING PREVENTIVE REPLACEMENT AGE
C
C
        DIMENSION  EX(1500),RL(1500), CRL (1500),  COST (1500)
        COMMON RL,CRL,N,B,ETA
        ETA=100.0
        B=1.1
503     FM=YY(1.0+(1.0/B))
        SDI=YY(1.0+(2.0/B))
        FMT=ETA*FM
        SD=SQRT(((ETA**2)*SDI)-(FMT**2))
        ASSD=FMT/SD
        WRITE  (3,99)  B,ETA,FMT,SD,ASSD
99      FORMAT (6X,'B=',F3.1,5X,'ETA=',F5.1,5X,'FMT=',F8.2,5X,'SD=',F8.2,
       .5X,'ASSD=',F9.5)
        CALL   ZK1
        WRITE  (3,100)
100     FORMAT (6X,17H RELATIVE COST OF,8X,8H OPTIMAL, 7X,10H STD. COST,/,
       .5X,25H FAILURE REPLACEMENT,5X,4H AGE,9X,6H RATIO,/)
        R=1.1
504     IT=1
        COST( IT )=(RL (IT)+(R*(1.0-RL(IT))))/CRL(IT)
        COPA=COST(IT)
        IT=2
181     COST( IT )=(RL(IT)+(R*(1.0-RL(IT))))/CRL(IT)
        IF  (COST(IT)-COPA)  3(,31),31
3       COPA=COST(IT)
        IF  (IT-N)    190,181,181
190     IT=IT+1
        GO  TO    181
31      ITP=IT-1
        STA=(FLOAT (ITP)-ETA)/ETA
        COPA=  (COPA*FMT)/R
        SVGA=1.0-COPA
        IF  (SVGA)  34,34,35
35      CONTINUE
        WRITE  (3,151)   R,STA,COPA
151     FORMAT  (12X,F7.3,12X,F8.4,12X,F8.4)
        GO  TO    501
34      CONTINUE
181     WRITE  (3,152)    R
152     FORMAT  (1X,T13,F7.3,12X,'REPLACE ON FAILURE ONLY')
501     IF  (R-1.99)    112,113,113
112     R=R+0.1
C
C
C
        GO TO 504
113     IF(R-9.99)  114,115,115
114     R=R+1.0
        GO TO 504
115     IF (R-39.99)  116,117,117
116     R=R+5.0
        GO  TO  504
117     IF  (R-99.99)  118,119,119
118     R=R+10.0
        GO  TO  504
119     CONTINUE
        IF (B-4.99)  110,111,111
110     B=B+0.1
        GO TO 503
111     CONTINUE
        STOP
        END
```

APPENDIX-D


```
C
C
C
      SUBROUTINE   ZKI
C     SUBROUTINE FOR WEIBULL ANALYSIS
      DIMENSION  EX(1500),RL(1500),CRL(1500)
      COMMON  RL,CRL,N,B,ETA
C     FINDING WEIBULL PROBABILITIES
      N=1
72    EX(N)=(FLOAT(N)/ETA)**B
      RL(N)=EXP(-EX(N))
      IF (RL(N)-0.005)  71,70,70
70    N=N+1
      GO TO 72
71    CONTINUE
      CRL(1)=(1.0+RL(1))/2.0
      DO  83  LJ=2,N
83    CRL(LJ)= CRL (LJ-1) + ((RL(LJ-1) + RL(LJ))/2.0)
      RETURN
      END
```



```
C
C
      FUNCTION   YY(T)
      YY=(T**T)*EXP(-T)*SQRT((2.0*3.1416)/T)*(1.0+(1.0/(12.0*T))+(1.0/
     .288.0*(T**2))-(139.0/(51840.0*(T**3)))-(571.0/(2488320*(T**4))))
      RETURN
      END
```

Program parameter specifications

B	- Shape parameter of the fitted Weibull distribution (β)
ETA	- Scale parameter of the failure distribution (η)
YY	- A function sub-program to evaluate the gamma function.
FM	- Gamma function of (1 + $1/\beta$).
SD1	- Gamma function of ($1 + 2/\beta$)
FMT	- The mean of the failure time distribution (Weibull).
SD	- Standard daviation (σ)
ASSD	- Average service life ($v = \mu/\sigma$) in standard deviation unit.
SQRT	- Square root ($\surd$)
Subroutine ZK1	- A subprogram which solves the age-based models and gives the results in tabular form.
$R=(k = C_f/C_p)$	- Relative cost of failure replacement
IT	- Denoting time periods or class intervals for age replacement policy.
RL	- Reliability function of a component = $\exp\{-(t/\eta)^{\beta}\}$
CRL	- Integral value of reliability function (or an array storing truncated mean-lives for various class intervals).
COST(IT)	- Arrays storing the cost figures for various preventive replacement intervals relative to age replacement policy.
COPA	- Variables which help to find a minimum replacement cost and an optimal interval for age replacement policy.
N	- Number of class intervals (or No. of periods or years).

ITP	- Represent the optimal interval (t_p) or age for a certain value of R in age replacement model.
STA	- Deviation from one policy to other policy in age replacement model.
SVGA	- Percentage savings from the chosen maintenance action (or percentage savings for optimal age or interval at a given value of R in age replacement policy).
EXP	- Exponential distribution function.
EX	- An array containing the values of the arguments of the exponential function in the calculation of BF.
BF	- An array representing the Weibull cumulative distribution function (c.d.f)
LJ	- Loop Index variable.
T	- A dummy variable used in the function subprogram YY.

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APPENDIX-E

TABLE FOR DETERMINATION OF MEAN (μ) AND SIGMA (σ) FOR β VALUES

APPENDIX-E

// EXEC

B	FM	BFS	SD	ASSD
0.5	1.999989	23.999924	4.472132	0.44721[illegible]
0.6	1.504565	9.26[illegible]457	2.645137	0.56[illegible]
0.7	1.265810	5.029115	1.951172	0.68[illegible]
0.8	1.132989	3.32334[illegible]	1.428173	0.79[illegible]
0.9	1.052164	2.478578	1.171123	0.89[illegible]
1.0	0.999978	1.999989	1.000915	0.99[illegible]
1.1	0.964[illegible]5	1.702418	0.878303	1.09[illegible]
1.2	0.940624	1.504565	0.797269	1.1[illegible]47
1.3	0.923541	1.366265	0.716475	1.2[illegible]
1.4	0.911382	1.265813	0.659694	1.381[illegible]
1.5	0.902699	1.190627	0.61259[illegible]	1.472[illegible]
1.6	0.896523	1.132789	0.573790	1.5624[illegible]
1.7	0.892189	1.08794[illegible]	0.540315	1.[illegible]512[illegible]
1.8	0.889225	1.052165	0.511315	1.73[illegible]
1.9	0.887296	1.023386	0.485354	1.8261[illegible]
2.0	0.886155	0.99998[illegible]	0.463367	1.9124[illegible]
2.1	0.885616	0.980779	0.443233	1.99[illegible]
2.2	0.885541	0.964885	0.4252[illegible]	2.0[illegible]
2.3	0.88[illegible]	0.951673	0.408637	2.167[illegible]
2.4	0.8[illegible]5	0.940624	0.393624	2.2[illegible]1[illegible]
2.5	0.887165	0.931351	0.379554	2.3[illegible]
2.6	0.88[illegible]1[illegible]3	0.923541	0.3[illegible]71[illegible]5	2.41[illegible]
2.7	0.8[illegible]172	0.916[illegible]	0.35[illegible]42[illegible]	2.5[illegible]17[illegible]
2.8	0.890337	0.911392	0.344594	2.5[illegible]
2.9	0.891577	0.90[illegible]7[illegible]	0.334734	2.56[illegible]
3.0	0.892854	0.902[illegible]	0.324825	2.74[illegible]
3.1	0.894175	0.8993[illegible]	0.315927	2.8[illegible]
3.2	0.895519	0.896523	0.307521	2.912[illegible]
3.3	0.896877	0.894159	0.299618	2.99[illegible]
3.4	0.898239	0.892152	0.292157	3.074[illegible]
3.5	0.899600	0.890559	0.285094	3.15[illegible]
3.6	0.900953	0.889225	0.2784[illegible]3	3.23[illegible]
3.7	0.902257	0.888150	0.27204[illegible]	3.316[illegible]
3.8	0.903625	0.887296	0.266002	3.3[illegible]7[illegible]
3.9	0.904937	0.886643	0.260243	3.47[illegible]
4.0	0.906224	0.886155	0.254745	3.557[illegible]
4.1	0.9075[illegible]	0.885819	0.249497	3.637[illegible]
4.2	0.908760	0.885616	0.24449[illegible]	3.717[illegible]
4.3	0.909991	0.885528	0.239674	3.7[illegible]7[illegible]
4.4	0.911200	0.885541	0.235[illegible]67	3.876[illegible]
4.5	0.912385	0.885644	0.230646	3.95[illegible]
4.6	0.913546	0.885827	0.226439	4.[illegible]245[illegible]
4.7	0.914685	0.886076	0.222322	4.114[illegible]
4.8	0.915801	0.886388	0.218355	4.1[illegible]
4.9	0.916893	0.886753	0.214615	4.2722[illegible]
5.0	0.917963	0.887165	0.210572	4.3511[illegible]
5.1	0.919009	0.887617	0.207461	4.4297[illegible]

B $=\beta$; FM $= \mu/\eta = \Gamma(1+1/\beta)$; BFS $= \Gamma(1+2/\beta)$; SD $= \sigma/\eta$; ASSD $= \mu/\sigma$

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