A THEORETICAL STUDY OF DIELECTRIC LOADED ANTENNAS.

A
THESIS
SUBMITTED TO
THE DEPARTMENT OF ELECTRICAL ENGINEERING
EAST PAKISTAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, DACCA,
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE (ENGINEERING) IN ELECTRICAL ENGINEERING.

By
RAMESH CHANDRA ROY.

DEPARTMENT OF ELECTRICAL ENGINEERING,
EAST PAKISTAN UNIVERSITY OF ENGINEERING & TECHNOLOGY,
DACCA.
1966

E. P. UNIVERSITY OF ENGINEERING & TECHNOLOGY LIBRARY
No.
Date.
DACCA

ACCEPTED AS SATISFACTORY FOR PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF M. Sc. (ENGINEERING) IN ELECTRICAL ENGINEERING.

Examiners (i) ....
31/5/67

(ii) ....
31/5/67

(iii) M. A. Islam ....
31/5/67

CERTIFICATE

THIS IS TO CERTIFY THAT THIS WORK WAS DONE BY ME AND IT HAS NOT BEEN SUBMITTED ELSEWHERE FOR THE AWARD OF ANY DEGREE OR DIPLOMA.

Countersigned,

Ramesh Chandra Roy

===============

(Signature of the Candidate.)

(Superviser)

ACKNOWLEDGEMENTS

The author wishes to express his gratitude to Dr. Md. Azizul Islam, who supervised this work for his constant interest, inspiration and advice. The author wishes to express his thanks to Dr. M.N. Azam, Head of the Department of Electrical Engineering, for his useful suggestions.

Thanks are also given to Mr. Sk. Rezaul Karim and Mr. Ganesh Chandra Das, post-graduate students of the Department of Electrical Engineering, for their ready help and co-operation, in different stages of the work.

C O N T E N T S

CHAPTER-I

INTRODUCTION

ENGINEERING & TE... OF ... UNIVERSITY E. P. LIBRARY ... TGY
No....................
Date.....................
DACCA

The problems of dielectric coated linear antennas have been investigated by many authors. In this respect numerous works havenbeen performed. Some of these works are given in the referrences in bibliography. With the basis of these referrences the present work has been performed by the author. The specialized thing for the author is to study some of these articles. So without enterning into the vast field of works the author only reflects some portion of these works. The author has studied certain articles as dielectric loading of dipole antennas in spherical and cylindrical medium. He has also studied the behaviour of electromagnetic waves in dissipative or lossy media. He has diverted his special interest in these works because the recent scientific activities in space exploration and guided missiles have presented many new and radically different communication and instrumentation problem. Also the work on this subject has become necessary to communicate between underground or underwater terminals. Propagation between burried transmitting and receiving antannas is usually via a path which can be described as "up, and down". The flow of energy is essentially from the source towards the surface, and then down to the receiver. The direct path through the medium is not usually considered as the attenuation would be prohibitive, particularly in the case of sea water. However, the situation may be changed when the antennas are placed within or near a poorly conducting stratum. Therefore, it becomes essential to have an accurate knowledge about the behaviour of an antenna in the medium so as to be able to interprete logically the results of the antenna measurements.

In the study of linear antenna, a convenient starting point is to study the effect of an incident electromagnetic field on an antenna structure. For this purpose it will be assumed that the antenna structure is a circular-cylinder of some essentially non-lossy material, covered by a coaxial cylinder or by a sphere of some lossy material. The medium affects the antenna performance severely and causes communication "blackout" by attenuating the signal appreciably. Therefore, it is important to understand the effect of this dielectric loading in order to spedify the system requirements. It is also important to know some of the results from the dielectric loading of the antennas. Under certain conditions a wave may be guided without loss of energy by a slab of perfect dielectric having no metal boundaries. The conductivity of sea water is relatively high than that of fresh water. This explains good reception at sea. There is another factor which may be considered in this analysis that the attenuation factor becomes high at millimeter or submillimeter wavelengths. The dielectric coated wire or dielectric rod may be used for low loss guiding device. But one of the most useful thing is that

~~is that of~~

if we want to increase the radiation resistance it is more helpful to increase the dielectric constant and premeabil-ily of the medium than to increase the physical size of the structure. Because by increasing the physical structure there is a certain range beyond which there will be no extra benefit. For $\epsilon_r > 1$, the radiation power factor is also decreased. Yet it is seen in the Chapter-I that moder-ate relative dielectric constants of the sphere ($1 < \epsilon_r < 4$) provide a slight improvement of the power factor if the boundary of the dielectric sphere is within the near zone of the dipole. This is contrary to the expectations.

CHAPTER- II

DIELECTRIC LOADING OF ELECTRICCDIPOLE ANTENNAS SURROUNDED BY A SPHERE[12]

I. INTRODUCTION

It has been indicated by Wheeler[8] that dielectric loading ($\epsilon_r > 1$) decreases the radiation power factor of electric dipole antennas. However, there are exceptions for an infinitesimal dipole surrounded by a finite dielectric sphere and also for dielectrically loaded small-angle biconical antennas, where a moderate increase of ϵ_r provides a slight increase of the radition power factor. In the latter two cases the dielectric surface is not tangential to the electric field lines. So the results of Wheeler are not strictly applicable.

According to him the dielectric loading of the antenna increases the susceptance B and does not alter the radiation conductance G. so the antenna radiation power factor $p=\frac{G}{B}$ decreases. The specific geometries considered here involve an infinitesimal electric dipole in a finite dielectric sphere. The field components of the small dipole can be obtained from a scalar solution of the wave equation. It is shown in the discussion of electric dipole that there is a slight improvement of the power factor p for a moderate relative dielectric (i.e. $1 < \epsilon_r < 4$), if the boundary of the dielectric sphere is within the near zone of the diepole. This is contrary to the expectations.

2. THE ELECTRIC DIEPOLE SURROUNDED BY A SPHERE

An electric dipole is located in the center of a dielectric sphere of radius "a" which is shown in fig. 1 and of dielectric constant ϵ_1, magnetic permeability μ_0, and conductivity δ_1 of the region I. The outer region II is

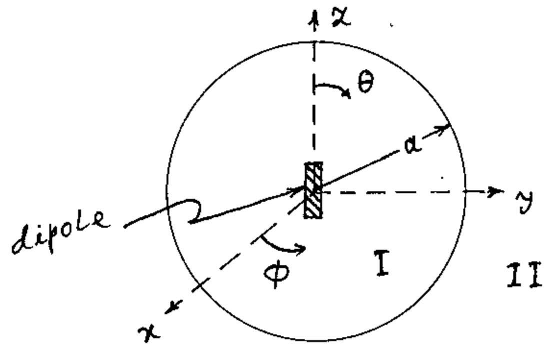


Fig. 1. An Electric Dipole Surrounded by a Sphere.

characterized by ϵ_2, μ_0, and δ_2. For a suppressed $e^{j\omega t}$ time variation of the fields, the field components are given by

$$E_{\theta i} = \frac{1}{r}\frac{\partial^2 (r u_i)}{\partial\theta\,\partial r} \tag{1}$$

$$H_{\phi i} = -(\delta_1 + i\omega\epsilon_i)\frac{\partial u_i}{\partial\theta} \tag{2}$$

$$E_{ri} = -\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial u_i}{\partial\theta}\right) \tag{3}$$

$$E_{\phi i} = H_{ri} = H_{\theta i} = 0 \tag{4}$$

where u satisfies the scalar wave equation

$$\nabla^2 u_i + k_i^2 u_i = 0 \tag{5}$$

$$k_i^2 = \omega^2\mu_0\epsilon_i - i\omega\mu_0\delta_i . \tag{6}$$

To evaluate the above equations, let us consider

$$E = -\nabla\phi$$

where ϕ is a scalar potential.

In the spherical co-ordinate systems

$$E = a_r E_r + a_\theta E_\theta + a_\phi E_\phi$$

$$\nabla\phi = a_r \frac{\partial\phi}{\partial r} + a_\theta \frac{1}{r}\frac{\partial\phi}{\partial\theta} + a_\phi \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\phi}.$$

Writing $\phi = -\frac{\partial}{\partial r}(ru)$, where u is also a function, we get by comparing

$$E_\theta = -\frac{1}{r}\frac{\partial\phi}{\partial\theta} = \frac{1}{r}\frac{\partial^2(ru)}{\partial\theta\,\partial r}$$

or $$E_{\theta i} = \frac{1}{r}\frac{\partial^2(ru_i)}{\partial\theta\,\partial r}$$

Again $$\nabla\times H = \delta E + \frac{\partial D}{\partial t} = \delta E + \epsilon\frac{\partial E}{\partial t}$$

$$= \delta E + i\omega\epsilon E = (\delta + i\omega\epsilon)E$$

or $$\nabla\times H = (\delta + i\omega\epsilon)(a_r E_r + a_\theta E_\theta + a_\phi E_\phi).$$

Also, $$\nabla\times H = \frac{a_r}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(H_\phi\sin\theta) - \frac{\partial H_\theta}{\partial\phi}\right] + \frac{a_\theta}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial}{\partial r}(rH_\phi)\right] + \frac{a_\phi}{r}\left[\frac{\partial}{\partial r}(rH_\theta) - \frac{\partial H_r}{\partial\theta}\right].$$

By comparison we get

$$(\delta + i\omega\epsilon)E_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial}{\partial r}(rH_\phi)\right].$$

As the variation in the ϕ-direction is zero, we get

$$H_\phi = -rE_\theta(\delta + i\omega\epsilon) = -(\delta + i\omega\epsilon)\frac{\partial(u)}{\partial\theta}$$

or $$H_{\phi i} = -(\delta_i + i\omega\epsilon_i)\frac{\partial(u_i)}{\partial\theta}.$$

Also $$(\delta + i\omega\epsilon)E_r = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(H_\phi\sin\theta) - \frac{\partial H_\theta}{\partial\phi}\right]$$

$$E_r = -\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right)\right]$$

or $$E_{ri} = -\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u_i}{\partial\theta}\right)\right]$$

and $$E_{\phi i} = 0;\quad H_{\phi i} = 0;\quad H_{ri} = 0.$$

To solve the equation (5) let us take the wave equation in spherical co-ordinates. Then $\nabla^2 u + k^2 u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)$

$$+ \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial\phi^2} + k^2 u.$$

With the symmetry about the axis the variation with azimuthal angle ϕ may be neglected. Then

$$\nabla^2 u + k^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r^2 \tan\theta}\frac{\partial u}{\partial \theta} + k^2 u$$

$$= 0 \, .$$

Let us assume a product solution of the form

$$u = R\Theta \, ,$$

where R is a function of r alone, and Θ is a function of θ alone. Then

$$\frac{\partial u}{\partial r} = R'\Theta \, , \quad \frac{\partial^2 u}{\partial r^2} = R''\Theta \qquad \text{where} \qquad R'' = \frac{d^2 R}{dr^2}$$

$$\frac{\partial u}{\partial \theta} = R\,\Theta' \, , \quad \frac{\partial^2 u}{\partial \theta^2} = R\,\Theta'' \qquad\qquad \Theta'' = \frac{d^2\Theta}{d\theta^2}$$

So, $R''\Theta + \frac{2}{r}R'\Theta + \frac{1}{r^2}R\Theta'' + \frac{1}{r^2\tan\theta}R\Theta' + k^2 R\Theta = 0$

or $r^2\frac{R''}{R} + 2r\frac{R'}{R} + \frac{\Theta''}{\Theta} + \frac{1}{\tan\theta}\frac{\Theta'}{\Theta} + k^2 r^2 = 0$

or $\left(r^2\frac{R''}{R} + 2r\frac{R'}{R} + k^2 r^2\right) = -\left(\frac{\Theta''}{\Theta} + \frac{1}{\tan\theta}\frac{\Theta'}{\Theta}\right)$.

The two sides of the above equation are equal to each other for all values of r and θ , and as R is a function of r only, and Θ is a function of θ only, both sides can be equal only to a constant. Since the constant may be expressed in any non-restrictive way, let it be n(n+1). Then the two resulting ordinary differential equations are

$$r^2\frac{d^2R}{dr^2} + 2r\frac{dR}{dr} + \{k^2r^2 - n(n+1)\}R = 0$$

and $\dfrac{d^2\Theta}{d^2\theta} + \dfrac{1}{\tan\theta}\dfrac{d\Theta}{d\theta} + n(n+1)\,\Theta = 0$.

It is seen that the solution of the first equation is the Bessel's function. So

$$R = A\,J_n(kr) \, .$$

The solution of the second equation in terms of simple functions is not obvious, so a series solution may be assumed. the co-efficients of this series may be determined so that the differential equation is satisfied and the resulting series made to define a new function. For n is an integer the proper selection of the arbitrary constants will make the series for the new function terminate in a finite number of terms. Thus, for any integer n, the polunomial defined by

$$P_n(\cos\theta) = \frac{1}{2^n n!}\left[\frac{d}{d(\cos\theta)}\right]^n (\cos^2\theta - 1)^n$$

is a solution to the above differential equation.The equation is known as Legendre's equation; the solutions are called Legendre polynomials of order n. It is evident that, since they are polynomials and not infinite series, their values can be calculated exactly if desired . The values are tabulated below

$$P_0(\cos\theta) = 1$$
$$P_1(\cos\theta) = \cos\theta$$
$$P_2(\cos\theta) = \tfrac{1}{2}(3\cos^2\theta - 1)$$
$$P_3(\cos\theta) = \tfrac{1}{2}(5\cos^3\theta - 3\cos\theta)$$
$$P_4(\cos\theta) = \tfrac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3)$$
$$P_5(\cos\theta) = \tfrac{1}{8}(63\cos^5\theta - 70\cos^3\theta + 15\cos\theta).$$

Then one solution is written

$$\Theta = C\, P_n^m(\cos\theta)$$

and the function defined by the above solution is called an associated Legendre function of the first kind, order n, degree m. These are actually related to the ordinary Legendre functions by the equation

$$P_n^m(x) = (1-x^2)^{m/2} \frac{d^m P_n(x)}{dx^m} .$$

By differentiations according to this equation, the polynomials of the first few orders are found to be

$$P_0'(\cos\theta) = 0$$

$$P_1'(\cos\theta) = \sin\theta$$

$$P_2'(\cos\theta) = 3\sin\theta\cos\theta$$

$$P_3'(\cos\theta) = \frac{3}{2}\sin\theta(5\cos^2\theta - 1)$$

$$P_4'(\cos\theta) = 5/2\,\sin\theta(7\cos^3\theta - 3\cos\theta) .$$

So, the solutions of the equation (5) may be written as

$$f_n^{(1)} = A_n j_n(k_i r) P_n(\cos\theta) \quad \text{------} \quad (7)$$

$$f_n^{(4)} = A_n h_n^{(2)}(k_i r) P_n(\cos\theta) \qquad (8)$$

where $j_n(z)$ and $h_n^{(2)}(z)$ are the spherical Bessel functions of the first and fourth kind respectively, and where $P_n(\cos\theta)$ is a Legendre polynomial of order n. The fields in a homogeneous infinite medium may be drived from the equation(8) after letting n=1. Thus, the H_{ϕ_1} component of the dipole moment Idh is given by

$$H_{\phi_1} = \frac{I\,dh}{4\pi r^2}(1 + i k_1 r)\sin\theta\, e^{-ik_1 r} \qquad (9)$$

From equation (8), $\quad h_1 = A_1 h_1^{(2)}(k_1 r) P_1(\cos\theta)$

$$\frac{\partial h_1}{\partial\theta} = A_1 \frac{\partial}{\partial\theta}[h_1^{(2)}(k_1 r) P_1(\cos\theta)] .$$

So from equation (2),

$$H_{\phi_1} = -(\delta_1 + i\omega\epsilon_1) A_1 \frac{\partial}{\partial\theta}\left[h_1^{(2)}(k_1 r) P_1(\cos\theta)\right] \tag{10}$$

where $h_1^{(2)}(k_1 r) = h_1^{(2)}(z) = \left(\frac{i}{z^2} - \frac{1}{z}\right) e^{-iz}$ (11)

and $P_1(\cos\theta) = \cos\theta$. (12)

Carrying out the differentiation in equation (10) it follows that

$$\begin{aligned} H_{\phi_1} &= (\delta_1 + i\omega\epsilon_1) A_1 \left(\frac{i}{z^2} - \frac{1}{z}\right) e^{-iz} \sin\theta \\ &= (\delta_1 + i\omega\epsilon_1) A_1 \frac{(i-z)}{z^2} e^{-iz} \sin\theta, \end{aligned}$$

and from equation (6)

$$(\delta_1 + i\omega\epsilon_1) = -\frac{k_1^2}{i\omega\mu_0} \quad ,$$

we get

$$-\frac{k_1^2}{i\omega\mu_0} A_1 \frac{(i-z)}{k_1^2 r^2} e^{-iz} \sin\theta = \frac{I\,dh}{4\pi r^2}(1+iz)\sin\theta\, e^{-iz}$$

$$\frac{A_1(-1-iz)}{\omega\mu_0} e^{-iz} \sin\theta = \frac{I\,dh}{4\pi}(1+iz)\sin\theta\, e^{-iz}$$

$$\therefore \quad A_1 = -\frac{I\,dh\,\omega\mu_0}{4\pi} \; . \tag{13}$$

Assuming (8) as the primary excitation, suitable solutions for the two regions are

$$u_1 = A\left[h_1^{(2)}(k_1 r) + B\, j_1(k_1 r)\right]\cos\theta \qquad \text{for } 0 < r \leq a \tag{14}$$

$$u_2 = C\, h_1^{(2)}(k_2 r) \cos\theta \qquad \text{for } a \leq r < \infty \; . \tag{15}$$

For the continuity of the tangential electric and magnetic field it is required that at r=a the boundary conditions are,

$$E_{\theta_1} = E_{\theta_2}$$

$$H_{\phi_1} = H_{\phi_2} \; .$$

Thus $(\delta_i + i\omega\epsilon_i)u_i$ and $\frac{\partial}{\partial r}(r u_i)$ are continuous. From the above boundary conditions we may write,

$$(\delta_1 + i\omega\epsilon_1)u_1 = (\delta_2 + i\omega\epsilon_2)u_2$$

or $(\delta_1 + i\omega\epsilon_1)A[h_1^{(2)}(k_1a) + Bj_1(k_1a)]\cos\theta = (\delta_2 + i\omega\epsilon_2)C h_1^{(2)}(k_2a)\cos\theta$

or $h_1^{(2)}(k_1a) + Bj_1(k_1a) = \frac{(\delta_2 + i\omega\epsilon_2)}{(\delta_1 + i\omega\epsilon_1)}\frac{C}{A}h_1^{(2)}(k_2a)$

or $h_1^{(2)}(x) + Bj_1(x) = \frac{(\delta_2 + i\omega\epsilon_2)}{(\delta_1 + i\omega\epsilon_1)}\frac{C}{A}h_1^{(2)}(y)$, (16)

also for $\frac{\partial}{\partial r}(r u_i)$ i.e. $\frac{\partial}{\partial x}(x u_i)$, we get

$$\frac{\partial}{\partial x}[x h_1^{(2)}(x) + Bx j_1(x)] = \frac{C}{A}\frac{\partial}{\partial y}[y h_1^{(2)}(y)] \tag{17}$$

where

$$x = k_1 a \tag{18}$$

$$y = k_2 a \, . \tag{19}$$

The spherical boundary between the two regions at r=a is assumed first to lie in the near fields of the dipole. For $k_1a, k_2a \ll 1$, small argument approximations of the spherical Bessel functions are used in (16) and (17). Then

$$j_1(x) \simeq \frac{x}{3} \tag{20}$$

$$h_1^{(2)}(x) \simeq \frac{x}{3} + \frac{i}{x^2} \tag{21}$$

and letting $\delta_i = 0$, we can solve (16) and (17) as follows. From equation (16) we get

$$\frac{x}{3} + \frac{i}{x^2} + B\frac{x}{3} = \frac{\epsilon_2}{\epsilon_1}\frac{C}{A}\left(\frac{y}{3} + \frac{i}{y^2}\right)$$

or $$B\frac{x}{3} = \frac{\epsilon_2}{\epsilon_1}\frac{C}{A}\left(\frac{y}{3} + \frac{i}{y^2}\right) - \left(\frac{x}{3} + \frac{i}{x^2}\right)$$

and from the equation (17) we get

$$\frac{\partial}{\partial x}\left(\frac{x^2}{3} + \frac{i}{x} + B\frac{x^2}{3}\right) = \frac{C}{A}\frac{\partial}{\partial y}\left(\frac{y^2}{3} + \frac{i}{y}\right)$$

or $\frac{2x}{3} - \frac{i}{x^2} + B\frac{2x}{3} = \frac{C}{A}\left(\frac{2y}{3} - \frac{i}{y^2}\right)$

or $\frac{2x}{3} - \frac{i}{x^2} + \frac{2\epsilon_2}{\epsilon_1}\frac{C}{A}\left(\frac{y}{3} + \frac{i}{y^2}\right) - 2\left(\frac{x}{3} + \frac{i}{x^2}\right) = \frac{C}{A}\left(\frac{2y}{3} - \frac{i}{y^2}\right)$.

Since k_1a, $k_2a \ll 1$, only the following terms are considered i.e.

$$-\frac{i}{x^2} + \frac{2\epsilon_2}{\epsilon_1}\frac{C}{A}\frac{i}{y^2} - \frac{2i}{x^2} = -\frac{C}{A}\frac{i}{y^2}$$

$$-\frac{3}{x^2} = -\frac{2\epsilon_2}{\epsilon_1}\frac{C}{A}\frac{1}{y^2} - \frac{C}{A}\frac{1}{y^2} = -\frac{C}{A}\frac{1}{y^2}\left(1 + \frac{2\epsilon_2}{\epsilon_1}\right)$$

or $$\frac{3}{\epsilon_1} = \frac{C}{A}\frac{1}{\epsilon_2}\left(\frac{\epsilon_1 + 2\epsilon_2}{\epsilon_1}\right)$$

or $$\frac{C}{A} = \frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2}$$

$$\therefore C = A\frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2}$$

as $x^2 = k_1^2a^2 = \omega^2\mu_0\epsilon_1 a^2$

and $y^2 = k_2^2a^2 = \omega^2\mu_0\epsilon_2 a^2$.

Putting the value from (13)

$$C = -\frac{\omega\mu_0 I\,dl}{4\pi} \cdot \frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2} \quad (22)$$

Now determining C and A the fields of the dipole are computed by substituting (15) in (1). Thus we get

$$E_{\theta_2} = \frac{1}{r}\frac{\partial^2(r\pi_2)}{\partial\theta\partial r} = \frac{1}{r}\frac{\partial^2}{\partial\theta\partial r}\left\{C\, r\, h_1^{(2)}(k_2 r)\cos\theta\right\}.$$

Using the large argument approximation of the spherical Bessel function

$$h_1^{(2)}(x) \simeq -\frac{1}{x}e^{-ix}, \quad (23)$$

we get

$$E_{\theta_2} = \left(-\frac{\omega\mu_0 I\,dl}{4\pi}\frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2}\right)\frac{1}{r}\frac{\partial^2}{\partial\theta\partial r}\left\{r\left(-\frac{1}{x}e^{-ix}\right)\cos\theta\right\}$$

$$= \left(\frac{\omega \mu_0 I dl}{4\pi r} \cdot \frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2} \right) \frac{\partial^2}{\partial\theta \partial r} \left\{ \frac{1}{k_2} e^{-ik_2 r \cos\theta} \right\}$$

$$\therefore E_{\theta_2} = \frac{i\omega\mu_0 I dl}{4\pi r} \cdot \frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2} \sin\theta \, e^{-ik_2 r} . \tag{24}$$

The radiated power is

$$W = \frac{1}{2}\sqrt{\frac{\epsilon_2}{\mu_0}} \int_0^{2\pi}\int_0^{\pi} |E_{\theta_2}|^2 r^2 \sin\theta \, d\theta \, d\phi \tag{25}$$

$$= \frac{1}{2}\sqrt{\frac{\epsilon_2}{\mu_0}} \int_0^{2\pi}\int_0^{\pi} \left(\frac{\omega\mu_0 I dl}{4\pi} \cdot \frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2} \right)^2 \sin^3\theta \, d\theta \, d\phi$$

$$= \pi\sqrt{\frac{\epsilon_2}{\mu_0}} \left(\frac{\omega\mu_0 I dl}{4\pi} \cdot \frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2} \right)^2 \cdot \frac{1}{4}\int_0^{\pi} (3\sin\theta - \sin 3\theta) d\theta$$

$$= \frac{1}{12\pi}\sqrt{\frac{\epsilon_2}{\mu_0}} (\omega\mu_0 I dl)^2 \left(\frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2} \right)^2 .$$

Now

$$\omega^2\mu_0^2 = \frac{k_2^2 \mu_0^2}{\mu_0 \epsilon_2} = k_2^2 \frac{\mu_0}{\epsilon_2}$$

$$\therefore W = \frac{k_2^2}{12\pi}\sqrt{\frac{\mu_0}{\epsilon_2}} (I dl)^2 \left(\frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2} \right)^2 .$$

Then the radiation resistance follows from this equation as

$$R = \frac{2W}{I^2} = \frac{k_2^2}{6\pi}\sqrt{\frac{\mu_0}{\epsilon_2}} (dl)^2 \left(\frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2} \right)^2 . \tag{26}$$

The capacitance of the dipole antenna is

$$C = \frac{\epsilon_1 A}{dh} \tag{27}$$

where A is the effective area. The radiation power factor is found as

$$p = \frac{R}{X} = R\omega C = \frac{9\epsilon_1\epsilon_2}{(\epsilon_1+2\epsilon_2)^2}\cdot\frac{k_2^3 A\,dh}{6\pi}. \tag{28}$$

Let the effective antenna volume Adh is V and let $\epsilon_r = \frac{\epsilon_1}{\epsilon_2}$, the equation (28) becomes

$$p = \frac{9\epsilon_r}{(\epsilon_r+2)^2}\;\frac{k_2^3 V}{6\pi}. \tag{29}$$

Now from this equation (29) we can see that the radiation power factor p is increased slightly for $1 < \epsilon_r < 4$, but is decreased for larger values of ϵ_r.

If the spherical boundary between the two regions at r=a lies in the far fields of the dipole, k_1a and $k_2a \gg 1$ and large argument approximations of the spherical Bessel functions are used in (16) and (17). Then

$$\begin{aligned} j_1(x) &\simeq -\frac{1}{x}\cos x \\ h_1^{(2)}(x) &\simeq -\frac{1}{x}e^{-ix} \end{aligned} \tag{30}$$

From (16) we get

$$-\frac{1}{z}e^{-iz} + B\left(-\frac{1}{z}\cos x\right) = \frac{\epsilon_2}{\epsilon_1}\frac{C}{A}\left(-\frac{1}{y}e^{-iy}\right),$$

and from (17) we get

$$\frac{\partial}{\partial x}\left[-e^{-iz} + B(-\cos x)\right] = \frac{C}{A}\frac{\partial}{\partial y}\left(-e^{-iy}\right)$$

or $$i e^{-ix} + B \sin x = i \frac{C}{A} e^{-iy}$$

$$\therefore B = \frac{i \frac{C}{A} e^{-iy} - i e^{-ix}}{\sin x}.$$

Then we may write

$$-\frac{1}{x} e^{-ix} - \frac{1}{x} \cos x \left\{ \frac{i \frac{C}{A} e^{-iy} - i e^{-ix}}{\sin x} \right\} = \frac{\epsilon_2}{\epsilon_1} \frac{C}{A} \left(-\frac{1}{y} e^{-iy}\right)$$

$$e^{-ix}\left(-1 + i \frac{\cos x}{\sin x}\right) = -\frac{\epsilon_2}{\epsilon_1} \frac{C}{A} \frac{x}{y} e^{-iy} + i \frac{\cos x}{\sin x} \frac{C}{A} e^{-iy}$$

$$(\cos x - i \sin x) \frac{(i \cos x - \sin x)}{\sin x} = \frac{C}{A} e^{-iy}\left(i \frac{\cos x}{\sin x} - \frac{\epsilon_2}{\epsilon_1} \sqrt{\frac{\epsilon_1}{\epsilon_2}}\right)$$

$$i = \frac{C}{A} e^{-iy} \left(\frac{i\sqrt{\epsilon_1} \cos x - \sqrt{\epsilon_2} \sin x}{\sqrt{\epsilon_1}} \right)$$

or $$\frac{C}{A} e^{-iy} = \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} \cos x + i\sqrt{\epsilon_2} \sin x}; \quad \text{or } C = A e^{iy} \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} \cos x + i\sqrt{\epsilon_2} \sin x}.$$

So
$$C = A e^{iy} \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} \cos x + i\sqrt{\epsilon_2} \sin x}. \tag{31}$$

From (1) we get the radiated electric fields

$$E_{\theta_2} = \frac{1}{r} \cdot \frac{\partial^2}{\partial\theta \partial r}(r u_2)$$

$$= \frac{1}{r} \frac{\partial^2}{\partial\theta \partial r} \left\{ r C h_1^{(2)}(k_2 r) \cos\theta \right\}$$

$$= \frac{1}{r} \frac{\partial^2}{\partial\theta \partial r} \left\{ r A e^{iy} \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} \cos x + i\sqrt{\epsilon_2} \sin x} \left(-\frac{1}{k_2 r}\right) e^{-ik_2 r} \cos\theta \right\}$$

$$= \frac{1}{r} \frac{\partial^2}{\partial\theta \partial r} \left\{ A e^{iy} \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} \cos x + i\sqrt{\epsilon_2} \sin x} \left(-\frac{1}{k_2}\right) e^{-ik_2 r} \cos\theta \right\}$$

$$= \frac{1}{r} \frac{\partial^2}{\partial\theta \partial r} \left\{ A \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} \cos(k_1 a) + i\sqrt{\epsilon_2} \sin(k_1 a)} \left(-\frac{1}{k_2}\right) e^{-ik_2(r-a)} \cos\theta \right\}$$

$$= \frac{1}{r}\frac{\partial}{\partial\theta}\left\{ A \frac{i\sqrt{\epsilon_1}}{\sqrt{\epsilon_1}\cos(k_1a)+i\sqrt{\epsilon_2}\sin(k_1a)} e^{-ik_2(r-a)} \cos\theta \right\}$$

$$= \frac{i\omega\mu_0 I dh}{4\pi r} \cdot \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_1}\cos(k_1a)+i\sqrt{\epsilon_2}\sin(k_1a)} e^{-ik_2(r-a)} \sin\theta$$

$$= \frac{i\omega\mu_0 I dh \sin\theta}{4\pi r} e^{-ik_2(r-a)} \left[\cos k_1a + i\sqrt{\frac{\epsilon_2}{\epsilon_1}} \sin k_1a\right]^{-1} \qquad (32)$$

As before the dadiated power is

$$W = \tfrac{1}{2}\sqrt{\frac{\mu_0}{\epsilon_2}} \frac{k_2^2 (I dh)^2}{6\pi} \cdot \frac{1}{\cos^2 k_1a + \frac{\epsilon_2}{\epsilon_1}\sin^2 k_1a}$$

The radiation resistance is

$$R = \frac{2W}{I^2} = \sqrt{\frac{\mu_0}{\epsilon_2}} \frac{k_2^2 (dh)^2}{6\pi} \frac{1}{\cos^2 k_1a + \frac{\epsilon_2}{\epsilon_1}\sin^2 k_1a} \qquad (33)$$

The radiation power factor p is obtained from (27) and (33) as

$$p = wRC = \frac{(\epsilon_1/\epsilon_2)}{\cos^2 k_1a + \frac{\epsilon_2}{\epsilon_1}\sin^2 k_1a} \cdot \frac{k_2^2 (A\, dh)}{6\pi} \qquad (34)$$

The radiation power factor p is increased with the increasing values of $\epsilon_r = \frac{\epsilon_1}{\epsilon_2}$. This increase is quadratic with ϵ_r, if $k_1a = (n+\frac{1}{2})\pi$, and linear if $k_1a = n\pi$.

3. DISCUSSION

The short dipole is also termed as condenser-type antenna. The characteristics of the capacitor antenna for the dielectric loading ϵ_r can be deduced from simple reasoning. Dielectric is assumed to be inserted with rotational symmetry in a cpacitor antenna of same symmetry. The boundary of the dielectric is made to follow the electric field lines between the two plates of the capacitor. The electric fields in and near the capacitor and the magnetic fields outside the dielectric remain unaltered if the voltage V of the capacitor is maintained constant. The fields outside the volume occupied by the antenna and the dieledtric depend on the electric and magnetic fields on the boundary surface. The essential part of the argument is that the dielectric boundary is tangential to the electric field E. This requirement is not met with the small dipole, which exhibit sizeable radial electric field conponents at the dielectric boundary.

CHAPTER-III

ELECTRIC DIPOLE SURROUNDED BY AN INFINITELY LONG CYLINDER(1)

The short dipole is located at the centre of an infinitely long cylinder as shown in the fig.1. The electric dipole of moment Ih is assumed to be z-oriented. For a z-oriented electric dipole, the vector potential is given by

$$\vec{A} = \hat{z}\frac{\mu}{4\pi}\int_{-h/2}^{h/2} I \frac{e^{ikR(z,z')}}{R(z,z')}\, dz' \simeq \hat{z}\frac{\mu I h}{4\pi}\frac{e^{ikR}}{R} \tag{1}$$

where $R^2=(x-x')^2+(y-y')^2+(z-z')^2$,

x',y',z' is the location of the dipole

and x,y,z is the point in space where A_z is desired.

The term $\frac{e^{ikR}}{R}$ can be expressed as a combination of the functions involving the cylindrical Hankel function of zeroth order. There being no restriction on the eigen-value, it will be continuous, and therefore $\frac{e^{ikR}}{R}$ should be expressed as an integral over all α's . Writing Weyrich's formula as

$$\frac{e^{ikR}}{R} = \frac{i}{2}\int_{-\infty}^{\infty} e^{i\alpha z}\, H_0^{(1)}(\sqrt{k^2-\alpha^2}\, r)\, d\alpha \tag{2}$$

and subjected to the following conditions

(a) r and z real; (b) $0 \leq \arg\sqrt{k^2-\alpha^2} < \pi$;(c) $0 \leq \arg k < \pi$

the condition (a) is obviously satisfied, and so is (c), since $k=w\sqrt{\mu}\sqrt{\pm\epsilon + i\frac{\sigma}{\omega}}$. The condition (b) will be used to choose the contour of integration. The equation (1) is reduced to

$$A_z = \frac{i\mu I k}{8\pi}\int_{-\infty}^{\infty} H_0\left(\sqrt{k^2-\alpha^2}\,r\right) e^{i\alpha z}\, d\alpha . \tag{3}$$

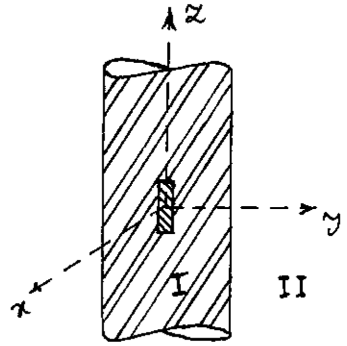


Fig.1- Electric dipole and an infinitely long cylinder.

Again

$$\vec{H} = \frac{1}{\mu}\nabla\times\vec{A} = -\hat{\phi}\,\frac{1}{\mu}\,\frac{\partial A_z}{\partial r}$$

$$\vec{E} = \frac{i\omega}{k^2}\,\nabla(\nabla\cdot\vec{A}) + i\omega\vec{A} \tag{4}$$

From above we get the following relations

$$E_r = \frac{i\omega}{k^2}\,\frac{\partial^2 A_z}{\partial r\,\partial z}$$

$$E_\phi = 0$$

$$E_z = \frac{i\omega}{k^2}\left[k^2 + \frac{\partial^2}{\partial z^2}\right] A_z . \tag{5}$$

The boundary conditions are

$$\left.\begin{aligned} H_\phi^{(1)} &= H_\phi^{(2)} \\ E_z^{(1)} &= E_z^{(2)} \end{aligned}\right\} \quad \text{at } r = a . \tag{6}$$

In region I, the vector potential wil consist of the homogeneous solution of the wave equation and that due to the source. In the region II, it will be only of the homogeneous solution of the wave equation. So the vector potentials in the two regions will be

$$A_x^{(1)} = \frac{i\mu I h}{8\pi}\int_{-\infty}^{\infty} H_0(\sqrt{k^2-\alpha^2}\,r)\, e^{i\alpha z}\, d\alpha + \int_{-\infty}^{\infty} a(\alpha) J_0(\sqrt{k^2-\alpha^2}\,r)\, e^{i\alpha z}\, d\alpha$$

$$A_x^{(2)} = \int_{-\infty}^{\infty} b(\alpha)\, H_0(\sqrt{k_0^2-\alpha^2}\,r)\; e^{i\alpha z}\, d\alpha \quad \ldots\ldots \tag{7}$$

where $a(\alpha)$ and $b(\alpha)$ can be determined from the boundary conditions.

To solve for $a(\alpha)$ and $b(\alpha)$ we use the following properties of Dirac-delta function

$$\int_{-\infty}^{\infty} e^{i(\alpha_1-\alpha_2)z}\, dz = 2\pi\,\delta(\alpha_1-\alpha_2)$$

and $$\int_{-\infty}^{\infty} g(\alpha_1)\,\delta(\alpha_1-\alpha_2)\, d\alpha_1 = g(\alpha_2) \tag{8}$$

From the boundary conditions we get

$$\frac{1}{\mu}\,\frac{\partial A_x^{(1)}}{\partial r}\Big|_{r=a} = \frac{1}{\mu_0}\,\frac{\partial A_x^{(2)}}{\partial r}\Big|_{r=a}$$

or $$\frac{1}{\mu}\left[\frac{i\mu I h}{8\pi}\int_{-\infty}^{\infty} H_0'(la)\, l\, e^{i\alpha z}\, d\alpha + \int_{-\infty}^{\infty} J_0'(la)\, a(\alpha)\, l\, e^{i\alpha z}\, d\alpha\right]$$

$$= \frac{1}{\mu_0}\left[\int_{-\infty}^{\infty} b(\alpha)\, H_0'(l_0 a)\, l_0\, e^{i\alpha z}\, d\alpha\right].$$

Multiplying both sides by $\int_{-\infty}^{\infty} e^{-i\beta z}\, dz$, we get

$$\frac{1}{\mu}\Big[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{i\mu I h}{8\pi} H_0'(la)\, l\, e^{i(\alpha-\beta)z}\, dz\, d\alpha + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J_0'(la)\, a(\alpha)\, l\, e^{i(\alpha-\beta)z}\, dz\, d\alpha\Big]$$

$$= \frac{1}{\mu_0}\Big[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} b(\alpha)\, H_0'(l_0 a)\, l_0\, e^{i(\alpha-\beta)z}\, dz\, d\alpha\Big],$$

where $l = \sqrt{k^2-\alpha^2}$, $l_0 = \sqrt{k_0^2-\alpha^2}$.

The prime sign with J_0 and H_0 indicates differentiation with respect to the argument.

Using the first condition of the equation (8) we get

$$\frac{1}{\mu}\Big[\int_{-\infty}^{\infty} \frac{i\mu I h}{8\pi} H_0'(la)\, l\cdot 2\pi\delta(\alpha-\beta)\, d\alpha + \int_{-\infty}^{\infty} J_0'(la)\, a(\alpha)\, l\cdot 2\pi\delta(\alpha-\beta)\, d\alpha\Big]$$

$$= \frac{1}{\mu_0}\Big[\int_{-\infty}^{\infty} b(\alpha)\, H_0'(l_0 a)\, l_0\cdot 2\pi\delta(\alpha-\beta)\, d\alpha\Big],$$

and from the second condition we get

$$\frac{1}{\mu}\Big[H_0'(la)\, 2\pi l\cdot \frac{i\mu I h}{8\pi} + J_0'(la)\, l\cdot 2\pi\cdot a(\alpha)\Big]$$

$$= \frac{1}{\mu_0}\Big[b(\alpha)\, H_0'(l_0 a)\, l_0\cdot 2\pi\Big]$$

or

$$\frac{l}{l_0}\Big[H_0'(la)\cdot \frac{i\mu I h}{8\pi} + J_0'(la)\, a(\alpha)\Big]$$

$$= \frac{\mu}{\mu_0}\Big[b(\alpha)\, H_0'(l_0 a)\Big]. \tag{9}$$

Again using the boundary condition $E_z^{(1)}=E_z^{(2)}$, we get

$$\frac{i\omega}{k^2}\left[k^2+\frac{\partial^2}{\partial z^2}\right]_{r=a} A_z^{(1)} = \frac{i\omega}{k_0^2}\left[k_0^2+\frac{\partial^2}{\partial z^2}\right]_{r=a} A_z^{(2)} .$$

Now, $$\frac{\partial A_z^{(1)}}{\partial z} = \frac{i\mu I h}{8\pi}\int_{-\infty}^{\infty} H_0(\sqrt{k^2-\alpha^2}\, r)\frac{\partial}{\partial z} e^{i\alpha z}\, d\alpha$$
$$+\int_{-\infty}^{\infty} a(\alpha) J_0(\sqrt{k^2-\alpha^2}\, r)\frac{\partial}{\partial z} e^{i\alpha z}\, d\alpha$$

or $$\frac{\partial^2 A_z^{(1)}}{\partial z^2} = \frac{i\mu I h}{8\pi}\int_{-\infty}^{\infty} H_0(\sqrt{k^2-\alpha^2}\, r)(-\alpha^2) e^{i\alpha z}\, d\alpha$$
$$+\int_{-\infty}^{\infty} a(\alpha) J_0(\sqrt{k^2-\alpha^2}\, r)(-\alpha^2) e^{i\alpha z}\, d\alpha = -\alpha^2 A_z^{(1)},$$

and similarly $$\frac{\partial^2 A_z^{(2)}}{\partial z^2} = -\alpha^2 A_z^{(2)} .$$

So we may get

$$\frac{i\omega}{k^2}\left[k^2-\alpha^2\right]_{r=a} A_z^{(1)} = \frac{i\omega}{k_0^2}\left[k_0^2-\alpha^2\right]_{r=a} A_z^{(2)}$$

or $$l^2 A_z^{(1)}\Big|_{r=a} = \frac{k^2}{k_0^2}\, l_0^2 A_z^{(2)}\Big|_{r=a}$$

or $$\frac{l^2}{l_0^2}\left[\frac{i\mu I h}{8\pi} H_0(la) + a(\alpha) J_0(la)\right] = \frac{k^2}{k_0^2}\left[b(\alpha) H_0(l_0 a)\right]. \qquad (10)$$

Dividing (9) by (10) we get

$$\frac{\left[\frac{i\mu I h}{8\pi} H_0'(la) + J_0'(la)\, a(\alpha)\right]}{\frac{l}{l_0}\left[\frac{i\mu I h}{8\pi} H_0(la) + J_0(la)\, a(\alpha)\right]} = \frac{\frac{\mu}{\mu_0} H_0'(l_0 a)}{\frac{k^2}{k_0^2} H_0(l_0 a)} .$$

Putting $\frac{\mu}{\mu_0} = K_m$, $N = \frac{k}{k_0}$, we get

$$N^2 H_0(la)\left[\frac{i\omega I h}{8\pi} H_0'(la) + J_0'(la)\, a(\alpha)\right]$$

$$= K_m \frac{l}{l_0} H_0'(l_0 a)\left[\frac{i\omega I h}{8\pi} H_0(la) + J_0(la)\, a(\alpha)\right]$$

or $$a(\alpha)\left[N^2 H_0(l_0 a) J_0'(la) - K_m \frac{l}{l_0} H_0'(l_0 a) J_0(la)\right]$$

$$= \frac{i\omega I h}{8\pi}\left[H_0(l_0 a) H_0(la) . K_m \frac{l}{l_0} - N^2 H_0(l_0 a) H_0'(la)\right]$$

or $$a(\alpha) = \frac{\frac{i\omega I h}{8\pi}\left[H_0(l_0 a) H_0'(la) - \frac{K_m}{N^2}\frac{l}{l_0} H_0(la) H_0'(l_0 a)\right]}{\frac{K_m}{N^2}\frac{l}{l_0} J_0(la) H_0'(l_0 a) - J_0'(la) H_0(l_0 a)}.$$

Putting the value of $a(\alpha)$ in (9) we get

$$\frac{l}{l_0}\left[\frac{i\omega I h}{8\pi} H_0'(la) + J_0'(la) . \left\{\frac{\frac{i\omega I h}{8\pi}\left[H_0(l_0 a) H_0'(la) - \frac{K_m}{N^2}\frac{l}{l_0} H_0(la) H_0'(l_0 a)\right]}{\frac{K_m}{N^2}\frac{l}{l_0} J_0(la) H_0'(l_0 a) - J_0'(la) H_0(l_0 a)}\right\}\right].$$

$$= K_m\, b(\alpha)\, H_0'(l_0 a)$$

or $$b(\alpha) = \frac{\frac{K_m}{N^2}\frac{l}{l_0}\;\frac{i\omega I h}{8\pi}\left[H_0'(la) J_0(la) - J_0'(la) H_0(la)\right]}{K_m\left[\frac{K_m}{N^2}\frac{l}{l_0} J_0(la) H_0'(l_0 a) - J_0'(la) H_0(l_0 a)\right]}.$$

But $J_0(la)H_0'(la) - J_0'(la)H_0(la) = \frac{2i}{\pi(la)}$.

So $b(\alpha) = \frac{l \mu I h}{4N^2 l_0^2 \pi^2 a} \cdot \frac{1}{\left[J_0'(la)H_0(l_0 a) - \frac{K_m}{N^2}\frac{l}{l_0} J_0(la) H_0'(l_0 a)\right]}$.

Hence the solution of the vector potential is formally given by (7) with the values of $a(\alpha)$ and $b(\alpha)$.

With $K_m=1$, $N=1$,

$a(\alpha)=0$

and $b(\alpha) = \frac{l_0 \mu_0 I h}{4 l_0^2 \pi^2 a} \cdot \frac{1}{\left[J_0'(l_0 a)H_0(l_0 a) - J_0(l_0 a)H_0'(l_0 a)\right]}$.

Then $b(\alpha) = \frac{i \mu_0 I h}{8\pi}$.

This is exactly what is expected because in the absence of the cylinder, the solution for A_z must be given by (3).

Radiated Power

The Poyinting vector is given by

$$\bar{P} = \tfrac{1}{2} E \times H^* = \tfrac{1}{2}\begin{vmatrix} \frac{a_r}{r} & a_\phi & \frac{a_z}{r} \\ E_r & r E_\phi & E_z \\ H_r^* & r H_\phi^* & H_z^* \end{vmatrix}$$

$$\bar{P} = \tfrac{1}{2}\left[a_r(E_\phi H_z^* - E_z H_\phi^*) - a_\phi(E_r H_z^* - E_z H_r^*) + a_z(E_r H_\phi^* - E_\phi H_r^*)\right].$$

Since $E_\phi = 0$ and no power in the ϕ-direction, we get

$$\bar{P} = \tfrac{1}{2}\left[a_z E_r H_\phi^* - a_r E_z H_\phi^*\right].$$

To find the radiated energy we may find the poynting vector along the infinitely long cylinder. So the energy will be $W=\int_{-\infty}^{\infty} P_r 2\pi r\, dz$ plus some contributions of ends of the cylinder located at $\pm\infty$. This flow of energy across the two ends of the cyclinder at $\pm\infty$ does not contribute to the field radiated away from the cylinder and will hot be considered here. So the radiated energy will be

$$W \simeq \int_{-\infty}^{\infty} P_r 2\pi r \quad dz$$

$$W = -\int_{-\infty}^{\infty} E_z H_\phi^* \pi r \quad dz \qquad \text{, since } \quad \bar{P}_r = -E_z H_\phi^* .$$

For the second region

$$E_z = \frac{i\omega}{K_0^2}\left[K_0^2 + \frac{\partial^2}{\partial z^2}\right] A_z^{(2)}$$

$$= \frac{i\omega}{K_0^2}\left[K_0^2 - \alpha^2\right] A_z^{(2)} .$$

$$= \frac{i\omega}{K_0^2}\left[K_0^2-\alpha^2\right] \int_{-\infty}^{\infty} b(\alpha)\, H_0(l_0 r)\, e^{i\alpha z} d\alpha$$

$$= \frac{i\omega l_0^2}{K_0^2} \int_{-\infty}^{\infty} b(\alpha)\, H_0(l_0 r)\, e^{i\alpha z}\, d\alpha$$

$$H_\phi = -\frac{1}{\mu_0}\frac{\partial A_z^{(2)}}{\partial r} = -\frac{1}{\mu_0}\int_{-\infty}^{\infty} b(\alpha)\, l_0\, H_0'(l_0 r)\, e^{i\alpha z}\, d\alpha .$$

So $$W = \int_{-\infty}^{\infty} \pi r\, dz \int_{-\infty}^{\infty} \frac{i\omega l_0^2}{K_0^2}\, b(\alpha)\, H_0(l_0 r)\, e^{i\alpha z} d\alpha$$

$$\cdot \int_{-\infty}^{\infty} \frac{1}{\mu_0}\left[b(\alpha)\, l_0\, H_0'(l_0 r)\right]^* e^{-i\alpha z}\, d\alpha$$

$$= \pi r \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} \frac{i\omega}{K_0^2}\, l_0^2\, b(\alpha)\, g(\alpha)\, e^{i\alpha z}\, d\alpha$$

$$\cdot \int_{-\infty}^{\infty} \frac{1}{\mu_0}\left[b(\alpha)\, l_0\, g(\beta)\right]^* e^{-i\beta z}\, d\beta$$

$$= \pi r \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \cdot \frac{i\omega l_0^2}{k_0^2 \mu_0} \cdot b(\alpha)\, g(\alpha) [l_0 b(\alpha)\, g(\beta)]^* \int_{-\infty}^{\infty} e^{i(\alpha-\beta)z} dz$$

From the condition of Dirac-delta function of (8) we get

$$W = \pi r \int_{-\infty}^{\infty} d\beta \cdot \frac{i\omega l_0^2}{k_0^2 \mu_0} \cdot b(\alpha) [l_0 b(\alpha) g(\beta)]^* 2\pi \int_{-\infty}^{\infty} g(\alpha)\, \delta(\alpha-\beta)\, d\alpha$$

$$= \pi r \int_{-\infty} d\beta \cdot \frac{i\omega l_0^2}{k_0^2 \mu_0} \cdot b(\alpha) [l_0 b(\alpha) g(\beta)]^* \cdot 2\pi g(\beta)$$

$$= \frac{2 i\omega \pi^2 r}{\mu_0 k_0^2} \int_{-\infty}^{\infty} b(\alpha)\, l_0^2 [l_0 b(\alpha) g(\beta)]^* g(\beta)\, d\beta$$

$$= \frac{2 i\omega \pi^2 r}{\mu_0 k_0^2} \int_{-\infty}^{\infty} b(\alpha)\, b^*(\alpha)\, l_0^2 \cdot l_0^* H_0(l_0 r) [H_0'(l_0 r)]^* d\alpha$$

We have
$$H_0^{(1)} = J_0 + j N_0$$
$$H_0^{(2)} = J_0 - j N_0$$

that is, the congugate of $H_0^{(1)}$ is $H_0^{(2)}$. So we have

$$W = \frac{2 i\omega \pi^2 r}{\mu_0 k_0^2} \int_{-\infty}^{\infty} b(\alpha)\, b^*(\alpha)\, l_0^2\, l_0^* H_0^{(1)}(l_0 r)\, H_0'^{(2)}(l_0 r)\, d\alpha \qquad (11)$$

The equation (11) is the complete expression for the radiated energy both in the near and far field. For evaluating the far field an approximate method will be discussed here.

Asymptotic evaluation of the electromagnetic fields by the method of steepest descent.

An asymptotic evaluation of the following form is required.. $A_z^{(2)} = \int_{-\infty}^{\infty} b(\alpha)\, H_0(\sqrt{k_0^2-\alpha^2}\, r)\, e^{i\alpha z}\, d\alpha$.

The variable of integration α runs along the real axis from $-\infty$ to $+\infty$. There are two branch points at $\alpha = \pm k_0$ which lie on the real axis. This difficulty may be avioded if k_0 is first assumed to have a small imaginary part corresponding to a small but finite conductivity of air and then let this imaginary part approach zero. With large value of r it is assumed that $(\sqrt{k_0^2-\alpha^2}\, r) \gg 1$.

Then

$$A_z^{(2)} \underset{r\to\infty}{\simeq} \int_{-\infty}^{\infty} b(\alpha) \sqrt{\frac{2}{\pi\sqrt{k_0^2-\alpha^2}\, r}}\, e^{-i\pi/4} e^{i[\alpha z + \sqrt{k_0^2-\alpha^2}\, r]}\, d\alpha \tag{12}$$

To integrate the above equation by the method of steepest descent it is necessary to make the following substitution:

$$\left.\begin{aligned} \alpha &= k_0 \sin\phi \\ \phi &= \phi_1 + i\phi_2 \\ z &= R \sin\theta \\ r &= R \cos\theta \end{aligned}\right\} \tag{13}$$

The angle θ is measured from the r-co-ordinates and from the z-axis.

Now to evaluate the integral by the method of steepest descent the integral may be written

$$I = \int_{-\infty}^{\infty} f(\alpha)\, e^{i[\alpha z + \sqrt{k_0^2-\alpha^2}\, r]}\, d\alpha \tag{14}$$

where $f(\alpha)$ is a complicated function. The condition of the equation (13) are substituted in (14) and we get

$$I = \int_C f(K_0 \sin\phi)\, e^{i(K_0 R \sin\theta \sin\phi + K_0 R \cos\theta \cos\phi)} K_0 \cos\phi \, d\phi$$

$$= \int_C f(K_0 \sin\phi)\, e^{i\{K_0 R \cos(\theta-\phi)\}} \cdot K_0 \cos\phi \, d\phi \tag{15}$$

where C is a suitable contour.

Now the transformation of the equation (14) from the α-plane to the ϕ-plane changes the contour from along the real axis between $-\infty$ to $+\infty$ to ABC as shown in fig.-2. It is desired to deform this contour along which the imaginary part of $iK_0 R \cos(\theta-\phi)$ is constant in the region where its real part is the largest. At the maximum point of $iK_0 R \cos(\theta-\phi)$, namely at $\phi = \theta - i0$, $\mathrm{Re}[iK_0 R \cos(\theta-\phi)]$ is largest. This is the so called "saddle point". The equation to the deformed contour is obtained by evaluating $I_m[iK_0 R \cos(\theta-\phi)]$ at $\phi = \theta - i0$. This gives

$$\cos(\theta-\phi) \cosh\phi_2 = 1. \tag{16}$$

The plot of (16) within the allowed Riemann sheet is A'B'C' as shown in fig.-2. Along this contour i.e. ϕ_1 and ϕ_2 satisfying equation (16)

$$e^{iK_0 R \cos(\theta-\phi)} = e^{iK_0 R \frac{\sinh^2\phi_2}{\cosh\phi_2}}. \tag{17}$$

The imaginary part of the exponent in (17) is constant along the new contour. For large R, the quantity has a maximum at $\phi_2=0$ and rapidly decreases on both sides of the saddle point.

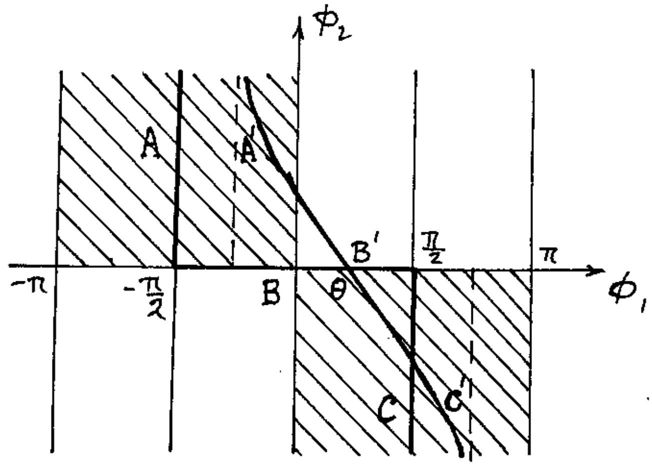


Fig.-2. Contours of Integration in the Complex ϕ-plane.

The poles on the proper Riemann sheet are the surface wave poles and those an the improper sheet are the leaky-wave poles. Without explicitly evaluating the residues at these poles, it may be observed that from the physical point of view, these contributions must be very small if the field far away from the surface of the cylinder is desired, because all such contributions will decay exponentially. Therefore for the field far away from the surface the significant contribution will be from the saddle point. About the saddle point

$$\frac{\sinh^2\phi_2}{\cosh\phi_2} \simeq \phi_2^2 \tag{18}$$

and along the new contour

$$d\phi = \sqrt{2}\, e^{-i\pi/4}\, d\phi_2 . \tag{19}$$

Then from (15) and (19) we get

$$I=\int_{-u}^{u}\sqrt{2}\, f(K_0\sin\phi)\, e^{iK_0R}\, e^{-iK_0R\phi_2^2}\, e^{-i\pi/4}.\ K_0\cos\phi\ d\phi_2 \tag{20}$$

where u is a point on the contour close to the saddle point. Within this region $f(K_0\sin\phi)\cos\phi \simeq f(K_0\sin\theta)\cos\theta$, independently of ϕ_2. Also, if the limit of the integration is changed to $-\infty$ to $+\infty$, very little is added to the orignal value, Then

$$I \simeq \sqrt{2} f(K_0\sin\theta) K_0\cos\theta\, e^{-i\pi/4} e^{iK_0R}\int_{-\infty}^{\infty} e^{-K_0R\phi_2^2}\, d\phi_2$$

$$\simeq \sqrt{\frac{2\pi}{K_0R}}\, f(K_0\sin\theta) K_0\cos\theta\, e^{-i\pi/4} e^{iK_0R}. \tag{21}$$

Then the equation (12) may be written as

$$A_{z\ \substack{r\to\infty\\ R\to\infty}}^{(2)} \simeq b(K_0\sin\theta)\sqrt{\frac{2}{\pi\sqrt{K_0^2-\alpha^2}\,r}}\ e^{-i\pi/4}\sqrt{\frac{2\pi}{K_0R}}\cdot K_0\cos\theta\, e^{-i\pi/4} e^{iK_0R}$$

$$\simeq 2b(K_0\sin\theta)\frac{K_0\cos\theta}{\sqrt{K_0\cos\phi\, r\cdot K_0R}}\, e^{-i\pi/2}\, e^{iK_0R}$$

$$\simeq 2b(K_0\sin\theta)\ \frac{K_0\cos\theta}{K_0 r}\ e^{-i\pi/2} e^{iK_0R}$$

or $$A_{z\ \substack{r\to\infty\\ R\to\infty}}^{(2)} \simeq 2b(K_0\sin\theta)\, e^{-i\pi/2}.\ \frac{e^{iK_0R}}{R}. \tag{22}$$

Then the electromagnetic far fields are

$$H_{\phi\ \substack{r\to\infty\\ R\to\infty}}^{(2)} \simeq -\frac{2}{\mu_0}\, b(K_0\sin\theta)\, K_0\cos\theta\ \frac{e^{iK_0R}}{R}$$

$$E_{\theta\ \substack{r\to\infty\\ R\to\infty}}^{(2)} \simeq \sqrt{\frac{\mu_0}{\epsilon_0}}\ H_\phi^{(2)} \tag{23}$$

where $b(k_0 \sin\theta)$ is the value of $b(\alpha)$ with $\alpha = k_0 \sin\theta$.

If we can assume that $l_0 a$, $la \ll 1$, then the equation (23) can be reduced to a simple form. There is, however, one restriction since α runs from $-\infty$ to $+\infty$, l_0 and l also run from $-\infty$ to $+\infty$. If z is assumed to be large, then the major contribution to the integral (12) will be from regions where α is not too large. Thus for large z, $l_0 a$ and la may assumed values small compared to unity.

For this special case, $b(\alpha)$ reduces to

$$b(\alpha) \simeq \frac{\mu_0 I h}{4\pi^2} \cdot \frac{1}{\frac{l_0^2 a^2}{2}(1-\epsilon_r) - \frac{2i}{\pi}} \tag{24}$$

and

$$\left.\begin{aligned} H_\phi^{(2)} &\simeq H_{\phi_0}\left[\frac{1}{1-\frac{i\pi k_0^2 a^2 \cos^2\theta(\epsilon_r - 1)}{4}}\right] \\ H_{\phi_0} &= -\frac{i\mu I h k_0 \cos\theta}{4\pi} \cdot \frac{e^{ik_0 R}}{R} \end{aligned}\right\} \text{------} \tag{25}$$

where H_{ϕ_0} is the magnetic field in free space due to a dipole alone. If $\epsilon_r = 1$, the term $\left[\frac{1}{1-\frac{i\pi k_0^2 a^2 \cos^2\theta(\epsilon_r - 1)}{4}}\right]$ becomes unity as expected, and the equation (25) reduces to the far zone magnetic field due to a dipole alone. The electric field is obtained by multiplying the equation (25) with $\sqrt{\mu_0/\epsilon_0}$.

It is possible to use the large argument approximation of the Bessel and Hankel functions if the diameter of the cylinder is assumed larger such that la and $l_0 a$ are both much greater than unity. Then we get

$$b(\alpha) \simeq \frac{i\mu I h}{8\pi}\left[\frac{l}{l_0}\right]^{1/2} \frac{e^{-iS_0}}{\cos s - i\frac{\epsilon_r l_0}{l}\sin s} \tag{26}$$

where $s = la - \pi/4$, $S_0 = l_0 a - \pi/4$.

Then

$$H_{\phi}^{(2)} \underset{\substack{r\to\infty \\ R\to\infty}}{\simeq} H_{\phi_0} \frac{(l/l_0)^{1/2} e^{-iS_0}}{\cos S - i\,\frac{\epsilon_r l_0}{l}\sin S} \tag{27}$$

where the value of H_{ϕ_0} has been given above.

As it is difficult to carry out integration about an infinite cylinder to obtain total radiated power, only the poynting vector will be considered for the purpose of comparison. The poynting vector for the medium,

$$P_r = \tfrac{1}{2}\, E_\theta^{(2)} \times H_\phi^{(2)} = \frac{1}{2}\sqrt{\frac{\mu_0}{\epsilon_0}}\left|H_\phi^{(2)}\right|^2$$

The poynting vector P_0 for the case of dipole in free space

$$P_0 = \tfrac{1}{2} E_\theta^{(2)} \times H_\phi^{(2)} \qquad = \tfrac{1}{2}\sqrt{\frac{\mu_0}{\epsilon_0}}\left|H_\phi^{(2)}\right|^2_{\text{at } \epsilon_r = 0.}$$

For free space i.e. at $\epsilon_r = 0$

$$H_\phi^{(2)} = H_{\phi_0} \tag{28}$$

Then putting the values of (27) and (28) we get

$$P_r = \tfrac{1}{2}\sqrt{\frac{\mu_0}{\epsilon_0}}\, H_{\phi_0}^2 \left|\frac{(l/l_0)^{1/2}\, e^{-iS_0}}{\cos S - i\,\frac{\epsilon_r l_0}{l}\sin S}\right|^2$$

and

$$P_0 = \tfrac{1}{2}\sqrt{\frac{\mu_0}{\epsilon_0}}\, H_{\phi_0}^2 .$$

So the ratio of the magnitudes of the two poynting vectors is given by

$$\frac{P_r}{P_0} \simeq \left|\frac{(l/l_0)^{1/2}}{\cos S - i\,\frac{\epsilon_r l_0}{l}\sin S}\right|^2 . \tag{29}$$

So far we have, there was no restriction on the value of ϵ and μ. In the following discussion both ϵ and μ will be assumed real. We take

$$\frac{\ell}{\ell_0} = \frac{\sqrt{k^2 - k_0^2 \sin^2\theta}}{k_0 \cos\theta} = \frac{\sqrt{N^2 - \sin^2\theta}}{\cos\theta} .$$

Therefore, with ϵ and μ real we have

$$\frac{P_r}{P_0} \simeq \frac{\dfrac{\sqrt{N^2-\sin^2\theta}}{\cos\theta}}{1+\left[\dfrac{\epsilon_r^2 \cos^2\theta}{N^2-\sin^2\theta} - 1\right]\sin^2 s} . \tag{30}$$

From the above expression if ϵ_r and N are unity, P_r/P_0 reduces to unity and it is expected. Consider the case when $\theta = 0$. Then

$$\frac{P_r}{P_0} \simeq \frac{N}{1+\left(\frac{\epsilon_r^2}{N^2}-1\right)\sin^2 s} ,$$

but $N = \frac{k}{k_0} = \sqrt{\frac{\mu}{\mu_0}\cdot\frac{\epsilon}{\epsilon_0}} = \sqrt{\epsilon_r k_m}$, $\quad \frac{\epsilon_r^2}{N^2} = \frac{\epsilon_r}{k_m}$

and $\quad s = \sqrt{k^2-\alpha^2}\,a - \pi/4 = ka - \pi/4 = \sqrt{\epsilon_r k_m}\, k_0 a$.

$$\text{So} \quad \frac{P_r}{P_0} \simeq \frac{\sqrt{\epsilon_r k_m}}{1+\left(\frac{\epsilon_r}{k_m}-1\right)\sin^2\left[\sqrt{\epsilon_r k_m}\, k_0 a - \pi/4\right]} . \tag{31}$$

Let us consider a few simple cases.

<u>Case I.</u>

$k_m = 1$, $(\sqrt{\epsilon_r}\, k_0 a)$ varies, then

$$\frac{P_r}{P_0} \simeq \frac{\sqrt{\epsilon_r}}{1+(\epsilon_r - 1)\sin^2\left[\sqrt{\epsilon_r}\, k_0 a - \pi/4\right]} . \tag{32}$$

The maximum value of P_r/P_0 is about

$$(\sqrt{\epsilon_r}\,k_0 a - \pi/4) = n\pi, \text{ where } n=0,1,2,\ldots\ldots\ldots$$

At these points

$$\left.\frac{P_r}{P_0}\right|_{max.} \simeq \sqrt{\epsilon_r} .$$

Again the minimum value of P_r/P_0 is about

$$(\sqrt{\epsilon_r}\,K_0 a - \pi/4) = (2m+1)\pi/2, \text{where } m=0,1,2,\ldots\ldots$$

At these points

$$\left.\frac{P_r}{P_0}\right|_{min.} \simeq \frac{\sqrt{\epsilon_r}}{1+(\epsilon_r-1)} \simeq \frac{1}{\sqrt{\epsilon_r}} .$$

The nature of variation of Pr/P_0 with ϵ_r & $k_0 a$ is shown in fig.-3.

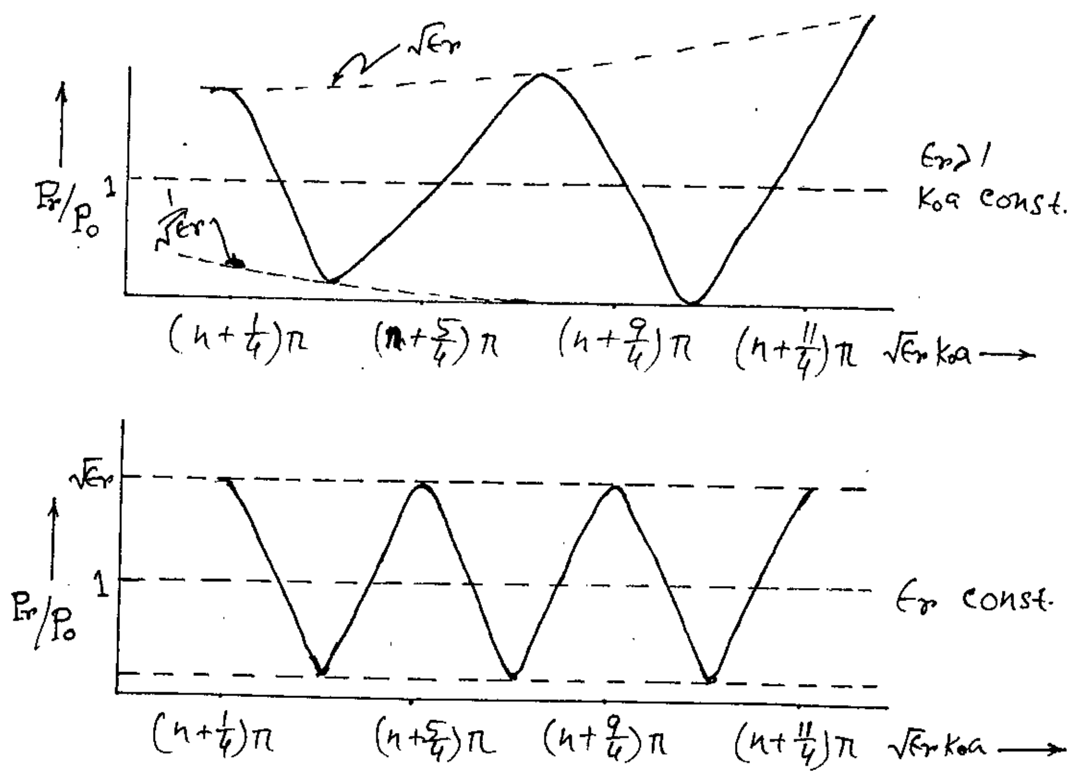


Fig.-3. Variation of radiated power with ϵ_r and $k_0 a$ (not to scale).

Case-II.

k_m varies, $\epsilon_r=1$, k_0a=Constant, then

$$\frac{P_r}{P_0} \simeq \frac{\sqrt{K_m}}{1+\left(\frac{1}{K_m}-1\right)\sin^2\left[\sqrt{K_m}\,K_0a-\pi/4\right]}$$

The maximum points are about

$$(\sqrt{K_m}\,K_0a - \pi/4) = (2n+1)\pi/2, \text{ where } n=0,1,2,\ldots\ldots.$$

At these points

$$\left.\frac{P_r}{P_0}\right|_{max.} \simeq (K_m)^{3/2}.$$

The minimum points are about

$$(\sqrt{K_m}\,K_0a - \pi/4) = m\pi, \text{ where } m=0,1,2,\ldots\ldots\ldots\ldots.$$

At these points

$$\left.\frac{P_r}{P_0}\right|_{min.} \simeq \sqrt{K_m}.$$

This is shown in fig-4.

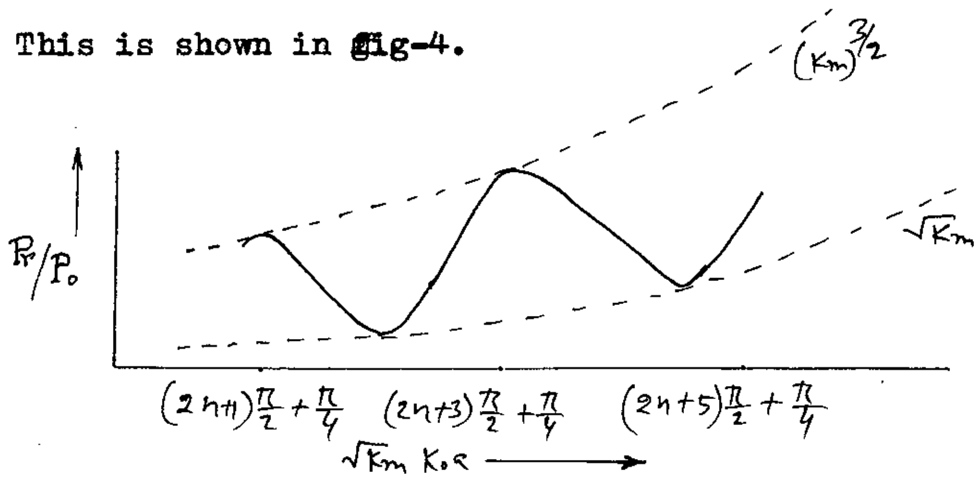


Fig.-4. Variation of radiated power with k_m (not to scale).

DISCUSSION

From the plots of fig.3 and fig.4, it is appeared that in the case of large cylinder, the radiation depends on k_m and ϵ_r, and there are resonant and antiresonant points. According to the pattern shown in figures the radiation increases with the increase of k_m and ϵ_r. It may be pointed out that the cylinder behaves like a "resonance cavities". This is the cause of the existence of the resonance and antiresonance points, with the resonance occuring at intervals $k_a = \pi$ radians. This is very helpful to design the antenna structure. These curves also reveal that if maximization of the radiation resistance is the criterian of the design, then it is suitable to increase the values of k_m or ϵ_r than to increase the physical structure of the antenna. Because there is a range beyond which there will be no extra benefit for the larger size of the antenna structure. Another point of discussion is that if ϵ_r is less than unity the value of $H_\phi^{(2)}$ does not increase. And for all other values of ϵ_r except unity, it decreases. This is not surprising because in this case, the infinitely long cylindrical dielectric rod behaves as a wave guide and some energy will be flowing along the rod.

CHAPTER-IV

DIPOLES IN A CONDUCTING MEDIUM[9]

I. INTRODUCTION

In this section we shall investigate an electric dipole located in a semiinfinite dissipative medium. Sommerfield was the first to formulate the problem of an electric dipole over a conducting earth, obtaining an asymptotic sotution when the dipole and the observer were on the interface. Modified saddle-point integration techniques have been umsed for the more general case of the dipole over the conducting earth. In the present investigation we shall apply a new technique to the Sommerfield integrals for asymptotic solutions of Fourier integrals. We shall consider the case for vertical and horizontal electric dipoles and it will be easy to treat the megnetic dipoles.

II. Case A: Vertical Electric Dipole

The geometry has been shown in fig.1. The dipole is located at $z=z_0$ above the interface. The upper region $z>0$ will be designated as region 1 and the lower region $z<0$ will be designated as region 2. The plane interface is at $z=0$. An e^{-iwt} time dependence is assumed and also it is assumed the permeability μ is identical in both the media.

For cylindrical symmetry we can express the entire electromagnatic fields for the vertical dipole in terms of

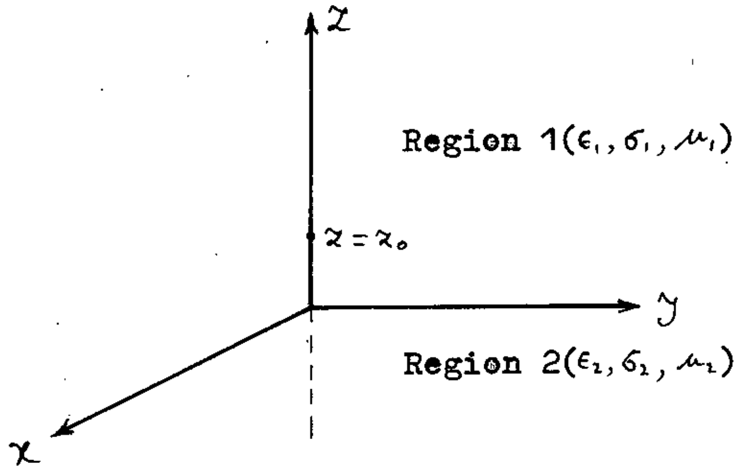


Fig.1. Co-ordinate system.

the z-component of the Hertzian vector Π_z. Let us take

$$E = \frac{i\omega}{\beta^2} \nabla(\nabla \cdot A) + i\omega A$$

$$B = \nabla \times A$$

$$A = -\hat{z}\, \frac{i\beta^2}{\omega} \Pi_z = -\hat{z}\, i\omega\mu\epsilon\, \Pi_z$$

where $\beta^2 = \omega^2\mu\epsilon$.

So

$$\Pi_z = -\frac{A}{i\omega\mu\epsilon} = \frac{iA}{\omega\mu\epsilon}$$

$$\nabla \cdot A = -\hat{z}\, i\omega\mu\epsilon(\nabla \cdot \Pi_z) = -i\omega\mu\epsilon\left(\frac{\partial \Pi_z}{\partial z}\right).$$

So we get

$$E = \frac{i\omega}{\beta^2} \nabla\left\{-i\omega\mu\epsilon\left(\frac{\partial \Pi_z}{\partial z}\right)\right\} + i\omega(-\hat{z}\, i\omega\mu\epsilon\, \Pi_z)$$

or $$E = \nabla\left(\frac{\partial \Pi_z}{\partial z}\right) + \hat{z}\beta^2 \Pi_z \qquad (1)$$

and $$B = \nabla \times A = -\frac{i\beta^2}{\omega} \nabla \times (\hat{z}\Pi_z)$$

where the complex propagation constant

$$\beta^2 = \omega^2 \mu \epsilon \tag{2}$$

Again $(\nabla^2 + \beta^2) A_z = -\mu J_z$ and considering the infinitesimal dipole strength p_z, we get

$$J_z = \frac{\partial p_z}{\partial t} = -i\omega p_z .$$

Then $$(\nabla^2 + \beta^2) A_z = i\omega\mu p_z$$

or $$(\nabla^2 + \beta^2)(-i\omega\mu\epsilon \Pi_z) = i\omega\mu p_z$$

$$(\nabla^2 + \beta^2)\Pi_z = -\frac{p_z}{\epsilon} .$$

Then with subscripts 1 and 2 for the regions, the field equations for an infinitesimal dipole of strength p_z in region 1 and 2 are

$$\left.\begin{aligned} &\nabla^2 \Pi_{1z} + \beta_1^2 \Pi_{1z} = -\frac{p_z}{\epsilon}\delta(x)\delta(y)\delta(z-z_0) \quad , z \geq 0 \\ &\nabla^2 \Pi_{2z} + \beta_2^2 \Pi_{2z} = 0, \qquad\qquad z \leq 0 \end{aligned}\right\} \tag{3}$$

But the boundary conditions are

$$E_{t1} = E_{t2} , \quad H_{t1} = H_{t2}$$

$$\epsilon_1 E_{n1} = \epsilon_2 E_{n2} \quad , \mu_1 H_{n1} = \mu_2 H_{n2} .$$

We may write

$$B_1 = -\frac{i\beta_1^2}{\omega} \nabla \times (\hat{z}\Pi_{1z})$$

or $$H_1 = -\frac{i\beta_1^2}{\omega\mu_1} \nabla \times (\hat{z}\Pi_{1z}) = -i\omega\epsilon_1 \nabla \times (\hat{z}\Pi_{1z})$$

and $$H_2 = -\frac{i\beta_2^2}{\omega\mu_2} \nabla \times (\hat{z}\Pi_{2z}) = -i\omega\epsilon_2 \nabla \times (\hat{z}\Pi_{2z}) .$$

So $\epsilon_1 \Pi_{1z} = \epsilon_2 \Pi_{2z}$.

Again $E_1 =$

Again $$E_1 = \nabla\left(\frac{\partial \Pi_{1z}}{\partial z}\right) + \hat{z}\beta_1^2 \Pi_{1z}$$

$$E_2 = \nabla\left(\frac{\partial \Pi_{2z}}{\partial z}\right) + \hat{z}\beta_2^2 \Pi_{2z} ,$$

that is , $$\nabla\left(\frac{\partial \Pi_{1z}}{\partial z}\right) - \nabla\left(\frac{\partial \Pi_{2z}}{\partial z}\right) = \hat{z}\beta_2^2\Pi_{2z} - \hat{z}\beta_1^2\Pi_{1z}$$
$$= \hat{z}\,\omega^2\epsilon_1\,\Pi_{1z}(\mu_2-\mu_1) .$$

As we have considered before that the permeability μ is identical in both the media. So we get

$$\frac{\partial \Pi_{1z}}{\partial z} = \frac{\partial \Pi_{2z}}{\partial z} .$$

So the boundary conditions are

$$\epsilon_1\Pi_{1z} = \epsilon_2\Pi_{2z}$$
$$\frac{\partial \Pi_{1z}}{\partial z} = \frac{\partial \Pi_{2z}}{\partial z} \qquad (4)$$

also at $z \to \infty$, $\Pi_{1z} \to 0$

and $z \to \infty$, $\Pi_{2z} \to 0$. (5)

These equations can be solved by the straight forward means of the two-dimensional Fourier transform $\bar{\Pi}$, where

$$\bar{\Pi}(\xi,\eta,z) = \iint_{-\infty}^{\infty} \Pi(x,y,z)\, e^{-i(\xi x+\eta y)}\, dx\, dy \qquad (6)$$

and the inverse Π is given by

$$\Pi(x,y,z) = \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} \bar{\Pi}(\xi,\eta,z)\, e^{i(\xi x+\eta y)}\, d\xi\, d\eta . \qquad (7)$$

Applying the two-dimensional Fourier transform in (3) we get,

$$\iint_{-\infty}^{\infty}\left[\nabla^2\pi_{1z}+\beta_1^2\pi_{1z}\right]e^{-i(\xi x+\eta y)}\,dx\,dy$$

$$=\iint_{-\infty}^{\infty}-\frac{p_z}{\epsilon_1}\,\delta(x)\,\delta(y)\,\delta(z-z_0)\,e^{-i(\xi x+\eta y)}\,dx\,dy.$$

Using rectangular co-ordinate system the left hand side becomes

$$\iint_{-\infty}^{\infty}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+\beta_1^2\right)\pi_{1z}e^{-i(\xi x+\eta y)}\,dx\,dy$$

$$=\iint_{-\infty}^{\infty}\left[\frac{\partial^2\pi_{1z}}{\partial x^2}\cdot e^{-i\xi x}\cdot e^{-i\eta y}+\frac{\partial^2\pi_{1z}}{\partial y^2}\cdot e^{-i\xi x}\cdot e^{-i\eta y}+\left(\frac{\partial^2}{\partial z^2}+\beta_1^2\right)\pi_{1z}e^{-i(\xi x+\eta y)}\right]dx\,dy$$

Integrating by parts, the first term will be

$$\int_{-\infty}^{\infty}e^{-i\eta y}\,dy\left\{e^{-i\xi x}\,\frac{\partial\pi_{1z}}{\partial x}\Big|_{-\infty}^{\infty}+i\xi\int_{-\infty}^{\infty}e^{-i\xi x}\,\frac{\partial\pi_{1z}}{\partial x}\,dx\right\}$$

$$=\int_{-\infty}^{\infty}e^{-i\eta y}\,dy\left[i\xi\int_{-\infty}^{\infty}e^{-i\xi x}\,\frac{\partial\pi_{1z}}{\partial x}\,dx\right]$$

$$=\int_{-\infty}^{\infty}e^{-i\eta y}\,dy\left[i\xi\left\{e^{-i\xi x}\pi_{1z}\Big|_{-\infty}^{\infty}+i\xi\int_{-\infty}^{\infty}e^{-i\xi x}\pi_{1z}\,dx\right\}\right]$$

$$=-\xi^2\iint_{-\infty}^{\infty}\pi_{1z}\,e^{-i\xi x}\cdot e^{-i\eta y}\,dx\,dy$$

$$=-\xi^2\,\bar{\pi}_{1z}(\xi,\eta,z).$$

Similarly the second term will be $-\eta^2\bar{\pi}_{1z}(\xi,\eta,z)$. Then the equation will be

$$-(\xi^2+\eta^2)\,\bar{\pi}_{1z}(\xi,\eta,z)+\iint_{-\infty}^{\infty}\left(\frac{\partial^2}{\partial z^2}+\beta_1^2\right)\pi_{1z}\,e^{-i(\xi x+\eta y)}\,dx\,dy$$

$$=\left[-(\xi^2+\eta^2)+\left(\frac{\partial^2}{\partial z^2}+\beta_1^2\right)\right]\bar{\pi}_{1z}(\xi,\eta,z)$$

$$=\left(\frac{\partial^2}{\partial z^2}+\gamma_1^2\right)\bar{\pi}_{1z}(\xi,\eta,z)$$

where $\gamma_1^2=\beta_1^2-(\xi^2+\eta^2)$.

Then the differential equation will be

$$\left(\frac{\partial^2}{\partial z^2}+\gamma_1^2\right)\bar{\pi}_{1z}(\xi,\eta,z)$$

$$=-\frac{p_z}{\epsilon_1}\iint_{-\infty}^{\infty}\delta(x)\,\delta(y)\,\delta(z-z_0)\,e^{-i(\xi x+\eta y)}\,dx\,dy.$$

Similarly applying Fourier transform to the other equation of (3) we get

$$\left(\frac{\partial^2}{\partial z^2}+\gamma_2^2\right)\bar{\pi}_{2z}(\xi,\eta,z)=0$$

where $\gamma_2^2=\beta_2^2-(\xi^2+\eta^2)$.

The solutions of these ordinary differential equations satisfying the boundary conditions in (4) are given in terms of the inverse transform .

Thus for $z \geqslant 0$

$$\Pi_{1z}(x,y,z) = \frac{1}{4\pi^2}\left(\frac{i}{2}\frac{p_z}{\epsilon_1}\right)\iint\limits_{-\infty}^{\infty}\left[\frac{e^{i\gamma_1|z-z_0|}}{\gamma_1} + \frac{\beta_2^2\gamma_1-\beta_1^2\gamma_2}{\beta_2^2\gamma_1+\beta_1^2\gamma_2}\cdot\frac{e^{i\gamma_1(z+z_0)}}{\gamma_1}\right]\cdot$$

$$\cdot\, e^{i(\xi x+\eta y)}\,d\xi\,d\eta\,. \tag{8}$$

For $z \leqslant 0$

$$\Pi_{2z}(x,y,z) = \frac{1}{4\pi^2}\left(\frac{i}{2}\frac{p_z}{\epsilon_2}\right)\iint\limits_{-\infty}^{\infty}\left(1+\frac{\beta_2^2\gamma_1-\beta_1^2\gamma_2}{\beta_2^2\gamma_1+\beta_1^2\gamma_2}\right)\cdot\frac{e^{i\gamma_1 z_0}\cdot e^{-i\gamma_2 z}}{\gamma_1}$$

$$\cdot\, e^{i(\xi x+\eta y)}\,d\xi\,d\eta \tag{9}$$

where

$$\left.\begin{aligned}\gamma_1^2 &= \left\{\beta_1^2-(\xi^2+\eta^2)\right\}\\ \gamma_2^2 &= \left\{\beta_2^2-(\xi^2+\eta^2)\right\}\,.\end{aligned}\right\} \tag{10}$$

The equations (8) and (9) are simplified to the form given below by changing to polar co-ordinates (λ,θ) and substituting the integral representation of Bessel functions. For $z \geqslant 0$

$$\Pi_{1z} = \frac{p_z}{\epsilon_1}\left[\frac{e^{i\beta_1R_1}}{4\pi R_1} - \frac{e^{i\beta_2R_2}}{4\pi R_2}\right.$$

$$\left. + \frac{i}{4\pi}\int_{-\infty}^{\infty}\frac{\beta_2^2}{\beta_2^2\gamma_1+\beta_1^2\gamma_2}\cdot e^{i\gamma_1(z+z_0)}H_0^{(1)}(\lambda r)\,\lambda\,d\lambda\,.\right. \tag{11}$$

For $z\leq 0$

$$\Pi_{2z} = \frac{p_z}{\epsilon_2}\frac{i}{4\pi}\int_{-\infty}^{\infty}\frac{\beta_2^2}{\beta_2^2\gamma_1+\beta_1^2\gamma_2}\cdot e^{i\gamma_1 z_0}\cdot e^{-i\gamma_2 z} H_0^{(1)}(\lambda r)\,\lambda\, d\lambda, \tag{12}$$

where $R_1^2 = x^2+y^2+(z-z_0)^2 = r^2+(z-z_0)^2$

and $R_2^2 = x^2+y^2+(z+z_0)^2 = r^2+(z+z_0)^2$.

Case B: Horizontal Electric Dipole

It is necessary to have two components of the Hertzian vector Π_x and Π_z for the horizontal dipole in the medium. Then the electromagnetic fields for this case are given by

$$\left.\begin{aligned} E &= \nabla\left(\frac{\partial \Pi_x}{\partial x}+\frac{\partial \Pi_z}{\partial z}\right)+\beta^2\left(\hat{x}\Pi_x+\hat{z}\Pi_z\right) \\ B &= -\frac{i\beta^2}{\omega}\left[\nabla\times\left(\hat{x}\Pi_x+\hat{z}\Pi_z\right)\right]. \end{aligned}\right\} \tag{13}$$

Similarly before we can deduce the field equations for $z\geq 0$

$$\left.\begin{aligned} \nabla^2\Pi_{x_1}+\beta_1^2\Pi_{x_1} &= 0 \\ \nabla^2\Pi_{z_1}+\beta_1^2\Pi_{z_1} &= -\frac{p_z}{\epsilon_1}\delta(x)\,\delta(y)\,\delta(z-z_0) \end{aligned}\right\} \tag{14}$$

and for $z\leq 0$

$$\left.\begin{aligned} \nabla^2\Pi_{x_2}+\beta_2^2\Pi_{x_2} &= 0 \\ \nabla^2\Pi_{z_2}+\beta_2^2\Pi_{z_2} &= 0 \end{aligned}\right\} \tag{15}$$

Again the boundary conditions will be as before

$$\left.\begin{aligned}
&\beta_1^2 \Pi_{x_1} = \beta_2^2 \Pi_{x_2} \qquad && \frac{\partial \Pi_{x_1}}{\partial x} + \frac{\partial \Pi_{z_1}}{\partial z} = \frac{\partial \Pi_{x_2}}{\partial x} + \frac{\partial \Pi_{z_2}}{\partial z} \\
&\beta_1^2 \Pi_{z_1} = \beta_2^2 \Pi_{z_2} && \beta_1^2 \frac{\partial \Pi_{x_1}}{\partial z} = \beta_2^2 \frac{\partial \Pi_{x_2}}{\partial z}
\end{aligned}\right\} \tag{16}$$

and at

$$z \to +\infty \qquad \Pi_1(x, y, z) \to 0$$
$$z \to -\infty \qquad \Pi_2(x, y, z) \to 0$$

As before, the equations (14) and (15) are reduced to ordinary differential equations by means of Fourier transform. Solving these differential equations with the boundary conditions in (16), the following inverse transforms can be written as

$$\Pi_{x_1} = \frac{ip_z}{8\pi^2\epsilon_1} \iint_{-\infty}^{\infty} \left(\frac{e^{i\gamma_1(z-z_0)}}{\gamma_1} - \frac{e^{i\gamma_1(z+z_0)}}{\gamma_2} + \frac{2e^{i\gamma_1(z+z_0)}}{\gamma_1+\gamma_2} \right) \cdot e^{i(\xi x+\eta y)} d\xi d\eta$$

$$\Pi_{z_1} = -\frac{ip_z}{8\pi^2\epsilon_1} \frac{\partial}{\partial x} \iint_{-\infty}^{\infty} \frac{2(\gamma_1-\gamma_2)}{\beta_2^2\gamma_1+\beta_1^2\gamma_2} e^{i\gamma_1(z+z_0)} \cdot e^{i(\xi x+\eta y)} d\xi d\eta$$

$$\Pi_{x_2} = \frac{ip_z}{8\pi^2\epsilon_2} \iint_{-\infty}^{\infty} \frac{2}{(\gamma_1+\gamma_2)} e^{i(\gamma_1 z_0 - \gamma_2 z)} \cdot e^{i(\xi x+\eta y)} d\xi d\eta$$

$$\Pi_{x2} = -\frac{i p_x}{8\pi^2\epsilon_2}\frac{\partial}{\partial x}\iint_{-\infty}^{\infty}\frac{2(\gamma_1-\gamma_2)}{\beta_2^2\gamma_1+\beta_1^2\gamma_2}\cdot e^{-i(\gamma_2 z-\gamma_1 z_0)}\cdot e^{i(\xi x+\eta y)}\,d\xi\,d\eta \qquad (17)$$

where $\gamma_1^2 = \beta_1^2-(\xi^2+\eta^2)$

$\gamma_2^2 = \beta_2^2-(\xi^2+\eta^2)$.

Similarly as before by transforming to polar co-orcinates (λ,θ) and substituting the integral representation of the Bessel functions, the equations (17) reduces to the following form : for $z\geqslant 0$

$$\Pi_{x_1} = \frac{p_x}{\epsilon_1}\left[\frac{e^{i\beta_1 R_1}}{4\pi R_1}-\frac{e^{i\beta_2 R_2}}{4\pi R_2}+\frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{2i}{\gamma_1+\gamma_2}H_0^{(1)}(\lambda r)e^{i\gamma_1(z+z_0)}\lambda\,d\lambda\right]$$

$$\Pi_{z_1} = -\frac{p_x\cos\phi}{4\pi\epsilon_1}\frac{\partial}{\partial r}\int_{-\infty}^{\infty}\frac{(\gamma_1-\gamma_2)}{\beta_1^2\gamma_2+\beta_2^2\gamma_1}\cdot e^{i\gamma_1(z+z_0)}H_0^{(1)}(\lambda r)\lambda\,d\lambda$$

and for $z\leqslant 0$ --- (18)

$$\Pi_{x_2} = \frac{p_x}{4\pi\epsilon_2}\left[\int_{-\infty}^{\infty}\frac{2i}{\gamma_1+\gamma_2}\cdot e^{i\gamma_1 z_0}\cdot e^{-i\gamma_2 z}H_0^{(1)}(\lambda r)\lambda\,d\lambda\right.$$

$$\Pi_{z_2} = -\frac{p_x\cos\phi}{4\pi\epsilon_2}\frac{\partial}{\partial r}\int_{-\infty}^{\infty}\frac{(\gamma_1-\gamma_2)}{\beta_1^2\gamma_2+\beta_2^2\gamma_1}\cdot e^{-i\gamma_2 z}\cdot e^{i\gamma_1 z_0}H_0^{(1)}(\lambda r)\lambda\,d\lambda,$$

where $R_1^2 = x^2+y^2+(z-z_0)^2 = r^2+(z-z_0)^2$

$R_2^2 = x^2+y^2+(z+z_0)^2 = r^2+(z+z_0)^2$.

To get the complete description of the electromagnetic fields the integrals (11), (12) and (18) have to be evaluated. Asymptotic solutions of the following two integrals will be given :

$$I_1 = \int_{-\infty}^{\infty} \frac{\gamma_2-\gamma_1}{\beta_1^2-\beta_2^2} \cdot e^{-\gamma_1 z_0} e^{\gamma_2 z} H_0^{(1)}(\lambda r)\, \lambda \, d\lambda \tag{19}$$

$$I_2 = \int_{-\infty}^{\infty} \frac{\beta_1^2\gamma_2-\beta_2^2\gamma_1}{(\beta_1^4-\beta_2^4)\left(\lambda^2-\frac{\beta_1^2\beta_2^2}{\beta_1^2+\beta_2^2}\right)} \cdot e^{-\gamma_1 z_0} e^{\gamma_2 z} H_0^{(1)}(\lambda r)\, \lambda \, d\lambda \,, \tag{20}$$

where $\gamma_1^2 = (\lambda^2-\beta_1^2)^{1/2}$

$\gamma_2^2 = (\lambda^2-\beta_2^2)^{1/2}$

and the rest of the integrals are obtained from them by simple differentiation. Thus the asymptotic solution to the second and fourth integral is obtained by taking

$$-\left(\frac{\partial}{\partial z_0}+\frac{\partial}{\partial z}\right) I_2 \,.$$

III. A Short Description of the Asymptotic Method

The asymptotic evaluation of an integral $\int_{-\infty}^{\infty} f(\lambda)\, e^{i\lambda r} d\lambda$ for large values of r is obtained in the following manner. The function $f(\lambda)$ should be a generalized function and should possess only finite number of singularities like poles, zeros and branchpoints. It is known from the complex-variable theory that the contributions to the solution come form these singularities of the function. The asymptotic solution is obtained by expanding the function $f(\lambda)$ around the singularity, e.g., if the function has a branch point of the type $(\lambda-\lambda_1)^{-1/2}$ at $\lambda=\lambda_1$, then $(\lambda-\lambda_1)^{1/2}f(\lambda)$ is expanded around the singularity λ_1. After the expansion the complex integral is reduced to a linear combination of simple integrals. After splitting off the complex function $f(\lambda)$ into a linear combination of simple functions, the inverse Fourier transform will be asymptotically equal to

$$\left[f_1(r,\lambda_1)\, e^{i\lambda_1 r} + f_2(r,\lambda_2)\, e^{i\lambda_2 r} + \cdots\cdots \right],$$

where λ_1, λ_2 are the singularities of the function $f(\lambda)$, and $f_1(r,\lambda_1)$ and $f_2(r,\lambda_2)$ are simple functions of r. So from the nature of the solution, the major significant contribution will be from the singularities close to the real axis. The further a singularity is from the real axis, the greater will be its exponential decay term for increasing r, and so the less significant will be its contribution. There is another difficulty when the distance seperating the two singualrities of a complex function $f(\lambda)$ is so small that the product of the distance r and the distance of seperation of the two singularities is small. This difficulty may be overcome by grouping these two singularities and expanding the function around one of them. That is, if a function has a branch point of the form $(\lambda-\lambda_1)^{1/2}$ and a pole $(\lambda-\lambda_2)^{-1}$ and the two singularities are close together, then to evaluate the function asymptotically, the function $\frac{(\lambda-\lambda_2)}{(\lambda-\lambda_1)^{1/2}}f(\lambda)$ is expanded around λ_2 or λ_1. The direct method is applied to the equation (19) and the modified method to that of (20).

IV. The Asymptotic Evaluation of the Integrals

Now we shall evaluate the integrals (19) and (20) for the special case when $|\beta_1^2| \gg |\beta_2^2|$ and z and $z_0 < r$ and $\beta_2 r > 1$. Thus $H_0^{(1)}(\lambda r) = \sqrt{\frac{2}{\pi i r}} \cdot \frac{e^{i\lambda r}}{\sqrt{\lambda}}\left[1 + \frac{1}{8ir\lambda} + \frac{9}{128(ir)^2\lambda^2} + \cdots\right]$.

A. Evaluation of I_1

$$I_1 = \int_{-\infty}^{\infty} \frac{\gamma_2-\gamma_1}{\beta_1^2-\beta_2^2} e^{-\gamma_1 z_0} e^{\gamma_2 z} H_0^{(1)}(\lambda r)\lambda\, d\lambda$$

$$= \int_{-\infty}^{\infty} \frac{\gamma_2-\gamma_1}{\beta_1^2-\beta_2^2} e^{-\gamma_1 z_0} e^{\gamma_2 z} \sqrt{\frac{2}{\pi i r}} \cdot \frac{e^{i\lambda r}}{\sqrt{\lambda}}\left[1 + \frac{1}{8ir\lambda} + \frac{9}{128(ir)^2\lambda^2} + \cdots\right]\lambda\, d\lambda$$

$$= \sqrt{\frac{2}{\pi i r}} \int_{-\infty}^{\infty} \frac{\gamma_2-\gamma_1}{\beta_1^2-\beta_2^2} \cdot e^{-\gamma_1 z_0} e^{\gamma_2 z} e^{i\lambda r}\left[\sqrt{\lambda} + \frac{1}{8ir\lambda^{1/2}} + \frac{9}{128(ir)^2\lambda^{3/2}} + \cdots\right] d\lambda \quad (21)$$

The exponential $e^{\gamma_2 z}$ is expanded as a series around z and is given by

$$e^{\gamma_2 z} = 1 + \gamma_2 z + \frac{(\gamma_2 z)^2}{2!} + \frac{(\gamma_2 z)^3}{3!} + \frac{(\gamma_2 z)^4}{4!} + \frac{(\gamma_2 z)^5}{5!} + \cdots$$

Again writing

$$(\gamma_2-\gamma_1) e^{\gamma_2 z} = (\gamma_2-\gamma_1)\left\{1 + \gamma_2 z + \frac{(\gamma_2 z)^2}{2!} + \frac{(\gamma_2 z)^3}{3!} + \frac{(\gamma_2 z)^4}{4!} + \frac{(\gamma_2 z)^5}{5!} + \cdots\right\}$$

$$= \gamma_2\left\{1 - \gamma_1 z + \gamma_2^2\left(\frac{z^2}{2!} - \frac{\gamma_1 z^3}{3!}\right) + \gamma_2^4\left(\frac{z^4}{4!} - \frac{\gamma_1 z^5}{5!}\right) + \cdots\right\}$$

$$+ 1\left\{-\gamma_1 + \gamma_2^2\left(z - \frac{\gamma_1 z^2}{2!}\right) + \gamma_2^4\left(\frac{z^3}{3!} - \frac{\gamma_1 z^4}{4!}\right) + \cdots\right\}.$$

By rearranging the equation (21) in odd and even series of γ_2 we get

$$I_1 = \sqrt{\frac{2}{\pi i r}} \int_{-\infty}^{\infty} \Big[\frac{\gamma_2}{\beta_1^2-\beta_2^2} \Big\{ 1-\gamma_1 z+\gamma_2^2\Big(\frac{z^2}{2!}-\frac{\gamma_1 z^3}{3!}\Big)+\gamma_2^4\Big(\frac{z^4}{4!}-\frac{\gamma_1 z^5}{5!}\Big)+\cdots \Big\}$$

$$\cdot e^{-\gamma_1 z_0} e^{i\lambda r} \Big\{ \sqrt{\lambda}+\frac{1}{8ir\lambda^{1/2}}+\frac{9}{128(ir)^2\lambda^{3/2}}+\cdots \Big\} d\lambda$$

$$+\sqrt{\frac{2}{\pi i r}} \int_{-\infty}^{\infty} \frac{1}{\beta_1^2-\beta_2^2} \Big\{ -\gamma_1+\gamma_2^2\Big(z-\frac{\gamma_1 z^2}{2!}\Big)+\gamma_2^4\Big(\frac{z^3}{3!}-\frac{\gamma_1 z^4}{4!}\Big)+\cdots \Big\}$$

$$\cdot e^{-\gamma_1 z_0} e^{i\lambda r} \Big\{ \sqrt{\lambda}+\frac{1}{8ir\lambda^{1/2}}+\frac{9}{128(ir)^2\lambda^{3/2}}+\cdots \Big\} d\lambda . \tag{22}$$

In the above equation (22) the singularities of the first part are the branch points at $\lambda=\pm\beta_1$, $\lambda=\pm\beta_2$ and $\lambda=0$, and the singularities of the second part are the branch points at $\lambda=\pm\beta_1$, and $\lambda=0$ and a zero at $\lambda=\pm\beta_2$. To satisfy the Sommerfield radiation condition, only the singularities in the upper half plane above the real axis are considered. Also as $|\beta_1| \gg |\beta_2|$ and $|\beta_1|$ is complex and highly dissipative, only the branch points at $\lambda=\beta_2$ is going to contribute significantly to the asymptotic value of the integral. In the far zone the zero will not contribute to the integral. So only the first part of the equation (22) will be considered. The expansion of the function

$$e^{-\gamma_1 z_0}(\lambda+\beta_2)^{1/2}\Big(\sqrt{\lambda}+\frac{1}{8ir\lambda^{1/2}}+\frac{9}{128(ir)^2\lambda^{3/2}}\Big)\big(1-\gamma_1 z+\cdots\big)$$

around $\lambda=\beta_2$ to the first two terms is given below:

$$\sqrt{\frac{2}{\pi i r}} \int_{-\infty}^{\infty} \frac{(\lambda-\beta_2)^{1/2}}{\beta_1^2-\beta_2^2} \cdot e^{i\beta_1\sqrt{1-\eta^2}\, z_0} e^{i\lambda r} \Big\{ \Big[\sqrt{2}\beta_2+\frac{\sqrt{2}}{8ir}+\frac{9}{128(ir)^2\lambda^{3/2}} \cdot \frac{1}{\beta_2} \Big]$$

$$\cdot\left[1-i\beta_1\sqrt{1-\eta^2}\,z\right]+(\lambda-\beta_2)\cdot\left\{\left[\frac{3}{2\sqrt{2}}-\frac{\sqrt{2}}{16ir}\cdot\frac{1}{\beta_2}+\sqrt{2}\beta_2 z^2\left(1+\frac{1}{8ir\beta_2}+\cdots\right)\right.\right.$$

$$\left.\left.-\frac{i\beta_2^2\sqrt{2}\,z_0}{\sqrt{\beta_1^2-\beta_2^2}}\right]\cdot\left[1-i\beta_1\sqrt{1-\eta^2}\,z\right]+\frac{i\beta_2^2\sqrt{2}\,z}{\beta_1\sqrt{1-\eta^2}}\right\}\cdots\Big\} \qquad (23)$$

where $\eta^2 = \frac{\beta_2^2}{\beta_1^2}$.

After an inverse transformation of (23) the resulting expression is given by

$$-\sqrt{\frac{2}{\pi i r}}\;\frac{(i\pi)^{1/2}}{r^{3/2}}\cdot\frac{e^{i\beta_2 r}e^{i\beta_1\sqrt{1-\eta^2}\,z_0}}{\beta_1^2-\beta_2^2}\sqrt{2}\left\{\beta_2\left[1-i\beta_1\sqrt{1-\eta^2}\,z\right]+\right.$$

$$\left.\frac{1}{ir}\left[1-i\beta_1\sqrt{1-\eta^2}\,z+\frac{3}{2}\frac{i\beta_2^2 z_2}{\beta_1\sqrt{1-\eta^2}}-\frac{3}{2}\frac{i\beta_2^2 z_0}{\beta_1\sqrt{1-\eta^2}}+\frac{3}{2}i^2\beta_2^2 z z_0-\cdots\right]\cdots\right\}. \qquad (24)$$

B. Evaluation of I_2

Again assuming the series of $H_0^{(1)}(\lambda r)$, the integral

$$I_2 = \frac{1}{\beta_1^4-\beta_2^4}\sqrt{\frac{2}{\pi i r}}\int_{-\infty}^{\infty}\frac{\beta_1^2\gamma_2-\beta_2^2\gamma_1}{\lambda^2-\frac{\beta_1^2\beta_2^2}{\beta_1^2+\beta_2^2}}\cdot e^{-\gamma_1 z}e^{\gamma_2 z}e^{i\lambda r}$$

$$\cdot\left\{\sqrt{\lambda}+\frac{1}{8ir\lambda^{1/2}}+\frac{9}{128(ir)^2\lambda^{3/2}}+\cdots\right\}d\lambda. \qquad (25)$$

Similarly the exponential $e^{\gamma_2 z}$ is expanded as Taylor's series in z we get

$$I_2 = \frac{1}{\beta_1^4-\beta_2^4}\sqrt{\frac{2}{\pi i r}}\int_{-\infty}^{\infty}\frac{\beta_1^2\gamma_2-\beta_2^2\gamma_1}{\lambda^2-\frac{\beta_1^2\beta_2^2}{\beta_1^2+\beta_2^2}}\,e^{-\gamma_1 z_0}e^{i\lambda r}\left\{\sqrt{\lambda}+\frac{1}{8ir\lambda^{1/2}}+\right.$$

$$\left.\frac{9}{128(ir)^2\lambda^{3/2}}+\cdots\right\}\cdot\left\{1+\gamma_2 z+\frac{(\gamma_2 z)^2}{2!}+\frac{(\gamma_2 z)^3}{3!}+\frac{(\gamma_2 z)^4}{4!}+\cdots\right\}d\lambda.$$

Rearranging the above equation in odd and even series of γ_2, we get

$$I_2 = \sqrt{\frac{2}{\pi i r}}\,\frac{1}{(\beta_1^4-\beta_2^4)}\left\{\int_{-\infty}^{\infty}\frac{(\lambda^2-\beta_2^2)^{1/2}}{\lambda^2-\frac{\beta_1^2\beta_2^2}{\beta_1^2+\beta_2^2}}\cdot e^{-\gamma_2 x_0}e^{i\lambda r}\left[\sqrt{\lambda}+\frac{1}{8ir\lambda^{1/2}}+\frac{9}{128(ir)^2\lambda^{3/2}}+\cdots\right]\right.$$

$$\cdot\left[(\beta_1^2-\beta_2^2\sqrt{\lambda^2-\beta_1^2})x+(\lambda^2-\beta_2^2)\left(\beta_1^2\frac{x^2}{2!}-\beta_2^2\sqrt{\lambda^2-\beta_1^2}\,\frac{x^3}{3!}\right)+(\lambda^2-\beta_2^2)\right.$$

$$\left.\cdot\left(\beta_1^2\frac{x^4}{4!}-\beta_2^2\sqrt{\lambda^2-\beta_1^2}\,\frac{x^5}{5!}\right)\cdots\right]d\lambda+\int_{-\infty}^{\infty}\frac{e^{-\gamma_1 x_0}e^{i\lambda r}}{\lambda^2-\frac{\beta_1^2\beta_2^2}{\beta_1^2+\beta_2^2}}\left[-\beta_2^2\sqrt{\lambda^2-\beta_1^2}+\right.$$

$$\left.(\lambda^2-\beta_2^2)\left(\beta_1^2x-\beta_2^2\frac{x^2}{2!}\right)\cdot(\lambda^2-\beta_1^2)^{1/2}+(\lambda^2-\beta_2^2)^2\left(\beta_1^2\frac{x^3}{3!}-\beta_2^2\sqrt{\lambda^2-\beta_1^2}\,\frac{x^4}{4!}\right)\cdots\right]$$

$$\left.\cdot\left[\frac{1}{\sqrt{\lambda}}+\frac{1}{8ir\lambda^{3/2}}+\frac{9}{128(ir)^2\lambda^{5/2}}+\cdots\right]d\lambda\right\}. \qquad (26)$$

Now let us consider the first half of the equation (26). The singularities are the branch points at $\lambda=\pm\beta_1$, $\lambda=\pm\beta_2$ and $\lambda=0$ and poles at $\lambda=\pm\frac{\beta_1\beta_2}{\sqrt{\beta_1^2+\beta_2^2}}$. As before the singularities in the upper half plane are considered to satisfy the radiation condition at infinity. Also, as $|\beta_1|\gg|\beta_2|$, and $|\beta_1|$ is complex and highly dissipative the main contribution to the integral is going to come from the singularities, the branch point at $+\beta_2$ and pole at $+\frac{\beta_1\beta_2}{\sqrt{\beta_1^2+\beta_2^2}}$. The distance between the branch point and the pole is

$$\left(\beta_2-\frac{\beta_1\beta_2}{\sqrt{\beta_1^2+\beta_2^2}}\right)=\beta_2\left[1-1/\sqrt{1-\eta^2}\right],$$

and is very small. So when r is moderately large, the modified method will be applied as suggested before. The singularities $\frac{(\lambda-\beta_2)^{1/2}}{\lambda-\frac{\beta_1\beta_2}{\sqrt{\beta_1^2+\beta_2^2}}}$ are grouped together, and the rest of the function

$$e^{-\gamma_1 z_0}\cdot\frac{(\lambda+\beta_2)^{1/2}}{\lambda+\frac{\beta_1\beta_2}{\sqrt{\beta_1^2+\beta_2^2}}}\Big[\sqrt{\lambda}+\frac{1}{8ir\lambda^{1/2}}+\frac{9}{128(ir)^2\lambda^{3/2}}+\cdots\Big]\Big[\beta_1^2-\beta_2^2\gamma_1 z+\cdots\Big]$$

is expanded around $\beta_2/\sqrt{1+h^2}$. After this expansion and rearrangement, the first part of (26) reduces to the following with $O(\beta_2 z)$, $O(\beta_2 z_0)$ neglected :

$$\begin{aligned}
&\sqrt{\frac{2}{\pi i r}}\cdot\frac{1}{\beta_1^4-\beta_2^4}\int_{-\infty}^{\infty}\frac{(\lambda-\beta_2)^{1/2}}{\lambda-\frac{\beta_2}{\sqrt{1+h^2}}}\, e^{i\beta_1 z_0/\sqrt{1-n^2}}\, e^{i\lambda r}\Big\{\beta_1^2\Big[1-\frac{ih\beta_2 z}{\sqrt{1+h^2}}+\\
&\Big(\frac{ih\beta_2 z}{\sqrt{1+h^2}}\Big)^2\frac{1}{2!}-\Big(\frac{ih\beta_2 z}{\sqrt{1+h^2}}\Big)^3\frac{1}{3!}+\cdots\Big]\cdot\Big[\frac{Q}{2}\Big(1+\frac{1}{16i\beta_2 r}(1+h^2)^{1/2}+\\
&\frac{9}{128(ir)^2}\cdot\frac{(1+h^2)Q}{2\beta_2}\Big)-\Big(\lambda-\frac{\beta_2}{\sqrt{1+h^2}}\Big)\Big(\frac{1+h^2}{4\beta_2 Q}-\frac{\beta_2}{\beta_1^2}Q(1+h^2)^{1/2}-\frac{(1+h^2)^{1/2}Q}{4\beta_2}\\
&-\frac{i\beta_2}{\beta_1}z_0 Q+\frac{1}{8ir}\frac{(1+h^2)}{4\beta_2^2 Q}+\frac{(1+h^2)Q}{32ir\beta_2^2}+\frac{iz_0}{2\beta_1}\frac{(1+h^2)^{1/2}Q}{8ir}+\frac{(1+h^2)Q}{16\beta_1^2 i^2 r}\Big)\\
&+\Big(\lambda-\frac{\beta_2}{\sqrt{1+h^2}}\Big)^2\Big(\frac{(1+h^2)(2h^2+1)+2(1+h^2)^{1/2}}{16\beta_2^2\, Q^3}\Big)\Big(-\frac{(1+h^2)Q}{16\beta_2^2}-\frac{(1+h^2)}{8\beta_2^2 Q}\\
&+\frac{iz_0(1+h^2)}{4\beta_1 Q}+\frac{z_0(1+h^2)^{3/2}}{4\beta_1^2 Q}-\frac{iz_0 Q(1+h^2)^{1/2}}{4\beta_1}\Big)\cdots\Big]d\lambda\Big\}\\
&+\sqrt{\frac{2}{\pi i r}}\cdot\frac{1}{\beta_1^4-\beta_2^4}\int_{-\infty}^{\infty}\frac{(\lambda-\beta_2)^{1/2}}{\lambda-\frac{\beta_2}{\sqrt{1+h^2}}}\cdot e^{i\beta_1 z_0/\sqrt{1+h^2}}\, e^{i\lambda r}\Big[\Big(\lambda-\frac{\beta_2}{\sqrt{1+h^2}}\Big)\\
&\cdot\frac{\beta_1^2\beta_2^2 z^2}{\sqrt{1+h^2}}\Big(\frac{Q}{2}+\frac{1}{8ir}\frac{(1+h^2)^{1/2}Q}{2\beta_2}+\frac{9}{128}\frac{(1+h^2)Q}{(ir)^2 2\beta_2^2}+\Big(\lambda-\frac{\beta_2}{\sqrt{1+h^2}}\Big)\\
&\cdot\beta_1^2\frac{z^2}{2!}\Big[\frac{Q}{2}+\frac{1+h^2}{4Q}+\frac{(1+h^2)^{1/2}Q}{4}\Big]\cdots\Big]d\lambda,
\end{aligned}\tag{27}$$

where $Q^2=[(1+h^2)^{1/2}+1]$ and $h^2=\frac{\beta_2^2}{\beta_1^2}$.

After an inverse Fourier tfansformation of the equation (27) and a rearrangement in ascending power of $\frac{1}{r}$, the first three terms are (the terms $O(n^2)$ are neglected)

$$\sqrt{\frac{2}{\pi i}}\cdot\frac{\beta_1^2}{\beta_1^4-\beta_2^4}\, e^{i\beta_2 r}\, e^{i\beta_1 z_0/\sqrt{1+n^2}}\, e^{-in\beta_2 z/\sqrt{1+n^2}}\Big\{\sqrt{i}\,\frac{Q}{r}+\sqrt{\frac{\pi}{i}}\cdot\frac{1}{r^2 Q}$$

$$\cdot\left(\frac{n^2}{16\beta_2}+in z_0+\beta_2 z^2+O(n^2)\right)+\frac{\sqrt{\pi}}{i^{3/2} r^3 Q^3}\left(\frac{7n^2}{32\beta_2^2}-\frac{2iz_0}{\beta_1}+z^2+O(n^2)\right)$$

$$+\cdots+\pi i\,\frac{n\beta_1^{1/2}}{(1+n^2)^{1/4}}\cdot\frac{e^{i(\beta_0-\beta_2)r}}{\sqrt{r}}\,\mathrm{erfc}\left[i^{1/2}x_0 r^{1/2}\right]\Big\} \tag{28}$$

where $x_0^2=\beta_2\left(\frac{1}{\sqrt{1+n^2}}-1\right)$ and $\beta_0=\frac{\beta_2}{\sqrt{1+n^2}}$.

In the second half of (26) the function has poles at $\lambda=\pm\frac{\beta_2}{\sqrt{1+n^2}}$ and branch points at $\lambda=0$ and $\lambda=\pm\beta_2$. Due to the same reason as said before, the only singularity that lies **in** the allowable plane that is of any significance is the pole at $\lambda=\frac{\beta_2}{\sqrt{1+n^2}}$. An expansion around $\lambda=\beta_2/\sqrt{1+n^2}$ leads to

$$\sqrt{\frac{2}{\pi i r}}\cdot\frac{1}{\beta_1^4-\beta_2^4}\int_{-\infty}^{\infty}\frac{e^{i\beta_1 z_0/\sqrt{1+n^2}}\cdot e^{i\lambda r}}{\lambda-\frac{\beta_2}{\sqrt{1+n^2}}}\Big[\frac{\beta_1\beta_2^{3/2}}{2(1+n^2)^{1/4}}\Big\{1-\frac{in^2\beta_2 z}{\sqrt{1+n^2}}+\left(\frac{in^2\beta_2 z}{\sqrt{1+n^2}}\right)\frac{1}{2!}\cdots\Big\}$$

$$+\left(\lambda-\frac{\beta_2}{\sqrt{1+n^2}}\right)[\cdots]\cdots\Big]\left[1+\frac{(1+n^2)^{1/2}}{8\beta_2 ir}+\frac{9}{128(ir)^2}\cdot\frac{(1+n^2)}{\beta_2^2}\cdots\right]d\lambda. \tag{29}$$

Only the first term of the expansion is written, since the Fourier transform of all others is zero. Inverting (29), the following expression is obtained:

$$\sqrt{\frac{2}{\pi i r}}\cdot\frac{\beta_1\beta_2^{3/2}}{\beta_1^4-\beta_2^4}\,\frac{1}{(1+n^2)^{1/4}}(\pi i)\, e^{i\beta_1 z_0/\sqrt{1+n^2}}\cdot e^{-n^2\beta_2 z/\sqrt{1+n^2}}\cdot e^{i\beta_2/\sqrt{1+n^2}\, r}. \tag{30}$$

The leading terms in the asymptotic solution of (26) are obtained by adding (28) and (30). The result after rearrangement is

$$\sqrt{\frac{2}{\pi i}} \cdot \frac{\beta_1^2}{\beta_1^4-\beta_2^4} \cdot \frac{e^{i\beta_2 r}}{r} \cdot e^{i\beta_1 z_0/\sqrt{1+n^2}} \cdot e^{-in^2\beta_1 z/\sqrt{1+n^2}} \sqrt{\pi i}\left\{-\frac{n\sqrt{\beta_2 r \pi i}}{(1+n^2)^{1/4}}\right.$$

$$\cdot\, \mathrm{erfc}\left[i^{1/2} x_0 r^{1/2}\right] e^{i\beta_2 r\left(\frac{1}{\sqrt{1+n^2}}-1\right)} + Q + \frac{n}{Q\, i\beta_1 r}\left[\frac{1}{16} + i\beta_1 z_0 - (i\beta_1 z)^2 + O(n^2)\right]$$

$$\left. + \frac{1}{Q^3 (i\beta_1 r)^2}\left[\frac{7}{32} - 2i\beta_1 z_0 - (i\beta_1 z)^2 + O(n^2)\right]\right\}. \tag{31}$$

V. Results

From the above evaluated two integrals, the Hertzian vectors π for both the vertical and horizontal dipoles can be written in the following form.

A. Vertical Dipole

$$\pi_{1z} = \frac{p_z}{4\pi\epsilon_1 r} \cdot \frac{\sqrt{2}\,\beta_2^2}{\beta_1^2(1-n^2)}\, e^{i\beta_2 r} \cdot e^{i\beta_1(z+z_0)/\sqrt{1+n^2}} \left\{-\frac{n\sqrt{\pi i \beta_2 r}}{(1+n^2)^{1/4}}\right.$$

$$\cdot\, \mathrm{erfc}\left[i^{1/2} x_0 r^{1/2}\right] e^{i\beta_2 r\left[\frac{1}{\sqrt{1+n^2}}-1\right]} + Q\left[1 + \frac{n}{Q^2 i\beta_1 r}\left(\frac{1}{16} + i\beta_1(z+z_0) + O(n^2)\right)\right.$$

$$\left.\left. + \frac{1}{Q^4 (i\beta_1 r)^2}\left(\frac{7}{32} - 2i\beta_1(z+z_0) \;\cdots\right)\right]\right\} \qquad z \geq 0 \tag{32}$$

$$\pi_{2z} = \frac{p_z}{4\pi\epsilon_2 r} \cdot \frac{\sqrt{2}\,\beta_2^2}{\beta_1^2(1-n^2)}\, e^{i\beta_2 r} \cdot e^{i\beta_1 z_0/\sqrt{1+n^2}} \cdot e^{-in^2\beta_1 z/\sqrt{1+n^2}}$$

$$\left\{-\frac{n\sqrt{\pi i \beta_2 r}}{(1+n^2)^{1/4}}\, \mathrm{erfc}\left[i^{1/2} x_0 r^{1/2}\right] e^{i\beta_2 r\left[\frac{1}{\sqrt{1+n^2}}-1\right]} + Q\left[1 + \frac{n}{Q^2 i\beta_1 r}\right.\right.$$

$$\left.\left.\cdot\left(\frac{1}{16} + i\beta_1 z - (i\beta_1 z)^2 + O(n^2)\right) + \frac{1}{Q^4 (i\beta_1 r)^2}\left(\frac{7}{32} - 2i\beta_1 z_0 - (i\beta_1 z)^2 \cdots\right)\right]\right\} \quad z \leq 0. \tag{33}$$

B. Horizontal Dipoles

$$\Pi_{1x} = \frac{p_x}{4\pi\epsilon_1}\cdot\frac{2}{r^2}\cdot\frac{e^{i\beta_2 r+i\beta_1\sqrt{1+n^2}(z+z_0)}}{\beta_1^{\,2}(1-n^2)}\left\{n+\frac{1}{i\beta_1 r}\left(1-\frac{3}{2}in^2\beta_1(z+z_0)\right)\cdots\right\}$$

$$\Pi_{1z} = -\frac{ip_x\cos\phi}{4\pi\epsilon_1 r}\cdot\frac{\sqrt{2}}{\beta_1^{\,2}(1-n^2)}\cdot e^{i\beta_2 r}e^{i\beta_1(z+z_0)/\sqrt{1+n^2}}\left\{\left(i\beta_2-\frac{1}{r}\right)\right.$$

$$\cdot\left(\frac{i\beta_1}{\sqrt{1+n^2}}-\frac{in^2\beta_1}{\sqrt{1+n^2}}\right)\left[-\frac{n\sqrt{\beta_2 r\pi i}}{(1+n^2)^{1/4}}\,\mathrm{erfc}\left[i^{1/2}x_0\,r^{1/2}\right]\cdot e^{i\beta_2 r\left(\frac{1}{\sqrt{1+n^2}}-1\right)}+Q+\right.$$

$$\left.\left.\frac{n}{Q\,i\beta_1 r}\left[\frac{1}{16}+i\beta_1(z+z_0)+\cdots\right]+\frac{1}{Q^3}\cdot\frac{1}{(i\beta_1 r)^2}\left[\frac{7}{32}-2i\beta_1(z+z_0)\cdots\right]\cdots\right]-\frac{3n}{Qr^2}\cdots\right\}\quad z\geq 0 \quad (34)$$

$$\Pi_{2x} = -\frac{p_x}{4\pi\epsilon_2}\cdot\frac{2}{r^2}\cdot\frac{e^{i\beta_2 r}e^{i\beta_1\sqrt{1-n^2}\,z_0}}{\beta_1(1-n^2)}\left\{n\left[1-i\beta_1\sqrt{1-n^2}\,z\right]+\right.$$

$$\left.\frac{1}{i\beta_1 r}\left[1-i\beta_1\sqrt{1-n^2}\,z+\frac{3}{2}\frac{in^2}{\sqrt{1-n^2}}i\beta_1 z-\frac{3}{2}\cdot\frac{in^2\beta_1 z_0}{\sqrt{1-n^2}}+\frac{3}{2}i^2n^2\beta_1^{\,2}zz_0\cdots\right]\cdots\right\}$$

$$\Pi_{2z} = -\frac{ip_x\cos\phi}{4\pi\epsilon_2}\,\frac{\sqrt{2}}{\beta_1^{\,2}(1-n^2)}\cdot\frac{e^{i\beta_2 r}\cdot e^{i\beta_1 z_0/\sqrt{1+n^2}}\cdot e^{in^2\beta_1 z/\sqrt{1+n^2}}}{r}$$

$$\cdot\left\{\left(i\beta_2-\frac{1}{r}\right)\left(\frac{i\beta_1}{\sqrt{1+n^2}}-\frac{in^2\beta_1}{\sqrt{1+n^2}}\right)\left[-\frac{n\sqrt{\beta_2 r\pi i}}{(1+n^2)^{1/4}}\,\mathrm{erfc}\left[i^{1/2}x_0\,r^{1/2}\right]\right.\right.$$

$$\cdot\,e^{i\beta_2 r\left(\frac{1}{\sqrt{1+n^2}}-1\right)}+Q+\frac{n}{Q(i\beta_1 r)}\left[\frac{1}{16}+i\beta_1 z_0-(i\beta_1 z)^2\right]+$$

$$\left.\left.\frac{1}{Q^3}\cdot\frac{1}{(i\beta_1 r)^2}\left[\frac{7}{32}-2i\beta_1 z_0-(i\beta_1 z)^2+O(n^2)\right]\cdots\right]-\frac{3n\beta_1}{Q\,i\beta_1 r^2}\cdots\right\}\quad z\leq 0. \quad (35)$$

DISCUSSION

The asymptotic solution of a dipole immersed in a semi-infinite dissipative medium has been obtained in order to study the communication between submerged submarines or between a transmitter and receiver burried in the ground in geophysical explorations. These investigations have been made under the assumption that the curvature of the earth and the ionosphere may be neglected. Since the present interest is only in sending signals in a single direction, isotropic sources are wasteful and inefficient. From the theory of dipoles in air it is known that, by a proper choice of parameters most of the radiated power can be confined to a narrow beam in a desired direction.

CHAPTER - V

STUDY OF SLOT ANTENNA

INTRODUCTION

observed

It is known that a wave guide[4] or cavity completely closed by perfect conductor can not radiate. If holes or slots are provided then there will in general be radiation. Fields excited in the slot from the interior in turn excite waves in space which may act to carry energy away. If the h holes are small compared to the wave length such radiation will be small. If the slots are large the fields may be built up to such an extent that radiation is appreciable and the slot becomes a useful antenna.

Slot Aerials

Let us consider that we cut a slot, approximately $\lambda/2$ long, in a large sheet of metal and feed current into the sheet by a parallel-wire line as shown in fig.-1. Then it is seen that the two halves of the slot form $\lambda/4$ short-circuited lines and currents will flow in the sheet.[5]

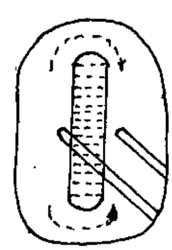


Fig.-1. Slot Aerials.

Thus there will be a potential difference across the slot and an electric field across the slot will be maximum at the centre and zero at the ends. This field will also fringe out on both sides of the sheet. A comparison of the field distribution with that of a $\lambda/2$ aerials, will show that the magnetic and electric fields are interchanged but in different form. Therefore the slot acts as an aerial but the radiation is polarised at right angles to that of a $\lambda/2$ aerial.

Normally we wish the slot to radiate on only one side of the sheet. This can be arranged by boxing up one side of the slot, a hemisphere about $\lambda/4$ in radius being suitable. The impedance is then doubled.

If a slot is cut so that it is across the original current flow on the face of the guide, then it can be seen that the new current distribution will approximate to that shown in fig.-1, and therefore the slot will radiate in the same manner.

Slot Arrays

The behaviour of slot aerials has already been discussed briefly above. So it is evident that an array can be formed from carefully spaced slots, suitably fed, the most practical arrangement being formed by cutting $\lambda/2$ slots in the walls of a waveguide. Such arrays fall into two main classes, depending upon whether the guide is closed by an adjustable piston so that it is carrying a large stationary wave or whether it is correctly terminated and therefore carrying a travelling wave. In the former case the phase of the wave will be the same all along the guide, except that it will be reversed in sign at each half-wave length. In the latter case, on the other hand, there will be a progressive change of phase depending upon the phase velocity in the guide. For example, we cut slots on the broad face of a guide, one-half of the guide wavelength apart, and displace each alternate slot on opposite sides of the centre-line. If the guide carries a stationary wave, then the amplitude of the guide wave at each slot will be the same but the phases at adjascent slots will be opposite. The spacing on alternate sides of the centre-line will, however, correct for this and we have a line of radiators all of the same phase. The arrangement will, therefore, be a broadside array.

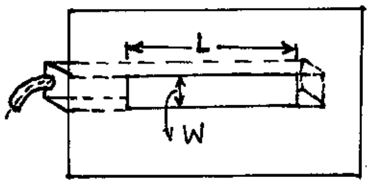
If a travelling wave in the guide is used then the spacing of slots is usually about 200 electrical degrees instead of 180 electrical degrees at the mean frequency to be used. As the phase velocity within the guide is different from that in space, the beam formed makes an angle with axis of the guide. This arrangement can be used satisfactory over a wide frequency-band. Since the wave in the guide is being attenuated, it is necessary to cut the slots such that the coupling of the slots further from the feedpoint of the guide is greater.

Endfire Slot Antenna

The endfire slot antenna((7),page 81) is a travelling-wave antenna characterized by a phase velocity equal to or less than the velocity of light in free space. Here we have discussed the type of aperture excitation required to produce endfire radiation. The apertures of the practical, broadband, endfire antennas may be made wide enough to obtain pencil-beam radiation without the need for an array of slots.

To determine the aperture excitation, it is convenient to think in terms of the equivalent current distribution in the aperture. Thus, to obtain a horizontally-polarized endfire beam, we need a distribution of horizontal electric current elements oriented at right angles to the endfire direction, or a distribution of vertical magnetic current elements. To obtain a vertically-polarized endfire beam we need a distribution of horizontal magnetic current elements oriented at right angles to the endfire direction, or a distribution of vertical electric current elements. But the signal in the plane of a horizontal ground plane is greatly reduced by the shorting-action of the ground plane for horizontally-polarized radiation. So only the vertically-polarized endfire antennas will be considered.

In practice, vertically-polarized endfire radiation can be obtained by either TE or TM excitation of slotted, dielectric-field waveguides.



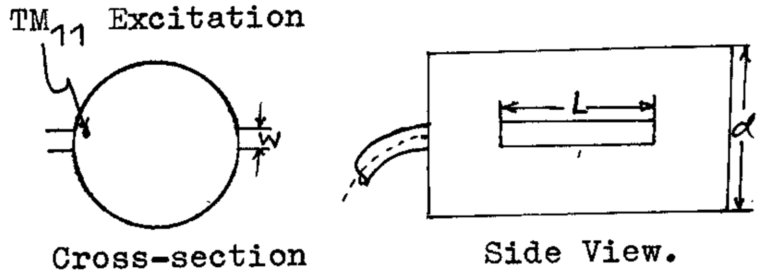


Fig.-2. Fig.-3.

A practical endfire slot antenna is shown in fig.-2. Fig.-3. shows an endfire slot antenna of diametrically opposed slots in a cylinder.

<u>Analysis of the aperture field</u>

An analysis based on the partially-filled waveguide of fig.-4 is quite useful and easy to apply. In this method it is assumed that the phase velocity is not appreciably altered when the air-filled part of the guide is removed. From

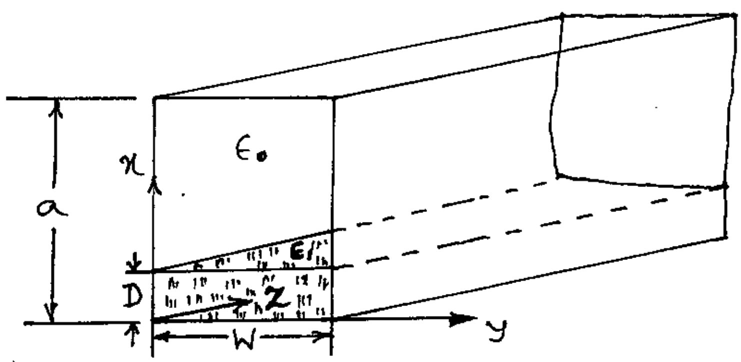


Fig-4. Partially-filled waveguide used in endfire antenna analysis.

the partially-filled waveguide analysis two types of hybrid mode operation are possible. One type corresponds to $H_x = 0$ and the other to $E_x = 0$. In practice it is found that something close to the $H_x = 0$ mode is produced by coventio-

nal

nal TE excitation of a dielectric-filled guide with a side normal to the E-field removed.

The phase velocity in the partially-filled waveguide is given by

$$c/v = \sqrt{1 - \left(\frac{k_0}{\beta}\right)^2 - \left(\frac{M}{2W/\lambda}\right)^2}, \qquad (1)$$

where

c = velocity in free space

v = phase velocity in the z-direction

$\beta = 2\pi/\lambda$

λ = free-space wave length

M = mode number

and k_0 is the propagation constant in the x-direction in mediumair.

For the $H_x = 0$ mode, k_0 satisfies the equation

$$k_1 \tan k_1 D = \left(\frac{\epsilon_1}{\epsilon_0}\right) k_0 \tan k_0 (D - a), \qquad (2)$$

where k_1 is the propagation constant in the x-direction in the dielectric medium 1.

<u>Analysis of a Terminated-Waveguide slot Antenna by an Equivalent Circuit Method ((13), page 16)).</u>

The terminated-waveguide slot antenna consists of a slot cut in the wall of a waveguide terminated in a known impedance. The slot field, therefore, is a function of the waveguide termination and may change greatly for different impedance values. The determination of the slot field is usually an extremely difficult task. So it is desirable to employ a technique to find the radiated field without a detailed knowledge of the slot field. If the slot radiates into a half-space i.e. the slot is contained in an "infinite baffle", the radiated field are analyzed very conveniently in terms of spherical transmission line theory. A single spherical transmission line is sufficient to represent approximately the far fields radiated from a slot excited with symmetric and antisymmetric incident electric fields. So that the half-space is represented approximately by two spherical tranmission lines, and the slot is viewed as a network coupling the wave guide with the half-space region. Now if the equivalent circuit parameters of the slot are known, the far spherical mode voltages and currents for a given waveguide excitation and termination may be computed by simple calculations.

The specific configurations considered here are symmetric rectangular E-plane and H-plane slots radiating from the broad or narrow face of rectangular waveguide. The coupling properties of the slot are represented by a lossless four terminal-pair equivalent network whose parameters, at suitable reference planes, may be expressed approximately in terms of those of the lossy two terminal-pair equivalent circuit for the radiating slot when considered as a dissipative element. The parameters for the latter network can be determined either theoretically from a variational calculation or from direct measurement in the feeding guide.

E-plane slot

(a) Four terminal-pair Equivalent Network:-

The E-plane slot configuration is shown in fig.-5, when the rectangular waveguide is assumed to propagate only the

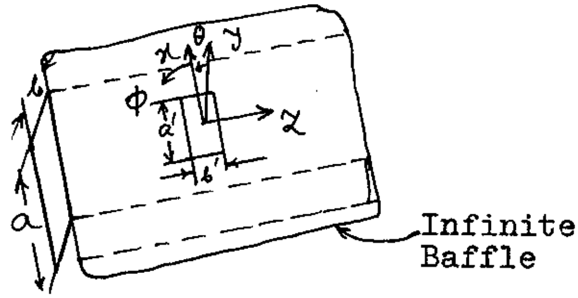


Fig.-5-Slot structure and orientation of co-ordinates for E-plane slot.

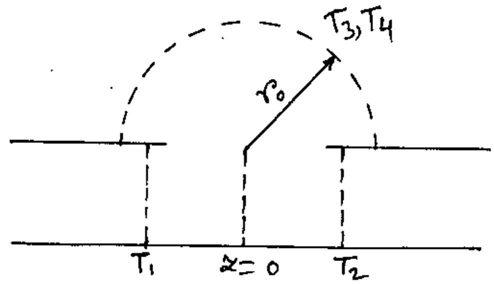


Fig.-6- E-plane slot structure(side view) and terminal plane locations.

dominant H_{10} mode. If the slot is excited by two waves having identical electric field components E at the symmetric, "far" terminal planes T_1 and T_2 (fig.-6), a magnetic wall ($H_{tangential} = 0$) exists at the z=0 plane; the corresponding slot field is shown in fig.-7(a). For convenience, planes T_1 and T_2 are choosen a "large" integral number of guide wavelengths away from the symmetry plane. If is seen from the symmetry of the slot field configuration that the fields ra

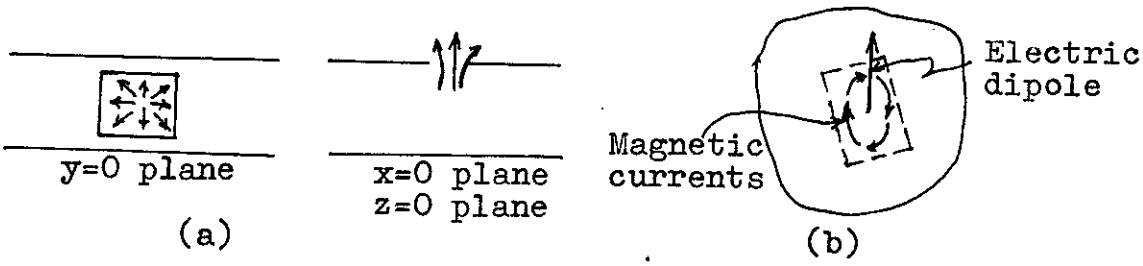


Fig.-7- Symmetrically excited slot (E-plane);
(a) Slot electric field,
(b) Equivalent source.

radiated into the half-space region resemble those due to a radiating electric dipole placed perpendicularly to the infinite plane at the slot centre as shown in fig.7(b). For anti-symmetric electric field excitation of the slot an electric wall ($E_{tangential}=0$) exists at the z=0 plane; the approximate form of the slot field is shown in fig.-8(a). The radiated fields behaves essentially like the field of a

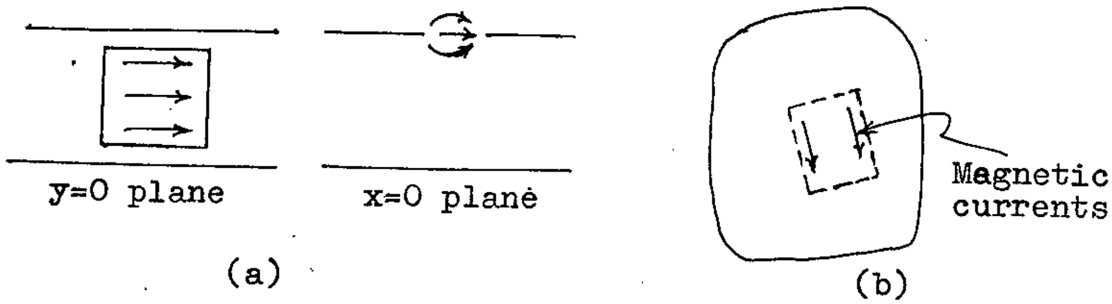


Fig.-8- Anti-symmetrically excited slot (E-plane);
(a) slot electric field,
(b) equivalent source.

magnetic dipole lying in the slot plane and directed perpendicular to the electric field lines as shown in fig.-8(b). The electric and magnetic dipole modes are sufficient to represent the far fields radiated by symmetrically and anti-symmetrically excited small slot respectively.

Since the slot can be represented as a superposition of symmetric and anti-symmetric field excitations, propagation in the half-space region may be approximated in terms of two propagating modes. To derive the parameters of the equivalent for the slot, the electric and magnetic fields on each side of the slot are represented in terms of a complete set of vector modes, whose amplitude co-efficients satisfy transmission line equations, and can be interpreted as transmission line voltages and currents. The relationship between the mode voltages and currents at the "far" terminal planes as shown in fig.-6 is expressed in terms of an equivalent network for the slot. Let V_3, I_3 denote the spherical E (vertical electric dipole) mode voltage and current at the far terminal plane T_3, and V_4, I_4 the spherical H (tangential magnetic dipole) mode voltage and current at the far terminal plane T_4; then the equivalent circuit for the slot structure in fig.-6 is that shown schematically in fig.-9. The network terminal paris T_3 and T_4 in fig.9

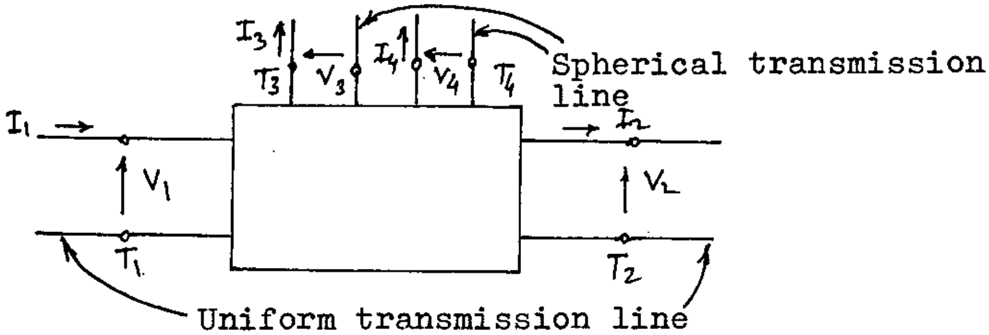


Fig.-9- Network equivalent of slot structure.

have been choosen to concide at the single hemispheric terminal surface (T_3, T_4) in fig.-6. The uniform transmission lines are characterized by the dominant mode propagation constant k and the characteristic admittance Y. Since the spherical transmission lines are situated in a "far" region, their characteristics become identical with those of uniform transmission lines in free space. Thus their propagation constants and characteristic admittances are equal to the free-space values k and $\eta = \sqrt{\frac{\epsilon}{\mu}}$, respectively.

In view of the symmetry of the configuration about z=0 plane, the far fields satisfy certain symmetry requirements. Expressed in terms of voltage-current relations on the equivalent network the symmetry requirements are the following : (1) symmetric voltage (anti-symmetric current) excitation at ports T_1 and T_2 excites only T_3 (fig.-7); (2) anti-symmetric voltage (symmetric current) excitation at ports T_1 and T_2 excits only port T_4 (fig.8); (3) when indetical terminations are placed at ports T_1 and T_2, excitation at port T_4 does not couple to T_3,and vice-versa. Thus the structure behaves like a hybrid junction. Thus the overall approximate equivalent circuit is shown in fig.-10.

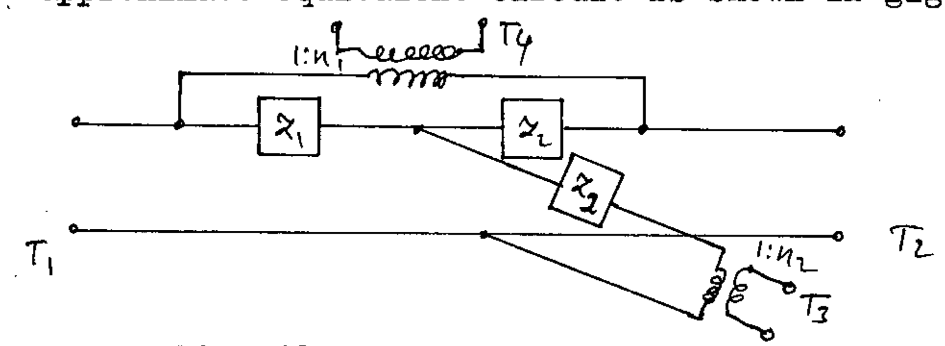


Fig.-10- Approximate network representation.

(b) Relationship with the Two-Terminal pair Equivalent Circuit:-

When the slot radiates into an empty half-space region, impedances equal to the characteristic impedance $\zeta \left(= \sqrt{\frac{\mu}{\epsilon}} = \frac{1}{\eta}\right)$ of free space are connected to terminal paris T_3 and T_4 of the equivalent circuit in fig.-10 to yield a bridged Tee-network, which may alternatively be expressed as the admittance Pi-circuit in fig.11. A Pi-circuit rather than a Tee-circuit is choosen since a narrow slot behaves like a series network so that the shunt admittances are very small. The equivalent four terminal pair circuit for the

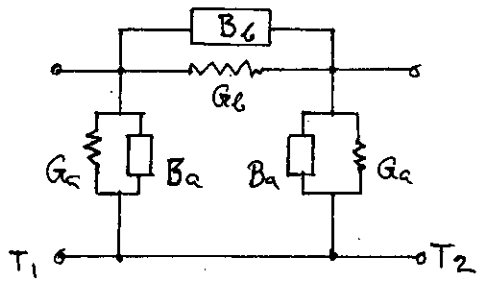


Fig.-11- Two terminal-pair Pi-network.

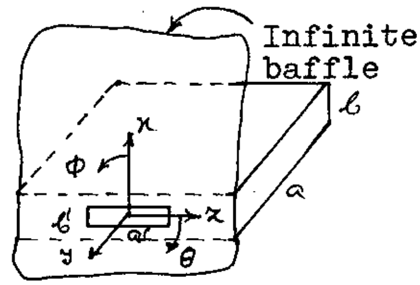


Fig.12- Physical structure and choice of co-ordinates (H plane slot).

slot is approximately lossless; i.e. the elements z_1 and z_2 are pure reactances, and η_1 and η_2 are real. Since both circuits contain four independant elements it is possible to express the parameter values of one in terms of those of the other. We obtain (letting $z_{1,2} = X_{1,2}$)

$$\left.\begin{aligned} \eta_2^2 &= \frac{\wp}{z}\,\frac{2}{R'_a}\,, \\ \eta_1^2 &= \frac{z}{\wp}\,\frac{2}{G'_a+2G'_\ell}\,, \\ -\frac{1}{X'_1} &= B'_a + 2B'_b\,, \\ X'_2 &= \tfrac{1}{2}\,(X'_a - X'_1), \end{aligned}\right\} \qquad (3)$$

where $\wp/z = \lambda/\lambda_g$, λ_g = guide wavelength

$$R'_a = \frac{G'_a}{G'^2_a + B'^2_a}\,, \quad X'_a = -\,\frac{B'_a}{G'^2_a + B'^2_a} \qquad (4)$$

and the prime on the impedance (admittance) parameters denote normalization with respect to the characteristic impedance z(admittance Y). The algebric sign of η_1 and η_2 can be determined from a field analysis.

H-plane slot

Derivation of the Coupling Equivalent Circuit Parameters:-

The physical configuration for this case is shown in fig.-12. A rectangular slot of dimensions a'xb' is located symmetrically in the narrow-face of a rectangular waveguide and radiates into a half-space region. The approximate constituent slot fields induced by symmetric and anti-symmetric electric field excitation of the slot at terminal

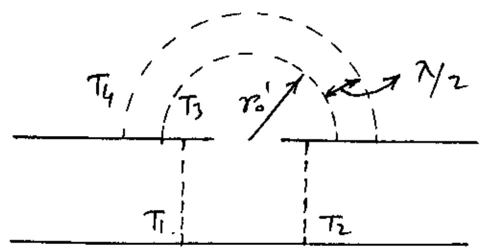


Fig.-13- Location of terminal planes (H plane slot).

planes T_1 and T_2 in fig.-13 are shown in fig.-14 and fig.-15; the dominant modes excited in the half-space by these magnetic current distributions are H_{01} (magnetic dipole mode) and H_{02} (axial magnetic quadruple mode), respectively,

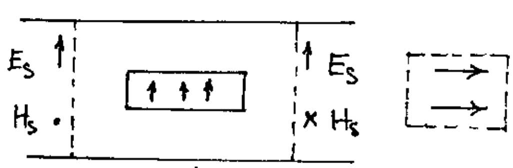


Fig.-14- Symmetrically excited slot.

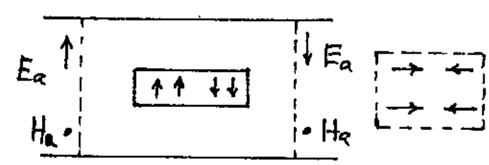


Fig.-15- Anti-symmetrically excited slot. (H-plane)

if the z-axis in fig.-12 is choosen as the polar axis of the spherical co-ordinate system. The reference planes T_3 and T_4 in fig.-13 have been choosen so as to make the field formulation in this problem indentical with that for the E-plane slot and permit directly the utilization of the equivalent circuit derived before. Moreover, the subscripts 3 and 4 refer, as in the E-plane case, to the modes excited by symmetric and anti-symmetric electric field excitation of the slot, respectively. If the terminal plane T_3 is located at a radius,

$$r_0' = m\lambda\,, \quad m = \text{large integer}, \tag{5}$$

the theoretical analysis proceeds exactly as in the E-plane case, and the approximate equivalent circuit is that shown in fig.-10. In relating the parameters of the lossless four terminal-pair equivalent circuit to those of the two terminal-pair network, it is convenient to use not the Pi-network but the Tee-network as shown in fig.-16, since the H-plane slot

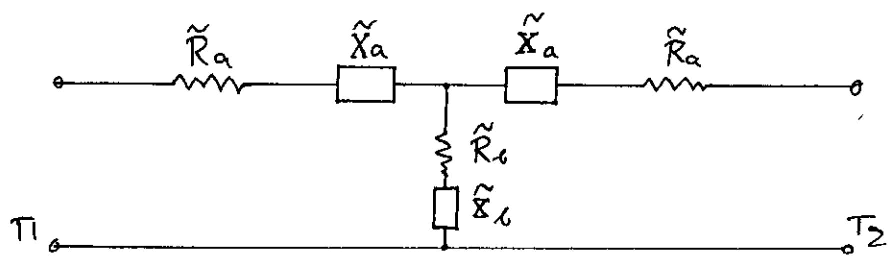


Fig.-16- Two terminal-pair Tee-network for H-plane radiating slot.

is essentially shunt so that the series impedances of the Tee-circuit are small. The sign $\sim$ serve to designate H-plane slot quantities. The network parameters in fig.-10 and fig.-16 are related as follows:

$$\tilde{\eta}_2^2 = \frac{2\lambda/\lambda_g}{\tilde{R}_a' + 2\tilde{R}_b'} ,$$

$$\tilde{\eta}_1^2 = \frac{2\lambda_g/\lambda}{\tilde{G}_a'} , \qquad (6)$$

$$\tilde{X}_1' = -\frac{1}{B_a'} ,$$

$$\tilde{X}_2' = \tfrac{1}{2}\left(\tilde{X}_a' + 2\tilde{X}_b' + \frac{1}{\tilde{B}_a'}\right), \qquad (7)$$

$$\tilde{G}_a' = \frac{\tilde{R}_a'}{\tilde{R}_a'^2 + \tilde{X}_a'^2} , \quad \tilde{B}_a' = -\frac{\tilde{X}_a'}{\tilde{R}_a'^2 + \tilde{X}_a'^2} .$$

The primes denote normalization with respect to the characteristic impedance (admittance) of the rectangular waveguide.

<u>A Two-Dimensional Slotted Array</u> ((14), page 26)

From the knowledge acquired so far, it has been realized that a slot in a cavity or waveguide could be an efficient radiator. A two-dimensional slotted array then could be obtained by placing slotted waveguides parallel to each other; and such a device has been used in practical antenna. A basic difficulty in an array of this kind is the exterior coupling among the elements. The use of quarter-wave decoupling chokes has been considered.

Here the two-dimensional array of slots are fed not from praallel guides but from a single waveguide of width sufficient to propagate a number of modes. Fig.-17 shows a broad waveguide excited in a higher mode, say TE_{01}. It will be observed that the fields in each of the four regions, AB, BC, CD and DE, are the same as for a guide of one quarter the width operating in its lowest mode. The only physical difference is along the imaginary partitions BB',CC',DD', where the currents are zero for the higher mode device.

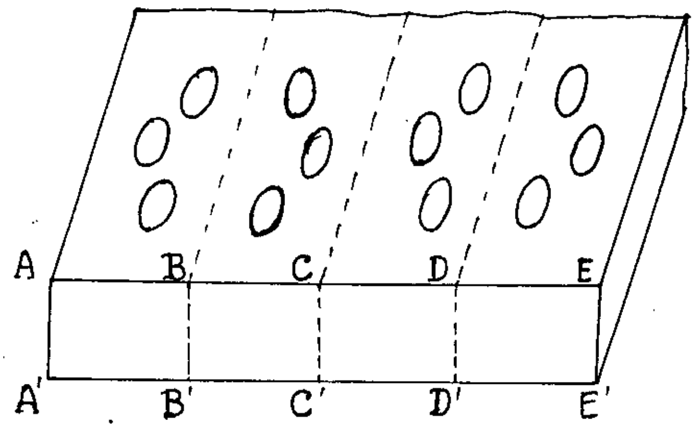


Fig.-17- Broad waveguide excited in a higher mode.

Rows of slots are disposed along the lines of maximum field intensity as shown. Slots must be oppositely offset in adjascent rows on account of the phase reversal of the field. Slot admittances may be adjusted. Now we are to consider the questions of mode stability and slot interaction.

It is supposed that initially only the desired mode is excited. A mixture of propagating modes is obviously undesirable. Their propagation constants are different and the slot spacing can be related to only one of them. Hence if more than one mode were present the radiation from each would be directed toward different angles in space. It is evident that in order to maintain only the desired mode along the course of the guide, energy must be abstracted equally in magnitude and phase from all parts of the regions AB,BC,etc.. In brief, the necessary and sufficient condition for the maintainance of the desired mode is that slot admittances be equalized laterally across the array. In this regard, the factor of primary importance is that of slot interaction.

The electric field in a slot is essentially perpendicular to the slot. As such it csn be conceived of as origi-nating from positive charges along one of its sides and equal negative charges along the other side. If offsets are equal and slot voltages equal laterally, it will be observed that the slot charges are symmetrically disposed, except for a sign reversal, about each of the modal planes BB',CC', etc.. That is, each of these planes acts as a virtual image plane. This arrangement is applicable both internally and externally. Therefore, for an infinitely long array, slot interaction is adequately accounted for by virtual conducting planes erected internally and externally along the lines BB',CC', etc. Thus the interior mode is effectively that which would be obtained with parallel guides, while each row of slots feeds into what is effectively a parallel-plate region having a width equal to the seperation of the slot rows.

For the exterior region, the sides AA',EE' are themselves the necessary image planes. In the exterior region, flanges must be erected to effect the lateral termination of the image plane system. Theoretically, for an ideally imaged system the flanges should be infinite height. It is found in practice that the deterioration in the performance of the array may not be serious for flange heights as small as one-half wavelength.

The requirements so far imposed is that slot offsets and admittances be equal laterally. In the longitudinal direction they can be given any desired distribution; that is, the array amplitude will be uniform laterally. It should be noted, however, that the finite length of the flanges introduces errors in the admittances of slots near the two ends of the array. Therefore, it is to be expected that array requiring the cirtical adjustment of elements near the ends will be difficult to attain in practice.

DISCUSSIONS

The slot type antennas are particularly useful in aircrafts and ships where flush mounting is required. Slot aircrafts have been found considerable application on centimetric waves and may be used in very high-speed aircraft as slots cut in the fuselage and filled with solid dielectric.

The basic problem of communication to and from high-speed aircraft has stimulated research work into the theory and application of slotted-cylinder antennas. Antennas having a single, smooth beam of moderate directivity are finding increasing use in aircraft and airport installations in connection with homing or ground-directed approach systems. Flared-slot antenna is su suitable for this purpose. The Yagi arrays and horns are sometimes inapplicable to high-performance aircraft or to airport installations where the only avoidable locations often are flat surfaces, such as the underside of the fuselage, wings, or horizontal stabilizers, or the ground near or even the runways.

BIBLIOGRAPHY

(1) Theory of Dielectric Coated Linear Antennas
-by- Mohammed Azizul Islam.
Northern University, Boston, Mass.

(2) Numerical Evaluation of Bessel and Hankel functions
-by- Md. Mozaharul Islam
Atomic Energy Centre, Dacca, East Pakistan.
October, 1965.

(3) Technical Report EE-44
The fields of Electric dipoles in sea water
The Earth-air-Ionosphere Problem
-by- Wallace L. Anderson, February, 1961.

(4) Fields and Waves in Modern Radio
-by- Simon Ramo & John R. Whinnery.

(5) Short Wave Wireless
-by- A.W. LADNER.

(6) Partial Differential Equations in Physics
-by- Arnold Sommerfield. (1949).

(7) IRE Transaction on Antennas & Propagation,
VOLUME AP-3, No.2, APRIL, 1955.

(8) Proceedings of IRE. 35, NO-12, 1479-1484(Dec. 1947)
Wheeler, H.A., Fundamental limitations of small antennas.

(9) IEEE Transactions on Antennas and Propagation
VOLUME AP-11, NO-2. March, 1963.

(10) Advanced Mathematics for Engineers
-by- H. W. Reddick & F. H. Miller.

(11) Electromagnetics
-by- John D. Kraus.

(12) JOURNAL OF RESEARCH OF THE NATIONAL BUREAU OF STANDARDS- D.Radio Propagation.
VOLUME 66D, NO. 5, September- October, 1962.

(13) IRE Transactions on Antennas and Propagation
VOLUME AP-4, NO-1, January, 1956.

(14) IRE Transactions on Antennas and Propagation
VOLUME AP-6, NO-1, January, 1958.

E. P. UNIVERSITY OF ENGINEERING & TECHNOLOGY LIBRARY
No.
Date.
DACCA