AN ALTERNATE ROUTING STRATEGY TO ENHANCE CALL HANDLING CAPACITY OF A MULTIEXCHANGE TELECOMMUNICATION NETWORK SYSTEM

BY

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A THESIS

SUBMITTED TO THE DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN ENGINEERING (ELECTRICAL AND ELECTRONIC).

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY
DHAKA, BANGLADESH

SEPTEMBER ,1986

CERTIFICATE

THIS IS TO CERTIFY THAT THIS WORK WAS DONE BY ME AND IT HAS NOT BEEN SUBMITTED ELSEWHERE FOR THE AWARD OF ANY DEGREE OR DIMPLOMA.

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ACKNOWLEDGEMENT

The author expresses his profound indebtedness and deep sense of gratitude to his reverend supervisor, Dr. A.K. M. Mahfuzur Rahman Khan, Professor of Electrical and Electronic Engineering Department, Bangladesh University of Engineering and Technology,Dhaka for his constant encouragement and valuable guidance throughout the course of this work.

The author wishes to express his thanks and deep sense of gratitude to Dr. Shamsuddin Ahmed, Professor and Dean, Faculty of Electrical and Electronic Engineering and Dr. Syed Fazl-i-Rahman, Professor and Head, Department of Electrical and Electronic Engineering, BUET, Dhaka for their kind encouragement during the course of the research work.

The author offers his sincere thanks to the staff of Computer Centre of BUET for their full co-operation in using the computer. The author also expresses his sincere thanks to Mr. Md. Hasan Ali for taking patience care while typing this thesis.

Finally, thanks to all who were involved directly or indirectly in successful completion of this research work.

ABSTRACT

Multiexchange networks with direct routing (simple), alternate routing (hierarchical) and alternate routing (symmetrical) networks are studied in this work.

Network systems are studied by digital simulation. Roulette-model simulation method is used in this case. The computer program is written in Fortran IV and run in IBM 4331. The validity of the simulation model checked with the similiar results obtainedfrom analytical or exact solution method (which is derived on the basis of classical probability theory) for small systems . The equations derived for different simple systems in exact solution method are solved with Gaussian-elimination technique.

Three, four and five exchange networks are studied in the present work. Maximum number of links considered here are ninety (90) and the traffic considered from 0.5 to 0.85 erlangs. 'Congestion probability' and 'Traffic handling capacity' are considered as performance indices of the network systems. Results obtained from exact solution and simulation methods are presented graphically. The performance of symmetrical network is the best among the three systems studied followed by direct routing (simple) and alternate routing networks (hierarchical).

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CHAPTER - 1
INTRODUCTION

1.1 General:

The technique of alternate routing has become firmly established in most large communication networks. Communication networks can be efficiently utilized by introducing the alternate routing system, where the overflow traffic from the direct route is carried through alternate routes. In a multi-exchange network system spreaded over a wide area,network busy conditions become increasingly noncoincidental and hence overflow call from a direct route can find idle paths along alternate routes through other exchanges. Implementation of such strategies in network systems become easier with the introduction of digital exchange. With computer controlled exchanges sophisticated routing strategies can be realized through software control. Study of alternate routing system for a multi-exchange network system is important for efficient planning and management of telecommunication system. The greater traffic efficiency of networks with automatic routing facilities calls for fewer circuits being required. Modern electronic equipment and highspeed signalling devices has made it possible to implement alternate routing doctrines dependent on network state feasible.

A planned alternate routing system can never be considered as final one, since the traffic flow in network as well as its pattern varies from day to day and fore casts are uncertain.

Provision should be kept in system so that it may further be modified according to future traffic characteristics.

1.2 Multiexchange Networks in Modern Telecommunication System:

Multiexchange networks play an important role in modern telecommunication system. There are over 400 million telephones in the world. It is uneconomical for every subscriber to be given telecommunication services to have an exclusive line to every other subscriber. A simple solution to this is to connect subscribers line to a central switching station, an exchange which can provide the same facility as and when needed. As it is impracticable to provide individual direct circuits between subscribers, so, it is impracticable for a single exchange to be connected to all subscribers. Thus there is a need for a number of exchanges and for circuits to carry calls between them. There is an economic conflict between the number of exchange and the cost of subscribers' line. If there are few exchanges and many of the subscribers lines are long, and therefore expensive. On the other hand, if the subscribers lines are short and many exchanges, but a large number of circuits required between exchanges, and the cost of providing these and the associated switching equipment may be too great. In designing a network system, these factors should be taken into account carefully to make an optimum network with minimum cost. Usually, sparsely populated areas contain small exchanges even though subscribers lines are long. Densely populated areas contain large exchanges and short subscribers lines.

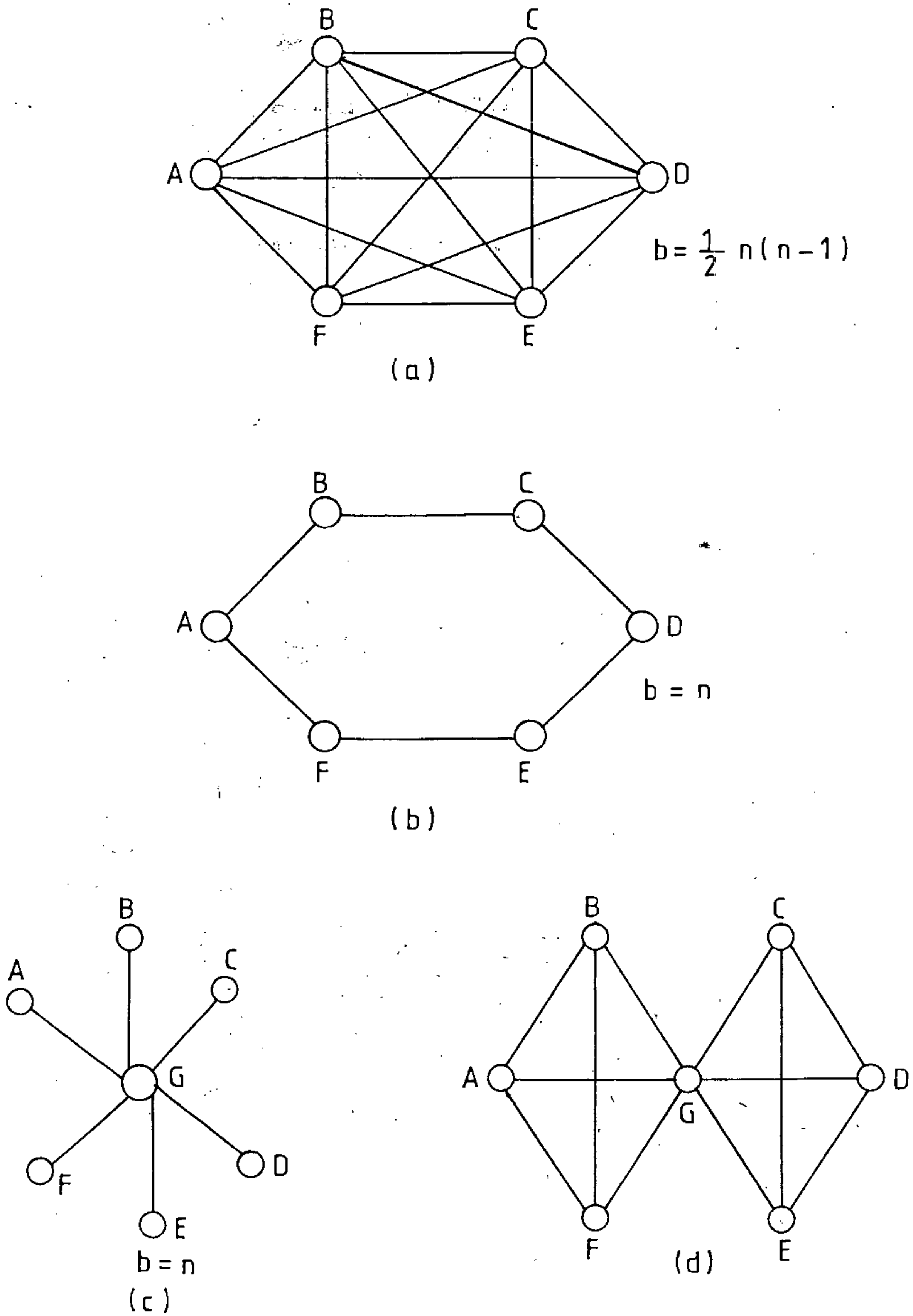


Fig. 1.1 Network configurations.

a. Full inter connected Network.

b. Mesh

c. Star

d. Partially interconnected.

Some of the ways, in which networks are connected described as follows:[8]

i) Fully inter-connected network:

Shown in fig. 1.1a. Each of the exchanges (A to F) is called a node. The routes between the exchanges or nodes (AB,CD,BC ....etc.) are called branches. The network shown in fig. 1.1a is called fully inter connected network, because each node is connected to every other node. If there are n nodes and b branches, then the number of branches

$$b = \frac{1}{2} n (n-1)$$

ii) Mesh network: Number of branches between the nodes reduced to a minimum. The number is reduced to a single mesh and is therefore called a mesh network. The number of branches b = n. (Fig. 1.1b).

iii) Star network: In mesh network, with originating and terminating exchange, to transit a call other exchanges may be involved. This is an uneconomical use of circuits. For example, in Fig.1.1b call from A to D involves nodes B and C. Sometimes, a circuit on route AB may be unable to carry call from A to B because it is occupied by a call from F to C. This is overcome by star network (as shown in fig. 1.1c) by adding an (n+1)th node, containing a switching centre G. This switching centre G is called Tandem exchange. Here

iv) Partially interconnected network: A practical telecommunications network usually contains a combination of star and mesh configurations, as shown in fig. 1. 1d. This is because community of interest between subscribers at adjacent nodes usually generates enough traffic to make it economical to provide direct routes. Thus, traffic between A and B or between A and F is routed directly, but traffic between A and C, D or E is routed via G.

Sophisticated switching techniques and cheaper trunking facilities have increased the importance of multiexchange network system. Modern switching systems provide greater flexibility of routing and automatic alternate routing has become a feature of modern network systems. Efficiency of a network system can be increased with the routing facilities.

In Dhaka, there are 78,000 telephones served by 10 exchanges. In Bangladesh the total numbers of telephone are 1,60,000. Existing network configuration is the combination of star and mesh network. The demands of telephone are increasing day by day with the development of socio- economic conditions of people. Efficient network system will be able to meet future demands. Before planning and designing a system, its parameters should be studied statistically and theoretically.

1.3 Overflow traffic and overflow system:

Study of overflow traffic characteristics is essential for study of overflow systems like alternate network system. Overflow traffic differs from the pure chance traffic. It has more noticable fluctuations. Overflow traffic is characterized by two parameters mean M and variance V , where V>M. But in case of purechance traffic or random traffic M=V (i.e. Mean equals to variance). Because of the 'Peaky' nature of overflow traffic, variance must be taken into account.

If an exchange has direct junctions to other exchanges and an overflow route to a tandem exchanges or other exchanges, the traffic on the overflow route will be the sum of the traffic lost from all the direct junction groups.

An under-provided direct route is called a high-usage route and the fully-provided indirect route to which its traffic finally overflows is called a final route. If the traffic offered to high usage routes in a multiexchange network is random, simultaneously infrequent peak loads in all direct routes will occur on all of them. Thus, fewer additional circuits are required to carry the overflow traffic than would be needed if additional circuits were added seperately to each of the direct routes to provide the required grade of service.

The traffic that is lost from each of the high-usage routes can be calculated from Erlangian formula, and the sum of these losses comprises the traffic offered to the overflow route. However, the number of circuits required to carry this traffic cannot be calculated from Erlang's loss formula, because the traffic is not as the same type as that offered to the direct routes.

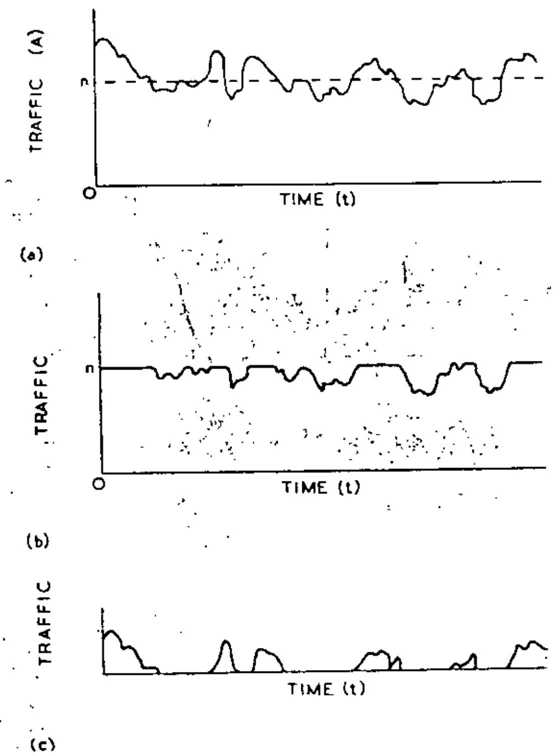


Fig. 1.2:Character of traffic for high-usage route with overflow.

a. Traffic offered to high-usage route.
b. Traffic carried by high-usage route .
c. Overflow traffic.

Fig. 1.2a[8] shows an example of the traffic A(t) to a high usage groups of n circuits. The traffic carried by this route is A(t) when A(t) $<$ n, and n when A(t) $>$ n as shown in fig. 1.2b. The traffic which is lost from this route and offered to the overflow route is A(t) - n when A(t) $>$ n, and 0 when A(t) $<$ n, as shown in fig. 1.2c. The total traffic offered to the over flow route is the sum of number of components of the kind shown in fig. 1.2c. The character of the overflow traffic will be obviously different from traffic carried by the direct routes. R.I. Wilkinson[1], derived an approximate formula to determine the number of circuits required for an overflow route. He determined an approximate probability distribution for the number of simultaneous calls for the overflow traffic and developed a method using 'equivalent random traffic' to determine the over all grade of service.

1.4 Importance of Routing:

The purpose of routing is to carry the overflow traffic under certain overload conditions in a system. Routing a call in the proper way helps to reduce the probability of blocking in a system. The following factors are important in providing alternate routing facilities-

- The advantage of the routing is the high utilization of the network. The high usage routes carry a greater part of the traffic offered between the exchanges and the traffic rejected is assembled on common tandem routes which permits a high utilization.

- If one link in the direct route of the network system fails, then the call can be rerouted to alternate route, thus it will not be interrupted. Hence, routing decreases the risk of interruption by technical faults and thus provides a better service.

- If a particular section of the network is subjected to heavy traffic under some catastrophical conditions like local disasters, such as storms , earth quakes, etc. then routing helps to reduce the call congestion towards the affected area.

- Routing facilities helps in efficient network management.

One sophisticated routing is the dynamic routing or adaptive routing, where calls progress in the final route can be rerouted to high usage route after call hangs up in that route. In the present work static routing technique will be used.

1.5 Types of Routing and Network.

There are three types of routing.

- Direct routing
- Tandem routing
- Alternate routing

According to routing facilities Network system can be classified as follows-

- Simple network
- Hierarchical network
- Symmetrical network.

1.6 Types of Study:

The followings are commonly considered as performance indices of a network system.

1. Congestion probability
2. Traffic handling capacity
3. Network cost for providing a given grade of service at given traffic level.
4. Reliability.

'Congestion probability' and 'Traffic handling capacity' of different network systems will be studied in this work.

1.7 Methods of Study:

The following methods are generally followed to study the performance indices of network systems-

1. Statistical equilibrium equation.
   (Derived on the basis of classical Theory of probability)
2. Approximate methods
   (on the basis of assumed distribution and combinatorial mathematics).
3. Simulation.

In the present work, statistical equilibrium equation method and simulation method will be followed to study the network systems.

1.8 Main Idea of the Work:

The present work concerns the study of some simple and alternate routing networks. Following three types of routing system will be studied in the present work-

- Direct routing (Simple Network).
- Alternate routing (Hierarchical Network).
- Alternate routing (Symmetrical Network).

Statistical equilibrium equations are derived for some simple Network systems. These equations form the mathematical model of the system. With the help of computer, these equations can be solved and necessary parameters can be calculated. This method is known as 'Exact solution method'.

Exact solution method is more accurate than any other method. But this method is complicated, laborious and need a lot of computational time. Though these make good starting point for approximate analysis of larger networks, these are of little practical as yet.

As mentioned above, numerical solution of a system is impossible when it is large one. One of the main techniques in common use is that of 'Simulation'. Simulation techniques have proved successful in solving practical problems. This involves creating valid model of the system in logical terms while omitting irrelevant detail and then using this model for a statistical analysis of the behaviour of the system. There are two types of Simulations-

- Time-true simulation.
- Roulette simulation.

In time true simulation, stochastic events are inter arrival time and holding time. Duration of these times are determined at their beginning by sampling from the prescribed distributions. Holding times 't' assumed to be negative exponentially distributed with mean 'h' for a time 't' probability distribution function f(t) is given by-

$$f(t) = \frac{1}{h} e^{-t/h}.$$

In Roulette Simulation, events are attributed space distributions according to their probability of occurance as determined from the rates of call arrival and call hangups. Random numbers with uniform distribution are then used to choose the events- call arrival or call hang up. The concept of time is absent in the process and this can be used in cases where measurements of time is not necessary.

In this work network systems will be studied with Roulette model simulation concept.

In the present work the following works will be performed:

- Analytical calculation of the performance index of a three exchange simple network, a four exchange (with a tandem) hierarchical and a four exchange simple network.

- Simulation study of the same networks, to check the validity of the simulation model.

- Simulation study of three, four and five exchange networks with and without alternate routing facilities (with same number of links) and comparision of their congestion probabilities.

- Effect of percentage increase in Traffic handling capacity with percentage increase in number of links.

Network systems will be studied with uniform load (i.e. with balanced traffic).

CHAPTER - 2
LITERATURE REVIEW

2.1 Introduction

Alternate routing strategy is an well established technique. The objective of alternate routing network is to carry traffic at a minimum cost for a given grade of service. Efficiency of such system can be studied by various parameters like mean,variance etc. There are many techniques available regarding the system. In the following section, a brief review is made of the relevant works in the recent past.

2.2 Review of the Previous Works on Alternate Routing System

The presence of non random or overflow traffic from high usage groups complicates the estimation of correct quantities in alternate paths. R.I. Wilkinson[1], in 1956, studied some alternate routing arrangements and developed an ingenious method of solving overflow problems. He determined an approximate probability distribution for the number of simultaneous calls for the overflow traffic and developed a method using 'equivalent random traffic' to determine the overall grade of service for an alternate routing arrangement. The method uses the following formula for the traffic rejected from a full availability group, with offered traffic A and n devices:

Mean of overflow traffic:

$$M = AE_n(A)$$

Variance of overflow traffic:

$$V = M \{1 - M + \frac{A}{n+1+M-A}\}$$

where $E_n(A)$ is Erlangs first formula.

These expressions permit straight forward calcualtion of M and V for given A and n. Calculation of A and n from given M and V requires a trial and error process. The equivalent random traffic A^* is given by

$$A^* = V + 3 \frac{V}{M} (\frac{V}{M} - 1)$$

The traffic A^* is considered to be offered to a full-availability group of $m+n^*$ circuits, where m is the number of tandem circuits and n^* is a number of circuits carrying traffic equivalent to that on the high-usage routes. It can be shown that:

$$n^* = \frac{A^*}{q} - M - 1$$

where, $q = 1 - 1 / (M + V/M)$

J.H. Weber[2], in 1961 studied traffic characterstics of communication networks with alternate routing. He developed a simulation program and with the help of digital computer

(IBM - 7090) studied the behaviour of small networks of various sizes, configurations and alternate routing doctrines under normal and abnormal conditions of load. In his work, he calculated the efficiency E and performance measure M = E(1-B) (B is the blocking of the network) for different networks. He studied simple, hierarchical alternate routing and symmetrical alternate routing with heavy and light traffic and compared the relative performance measures and relative efficiencies of these networks.

Alternate routes are in general more expensive, but by carrying larger quantities of traffic than direct routes, their utilization per circuit may be higher. Thus direct routes and alternate routes differes both in utilization per circuit and in cost per circuit. Mr. Eldin[3] studied the profitable use of alternate routing and developed a method in order to evaluate where and when the introduction of direct route is economically justified and how the traffic should be divided between the direct route and one or more alternate routes.

H. Akimura, H. Tokushima and T. Niahimura[4] of Japan studied alternate routing system and derived an optimum design technique which mimimizes the system cost. On the basis of Wilkinsons formula of equivalent random theory, they studied optimum condition for the basic triangular model. They also studied two stage overflow model. Derivatives of Wilkinsons formula, which gives the mean and variance of the overfow system are derived and a table of these are prepared by them. Applying the derivatives of these parameters they analysed the optimum conditions for the basic triangular model. On the basis of fundamental analysis they also developed optimal design condition for the two stage over flow model.

Mr. Sagerhom[5] derived a method to calculate the traffic offered to an alternate routing arrangement. Based on the Wilkinsons equivalent random method, he designed a new routine which gives much better result than the old manual methods. The routine works in two methods. First, he assumed that all routes have direct offered traffics. He calculated such values of these direct offered traffics that calculated traffics for each route exactly corresponds with the measured carried traffics for the routes. In the second step direct offered traffics are calculated which minimize the sum of squared relative differences between calculated and measured carried traffic per route. Both of the steps are performed with iterative calculations.

For supervision of overload in an alternate routing system it is possible to use the time congestion in the final route, i.e. the proportion of time when all trunks in this route are simultaneously occupied. When estimating this system parameter from measurements, it is desirable to have an idea of the precision of the estimate. The problem of calculating mean and variance of this estimator therefore arises.

K.M. Olsson [6] derived an approximate method to determine the mean and variance of the time congestion in the final route of an alternate routing system. He considered a system consisting of a number of primary routes and a common final route. The traffic offered to each of the primary route was assumed poissonian traffic. The total traffic overflowing from the primary route was approximated by an interrupted poisson process. On the basis of interrupted poisson process (IPP) he calculated the mean and variance of time congestion of the system.

B. Wallstrom, L. Reneby [21] presented a method for computing approximate individual traffic losses in alternate routing networks by using interrupted poisson process. For this purpose they developed, (i) a modified IPP generator based on the five first traffic moments. and (ii) a generator of dependent traffic process based on mixed traffic moments. In their work, they considered:

(i) hierarchical network structure (no call can occupy more than one high usage link)

(ii) the links of the final or back bone routes are all low loss links.

Mr. J. Rubas [7] of Australia studied alternate routing system with binomially distributed traffic, where, the traffic offered is smoother than pure chance traffic. Instead of the poisson based "equivalent random" method he proposed a method more accurate in computation of traffic overflow moments. In his work, he showed the profitable use of alternate routing systems in situations, where traffic generated by a limited number of sources.

A. Ruckriski and A. Klimontowiz [23] studied the estimation of service efficiency of component flows in overfow systems with full availability groups and poisson input flows. On the basis of simulation they determined the effect of factors conditioning service efficiency of component flows.

Schehrer[24] worked for the optimal design of alternate routing system. Most of the known methods for the economically optimal design of alternate routing systems neglect the variance of overflow traffic, especially in case of gradings (for reasons of simplicity). Therefore, in some casesthese methods yield results which differs remarkably from the actual economic optimum group sizes. Schehrer presented some methods more accurate and laborious ones upto simplified and easier ones for the calculation of such networks. They regard the variance of overflow traffic, too, and are suitable for groups with full access as well as for gradings. German research society helped in Schehrer's investigation.

Mr. A. Alim,[25] in 1980, worked on overflow system with special reference to Bangladesh. He studied overflow system, with finite number of sources. In his work he derived some equations on the basis of Schehrers formula. He used exact solution, approximate and simulation (Time- True simulation) method to study the overflow system.

CHAPTER -3

ROUTING PLAN AND NETWORK STRUCTURE

3.1 Introduction

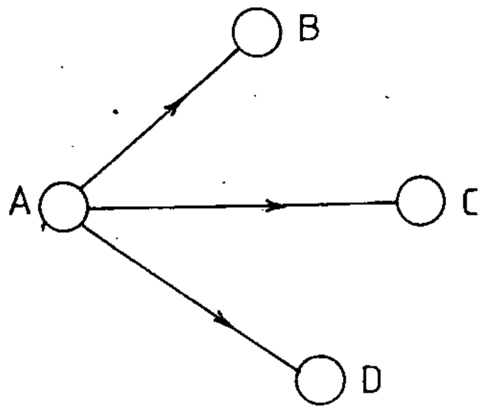
Growing demands of telephone connection and system management call for different network structure. With the advent of sophisticated technology, forms of network structure changing day by day and continuously bringing the problems of network design. Along with complex network structure various routing plan to reap the benefit of the structure are being investigated for the sake of efficency. This chapter includes the description of network structure and routing plan.

3.2 Classification of Routing

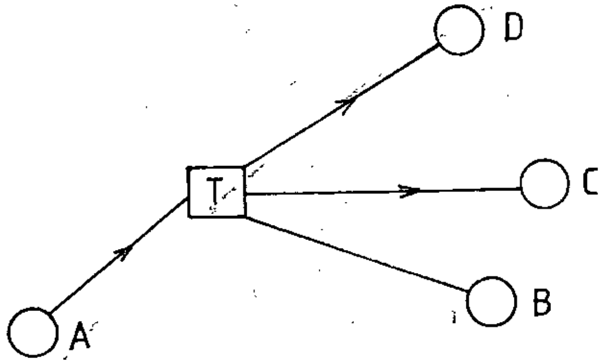
On the basis of connection between two exchanges, routing may be classified as follows-[17]

- Direct routing.
- Tandem routing.
- Alternate routing.

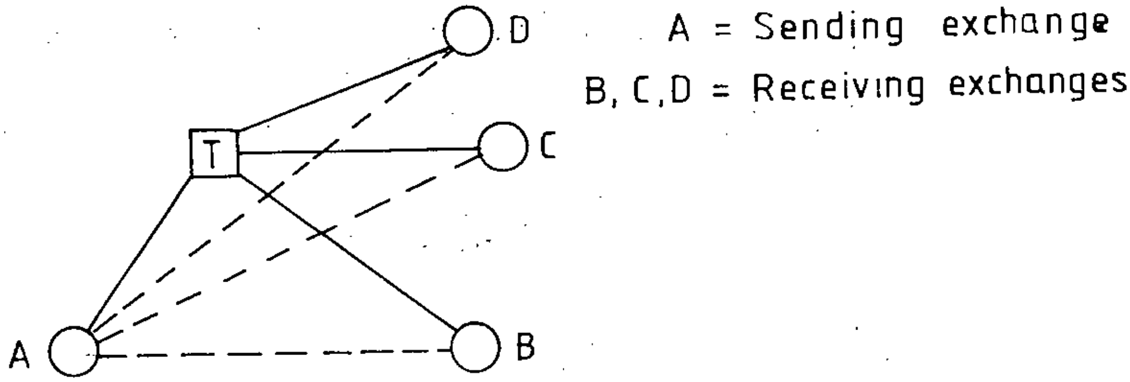
- In direct routing, the entire traffic is carried directly from one exchange to another. In fig. 3.1a, AB, AC and AD are the direct routes between two exchanges.



(a). Direct routing



(b). Tandem routing



(c). Alternate routing, one tandem stage.

Fig. 3.1 Direct, Tandem and alternate routing in a junction network.

- In tandem routing, the entire traffic between two exchanges is routed via one or more tandem stages to other exchanges. In fig. 3.1b, a call between A to B, C or D can be set up via tandem exchange T.

- In alternate routings, the traffic is primarily routed through a direct routes, called high usage routes, calls lost from the direct routes are passed over alternate routes, through one or more tandem stage. In fig. 3.1c, a call from A to B is chosen first through direct route AB, if not available then the route AT and BT are selected.

Routing may also be classified into following types-

- Dynamic routing
- Static routing.

- Dynamic routing: In an alternate routing network, when a call in a high usage (or direct route) hangs up, then the call in progress in the final route can be rerouted through the high usage route. Networks with this type of routing is known as rearrangable network.

3.3 Routing Plan

3.3.1 Basic Principles

Network with minimum configuration is the star connected network. It is usually augmented by interconnecting the exchanges through a intermediate switching exchange called tandem exchange. This network carries sufficient traffic and serves a high community interest. Exchanges connected with each other through back bone route via tandem exchange from lowest to the highest level. Star connected network configuration can be changed by connecting the exchanges through direct routes.

The routes connecting the exchanges directly, called high usage route and the routes, which carries the overflow traffic called final routes. Large groups of circuits are more efficient than small groups because of their higher occupancy (i.e. traffic per circuit). When there is a large amount of traffic between two exchanges, then it is economical to provide direct routes between them[8]. But when there is less traffic then it is economical to use automatic alternate routing; the direct circuits are used efficently and the overflow traffic shares tandem circuits with traffic to other destinations. If the traffic is very little, then it is uneconomical to provide direct routes between two exchanges, all traffics should be routed via tandem exchange. The traffic levels for which these three routing should be used depends on the relative costs of the direct and tandem circuits (including associated switching and signalling equipment).

Both way working in low traffic route is much more economic and efficient than that of unidirectional working, because in both way working traffic carrying capacity is more higher with fewer circuits.

But the disadvantages of both way working are

- the increased cost of signalling equipment and

- the possibilities of calls failing due to 'collisions'. (when simultaneous calls are made to setup connections from both ends of circuit). Both way working is normally used when small groups of circuits are required over routes that are long and expensive.

Transmission and signalling plan influences on routing plan besides the cost and traffic considerations. Numbering plan is also an important factor in routing plan.

After deciding the location of the local-exchange and location of the exchanges the following routing plan should be developed to determine-

- which local exchanges should be connected directly and which should be connected indirectly via tandem exchange.

- the number and location of tandem switching centres.

- the number of levels of tandem switching to be used in the network.

- whether automatic alternate routing is to be used and if so, under what conditions.

3.3.2 Local Area Routing

With in a number plan area, the following factors influences on routing-

- the size of the area.
- the volume of the traffic to be handled.
- the community of interest between them and
- the type of switching system.

In step by step switching system, the routing that can be used are limited by the number of digits in the numbering plan for the area. In a small isolated town there may be a single telephone exchanges. If this exchange is small, its trunk office will be routed to group switching centre or toll centre in a larger town, which will also provide better service. The group switching centre will also provide tandem switching for majority of traffic from the local exchange, but there may also be a small number of direct routes to other local exchanges.

3.3.3 Trunk-Network Routing

In a trunk network, all routes may be fully provided or automatic alternate routing may be used. The greater traffic efficiency of networks with automatic alternate routing results in fewer circuits being required. However, the switching equipment is more complex and the task of network management more complicated. Alternate routing is most effective when busy hours on different routes donot coincide. This happens in large countries (like USA) because of the time difference between east and west.[8]

Since the majority of calls are completed over high-usage routes that by-pass portions of the final route, only a small proportion of traffic uses the maximum permitted number of links. Thus, very few connections have the maximum attenuation permitted by the transmission plan and the largest setting-up time permitted by the signalling plan,

3.3.4 International routing

A world network for international subscriber dialling (i.s.d) have planned and becoming operational. It has hierarchy with three levels of international exchanges (designated CT_1, CT_2 & CT_3) as shown in Fig. 3.2.

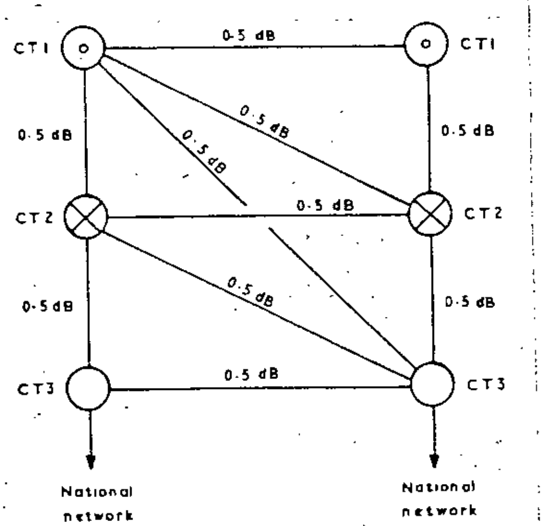


Fig. 3.2: The international routing plan.

Centres of the lowest level of this hierarchy are called CT3s (CT stands for centre du Transit). The CT3 is the interface between the national and international systems. There could be a number of CT3s in any one country. A CT3 may have a direct route to the destination CT3 or, failing that, it may be connected via a CT2 which serves a defined part of the world. The highest level in the hierarchy is the CT1s, and seven CT1s are planned to cover the whole world. These are in (or planned to be in) London, New York, Sydney, Singapore, Moscow, Tokyo and some where in India or Pakistan. The CT1s are planned to be fully interconnected.

3.4 Networks used in this work

Following types of network are studied in the present work-

- simple network.
- Hierarchical network.
- Symmetrical network.

Detail descriptions of these network systems are given below-

Simple network - A simple network has only final routes (direct routes) i.e. no alternate route is allowed.

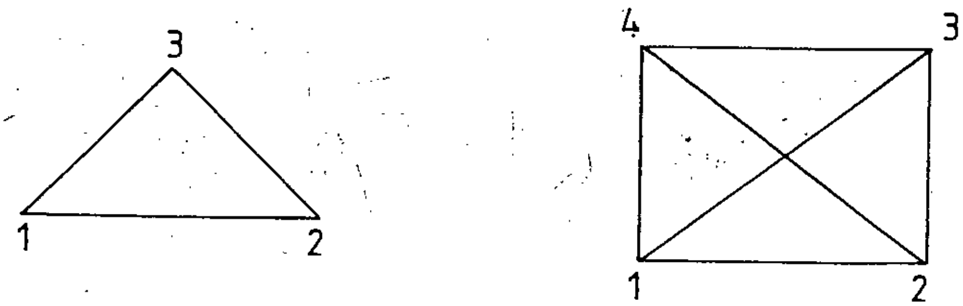


Fig. 3.3 Three and four exchange simple network.

In fig. 3.3, a three and four exchange simple network is shown 1-2, 1-3, 1-4, 2-3, 2-4, 3-4 are the final routes. These routes have no alternate routes.

Hierarchical network - (alternate routing) - is one in which at least some of the network groups are high usage i.e. the traffic which is not carried can be overflowed to other groups, at least some of which are finals, which have no alternate routes.

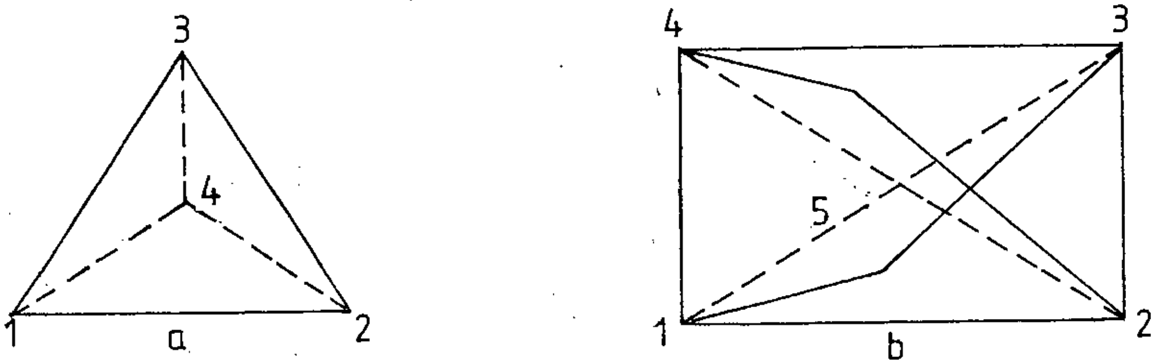


Fig. 3.4 Four and five exchange hierarchical network.

In fig. 3.4a, 1-2 is the high usage route and 1-4,4-2 are the final routes from 1 to 2. In fig. 3.4b, 5 exchange network 1-2, 1-3, 1-4, 2-3, 2-4, 3-4 are the high usage routes and 1-5, 5-2, 3-5, 4-5 routes are used in the final route.

Symmetrical (Alternate route) network - A symmetrical (alternate route) network has only high usage groups.

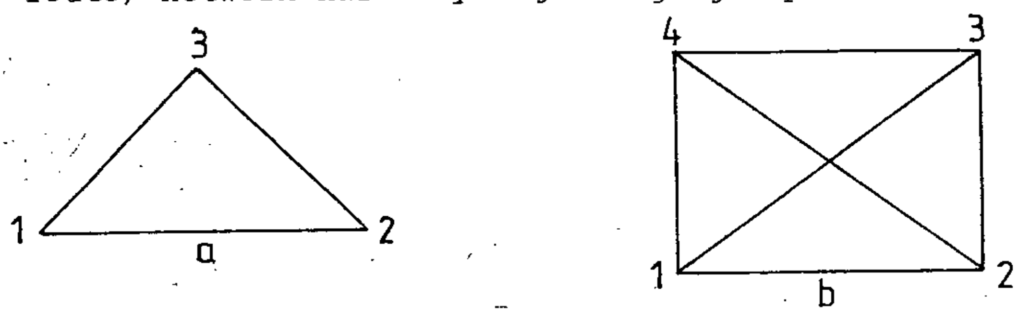


Fig. 3.5 Three and four exchange symmetrical network.

In this routing system high usage routes are used as direct routes and as well as alternate routes. In fig. 3.5a, 1-2 is the direct route (high usage route) and 1-3, 3-2 are the final routes from 1 to 2.

CHAPTER - 4

ANALYTICAL EVALUATION OF SMALL MULTIEXCHANGE NETWORKS.

4.1 Introduction:

This chapter includes the exact solution of some simple network system. In this respect, some steady state statistical equilibrium equations are derived. These equations form the mathematical model of the system. Some simple network systems are presented here, because the solution of a large system is complicated, computationally intractable, labourious and also need a lot of computational time. The advantage of the equations of state is that their basic assumptions are sufficiently realistic and that they provide a strictly mathematical tool which is adaptable to the physical conditions. Mathematical model represented by the equations of stationary stochastic process where the probabilities of the states are usually obtained as a solution of a linear equation system. In this chapter 3- and 4- exchange network analysis and their corresponding state diagram is shown.

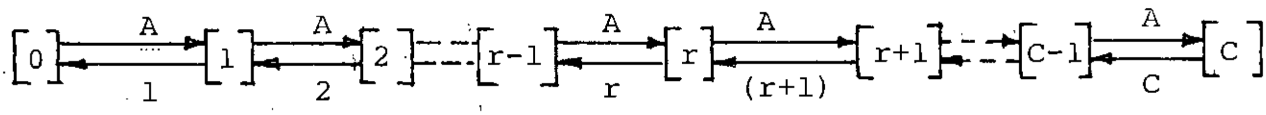
4.2 General Equation of State:

The arrival process is poissonian with arrival (or birth) rate A. There are C servers. The holding time is exponential. There is no waiting facility. If all servers are occupied, arrival demands are discarded.

When the numbers of busy server is r ($r = 0, 1, 2 \ldots\ldots c$), the system will be said to be in "State [r]". It is assumed that stationarity exists. This means that the probability of the system being in [r] is time- independent.

At an arrival (rate = A) the index r of the state of the system is increased by 1 as long as $r < c$. When the system is in [r] , the arrival rate remains A. The probability of state and rates of arrival and termination (collectively called transitions) may be summerised in the following scheme:

States:



Probability of States:

P_0 P_1 P_2 P_{r-1} P_r P_{r+1} P_{c-1} P_c

Now, stationarity require that the probability of formation of [r] in $\Delta\tau$ should equal the probability of a departure from [r] during $\Delta\tau$. The formation is possible from state [r-1] , which has probability P_{r-1}, by an arrival, which has the probability $A.\ \Delta\tau$. The joint probability of these events is $P_{r-1} .\ A\ \Delta\tau$. When $r < c$, another possibility is from

$[r+1]$ (probability P_{r+1}) by a death (conditional probability $(r+1)\ \Delta\tau$); joint probability $P_{r+1} \cdot (r+1)\ \Delta\tau$. When the system is in $[r]$, where $r<c$, both arrival and termination would mean departure from $[r]$; hence the probability of a departure from $[r]$ in $\Delta\tau$ is: $P_r\ (A+r)\Delta\tau$. So

$$P_{r-1}\ A\Delta\tau + P_{r+1}\ (r+1)\Delta\tau = P_r\ (A+r)\ \Delta\tau$$

or $AP_{r-1} - (A+r)\ P_r + (r+1)\ P_{r+1} = 0 \quad (\ r=0,1,\ldots\ldots\ c-1)$

... (4.1.1)

For the state $[c]$ since there is no transition from $[c]$ to $[c+1]$

then,

$$AP_{c-1} - CP_c = 0 \quad \ldots\ldots \quad (4.1.2)$$

The defined states from a complete set of mutually exclusive events, and hence,

$$\sum_{r=0}^{c} P_r = 1 \quad \ldots \quad (4.1.3)$$

The equations (4.1.1) & (4.1.2) state that the rate at which system assume the $[r]$ (or $[c]$) state with in the hypersystem equals the rate at which they leave that state. Thus they may be said to describe the state of statistical equilibrium, a notion from statistical mechanics.

4.3 The Assumptions Necessary for Equations of State:

In order that a mathematical process shall develope into equations of state, the following conditions regarding the nature of telephone traffic are necessary:[9]

1) The occupations in the selectors or of the connecting devices are generated by individual traffic sources . A single sources remains in an engaged state as long as it is occupied and cannot initiate a new call before this occupation ceases.

2) The individual sources are mutually indpendent. They generate calls independently of each other, and the holding times are independent of other occupation.

3) Only one event occurs at exactly the same instants of time.

A further assumption could be that the process is Markovian i.e. only dependent on the momentary state and independent of what has occured before the moment considered.

4.4 Analysis of Network Systems:

4.4.1 A Three Exchange Network System Analysis:

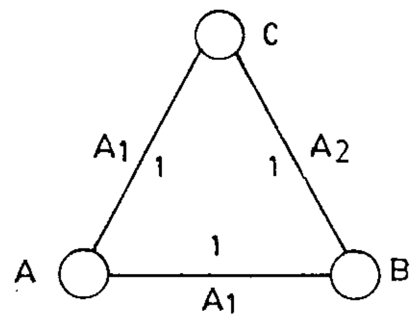


Fig. 4.1 A three exchange network.

A,B,C are three calling exchanges. Bothway traffic return between AB,BC and CA are considered A_1, A_2 and A_3 respectively. All routes are considered to have one link each.

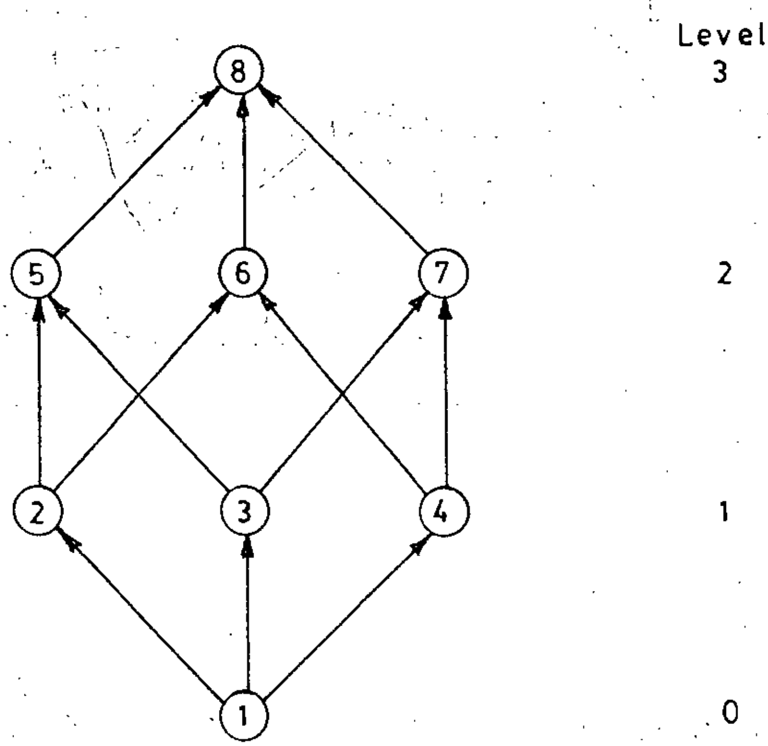


Fig. 4.2 Transition state diagram of a 3-exchange network.

A diagramatic representation is shown in Fig. 4.2. In this diagram the encircled numbers, represents the possible states of the system. Table-1 (appendix - 1A) gives the network states generated by the routing method. State equations developed for this system is shown in Appendix-1B.

Here, the total number of state is $2^3=8$

Over all call congestion B = P(5) + P(6) + P(7) + P(8)

4.4.2 A Four Exchange Network (with dedicated tandem exchange) system:

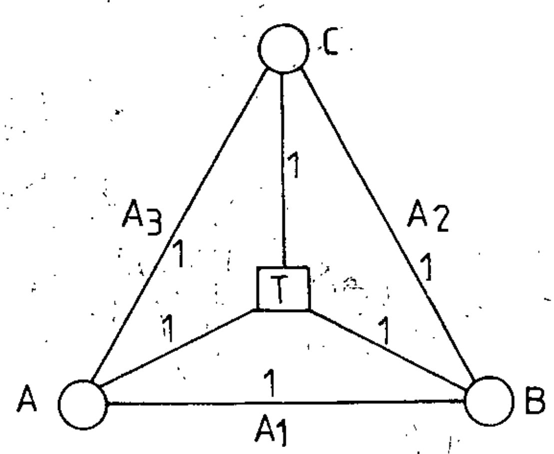


Fig. 4.3 A Four exchange network (with dedicated tandem exchange).

A four exchange network is considered, one of these T is considered dedicated tandem with in traffic of its own. Terminal exchanges A, B and C are connected with each other and with the tandem exchange T. All routes are considered

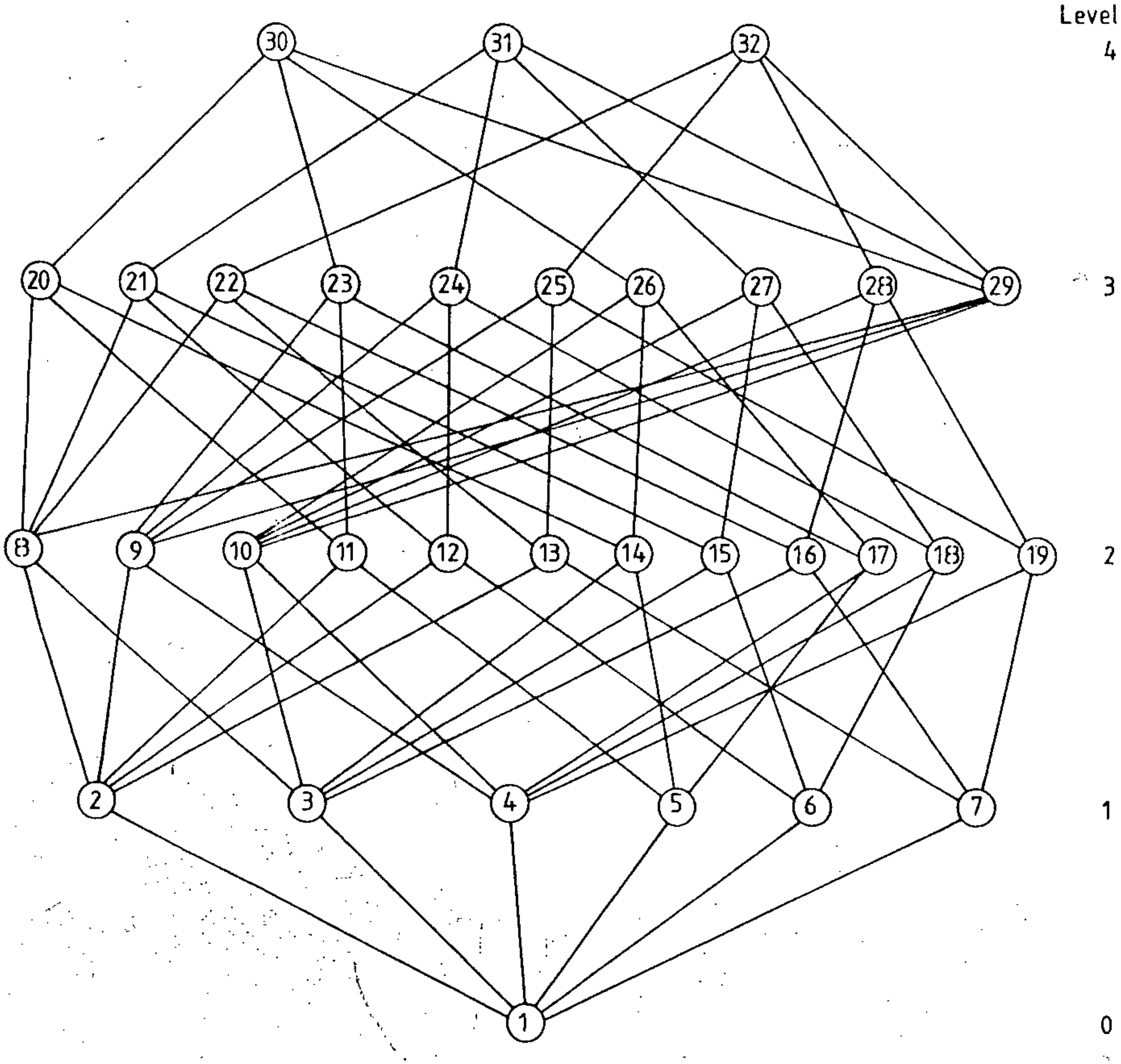


Fig. 4.4 Transition state diagram of four exchange network (with dedicated tandem exchange).

to have one link each. Both way traffic between AB,BC and CA are A_1, A_2 and A_3 respectively.

A diagramatic representation is shown in Fig. 4.4. Table-2A (shown in appendix-2A) gives the network states generated by the routing method. State equations are also shown in appendix -2B.

Over all congestion of the system

$$B = P(11) + P(15) + P(19) + P(20) + P(21) + P(22) + P(23) + P(24) + P(25) + P(26) + P(27) + P(28) + P(30) + P(31) + P(32)$$

4.4.3 A Four Exchange Network System:

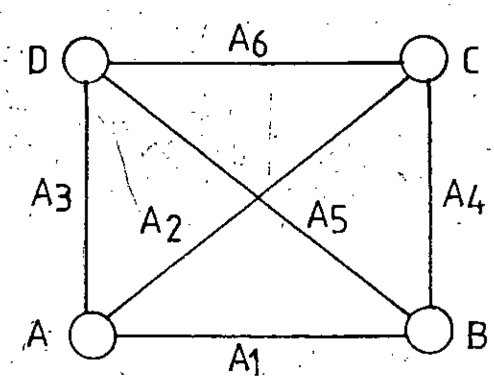


Fig. 4.5 A Four exchange (simple) network.

A,B,C,D four exchanges are considered. AB,AC,AD,BC,BD, CD are the routes and the traffic in these routes are A_1, A_2,A_3, A_4, A_5, A_6. All these routes are considered to have one link each.

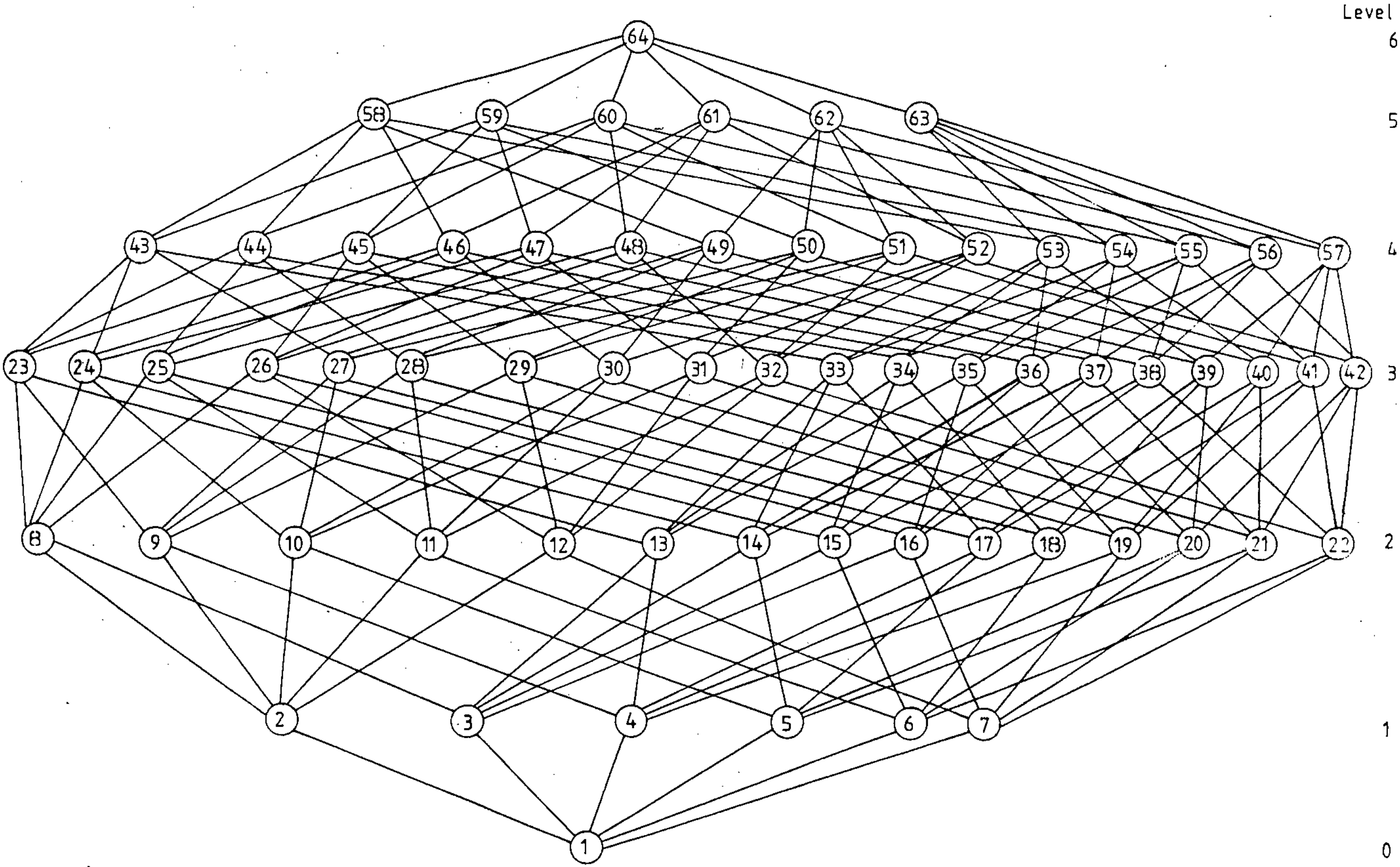


Fig. 4.6 Transition state diagram of four exchange network. (simple network).

Diagramatic representation of the states generated by the system is shown in Fig. 4.6. Table-3 (shown in appendix-3A) gives the network states. State equations are derived for this network are shown in appendix -3B.

Overall congestion B= P(23) + P(30) + P(37)

+ P(41) + P(43) + P(44) + P(45) + P(52)

+ P(56) +P(57) +P(58) + P(59) + P(60)

+ P(61) + P(62) + P(63) + P(64).

CHAPTER - 5
SIMULATION

5.1 Introduction:

Statistical modelling and analytical solutions are possible for small systems. But for a complicated practical-system, such theoretical analysis is impossible. Analytic models are more rough, account for lesser number of variables and always call for some allowances and simplifications. Statistical models are bulky, poorly analyzable, need a lot of computational time. To overcome these disadvantages, simulation technique is adopted to solve problems of practical interest. Simulation is a powerful tool for designing, dimensioning and predicting the behaviour of a telephone traffic system.

A simulation program is developed on the assumption of mathematical model of a telephone network system. The program is tested with data from small and simple network systems. Results obtained from simulation program are compared with analytic results. Test results show good agreement with each other. Then various network systems are studied with simulation program.

5.2 Earlier Works on Simulation in Traffic Study:

C 40

The earliest known work on artificial traffic was due to Fraser and Elliman[32] who convinced the idea of artificial traffic machine in production, distribution and analysis of telephone traffic. In Bell laboratory, traffic load capacity of No.1. Crossbar network was determined by simulation of link and junctor patterns[33]. Broadhurst and Harmston[34] published a paper on traffic to work on roulette principles using noise voltages of neon tube as the the source of random numbers. Neovious[35], introduced a method of computer generation of Pseudorandom numbers to represent events in a process of simulation. Wallastrom[36] worked on artificial traffic trials in a two stage link system to see the influence of the number of columns and routes and the method of hunting for a path. Dietrich and Wagner gave general outline for simulating a telephone connection network with realistic considerations for the input processes. Grantges introduced a general purpose simulation language- NEASIM based on less mathematical model of a network system. This includes a number of realistic considerations for improving the accuracy of simulation results. Boess study on the comparison of routing rules in switching networks, is a major contribution towards telephone traffic research by simulation technique. Besides these, valuable contributions are due to Kosten, Mozoz and Las, Nivertand Shandtz and Kosten in the use simulation techniques for studying telephone network performance.

5.3 Development of Simulation Technique:

There are three main tasks to be performed, as shown in fig. 5.1. The first, is to generate a model and initialize it.

From the description of the system, a set of numbers must be created to represent the state of the system. This set of numbers will be called the'system image', since its purpose is to reflect the state of the system at all times. The activities of the system must be represented as routines that create the discrete events making the change of the system image. The second task is to program the procedure that execute the cycle of action involved in carrying out the simulation. The third task, is the generation of an output report. The statistics, gathered during the simulation will usually be organised by a report generator.

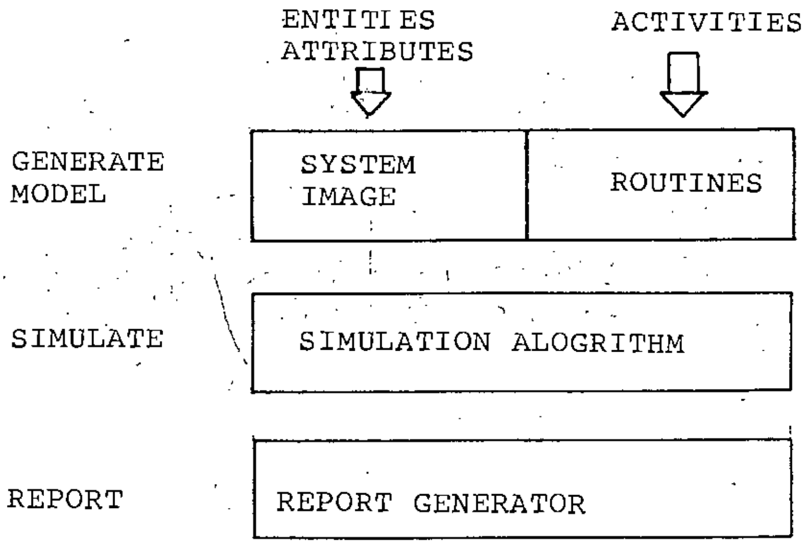


Fig. 5.1 Simulation Prog. tasks.

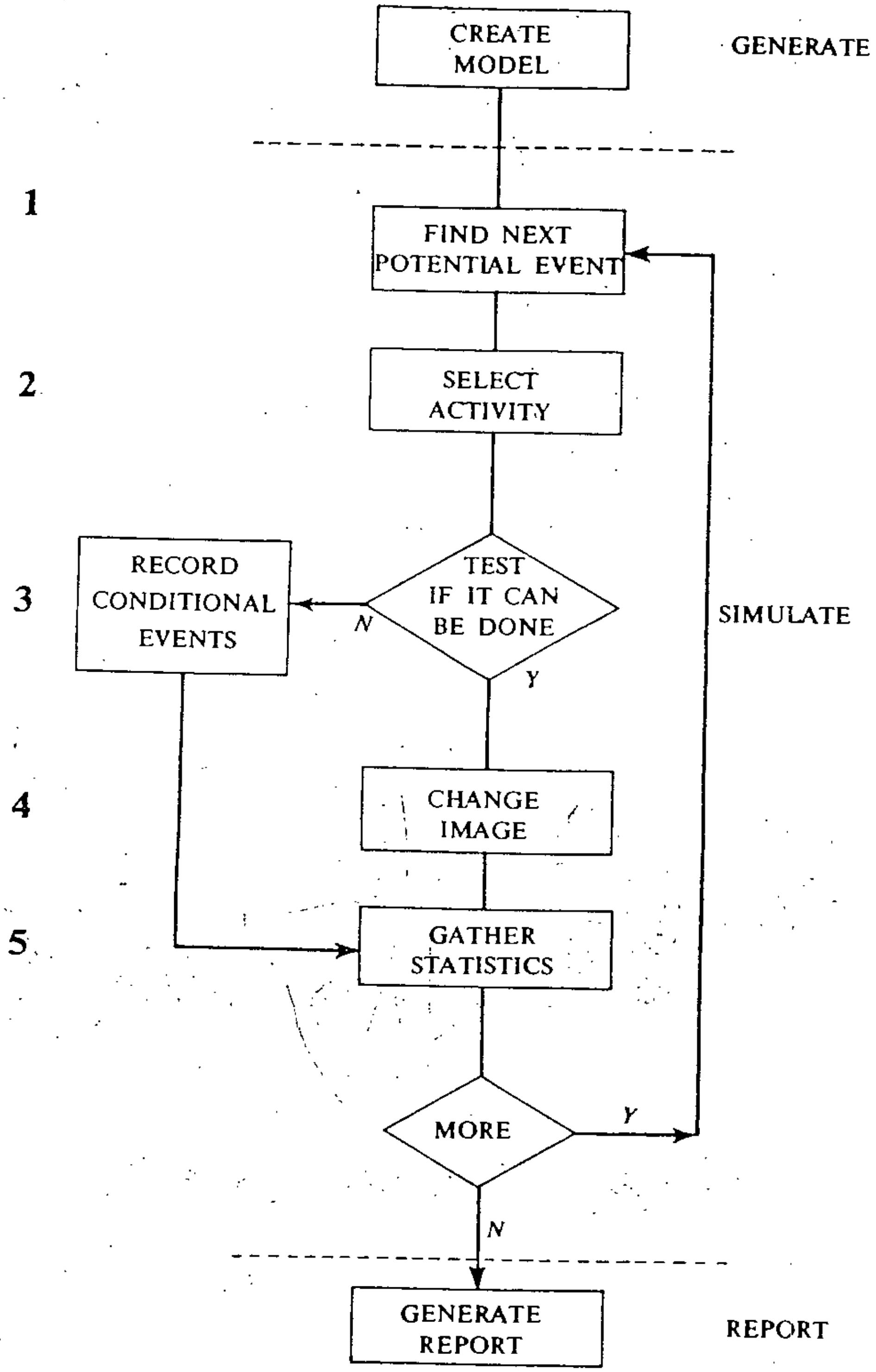


Fig. 5.2 Execution of a simulation alogrithm.

The general flow of control during the execution of a simulation program is illustrated in Fig. 5.2 shown at the previous figure is the task which is usually executed once at the end of the simulation, although it is not unusual to print-out intermediate results as the simulation proceeds. Carrying out the simulation alogrithm involves repeated execution of five steps described previously.

These steps are-

1. Find the next potential event.
2. Select an activity.
3. Test if the event can be executed.
4. Change the system image.
5. Gather statistics.

5.4 Classification of Simulation Languages:

Depending on the physical process involved in a system, varities of simulation programmes are possible. A few names considered to be useful for all catagories of works are General purpose system simulation (GPSS) by IBM, SIMSCRIPT and DYNAMO. Because of their general nature, they are large in size and costly for special purpose use. Elliat simulation package(ESP) is a special purpose language suitable for telephone systems simulation. It is a discrete system simulation language.

Fig. 5.3 gives a classification of presently available simulation programs.[14]

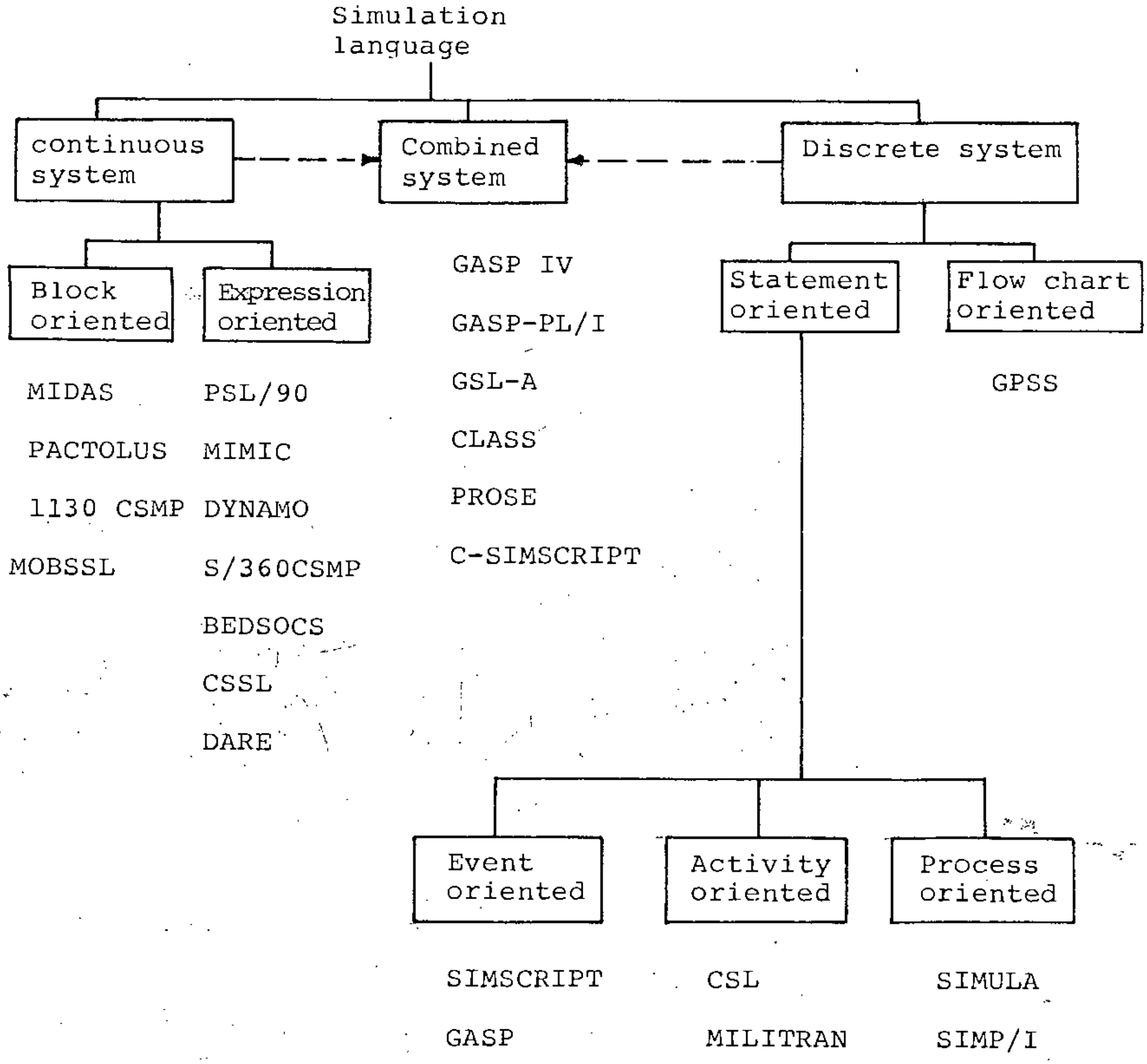


Fig. 5.3. Classification of Simulation Language.

5.5 Comparision of Simulation and Analytical Methods:

The main drawback of simulation is apparent, it gives specific solution rather than general solution. The mathematical solution is obviously preferable, particularly when the solution being sought is some maximizing conditions. A single mathematical solution might give such a condition where as many simulation runs may be needed to find a maximum, and yet still have undecided question of whether it is a local or global maximum.

In approaching a system study, it is essential to consider first what mathematical techniques might be applied to derive analytical solutions. It is a matter of judgement to decide whether the degree of abstraction required to apply analytical methods is too severe. To make the judgement it is necessary to consider carefully what questions need to be answered in the system study, and to what degree of accuracy the answers need to be known.

When the decision is made to simulate in order to use a more realistic model, it is still important to limit the amount of details in the model to the minimum level necessary. The step-by-step nature of simulation technique means that the amount of computation increases very rapidly as the amount of detail increases. Coupled with the need to make many runs to explore the range of conditions. The extra realism of simulation models can result in a very extensive amount of computing.

In many ways, the ideal way of using simulation is an extension of mathematical solutions that might have been obtained at the cost of too much simplifications. There are many simple limitations on system, such as physical stops, finite time delays, or non-linear forces, which render what would otherwise be a soluble mathematical model insoluble. Simulation easing removes these limitations, and it can then provide a powerful extension of known mathematical solution.

Even when a model with a known solution is considered adequate, there are still times when a simulation will provide a quicker or more convenient way of deriving results. Many analytical results occur in the form of complex series or integrals that still require extensive evaluation. It is often more convenient to use simulation to obtain results directly from a model with specific values rather than to perform the numerical evaluation of analytical solution.

5.6 Types of Simulation:

The type of a simulation method is determined by the type of the system is being simulated. There are two types of simulation

- (a) continuous system simulation.
- (b) discrete event simulation.

A continuous static system is modeled by linear or non-linear algebric equations with continuous variables and continuous dynamic system modeled by differential equations of the same variables.

In discrete event simulation, events are generated at instants of time decided by the process. A deterministic process follow deterministic rules to decide the type and the time of event; a process with a degree of stochastic elements in it, generated the types and instants of events according to accepted probability distributions.

Call arrival and call release in a telephone connection occur as discrete events. Practical experience show that the distribution of both interarrival and call holding times most commonly follow negative exponential distributions.

5.6.1 Discrete System Simulation

In discrete system simulation a set of numbers are set to represent the state of the system. These numbers are called state descriptors. Some of the state descriptors may range over values that have physical significance and other represents conditions or flags. A discrete event is a set of circumstances that cause an instantaneous change in one or more system state descriptors. Depending on the way the variable time is considered, a discrete system simulation may be classified as

- (a) - Time-True Simulation.
- (b) - Roulette model simulation.

The general concept of Time-True and Roulette Simulation can be understood with the help of the following example[10]. There are two groups of sources (A and B), producing poisson flows of demands with rates λ_a and λ_b, respectively. There are n servers. Demands from group B are allowed to seize any free servers. Demands from group B are allowed to seize any free server; those demands are lost for which no free server is available on their arrival. Demands stemming from group A cannot seize a server, unless they leave at least m (>0) servers unoccupied. Once the processing of a group A demand begins, however, this action is completed. Contrary to group B demands the demands of group A have queueing facility. One can think of the following interpretation. Let the demands of group A and B patients to be taken into a hospital. Group B are emergency cases that may "seize" any free bed, in the case where no bed is available the patient goes to another hospital (i.e. lost for the hospital under consideration). Group A are patients on a waiting list (i.e. in a queue). Their admission to hospital is made possible only in the case where m beds are still left free for emergency cases (n=3, m=1). The model is completed by stating that the c.d.f's of holding times for group A and B are $F_a(t)$ and $F_b(t)$, respectively. It is required to determine the fraction of group B demands that is lost and the average waiting time for group A demands. Needless to say this is a gross oversimplification of the situation at hand, but it is a good example.

5.6.1a) Time-True Simulation

The diagram of fig. 5.4 consists of lapses of time (holding times and inter-arrival times) that are"tied together" instants, to be called events. The events here are arrivals and ends of occupations. The state of the system specifies which of the servers are busy at a certain amount as well as the number of iteams in the queue. The state can changes at events only. The way in which the state changes at events is determined by (i) the type of event and (ii) the configuration of the facilities (servers and queues). Hence, the only stochastic elements are the holding and interarrival times. Now, in time-true simulation their durations will be determined at their begining by sampling from the prescribed distributions. The process of reconstruction develops according to a fictitious time, called CLOCK (Fig. 5.4). It seperates the PAST from the future. At any CLOCK-time the state is supposed to be known as well as the endpoints of interarrival and holding time that at CLOCK have not yet completely ended (together called "current intervals" denoted by thick segments). At the begining

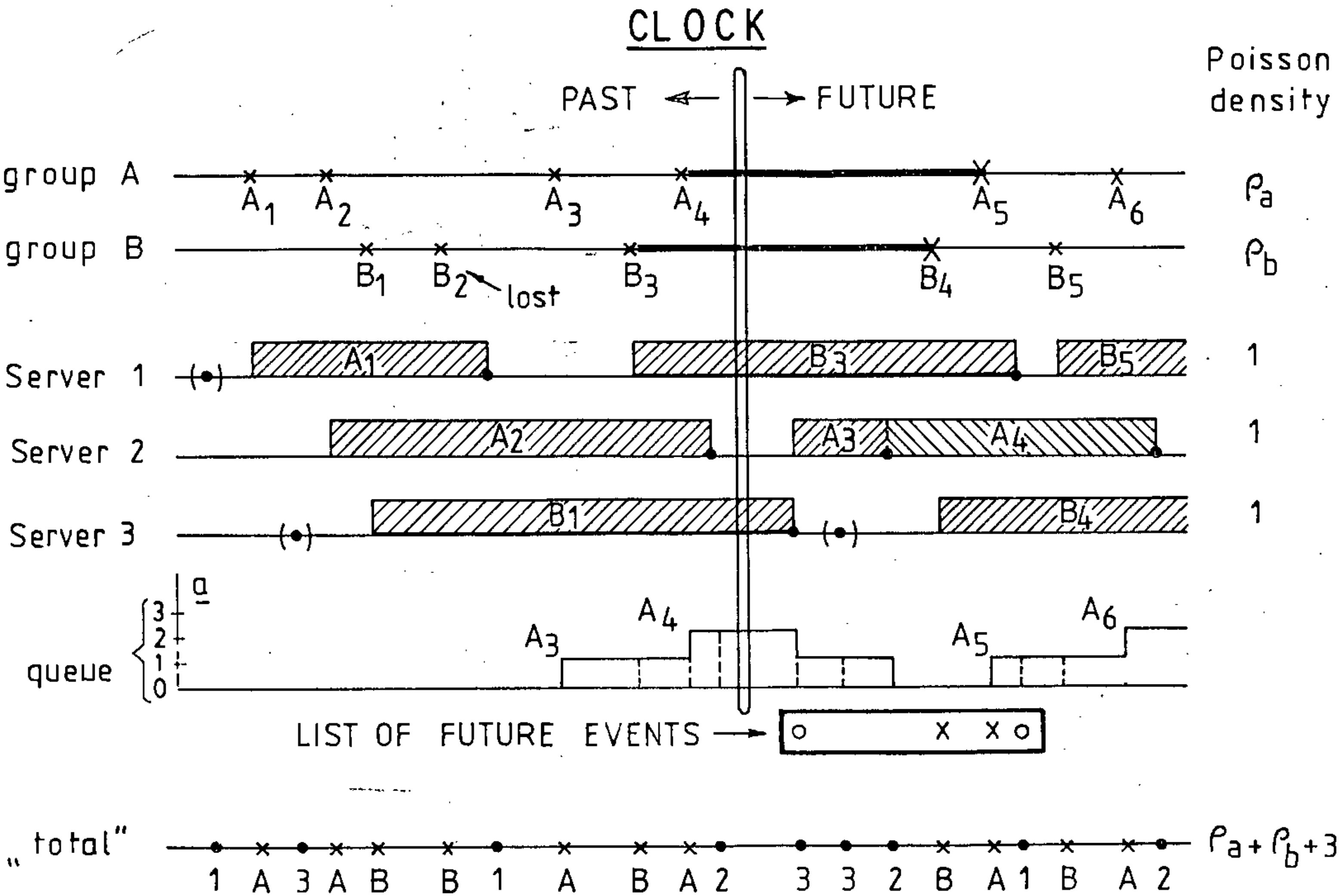


Fig. 5.4 Realization of the process of arrivals and occupations.

(CLOCK-time zero) this is satisfied, for example, by an empty system and known first arrivals in both groups. Nothing happens until clock meets the earliest of there end points, the next event. The type of this event, the state and the configuration then determine the change of the state, if any, at this events. Any such a change may entail the start of one or more new current intervals, the length of which are immediately determined by sampling. The end of those new current intervals mark possible new events. Hence, every event that is passed by clock, may generate new future events. Mostly, a chronological LIST OF FUTURE EVENTS is constructed. In time-true simulation one can count the demands in group B as well as those that were lost. A simple division of those totals yields an estimate of the probability of loss in group B. The denominator (the total of all demands) can be replaced by its (known) expected value. As all durations between consecutive events are also known, it is relatively easy to keep a record of the total waiting-time of group A, demands during a simulation run (the area under the q-line in Fig. 5.4). Dividing this total by the total of A demands (either observed or expected) yields an estimate for the expected waiting-time or group A demands. From waiting-times of individual demands-which can be obtained with slightly more effort- a histogram of waiting-times can constructed. This then offers an approximation to the distribution of waiting-times.

The time-true simulation has three disadvantages: (i) the implementation of the sampling from the given distributions may be difficult and time-consuming; (ii) when the system is large, much information about holding and interarrival-times (the current intervals) should be memorized; (iii) when the system is large, the filing of the future events on their chronological list may be cumbersome or time consuming.

5.6.1b) Roulette Simulation: does not posses, the disadvantages mentioned in the case of time-true simulation. In Fig. 5.4 the arriving demands in group A and B are given by poisson point - processes (the crosses). Now let the holding- times be exponentially distributed with average 1. Consequently, the arrival rates will be given in normal notation ρ_a and ρ_b: respectively. Not, only the interarrival- times, but also the holding - times, will now posses the property of forgetfulness (appendix -4B). A statistically true picture of holding- times may be obtained by " chopping them off" by unit density poisson point- processes. The dots on server- occupation lines in Fig. 5.4 when such breakdown points occur at instances at which the associated server is not occupied (dots between brackets) nothing happens. The five poisson point- processes (two arrival process and three break down processes), may be merged into one poisson point- processes, called "total". Its density is $\rho_a + \rho_b + 3$. In each of the point (A,B, 1,2,or 3) may be obtained by drawing lots with probabilities proportional to ρ_a: ρ_b: 1: 1: 1, respectively. Let, it be considered that the class indices of the consecutive points of the "total" process (mark A,B,1,2, or 3 on the "total line) is known. It is evident that one is able then to construct the realization of the complete stochastic process in the system as far as the sequence of events concerned: the concept "time" disappears. What remain is a so called sequence- true resume (Roulette simulation) of the realization. The class indices in question can be determined by the use of a roulette with positions marked A,B,1,2 and 3 in the correct proportion ρ_a: ρ_b: 1: 1: 1. In a general, case the roulette should posses the following sets of differently marked equiprobable positions: i) break down positions: one set per server; the number of the positions of these sets are equal; (ii) build-up positions: one set per group of sources; when ρ_i is the arrival rate of such a group, the corresponding set of build-up positions as the sets in (i). When the roulette stops at a break down position associated with a non occupied server this roulette point simply is ineffective. The roulette simulation ceases to be applicable when the concept 'time' enters as an explicit variable. It is impossible, for example to deal with the probability of a delay in excess of a certain prescribed duration. Further more, cases where the system behavior depends on some holding or waiting time exceeding some value and not covered by the roulette simulation principle. The roulette model does not need a list of future events, so there are no filing difficulties. There is no need for sampling from arbitrary distributions, simple (pseudo) random numbers are sufficient.

In this work, Roulette simulation run has been applied to find the various parameters of the network systems.

5.7 Random Event Generation:

In a statistical process such as the flow of telephone traffic, many individual events (e.g. arrival of a new call or the termination of an existing call) occur in a random fashion. If such a process is to be simulated, then it is necessary to have some means of generating random events. In digital simulation this means a source of random numbers, which are drawn one by one and used to determine various random events occuring in the simulated process. To satisfy the requirement of randomness the successive numbers drawn must be uncorrelated and uniformly distributed over the range of interest. By a continuous uniform distribution we mean that the probability of a variable x falling in any interval within a certain range of values is proportional to the ratio of the interval size to the total range; that is, every number in the range is equally likely to be chosen. Suppose, the possible range of the value is the interval (a,b) then the probability that x will fall in an interval of length Δx is $\Delta x/_{(b-a)}$.

For our purposes we can assume the interval is between the numbers 0 and 1, since a number (y) in any other range can be obtained from x in (0,1) using the following formula:

$$y = (b-a)\ x+a$$

In computer applications we must be careful when applying this result, since a floating point number can only be represented by a certain number of digits and thus if $(b-a) > 1$, then this may increase the "granualarity" of the random numbers used, and hence we may not be able to regard these numbers as being "continuous".

In situations where we require random numbers for a small applications, a good source is the telephone directory. If subscribers with more than one exchange line are excluded because of possible correlation between their numbers, the last two or three digits of the telephone numbers of consecutive entries are randomly distributed and can be used for simulation. An alternative method for obtaining random numbers uses tables which have been published for statistical sampling and simulation applications. However, the most widely used method for obtaining random number is to use a mathematical alogrithm to generate so called, "Pseudo-random" numbers, we shall deal with this method in the next section.

5.8 Computer Generation of Random Numbers

While it would be possible to input a predetermined series of random numbers into a computer as data, it is more efficient to program the computer to generate the numbers mathematically. Since the result of any mathematical process is in principle completely, predictable, number generated in this way are known as, 'Pseudo-random' numbers. For practical purposes, however, they are indistinguishable from those produced by a physical random process, provided, they satisfy the usual statistical tests for randomness.

The methods are used to generate pseudo-random numbers are "congruence" or "residue" method. In this method we start with an initial number (N) called a seed, and derive a sequence of numbers, each of which can be used to give numbers between zero and one, this is achieved in the following way:

Given three constants a,b,c number from the ith number by multiplying by a, adding b and taking the remainder modulo 2^c.

$$N_{i+1} = aN_i + b(\text{mod}2^c)$$

Where a and b are carefully chosen parameters and c is the number of bits in the computer word. To obtain a random number in the range (0,1) we simply divide N_i by 2^c. It will be appearent that is impossible to produce a non-repeating sequence of numbers from this procedure; however, careful selection of the constants results in a very long sequence before such a repetition occurs, so that for practical purposes the sequence can be said to produce random numbers.

The general formula is known as a congruential random number generator. There are two special cases:

- If a=1, and b≠0, then the method is said to be additive.
- If a≠0 and b=0, then the method is said to be multiplicative.

In all other cases the method is said to be mixed. Research has suggested that the multiplicative method is superior to the additive method and that the mixed methods are not noticeably better than simple multiplicated method.

5.9 Simulation Program Used in this Work

The simulation program is a mathematical logical description of the telephone network system. In the present work, a program is developed on the basis of roulette-model simulation concept. 'Call setup' and'Call release' are the events in the system. At a particular instant, an event happens with a probability value sampled out of an accepted probability distribution of events. Inlets, outlets and links associated with a call occur at random. Similarly release of a call occur at random. To implement random choice of these, a pseudo-random number generator is used. The program consists of a main program and a number of subroutines. The main program performs the operation of the system. The alogrithmic development of the program is described as follows.

Network Configuration

Network configuration is presented in the 'SUBROUTINE NETWORK'. The structure of the network, that is, the number of nodes, links between nodes, overflow node between two nodes, call intensity of each link are stated in this subroutine program. In this sub-program-

NEX- represents the total number of exchanges

NCEX- total number of calling exchanges.

NTEX- Number of tandem exchanges.

NLF(NN) - Number of direct routes between two exchanges.

NOF(I)- Number of overflow node between two exchanges.

LP(NN)- Link path between two exchanges.

A(NN)- Traffic intensity of each link.

Different network systems are studied with different values of the above mentioned descriptors. In this work, the maximum of exchanges were considered five(5), number of calling exchanges four(4), number of links ninety(90), and the traffic intensity from 0.5 to 0.8 erlang.

Initialization of the Program

The program needs initialization of its counters and array elements that store the state descriptors. Before starting operation of a system, a system is in empty state, so initialization is necessary. 'SUBROUTINE INITAL' describes the process of initialization in this present work. The following descriptors are to be initialized-

NOCC - Number of occupied calls.

NCALLS - Number of calls carried by the system.

NOFLO - Total number of calls offered to the system.

NCDR - Number of calls offered to the direct route.

NFLOI - Number of calls offered to the first-overflow node.

NLOST - Number of calls lost.

NFREED - Number of calls freed.

Call Setup Process

Call setup process is a random process. The setup process depends on the state and configuration of the system. The call setup process is described below:

In this process an inlet is chosen at random and tested to see whether it can accept a call. If at least one of the links from the inlet is free, then an outlet is chosen at random from the rest of the nodes otherwise the call is considered lost. If the chosen outlet does not have any of the free links, then the call is also considered lost. If the call is accepted by the inlet and outlet node, then the direct link LL = LNKG (Inlet, outlet), joining the inlet and outlet node is looked after. The link is selected according to network configuration (example shown in fig. 5.5) in the 'SUBROUTINE NETWRK',

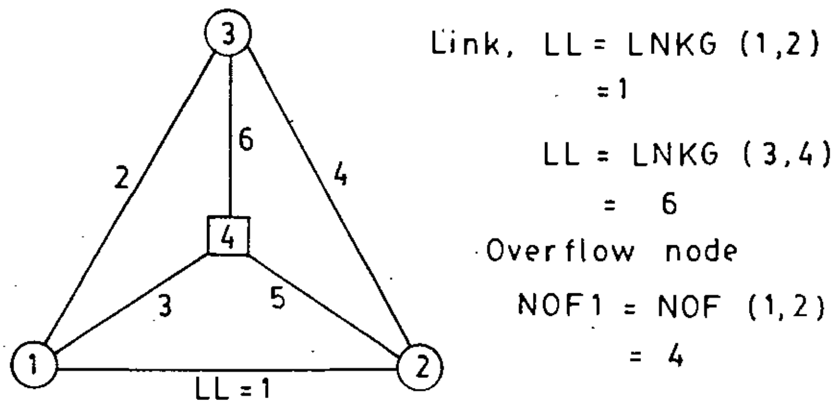


Fig. 5.5 A Four exchange network (with Tandem).

and the call is offered to the direct route (NCDR = NCDR + 1). For direct routing network system, if the link in the direct route is free, then the call will be setup in this route. The number of the occupied calls (NOCC) and the number of calls carried by the system are increased by 1. (NOCC = NOCC + 1, NCALLS = NCALLS + 1),. On the otherhand, if any of the links in the direct NLF(LL) is not free then the call will be lost. In case of overflow system, if direct route is busy, (NLE(LL) = 0) then the call will be routed to the first overflow node. NOF1 = NOF (Inlet, Outlet) according to network configuration. Then the link joining the inlet and overflow node L_1 = LNKG (inlet, NOF1) and the link joining the overflow node and outlet L_2 = LNKG (NOF1, outlet) is selected as network configuration. (In fig. 5.5 L_1 = LNKG (1,4) = 3 and L_2 = LNKG (2,4)= 5). If any of these links L_1 and L_2, is busy then the call will be lost. Otherwise, the call will be carried by the system and the number of occupied calls will be increased by 1. For each call setup, the associated inlet and outlet node and the links are recorded.

Call release process

In call release an existing call is chosen at random, then the inlet and outlet nodes associated with the call are selected. The link is to be freed, LL = LNKG (Inlet, Outlet), as stated in the 'SUBROUTINE NETWRK'. Then one of the links from inlet and one from outlet are freed. Link associated with the call is thus freed (NLF(LG) = NLF(LG) + 1). If the call is in the overflow route, then it is freed in the same process. Number of occupied calls is then reduced by 1. (NOCC = NOCC - 1, NFREED = NFREED + 1).

Sampling and sampled data

Sampling is necessary in simulation work to perform accurate result. In this work, 20 samples and 1000 events per sample

(reasonable for sampling) are considered. Total 20,000 events are generated in each operation. Sampled data includes the following parameters-

NOFLO - Number of offered calls.

NCDR - Number of calls offered to the direct route.

NFLO1 - Number of calls offered to the first overflow route.

NCALLS - Number of calls carried by the system.

NLOST - Number of calls lost by the system.

NFREED - Number of calls freed.

N1 - Number of calls carried by the over flow route.

5.10 Accuracy of simulation

Accuracy is needed in simulation work. The accuracy of the simulation work depends on the following factors mainly.

Consideration of the warm up period

The system starts in empty state. To attain statistical equilibrium, the system should be provided with some calls in progress, before measurements are started.[19] This work considers a warm up period of 8 times of minimum of total traffic or total number of branches in the system.

Random number generator

A suitable random number generator is used which generates numbers between 0-1. In the present a subroutine program is used to generate non-repeating uniform random number between 0 and 1.

Choice of confidence interval

Instead of representing by Exact values, simulation results are better expressed as sample means with confidence intervals calculated at a certain level of signifance. Details of confidence interval calculation shown in appendix- 6A. . In the present program, 'SUBROUTINE CONFID' states the process of confidence interval calculation.

CHAPTER 6

RESULTS AND DISCUSSIONS

6.1 Graphical Representation of Results.

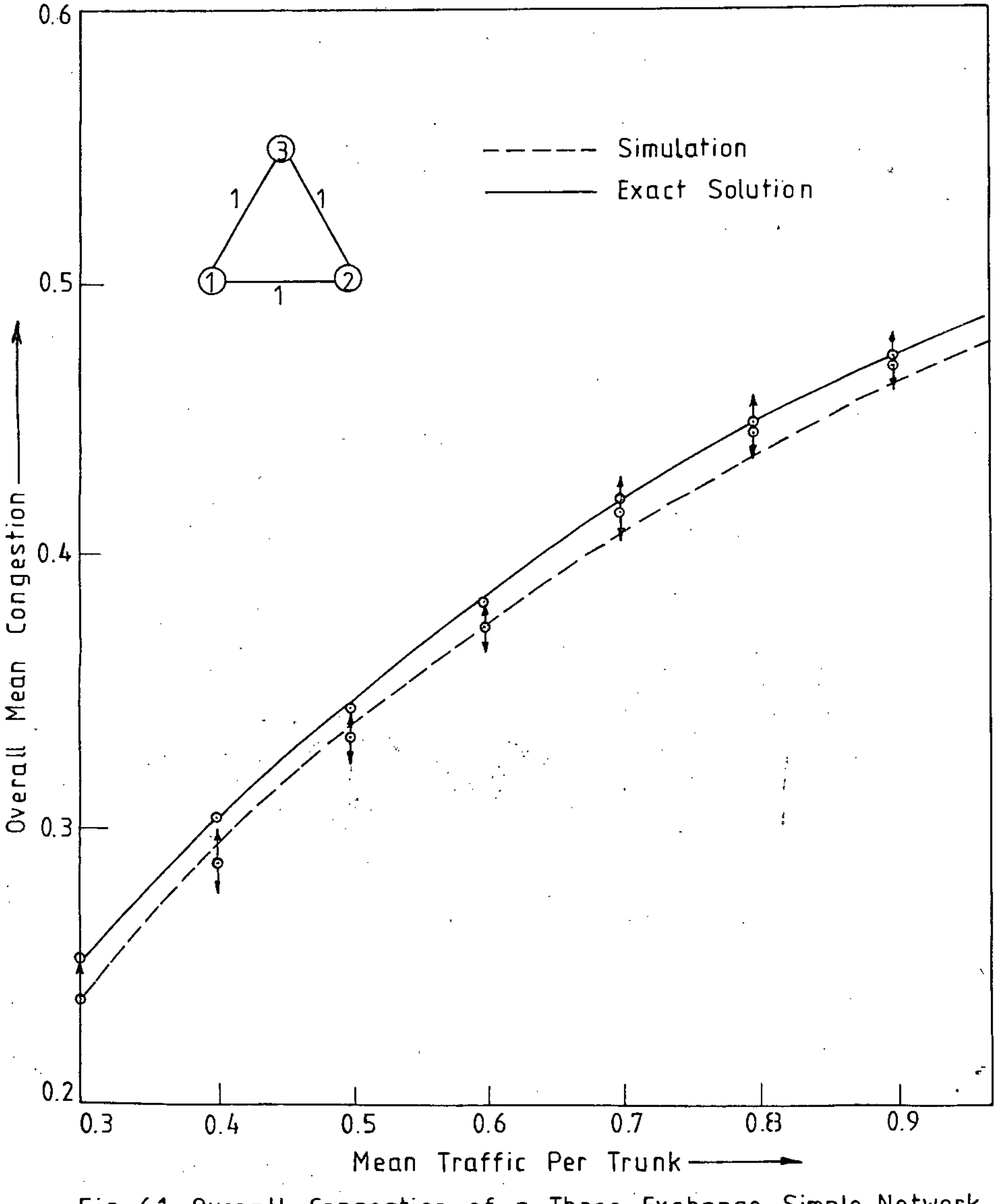


Fig. 6.1 Overall Congestion of a Three Exchange Simple Network by 'Exact Solution' and 'Simulation'.

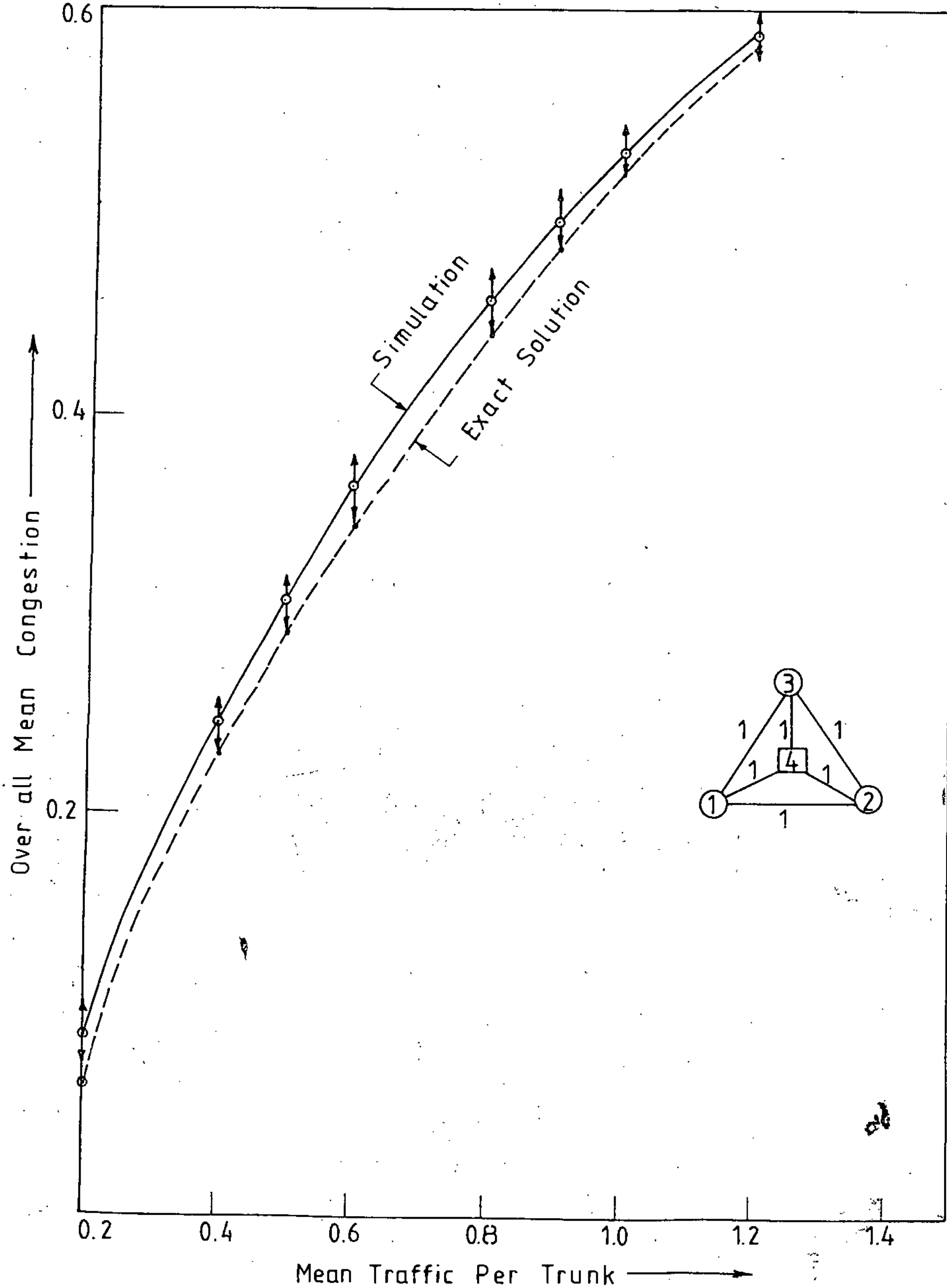


Fig. 6.2 Over all Congestion of a Four Exchange (with Tandem) Hierarchical Network by 'Exact Solution' and 'Simulation'.

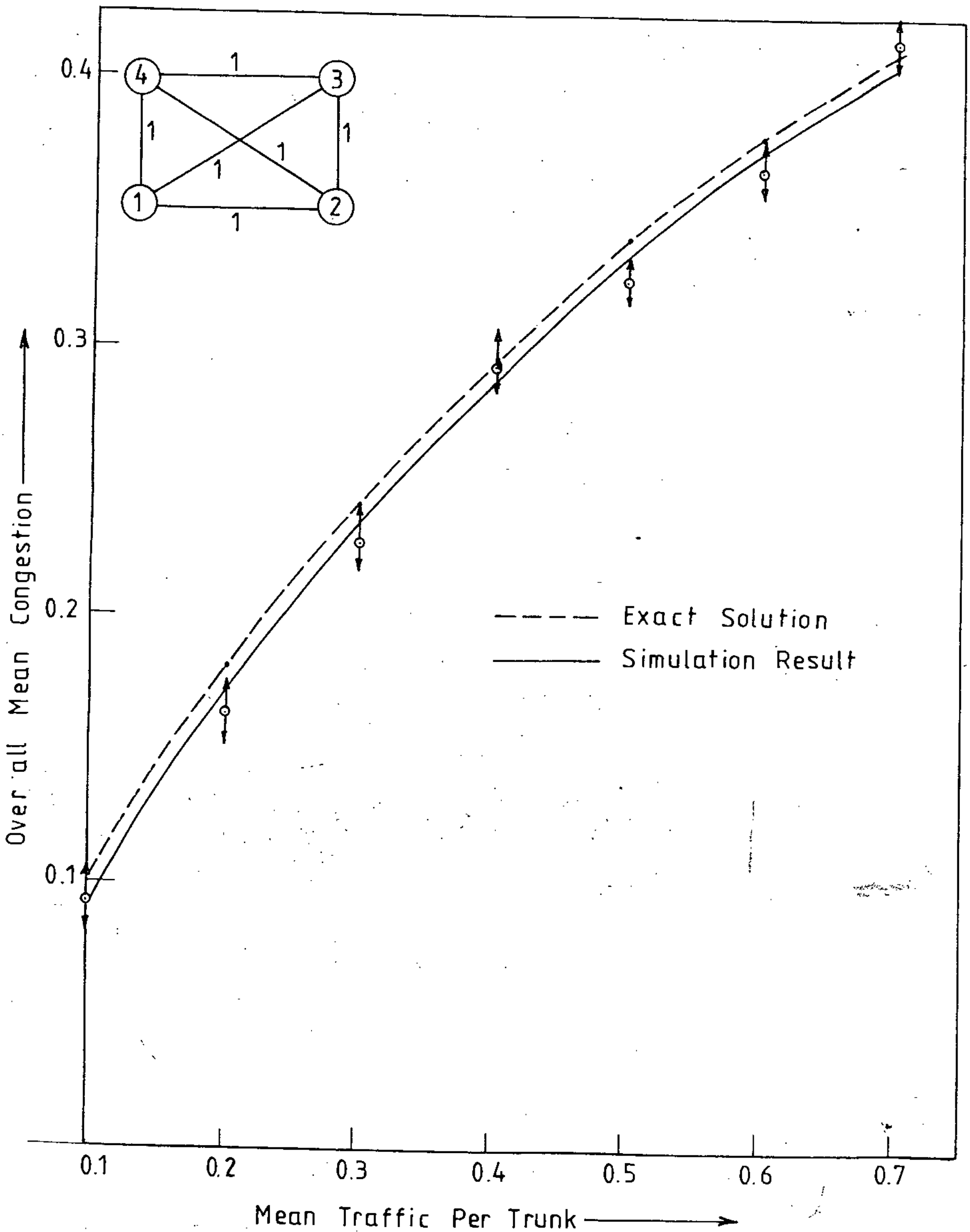


Fig. 6.3 Overall Congestion of Four Exchange Simple Network by 'Exact Solution' and 'Simulation'.

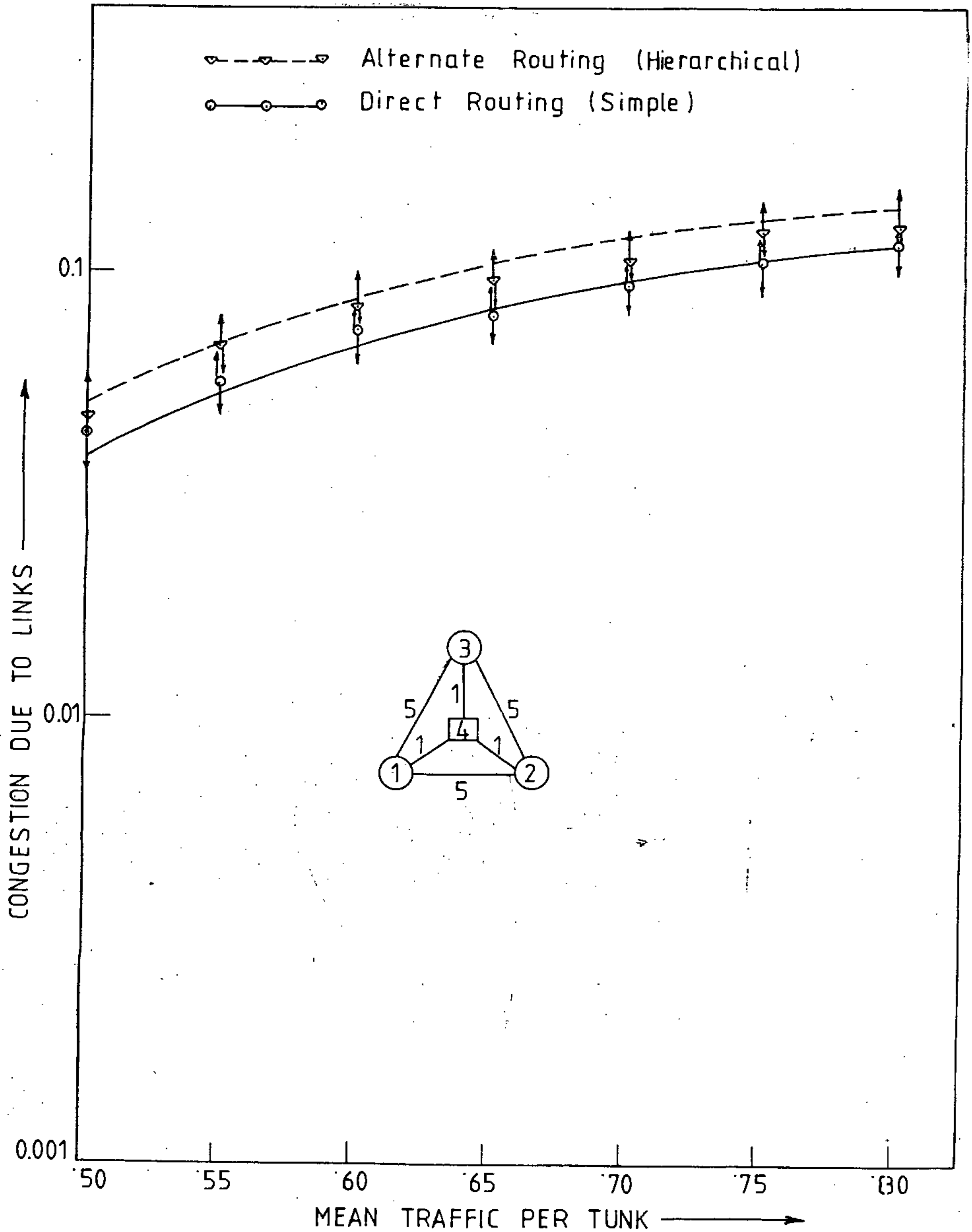


Fig. 6.4 Congestion due to links Vs. Traffic offered to the system for a four Exchange Alternate routing (Hierarchical) and Direct routing Network. (Total no. of links = 18)

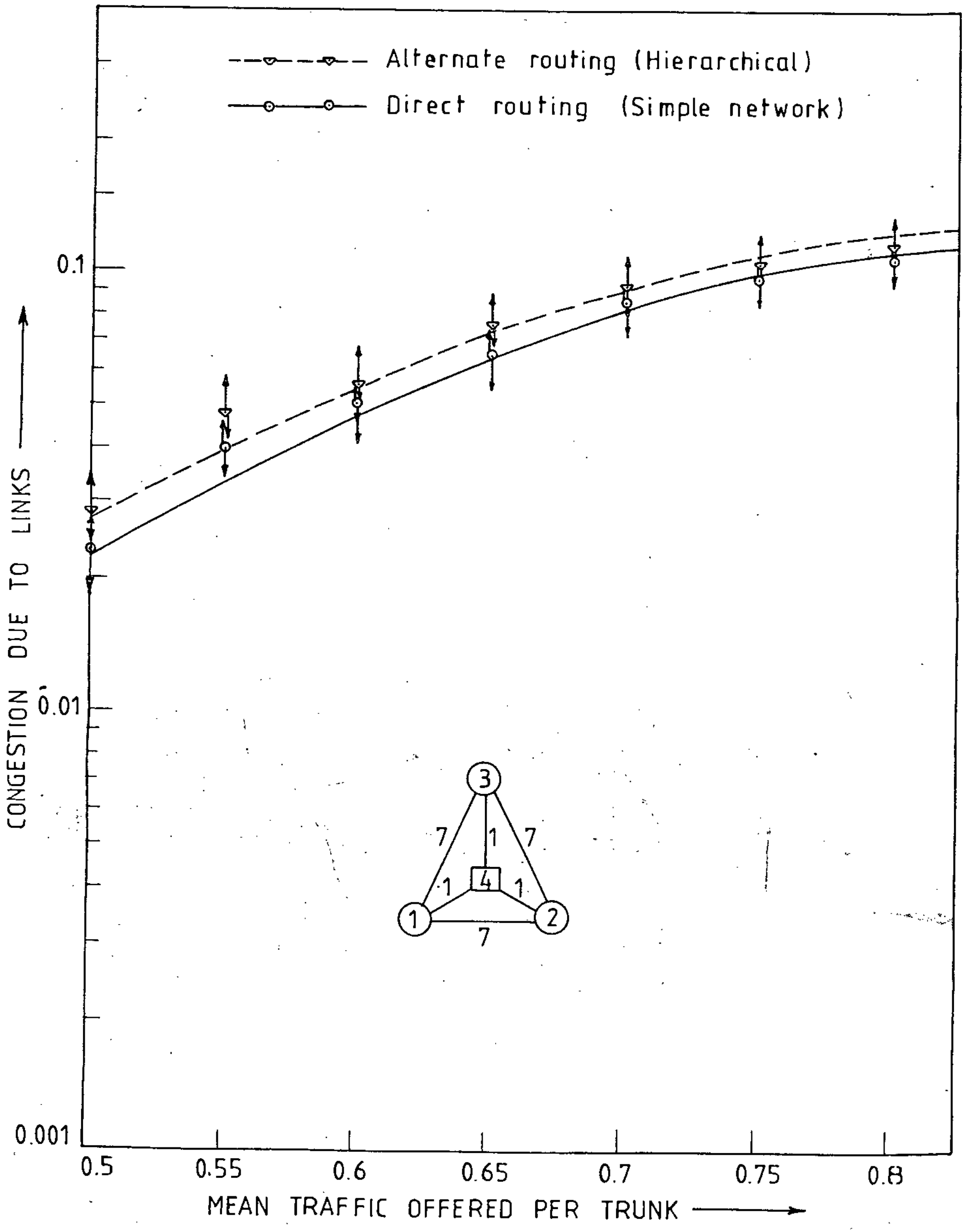


Fig. 6.5 Congestion due to links vs. traffic offered to the system for a four exchange alternate routing (hierarchical) and direct routing networks. (Total no. of links = 24).

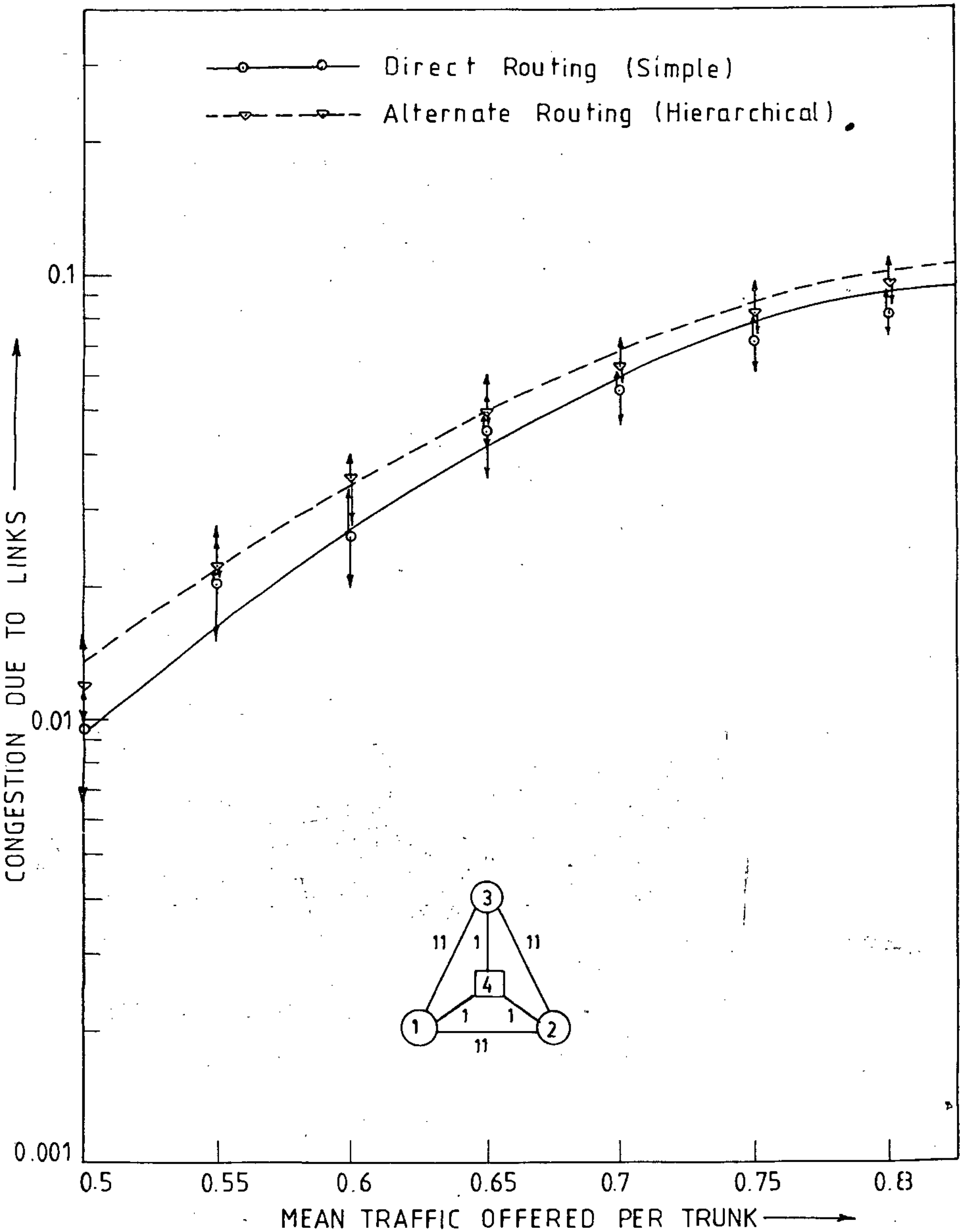


Fig. 6.6 Congestion due to links vs. traffic offered to the system for a four exchange alternate routing (Hierarchical) and direct routing networks. (Total number of links = 36).

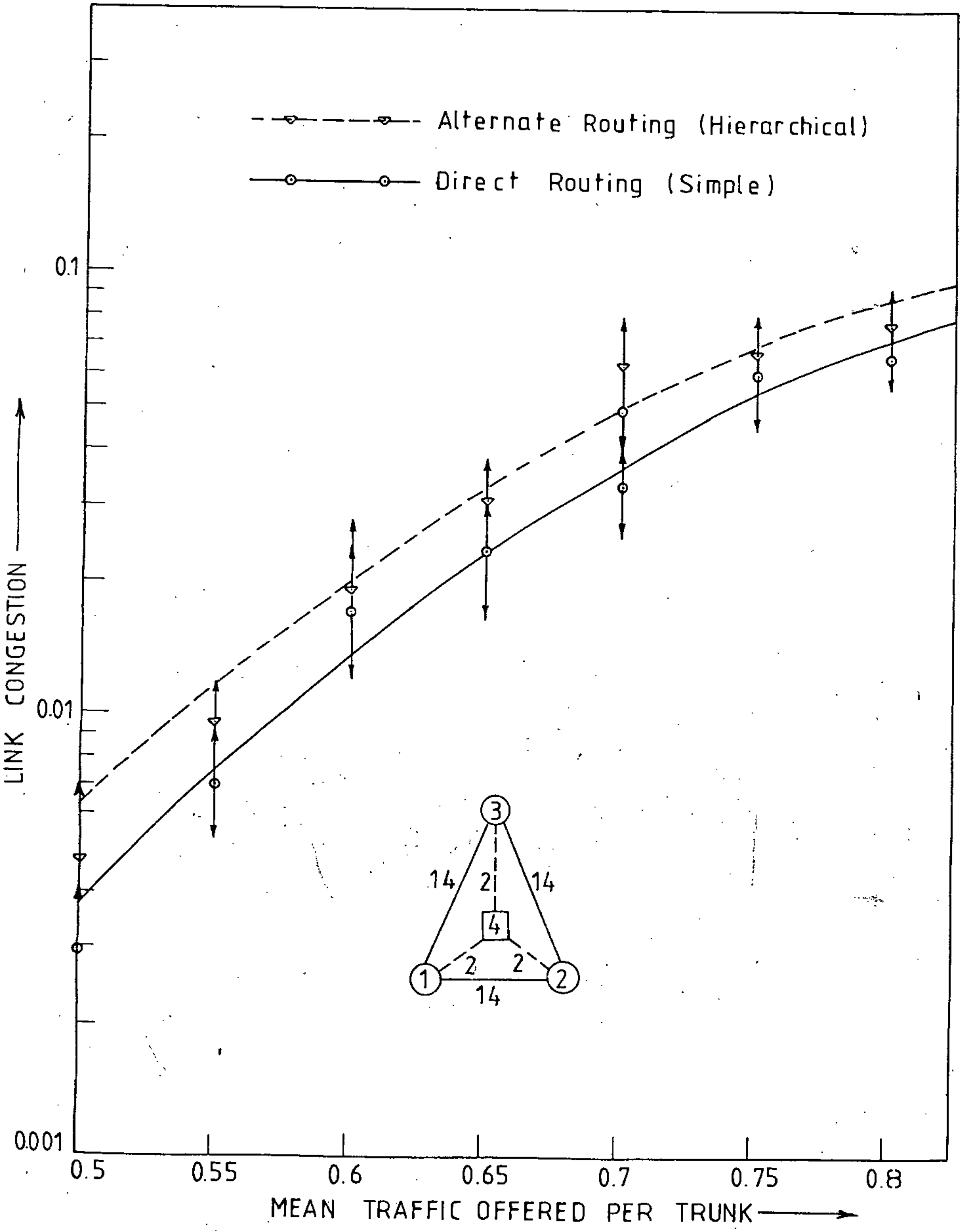


Fig. 6.7 Congestion due to links vs. traffic offered to the system for a four exchange alternate routing (hierarchical) and direct routing networks. (Total no. of links = 48)

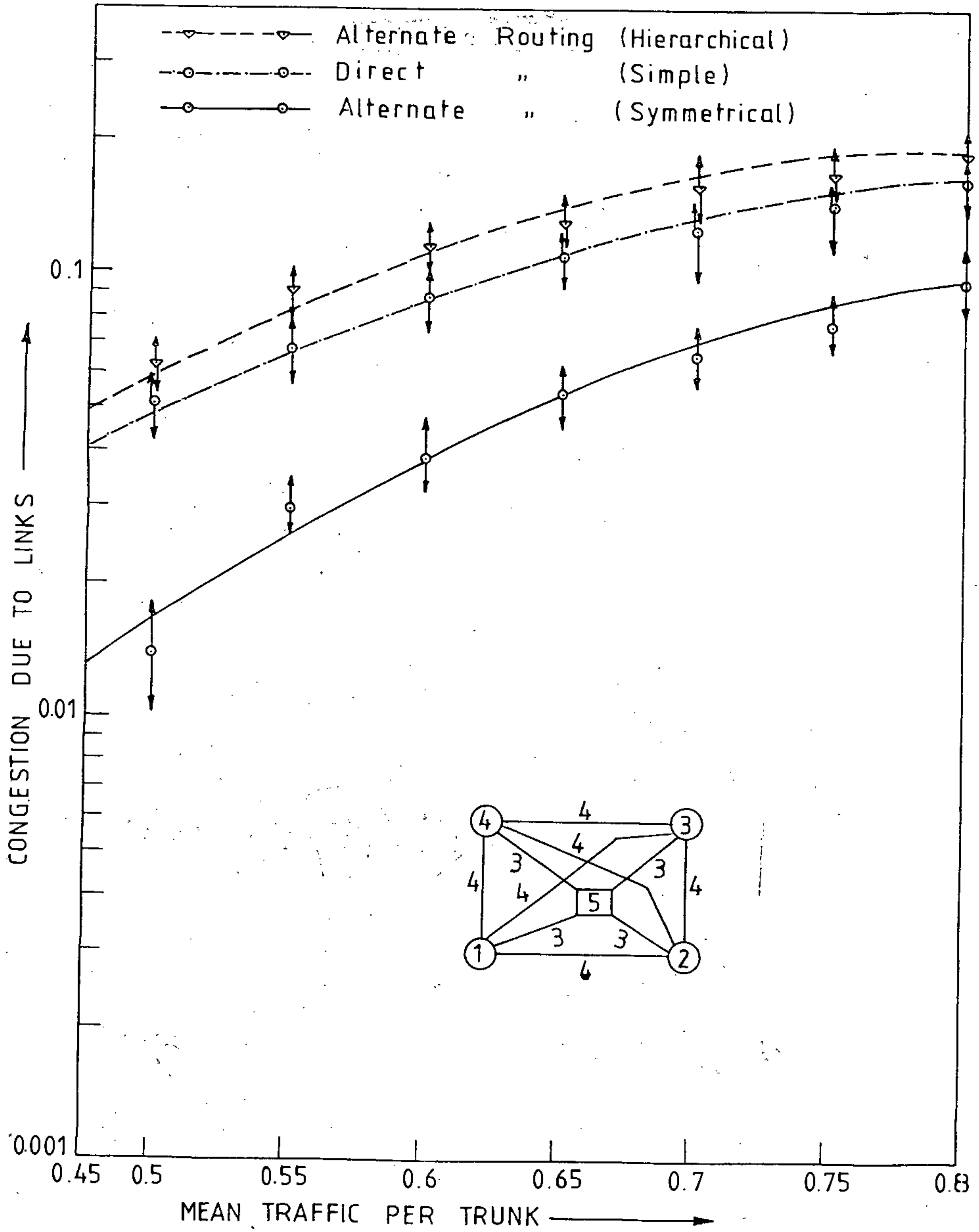


Fig. 6.8 Congestion due to links vs. traffic offered per truk for a five exchange alternate routing (Hierarchical) and direct routing, alternate routing (Symmetrical) networks. (for total links = 36)

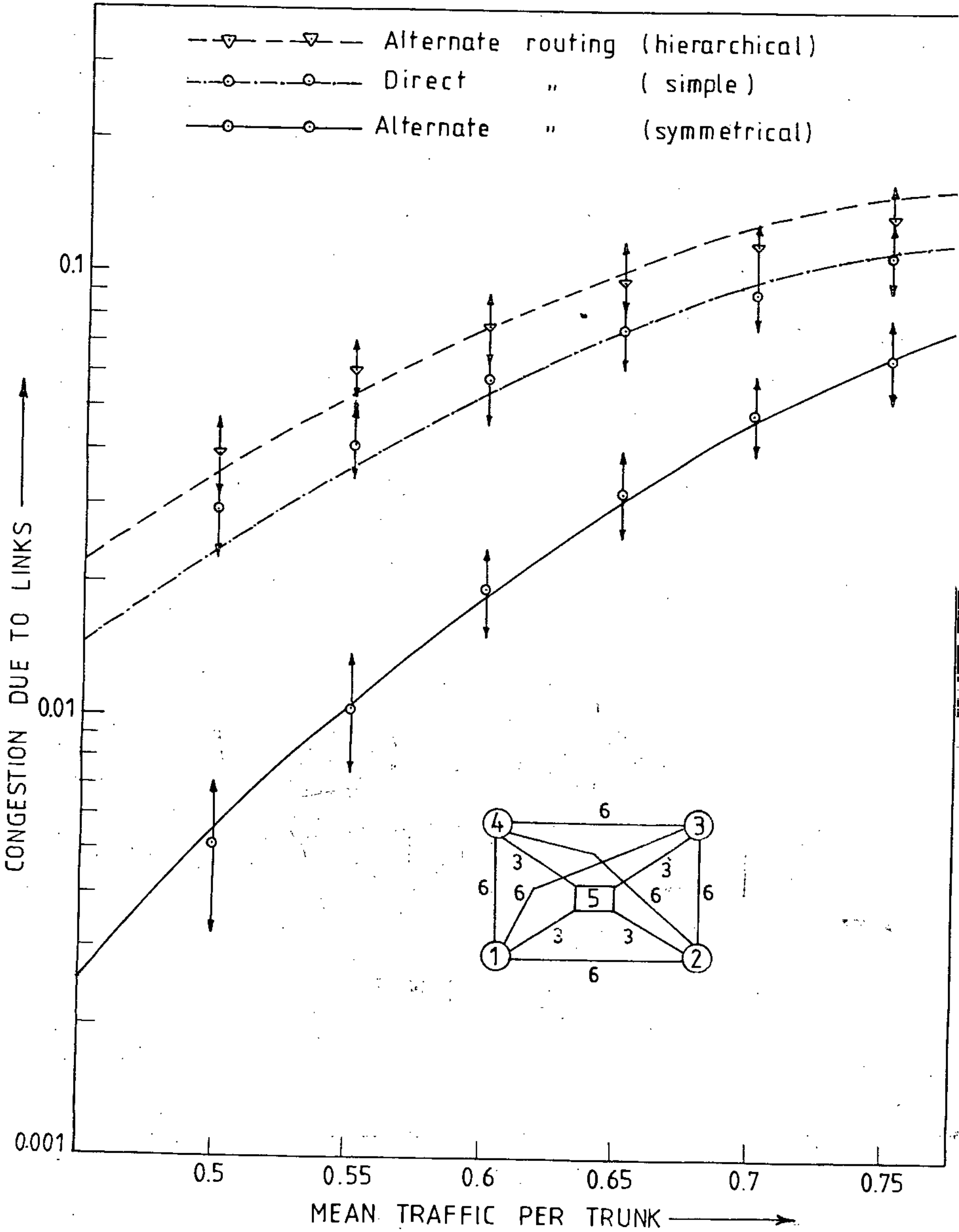


Fig. 6.9 Congestion due to links vs. traffic offered per truk for a five exchange alternate routing (Hierarchical) and direct routing, alternate routing (Symmetrical) networks. (for 48 links).

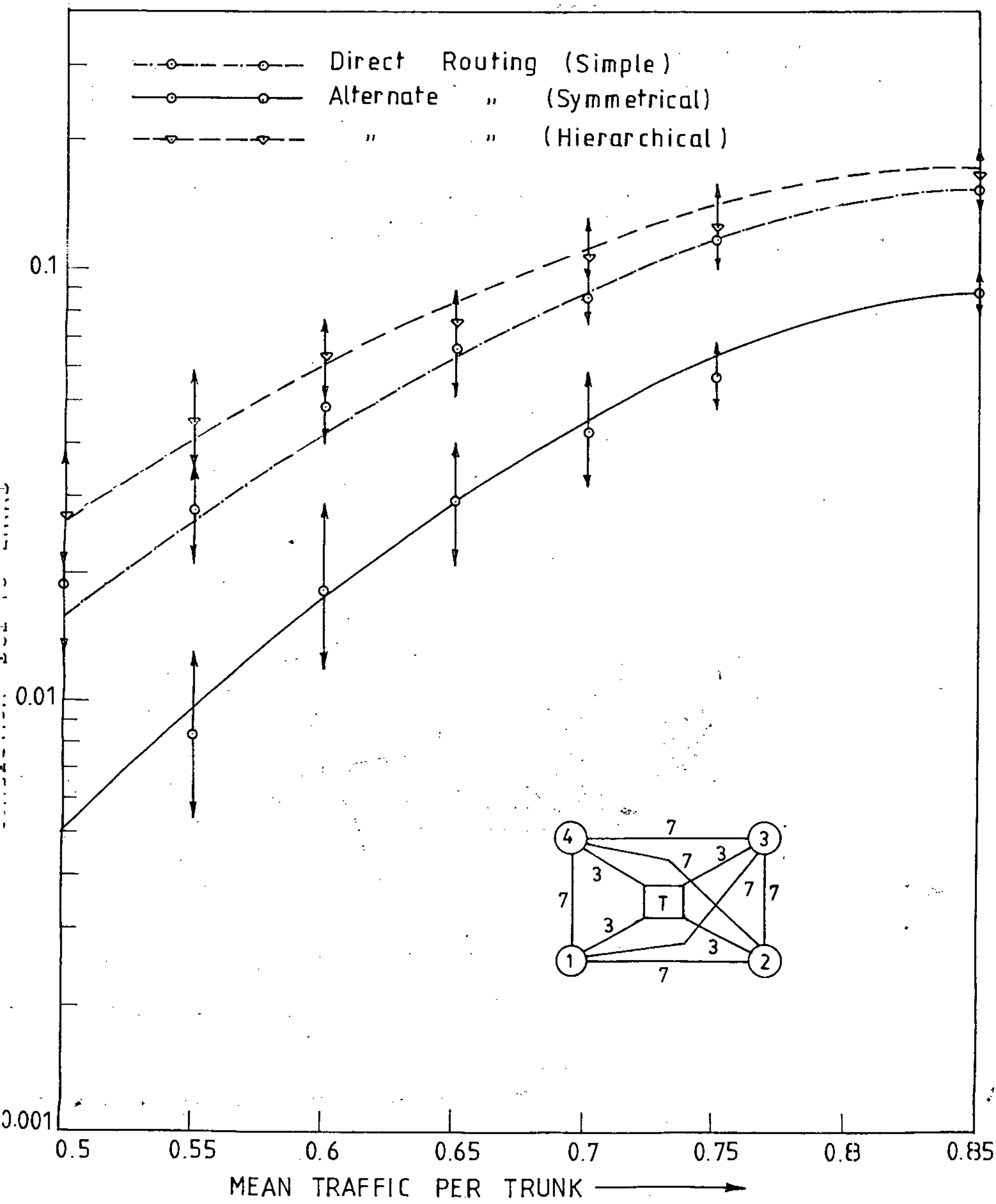


Fig. 6.10 Congestion due to links vs. traffic offered per truk for a five exchange alternate routing (Hierarchical) and direct routing, alternate routing (Symmetrical) networks. (for 54 links).

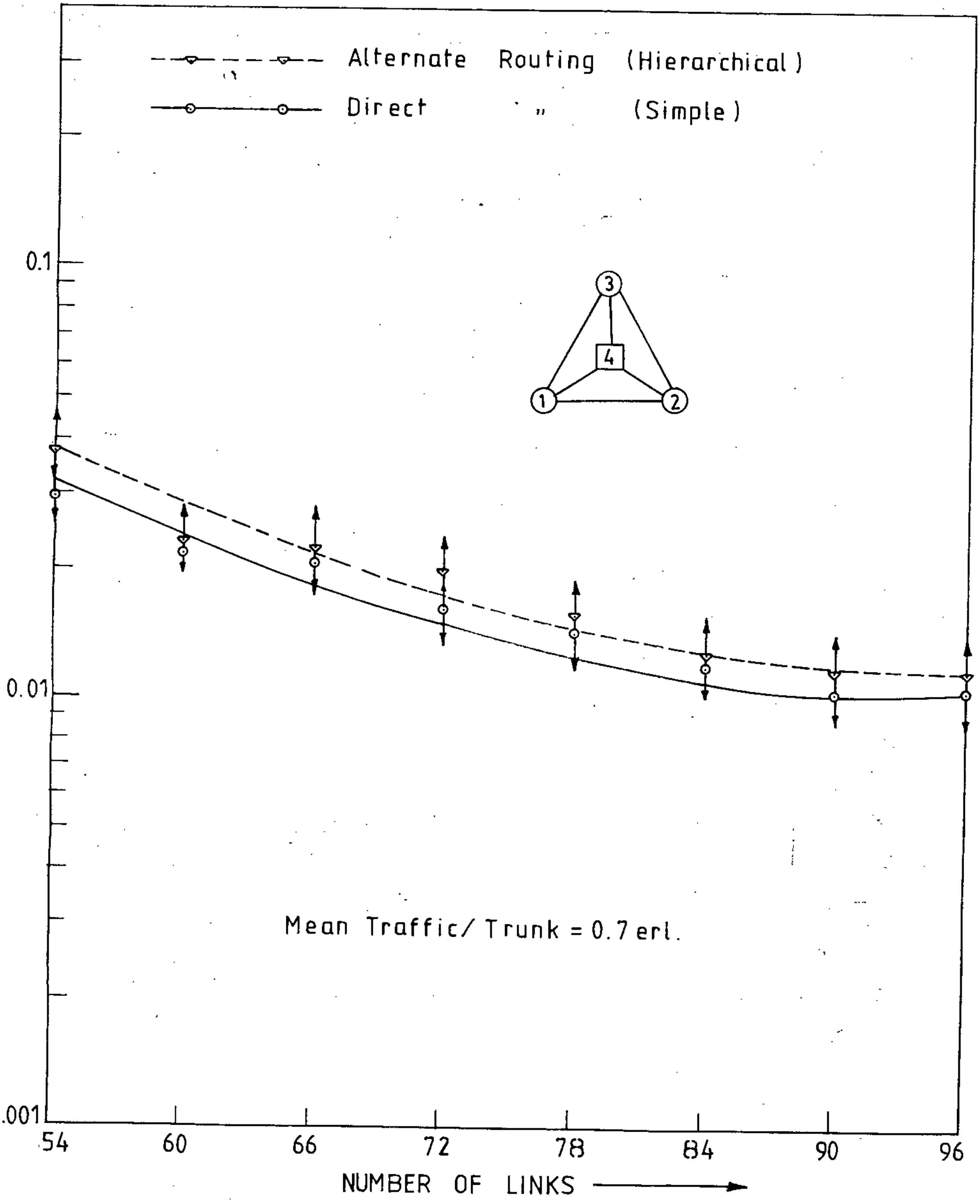
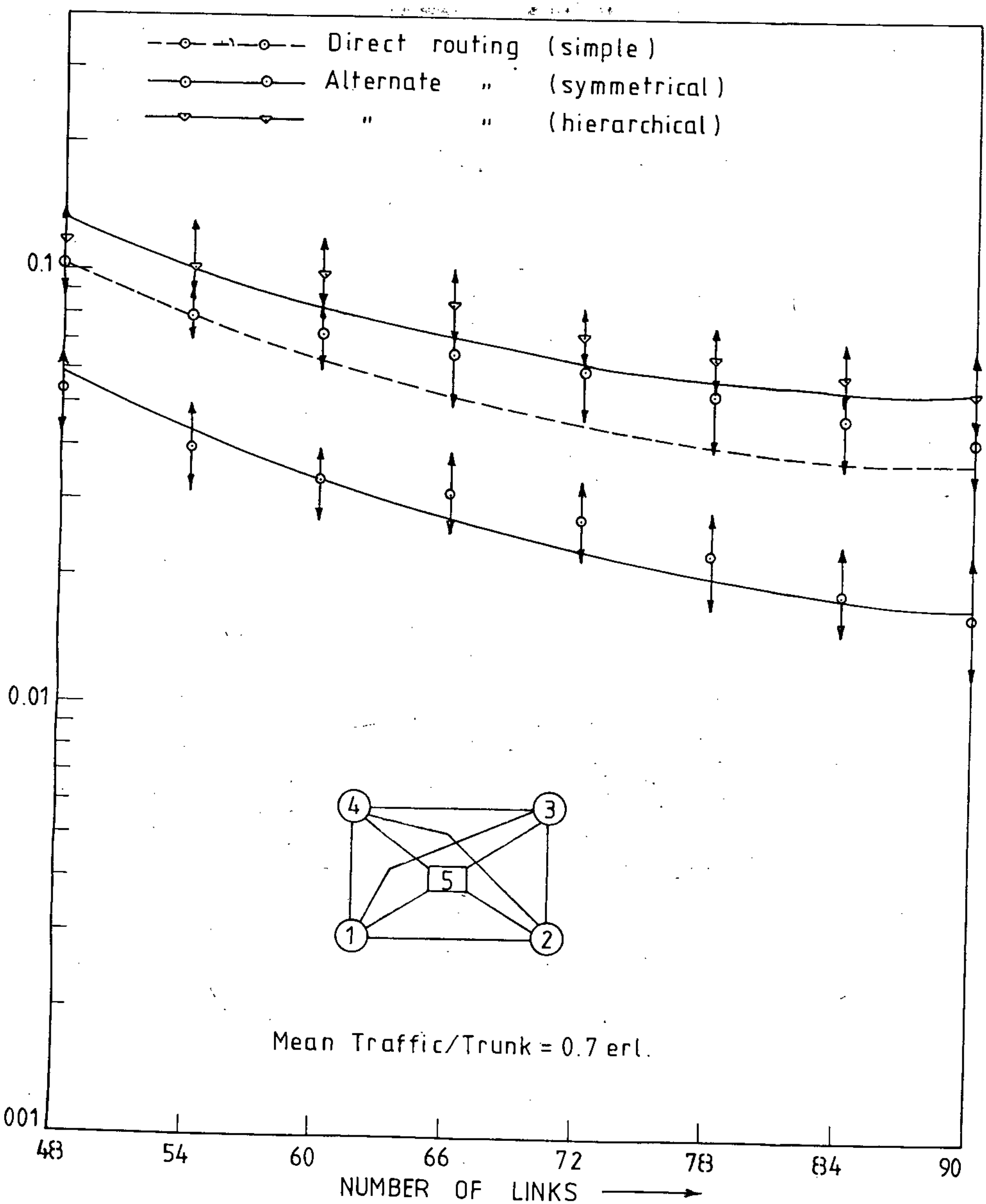


Fig. 6.11 Congestion due to links vs. number of links for a four exchange alternate routing (Hierarchical), and direct routing (simple network).



ȝ. 6.12 Congestion due to links vs. number of links for a five ,exchange alternate routing (Hierarchical), direct routing (simple) and alternate routing (symmetrical).

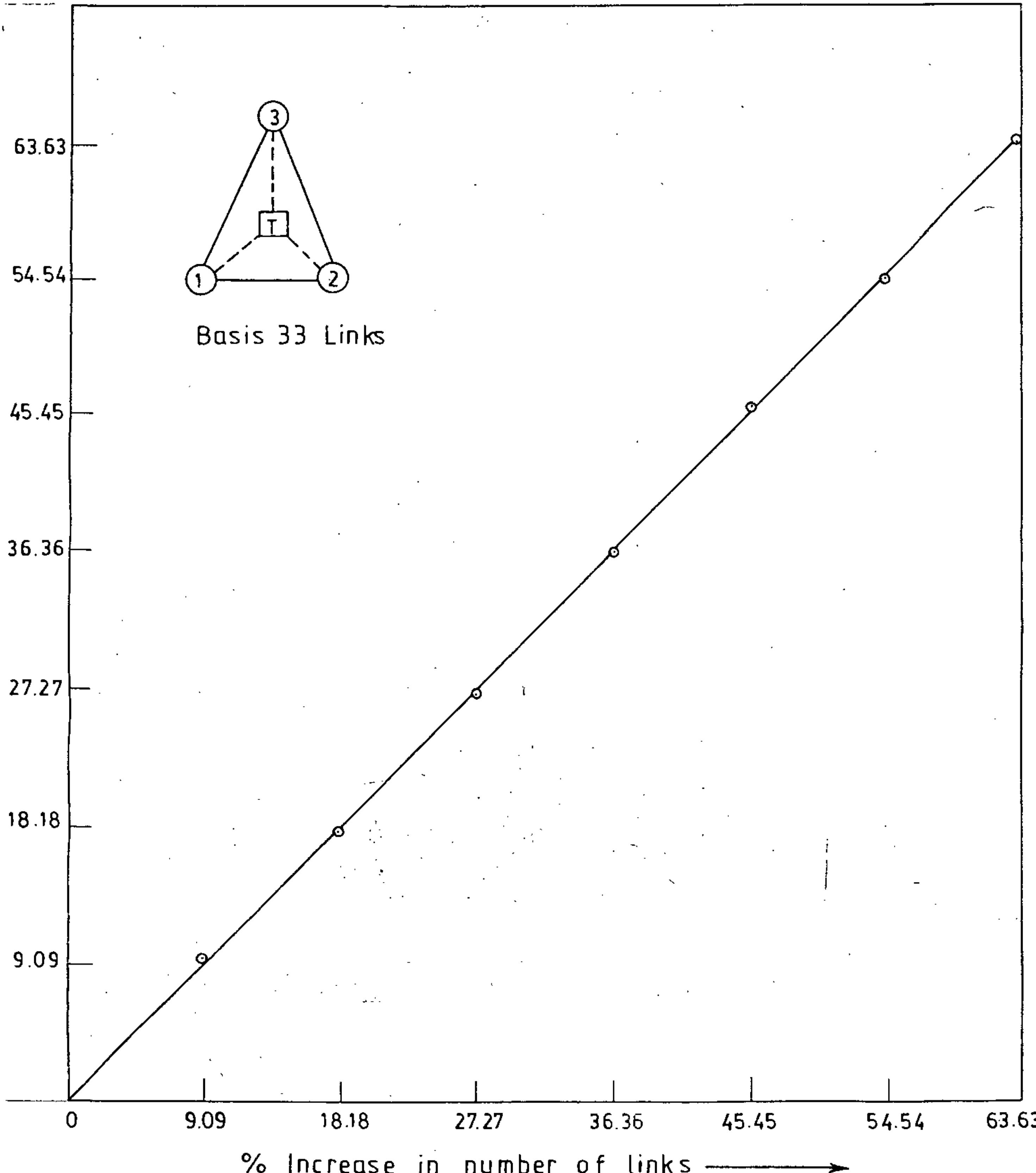


Fig. 6.13 Percentage increase in Traffic handling Capacity VS. Percentage increase in number of links (for a four exchange Network).

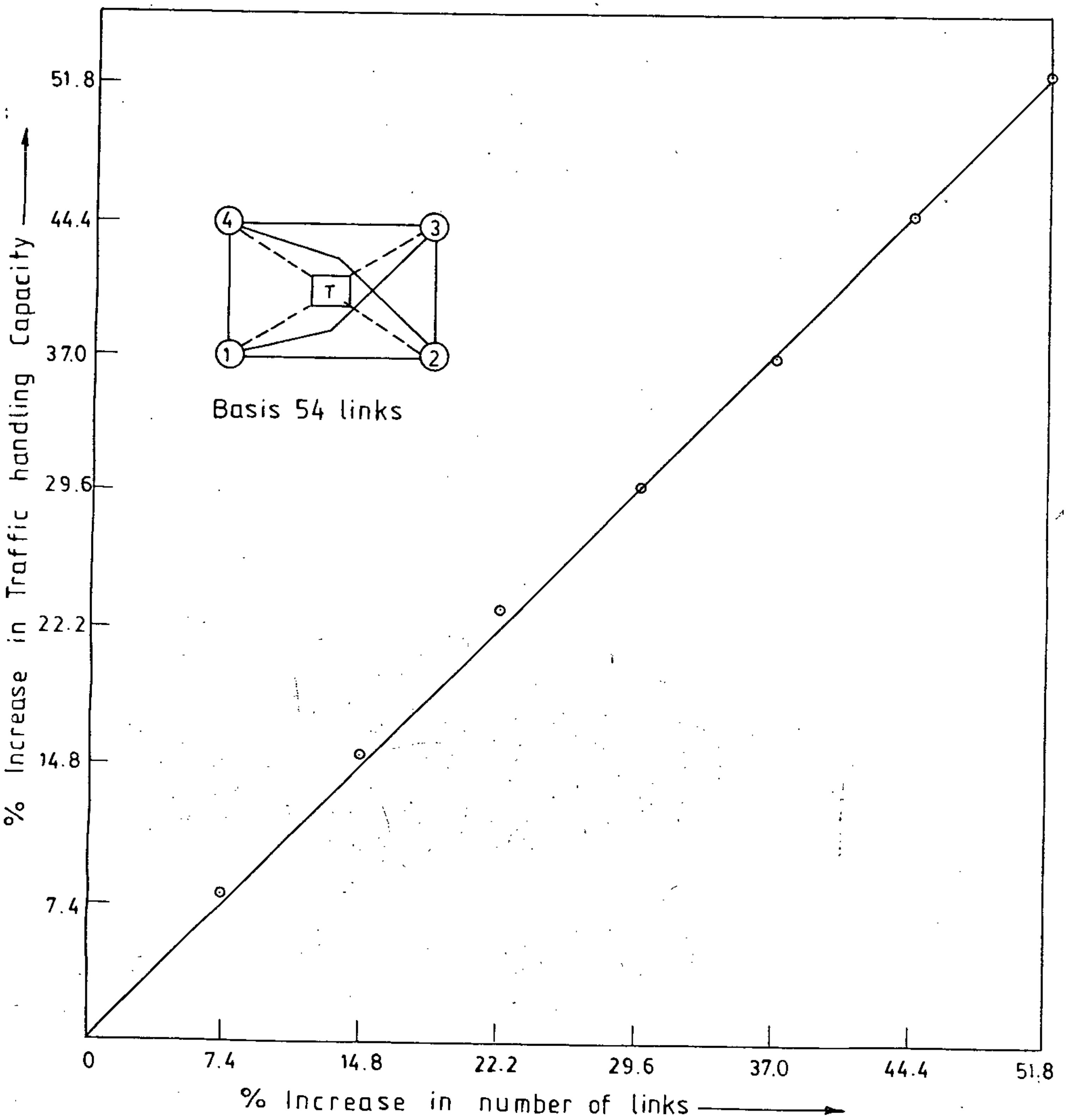


Fig. 6.14 Percentage increase in Traffic handling Capacity Vs. Percentage increase in number of links (for a five Exchange Network).

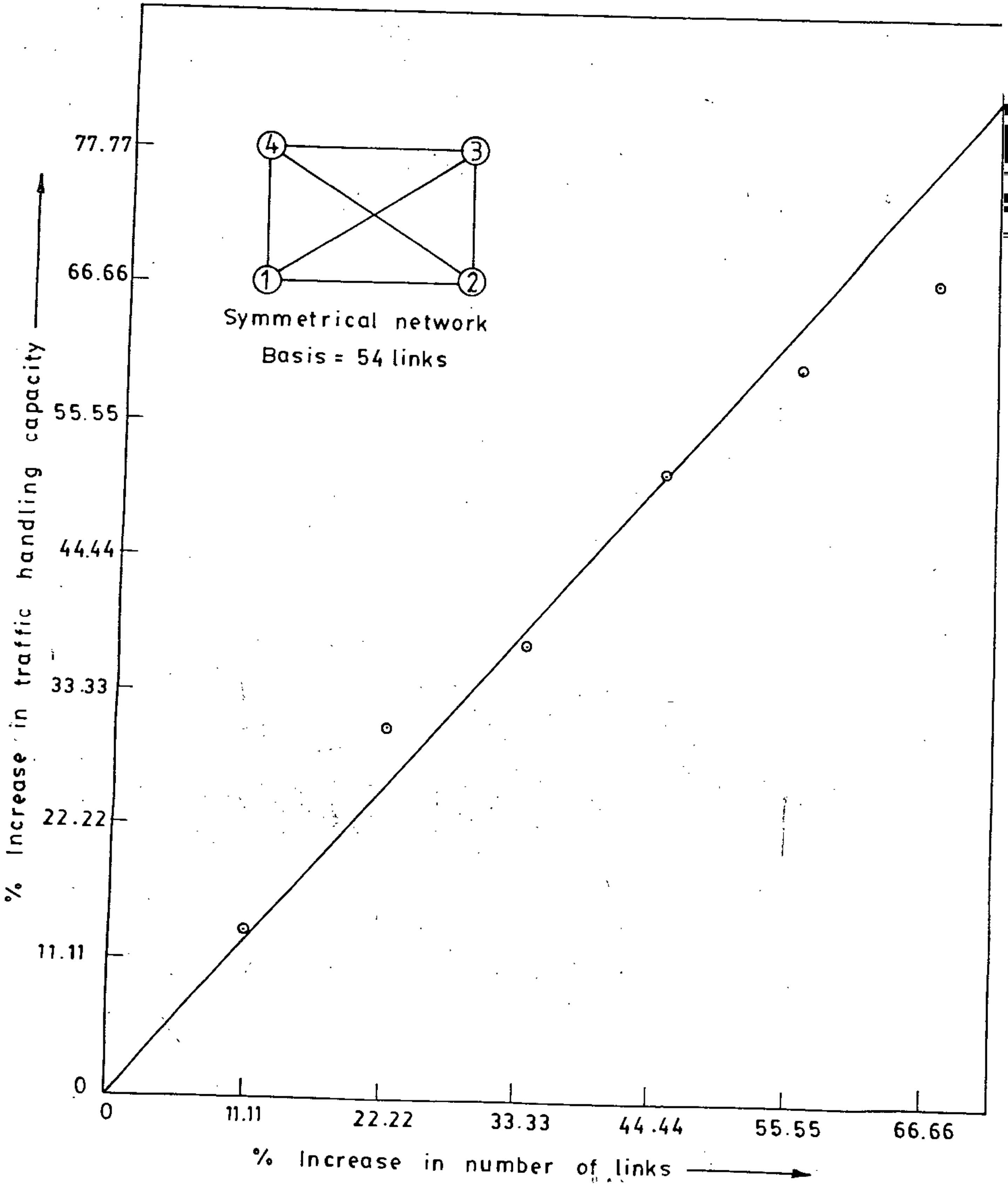


Fig. 6.15 % Increase in Traffic handling capacity Vs. % increase in number of links.

6.2 Discussion

6.2.1 General Remarks

In the present work, network systems have been studied with 'Exact solution method' and 'Simulation' method. Congestion of different network systems are plotted against offered traffic per trunk. For simple systems, the simulations results are compared with analytic results (i.e. exact solution method) obtained by classical probabilistic theory. Within accepted confidence interval the simulation results seem to agree with analytic results to a considerable extent. The coincidence between the results proves the validity of the simulation model. Complex practical networks are then studied with simulation method. All the network systems are studied with balanced or uniform load. Negative exponential distribution is considered in the simulation model. Network systems are studied with traffic between 0.5 and 0.85 erlangs. Simple, Hierarchical and symmetrical network systems are studied in this work. The performance of these network systems are compared with each other by 'Congestion Probability'. The effect of traffic handling capacity of network system is tested with the increase in number of links. All these results are plotted graphically from fig. 6.1 to fig. 6.15.

6.2.2 Presentation of Results.

In chapter-4, exact solution method for calculating overall congestion of a three exchange simple network, four exchange simple network systems have been discussed. Equations derived for these network systems are shown in appendix-1B, appendix-2B and appendix-3B respectively. The equations derived for the simple systems leads to a large set of equations, which will be more complex incase of large system.

All these simple systems considers one link between two exchanges. The over all congestion of three exchange simple, four exchange hierarchical and four exchange simple network systems 'exact solution method at different traffic are plotted in figures 6.1, 6.2, 6.3 respectively.

The over all congestion of the same networks are studied with'simulation method' and plotted in the same figures 6.1, 6.2, & 6.3 respectively.

Link congestion of three exchange direct routing (simple) network, four exchange alternate routing (hierarchical) network, with same number of links against different traffic intensities are shown in figures 6.4, 6.5, 6.6 and 6.7 respectively. Congestion for links 18,24,36 and 48 of these two network systems are shown in fig. 6.4, 6.5, 6.6 and 6.7 respectively.

In figures 6.8, 6.9 and 6.10 the link congestions for four exchange direct routing (simple) network, five exchange alternate routing (hierarchical) and four exchange alternate routing (symmetrical) networks are plotted against traffic offered per trunk.

In each figure the network systems are compared one other with same number of links. These systems are studied with links 36, 48 and 54 shown in figures 6.8, 6.9 and 6.10 respectively.

Fig. 6.11 shows the link congestion of a three exchange direct routing network and four exchange alternate routing (hierarchical) network for different number of links.

Fig. 6.12 shows the link congestion of four exchange direct routing (simple), five exchange alternate routing (hierarchical) and four exchange alternate routing (symmetrical) networks for different numbers of links.

Percentage increase in traffic handling capacity are plotted against percentage increase in number links for four exchange alternate routing (hierarchical), five exchange alternate routing (hierarchical) and four exchange alternate routing (Symmetrical) networks shown in figures 6.13, 6.14 and 6.15 respectively.

6.2.3 Observations:

1. Comparison between the results by 'Exact Solution' and 'Simulation' method:

The over all congestions of three exchange simple, four exchange hierarchical and four exchange simple networks by 'exact solution' and 'simulation' are plotted in figures 6.1, 6.2, 6.3. These results shows good agreement with each other. Thus the validity of simulation model is checked.

2. Comparision in link congestion between three exchange direct routing (simple), and four exchange alternate routing (hierarchical) networks:-

Link congestion for threee exchange direct routing (simple) network and four exchange alternate routing (hierarchical) network at different traffic intensities are plotted in fig. 6.4, 6.5, 6.6 and 6.7. Congestion for links 18,24,36 and 48 are plotted in figures 6.4, 6.5, 6.6 and 6.7 respectively. With same links the link congestion of four exchange alternate routing (hierarchical) network is always higher than three exchange direct routing (simple) network.

3. Comparision in link congestion between four exchange direct routing (simple), five exchange alternate routing (hierarchical) and four exchange alternate routing (symmetrical) networks at different traffic intensities:

The congestion due to links of four exchange direct routing (simple), five exchange alternate routing (hierarchical) and four exchange alternate routing (symmetrical) routing at different traffic intensities are plotted in figures 6.8,6.9 and 6.10 for links 36,48 and 54. The congestion of alternate routing (hierarchical) network is higher than the direct routing (simple) network and the congestion of direct routing (simple) network is higher than alternate routing (symmetrical) network.

4. Congestions due to links at different number of links for alternate routing (hierarchical), direct routing (simple) and alternate routing (symmetrical) networks:

Link congestions of four exchange alternate routing (hierarchical) and three exchange direct routing (simple) networks at different number of links are plotted in fig. 6.11. Congestion of direct routing (simple) network is always lower than the alternate routing (hierarchical) network.

Link congestions of four exchange direct routing (simple) five exchange alternate routing (hierarchical) and four exchange alternate routing (symmetrical) networks are plotted in figure 6.12 (for different number of links). The link congestion of five exchange alternate routing (hierarchical) network is higher than the direct routing (simple) network and the congestion of direct routing (simple) network is higher than the alternate routing (symmetrical)network.

5. Effect on traffic handling capacity with increase in number of links:

The traffic handling capacity of a system is mathematically defined as (1-B)xA, where B is the congestion of the system and A is the traffic intensity.

The percentage increase in traffic handling capacity of four exchange alternate routing (hierarchical) network, five exchange alternate routing (hierarchical) network and four exchange alternate routing (symmetrical) against percentage increase in number of links are plotted in figures 6.13, 6.14 and 6.15. In each figure percentage increase in traffic handling capacity varies linearly with percentage increase in number of links. In fig. 6.15, for symmetrical network the rate of increase in percentage increase in traffic handling capacity is more higher with increase in percentage increase in number of links.

6.2.4 Simulation Results

Simulation study (with digital computer) is helpful in understanding various complex networks. The numerical results produced by simulation method gives experimentally measured values of system performance and other parameters. Simulation results giving the congestion at different loads are presented with confidence interval at 95% significance level. 1000 events per sample and 20 samples per run are used in the present program.

Number of samples can be increased to get more accurate results but the required computer time will be higher. This study is performed under idealized assumptions of statistical equilibrium poissions input, independent events, exponential holding time distributions etc.

CHAPTER - 7
CONCLUSION

Multiexchange networks with direct routing (simple) alternate routing (hierarchical) and alternate routing (symmetrical) networks are studied in the present work. The study shows that the symmetrical network is the best among all the networks because it suffers least link congestion due to the fact that here high usage routes are also used as final routes. However, the control and management of symmetrical network is complex and difficult. Between the other two networks the direct one performs better in terms of congestion but it shows less reliability than the hierarchical network. In alternate route (hierarchical) network the congestion is higher, because in this network more than one link is used per call and this is the source of inefficiency in such systems[2]. Reliability of alternate routing (hierarchical) network is more than direct routing networks, because call fails in the direct route due to technical faults or local disasters (such as storms, earth quakes etc.) can be rerouted to the final route.

In this study the traffic handling capacity criterion was tested for the symmetrical and hierarchical networks. Here also the symmetrical network showed better performance. With the percentage increase in number of links, the percentage increase in traffic handling capacity showed significant improvement over that of hierarchical network.

In this work, 3-,4- and 5- exchange networks are studied with simulation method. The validity of the simulation model is checked with the help of Exact solution method. There is a fair scope of study for more large and complex network systems with present simulation program. Dynamic or adaptive control routing can be introduced in alternate routing systems, which will work more efficiently with better performance. Dynamic routing can also be studied with the present program.

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Appendix 1A. Table - 1 (For a Three Exchange Network)

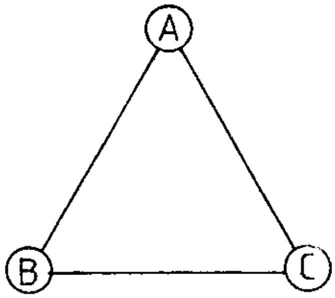

Fig. 1A. A three exchange network

No. of States	Calls present
1	0
2	AB
3	BC
4	CA
5	AB,BC
6	AB,CA
7	BC,CA
8	AB,BC,CA

Appendix 1B. State Equations for a three Exchange Network System:

1. $(A_1 + A_2 + A_3)\ P(1) = P(2) + P(3) + P(4)$

2. $(A_2 + A_3 + 1)\ P(2) = A_1\ P(1) + P(5) + P(6)$

3. $(A_1 + A_3 + 1)\ P(3) = A_2\ P(1) + P(5) + P(7)$

4. $(A_1 + A_2 + 1)\ P(4) = A_3\ P(1) + P(6) + P(7)$

5. $(A_3 + 2)\ P(5) = A_2\ P(2) + A_1\ P(3) + P(8)$

6. $(A_2 + 2)\ P(6) = A_3\ P(2) + A_1\ P(4) + P(8)$

7. $(A_1 + 2)\ P(7) = A_2\ P(4) + A_3\ P(3) + P(8)$

8. $\sum_{i=1}^{8} P(i) = 1$

Appendix- 2A. Table-2 (A Four Exchange Network with a Tandem Exchange)

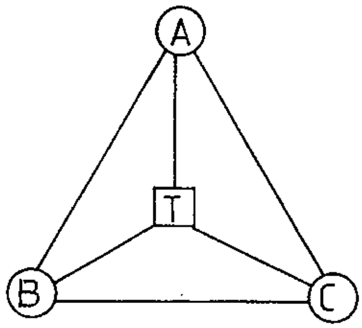

Fig.2A. A Four exchange Hierarchical(with tandem) network.

State No.	Calls present	State No.	Calls present
1	No call	19	CA,CTA
2	AB	20	AB,BC,ATB
3	BC	21	AB,BC,BTC
4	CA	22	AB,BC,CTA
5	ATB	23	AB,CA,ATB
6	BTC	24	AB,CA,BTC
7	CTA	25	AB,CA,CTA
8	AB,BC	26	BC,CA,ATB
9	AB,CA	27	BC,CA,BTC
10	AB,BC	28	BC,CA,CTA
11	AB,ATB	29	AB,BC,CA
12	AB,BTC	30	AB,CA,BC,ATB
13	AB,CTA	31	AB,BC,CA,BTC
14	BC,ATB	32	AB,BC,CA,CTA.
15	BC,BTC		
16	BC,CTA		
17	CA,ATB		
18	CA,BTC		

APPENDIX: 2B

State Equations (For 4 Exchanges with one Tandem exchange)

1. $(A_1+A_2+A_3)\ P(1) = P(2)+P(3)+P(4)+P(5)+P(6)+P(7)$

2. $(A_1+A_2+A_3+1)\ P(2) = A_1P(1)+P(8)+P(9)+P(11)+P(12)+P(13)$

3. $(A_1+A_2+A_3+1)\ P(3) = A_2P(1)+P(8)+P(10)+P(14)+P(15)+P(16)$

4. $(A_1+A_2+A_3+1)\ P(4) = A_3P(1)+P(9)+P(10)+P(17)+P(18)+P(19)$

5. $(A_1+A_2+A_3+1)\ P(5) = P(11)+P(14)+P(17)$

6. $(A_1+A_2+A_3+1)\ P(6) = P(12)+P(15)+P(18)$

7. $(A_1+A_2+A_3+1)\ P(7) = P(13)+P(16)+P(19)$

8. $(A_1+A_2+A_3+2)\ P(8) = A_2\ P(2)+A_1\ P(3)+P(20)+P(21)+P(22)+P(29)$

9. $(A_1+A_2+A_3+2)\ P(9) = A_3\ P(2)+A_1\ P(4)+P(23)+P(24)+P(25)+P(29)$

10. $(A_1+A_2+A_3+2)\ P(10) = A_3\ P(3)+A_2P(4)+P(26)+P(27)+P(28)+P(29)$

11. $(A_2+A_2+2)\ P(11) = A_1\ P(2)+A_1\ P(5)+P(20)+P(23)$

12. $(A_2+A_3+2)\ P(12) = A_1\ P(6)+P(21)+P(24)$

13. $(A_2+A_3+2)\ P(13) = A_1\ P(7)+P(22)+P(25)$

14. $(A_1+A_3+2)\ P(14) = A_2\ P(5)+P(20)+P(26)$

15. $(A_1+A_3+2)\ P(15) = A_2\ P(3)+A_2\ P(6)+P(21)+P(27)$

16. $(A_1+A_3+2)\ P(16) = A_2\ P(7)+P(22)+P(28)$

17. $(A_1+A_2+2)\ P(17) = A_3\ P(5)+P(23)+P(26)$

18. $(A_1 + A_2 + 2)\ P(18) = A_3\ P(6) + P(24) + P(27)$

19. $(A_1 + A_2 + 2)\ P(19) = A_3\ P(4) + A_3\ P(7) + P(25) + P(28)$

20. $(A_3 + 3)\ P(20) = A_1\ P(8) + A_2\ P(11) + A_1\ P(14) + P(30)$

21. $(A_3 + 3)\ P(21) = A_2\ P(8) + A_2\ P(12) + A_1\ P(15) + P(31)$

22. $(A_3 + 3)\ P(22) = A_2\ P(13) + A_1\ P(16) + P(32)$

23. $(A_2 + 3)\ P(23) = A_3\ P(11) + A_1\ P(17) + P(30) + A_1\ P(9)$

24. $(A_2 + 3)\ P(24) = A_3\ P(12) + A_1\ P(18) + P(31)$

25. $(A_2 + 3)\ P(25) = A_3\ P(9) + A_3\ P(13) + A_1\ P(19) + P(32)$

26. $(A_1 + 3)\ P(26) = A\ P(10) + A_3\ P(14) + A_2\ P(17) + P(30)$

27. $(A_1 + 3)\ P(27) = A_3\ P(15) + A_2\ P(18) + P(31) + A_2\ P(10)$

28. $(A_1 + 3)\ P(28) = A_3\ P(16) + A_2\ P(19) + P(32) + A_3\ P(10)$

29. $(A_1 + A_2 + A_3 + 3)\ P(29) = A_3\ P(8) + A_2\ P(9) + A_1\ P(10) + P((30) + P(31) + P(32)$

30. $4P(30) = A_3\ P(20) + A_2\ P(23) + A_1\ P(26) + A_1\ P(29)$

31. $4P(31) = A_3\ P(21) + A_2\ P(24) + A_1\ P(27) + A_2\ P(29)$

32. $4P(32) = A_3\ P(22) + A_2\ P(25) + A_1\ P(28) + A_3\ P(29)$

33. $\sum_{i=1}^{32} P(i) = 1$

Appendix: 3A: Table-3 (A Four Exchange Network)

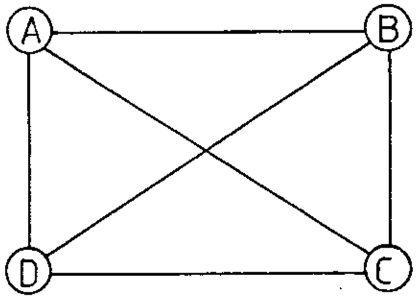

Fig. 3A. A Four exchange simple network

No. of States	Calls present	No. of States	Calls present
1	0	17	AD,BC
2	AB	18	AD,BD
3	AC	19	AD,CD
4	AD	20	BC,BD
5	BC	21	BC,CD
6	BD	22	BD,CD
7	CD	23	AB,AC,AD
8	AB,AC	24	AB,AC,BC
9	AB,AD	25	AB,AC,BD
10	AB,BC	26	AB,AC,CD
11	AB,BD	27	AB,AD,BC
12	AB,CD	28	AB,AD,BD
13	AC,AD	29	AB,AD,CD
14	AC,BC	30	AB,BC,BD
15	AC,BD	31	AB,BC,CD
16	AC,CD	32	AB,BD,CD

No.of States	Calls present	No.of States	Calls present
33	AC,AD,BC	49	AB,AD,BC,BD
34	AC,AD,BD	50	AB,AD,BC,CD
35	AC,AD,CD	51	AB,AD,BD,CD
36	AC,BC,BD	52	AB,BC,BD,CD
37	AC,BC,CD	53	AC,AD,BC,BD
38	AC,BD,CD	54	AC,AD,BC,CD
39	AD,BC,BD	55	AC,AD,BD,CD
40	AD,BC,CD	56	AC,BC,BD,CD
41	AD,BD,CD	57	AD,BC,BD,CD
42	AC,BD,CD	58	AB,AC,AD,BC,BD
43	AB,AC,AD,BC	59	AB,AC,AD,BC,CD
44	AB,AC,AD,BD	60	AB,AC,AD,BD,CD
45	AB,AC,AD,CD	61	AB,AC,BC,BD,CD
46	AB,AC,BC,BD	62	AB,AD,BC,BD,CD
47	AB,AC,BC,CD	63	AC,AD,BC,BD,CD
48	AB,AC,BD,CD	64	AB,AC,AD,BC,BD,CD

Appendix: 3B : State Equations for a Four Exchange Network.

1. $(A_1+A_2+A_3+A_4+A_5+A_6)\ P(1) = P(2) + P(3) + P(4) + P(5) + P(6) + P(7)$

2. $(A_2+A_3+A_4+A_5+A_6+1)\ P(2) = A_1\ P(1) + P(8) + P(9) + P(10) + P(11)$

3. $(A_1+A_3+A_4+A_5+A_6+1)P(3) = A_2\ P(1) + P(8) + P(13) + P(14) + P(15) + P(16)$

4. $(A_1+A_2+A_4+A_5+A_6+1)\ P(4) = A_3\ P(1) + P(9) + P(13) + P(17) + P(18) + P(19)$

5. $(A_1+A_2+A_3+A_5+A_6+1)\ P(5) = A_4\ P(1) + P(10) + P(14) + P(17) + P(20) + P(21)$

6. $(A_1+A_2+A_3+A_4+A_6+1)\ P(6) = A_5\ P(1) + P(11) + P(15) + P(18) + P(20) + P(22)$

7. $(A_1+A_2+A_3+A_4+A_5+1)\ P(7) = A_6P(1) + P(12) + P(16) + P(19) + P(21) + P(22)$

8. $(A_3+A_4+A_5+A_6+2)\ P(8) = A_2\ P((2) + A_1P(3) + P(23) + P(24) + P(25) + P(26)$

9. $(A_2+A_4+A_5+A_6+2)\ P(9) = A_3\ P(2) + A_1P(4) + P(23) + P(27) + P(28) + P(29)$

10. $(A_2+A_3+A_5+A_6+2)\ P(10) = A_4\ P(2) + A_1P(5) + P(24) + P(27) + P(30) + P(31)$

11. $(A_2+A_3+A_4+A_6+2)\ P(11) = A_5\ P(2) + A_1P(6) + P(25) + P(28) + P(30) + P(32)$

12. $(A_2+A_3+A_4+A_5+2)\ P(12) = A_6P(2) + A_1P(7) + P(26) + P(29) + P(31) + P(32)$

13. $(A_1+A_4+A_5+A_6+2)\ P(13) = A_3P(3) + A_4P(4) + P(23) + P(33) + P(34) + P(34)$

14. $(A_1+A_3+A_5+A_6+2)\ P(14) = A_4P(3) + A_2P(5) + P(24) + P(33) + P(36) + P(37)$

15. $(A_1+A_3+A_4+A_6+2)\ P(15) = A_5P(3) + A_2P(6) + P(25) + P(34) + P(36) + P(38)$

16. $(A_1+A_3+A_4+A_5+2)\ P(16) = A_6P(3) + A_2P(7) + P(26) + P(35) + P(37) + P(38)$

17. $(A_1+A_2+A_5+A_6+2)\ P(17) = A_4P(4) + A_3P(5) + P(27) + P(33) + P(39) + P(40)$

18. $(A_1+A_2+A_4+A_6 + 2)\ P(18) = A_5P(4) + A_1P(6) + P(28) + P(35) + P(39) + P(41)$

19. $(A_1+A_2+A_4+A_5 + 2)\ P(19) = A_6P(4) + A_3P(7) + P(29) + P(35) + P(40) + P(41)$

20 $(A_1+A_2+A_3+A_6 + 2)\ P(20) = A_5P(6) + A_4P(5) + P(30) + P(36) + P(39) + P(42)$

21. $(A_1+A_2+A_3+A_5 + 2)\ P(21) = A_6P(5) + A_5P(7) + P(31) + P(37) + P(40) + P(42)$

22. $(A_1+A_2+A_3+A_4 + 2)\ P(22) = A_5P(7) + A_6P(6) + P(32) + P(38) + P(41) + P(42)$

23. $(A_4+A_5+A_6 + 3)\ P(23) = A_3P(8) + A_2P(9) + A_1P(13) + P(43) + P(44) + P(45)$

24. $(A_3+A_5 +A_6 + 3)\ P(24) = A_3P(8) + A_2P(10) + A_1P(14) + P(43) + P(46) + P(47)$

25. $(A_3 + A_4 + A_6 + 3)\ P(25) = A_5P(8) + A_2P(11) + A_1P(15) + P(44) + P(46) + P(48)$

26. $(A_3 +A_4+A_5+ 3)\ P(26) = A_6P(8) + A_2P(12) + A_1P(16) + P(45) + P(47) + P(48)$

27. $(A_2 +A_5 + A_6+3)\ P(27) = A_4P(9) + A_3P(10) + A_1P(17) + P(43) + P(49) + P(50)$

28. $(A_2 +A_4 +A_6 + 3)\ P(28) = A_5P(9) + A_3P(11) + A_1P(18) + P(44) + P(49) + P(51)$

29. $(A_2+A_4 +A_5+ 3)\ P(29) = A_6P(9) + A_3P(12) + A_1P(19) + P(45) + P(50) + P(51)$

30. $(A_2+A_3 +A_6 + 3)\ P(30) = A_5P(10) + A_4P(11) + A_1P(20) + P(46) + P(49) + P(52)$

31. $(A_2 +A_3 +A_5 + 3)\ P(31) = A_6P(10) + A_4P(12) + A_1P(21) + P(47) + P(50) + P(52)$

32. $(A_2+A_3+A_4+3)P(32) = A_6P(11) + A_5P(12) + A_1P(22) + P(48) + P(51) + P(52)$

33. $(A_1+A_5+A_6 + 3)\ P(33) = A_4P(13) + A_3P(14)+A_2P(17) + P(43) + P(53)+ P(54)$

34. $(A_1+A_4+A_6 + 3)\ P(34) = A_5P(13) + A_4p(15)+A_2P(18)+P(44) + P(53) + P(55)$

35. $(A_1+A_4+A_5 + 3)\ P(35) = A_6P(13) + A_3P(16) + A_2P(19) + P(45) + P(54) + P(55)$

36. $(A_1+A_3+A_6 + 3)\ P(36) = A_5P(14) + A_4P(15) + A_2P(20) + P(46) + P(53) + P(56)$

37. $(A_1+A_4+A_5 + 3)\ P(37) = A_5P(14) + A_3P(16) + A_2P(21) + P(47) + P(54)+ P(56)$

38. $(A_1+A_3 + A_4 + 3)\ P(38) = A_6P(15) + A_5P(16)+A_2P(21) + P(48) + P(55) + P(56)$

39. $(A_1 + A_2 + A_6+ 3)\ P(39) = A_5P(17) + A_4P(18) + A_3P(20) + P(49) + P(53)+P(57)$

40. $(A_1+A_2+A_5+ 3)\ P(40) = A_5P(17) + A_4P(19) + A_3P(21) + P(50)+ P(54) + P(57)$

41. $(A_1+A_2+A_3 + 3)\ P(41) = A_6P(18) + A_5P(19) + A_3P(22) + P(51) + P(55) + P(57)$

42. $(A_1+A_2+ A_3 + 3)\ P(42) = A_6(20) + A_5P(21) + A_4P(22) + P(52) + P(56) + P(57)$

43. $(A_5+ A_6 + 4)\ P(43) = A_4P(23) + A_3P(24) + A_2P(27) + A_1P(33) + P(58) + P(59)$

44. $(A_6+A_6 + 4)\ P(44) = A_5P(23) + A_3P(25) + A_2P(28) + A_1P(34) + P(58) + P(60)$

45. $(A_4+A_5 + 4)\ P(45) = A_6P(23) + A_3P(26) + A_2P(29) + A_1P(35) + P(59) + P(60)$

46. $(A_3+A_6 + 4)\ P(46) = A_5P(24) + A_4P(25) + A_2P(30) + A_1P(36) + P(58) + P(61)$

47. $(A_3+A_6+ 4)\ P(47) = A_6P(24) + A_4P(26) + A_2P(31) + A_1P(37) + P(59) + P(61)$

48. $(A_4 + A_5 + 4)\ P(48) = A_6P(25) + A_5P(26) + A_2P(32) + A_1P(38) + P(60) + P(61)$

49. $(A_2+A_6+4)\,P(49) = A_5P(27) + A_4P(28) + A_3P(30) + A_1P(39) + P(58) + P(62)$

50. $(A_2+A_5+4)\,P(50) = A_6P(27) + A_4P(29) + A_3P(31) + A_1P(40) + P(59) + P(62)$

51. $(A_2+A_4+4)\,P(51) = A_6P(28) + A_5P(29) + A_3P(32) + A_1P(41) + P(60) + P(62)$

52. $(A_2+A_3+4)\,P(52) = A_6P(30) + A_5P(31) + A_4P(32) + A_1P(42) + P(61) + P(62)$

53. $(A_1+A_6+4)\,P(53) = A_5P(33) + A_4P(34) + A_3P(36) + A_2P(49) + P(58) + P(63)$

54. $(A_1+A_5+4)\,P(54) = A_5P(33) + A_4P(35) + A_3P(37) + A_2P(40) + P(59) + P(63)$

55. $(A_1+A_4+4)\,P(55) = A_6P(34) + A_5P(35) + A_3P(38) + A_2P(41) + P(60) + P(63)$

56. $(A_1+A_3+4)\,P(56) = A_5P(36) + A_5P(37) + A_3P(38) + A_2P(42) + P(61) + P(63)$

57. $(A_1+A_2+4)\,P(57) = A_6P(39) + A_5P(40) + A_4P(41) + A_3P(42) + P(62) + P(63)$

58. $(A_6+5)\,P(58) = A_5P(43) + A_4P(44) + A_3P(46) + A_2P(49) + A_1P(53) + P(64)$

59. $(A_5+5)\,P(59) = A_6P(43) + A_4P(45) + A_3P(47) + A_2P(50) + A_1P(54) + P(64)$

60. $(A_4+5)\,P(60) = A_6P(44) + A_5P(45) + A_3P(48) + A_2P(51) + A_1P(55) + P(64)$

61. $(A_3+5)\,P(61) = A_6P(46) + A_5P(47) + A_4P(48) + A_2P(52) + A_1P(56) + P(64)$

62. $(A_2+5)\,P(62) = A_6P(49) + A_5P(50) + A_4P(51) + A_3P(52) + A_1P(57) + P(64)$

63. $(A_1+5)\,P(63) = A_6P(53) + A_5P(54) + A_4P(55) + A_2P(56) + A_2P(57) + P(64)$

64. $\sum_{i=1}^{64} P(i) = 1$

Appendix 4A: Characteristics of holding times.

The holding time $\underline{h}$ is a so-called stochastic variable i.e. its value cannot be predicted with certainty but only in a statistical way. The prediction is based on statistical information that has been obtained in situations in the past that are supposed to be identical to the present situation in the statistical sense. One way of specifying this statistical knowledge about the holding time $\underline{h}$ is by giving its probability density function (p.d.f.) f(t). This means that f(t) dt is the probability that $\underline{h}$ has a value between t and t + dt, or, in a notation that will be used in the sequel, that "$\underline{h}$ has a value t(+dt)".

The symbol := will be used as a definition symbol. The expression α is defined as being equal to β will be denoted by α := β. In this shorthand notation we then have,

$$f(t)\ dt := P_r\ \{\underline{h} = t\ (+dt)\} \qquad (4A.1)$$

From this definition it follows that

$$P_r\{\ a < \underline{h} < b\} = \int_a^b f(t)\ dt. \qquad (4A.2)$$

When a= 0 and b =α this a should be equal to unity:

$$\int_0^\alpha f(t)\ dt = 1 \qquad (4A.3)$$

Another way of specification is by means of the cumulative distribution function (c.d.f.) F(t):

$$F(t) = \int_0^t f(u)\, du = P_r \{\underline{h} < t\} \qquad (4A.4)$$

Equivalently, we have

$$P_r \{\underline{h} > t\} = 1 - F(t) = \int_t^\alpha f(u)\, du$$

The c.d.f is zero for t=0 and is non-decreasing, tending to unity for $t \to \alpha$

The average holding-time h is given by the first moment

$$h := \int_0^\alpha t\, f(t)\, dt \qquad (4A.5)$$

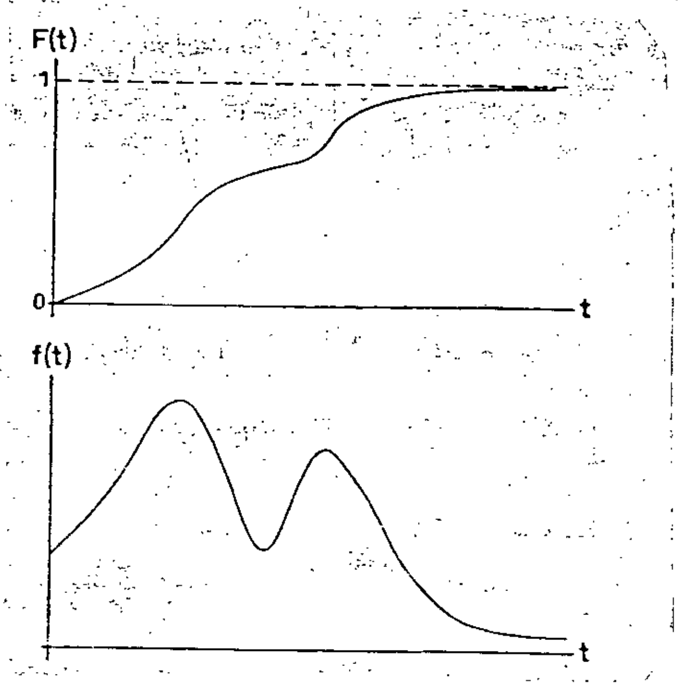

Fig. 4A. Cummulative distribution function (c.d.f) and probability distribution function (p.d.f) of holding time.

When no specification of the p.d.f (or c.d.f) is made, we shall speak about the general holding time case. The only assumption is to be made is that the c.d.f is non-decreasing between 0 and 1. Further restrictions may lead to special clases of c.d.f's. The restrictions should preferably be such that:

i) the restricted class contains c.d.f's that cover any available statistical material to a sufficient degree.

ii) the analysis using a c.d.f from the restricted class should be simpler than when it is unrestricted.

Usually c.d.f's will be supposed to be taken from the CM-class which is defined by the properties:

(C) the c.d.f is continuous.

(M) the moments are finite ($\int_0^{\alpha} t^r f(t)\,dt<\alpha$ for +ve integral r).

The property (M) can be realized in easy way, e.g., by letting the corresponding p.d.f. be equal to zero for $t > T$, where T is a prescribed upper bound.

Statistical material is always finite in number of items of data and their values. Therefore, it is never possible to conclude from any statistical material available that the c.d.f must necessarily be discontinuous. Moreover, as no observed holding-time can exceed, let us say, 100 years, some suitable upper bound T may always be found. Therefore, there will always be c.d.f's in the CM-class that are compaitable with any given statistical material. Consequently, the CM-class may be said to be completely general. Any attempt to generalize further than this class should be rejected as being "art for arts sake".

A special distribution from the CM-class that is frequently used is the:

Exponential distribution with mean h

$$f(t) = \frac{1}{h} e^{-t/h}$$

$$F(t) = 1 - e^{-t/h}$$

The fact that a quantity has an exponential distribution with mean h will often be described by stating that this quantity "is exponential (h)".

Though generalization further than CM-class is not necessary, it may occassionally be useful to adopt a special c.d.f. not included in the CM-class, such as, for example, the case of:

Constant holding-time:

$$\underline{h} = \text{const} = h,$$

$$F(t) = \begin{cases} 0 & (t<h) \\ 1 & (t>h) \end{cases}$$

In the constant holding-time case the p.d.f degenerates to an improper function (Dirac function). In some practical cases it is clear that the holding times are nearly constant. In other cases, however, there is sometimes a surprisingly good agreement with an exponential distribution, e.g., in the case of duration of telephone calls (Erlang, 1925, Molina,1927). From an analytical point of view the exponential assumption is very appropriate, as will be made clear in the following pages. For this reason this assumption is made whenever reality is far away from this distribution.

When an occupation is observed at some instant, this instant divides the holding time $\underline{h}$ into past duration or age p and a future or residual duration $\underline{r}$ of course $\underline{h} = p + \underline{r}$.

One may now investigating the p.d.f. of the residual duration $\underline{r}$, both unconditional and conditional, subject to the condition p=u.

Let f(t) be the unconditional p.d.f. of the residual duration, i.e. $f(t) . dt := P_r \{\underline{r} = t\ (+dt)\}$. The residual duration exceeds t if the observed occupation has a holding - time $\underline{h} > t$ and, moreover, the observation is made within a lapse of time $\underline{h} - t$ following its begining. Hence, $P_r \{\underline{r} > t\}$ equals the expectation of the excess of $\underline{h}$ over t (when positive), divided by the average holding-time:

$$P_r\{\underline{r} > t\} = \int_t^{\alpha} (u-t)\ f(u)\ du/h.$$

The unconditional p.d.f. of the residual duration is the inverse of the derivative of this expression,

$$f(t) = \frac{1}{h} \int_t^{\alpha} f(u)\ du = \frac{1}{h}\ \{1-F(t)\}$$

where, F(t) is the c.d.f. of the holding-time $\underline{h}$. The average residual duration is

$$E(\underline{r}) = \frac{1}{h}\int_0^{\alpha} t\ \{1-F(t)\}\ \frac{dt}{h} = \frac{t^2}{2h}\ \{1-F(t)\}\Big|_0^{\alpha} + \frac{1}{2h}\int_0^{\alpha} t^2.f(t)\ dt.$$

For, distributions from the CM-class the integrated part vanishes as

$$\lim_{t\to\alpha} t^2\ \{1-F(t)\} = \lim_{t\to\alpha} t^2 \int_t^{\alpha} f(u)\,du = \lim_{t\to\alpha} \int_t^{\alpha} u^2 f(u)\,du = 0$$

(Property M). Hence,

$$E(\underline{r}) = E\ (\underline{h}^2)/2h$$

Now, let f(t,u) be the conditional p.d.f. of the residual duration $\underline{r}$, subject to the condition

$P = u$, where $f(t,u)\ dt = P_r\ \{\underline{r} = t\ (+dt)\ \ P=u\}$.

The matter of interest is the (conditional) probability that the residual duration $\underline{r}$ should exceed a stated value t, when it is given that the present age is u, that is that the total duration is at least u. we have

$$P_r\{\underline{r} > t \mid \underline{h} > u\} = P_r\{\underline{h} > t + u \mid \underline{h} > u\}$$

$$= \frac{P_r\{\underline{h} > t + u\}}{P_r\{\underline{h} > u\}}$$

$$= \frac{1 - F(t+u)}{1 - F(u)}$$

The corresponding conditional p.d.f. $f(t,u)$ of the residual duration $\underline{r}$, given that the age is u, is obtained by differentiating this expression with respect to t, yielding the result

$$f(t,u) = \frac{f(t+u)}{1-F(u)}$$

Thus $f(t,u)\,dt$ is the probability that an occupation of age u will end after a further duration of between t and t+dt. The probability that an occupation of age t will end "immediately", that is within the interval (t,,t+dt), is $f(0,t)\,dt$. The quantity

$$g(t) := f(0,t)$$

will be called the (conditional) termination rate for occupation of age t. Thus

$$g(t) = \frac{f(t)}{1 - F(t)}$$

When the distribution is exponential $\tilde{f}(t)$, $f(t,u)$ and $f(t)$ all are equal to $e^{-t/h}/h$. Moreover, $g(t) = \frac{1}{h}$. Verbally: When the distribution of the holding-time is exponential, the residual duration has the same distribution both unconditionally and conditionally, the conditional termination rate equals the unconditional termination rate.

This is some times called the property of forgetfulness (saaty, 1961). In the early, days of life insurance the advantage of the property of foregetfulness has already been recognized, as it render analysis is very simple. It means that, when the exponential distribution holds, all predictions as to the future life chances of all individuals would be independent of age. Therefore preminus could be independent of age. Unfortunately, for insurance practice, however, this assumption has proved to be too far from reality.

If at a certain moment r occupations are obserbed which all posses the same distribution of holding-times and if they have past durations $t_1, \ldots\ldots\ldots, t_r$, the probability that one of them will end during the next interval Δt is

$$\sum_{i=1}^{r} g(t_i)\ \Delta t. \prod_{j=1, \neq i}^{r} \{1-g(t_j)\Delta t\} \approx \Delta t \sum_{j=1}^{r} g(t_i)$$

The quantity $\sum_{i=1}^{r} g(t_i)$ may be termed the total conditional termination rate. If the holding-time exponential (1), this rate reduces to r. In the exponential case knowledge, partial or complete, of the history of occupations does not influence the termination rate.

We shall need some results which are connected with g(t). Let

$$G(t) := \int_{o}^{t} g(u)\ du$$

The following relations can be easily verified:

$$g(t) = f(t) / \int_{t}^{\alpha} f(u).du$$

$$G(t) = - l_n \int_{t}^{\alpha} f(u).due$$

$$e^{-G(t)} = \int_{t}^{\alpha} f(u).du$$

$$\int_{0}^{\alpha} e^{-G(t)}\ dt = h$$

From now on we shall normally use time-scaling by reducing quantities with time dimension by a factor h. The Greek character τ will be reserved to denote "normed times": $\tau = t/h$. The characters u and v will be used to denote normed and unnormed quantities with time dimension, but only in places where the context unambiguously shows the meaning. When the p.d.f. and c.d.f in the normed time notation are given by

$f^*(\tau)$ and $F^*(\tau)$, respectively, we get

$$f^*(\tau) = h\ f(\tau h) \text{ and } F^*(\tau) = F(\tau h)$$

Moreover,

$$\int_0^\alpha \tau f^*(\ )\ d\ = 1$$

The asterisks will be dropped when there is no danger of confusion. It should be noted that

$$\int_0^\alpha e^{-G(\tau)} d\tau = 1$$

All the other foregoing results remain valid when t is replaced by τ and h by 1. In particular we may note for the case of an exponential distribution with average(1):

i) the p.d.f. and c.d.f are given by

$$f(\tau) = e^{-\tau},\ F(\tau) = 1-e^{-\tau}$$

ii) both the conditional and unconditional termination rate of single occupation are 1;

iii) the total conditional termination rate of r occupations reduces to r.

Appendix 5A: Choice of a Time Unit for the Negative Exponential Procedure.

Call generation and call release are represented by random events genrated by using a negative exponential distribution with parameter λ equal to the interarrival time and mean holding time respectively.

A set of random numbers

$$\{ \frac{u}{w} : \quad 1< u <w - 1\}$$

When $w = 2^{31} - 1 = 2147483647$ and having fair uniform distribution is generated by a linear congruence method producing the next value of u in the psendo-random number sequence from the present value of u. With appropriate λ, the mean of the negative exponential distribution, the output

$$\lambda - \quad l_n (\frac{u}{w})$$

is rounded to the nearest integer which is a necessity in the simulation program. This rounding, besides introducing error in the expected mean and variance, sets practical limitations to the value λ. This is clear from the analysis below.

If $-\lambda l_n\left(\frac{u}{w}\right)$ is rounded to the nearest integer then-

$$\left| -\lambda l_n\left(\frac{u}{w}\right) < 0.5 \right| = 0$$

So, the output gives zero with a probability

$$P_r\{-\lambda l_n\left(\frac{u}{w}\right) < 0.5\}$$

$$= 1 - e^{-\frac{1}{2\lambda}}$$

$\frac{1}{2\lambda}$ for large value of .

for λ= 1000, approximately 1 in 2000 numbers produced will be zero which is not disirable. To avoid the production of a zero completely, λ needs to be more than 2^{30} when $-\lambda l_n\left(\frac{u}{w}\right)$ is too large for the computer maximum storage 8388607. For the generation of interarrival time and holding time in this work it is recommended that λ should near about 10,000

To get this requirement fulfilled a time unit of 1000 is chosen in this work.

Appendix - 5B: The Implementation of Randomness.

Sampling from given distribution is needed in time-true simulation and multi-position roulette is necessary in roulette simulation. Both requirements are met when sampling can be done from uniformly distributed population. This goal will first be considered.

In the past gambling devices (roulette, dice, etc.) have been used, as well as table of random numbers. These methods are very unsuitable in connection with computers. In current place random numbers are placed by pseudo-random numbers. In many we call numbers "random" when we are unable or unwilling to look into the mechanism that produces those numbers. In this respect it is immaterial whether those numbers are produced by a physical randomizing device or in a mathematical way. For example,'Fisher and Yates' tables consisted of 15th ..... 19th digits of logarithms. Though there is nothing random in the actual sequence of logarithms, the mere fact that we ignore the part of the table from which those five decimal numbers stem makes us accept them as "random numbers". Computers usually handle numbers of some fixed length, say of m digits (e.g., m=9 decimals or n= 32 binary digits). Those standard length numbers are called "words" (in view of the fadt that they also may represent non- numerical information). Computers mostly have one or more subprogrammes called pseudo-random number generators. Each time such a pseudo-random number generator is called for, it produces one workd of standard length to be used as a substitute for a random number.

Many of these pseudo-random number generator incorporate a "black box" carrying out an operation of the type:

$$X_n = f(X_{n-1}) \quad \ldots \qquad (5A.1)$$

This means that any new result x_n of the generator is produced by performing some given, operation f(· · · ·) on the former result x_{n-1}. The generator is set initially by procuring a so-called seed x_o. In other pseudo-random number generators new pseudo-random numbers are constructed from two predcessors,

$$X_n = f\ (X_{n-1}, X_{n-2}) \quad \ldots \qquad (5A.2)$$

in which case two seeds are necessary.

Usually pseudo-random numbers are taken to be non- negative. Single length words consits of a finite number (m) of digits (decimal or binary). Hence, there are a finite number of different words ($M=10^m$ or 2^m respectively). This means that a generator of type (5A.1) in at most M operations will produce a word that has already been encountered. Then cycling occurs, i.e. exact repetition of the history. The length of the cycle p, called the period is < M. For generator of type (5A.2) the period is $< M^2$, as there are M^2 different word-pairs. It will be clear that too short a period is inacceptable.

The main conditions to be satisfied by a pseudo-random number generator are:

i) a large period

ii) uniformity of distribution.

iii) freedom from successive outcomes.

If the generator has maximum period M for (5A.1), M^2 for (5A.2) , each number 0, ..., M-1 appears with equal frequency and the uniformity of the distribution is assured. If not, the uniformity must be tested. One can split up the range (0,...... ,M-1) in, say, 100 intervals of (approximately) equal length and test for uniformity, e.g. by the x^2-test, on these 100 classes. A test on correlation should always be carried out. The pseudo- random number generator that have found wide spread use are:

i) The mixed congruence generator:

$$X_n = ax_{n-1} + c \pmod{M} \quad (a,c > 0) \qquad \ldots \quad (5A.3)$$

The former number X_{n-1} is multiplied by a fixed single length number a, a constant c is added and the "tail" of the result is taken X_n. The generator described has the great advantage that under favourable conditions the period is maximal (M), thus ensuring uniformity. Those conditions are:

C and M are mutually prime.

a = 1 (mod. p) for every prime factor P of M,

a = 1 (mod. 4) if 4 divides M.

ii) The Fibonacci generator:

$$X_n = X_{n-1} + X_{n-2} \ (\text{mod. } M) \qquad \ldots \qquad (5A.4)$$

This generator is very fast, as the operation of addition may be very much quicker on a computer than multiplication. Under favourable conditions (e.g. $x_o = x_1 = 1$) the period is $\frac{3}{2}$ M. Uniformity is met reasonably well.

Freedom from correlation is a condition that is not always met so very well (especially, the Fibonacci generator known to posses some correlation). The following measure has been shown to remove any co-relation drastically. A buffer with (say) 128 single word memory location is inserted between the generator to be improved and the outer world. Initially there 128 positions are filled by generator- words. From then on pairs of generator- words (say X_{2n} and X_{2n+1}) are used in combination to form pseudo-random numbers by the following two-stroke operation:

i) a word from the buffer is forwarded to the outer world as a pseudo-random number, the buffer location i (i=0,...., 127) from which to take this word is designated by a simple operation (in a binary computer: a shift of seven positions):

$$i = 128\, X_{2n}/M \quad ; \qquad \ldots \qquad (5A.5)$$

ii) This ith location is refilled with X_{2n+1}.

Assuming that computing the generator words takes most of the time, we can say that correlation has been practically removed at athe expense of some memory space and a reduction in speed by a factor 2.

From now on we shall consider the words issued by generator (either directly or via a buffer) as representing m digits of a true fraction (i.e. pseudo-random numbers are " reduced" by division by M). Hence these (reduced) pseudo-random numbers are further more >0 and <1.

A roulette is implemented very easily. Let the "total" point process consist of point of s classes with probabilities $P_1, \ldots\ldots\ldots\ldots\ldots P_s$ ($\Sigma P_i = 1$).

Then the interval 0,1 is divided into the following S subintervals I_i:

$$I_i = \sum_1^{i-1} P_j , \sum_1^{i} P_i \qquad (i=1, \ldots\ldots, s) \quad \ldots \quad (5A.6)$$

Whenever a reduced pseudo-random number is in the interval I_i, it marks a point of class i.

The pseudo-random numbering may also be used to implement sampling from some given population with c.d.f. F(t). Let, u be some reduced pseudo-random number. Then, the sample value t is determined in such a way that u= F(t). For when u and t are related in this way, the fraction of pseudo-random numbers not exceeding u and the fraction of sample values not exceeding t are equal (viz. u for any value of u). Determining t from u = f(t) is a process of inverse interpolation. This is a rather time consuming numerical process. It can best be implemented by a piecewise linearization of the c.d.f. the computer then needs a set of co-ordinate-pairs for the needs of the linear pieces and a subprogramme for carrying out the interpolation on the pieces.

L.Kosten gave the following scheme which is very adequate. The continuous distribution in question is substituted for by a discrete distribution with equal probabilities. For this purpose the interval t = 0, is, for instance, divided in the ten decile classes, all of which have the calss-probability $\frac{1}{10}$ (Fig. 5A). The averages of the ten decile classes d_o, ... d_9 are determined. Now, a reduced pseudo-random number u is drawn. Then an index i is determined from

$$i = 10\,u \qquad \ldots \qquad (5A.7)$$

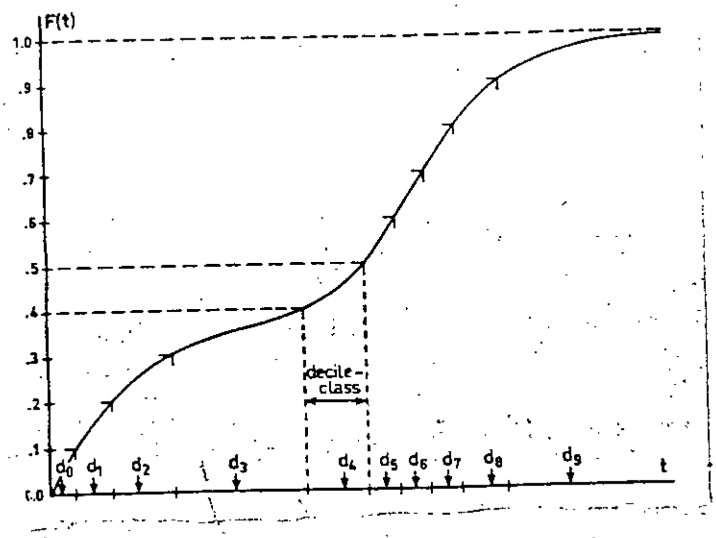

Fig. 5A: Substitution for a continuous by the uniform discrete distribution of decile-class average.

This index i takes on values 0,.....,9 with equal probabilities. The sample value for the discretized distribution then is put on t=di.

It will be obvious that, the process decribed may be very much quicker than inverse interpolation on the (linearized) c.d.f. Moreover, the discretized distribution has the same average as the original distribution, which is not the case with the piece-wise linearized function unless special measures have been taken.

If a discretization in ten decile classes is thought to be too crude, 100 (per) centile classes can be introduced at the mere expense of some memory space.

Appendix 6A: Calculation of Confidence interval

A confidence interval, calculated from the sample parameters of a random variable, earmarks a range of values, which will include the expected population mean with a certain level of certainity called the significance level. If M_i and N_i are the lost and offered calls to a connexion system in the ith sample measurement, for n such sample taken during the whole simulation run, the mean grade of service for the run is

$$G_2 = \frac{\sum_{i=1}^{n} M_i}{\sum_{i=1}^{n} N_i} \qquad (6A.1)$$

and the sample mean grade of service is

$$G_1 = \frac{1}{n} \sum_{i=1}^{n} \frac{M_i}{N_i} \qquad (6A.2)$$

With constant time sampling, both G_1 and G_2 have bias doward, the amount of the bias depending on the length of the sampling interval. This can be avoided by constant call sampling when $G_1 = G_2$.

With n-1 degrees of freedom the measured sample variance is

$$V = \frac{1}{n-1} \quad X_i - n\, G_1^2 \qquad (6A.3)$$

When $X_i = \frac{M_i}{N_i}$, $i = 1,2, \ldots\ldots, n$. The random variable X_i generally have a positive serial correlation. If the sequence $X_1, X_2, X_3 \ldots\ldots\ldots\ldots, X_n$ is considered to be Markovian with a serial correlation co-efficient PK for K lag, the sample variance correlation corrections is

$$\sigma^2 = \frac{V}{1 - \frac{2}{n(n-1)} \quad P^{n+1} - \quad \frac{P(P - (n-1)\ (1-P))}{(1-p)^2}} \qquad (6A.4)$$

Where P is given by R, the sample correlation of lag 1

$$R = \frac{n}{v(n-1)^2} \sum_{i=1}^{n-1} \frac{M_i}{N_i} - G_1 \left(\frac{M_{i+1}}{N_{i+1}} - G_2 \right) \qquad (6A.5)$$

when

$$G_1 = \frac{1}{n-1} \sum_{i=1}^{n} \frac{M_i}{N_i}, \quad G_2 = \frac{1}{n-1} \sum_{i=2}^{k} \frac{M_i}{N_i}$$

Students Distribution

Now, if μ is the expected mean grade of service, the stochastic

$$t = \frac{(G_2 - \mu)}{\sigma^2/n} \qquad (6A.6)$$

has got a probability distribution called the students distribution of n-1 degree of freedom.

Characteristic features of students distribution are

- it uses the sample variance instead of population variance.
- it approximates normal distribution for large value of n more than 100
- it depends on the number of degree of freedom.

With a two tailed test of significance if $t_{n-1}(a)$ is the value t in equation (6A.6) at the a- percentage point for n-1 degrees of freedom than 100 (1-a) percentage of the distribution lies between point $- t_{n-1}$ (a) and $t_{n-1}(a)$ because the distribution is symmetric about the zero.

Therefore,

$$P_r \{- t_{n-1}(a) < \frac{G_2 - \mu}{\sqrt{\frac{\sigma^2}{n}}} \; t_{n-1}(a)\} = 1-a \qquad (6A.7)$$

or

$$P_r \{ G_1 - t_{n-1}(a) \frac{\sigma}{\sqrt{n}} \leqslant \mu \leqslant G_2 + t_{n-1}(a) \frac{\sigma}{\sqrt{n}} \} = 1-a$$

i.e. with a probability 1-a, the random variable $G_2 - t_{n-1}(a) \frac{\sigma}{\sqrt{n}}$ lie on the opposite sides of the true mean. If a=0.05, (1-a) = 0.95, then in 95% cases the interval $G_2 \pm t_{n-1}(a) \frac{\sigma}{\sqrt{n}}$ includes the true mean.

Calculation of t

For a significance level a, $t_{n-1}(a)$ can be approximated by a polynomial

$$X_p = T - \frac{C_o + C_1 T + C_2 T^2}{1 + D_1 T + D_2 T^2 + D_3 T^3}$$

when

$$T = \sqrt{l_n} \left(\frac{1}{p} 2 \right)$$

$$P = \frac{d}{200}$$

$C_o = 2.515517$ $\quad$ $D_1 = 1.432788$

$C_1 = 0.802853$ $\quad$ $D_2 = 0.189269$

$C_2 = 0.010328$ $\quad$ $D_3 = 0.001308$