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**SIMULATION OF SUBSURFACE WATER FLOW
BY GALERKIN FINITE ELEMENT METHOD
IN THE DHAKA CITY AQUIFER**

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SIMULATION OF SUBSURFACE WATER FLOW BY GALERKIN FINITE ELEMENT METHOD IN THE DHAKA CITY AQUIFER

Submitted by

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In partial fulfillment of the requirements for the
Degree of Master of Science in Engineering (Water Resources)



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August 1998



CERTIFICATE

This is to certify that this thesis work **Simulation of Subsurface Water Flow by Galerkin Finite Element Method in the Dhaka City Aquifer** has been done by me. Neither of this thesis nor any part thereof has been submitted elsewhere for the award of any degree or diploma.



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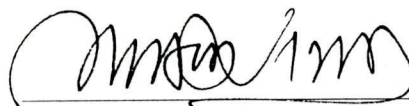
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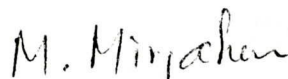
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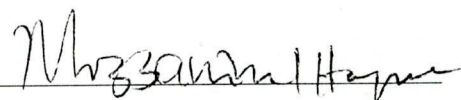
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ABSTRACT

A two-dimensional mathematical model has been developed using Galerkin finite element method for simulation of transient saturated-unsaturated groundwater flow in multiaquifer system with window based, menu-driven, user friendly interface to provide on screen input data interpretation, to guide user through steps of data processing , and to generate reports and graphs from output results. The model is able to handle combined and system-dependent boundary conditions that can be used in practical situation of groundwater flow in the complex hydrogeological conditions of a country like Bangladesh in general and particularly for Dhaka City. Due to less complexity in 2-D simulation and the availability of GUI the model is easier to use and time efficient.

The program deals with a variety of boundary conditions encountered in real hydrogeological situation consisting of simple constant or time dependent prescribed head (surface water bodies) or prescribed flux boundaries (pumping well, recharging well etc.), complicated system-dependent and atmosphere-controlled boundary including the near-surface activities i.e., precipitation, evaporation, root water extraction and surface ponding caused by excess precipitation and also boundary along the riverside having variable seepage face conditions. With a view to make the model more user friendly and efficient in operation a Graphical User Interface (GUI) is designed to handle the user input, reporting the output and graph generation. This GUI is designed and implemented using Object Oriented Programming language Visual Basic that will make the Fortran program operable in the 32-bit faster windows environment.

To test the different features incorporated in the model, a number of published schematized problems have been simulated by the program. Five such problems are presented in detail in this thesis with simulated results. The program and the modification made were tested on the following examples:

1. One-dimensional vertical soil column with the variable atmospheric boundary condition and with water extraction by plant roots,

2. One-dimensional vertical soil column without the variable atmospheric boundary condition
3. two-dimensional rectangular vertical flow domain with the variable seepage-face boundary condition, without influence of atmosphere.
4. Two-dimensional vertical flow domain representing a simplified river bank situation with both variable atmospheric and variable seepage-face boundary conditions (Coarser Grid)
5. Two-dimensional vertical flow domain representing a simplified river bank situation with both variable atmospheric and variable seepage-face boundary conditions (Finer Grid) The effect of grid fineness is tested in examples 4 and 5.

To verify the model responses in simulating a complex real hydrogeological situation Dhaka City aquifer is studied. The boundary conditions of the domain are derived from the river water level and piezometric level data. Model calibration of the parameters has been performed under this study. Simulated water balance components identify the river effect as the most dominating recharge mechanism of the area, which contributes around 60 percent of the total volume of abstraction. Urban recharge (taken 30% of the total abstraction) also contributes to the abstraction volume of the area. The results of simulation show that the approach is fruitful and can be used for detailed study of the groundwater status of the Dhaka City aquifer in future.

The program is proved to work well but have a disadvantage of being associated with the time step of the atmospheric input data, which is usually 1 day. Shorter and/or automatically set time step for the atmospheric input data, as well as for the boundary-condition subroutines, is therefore recommended.

A need has been also identified for yet more flexible treatment of combined and system-dependent boundary conditions (e.g. a combination of the variable atmospheric boundary condition with the variable seepage-face one) and for the coupling of the present model with a two-dimensional model of surface runoff in a flood plain.

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LIST OF ABBREVIATIONS

BGS	British Geological Survey
BWDB	Bangladesh Water Development Board
BUET	Bangladesh University of Engineering & Technology
DWASA	Dhaka Water Supply and Sewerage Authority
IFCDR	Institute of Flood Control and Drainage Research
PWD	Public Works Department
SWMC	Surface Water Modeling Center
USGS	United States Geographical Survey

CHAPTER 1

INTRODUCTION



1.1 General

Groundwater is an often unnoticed and unacknowledged cornerstone in the foundation of many regional economic and environmental systems. It is far more reliable as source of supply than the surface water and if protected can provide potable water of quality. Groundwater serves as critical source of water supply for agriculture and municipal uses, which is related to poverty alleviation, health and social vulnerability. Groundwater provides base-flow in many streams and rivers. It is the major source of water for many surface vegetation communities and critical to the maintenance of most wetlands and linked freshwater ecosystems. Groundwater is also the primary buffer against drought.

Though most of the groundwater is located in the saturated zone, the infiltration and evaporation and hence the recharge of groundwater is controlled by the unsaturated zone. The processes that occur in the unsaturated zone are more complicated, both physically and mathematically, than those in the saturated zone due to the nonlinearity of the equations governing the unsaturated flows. The heterogeneity of soil together with the spatial and temporal variability of complicated boundary conditions compounds the matter even further. Atmospheric boundary condition associated with seepage faces, infiltration and evaporation are the main processes occurring there which make frequent switching between different types of boundary conditions. Although certain simple flow problems in the saturated and unsaturated zones can be solved analytically for homogeneous soils, such solutions of most of flow problems for heterogeneous soils are not available; numerical methods are necessary to approximate them.

The geologic, hydrologic and climatic conditions of Bangladesh are favorable for occurrence and development of large quantities of groundwater. Groundwater is the principal source of freshwater drinking and sanitation purposes in Bangladesh. Also the dry season irrigation is predominantly based on groundwater abstraction. The

period of rapid groundwater exploitation for irrigation started in early seventies and continued by increasing the wells at a compound rate. The consequences of these practices are now apparent. In the area of overexploitation, the lowering of groundwater level is occurring at a rate such that rainy season replenishment is not sufficient to cover up this declining trend ultimately creating an adverse affect on the environment (MPO, 1989).

In case of densely populated urban areas like Dhaka city, a huge amount of water is required for drinking and sanitation purposes and the demand is exclusively (95% of total abstraction) fulfilled from the tubewell abstractions of groundwater from the underlying aquifers of Dhaka city. There are approximately six hundred deep tubewells installed in the main aquifer. The situation of Dhaka City has become more critical due to unprecedented growth rate of Dhaka City both in terms of population and size. The high demand of water by this densely habitat city is reflected from the fact that the respective water supply authority (DWASA) has to increase the production of drinking water supply by more than six times since the sixties but still the supply is far lagging behind the demand. As a result, beneath Dhaka City, water levels in the aquifers have fallen steadily over the last twenty-five years in response of ever increasing abstraction. So the groundwater resource evaluation, prediction and management bear much attention at this time in Bangladesh where mathematical modeling as a useful, sophisticated and modern tool has been used by engineers and planners to some extent.

Mathematical modeling of flow and transport phenomena in porous media is an increasingly important tool in the management of water resources. Groundwater model used for different development scenario serves to study the probable management policies. These models are the abstractions of the complex real world situations achieved by making a considerable number of simplifying assumptions about the geology, hydraulic properties and water balance. It is obvious that an aquifer system or a part of the system can be modeled in a great number of ways depending on the assumptions made in order to simplify the real physical system starting from basic mathematics involved in it. The conceptual simple water balance, the finite difference method and the finite element method are the most frequently used methods for this purpose. Finite element is the most sophisticated method now a

days used because of its some definite advantages. Starting from a couple of decades back, at present much numerical experimentation and researches are devoted to this method.

A real aquifer system is composed of irregular geometry and multiple layers with different geophysical and hydraulic characteristics. The aquifer excitations are also different in different parts of the aquifer depending on the land use pattern and other human activities and the resulting aquifer responses are subjected to spatial and temporal variations depending on the boundary and atmospheric conditions. In case of Bangladesh, the aquifers are further complicated with respect to physical structure and boundary conditions. In monsoon period there is heavy rainfall over the country upto a magnitude of 2200 mm per annum. This heavy rainfall associated with the external cross-border flow of water through the rivers causes flooding of the lands and long term submergence of a considerable portion of the country. So aquifers are replenished directly from the precipitation-infiltration process, from the accumulated standing water over the submerged lands, the occurrence and distribution of which vary with time. On the other hand, the rivers, which partially or fully penetrate the aquifers or overlying aquitards, fed the aquifers naturally, or in induced way when heavy pumping near the river occurs. The aquifers are also of compound nature with variable properties of soil creating multiple resistance layers partially confining the high yielding porous materials those serves as good storage. In case of urban areas, like densely populated Dhaka City, human activities in the area reduces the natural replenishment from the surface but contributed considerably in other way by return flows from the sanitary and industrial water supplies.

Although till now, a large number of numerical models in the finite element method have been developed, the number of models handling real problems are very few. A finite element modeling that deals with three-dimensional simulation of saturated-unsaturated flow for a strip of Dhaka City besides the Buriganga River has been carried out by Bhuiyan (1995) for his M. Sc. thesis. This work was not incorporated with user friendly interface to handle Input-Output and also was cumbersome and time consuming due to complexity of 3-D simulation. A two-dimensional groundwater model for a multiaquifer system has been developed by Hoque (1988) using Galerkin approach of finite element method. The model is able to simulate a

variety of problems but only for saturated flow conditions. A test application of the model has been done in Ganges Kobadak irrigation project using the available hydrological and hydrogeological data.

The present work is devoted to develop a two-dimensional groundwater model that can handle a situation like Dhaka City. The saturated and unsaturated flows are treated simultaneously. Galerkin finite element method is used to develop a two-dimensional numerical algorithm for subsurface flow by solving the governing flow equation – the Richards equation. A modified form of the van Genuchten's (1985) closed-form functions as proposed by Vogel and Cislerova (1988) is used to identify soil moisture characteristics in unsaturated zone.

The flow domain is sub-divided into a network of triangular elements with the corners of the triangles as nodes. A fully implicit finite difference scheme is used for time discretization and special treatment of the time derivative term is given to ensure mass conservation at each time step as proposed by Celia et al. (1990). The spatial discretization transforms the continuous partial differential equation into semidiscrete system of ordinary differential equations, which is transformed into an algebraic set of highly nonlinear equations employing time discretization. These equations are linearized by Picard iteration scheme and then solved by Gaussian elimination to find the unknown values. Prior to solving the set of equations proper boundary conditions – system independent Dirichlet, Neuman and Mixed type boundary conditions and system dependent atmospheric and seepage face boundary conditions are incorporated in the model.

With a view to make the model more user friendly and efficient in operation a Graphical User Interface (GUI) is designed to handle the user input, reporting the output and graph generation. This GUI is designed and implemented using Object Oriented Programming language Visual Basic that makes the Fortran program operable in the 32-bit faster windows environment.

1.2 Objectives and Scope of the Study

The objectives of the study are as follows:

- (a) to develop a two-dimensional model to simulate the subsurface flow of water in aquifer system using the Galerkin-type finite element method;
- (b) to make the model interactive, user-friendly and efficient in operation, a Graphical User Interface (GUI) is designed using Object Oriented Programming language Visual Basic for handling the user input, output and graph generation;
- (c) to apply the proposed model to some published subsurface flow problems to examine its numerical efficiency and accuracy and to test the GUI developed and then to a real case – the Dhaka city region.

The outcome of this work is to develop a 2-D subsurface flow model with window based, menu-driven, user friendly interface to provide on screen input data interpretation, to guide user through steps of data processing , and to generate reports and graphs from output results. The model is able to handle combined and system-dependent boundary conditions that can be used in practical situation of groundwater flow in the complex hydrogeological conditions of a country like Bangladesh in general and particularly for Dhaka City. Due to less complexity in 2-D simulation and the availability of GUI the model is easier to use and time efficient.

1.3 Organization of the Thesis

The thesis is composed of eight chapters in total. Up to chapter Five, related literature and theoretical aspects are discussed. Chapter Six presents the Graphical User Interface (GUI) description. The results of model testing and applications are included in chapter Seven. The conclusions and recommendations with respect to the published example testing and Dhaka City modeling are presented in chapter Eight.

The review of literature with special focus on the finite difference and finite element models are presented in chapter Two. A recapitulation of the mathematical

background of the model development ,which describes Galerkin finite element formulation with the adopted solution scheme and iteration strategy, is given in Chapter Three. In chapter Four and Five the implemented auxiliary features of the model are discussed in brief. Chapter Six describes in details how the GUI is developed and how it works.

Chapter Seven consists mainly of two parts; one dealing with the results of simulation for model testing for different schematized problems and the subsequent part describes application of the model to Dhaka City aquifer. The results obtained from each such application are discussed and accompanied with generated graphs, as well as basic data that are indispensable for the definition of problems studied and for understanding the results, in the corresponding subsections in a sequential order.

CHAPTER 2

REVIEW OF LITERATURE

2.1 Introduction

Groundwater or subsurface water, is a term used to denote all the waters found beneath the surface of the ground in the void spaces of the soil. Groundwater flow includes the much deeper, saturated regime that have an important influence on many geologic processes and also encompasses the near-surface, unsaturated, soil-moisture regime that plays an important role in the hydrologic cycle. Practically all groundwater can be thought of as part of the hydrologic cycle, which in simplest way is the earth's endless water circulatory system between ocean, atmosphere and land (Figure. 2.1).

The science of occurrence, distribution and movement of water below the earth surface is called groundwater hydrology. The study of groundwater requires knowledge of many of the basic principles of geology, physics, chemistry and mathematics. Geology provides a qualitative knowledge of the framework of flow, but it is physics and chemistry that provides the tools for quantitative analysis. Groundwater hydrology is a quantitative science, so it should come as no surprise to find that mathematics is its language, or at the very least one of its principal dialects.

Groundwater development dates back to the ancient times. Starting from the seventeenth century when a clear understanding of groundwater as a hydrological cycle was developed, up to now, a large number of scientists and engineers have devoted their research in this field. The study of groundwater is germane to geologists, hydrologists, soil scientists, foresters, geographers, ecologists, engineers, chemists, mathematicians, petroleum and agricultural scientists and many others. So motivated, the present chapter is an attempt to take an integrated view of concepts, ideas and methods developed in agricultural engineering, soil physics, hydrogeology, petroleum engineering, civil engineering, geophysics and related fields.

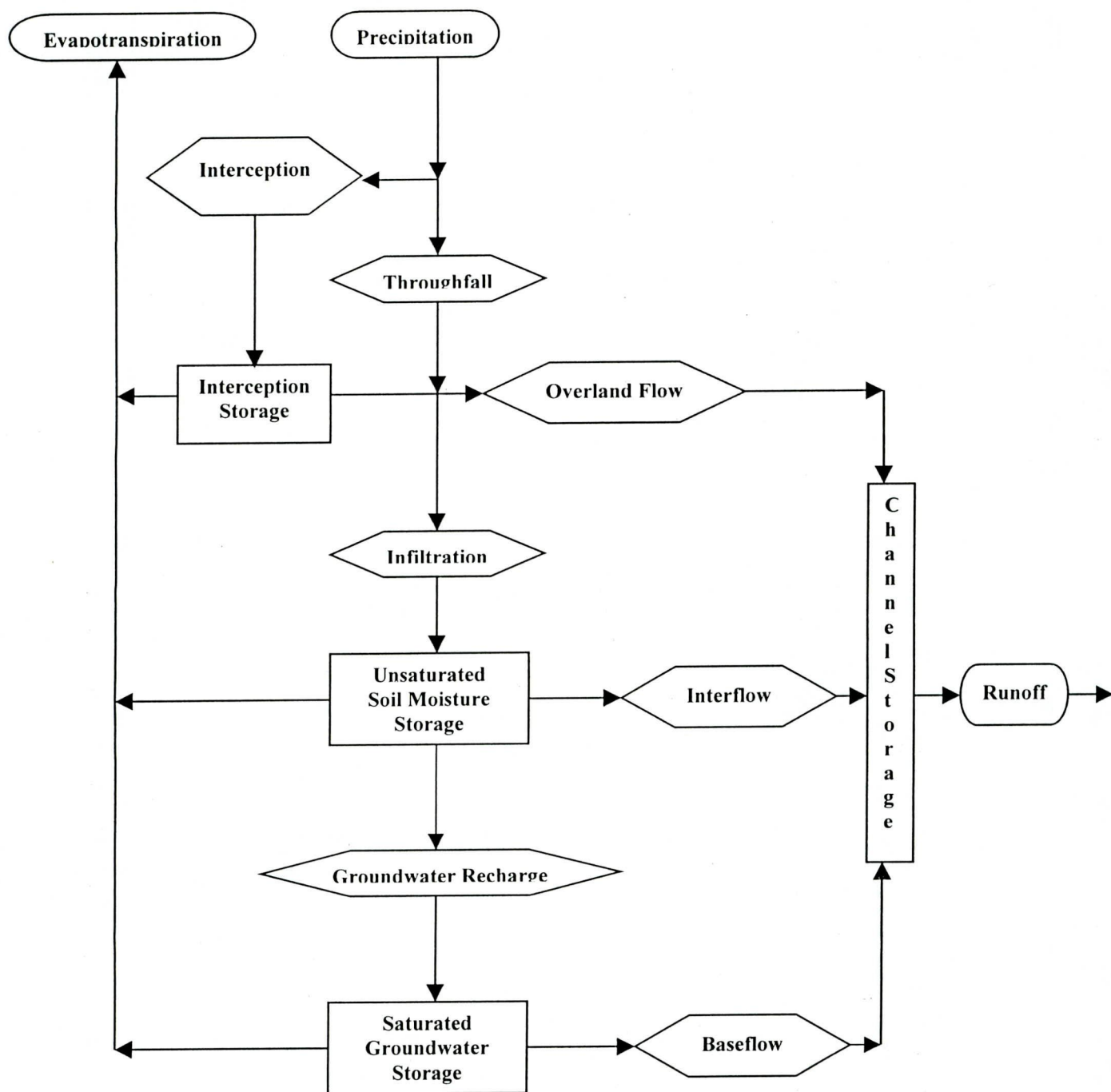


Figure 2.1 Systems Representation of the Hydrologic Cycle

2.2 Development of Groundwater Theories

Groundwater exploitation preceded the establishment of the rudiments of groundwater science by many centuries. The source of groundwater remained unproven, if not undisclosed, until the later part of the 17th century when some European scientists

(e.g., *Pierre Perrault*, 1674; *Edone Mariotte*, 1686) concluded from rainfall-runoff measurements that yearly precipitation volume was high enough with respect to river flow which can contribute to groundwater body and other surface water bodies. Before that it was widely believed that earth is practically impermeable by rainwater.

2.2.1 Darcy in Context

The birth of groundwater hydrology as a quantitative science can be traced to the year 1856. It was the year that a French hydraulic engineer named *Henry Darcy* published his report on the water supply of the city of Dijon , France. In the report Darcy described a laboratory experiment that he had carried out to analyze the flow of water through sands. The results of his experiment can be generalized into the empirical law that now bears his name. *Darcy's law* has come into play such a central role in the study of flow of fluids in the Earth's subsurface that Darcy's work gives the appearance of the start of a new era. Darcy's work was unique in that it dealt with a natural material rather than an engineered material and that he accounted for gravity. Darcy's (1856) law, in its generalized form, remains today the basis of the physical theory of soil water movement both in saturated and in unsaturated zone.

Other noteworthy developments were taking place in the sciences during the 18th century and the first half of the 19th century. In his most important work, *Hydrodynamica*, published in 1738, *Daniel Bernoulli* of Switzerland identified the three components that make up the mechanical energy of a moving fluid; potential energy due to gravity, elastic energy due to fluid pressure, and kinetic energy. *Poiseuille* (1842), a medical doctor interested in the mechanisms of blood flow through human and animal veins, derived the equations governing the flow of liquids in fine capillary tubes. In Germany *Hagen* (1939) had experimentally obtained the same results.

2.2.2 Second Half of the 19th Century

Immediately following Darcy's insightful contribution, the 19th century saw the development of the basis for quantitative description of groundwater motion. By integrating the equation of motion over a radial domain, *Jules-Juvenal Dupuit* (1863)

derived solutions for steady flow in a confined aquifer and in an unconfined aquifer. The groundwater movement in shallow unconfined aquifers is practically horizontal due to almost vertical equipotentials that give the driving force for the flow as the slope of the water table with little error. This assumption of horizontal flow he made in the case of a gravity well in an unconfined aquifer, while yet accounting for the variation in the saturated thickness of the aquifer, is used even now and is referred to as the Dupuit assumption (*Dupuit*, 1863).

Another important figure of this period was *Joseph Boussinesq*, who, in investigating the role of friction in laminar flow of fluids, derived the expression for the flow of water in an idealized parallel-plate fracture, now referred to as the cubic law (*Boussinesq*, 1968).

In Germany, *Adolf Thiem* (1870) independently derived the expressions for steady radial flow of water in confined and unconfined aquifers. In the field of groundwater hydrology, *Gunther Thiem* (*Thiem*, 1906) is widely known for the equation describing the steady radial flow of water in a confined aquifer, although the solution was derived earlier by Dupuit. *Philipp Forchheimer* (1886) introduced the concepts of isopotential lines and streamlines, conformal mapping and the construction of flow nets, the method of images, and the theory of functions of the complex variable. Forchheimer formally wrote down the Laplace Equation (*Forchheimer*, 1898) to describe the steady flow of groundwater and went on to use mathematical techniques such as conformal mapping to solve problems. He was the first to solve the problem of groundwater flow in a semi-infinite water-yielding formation bounded by a perennial stream and the problem of a well discharging from a water-yielding formation that is supplied by uniform recharge. In addition to formally explaining the results of earlier workers such as Dupuit and Theim, Forchheimer presented new results for single wells as well as groups of wells and sloping aquifers.

In the United States *Slichter* (1899) pioneered the study of groundwater systems by mathematically analyzing the steady flow of water through geologic media. In particular, he investigated mutual interference between artesian wells and the perturbation of the regional steady state groundwater flow field by producing water well. Another important contribution of Slichter was that he investigated the physical

significance of hydraulic conductivity, which was merely treated as an empirical coefficient by Darcy.

2.2.3 The Turn of the 20th Century

In the twentieth century, increased activity in all phases of groundwater hydrology has occurred. *Boussinesq* (1903-4) first addressed the problem of the falling water table and introduced the concept of specific yield to take into account the contribution of drainage from the unsaturated zone to the flux through the water table.

Boussinesq equation, however, does not consider the physics of water movement in the unsaturated soil. *Edgar Buckingham* (1907), a leading physicist of the time, appears to be the first worker to address the transient migration of fluids in the Earth's subsurface. Studying the movement of moisture in unsaturated soils theoretically and experimentally, he defined capillary potential and proposed an equation similar in form to Darcy's law to describe the movement of moisture in unsaturated soils. A special of this formulation was that the hydraulic conductivity was treated as a function of capillary potential rather than simply being a constant, as in Darcy's law.

The early 20th century saw the simultaneous recognition of the importance of time in many Earth science disciplines. Researchers in these fields recognized that almost all subsurface fluid flow systems are dynamic in nature. In the field of soil physics *Green and Ampt* (1911) proposed a simple approximation to quantify the vertical infiltration of water into an unsaturated soil. The Green and Ampt idealization assumes that as water infiltrates into a soil, a sharp, piston-like zone of saturation advances with time. This approximation is still used in interpreting field data from infiltrometer tests to estimate in situ hydraulic conductivity of soils.

2.2.4 The Remarkable Era: 1920-1940

In the evolution of ideas pertaining the flow of fluids in geological media, the period 1920-1940 must rank as truly remarkable. Conceptual as well as mathematical contributions of fundamental importance relating to the dynamic behavior of fluid

flow systems was made simultaneously in the fields of soil physics, petroleum engineering, groundwater hydrology and civil engineering.

Willard Gardner was among the earliest (*Gardner and Widtsoe*, 1921) to attempt to quantify nonsteady moisture movement in unsaturated soils in terms of a transient diffusion equation analogous to Fourier's transient heat conduction equation. A major contribution of the 1920s was the paper by *Meinzer* (1928), whom many would consider to be the founder of the discipline of groundwater hydrology in the United States. Meinzer evaluated early studies on the framework of principles and methodologies for investigations of the occurrence and distribution of groundwater and provided the first manual for groundwater hydrologists. In his paper, Meinzer had described the physics of hydraulic capacitance in saturated geologic materials, which would be described as "storage coefficient" by Theis a decade later.

Combining Buckingham's (1907) work on the equation of water motion in unsaturated soils with the newly available soil moisture retention curves, *Richards* (1931) formally wrote down, for the first time, the nonlinear partial differential equation describing transient flow of water in unsaturated soils. At present most of the studies of soil water movement is based on *Richards equation* (1931).

In the field of groundwater hydrology, *Theis* (1935) set up and obtained a solution to the parabolic equation describing non-steady flow to a laterally infinite aquifer with a well of vanishingly small radius (line-source well) producing at a constant rate. Theis' work proved to be a milestone not only in groundwater hydrology but also in the Earth sciences in general. In addition to constituting the basic and simplest technique used widely for interpreting data from transient aquifer tests, the Theis' model is also frequently used as the standard against which the transient behavior of more complex aquifers is studied for comparison.

2.2.5 The Second Half of the 20th Century

By the early 1940s the physical and mathematical foundation for transient movement of fluids in subsurface geological systems involving porous medium deformation, desaturation, and fluid expansion had been clearly formalized. The succeeding half a

century, extending to the present, has seen the application of these basic ideas and tools to field conditions involving a wide variety of flow geometry, initial conditions and forcing functions.

Because of the difficulties of obtaining closed-form solutions to the nonlinear differential equations, Richards equation remained unsolved for nearly two decades. It would be the early 1950s before *Childs and Collis-George* (1950) showed that the severity of nonlinearity of the parabolic equation could be lessened by using volumetric moisture content as the dependent variable, rather than capillary potential. Following the suggestions, *Klute* (1952), *Philip* (1955), and others began obtaining solutions for Richards' equation under highly simplified conditions using numerical methods.

Because of the nonlinearity of Richards' equation it was evident that inversion of field data on unsaturated hydraulic conductivity would not be possible unless some simple relationship between hydraulic conductivity and gauge pressure head could be assumed. On the basis of available data, *Gardner* (1957) found that over a limited range of gauge pressure, one could reasonably assume an exponential relationship between hydraulic conductivity and gauge pressure head. Gardner then went out to point out that an exponential relation helps to obtain closed-form solution for the one-dimensional problem. This work of Gardner and the work of *Philip* (1955) continue to influence present day research relating to hydraulic characterization of unsaturated soils in the field.

Boulton (1954) initiated investigation of the transient flow of water to a well in an unconfined aquifer. Instead of venturing to rigorously solve the highly complex flow process above the water table as embodied in Richards' equation, Boulton simplified the effect of the unsaturated zone by introducing the approximation of delayed yield in conjunction with the notion of specific yield. Boulton's model still continues to be used by groundwater hydrologists as the basis for estimating parameters of an unconfined aquifer.

Childs (1945) and *Youngs* (1957) described that both the soil water pressure head and the soil moisture content approach constant values during a prolonged vertical

infiltration in long columns, in which the hydraulic conductivity therefore equals to vertical downward flux. This effect has been used for the measurement of hydraulic conductivity on unsaturated porous materials (*Childs and Collis-George*, 1950). For upward movement caused by the evaporation at the soil surface or by root water uptake, it was found that the water movement can be limited by the soil conditions, being dependent on the depth of the water table as well as the soil hydraulic properties (*Gardner*, 1958; *Gardner and firemen*, 1958).

For nonsteady-state conditions, the mechanisms of water flow and water retention in unsaturated soils interfere, which is reflected in diffusion formalism of the underlying mathematical theory. *Childs and Collis-George* (1950) defined the soil water diffusivity by rearranging the terms in Richards equation and suggested the Diffusion-type Richards equation as suitable for possible solution of vertical infiltration from a ponded soil surface into a uniform porous material when the effect of gravity is negligible. *Philip* (1955), utilizing the results of *Crank and Henry* (1949), recognized that the flow equation with the earth gravity neglected becomes a nonlinear diffusion equation, the solution of which with the ponded infiltration boundary conditions can be obtained using a numerical iterative method.

A significant research direction of the 1960s was the development of numerical models. The digital computer provided the possibility of solving transient fluid flow problems in complex geological systems which are far beyond the reach of closed-form solutions. The finite element method (*Clough*, 1960) which was initially designed for solving structural engineering problems, was soon adopted to solve steady state and transient problems of groundwater flow (*Javandel and Witherspoon*, 1968). *Fayers and Sheldon* (1962) illustrated the use of a digital computer to solve fluid flow problems in three dimensions using the classical finite difference approximations. *Tyson and Weber* (1964) presented an integral form of finite difference method which could efficiently handle groundwater systems with complex geometry.

The 1970s witnessed a shift in research emphasis among Earth scientists from issues based on resource development to issues related to environmental degradation. During the 1980s groundwater contamination arising from leak gasoline tanks and from

uncontrolled disposal of industrial hydrocarbons came into unexpectedly sharp focus. The 1980s also saw active research designed to understand the role of water in influencing natural earthquakes.

In the case of unsaturated flow theory existing paradoxes and realities are discussed by *Gray and Hassanizadeh* (1991) and they gave unsaturated flow theory including interfacial phenomena and advanced theory of multiphase flow in general. In recent decades, much attention has been paid to hill-slope hydrology, as attested by volumes edited by *Kirby* (1978) and *Abrahams* (1986). In this communication *Philip* (1991) developed extensions to infiltration theory for horizontal land surfaces needed to embrace hill-slope conditions.

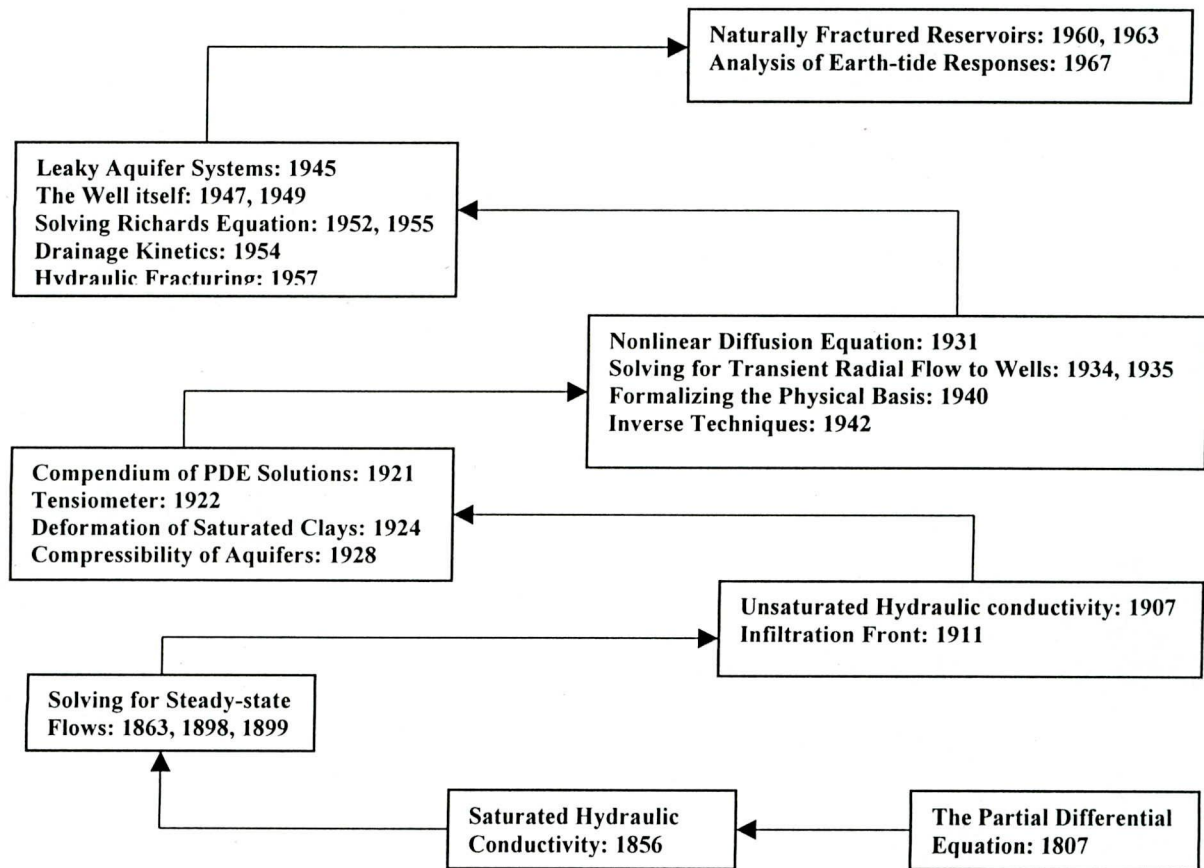


Figure 2.2. Chronology of Important Developments

Presently, hydrogeologic systems are studied by researchers with interests varying widely, from the shallow soils in which plant roots thrive to the oceanic crust beneath the deep oceans in which hydrothermal fluids circulate. The motivations of these studies vary from engineering design to pure curiosity. And till now methods and

theories are being developed to quantify the movement of the air, gases and vapors through the vadose in addition to moisture movement.

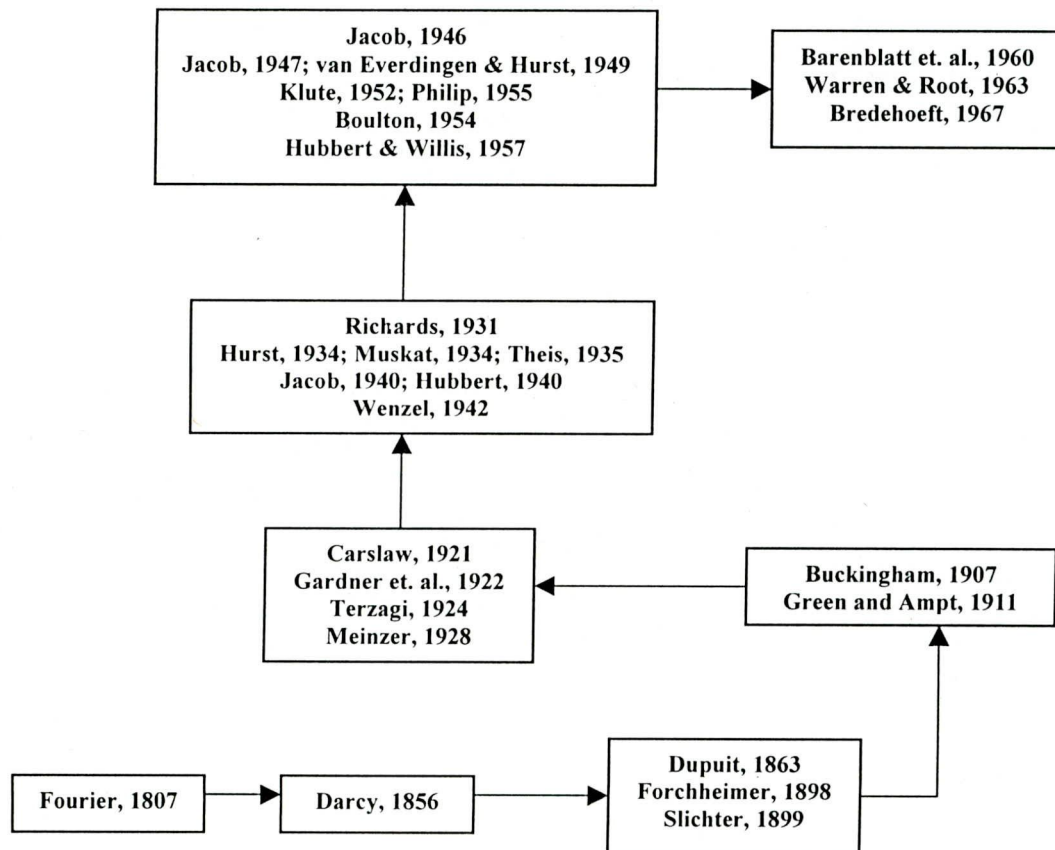


Figure 2.3. Chronology of Important Papers

2.3 Theory of Multidimensional Flow

Two- and three-dimensional soil-water flow problems that arise, for example, when water is supplied from sources of finite areal dimensions on the surface, present a more difficult problem for analysis than the one-dimensional flow. In these cases analytical solutions to Richard's equation have been possible only for particular mathematical forms of the relationships between soil-water properties and are as good as those relationships which describe the properties of the given soil (e. g., *Wooding*, 1968; *Philip*, 1969). *Weachter and Philip* (1985) recognized that an analogy exists between the quasi-steady absorption of water from cavities and the scattering of the plane acoustic waves around soft obstacles. Large series of analytical solutions has resulted from this recognition of absorption and infiltration from the cavities of

different shapes, as well as for water exclusion from empty subterranean holes (e. g., *Philip*, 1968; *Philip*, 1989; *Philip et al*, 1989).

2.4 Solution of Groundwater Flow Problems

Groundwater model is an integrated mathematical representation of the hydrogeologic processes taking place within the basins and aquifer systems. The elementary need of the model is to know some unknown values (e. g., water level, volume of water that can be abstracted safely, etc.) incorporating some known values (e. g., properties of the water bearing or transferring stratum) in the equations which are established as representation of the physical hydrogeological processes or mechanisms for a set of field conditions (*Thomas*, 1973).

Groundwater flow problems are initial and final boundary value problems. And this aspect necessitates development of different types of model. In principle three classes of methods exist for solving such problems - analytical methods, methods based on the use of physical models and analogs and numerical methods which are mathematical models.

For a number of problems involving linear or quasi linear equations and region of simple and regular geometry, it is possible to obtain exact solutions by analytical methods. Scale models like sand box model and analogs or electric analog, membrane analogs were used for some special purposes. For most regional studies of practical interest of irregular shaped aquifer with non-homogeneous or anisotropic materials and variable boundary none of the first two methods are suitable. Computer based numerical methods are, nowadays, the major tool for solving large scale groundwater forecasting problems as encountered in practice. In last few decades, parallel to the advance in computer technology, much effort has been devoted, in many parts of the world, to the development of the methodology and techniques for numerical solution of the mathematical formulation of the problem.

As per the conceptual basis of construction of models, groundwater model can be single cell model or multi-cell model. Single cell model is perhaps the simplest model, which visualizes an entire basin as a single cell. The underlying assumption is

that average conditions (e. g., an average water table elevation) suffice to describe the behavior of the aquifer cell. The single cell model comprises a horizontal area bounded by impervious boundary. In case of multi-cell model the investigated aquifer is divided into a number of cells with the considerations of the basic ideas of single cell model. The major difference lies in the assumption that the horizontal boundaries of each cells are pervious and flows occur across the boundaries (*Bear*, 1978).

2.4.1 Analytical Solutions

Although limited in application, analytical models are useful in aquifer test analysis; evaluation of simple aquifer systems; design, calibration, or verification of numerical models; and understanding basic principles of groundwater flow and transport. Advances in analytical modeling virtually dominated groundwater literature through the 1950s. A large number of analytical models structured to solve partial differential equations governing the flow of groundwater to and from wells have been developed. Among others, contributors for non-steady artesian aquifer are *Theis* (1835), *Hantush* (1959, 1964), *Papadopoulos* (1965, 1967). *Boulton and Streltsova* (1977 a, b), for leaky artesian aquifer with negligible aquitard storage and isotropic aquifers are *Hantush* (1959), *Witherspoon* (1967), for water table aquifers are *Boulton* (1963), *Neuman* (1975), *Boulton and Streltsova* (1976, 1978). A few analytical models involving finite single boundary and multiboundary aquifer systems have been developed (e. g., *Stallman*, 1963; *Vandenberg*, 1977 etc.). A brief review of the models developed upto 1979 is given by *Walton* (1979).

For unsaturated flow case, initially, most solutions were applied only to one-dimensional, horizontal or vertical systems. Highly significant is the early work of *Philip* (1955, 1957) that provided much of the physical and mathematical groundwork for subsequent analysis by others. Numerous studies have followed for both one- and multi-dimensional problems (*Parlange*, 1971; *Warrick*, 1974; *Babu*, 1976; *Raats*, 1976; *Lomen and Warrick*, 1976; *Batu*, 1979; among many others).

The Richards equation describing a one-dimensional steady-state flow is a non-linear, but simple, ordinary differential equation of the second order, which can be easily integrated once with respect to the spatial coordinate. The second integration requires

that a certain parametric relation be adopted between hydraulic conductivity and the soil water metric head (e. g. the *Gardner's*(1958) *exponential formula*). Based on this iterative concept, *Philip* (1957a) has derived his series solution for one-dimensional vertical infiltration. Since the series does not converge for large time, it was rather the simplified semi-empirical two-term *Philip's* (1957b) *infiltration equation*, which became popular. *Philip* (1960a, b, c) has shown that the nonlinear diffusion problem can be solved analytically if the diffusivity-moisture content relation belongs to a certain class of functions. Later, new quasi-analytical techniques have been suggested by several authors for the solution of nonlinear diffusion and vertical infiltration problems (e.g., *Parlange*, 1971a, b; *Philip and Knight*, 1974), with an increasing accent on infiltration from rain of constant or variable intensity, i. e. on the Neuman-type boundary condition (e. g., *Smith and Parlange*, 1978; *White et al.*, 1979; *Clothier et al.*, 1981; *Broadbridge and White*, 1988). Another quasi-analytical solution for infiltration on planar slopes was developed by *Philip* in 1991.

Other one-dimensional processes, like the water redistribution after infiltration (*Youngs and Pouloussilis*, 1976) and transient or quasi-steady column drainage (*Swartzendruber*, 1969; *Raats and Gardner*, 1974) received less theoretical attention, particularly because they are less apt to an analytic solution. The effect of hysteresis in the soil moisture content - matric head relation must be inevitably taken into account when treating the process of water redistribution after infiltration.

2.4.2 Numerical Methods

During the past several decades, there has been considerable progress in the conceptual understanding and mathematical description of soil water movement in the unsaturated zone. The development of high-speed computers made it possible to solve the complicated mathematical governing equations of soil water movement using numerical approximate and iterative techniques. A variety of numerical models are now available to predict water transfer processes between the soil surface and groundwater table.

Regarding the techniques of solution there are currently five widely used numerical methods exist (*Huyakorn and Pinder*, 1983). These are finite difference methods,

Galerkin or variational finite element methods, collocation methods, method of characteristics and boundary element methods. These methods are closely related. In several cases the finite difference, finite element and the collocation methods yield the same approximation. The method of characteristics is a variant of the finite difference method and is particularly suitable for solving hyperbolic equations. Finally the boundary element method, a variant of the conventional finite element method, is especially useful in the solution of elliptic equations for which Green's functions exist.

Each numerical approximation leads to a form of matrix equation, by combining the resultant algebraic equations for each node point, which can be solved numerically by one of the two basic methods: direct or iterative.

Direct methods can be further subdivided into: (1) solution by determinants, (2) Solution by successive elimination by the unknowns, and solution by matrix inversion. According to *Narasimhan and Witherspoon* (1977), perhaps the most widely used direct approach for transient problems is that of successive elimination and back substitution, which includes the Gaussian elimination methods (*Scarborough*, 1966) and the Cholesky decomposition method (*Weaver*, 1967). Large storage requirements of the sparse matrices (containing many zero values) and accumulation of round off are the main disadvantages of direct methods.

Iterative schemes avoid the need for storing large matrices, which make them attractive for solving problems with many unknowns. Numerous schemes have been developed; a few of the more commonly used ones include successive over-relaxation methods (*Verga*, 1962), alternating direction implicit procedure (*Wachpress and Habtler*, 1960), and the strongly implicit procedure (*Stone*, 1968). The iterative approach is less desirable for solving steady-state problems (*Narasimhan and Witherspoon*, 1977), because its efficiency depends on the initial estimate of the solution. The same authors also point out that the greatest limitation of the iterative schemes is the requirement that the matrix be well conditioned. An ill-conditioned matrix can drastically affect the rate of convergence or even prevent convergence.

More recently, a semi-iterative method has gained popularity (*Gresho*, 1968). This method, or class of methods, is known as conjugate gradient methods. It is widely

used to solve linear algebraic equations where the coefficient matrix is sparse and square (*Concus et al.*, 1976). *Kuiper* (1987) compared the efficiency of seventeen different iterative methods for solution of the nonlinear three-dimensional groundwater flow equation. He concluded that, in general, the conjugate gradient method did the best.

The two most widely accepted numerical methods for solving the groundwater flow problems are the finite-difference method and the finite-element method. A comprehensive treatment of the application of these numerical methods to groundwater flow problems is presented by *Remson et al.*(1971) and *Wang and Anderson*(1982). An overview of simulation modeling of soil water flow in the unsaturated zone is presented by *Feddes et al.*(1988).

2.4.2.1 Finite Difference Method

Finite difference methods (*Remson et al.*, 1971), with explicit, implicit or mixed difference schemes, belong to the most frequently used techniques in modeling unsaturated flow. The fundamental idea here is to replace all derivatives by finite differences and thus reduce the original continuous boundary value problem to a discrete set of simultaneous algebraic equations. The advantage of the finite difference method is in its simplicity and efficiency in treating the time derivatives. On the other hand, the method is rather incapable to deal with complex geometries of flow regions. A slow convergence, a restriction to bilinear grids and difficulties in treating moving boundary conditions are other serious drawbacks of the method. In dealing with nonuniform soils of complex geometry and arbitrary anisotropy, the finite difference approach is often difficult to apply. The treatment of system-dependent atmospheric boundaries, such as seepage faces and evaporation surfaces, is difficult with this approach, too, because the handling of prescribed flux boundary conditions leads to complex expressions and to non-symmetric matrices (*Forsythe and Wasow*, 1960, p. 202). A review of finite difference techniques and a survey of pertinent literature are contained also in *Braester et al.* (1971).

Finite difference approximations have been used in a large number of studies solving one-dimensional (vertical), variably saturated flow problems (e. g., *Day and Luthin*,

1956; *Whisler and Watson*, 1968; *Freeze*, 1969; *Brandt et. al.*, 1971; *Haverkamp et. al.*, 1977; *Dane and Mathis*, 1981; *Haverkamp and Vauclin*, 1981). Fewer researchers have attempted finite differences to solve variably saturated flow problems in higher dimensions (e. g., *Rubin*, 1968, *Cooley*, 1971; *Freeze*, 1971a, b; *Narashimhan et. al.*, 1978; *Healy et. al.*, 1990; *Kirkland et. al.*, 1992). Most of the existing two-dimensional finite difference solutions to variably saturated flow problems have limitations. The finite difference models of *Freeze* (1971 a, b) and *Cooley* (1971) are not robust as they incur numerical instabilities and convergence difficulties. These problems arise primarily from the inefficiencies of the line successive over-relaxation and alternating directional implicit schemes used in solving the two-dimensional, nonlinear equations. *Kirkland et. al.* (1992) presented an efficient algorithm of a finite difference solution to two-dimensional variably saturated flow problems. However, the objective of *Kirkland et. al.* (1992) was to develop competitive numerical procedure to solve infiltration problems in dry soils. The fundamental flow equation solved by *Kirkland et. al.* (1992) (Richards' equation) does not account for the effects of specific storage and consequently it can not be used to model accurately a wide variety of variably saturated flow problems, including many transient drainage and seepage face phenomena in large domains. *Clement et. al.* (1994) mentioned some of the limitations of the existing models and developed a physically based two-dimensional finite difference algorithm for variably saturated flow.

2.4.2.2 Finite Element Method

The finite element method can easily handle irregularly shaped flow regions composed of nonuniform soils with arbitrary anisotropy. The method was first applied to a problem involving saturated-unsaturated flow in porous media by *Neuman* (1973). With finite element methods (e.g., *Zienkiewicz and Parekh*, 1970; *Zienkiewicz*, 1971, *Neuman*, 1973) the domain is divided into a number of grid elements. Each element is characterized by local coordinate functions (*Neuman et. al.*, 1975). This permits the application of variational or weighted residual principles (*Wang and Anderson*, 1982). In this method the objective is to transform the partial differential equation into an integral equation which includes the derivatives of the first order only. Then the integration is performed numerically over elements into

which the considered domain is divided. The Galerkin-type finite element approach (*Zienkiewicz*, 1977) is most widely used today. *Huyakorn and Pinder* (1983) reviewed and presented the application of finite-element methods to groundwater problems.

Finite element method has been successfully used by many researchers to solve the flow in saturated, unsaturated and variably saturated conditions. The classic papers by *Javandel and Witherspoon* (1968) and *Zienkiewicz et. al.* (1966) appears to be first two publications describing the use of the triangular finite element in porous media flow. Shortly thereafter *Neuman and Witherspoon* (1970, 1971) and *Taylor and Brown* (1967) demonstrated the application of this methodology to the analysis of free surface groundwater flow. Models for infiltration in unsaturated soil have been developed by several researchers. *Abriola* (1986) developed a finite element model for unsaturated flow using hierarchic basis functions. *Hayoe* (1978) studied the relative efficiencies of finite difference and Galerkin techniques for modeling soil-water transfer. *Celia et al.* (1990) developed a general mass-conservative numerical solution for the unsaturated flow equation; a modified numerical approach with mixed form of the unsaturated flow equation was used. *Gottardi et. al.* (1992) used moving finite element method for one-dimensional infiltration into homogeneous unsaturated soil and concluded that this method is faster than finite difference and finite element method for this specific case.

Finite element techniques to two-dimensional saturated-unsaturated flow problems were applied by several researchers. Among them are *Neuman* (1973), *Neuman et. al.* (1974), *Reeves and Duguid* (1976), *Bruch* (1977), *Frind et. al.* (1977), *Yeh and Ward* (1980). In all of these studies nonlinearities were treated using standard Picard solution algorithm and the models were developed by Galerkin finite element method. To deal with wider range of field conditions, an improved Picard algorithm and a Newton-Raphson algorithm were used by *Huyakorn et. al.* (1984). *Cooley* (1983) used somewhat different sub-domain finite element method in combination with Newton-Raphson and strongly implicit procedure. Allen and *Murphy* (1986) introduced a finite element collocation technique for solving variably saturated flow in two space dimensions. The scheme used a mass-conservative formulation of Richards' equation as the basis for the finite difference time stepping method. *Panday*

et. al. (1993) discussed recent advances in finite element modeling techniques for variably saturated flow problems.

2.5 Stochastic Models of Soil Water Movement

To overcome problems with spatial and temporal variability of soil hydraulic parameters, as well as the difficulties associated with their determination, various stochastic approaches to soil water modeling have been attempted. The literature review that follows is partly based on *Vogel's* (1990) thesis.

Freeze (1975) applied stochastic input parameters into a deterministic numerical model, which yielded a statistical distribution of outputs. The same approach was later applied e. g. by *Peck et al.*, (1977), *Smith and Freeze* (1979), *Warrick and Amoozegar-Fard* (1979), *Russo and Bresler* (1981), *Bresler and Dagan* (1983), *Dagan and Bresler* (1983) and *Hopmans and Stricker* (1988).

Another approach is to regard the Richards equation itself a stochastic one, with all dependent variables as random variables (e. g., *Yeh et al.*, 1985a, b, c; *Mantoglou and Gelher*, 1987a,b,c).

2.6 Status of Groundwater Modeling in Bangladesh

The first groundwater model used in Bangladesh is in 1977 under BADC/IDA Tubewell Project. Afterward a variety of groundwater modeling studies have been carried out. Between 1977-1984, eight groundwater modeling exercises have been undertaken. The water balance study of the BWDB/UNDP (1984) project also modeled the groundwater component. According to MPO (1987), these models can be classified into three types; Single Cell models, Multiple Single Cell models and Multiple Cell Integrated Finite-difference models. A brief description about these models is summarized next.

2.6.1 BADC/IDA Tubewell Project (1977)

Under this project, the main model covered most of northwest region at north of Atrai basin and another sub-model covered the area between the rivers Atrai and Ganges. Both of the models were finite difference single layer model with aquifer system of semi-confined or unconfined type. The objectives of the models were to forecast the development of DTW. But the quality of database was not sufficient for accurate simulation. The calibration period for main model was November to June and that for the sub-model was September to May.

2.6.2 ADB Tubewell Project : North Bangladesh (1980)

The model was single layer finite difference type with unconfined aquifer system. The model area covered parts of Dinajpur and Rangpur districts. The objectives of the models were to forecast the development of DTW and refinement of BADC/ IDA (1977) main model. The quality of data base was reasonable and the calibration period was dry season.

2.6.3 Rajshahi, Pabna Groundwater Model (1980)

The model was two layer finite difference type with the aquifer system consisting of confined aquifer overlain by a semi-confining layer. The objectives of the models were to forecast the development level of DTW. The calibration period was dry season. The quality of database was reasonable for aquifer geometry but poor for other parameters. The simulation being based on sparse data, results were not published widely.

2.6.4 Development Plan for Waste Water Systems for the Dhaka Metropolitan Area (1980)

The model used was of single layer and of finite difference type. The aquifer system considered consist of semi-confined aquifer. The objective of the model was assessment of piezometric head declined under different future abstraction schemes

for the period from 1980 to 2010. The calibration period was from 1966 to 1978. But the quality of data base was poor.

2.6.5 ADB-II DTW Project (1982)

Under this project, a single cell model was developed for the Northwest region to simulate aquifer responses in six typical areas of six major physiographic units. To test the effect of river level fluctuations on the aquifer, strip models were used. The simulated flow balance was thus consisted of vertical flow and lateral flow as in single cell model.

2.6.6 Southwest Rural Development Project (1984)

The model was single layer finite difference type with a semi-confined aquifer. The modeled area was southwest region of Bangladesh comprising the districts of greater Jessore, Kushtia and Faridpur. The objective of the model was to forecast STW/DTW development potential. The components of flow balance equation consisted of leakage, recharge, discharge, inflow/ outflow across boundaries which were computed explicitly with no integrated interaction being taken care between aquifer and semi-confining layer thus affecting the accuracy of simulation.

2.6.7 The Northwest Bangladesh Groundwater Model

The prime objective of the model was to obtain realistic estimates of STW development levels in Rajshahi division of Bangladesh. Two criteria were set to determine the optimum development level of STW. Firstly, the maximum depth to pumping level should be less than 7.5 m and secondly, the system should be adequately refilled during wet season. So the complete hydrologic cycle was incorporated in the model simulation unlike for the dry period. The model was finite difference type with internal boundaries primarily based on major physiographic unit and internal rivers whereas the external boundaries were formed by rivers Ganges and Jamuna and geographical boundary with India. The aquifer system was simulated as a single aquifer of variable thickness overlain by a semi-confining layer.

2.6.8 Water Balance Studies (1983) BWDB/ UNDP

For the northwest and northeast regions of Bangladesh, wet and dry deep percolation rates for 27 aquifer units were determined from this model simulation. But they did not represent actual field values. Considering a minimum and maximum recharge periods during monsoon to be 90 to 150 days, a minimum and maximum potential recharge quantity based on wet land deep percolation rate were calculated. As only 23 percent of the total land area was cultivated by T. Aman (wet land); the cumulative recharge assuming wet land was less than the weighted average recharge considering both wet and dry conditions. However, this recharge value could represent potential recharge where no rejection of recharge occurs due to aquifer full condition.

2.6.9 Model used by MPO (1987 & 1991)

The setup of MPO in the study of groundwater availability and planning for development is basically equipped with a network of groundwater models. The outputs from one model acting as an input for another model and depending on the stage of formulation or implementation of the plan; different models of qualitatively different status are supposed to be used as needed. From 1983 to 1986, MPO within the National Water Plan (NWP) preparation schedule, model was applied in eight special study areas, each with an area of 100 square km, dispersed across Bangladesh. These areas were intensively monitored and data acquired to calibrate the models. In catchment recharge simulation, all catchment which had data were modeled as a multiple single cell model in which only the vertical components of flow were considered. In the second phase of the NWP, the existing models of the first phase were adapted for national assessment and future incremental resource available for development. There are four models in the network. The models are Recharge Model, Depth Storage Model, Resource Potential Model and Multi Cell Model. The detailed description of the models are given in the following sections.

2.6.10 Recharge Model

Recharge model calculates the maximum possible recharge (unobstructed by raised groundwater table), and limited only by the rate of deep percolation of the top or

subsoil. The conceptual basis is similar to Karim's (1984) study in calculation of potential recharge with provision for lowering the water table through irrigation during dry season.

2.6.11 Depth/ Storage Model

This model is used to compute the net available storage in the aquifer system available for groundwater abstraction. This is an adaptation of the model used during the first phase. The methodology has been slightly modified in second phase and a range of specific yield values rather than single value are used in the simulation. The specific yield value and water table decline are the two determining factors of net available storage.

2.6.12 Resource Potential Model

The model combines results obtained from the recharge model and the depth/ storage model and compute groundwater resource development potential for different flood phases and for different modes of pumping.

2.6.13 Multi Cell Model

This model is used for the detailed analysis of groundwater behavior in the MPO special study areas, and can also be used in a single cell mode for individual upazillas. The main purpose of the model is to obtain a better definition of the parameters, which most strongly affect the computation of potential recharge and the net available storage.

2.6.14 Groundwater Models for Dhaka WASA

Three groundwater model have been used in the past to assess the groundwater resource availability in the Dhaka city region. RMP/ Montgomery developed a groundwater model for Dhaka WASA in 1980, *Solomon and Chidley* review this model in 1986 and lastly in 1989 Dhaka region groundwater and subsidence model was developed. These are discussed briefly below.

2.6.15 RMP/ Montgomery (1980)

The model was used to assess the impact of additional groundwater abstraction on the aquifer system. The model based on finite difference method simulate the aquifer system as a single aquifer unit overlain by a semi-confining layer. The study area being surrounded by major rivers, it was expected to prevail a good interaction between the rivers and aquifer system. But when the model was run in predictive mode to assess the impact of increased abstraction from 1980 to 2010, the results of the model showed poor relation between the rivers and the aquifer.

2.6.16 Solomon and Chidley's Review of the Parsons' Model (1986)

The limitations of the *Parson's Model*, as described by *Solomon and Chidley*, were due to poor representation of boundary conditions and recharge mechanism. They used a USGS (United States Geological Survey) model to assess the groundwater resource considering the effect of rivers and the potential recharge values from MPO and the hydrogeological parameters from BWDB. But still the results were not satisfactory, that is, similar to those of previous models of *Tames M. Montgomery and R. M. Parsons*. They did not calibrate their USGS model against historical data before predictive run. However, they came to a conclusion that a detailed modeling of Dhaka city is necessary with special emphasis on extending the model boundaries to the major rivers and improved simulation of recharge mechanism.

2.6.17 Dhaka Region Groundwater and Subsidence Model (1991)

In 1989, the Dhaka region groundwater and subsidence model study was started with the purpose of assessing the impact of the proposed installation of 50 new and 10 replacement tubewells. The model was based on integrated finite difference method. A basic feature of the model is the resolution of three-dimensional groundwater flow into its horizontal and vertical components.

The areal extent of the model is bounded by three major rivers, Jamuna, Ganges and Meghna in three sides and the remaining northern part is bounded partly by Old Brahmaputra and partly by northern edge of Gazipur and Manikganj districts. The

groundwater resources model consists of two integrated units. The regional model evaluated the consequences of groundwater development on a regional basis. The Dhaka submodel considered the Dhaka metropolitan area and immediate surroundings. This model is sufficiently refined to take into account the spatial variability of various parameters such as local relief, geology, physiography and distribution of abstraction.

The model developed is a four layer model; lower aquifer (main aquifer of abstraction), lower aquitard, upper aquifer and upper aquitard. The two fundamental principles used are continuity of mass and Darcy's law. The flows are horizontal in aquifers and vertical in aquitards. Both steady and transient state of flows are encountered in the model. The geometry of aquifer system is defined by the elevations of the top and bases of different layers.

CHAPTER 3

MODEL PROCESSES AND PARAMETERS

3.1 Introduction

The success of the numerical model depends on the real hydrogeological characterization of the simulated system. In case of saturated-unsaturated flow the governing flow equation can be written as follows

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x_i} \left[K \left(K_{ij}^A \frac{\partial h}{\partial x_j} + K_{iz}^A \right) \right] - S \quad (3.1)$$

where θ (dimensionless) is the volumetric moisture content, h [L] is the pressure head, S [T⁻¹] is a sink term, x_i ($i=1,2$) [L] are the spatial coordinates, t [T] is the time, K_{ij}^A (dimensionless) are components of the anisotropy tensor K^A , and K [LT⁻¹] is a scalar unsaturated hydraulic conductivity function given by

$$K(h, x, z) = K_s(x, z) K_r(h, x, z) \quad (3.2)$$

Where K_r (dimensionless) is the relative hydraulic conductivity and K_s is the saturated hydraulic conductivity. The anisotropy tensor K_{ij}^A in (3.1) is used to account for the direction-dependent hydraulic conductivity of the porous medium.

In order to solve the governing flow equation one must specify the retention curve $\theta(h)$ and the hydraulic conductivity function $K(h)$ related to negative pressure head. The equation also contains saturated hydraulic conductivity, a sink term and a root water extraction term. The parameters related with these are explained briefly in the following sections. Special attention is paid to the submodel of water extraction by plant roots and to the parameterization of the hydraulic conductivity - pressure head and moisture content - pressure head relations.

3.2 Model Parameters

3.2.1 Water Extraction by Roots

In the field, the living plant root system is dynamic (dying roots are constantly replaced by new ones), geometry is time dependent, water permeability varies with position along the root and with time. Root water uptake is most effective in young root material, but the length of young roots is not directly related to total root length (Feddes et al., 1988). In addition, experimental evaluation of root properties is hardly practical, and often impossible. Thus instead of considering water flow to and in single roots, a more suitable approach is the macroscopic one, in which a sink term $S[T^{-1}]$ (volume of water per volume of soil per unit of time) representing water extraction by macroscopically homogeneous element of the root system in a unit volume of soil, is added to the continuity equation for water, as it is done in (3.1).

The rate of water intake by plant roots depends on the atmospheric conditions (expressed as the potential rate of transpiration) as well as on conditions prevailing in the underground, and of course on the state and properties of the root system itself. The rate of water extraction $S [T^{-1}]$ is usually believed to depend in a linear way upon the difference in water potentials between the bulk soil (not in the vicinity of the roots) and at some reference point in the plants (e. g., at the base of the stem) (Hillel, 1980, p. 181). An alternative formulation more suitable for computations but less explanatory from the physical point of view makes the extraction rate S dependent directly upon the vertical depth below soil surface and upon the transition rate, without feedback (e. g., Molz and Remson, 1970; Nimah and Hanks, 1973a, b; Feddes et al., 1974). Feddes (1981), Molz (1981) and Campbell (1985) gave an overview of possible S -functions.

Feddes et al. (1978) described S semi-empirically as

$$S(h) = a(h) S_p \quad (3.3)$$

where the water stress response function $a(h)$ is a prescribed dimensionless function (Fig. 3.1) of the soil water pressure head ($0 < a < 1$), and $S_p [T^{-1}]$ is the potential water uptake rate. Fig. 3.1 gives a schematic plot of the stress response function as used by

Feddes et al. (1978). In this case, water uptake is assumed to be zero close to soil saturation (i. e., when the soil is wetter than some arbitrary “anaerobiosis point”, which in terms of pressure head is h_1). For the soil drier than the wilting point pressure head ($h < h_4$), water uptake is also assumed to be zero. Optimal water uptake is considered between pressure heads h_2 and h_3 , whereas for pressure head between h_3 and h_4 a linear variation and between h_1 and h_2 a linear or hyperbolic variation is assumed. The value of h_3 is dependent on the demand of the atmosphere and varies with potential transpiration rate. S_p is equal to the water uptake rate during periods of no water stress when $a(h) = 1$.

When the potential water uptake rate is equally distributed over a two-dimensional rectangular root domain, S_p becomes

$$S_p = \frac{1}{L_x L_z} L_t T_p \quad (3.4)$$

where T_p is the potential transpiration rate [LT^{-1}], L_z is the depth [L] of the root zone, L_x is the width [L] of the root zone and L_t is the width [L] of the soil surface associated with the transpiration process.

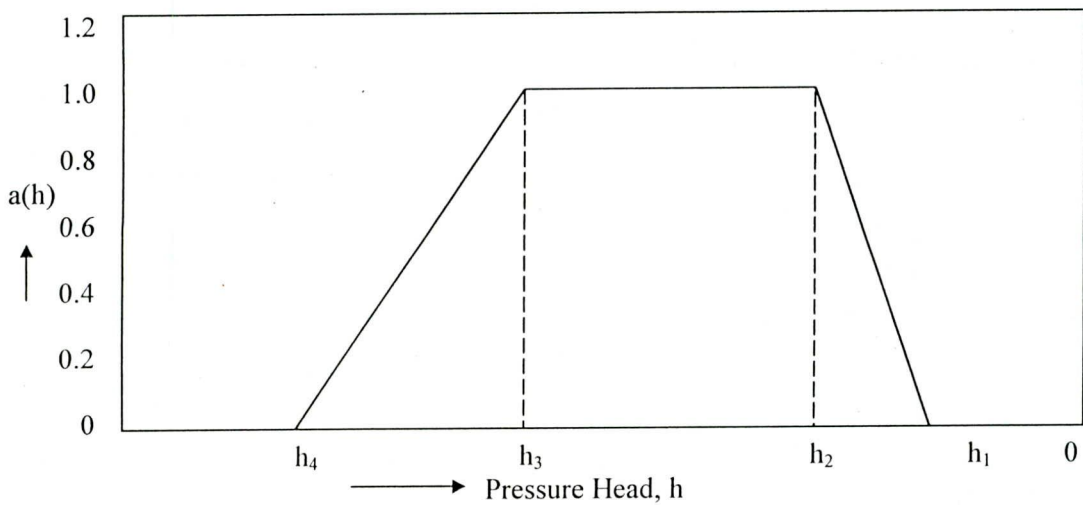


Figure 3.1. Schematic of the Plant Water Stress Response Function, $a(h)$, as used by Feddes et al. (1978)

Notice that S_p reduces to T_p/L_z when $L_t = L_x$. The above equation (3.4) may be generated by introducing a non-uniform distribution of the potential water uptake rate over a root zone of arbitrary shape [Vogel, 1987]:

$$S_p = b(x,z)L_tT_p \quad (3.5)$$

where $b(x,z)$ is the normalized water uptake distribution function [L^{-2}]. This function describes the spatial variation of the potential extraction term S_p over the root zone (Fig. 3.2), and is obtained from $b'(x,z)$ as follows

$$b(x,z) = \frac{b'(x,z)}{\int_{\Omega_R} b'(x,z)d\Omega} \quad (3.6)$$

where Ω_R is the region occupied by the root zone, and $b'(x,z)$ is an arbitrary prescribed distribution function. Normalizing the uptake distribution ensures that $b(x,z)$ integrates to unity over the flow domain belonging to the transpiration associated width L_t , i. e.,

$$\int_{\Omega_R} b(x,z)d\Omega = 1 \quad (3.7)$$

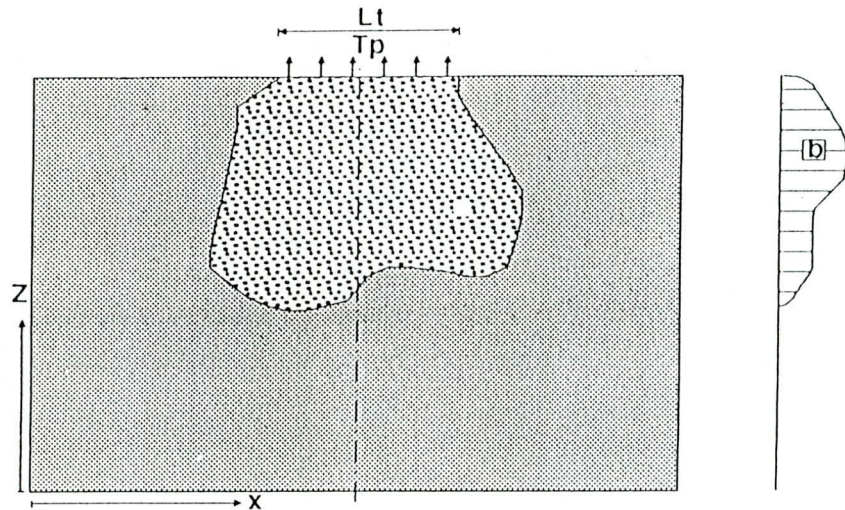


Figure 3.2. Schematic of the Potential Water Uptake Distribution Function, $b(x,z)$, in the Soil Root Zone

From (3.5) and (3.7) it follows that S_p is related to T_p by the expression:

$$\frac{1}{L_i} \int_{\Omega_R} S_p d\Omega = T_p \quad (3.8)$$

The actual water uptake distribution is obtained by substituting (3.5) into (3.3):

$$S(h, x, z) = a(h, x, z) b(x, z) L_i T_p \quad (3.9)$$

whereas the actual transpiration rate, T_a , is obtained by integrating (3.9) as follows

$$T_a = \frac{1}{L_i} \int_{\Omega_R} S d\Omega = T_p \int_{\Omega_R} a(h, x, z) b(x, z) d\Omega \quad (3.10)$$

3.2.2 Hydraulic Properties of the Unsaturated Soil

The hydraulic properties of an unsaturated soil are characterized, within the framework of the underlying theory, by the retention curve (the soil moisture characteristic) relating the volumetric soil moisture content to the pressure head (more exactly to the matric head) and by the hydraulic conductivity function expressing the dependence of the unsaturated hydraulic conductivity upon either pressure head or volumetric moisture content. A number of equations can be found in the literature for approximating these relationships. The present work makes use of a set of closed form equations resembling those of van Genuchten (1980), which are later modified by Vogel and Cislerova (1988).

Van Genuchten (1980) used the statistical pore-size distribution model of Maulem (1976) to obtain a predictive equation for the unsaturated hydraulic conductivity function. The original van Genuchten equation for the retention curve is:

$$\begin{aligned} S_e &= [1 + |\alpha h|^n]^{-m} & \text{for } h < 0 \\ S_e &= 1 & \text{for } h \geq 0 \end{aligned} \quad (3.11)$$

where S_e [dimensionless] is the degree of saturation, h [L] is the pressure head, α [L⁻¹] an empirical parameter, n [dimensionless] an empirical exponent, and m [dimensionless] is another empirical exponent, which is almost without exception taken equal to $m = 1 - 1/n$.

The degree of saturation is equal to:

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}$$

where θ [dimensionless] is the volumetric moisture content, θ_s [dimensionless] is the saturated volumetric moisture content and θ_r [dimensionless] is residual (immobile) volumetric moisture content.

By combining equation (3.11) with the Maulem's (1976) theory and assuming (3.14) $1/n$, van Genuchten (1980) arrived at a hydraulic conductivity function:

$$\begin{aligned} K(h) &= K_s K_r(h) & \text{for } h < 0 \\ K(h) &= K_s & \text{for } h \geq 0 \end{aligned}$$

where K [LT^{-1}] is the unsaturated hydraulic conductivity, K_s [LT^{-1}] is the saturated hydraulic conductivity and K_r [dimensionless] is the relative hydraulic conductivity, which equals to:

$$K_r = S_e^{1/2} [1 - (1 - S_e^{1/m})^m]^2 \quad (3.12)$$

Vogel and Cislerova (1988; Also Luckner et. al., 1989) suggested some improvements to the van Genuchten formulae, allowing for the non-zero air-entry value as well as for an abrupt transition from the mobile state of the soil moisture to the immobile one during desaturation. Their formula for the retention curve reads:

$$\theta(h) = \begin{cases} \theta_a + \frac{(\theta_m - \theta_a)}{(1 + |\alpha h|^n)^m} & \text{for } h_r < h < h_s \\ \theta_s & \text{for } 0 \geq h \geq h_s \\ \theta_r & \text{for } h \leq h_r \end{cases} \quad (3.13)$$

where, in addition to the previous parameters, θ_m and θ_a are imaginary upper and lower limit, respectively, of the volumetric moisture content. These parameters have no direct physical meaning; θ_m can be even greater than porosity and θ_a can be even less than zero. h_s [L] is the air-entry value and h_r [L] is the highest value of the pressure head for which the soil moisture is still immobile. The approach maintains the physical meaning of θ_s and θ_r as measurable quantities.

A provision was also made by Vogel and Cislerova (1988) for the possibility of matching the hydraulic conductivity function to a measured non-saturated value of

hydraulic conductivity K_k at some moisture content θ_k for which also the value of pressure head h_k is known ($\theta_k < \theta_s$, $K_k < K_s$). The resulting hydraulic conductivity function is:

$$K(h) = \begin{cases} K_s K_r(h) & h \leq h_k \\ K_k + \frac{(h - h_k)(K_s - K_k)}{h_s - h_k} & h_k < h < h_s \\ K_s & h \geq h_s \end{cases} \quad (3.14)$$

where

$$K_r = \frac{K_k}{K_s} \left(\frac{S_e}{S_{ek}} \right)^{1/2} \left[\frac{F(\theta_r) - F(\theta)}{F(\theta_r) - F(\theta_k)} \right]^2 \quad (3.15a)$$

$$F(\theta) = \left[1 - \left(\frac{\theta - \theta_a}{\theta_m - \theta_a} \right)^{1/m} \right]^m \quad (3.15b)$$

$$m = 1 - \frac{1}{n}; \quad n > 1 \quad (3.15c)$$

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \quad (3.15d)$$

$$S_{ek} = \frac{\theta_k - \theta_r}{\theta_s - \theta_r} \quad (3.15e)$$

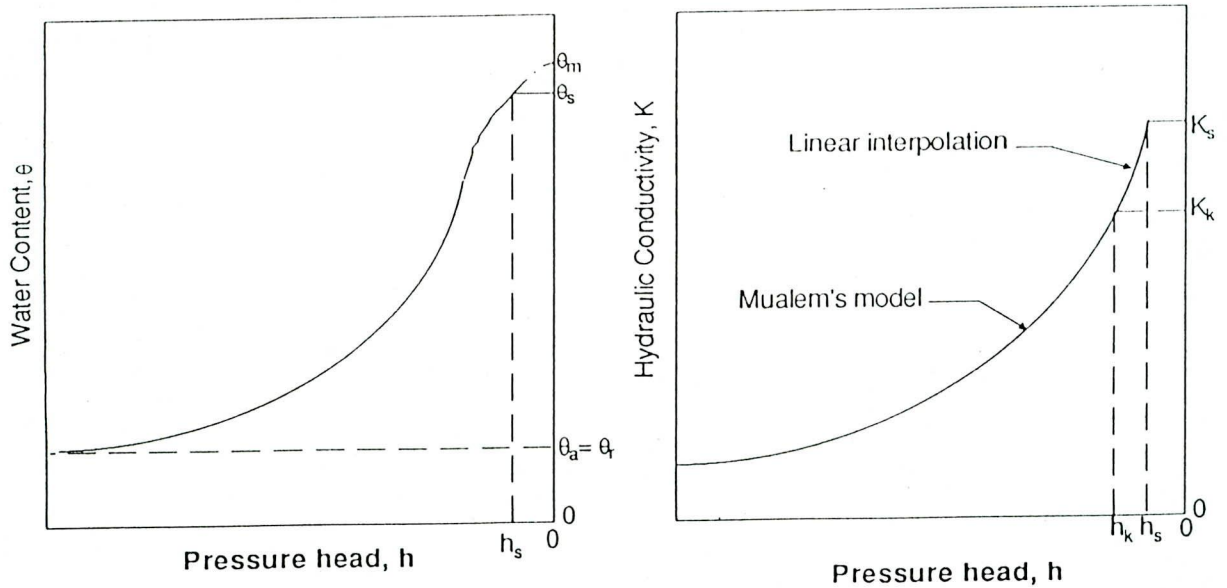


Fig. 3.3 : Schematics of the (a) soil water retention (b) and hydraulic conductivity functions as given by the equations (3.13) and (3.14) respectively

3.2.3 Scaling of Soil Hydraulic Properties

Three independent scaling factors are embodied in the program which may be used to define a linear model of the actual spatial variability of the soil hydraulic properties as follows (Vogel et. al., 1991):

$$\begin{pmatrix} K(h) = \alpha_k K^*(h^*) \\ \theta(h) = \theta_r + \alpha_\theta [\theta^*(h^*) - \theta_r^*] \\ h = \alpha_h h^* \end{pmatrix} \quad (3.16)$$

where, in the most general case, α_k , α_θ and α_h are mutually independent dimensionless scaling factors for hydraulic conductivity, volumetric moisture content and pressure head, respectively. These scaling factors are generally functions of coordinates (x, z).

CHAPTER 4

METHODOLOGY

4.1 Introduction

The physical processes involved in groundwater flow can be described mathematically coupling Darcy's law of groundwater flow with the equation of continuity that describes the conservation of fluid mass during flow through a porous medium. With some assumptions the resulting mathematical statement of the problem is a second order partial differential equation which is in generalized form called Richards' equation (1931). It is the Richards' equation, which governs the soil water flow phenomena and is used as a basis of numerical modeling in the program Ground Water Model (GWm). Under different conditions (steady or unsteady flow, one-, two- or three-dimensional flow problems, purely unsaturated or saturated-unsaturated flow, etc.), different forms of Richards' equation are pertinent and suitable to the specific situation (Dolezal, 1992) and, if as need arises, have to be supplemented by additional equations for accompanying physical processes. In this case, a two-dimensional isothermal Darcian flow of water is considered, in a variably saturated rigid porous medium, assuming that the air phase plays an insignificant role in the liquid flow process. To solve this equation numerically a class of weighted residual finite element method the Galerkin method is used here. In the following sections the theoretical background of the equation of flow and the formulation of the groundwater flow model has been described.

4.2 The Governing Flow Equation

4.2.1 Darcy's Law

Darcy's law can be written for multidimensional flow under anisotropic and non-uniform conditions as:

$$q_i = -K_{ij} \frac{\partial \phi}{\partial x_j} \quad \text{with } i, j = 1, 2, 3 \text{ (or } x, y, z) \quad (4.1)$$

where q is the volumetric flux of water [LT^{-1}], K_{ij} is the hydraulic conductivity written as a tensor of second rank [LT^{-1}], ϕ [L] is the total soil water potential (hydraulic head).

For isotropic conditions, when the potential is expressed in terms of pressure head h and gravitational head z , the Darcy's law takes the form:

$$q_i = -K \frac{\partial}{\partial x_j} (h + z) \quad (4.2)$$

4.2.2 Conservation of Mass

For conservation of mass, from continuity equation, the rate of mass inflow equals the rate of increase of mass in a control volume. The continuity equation describing the transient flow of water in a saturated-unsaturated soil can be written as:

$$\text{div } \vec{q} = -\frac{\partial \theta}{\partial t} - S \quad (4.3)$$

where θ is the volumetric soil moisture content [dimensionless], t [T] is time, and S [T] is the intensity of local sink (or negative source) or soil water (e. g., rate of water extraction by roots).

4.2.3 Flow Equation

The Richards' flow equation is then obtained by combining eqn. 4.1 with eqn. 4.3, which gives

$$\frac{\partial}{\partial x_i} \left(K_{ij} \frac{\partial \phi}{\partial x_j} \right) = \frac{\partial \theta}{\partial t} + S \quad (4.4)$$

For a two-dimensional vertical soil p[rofile, with the hydraulic conductivity decomposed, it reads:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x_i} \left[K \left(K_{ij}^A \frac{\partial h}{\partial x_j} + K_{iz}^A \right) \right] - S \quad (4.5)$$

where θ (dimensionless) is the volumetric moisture content, h [L] is the pressure head, S [T^{-1}] is a sink term, x_i ($i=1,2$) [L] are the spatial coordinates, t [T] is the time, K_{ij}^A

(dimensionless) are components of the anisotropy tensor K^A , and $K [LT^{-1}]$ is a scalar unsaturated hydraulic conductivity function given by

$$K(h, x, z) = K_s(x, z) K_r(h, x, z) \quad (4.6)$$

where K_r (dimensionless) is the relative hydraulic conductivity and K_s is the saturated hydraulic conductivity. The anisotropy tensor K^A_{ij} in (4.6) is used to account for the direction-dependent hydraulic conductivity of the porous medium. The diagonal entries of K^A_{ij} equal one and the off-diagonal entries zero for an isotropic medium. Einstein's summation convention is used in (4.5): summation of terms in which a subscript appears twice is assumed over all values of that subscript.

This mixed form of equation consists of two state variables. One is the moisture content θ , which is normally used in unsaturated flow case and another is h , the pressure head, used in saturated flow case. In the unsaturated or variably saturated case, because of the nonlinear nature of the relations between relative permeability, water saturation and pressure head, the governing equation is highly nonlinear.

4.3 Mathematical Formulation of the Solution Scheme - The Basic Approach

Numerical methods are based on subdividing the flow region into finite segments bounded and represented by a series of nodal points at which a solution is obtained. This solution depends on the solutions in the surrounding segments and on an appropriate set of auxiliary conditions. With finite element methods, the domain is divided into a number of grid elements. In modeling soil water flow problems, triangular elements can be efficiently used to represent difficult geometries and to concentrate coordinate functions in regions where rapid changes are anticipated such near soil surface or wetting fronts. The corners of such triangular elements (Fig 4.1) are designated as nodal points. These nodes serve the purpose of locating the state variables, e. g. pressure heads. Each element is characterized by local coordinate functions (Nueman et. al., 1975). This permits the application of variational or weighted residual principles (Wang and Anderson, 1982). Now a days, the Galerkin type finite element approach (Zienkiewicz, 1977) is most widely used.

4.3.1 Discretization of the Flow Domain

Different types of two-dimensional isoparametric finite elements used for space discretization are linear and quadretic quadrilateral elements and linear and quadretic triangular elements. The flow domain is subdivided into a network of triangular elements as shown in Fig. 4.1. Following Nueman et. al. (1975), the corners of these elements are designated as nodal points and the global space coordinates (x, z) of the n -th node can be designated as x_i^n ($i=1,2$). Also, each element, e , is associated with a set of so called local coordinate functions or basis functions, $\xi_n^e(x_i)$, which are used to express the variation of unknown variables within an element and also coordinate transformation from global (domain, x, z) to local (element) coordinates. These shape functions vary linearly inside the element, vanish outside the element and satisfy the requirement $\xi_n^e(x_i^m) = \delta_{mn}$; where δ_{mn} is the Kronekar delta having values of either 1, if $m=n$ or zero, if $m \neq n$. For all elements having a common node n a union of these functions (which can be actually regarded also as their sum) yields a piecewise linear basis function $\phi_n(x_i) = U_e \xi_n^e(x_i)$ for which the same condition $\phi_n(x_i^n) = \delta_{nn}$ is valid.

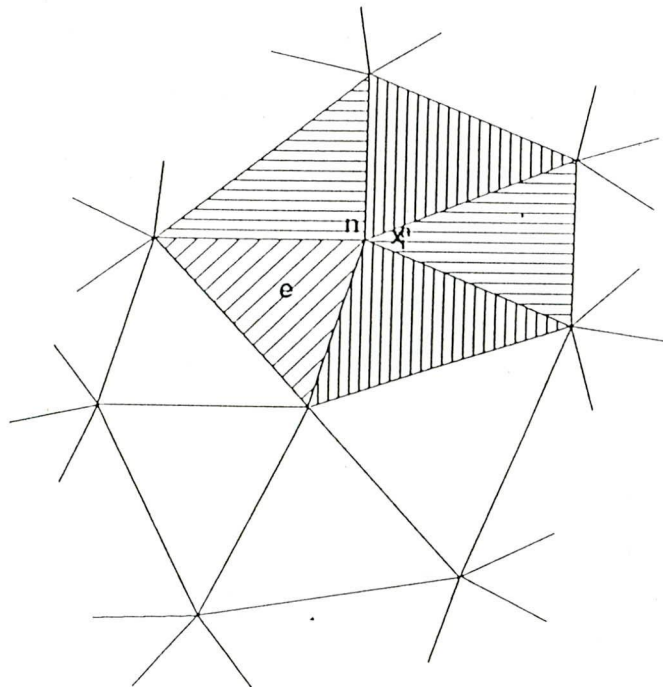


Fig. 4.1 Network of triangular elements

4.3.2 The Galerkin Approximation

The governing flow equation is

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x_i} \left[K \left(K_{ij}^A \frac{\partial h}{\partial x_j} + K_{iz}^A \right) \right] - S \quad (4.7)$$

According to the Galerkin method, the approximate or trial solution in terms of pressure head $h(x, z)$ at any given time t can be expressed as a series summation of the products of the nodal pressure heads, h_n , and the associated nodal basis functions $U_e \xi_n^e(x, z)$:

$$\begin{aligned} \hat{h}(x, z, t) &= \sum_{n=1}^N h_n(t) U_e \xi_n^e(x, z); \quad n = 1, 2, \dots, N \\ &= \sum_{n=1}^N h_n(t) \phi_n(x_i) \end{aligned} \quad (4.8)$$

where N is the total number of nodes, $h_n(t)$ is the value of solution at the node n , and the union sign is taken over all elements (in fact, over all elements having the common node n), $\xi_n^e(x_i)$ is a set of local coordinate functions or basis functions associated with each element e and $\phi_n(x_i) = U_e \xi_n^e(x_i)$ is a piecewise linear basis function for all elements having a common node n .

The values of h_n must be determined so as to satisfy the initial and boundary conditions of the problem together with the orthogonality requirement:

$$\sum_e \int_{\Omega^e} L(\hat{h}) \xi_n^e d\Omega = 0; \quad n = 1, 2, \dots, N \quad (4.9)$$

where Ω^e is the interior of e and L is differential operator of the governing flow equation.

Substituting the trial solution in the governing equation will result in a residual, R , which would be zero everywhere for all x, z and t , if the trial solution were the exact solution. In the method of weighted residuals, an attempt is made to force this residual to be zero, in an average sense over the entire domain, through the selection of the unknown values of h_n ($n=1, 2, \dots, N$). This is accomplished by forming a weighted

integral of R over the entire solution domain and then setting this integral (weighted residual) to zero. In Galerkin method, the weighing functions are chosen to be identical to the basis functions, $\phi_m(x, z)$. Thus the values of h_n are determined such that these satisfy the initial and boundary conditions and also the orthogonality requirement of the expression $R(x, z)$ to all of the functions $\phi_m(x, z)$ ($m = 1, 2, \dots, N$).

In the Galerkin method, the residuals of the governing flow equation weighted by each of the nodal basis functions are required to be zero when integrated over the entire flow domain:

$$\sum_e \int_{\Omega^e} [L(\sum_{n=1}^N h_n U_e \xi_n^e) - S] \xi_m^e d\Omega = 0; \quad m = 1, 2, \dots, N \quad (4.10)$$

where the differential operator L is

$$L = \bar{C} \frac{\partial}{\partial t} - \frac{\partial}{\partial x_i} [K(K_{ij}^A \frac{\partial}{\partial x_j} + K_{iz}^A)]; \quad \bar{C} = \frac{\partial \theta}{\partial \hat{h}} \quad (4.11)$$

and the integral is

$$\begin{aligned} \sum_e \sum_{n=1}^N \{ h_n \int_{\Omega^e} \nabla \cdot [K(K_{ij}^A \nabla \xi_n^e + K_{iz}^A \xi_n^e)] \xi_m^e d\Omega \\ - \int_{\Omega^e} \bar{C} \xi_n^e \xi_m^e \frac{\partial h_n}{\partial t} d\Omega - \int_{\Omega^e} S \xi_m^e d\Omega \} = 0 \end{aligned} \quad (4.12)$$

The divergence theorem gives

$$\int_{\Omega^e} \nabla \cdot q d\Omega = \int_{\Gamma} q \cdot \eta d\Gamma$$

Let $q = UVV$; then

$$\nabla \cdot q = U \nabla^2 V + \nabla U \cdot \nabla V$$

and

$$\int_{\Omega^e} U \nabla^2 V d\Omega = \int_{\Gamma} U \frac{\partial V}{\partial \eta} d\Gamma - \int_{\Omega^e} \nabla U \cdot \nabla V d\Omega$$

which is Green's first identity (Green's theorem).

Applying Green's theorem to the first integral of (4.12) yields:

$$\begin{aligned} \sum_e \sum_n h_n \{ \int_{\Gamma} K K_{ij}^A \xi_m^e \frac{\partial \xi_n^e}{\partial \eta} d\Gamma - \int_{\Omega^e} (K K_{ij}^A \nabla \xi_m^e \cdot \nabla \xi_n^e) d\Omega \} + \\ \sum_e \sum_{n=1}^N [h_n \int_{\Omega^e} \nabla (K K_{iz}^A) d\Omega - \int_{\Omega^e} \bar{C} \xi_m^e \xi_n^e \frac{\partial h_n}{\partial t} d\Omega - \int_{\Omega^e} S \xi_m^e d\Omega] = 0 \end{aligned} \quad (4.13)$$

In applying Green's theorem the derivatives of K has disappeared and only first order derivatives of the interpolation functions appear in the equation, thus making the problem more tractable.

The first boundary integral in the equation (4.13) can be written as

$$\sum_e \sum_{n=1}^N h_n \int_{\Gamma^e} K K_{ij}^A \xi_m^e \frac{\partial \xi_n^e}{\partial \eta} d\Gamma = \sum_e \int_{\Gamma^e} \xi_m^e [K K_{ij}^A \sum_{n=1}^N \frac{\partial}{\partial \eta} (h \xi_n^e)] d\Gamma = - \sum_e \int_{\Gamma^e} \xi_m^e q d\Gamma$$

in which q is the outflow intensity across the element boundary. Finally, the equation (4.13) can be written in a matrix form as

$$[A_{nm}] \{h_m\} + F_{nm} \frac{d\{\theta_m\}}{dt} = \{Q_n\} - \{B_n\} - \{D_n\}; \quad n, m = 1, 2, \dots, N \quad (4.14)$$

in which the elements of the matrices are

$$A_{nm} = \sum_e K K_{ij}^A \int_{\Omega^e} \frac{\partial \xi_n^e}{\partial x_i} \frac{\partial \xi_m^e}{\partial x_j} d\Omega \quad (4.15a)$$

$$F_{nm} = \delta_{nm} \sum_e \int_{\Omega^e} \xi_n^e d\Omega \quad (4.15b)$$

$$Q_n = - \sum_e \int_{\Gamma^e} \xi_n^e q d\Gamma \quad (4.15c)$$

$$B_n = \sum_e K K_{iz}^A \int_{\Omega^e} \frac{\partial \xi_n^e}{\partial x_i} d\Omega \quad (4.15d)$$

$$D_n = \sum_e \int_{\Omega^e} S \xi_n^e d\Omega \quad (4.15e)$$

4.3.3 A Triangular Element in the Flow Domain

The triangular element (Fig. 4.2) has node numbers i, j and k in counterclockwise order. The coordinates of nodes i, j and k are designated as (x_i, z_i) , (x_j, z_j) and (x_k, z_k) respectively. The unknown pressure heads at the nodes are:

$$h_i = h(x_i, z_i), h_j = h(x_j, z_j) \text{ and } h_k = h(x_k, z_k). \quad (4.16)$$

The trial solution $h_e(x, z)$ throughout the triangular element is defined by linear interpolation of the nodal values h_i, h_j and h_k as:

$$h_e(x, z) = a_0 + a_1 x + a_2 z \quad (4.17)$$

The coefficients can be determined by setting up three equations, which require that the nodal values be obtained at the nodal coordinates:

$$\begin{cases} h_i^{(i)} = a_0^{(i)} + a_1^{(i)} x_i + a_2^{(i)} z_i \\ h_j^{(i)} = a_0^{(i)} + a_1^{(i)} x_j + a_2^{(i)} z_j \\ h_k^{(i)} = a_0^{(i)} + a_1^{(i)} x_k + a_2^{(i)} z_k \end{cases} \quad (4.18)$$

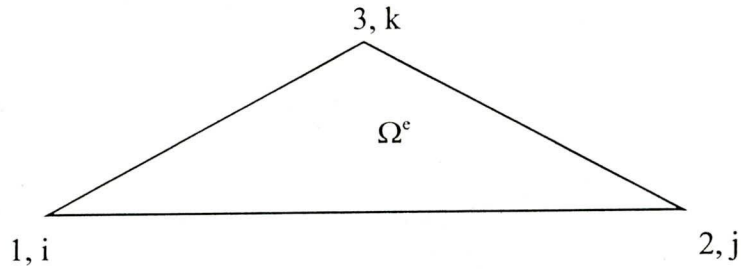


Fig. 4.2 Single Triangular Element

Now a_0 , a_1 and a_2 are obtained by solving the above equations:

$$\begin{cases} a_0^{(i)} = x_j z_k - x_k z_j & a_1^{(i)} = z_j - z_k & a_2^{(i)} = x_k - x_j \\ a_0^{(j)} = x_k z_i - x_i z_k & a_1^{(j)} = z_k - z_i & a_2^{(j)} = x_i - x_k \\ a_0^{(k)} = x_i z_j - x_j z_i & a_1^{(k)} = z_i - z_j & a_2^{(k)} = x_j - x_i \end{cases} \quad (4.19)$$

If coefficients a_1 and a_2 are denoted b and c respectively and these expressions are substituted into (4.17), then it can be written as :

$$\hat{h}^e(x, z) = \xi_i^e(x, z)h_i + \xi_j^e(x, z)h_j + \xi_k^e(x, z)h_k \quad (4.20)$$

where

$$\xi_i^e(x, z) = \frac{1}{2\Delta^e} [(x_j z_k - x_k z_j) + (z_j - z_k)x + (x_k - x_j)z] \quad (4.21a)$$

$$\xi_j^e(x, z) = \frac{1}{2\Delta^e} [(x_k z_i - x_i z_k) + (z_k - z_i)x + (x_i - x_k)z] \quad (4.21b)$$

$$\xi_k^e(x, z) = \frac{1}{2\Delta^e} [(x_i z_j - x_j z_i) + (z_i - z_j)x + (x_j - x_i)z]$$

and

$$2\Delta_e = (x_i z_j - x_j z_i) + (x_k z_i - x_i z_k) + (x_j z_k - x_k z_j) \quad (4.22)$$

The Δ_e in the above equation is the area of triangle ijk expressed in terms of coordinates of its corners. The functions $\xi_i^e(x, z)$, $\xi_j^e(x, z)$ and $\xi_k^e(x, z)$ are the element interpolations or local coordinate functions or basis functions.

In addition, we can also derive the derivatives in equation (4.15):

$$\frac{\partial \xi_n^e}{\partial x} = \frac{b_n}{2\Delta}; \quad \frac{\partial \xi_n^e}{\partial z} = \frac{c_n}{2\Delta} \quad (4.23)$$

Now the element matrices in equation (4.14) can be written as below :

$$\begin{aligned} A_{nm} &= \sum_e K K_{ij}^A \int_{\Omega^e} \xi_l^e \frac{\partial \xi_n^e}{\partial x_i} \frac{\partial \xi_m^e}{\partial x_j} d\Omega \\ &= \sum_e \frac{k}{4\Delta_e} \bar{K} [K_{xx}^A b_m b_n + K_{xz}^A (c_m b_n + b_m c_n) + K_{zz}^A c_n c_m] \end{aligned} \quad (4.24a)$$

$$\begin{aligned} F_{nm} &= \delta_{nm} \sum_e \int_{\Omega^e} \xi_n^e d\Omega \quad (\delta_{nm} \dots \text{Kronecher delta}) \\ &= \begin{pmatrix} \sum_e \frac{k\Delta_e}{3} & \text{if } n = m \\ 0 & \text{if } n \neq m \end{pmatrix} \end{aligned} \quad (4.24b)$$

$$Q_n = -\sum_e \int_{\Gamma^e} \xi_n^e q d\Gamma = -\sum_e \lambda_n q_n \quad (4.24c)$$

$$B_n = \sum_e K K_{iz}^A \int_{\Omega^e} \xi_n^e \frac{\partial \xi_n^e}{\partial x_i} d\Omega = \sum_e \frac{k}{2} \bar{K} (K_{xz}^A b_n + K_{zz}^A c_n) \quad (4.24d)$$

$$D_n = \sum_e \int_{\Omega^e} S \xi_n^e d\Omega = \sum_e \frac{k}{12} \Delta_e (3\bar{S} + S_n) \quad (4.24e)$$

Equations (4.24a) to (4.24e) hold for flow in a two-dimensional Cartesian (x, z) domain, as well as for flow in axisymmetric (x, z) system in which x is used as a radial coordinate. For plane flow we have

$$k = 1 \quad \lambda_n = \frac{L_n}{2} \quad (4.24f)$$

while for axisymmetric flow

$$k = 2\pi \frac{x_i + x_j + x_k}{3} \quad \lambda_n = L_n \pi \frac{x'_n + 2x_n}{3} \quad (4.24g)$$

The subscripts i, j and k refer to the three corners of each triangle as shown in figure (4.2). The average hydraulic conductivity K and the average rate of water extraction S are, respectively, $K = (K_i + K_j + K_k)/3$, $S = (S_i + S_j + S_k)/3$. L_n is the length of boundary segment connected to node n, q_n stands for the flux [LT^{-1}] across the boundary in the vicinity of a boundary node n (positive when directed outward of the system), and x'_n is the x-coordinate of a boundary node adjacent to node n. The boundary flux is assumed to be uniform over each boundary segment.

4.3.4 Time Discretization

To solve the ordinary differential equation (4.14), a proper time discretization is necessary. In saturated flow when the specific storage is zero the time derivative disappears. At this condition the governing equation becomes elliptic type and in other cases this is parabolic type. Change in boundary conditions of the saturated flow has an instantaneous effect on the entire saturated region. The solution is no longer a continuous function of time. Pressure of current time level is independent of the pressure head of the previous time level. For these reasons and to get a correct specification of the time dependent part or fully implicit difference scheme has been adopted here.

Integration of equation (4.14) in time is achieved by discretizing the time domain into a sequence of finite intervals and replacing the time derivatives by finite differences. An implicit (backward) finite difference scheme is used for both saturated and unsaturated conditions:

$$[F] \frac{\{\theta\}_{j+1} - \{\theta\}_j}{\Delta t_j} + [A]_{j+1} \{h\}_{j+1} = \{Q\}_{j+1} - \{B\}_{j+1} - \{D\}_{j+1} \quad (4.25)$$

where $(j+1)$ denotes the current time level at which the solution is being considered, j refers to the previous time level, and $\Delta t_j = t_{j+1} - t_j$. Equation (4.25) represents the final set of algebraic equations to be solved. Since the coefficients θ , A , B and D are functions of the dependent variable h , the set of equations is generally highly nonlinear.

4.4 Computational Strategy

4.4.1 Iteration Process

After applying Galerkin approximation, the system of the time dependent integro-differential equations are nonlinear because the matrix elements, A_{nm} and B_{nm} , are functions of nodal unknowns h_n . So the first estimation of the solution must be improved by an iterative process leading to a convergent solution. Picard iteration

method and Newton-Raphson method are the two methods to treat this nonlinearly. Although Newton-Raphson method is second order convergent and Picard method is first order (i. e., error decreases linearly with the error at the previous iteration in Picard method and in Newton-Raphson method the error in present iteration is approximately proportional to the square of the previous iteration's error), Newton-Raphson method, unlike Picard method, requires continuity of the gradient of hydraulic diffusivity ($dD/d\theta$), for convergence (Huyakorn, 1983).

In the present case, Picard iteration method is used. For each iteration, the resulting set of simultaneous linear algebraic equations is solved by the Gauss elimination algorithm, which takes advantage of the banded and symmetric features of the coefficient matrices in equation (4.24). Next, the coefficients in equation (4.24) are reevaluated using the first solution, and the new equations are again solved. The iterative procedure continues as long as it is necessary to achieve a satisfactory degree of convergence i. e., until at all the nodes the absolute change in pressure head between two successive iterations becomes less than some small value determined by the imposed relative and absolute pressure head tolerances. The first estimate (at zero iteration) of the unknown pressure heads at each time step is obtained by extrapolation from the pressure head values at the previous two time levels. In order to reduce the computational effort, the entire system of equations is solved only for the first iteration (Staufer, 1984). Subsequent iterations are limited to those parts of the flow region which exhibited changes in the nodal values of the pressure head during the first iteration that exceeded a prescribed small tolerance. These parts include all elements surrounding nodes for which changes in the pressure head were recorded. The procedure was found to save a considerable amount of computer time, especially when water is confined only to a small part of the flow domain (e. g., for infiltration in relatively dry soil).

4.4.2 Time Control

For transient problem, computation is started from initial time ($t = 0$). After that computation is proceeded by time marching through the accretion of time steps with

the old time. This is the time level of the numerical scheme. Also we have some time dependent input (such as river water level , rainfall precipitation) which should be read in specified time levels. And finally the required output is sought in certain time levels. Thus three different time discretization levels are distinguished in the program:

- (1) the time discretization associated with the numerical solution;
- (2) the time discretization associated with the implementation of variable boundary conditions; and
- (3) the time discretization, which provides printed output of the simulation results (e. g., nodal values of dependent variables, water and solute mass balance components, and other information about the flow regime).

During computation the first one is reforming as computation proceeded while the second and the third time levels are predetermined by the user to input data and store output.

Time Control in Numerical Scheme

Owing to severe nonlinearities encountered in problems of flow in variably saturated soils, the iterative procedure in the scheme may not converge unless time step (Δt) is unusually small, although this condition will be improved when a larger part of the flow region becomes saturated and thereby Δt can be increased. So a automatic time-stepping control is incorporated in the program as suggested by Huyakorn et. al. (1983).

Time discretization associated with the numerical solution starts with a prescribed initial time increment, Δt . This time increment is automatically adjusted at each time level according to the following rules (MIs, 1982; Vogel, 1987):

- a. Discretization (1) must coincide with the time values resulting from discretization (2) and (3) as specified above.
- b. Time increments can not become less than a preselected minimum time step, $\Delta t_{\min}$, nor exceed a maximum time, $\Delta t_{\max}$ (i. e., $\Delta t_{\min} \leq \Delta t \leq \Delta t_{\max}$).

- c. If, during a particular time step, the number of iterations necessary to reach convergence is ≤ 3 , the time increment for the next time step is increased by multiplying Δt by a predetermined constant > 1 (usually between 1.1 and 1.5). If the number of iterations ≥ 7 , Δt for the next time level is multiplied by a constant < 1 (usually between 0.3 and 0.9).
- d. If, during a particular time step, the number of iterations at any time level becomes greater than a prescribed maximum (usually between 10 and 50), the iteration process for that time level is terminated. The time step is subsequently reset to $\Delta t/3$, and the iterative process restarted.

Time Control of Time Dependent Input

The input data for time dependent input variables such as river water level, rainfall, evaporation are generally found in hourly, daily or monthly basis. The time marching in the numerical scheme should coincide with the input time level to feed the data in proper time. So when solution time is approaching to the input time level, then an adjustment in the time stepping is made depending on the existing time step and the difference between the time level in the numerical scheme and the next time level of the input data.

Time Control for Output

During the data preparation of the program, a number of time levels is specified to store the outputs of the simulated unknowns such as pressure head, moisture content, inflow or outflow through boundary, flux etc. Here also, an adjustment of the time stepping is required to coincide with the desired print levels.

It has appeared during the work of this thesis that an internal adjustment of the time step for variable boundary conditions (discretization (2)) in accordance with changes of the time step of numerical solution (discretization (1)) would be desirable, especially if it were connected with a linear or other interpolation of the boundary

values provided in the input files. However, this modification has not yet been accomplished.

4.4.3 Treatment of the Water Capacity term

The iteration process is extremely sensitive to the method used for evaluating the water content term ($\Delta\theta/\Delta t$) in equation (4.14). The program uses the "mass-conservative" method proposed by Celia et. al. (1990). Their method has been shown to provide excellent results in terms of minimizing the mass balance error. The mass-conservative method proceeds by separating the water content term into two parts:

$$[F] \frac{\{\theta\}_{j+1} - \{\theta\}_j}{\Delta t_j} = [F] \frac{\{\theta\}_{j+1}^{k+1} - \{\theta\}_{j+1}^k}{\Delta t_j} + [F] \frac{\{\theta\}_{j+1}^k - \{\theta\}_j}{\Delta t_j} \quad (4.26)$$

where $k+1$ and k denote the current and previous iteration levels, respectively; and $j+1$ and j the current and previous time levels, respectively. The second term on the right-hand side of the equation (4.26) is known prior to the current iteration. The first term on the right-hand side can be expressed in terms of the pressure head, so that (4.26) becomes

$$[F] \frac{\{\theta\}_{j+1} - \{\theta\}_j}{\Delta t_j} = [F][C]_{j+1} \frac{\{h\}_{j+1}^{k+1} - \{h\}_{j+1}^k}{\Delta t_j} + [F] \frac{\{\theta\}_{j+1}^k - \{\theta\}_j}{\Delta t_j} \quad (4.27)$$

where $C_{nm} = \delta_{nm} C_n$, in which C_n represents the nodal value of the soil water capacity. The first term on the right-hand side of the equation (4.27) should vanish at the end of the iteration process if the numerical solution converges. This particular feature guarantees relatively small mass balance errors in the solution.

4.4.4 Water Balance Computations

This program performs water balance computations at prescribed times for several preselected subregions of the flow domain. The water balances information for each subregion consists of the actual volume of water, V , in that subregion, and the rate, O , of inflow or outflow to or from the subregion. V and O are given by

$$V = \sum_e k \Delta_e \frac{\theta_i + \theta_j + \theta_k}{3} \quad (4.28)$$

and

$$O = \frac{V_{new} - V_{old}}{\Delta t} \quad (4.29)$$

respectively, where θ_i , θ_j and θ_k are water contents evaluated at the corner nodes of the element e and V_{new} and V_{old} are volumes of water in the subregion computed at the current and previous time levels, respectively. Δ_e is the area of the triangular element and the coefficient k is according to the equations (4.24f) or (4.24g). The summation in equation (4.28) is taken over all elements within the region.

The absolute error in the mass balance is computed as

$$\varepsilon_a^w = V_t - V_0 + \int_0^t T_a L_t dt - \int_0^t \sum_{n_r} Q_n dt \quad (4.30)$$

where V_t and V_0 are the volumes of water in the flow domain at time t and zero, respectively, as calculated with equation (4.28). The third term on the right hand side of the equation represents the cumulative root water uptake amount, while the fourth term gives the cumulative flux through nodes, n_r , located along the boundary of the flow domain, or at internal source and sink nodes.

The accuracy of the numerical solution is evaluated by the relative error ε_r^w , [%] in the water mass balance as follows:

$$\varepsilon_r^w = \frac{|\varepsilon_a^w|}{\max(\sum_e |V_t^e - V_0^e| + \int_0^t T_a L_t dt - \int_0^t \sum_{n_r} |Q_n| dt)} \quad (4.31)$$

where V_t^e and V_0^e are volumes of water in element, e , at time t and zero, respectively. It should be noted that the program does not relate the absolute error to the volume of water in the flow domain, but instead to the maximum value of two quantities. The first quantity represents the sum of the absolute changes in water content over all elements, whereas the second quantity is the sum of the absolute values of all fluxes in and out of the flow domain.

4.4.5 Computation of Nodal Fluxes

Components of the Darcian flux are computed in the model only at selected print times (corresponding to the discretization (3) in article 4.4.2) during the simulation when the water flow equations are solved. The x- and z-components of the nodal fluxes are computed for each node n according to :

$$\begin{aligned}
 q_x &= -\frac{K_n}{N_e} \sum_{e_n} \left[\frac{\gamma_i^x h_i + \gamma_j^x h_j + \gamma_k^x h_k}{2 \Delta_e} + K_{xz}^A \right] \\
 q_z &= -\frac{K_n}{N_e} \sum_{e_n} \left[\frac{\gamma_i^z h_i + \gamma_j^z h_j + \gamma_k^z h_k}{2 \Delta_e} + K_{xz}^A \right] \\
 \gamma_n^x &= K_{xx}^A b_n + K_{xz}^A C_n \\
 \gamma_n^z &= K_{xz}^A b_n + K_{zz}^A C_n
 \end{aligned} \tag{4.32}$$

where N_e is the number of (triangular) elements e_n adjacent to node n , and formulae for the element area is Δ_e and the geometric coefficients are $b_n = a_1^{(n)}$ and $c_n = a_2^{(n)}$. Einstein's summation convention is not used in the above equation.

4.4.6 Water Uptake by Plant Roots

The model considers the root zone to consist of all nodes, n , for which the potential root water uptake distribution, b , is greater than zero. The root water extraction rate is assumed to vary linearly over each element, which leads to the approximation for the root water extraction term D_n in the global matrix equation. The values of the actual root extraction rate S_n is evaluated using eq. (3.9). The model calculates the total transpiration rate per unit soil surface length using the equation

$$T_a = \frac{1}{L_t} \sum_e k \Delta_e \bar{S} \tag{4.33}$$

In which the summation takes place over all elements within the root zone.

4.4.7 Evaluation of Soil Hydraulic Properties

At the beginning of numerical simulation, the model generates a table of water contents, hydraulic conductivities and specific water capacities from the specified set of hydraulic parameters for each soil type in the flow domain. The tabulated values of θ_i , K_i and C_i are evaluated at prescribed pressure heads, h_i , within a specified interval (h_a, h_b) . The entries in the table are generated such that

$$\frac{h_{j+1}}{h_j} = \text{const.} \quad (4.34)$$

which means that the spacing between two consecutive pressure head values increases in a logarithmic fashion. Values for the hydraulic properties, $\theta(h)$, $K(h)$ and $C(h)$, are computed during the iteration solution process using linear interpolation between the entries in the table. If an argument, h , falls outside the prescribed interval (h_a, h_b) , the hydraulic characteristics are evaluated directly from the hydraulic functions, i. e., without interpolation. The above interpolation technique was found to be much faster computationally than direct evaluation of the hydraulic functions over the entire range of pressure heads, except when very simple hydraulic models were used.

4.4.8 Implementation of the Hydraulic Conductivity Anisotropy

Since the hydraulic conductivity anisotropy tensor, K^A , is assumed to be symmetric, it is possible to define at any point in the flow domain a local coordinate system for which the tensor K^A is diagonal (i. e., having zeros everywhere except on the diagonal). The diagonal entries K_1^A and K_2^A of K^A are referred to as the principal components of K^A .

The model code permits one to vary the orientation of the local principal directions from element to element. For this purpose, the local coordinate axes are subjected to a rotation such that they coincide with the principal directions of the tensor K^A . The principal components K_1^A and K_2^A , together with the angle ω between the principal direction of K_1^A and x-axis of the global coordinate system, are specified for each

element. Each locally determined tensor K^A is transformed to the global (x,z) coordinate system at the beginning of the simulation using the following rules:

$$\begin{aligned} K_{xx}^A &= K_1^A \cos^2 \omega + K_2^A \sin^2 \omega \\ K_{zz}^A &= K_1^A \sin^2 \omega + K_2^A \cos^2 \omega \\ K_{xz}^A &= (K_2^A - K_1^A) \sin \omega \cos \omega \end{aligned} \quad (4.35)$$

4.4.9 Steady-state Analysis

All transient flow problems are solved by time marching until a prescribed time is reached. The steady-state problem can be solved in the same way, i. e., by time marching until two successive solutions differ less than some prescribed pressure head tolerance. The model implements a faster way of obtaining the steady-state solution without having to go through a large number of time steps. The steady-state solution for a set of imposed boundary conditions is obtained directly during one set of iterations at the first time step by equating the time derivative term in the Richards equation to zero.

CHAPTER 5

TREATMENT OF AUXILIARY CONDITIONS

5.1 Introduction

The governing differential equation of flow through porous medium describes a class of phenomena and does not represent any specific case. From infinite number of possible solutions, to find out the specific one, it is necessary to provide the supplementary information that is not contained in the equation. Except the geometry of the flow domain, the most important information are initial and boundary condition of the flow. Boundary and initial conditions are motivated by the physical reality of the flow. Hence they are first assumed or determined on the basis of the available information and the past experience in the field and then expressed in mathematical forms. As different boundary conditions lead to different solutions correct determination of the boundary conditions is very important.

5.2 Initial Conditions

The initial conditions are usually defined in terms of the dependent variable of the Richards equation, which can be, e. g., the volumetric moisture content θ , or the pressure head h , or other suitable variable derived from them. Initial conditions include the specification of pressure head h or moisture content θ , at all points within the domain Ω at some initial time, usually denoted as $t = 0$. This can be written schematically as specifying

$$\psi = f(x, y, z, 0) \quad ; \quad \text{all } (x, y, z) \in \Omega \quad (5.1)$$

where f is a known function.

Initial conditions must be defined when a transient soil water flow is modeled. In the finite element approach, the values of pressure head or soil moisture content at each nodal point within the flow region are required. When detailed data are not available then sometimes the soil moisture contents or pressure heads at field capacity or those

in equilibrium with the boundary conditions and with the groundwater table might be a realistic initial condition.

5.3 Boundary Conditions

The boundary conditions define the situation at the boundaries of the flow region during the whole time period considered. Three basic types of boundary conditions are usually distinguished (e. g., Dennemeyer, 1968):

- (i) Dirichlet type (which in this case means a prescribed pressure head or a prescribed moisture content),
- (ii) Neumann type (in this case: a prescribed flux),
- (iii) Mixed type (in this case: a prescribed relation between the flux and the pressure head or moisture content).

In the present context the boundary conditions can also be distinguished as:

- (i) System-independent boundary conditions, when the condition in a given node is determined a priori (including its temporal variability, if such is the case),
- (ii) System-dependent boundary conditions, which change either its numerical value or its structure and type or both depending upon the instantaneous state of the flow region or of its part.

The present model implements all three (Dirichlet, Neumann and mixed) types of the system-independent and four types of system-dependent boundary conditions which are described in the following text.

5.4 System-independent Boundary Conditions

5.4.1 Dirichlet Condition

A certain value of a scalar quantity is prescribed at every time instant in each nodal point of the boundary. In GWm, it is the pressure head, h , which must be known at all

boundary nodes, defined by

$$H(x, z, t) = \psi(x, z, t) \text{ for } (x, z) \in \Gamma_D \quad (5.2)$$

where Γ_D is a Dirichlet type boundary segment, and ψ is a prescribed function of x , z and t . Generally all boundary nodal points on the river bottom and on the permanently submerged sides can be provided with a Dirichlet boundary condition. However, in the regions that can be variably under water or above water, a combined system-dependent Dirichlet/ Neumann condition is to be defined.

Finite element equations corresponding to Dirichlet nodes where the pressure head is prescribed can, at least in principle, be eliminated from the global matrix equation. An alternative and numerically simpler approach is to replace the Dirichlet finite element equations by dummy expression of the form (Neumann et al., 1974)

$$\delta_{nm} h_m = \psi_n \quad (5.3)$$

where δ_{nm} is the Kronecker delta and ψ_n is the prescribed value of the pressure head at node n . The values of h_n in all other equations are set equal to ψ_n and the appropriate entries containing ψ_n in the left-hand side matrix are incorporated into the known vector on the right-hand side of the global matrix equation. When done properly, this rearrangement will preserve symmetry in the matrix equation. After solving for all pressure heads, the value of the flux Q_n can be calculated explicitly and accurately from the original finite element equation associated with node n (e. g., Lynch, 1984).

5.4.2 Neumann Condition

A certain value of vector quantity such as the Darcian flux q is prescribed at every time instant in each nodal point of the boundary, as defined by

$$q_x n_x + q_z n_z = \sigma(x, z, t) \text{ for } (x, z) \in \Gamma_N \quad (5.4)$$

where Γ_N indicates a Neumann-type boundary segment, σ is a prescribed function of x , z and t ; n_x and n_z are, respectively, the horizontal and vertical components of the outward unit vector normal to the boundary Γ_N , and q_x and q_z are the horizontal and vertical components of the Darcian flux at the point (x, z) . A trivial but very frequently used variant of the Neumann condition is the condition of the zero flux (no flow condition); in that case, the function $\sigma(x, z, t)$ is identically equal to zero. This type of condition is the most natural boundary condition of the finite-element method.

Within the GWm program, the nodes with the no-flow condition are treated as internal nodes without sinks and sources. Non-zero system-independent Neumann conditions may be postulated, e. g., on the soil surface, when the maximum infiltration rate will not exceeded surely and the evaporation will occur at the potential rate. It was not used during computations carried out in this thesis. The values of the fluxes Q_n at the nodal points along the prescribed flux boundaries are computed according to equation (4.24c). Internal nodes which act as Neumann type sources or sinks have values of Q_n equal to the imposed fluid injection or extraction rate.

5.4.3 Mixed Type Boundary Condition

A relation is prescribed between a scalar quantity and a vector quantity at every time instant in each node of the boundary. If the pressure head h and the Darcian flux q are these quantities, then an equation is to be fulfilled which may be typically of the form

$$F(h, \text{grad } h, q) = 0 \quad \text{for } (x, z) \in \Gamma_M \quad (5.5)$$

where Γ_M is the mixed-type boundary condition segment and F is a scalar function of one scalar and two vector arguments; it must be, of course, different from the Darcy law, which is of the same form.

A characteristic example is the bottom-flux (deep-drainage) boundary condition as defined by the equation (5.13) below. A simpler case is so-called free-drainage boundary condition when the gradient of hydraulic head is assumed unity at the bottom of a soil profile, so that the Darcian flux is equal to the instantaneous hydraulic conductivity in the node which, in turn, is a function of the nodal pressure head or moisture content. Simunek et al. (1992, p. 14-15) speak in this context about the Cauchy boundary condition.

A characteristic example of mixed type of boundary is the bottom-flux (deep-drainage) boundary condition as defined by a semi-empirical equation suggested by Hopmans and Stricker (1988):

$$q(\psi) = -A_{q\psi} \exp(B_{q\psi} |\psi - RGL|) \quad (5.6)$$

where $A_{q\psi}$ and $B_{q\psi}$ are empirical parameters, RGL represents the reference position of the groundwater level, which should be set equal to or higher than the z-coordinate of the soil surface. The discharge rate $Q(n)$ assigned to a node n is determined by multiplying $q(\psi)$ with the boundary area related to that node.

5.5 System-dependent Boundary Conditions

In the simulation of a real situation, this type of boundary conditions play an important role in the overall process. These also arise complicity in the solution algorithm. The faces of the boundary, which are in direct contact with the atmosphere and the influences of rainfall and evaporation are dominant, is one of such kind of boundary that may be called atmospheric boundary. Other one is the seepage face boundary, through which water moves out of the system in atmospheric pressure (e. g., in a riverbank when water level of river is lower than the groundwater table). System-dependent boundary conditions of these types and their implementation in the program are described below.

5.5.1 Atmospheric Boundary Condition

The potential fluid flux across the soil-air interface is controlled exclusively by atmospheric conditions. However, the actual flux depends also on the prevailing soil moisture and pressure head conditions. Soil surface boundary conditions may change from prescribed-flux to prescribed-head type conditions (and vice-versa). In the absence of surface ponding, the numerical solution of (3.1) is made possible by limiting the Darcian flux q and the pressure head h by the following two conditions (Neumann et al., 1974):

$$-I_s^* \leq q_x n_x + q_z n_z \leq E_s^* \quad (5.7)$$

and

$$h_A \leq h \leq h_S \quad (5.8)$$

where E_s^* [LT^{-1}] is the maximum potential rate of evaporation and I_s^* [LT^{-1}] is the maximum infiltration rate under the current atmospheric condition, q_x and q_z are components of the vector of Darcian flux at the soil surface, n_x and n_z are components of the unit vector of the outer normal to the soil surface, and h_A and h_S are,

respectively, minimum and maximum pressure heads allowed under the prevailing soil properties.

The value for h_A is determined from the equilibrium conditions between soil water and atmospheric water vapor, whereas h_S is usually set equal to zero. When one of the endpoints, that is h_A or h_S , is reached, a prescribed head boundary condition (equal to the end value reached) will be used. Methods of calculating E_s^* and h_A on the basis of atmospheric data have been discussed by Feddes et al. (1974). They suggest that the total potential evapotranspiration from both soil and crop, E^* [LT^{-1}], be calculated according to the combined-type equation, which reads, after a revision:

$$E^* = \frac{\delta (R_n - G) + \rho_a c_p (e_z^* - e_z) / r_a}{(\delta + \gamma) L \rho_w} \quad (5.9)$$

where δ [$ML^{-1}T^{-2}\Theta^{-1}$] is slope of saturation vapor pressure curve, R_n [MT^{-3}] is net radial flux, G [MT^{-3}] is heat flux into soil, ρ_a [ML^{-3}] is the density of moist air, c_p [$L^2T^{-2}\Theta^{-1}$] is the specific heat of air at constant pressure, e_z^* [$ML^{-1}T^{-2}$] is saturated vapor pressure and e_z [$ML^{-1}T^{-2}$] is measured vapor pressure at elevation z and at ambient temperature, r_a [$L^{-1}T$] is the resistance to turbulent vapor diffusion through the air layer below the elevation z , γ [$ML^{-1}T^{-2}\Theta^{-1}$] is psychrometric constant, L [L^2T^{-2}] is latent heat of water vaporization and ρ_w [ML^{-3}] is the water density. The symbol " Θ " is used here to indicate the unit of temperature. The potential evaporation, E_s^* , is then calculated as given below from a simplified form of the above equation (5.9) by neglecting the second (aerodynamic) term and taking into account only that fraction of R_n which reaches the soil surface (Neuman et al., 1975; with reference to Ritchie, 1972).

$$E_s^* = \frac{\delta R_n \exp^{-0.39(LAI)}}{(\delta + \gamma) L \rho_w} \quad (5.10)$$

where LAI is the leaf-area index.

Feddes et al. (1974) suggest the following formula to determine the h_A in (5.8) :

$$h_A = (RT/Mg) \ln(f) \quad (5.11)$$

Where R is the universal gas constant, T is the absolute temperature, M is molecular weight of water, g is acceleration due to gravity, and f is the relative humidity of air.

Formulae of this kind are, however, not included in GWm where it is assumed that the potential evaporation, as well as potential infiltration (= precipitation) is provided as the input data. It is in fact only the second of the conditions (5.7)-(5.8) which is used in GWm. If the soil surface is very wet or very dry, the Dirichlet boundary condition with either $h=h_S$ or $h=h_A$ is maintained and the flux is computed by the finite element algorithm. As soon as this flux becomes algebraically either smaller or larger, respectively, than the potential flux suggested by the atmospheric input data, a reverse switch takes place to the Neumann boundary condition with the flux dictated by the atmosphere. Surface fluxes during this computation are positive out of and negative into the soil.

5.5.2 Seepage Face Boundary Condition

A seepage face through which water seeps out from the saturated portion of porous medium is defined as a system-dependent boundary condition in GWm. The length of the seepage face varies with time in a manner that can never be predicted a priori. GWm assumes that the pressure head along the seepage face must be uniformly zero (because the atmospheric pressure head is also considered zero). This condition (the Dirichlet one) persists as long as the nodal pressure head is non-negative. When it becomes negative (i.e., when the node becomes unsaturated), the Neumann condition of no flow is imposed on this node, until the node becomes saturated again, with the nodal pressure head greater than or equal to zero. Then the original Dirichlet condition becomes again effective.

5.5.3 Variable Atmospheric Boundary Condition

The above described mechanism of switching between the Dirichlet condition (when the soil surface is either too wet at the pressure head h_S or too dry at the pressure head h_A) and the Neumann condition (when the precipitation is infiltrated without rest or the evaporation corresponds to the potential level) is maintained. However, at the instant when new atmospheric data enter the computation, the subroutine Abound calculates the difference between precipitation, evaporation and surface Darcian flux (=infiltration) for the previous time step (time steps mentioned here correspond to the discretization level (2) as defined before). The calculation is done for the whole soil

surface as a whole, i. e., a sum of infiltration over all "atmospheric" nodes is determined. The difference mentioned is algebraically added to the depth of water on the soil surface at the beginning of the previous time step (zero depth must be assumed at the beginning of the calculation). If zero or negative new depth results (at the end of the previous time step), the boundary condition corresponding to the above described "atmospheric boundary condition" is set for the whole following time step at the whole atmospheric boundary. If a positive new depth results from the water balance at the soil surface during the previous time step, a constant pressure head Dirichlet boundary condition is set for the whole following time step at all "atmospheric " boundary nodes (it is assumed that if a water surplus occurs on a part of the soil surface, it will be immediately redistributed over the whole horizontal soil surface). No water from the soil surface is assumed to be discharged through surface runoff.

5.5.4 Variable Seepage Face Boundary Condition

The above described mechanism of switching between saturated (actual) seepage face with zero pressure head Dirichlet boundary condition and unsaturated (potential) seepage face with no-flow boundary condition is let to operate between the time steps of the atmospheric input data (discretization level (2) as per article 4.2.2). The atmospheric input file (ATMOSPH.IN) now contains also daily data of the river water stage (with respect to the reference datum equal to the origin of coordinates of the finite element grid). As soon as new water stage enters, the vertical coordinates of all the nodes on the potential variable seepage face are compared with it. Those lying below or at the new river water level are ascribed a Dirichlet boundary condition of constant pressure head corresponding to the vertical distance of the node below the river level. The nodes lying above the new river water level are given either no-flow boundary condition, if they lie also above the previous water level, or zero pressure-head Dirichlet boundary condition, if they lie below or at the previous water level.

5.6 Coding of Boundary Conditions

The types of boundary conditions set and the initial and boundary information itself must be defined in a specified way before starting the numerical simulation. The code

numbers [kode(n)] identify the boundary conditions assigned to each node according to the following Table 5.1.

Table 5.1: Nodes Identification Code [Kode(n)]

Node type	Kode(n)
Internal; not sink/ source	0
Internal; sink/ source (Dirichlet condition)	1
Internal; sink/ source (Neumann condition)	-1
Impermeable head boundary	0
Specified head boundary	1+
Specified flux boundary	-1++
Atmospheric boundary condition	± 4
Variable pressure head boundary condition	+3
Variable flux condition	-3
Seepage face (initially saturated)	+2
Seepage face (initially unsaturated)	-2

+5 and/ or 6 may also be used

+-5 and/ or -6 may be also used

5.6.1 Coding of the Atmospheric Boundary Condition

Atmospheric boundary condition can be implemented if there are some nodes with Kode (n)=4, in which case time-dependent input data for the water level Wl, precipitation rate Prec and evaporation rate rSoil must be specified in the input file ATMOSP.HIN. THE potential fluid flux across the soil surface is determined as $r_{Atm}=r_{Soil}-Prec$. The actual surface flux is calculated internally by the program. Two limiting values of surface pressure head must also be provided: hCritS which specifies the maximum allowed pressure head at the soil surface (usually 0.0), and hCritA which specifies the minimum allowed surface pressure head (defined from equilibrium conditions between soil water and atmospheric vapor). The program automatically switches the value of Kode(n) from -4 to +4 if one of these limiting points is reached. Table 5.2(a) shows the initial setting of the variables Kode(n), Q(n) and h(n) for the time-dependent boundary conditions. Table 5.2(b) summarizes the

use of the variables r_{Atm} , h_{CritS} and h_{CritA} during program execution. $Width(n)$ in the table denotes the length of the boundary segment associated with node n .

**Table 5.2a: Initial Setting of Kode(n), Q(n) and h(n)
for Atmospheric Boundary Condition**

Nodal point	Kode(n)	Q(n)	h(n)
n	-4	0.0	Initial value

**Table 5.2b: Definition of the Variables Kode(n), Q(n) and h(n)
when an Atmospheric condition is applied**

Kode(n)	Q(n)	h(n)	Event
-4	$-width(n) \cdot r_{Atm}$	Unknown	$r_{Atm} = r_{Soil-Prec}$
+4	Unknown	h_{CritA}	Evaporation capacity is exceeded
+4	Unknown	h_{CritS}	Infiltration capacity is exceeded

The subroutine Abound has been included to specify a variable atmospheric boundary condition. This subroutine computes the depth of water on the soil surface at the beginning of a time step of atmospheric input data, using simple continuity equation

$$\Delta d = (Q + r_{Soil-Prec}) (T_{j+1} - T_j) \quad (5.12)$$

where Q is the simulated average Darcian flux across the atmospheric boundary in the previous time step. The program specifies at the soil surface a Dirichlet boundary condition if the computed water depth is greater than zero (i. e., if surface ponding occurs). This type of variable atmospheric boundary must be prescribed with a logical variable $Vatms$ set to .true. in the input file SELECTOR.IN.

5.6.2 Coding of Variable Pressure Head and Variable Flux Boundary Conditions

Variable pressure head or variable flux boundary condition along a certain part of the boundary is implemented when $Kode(n) = 3$ or -3 , respectively. The initial settings of $Kode(n)$, $Q(n)$ and $h(n)$ are shown in Table 5.3(a). In that case, the input file ATMOSPH.IN must contain the prescribed time-dependent values of the pressure head, ht , or the flux, rt , imposed along the boundary. The values of ht or rt are

assigned to particular nodes at specified times according to rules given in Table 5.3(b).

Table 5.3a: Initial Setting of Kode(n), Q(n) and h(n) for Variable Pressure Head and Variable Flux Boundary Conditions

Node type	Kode(n)	Q(n)	h(n)
Variable head boundary	+3	0.0	Initial value
Variable flux boundary	-3	0.0	Initial value

Table 5.3b: Definition of the Variables Kode(n), Q(n) and h(n) when Variable Head or Flux Boundary Conditions are applied

Node type	Kode(n)	Q(n)	h(n)
Variable head boundary	+3	Unknown	Initial value
Variable flux boundary	-3	-width(n)*rt	Initial value

5.6.3 Coding of Seepage face boundary conditions

A subroutine VBound has been included which treats the nodal points on the river bank face either as nodes with the Dirichlet boundary condition or as elements of a saturated or unsaturated seepage face, based on the river water level. The nodal points above the water level in the river are specified as saturated seepage nodal points, if they have possessed the Dirichlet boundary condition in the previous time step of the atmospheric input data. The user must provide a list of all nodes along each variable seepage face boundary at the start of simulation, in the input file VSEEP.IN. The initial setting of the variables Kode(n), Q(n) and h(n) for nodes along a seepage face is summarized in the following Table 5.4.

Table 5.4: Initial Setting of Kode(n), Q(n) and h(n) for Seepage Faces

Node type	Kode(n)	Q(n)	h(n)
Seepage face (initially saturated)	+2	0.0	0.0
Seepage face (initially unsaturated)	-2	0.0	Initial value

5.7 Water Uptake by Plant Roots

Plant extraction of water from the soil root zone is calculated by evaluating the term D_n in the finite element scheme. The values of the potential transpiration rate, r_{Root} , must be specified at preselected times in the input file ATMOSP.H.IN. The actual transpiration rates are calculated internally. The root uptake parameters are taken from a separate input file SINK.in. Values of Beta (n), which describes the potential water uptake distribution over the root zone (equations 3.5 to 3.7), must be specified for each node in the flow domain. All parts of the flow region where Beta (n)>0 are treated as the soil root zone, even if the potential transpiration values in ATMOSP.H.IN are zero.

5.8 Deep Drainage from the Soil Profile

The GWm code calculates the vertical drainage across the lower boundary of the saturated flow domain, if Kode (n)=-3 is specified on this boundary and if the logical variable gGWLF in the input file ATMOSP.H.IN is set to .true. The discharge rate $Q(n)$ assigned to a node n is determined by the program as $Q(n) = -\text{Width} \cdot q(h)$, where h is the local value of the pressure head and $q(h)$ is calculated from the semi-empirical equation suggested by Hopmans and Stricker (1988):

$$q(h) = -A_{qh} \exp(B_{qh} |h - \text{GWL0L}|) \quad (5.13)$$

where A_{qh} and B_{qh} are empirical parameters, which must be specified in the input, file ATMOSP.H.IN, together with GWL0L, which represents the reference position of the groundwater level. The latter should be equal to or higher than the z-coordinate of the soil surface.

CHAPTER 6

GRAPHICAL USER INTERFACE (GUI)

6.1 About GUI

A GUI could be defined as a window that incorporates different objects in a graphical way to control the program flow. In conventional DOS based applications, developed using procedural language, user has no control over program flow. Once it is executed the program follow the logic embedded in it until program terminates normally or for an error. But GUI driven application provides user flexibility for choosing a set of activities through menu option, Toolbar or command button. User has control over the different functions of the program.

6.2 Usefulness of GUI

Efficient, reliable, and user-friendly software helps users solve complex problem comfortably, and increase productivity. Alternately, a user-unfriendly software can result in considerable losses both from person-hours invested and reliability as it could yield misleading or incorrect results after several hours or days of operation. The goal of the Graphical User Interface (GUI) for the Ground Water model (GWm) is to help user comfortably and conveniently handle data input to the model, run the model and print and graphically represent output results.

The Ground Water model (GWm) is implemented using procedural language Fortran 77. Model takes input from several ASCII text files in which data have to be written according to the format statements of the Fortran programs. Each of these input files contains records of differing structure and varying length. Creating an input file or editing values of the variables in an existing file by an ASCII Text Editor is cumbersome and erroneous as the activities are not at all user friendly. Supposing any user wants to change a value of any input variable (s)he must know the records' structure of that file and format of the variables in order to identify it inside the particular record of the input file and to change the value according to the defined

format in the Fortran program. With a view to making the model more user-friendly and efficient in operation an Interface has been designed Graphically to handle the user input, report the output and generate graphs.

6.3 Functionality of GUI

The GUI facilitates user to provide input in several Form Based Windows and user need not to care about the format of the input data and how the data is written in the file. Some Error Trapping and Logical Features could also be programmed with the Form Based Graphical Input Window so that a valid range of value for a particular input variable could be checked and notified immediately. Also based on value of a variable, value for another variable in the same or different file could be accepted or rejected. Besides, data input form can be labeled with user friendly comments so that user can easily comprehend which variables' values are to be input in the window. After completing the input or editing existing values in all the files user shall be able to run the program by clicking a mouse button. On successful execution of the Fortran program the output files will be available to the user. Using the GUI, user shall be able to see easily and print each output files. In addition to these facilities user shall be able to generate various graphs from the output files by selecting option from menu. This GUI has been designed and implemented using Object Oriented Programming language Visual Basic (VB) that has made the Legacy software operable in the 32-bit faster window's environment. The benefits of the GUI could be attributed as savings of time by introducing user-friendly interface for the old-fashioned software, increased reliability due to reduction of chances of error in data input, easier output reporting and graph plotting.

6.4 Evaluating Suitability of Visual Basic

Visual Basic (VB) allows creating robust and useful applications that fully make use of the Graphical User Interface. Using VB the GUI can be created for application by drawing objects in a graphical way. Properties of these objects can be set to refine their appearance and behavior. The Object Oriented Programming language VB can manipulate files of variable record length that are used as input data files for Ground Water model. Besides, several tools are available in VB to create graphical objects for

data input and to draw graph from output files. Using VB interface can be made to react to the user by writing code that responds to events that occur in the interface.

6.5 GUI Elements

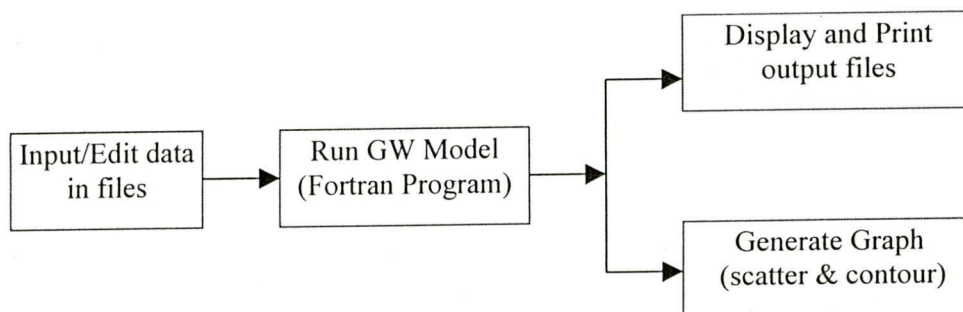


Fig. 6.1 Flow Diagram of User's Activities

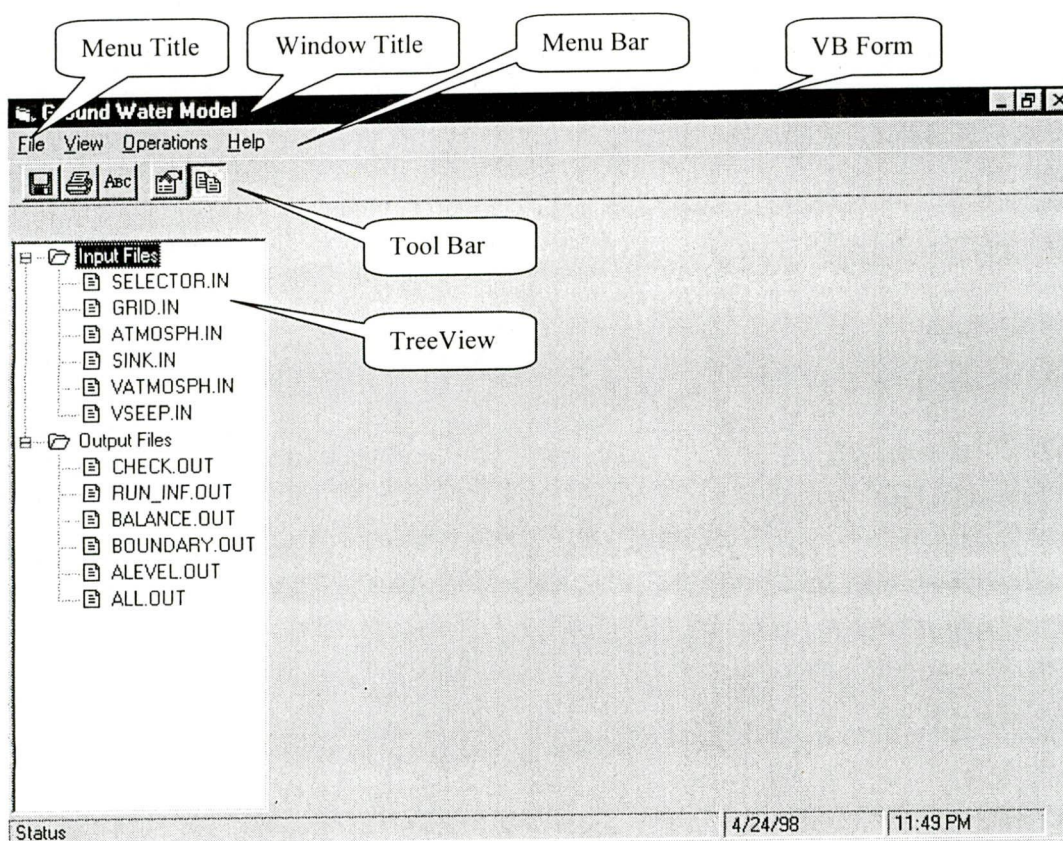


Fig. 6.2 Interface Window that Triggers Different Function depending on User Event

User activities related to Ground Water model shown in the flow diagram (Fig.6.1) can be controlled by a GUI consisting of a window with the following interface elements:

1. Menu Bar
2. Toolbar
3. Form
4. TreeView control

Fig. 6.2 in incorporates the interface element in a window that triggers different function depending on user event.

6.5.1 Menu Bar

Menu offers a convenient and consistent way to group commands and an easy way for users to access them. The Menu Bar appears immediately below the Title Bar on the form and contains one or more menu titles. When user click a menu title (such as File), a menu containing a list of menu items drops down. File, View, Operations & Help are included in the Menu Bar. Underlined character in the menu title is called a hot-key, instead of mouse click pressing of **Alt** + hot-key could also display the drop-down menu list. Following windows display drop-down list of each of the menu titles:

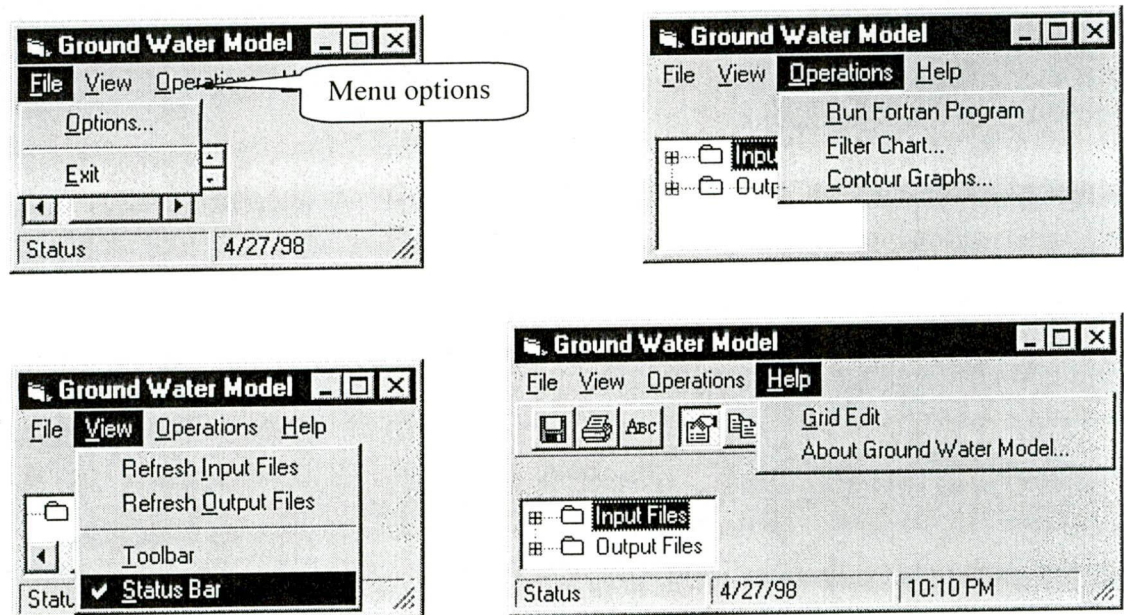


Fig. 6.3 Drop-down lists of Menu Titles

6.5.2 Toolbar

Toolbar provides quick access to commonly used commands in the programming environment. A Toolbar contains a collection of button objects used to create a Toolbar that is associated with an application. Typically click over a button on the Toolbar once to carry out the action represented by that button. In our GUI some toolbar button acts as a switch, once switched on by click over it will provide the function until it is not off by click over another button. The Fig. 6.4 provides the description of the Toolbar buttons in the GW model GUI:

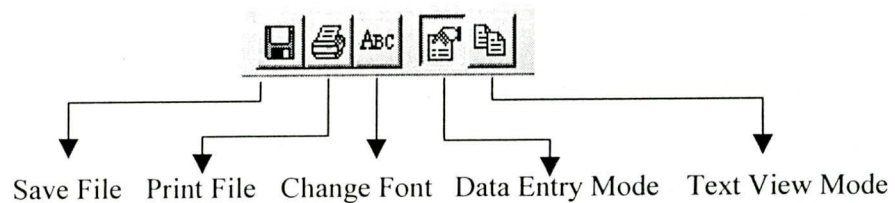


Fig. 6.4 Buttons of the Toolbar

6.5.3 Form

Form is the foundation for creating the interface of an application. Forms can be used to add windows and dialog boxes to the application. It can also be used as containers for items that are not a visible part of the application's interface. Controls, graphics, and pictures are added to a form to create the look we want for the GUI.

The first step of creating an application with VB is to design the interface: the forms, controls, and other objects the user will see and use. Then the properties are set and the VB code to activate the interface is written. VB controls are incorporated to get user input and to display output. Some of the controls that are used in the GUI include SSTab (collection of Tab Pages), Text box, Option button, Check box, Command button, Grid and TreeView controls.

6.5.4 TreeView Control

A TreeView is typically used to display the headings in a document, the entries in an index, the files and directories on a disk, or any other kind of information that might

usefully be displayed as a hierarchy. A TreeView control displays a hierarchical list of Node objects, each of which consists of a label and an optional bitmap. After creating a TreeView control, Node objects can be added, removed, arranged, and otherwise manipulated by setting properties and invoking methods. Node objects can be programmatically expanded and collapsed to display or hide all child nodes. Input files of the model are added as child nodes of Input Files Node and output files are added as child nodes of Output Files Node and shown in Fig. 6.2.

6.6 Menu Bar Options

6.6.1 File Menu

6.6.1.1 Options

Provide a Tab-page "Options" window where user can define the names of input files and their record structure and also the name of output files. There are three Tab-pages: Input Files, Output Files and Input Scripts. Name of the input files can be entered or edited in "Input Files" tab-page. Similarly output files' information are entered through "Output Files" tab-page.

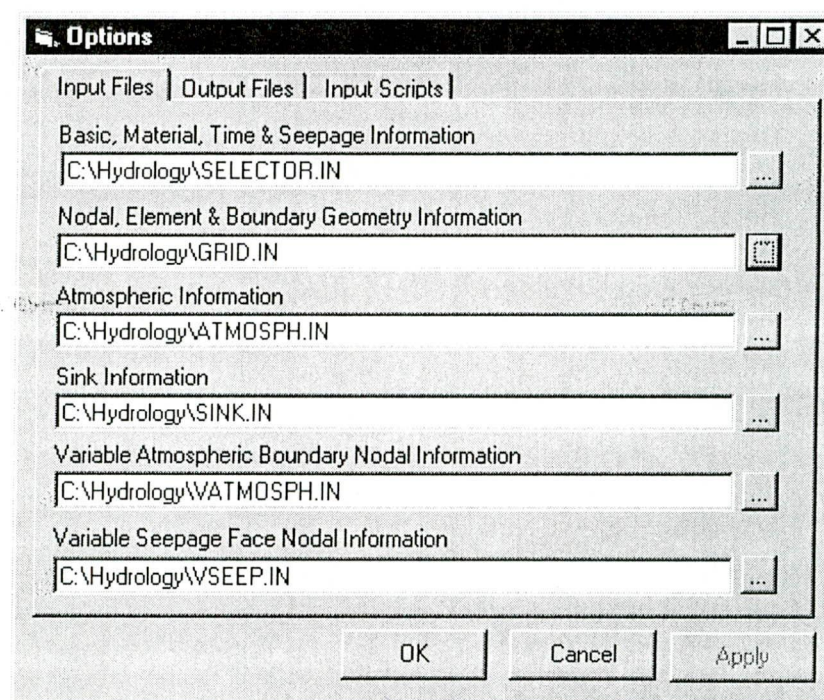


Fig. 6.5 'Input Files' Tab-page of File menu's "Options" option

Ground water model's input files consist of variable length data records having integer, floating point, exponential and logical data fields together with comment lines. User can define the record structure of the input file in "Input Scripts" tab-page. Through this window user can easily define the data type (integer, character, logical etc) and variable name assigned to the data fields in each record. Figs. 6.5 and 6.6 display the "Input Files" and "Input Script" tab-pages of "Options" window.

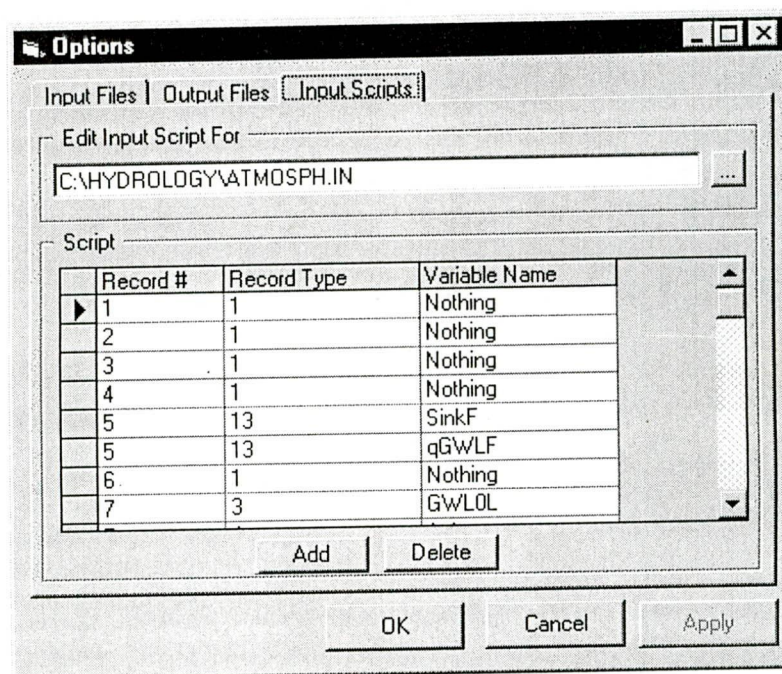


Fig. 6.6 'Input Scripts' Tab-page of File menu's "Options" option

6.6.1.2 Exit

To quit from the GUI of Ground Water model.

6.6.2 View Menu

6.6.2.1 Refresh Input Files

If an input file is selected for editing, the file content is loaded into the memory from the hard disk. After commencing a change if a mistake is realized, user can always reload the original content from the hard disk through this option. Refresh will not take effect if the change is already confirmed by save Toolbar option.

6.6.2.2 Refresh Output Files

Similar to the "Refresh Input Files" menu option but applies to the output files.

6.6.2.2 Toolbar

Mouse click or hot key selection of this option will result in toggle the Toolbar on and off beneath the Menu Bar.

6.6.2.4 Status Bar

Like the Toolbar option this toggles the Status Bar on and off at the bottom of the window.

6.6.3 Operations Menu

6.6.3.1 Run Fortran Program

When input to the data files is completed error can be checked in those files through selecting Text View mode button in Toolbar. Ground Water model is written in Fortran 77 that runs in DOS environment, where as the GUI written in VB5 runs in Windows 95 environment. To execute the model program user has to quit from Win95 and start DOS session. Selecting this option first display a message in a Message Box Window and then ends the Win95, loads DOS and executes a batch file to run the model. The message box is shown in the Fig. 6.7. Running of Fortran program can be terminated anytime by pressing the Ctrl and Break keys together. On successful completion or termination of the program user has to exit from the DOS, which automatically loads the Win 95. But to work with the GUI it has to be executed again or it can be put in the window's startup so that each start of Win95 session launches the GUI. To view the output results user can select the desired file from the TreeView and select the Text View Mode button in the Toolbar.

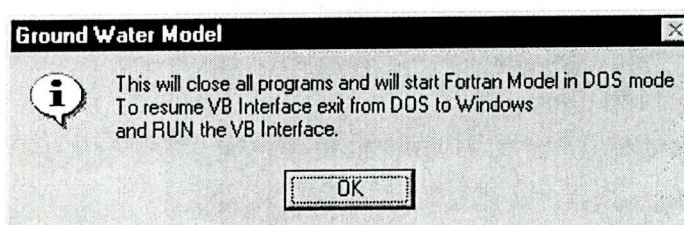


Fig. 6.7

Message Box appears when 'Run Fortran Program' is selected

6.6.3.2 Filter Chart

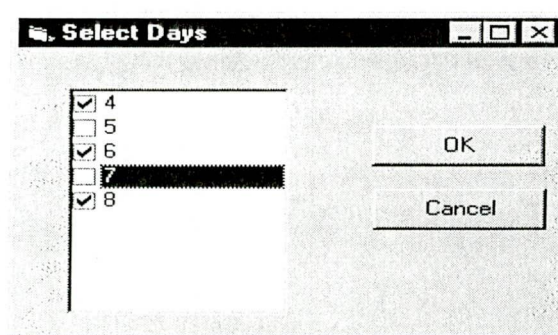


Fig. 6.8 'Select Days' window to select particular day(s)

Scatter graph can be created by selecting days from the 'Select Days' window shown in Fig. 6.8, which will appear when this option is selected. Graphs are drawn from the output files ALEVEL.OUT & ALL.OUT. Later file contains the time series data.

6.6.3.3 Contour Graphs

The contour graph is created from the output file ALL.OUT. Selecting this option will display the Generate Contour Graph window. A particular day can be selected from this window and the contour graph can be generated. Afterwards if OK button is clicked SPYGLASS application is launched and then a macro is executed to generate the contour graph for that day.

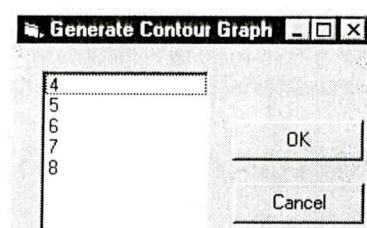


Fig. 6.9 List of days in a window for generating Contour graph

6.6.4 Help Menu

6.6.4.1 Grid Edit

If a change is made in a grid cell that will not be effective until the user press *Enter* key or move to another cell of the grid or another control on the same window. The following message is displayed when this option is selected:

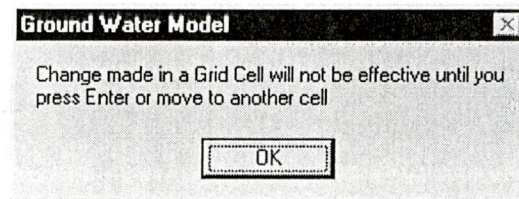


Fig. 6.10 Help message box for Grid control

6.6.4.2 About Ground Water Model

The window in Fig. 6.11 will appear which will provide information of the version number and the description of the model.

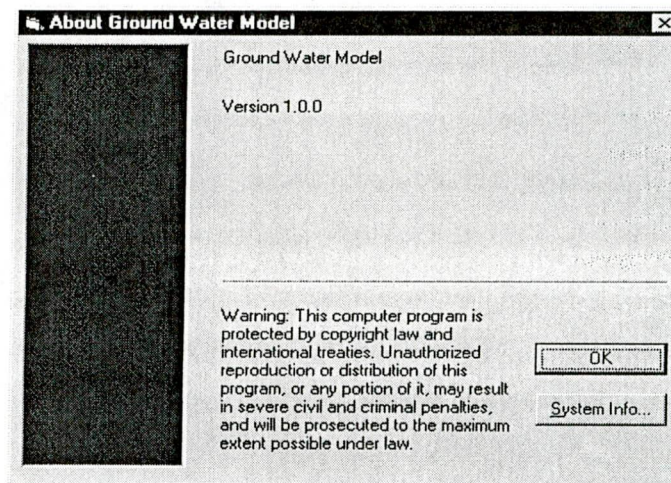


Fig. 6.11 Window containing Release and System Information

6.7 Toolbar Buttons

6.7.1 Save File



This button is used to commence the save in disk after a change is made in Input

6.7.2 Print File



When selected it prints the input and output files.

6.7.3 Change Font



Mouse button click over it will display a font window. Change of font name and size will be visible in text view and printout of input and output files.

6.7.4 Data Entry Mode



To input or edit data in input files this button has to be clicked. When an input file from the TreeView is selected the Tab-page window of that file is available to edit.

6.7.5 Text View Mode



When this button is clicked the contents of input or output files selected in the TreeView is displayed as read only in a window. The font and the size of the displayed text ensure the settings that have been selected by Change Font button.

6.8 Entering Data

The input files for the Ground Water model are: (i) SELECTOR.IN, (ii) GRID.IN, (iii) ATMOSPH.IN, (iv) SINK.IN, (v) VATMOSPH.IN and (vi) VSEEP.IN . The name of these files appears in the explorer like TreeView under the 'Input Files' node. When a file is selected and 'Data Entry Mode' button in the Toolbar is clicked by mouse a tab-page window will appear in the GUI. This window contains several VB graphical objects, also known as controls, to get input from the user. These objects are properly labeled to reduce confusion of user on what inputs are to be entered in each of them. A SSTab container object, which gives the tab-pages, contains several objects consisting of Text box, Option button group in Frame, Check book, DBGrid and Command button.

6.8.1 SSTab Control

The SSTab control provides a group of tabs, each of which acts as a container for other controls. Only one tab is active in the control at a time, displaying the controls it contains to the user while hiding the controls in the other tabs. An SSTab control is

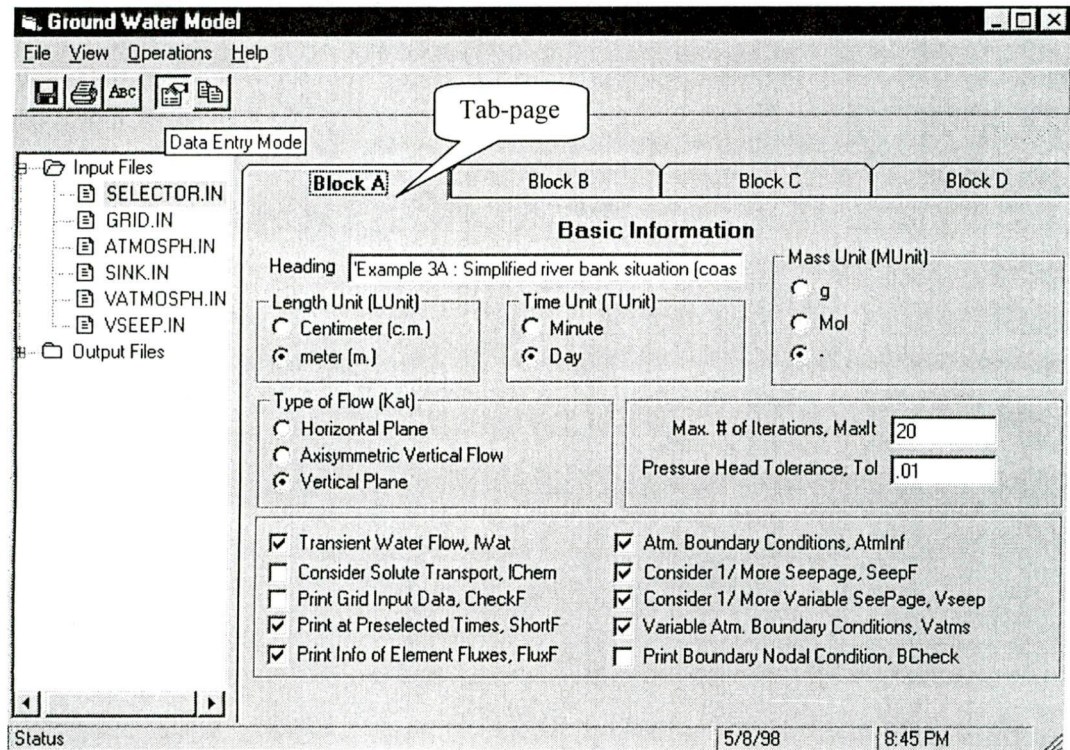


Fig. 6.12 Input of Basic Information in SELECTOR.IN

like the dividers in a notebook or the labels on a group of file folders. Using an SSTab control, multiple pages can be defined for the same area of a window of GUI. At design time by clicking each tab the controls are drawn over it which are displayed at run time when the user selects that tab. At run time, users can navigate between tabs by either pressing CTRL+TAB or by pressing left mouse button. Fig. 6.12 shows 4 tab-pages for data entry in SELECTOR.IN file. Each block of information is grouped together in a tab-page.

6.8.1.1 Text box

Text box is versatile controls that can be used to get input from the user or to display text. In the GUI it is used to accept integer, real, exponential and character data from the user.

6.8.1.2 Option Group

Option buttons present a set of two or more choices to the user. Unlike check boxes, however, option buttons should always work as part of a group; selecting one option button immediately clears all the other buttons in the group. Defining an option button group tells the user, "Here is a set of choices from which he can choose one and only one." In the Fig. 6.12 user can select either centimeter (cm.) or meter (m.) from the option group 'Length Unit' and the selected unit is automatically updated in the SELECTOR.IN file for the variable *LUnit*.

6.8.1.3 Check Box

A check box indicates whether a particular condition is on or off. Check boxes in an application can be used to give users true/false or yes/no options. Because check boxes work independently of each other, a user can select any number of check boxes at the same time. In the Fig. 6.12 check boxes are used to input true/false value for the logical variable *lWat*, *lChem* etc.

6.8.2 DBGrid

The DBGrid control displays data in a grid or table. When used in conjunction with a data control, it presents fully editable data from multiple fields in an external database. For the GUI we need variable Grid cells which is related to value of another variable. When the value of the variable(s), such as *NMat* or *NPar*, is changed that should affect the dimension of the DBGrid for Material Information shown in the following Fig. 6.13. The value of variable which is related to DBGrid's dimension is colored blue in the tab-page, and the change of its value will not be reflected in the DBGrid's dimension until the Apply command button is pressed. 'Current Position', a read only text box shows the present position of the cursor over a grid cell.

Ground Water Model

File View Operations Help

Data Entry Mode

Input Files

- SELECTOR.IN
- GRID.IN
- ATMOSPH.IN
- SINK.IN
- VATMOSPH.IN
- VSEEP.IN

Output Files

- CHECK.OUT
- RUN_INF.OUT
- BALANCE.OUT
- BOUNDARY.OL
- ALEVEL.OUT
- ALL.OUT

Block A Block B Block C Block D

Material Information

Lower Limit of Pressure Head Interval, hTab1 .00001

Upper Limit of Pressure Head Interval, hTabN 10.

No. of Subregions, N Lay 3

of Soil Materials, NMat 6 # of Parameters, NPar 9

Apply

Material Information

thr	ths	tha	thm	Alfa
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
9595	78	0	0	0
90	0	0	0	0
9999	0	0	0	0

DBGrid Cursor location Current Position 1,1

Status 5/9/98 12:33 AM

Fig. 6.13 Input of Material Information in SELECTOR.IN

6.8.3 Data Input Forms

Following are the forms of GUI for data input / edit to different blocks of input files.

Ground Water Model

File View Operations Help

Data Entry Mode

Input Files

- SELECTOR.IN
- GRID.IN
- ATMOSPH.IN
- SINK.IN
- VATMOSPH.IN
- VSEEP.IN

Output Files

- CHECK.OUT
- RUN_INF.OUT
- BALANCE.OUT
- BOUNDARY.OL
- ALEVEL.OUT
- ALL.OUT

Block A Block B Block C Block D

Seepage Information

of Seepage Faces, NSeep 3

Apply

Seepage Information NSP(1),NSP(2),.....,NSP(NSeep)

Col 0	Col 1	Col 2	Col 3	Col 4
7				
3				

Current Position 1

of Total Nodes 14

Apply

Nodal Numer Info NP(1),NP(2),.....,NP(NSeep)

Col 0	Col 1	Col 2	Col 3	Col 4
26	51	76	101	126
0	0	66	3	0
0	77			

Current Position 1

Status 5/9/98 12:33 AM

Fig. 6.14 Input of Time Information in SELECTOR.IN

Ground Water Model File View Operations Help

Icons: [Save] [Print] [ABC] [Data Entry] [Help]

Data Entry Mode

Input Files

- SELECTOR.IN
- GRID.IN
- ATMOSPH.IN
- SINK.IN
- VATMOSPH.IN
- VSEEP.IN

Output Files

- CHECK.OUT
- RUN_INF.OUT
- BALANCE.OUT
- BOUNDARY.OUT
- ALEVEL.OUT
- ALL.OUT

Block A Block B **Block C** Block D

Time Information

Initial Time Increment, dt: 0.02 Multiplying factor for dt when iterations < or = 3, dMul: 1.3

Min. Time Increment, dtMin: 0.001 Multiplying factor for dt when iterations, or = 7, dMul2: 3

Max. Time Increment, dtMax: 0.50

of Specified Print-Times, MPL: 38 Apply

Time Information TPrint(1), TPrint(2), ..., TPrint(MPL)				
Col 0	Col 1	Col 2	Col 3	Col 4
9100	9009	0	0	0
0	0	0	0	0
0	0	0	0	0
0	99			

Current Position: 1

Status: 5/9/98 12:33 AM

Fig. 6.15 Input of Seepage Information in SELECTOR.IN

Ground Water Model File View Operations Help

Icons: [Save] [Print] [ABC] [Data Entry] [Help]

Data Entry Mode

Input Files

- SELECTOR.IN
- GRID.IN
- ATMOSPH.IN
- SINK.IN
- VATMOSPH.IN
- VSEEP.IN

Output Files

- CHECK.OUT
- RUN_INF.OUT
- BALANCE.OUT
- BOUNDARY.OUT
- ALEVEL.OUT
- ALL.OUT

Block F **Block G** Block H

Nodal Information

Max. Nodes on Transverse Line, IJ: 2

Total Nodal Points, NumNP: 2 Apply

Nodal Information (n, Code, x, z, h, Conc, Q, M, B, Axz, Bxz, Dxz)			
n	Code	x	z
0	0	0	0
0	0	0	0

Current Position: 1.3

Status: 5/9/98 12:33 AM

Fig. 6.16 Input of Nodal Information in GRID.IN

Ground Water Model

File View Operations Help

Data Entry Mode

Input Files

- SELECTOR.IN
- GRID.IN
- ATMOSPH.IN
- SINK.IN
- VATMOSPH.IN
- VSEEP.IN

Output Files

- CHECK.OUT
- RUN_INF.OUT
- BALANCE.OUT
- BOUNDARY.OUT
- ALEVEL.OUT
- ALL.OUT

Block F Block G Block H

Element Information

Total Elements, NumEl Apply

Element Information (e, i, j, k, l, Angle, Aniz1, Aniz2, LayNum)			
e	i	j	k
1	2005	206	207
2	201	207	208
3	203	209	210
4	204	210	211
5	205	211	212
6	206	212	213

Current Position

Status 5/9/98 12:33 AM

Fig. 6.17 Input of Element Information in GRID.IN

Ground Water Model

File View Operations Help

Data Entry Mode

Input Files

- SELECTOR.IN
- GRID.IN
- ATMOSPH.IN
- SINK.IN
- VATMOSPH.IN
- VSEEP.IN

Output Files

- CHECK.OUT
- RUN_INF.OUT
- BALANCE.OUT
- BOUNDARY.OUT
- ALEVEL.OUT
- ALL.OUT

Block F Block G Block H

Boundary Geometry Information

Total Boundary Nodes for which Kode(N) <> 0, NumBP Apply

Node Number Array:		
Col 3	Col 4	Col 5
1	1	1

Current Position

Width Array:		
Col 3	Col 4	Col 5
8	8	8

Current Position

Length

Status 5/9/98 12:33 AM

Fig. 6.18 Input of Boundary Geometry Information in GRID.IN

Ground Water Model

File View Operations Help

Data Entry Mode

Block I

Atmospheric Information

☒ Plant Water Uptake ☒ Discharge-Ground Water Level used as Bottom Boundary Condition

Ref. of GroundWater Table Simulation Start Time [T]

Value of Parameter Aqh Max. Pressure Head at Soil Surface

Value of Parameter Bqh

of Atmospheric Data Records (MaxAL)

Atmospheric Evaporation & Flux (tAtm, wL, Prec, cPrec, rSoil, rRoot, hCrit, rt, ht, ctt, cht)				
	tAtm	wL	Prec	cPrec
▶ 356.0	32.005		0.00E+00	0
357.0	32.011		0.00E+00	0
358.0	32.007		0.00E+00	0
359.0	32.005		0.00E+00	0

Current Position

Status 5/9/98 12:33 AM

Fig. 6.19 Input of Atmospheric Information in ATMOSPH.IN

Ground Water Model

File View Operations Help

Data Entry Mode

Block NEW1

Variable Atmospheric Nodal Information

of Nodes in Variable Atmospheric Face (NPatmD)

Sequential Global # of Nodes			
Col 0	Col 1	Col 2	Col 3
▶ 1	2	3	4
1222	777		

Current Position

Status 4/24/98 11:49 PM

Fig. 6.20 Input of Variable Atmospheric Nodal Information in VATMOSPH.IN

Ground Water Model

File View Operations Help

Data Entry Mode

Input Files

- SELECTOR.IN
- GRID.IN
- ATMOSPH.IN
- SINK.IN
- VATMOSPH.IN
- VSEEP.IN**

Output Files

- CHECK.OUT
- RUN_INF.OUT
- BALANCE.OUT
- BOUNDARY.OU
- ALEVEL.OUT
- ALL.OUT

Block NEW2

Variable Seepage Face Nodal Information

of Variable Seepage Face, NVFace Apply

# of Nodes on Variable Seepage Face	
Col 0	
12	
5	

Current Position

Total # of Nodes Apply

Sequential Global # of Nodes on the Variable Seepage Face			
Col 0	Col 1	Col 2	Col 3
2600	51	76	101
155	156	88	7

Current Position

Status 4/24/98 11:49 PM

Fig. 6.21 Input of Variable Seepage Face Nodal Information in VSEEP.IN

Ground Water Model

File View Operations Help

Text View mode

Input Files

- SELECTOR.IN**
- GRID.IN
- ATMOSPH.IN
- SINK.IN
- VATMOSPH.IN
- VSEEP.IN

Output Files

- CHECK.OUT
- RUN_INF.OUT
- BALANCE.OUT
- BOUNDARY.OUT
- ALEVEL.OUT
- ALL.OUT

Text View :

```

*** BLOCK A : BASIC INFORMATION *****
Heading
'Example 3A : Simplified river bank situation (coarser grid)'
*****
'm' 'day' ':'
Kat (0:horizontal plane, 1:axisymmetric vertical flow, 2:vertical plane)
2
MaxIt Tol (maximum number of iterations and p. head tolerance)
20.01
IWat IChem CheckF ShortF FluxF AtmInF SeepF VSeep Vatms Bcheck
tffffff
*** BLOCK B : MATERIAL INFORMATION *****
NMat NLayer hTab1 hTabN NPar
63.00001 10.9
thr ths tha thm Alfa n Ks Kk thk
000000000
000000000
000000000
9595780000000
9000000000
999900000000
*** BLOCK C : TIME INFORMATION *****
00000000000000000000

```

Status 5/9/98 12:33 AM

Fig. 6.22 Text View of input file SELECTOR.IN

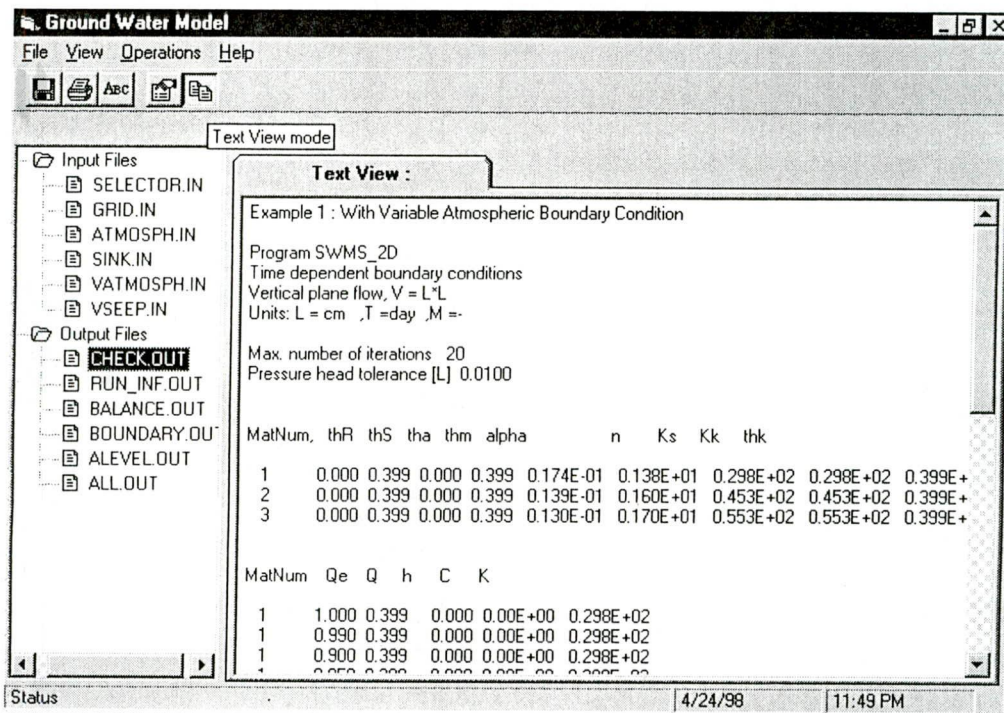


Fig. 6.23 Text View of output file CHECK.OUT

6.9 Graph Plotting

Results of simulation for some selected typical time instances are presented graphically in the following Figures. Line and contour graphs are plotted from the

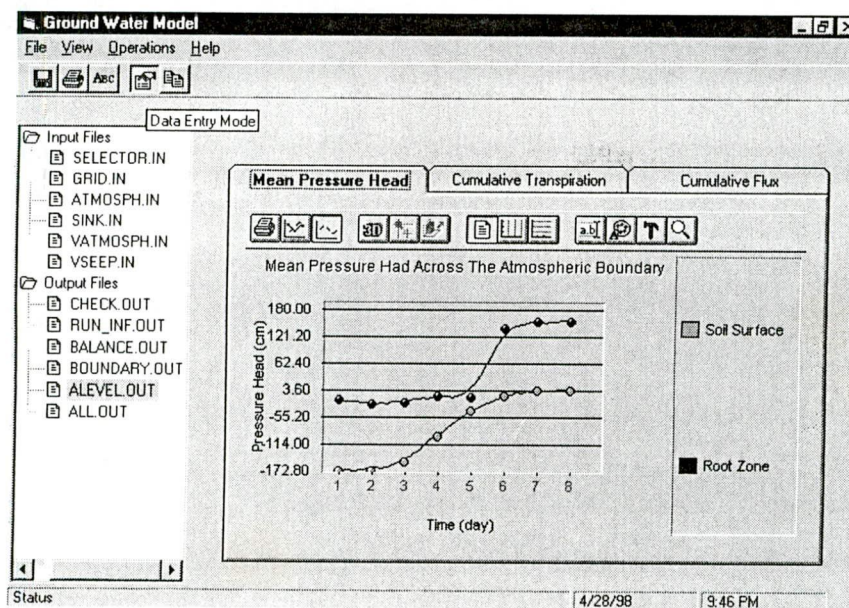


Fig. 6.24 Mean Pressure Head at the Soil Surface and Root Zone

simulated output results available in ALEVEL.OUT & ALL.OUT files. In the 'Data Entry Mode' when either of the files is selected a tab-page form will appear to display graphs. User has to go through the menu option 'Filter Chart' to select one or more days as illustrated in Fig. 6.8 for plotting graphs by using results in ALL.OUT.

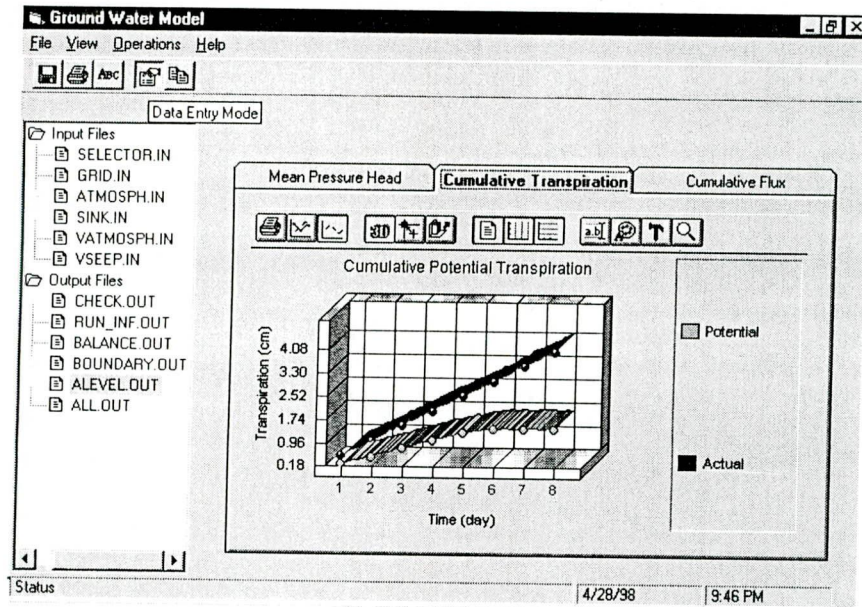


Fig. 6.25 Cumulative Potential and Actual Transpiration

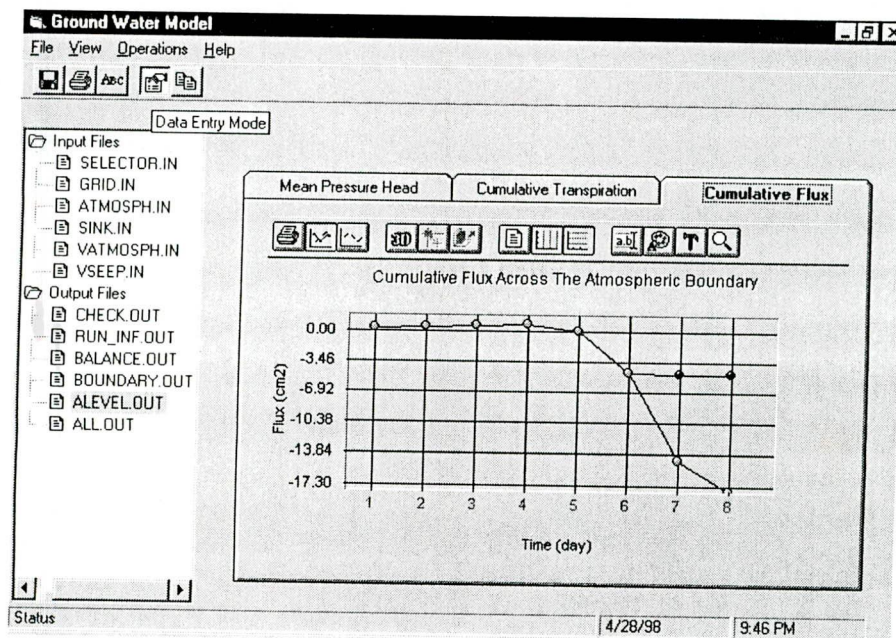


Fig. 6.26 Cumulative Flux across the Atmospheric Boundary

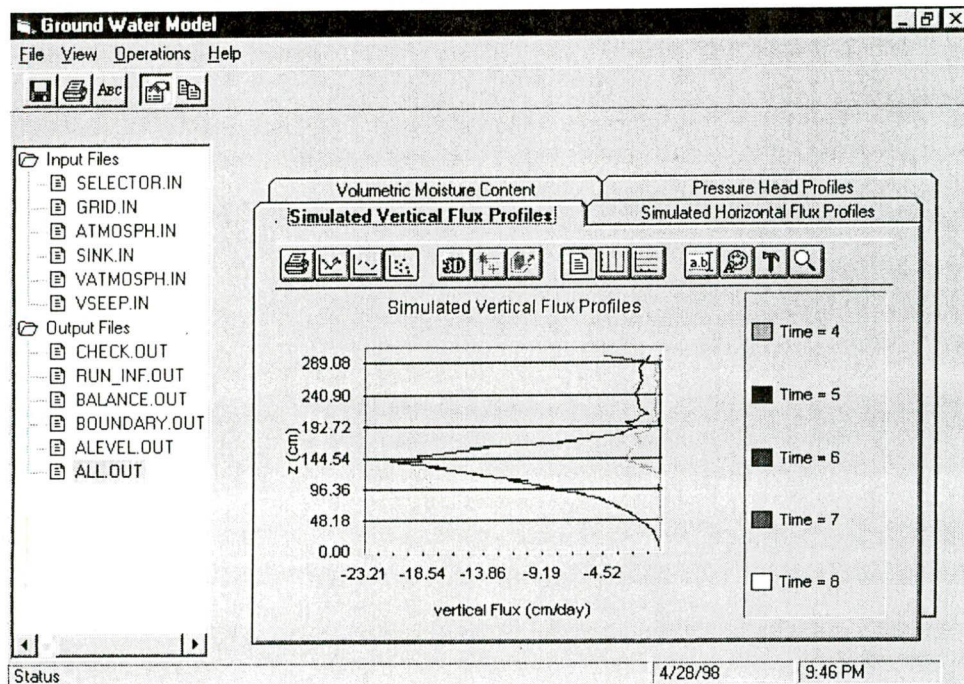


Fig. 6.27 Simulated Vertical Flux Profiles for Selected Days

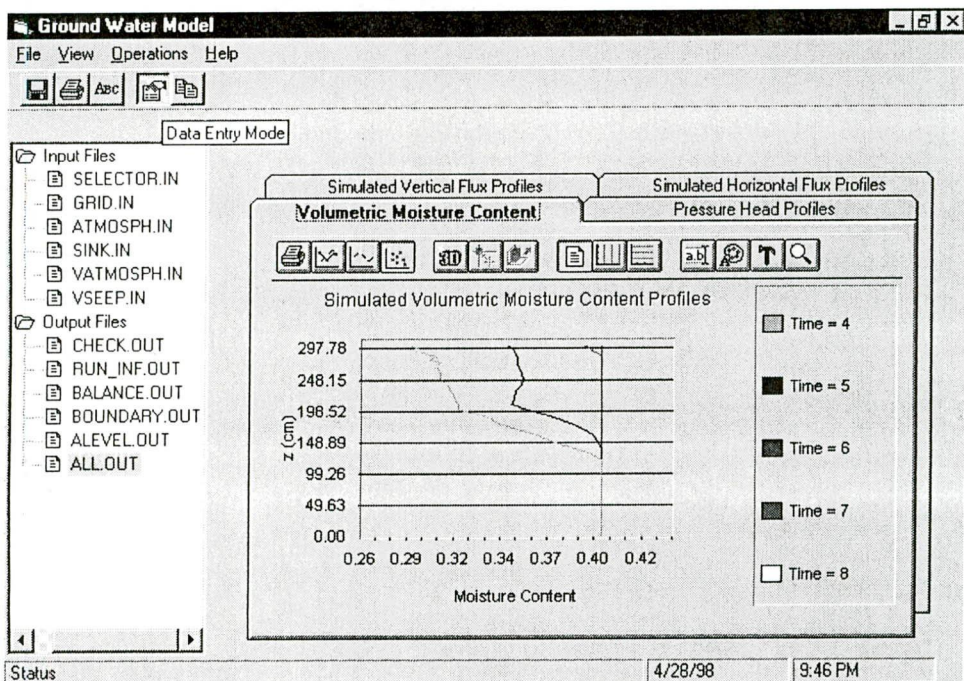


Fig. 6.28 Simulated Volumetric Moisture Content profiles for various days

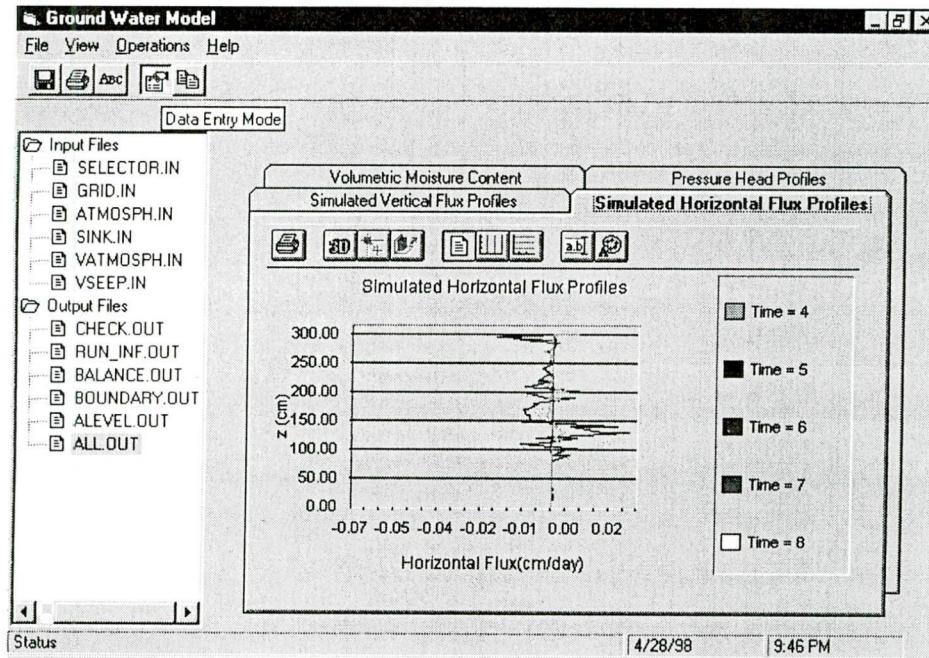


Fig. 6.29 Simulated Horizontal Flux profiles for selected days

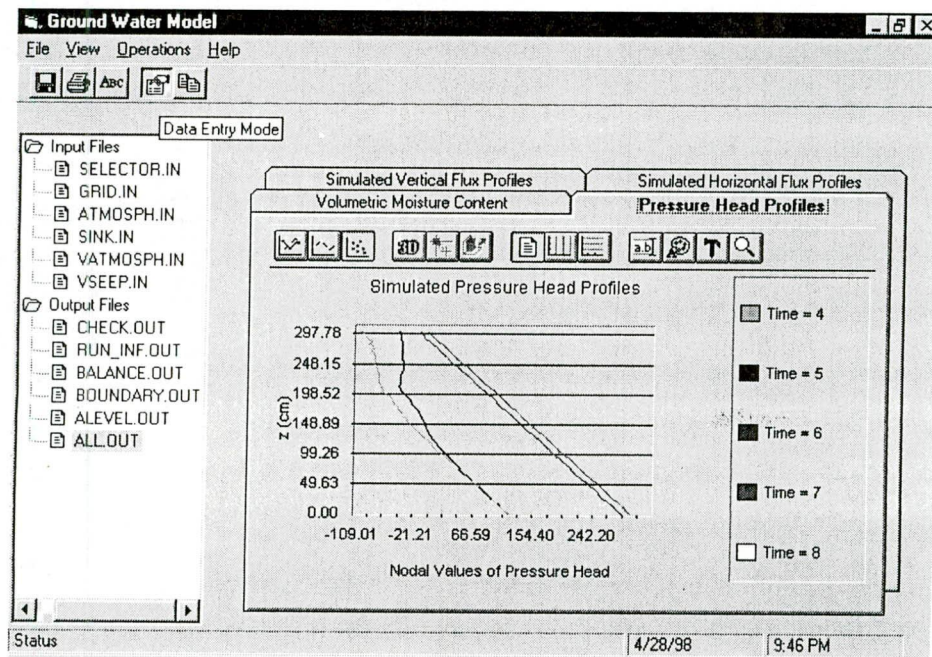


Fig. 6.30 Simulated Pressure Head profiles for various days

6.10 Contour Graph Plotting

To draw the contour a particular day has to be selected from 'Generate Contour Graph' window shown in Fig. 6.9. Data is passed for that particular day to SPYGLASS application and the contour is generated as shown in the following Figure:

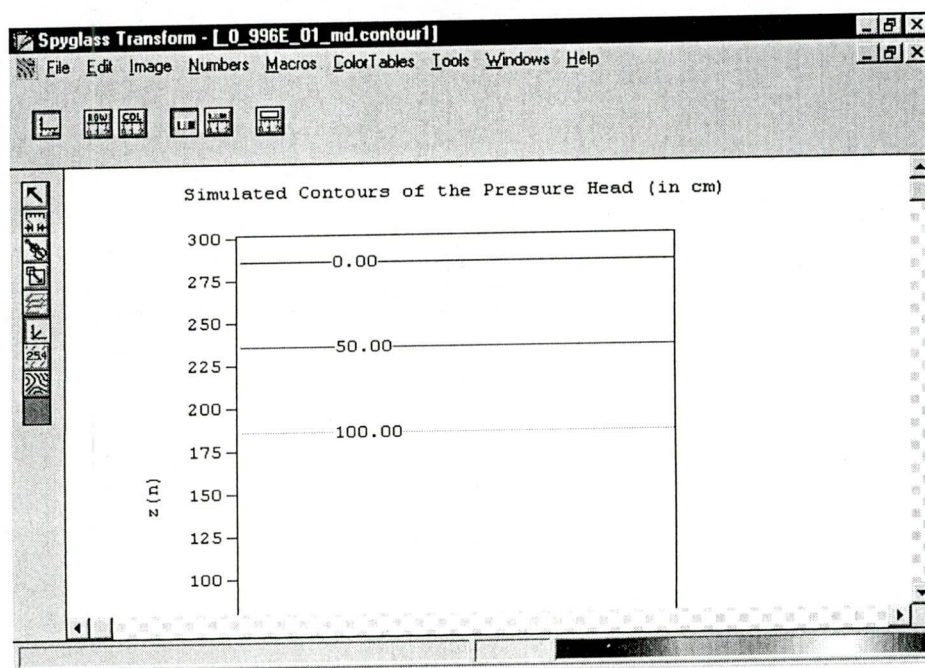


Fig. 6.31 Contour Graph by SpyGlass software

CHAPTER 7

APPLICATION OF THE MODEL

7.1 Introduction

Based on the methodology described in the earlier chapters, in combination with the computational strategy and techniques, the model developed to solve variably saturated flow problems is capable of handling a wide variety of problems including multi-aquifer system of real situation with non-homogeneous material properties and any combination of boundary conditions. The algorithm of the present model is adapted from the Program Code SWMS_2D (Simunek et. al.,1992). During the development and modifications of the different features of the model a number of familiar published problems have been used to verify the algorithm. In this chapter a few of these problems are presented for demonstration purposes. All the examples solved here is taken from the report by Simunek et. al. (1992). After having required performance of all the cases, a real situation in the Dhaka City is simulated to analyze the model behavior. In the subsequent sections these applications and results are discussed.

7.2 Model Testing under Extreme Hypothetical Conditions

Two example problems are presented in this section designed to verify the modified model and program under extreme atmospheric conditions:

Example 1 simulates one dimensional water movement in a soil column in order to illustrate the effect of ponding that can occur on the soil surface. Hypothetical 60-days atmospheric data are created to provoke the effect.

Example 2 is to illustrate the effect of variable seepage face of a two-dimensional rectangular flow domain. In this example, hypothetical fluctuations of surface water level on one side of the flow domain are the driving forces of water flow, in absence of the atmospheric boundary condition.

7.3 Example 1 : One-dimensional Vertical Flow Domain under Variable and Constant Atmospheric Boundary Condition

A hypothetical 300 cm long vertical column with three-layered soil is considered for the study of differences caused by setting either variable or constant atmospheric boundary condition. The root zone depth is assumed 50 cm (Fig. 7.1). Initial and boundary conditions are shown in Fig. 7.1.

Soil hydraulic properties are assumed homogeneous and isotropic within each layer, according to Tables 7.1b and 7.2b. The initial soil moisture profile (only the above-groundwater part) can be seen in Fig. 7.2a, together with the graphs of retention curves and hydraulic conductivity curves for all three layers. The soil parameters are taken from the report by Simunek et al. (1992).

The soil surface boundary is exposed, after a short period of drought, to very high precipitations for several days (with the peak daily maximum 100 mm) to create surface ponding. This is followed by a longer dry spell. In the second half of the simulation period, after the ponding has disappeared, another similar group of rains is imposed, to repeat the effect, followed by another dry spell till the end of simulation. The course of weather can be seen from Fig. 7.2b. Constant rates of potential evaporation (2 mm/day) and transpiration (5mm/day) are assumed during the whole simulation period. The surface fluxes (infiltration and evaporation), as well as the transpiration rate, are assumed uniformly distributed during 24 hours of each day. The bottom boundary condition consists of a prescribed drainage flux-groundwater level relationship, $q(h)$, as given by equation (5.12). The groundwater level is initially set at 150cm below the soil surface. The initial moisture content and pressure head profiles are assumed in equilibrium with the initial ground water level.

The finite element mesh is constructed by dividing the flow region into 39 quadrilateral elements with 80 nodal points. The sequence of nodal and element numbering is shown in Fig.7.1.

In the following sections, both variants (with and without variable atmospheric boundary conditions) are described and discussed in detail.

7.3.1 Variant with Variable Atmospheric Boundary Conditions

The atmospheric boundary conditions is referred to as variable in this thesis if the nodal points on the atmospheric boundary are capable of switching between Dirichlet boundary condition and atmospheric boundary condition. The atmospheric boundary can be defined as variable in the program GWm by setting the logical variable **Vatms** in the input file **SELECTOR.IN** equal to **".true."**.

In this variant of the example 1, three types of boundaries are defined:

The bottom boundary face is identified as a variable flux boundary for which **Kode(n)= -3**. This boundary condition can be implemented in the model by setting the logical variable **qGWLF** in the input file **ATMOSPH.IN** equal to **".true."**.

The atmospheric surface can have either constant head Dirichlet boundary condition when surface ponding occurred [**Kode(n) = 1**] or the atmospheric boundary condition [**Kode(n)=4**] in the absence of ponding. Distinction between both cases is done by a subroutine that computes water depth on the soil surface (employing simple water balance and neglecting surface runoff). Initial value of **Kode(n)** is **-4**.

The sides of the column are set to Neuman no-flow boundary condition with **Kode(n)= 0** and behaved in the program like internal nodes.

Basic parameters of simulation and soil hydraulic properties are summarized in the following Tables 7.1a, 7.1b and 7.1c. Resulting mean pressure heads at the soil surface and in the root zone, cumulative flux (evaporation - precipitation) across the atmospheric boundary and cumulative transpiration fluxes are shown in Fig. 7.3. Simulated profiles of volumetric moisture content, pressure head and vertical volumetric flux can be seen in Fig. 7.4a & b, 7.5a & b and 7.6a & b respectively. The

discussion of the results of simulation, in relation to those obtained with constant atmospheric boundary condition, is contained in article 7.3.3.

Table 7.1a : Basic Information

Type of Flow System	One dimensional vertical soil column
Data Source	Hypothetical extreme precipitation events combined with constant evaporation and transpiration

Table 7.1 b : Input Hydraulic Parameters

Parameters	1st layer (205 - 300 cm)	2nd layer (105 - 205 cm)	3rd layer (0.0 - 105 cm)
$\theta_s = \theta_m = \theta_k$	0.399	0.399	0.399
$\theta_r = \theta_a$	0.0001	0.0001	0.0001
$K_s = K_k$ (m/day)	0.2975	0.4534	0.5534
α (1/m)	1.740	1.390	1.300
n (-)	1.3757	1.6024	1.7024

Table 7.1c : Numerical Simulation Information

Time Information	
Initial time increment : dt	0.02 day
Minimum time increment : $\Delta t_{\min}$	1e-10 day
Maximum time increment : $\Delta t_{\max}$	0.50 day
Iteration Information	
Maximum number of iteration : Max It	20
Absolute pressure head tolerance : Tol	0.01 cm
Some logical switch setting	
Atmospheric boundary : AtmInf	.true.
Seepage face : SeepF	.false.
Variable seepage face : Vseep	.false.
Variable atmospheric boundary : Vatms	.true.

7.3.2 Variant without Variable Atmospheric Boundary Condition (with Constant Atmospheric Boundary Condition)

In this variant, only two types of boundary nodes are to be distinguished (not speaking about the sides of the column with **Kode(n) = 0**):

The bottom face of the soil profile is specified as variable flux boundary condition for which **Kode(n) = -3**.

The atmospheric boundary nodal points have **Kode(n) = 4** (with initial setting to -4).

Basic simulation parameters and soil hydraulic properties are summarized in the following tables 7.2a, 7.2b and 7.2c. Resulting mean pressure heads at the soil surface and in the root zone, cumulative flux(evaporation-precipitation) across the atmospheric boundary and cumulative transpiration flux are shown in Fig. 7.7. Simulated profiles of volumetric moisture content, pressure head and vertical volumetric flux can be seen in Fig. 7.8a & b, 7.9a & b and 7.10a & b respectively. The discussion of the results of simulation, in relation to those obtained with variable atmospheric boundary condition, is contained in article 7.3.3.

Table 7.2a : Basic Information

Type of Flow System	One dimensional vertical soil column
Data source	Hypothetical extreme precipitation events combined with constant evaporation and transpiration

Table 7.2 b : Input Hydraulic Parameters

Parameters	1st layer (205 - 300 cm)	2nd layer (105 - 205 cm)	3rd layer (0.0 - 105 cm)
$\theta_s = \theta_m = \theta_k$	0.399	0.399	0.399
$\theta_r = \theta_a$	0.0001	0.0001	0.0001
$K_s = K_k$ (m/day)	0.2975	0.4534	0.5534
α (1/m)	1.740	1.390	1.300
n (-)	1.3757	1.6024	1.7024

Table 7.2c : Numerical Simulation Information

Time Information		
Initial time increment : dt		0.02 day
Minimum time increment : $\Delta t_{\min}$		1e-10 day
Maximum time increment : $\Delta t_{\max}$		0.50 day
Iteration Information		
Maximum number of iteration : Max It		20
Absolute pressure head tolerance : Tol		0.01 cm
Some logical switch setting		
Atmospheric boundary : AtmInf		.true.
Seepage face : SeepF		.false.
Variable seepage face : Vseep		.false.
Variable atmospheric boundary : Vatms		.false.

7.3.3 Discussion of the Results

Figs. 7.3 and 7.7 present the time course of simulated mean pressure heads at the soil surface and in the root zone, as well as the time course of cumulative transpiration fluxes and cumulative fluxes across the atmospheric boundary (i.e. the cumulative differences evaporation - infiltration), for the variants, respectively, with variable and constant atmospheric boundary condition.

For both variants, Fig. 7.3 and 7.7 show that the mean pressure heads, both on the soil surface and in the root zone, are decreasing during the first several days of simulation (parts (a) of the figures mentioned), as the soil profile is dried and drained simultaneously (there is no precipitation during the first four days). Evaporation took place from the soil surface (note the positive cumulative flux in (c)) and water is extracted from the root zone by transpiration equal to the potential one (solid and dashed lines coincide in (b)).

Beginning from the 5th day, rainfall has reversed the situation and ponding finally occurred during the 8th day. After that date, the average pressure heads are positive with variable atmospheric boundary condition (Fig. 7.3 (a)). The average pressure head in the root zone is larger than the average surface pressure head because the root zone is mostly saturated and lay under the temporary ground water table. Infiltration took place under positive hydraulic head (with Dirichlet boundary condition) until all water on the soil surface has been depleted by both infiltration and evaporation. This took place on the 33rd day. Infiltration is rapid as long as there is some unsaturated space in the soil profile (Fig. 7.3a). After total saturation, which occurred during the 8th day, water is only drained from the soil profile by some (unspecified) natural or artificial drainage system according to the equation (5.12); the infiltration could not therefore be higher than the drainage flux at the bottom of the profile. The dashed line (actual cumulative surface flux) in Fig. 7.3(c) departs from the solid line (potential cumulative surface flux) beginning from the 8th day. It is still decreasing (infiltration continues) but its slope is much less than that corresponding to potential infiltration. After the 33rd day, evaporation from the soil surface took place again, and the slope of the dashed line in Fig. 7.3 (c) is again increasing, until new rainfall came on 41st day. The whole situation then repeated with water remaining on the soil surface till the end of the simulation period. No evaporation from the soil surface takes place during the periods of ponding (instead, the potential evaporation occurs from the water layer on the soil surface but this is not comprised in Fig. 7.3(c)). Transpiration is totally suppressed during the periods when the soil profile is saturated (the dashed line of actual cumulative transpiration in Fig. 7.3 (b) is horizontal). Most plants would be heavily damaged by anaerobiosis during the surface ponding. This model,

however, does not comprise a plant submodel. Therefore, as soon as the soil profile became aerated again (on about 34th day), transpiration recovers with almost potential intensity, until the new ponding.

On the contrary, in the variant with constant atmospheric boundary condition (Fig. 7.7), water is not allowed to cumulate on the soil surface (we can imagine that it is immediately removed by surface outflow). After the cessation of rain, infiltration ceases immediately, too, and evaporation begins to be the only component of the surface flux. As it can be seen from Fig. 7.7(c), evaporation takes place with potential rate until the soil surface becomes dry enough (on 33rd day) to reduce the evaporation intensity. Between 9th and 33rd day, the dashed line (actual cumulative surface flux) in Fig. 7.7(c) is parallel to the solid one (potential flux) and the distance between them is equal to the surface outflow that took place during the period of rainfall. After the 33rd day, as well as after the 44th day, the evaporation intensity is practically zero because the soil surface is dry (the dashed line in Fig. 7.7(c) is almost horizontal). The root zone also gets dry enough for the transpiration to be reduced, beginning with the 33rd day, after which the dashed line of actual cumulative transpiration in Fig. 7.7(b) assumes less slope. Transpiration is blocked during the first rain (the dashed line in Fig. 7.7(b) is almost horizontal between 6th and 12th day) while the second rain (on 41st to 44th day) has no pronounced effect, regardless of the large amount of water infiltrated (the soil is already too dry).

The events described above can be equally well followed from the moisture content profiles in Figs. 7.4a, b and 7.8a, b. Figures 7.4a and 7.8a (for 4th to 9th day) are the same for both cases (note that the soil profile is saturated at the end of 8th and 9th day even without variable atmospheric boundary condition) but the soil appears to be much drier in Fig. 7.8b, because of the missing water from the first rain (lost by surface runoff). The soil heterogeneity (remember that the soil profile is composed of three layers) is most explicitly expressed in the heights 200-210 cm (measured from the profile bottom) where the saturated moisture content is changing. It is the property of the model that a boundary between two different soil materials is not abrupt but occupies a strip of finite width (10 cm is our case) in which the soil properties are

changing in a linear manner (also Fig. 7.2c - initial moisture content profile, of which only the part above initial ground water table is plotted).

Figs. 7.5a, b and 7.9a, b allow us to follow the time development of pressure heads and of the ground water table (indicated by the zero pressure head). The ground water table is falling slightly during the initial dry period (due to both bottom drainage and transpiration) but the main part of water for evaporation and transpiration is extracted from the upper, unsaturated part of the profile. When the rain begins, water content is also replenished from above (on 6th day, we have yet no sensible influence of infiltration upon the ground water table). A distinct difference can be observed between both variants on 12th day when the whole profile is still saturated and with a small positive pressure head on the surface for the variable atmospheric boundary condition (Fig. 7.9b, beginning from 40th day). The absence of any response to the second rain (on 41st to 44th day) in Figs. 7.9a and 7.9b (without variable atmospheric boundary condition) is striking.

Finally, we can observe the time course of the variable volumetric fluxes (Darcy velocities) in Figs. 7.6a, b and 7.10a, b. The difference in hydraulic properties of individual layers is well pronounced only on 7th day (during unsaturated infiltration), while the flux is and must be (from the requirement of continuity) constant throughout the soil profile if the latter is saturated.

Because of the hampered bottom drainage, the saturated flux is also very low (almost indistinguishable from zero in the figures). On 42nd day, relatively large downwards flux (infiltration) is visible in Fig. 7.6b (with variable atmospheric boundary condition) while nothing can be seen in Fig. 7.10b.

The time step of one day for the atmospheric boundary condition which is also used in the subroutine for surface water retention is too large to give a true picture of the phenomena caused by intensive rainfalls (infiltration, surface runoff). It will be desirable in the future to shorten this time step or to make a provision in the program for automatic adjustment of the length of this step depending upon weather and surface water retention.

FLOW DOMAIN 1

Soil water movement in a grass field with and without variable atmospheric boundary condition.

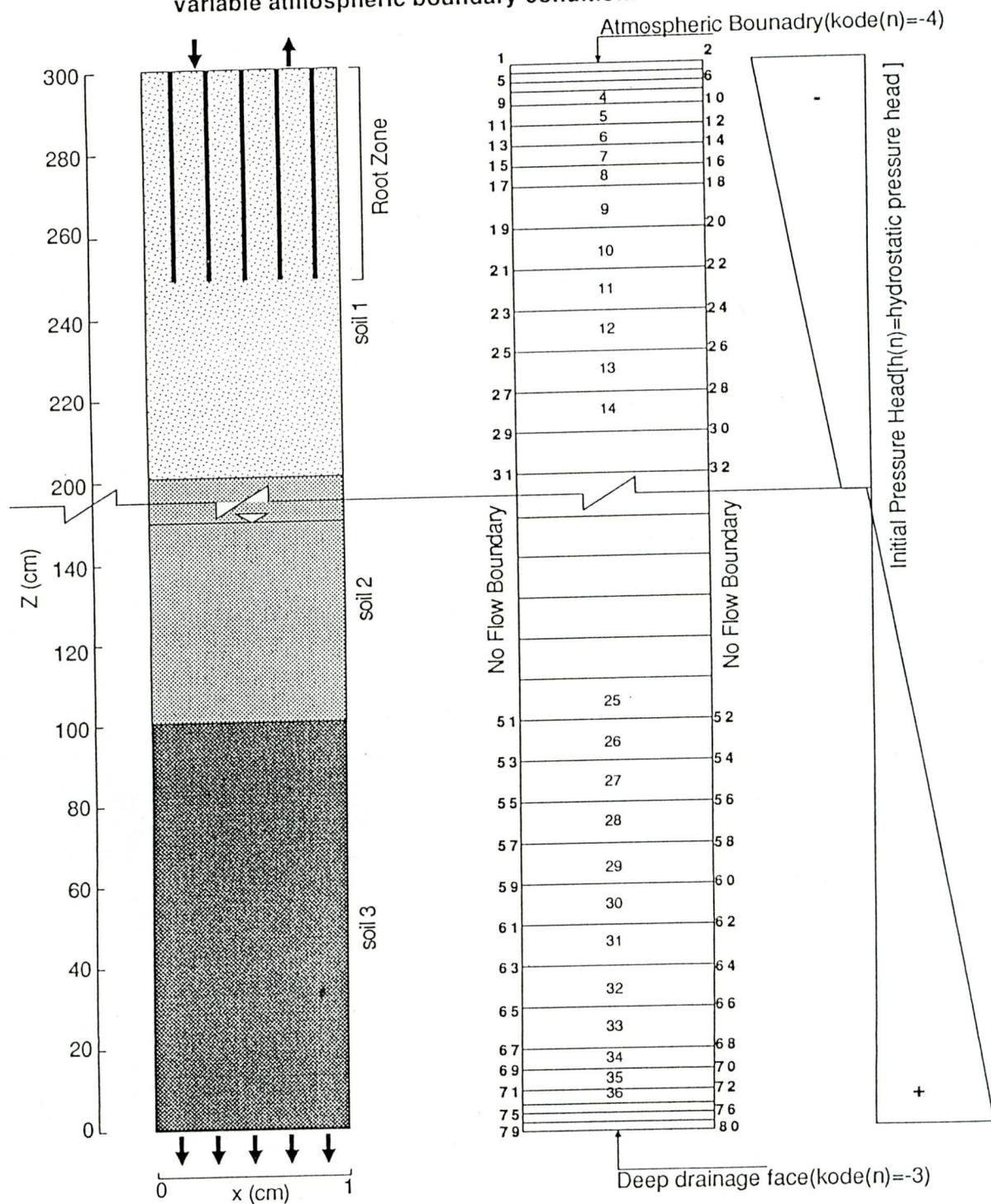
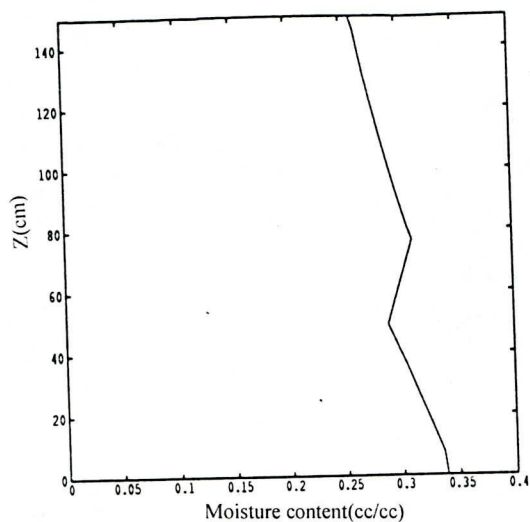
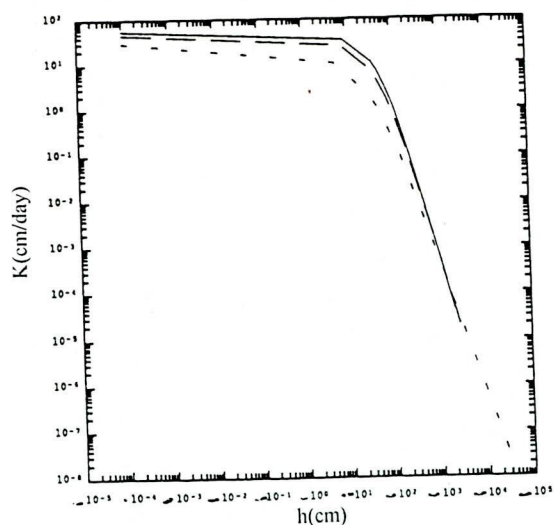


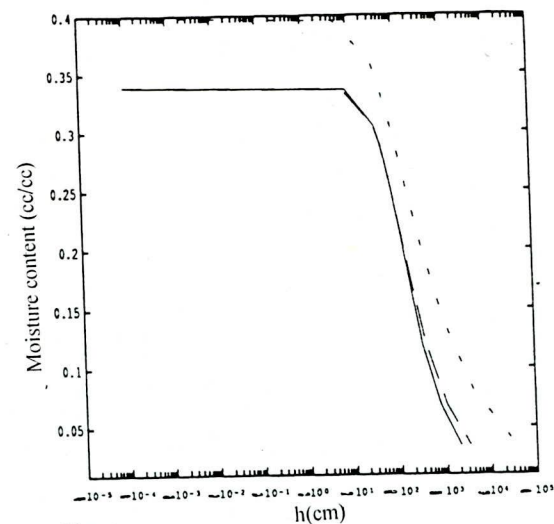
Fig. 7.1 : Flow system and finite element mesh with initial and boundary conditions for one dimensional vertical flow (Example 1)



(i) Initial moisture contents in the unsaturated zone



(ii) Hydraulic conductivity of the 1st (---), 2nd (---) and 3rd (solid) layers



(iii) Volumetric moisture content of the 1st (---), 2nd (---) and 3rd (solid) layers

Fig. 7.2a : Initial volumetric moisture content profile and hydraulic properties of the 1st, 2nd and 3rd soil layers

Initial moisture content (i) is given for example 1 and variation of hydraulic properties (ii & iii) of the 1st, 2nd and 3rd soil layers remain the same for all examples (1,2,3A & 3B)

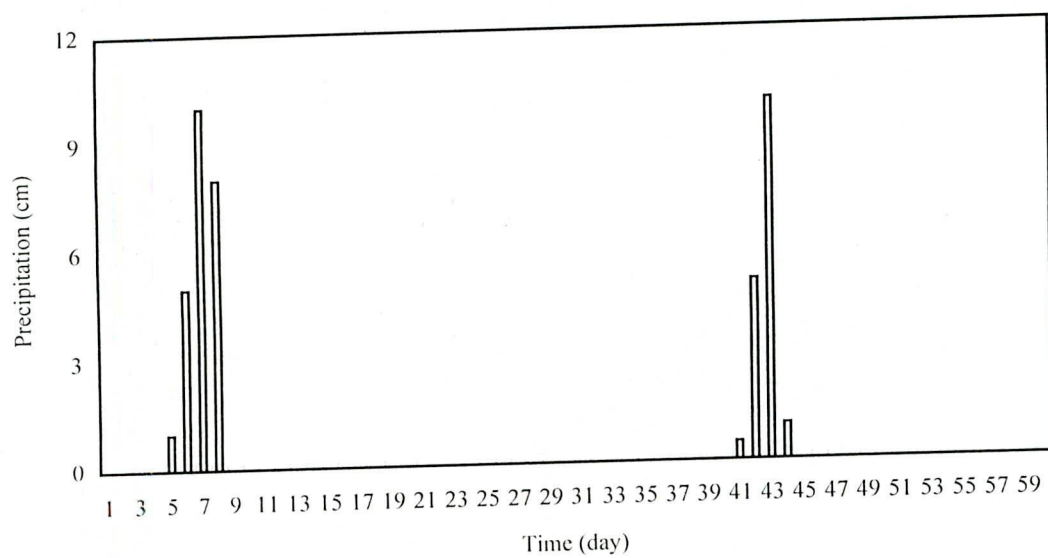
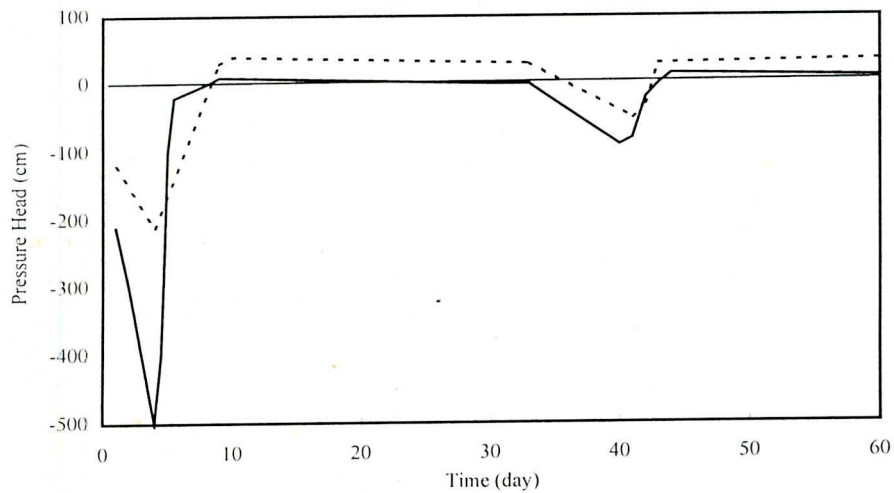


Fig. 7.2b : Hypothetical precipitation for Example 1

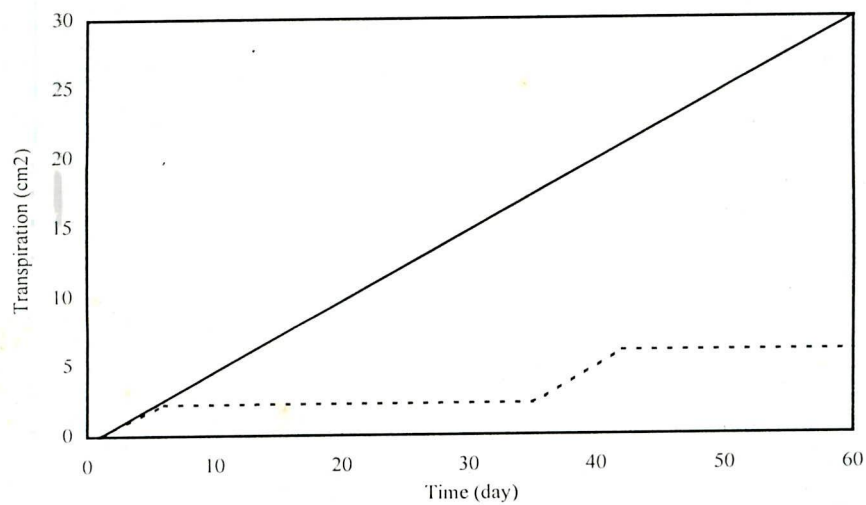
The Simulated Results

for

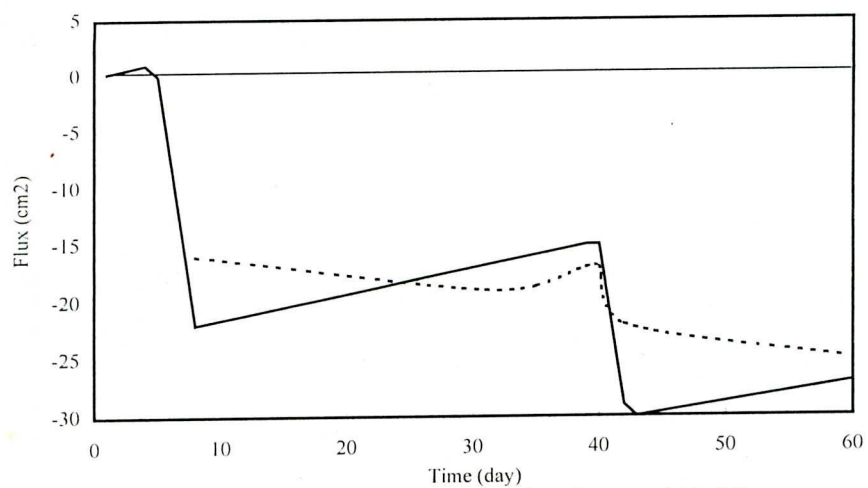
Example 1 : With Variable Atmospheric Boundary Condition



(a) Mean pressure head at the soil surface (solid) and the root zone (---)



(b) Cumulative potential (solid) and actual (---) transpiration



(c) Cumulative flux across the atmospheric boundary potential (solid) and actual (---)

Fig. 7.3 : Simulated mean pressure head, potential and actual fluxes across the atmospheric boundary (+ve upward & -ve downward) (Example 1, with variable atmospheric boundary condition)

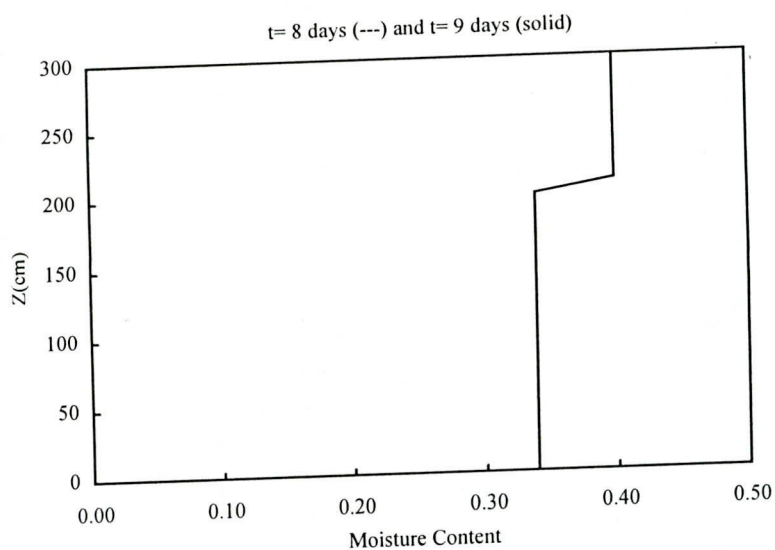
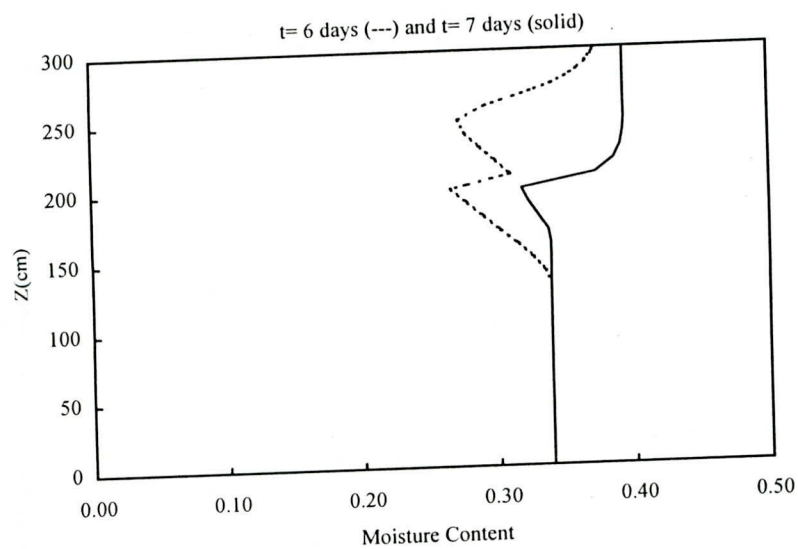
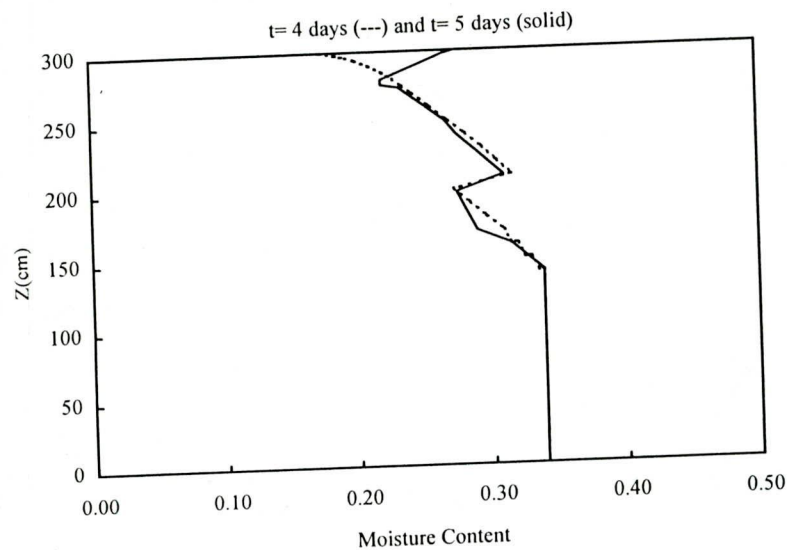


Fig 7.4a : The simulated volumetric moisture content profiles at the end of the days indicated (Example-1, with variable atmospheric boundary condition)

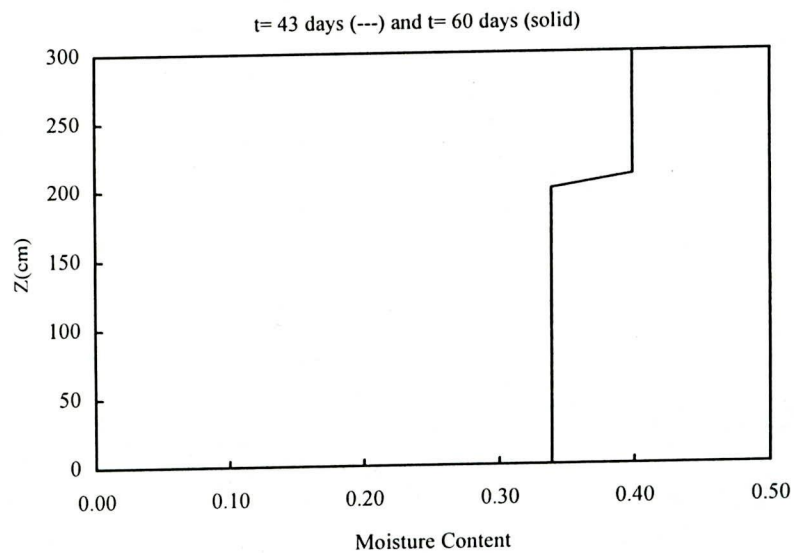
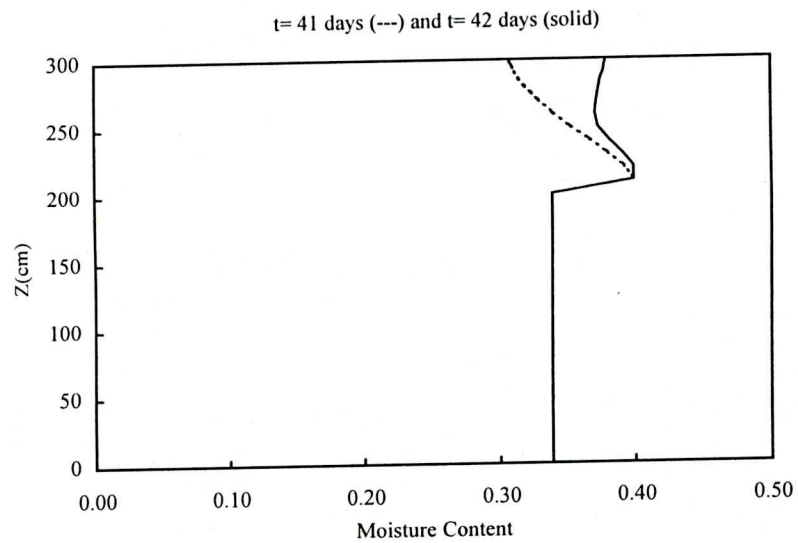
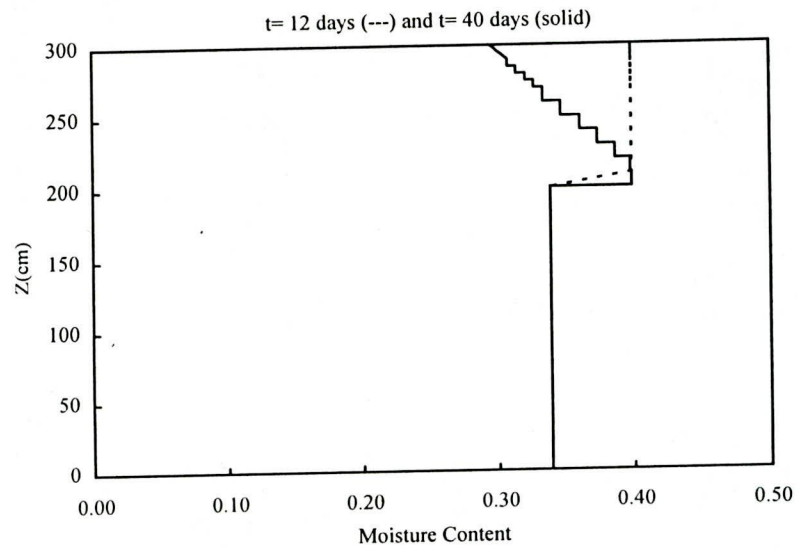
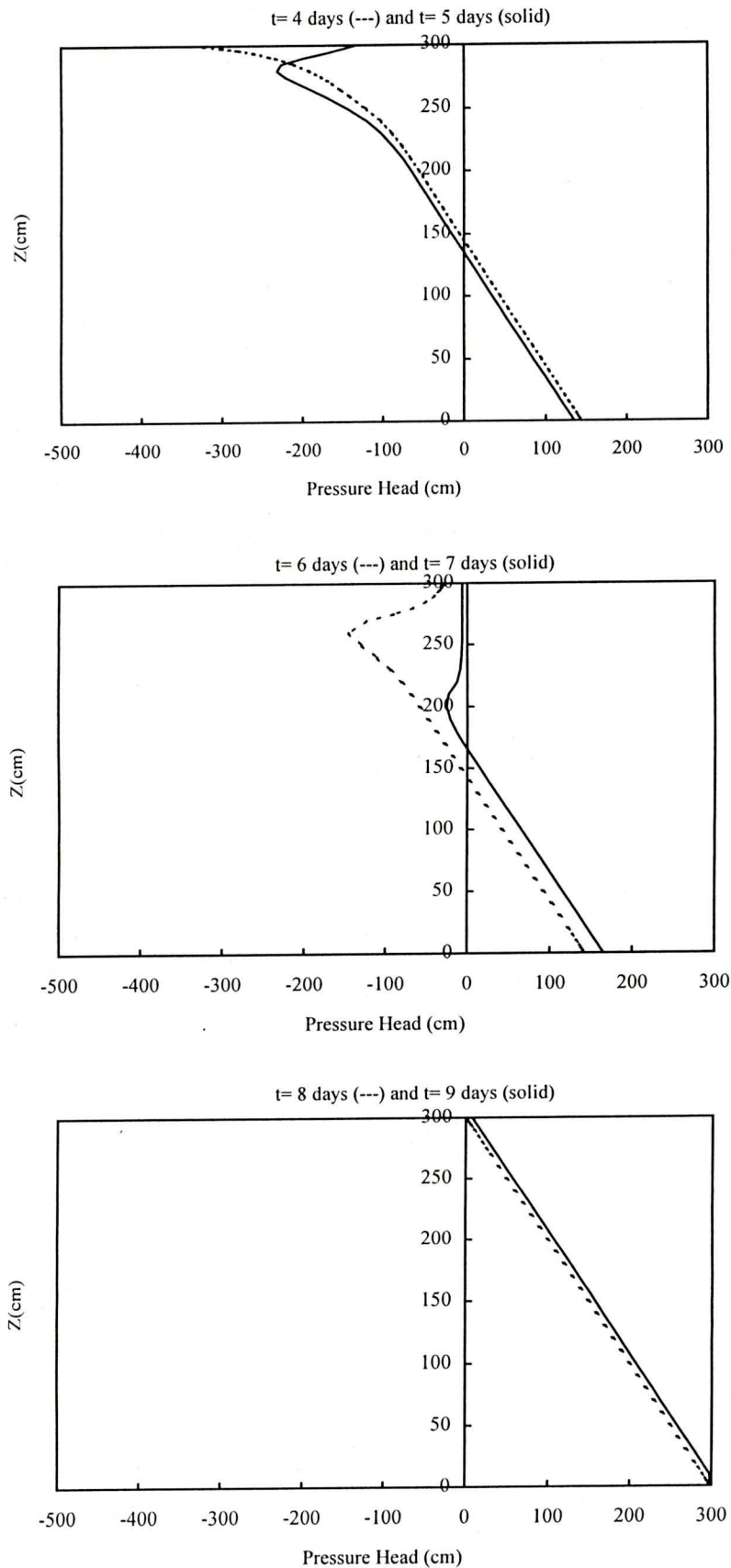


Fig 7.4b : The simulated volumetric moisture content profiles at the end of the days indicated (Example-1, with variable atmospheric boundary condition)



**Fig 7.5a : The simulated pressure head profiles at the end of the days indicated
(Example-1, with variable atmospheric boundary condition)**

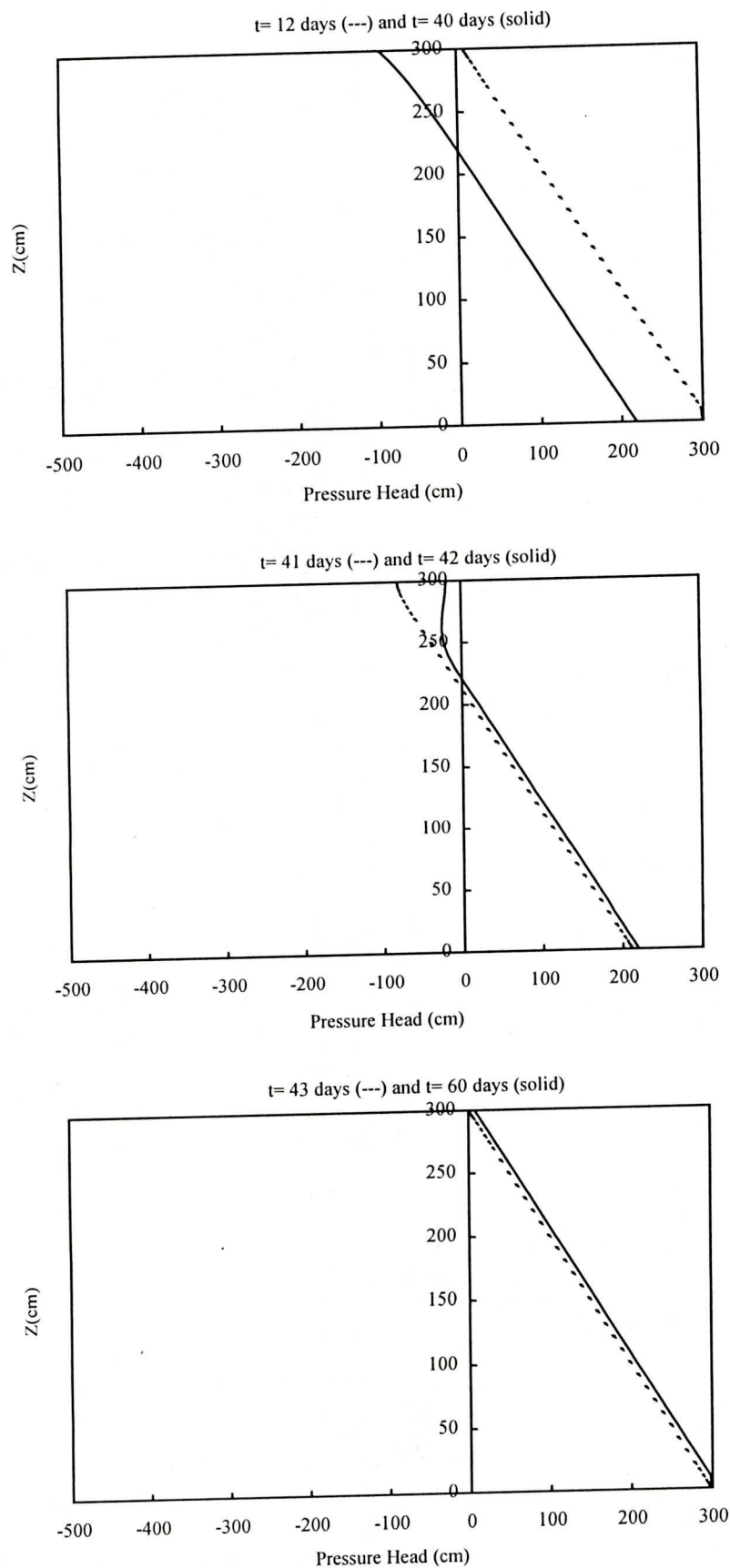


Fig 7.5b : The simulated pressure head profiles at the end of the days indicated (Example-1, with variable atmospheric boundary condition)

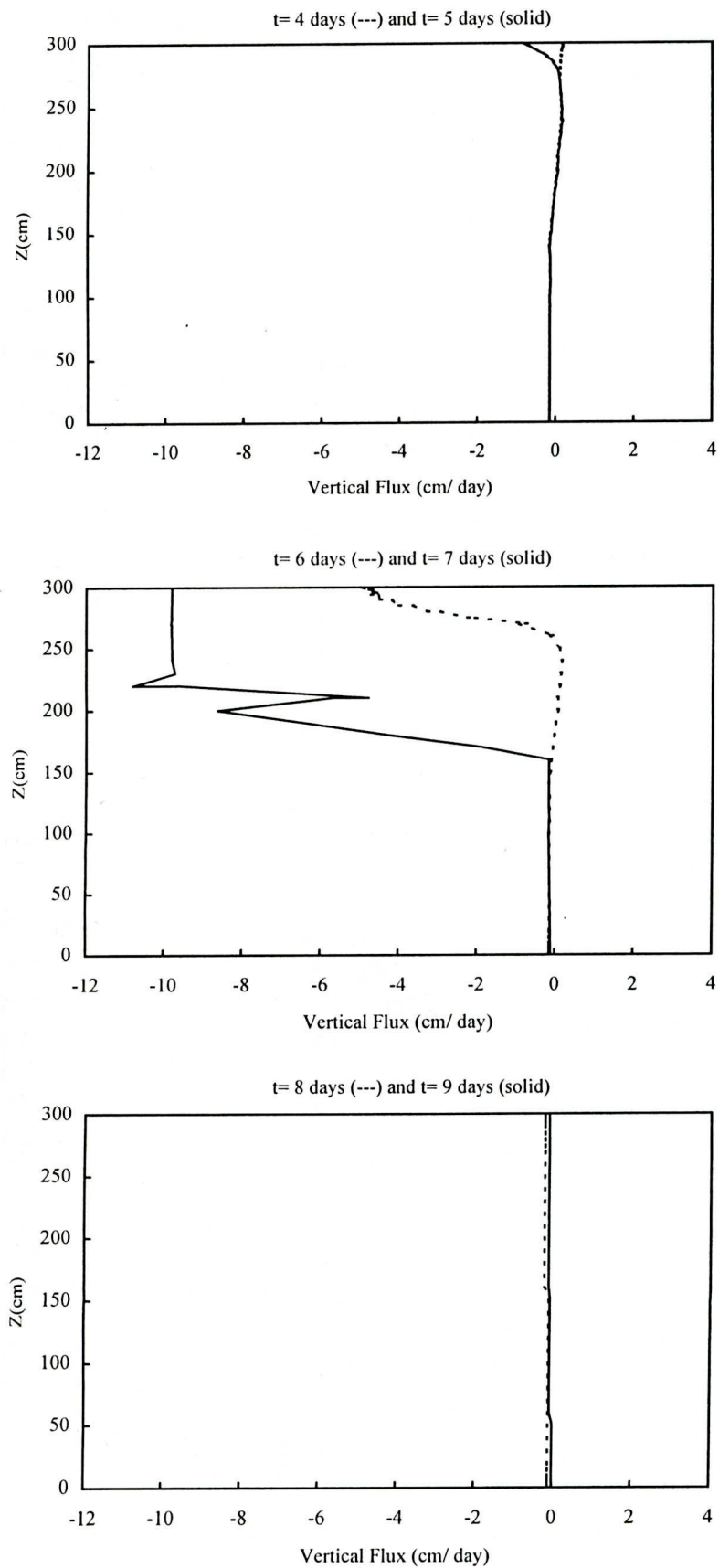
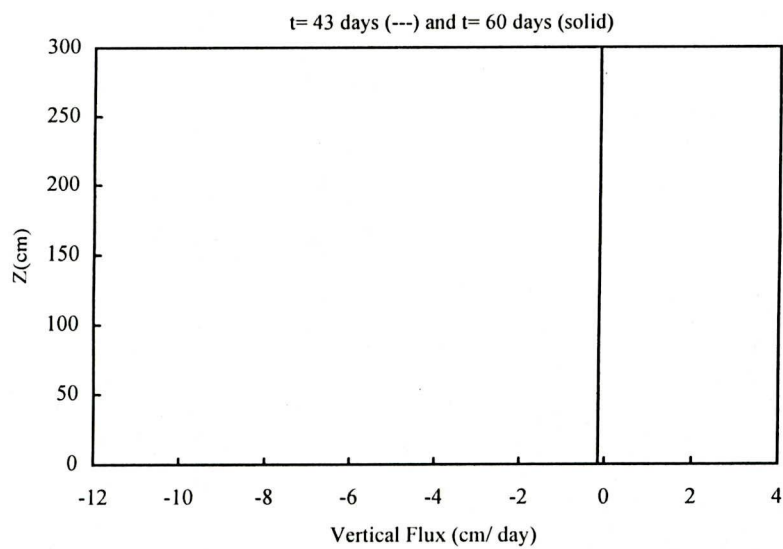
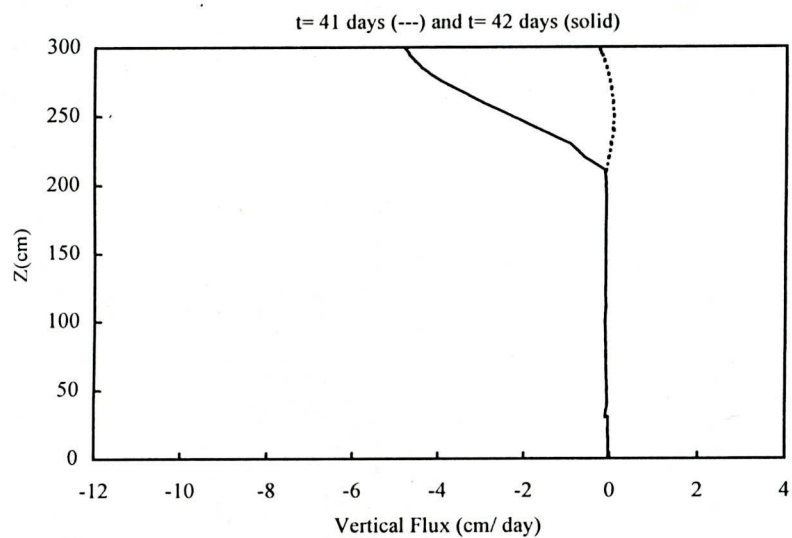
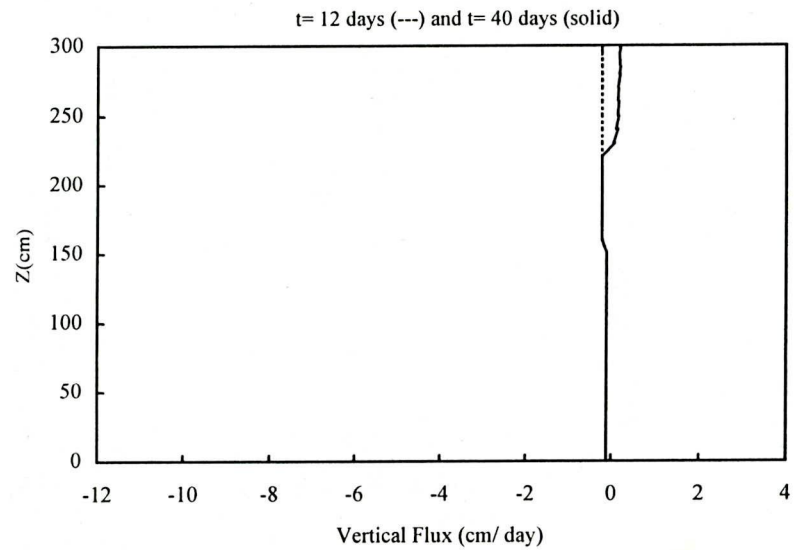
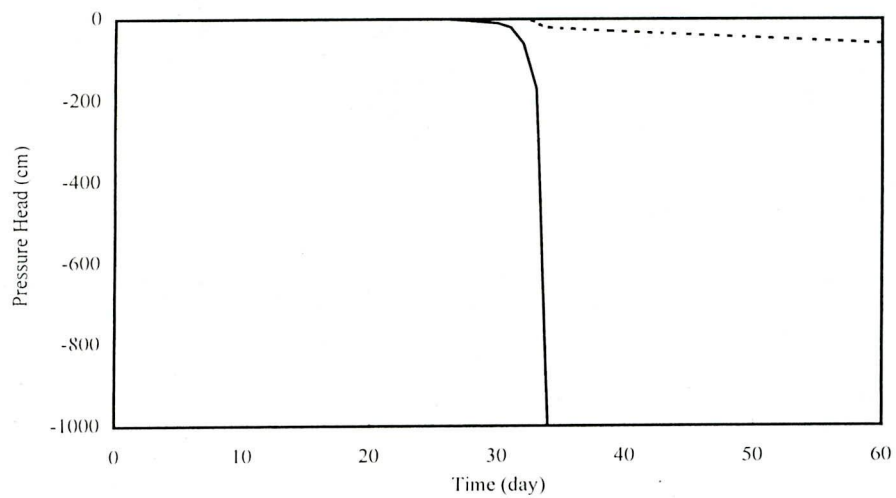


Fig 7.6a : The simulated vertical flux profiles at the end of the days indicated (Example-1, with variable atmospheric boundary condition)

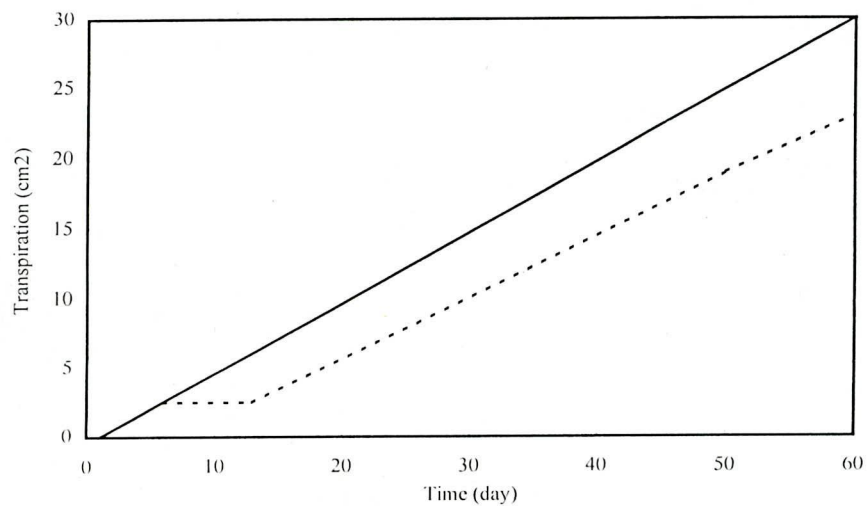


**Fig 7.6b : The simulated vertical flux profiles at the end of the days indicated
(Example-1, with variable atmospheric boundary condition)**

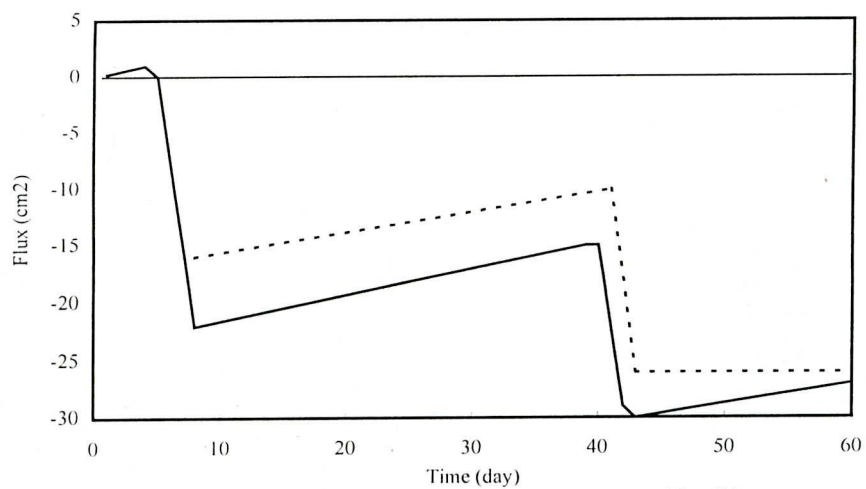
The Simulated Results
for
Example 1 : Without Variable Atmospheric Boundary Condition



(a) Mean pressure head at the soil surface (solid) and the root zone (---)



(b) Cumulative potential (solid) and actual (---) transpiration



(c) Cumulative flux across the atmospheric boundary potential (solid) and actual (---)

Fig. 7.7 : Simulated mean pressure head, potential and actual fluxes across the atmospheric boundary (+ve upward & -ve downward) (Example 1, without variable atmospheric boundary condition)

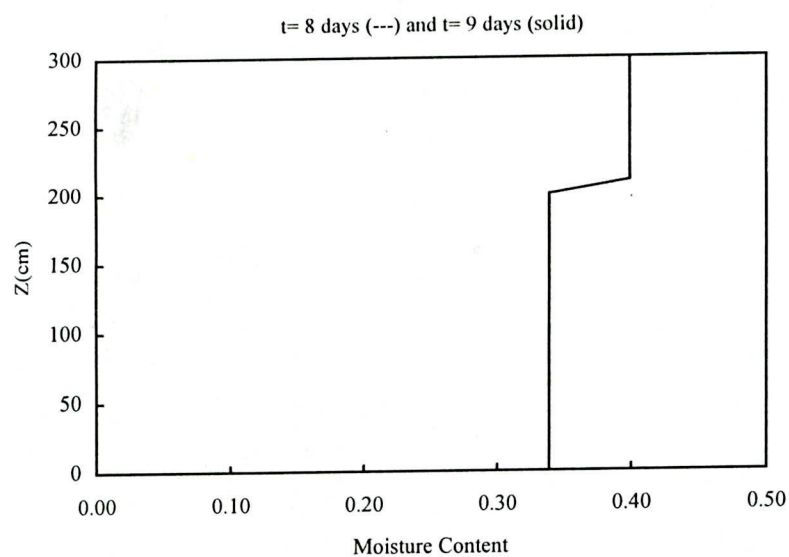
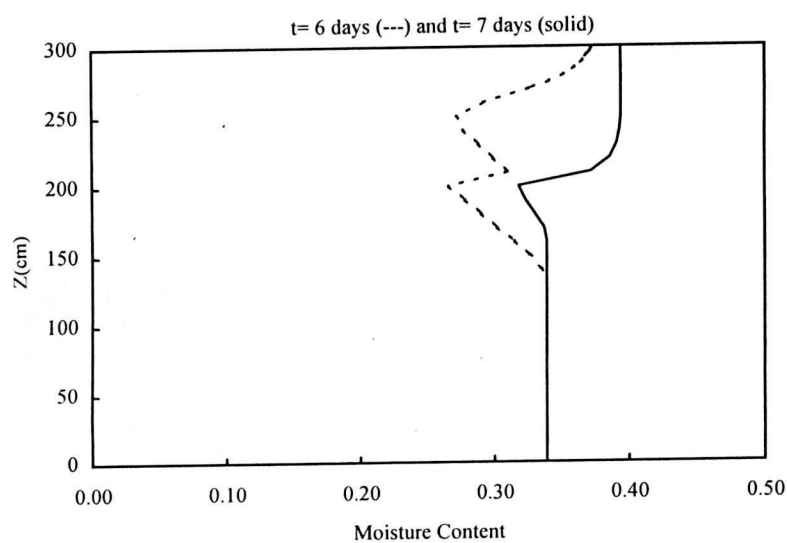
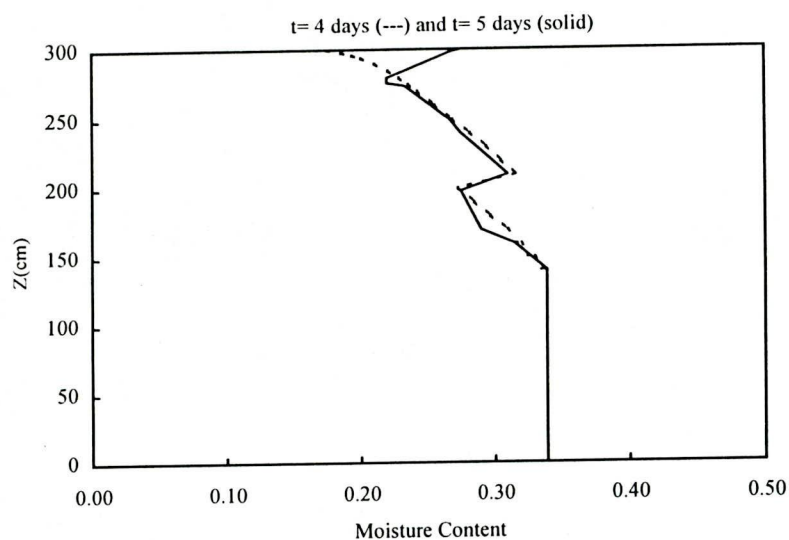


Fig 7.8a : The simulated volumetric moisture content profiles at the end of the days indicated (Example-1, without variable atmospheric boundary condition)

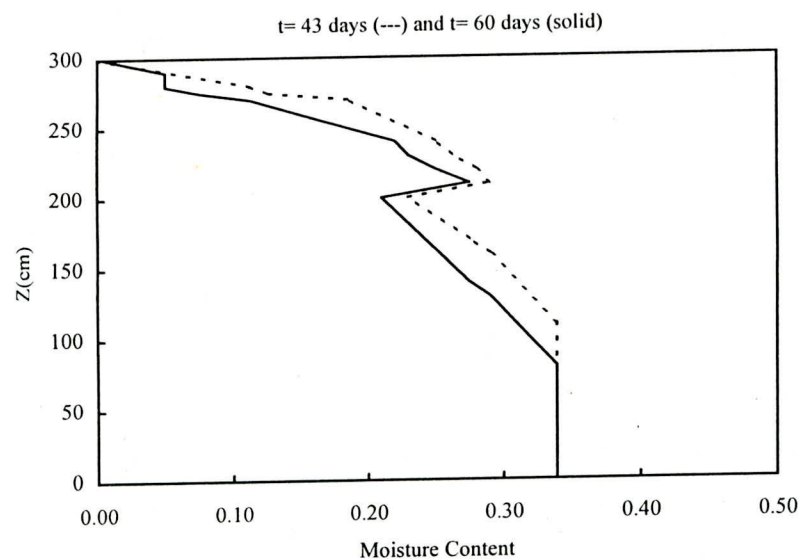
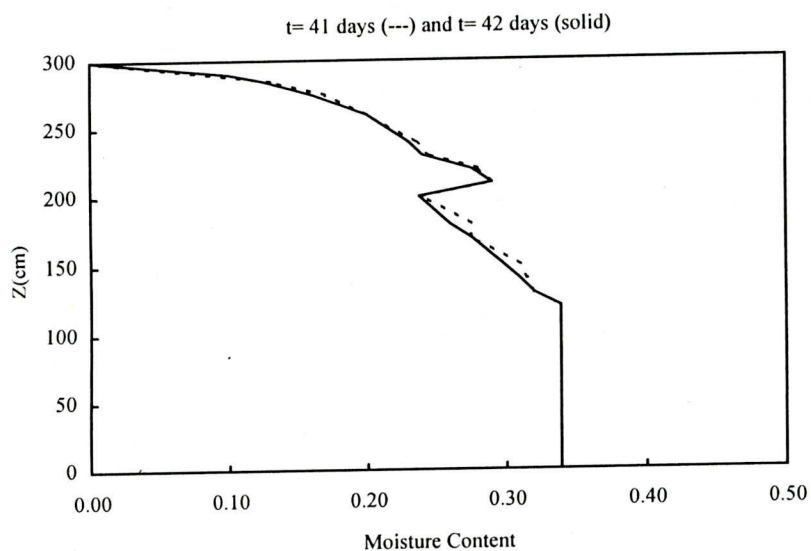
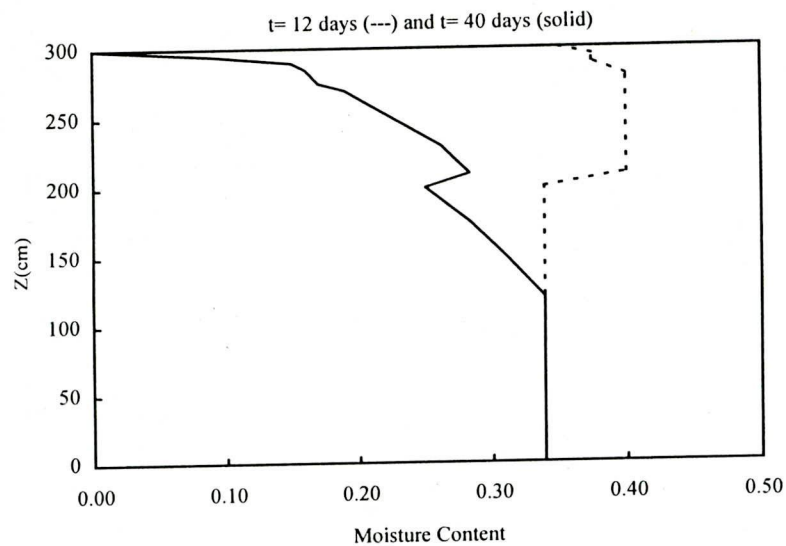
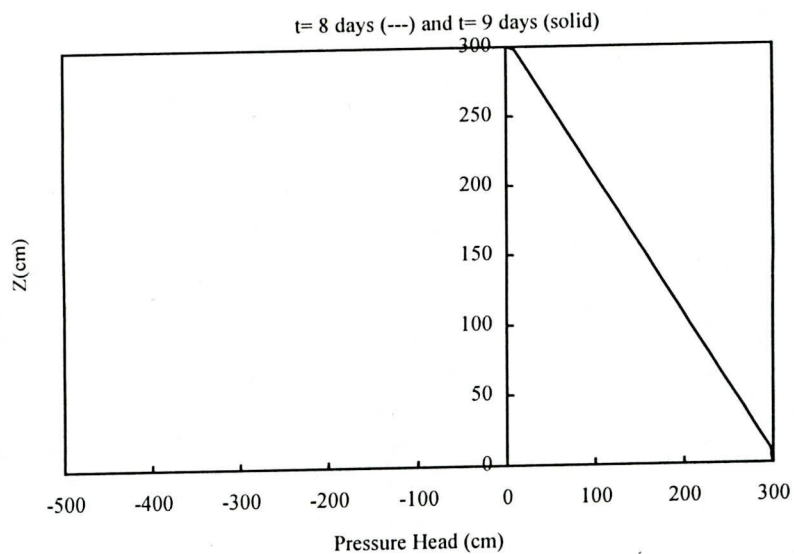
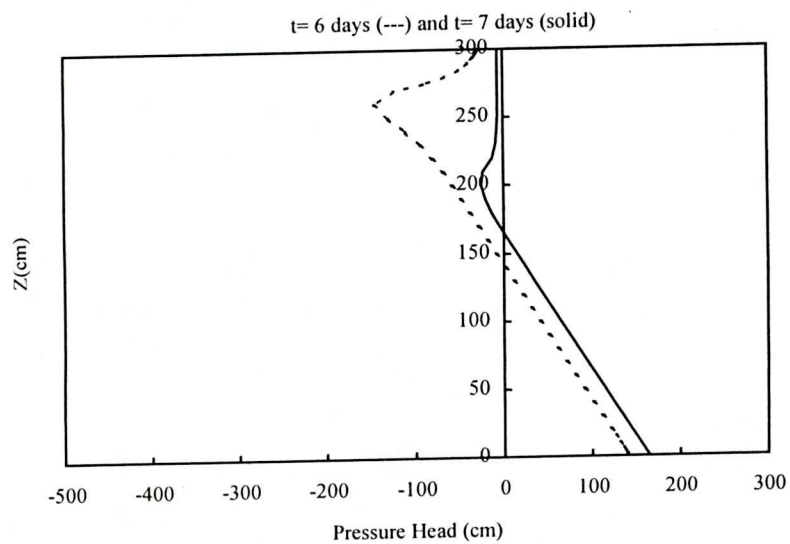
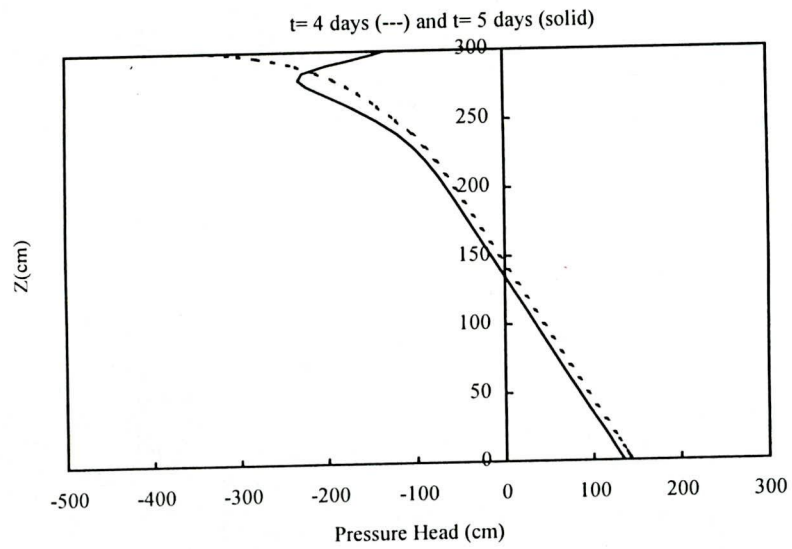


Fig 7.8b : The simulated volumetric moisture content profiles at the end of the days indicated (Example-1, without variable atmospheric boundary condition)



**Fig 7.9a : The simulated pressure head profiles at the end of the days indicated
(Example-1, without variable atmospheric boundary condition)**

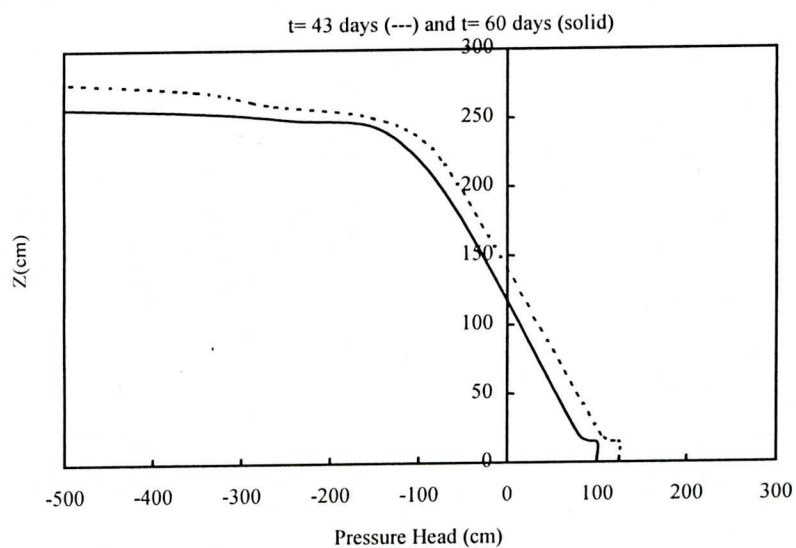
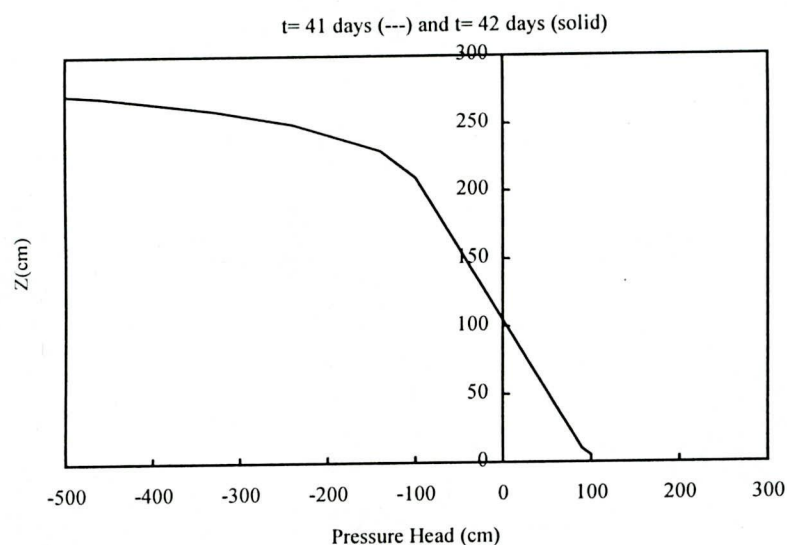
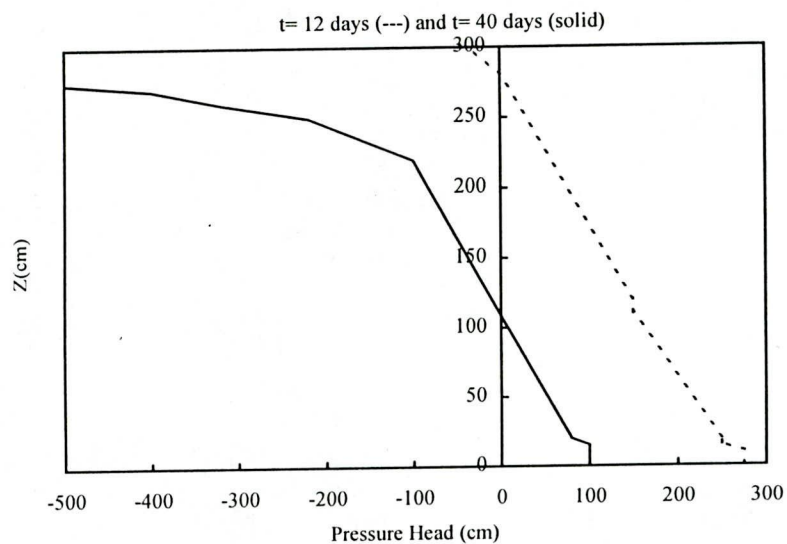
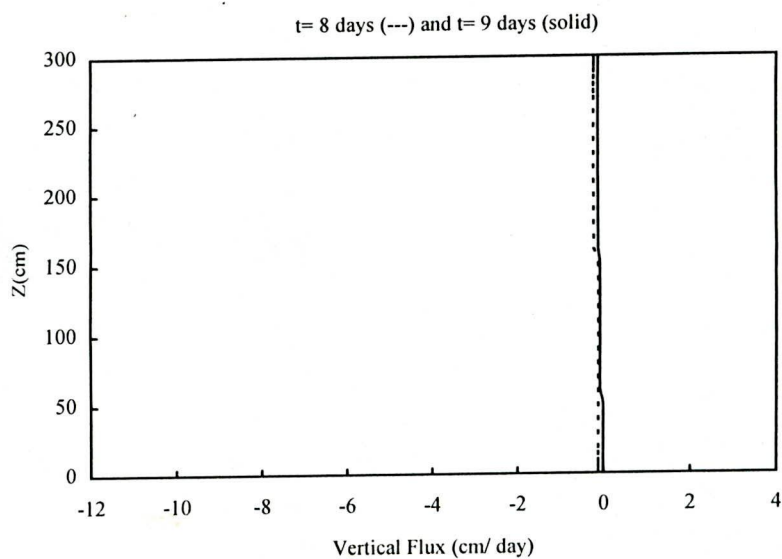
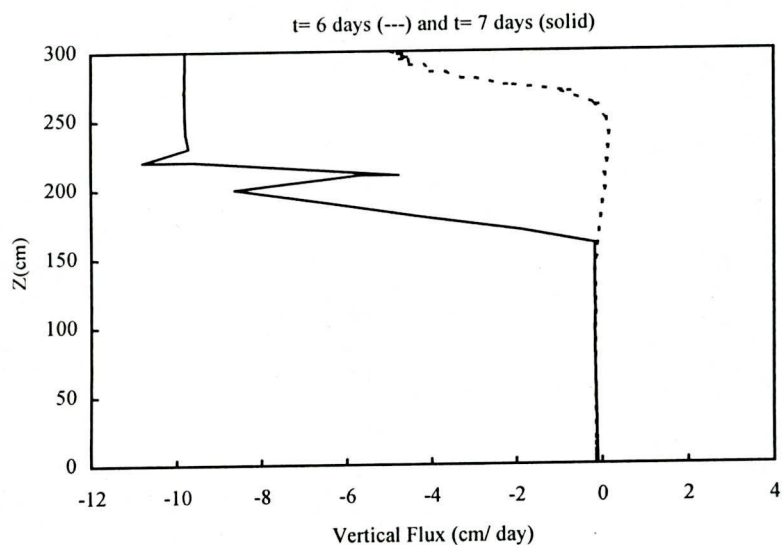
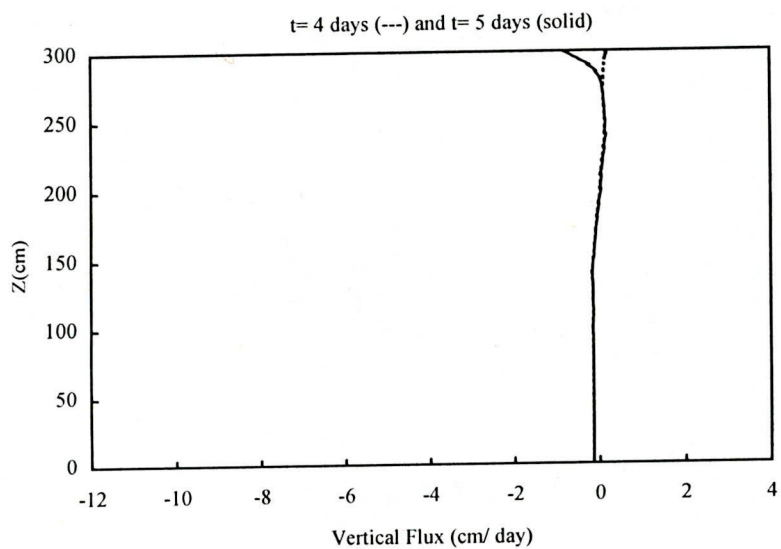
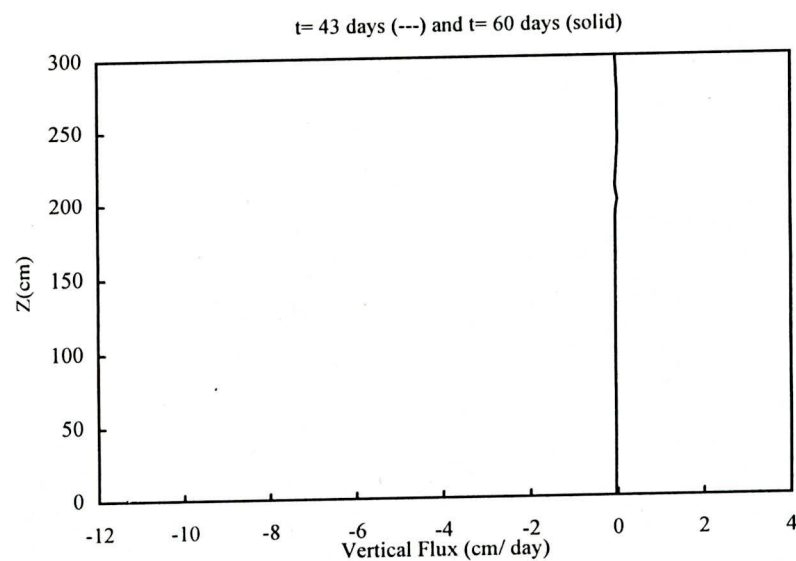
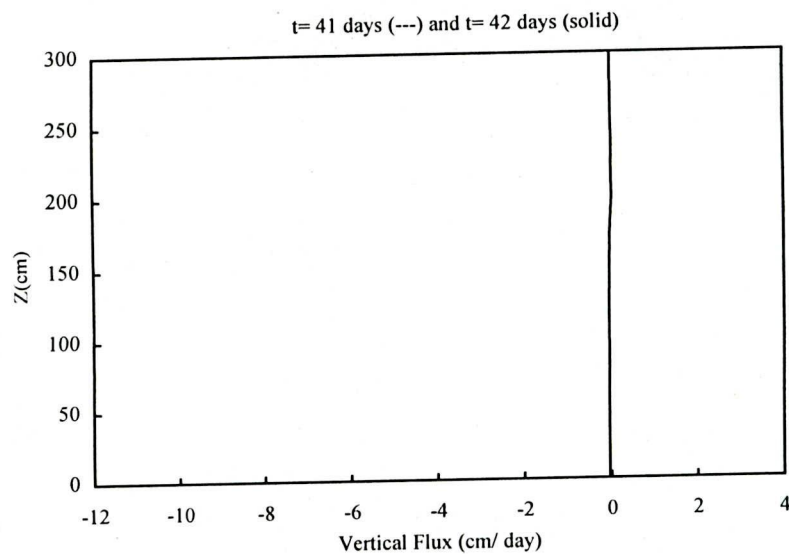
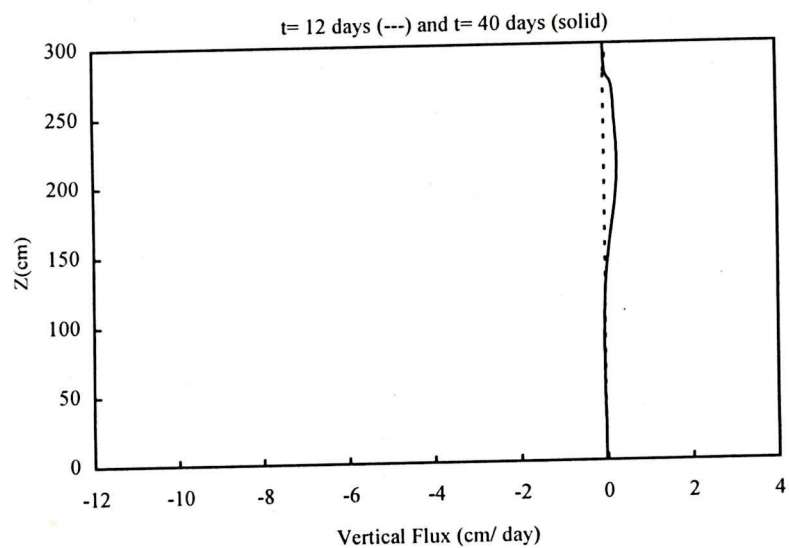


Fig 7.9b : The simulated pressure head profiles at the end of the days indicated (Example-1, without variable atmospheric boundary condition)



**Fig 7.10a : The simulated vertical flux profiles at the end of the days indicated
(Example-1, without variable atmospheric boundary condition)**



**Fig 7.10b : The simulated vertical flux profiles at the end of the days indicated
(Example-1, without variable atmospheric boundary condition)**

7.4. Example 2: Two-dimensional Rectangular Flow Domain with a Variable Seepage Face

This example is designed to illustrate water movement to illustrate water movement in a block of soil adjacent to a riverbank, as induced by the fluctuating water level in the river, with seepage face of variable size and position above the river water level. Two-dimensional rectangular flow domain (10 m x 10 m) suggested as depicted in Fig. 7.11a and 7.11b. The upper ("atmospheric") boundary face of the flow region is defined as a no-flow boundary ($Kode(n) = 0$), to prevent any disturbing influence of atmospheric events. The same boundary condition is postulated also for the off-river face of the flow region (the right edge of the rectangle). The nodal points above water level on the bank face (on the left edge of rectangle) are assigned seepage-face boundary condition with $Kode(n) = 2$, while the Dirichlet boundary condition of variable pressure head is imposed on the points below water level on the same bank face, with $Kode(n) = 1$ and the pressure head calculated from the river water level. The subroutine VBound is written to calculate the pressure head, as well as to decide, on the basis of river water level fluctuations, which nodes are to acquire $Kode(n) = +1$ and which will belong to the seepage-face, at each time step on the level(2) according to article 4.4.2. The nodes that have newly become a part of the seepage face are assigned $Kode(n) = +2$ (saturated, actual seepage face), while the initial assignments to all seepage face nodes (above initial water level) is $Kode(n) = -2$ (unsaturated, potential seepage face). The bottom face of the flow domain is considered as variable drainage flux boundary with $Kode(n) = -3$. Initial ground water table within the flow region is assumed horizontal and in equilibrium with the initial water level in the river, 5 meter above the bottom surface (Fig. 7.11b). Initial pressure head and moisture content values within the flow region are assumed in equilibrium with the ground water table.

The rectangular flow region is discretized into 621 nodal points and 500 elements (Fig 7.7b). the element dimensions are smaller along the top, bottom and river faces, while larger pressure gradients could be expected. The region is composed of 3 layers with slightly different hydraulic parameters (Fig 7.7b). The soil parameters are taken over

from the Simunek et al (1992) and are the same as in example 1 but the thickness of the individual layers are different from example 1. Root zone occupied the upper 0.5 meter of the flow region but this is only a formal definition, as the fluxes through the atmospheric boundary as well as the transpiration are zero.

Water level fluctuation in the hypothetical river left to the flow domain is imposed to test the functioning of the variable seepage face subroutine (Fig 7.8a). An extreme trapezoidal flood wave is supposed to take place during the first 15 days with the peak level 10 m (just at the level of soil surface so that spillage did not occur) held constant during the 6th, 7th and 8th day; after that, low water level (2 m above bottom) is maintained till the 40th day. Finally, water level increased gradually to 5 m on 41st to 43rd day and is maintained at the latter level till the end of simulation. Water level is kept constant during each day; the whole daily change of level is assumed to occur suddenly at midnight.

Basic parameters of simulation and soil hydraulic parameters are summarized in the following Tables 7.3a, 7.3b and 7.3c. results of simulation for some selected typical time instants are presented in detail in the following pages.

Table 7.3a : Basic Information

Type of Flow System	Two dimensional transient flow in a rectangular domain
Data source	Hypothetical water level fluctuation in the river(2-10 m above bottom) without atmospheric influences

Table 7.3 b : Input Hydraulic Parameters

Parameters	1st layer (1000-700 m)	2nd layer (700-400 m)	3rd layer (0.0-400 m)
$\theta_s = \theta_m = \theta_k$	0.399	0.399	0.399
$\theta_r = \theta_a$	0.0001	0.0001	0.0001
$K_s = K_k$ (m/day)	0.2975	0.4534	0.5534
α (1/m)	1.740	1.390	1.300
n (-)	1.3757	1.6024	1.7024

Table 7.3c : Numerical Simulation Information

Time Information	
Initial time increment : dt	0.02 day
Minimum time increment : $\Delta t_{\min}$	1e-10 day
Maximum time increment : $\Delta t_{\max}$	0.50 day
Iteration Information	
Maximum number of iteration : Max It	20
Absolute pressure head tolerance : Tol	0.01 cm
Some logical switch setting	
Atmospheric boundary : AtmInf	.false.
Seepage face : SeepF	.true.
Variable seepage face : Vseep	.true.
Variable atmospheric boundary : Vatms	.false.

7.4.1 Discussion of the Results

Results obtained in Example 2, in spite of the schematic nature of the example, can throw some light on the pattern of soil moisture and ground water movement in river banks during real hydrology events.

The results presented can be divided into four groups:

- i) time course of average pressure head in the root zone and of cumulative bottom flux (fig 7.12b),
- ii) vertical profiles of volumetric moisture content at the end of selected days, in the middle of the flow region (Figs. 7.13a to 7.13d),
- iii) vertical profiles of the vertical component of darcian flux at the end of selected days, in the middle of the flow region (Figs. 7.14a to 7.14d),
- iv) contours of pressure head at the end of selected days (Figs. 7.15a to 7.15h),

The days selected for presentation are:

Day	Water level (m)	Commentary
1	5	Initial state kept, without movement
2	6	Flood wave begins
3	7	Flood wave rises further
4	8	Flood wave rises further
5	9	Flood wave rises further
6	10	Beginning of the top flood
7	10	Middle of the top flood
8	10	Last day of the top flood
9	9	Flood begins to recede
10	8	Recession continues
11	7	Recession continues
12	6	Recession continues
13	5	Recession continues
14	4	Recession continues
15	3	Recession continues
16	2	End of flood, minimum level
17-20	2	Minimum level kept
40	2	Last day with minimum level
41	3	Water level begins to rise
42	4	The rise continues
43	5	End of the rise, medium level

Fig. 7.12b shows that the average pressure head in the root zone increased very rapidly from initial-475 cm to about-200 cm, reaching its maximum after the flood had receded (on about 20th to 26th day). After that, the influence of low water level in the river began to pronounce, causing relatively slow drainage of the soil profile (the bottom drainage acted, too, but see the following section). This slow drainage is not sensibly influenced by the rising water table on 41st-43rd day. The system modeled (given by the soil parameters, by geometry and by boundary conditions) seems to

accept very quickly (with some delay) water from the river but releases this water back only very slowly.

In Figs. 7.15a to 7.15h, one can observe how the ground water table (a contour coded 0) rose and fell with the river water level, sloping off-river (to the right) in the period of rising hydrograph and shortly after (2nd to 9th day and 41st to 43rd day), and to the river bank (to the left) in the periods when the hydrograph is falling for several days or constant at a low level (10th to 40th day). On 1st day, we observe an obvious static equilibrium with all contours horizontal. The contours in the saturated zone below ground water table are at all times effectively parallel to the ground water table at distances corresponding to their nominal values. This means that the vertical gradient of hydraulic head in the saturated zone differs little from the hydrostatic one (validity of the well-known Dupuit-Forchheimer assumption of vertical equipotential lines in unconfined aquifers is thus demonstrated again). The contours in the unsaturated zone behave in a different manner. They become very narrow to each other in the period of rising ground water table (especially after 4th day) and remain so during the whole simulation period. A through observer may recognize that the contours 2 and 3, begin most narrow to each other probably at the end of the 8th day (at the top of the flood), become more and more distant between 9th and 20th day. In the final period of simulation (40th to 43rd day), both unsaturated contours are almost without movement, contour -300 begin situated sensibly lower and contour -200 sensibly higher and more to the right on 20th day. This behavior of unsaturated contours demonstrates: (a) tremendous difference in time scale between saturated and unsaturated movement (the unsaturated movement being much slower), (b) the process of wetting continues to take place in the upper and right part of the flow region, even if the rectangles as a whole undergoes intensive drainage, and even if the root zone (the upper half-meter strip) as a whole undergoes drainage (Fig. 7.12b), (c) the lower part of the unsaturated zone (between contours 0 and -200 and slightly above the contour -200) undergoes drainage which is very slow even if hysteresis of the retention curve is not allowed for (the model does not comprise a subroutine for hysteresis); this rehabilitates the concept of field capacity.

The volumetric moisture contents and vertical darcian fluxes are illustrated in Figs. 7.13a to 7.13d and 7.14a to 7.14d. The interface between soil layers in the height 650-700 cm is clearly visible in moisture content profiles. These profiles also indicate that drainage occurs in the most of the unsaturated zone between 12th and 20th day. Much less distinct is the continuous wetting that takes part in the uppermost part of the soil (compare the upper ends of profiles on 20th and 40th day).

The profile of vertical fluxes in the same figures demonstrate again the local maxima of flux near the ground water table, this time with algebraic sign (upward fluxes are positive). Horizontal scales of the graphs are not constant. Bottom flux is invisible (it is too small).

FLOW DOMAIN 2

Two dimensional vertical flow domain with variable seepage boundadry condition

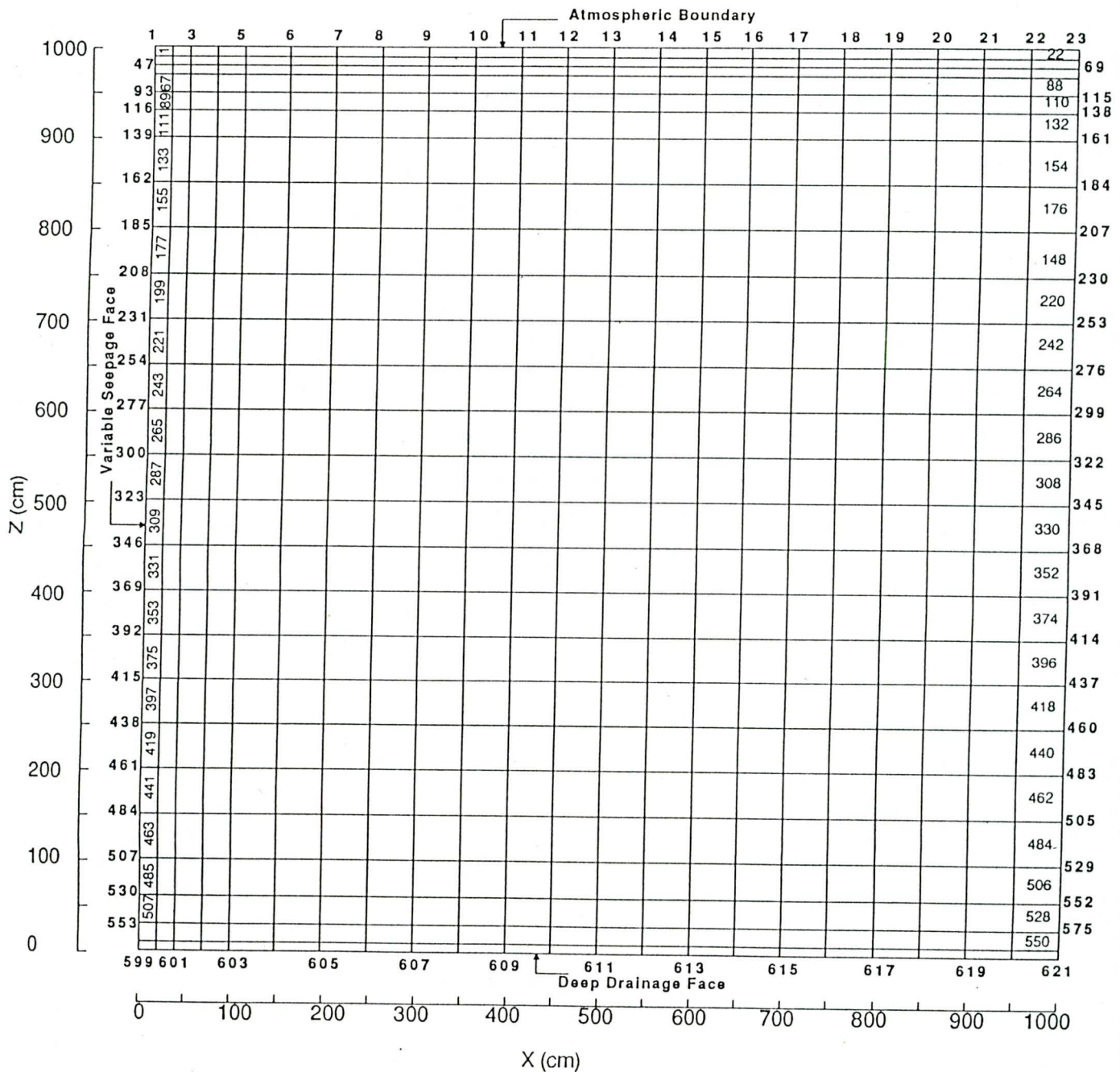


Fig. 7.11a : Finite element mesh for example 2 with variable seepage boundary condition

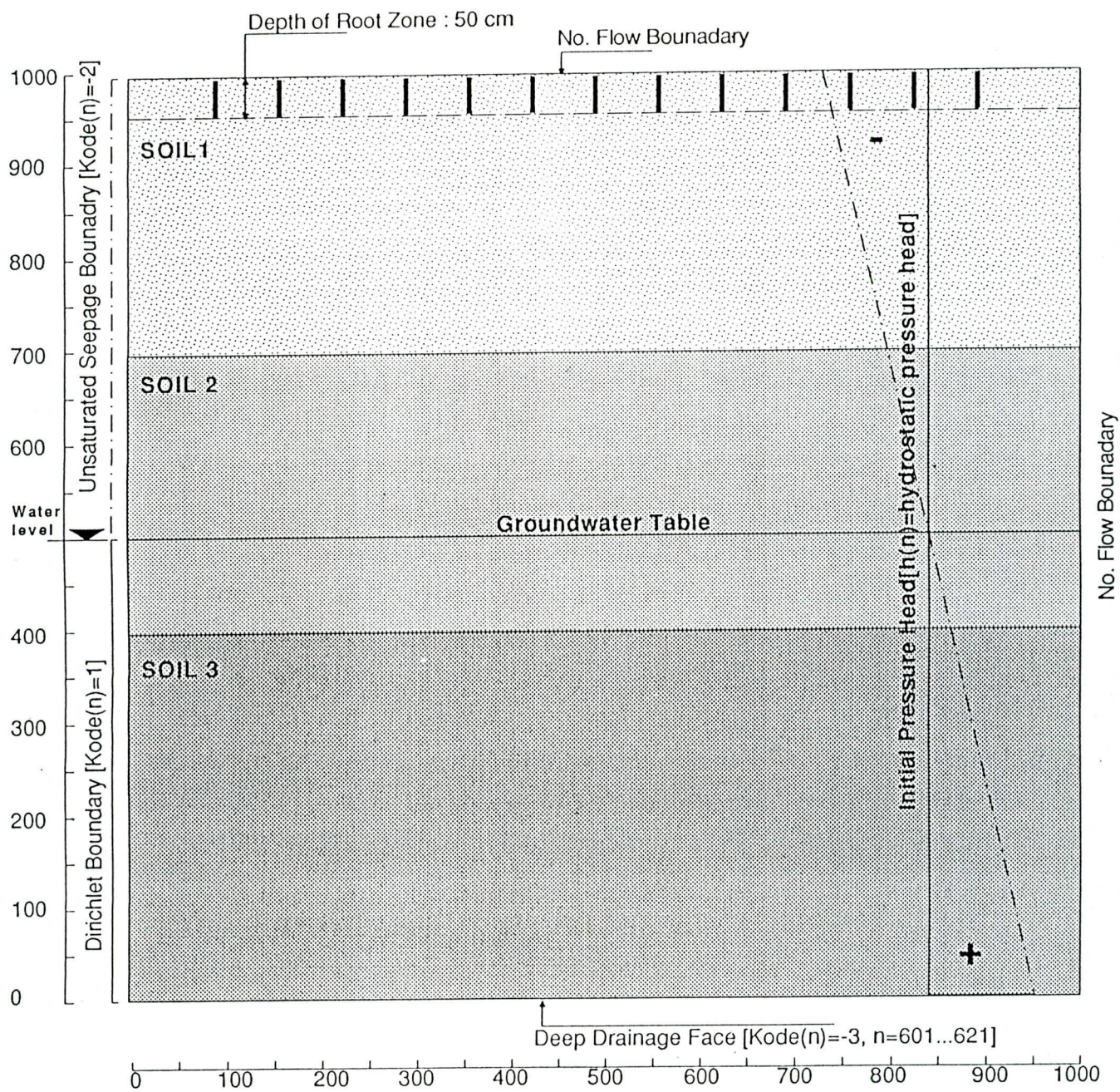


Fig. 7.11b : Flow system with initial and boundary conditions (Example 2)

(Dirichlet boundary condition on the left side will switch to seepage face [Kode(n)=+2] if water level in the river falls down; seepage face code ± 2 will switch to +1 if water level rises)

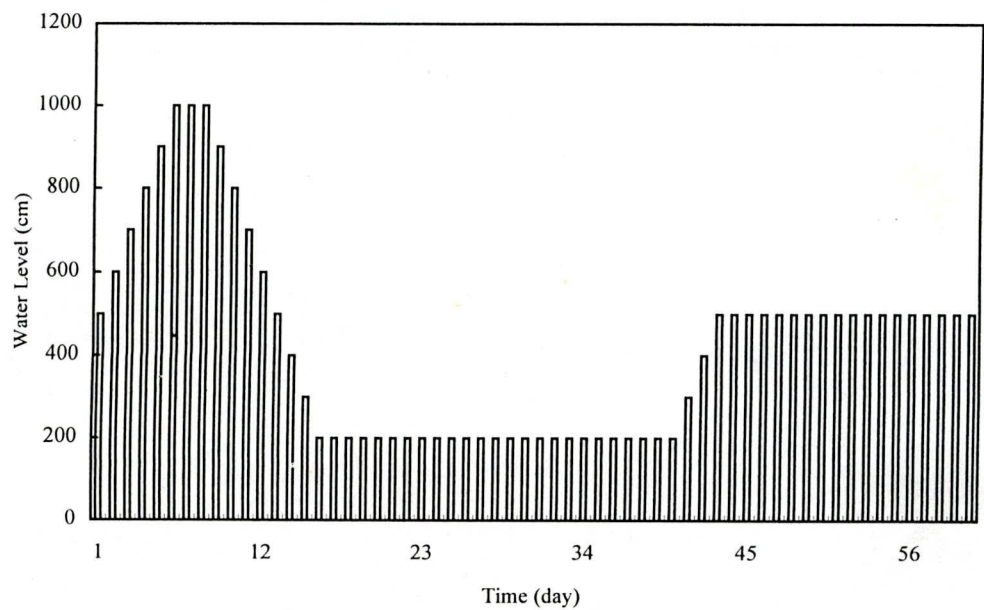


Fig 7.12a : Hypothetical daily water level fluctuations in the river for Example 2

The Simulated Results

for

**Example 2 : Two-dimensional Rectangular Flow Domain with a
Variable Seepage Face**

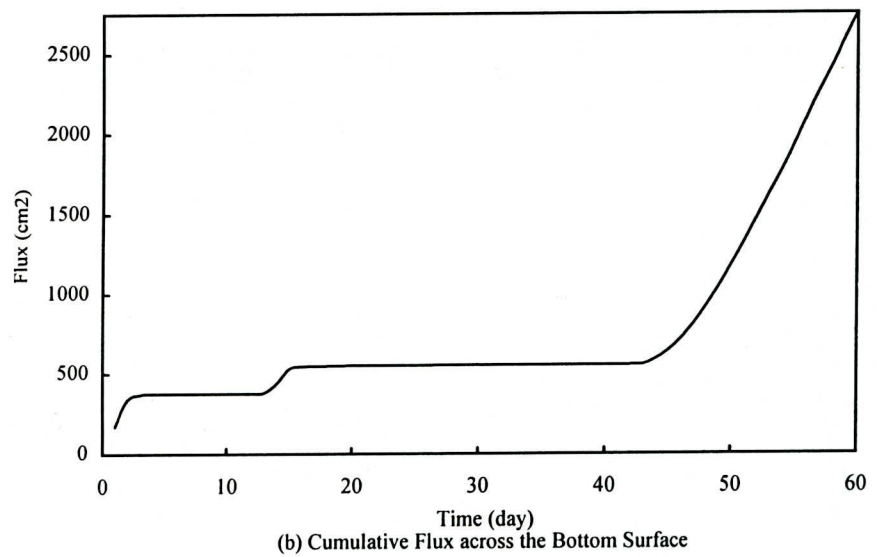
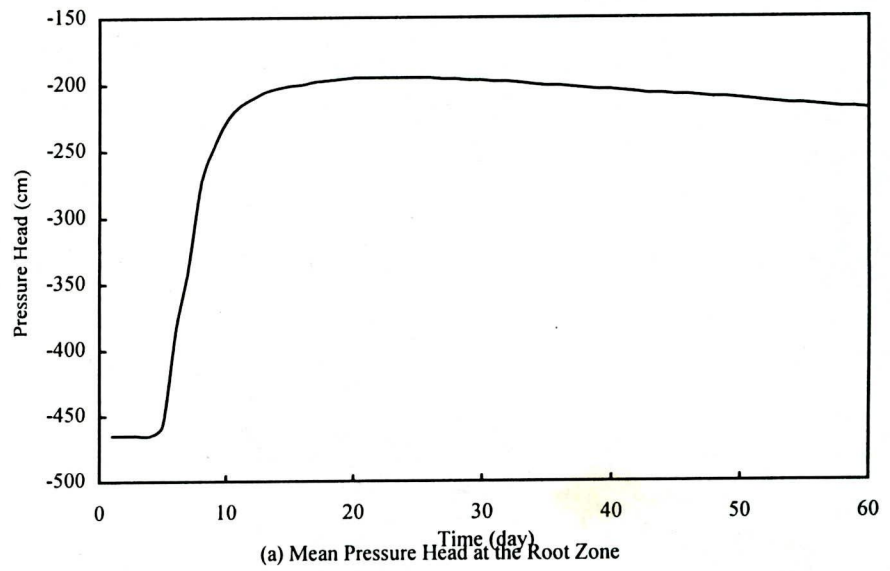


Fig. 7.12b : Simulated mean pressure head (cm) and bottom flux (cm², +ve downward) for Example 2

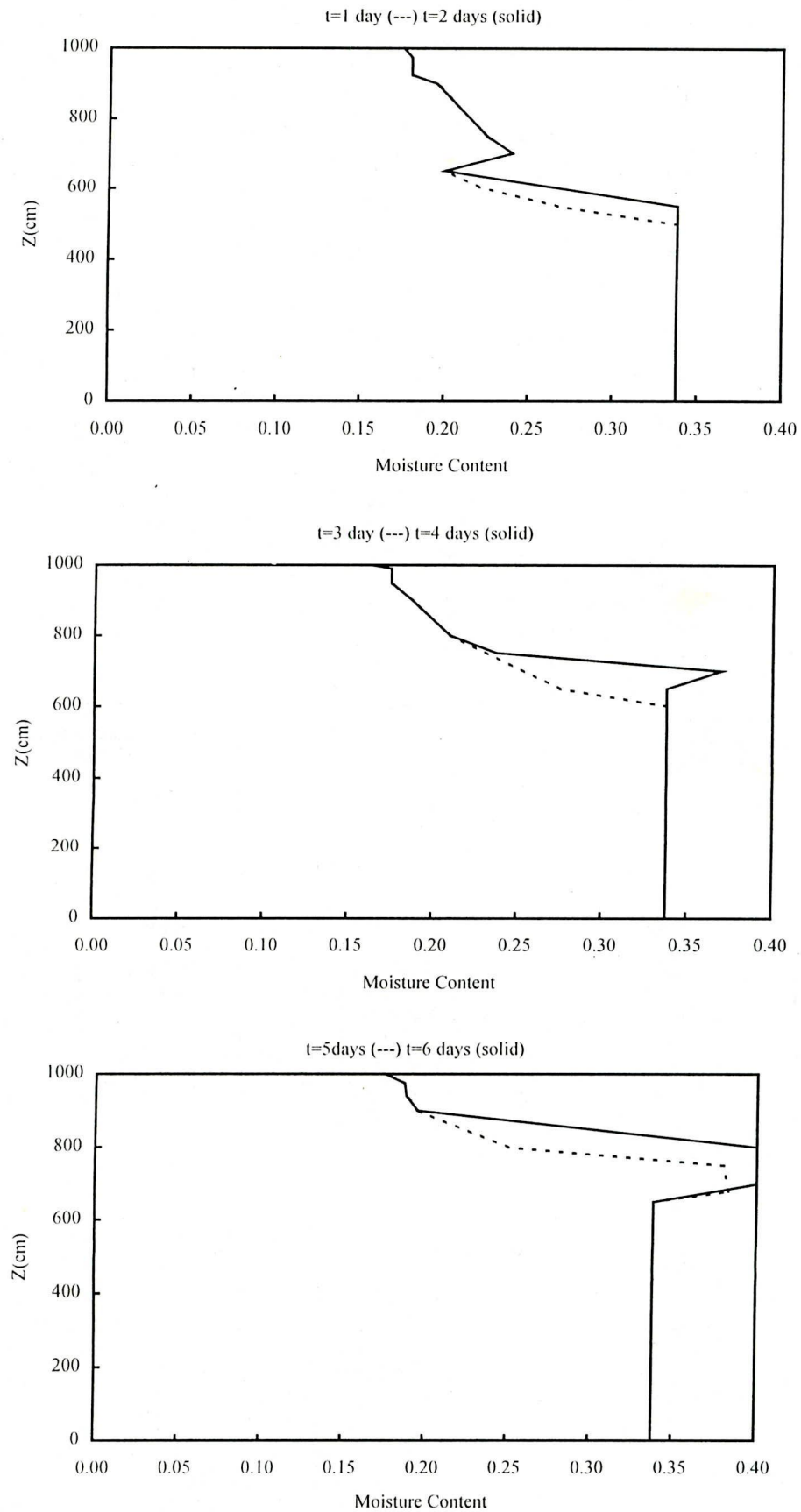


Fig. 7.13a : The simulated volumetric moisture content profiles at the end of days indicated (Example 2)

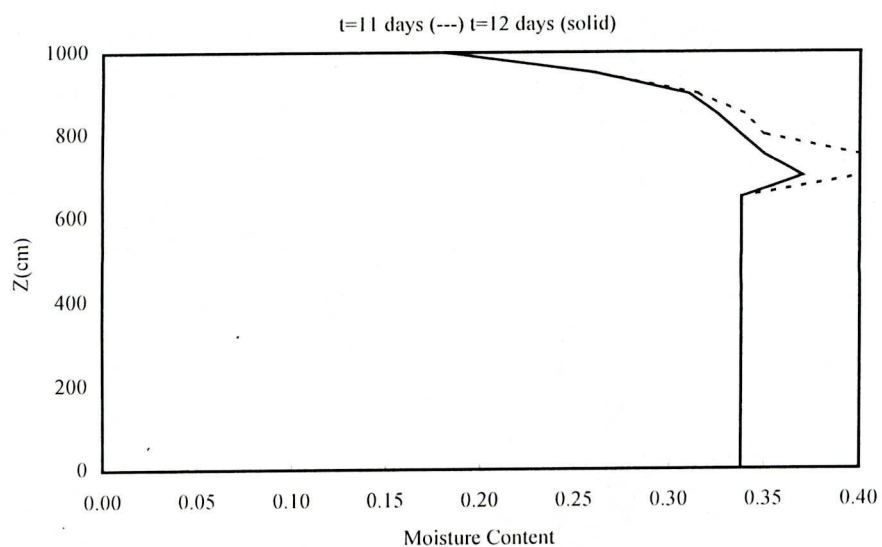
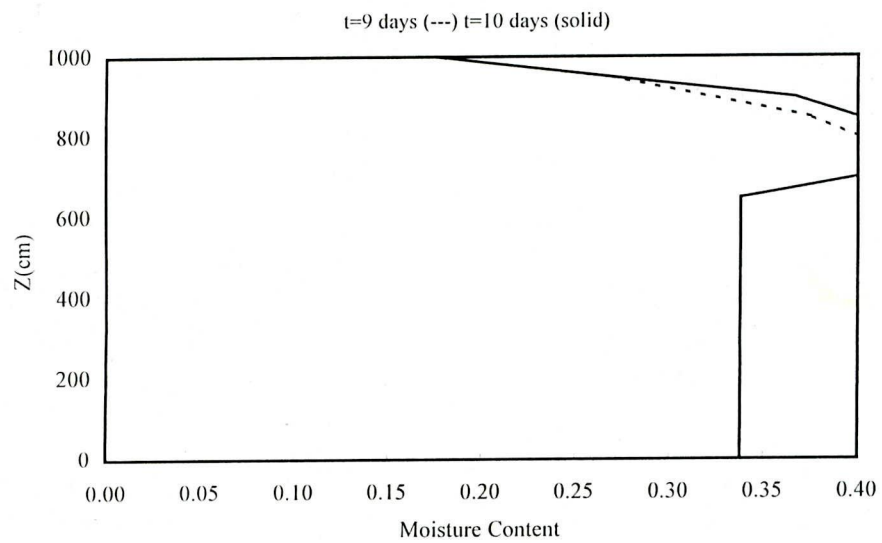
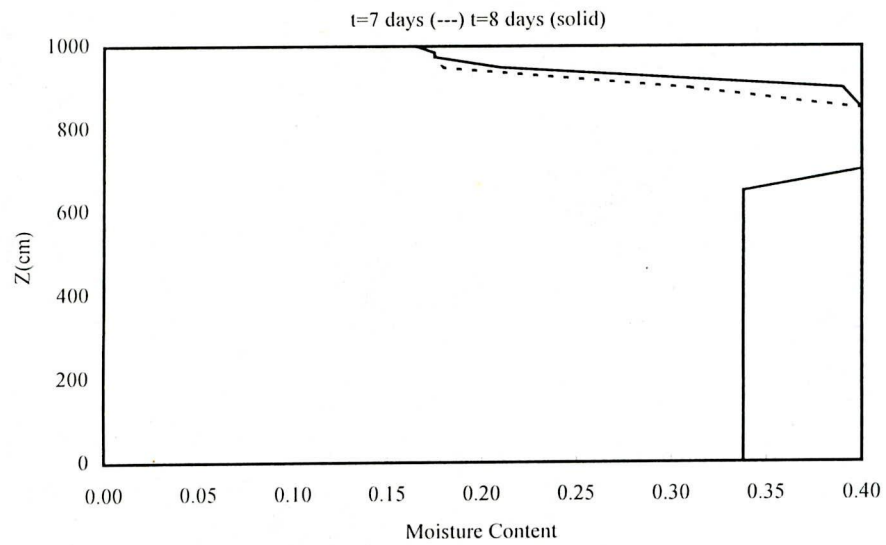


Fig. 7.13b : The simulated volumetric moisture content profiles at the end of days indicated (Example 2)

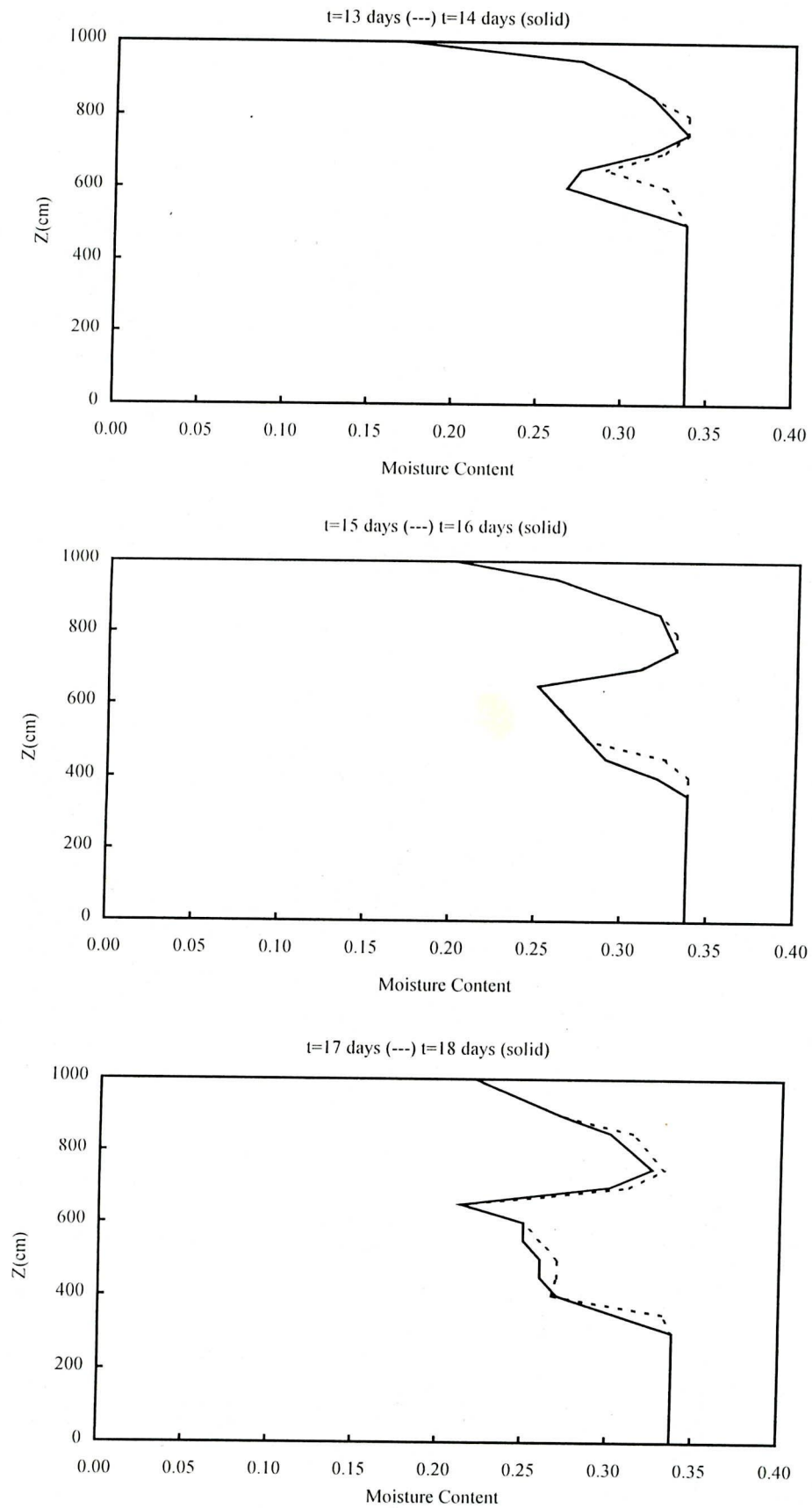


Fig. 7.13c : The simulated volumetric moisture content profiles at the end of days indicated (Example 2)

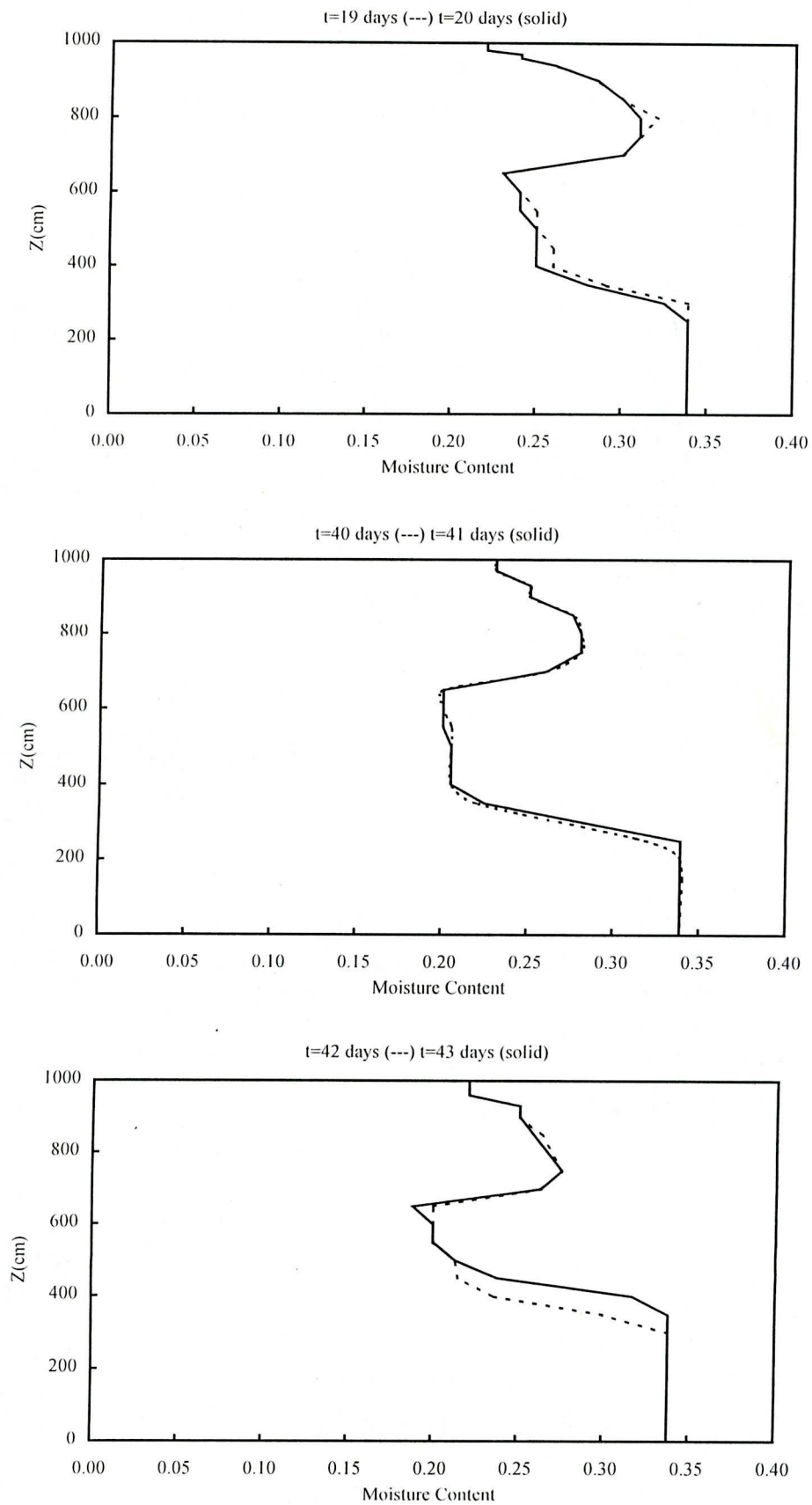


Fig. 7.13d : The simulated volumetric moisture content profiles at the end of days indicated (Example 2)

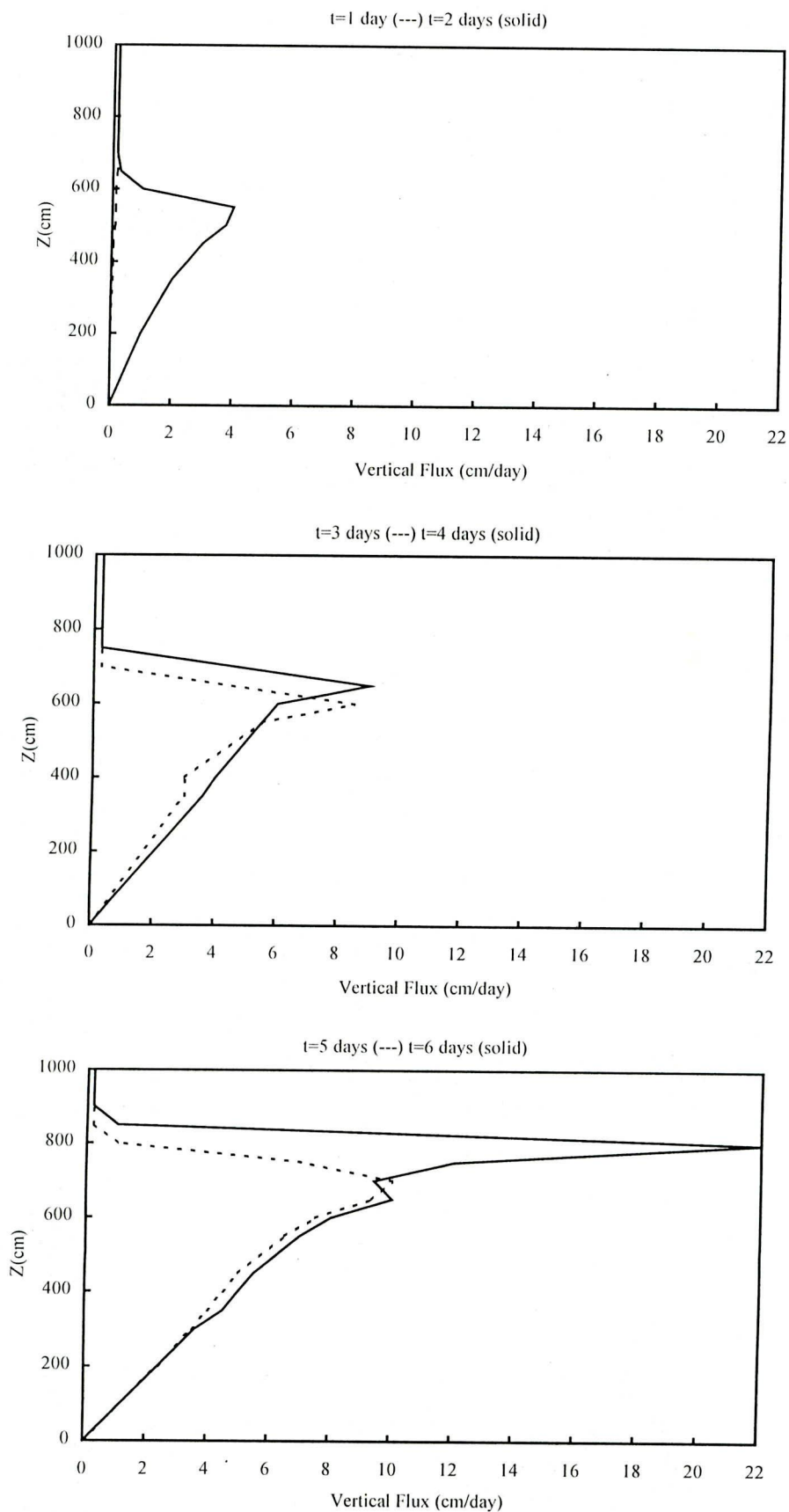


Fig. 7.14a : The simulated vertical darcian flux profiles (+ve upward, -ve downward) at the end of days indicated (Example 2)

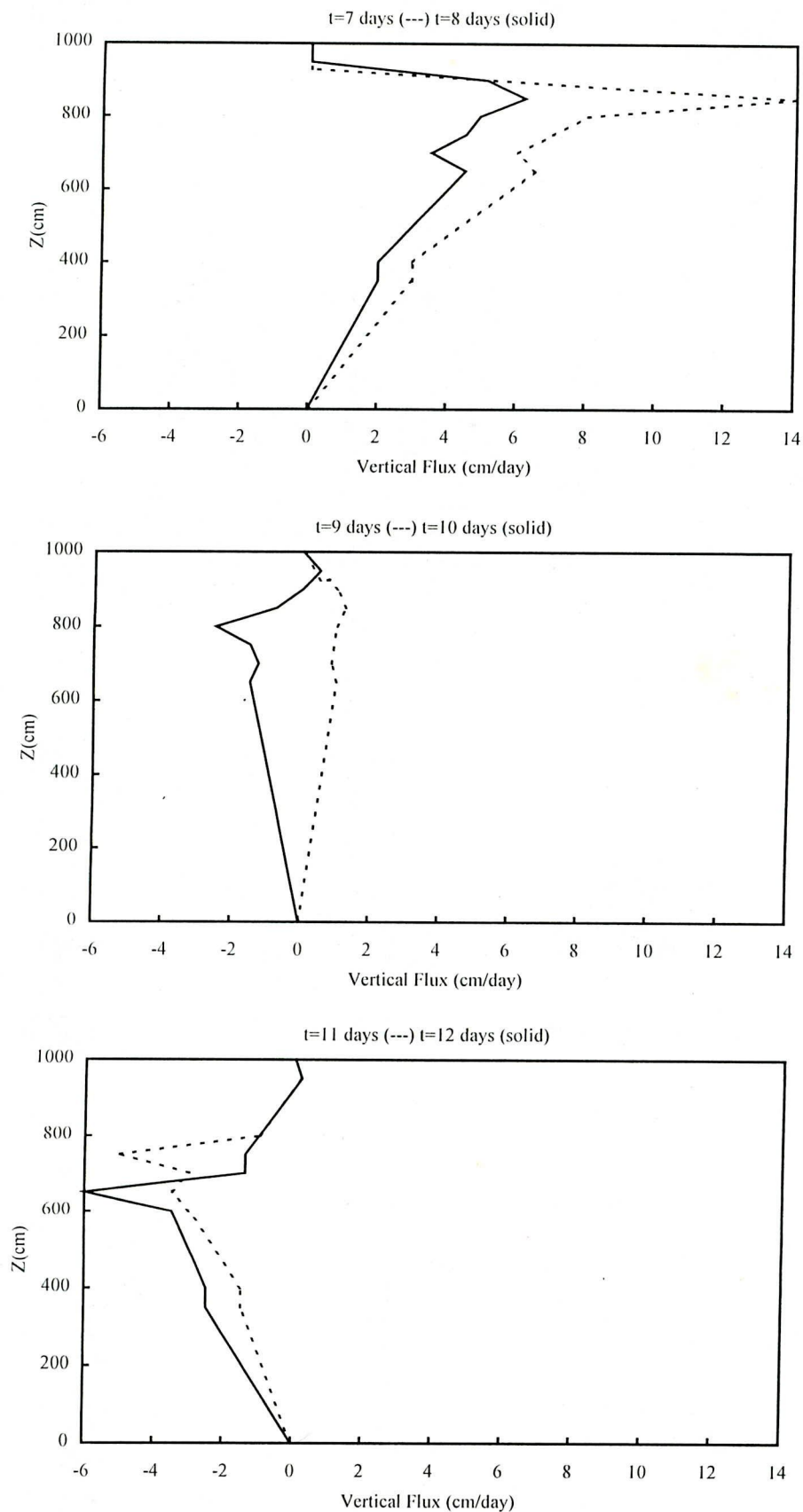


Fig. 7.14b : The simulated vertical darcian flux profiles (+ve upward, -ve downward) at the end of days indicated (Example 2)

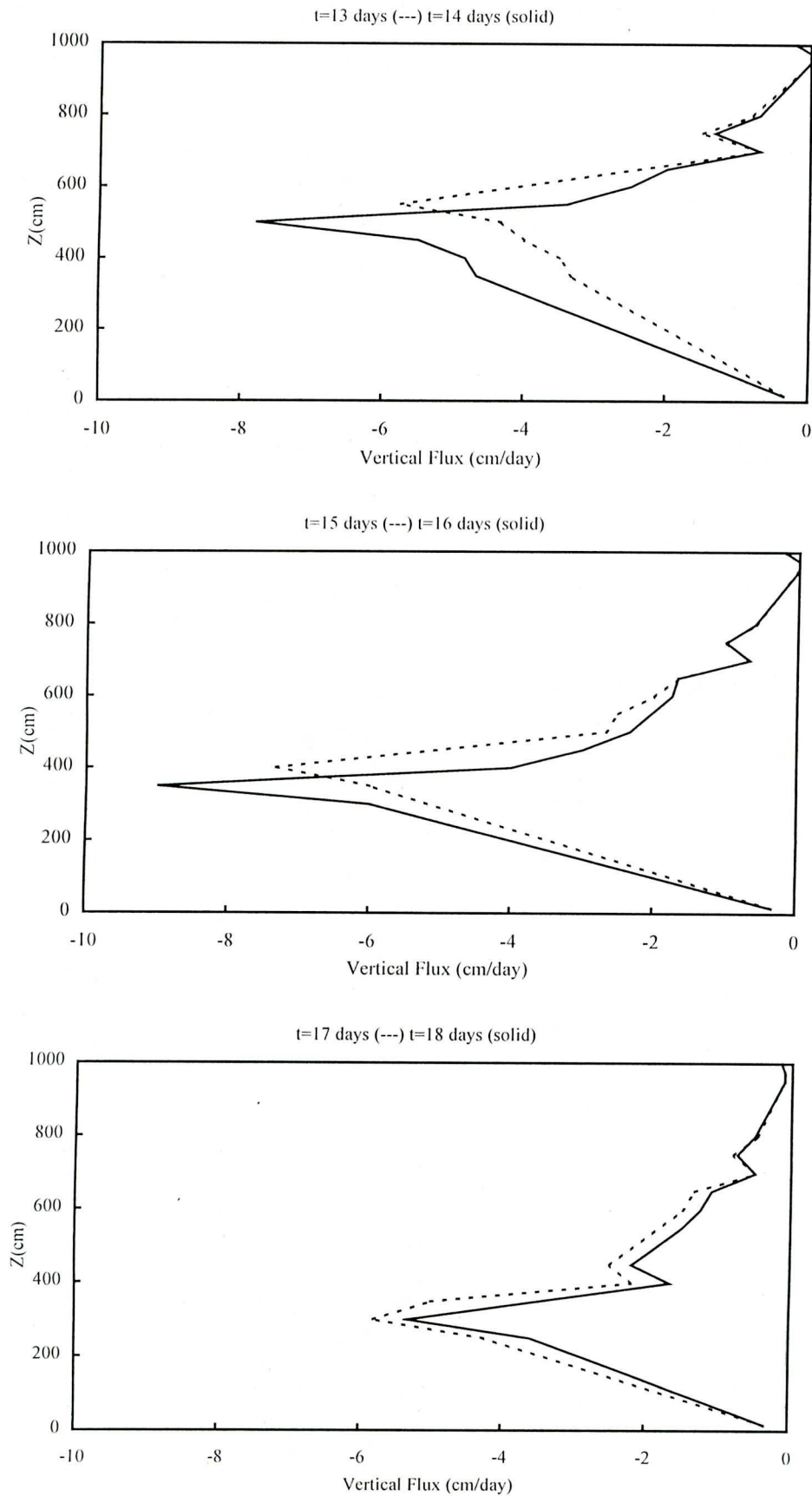


Fig. 7.14c : The simulated vertical darcian flux profiles (+ve upward, -ve downward) at the end of days indicated (Example 2)

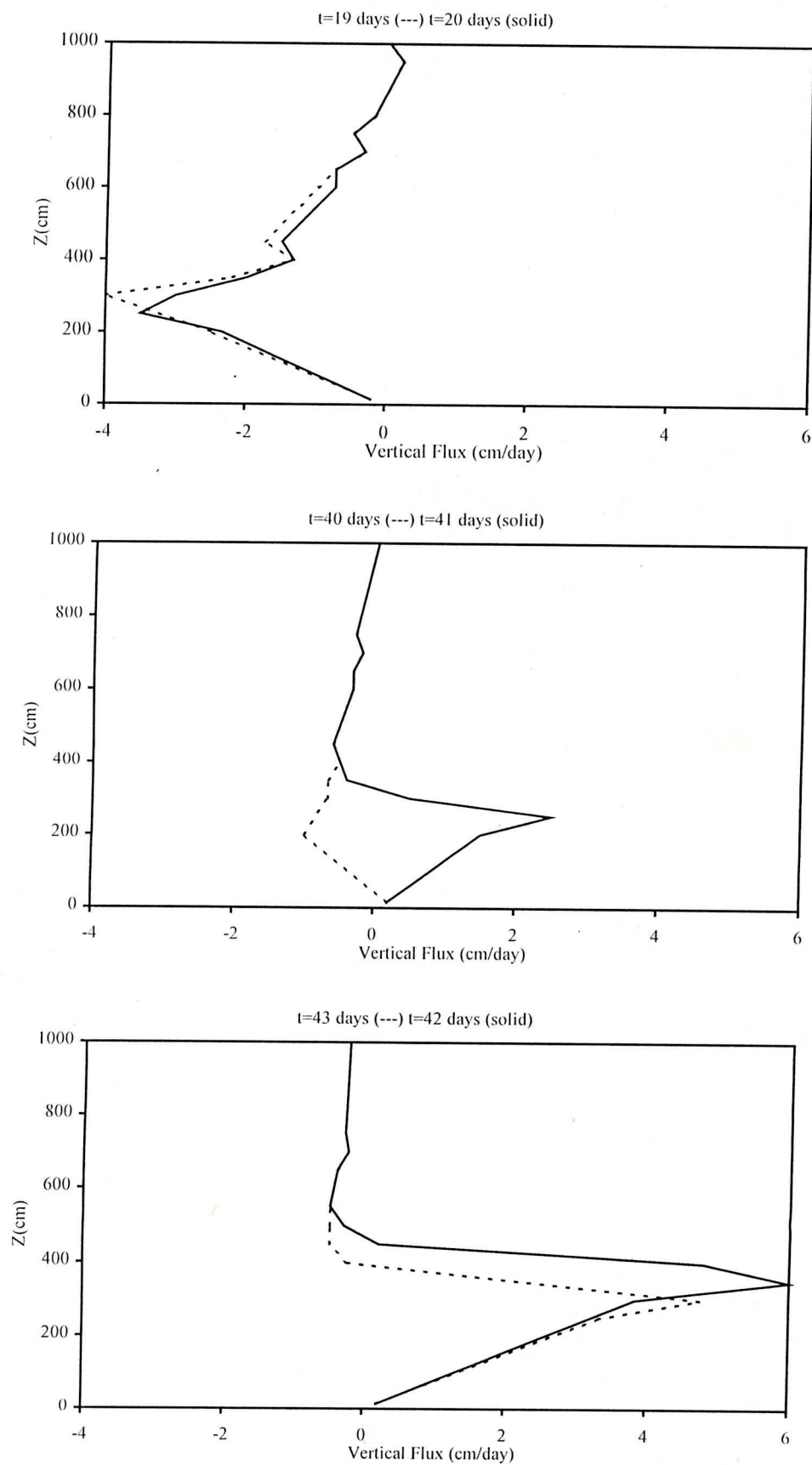


Fig. 7.14d : The simulated vertical darcian flux profiles (+ve upward, -ve downward) at the end of days indicated (Example 2)

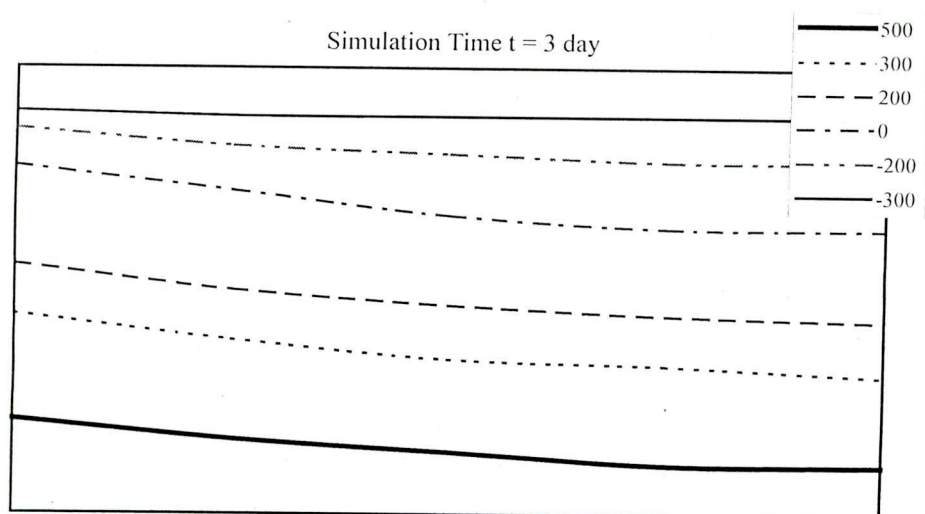
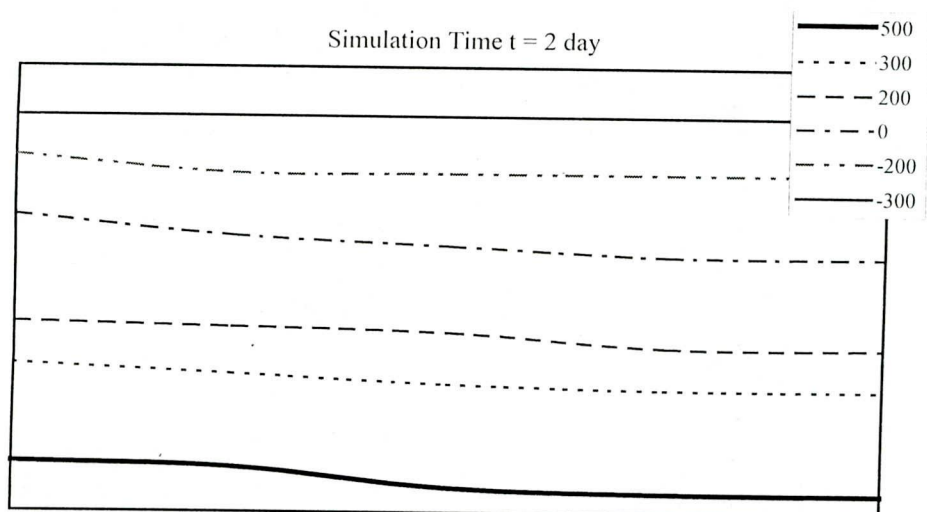
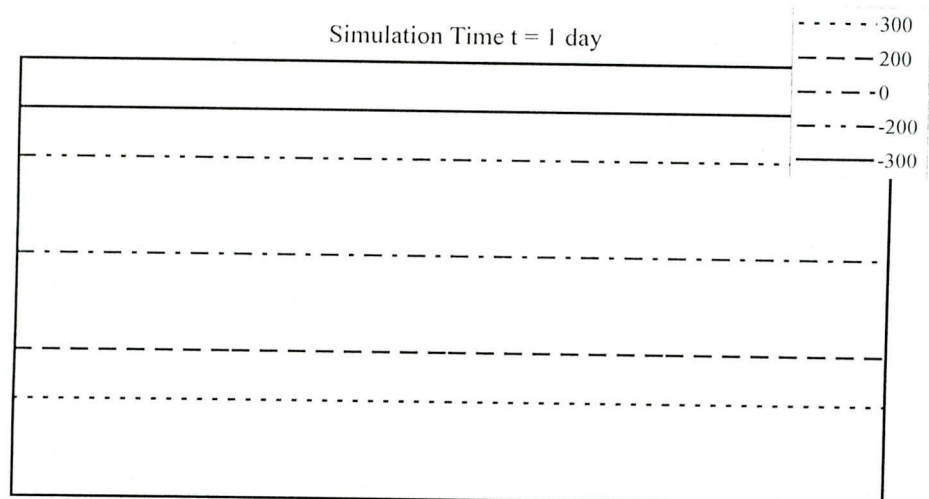


Fig. 7.15a : The simulated contours of the pressure head (in cm) corresponding to instants at the end of the days indicated (Example 2)

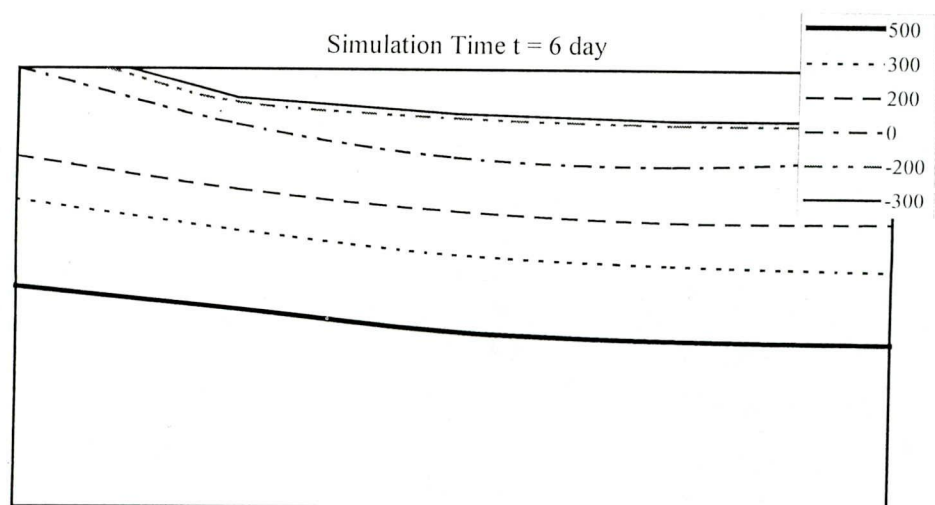
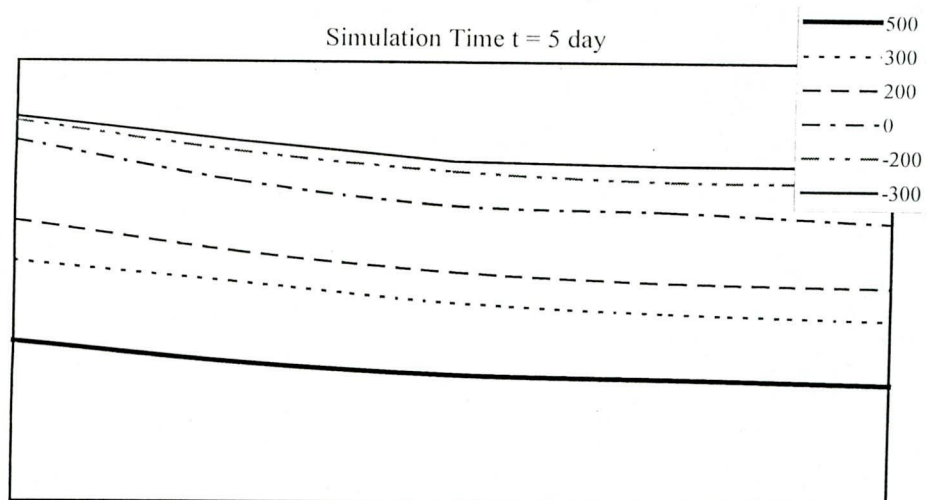
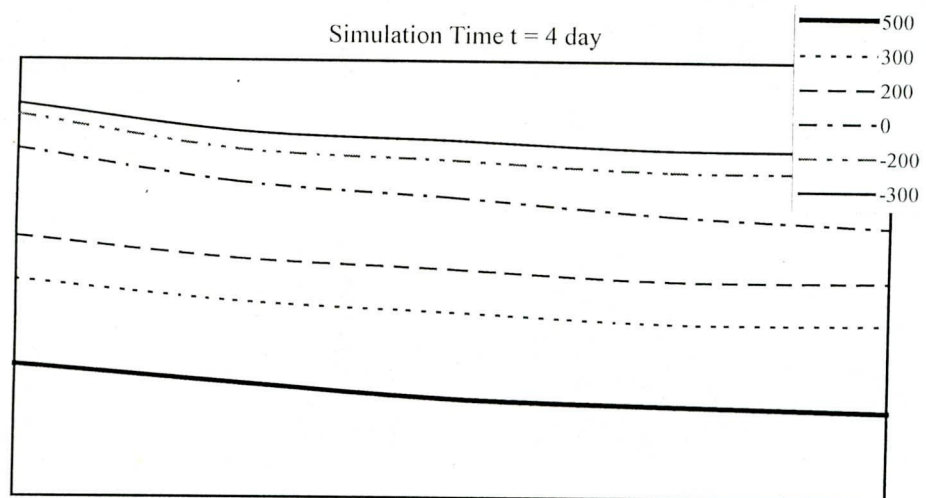


Fig. 7.15b : The simulated contours of the pressure head (in cm) corresponding to instants at the end of the days indicated (Example 2)

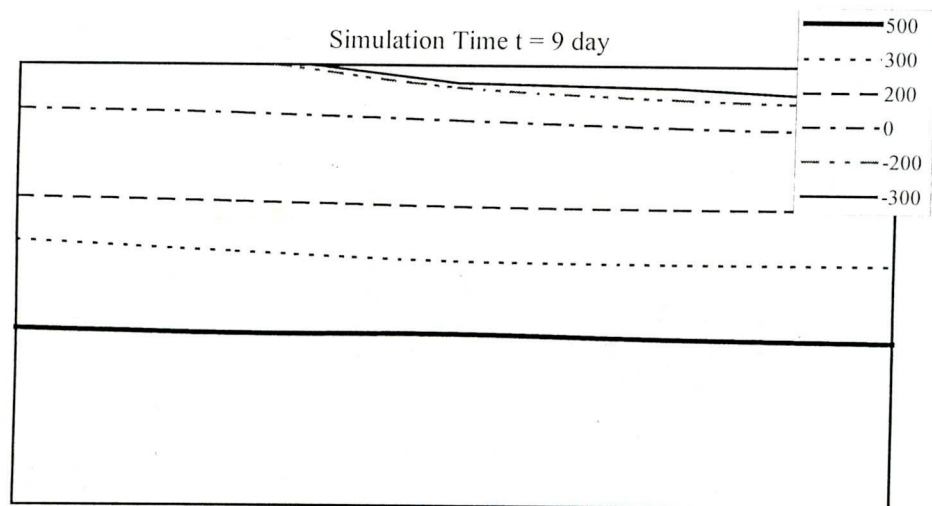
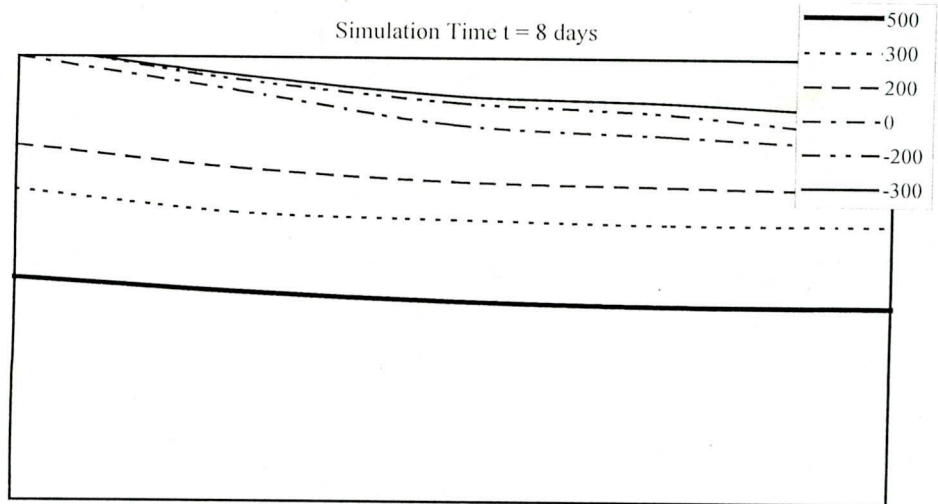
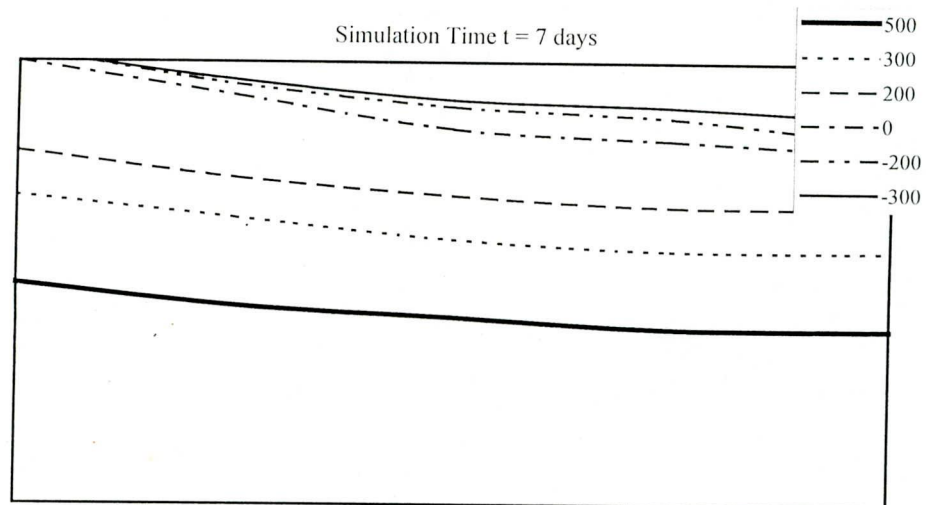


Fig. 7.15c : The simulated contours of the pressure head (in cm) corresponding to instants at the end of the days indicated (Example 2)

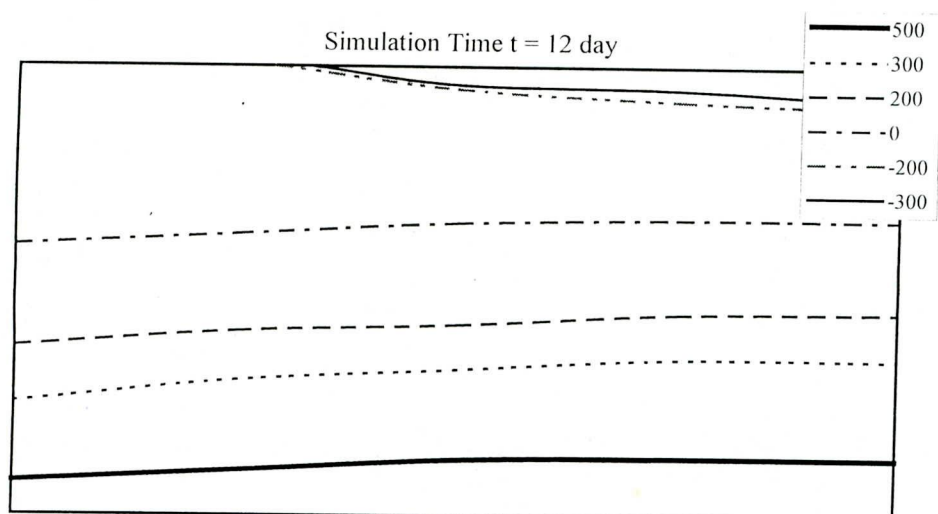
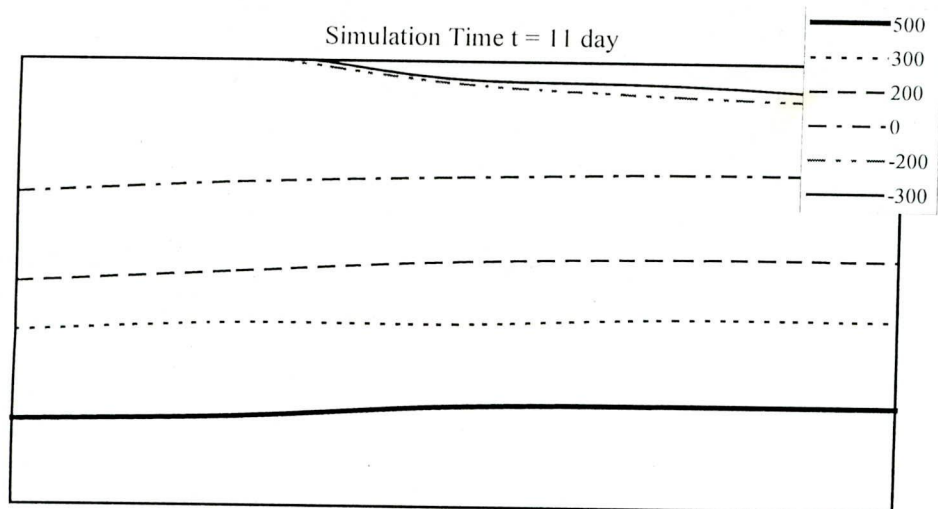
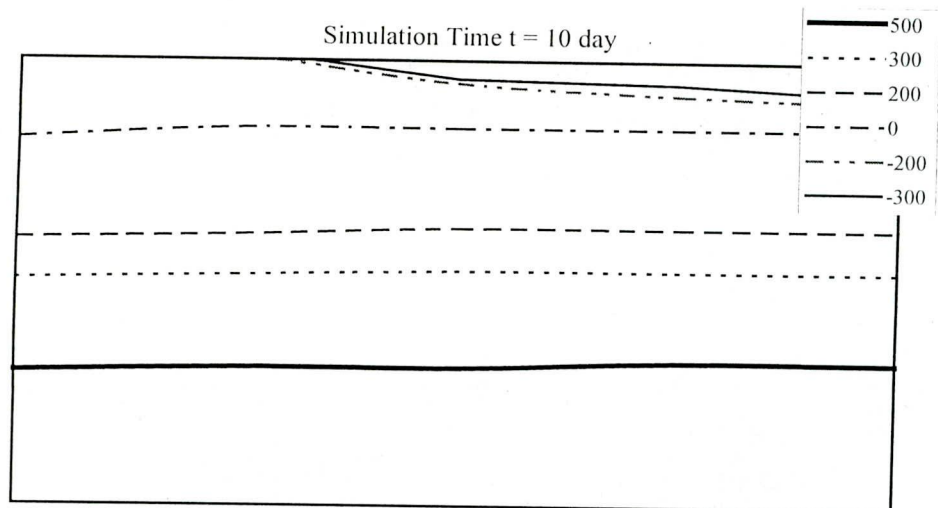


Fig. 7.15d : The simulated contours of the pressure head (in cm) corresponding to instants at the end of the days indicated (Example 2)

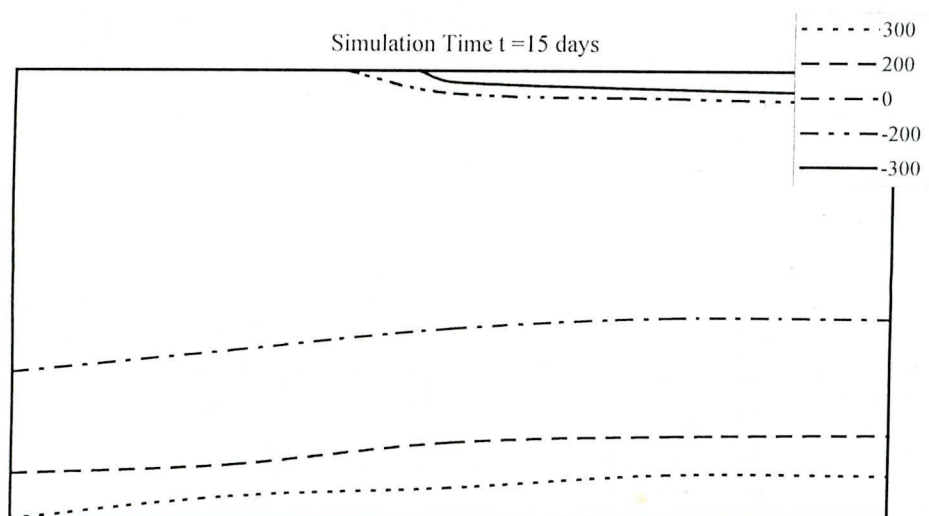
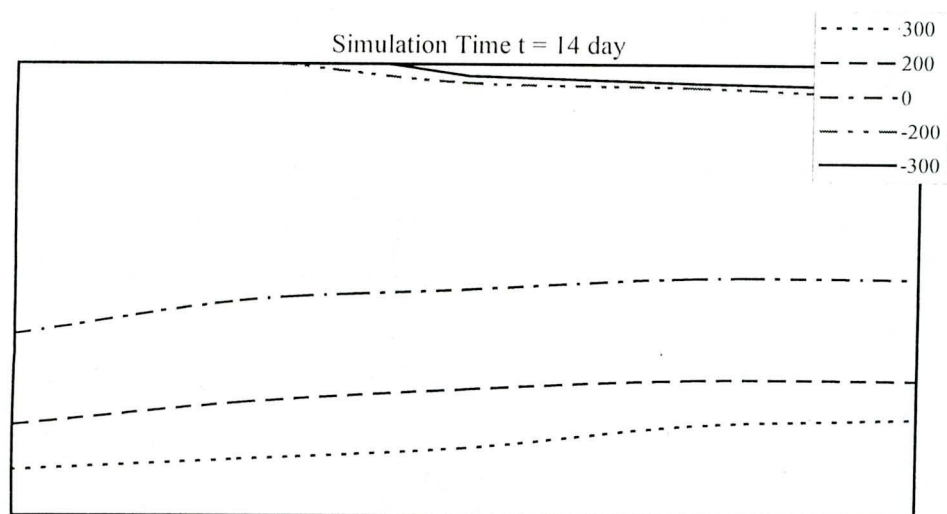
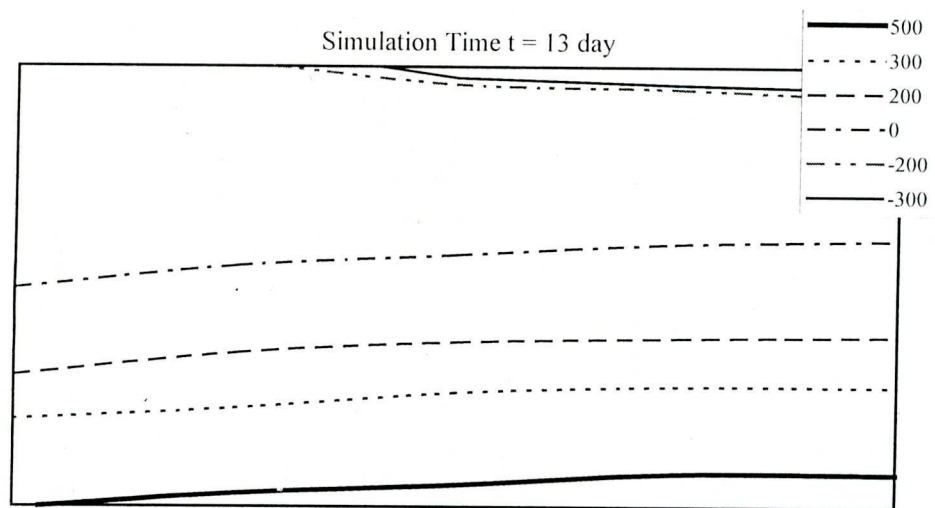


Fig. 7.15e : The simulated contours of the pressure head (in cm) corresponding to instants at the end of the days indicated (Example 2)

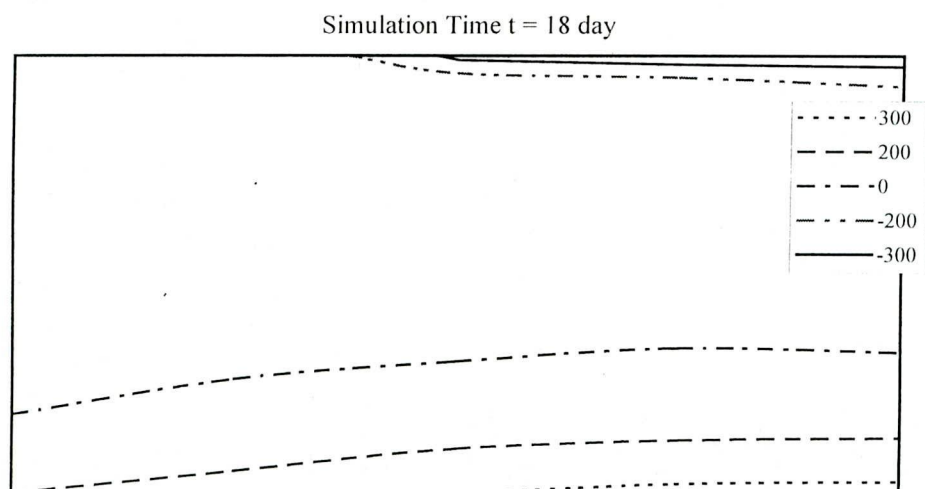
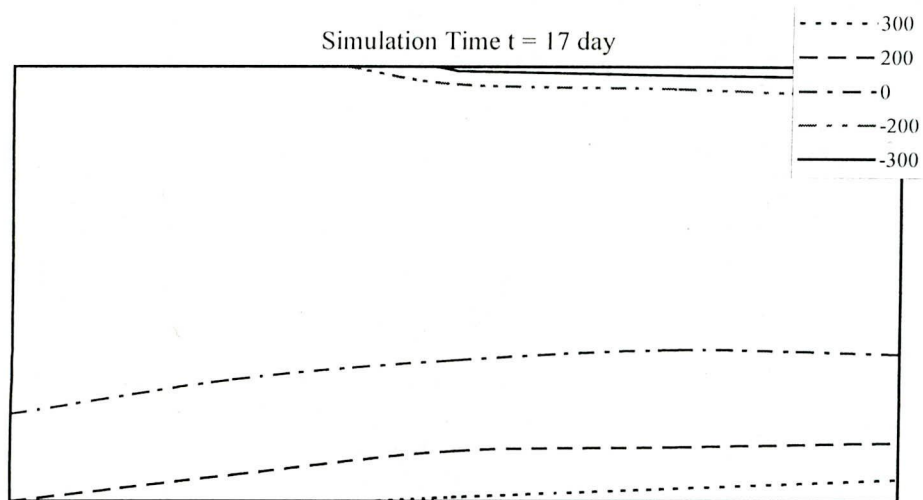
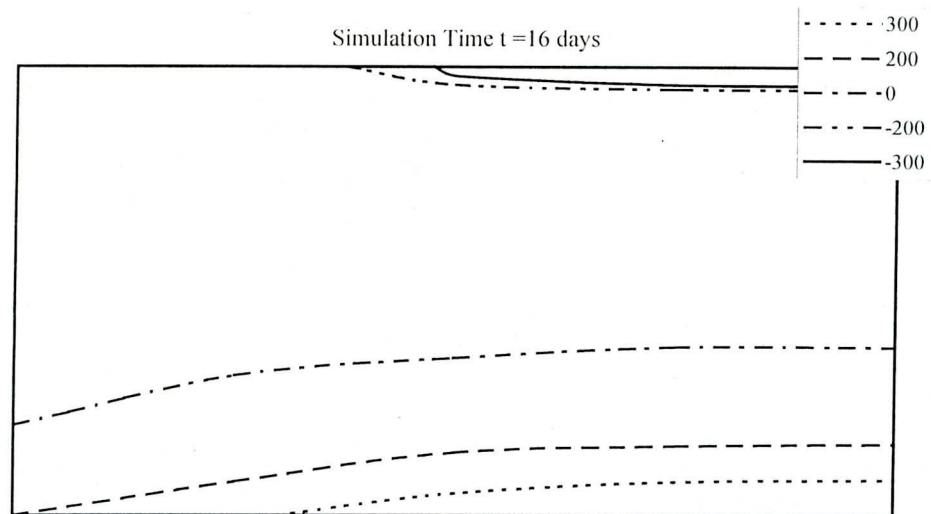


Fig. 7.15f : The simulated contours of the pressure head (in cm) corresponding to instants at the end of the days indicated (Example 2)

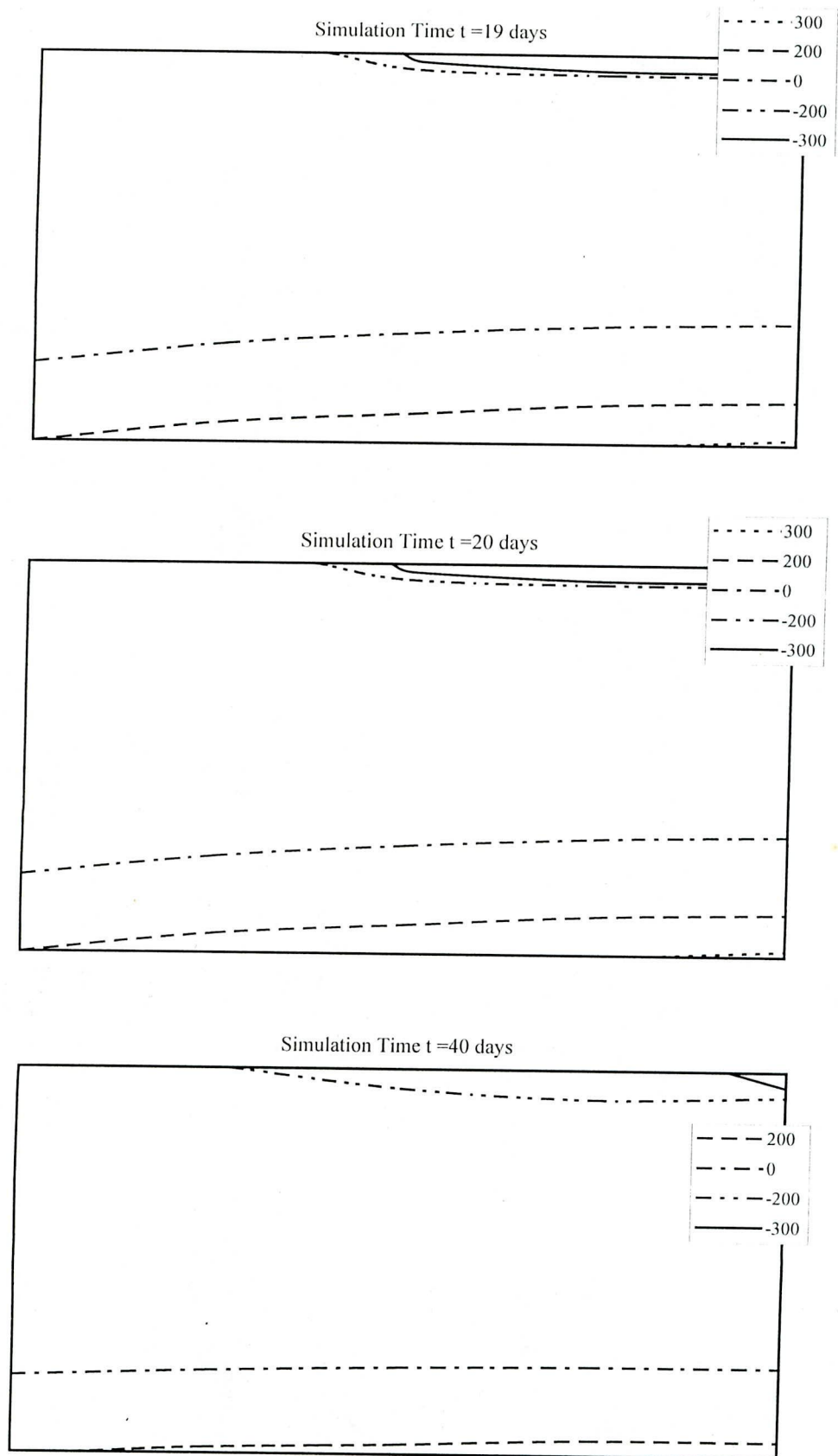


Fig. 7.15g : The simulated contours of the pressure head (in cm) corresponding to instants at the end of the days indicated (Example 2)

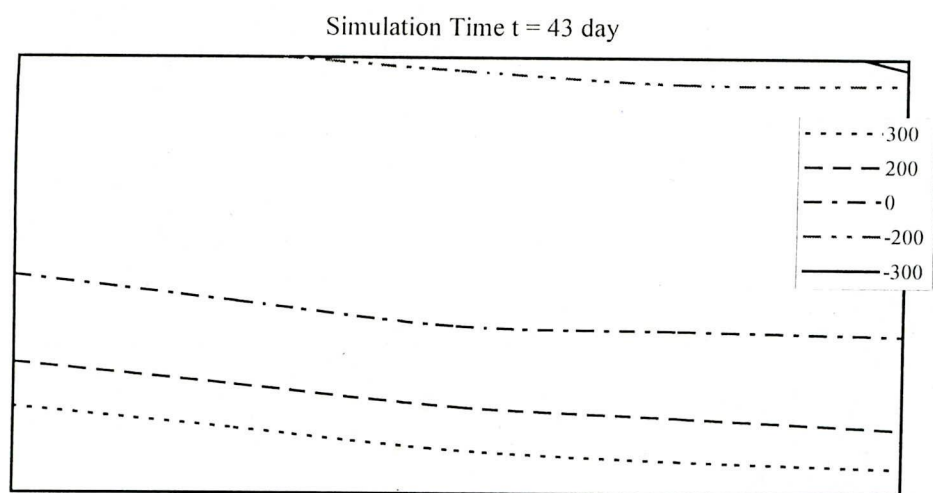
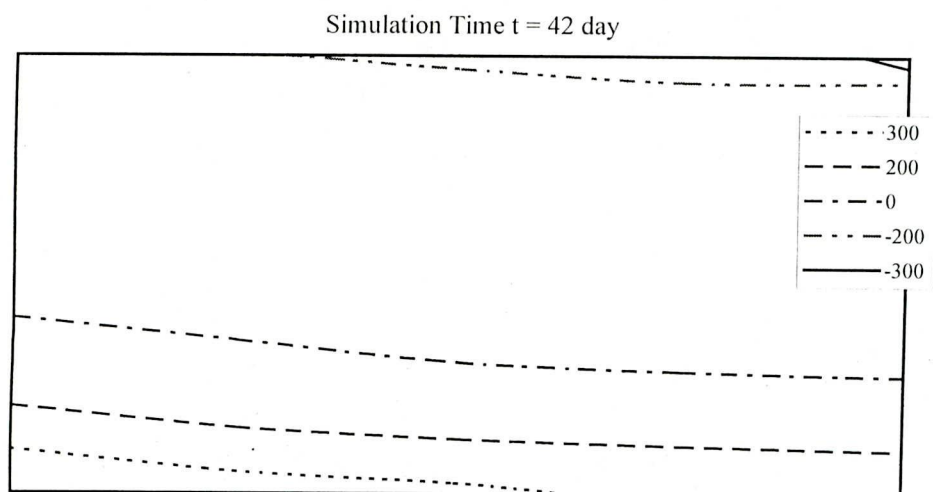
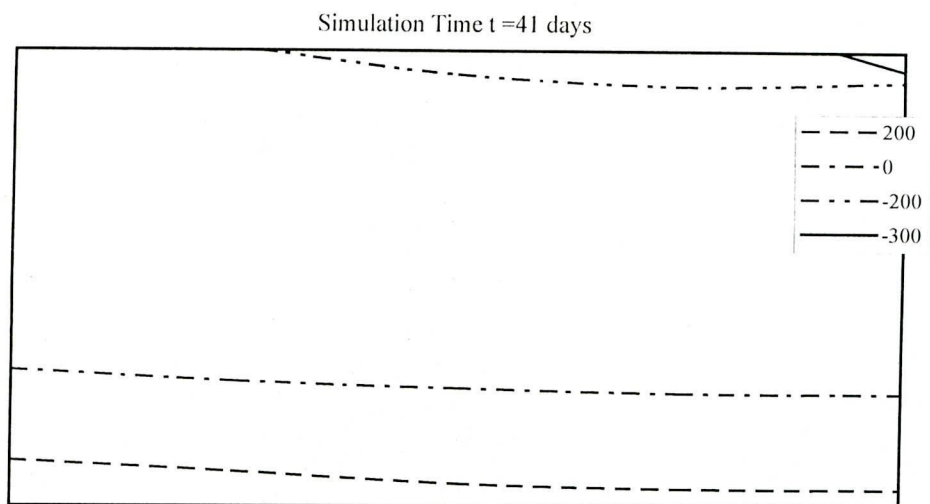


Fig. 7.15h : The simulated contours of the pressure head (in cm) corresponding to instants at the end of the days indicated (Example 2)

7.5 Simplified River Bank Situation

Two-dimensional flow domain (Fig. 7.16b) is discretized in two different ways producing a coarse grid with approximately equilateral elements (Example 3A, fig. 7.16a) and a fine grid with elements that are equilateral in the lower (saturated) part but had horizontal dimensions much larger (up to 10 times) than vertical ones in the upper part of the flow region (potentially unsaturated), to allow for the expected steep vertical gradients of pressure head in the unsaturated zone (Example 3B, Fig. 7.17). The results of simulation are presented below in graphical form. Both subexamples are fed by atmospheric and river stage data shown in graphs 7.18a and 7.18b. The results of simulation are discussed in article 7.5.3.

7.5.1 Example 3A: Flow domain discretized with Coarser Grid

The whole flow domain of this example problem is divided into almost equidimensional quadrilateral and triangular elements (Fig. 7.16a). The equidimensionality of elements should have favorable effect in minimizing numerical errors (according to Šimunek et al., 1992, p. 39). The total of 214 nodal points are numbered sequentially from 1 to NumNP (total number of nodes) by proceeding along each transverse (horizontal) line in the same direction. Elements (217 in total) are numbered in a similar manner. Maximum number of nodes along a transverse line, IJ , is 25, which determines the effective size of the finite element matrix (i.e., its bandwidth).

Boundary conditions are summarized in Fig. 7.16b. Dirichlet boundary condition ($Kode(n)=1$) with variable pressure head determined from the river water level is set on the river bottom whereas a variable seepage face ($Kode(n)$ whether 1 or 2, adjusted automatically by the subroutine) is assumed on the vertical river bank. Variable atmospheric boundary condition (with $Kode(n)$ either 4 or 1, adjusted also automatically) is defined on the horizontal soil surface allowing for accumulation of rainwater without surface outflow. Only physical evaporation from soil surface is assumed, transpiration is set to zero (bare soil is assumed) but formally, root zone

depth is set to 100 cm to obtain average pressure head in this layer as standard output. Initial ground water level is set equal to the initial water in the river (7 m above the river bottom) and initial pressure heads in the whole flow domain are adjusted as being in equilibrium with this ground water table. Parameters of the deep drainage boundary condition are taken over from Example 2 of Simunek et al. (1992), similarly as in all other examples in this thesis.

Basic simulation data and soil hydraulic properties for this subexample are summarized in the following tables 7.4a, 7.4b and 7.4c. The discussion of results in comparison with the results from the finer grid (Example 3B) are given in article 7.5.3.

Table 7.4a : Basic Information

Type of Flow System	Two dimensional river bank situation, coarser grid
Data source	Hypothetical daily rainfall and river water level, monthly potential evaporation, no transpiration assumed.

Table 7.4 b : Input Hydraulic Parameters

Parameters	1st layer (31.5-40 m)	2nd layer (22-31.5 m)	3rd layer (0.0-22 m)
$\theta_s = \theta_m = \theta_k$	0.399	0.339	0.339
$\theta_r = \theta_a$	0.0001	0.0001	0.0001
$K_s = K_k$ (m/day)	0.2975	0.4534	0.5534
α (1/m)	1.740	1.390	1.300
n (-)	1.3757	1.6024	1.7024

Table 7.4c : Numerical Simulation Information

Time Information		
Initial time increment : dt		0.02 day
Minimum time increment : $\Delta t_{\min}$		1e-10 day
Maximum time increment : $\Delta t_{\max}$		0.50 day
Iteration Information		
Maximum number of iteration : Max It		20
Absolute pressure head tolerance : Tol		0.01 cm
Some logical switch setting		
Atmospheric boundary : AtmInf		.true.
Seepage face : SeepF		.true.
Variable seepage face : Vseep		.true.
Variable atmospheric boundary : Vatms		.true.

7.5.2 Example 3B: Flow Domain discretized with Finer Grid

The same flow domain as in the previous example (Fig. 7.16b) is discretized by a relatively finer grid (Fig. 7.17). The elements are euidimensional (quadrilateral) in the lower part of the domain and smaller, horizontally elongated rectangles in the upper part of it, getting finer toward the soil surface where large vertical hydraulic gradients and nonlinear unsaturated flow are expected. This provision corresponds to the general requirement for finer grid in the unsaturated zone and in the zones of large gradients but stands in contradiction with another requirement for equidimensional elements "to decrease numerical errors" (Simunek et al., 1992, p. 39). It is not possible to comply with both requirements without increasing the number of elements drastically. The total of 1962 nodal points and 1896 elements are numbered sequentially by proceeding along each transverse (horizontal) line in the same direction. In this example problem, the maximum number of nodal point along a transverse direction is 41.

Boundary and initial conditions are the same as in previous example. Basic simulation data and soil hydraulic properties for this subexample are summarized in the

following tables 7.5a, 7.5b and 7.5c. The discussion or results in comparison with the results from the coarser grid (Example 3A) are given in article 7.5.3.

Table 7.5a : Basic Information

Type of Flow System	Two dimensional river bank situation, finer grid
Data source	Hypothetical daily rainfall and river water level, monthly potential evaporation, no transpiration assumed.

Table 7.5 b : Input Hydraulic Parameters

Parameters	1st layer (30.25-40 m)	2nd layer (20.5-30.25 m)	3rd layer (0.0-20.5 m)
$\theta_s = \theta_m = \theta_k$	0.399	0.339	0.339
$\theta_r = \theta_a$	0.0001	0.0001	0.0001
$K_s = K_k$ (m/day)	0.2975	0.4534	0.5534
α (1/m)	1.740	1.390	1.300
n (-)	1.3757	1.6024	1.7024

Table 7.5c : Numerical Simulation Information

Time Information	
Initial time increment : dt	0.02 day
Minimum time increment : $\Delta t_{\min}$	1e-10 day
Maximum time increment : $\Delta t_{\max}$	0.50 day
Iteration Information	
Maximum number of iteration : Max It	20
Absolute pressure head tolerance : Tol	0.01 cm
Some logical switch setting	
Atmospheric boundary : AtmInf	.true.
Seepage face : SeepF	.true.
Variable seepage face : Vseep	.true.
Variable atmospheric boundary : Vatms	.true.

7.5.3 Discussion of the Results

Figs. 7.19 and 7.23 present simulated mean pressure heads at the soil surface and in the (non-transpiring) root zone, as well as cumulative fluxes across both the atmospheric boundary and the bottom boundary, for coarse-grid and fine-grid example, respectively. The graphs reveal several surprising features which the present author has not yet been able to interpret fully: (i) suction on the soil surface is smaller (pressure is larger) than the average suction in the root zone, and sticks for a long time at about -200 m,

(ii) actual cumulative flux (cumulative infiltration) is in absolute value larger than the potential cumulative flux,

(iii) the difference in behavior between the coarse-grid subexample and the fine-grid one is substantial.

The time course of suction roughly responds to the development of weather, with rainy period from days 10 to 153, transitional period from days 154 to 214 and dry period from days 215 to 365. First dry spell that had a pronounced effect on soil suction came on the day 133. After that, last group of large rains (within the days 139-148) replenished soil water storage.

Beginning with about the day 150, the subexamples 3A and 3B behave differently: for example, the fine-grid subexample (3B) showed a pretty reaction upon last two groups of rainy days (on the days 188-191 and 198-200), while the coarse-grid (3A) is much more clumsy, reacting only on large and prolonged periods of either drought or rains.

Vertical scale (pressure head scale) on these figures is very compressed, reaching -800m at the bottom of the graph. In both subexamples the surface suction increases very rapidly to values about -200m, after which it almost stagnates and continues to increase only when a long dry spell occurs. This is however not connected with switching of the atmospheric boundary condition from Neuman-type condition to Dirichlet-type one at minimum pressure head at soil surface- $h_{critA(i)}$ (Simunek et al., 1992 p.41-42 and 78), because the value of $h_{critA(i)}$ is set in these examples to 10000

m. On the contrary, the average pressure head in the root zone (dashed lines in Figs. 7.19(a) and 7.23(a)) decreases (= suction increases) continuously without evident stagnation at the value -200 m or at any other value. This phenomenon remains unexplained. There are some indications that the value of the upper limit of pressure head at soil surface (value h_{CritS} , set in this example equal to zero) can influence this strange behaviors of the surface pressure head.

The time course of cumulative surface flux (evaporation minus infiltration) in Figs. 7.19(b) and 7.23(b) shows periods of intensive infiltration (steep, almost vertically decreasing parts of the curves), during some of which the dashed curves (actual fluxes) shift almost suddenly more downward from the solid curves (potential fluxes), i.e., actual infiltration is larger than the potential one. This water does not disappear through surface runoff but can infiltrate in the days following the high intensity rainfall. A similar situation is encountered also in Example 1 where, however, infiltration into saturated soil is drastically limited by the bottom-flux boundary condition. In the two-dimensional case, which we have now, infiltration can continue even if the soil is saturated because water can be discharged from the flow region through the left side, into the river. In the rest of the simulation period, the curves of actual and potential fluxes are practically parallel which indicates that evaporation from the soil surface is not reduced considerably even during the dry periods (despite of the high values of suction displayed on the graphs) and continued with potential rate. The above described phenomenon of increased infiltration due to surface water storage is more frequent with the finer-grid example (Fig. 7.23) while the coarse grid appeared again to be intensive to smaller inputs: only the two highest rainfall events (on the days 126 and 144) are able to provoke the effect. The cumulative bottom flux suffers from the same error as in Example 2 and must not therefore be taken serious. The influence of this error upon other results is probably small.

Figs. 7.22a to 7.22h and 7.26a to 7.26h present contours of the pressure head in the upper part of the flow region (the software available for plotting contours did not work well in other than rectangular regions). Almost hydrostatic equilibrium (corresponding to the initial water level 2 m above river bottom) persists in both

subexamples at the end of the day 10, because water level in the river is constant and precipitation is negligible. Further development on the days 26, 27 and 30 and 35 corresponds to the first remarkable period of intensive rains and increased water levels. the wetting front of infiltration rainwater is well visible. Coarser grid appears to be more stable but less sensitive as expected. For example, on the day 27, the finer grid signalizes saturated zone at the soil surface (note the presence of contour 0.0 in Fig. 7.26a) while the soil remains unsaturated when simulated with the coarser grid. On the day 35 (Figs. 7.22b and 7.26b), the wetting front is steeper and the capillary fringe more compressed with the finer grid (as, for example, at the upper contour 0.0 on the day 27, or later, at the same contour on the day 126) which witness about numerical instability, probably because of the elongated elements. The top event of the season is illustrated in the same figures (7.22b - d, and 7.26b -d) for days 124 to 127 and 130, with the subsequent effect of low water level and dry spell on the days 133 and 137. Water "attacks" our flow region both from the left and from above. The region is never completely saturated but is not far from saturation. The flood wave culminates on the day 126 and its peak reflects nicely within the flow region on the day 127. Subsequent drainage and drying of the soil profile is slow and is most visible at the upper left corner of the region (the effect of gravitational drainage is primary because the boundary conditions imposed do not allow evaporation from the side of the river bank). The differences between subexamples are visible: a strip of unsaturated soil persists in the middle part of the region with the coarser grid on the days 124 to 127 and the upper part of the region is saturated on the day 126 with the coarser grid, while the unsaturated zone is less expressed on these days with the finer grid but persists longer: the saturation of the upper part of the region is visible only on the day 127. It is however difficult to make conclusions from these differences. In the drainage stage (days 127 to 137), the coarser grid seems to be less stable (or simply too crude) and produces some peculiar contour patterns. Saturated zone is being drained quicker (the ground water table falls more rapidly) with the finer grid.

Finally, Figs. 7.20a & 7.20b and 7.24a & 7.24b contain vertical profiles of the volumetric moisture content and of the vertical component of volumetric flux (darcian flux). The profile is situated 12m from the edge of the river bank, i.e. in the middle of

the subregion for which the contours are plotted, and vertically comprises, just this subregion. The soil is homogeneous within most of this subregion. The strips in which soil properties are changing are between 30 and 33 m for the coarser grid between 30 and 30.5 m for the finer grid. The effect of this strip is visible mainly for the coarser grid. Progress of the wetting front can be well followed from the 26th to the 30th day. Drainage of the soil profile takes place between the 130th and 137th day. Differences between the grids are quantitative, not qualitative, as far as moisture content profiles are discussed. Much more pronounced are differences in the vertical flux profiles. Large downward fluxes during infiltration of rainwater are visible at the soil surface of the days 26 to 27. Combination of downward flux in the upper part of the region (due to rainfall infiltration) and of the upward flux in its lower part (due to flood wave) is distinctly visible on the day 126 (also 127 with the coarse grid). Low values of downward fluxes in the upper part of the region on the days 130 - 137 demonstrate how slow the drainage of soil profile is, in comparison with infiltration. the profiles of fluxes are again not much satisfactory. Large inaccuracies and inadequacies must be expected with them. Finer grid seems to be better.

FLOW DOMAIN 3

Flow system discretization with comparatively coarser grids

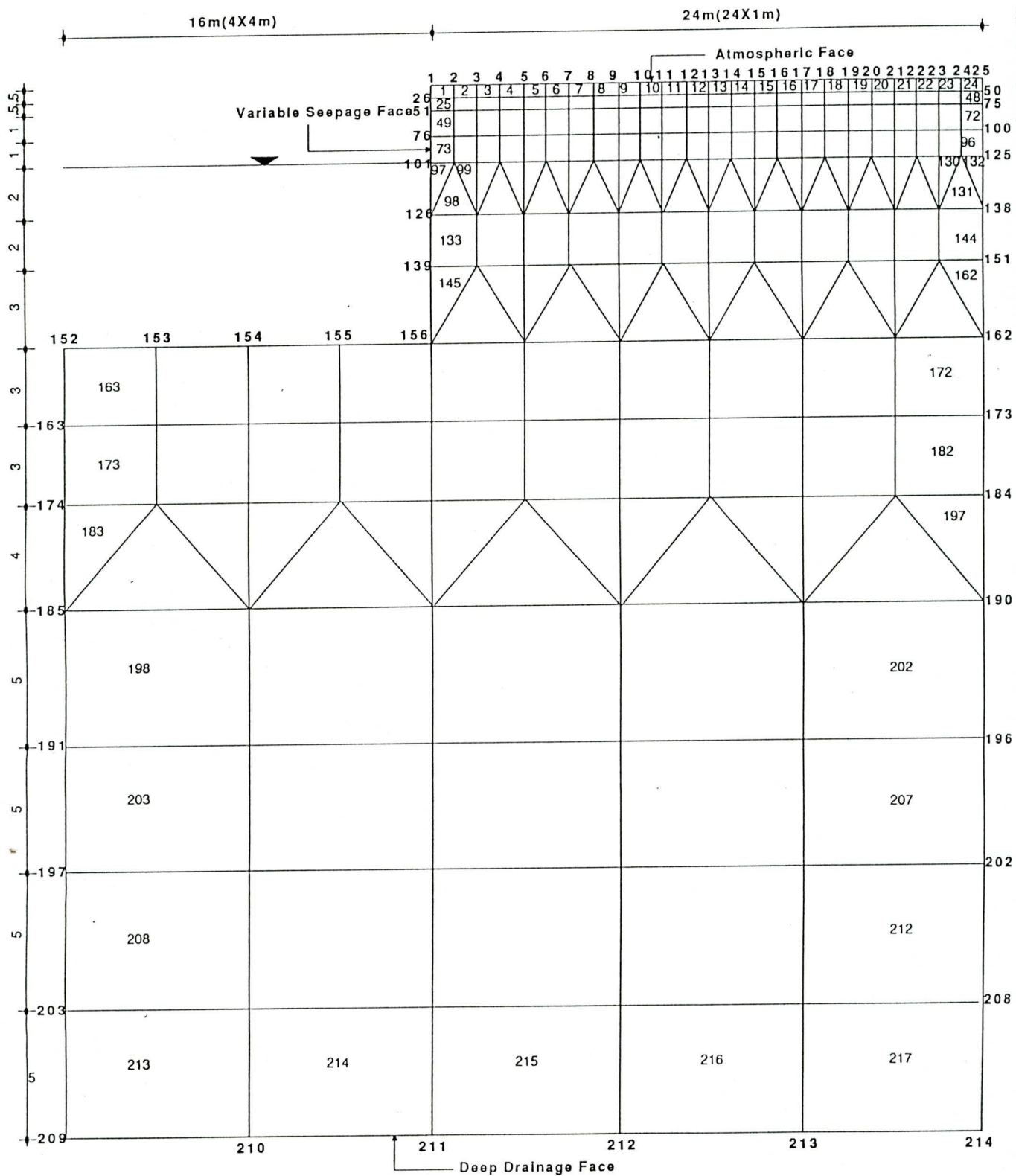


Fig. 7.16a : Finite element mesh for simplified river bank situation with coarser grid (Example 3A)

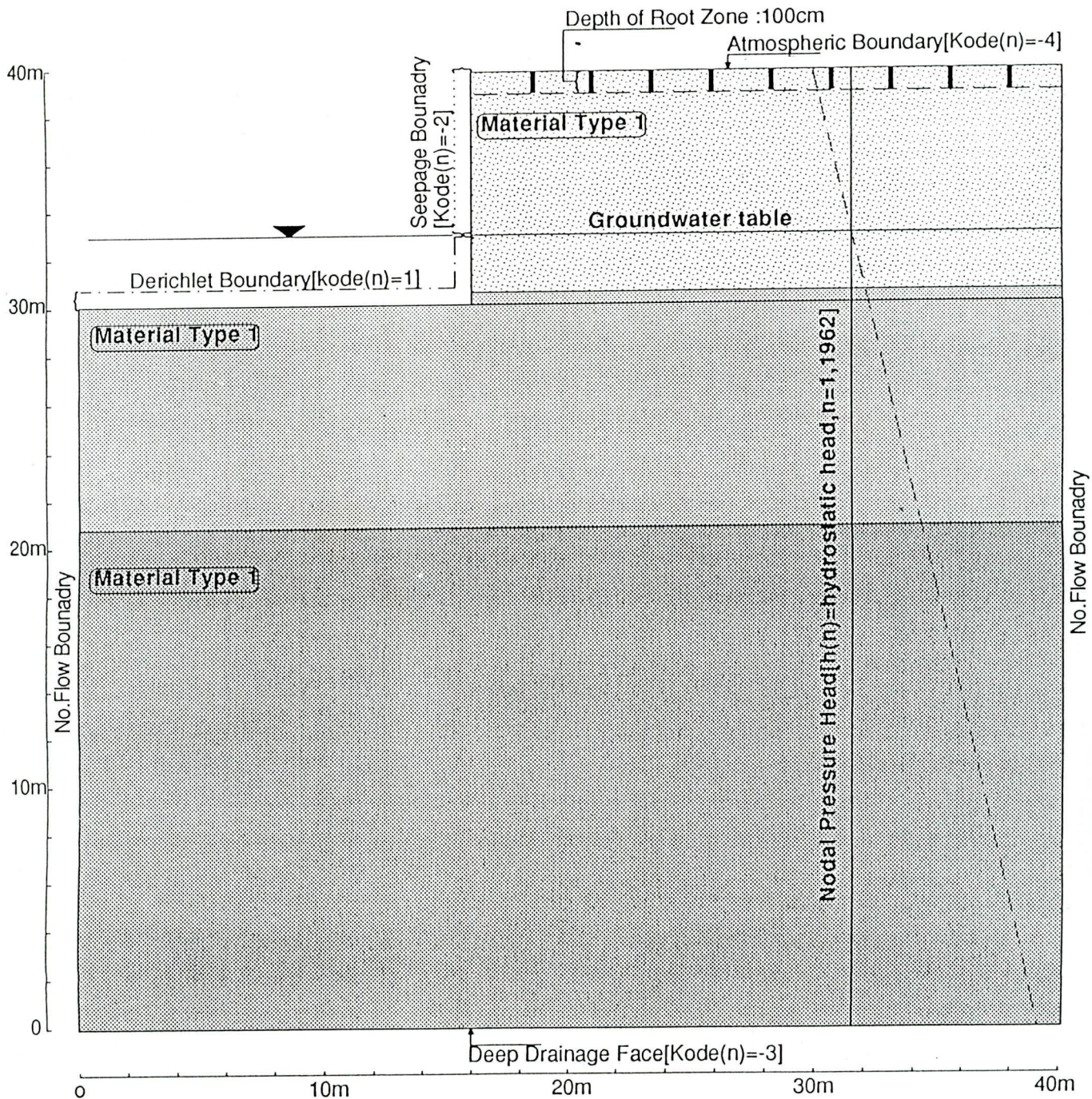


Fig. 7.16b : Flow system with initial and boundary conditions for Example 3A and 3B (simplified river bank situation)

FLOW DOMAIN 4

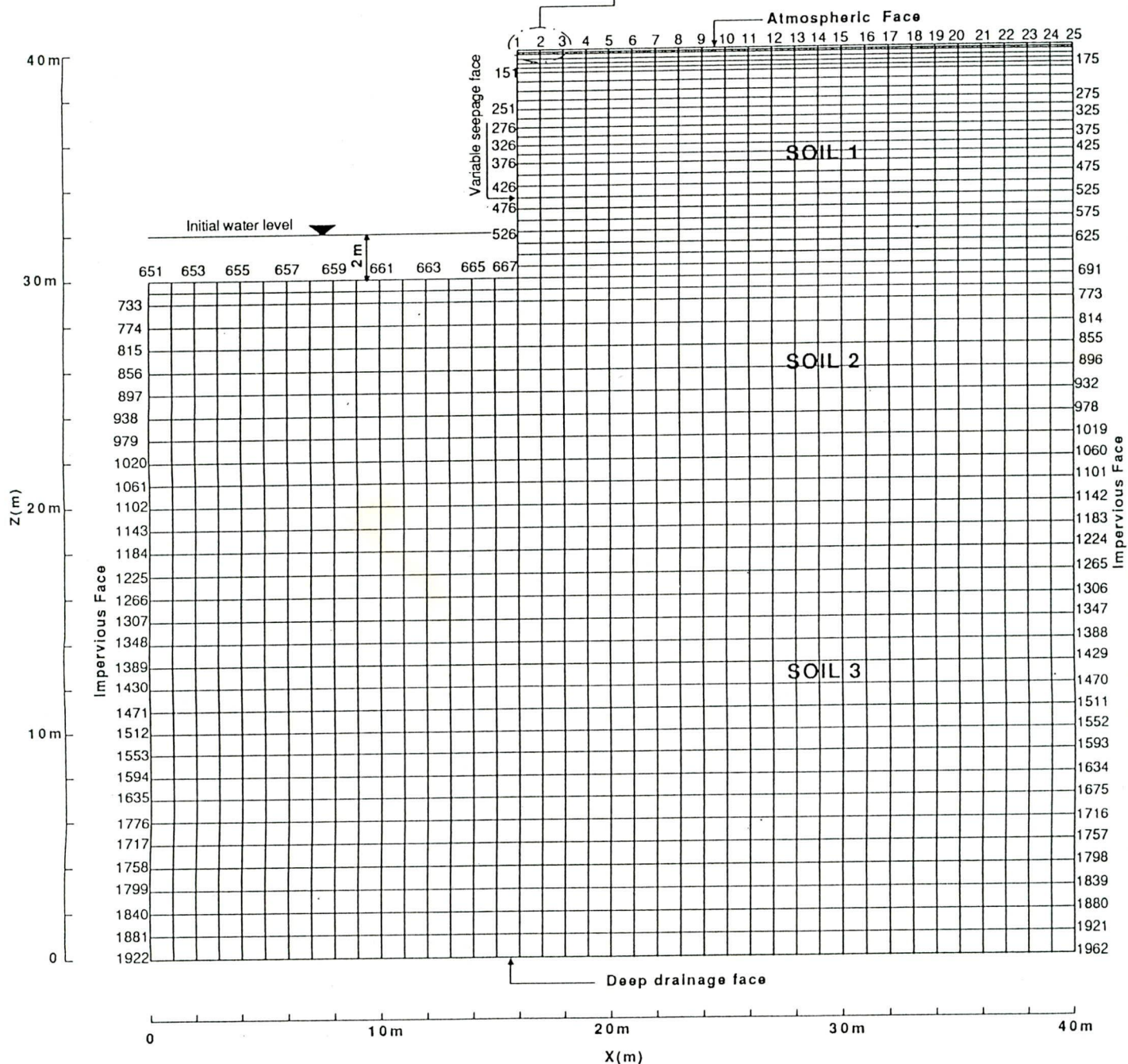
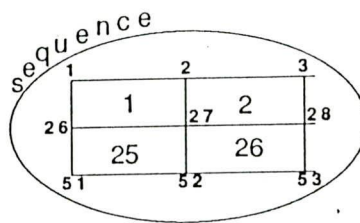
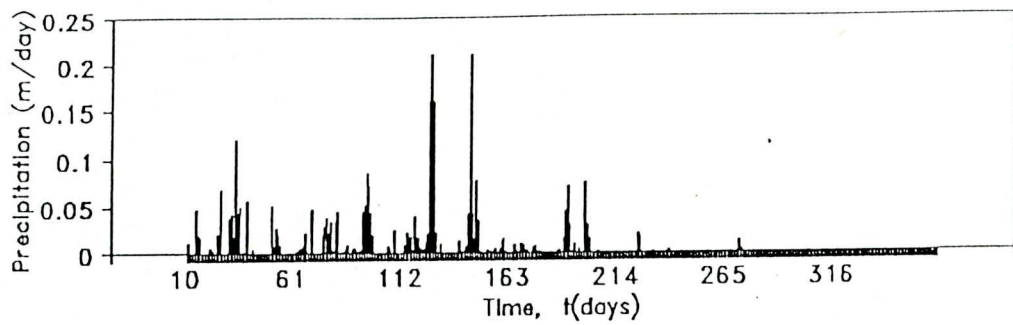
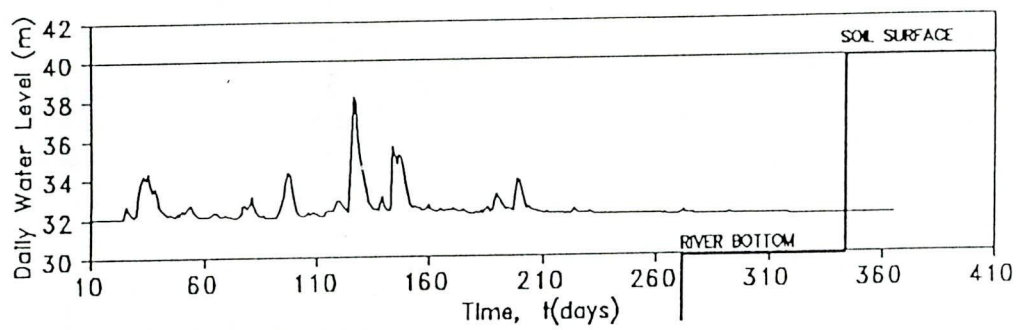


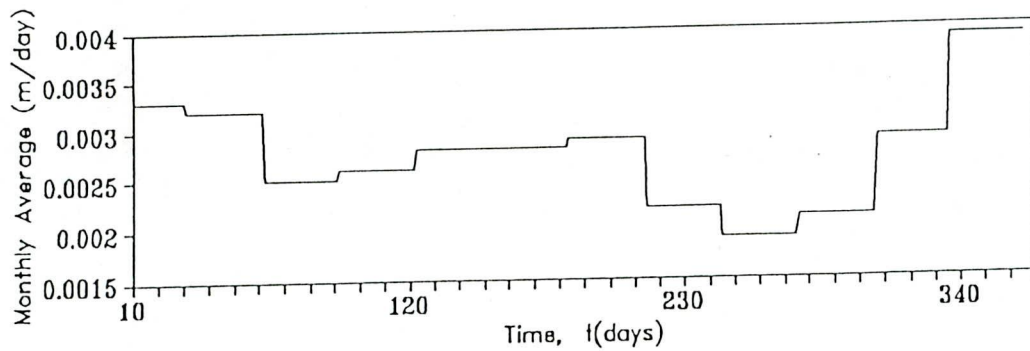
Fig. 7.17 : Finite element mesh for simplified river bank situation with finer grid (Example 3B)



(a) Daily precipitation

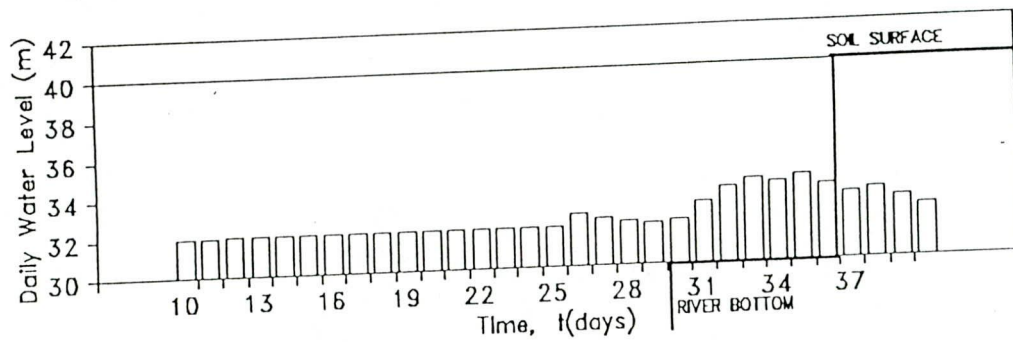
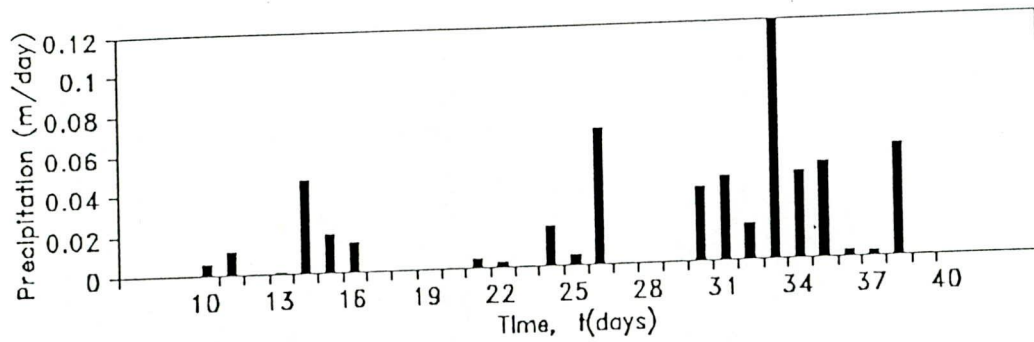


(b) Daily water level

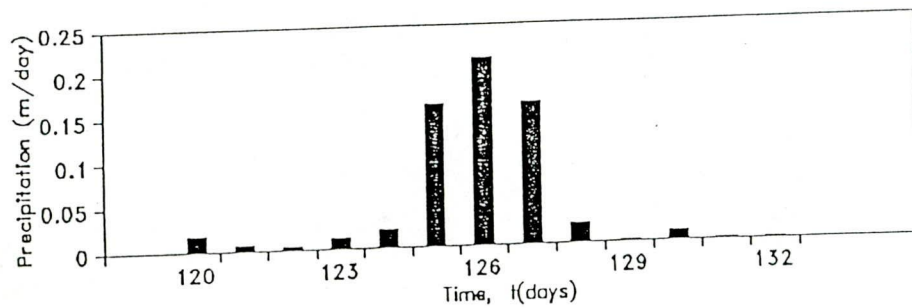


(c) Monthly average potential evaporation

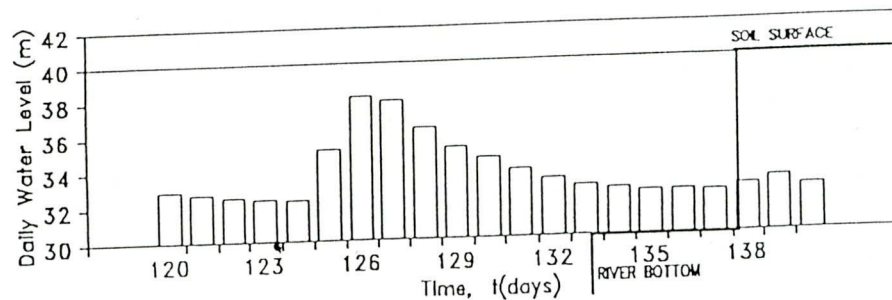
Fig. 7.18a : Daily precipitation (a), water level fluctuation (b) and monthly average potential evaporation (c)



(II) Daily water level variation



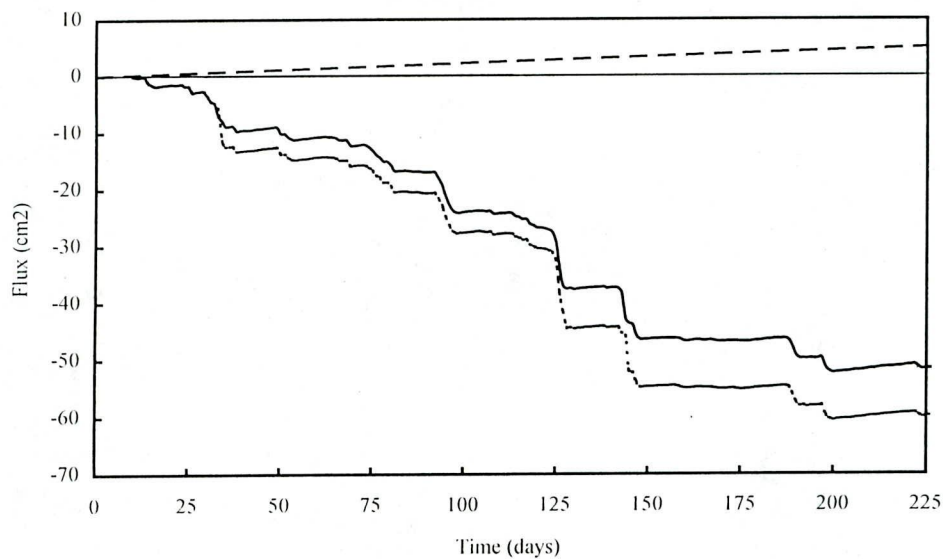
(I) Daily Precipitation



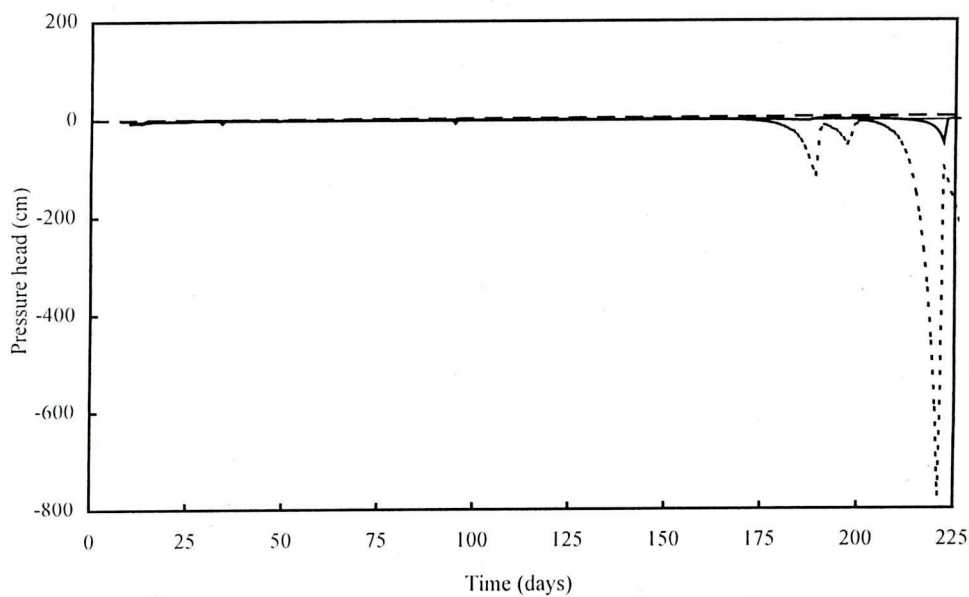
(II) Daily water level variation

Fig. 7.18a : Daily precipitation (a), water level fluctuation (b) and monthly average potential evaporation (c) for Example 3A and 3B

The Simulated Results
For
Example 3A : Simplified River Bank Situation with
Flow Domain discretized with Coarser Grid



(a) Cumulative flux across the atmospheric boundary (i) potential (solid) and actual (---); flux across the bottom surface(- - -)



(b) Mean pressure head at the soil surface (solid) and root zone (---)

Fig. 7. 19 : Simulated mean pressure head, flux across the atmospheric boundary (+ve upward, -ve downward) and bottom flux (+ve upward) (Example 3A)

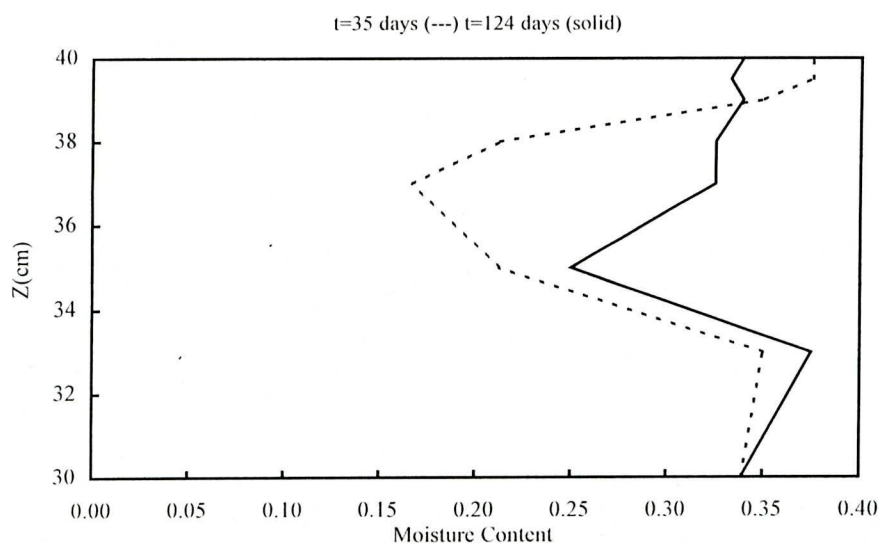
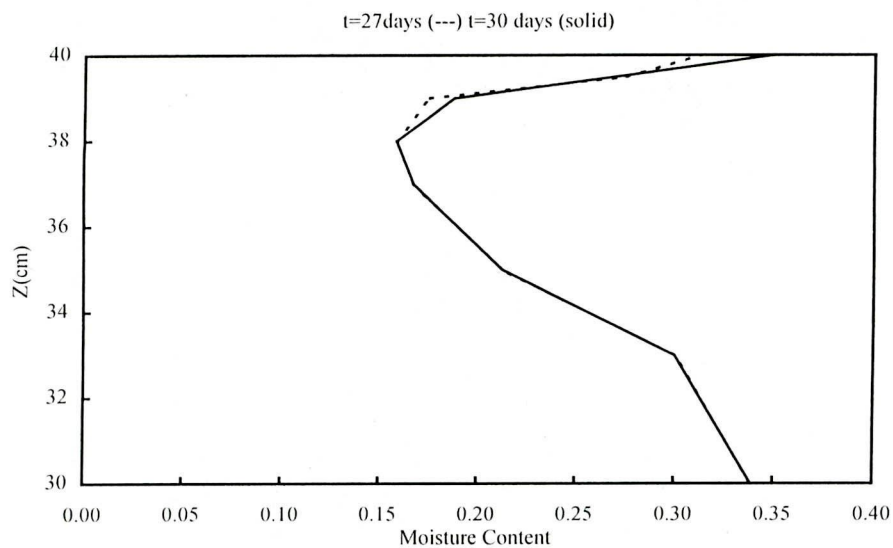
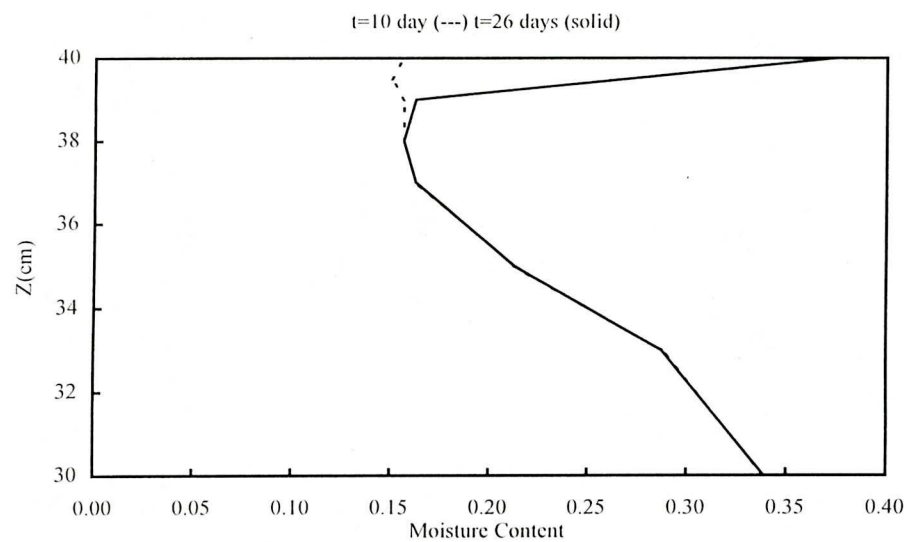


Fig. 7.20a : The simulated volumetric moisture content at the end of the days indicated in the vertical profile 12 m from the river bank (Example 3A)

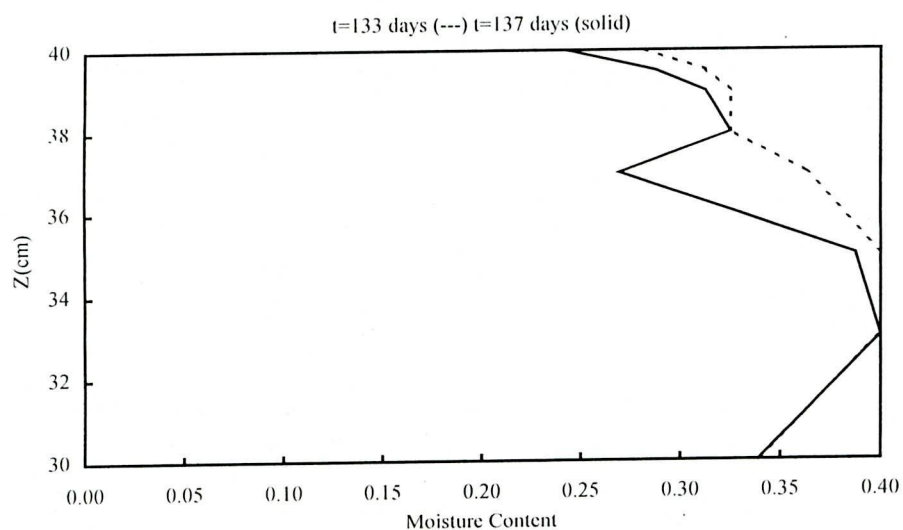
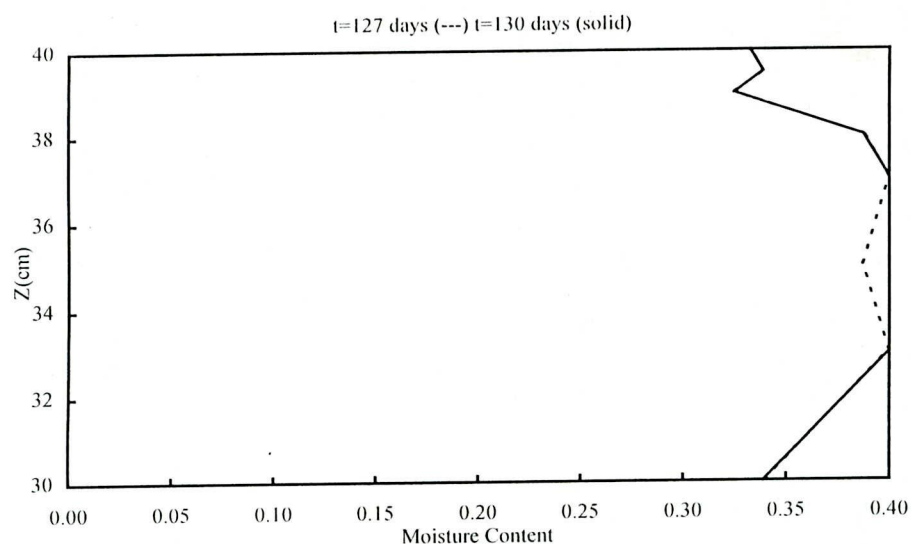
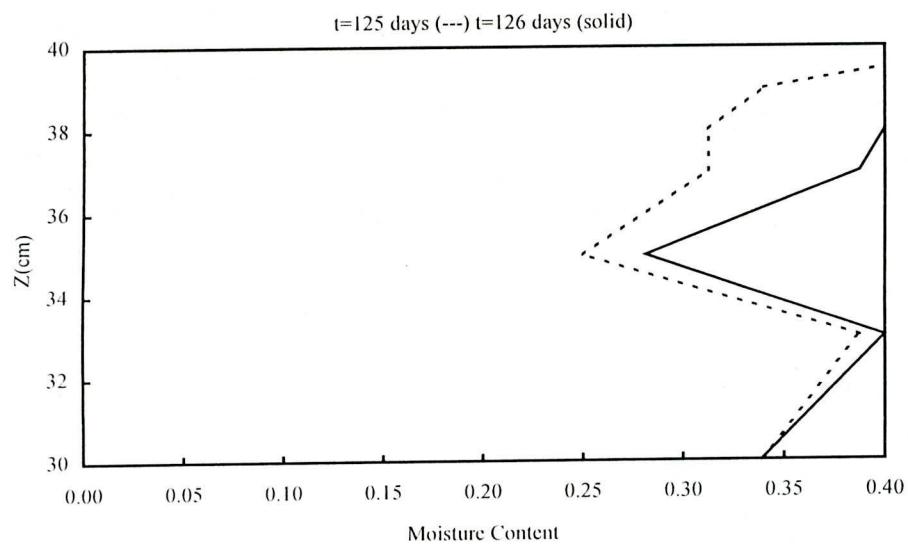


Fig. 7.20b : The simulated volumetric moisture content at the end of the days indicated in the vertical profile 12 m from the river bank (Example 3A)

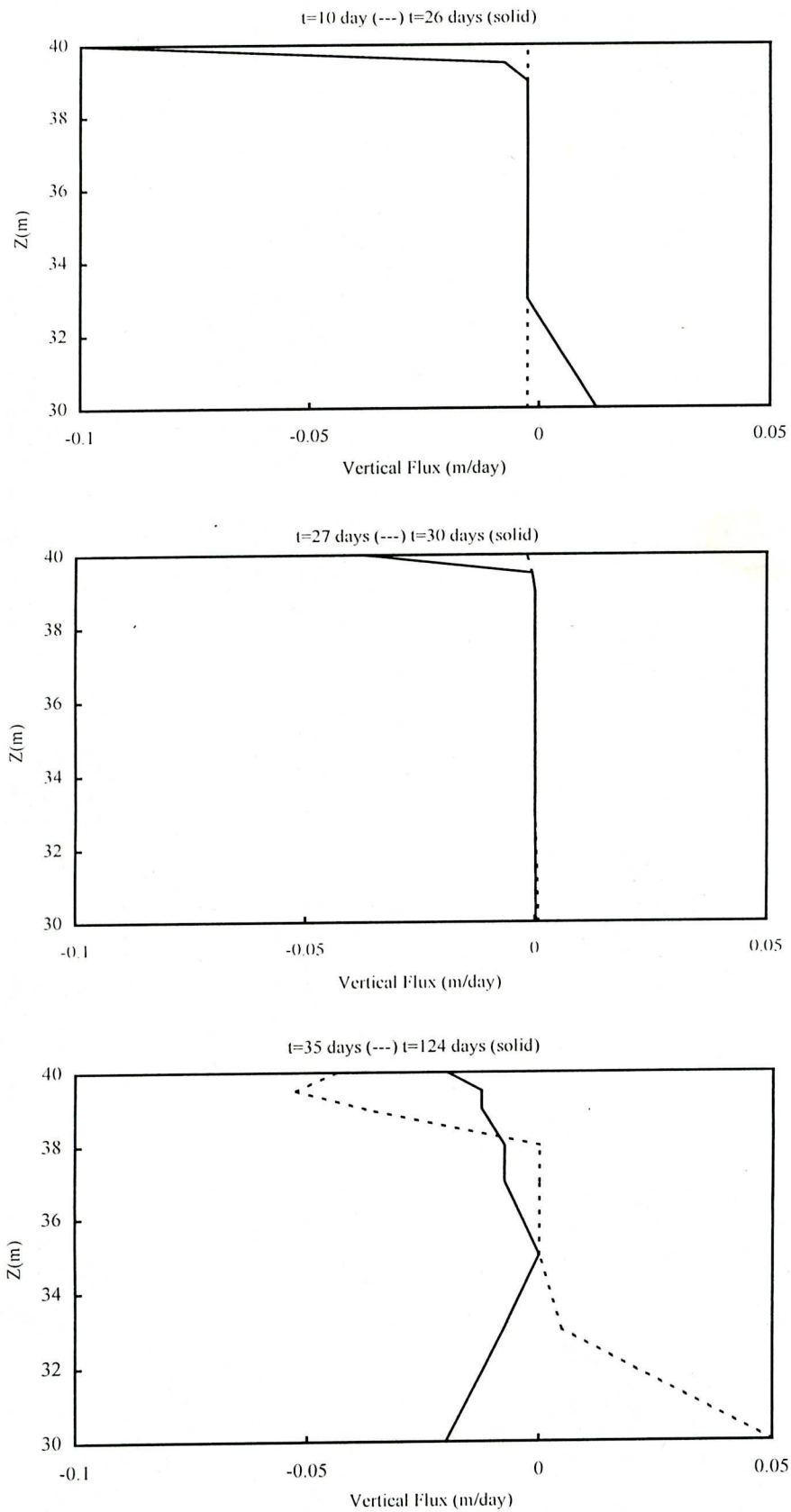


Fig. 7.21a : The simulated vertical darcian flux (m/day) (+ve upward, -ve downward) at the end of the days indicated in the vertical profile 12 m from the river bank (Example 3A)

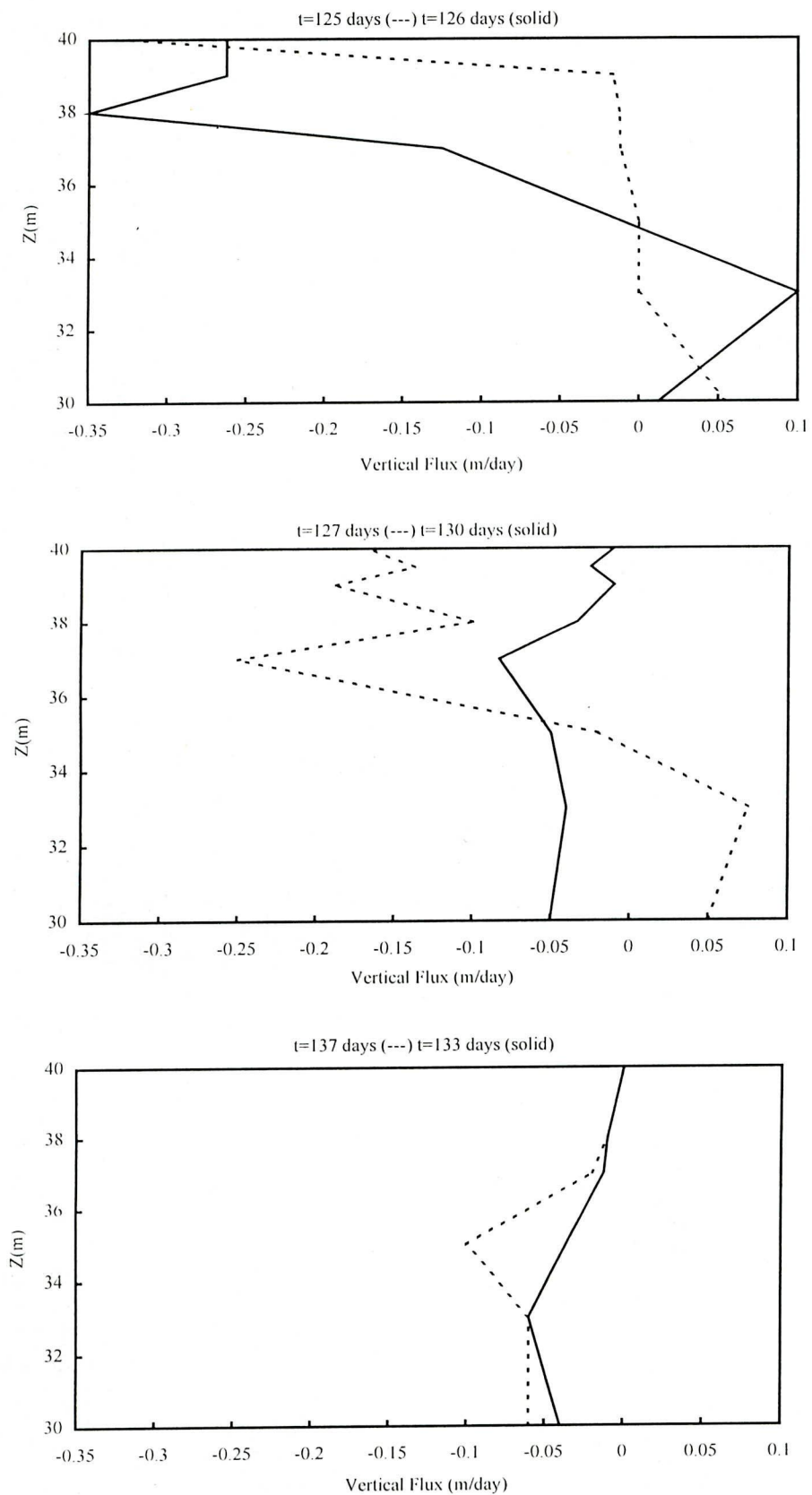


Fig. 7.21b : The simulated vertical darcian flux (m/day) (+ve upward, -ve downward) at the end of the days indicated in the vertical profile 12 m from the river bank (Example 3A)

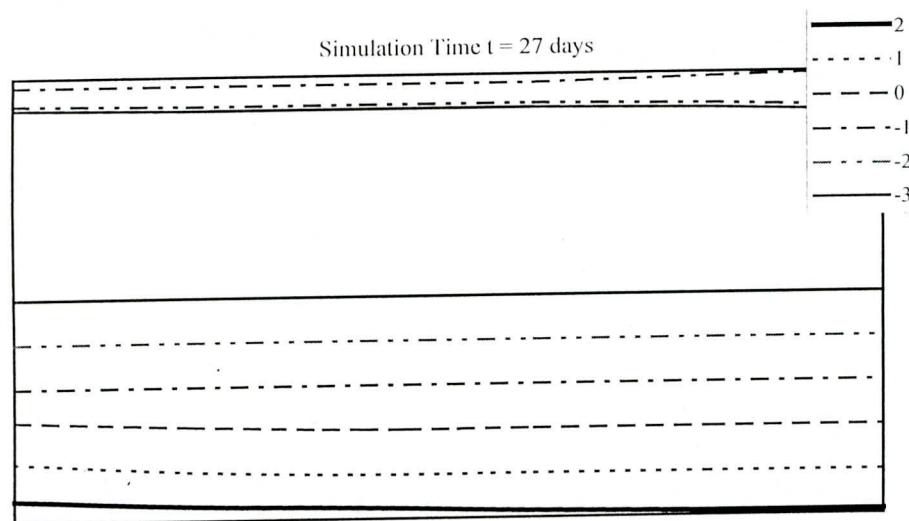
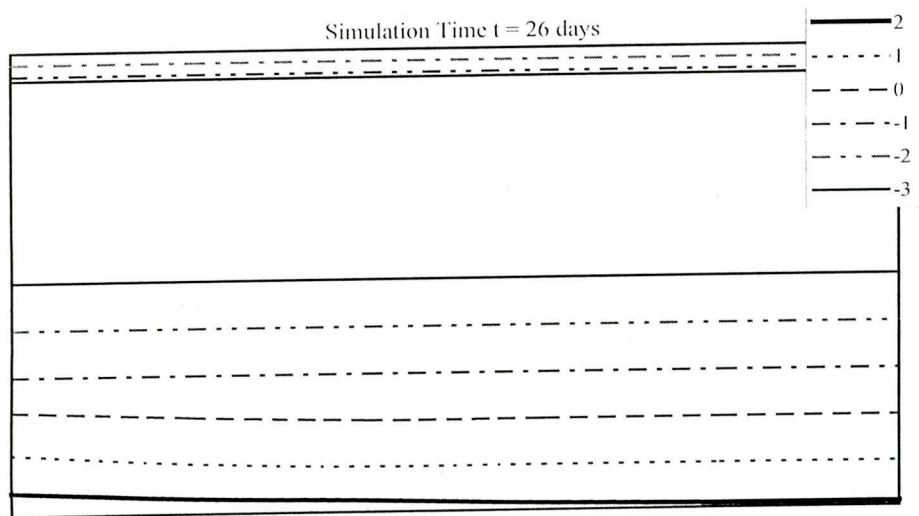
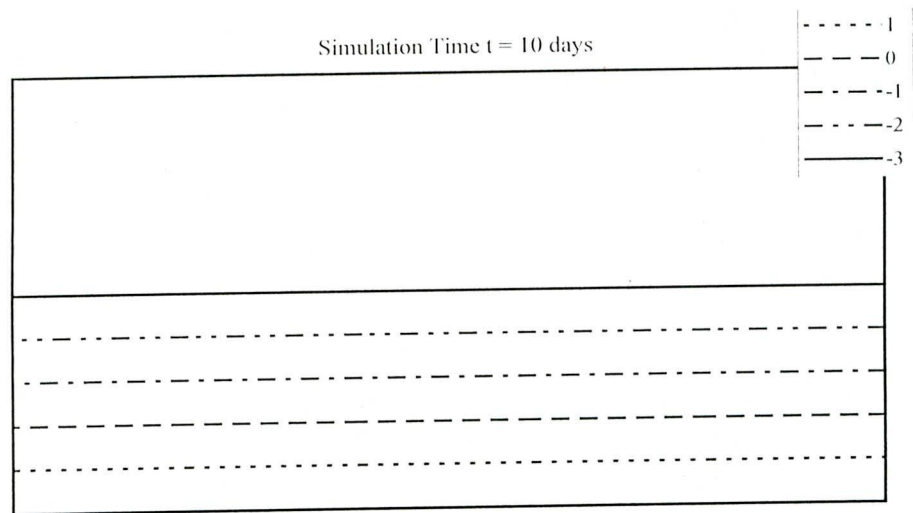


Fig. 7.22a : The simulated contours of pressure head (in m) at the end of the days indicated for the upper rectangular part of the flow region (Example 3A)

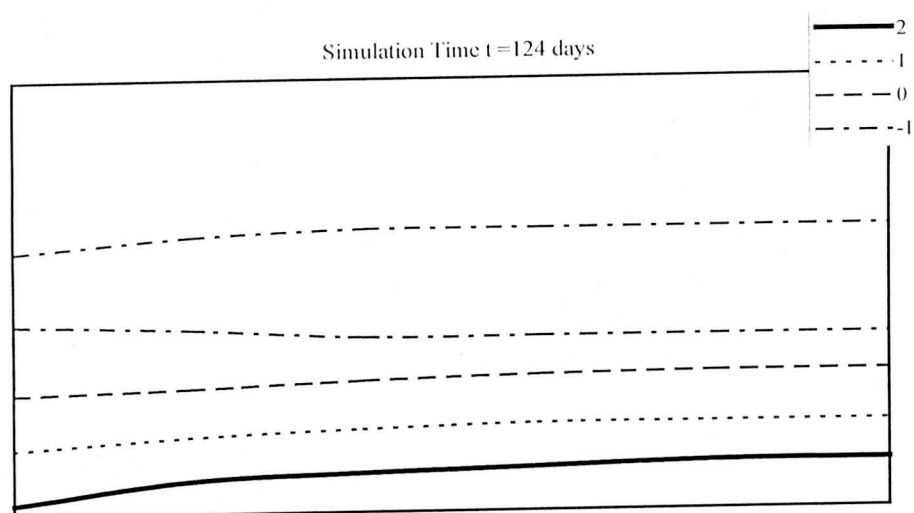
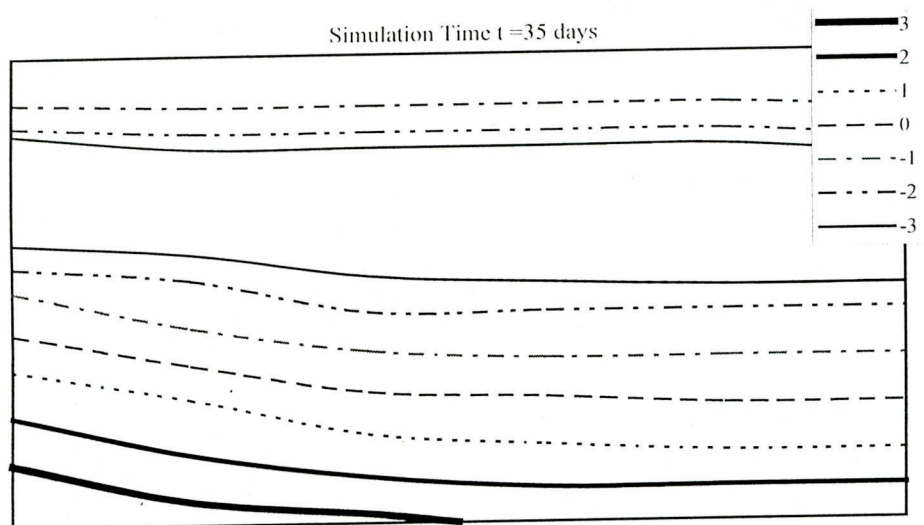
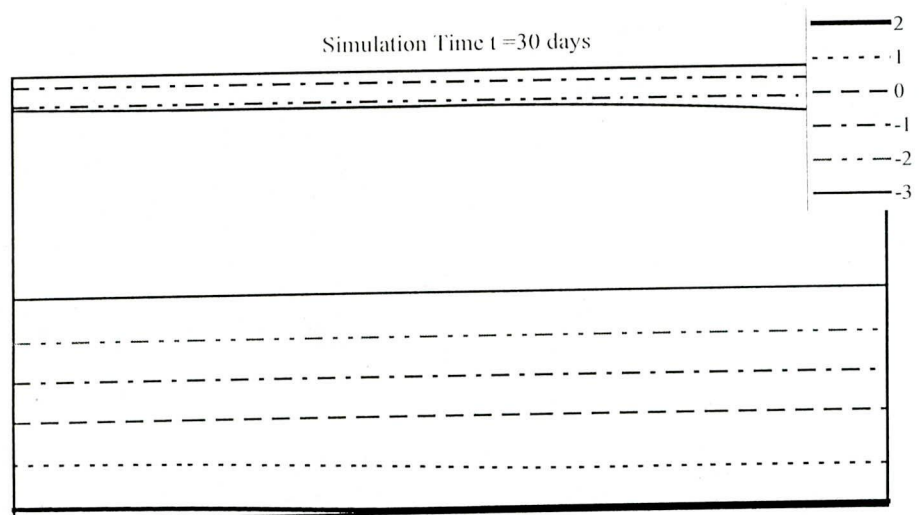


Fig. 7.22b : The simulated contours of pressure head (in m) at the end of the days indicated for the upper rectangular part of the flow region (Example 3A)

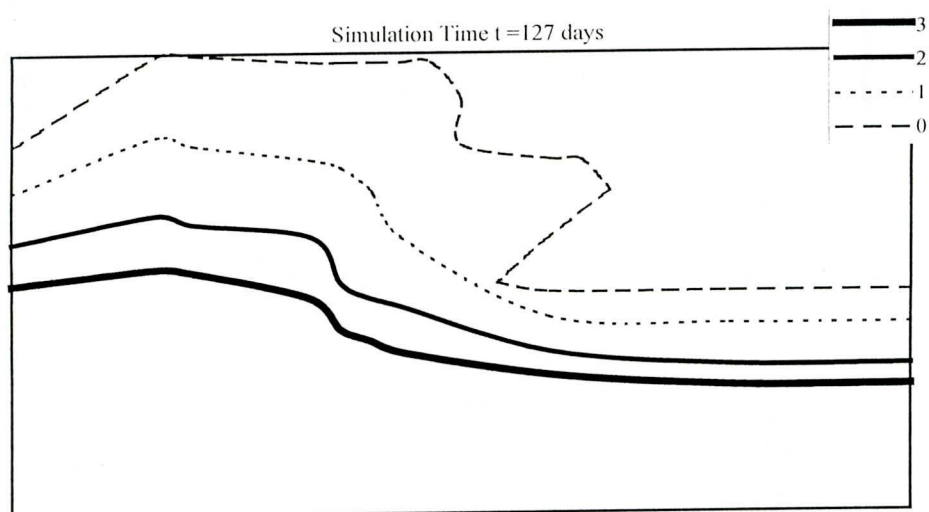
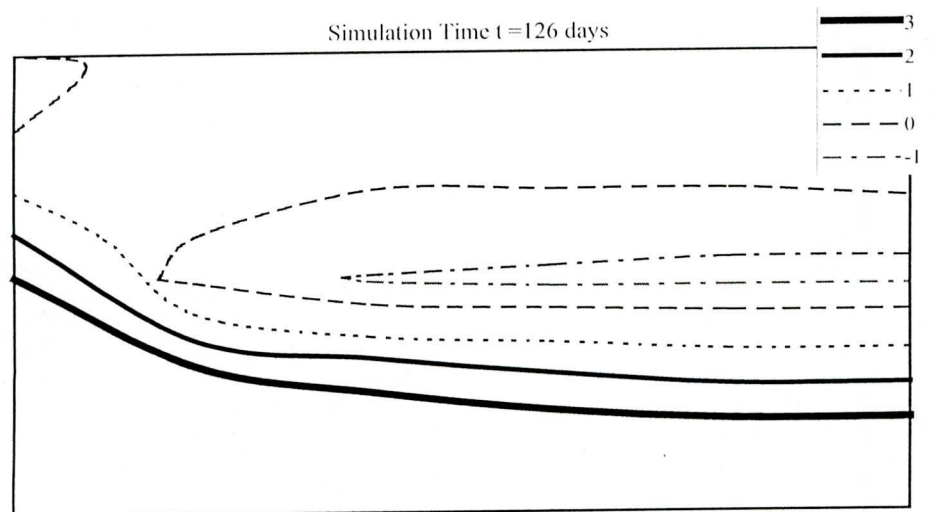
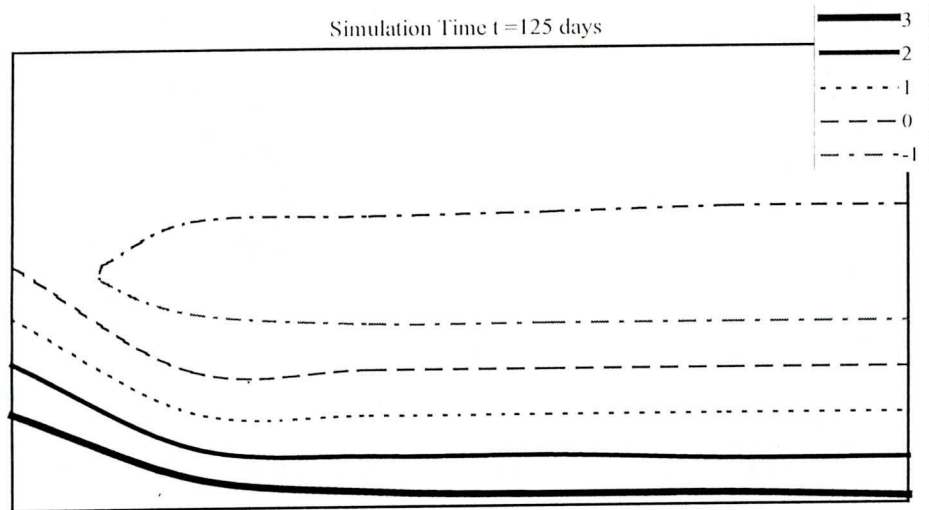


Fig. 7.22c : The simulated contours of pressure head (in m) at the end of the days indicated for the upper rectangular part of the flow region (Example 3A)

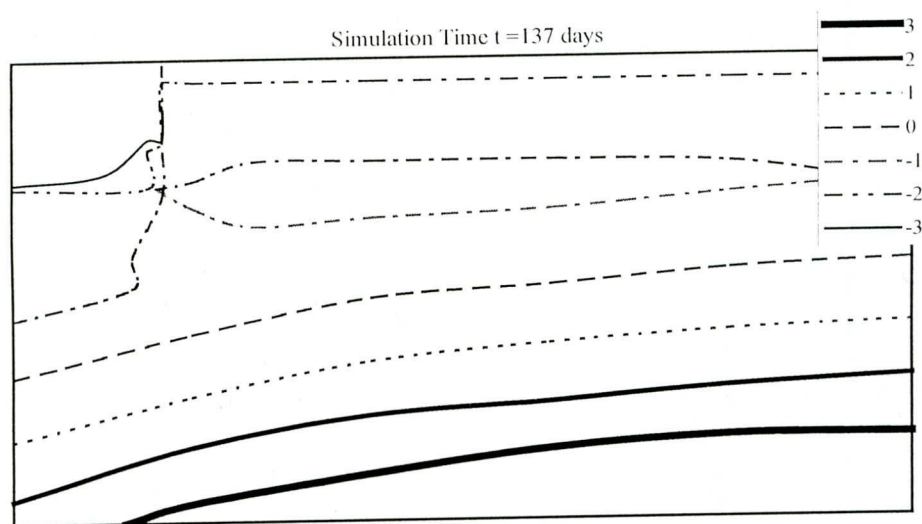
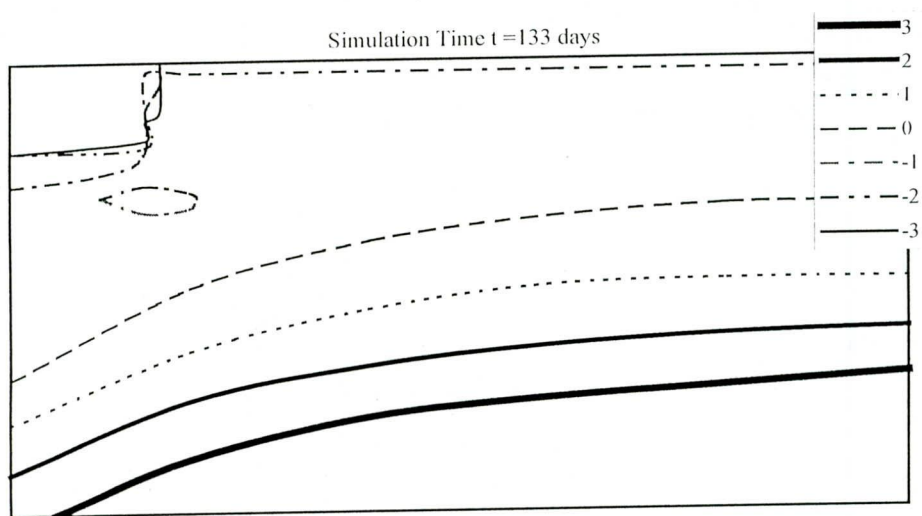
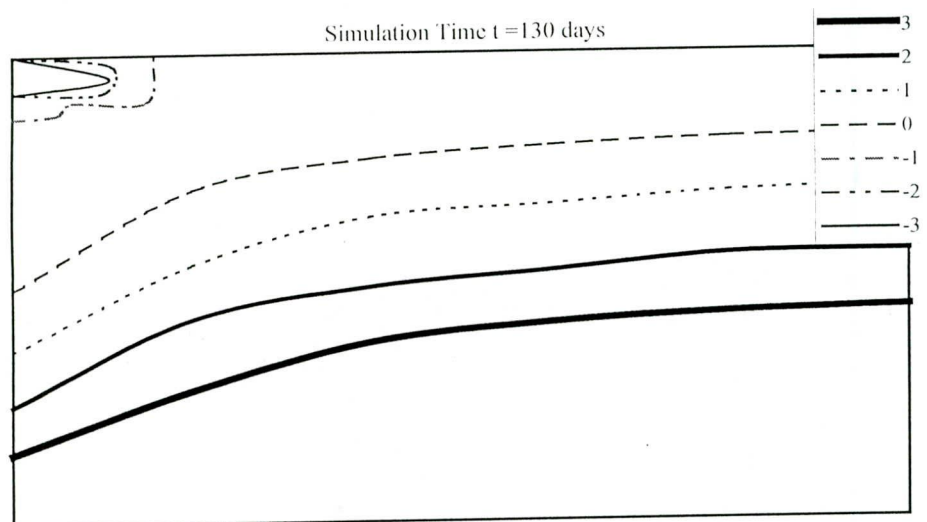
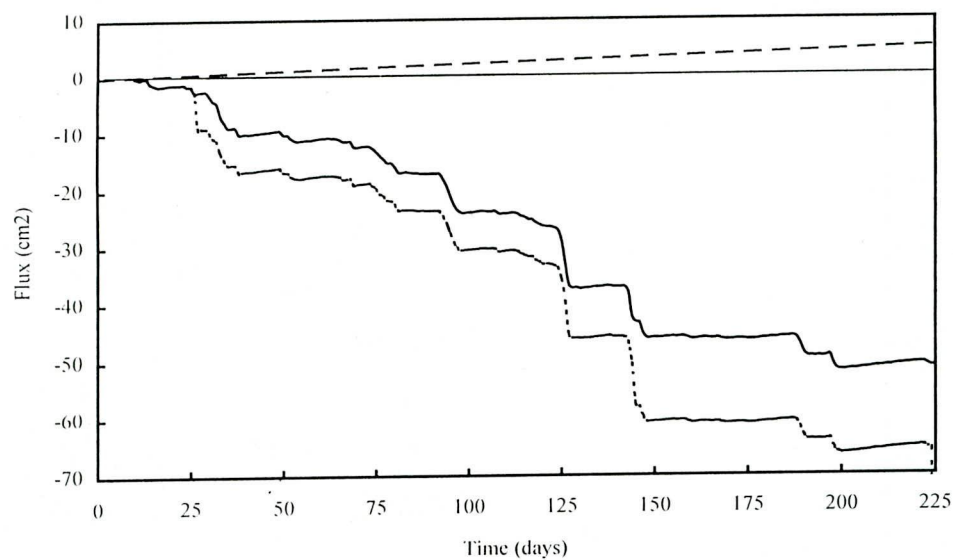
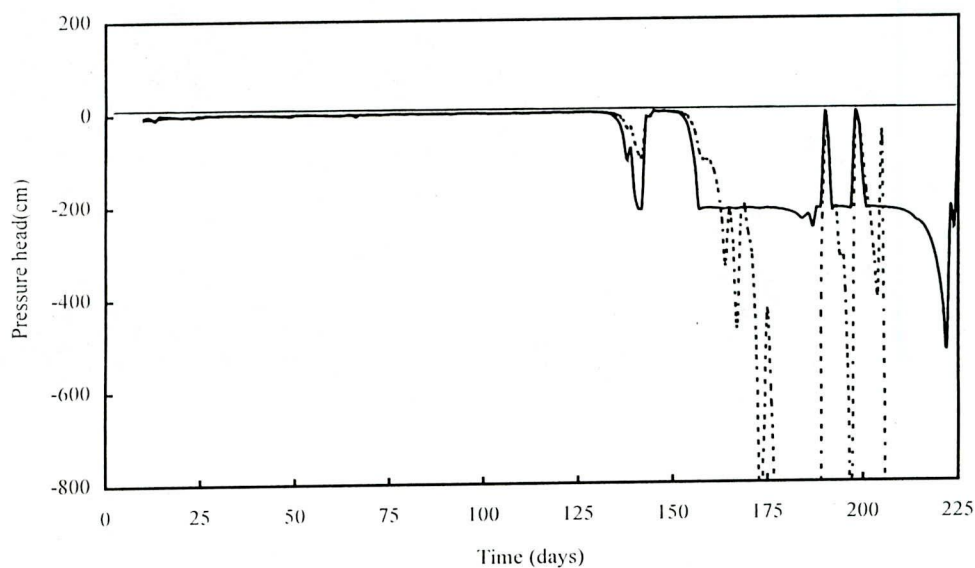


Fig. 7.22d : The simulated contours of pressure head (in m) at the end of the days indicated for the upper rectangular part of the flow region (Example 3A)

The Simulated Results
For
Example 3B : Simplified River Bank Situation with
Flow Domain descrtized with Finer Grid



(a) Cumulative flux across the atmospheric boundary (i) potential (solid) and actual (---);
flux across the bottom surface(- - -)



(b) Mean pressure head at the soil surface (solid) and root zone (---)

Fig. 7. 23 : Simulated mean pressure head, flux across the atmospheric boundary
(+ve upward, -ve downward) and bottom flux (+ve upward) (Example 3B)

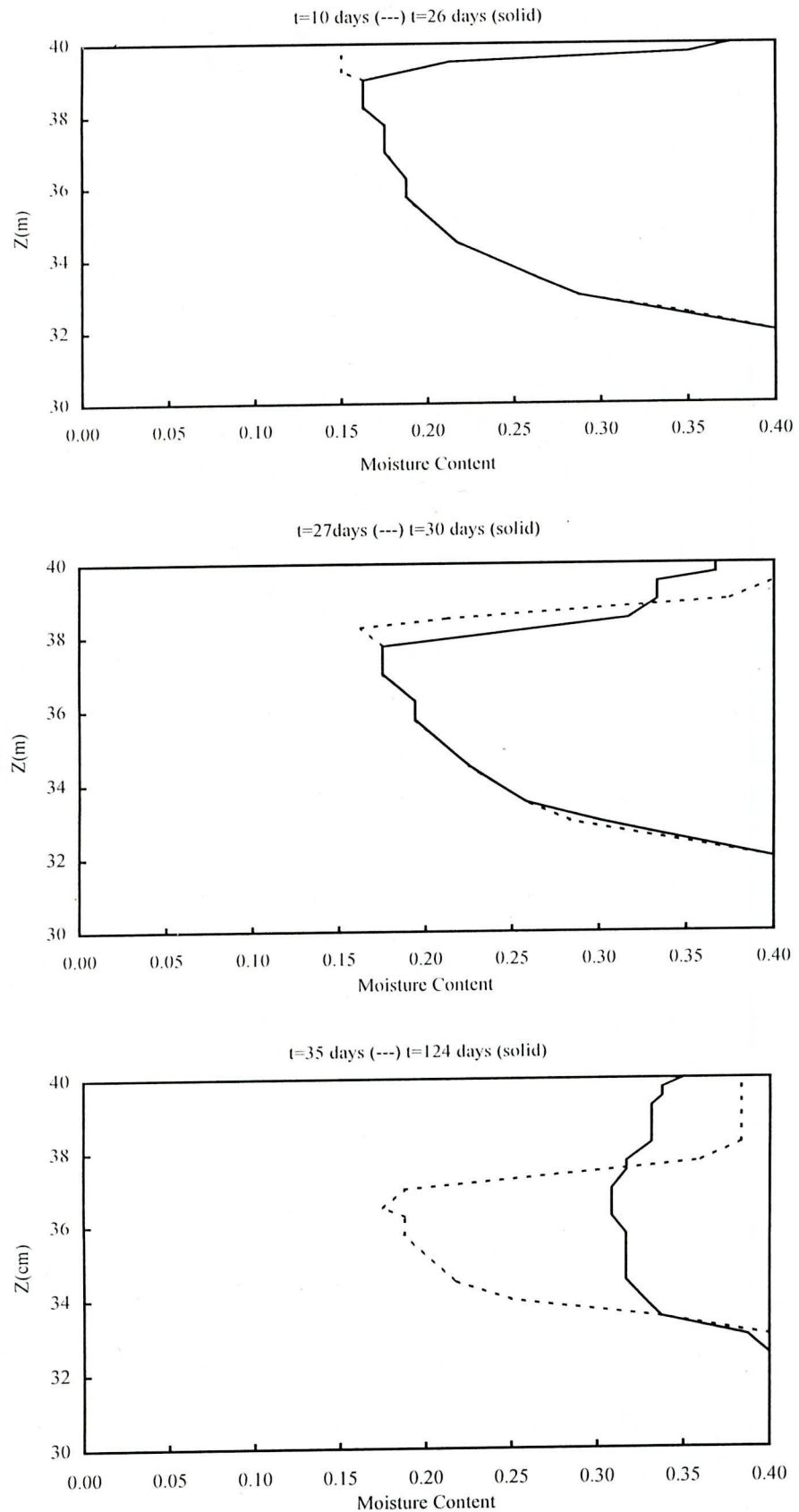


Fig. 7.24a : The simulated volumetric moisture content at the end of the days indicated in the vertical profile 12 m from the river bank (Example 3B)

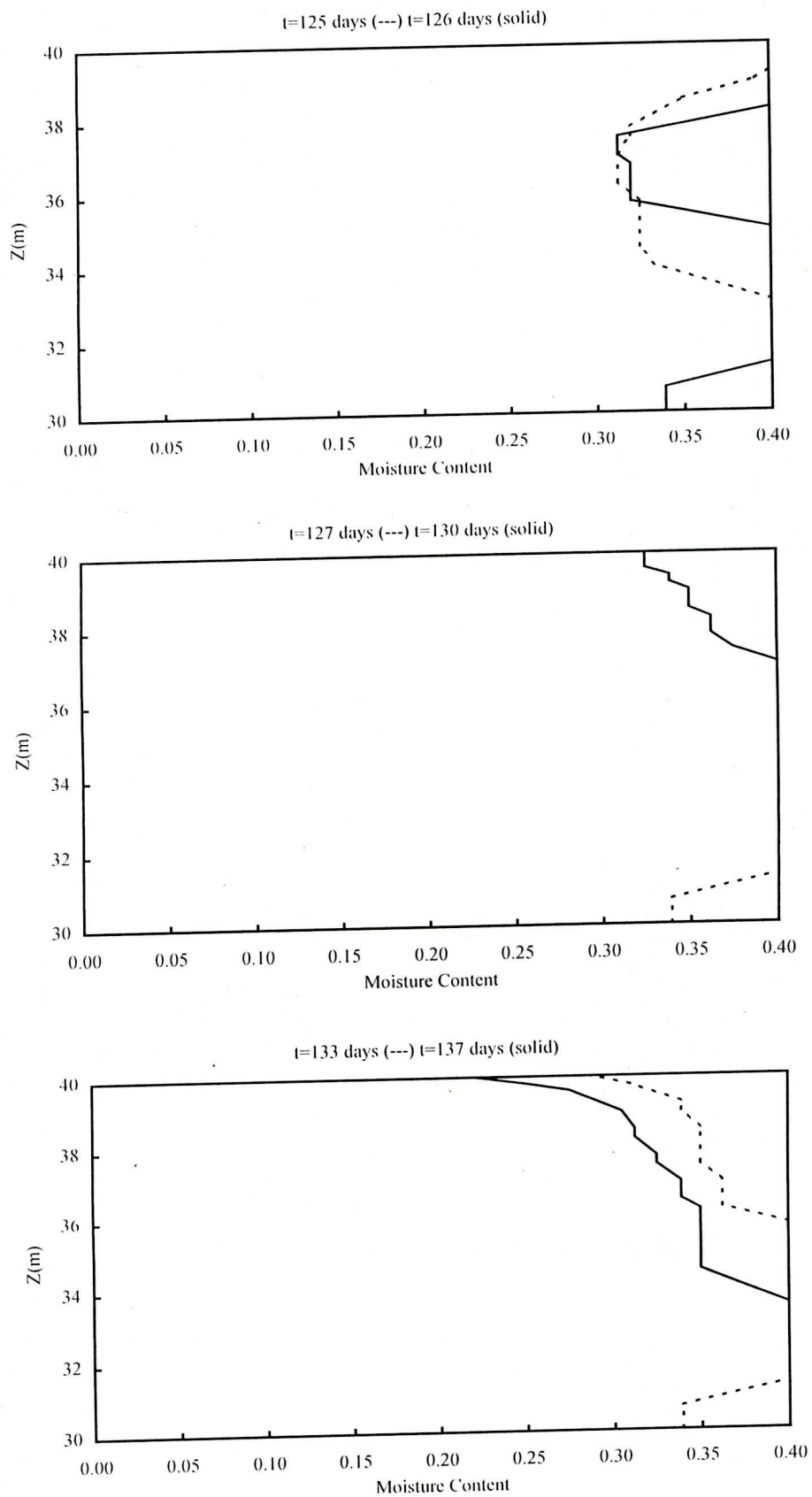


Fig. 7.24b : The simulated volumetric moisture content at the end of the days indicated in the vertical profile 12 m from the river bank (Example 3B)

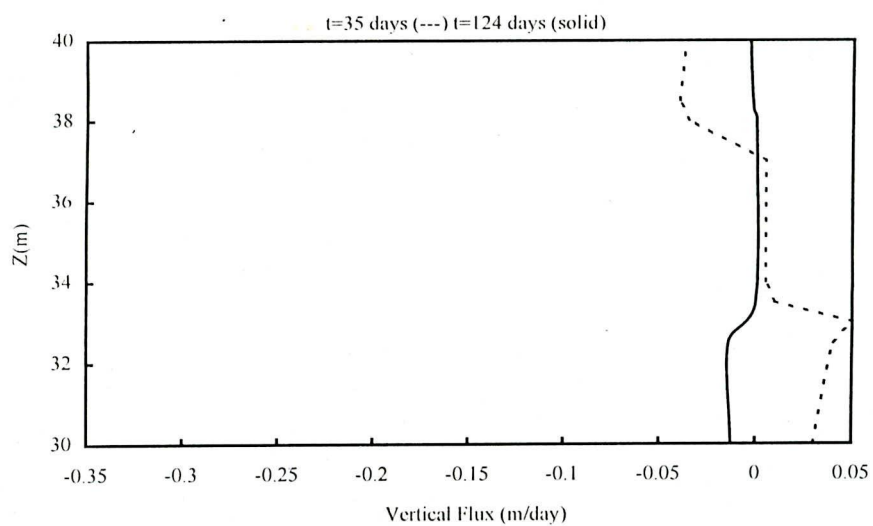
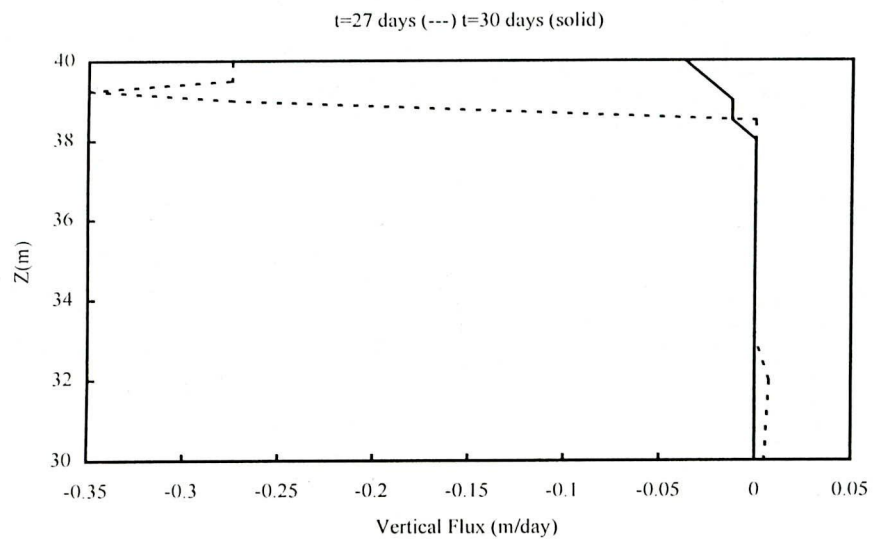
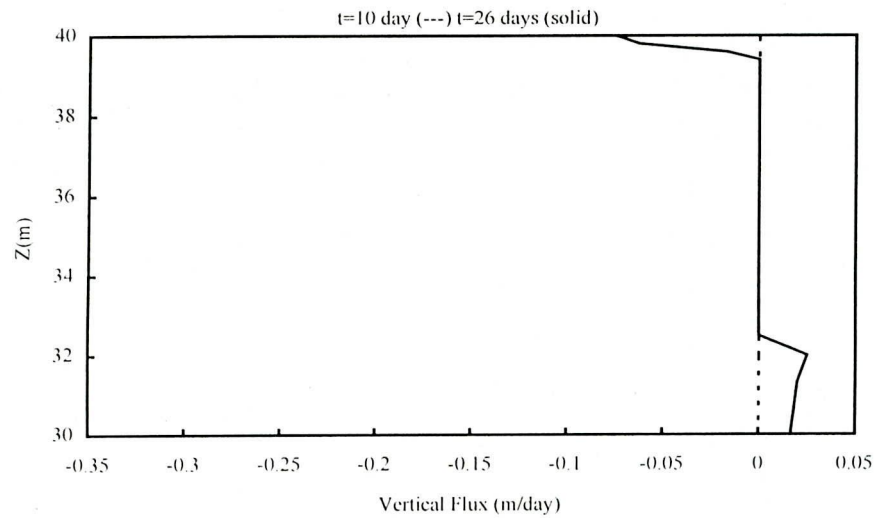


Fig. 7.25a : The simulated vertical darcian flux (m/day) (+ve upward, -ve downward) at the end of the days indicated in the vertical profile 12 m from the river bank (Example 3B)

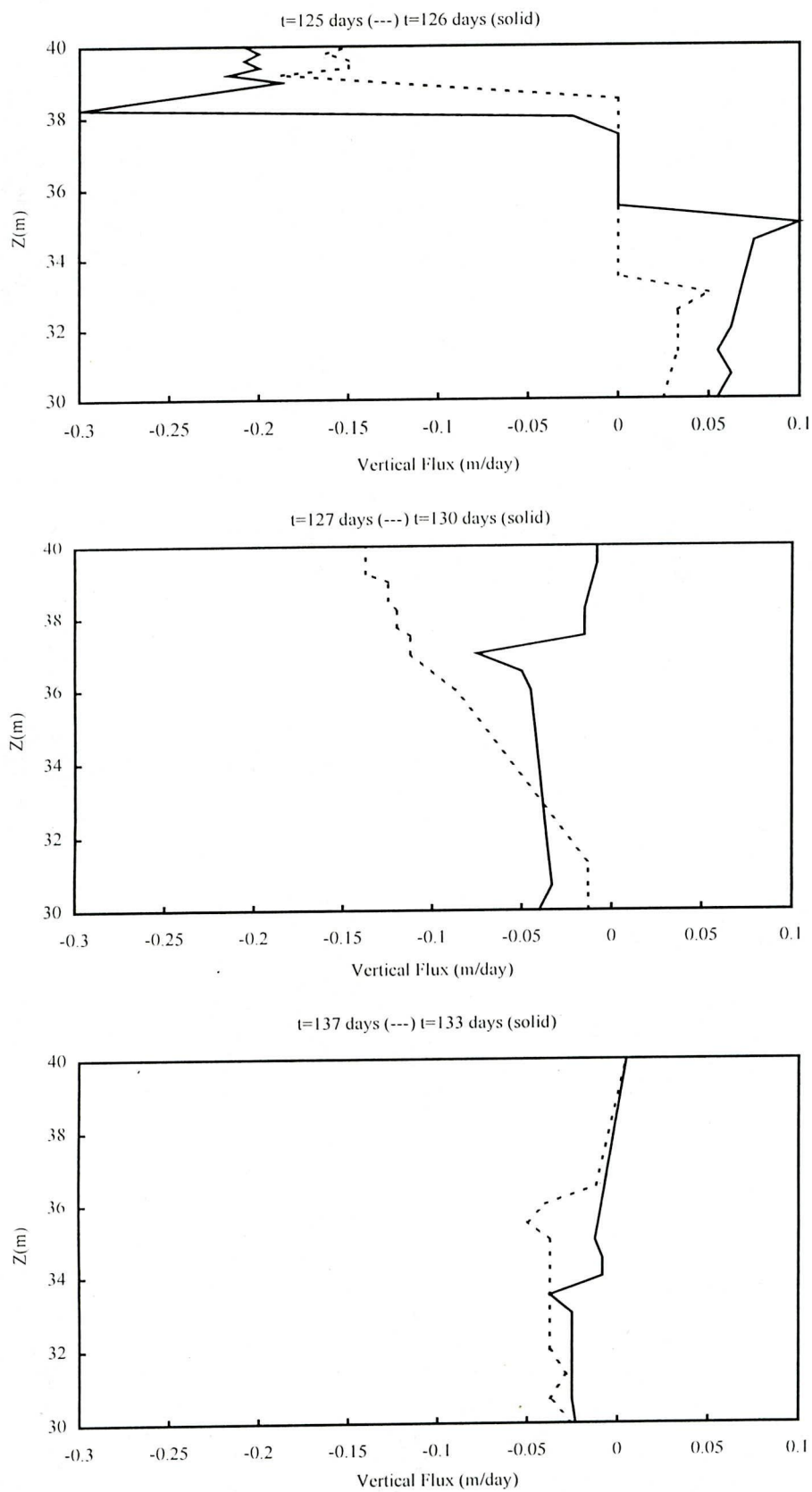


Fig. 7.25b : The simulated vertical darcian flux (m/day) (+ve upward, -ve downward) at the end of the days indicated in the vertical profile 12 m from the river bank (Example 3B)

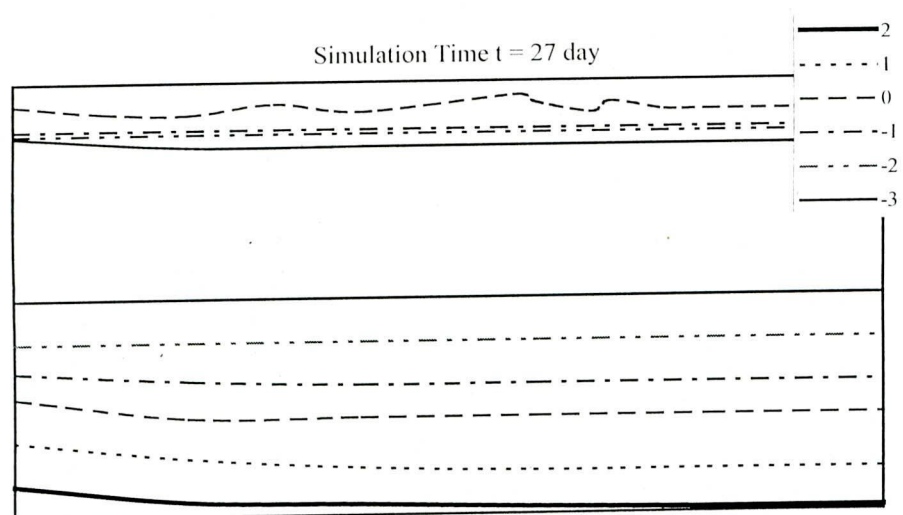
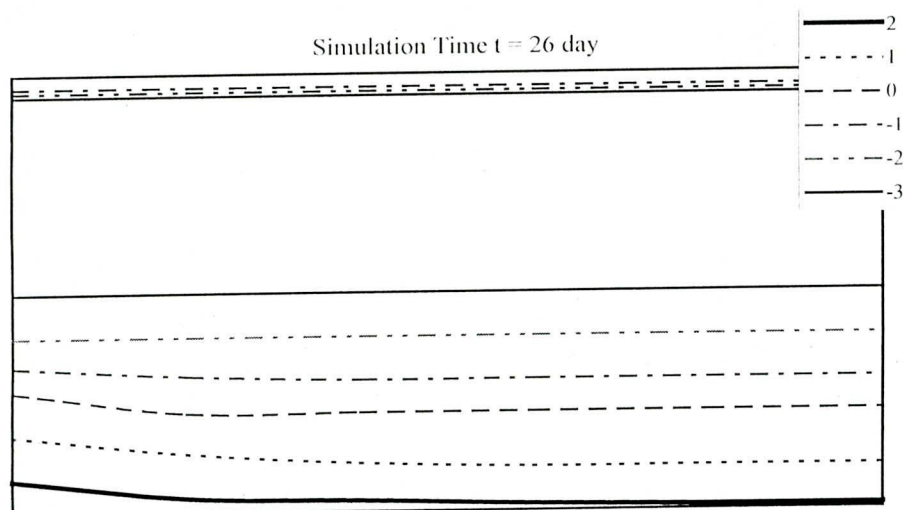
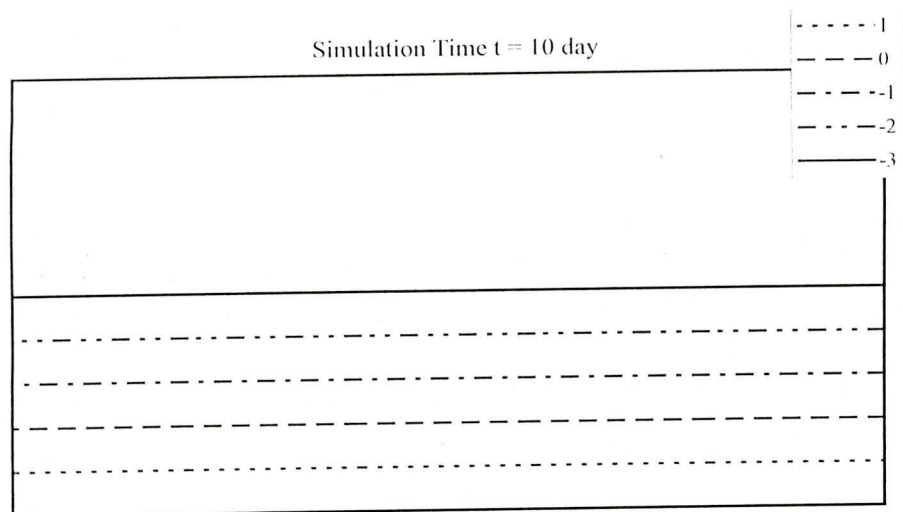


Fig. 7.26a : The simulated contours of pressure head (in m) at the end of the days indicated for the upper rectangular part of the flow region (Example 3B)

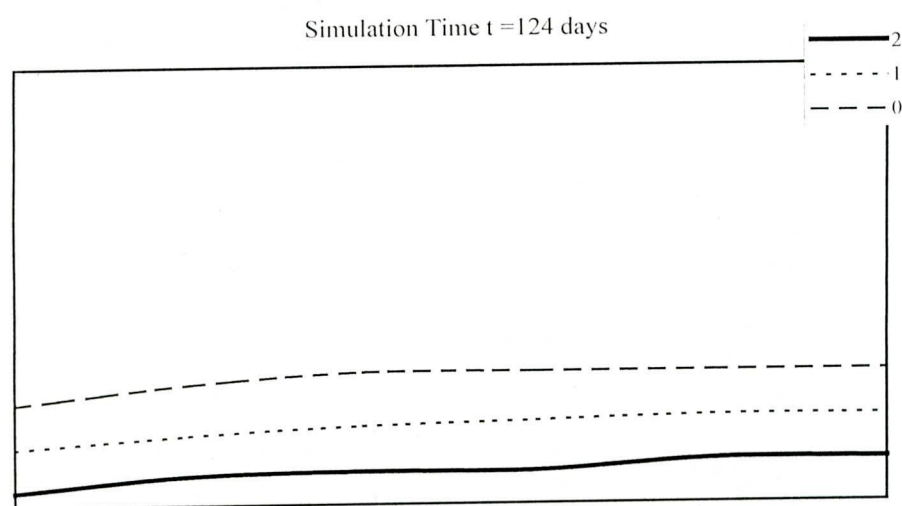
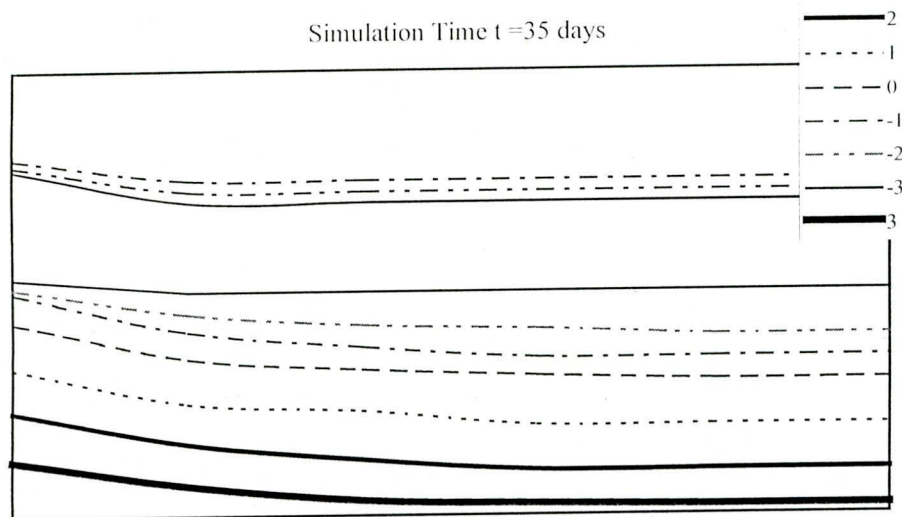
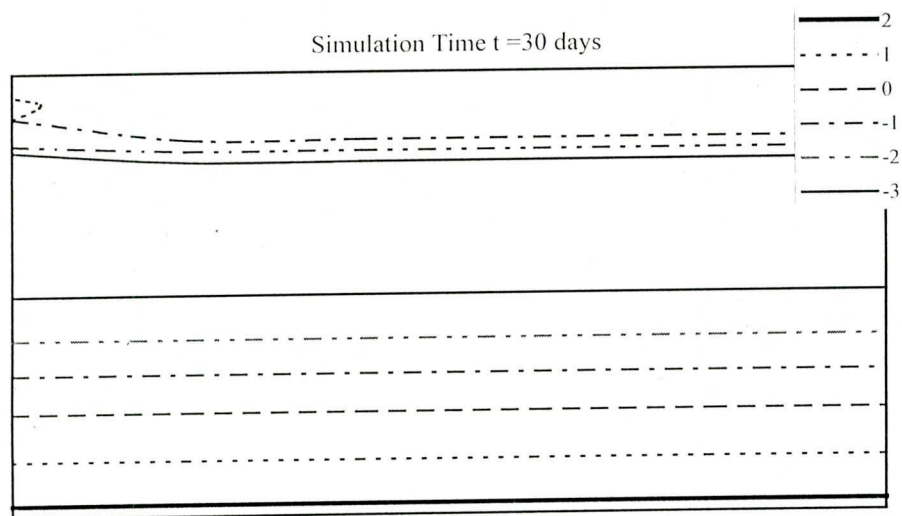
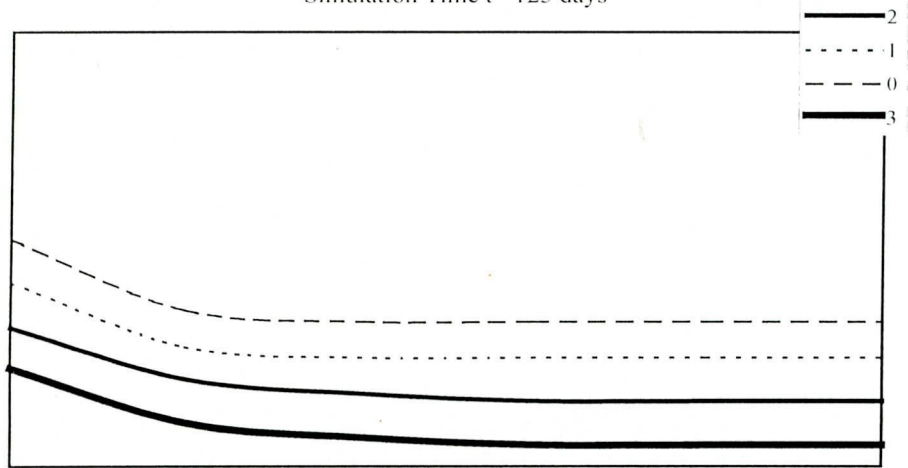
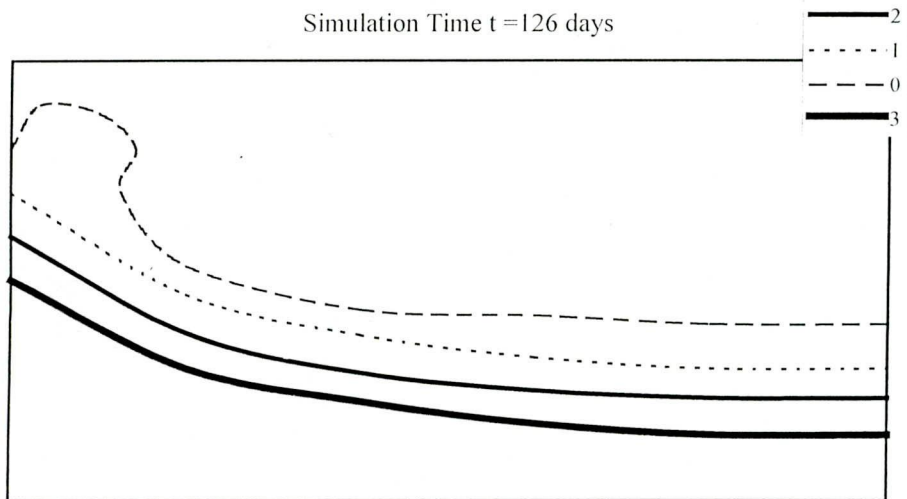


Fig. 7.26b : The simulated contours of pressure head (in m) at the end of the days indicated for the upper rectangular part of the flow region (Example 3B)

Simulation Time $t = 125$ days



Simulation Time $t = 126$ days



Simulation Time $t = 127$ days

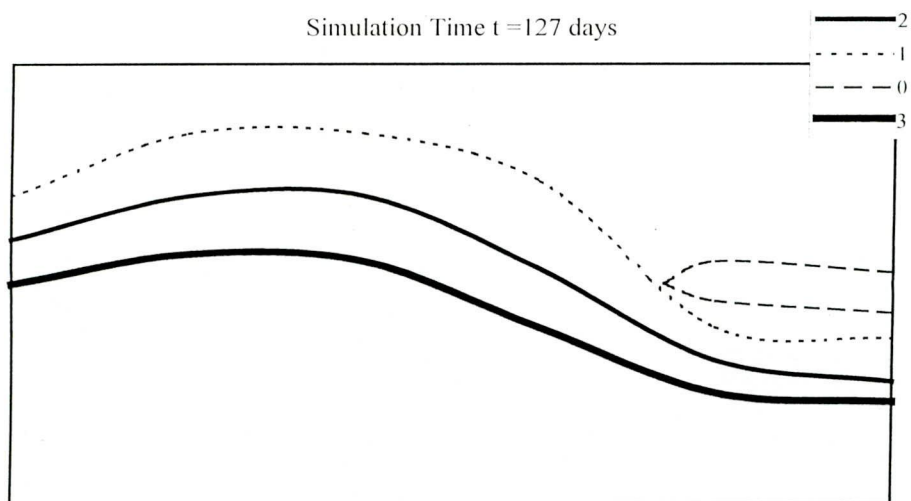


Fig. 7.26c : The simulated contours of pressure head (in m) at the end of the days indicated for the upper rectangular part of the flow region (Example 3B)

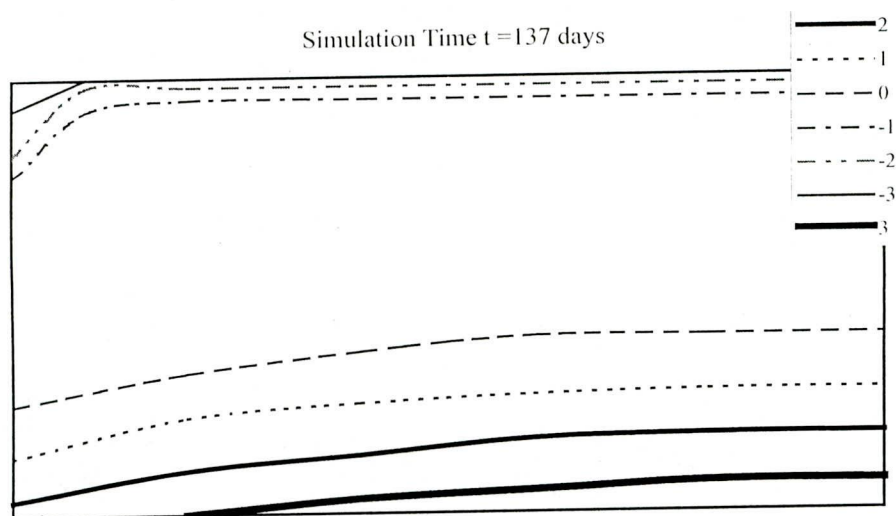
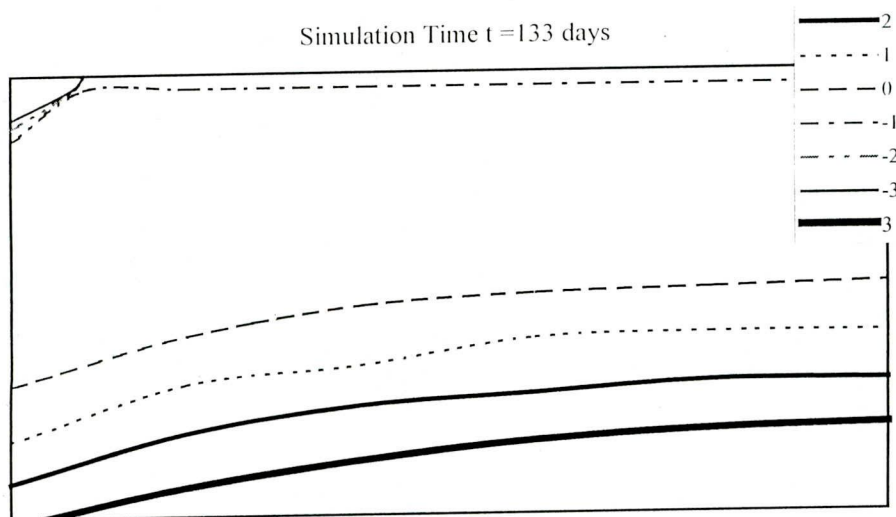
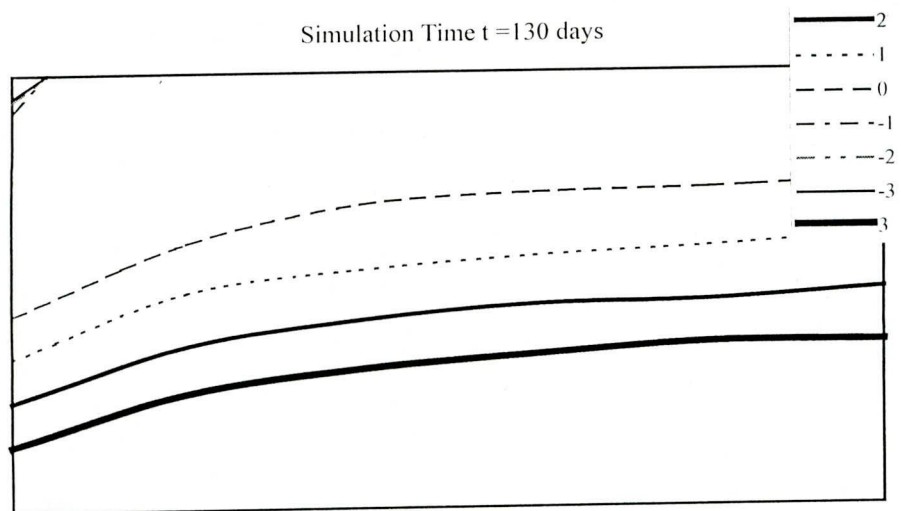


Fig. 7.26d : The simulated contours of pressure head (in m) at the end of the days indicated for the upper rectangular part of the flow region (Example 3B)

7.6 Application of the Model to a Real Hydrogeological Situation (Dhaka City Aquifer)

7.6.1 General Description of the Study Area

Location and Topography

The study area is within the north central region of Bangladesh. The north central region covers the region bounded by the Jamuna, Ganges, Meghna and Old Brahmaputra. The study area is bounded by Tongi khal on the north, Buriganga on the west, Balu River on the east and by the District boundary on the south (Fig. 7.27). The model area covers approximately 320 km². The model area has been selected to cover DWASA's supply area and adjacent areas influenced by the groundwater abstraction.

The average ground elevation of the study area is 4.5m PWD. It varies from 1m to 10m PWD. The land slopes gradually from north to south. The ground elevation of Dhaka metropolitan area varies from 1m to 8m PWD (Fig. 7.28).

Climate

The average yearly rainfall in the study area is 2000 to 2200 mm based on the period April 1976 to March 1997. The long term mean for Dhaka city is 2040 mm per annum (MPO, 1987). Most of the annual rainfall, 80 to 90 percent, falls during the monsoon season from May to September (EPC, 1991). Temperature ranges from lows of about 10°C at night during December and January to daytime highs of over 35°C during April and May (EPC, 1991). Average monthly potential evapo-transpiration of Dhaka City ranges from about 2.87 mm/day in January to about 6.25 mm/day in April with a yearly total of 1500 mm. During the monsoon season mean daily values are about 4.8 mm/day (EPC, 1991).

Geology and Hydrogeology

One of the important tasks for setting up a groundwater model is to define the geology i.e. the aquifer system of the model area. Lithological data of 66 discrete boreholes from BWDB

have been used to describe the geology in the model. Thirty six of those are located in the Dhaka Metropolitan area (Fig. 7.29). Borehole logs clearly show that the deposits underlying Dhaka are heterogeneous and a certain degree of conceptualization is required to derive an operational model. The main features of the geological layout must, however, be maintained in order to obtain a satisfactory representation of the flow.

Alluvial deposits form the geology of the Dhaka area. The deposits range from fine textured solid such as clay and silt to coarser material in terms of sand and gravel. Despite the occurrence of several interbedded layers it is possible to identify at least four major geological units (Fig.7.30):

1. Upper aquitard (clay and silt)
2. Upper aquifer (fine to medium sand)
3. Lower aquitard (clay and silt)
4. Lower aquifer (medium to coarse sand)

It is important to maintain the hydrological features of the aquifer system in order to correctly simulate the groundwater flow. For each of the four layers of the model hydraulic parameters in terms of vertical conductivity, horizontal conductivity, confined storage coefficient and unconfined storage coefficient must be specified. Each of the four parameters may exhibit spatial variation within the model area and distributed maps may be specified.

The hydraulic properties of the individual geological layers are not known. A distributed transmissivity map has been produced based on a number of aquifer tests carried out by DWASA (Hossain, 1997). The transmissivity values are mostly derived from specific yield tests. Values are found in the range of 300-1200 m²/day. Taking into account the saturated thickness of the layer, horizontal hydraulic conductivity and storage coefficient may be calculated and subsequently interpolated to cover the entire model area.

Existing Groundwater Condition

In the Dhaka city, annual abstraction of groundwater by DWASA and other private and industrial units has risen continuously since the early 1960s to reach approximately 280

million cubic meters today (Fig. 7.31). Outside Dhaka City the aquifers are fully recharged each year, with groundwater levels returning to their original levels a month or so before the end of monsoon. But beneath Dhaka city, the situation is fully different; here water levels in the aquifer have fallen steadily over the last 25 years in response to continually increasing abstraction and has reached a maximum of over 20 to 25 meters in some areas. Existing groundwater condition can be interpreted from observation well data. A total of 27 observation wells in and around the model area are available, out of which 11 are located within Dhaka City. Figure 7.32 shows the location of observation wells and Figure 7.33 show the time series plot of groundwater level of the stations that are within the study area.

The groundwater level of Dhaka City exhibits a large fluctuation between monsoon and the dry season. The groundwater level is declining at a rate of approximately 1.2 m/yr. in Dhaka City. The declination rate is higher in Sabujbagh area (DHA019, DHA020) which is about 1.7 m/yr (DWASA, 1998). This may be due to DTW's have been installed to keep pace with the rapid expansion of the residential areas. The lowest groundwater level was found in South Basabo (DHA020) which is about -26m PWD. The highest level was found in Maniknagar (DHA013) which is about -14m PWD. Both were found in the year 1996-97. The falling rate of water table is increasing gradually which indicates an alarming situation.

7.6.2 Discretization of the Model Area

The numerical method applied requires a sub-division of the model area into a numerical grid by means of Discretization. Input parameters are entered at nodal points. On one hand it is desirable to specify a fine grid resolution in order to obtain detailed model results. On the other hand the grid should not be finer than the available input data allows and a coarser grid may also reduce the time required to run the model.

The model area has been discretised into 500m grid squares in the horizontal plan (Fig. 7.34). The total number of elements is 1263 and nodes are 1295. The program automatically divides the square elements shown in figure into two equilateral triangular elements.

7.6.3 Model Boundaries

The west, east and north of the model area are bounded by the vertical planes passing through the axes of the Buriganga River, Balu river and Tongi khal respectively. As in the city side abstraction is enormously higher than the opposite side of the river obviously a high gradient towards the city from the opposite side of the river prevails. It is assumed that the river is hydraulically connected with the soil layers below its bed. Thus a time dependent Dirichlet boundary condition is imposed on west, east and north boundaries of the study area.

Problem is faced when attempt is made to select the off-river boundary, i. e., the south boundary of the model area. There is no distinct natural boundary along this face. So an artificial time dependent Dirichlet boundary condition is imposed based on the observation well data and their interpolation values in the intermediate nodes. Except boundaries all other internal nodes are treated with atmospheric boundary condition.

7.6.4 Data preparation and Parameter Fitting

Data required for the present simulation includes:

- (a) data defining the geometry of the aquifer system (e. g., average surface elevation)
- (b) geometry of the model grid network
- (c) data defining the hydrogeological characteristics of the aquifer system (e. g., saturated hydraulic conductivity, unsaturated characteristic curves and storage coefficients etc.)
- (d) data related to the boundary conditions (e. g., river water variations, known piezometric data of the observation wells, precipitation, evaporation, extent of root zone (if exists) etc.)
- (e) data related to the aquifer system excitations (e. g., abstraction of water by WASA and private wells, return flow from the municipal water supply).

The hydrogeological data are collected from DWASA. They have defined the saturated hydraulic conductivity of different zones of Dhaka region from a correlation between the results of pumping tests and lithological profiles. Storage coefficients are also determined

from pumping tests. The vertical permeabilities are derived from the horizontal permeabilities by assigning (K_h/K_v) a value of anisotropy. Based on the mean anisotropic ratio evaluated by BGS (1989), ratio of 2.5 for the lower aquifer is adopted.

For the characterization of the unsaturated soil behavior, van Genuchten's equations modified by Vogel and Cislerova (1988) is selected. The parameters related with this are θ_r , θ_s , θ_a , θ_m , θ_k , α , n , K_k and K_s which are defined in previous chapter. For accurate characterization, a set of field data is required for moisture content, soil water suction and unsaturated hydraulic conductivity. But in the present study, due to some limitations, collection of such type of field data is not possible and a set of arbitrary fitting parameters is taken.

Atmospheric data (mean daily precipitation and evaporation) (Fig. 7.35) and river water level data (Fig. 7.36) are collected from SWMC. In Bangladesh, BWDB and BMD maintain a well-established network for data collection. Data are collected on a daily basis. Six rainfall stations are available around the model area of which only one station (R009) is within the study area. Data for the simulation period are collected from the central database of the SWMC and BWDB. Pan evaporation data for the period has been extracted from the E13 station.

The piezometric and abstraction data for the area are collected from DWASA. (Figs. 7.32 & 7.37). There are 459 DTWs around the study area of which nearly 220 are located within the study area. A national DEM is available covering most of Bangladesh. The DEM is based on 500m grid elevation data surveyed during 1960-64. Dhaka metropolitan area is not well described in the model and consequently a topographic survey carried out in 1991 as part of the FAP8A study, has been utilized (fig. 7.28).

7.6.5 Model Calibration

Calibration of the model is necessary prior to applying it. Calibration ensures that the model is capable of simulating the present conditions in accordance with the field observations with sufficient accuracy. An inevitable question arises when the calibration is evaluated - what accuracy should be expected and required of the model? No universal criteria have been

formulated but the purpose of the model and the uncertainty of input data and measured calibration references should be considered. A very close agreement between simulation and observation obtained from uncertain and insufficient data may lead to misconceptions regarding the reliability of simulated results.

In the present study due to lack of time, no rigorous calibration method is followed. The model has been calibrated against 11 time series of observed potential head on trial and error basis. It has been aim to obtain:

- Deviations between mean levels of observed and simulated potential heads should be within 2 meters
- The overall long term trend (rate of decline in groundwater levels)
- The annual fluctuations (if any).

The major obstacles to calibration are uncertainty regarding abstraction rates and abstraction well locations. The uncertainty regarding range and distribution of hydraulic conductivities and storage coefficients of the aquifer system also affects the accuracy of the model. A comparison between simulated and observed heads inside the study area is shown in Fig. 7.38.

In Dhaka City the groundwater level in general is not affected by annual variations as the impermeable urbanized areas prevent any significant recharge by infiltration. Variations of up to 2m are, however, found for DHA009 at some points and for DHA010, DHA012 and DHA013 at one point each. The absence of annual variations is found both in observations and in simulations. The time series show a steady decrease as a response to increased abstraction in the area in general. The rate of decrease is in general captured very well by the model (DHA001, DHA005, DHA009, DHA011, DHA014, DHA019 and DHA020) while it is slightly underestimated in DHA010, DHA012, DHA013 and DHA021.

Given the constraints in data the model calibration is satisfactory. The model is capable of simulating a correct average level and a correct rate of decline in groundwater levels. If more time is spent a closer fit between simulation and observation can be achieved, but more important an effort should be made to verify and complete the input data.

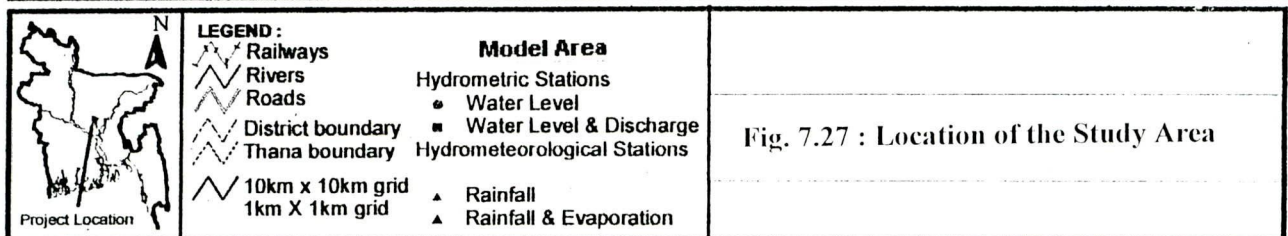
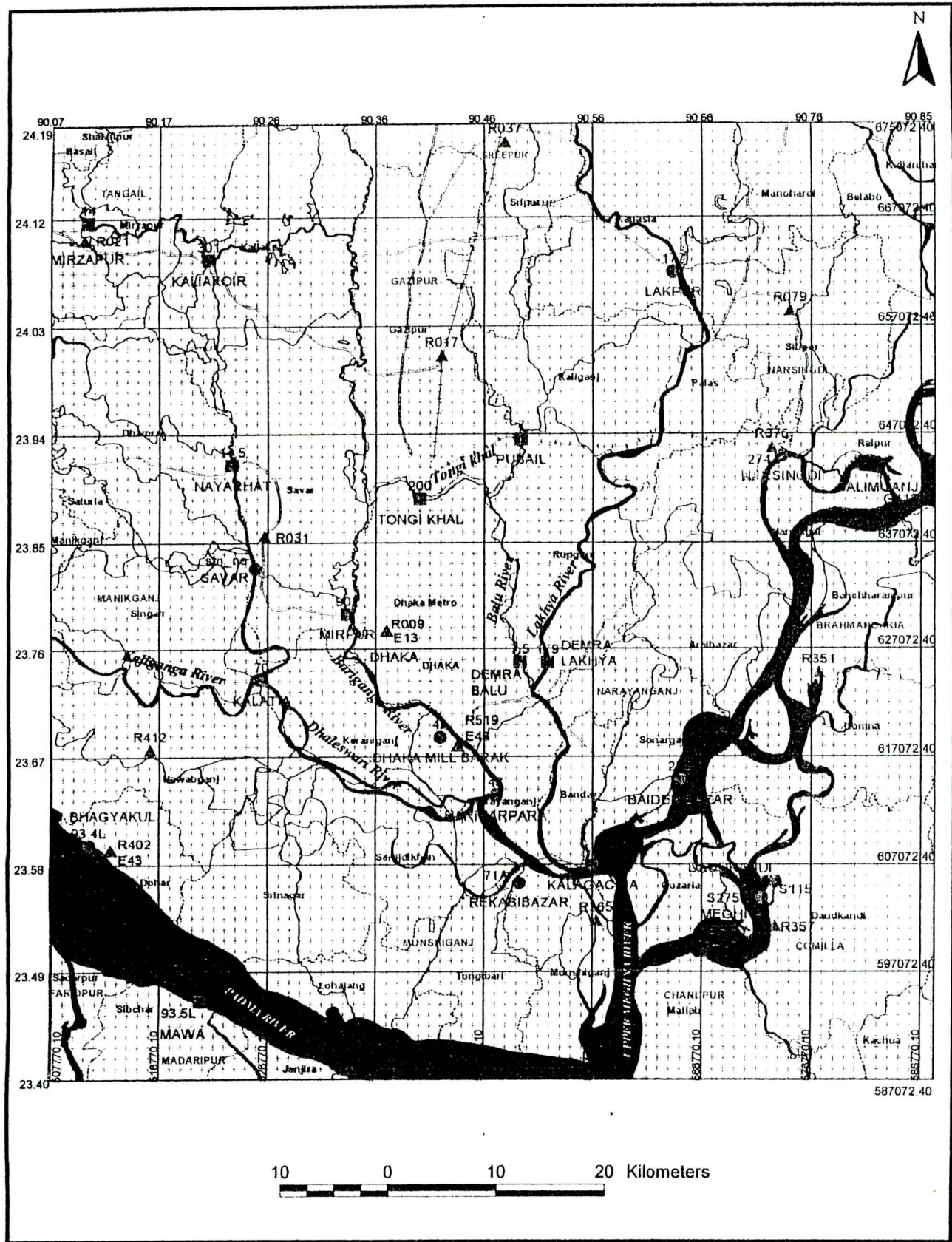
After fulfilling the calibration requirement the water balance of the domain is shown in Table 7.6 for the time period January, 1996 to December, 1996.

Table 7.6 : Simulated Water Balance for the Model Area, 1996

Components	Volume (m ³)
Abstraction	2652.71 X 10 ⁶
Atmospheric face (cumulative outward)	*1685.14 X 10 ⁶
Rainfall	3597.52 X 10 ⁶
Atmospheric face (cumulative inward)	255.92 X 10 ⁶
Recharge	1610 X 10 ⁶
Return flow from water supply	795.8 X 10 ⁶

*Value indicates loss of water from the surface when no ponding of the surface exists, i. e., evaporation loss from the ponding water instantaneously after heavy precipitation is excluded.

From the above water balance, it is seen that the abstraction of water is mainly compensated in the domain by the boundary flow from the riverside. This volume of water is not only contributed from the river itself but from the high water table of the surrounding area (i. e., from the other side of the river also). The volume of water entered to the soil domain from the direct precipitation is only 9.65 percent of the abstraction volume. Most part of the precipitation gets evaporated and lost by surface runoff due to the low hydraulic conductivity of the aquifer.



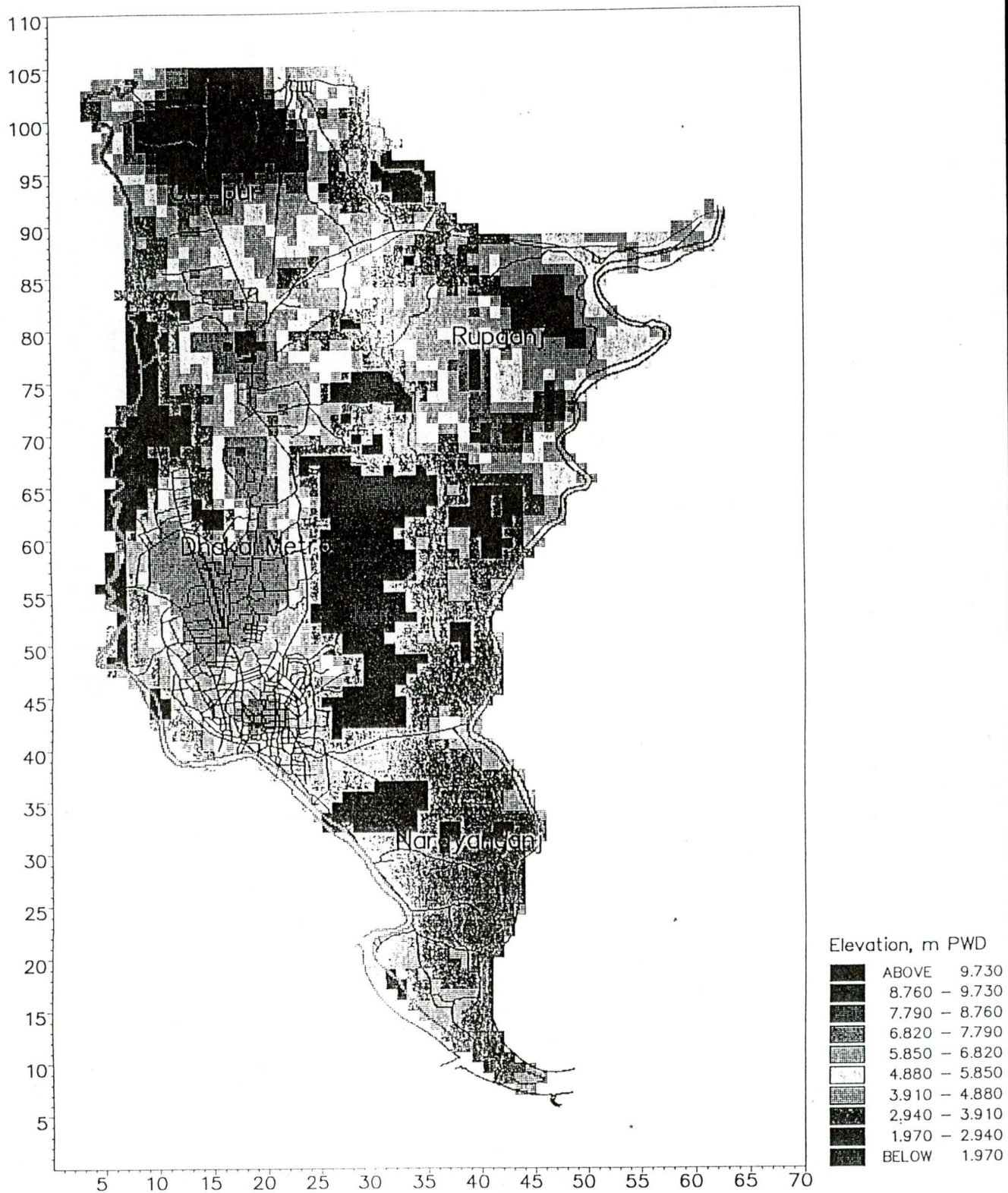


Fig. 7.28 : Topography of the Study Area

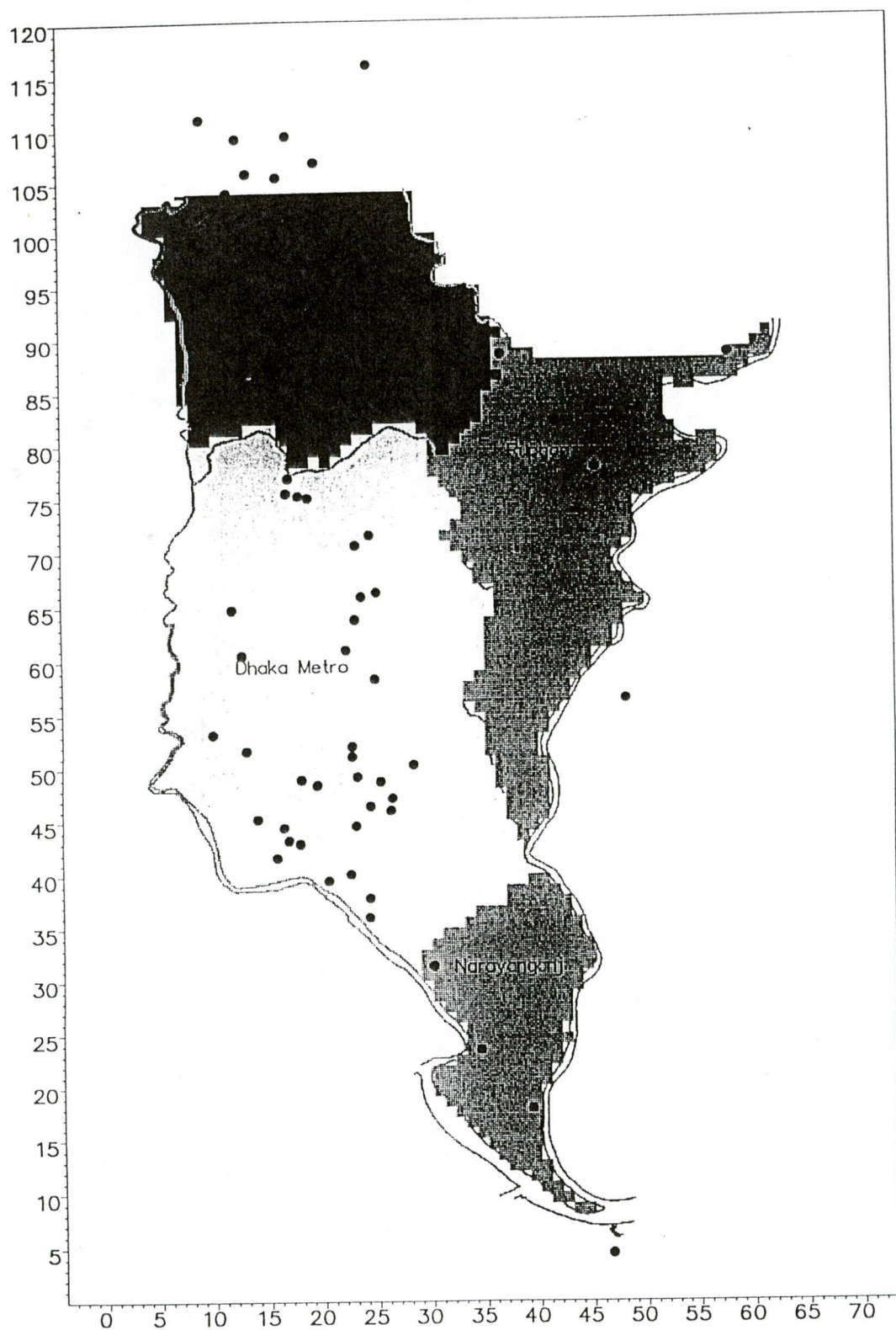


Fig. 7.29 : Location of Boreholes

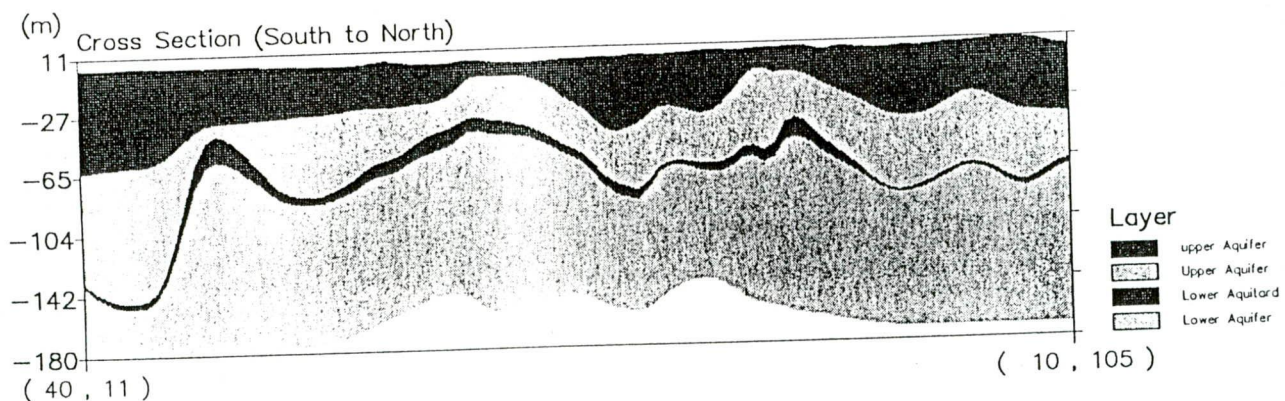
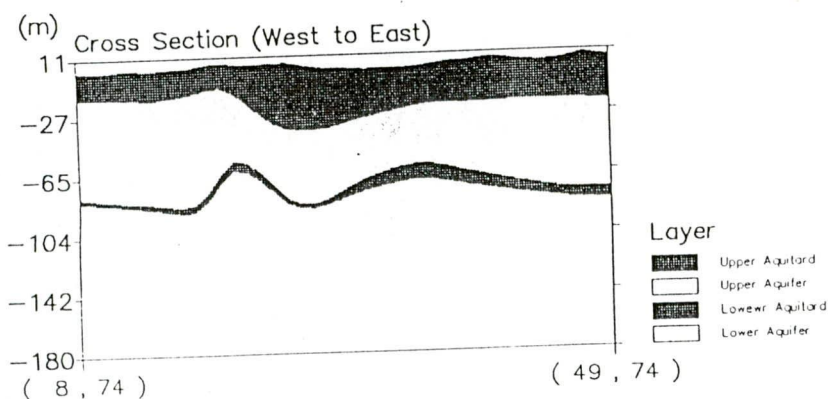
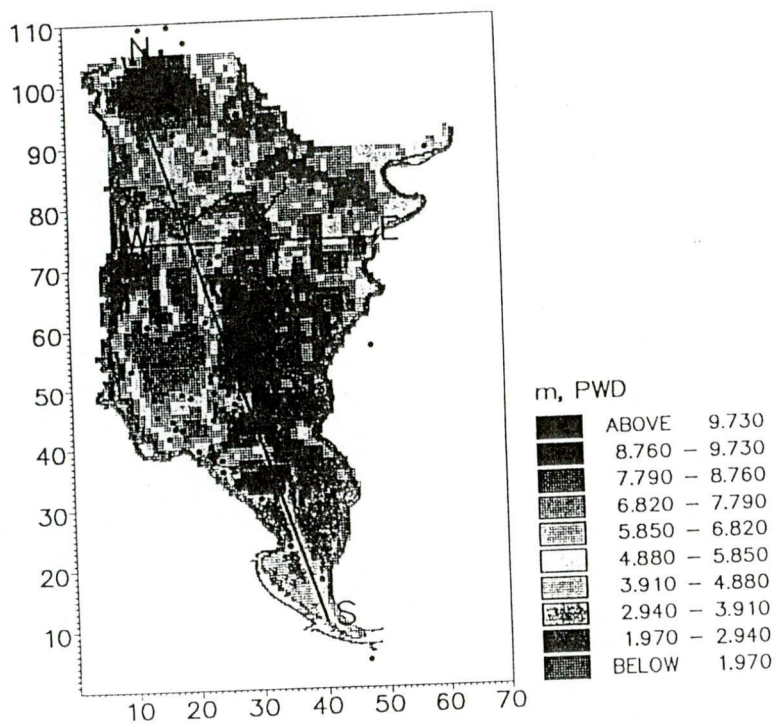


Fig. 7.30 : Geological Cross Section

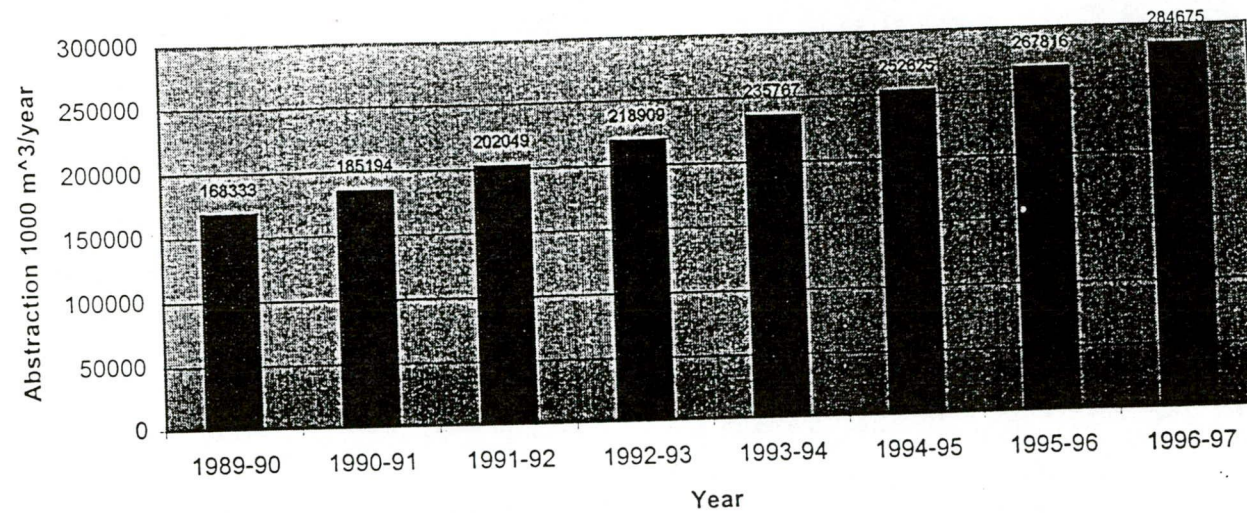


Fig. 7.31 : Abstraction for Municipal Supply in Dhaka City Controlled by DWASA

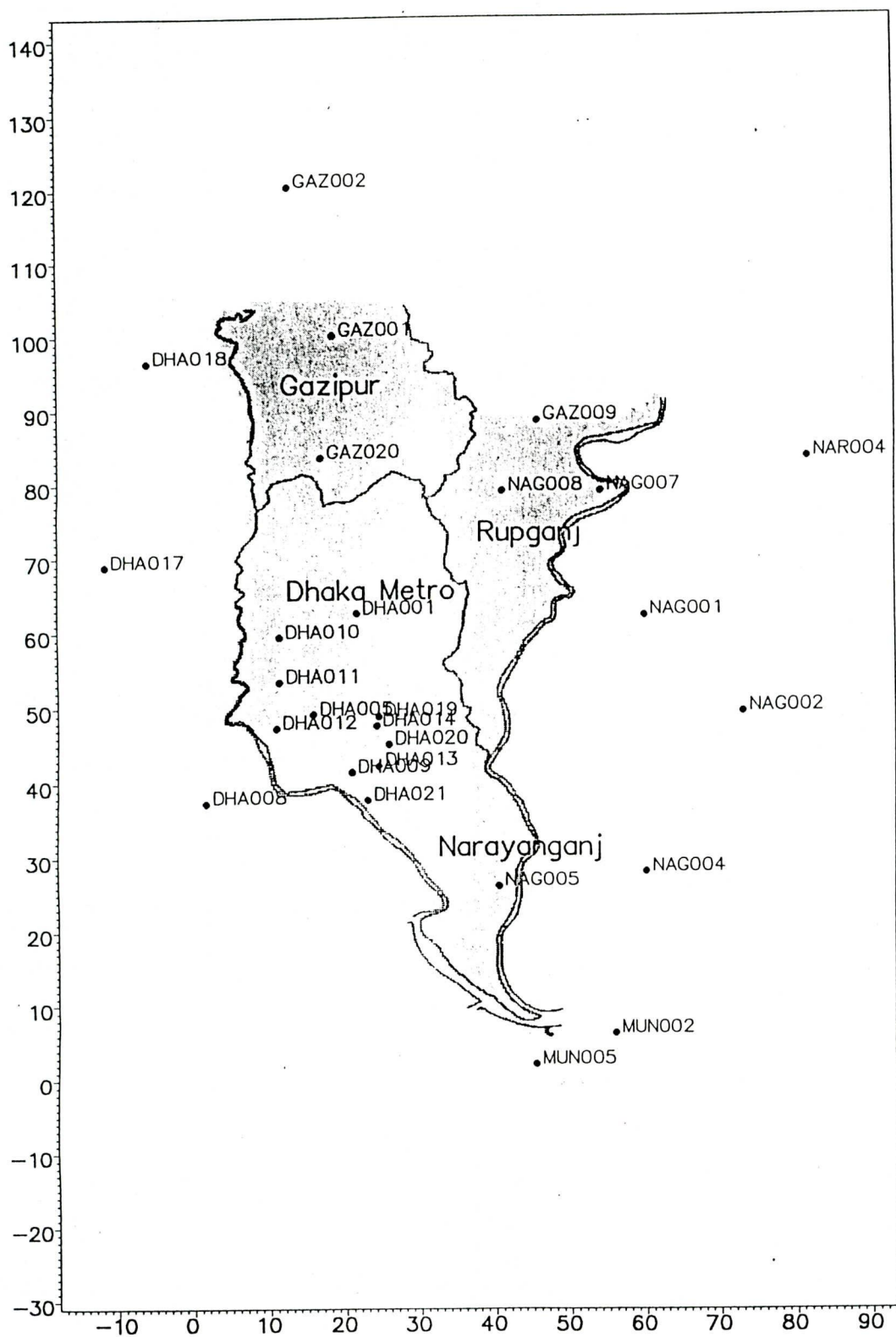


Fig. 7.32 : Location of Observation Wells

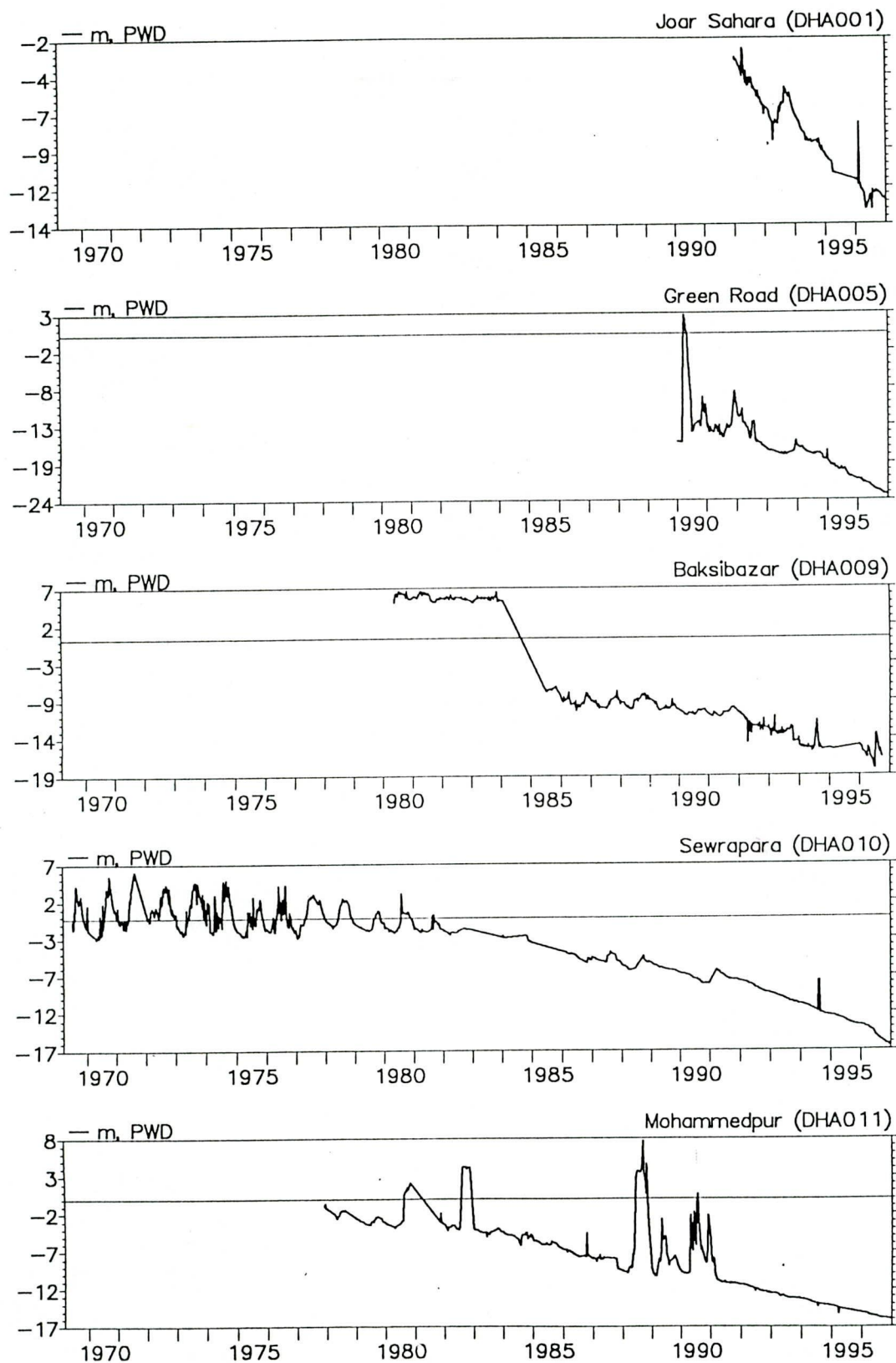


Fig. 7.33 : Potential Head of Groundwater within the Study Area

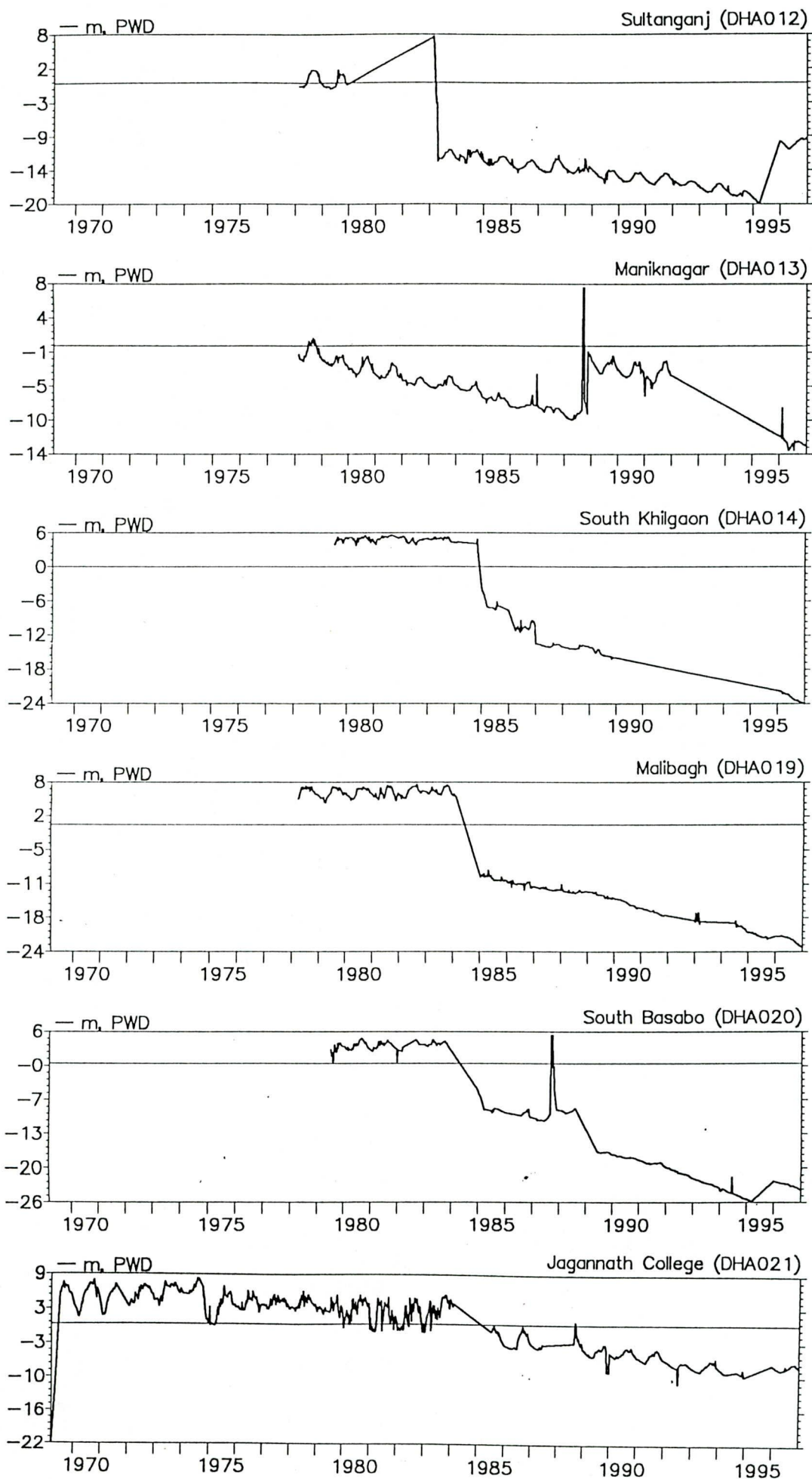


Fig. 7.33 : Potential Head of Groundwater within the Study Area

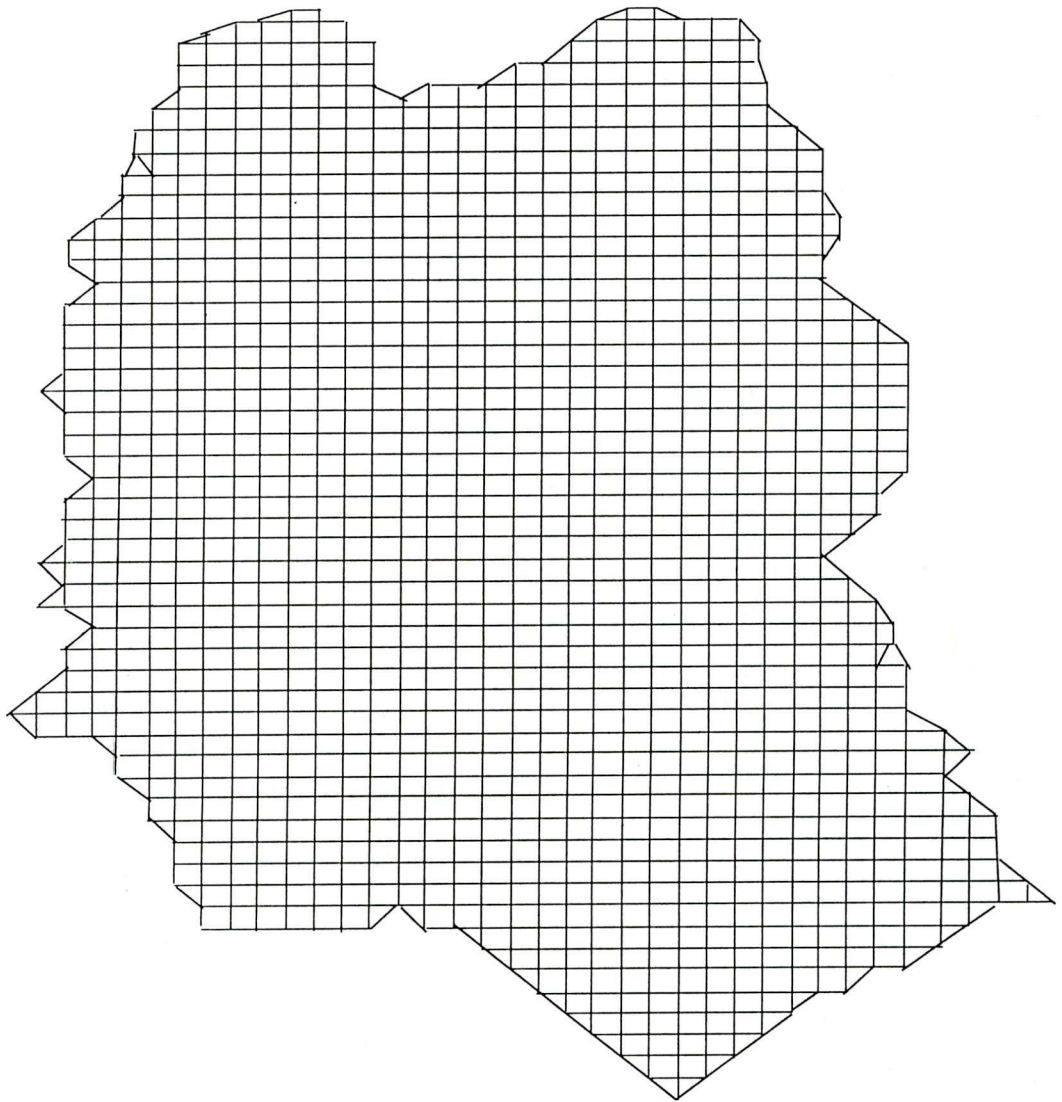


Fig. 7.34 : Finite Element Discretization of the Study Area

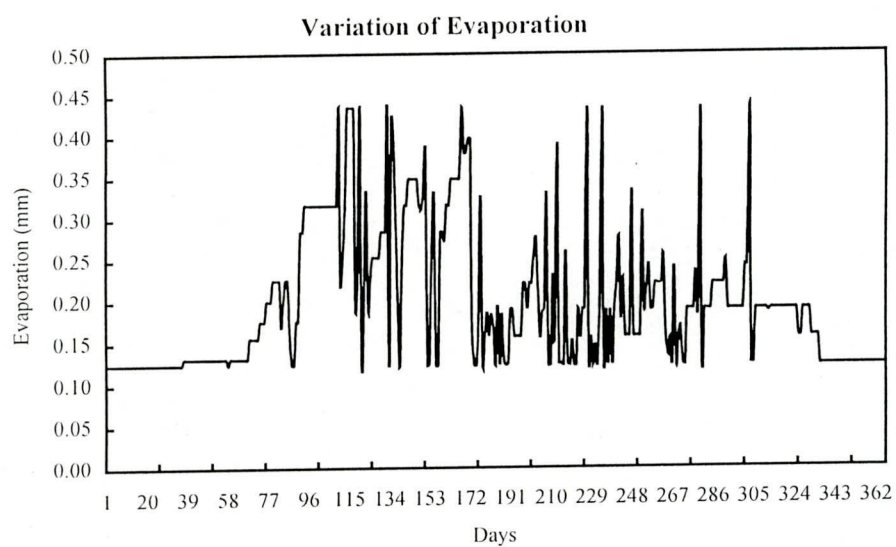
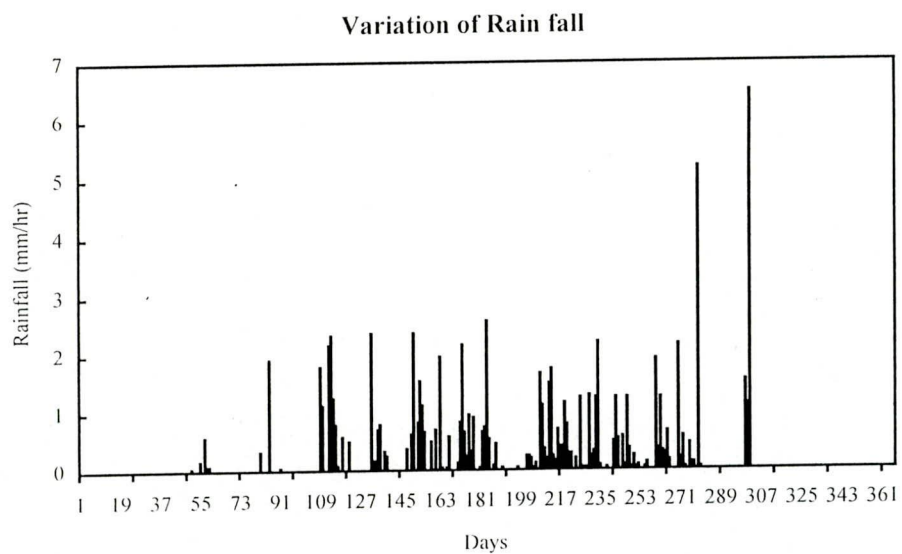


Fig. 7.35 : Hydrometeorological Input Data for the Study Area for the year 1996

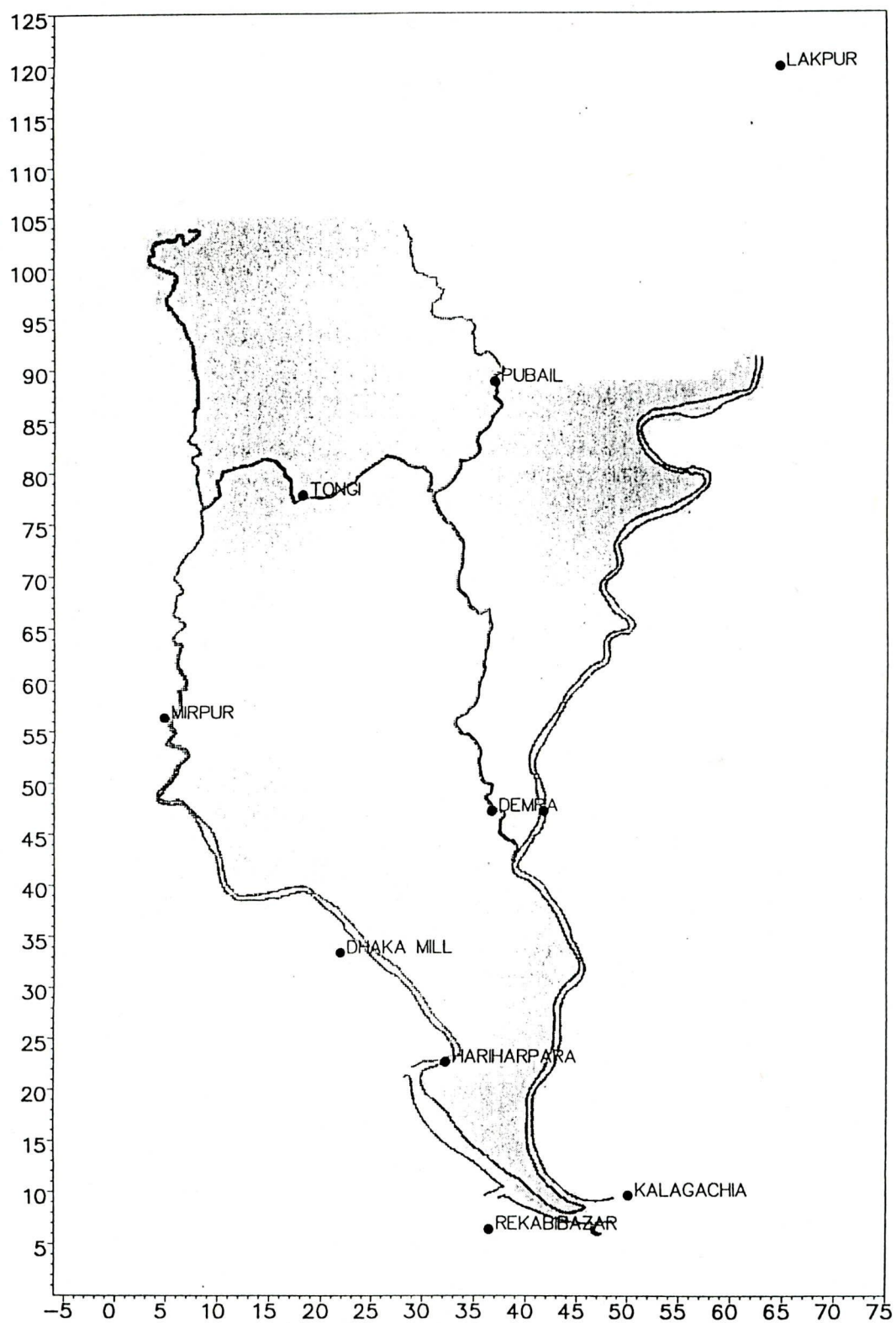


Fig. 7.36 : Location of Observed Water Level Stations

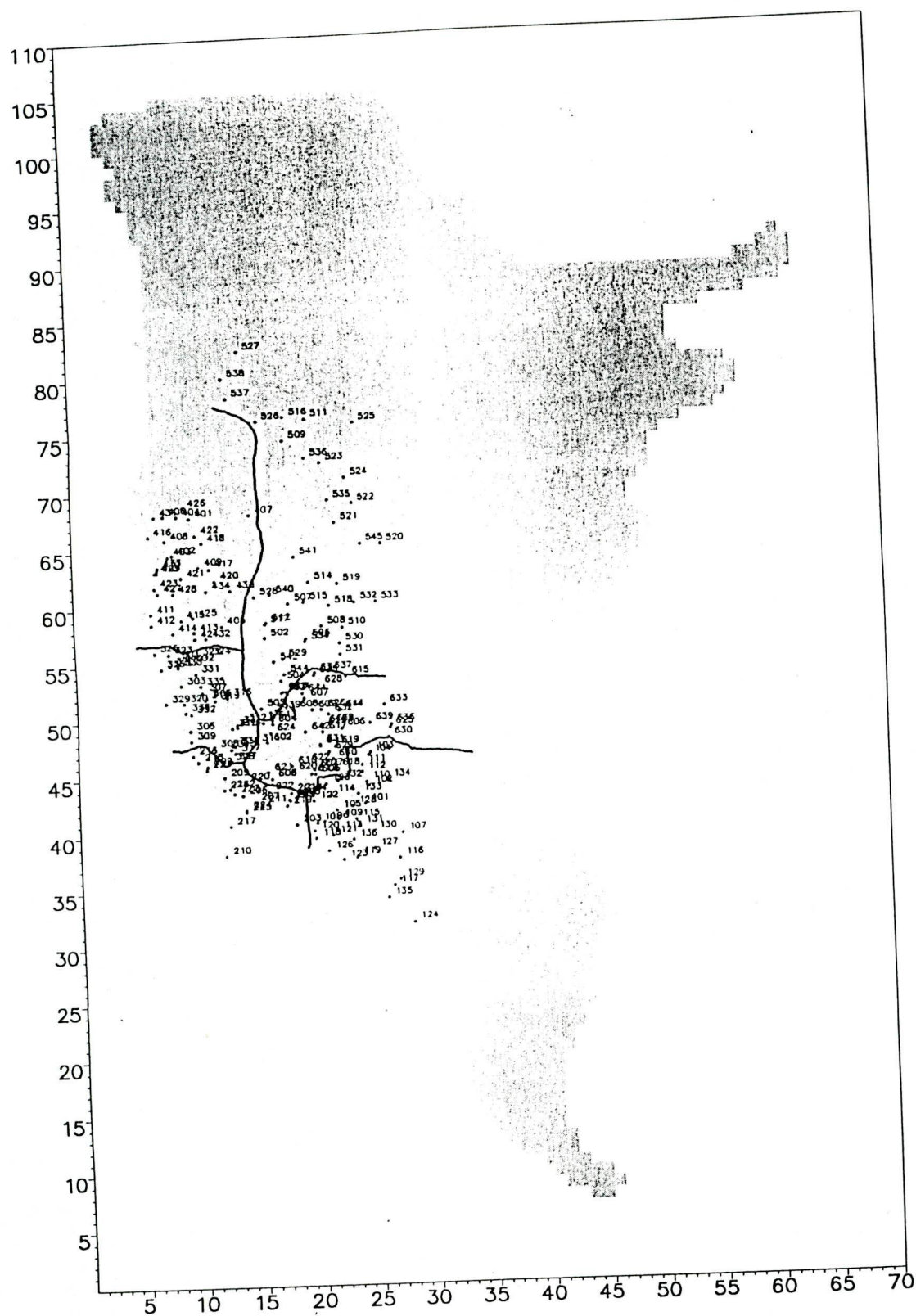


Fig. 7.37 : Location of DWASA Deep Tube Wells

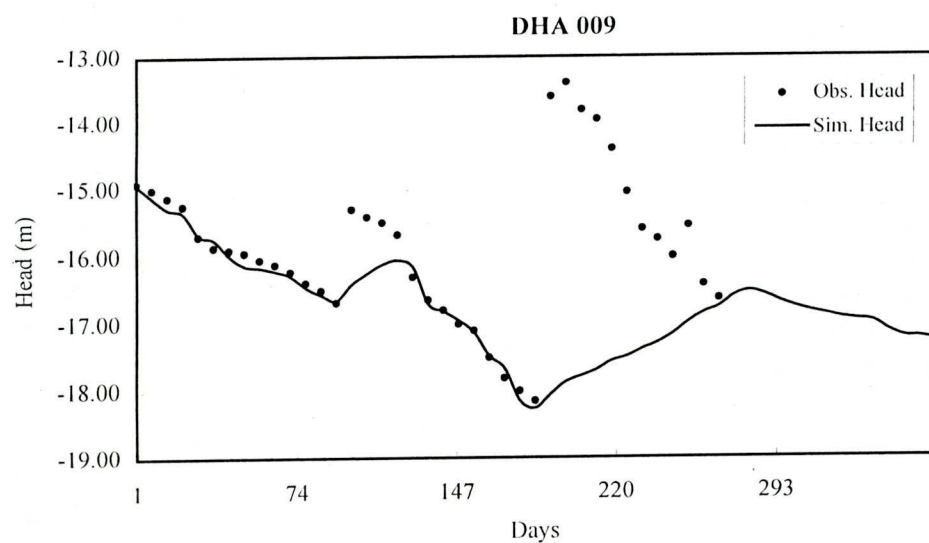
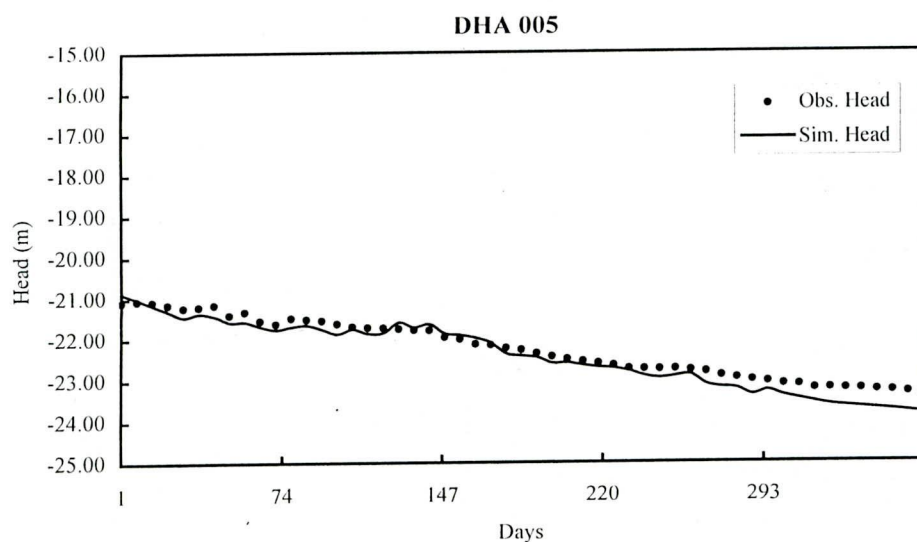
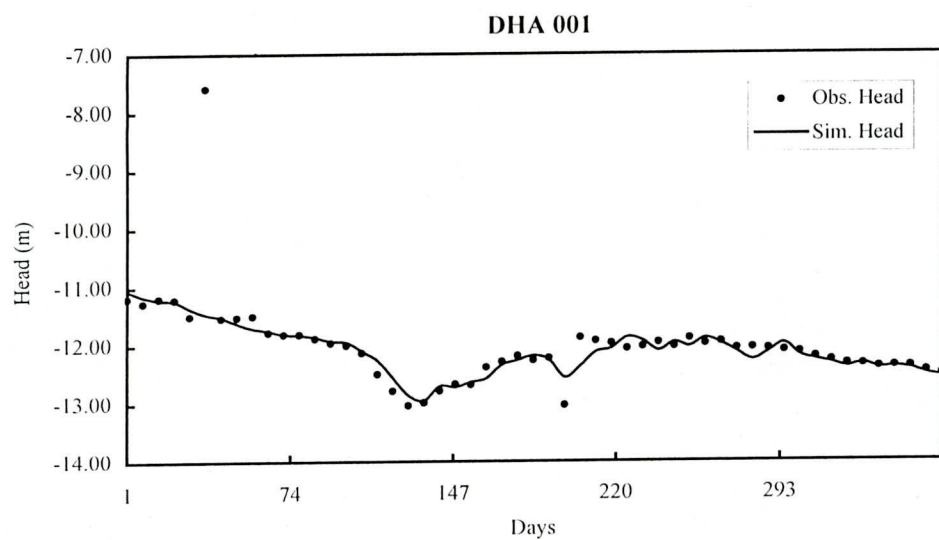


Fig. 7.38 : Comparison of Simulated and Observed Head inside the Study Area

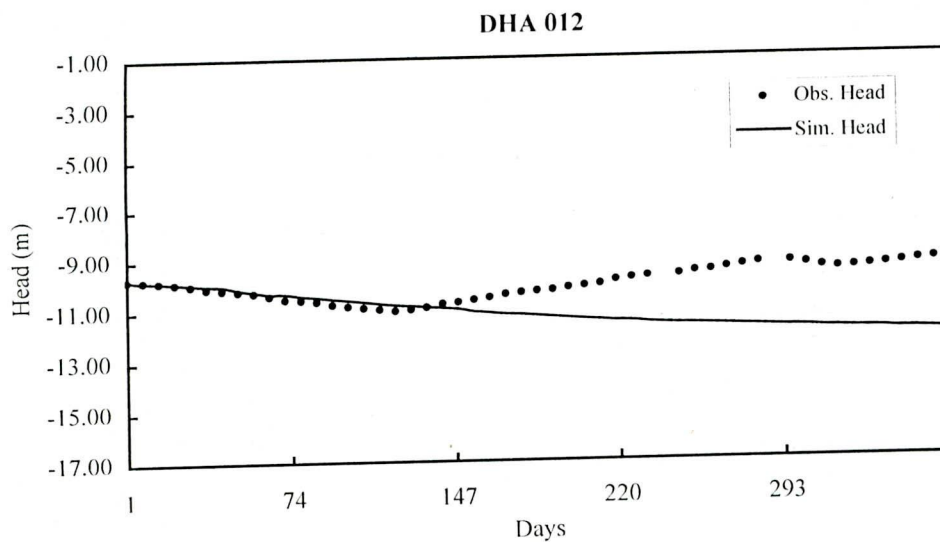
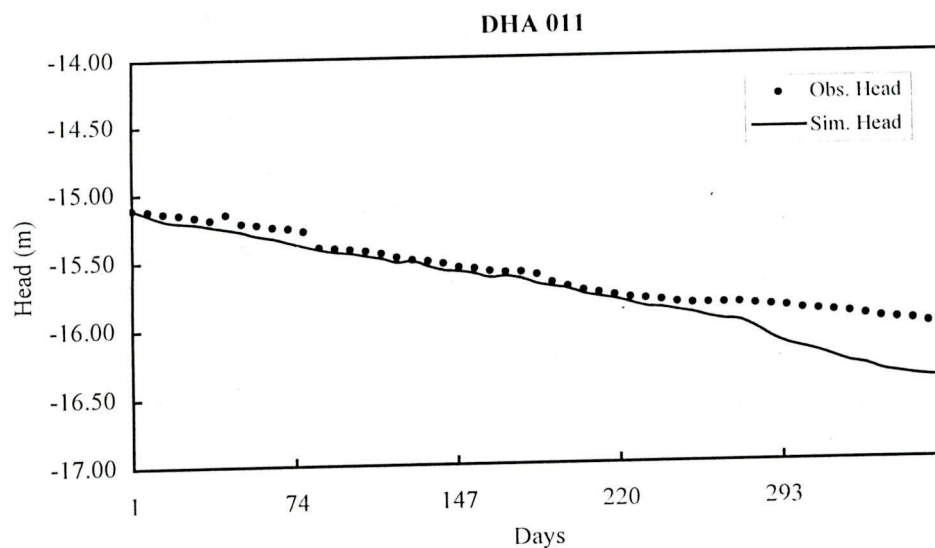
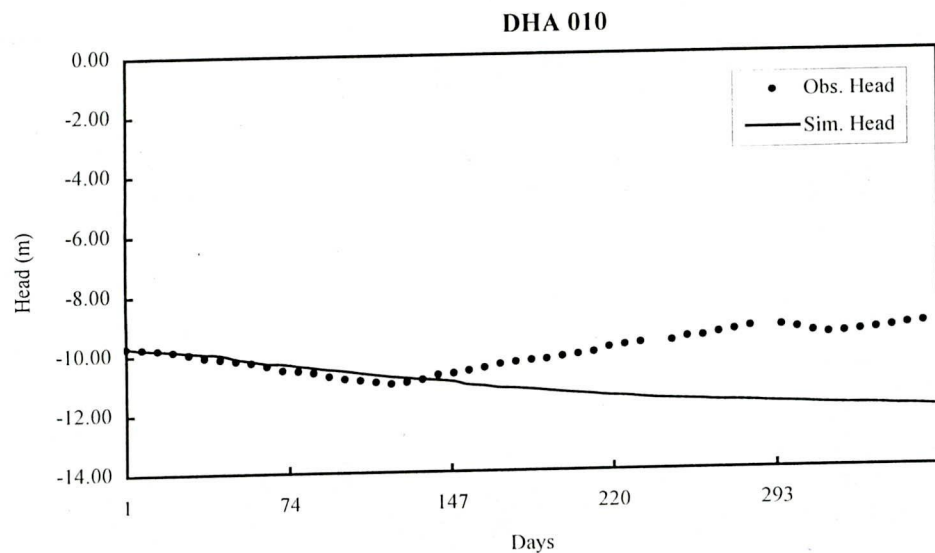


Fig. 7.38 : Comparison of Simulated and Observed Head inside the Study Area

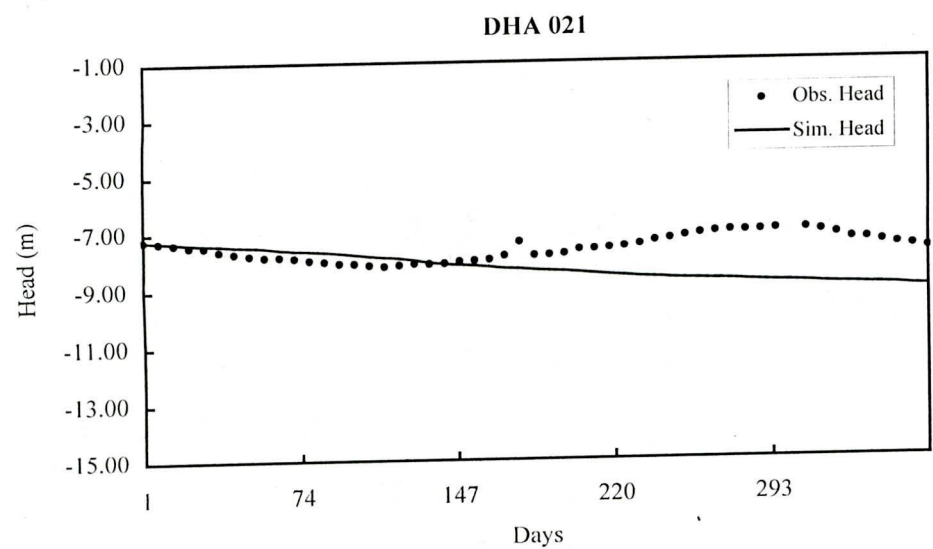
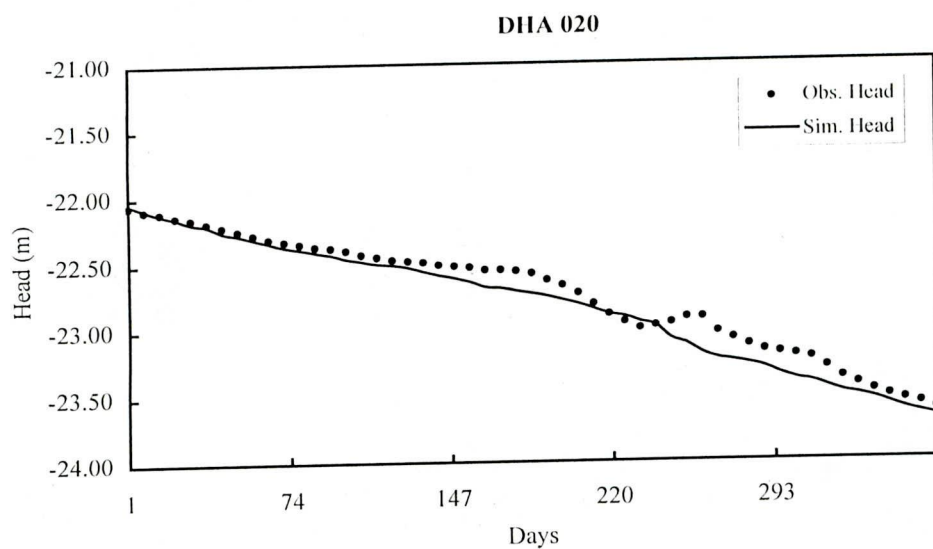
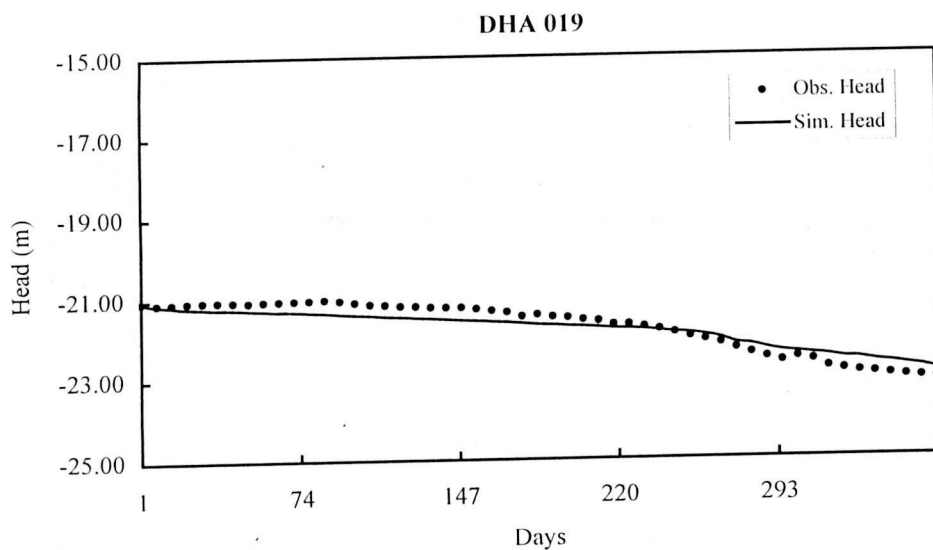


Fig. 7.38 : Comparison of Simulated and Observed Head inside the Study Area

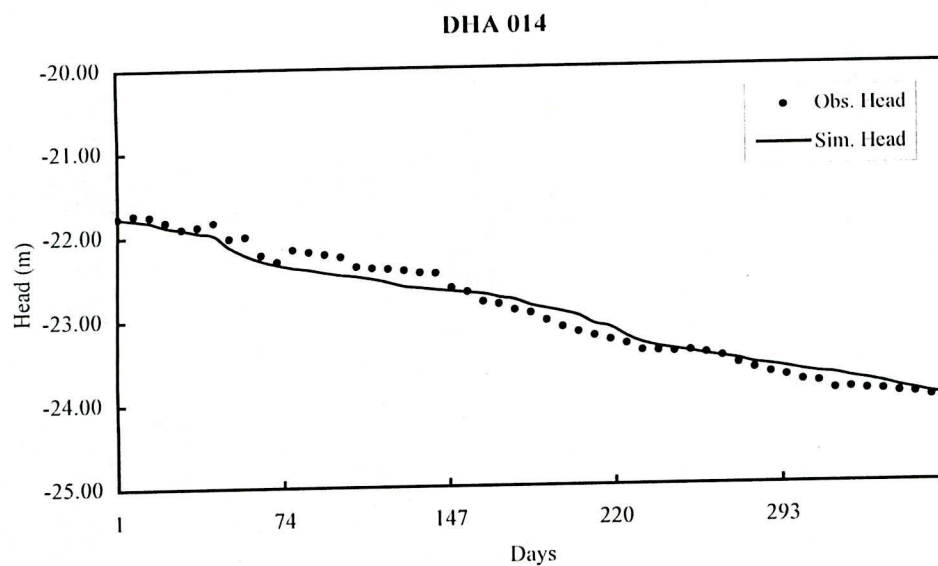
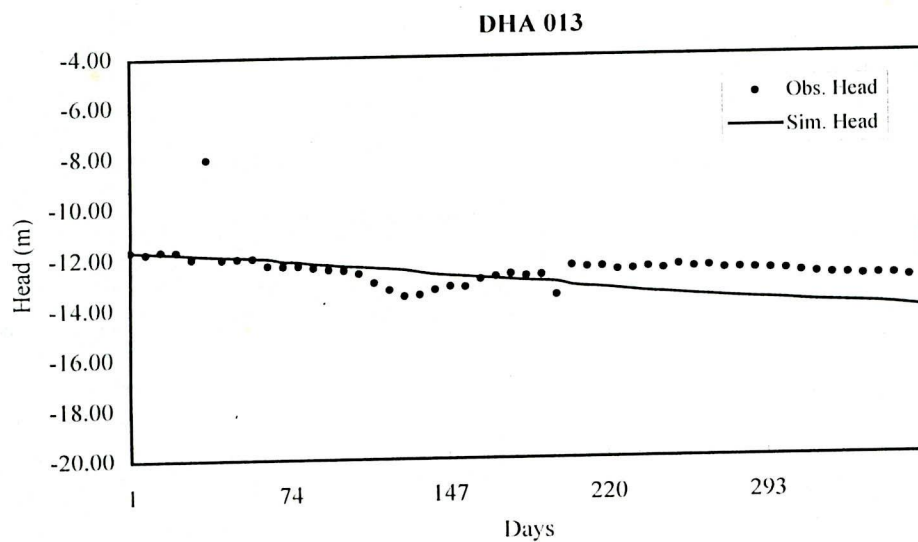


Fig. 7.38 : Comparison of Simulated and Observed Head inside the Study Area

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

Using Galerkin type finite element method a two-dimensional model has been developed for variably saturated groundwater flow simulation. To check the model behavior and accuracy of the algorithm under different auxiliary conditions five hypothetical problems are solved. Finally a real hydrogeological situation of Dhaka City has been analyzed. Conclusions from these applications and testing are stated below.

Example 1 : One-dimensional Vertical Flow Domain with and without Variable Atmospheric Boundary Condition (VABC)

- (a) Some distinct differences between results of simulation with and without VABC have been observed. Both variants seem to work well but both represent different hydrologic situations. Without VABC, a part of the rainwater is lost by surface outflow. A consequence is that the cumulative infiltration is lower and the soil profile drier most of the time, when compared to the variant with VABC. The VABC gives rise also to another effect, which seems reasonable: as soon as the soil profile becomes saturated, the only outflow possible is through the bottom drainage, as approximated by equation (5.13). This bottom drainage is very slow. Quite necessarily, there is an abrupt decrease of infiltration rate at the instant of soil saturation, followed by stagnation of water on the soil surface. The decrease of infiltration rate could be probably observed in nature, too, but would be certainly less abrupt. Nothing such can be seen without VABC. Here, the saturation of soil profile has much less opportunity to occur, since all surplus rainfall is immediately removed by surface outflow, as soon as pressure head on the surface reaches the maximum allowed value (zero in our case). The periods of low intensity infiltration into the saturated soil profile are therefore either absent or of very short duration.

- (b) The differences in hydraulic properties of three layers of which the flow domain is composed are visible in the darcian (volumetric) flux profiles during unsaturated infiltration but are masked when a saturated flow takes place (the flux must be constant throughout the saturated soil profile from the requirement of continuity).
- (c) The present model does not comprise a plant growth submodel. Therefore, as soon as the soil profile becomes aerated again after an extended period of submergence, transpiration recovers with potential intensity, which does not correspond to reality. Most plants would be heavily damaged by anaerobiosis during a prolonged surface ponding.
- (d) The time step of one day for the atmospheric boundary condition which is also used in the subroutine for surface water retention is too large to give a true picture of the phenomena caused by intensive rainfalls (infiltration, surface runoff). It will be desirable in the future to shorten this time step or to make a provision in the program for automatic adjustment of the length of this step depending upon weather and upon surface water retention.

Example 2: Two-dimensional Rectangular Vertical Flow Domain with the Variable-Seepage Face Boundary Condition (VSFBC)

- (a) The program works well for a variable seepage face, and the flood wave reflects within the flow region in a surprisingly smooth manner; almost no sensible effect of the step-wise (instead of continuous) river stage fluctuation can be observed. Of course, shorter time steps would be desirable, too, as stated already above, to make the river stage hydrograph more continuous.
- (b) The results of simulation, in spite of the highly schematized geometry, may be instructive by demonstrating a typical pattern of ground water movement in riverbanks during real hydrologic events. High darcian fluxes occur in the periods of rising or falling river hydrograph, especially in the vicinity of ground water table and slightly above it. This can be explained by the need of unsaturated zone to be saturated before ground water table can rise. Much water is needed to accomplish this saturation which must be brought to the spot by both horizontal and vertical

flux from the river. The intensive fluxes disappear when all parts of the flow region are either saturated or at least wet enough, which can happen during a prolonged high flood. Then it takes several days after the beginning of the recession phase of the flood before large fluxes re-appear, again in the vicinity of ground water table, later in the whole saturated profile near the river bank. This phase of receding hydrograph is most dangerous for stability of the river banks, even if there is no clear evidence for a saturated seepage face of non-negligible size above the river water level. The absence of seepage face may be, of course, a consequence of the forced resetting of boundary conditions on the potential seepage face at the end of each day. A secondary maximum of darcian flux can appear at the interface between layers of different hydraulic properties.

- (c) The pressure head contours in the saturated zone below ground water table are at all times effectively parallel to the ground water table, at distances corresponding to their nominal values. This means that the vertical gradient of the hydraulic head in the saturated zone differs little from the hydrostatic one (validity of the well-known Dupuit-Forchheimer assumption of vertical equipotential lines in unconfined aquifers is thus demonstrated).
- (d) The behavior of unsaturated contours indicates:
 - (i) a tremendous difference in time scale between saturated and unsaturated movement (with the unsaturated movement being much slower),
 - (ii) that the process of wetting continues in a part of the flow region even if the flow region as a whole undergoes intensive drainage, and even if the root zone as a whole undergoes drainage,
 - (iii) that the lower part of the unsaturated zone undergoes drainage which is very slow even if hysteresis of the retention curve is not allowed for (the model does not comprise a subroutine for hysteresis); this rehabilitates the concept of field capacity.

Example 3 : Two-dimensional Vertical Flow Domain with Simplified River Bank Situation (3A with Coarser Grid and 3B with Finer Grid)

- (a) Fine grid (Example 3B) generally works better, has better spatial resolution, and is more sensitive to small rain events and short dry spells, while the model with the coarse grid (Example 3A) reacts only on prolonged periods of drought or of intensive rains. During the falling limb of the hydrograph, the fine grid is drained more quickly. There are, however, more problems with numerical stability with the fine-grid model, caused probable by the presence of non-equidimensional finite elements (elongated in horizontal direction) at the soil surface. Some of the unsatisfactory contour graphs resulted when the allowed number of iteration had been exceeded and the iteration process terminated without reaching the required accuracy.
- (b) As a contrary to the one-dimensional column case simulated in Example 1, it was observed in the Example 3 that infiltration could continue with relatively high rate even when the soil was fully saturated, because water could be drained from the flow region into the river.
- (c) Large downward fluxes (vertical darcian fluxes) during infiltration of rain water are visible near the soil surface in the plots of vertical fluxes. The vertical flux profiles also demonstrate (similarly as is previous examples) how slow the drainage of soil profile is in comparison with infiltration.

Dhaka City Modeling

- (a) The program converges within a satisfactory maximum iteration at each time level for daily basis input data.
- (b) In one year cycle, the piezometric levels don't return to their original position in the remote areas from river and a maximum of 0.8 to 1.2 m of unrecoverable piezometric head decline is observed in areas of high abstractions.

- (c) Main component of recharge is by the flow from the riverside boundary (approximately 60 percent). Urban recharge contributes to the aquifer as near as 30 percent of the abstraction volume.
- (d) Direct effect of rainfall through infiltration has negligible effect on the piezometric head profiles of the area.
- (e) A hydrodynamic river flow model can be coupled with the resent one to simulate riverine area in an integrated approach.

8.2 Recommendations for Future Studies

About the Model

- (a) The present model can be expanded further into a more flexible one that would be able to cope with any type and any combination of boundary conditions by incorporating well-structured user's subroutines.
- (b) The model could be coupled with a more complex model of two-dimensional surface runoff during a flood. The hysteresis of the retention curve could be taken into account. A more realistic treatment of the behavior of vegetation in conditions of a prolonged flooding would be desirable, too, as well as a provision for the root water uptake pattern and rooting depth variable in time.
- (c) The graphical user interface developed as a part of thesis works performs well. It facilitates user easy and error free data entry to the input files. In addition to that several scatter graphs can be plotted through this interface which is developed using Visual Basic language. Visual Basic does not directly support the graph plotting functions. As the thesis work needs generation of scatter and contour graphs it would be efficient if a graph control in visual basic would be available that supports mentioned graphs plotting. To draw the scatter graph a visual basic control is used which does not support the contour. In order to draw the contour a copied version of SpyGlass for windows 3.11 is used. These two separate

modules for graph plotting has decreased the performance measure of the interface particularly the SpyGlass, which sometimes give wrong contour plot.

- (d) Again, the model is developed using DOS based Fortran 77 programming language. To run the model program if the particular option is selected from the interface the program control closes the interface from window 95 and loads DOS operating system and starts executing the program. This reduces the convenience and performance of the GUI substantially. When the execution of model is completed the program control loads the window operating system and user has to load the interface by clicking the mouse button. The GUI could act in harmony with the model if the later could be developed using window based Fortran compiler. This would definite enhance the convenience and functionality of GUI.
- (e) Besides, if there would be any visual basic control (such as TeeChart Pro) which provides contour and scatter graphs plotting features the GUI would accurately and conveniently draw the graphs from the results of the model run. A substantial improvement in contour plotting would have been possible if the interface could be coupled with GIS software such as ArcView. As the ArcView and TeeChart Pro are expensive software those were not available freely anywhere for this research purpose and due to the limitation of fund for the thesis works the plan for procuring those have been dropped.
- (f) Finally, the core of the model, the finite element method itself, could be gradually refined by incorporating higher order elements, as well as multigrid and superconvergence methods. It seems possible to incorporate some kind of space-weighted linear basis functions, which could minimize the error due to different shape and size of elements, perhaps by employing multigrid techniques.

About Dhaka City Modeling

- (a) A complete database of the area should be developed for soil layers of different locations or geotechnical subareas, their hydrogeological characteristics and the piezometric levels of the observation wells.

- (b) To determine accurately the river leakage to the aquifer system a number of wells should be established in both sides of the river.
- (c) For proper quantification of vertical leakage contribution from surface total paved and unpaved areas, the water level conditions in the lakes, and unsaturated soil characteristics of the aquifer layers should be examined and defined correctly from field investigation and data analysis.

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APPENDIX

Flow Diagram of the Model

